Finite β and Vacuum Field Studies for the Helias Stellarator

P. MERKEL, J. NÜHRENBERG, R. ZILLE
Max-Planck-Institut für Plasmaphysik
IPP-EURATOM Association
D-8046 Garching bei München

The achievement of magnetohydrodynamically stable stellarators with medium $\langle\beta\rangle$ values ($\gtrsim 0.05$) at medium aspect ratio $A(10 \lesssim A \lesssim 20)$ is a major challenge to stellarator research. In continuation of earlier configuration studies [1] a class of $\ell = 0, 1, 2, 3$ stellarators is described which is called Helias [2] because it combines features of W VII-AS and Heliac. It comprises stellarators with $A = 12$ which are stable to Mercier, resistive interchange, and ballooning modes at $\langle\beta\rangle = 0.05$ as is discussed below. In addition, Helias vacuum field studies are presented.

The geometry of Helias equilibria is given by their aspect ratio A, number of periods N, and 8 parameters which define the shape of the plasma boundary as

$$\begin{aligned} R =& A + R_{0,1}\cos V + (1 - \Delta_{1,0} - \Delta_0 \cos V)\cos U + \Delta_{2,0}\cos 2U \\ &- \Delta_{1,-1}\cos(U - V) + \Delta_{2,-1}\cos(2U - V) + \Delta_{2,-2}\cos(2U - 2V) \\ Z =& \quad Z_{0,1}\sin V + (1 + \Delta_{1,0} - \Delta_0\cos V)\sin U + \Delta_{2,0}\sin 2U \\ &+ \Delta_{1,-1}\sin(U - V) + \Delta_{2,-1}\sin(2U - V) - \Delta_{2,-2}\sin(2U - 2V) \end{aligned}$$

Here, R, Z, ϕ ($V = \phi N$) are cylindrical coordinates; U is the poloidal parametrization. Thus, $R_{0,1}$ and $Z_{0,1}$ define the radial and vertical displacements of the plasma column, i.e. the $\ell = 1$ content, Δ_0 the $\ell = 0$ content, $\Delta_{1,0}$ the $\ell = 2$ axisymmetric content, $\Delta_{1,-1}$ the $\ell = 2$ stellarator content (elliptical cross-section turning 180^0 per field period), $\Delta_{2,-2}$ the $\ell = 3$ stellarator content (triangular cross-section turning 240^0 per field period), $\Delta_{2,0}$ and $\Delta_{2,-1}$ the indentation.

Figures 1 and 2 show a Helias equilibrium with the above parameters given in the caption.

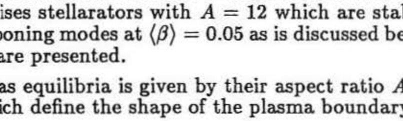

Fig.1: Flux surface cross-sections at $V = 0, \frac{\pi}{2}, \pi$ of a Helias equilibrium obtained with the BETA code [3] with $N = 5$, $A = 11.5$, $R_{0,1} = 0.8$, $Z_{0,1} = 0.4$, $\Delta_{1,0} = 0.1$, $\Delta_0 = 0.07$, $\Delta_{2,0} = 0.05$, $\Delta_{1,-1} = 0.29$, $\Delta_{2,-1} = 0.24$, $\Delta_{2,-2} = 0.07$. $\langle\beta\rangle = 0.05$. The pressure profile is characterized by $p = p_0(1 - s)$.

The equilibrium shown has no net toroidal current (more precisely, $J(s) \equiv 0$, where J is the toroidal current and s the flux label), $\langle\beta\rangle = 0.05$ with a parabolic (in radius) pressure profile so that the peak β-value is 0.1, and a finite-β well depth of about 0.09. The twist per period ι_P lies in the range $0.1 < \iota_P < 0.14$ so that low-order rational values of the

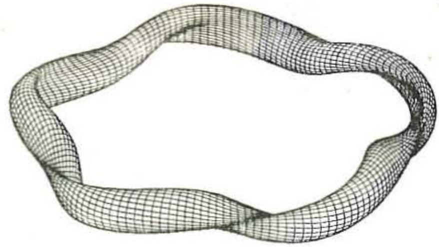

Fig.2: Perspective view of the Helias plasma boundary.

twist per period are avoided (only $\iota_P = \frac{1}{9}$ and $\frac{1}{8}$ are crossed), on the one hand, and the lowest-order rational values of ι_T $\frac{1}{2}$ and 1 are avoided as well, $0.5 < \iota_T < 0.7$. The parallel current density is strongly reduced as compared with an $\ell = 2$ stellarator, as evidenced by $\langle j_\parallel^2 / j_\perp^2 \rangle \lesssim 1$. The reduction is also significant if compared with W VII-AS, where the corresponding number is 4. The equilibrium is stable or marginally stable to all local stability criteria which have been evaluated hitherto. Figure 3 shows the evaluation of Mercier's criterion, which appears to be safely stable except for the narrow regions around $\iota_P = \frac{1}{9}$ and $\frac{1}{8}$.

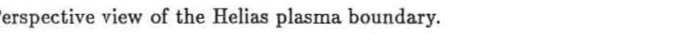

Fig.3: Values of the Mercier (solid line with circles) and the resistive interchange (broken line with circles) criteria as functions of $\sqrt{s}$, which represents the normalized average flux surface radius. The ordinate is taken as the exponent (shifted by $\frac{1}{2}$) occurring in the asymptotic theory of local ballooning modes; negative values represent imaginary exponents. The solid line without circles shows the Mercier criterion with the $\frac{1}{9}$ and $\frac{1}{8}$ resonance included. The BETA run evaluated here has NS, NU, NV = 30, 48, 36; extrapolation to zero mesh size just slightly lowers the curves.

The righthand side shows three pressure profiles as functions of $\sqrt{s}$. p_1 is the profile used for the results shown in the lefthand part; p_2 is the profile corresponding to marginal resistive interchange stability excluding resonance effects; p_3 is the profile including the $\iota_P = \frac{1}{8}, \frac{1}{9}$ resonances.

These formal violations of stability criteria involving the parallel current density are really manifestations of the existence problem of 3D equilibria and can be eliminated by small regions of flattened pressure profile. Helias configurations with smaller shear avoiding these resonances could also be realized if more refined MHD theory showed this to be of advantage. With the value of Mercier's criterion at $s = \frac{1}{2}$ as a reference value, it is concluded that a substantial improvement in stability is obtained in comparison with Heliac results [4] (in the normalization used in [4] the Helias and the Heliac values are 0.03 and -0.03, respectively). Also shown in Fig.3 is the resistive interchange criterion, which, of course, is more stringent but still approximately marginal in this configuration.

The resistive interchange criterion is, for the case of vanishing net longitudinal current, identical with the applicability condition of a sufficient stability criterion [5] and with the stability condition for peeling modes [6], which adds significance to its use for selecting viable finite-β stellarators. Moreover, it has been shown [7] that island growth (as a function of β) is connected with resistive instability. Thus, the occurrence of the resonances may be presumed to be harmless under these circumstances. As illustrative information three different pressure profiles are shown in Fig.3.

The profile p_1 is the one actually used in the equilibrium computation. The second profile is obtained from marginal resistive interchange stability in the following way: excluding resonance effects, we calculate $p_2 = \int_1^s ds'(V''/\langle \vec{j}^2_{nonres}/p_1'^2|\nabla s|^2\rangle)$. Thus, p_2 is too optimistic (pessimistic) for an unstable (stable) value of the resistive interchange criterion, because the decrease (increase) of the well depth is not taken into account in the above formula. Closeness of p_1 and p_2 indicates a marginal situation more clearly than the actual values of the criteria. The third profile p_3 is obtained by taking into account resonant effects in $\vec{j}$ and defining $p_3 = \int_1^s ds'(p_2'\langle \vec{j}^2_{nonres}/|\nabla s|^2\rangle/\langle \vec{j}^2_{res}/|\nabla s|^2\rangle)$. In the present context of evaluating stability this regularization of the parallel current density is more natural than Boozer's method based on the classical diffusion argument [8]. Both ways are of course closely related and lead to the same analytical behaviour of the pressure profile near the resonances. Narrowness of the flattened regions and, correspondingly, closeness of the profiles (and β-values) alleviates the doubts connected with the 3D nature of the equilibrium. The above arguments also rely on the dependence of ι on β. Figure 4 shows the $\langle\beta\rangle = 0$, 0.05 twist curves. Finite β has little effect on ι in contrast to the situation in ATF and W VII-AS.

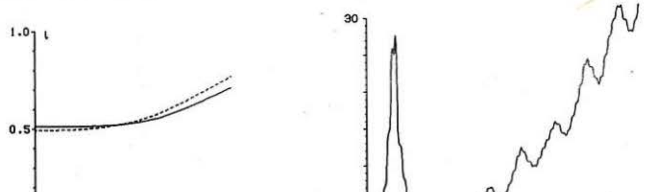

Fig.4: The total twist values ι_T as functions of $\sqrt{s}$ for the $\langle\beta\rangle = 0$ and $\langle\beta\rangle = 0.05$ cases.
The righthand side shows the solution F of the one-dimensional ballooning equation for the equilibrium of Fig.1 evaluated at $\iota_T = \frac{4}{7}$. The variable $\bar{\phi}$ is the contracted toroidal variable which varies between 0 and 1 as the field line closes on itself, which corresponds to 35 field periods of the equilibrium. The field line starting at $U = V = 0$ is considered. A zero of F would indicate ballooning instability.

Figure 4 (righthand side) shows the evidence for ballooning stability. Here, we evaluate the one-dimensional ballooning equation [9] at $\iota_T = \frac{4}{7}$, i.e. we consider a localized $m = 7, n = 4$ mode (which should not be influenced by resonance effects within one period) on the full torus. The potentially dangerous oscillatory curvature terms (apart from the favourable average magnetic well) manifest themselves in the minimum of the ballooning solution but are apparently not strong enough to drive a ballooning instability. This result is in accordance with our previous result [9] that ballooning instability occurs in stellarators only if the Mercier criterion is violated.

The choice of the parameters of the particular Helias configuration presented in Fig.1 may be characterized as follows: A decrease of any of the 9 parameters A, $R_{0,1}$, $Z_{0,1}$, $\Delta_{1,0}$, Δ_0, $\Delta_{2,0}$, $\Delta_{1,-1}$, $\Delta_{2,-1}$, $\Delta_{2,-2}$ decreases the Mercier and resistive interchange stability. Thus, while one may want to decrease all of these parameters, e.g. for easier realization, this imposes a penalty on the stability properties. Apparently, the nature of stellarator optimization is such that the optimum occurs at the boundary of the optimization domain, this boundary being given by side conditions, e.g. minimum acceptable β-value, maximum acceptable aspect ratio, maximum acceptable geometrical distortion.

Since finite-β 3D codes do not yet provide a reliable insight into the quality of magnetic surfaces, vacuum field calculations for Helias were performed with NESTOR [10]. Figure 5 shows the Poincaré plots of three Helias vacuum fields; despite the strong three-dimensionality of the configuration the quality of the surfaces appears to be very good and the radial extent of the detectable islands small if the occurence of the lowest order resonances (e.g. $\frac{1}{7}$) in the outer region of the confinement domain is avoided. In particular, a strong decrease of island size is observed for $\iota_p < \frac{1}{7}$.

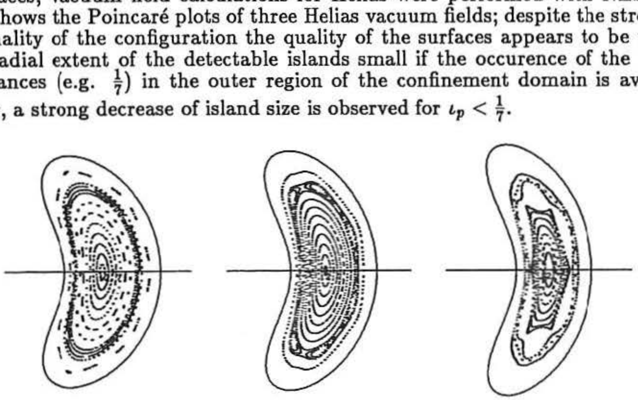

Fig.5: Poincaré plots of Helias vacuum fields with the surface parameters as given in Fig.1, except $\Delta_{1,-1} = 0.27, 0.32, 0.39$.

References

[1] J. Nührenberg, Proc. of Int. Symp. on Stellarators with Three-dimensional Magnetic Axis, Sendai, Japan (1979) 1; IPP Annual Report 1984

[2] J. Nührenberg, R. Zille, Phys. Lett. 114A, 129 (1986)

[3] F. Bauer, O. Betancourt, P. Garabedian, Equilibrium and Magnetohydrodynamic Equilibrium and Stability of Stellarators (Springer, New York, 1984)

[4] O. Betancourt and P. Garabedian, Phys.Fluids **28** (1985) 912.

[5] D. Lortz, E. Rebhan, and G. Spies, Nucl.Fusion **11** (1971) 583.

[6] D. Lortz, Nucl.Fusion **15** (1975) 49.

[7] J.R. Cary and M. Kotschenreuther, Phys.Fluids **28** (1985) 1392.

[8] A. Boozer, Phys.Fluids **24** (1981) 1999.

[9] J. Nührenberg, R. Zille, Proc. 12th European Conf. Contr. Fusion and Plasma Phys., Budapest, 1985, Part I, 445 - 448

[10] P. Merkel, Proc.5th Int.Workshop on Stellarators, Vol.I, 387