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Minimal $\mathcal{N} = 4$ topologically massive supergravity

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Abstract: Using the superconformal framework, we construct a new off-shell model for $\mathcal{N} = 4$ topologically massive supergravity which is minimal in the sense that it makes use of a single compensating vector multiplet and involves no free parameter. As such, it provides a counterexample to the common lore that two compensating multiplets are required within the conformal approach to supergravity with eight supercharges in diverse dimensions. This theory is an off-shell $\mathcal{N} = 4$ supersymmetric extension of chiral gravity. All of its solutions correspond to non-conformally flat superspaces. Its maximally supersymmetric solutions include the so-called critical (4,0) anti-de Sitter superspace introduced in [25], and well as warped critical (4,0) anti-de Sitter superspaces. We also propose a dual formulation for the theory in which the vector multiplet is replaced with an off-shell hypermultiplet. Upon elimination of the auxiliary fields belonging to the hypermultiplet and imposing certain gauge conditions, the dual action reduces to the one introduced in [20].

Keywords: Extended Supersymmetry, Supergravity Models, Superspaces

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JHEP03(2017)109

Contents

JHEP03(2017)109

1 Introduction

A unique feature of three spacetime dimensions (3D) is the existence of topologically massive Yang-Mills and gravity theories. They are obtained by augmenting the usual Yang-Mills action or the gravitational action by a gauge-invariant topological mass term. Such a mass term coincides with a non-Abelian Chern-Simons action in the Yang-Mills case [1–4] and with a Lorentzian Chern-Simons term in the case of gravity [3, 4]. Without adding the Lorentzian Chern-Simons term, the pure gravity action propagates no local degrees of freedom. The Lorentzian Chern-Simons term can be interpreted as the action for conformal gravity in three dimensions [3, 5, 6].[1]

Topologically massive theories of gravity possess supersymmetric extensions. In particular, $\mathcal{N} = 1$ topologically massive supergravity was introduced in [9] and its cosmological extension followed in [10]. The off-shell formulations for $\mathcal{N}$-extended topologically massive supergravity theories were presented in [11] for $\mathcal{N} = 2$ and in [12] for $\mathcal{N} = 3$ and $\mathcal{N} = 4$. In

[1]The usual Einstein-Hilbert action for 3D gravity with a cosmological term can also be interpreted as the Chern-Simons action for the anti-de Sitter group [7, 8].

all of these theories, the action functional is a sum of two terms, one of which is the action for pure $\mathcal{N}$-extended supergravity (Poincaré or anti-de Sitter) and the other is the action for $\mathcal{N}$-extended conformal supergravity. The off-shell actions for $\mathcal{N}$-extended supergravity theories in three dimensions were given in [13] for $\mathcal{N} = 1$, [14, 15] for $\mathcal{N} = 2$, and [14] for the cases $\mathcal{N} = 3,\ 4$. The off-shell actions for $\mathcal{N}$-extended conformal supergravity were given in [5] for $\mathcal{N} = 1$, [16] for $\mathcal{N} = 2$, and [17] for $\mathcal{N} = 3,\ 4$. The latter work made use of the formulation for $\mathcal{N}$-extended conformal supergravity presented in [18].

The off-shell structure of 3D $\mathcal{N} = 4$ supergravity [14] is analogous to that of 4D $\mathcal{N} = 2$ supergravity (see, e.g., [19] for a pedagogical review) in the sense that two superconformal compensators are required (for instance, two off-shell vector multiplets, one of which is self-dual and the other anti-self-dual) in order to realise pure Poincaré or anti-de Sitter (AdS) supergravity theories. We recall that the equations of motion for pure $\mathcal{N} = 4$ Poincaré or AdS supergravity are inconsistent if one makes use of a single compensator [12]. By construction, the off-shell $\mathcal{N} = 4$ topologically massive supergravity theory of [12] makes use of two compensators. However, in [20] the consistent system of dynamical equations was proposed for $\mathcal{N} = 4$ topologically massive AdS supergravity with a single compensating hypermultiplet, following earlier work in [21–23] on ABJ(M) models. A peculiar feature of this model, like those considered in [21–23], is that it has no free parameter. Consequently the dimensionless combination, $\mu\ell$, of mass μ and AdS radius ℓ takes a fixed value, $\mu\ell = 1$, as in chiral gravity [24]. In [24] it was argued that $\mu\ell = 1$ is the only value for the quantum theory to have a chance to be free of ghosts. It is thus interesting that the $\mathcal{N} = 4$ theory of [20] picks precisely this value.[2]

In [20] a supergravity action functional was also postulated to generate the dynamical equations given. This action was claimed to be off-shell without giving technical details. In this paper we propose a new off-shell model for $\mathcal{N} = 4$ topologically massive supergravity which is minimal in the sense that it makes use of a single compensating vector multiplet. The theory is consistent only if the term corresponding to $\mathcal{N} = 4$ conformal supergravity is turned on. An important maximally supersymmetric solution for this theory is the so-called critical (4,0) AdS superspace introduced in [25]. Our supergravity theory is first presented in a manifestly supersymmetric form, and then its action functional is reduced to components. By choosing appropriate gauge conditions at the component level and performing a duality transformation, we show how to reduce our off-shell supergravity action to the one postulated in [20].

This paper is organised as follows. In section 2 we recall the superspace geometry of the two $\mathcal{N} = 4$ vector multiplets and the corresponding locally supersymmetric actions. In section 3 we present two models for minimal $\mathcal{N} = 4$ topologically massive supergravity, analyse their equations of motion and give a brief discussion of the maximally supersymmetric solutions. Section 4 is devoted to the component structure of minimal $\mathcal{N} = 4$ topologically massive supergravity. Concluding comments are given in section 5. The main body of the paper is accompanied with three technical appendices. The essential details

[2]The only known models which pick precisely this value are the topologically gauged ABJ(M) models of [21–23].

JHEP03(2017)109

of the known superspace formulations for $\mathcal{N} = 4$ conformal supergravity are collected in appendices A and B. Some useful super-Weyl gauge conditions in SO(4) superspace and their implications are given in appendix C.

2 The $\mathcal{N} = 4$ vector multiplets

There are two inequivalent irreducible $\mathcal{N} = 4$ vector multiplets in three dimensions, self-dual and anti-self-dual ones, as discovered by Brooks and Gates [26]. In this section we review the superspace geometry of these supermultiplets in the presence of $\mathcal{N} = 4$ conformal supergravity [14, 18] and the corresponding locally supersymmetric actions [14].

Throughout this paper we make use of both the SO(4) superspace formulation of conformal supergravity, which was sketched in [27] and fully developed in [14], and the conformal superspace formulation presented in [18]. These formulations are related to each other since SO(4) superspace may be viewed as a gauge fixed version of the $\mathcal{N} = 4$ conformal superspace [18]. Due to this reason, we will first start by formulating vector multiplets in conformal superspace. We refer the reader to appendix A for the salient details of the conformal superspace formulation. The geometry of SO(4) superspace in briefly reviewed in appendix B.

2.1 Kinematics

To describe an Abelian vector multiplet in a curved superspace $\mathcal{M}^{3|8}$ parametrised by coordinates $z^M = (x^m,\ \theta^\mu_{\mathcal{I}})$, we introduce gauge covariant derivatives

$$\boldsymbol{\nabla} = E^A \boldsymbol{\nabla}_A\,, \quad \boldsymbol{\nabla}_A = (\boldsymbol{\nabla}_a, \boldsymbol{\nabla}^I_\alpha) := \nabla_A - V_A \boldsymbol{Z}\,, \quad [\boldsymbol{Z}, \boldsymbol{\nabla}_A] = 0\,, \tag{2.1}$$

with $E^A = \mathrm{d}Z^M E_M{}^A$ the superspace vielbein, ∇_A the superspace covariant derivatives (A.2) obeying the (anti-)commutation relations (A.4), and $V = E^A V_A$ the gauge connection associated with the generator $\boldsymbol{Z}$. The gauge transformation of V is

$$\delta V = \mathrm{d}\tau\,, \tag{2.2}$$

where the gauge parameter $\tau(z)$ is an arbitrary real scalar superfield.

The algebra of gauge covariant derivatives is

$$\begin{aligned}[\boldsymbol{\nabla}_A, \boldsymbol{\nabla}_B\} = &-T_{AB}{}^C \boldsymbol{\nabla}_C - \frac{1}{2} R(M)_{AB}{}^{cd} M_{cd} - \frac{1}{2} R(N)_{AB}{}^{PQ} N_{PQ} - R(\mathbb{D})_{AB} \mathbb{D} \\ &- R(S)_{AB}{}^\gamma_I S^I_\gamma - R(K)_{AB}{}^c K_c - F_{AB} \boldsymbol{Z}\,,\end{aligned} \tag{2.3}$$

where the torsion and curvatures are those of conformal superspace but with F_{AB} corresponding to the gauge covariant field strength $F = \frac{1}{2} E^B \wedge E^A F_{AB} = \mathrm{d}V$. The field strength F_{AB} satisfies the Bianchi identity

$$\mathrm{d}F = 0\,, \quad \nabla_{[A} F_{BC\}} + T_{[AB}{}^D F_{|D|C\}} = 0 \tag{2.4}$$

and must be subject to covariant constraints to describe an irreducible vector multiplet.

JHEP03(2017)109

In order to describe an $\mathcal{N}=4$ vector multiplet, the superform F is subject to the constraint (see [14] for more details)

$$F^{IJ}_{\alpha\beta} = -2\mathrm{i}\varepsilon_{\alpha\beta}G^{IJ}\,, \quad G^{IJ} = -G^{JI}\,, \tag{2.5a}$$

and then the Bianchi identity fixes the remaining components of F to be

$$F_{a\beta}^{\ J} = \frac{1}{3}(\gamma_a)_\beta{}^\gamma \nabla_{\gamma K} G^{JK}\,, \tag{2.5b}$$

$$F_{ab} = -\frac{\mathrm{i}}{48}\varepsilon_{abc}(\gamma^c)^{\alpha\beta}[\nabla^K_\alpha, \nabla^L_\beta]G_{KL}\,, \tag{2.5c}$$

where G^{IJ} is primary and of dimension 1,

$$S^I_\alpha G^{JK} = 0\,, \quad K_a G^{IJ} = 0\,, \quad \mathbb{D}G^{IJ} = G^{IJ}\ . \tag{2.6}$$

Moreover, the field strength G^{IJ} is constrained by the dimension-3/2 Bianchi identity

$$\nabla^I_\gamma G^{JK} = \nabla^{[I}_\gamma G^{JK]} - \frac{2}{3}\delta^{I[J}\nabla_{\gamma L}G^{K]L}\ . \tag{2.7}$$

It is well known (see [14] and references therein) that the constraint (2.7) defines a reducible off-shell supermultiplet.[3] The point is that the Hodge-dual of G^{IJ},

$$\tilde{G}^{IJ} := \frac{1}{2}\varepsilon^{IJKL}G_{KL}\,, \tag{2.8}$$

obeys the same constraint as G^{IJ} does,

$$\nabla^I_\gamma \tilde{G}^{JK} = \nabla^{[I}_\gamma \tilde{G}^{JK]} - \frac{2}{3}\delta^{I[J}\nabla_{\gamma L}\tilde{G}^{K]L}\,, \tag{2.9a}$$

where ε^{IJKL} is the Levi-Civita tensor. As a result one may constrain the field strength G^{IJ} to be self-dual, $\tilde{G}^{IJ} = G^{IJ}$ or anti-self-dual, $\tilde{G}^{IJ} = -G^{IJ}$. These choices correspond to two different irreducible off-shell $\mathcal{N}=4$ vector multiplets, which we denote by G^{IJ}_+ and G^{IJ}_-, respectively. In what follows we will make use of an (anti-)self-dual Abelian vector multiplet such that its field strength $G^{IJ}_\pm$ is nowhere vanishing, $G^2_\pm := \frac{1}{2}G^{IJ}_\pm G_{\pm IJ} \neq 0$.

When working with $\mathcal{N}=4$ supersymmetric theories, a powerful technical tool is the isospinor notation based on the isomorphism $\mathrm{SO}(4) \cong \big(\mathrm{SU}(2)_\mathrm{L} \times \mathrm{SU}(2)_\mathrm{R}\big)/\mathbb{Z}_2$, which allows one to replace each SO(4) vector index with a pair of isospinor ones. In defining the isospinor notation, we follow [14] and associate with a real SO(4) vector V_I a second-rank isospinor $V_{i\bar{i}}$ defined as

$$V_I \to V_{i\bar{i}} := (\tau^I)_{i\bar{i}}V_I\,, \quad V_I = \tau_I{}^{i\bar{i}}V_{i\bar{i}}\,, \quad (V_{i\bar{i}})^* = V^{i\bar{i}}\,, \tag{2.10}$$

where we have introduced the τ-matrices

$$(\tau^I)_{i\bar{i}} = (\mathbb{1}, \mathrm{i}\sigma_1, \mathrm{i}\sigma_2, \mathrm{i}\sigma_3)\,, \qquad I = 1, \cdots, 4\,, \quad i = 1, 2\,, \quad \bar{i} = \bar{1}, \bar{2}\ . \tag{2.11}$$

[3] Such a long $\mathcal{N}=4$ supermultiplet naturally originates upon reduction of any off-shell $\mathcal{N}>4$ vector multiplet to $\mathcal{N}=4$ superspace [28].

JHEP03(2017)109

The isospinor indices of $\mathrm{SU}(2)_{\mathrm{L}}$ and $\mathrm{SU}(2)_{\mathrm{R}}$ spinors ψ_i and $\chi_{\bar{i}}$, respectively, are raised and lowered using the antisymmetric tensors $\varepsilon^{ij}, \varepsilon_{ij}$ and $\varepsilon^{\bar{i}\bar{j}}, \varepsilon_{\bar{i}\bar{j}}$ (normalised by $\varepsilon^{12} = \varepsilon_{21} = \varepsilon^{\bar{1}\bar{2}} = \varepsilon_{\bar{2}\bar{1}} = 1$) according to

$$\psi^i = \varepsilon^{ij}\psi_j\,, \quad \psi_i = \varepsilon_{ij}\psi^j\,, \quad \chi^{\bar{i}} = \varepsilon^{\bar{i}\bar{j}}\chi_{\bar{j}}\,, \quad \chi_{\bar{i}} = \varepsilon_{\bar{i}\bar{j}}\chi^{\bar{j}}\,. \tag{2.12}$$

We then have the following dictionary:

$$V^I U_I = V^{i\bar{i}} U_{i\bar{i}}\,, \tag{2.13a}$$

$$A_{i\bar{i}j\bar{j}} := A_{IJ}(\tau^I)_{i\bar{i}}(\tau^J)_{j\bar{j}} = \varepsilon_{ij}A_{\bar{i}\bar{j}} + \varepsilon_{\bar{i}\bar{j}}A_{ij}\,, \quad A_{ij} = A_{ji}\,, \quad A_{\bar{i}\bar{j}} = A_{\bar{j}\bar{i}}\,, \tag{2.13b}$$

$$\frac{1}{2}A^{IJ}B_{IJ} = A^{ij}B_{ij} + A^{\bar{i}\bar{j}}B_{\bar{i}\bar{j}}\,, \tag{2.13c}$$

$$\varepsilon_{i\bar{i}j\bar{j}k\bar{k}l\bar{l}} = \varepsilon_{ij}\varepsilon_{kl}\varepsilon_{\bar{i}\bar{l}}\varepsilon_{\bar{j}\bar{k}} - \varepsilon_{il}\varepsilon_{jk}\varepsilon_{\bar{i}\bar{j}}\varepsilon_{\bar{k}\bar{l}}\,, \tag{2.13d}$$

where V^I and U^I are SO(4) vectors, A^{IJ} and B^{IJ} are anti-symmetric second-rank SO(4) tensors. The left-hand side of (2.13d) is the Levi-Civita tensor in the isospinor notation.

In the isospinor notation, the self-dual (G^{IJ}_+) and anti-self-dual (G^{IJ}_-) vector multiplets take the form

$$G^{i\bar{i}j\bar{j}}_+ = -\varepsilon^{ij}G^{\bar{i}\bar{j}}\,, \qquad G^{i\bar{i}j\bar{j}}_- = -\varepsilon^{\bar{i}\bar{j}}G^{ij}\,, \tag{2.14}$$

and the Bianchi identity (2.7) turns into

$$\nabla^{(i\bar{i}}_\alpha G^{kl)} = 0\,, \qquad \nabla^{i(\bar{i}}_\alpha G^{\bar{k}\bar{l})} = 0\,. \tag{2.15}$$

At this stage it is useful to introduce left and right isospinor variables $v_{\mathrm{L}} := v^i \in \mathbb{C}^2 \setminus \{0\}$ and $v_{\mathrm{R}} := v^{\bar{i}} \in \mathbb{C}^2 \setminus \{0\}$, which can be used to package the anti-self-dual field strength G^{ij} and the self-dual field strength $G^{\bar{i}\bar{j}}$ into fields without isospinor indices, $G^{(2)}_{\mathrm{L}}(v_{\mathrm{L}}) := G_{ij}v^iv^j$ and $G^{(2)}_{\mathrm{R}}(v_{\mathrm{R}}) := G_{\bar{i}\bar{j}}v^{\bar{i}}v^{\bar{j}}$, respectively. The same isospinor variables can be used to define two different subsets, $\nabla^{(1)\bar{i}}_\alpha$ and $\nabla^{(\bar{1})i}_\alpha$, in the set of spinor covariant derivatives $\nabla^{i\bar{i}}_\alpha$ by the rule

$$\nabla^{(1)\bar{i}}_\alpha := v_i\nabla^{i\bar{i}}_\alpha\,, \qquad \nabla^{(\bar{1})i}_\alpha := v_{\bar{i}}\nabla^{i\bar{i}}_\alpha\,. \tag{2.16}$$

It follows from (A.17) that the operators $\nabla^{(1)\bar{i}}_\alpha$ obey the anti-commutation relations:

$$\begin{aligned}\{\nabla^{(1)\bar{i}}_\alpha, \nabla^{(1)\bar{j}}_\beta\} &= 2\mathrm{i}\varepsilon_{\alpha\beta}\varepsilon^{\bar{i}\bar{j}}WL^{(2)} + \mathrm{i}\varepsilon_{\alpha\beta}\varepsilon^{\bar{i}\bar{j}}\nabla^{\gamma(1)}{}_{\bar{k}}WS^{(1)\bar{k}}_\gamma \\ &\quad -\frac{1}{4}\varepsilon_{\alpha\beta}\varepsilon^{\bar{i}\bar{j}}\nabla_\gamma{}^{(1)}{}_{\bar{k}}\nabla^{(1)\bar{k}}_\delta WK^{\gamma\delta}\,,\end{aligned} \tag{2.17}$$

where $L^{(2)} = v_iv_jL^{ij}$ and $S^{(1)\bar{i}}_\alpha$ is defined similarly to $\nabla^{(1)\bar{i}}_\alpha$. The rationale for the definitions given is that the constraints (2.15) now become the analyticity conditions

$$\nabla^{(1)\bar{i}}_\alpha G^{(2)}_{\mathrm{L}} = 0\,, \qquad \nabla^{(\bar{1})i}_\alpha G^{(2)}_{\mathrm{R}} = 0\,. \tag{2.18}$$

JHEP03(2017)109

which tell us that each of $G^{(2)}_{\rm L}$ and $G^{(2)}_{\rm R}$ depends on half the Grassmann coordinates. The constraints (2.18) do not change under re-scalings $v^i \to c_{\rm L} v^i$ and $v^{\bar{i}} \to c_{\rm R} v^{\bar{i}}$, with $c_{\rm L}, c_{\rm R} \in \mathbb{C} \setminus \{0\}$, with respect to which $G^{(2)}_{\rm L}(v_{\rm L})$ and $G^{(2)}_{\rm R}(v_{\rm R})$ are homogeneous polynomials of degree two. We see that the isospinor variables $v_{\rm L}$ and $v_{\rm R}$ are defined modulo the equivalence relations $v^i \sim c_{\rm L} v^i$ and $v^{\bar{i}} \sim c_{\rm R} v^{\bar{i}}$, and therefore they parametrise identical complex projective spaces $\mathbb{C}P^1_{\rm L}$ and $\mathbb{C}P^1_{\rm R}$. The superfields $G^{(2)}_{\rm L}(v_{\rm L})$ and $G^{(2)}_{\rm R}(v_{\rm R})$ are naturally defined on curved $\mathcal{N} = 4$ projective superspace $\mathcal{M}^{3|8} \times \mathbb{C}P^1_{\rm L} \times \mathbb{C}P^1_{\rm R}$ introduced in [14].

The field strengths $G^{(2)}_{\rm L}(v_{\rm L})$ and $G^{(2)}_{\rm R}(v_{\rm R})$ are examples of the covariant projective multiplets introduced in [14] in SO(4) superspace and later reformulated in [12] within the conformal superspace setting. There are two types of covariant projective multiplets, the left and right ones. A left projective multiplet of weight n, $Q^{(n)}_{\rm L}(v_{\rm L})$, is a superfield that is defined on some open domain of $\mathbb{C}^2 \setminus \{0\}$ and possesses the following four properties. Firstly, it is a primary superfield,

$$S^{i\bar{i}}_\alpha Q^{(n)}_{\rm L} = 0\,, \quad K_a Q^{(n)}_{\rm L} = 0\,. \tag{2.19}$$

Secondly, it is subject to the constraint

$$\nabla^{(1)\bar{i}}_\alpha Q^{(n)}_{\rm L} = 0\,. \tag{2.20}$$

Thirdly, it is a *holomorphic* function of $v_{\rm L}$. Fourthly, it is *homogeneous* function of $v_{\rm L}$ of degree n,

$$Q^{(n)}_{\rm L}(c\, v_{\rm L}) \;=\; c^n\, Q^{(n)}_{\rm L}(v_{\rm L})\,, \qquad c \in \mathbb{C} \setminus \{0\}\,. \tag{2.21}$$

Every left projective multiplet is inert with respect to $\mathrm{SU}(2)_{\rm R}$ and transforms under $\mathrm{SU}(2)_{\rm L}$ as

$$\delta_\Lambda Q^{(n)}_{\rm L} \;=\; \Lambda^{ij} L_{ij} Q^{(n)}_{\rm L}\,, \tag{2.22a}$$

$$\Lambda^{ij} L_{ij} Q^{(n)}_{\rm L} \;=\; -(\Lambda^{(2)}_{\rm L} \partial^{(-2)}_{\rm L} - n \Lambda^{(0)}_{\rm L}) Q^{(n)}_{\rm L}\,, \tag{2.22b}$$

where we have defined

$$\Lambda^{(2)}_{\rm L} := \Lambda^{ij} v_i v_j\,, \quad \Lambda^{(0)}_{\rm L} := \frac{v_i u_j}{(v_{\rm L}, u_{\rm L})} \Lambda^{ij} \tag{2.23}$$

and made use of the differential operator

$$\partial^{(-2)}_{\rm L} := \frac{1}{(v_{\rm L}, u_{\rm L})} u^i \frac{\partial}{\partial v^i}\,, \quad (v_{\rm L}, u_{\rm L}) = v^i u_i\,. \tag{2.24}$$

Here we have also introduced a second left isospinor variable $u_{\rm L} := u^i$ which is restricted to be linearly independent of $v_{\rm L}$, that is $(v_{\rm L}, u_{\rm L}) \neq 0$. One may see that $L^{(2)} Q^{(n)}_{\rm L} = 0$, and therefore the integrability condition $\{\nabla^{(1)\bar{i}}_\alpha, \nabla^{(1)\bar{j}}_\beta\} Q^{(n)} = 0$ for the constraint (2.20) holds, in accordance with (2.17). The right projective multiplets are defined similarly. The covariant projective multiplets $G^{(2)}_{\rm L}(v_{\rm L})$ and $G^{(2)}_{\rm R}(v_{\rm R})$ are known as the left and right $\mathcal{O}(2)$ multiplets, respectively.

JHEP03(2017)109

As shown in [14] the self-dual vector multiplet, $G^{(2)}_{\rm R}(v_{\rm R})$, can be described in terms of a gauge prepotential $\mathcal{V}_{\rm L}(v_{\rm L})$, which is a left weight-0 *tropical multiplet* and is real with respect to the analyticity preserving conjugation called the smile conjugation. The interested reader is referred to [14] for the technical details. Similar properties hold for the anti-self-dual vector multiplet except all 'left' objects have to be replaced by 'right' ones and vice versa.

2.2 Dynamics

General off-shell matter couplings in $\mathcal{N}=4$ supergravity were constructed in [14]. The action for such a supergravity-matter system may be represented as a sum of two terms (one of which may be absent),

$$S = S_{\rm L} + S_{\rm R} \ . \tag{2.25}$$

The left $S_{\rm L}$ and right $S_{\rm R}$ actions, are naturally formulated in curved $\mathcal{N}=4$ projective superspace. The left action has the form

$$S_{\rm L} = \frac{1}{2\pi} \oint (v_{\rm L}, {\rm d}v_{\rm L}) \int {\rm d}^{3|8}z \, E \, C^{(-4)}_{\rm L} \mathcal{L}^{(2)}_{\rm L} \,, \qquad E^{-1} = {\rm Ber}(E_A{}^M) \,, \tag{2.26}$$

where the Lagrangian $\mathcal{L}^{(2)}_{\rm L}(v_{\rm L})$ is a real left projective multiplet of weight 2, and ${\rm d}^{3|8}z$ denotes the full superspace integration measure, ${\rm d}^{3|8}z := {\rm d}^3x \, {\rm d}^8\theta$. Furthermore, the *model-independent* primary isotwistor superfield $C^{(-4)}_{\rm L}(v_{\rm L})$ has dimension -2, i.e. $\mathbb{D} C^{(-4)}_{\rm L} = -2C^{(-4)}_{\rm L}$. It is defined to be real with respect to the smile-conjugation defined in [14] and obeys the differential equation

$$\Delta^{(4)}_{\rm L} C^{(-4)}_{\rm L} = 1 \ . \tag{2.27}$$

Here $\Delta^{(4)}_{\rm L}$ denotes the following fourth-order operator[4]

$$\Delta^{(4)}_{\rm L} = \frac{1}{96}\Big(\nabla^{(2)\bar{i}\bar{j}}\nabla^{(2)}_{\bar{i}\bar{j}} - \nabla^{(2)\alpha\beta}\nabla^{(2)}_{\alpha\beta}\Big) = \frac{1}{48}\nabla^{(2)\bar{i}\bar{j}}\nabla^{(2)}_{\bar{i}\bar{j}} \,, \tag{2.28}$$

with $\nabla^{(2)}_{\bar{i}\bar{j}} := \nabla^{(1)}_{(\bar{i}}\nabla^{(1)}_{\gamma\bar{j})}$ and $\nabla^{(2)}_{\alpha\beta} := \nabla^{(1)\bar{k}}_{(\alpha}\nabla^{(1)}_{\beta)\bar{k}}$. The action (2.26) is independent of the representative $C^{(-4)}_{\rm L}$ in the sense that it does not change under an arbitrary infinitesimal variation of $C^{(-4)}_{\rm L}$ subject to the above conditions. The structure of $S_{\rm R}$ is analogous.

There are two equivalent action functionals to describe the dynamics of a single self-dual Abelian vector multiplet coupled to conformal supergravity. One of them is a right action formulated in terms of a right $\mathcal{O}(2)$ multiplet $G^{(2)}_{\rm R}(v_{\rm R}) = v_{\bar{i}}v_{\bar{j}}G^{\bar{i}\bar{j}}$, which is associated with the superfield strength $G^{\bar{i}\bar{j}}$ of the vector multiplet. This action, has the form[5] [14]

$$S^{(+)}_{\rm VM} := \frac{\sqrt{2}}{\pi} \oint (v_{\rm R}, {\rm d}v_{\rm R}) \int {\rm d}^{3|8}z \, E \, C^{(-4)}_{\rm R} G^{(2)}_{\rm R} \ln \frac{G^{(2)}_{\rm R}}{{\rm i}\Upsilon^{(1)}_{\rm R}\breve{\Upsilon}^{(1)}_{\rm R}} \,, \tag{2.29}$$

[4]The operator $\Delta^{(4)}_{\rm L}$ is a covariant projection operator. Given a covariant left projective multiplet $Q^{(n)}_{\rm L}(v_{\rm L})$ of weight n, it may be represented in the form $Q^{(n)}_{\rm L} = \Delta^{(4)}_{\rm L} T^{(n-4)}_{\rm L}$, for some left isotwistor superfield $T^{(n-4)}_{\rm L}(v_{\rm L})$, see [14] for details.

[5]We should emphasise that in this paper we have defined the vector multiplet actions with "wrong" sign, because in our approach they correspond to superconformal compensators.

JHEP03(2017)109

where the weight-one arctic multiplet $\Upsilon^{(1)}_{\rm R}$ and its smile conjugate $\breve{\Upsilon}^{(1)}_{\rm R}$ are pure gauge degrees of freedom. The action (2.29) is the 3D $\mathcal{N}=4$ counterpart of the projective-superspace action [29] for the 4D $\mathcal{N}=2$ improved tensor multiplet [30]. The other representation for $S^{(+)}_{\rm VM}$ makes use of a left tropical prepotential $\mathcal{V}_{\rm L}(v_{\rm L})$ of the self-dual vector multiplet with gauge transformations

$$\delta\mathcal{V}_{\rm L} = \lambda_{\rm L} + \breve{\lambda}_{\rm L} \ . \tag{2.30}$$

The gauge parameter $\lambda_{\rm L}$ is an arbitrary left arctic multiplet of weight zero. The gauge invariant field strength, $G^{\bar{i}\bar{j}}$, is related to $\mathcal{V}_{\rm L}$ through

$$G^{(2)}_{\rm R}(v_{\rm R}) = v_{\bar{i}}v_{\bar{j}}G^{\bar{i}\,\bar{j}} = \frac{\rm i}{4}v_{\bar{i}}v_{\bar{j}}\oint\frac{(v_{\rm L},{\rm d}v_{\rm L})}{2\pi}\frac{u_iu_j}{(v_{\rm L},u_{\rm L})^2}\nabla^{\alpha i\bar{i}}\nabla_\alpha{}^{j\bar{j}}\mathcal{V}_{\rm L}(v_{\rm L}) \ . \tag{2.31}$$

Here $u_{\rm L}=u^i$ is a constant isospinor such that $(v_{\rm L},u_{\rm L})\neq 0$ along the closed integration contour.[6] The action (2.29) can be recast as a left BF-type action [12]

$$S^{(+)}_{\rm VM} = -\frac{1}{2\pi}\oint(v_{\rm L},{\rm d}v_{\rm L})\int{\rm d}^{3|8}z\,E\,C^{(-4)}_{\rm L}\mathcal{V}_{\rm L}\boldsymbol{G}^{(2)}_{\rm L} \ , \tag{2.32}$$

where $\boldsymbol{G}^{(2)}_{\rm L}(v_{\rm L}) = v_iv_j\boldsymbol{G}^{ij}$ is the composite left $\mathcal{O}(2)$ multiplet [12]

$$\begin{aligned}\boldsymbol{G}^{(2)}_{\rm L} &= -\frac{\rm i}{\sqrt{2}}v_iv_j\oint\frac{(v_{\rm R},{\rm d}v_{\rm R})}{2\pi}\frac{u_{\bar{i}}u_{\bar{j}}}{(v_{\rm R},u_{\rm R})^2}\nabla^{\alpha i\bar{i}}\nabla_\alpha{}^{j\bar{j}}\ln\frac{G^{(2)}_{\rm R}}{{\rm i}\Upsilon^{(1)}_{\rm R}\breve{\Upsilon}^{(1)}_{\rm R}} \\ &= \frac{\rm i}{4}v_iv_j\nabla^{\alpha i\bar{i}}\nabla^{j\bar{j}}_\alpha\left(\frac{G_{\bar{i}\bar{j}}}{G_+}\right) \ .\end{aligned} \tag{2.33}$$

The composite left superfield $\boldsymbol{G}^{ij}$ can be equivalently realised as the anti-self-dual SO(4) bivector $\boldsymbol{G}^{IJ}_-$.

Similarly, the action for the anti-self-dual vector multiplet [14] can be recast as the right BF-type action [12]

$$S^{(-)}_{\rm VM} := -\frac{1}{2\pi}\oint(v_{\rm R},{\rm d}v_{\rm R})\int{\rm d}^{3|8}z\,E\,C^{(-4)}_{\rm R}\mathcal{V}_{\rm R}\boldsymbol{G}^{(2)}_{\rm R} \ , \tag{2.34}$$

where $\boldsymbol{G}^{(2)}_{\rm R}(v_{\rm R}) = v_{\bar{i}}v_{\bar{j}}\boldsymbol{G}^{\bar{i}\,\bar{j}}$ is the composite right $\mathcal{O}(2)$ multiplet [12]

$$\begin{aligned}\boldsymbol{G}^{(2)}_{\rm R} &= -\frac{\rm i}{\sqrt{2}}v_{\bar{i}}v_{\bar{j}}\oint\frac{(v_{\rm L},{\rm d}v_{\rm L})}{2\pi}\frac{u_iu_j}{(v_{\rm L},u_{\rm L})^2}\nabla^{\alpha i\bar{i}}\nabla_\alpha{}^{j\bar{j}}\ln\frac{G^{(2)}_{\rm L}}{{\rm i}\Upsilon^{(1)}_{\rm L}\breve{\Upsilon}^{(1)}_{\rm L}} \\ &= v_{\bar{i}}v_{\bar{j}}\frac{\rm i}{4}\nabla^{\alpha i\bar{i}}\nabla^{j\bar{j}}_\alpha\left(\frac{G_{ij}}{G_-}\right) ,\end{aligned} \tag{2.35}$$

and $\mathcal{V}_{\rm R}(v_{\rm R})$ is the tropical prepotential of the anti-self-dual vector multiplet. The composite right superfield (2.35) can be equivalently realised as the self-dual SO(4) bivector $\boldsymbol{G}^{IJ}_+$.

[6] One may show that the right-hand side of (2.31) is independent of $u_{\rm L}$.

JHEP03(2017)109

The composite $\mathcal{O}(2)$ multiplets can be expressed in terms of SO(4) vector indices as follows [12]

$$\boldsymbol{G}^{IJ}_{\pm} = X^{IJ}_{\mp} \pm \frac{1}{2}\varepsilon^{IJKL}X_{\mp KL}\,, \quad \frac{1}{2}\varepsilon_{IJKL}\boldsymbol{G}^{KL}_{\pm} = \pm\boldsymbol{G}_{\pm IJ}\,, \tag{2.36}$$

where we have defined

$$X^{IJ}_{\pm} := \frac{\mathrm{i}}{6G_{\pm}}\nabla^{\gamma[I}\nabla_{\gamma K}G^{J]K}_{\pm} + \frac{2\mathrm{i}}{9G^{3}_{\pm}}\nabla^{\alpha P}G_{\pm KP}\nabla_{\alpha Q}G^{Q[I}_{\pm}G^{J]K}_{\pm}\,. \tag{2.37}$$

To show that $\boldsymbol{G}^{IJ}_{\pm}$ is primary and satisfies the Bianchi identity, the following identities prove useful

$$G^{IK}_{\pm}G_{\pm JK} = \frac{1}{2}\delta^{I}_{J}G^{2}_{\pm}\,, \tag{2.38a}$$

$$\varepsilon^{IJKL}G_{\pm LP} = \mp 3\delta^{[I}_{P}G^{JK]}_{\pm}\,. \tag{2.38b}$$

It is worth mentioning that the two $\mathcal{N}=4$ linear multiplet actions (2.32) and (2.34) are universal [12] in the sense that all known off-shell supergravity-matter systems (with the exception of pure conformal supergravity) may be described using such actions with appropriately engineered composite $\mathcal{O}(2)$ multiplets $\boldsymbol{G}^{(2)}_{\mathrm{L}}$ and $\boldsymbol{G}^{(2)}_{\mathrm{R}}$.

3 Minimal topologically massive supergravity

In this section we present two new supergravity-matter systems as models for minimal topologically massive supergravity.

3.1 Action principle and equations of motion

Our models for minimal topologically massive supergravity are described by $\mathcal{N}=4$ conformal supergravity coupled to a vector multiplet, either self-dual or anti-self-dual, via the following supergravity-matter actions:

$$\kappa S_{\pm} := \frac{1}{\mu}S_{\mathrm{CSG}} + S^{(\pm)}_{\mathrm{VM}}\,, \qquad \kappa^{2} = 1\,, \tag{3.1}$$

where S_{CSG} denotes the conformal supergravity action [17]. We will refer to the theories with actions S_{+} and S_{-} as the self-dual and anti-self-dual topologically massive supergravity (TMSG) theories, respectively.

As shown in [12], the equation of motion for the vector multiplet derived from the action (3.1) is equivalent to

$$\boldsymbol{G}_{\mp}{}^{IJ} = 0\,, \tag{3.2}$$

while the equation of motion for the conformal supergravity multiplet (that is, the $\mathcal{N}=4$ Weyl supermultiplet) is

$$\frac{1}{\mu}W + T_{\pm} = 0\,. \tag{3.3}$$

JHEP03(2017)109

Here $T_\pm$ is the supercurrent, which corresponds to the action $S^{(\pm)}_{\rm VM}$,

$$T_\pm = \pm G_\pm \ . \tag{3.4}$$

One can check that the supercurrent $T_\pm$ obeys the conservation equation [31]

$$\nabla^{\alpha(I}\nabla^{J)}_\alpha T_\pm = \frac{1}{4}\delta^{IJ}\nabla^\alpha_K\nabla^K_\alpha T \tag{3.5}$$

when the matter equation of motion (3.2) is satisfied.

Making use of the Bianchi identity (2.7) as well as the equations of motion (3.2)–(3.4), one finds the following equations on $G_\pm$:

$$\Big(\nabla^{\gamma(I}\nabla^{J)}_\gamma - \frac{1}{4}\delta^{IJ}\nabla^\gamma_K\nabla^K_\gamma\Big)G_\pm = 0\,, \tag{3.6a}$$

$$\Big(\nabla^\gamma_K\nabla^K_\gamma \mp 8{\rm i}W\Big)G^{-1}_\pm = 0\,, \tag{3.6b}$$

$$\frac{1}{\mu}W \pm G_\pm = 0\,, \tag{3.6c}$$

$$\nabla^{[I}_{(\alpha}\nabla^{J]}_{\beta)}G^{-1}_\pm = \pm\frac{1}{2}\varepsilon^{IJKL}\nabla_{(\alpha K}\nabla_{\beta)L}G^{-1}_\pm \ . \tag{3.6d}$$

We now turn to an analysis of the consequences of the equations of motion (3.6).

3.2 Analysing the equations of motion

To analyse the equations of motion corresponding to the action (3.1) we need to fix the gauge freedom. Firstly, we use the special conformal transformations to make the dilatation connection vanish, $B_A = 0$. This corresponds to degauging of conformal superspace to SO(4) superspace [14] and gives rise to new torsion terms[7] which can be expressed in terms of superfields $\mathcal{S}^{IJ}$, $\mathcal{S}$, $C_a{}^{IJ}$ and their covariant derivatives. We refer the reader to [14] for details and provide a summary of the salient details of SO(4) superspace in appendix B.

Upon imposing the gauge $B_A = 0$ one can show that (3.6) is equivalent to

$$\Big(\mathcal{D}^{\gamma(I}\mathcal{D}^{J)}_\gamma - \frac{1}{4}\delta^{IJ}\mathcal{D}^\gamma_K\mathcal{D}^K_\gamma - 4{\rm i}\mathcal{S}^{IJ}\Big)G_\pm = 0\,, \tag{3.7a}$$

$$\Big(\mathcal{D}^\gamma_K\mathcal{D}^K_\gamma + 8{\rm i}(2\mathcal{S} \mp W)\Big)G^{-1}_\pm = 0\,, \tag{3.7b}$$

$$\frac{1}{\mu}W \pm G_\pm = 0\,, \tag{3.7c}$$

$$(\mathcal{D}^{[I}_{(\alpha}\mathcal{D}^{J]}_{\beta)} - 4{\rm i}C_{\alpha\beta}{}^{IJ})G^{-1}_\pm = \pm\frac{1}{2}\varepsilon^{IJKL}(\mathcal{D}_{(\alpha K}\mathcal{D}_{\beta)L} - 4{\rm i}C_{\alpha\beta KL})G^{-1}_\pm\,, \tag{3.7d}$$

where $\mathcal{D}^I_\alpha$ is the SO(4) superspace covariant derivative [14, 27] (see also [18]). In isospinor index notation, for the self-dual vector multiplet one obtains

$$\Big(\mathcal{D}^{\gamma i\bar{i}}\mathcal{D}_{\gamma i\bar{i}} + 8{\rm i}(2\mathcal{S} - W)\Big)G^{-1}_+ = 0\,, \tag{3.8a}$$

$$(\mathcal{D}^{(\bar{i}\bar{k}}_\alpha\mathcal{D}_\beta{}^{j)}{}_{\bar{k}} - 4{\rm i}C_{\alpha\beta}{}^{ij})G^{-1}_+ = 0\,, \tag{3.8b}$$

$$(\mathcal{D}^{\gamma(i(\bar{i}}\mathcal{D}^{j)\bar{j})}_\gamma - 4{\rm i}\mathcal{S}^{ij\bar{i}\bar{j}})G_+ = 0\,, \tag{3.8c}$$

$$W + \mu G_+ = 0\,, \tag{3.8d}$$

[7] See [17] for more details. It is important to note that the SO(4) connection of SO(4) superspace differs from the one of conformal superspace by a redefinition, for details see [18].

JHEP03(2017)109

while for the anti-self-dual vector multiplet one finds

$$\Big(\mathcal{D}^{\gamma i\bar{i}}\mathcal{D}_{\gamma i\bar{i}} + 8\mathrm{i}(2\mathcal{S}+W)\Big)G_-^{-1} = 0\,, \tag{3.9a}$$

$$(\mathcal{D}_\alpha^{k(\bar{i}}\mathcal{D}_{\beta k}{}^{\bar{j})} - 4\mathrm{i}C_{\alpha\beta}{}^{\bar{i}\bar{j}})G_-^{-1} = 0\,, \tag{3.9b}$$

$$(\mathcal{D}^{\gamma(i(\bar{i}}\mathcal{D}_\gamma^{j)\bar{j})} - 4\mathrm{i}\mathcal{S}^{ij\bar{i}\bar{j}})G_- = 0\,, \tag{3.9c}$$

$$W - \mu G_- = 0\,. \tag{3.9d}$$

One should keep in mind that the equations of motion for G_+ and G_- derived from the actions S_+ and S_-, respectively, were used in the above results.

Under super-Weyl transformations the SO(4)-covariant derivatives and the torsion terms transform as[8]

$$\mathcal{D}_\alpha^I \to \mathcal{D}'^I_\alpha = \mathrm{e}^{\frac{1}{2}\sigma}\Big(\mathcal{D}_\alpha^I + (\mathcal{D}^{\beta I}\sigma)M_{\alpha\beta} + (\mathcal{D}_{\alpha J}\sigma)N^{IJ}\Big)\,, \tag{3.10a}$$

$$\mathcal{S}^{IJ} \to \mathcal{S}'^{IJ} = \frac{\mathrm{i}}{4}\mathrm{e}^{2\sigma}\Big(\mathcal{D}^{\gamma(I}\mathcal{D}_\gamma^{J)} - \frac{1}{4}\delta^{IJ}\mathcal{D}^{\gamma K}\mathcal{D}_{\gamma K} - 4\mathrm{i}\mathcal{S}^{IJ}\Big)\mathrm{e}^{-\sigma}\,, \tag{3.10b}$$

$$\mathcal{S} \to \mathcal{S}' = -\frac{\mathrm{i}}{16}(\mathcal{D}_K^\gamma\mathcal{D}_\gamma^K + 16\mathrm{i}\mathcal{S})\mathrm{e}^\sigma\,, \tag{3.10c}$$

$$C'_a{}^{IJ} \to C_a{}^{IJ} = -\frac{\mathrm{i}}{8}(\gamma_a)^{\alpha\beta}(\mathcal{D}_\alpha^{[I}\mathcal{D}_\beta^{J]} - 4\mathrm{i}C_{\alpha\beta}{}^{IJ})\mathrm{e}^\sigma\,, \tag{3.10d}$$

$$W \to W' = \mathrm{e}^\sigma W\,, \tag{3.10e}$$

where σ is a real unconstrained superfield. Within the superconformal framework, all supergravity-matter actions are required to be super-Weyl invariant.

The super-Weyl gauge freedom may be used to impose useful gauge conditions. For instance, one can make use of the super-Weyl transformations to gauge away the self-dual or anti-self-dual part of $C_a{}^{IJ}$ such that the remaining torsion components are expressed directly in terms of the matter fields. For concreteness, let us consider the theory described by the action S_+, with corresponding equations of motion (3.8), and gauge away $C_a{}^{\bar{i}\bar{j}}$ via a super-Weyl transformation. We then find

$$W = -\mu G_+\,, \tag{3.11a}$$

$$\mathcal{S}^{ij\bar{i}\bar{j}} = -\frac{\mathrm{i}}{4}G_+^{-1}\mathcal{D}^{\gamma(i(\bar{i}}\mathcal{D}_\gamma^{j)\bar{j})}G_+\,, \tag{3.11b}$$

$$2\mathcal{S} - W = \frac{\mathrm{i}}{8}G_+\mathcal{D}^{\gamma i\bar{i}}\mathcal{D}_{\gamma i\bar{i}}G_+^{-1}\,, \tag{3.11c}$$

$$C_{\alpha\beta}{}^{ij} = -\frac{\mathrm{i}}{4}G_+\mathcal{D}_\alpha^{(i\bar{k}}\mathcal{D}_\beta{}^{j)}{}_{\bar{k}}G_+^{-1}\,, \tag{3.11d}$$

$$C_a{}^{\bar{i}\bar{j}} = 0\,. \tag{3.11e}$$

In this gauge, we see that the geometry is determined in terms of a single superfield, which is chosen to be the scalar G_+. After imposing this super-Weyl gauge condition it is possible to show that there is enough super-Weyl freedom left to impose the additional condition

$$2\mathcal{S} + W = 0\,, \tag{3.12}$$

[8]The infinitesimal form was given in [14, 25].

JHEP03(2017)109

see appendix C for the derivation. This condition proves to lead to the following nonlinear equation for G_+:

$$\mathcal{D}^{\gamma i\bar{i}}\mathcal{D}_{\gamma i\bar{i}}G_+^{-1} + 16\mathrm{i}\mu = 0 \ . \tag{3.13}$$

The main virtue of the super-Weyl gauge conditions imposed is that all the torsion and curvature tensors are descendants of the single scalar superfield G_+. However, this gauge choice is not particularly useful from the point of view of studying (maximally) supersymmetric backgrounds. A more convenient super-Weyl gauge fixing is $G_+ = \text{const}$. We spell out the implications of such a gauge condition below.

Given a vector multiplet with a superfield strength G^{IJ} such that G is nowhere vanishing, one can always make use of the super-Weyl transformations to choose a gauge where

$$G = \frac{1}{2}G^{IJ}G_{IJ} = 1 \,, \quad \mathcal{D}^I_\alpha G^{JK} = 0 \ . \tag{3.14}$$

Such a gauge condition has slightly different consequences on the superspace geometry for the two vector multiplets G^{IJ}_+ and G^{IJ}_- satisfying the equations of motion (3.2) and (3.3). In both cases the super-Cotton tensor is constant,

$$W = \text{const} \quad \Longrightarrow \quad \mathcal{S}^{IJ} = 0 \,, \tag{3.15}$$

while the constraints on the remaining torsion components differ. For the on-shell self-dual vector multiplet one finds the following consistency conditions

$$\frac{1}{2}\varepsilon_{IJKL}C_a{}^{KL} = C_{aIJ} \,, \quad 2\mathcal{S} - W = 0 \,, \tag{3.16}$$

while for the on-shell anti-self-dual vector multiplet one finds

$$-\frac{1}{2}\varepsilon_{IJKL}C_a{}^{KL} = C_{aIJ} \,, \quad 2\mathcal{S} + W = 0 \ . \tag{3.17}$$

In the case where C_a^{IJ} vanishes, the algebra of covariant derivatives coincides with that of $(4,0)$ AdS superspace in the critical case where $2\mathcal{S} \mp W = 0$, see [25].[9] In general, however, $C_a{}^{IJ}$ does not vanish and instead satisfies some differential conditions implied by the Bianchi identities

$$\begin{aligned}[[\mathcal{D}_A, \mathcal{D}_B\}, \mathcal{D}_C\} \ &+ \ (-1)^{\varepsilon_A(\varepsilon_B+\varepsilon_C)}[[\mathcal{D}_B, \mathcal{D}_C\}, \mathcal{D}_A\} \\ &+ \ (-1)^{\varepsilon_C(\varepsilon_A+\varepsilon_B)}[[\mathcal{D}_C, \mathcal{D}_A\}, \mathcal{D}_B\} = 0 \ .\end{aligned} \tag{3.18}$$

To analyse the Bianchi identities in detail it will be useful to convert to isospinor notation.

We consider in detail the self-dual TMSG theory. In the isospinor notation, the covariant derivative algebra which follows from the equations of motion is

$$\begin{aligned}\{\mathcal{D}^{i\bar{i}}_\alpha, \mathcal{D}^{j\bar{j}}_\beta\} \ = \ &2\mathrm{i}\varepsilon^{ij}\varepsilon^{\bar{i}\bar{j}}\mathcal{D}_{\alpha\beta} + 4\mathrm{i}\varepsilon_{\alpha\beta}\varepsilon^{\bar{i}\bar{j}}WL^{ij} + 4\mathrm{i}C_{\alpha\beta}{}^{\bar{i}\bar{j}}L^{ij} \\ &+2\mathrm{i}\varepsilon_{\alpha\beta}\varepsilon^{ij}C^{\gamma\delta\bar{i}\bar{j}}M_{\gamma\delta} - 2\mathrm{i}\varepsilon^{ij}\varepsilon^{\bar{i}\bar{j}}WM_{\alpha\beta} \ .\end{aligned} \tag{3.19a}$$

[9] The $\mathcal{N} = 4$ super-Cotton tensor is denoted by X in [14, 25].

JHEP03(2017)109

Analysing the Bianchi identities (3.18) determines the remainder of the covariant derivative algebra:

$$\begin{aligned}\Big[\mathcal{D}_{\alpha\beta},\mathcal{D}_{\gamma}^{k\bar{k}}\Big] &= -\varepsilon_{\gamma(\alpha}W\mathcal{D}_{\beta)}^{k\bar{k}}+(\varepsilon_{\gamma(\alpha}C_{\beta)\delta}{}^{\bar{k}\bar{j}}+\varepsilon_{\delta(\alpha}C_{\beta)\gamma}{}^{\bar{k}\bar{j}})\mathcal{D}^{\delta k}{}_{\bar{j}}\\ &\quad +2\varepsilon_{\gamma(\alpha}C_{\beta)\delta\rho}{}^{k\bar{k}}M^{\delta\rho}-2C_{\alpha\beta\gamma}{}^{j\bar{k}}L_{j}{}^{k}\,, \qquad (3.19b)\\ [\mathcal{D}_{\alpha\beta},\mathcal{D}_{\gamma\delta}] &= \mathrm{i}\varepsilon_{\gamma(\alpha}C_{\beta)\delta\rho k\bar{k}}\mathcal{D}^{\rho k\bar{k}}+\mathrm{i}\varepsilon_{\delta(\alpha}C_{\beta)\gamma\rho k\bar{k}}\mathcal{D}^{\rho k\bar{k}}\\ &\quad +\varepsilon_{\delta(\alpha}W^2M_{\beta)\gamma}+\varepsilon_{\gamma(\alpha}W^2M_{\beta)\delta}\\ &\quad +\frac{\mathrm{i}}{12}\varepsilon_{\delta(\alpha}\Big(\mathcal{D}_{\beta)}^{k\bar{k}}\mathcal{D}_{\gamma k}{}^{\bar{l}}C_{\rho\sigma\bar{k}\bar{l}}\Big)M^{\rho\sigma}+\frac{\mathrm{i}}{12}\varepsilon_{\gamma(\alpha}\Big(\mathcal{D}_{\beta)}^{k\bar{k}}\mathcal{D}_{\delta k}{}^{\bar{l}}C_{\rho\sigma\bar{k}\bar{l}}\Big)M^{\rho\sigma}\\ &\quad -\varepsilon_{\delta(\alpha}C_{\beta)\gamma\bar{k}\bar{l}}C^{\rho\sigma\bar{k}\bar{l}}M_{\rho\sigma}-\varepsilon_{\gamma(\alpha}C_{\beta)\delta\bar{k}\bar{l}}C^{\rho\sigma\bar{k}\bar{l}}M_{\rho\sigma}\,, \qquad (3.19c)\end{aligned}$$

as well as the following differential constraint on $C_a{}^{\bar{i}\bar{j}}$

$$\mathcal{D}_{\alpha}^{i\bar{i}}C_{\beta\gamma}{}^{\bar{j}\bar{k}}=2\varepsilon^{\bar{i}(\bar{j}}C_{\alpha\beta\gamma}{}^{i\bar{k})}\ . \qquad (3.20)$$

The above constraint implies, in turn,

$$\mathcal{D}_{\alpha}{}^{\gamma}C_{\beta\gamma}{}^{\bar{i}\bar{j}}+C_{(\alpha}{}^{\gamma}{}_{\bar{k}}{}^{(\bar{i}}C_{\beta)\gamma}{}^{\bar{j})\bar{k}}+2WC_{\alpha\beta}{}^{\bar{i}\bar{j}}=0\ . \qquad (3.21)$$

Since the $\mathrm{SU}(2)_{\mathrm{R}}$ curvature vanishes, we can completely gauge away the corresponding connection. Such a gauge condition is assumed in what follows. In this gauge, the field strength $G^{\bar{i}\bar{j}}$ becomes a constant symmetric isospinor subject to the normalisation condition $G^{\bar{i}\bar{j}}G_{\bar{i}\bar{j}}=1$. It is invariant under a U(1) subgroup of $\mathrm{SU}(2)_{\mathrm{R}}$.

We are now in a position to describe all maximally supersymmetric solutions of the theory. In accordance with the general superspace analysis of supersymmetric backgrounds in diverse dimensions [32–34], such superspaces have to comply with the additional constraint

$$\mathcal{D}_{\alpha}^{i\bar{i}}C_{\beta\gamma}{}^{\bar{j}\bar{k}}=0\,, \qquad (3.22)$$

which leads to the integrability conditions

$$(\mathcal{D}_a-WM_a)C_b{}^{\bar{j}\bar{k}}=0\,, \qquad (3.23a)$$

$$C^{\gamma}{}_{(\alpha}{}^{\bar{i}\bar{j}}C_{\beta)\gamma}{}^{\bar{k}\bar{l}}=0\ . \qquad (3.23b)$$

The general solution of (3.23b) is

$$C_{\alpha\beta}{}^{\bar{i}\bar{j}}=C_{\alpha\beta}C^{\bar{i}\bar{j}}\,, \qquad (3.24)$$

where $C^{\bar{i}\bar{j}}$ is a constant symmetric rank-2 isospinor. Without loss of generality, $C^{\bar{i}\bar{j}}$ can be normalised as $C^{\bar{i}\bar{j}}C_{\bar{i}\bar{j}}=1$. The covariant constancy conditions (3.22) and (3.23a) now amount to

$$\mathcal{D}_{\alpha}^{i\bar{i}}C_b=0\,,\quad (\mathcal{D}_a-WM_a)C_b=0\ . \qquad (3.25)$$

JHEP03(2017)109

We recall that the Lorentz generator with a vector index, M_a, acts on a three-vector by the rule $M_a C_b = \varepsilon_{abc} C^c$. The second condition in (3.25) implies that C_b is a Killing vector of constant norm,

$$\mathcal{D}_a C_b + \mathcal{D}_b C_a = 0 \ , \quad C^2 = C^a C_a = \text{const} \ . \tag{3.26}$$

Thus there are three types of backgrounds depending on whether the Killing vector C^a is chosen to be time-like, space-like or null. The algebra of covariant derivatives for such a background is

$$\{\mathcal{D}^{i\bar{i}}_\alpha, \mathcal{D}^{j\bar{j}}_\beta\} = 2\mathrm{i}\varepsilon^{ij}\varepsilon^{\bar{i}\bar{j}}(\mathcal{D}_{\alpha\beta} - W M_{\alpha\beta}) + 4\mathrm{i}\varepsilon_{\alpha\beta}\varepsilon^{\bar{i}\bar{j}} W L^{ij} + 4\mathrm{i} C^{\bar{i}\bar{j}} C_{\alpha\beta} L^{ij} + 2\mathrm{i}\varepsilon_{\alpha\beta}\varepsilon^{ij} C^{\bar{i}\bar{j}} C^{\gamma\delta} M_{\gamma\delta} \ , \tag{3.27a}$$

$$\Big[\mathcal{D}_{\alpha\beta}, \mathcal{D}^{k\bar{k}}_\gamma\Big] = -\varepsilon_{\gamma(\alpha} W \mathcal{D}^{k\bar{k}}_{\beta)} + (\varepsilon_{\gamma(\alpha} C_{\beta)\delta}{}^{\bar{k}\bar{j}} + \varepsilon_{\delta(\alpha} C_{\beta)\gamma}{}^{\bar{k}\bar{j}}) \mathcal{D}^{\delta k}{}_{\bar{j}} \ , \tag{3.27b}$$

$$[\mathcal{D}_{\alpha\beta}, \mathcal{D}_{\gamma\delta}] = W^2 \big(\varepsilon_{\delta(\alpha} M_{\beta)\gamma} + \varepsilon_{\gamma(\alpha} M_{\beta)\delta}\big) - \big(\varepsilon_{\delta(\alpha} C_{\beta)\gamma} + \varepsilon_{\gamma(\alpha} C_{\beta)\delta}\big) C^{\rho\sigma} M_{\rho\sigma} \ . \tag{3.27c}$$

One may think of this algebra as a Lie superalgebra.[10] By construction, the theory involves the constant symmetric isospinor $G^{\bar{i}\bar{j}}$ being invariant under a U(1) subgroup of the group $\mathrm{SU}(2)_{\mathrm{R}}$. If $C^{\bar{i}\bar{j}}$ does not coincide with $G^{\bar{i}\bar{j}}$, then the group $\mathrm{SU}(2)_{\mathrm{R}}$ is completely broken. This indicates that $C^{\bar{i}\bar{j}} = G^{\bar{i}\bar{j}}$.

The simplest maximally supersymmetric solution of the theory is characterised by (see also [20])

$$C_a{}^{\bar{i}\bar{j}} = 0 \ . \tag{3.28}$$

It corresponds to the critical (4,0) AdS superspace introduced in [25]. Its algebra of covariant derivatives is as follows:

$$\{\mathcal{D}^{i\bar{i}}_\alpha, \mathcal{D}^{j\bar{j}}_\beta\} = 2\mathrm{i}\varepsilon^{ij}\varepsilon^{\bar{i}\bar{j}}(\mathcal{D}_{\alpha\beta} - W M_{\alpha\beta}) + 4\mathrm{i}\varepsilon_{\alpha\beta}\varepsilon^{\bar{i}\bar{j}} W L^{ij} \ , \tag{3.29a}$$

$$[\mathcal{D}_a, \mathcal{D}^{j\bar{j}}_\beta] = \frac{1}{2} W (\gamma_a)_\beta{}^\gamma \mathcal{D}^{j\bar{j}}_\gamma \ , \qquad [\mathcal{D}_a, \mathcal{D}_b] = -W^2 M_{ab} \ . \tag{3.29b}$$

The last relation shows that the cosmological constant is $\Lambda = -W^2 = -\ell^{-2}$, in agreement with [20, 25]. Here ℓ is the radius of curvature in AdS_3. The latter relation is equivalent to $\mu\ell = 1$, which corresponds to chiral gravity [24].

More generally, the (p, q) AdS superspaces, $p + q = \mathcal{N}$, in three dimensions were classified in [25].[11] In the $\mathcal{N} = 4$ case, the (3,1) and (2,2) AdS superspaces are necessarily conformally flat, $W = 0$. The distinguished feature of (4,0) AdS supersymmetry is that the super-Cotton scalar W may have a non-zero value. The algebra of covariant derivatives is given by [25]

$$\{\mathcal{D}^{i\bar{i}}_\alpha, \mathcal{D}^{j\bar{j}}_\beta\} = 2\mathrm{i}\varepsilon^{ij}\varepsilon^{\bar{i}\bar{j}}\mathcal{D}_{\alpha\beta} + 2\mathrm{i}\varepsilon_{\alpha\beta}\varepsilon^{\bar{i}\bar{j}}(2\mathcal{S} + W) L^{ij} + 2\mathrm{i}\varepsilon_{\alpha\beta}\varepsilon^{ij}(2\mathcal{S} - W) R^{\bar{i}\bar{j}} - 4\mathrm{i}\mathcal{S}\varepsilon^{ij}\varepsilon^{\bar{i}\bar{j}} M_{\alpha\beta} \ , \tag{3.30a}$$

$$[\mathcal{D}_a, \mathcal{D}^{j\bar{j}}_\beta] = \mathcal{S} (\gamma_a)_\beta{}^\gamma \mathcal{D}^{j\bar{j}}_\gamma \ , \qquad [\mathcal{D}_a, \mathcal{D}_b] = -4\mathcal{S}^2 M_{ab} \ , \tag{3.30b}$$

[10] More precisely, (3.27) is isomorphic to the Lie superalgebra corresponding to the isometry supergroup of the background superspace under consideration.

[11] In three dimensions, $\mathcal{N}$-extended AdS supergravity exists in several incarnations [7] known as the (p, q) AdS supergravity theories, where the integers $p \geq q \geq 0$ are such that $\mathcal{N} = p + q$.

JHEP03(2017)109

where the positive constant $\mathcal{S}$ determines the curvature of AdS_3. For a generic value of W the entire SO(4) R-symmetry group belongs to the superspace holonomy group. But there are two special values of W for which either the $SU(2)_R$ or the $SU(2)_L$ curvature vanishes and the structure group is reduced. These are given by

$$W = \pm 2\mathcal{S} \tag{3.31}$$

and correspond to the critical (4,0) AdS superspaces. As briefly discussed in [35], the isometry group of (4,0) AdS superspace is isomorphic to $D(2,1;\alpha) \times SL(2,\mathbb{R})$ in the non-critical case $W \neq \pm 2\mathcal{S}$, where $D(2,1;\alpha)$ is one of the exceptional simple supergroups, with the real number $\alpha \neq -1, 0$, see e.g. [36, 37] for reviews. The supergroup parameter α is related to the (4,0) AdS parameter $q = 1 + \frac{W}{2\mathcal{S}}$ introduced in [35]. If the values of α are restricted to the range[12] $-1 < \alpha \leq -\frac{1}{2}$, then we can identify $-2\alpha = 1 + \frac{W}{2\mathcal{S}}$. The case $\alpha = -\frac{1}{2}$ corresponds to the conformally flat (4,0) AdS superspace, for which $W = 0$. Its isometry group is $OSp(4|2) \times SL(2,\mathbb{R})$. The limiting choice $\alpha = -1$ corresponds to one of the two critical (4,0) AdS cases, $W = 2\mathcal{S}$.[13] The isometry group of this (4,0) AdS superspace is $SU(1,1|2) \rtimes SU(2) \times SL(2,\mathbb{R})$, see also the discussion in [38].

If $C^a \neq 0$, the maximally supersymmetric background (3.27) describes a warped critical (4,0) AdS superspace. The bosonic body of such a superspace is warped AdS_3 spacetime associated with the Killing vector $c^a(x) = C^a(z)|_{\theta=0}$. Warped AdS_3 spacetimes have been discussed in detail in the literature, see [39–43] and references therein. In the $\mathcal{N} = 2$ supersymmetric case, the (super)space geometry of maximally supersymmetric warped (2,0) AdS backgrounds was described in [11] and further elaborated in [34]. Supersymmetric warped (1,1) AdS backgrounds, which are necessarily non-maximal, were thoroughly studied in [40].

It is worth giving a few general comments about maximally supersymmetric warped AdS backgrounds in $\mathcal{N}$-extended supergravity theories. Such backgrounds do not exist in the case of $\mathcal{N} = 1$ supergravity. This result was first demonstrated by Gibbons, Pope and Sezgin [44], and it follows trivially from the general superspace analysis of supersymmetric backgrounds in diverse dimensions [32–34].[14] However, maximally supersymmetric warped AdS backgrounds do exist in extended supergravity, $\mathcal{N} > 1$, if the structure group includes not only the Lorentz group $SL(2,\mathbb{R})$ but also a nontrivial R-symmetry group. For instance, the structure group for $\mathcal{N} = (2,0)$ AdS supergravity is $SL(2,\mathbb{R}) \times U(1)_R$, and thus this theory possesses maximally supersymmetric warped AdS backgrounds, which were described in [11, 34] using the superspace techniques, and some time later in [41, 42] using the component approach. On the other hand, the structure group for $\mathcal{N} = (1,1)$

[12] Not all values of α lead to distinct supergroups, since the supergroups defined by the parameters $\alpha^{\pm 1}$, $-(1+\alpha)^{\pm 1}$ and $-\alpha^{\pm 1}(1+\alpha)^{\mp 1}$ are isomorphic [36, 37].

[13] The isometry groups of the two critical (4,0) AdS superspaces are isomorphic.

[14] Indeed, the superspace geometry of $\mathcal{N} = 1$ supergravity is determined by two torsion tensors, a scalar S and a symmetric spinor $C_{\alpha\beta\gamma} = C_{(\alpha\beta\gamma)}$, see [13, 14] for more details. According to [33, 34], every maximally supersymmetric background is characterised by the conditions $C_{\alpha\beta\gamma} = 0$ and $S = \text{const}$, see also [45]. The resulting algebra of covariant derivatives corresponds to $\mathcal{N} = 1$ AdS superspace for $S \neq 0$, or Minkowski superspace for $S = 0$.

JHEP03(2017)109

AdS supergravity coincides with the Lorentz group, and therefore this theory possesses no maximally supersymmetric warped AdS backgrounds, see [11, 34] for more details.

We now linearise the equation (3.21) around the critical $(4,0)$ AdS superspace and let $C_a{}^{\bar{i}\bar{j}} = \delta C_a{}^{\bar{i}\bar{j}}$ where $\delta C_a{}^{ij}$ is a small disturbance. Eq. (3.21) turns into

$$\boldsymbol{\mathcal{D}}_\alpha{}^\gamma \delta C_{\beta\gamma}{}^{\bar{i}\bar{j}} - 2\mu \delta C_{\alpha\beta}{}^{\bar{i}\bar{j}} = 0 \quad \Longrightarrow \quad \boldsymbol{\mathcal{D}}^a \delta C_a{}^{\bar{i}\bar{j}} = 0\,, \tag{3.32}$$

where $\boldsymbol{\mathcal{D}}_a$ denotes the vector covariant derivative of the critical $(4,0)$ AdS superspace. After applying another vector derivative one finds the equation

$$(\boldsymbol{\mathcal{D}}^a \boldsymbol{\mathcal{D}}_a - 2\mu^2)\delta C_b{}^{\bar{i}\bar{j}} = 0\ . \tag{3.33}$$

One can also derive further equations on descendants of $\delta C_{\alpha\beta}{}^{\bar{i}\bar{j}}$ using the constraint (3.20). In particular, one finds

$$\Big(\boldsymbol{\mathcal{D}}_\alpha{}^\delta - \frac{3}{2}\mu\delta_\alpha^\delta\Big)\delta C_{\beta\gamma\delta}{}^{i\bar{i}} = 0\,, \qquad \delta C_{\alpha\beta\gamma}{}^{i\bar{i}} := \frac{1}{3}\boldsymbol{\mathcal{D}}_\alpha{}^{i}{}_{\bar{j}}\delta C_{\beta\gamma}{}^{\bar{i}\bar{j}}\,, \tag{3.34a}$$

$$(\boldsymbol{\mathcal{D}}_\alpha{}^\rho - \mu\delta_\alpha^\rho)\delta C_{\beta\gamma\delta\rho} = 0\,, \qquad \delta C_{\alpha\beta\gamma\delta} := \boldsymbol{\mathcal{D}}^{i\bar{i}}_{(\alpha}\delta C_{\beta\gamma\delta)i\bar{i}}\,, \tag{3.34b}$$

where $\boldsymbol{\mathcal{D}}^{i\bar{i}}_\alpha$ denotes the spinor covariant derivative of the critical $(4,0)$ AdS superspace. The component projection of $\delta C_{\alpha\beta\gamma}{}^{i\bar{i}}$ is proportional to the linearised gravitino field strength, while $\delta C_{\alpha\beta\gamma\delta}$ is proportional to the linearised Cotton tensor. These superfields can be shown to satisfy the following consequences of eqs. (3.34):

$$\Big(\boldsymbol{\mathcal{D}}^a\boldsymbol{\mathcal{D}}_a + \frac{1}{4}\mu^2\Big)\delta C_{\alpha\beta\gamma}{}^{i\bar{i}} = 0\,, \tag{3.35a}$$

$$(\boldsymbol{\mathcal{D}}^a\boldsymbol{\mathcal{D}}_a + 2\mu^2)\delta C_{\alpha\beta\gamma\delta} = 0\ . \tag{3.35b}$$

In the above we made use of the following result for a symmetric rank-$(2s)$ superfield $T_{\alpha_1\cdots\alpha_{2s}} = T_{(\alpha_1\cdots\alpha_{2s})}$ (with isospinor indices suppressed):

$$\Big(\boldsymbol{\mathcal{D}}_{\alpha_1}{}^\beta - \delta_{\alpha_1}{}^\beta\frac{\mu}{\eta}\Big)T_{\alpha_2\cdots\alpha_{2s}\beta} = 0 \quad \Longrightarrow \quad \Big(\boldsymbol{\mathcal{D}}^a\boldsymbol{\mathcal{D}}_a - \frac{\mu^2}{\eta^2} + (s+1)\mu^2\Big)T_{\alpha_1\cdots\alpha_{2s}} = 0\,, \tag{3.36}$$

with η a dimensionless parameter. Computing the bar-projection of the equations (3.32), (3.34a) and (3.34b), we can determine the representations of the AdS group SO(2,2) to which the fields $\delta C_{\alpha\beta}{}^{\bar{i}\bar{j}}|$, $\delta C_{\alpha\beta\gamma}{}^{i\bar{i}}|$ and $\delta C_{\alpha\beta\gamma\delta}|$ belong. We recall that the unitary representations of SO(2,2), denoted $D(E_0, \hat{s})$, are labelled by two real weights $(E_0, \hat{s})$, where E_0 is the lowest energy and $\hat{s}$ is the helicity, see e.g. [46]. The weights obey the unitarity bound $E_0 \geq |\hat{s}|$ for $\hat{s} > 0$, where the representations with $E_0 = |\hat{s}| > 0$ are called singleton representations. For a superfield $T_{\alpha_1\ldots\alpha_{2s}}$ obeying the first-order equation (3.36), its lowest component $T_{\alpha_1\ldots\alpha_{2s}}|$ transforms in the representation with

$$E_0 = 1 + \frac{1}{|\eta|}\,, \quad \hat{s} = \frac{s\eta}{|\eta|}\,, \tag{3.37}$$

JHEP03(2017)109

as follows from the analysis in [46] (see also [47]). Thus the gravitational field $\delta C_{\alpha\beta\gamma\delta}|$ is a helicity 2 singleton, while the spin-1 and spin-3/2 fields, $\delta C_{\alpha\beta}{}^{\bar{i}\bar{j}}|$ and $\delta C_{\alpha\beta\gamma}{}^{\bar{i}}|$, are massive.

In the above we worked with the self-dual TMSG theory, however the analysis of the equations of motion corresponding to the action S_- is completely analogous. There one finds the covariant derivative algebra is

$$\begin{aligned}\{\mathcal{D}_\alpha^{i\bar{i}}, \mathcal{D}_\beta^{j\bar{j}}\} &= 2\mathrm{i}\varepsilon^{ij}\varepsilon^{\bar{i}\bar{j}}\mathcal{D}_{\alpha\beta} - 4\mathrm{i}\varepsilon_{\alpha\beta}\varepsilon^{ij}WR^{\bar{i}\bar{j}} + 4\mathrm{i}C_{\alpha\beta}{}^{ij}R^{\bar{i}\bar{j}} \\ &\quad +2\mathrm{i}\varepsilon_{\alpha\beta}\varepsilon^{\bar{i}\bar{j}}C^{\gamma\delta ij}M_{\gamma\delta} + 2\mathrm{i}\varepsilon^{ij}\varepsilon^{\bar{i}\bar{j}}WM_{\alpha\beta}\,,\end{aligned} \tag{3.38}$$

where $C_a{}^{ij}$ satisfies the Bianchi identity

$$\mathcal{D}_\alpha^{i\bar{i}}C_{\beta\gamma}{}^{jk} = 2\varepsilon^{i(j}C_{\alpha\beta\gamma}{}^{k)\bar{i}}\,. \tag{3.39}$$

Using the above equation one finds

$$\mathcal{D}_\alpha{}^\gamma C_{\beta\gamma}{}^{ij} + C_{(\alpha}{}^\gamma{}_k{}^{(i}C_{\beta)\gamma}{}^{j)k} - 2WC_{\alpha\beta}{}^{ij} = 0\,. \tag{3.40}$$

The solution $C_a{}^{ij} = 0$ corresponds to $(4,0)$ AdS superspace in the critical case $2S = -W$. We now linearise around the $(4,0)$ AdS superspace and set $C_a{}^{ij} = \delta C_a{}^{ij}$ where $\delta C_a{}^{ij}$ is a small disturbance. It can be seen that $\delta C_a{}^{ij}$ obeys the equation

$$\boldsymbol{\mathcal{D}}_\alpha{}^\gamma \delta C_{\beta\gamma}{}^{ij} - 2\mu\delta C_{\alpha\beta}{}^{ij} = 0\,, \tag{3.41}$$

where $\boldsymbol{\mathcal{D}}_a$ corresponds to the vector covariant derivative of the $(4,0)$ AdS superspace. After applying another vector derivative one finds

$$(\boldsymbol{\mathcal{D}}^a\boldsymbol{\mathcal{D}}_a - 2\mu^2)\delta C_b{}^{ij} = 0\,. \tag{3.42}$$

4 Component actions

In this section we give the component results corresponding to the minimal $\mathcal{N} = 4$ topologically massive supergravity action (3.1).

4.1 The component conformal supergravity action

The complete component analysis of the $\mathcal{N}$-extended Weyl multiplet was given in [17]. Here we specialise to the $\mathcal{N} = 4$ case where the auxiliary fields coming from the super-Cotton tensor are defined as:

$$w := \frac{1}{4!}\varepsilon_{IJKL}w^{IJKL} = W|\,, \qquad y := \frac{1}{4!}\varepsilon_{IJKL}y^{IJKL} = -\frac{\mathrm{i}}{4}\nabla_I^\alpha\nabla_\alpha^I W|\,, \tag{4.1a}$$

$$w_{\alpha L} := \frac{1}{3!}\varepsilon_{IJKL}w_\alpha{}^{IJK} = -\frac{\mathrm{i}}{2}\nabla_{\alpha L}W|\,. \tag{4.1b}$$

The full $\mathcal{N} = 4$ conformal supergravity action was given in [17] and is

$$\begin{aligned}S_{\rm CSG} = \frac{1}{8}\int \mathrm{d}^3x\, e\Big\{&\varepsilon^{abc}\Big(\omega_a{}^{fg}\mathcal{R}_{bcfg} - \frac{2}{3}\omega_{af}{}^g\omega_{bg}{}^h\omega_{ch}{}^f - \frac{\mathrm{i}}{2}\Psi_{bcI}^\alpha(\gamma_d)_\alpha{}^\beta(\gamma_a)_\beta{}^\gamma\varepsilon^{def}\Psi_{ef\gamma}^I \\ &- 2\mathcal{R}_{ab}{}^{IJ}V_{cIJ} - \frac{4}{3}V_a{}^{IJ}V_{bI}{}^KV_{cKJ}\Big) \\ &- 32\mathrm{i}w_I^\alpha w_\alpha^I - 8wy - 16\mathrm{i}\,\psi_{aI}^\alpha(\gamma^a)_\alpha{}^\beta w_\beta^I w - 2\mathrm{i}\varepsilon^{abc}(\gamma_a)_{\alpha\beta}\psi_{bI}^\alpha\psi_c{}^{\beta I}w^2\Big\}\,,\end{aligned} \tag{4.2}$$

JHEP03(2017)109

where the component curvatures $\mathcal{R}_{ab}{}^{cd}$ and $\mathcal{R}_{ab}{}^{IJ}$ are defined as

$$\mathcal{R}_{ab}{}^{cd} := 2e_a{}^m e_b{}^n \partial_{[m}\omega_{n]}{}^{cd} - 2\omega_{[a}{}^{cf}\omega_{b]f}{}^{d}\,, \tag{4.3a}$$

$$\mathcal{R}_{ab}{}^{IJ} := 2e_a{}^m e_b{}^n \partial_{[m}V_{n]}{}^{IJ} - 2V_{[a}{}^{IK}V_{b]K}{}^{J}\ . \tag{4.3b}$$

4.2 The component vector multiplet actions

The component $\mathcal{N}=4$ linear multiplet actions were given in [12]. Making use of the results there, one can construct the left and right vector multiplet actions.

The component fields of the vector multiplets are defined as

$$g_{\pm}^{IJ} := G_{\pm}^{IJ}|\,, \tag{4.4a}$$

$$\lambda_{(\pm)\alpha}^{\ \ I} := \frac{2}{3}\nabla_{\alpha J}G_{\pm}^{IJ}|\,, \tag{4.4b}$$

$$h_{(\pm)}{}^{IJ} := \frac{\mathrm{i}}{3}\nabla^{\gamma[I}\nabla_{\gamma K}G_{\pm}^{J]K}|\,, \tag{4.4c}$$

$$f_{(\pm)ab} := -\frac{\mathrm{i}}{24}\varepsilon_{abc}(\gamma^c)^{\alpha\beta}\nabla_{\alpha}^{K}\nabla_{\beta}^{L}G_{\pm KL}| - \frac{1}{2}(\psi_{[a}{}^{K}\gamma_{b]}\lambda_{(\pm)K}) + \frac{\mathrm{i}}{2}\psi_a{}^{\gamma K}\psi_{b\gamma}^{\ \ L}\,g_{\pm KL}\,, \tag{4.4d}$$

where $g_{\pm}{}^{IJ}$ is (anti-)self-dual

$$\frac{1}{2}\varepsilon^{IJKL}g_{\pm KL} = \pm g_{\pm}{}^{IJ}\ . \tag{4.5}$$

The component gauge one-forms $v_{(\pm)a}$ are defined as

$$v_{(\pm)a} := e_a{}^m v_{(\pm)m}\,, \quad f_{(\pm)ab} = 2e_a{}^m e_b^n\,\partial_{[m}v_{(\pm)n]}\,, \quad v_{(\pm)m} := V_{\pm m}|\,, \tag{4.6}$$

where $V_{\pm}$ is the superspace gauge one-form associated with the field strength $G_{\pm}^{IJ}$.

It is useful to replace $h_{(\pm)}{}^{IJ}$ by the fields

$$\begin{aligned}\hat{h}_{\pm}{}^{IJ} &= \frac{1}{2}(h_{(\mp)}{}^{IJ} + \tilde{h}_{(\mp)}{}^{IJ})\\ &= h_{(\mp)}{}^{IJ} \mp 2w g_{\mp}{}^{IJ}\,,\end{aligned} \tag{4.7}$$

which proves to be (anti-)self-dual

$$\frac{1}{2}\varepsilon^{IJKL}\hat{h}_{\pm KL} = \pm\hat{h}_{\pm}^{IJ}\ . \tag{4.8}$$

The component self-dual vector multiplet action is

$$\begin{aligned}S_{\mathrm{VM}}^{(+)} = -\int \mathrm{d}^3x\, e\Big(&\varepsilon^{abc}v_{(+)a}\boldsymbol{f}_{(+)bc} + \frac{1}{4}\hat{\boldsymbol{h}}_{+}{}^{IJ}g_{+IJ} + \frac{1}{4}\hat{h}_{-}{}^{IJ}\boldsymbol{g}_{-IJ} - \frac{\mathrm{i}}{2}\lambda^{\alpha I}\boldsymbol{\lambda}_{\alpha I}\\ &- \frac{1}{2}(\gamma^a)_{\gamma\delta}\psi_a{}^{\gamma}_{I}(\lambda^{\delta J}\boldsymbol{g}_{-J}{}^{I} + \boldsymbol{\lambda}^{\delta J}g_{+J}{}^{I}) + \frac{\mathrm{i}}{2}\varepsilon^{abc}(\gamma_a)_{\gamma\delta}\psi_{bK}^{\ \ \gamma}\psi_{cL}^{\ \ \delta}\,g_{+}{}^{KP}\boldsymbol{g}_{-}{}^{L}{}_{P}\Big)\,,\end{aligned} \tag{4.9}$$

JHEP03(2017)109

where the bolded component fields correspond to those of the composite vector multiplet,

$$\boldsymbol{g}_-^{IJ} = \boldsymbol{G}_-^{IJ}|\,, \quad \boldsymbol{\lambda}_\alpha^I = \frac{2}{3}\nabla_{\alpha J}\boldsymbol{G}_-^{IJ}|\,, \quad \hat{\boldsymbol{h}}_+{}^{IJ} = \frac{\mathrm{i}}{3}\nabla^{\gamma[I}\nabla_{\gamma K}\boldsymbol{G}_-^{J]K}| + 2w\boldsymbol{g}_-{}^{IJ}\,, \tag{4.10a}$$

$$v_a = e_a{}^m V_m| = V_a| + \frac{1}{2}\psi_a{}^\alpha_I V_\alpha^I|\,, \tag{4.10b}$$

$$\boldsymbol{f}_{(+)ab} = -\frac{\mathrm{i}}{24}\varepsilon_{abc}(\gamma^c)^{\alpha\beta}\nabla_\alpha^K\nabla_\beta^L\boldsymbol{G}_{-KL}| - \frac{1}{2}(\psi_{[a}{}^K\gamma_{b]}\boldsymbol{\lambda}_K) + \frac{\mathrm{i}}{2}\psi_a{}^{\gamma K}\psi_{b\gamma}^L\,\boldsymbol{g}_{-KL}\,. \tag{4.10c}$$

The component anti-self dual vector multiplet action is

$$\begin{aligned} S^{(-)}_{\mathrm{VM}} = -\int \mathrm{d}^3x\, e\Big(&\varepsilon^{abc}v_{(-)a}\boldsymbol{f}_{(-)bc} + \frac{1}{4}\hat{h}_+{}^{IJ}\boldsymbol{g}_{+IJ} + \frac{1}{4}\hat{\boldsymbol{h}}_-{}^{IJ}g_{-IJ} - \frac{\mathrm{i}}{2}\boldsymbol{\lambda}^{\alpha I}\lambda_{\alpha I} \\ &- \frac{1}{2}(\gamma^a)_{\gamma\delta}\psi_a{}^\gamma_I(\boldsymbol{\lambda}^{\delta J}g_-{}^J{}^I + \lambda^{\delta J}\boldsymbol{g}_+{}^I{}_J) + \frac{\mathrm{i}}{2}\varepsilon^{abc}(\gamma_a)_{\gamma\delta}\psi_b{}^\gamma_K\psi_{cL}^\delta\,\boldsymbol{g}_+{}^{KP}g_-{}^L{}_P\Big)\,, \end{aligned} \tag{4.11}$$

where

$$\boldsymbol{g}_+^{IJ} = \boldsymbol{G}_+^{IJ}|\,, \quad \boldsymbol{\lambda}_\alpha^I = \frac{2}{3}\nabla_{\alpha J}\boldsymbol{G}_+^{IJ}|\,, \quad \hat{\boldsymbol{h}}_-{}^{IJ} = \frac{\mathrm{i}}{3}\nabla^{\gamma[I}\nabla_{\gamma K}\boldsymbol{G}_+^{J]K}| - 2w\boldsymbol{g}_+{}^{IJ}\,, \tag{4.12a}$$

$$v_a = e_a{}^m V_m| = V_a| + \frac{1}{2}\psi_a{}^\alpha_I V_\alpha^I|\,, \tag{4.12b}$$

$$\boldsymbol{f}_{(-)ab} = -\frac{\mathrm{i}}{24}\varepsilon_{abc}(\gamma^c)^{\alpha\beta}\nabla_\alpha^K\nabla_\beta^L\boldsymbol{G}_{+KL}| - \frac{1}{2}(\psi_{[a}{}^K\gamma_{b]}\boldsymbol{\lambda}_K) + \frac{\mathrm{i}}{2}\psi_a{}^{\gamma K}\psi_{b\gamma}^L\,\boldsymbol{g}_{+KL}\,. \tag{4.12c}$$

Plugging in the superspace expressions for $\boldsymbol{G}_\pm{}^{IJ}$ one one can construct the component fields of the composite vector multiplets. The component fields are found to be

$$\boldsymbol{g}_\pm{}^{IJ} = \frac{1}{g_\pm}\hat{h}_\pm^{IJ} - \frac{\mathrm{i}}{2g_\pm^3}\lambda_{\pm K}^\alpha\Lambda_{\pm\alpha}^{[I}g_\pm{}^{J]K} \pm \frac{\mathrm{i}}{4g_\pm^3}\varepsilon^{IJLP}\lambda_{\pm K}^\alpha\lambda_{\pm\alpha L}g_{\pm P}{}^K\,, \tag{4.13a}$$

$$\begin{aligned} \boldsymbol{\Lambda}_{(\pm)\alpha}^I = {}& \frac{2}{g_\pm}\nabla_\alpha{}^\gamma\lambda_{(\pm)\gamma}^I + \frac{2}{g_\pm^3}f_{\pm\alpha\beta}\lambda_{(\pm)}{}_J^\beta g_\pm{}^{IJ} + \frac{1}{3g_\pm^3}\hat{h}_{\mp JK}\lambda_{(\pm)\alpha}^I g_\pm{}^{JK} \\ &+ \frac{2}{3g_\pm^3}\hat{h}_{\mp JK}\lambda_{(\pm)\alpha}^J g_\pm{}^{KI} + \frac{1}{3g_\pm^2}\hat{h}_\mp{}^{IJ}\lambda_{(\pm)\alpha}^K g_{\pm JK} \\ &+ \frac{2}{3g_\pm^3}\nabla_{\alpha\beta}g_{\pm JK}\lambda_{(\pm)}{}^{\beta I}g_\pm{}^{JK} + \frac{4}{3g_\pm^3}\nabla_{\alpha\beta}g_{\pm JK}\lambda_{(\pm)}{}^{\beta J}g_\pm{}^{KI} \\ &- \frac{2}{g_{(\pm)}^3}\nabla_{\alpha\beta}g_\pm{}^{IJ}\lambda_{(\pm)}{}^{\beta K}g_{\pm JK} \\ &\pm \frac{1}{g_\pm}w\lambda_{(\pm)\alpha}^I \pm \frac{8\mathrm{i}}{g_\pm}w_{\alpha J}g_\pm{}^{IJ} + \mathcal{O}(\lambda^2)\,, \end{aligned} \tag{4.13b}$$

$$\begin{aligned} \hat{\boldsymbol{h}}_\pm{}^{IJ} = {}& \frac{4}{g_\pm}\Box g_\pm^{IJ} + \frac{2}{g_\pm^3}f_{\pm ab}f_\pm{}^{ab}g_\pm{}^{IJ} + \frac{4}{g_\pm^3}\varepsilon^{abc}f_{\pm ab}\nabla_c g_{\pm K}{}^{[I}g_\pm{}^{J]K} \\ &- \frac{1}{4g_\pm^3}\hat{h}_\mp{}^{KL}\hat{h}_{\mp KL}g_\pm{}^{IJ} - \frac{2}{g_\pm^3}g_\pm{}^{KL}\nabla_a g_{\pm KL}\nabla^a g_\pm{}^{IJ} \\ &+ \frac{1}{g_\pm^3}g_\pm{}^{IJ}\nabla_a g_{\pm KL}\nabla^a g_\pm{}^{KL} - \frac{2}{g_\pm}w^2 g_\pm{}^{IJ} \pm \frac{2}{g_\pm}y g_\pm{}^{IJ} \\ &+ \text{fermion terms}\,, \end{aligned} \tag{4.13c}$$

JHEP03(2017)109

$$
\begin{aligned}
\boldsymbol{f}_{(\pm)mn} &:= e_m{}^a e_n{}^b \boldsymbol{f}_{(\pm)ab} \\
&= \partial_{[m}\left(\frac{4}{g_\pm} f_{(\pm)n]} - \frac{2}{g_\pm} V_{n]}{}^{IJ} g_{\pm IJ}\right) - \frac{1}{g_\pm^3}\partial_{[m} g_\pm{}^{IK}\partial_{n]} g_\pm{}^J{}_K g_{\pm IJ} \\
&\quad + \text{fermion terms}\,, \qquad (4.13d)
\end{aligned}
$$

where

$$
f_{(\pm)}{}^m = \frac{1}{2}\varepsilon^{mnp} f_{(\pm)np}\ . \qquad (4.14)
$$

Here we have introduced the following:

$$
\nabla_a g_\pm{}^{IJ} := \mathcal{D}_a g_\pm{}^{IJ} + \frac{1}{2}\psi_a{}^{\alpha[I}\lambda_{\pm\alpha}^{J]} \pm \frac{1}{4}\varepsilon^{IJKL}\psi_{a\alpha K}\lambda_{\pm}^{\alpha L}\,, \qquad (4.15a)
$$

$$
\Box g_\pm{}^{IJ} := \mathcal{D}^a\mathcal{D}_a g_\pm{}^{IJ} + \frac{1}{4}\mathcal{R} g_\pm{}^{IJ} + \text{fermion terms}\,, \qquad (4.15b)
$$

and[15]

$$
\mathcal{D}_a := e_a{}^m\partial_m - \frac{1}{2}\omega_a{}^{bc}M_{bc} - \frac{1}{2}V_a{}^{IJ}N_{IJ} - b_a\mathbb{D}\ . \qquad (4.16)
$$

4.3 $\mathcal{N} = 4$ topologically massive supergravity in components

To simplify our results it is useful to make use of the gauge freedom to impose some gauge condition. One can always choose a gauge condition where

$$
B_A = 0\,, \quad G_\pm = 1\ . \qquad (4.17)
$$

At the component level these require

$$
g_\pm = 1\,, \quad \lambda_\alpha^I = 0\,, \quad b_m = 0\ . \qquad (4.18)
$$

The first gauge condition fixes the dilatation transformations, the second fixes the S-supersymmetry transformations and the third fixes the conformal boosts. For a right G^{ij} and left $G^{\bar{i}\bar{j}}$ vector multiplet we can use the respective SU(2) symmetry to fix their lowest components to a constant. This then gives

$$
\nabla_a g_\pm{}^{IJ} = 2V_a{}^{K[I} g_{\pm K}{}^{J]}\ . \qquad (4.19)
$$

With the above gauge conditons we find

$$
\begin{aligned}
\hat{\boldsymbol{h}}_\pm^{IJ} g_{\pm IJ} &= 2\mathcal{R} + 4 f_{\pm ab} f_\pm{}^{ab} - \frac{1}{2}\hat{h}_\mp{}^{IJ}\hat{h}_{\mp IJ} \\
&\quad -2V_{aKL}V^{aKL} + 4V_a{}^{IK}V^{aJL} g_{\pm IJ} g_{\pm KL} \\
&\quad -4w^2 \pm 4y + \text{fermion terms}\,, \qquad (4.20a)
\end{aligned}
$$

$$
\hat{h}_\pm^{IJ}\boldsymbol{g}_{\pm IJ} = \hat{h}_\pm^{IJ}\hat{h}_{\pm IJ}\,, \qquad (4.20b)
$$

$$
\boldsymbol{f}_{(\pm)mn} = \partial_{[m}\Big(4 f_{(\pm)n]} - 2V_{n]}{}^{IJ} g_{\pm IJ}\Big) + \text{fermion terms}\ . \qquad (4.20c)
$$

[15] We have denoted the component vector derivative $\mathcal{D}_a$ in the same way as the SU(2) superspace covariant derivative. It should be clear from context to which we are referring to.

JHEP03(2017)109

Using the above conditions one finds (upon integrating by parts) the self-dual vector multiplet action is

$$S_{\rm VM}^{(+)} = -\int {\rm d}^3x\, e\Big(\frac{1}{2}\mathcal{R} - f_{(+)ab}f_{(+)}^{ab} - 2f_{(+)}^{a}V_a{}^{IJ}g_{+IJ} - \frac{1}{2}V_{aKL}V^{aKL} + V_a{}^{IK}V^{aJL}g_{+IJ}g_{+KL} + \frac{1}{8}\hat{h}_-^{IJ}\hat{h}_{-IJ} - w^2 + y + \text{fermion terms}\Big)\,, \tag{4.21}$$

while the anti-self-dual vector multiplet action is

$$S_{\rm VM}^{(-)} = -\int {\rm d}^3x\, e\Big(\frac{1}{2}\mathcal{R} - f_{(-)ab}f_{(-)}^{ab} - 2f_{(-)}^{a}V_a{}^{IJ}g_{-IJ} - \frac{1}{2}V_{aKL}V^{aKL} + V_a{}^{IK}V^{aJL}g_{-IJ}g_{-KL} + \frac{1}{8}\hat{h}_+^{IJ}\hat{h}_{+IJ} - w^2 - y + \text{fermion terms}\Big)\,. \tag{4.22}$$

The complete component action for minimal $\mathcal{N}=4$ topologically massive supergravity (3.1) is then given by

$$\kappa S_\pm = \frac{1}{\mu}S_{\rm CSG} + S_{\rm VM}^{(\pm)}\,, \tag{4.23}$$

where $S_{\rm CSG}$ is the component action (4.2). As a simple check one can readily verify that the equation of motion on the field y gives

$$w = \mp\mu\,, \tag{4.24}$$

which is consistent with the supergravity equation of motion being $W = \mp\mu G_\pm$ in the presence of the vector multiplet compensator.

For completeness we will also give the component action in isospinor notation. The $\mathcal{N}=4$ conformal supergravity action (4.2) becomes

$$\begin{aligned} S_{\rm CSG} = \frac{1}{8}\int {\rm d}^3x\, e\,\Big\{&\varepsilon^{abc}\Big(\omega_a{}^{fg}\mathcal{R}_{bcfg} - \frac{2}{3}\omega_{af}{}^{g}\omega_{bg}{}^{h}\omega_{ch}{}^{f} \\ &- 4\mathcal{R}_{ab}{}^{ij}V_{cij} - \frac{8}{3}V_{ai}{}^{j}V_{bj}{}^{k}V_{ck}{}^{i} - 4\mathcal{R}_{ab}{}^{\bar{i}\bar{j}}V_{c\bar{i}\bar{j}} - \frac{8}{3}V_{a\bar{i}}{}^{\bar{j}}V_{b\bar{j}}{}^{\bar{k}}V_{c\bar{k}}{}^{\bar{i}}\Big) \\ &- 32{\rm i}w_{i\bar{i}}^{\alpha}w_{\alpha}^{i\bar{i}} - 8wy - 16{\rm i}\,\psi_{a\,i\bar{i}}^{\alpha}(\gamma^a)_\alpha{}^\beta w_\beta^{i\bar{i}}w - 2{\rm i}\varepsilon^{abc}(\gamma_a)_{\alpha\beta}\psi_{b\,i\bar{i}}^{\alpha}\psi_c{}^{\beta i\bar{i}}w^2\Big\}\,, \end{aligned} \tag{4.25}$$

where the component SU(2) curvatures $\mathcal{R}_{ab}{}^{ij}$ and $\mathcal{R}_{ab}{}^{\bar{i}\bar{j}}$ are

$$\mathcal{R}_{ab}{}^{ij} := 2e_a{}^m e_b{}^n\partial_{[m}V_{n]}{}^{ij} - 2V_{[a}{}^{ik}V_{b]k}{}^{j}\,, \tag{4.26a}$$

$$\mathcal{R}_{ab}{}^{\bar{i}\bar{j}} := 2e_a{}^m e_b{}^n\partial_{[m}V_{n]}{}^{\bar{i}\bar{j}} - 2V_{[a}{}^{\bar{i}\bar{k}}V_{b]\bar{k}}{}^{\bar{j}}\,. \tag{4.26b}$$

The self-dual vector multiplet action in isospinor notation is

$$S_{\rm VM}^{(+)} = -\int {\rm d}^3x\, e\Big(\frac{1}{2}\mathcal{R} - f_{(+)ab}f_{(+)}^{ab} - 4f_{(+)}^{a}V_a{}^{\bar{i}\bar{j}}g_{+\bar{i}\bar{j}} - V_{a\bar{i}\bar{j}}V^{a\bar{i}\bar{j}} + 2V_a{}^{\bar{i}\bar{k}}V^{a\bar{j}\bar{l}}g_{+\bar{i}\bar{j}}g_{+\bar{k}\bar{l}} + \frac{1}{4}\hat{h}_-^{ij}\hat{h}_{-ij} - w^2 + y + \text{fermion terms}\Big)\,, \tag{4.27}$$

JHEP03(2017)109

while the anti-self-dual vector multiplet action is

$$S^{(-)}_{\rm VM} = -\int {\rm d}^3x\, e\Big(\frac{1}{2}\mathcal{R} - f_{(-)ab}f^{ab}_{(-)} - 4f^{a}_{(-)}V_a{}^{ij}g_{-ij} - V_{aij}V^{aij} + 2V_a{}^{ik}V^{ajl}g_{-ij}g_{-kl} + \frac{1}{4}\hat{h}^{\overline{ij}}_{+}\hat{h}_{+\overline{ij}} - w^2 - y + \text{fermion terms}\Big)\,. \tag{4.28}$$

Having derived the component actions for minimal $\mathcal{N}=4$ topologically massive supergravity, it is worth elaborating on these results further. For instance, if we consider just one of the vector multiplet actions without the conformal supergravity action, one can see that the equation of motion for y leads to an inconsistency. This is equivalent to the fact that the superfield equations of motion for the $\mathcal{N}=4$ gravitational superfield[16] derived from the actions $S^{(+)}_{\rm VM}$ and $S^{(-)}_{\rm VM}$ are $G_+=0$ and $G_-=0$, respectively, and these equations are inconsistent with the requirements $G_\pm \neq 0$. However, one gets consistent equations of motion if one adds the left and right vector multiplets [12] and considers the action

$$S = S^{(+)}_{\rm VM} + S^{(-)}_{\rm VM}\,. \tag{4.29}$$

Now the superfield equation of motion for the $\mathcal{N}=4$ gravitational superfield is [12]

$$G_+ - G_- = 0\,, \tag{4.30}$$

which is completely consistent. Moreover, this equation is consistent with our gauge conditions because imposing the gauge $G_+=1$ implies $G_-=1$, which in turn implies that the auxiliary field y cancels. Furthermore, the fields w and $\hat{h}^{IJ}$ become auxiliary and their equation of motion is the requirement that they vanish. The equations of motion on the SU(2) connections requires $f_{(-)a}=f_{(+)a}=0$ and we are left with just the $\mathcal{N}=4$ Poincaré supergravity action (up to a normalisation factor)

$$S = -\int {\rm d}^3x\, e\,\mathcal{R} \;+\; \text{fermion terms}\,. \tag{4.31}$$

In the presence of the conformal supergravity action the gauge conditions $G_+=G_-=1$ are no longer consistent [12] and instead one has to use the results in subsection 4.2 in the general gauge. If one also adds to (4.29) the supersymmetric cosmological term [14], the resulting theory corresponds to (2,2) AdS supergravity as was described in detail in [12, 14].

It is worth mentioning some simplifications that can be made to the $\mathcal{N}=4$ topologically massive supergravity actions upon using the equations of motion. To illustrate this let us consider the theory with a self-dual vector multiplet. In this case the equation of motion for the $\mathrm{SU}(2)_{\rm L}$ gauge field is

$$\mathcal{R}_{ab}{}^{ij} = 0\,, \tag{4.32}$$

which tells us that the $\mathrm{SU}(2)_{\rm L}$ gauge field can be completely gauged away. The equation of motion for the auxiliary field $\hat{h}^{ij}$ sets the auxiliary field to zero and removes it from the

[16] The $\mathcal{N}=4$ gravitational superfield is a scalar prepotential describing the multiplet of $\mathcal{N}=4$ conformal supergravity. It is the 3D $\mathcal{N}=4$ counterpart of the $\mathcal{N}=2$ gravitational superfield in four dimensions [48].

JHEP03(2017)109

action. The equation of motion on y just sets $w = -\mu$ and gives rise to a cosmological term. The resulting action is

$$\begin{aligned}\kappa S_+ = \int \mathrm{d}^3x\, e \Bigg[& \frac{1}{8\mu}\Big\{\varepsilon^{abc}\Big(\omega_a{}^{fg}\mathcal{R}_{bcfg} - \frac{2}{3}\omega_{af}{}^{g}\omega_{bg}{}^{h}\omega_{ch}{}^{f} \\ & - 4\mathcal{R}_{ab}{}^{\bar{i}\bar{j}}V_{c\bar{i}\bar{j}} - \frac{8}{3}V_{a\bar{i}}{}^{\bar{j}}V_{b\bar{j}}{}^{\bar{k}}V_{c\bar{k}}{}^{\bar{i}}\Big)\Big\} \\ & - \frac{1}{2}\mathcal{R} + \mu^2 + f_{(+)ab}f^{ab}_{(+)} + 4f^a_{(+)}V_a{}^{\bar{i}\bar{j}}g_{+\bar{i}\bar{j}} + V_{a\bar{i}\bar{j}}V^{a\bar{i}\bar{j}} \\ & - 2V_a{}^{\bar{i}\bar{k}}V^{a\bar{j}\bar{l}}g_{+\bar{i}\bar{j}}g_{+\bar{k}\bar{l}} + \text{fermion terms}\Bigg] \,. \end{aligned} \tag{4.33}$$

Similar simplifications can be made for the anti-self dual vector multiplet action.

We can now show how to derive the supergravity action postulated in [20] from our theory S_-. The crucial observation is that the U(1) gauge field appears in the action (4.28) only via its field strength $f_{(-)ab}$, and therefore it may be dualised into a scalar field. To implement this, we replace (4.28) with an equivalent first-order action

$$\begin{aligned} S^{(-)}_{\rm FO} = - \int \mathrm{d}^3x\, e \Big(& \frac{1}{2}\mathcal{R} - f_{(-)ab}f^{ab}_{(-)} - 4f^a_{(-)}V_a{}^{ij}g_{ij} - V_{aij}V^{aij} + 2V_a{}^{ik}V^{ajl}g_{+ij}g_{+kl} \\ & + \frac{1}{4}\hat{h}^{\bar{i}\bar{j}}_+\hat{h}_{+\bar{i}\bar{j}} - w^2 - y + 2f^a_{(-)}\mathcal{D}_a\varphi + \text{fermion terms}\Big) \,, \end{aligned} \tag{4.34}$$

where $f_{(-)ab}$ is an unconstrained antisymmetric tensor field, and φ a Lagrange multipler. Varying φ gives $\mathcal{D}_a f^a_{(-)} = 0$, and therefore $f_{(-)ab}$ becomes the field strength of a U(1) vector multiplet. Then $S^{(-)}_{\rm FO}$ turns into the original action (4.28). On the other hand, we may integrate out $f_{(-)ab}$ from $S^{(-)}_{\rm FO}$ using its equation of motion

$$f_{(-)a} = V_a{}^{ij}g_{ij} - \frac{1}{2}\mathcal{D}_a\varphi \,. \tag{4.35}$$

Plugging this back into (4.34) gives the dual action

$$\begin{aligned} S^{(-)}_{\rm hyper} = - \int \mathrm{d}^3x\, e \Big(& \frac{1}{2}\mathcal{R} - \frac{1}{2}\mathcal{D}^a\varphi\mathcal{D}_a\varphi + 2\mathcal{D}_a\varphi V^{aij}g_{ij} - 2V_{aij}V^{aij} \\ & + \frac{1}{4}\hat{h}^{\bar{i}\bar{j}}_+\hat{h}_{+\bar{i}\bar{j}} - w^2 - y + \text{fermion terms}\Big) \,, \end{aligned} \tag{4.36}$$

where we used

$$V_a{}^{ik}V^{ajl}g_{ij}g_{kl} = V^{ij}_a V^{akl}g_{ij}g_{kl} - \frac{1}{2}V_{aij}V^{aij} \,. \tag{4.37}$$

If we impose a Weyl gauge $\varphi = 1$ and make use of the equation of motion for the auxiliary field $\hat{h}^{\bar{i}\bar{j}}_+$, which is $\hat{h}^{\bar{i}\bar{j}}_+ = 0$, we recover the bosonic matter sector of the topologically massive supergravity action in [20] up to conventions and fermion terms. Since the auxiliary field $\hat{h}^{\bar{i}\bar{j}}_+$ has been integrated out, the action given in [20] does not appear to be off-shell.

JHEP03(2017)109

5 Discussion

In this paper we constructed minimal $\mathcal{N} = 4$ topologically massive supergravity. It has several unique features that we summarise here.

- Unlike the other $\mathcal{N}$-extended TMSG theories with $\mathcal{N} \leq 4$ [9–12], its action cannot be viewed as the supergravity action (with or without a supersymmetric cosmological term) augmented by the conformal supergravity action playing the role of a topological mass term. The point is that the theory becomes inconsistent upon removing the conformal supergravity action, as was explained in section 4.3.

- Our theory makes use of a single superconformal compensator. We recall that all known Poincaré or AdS supergravity theories with eight supercharges in diverse dimensions require, in general, two such compensators in order for the corresponding dynamics to be consistent. One known exception is the off-shell formulation for 4D $\mathcal{N} = 2$ AdS supergravity given in [49], which makes use a single massive tensor compensator (described by an unconstrained chiral scalar prepotential) and no compensating vector multiplet.[17] In the case of higher derivative theories, two compensators are no longer required. This was observed in four dimensions for models involving the $\mathcal{N} = 2$ supersymmetric R^2 term [57], and in three dimensions for $\mathcal{N} = 4$ topologically massive supergravity [20].

- Our minimal TMSG theory does not allow any supersymmetric cosmological term. However, a cosmological term gets generated at the component level upon integrating out the auxiliary fields. This is manifested in the fact that the critical (4,0) AdS superspace [25] is a maximally supersymmetric solution of the theory.

- The theory has only one coupling constant.

- Our minimal TMSG theory is the first off-shell $\mathcal{N} = 4$ supergravity theory in three dimensions with the property that the critical (4,0) AdS superspace [25] is a solution of the theory. Upon integrating out the auxiliary fields we recover the model discussed in [20].

- Our theory is an off-shell $\mathcal{N} = 4$ supersymmetric extension of chiral gravity [24]. It is obvious that such an extension, which has never been constructed before, must involve a single conformal compensator.

The above features demonstrate the physical relevance of the theory proposed.

As mentioned in section 1, there exist $\mathcal{N} = 6$ and $\mathcal{N} = 8$ supersymmetric extensions of chiral gravity [24]. Unlike our theory, these TMSG theories are necessarily on-shell. The off-shell structure of our $\mathcal{N} = 4$ theory is indispensable for at least two reasons: (i) it allows for

[17] The vector multiplet has been eaten up by the tensor multiplet. The vector compensator acts as a Stückelberg field to give mass to the tensor multiplet. This is an example of the phenomenon observed originally in [50] and studied in detail in [29, 51–56].

JHEP03(2017)109

the general coupling to matter supermultiplets; and (ii) at the quantum level, it allows one to derive supersymmetric power-counting rules through the use of supergraph techniques.

In the on-shell construction of topologically gauged $\mathcal{N} = 6$ and $\mathcal{N} = 8$ ABJM type theories [21–23], a crucial role is played by a sixth order scalar potential. In the off-shell approach, such a scalar potential is automatically generated upon elimination of the auxiliary fields, as was demonstrated in [58] where the $\mathcal{N} = 6$ and $\mathcal{N} = 8$ ABJM models were realised in $\mathcal{N} = 3$ harmonic superspace. There is an analogous feature in our actions. Specifically, before imposing any gauge condition there is a term $w^2 g_\pm$ in our actions and upon eliminating the auxiliary fields the term $\mu^2 g_\pm^3$ is generated. This term plays a similar role as the sixth order polynomial in [21–23] in the sense that its coefficient is fixed by the equations of motion (in terms of the coupling coefficient of the conformal supergravity action) and the conformal coupling between the Einstein-Hilbert term and the $\mathcal{O}(2)$ multiplet. In this respect our model is akin to those of [21–23].

Both models for minimal $\mathcal{N} = 4$ topologically massive supergravity constructed in this paper possess dual formulations. They are obtained by replacing the vector multiplet actions $S^{(+)}_{\rm VM}$ and $S^{(-)}_{\rm VM}$ with off-shell hypermultiplet actions $S^{(+)}_{\rm HM}$ and $S^{(-)}_{\rm HM}$, respectively, such that

$$S^{(+)}_{\rm HM} := -\frac{\mathrm{i}}{2\pi} \oint (v_{\rm R}, \mathrm{d}v_{\rm R}) \int \mathrm{d}^{3|8} z\, E\, C^{(-4)}_{\rm R} \Upsilon^{(1)}_{\rm R} \breve{\Upsilon}^{(1)}_{\rm R} \,, \tag{5.1}$$

and similarly for the left hypermultiplet action $S^{(-)}_{\rm HM}$. In the dual formulation, its compensating multiplet is the so-called polar hypermultiplet described by the weight-one arctic multiplet $\Upsilon^{(1)}_{\rm R}$ and its smile conjugate $\breve{\Upsilon}^{(1)}_{\rm R}$. Duality between the theories with actions $S^{(+)}_{\rm VM}$ and $S^{(+)}_{\rm HM}$ can be shown in complete analogy with the 4D $\mathcal{N} = 2$ case [29].

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A The geometry of $\mathcal{N} = 4$ conformal superspace

Here we collect the essential details of the $\mathcal{N} = 4$ superspace geometry of [18]. We refer the reader to [14, 18] for our conventions for 3D spinors.

JHEP03(2017)109

We begin with a curved three-dimensional $\mathcal{N} = 4$ superspace $\mathcal{M}^{3|8}$ parametrized by local bosonic (x^m) and fermionic coordinates (θ^μ_I):

$$z^M = (x^m,\ \theta^\mu_I)\,, \tag{A.1}$$

where $m = 0, 1, 2$, $\mu = 1, 2$ and $I = 1, \cdots, 4$. The structure group is chosen to be $\mathrm{OSp}(4|4, \mathbb{R})$ and the covariant derivatives are postulated to have the form

$$\nabla_A = E_A - \omega_A{}^{\underline{b}} X_{\underline{b}} = E_A - \frac{1}{2}\Omega_A{}^{bc} M_{bc} - \frac{1}{2}\Phi_A{}^{PQ} N_{PQ} - B_A \mathbb{D} - \mathfrak{F}_A{}^B K_B\ . \tag{A.2}$$

Here $E_A = E_A{}^M \partial_M$ is the inverse vielbein, M_{ab} are the Lorentz generators, N_{IJ} are generators of the SO(4) group, $\mathbb{D}$ is the dilatation generator and $K_A = (K_a, S^I_\alpha)$ are the special superconformal generators.

The Lorentz generators obey

$$[M_{ab}, M_{cd}] = 2\eta_{c[a} M_{b]d} - 2\eta_{d[a} M_{b]c}\,, \tag{A.3a}$$

$$[M_{ab}, \nabla_c] = 2\eta_{c[a}\nabla_{b]}\,, \quad [M_{\alpha\beta}, \nabla^I_\gamma] = \varepsilon_{\gamma(\alpha}\nabla^I_{\beta)}\ . \tag{A.3b}$$

The SO(4) and dilatation generators obey

$$[N_{KL}, N^{IJ}] = 2\delta^I_{[K} N_{L]}{}^J - 2\delta^J_{[K} N_{L]}{}^I\,, \qquad [N_{KL}, \nabla^I_\alpha] = 2\delta^I_{[K}\nabla_{\alpha L]}\,, \tag{A.3c}$$

$$[\mathbb{D}, \nabla_a] = \nabla_a\,, \qquad [\mathbb{D}, \nabla^I_\alpha] = \frac{1}{2}\nabla^I_\alpha\ . \tag{A.3d}$$

The Lorentz and SO(4) generators act on the special conformal generators K_A as

$$[M_{ab}, K_c] = 2\eta_{c[a} K_{b]}\,, \quad [M_{\alpha\beta}, S^I_\gamma] = \varepsilon_{\gamma(\alpha} S^I_{\beta)}\,, \tag{A.3e}$$

$$[N_{KL}, S^I_\alpha] = 2\delta^I_{[K} S_{\alpha L]}\,, \tag{A.3f}$$

while the dilatation generator acts on K_A as

$$[\mathbb{D}, K_a] = -K_a\,, \quad [\mathbb{D}, S^I_\alpha] = -\frac{1}{2}S^I_\alpha\ . \tag{A.3g}$$

Among themselves, the generators K_A obey the algebra

$$\{S^I_\alpha, S^J_\beta\} = 2\mathrm{i}\delta^{IJ}(\gamma^c)_{\alpha\beta} K_c\ . \tag{A.3h}$$

Finally, the algebra of K_A with ∇_A is given by

$$[K_a, \nabla_b] = 2\eta_{ab}\mathbb{D} + 2M_{ab}\,, \tag{A.3i}$$

$$[K_a, \nabla^I_\alpha] = -\mathrm{i}(\gamma_a)_\alpha{}^\beta S^I_\beta\,, \tag{A.3j}$$

$$[S^I_\alpha, \nabla_a] = \mathrm{i}(\gamma_a)_\alpha{}^\beta \nabla^I_\beta\,, \tag{A.3k}$$

$$\{S^I_\alpha, \nabla^J_\beta\} = 2\varepsilon_{\alpha\beta}\delta^{IJ}\mathbb{D} - 2\delta^{IJ} M_{\alpha\beta} - 2\varepsilon_{\alpha\beta} N^{IJ}\ . \tag{A.3l}$$

JHEP03(2017)109

The covariant derivatives obey the (anti-)commutation relations of the form

$$[\nabla_A, \nabla_B\} = -T_{AB}{}^C\nabla_C - \frac{1}{2}R(M)_{AB}{}^{cd}M_{cd} - \frac{1}{2}R(N)_{AB}{}^{PQ}N_{PQ} - R(\mathbb{D})_{AB}\mathbb{D} - R(S)_{AB}{}^{\gamma}_{K}S^{K}_{\gamma} - R(K)_{AB}{}^{c}K_c\,, \tag{A.4}$$

where $T_{AB}{}^C$ is the torsion, and $R(M)_{AB}{}^{cd}$, $R(N)_{AB}{}^{PQ}$, $R(\mathbb{D})_{AB}$, $R(S)_{AB}{}^{\gamma}_{K}$ and $R(K)_{AB}{}^{c}$ are the curvatures corresponding to the Lorentz, SO(4), dilatation, S-supersymmetry and special conformal boosts, respectively.

The full gauge group of conformal supergravity, $\mathcal{G}$, is generated by *covariant general coordinate transformations*, $\delta_{\rm cgct}$, associated with a parameter ξ^A and *standard superconformal transformations*, $\delta_{\mathcal{H}}$, associated with a parameter $\Lambda^{\underline{a}}$. The latter include the dilatation, Lorentz, SO(4), and special conformal (bosonic and fermionic) transformations. The covariant derivatives transform as

$$\delta_{\mathcal{G}}\nabla_A = [\mathcal{K}, \nabla_A]\,, \tag{A.5}$$

where $\mathcal{K}$ denotes the first-order differential operator

$$\mathcal{K} = \xi^C\nabla_C + \frac{1}{2}\Lambda^{ab}M_{ab} + \frac{1}{2}\Lambda^{IJ}N_{IJ} + \Lambda\mathbb{D} + \Lambda^A K_A\ . \tag{A.6}$$

Covariant (or tensor) superfields transform as

$$\delta_{\mathcal{G}}T = \mathcal{K}T\ . \tag{A.7}$$

In order to describe the Weyl multiplet of conformal supergravity, some of the components of the torsion and curvatures must be constrained. Following [18], the spinor derivative torsion and curvatures are chosen to resemble super-Yang Mills

$$\{\nabla^I_\alpha, \nabla^J_\beta\} = -2\mathrm{i}\varepsilon_{\alpha\beta}\mathcal{W}^{IJ}\,, \tag{A.8}$$

where $\mathcal{W}^{IJ}$ is some operator that takes values in the superconformal algebra, with P_A replaced by ∇_A. In [18] it was shown how to constrain W^{IJ} entirely in terms of the super Cotton tensor (or scalar for $\mathcal{N}=4$). The super Cotton scalar W, is a primary superfield of dimension 1,

$$S^I_\alpha W = 0\,, \quad K_a W = 0\,, \quad \mathbb{D}W = W\ . \tag{A.9}$$

The algebra of covariant derivatives is

$$\{\nabla^I_\alpha, \nabla^J_\beta\} = 2\mathrm{i}\delta^{IJ}\nabla_{\alpha\beta} + \mathrm{i}\varepsilon_{\alpha\beta}\varepsilon^{IJKL}WN_{KL} - \mathrm{i}\varepsilon_{\alpha\beta}\varepsilon^{IJKL}(\nabla^\gamma_K W)S_{\gamma L} + \frac{1}{4}\varepsilon_{\alpha\beta}(\gamma^c)^{\gamma\delta}\varepsilon^{IJKL}(\nabla_{\gamma K}\nabla_{\delta L}W)K_c\,, \tag{A.10a}$$

$$[\nabla_a, \nabla^J_\beta] = \frac{1}{2}\varepsilon^{JPQK}(\gamma_a)_{\beta\gamma}(\nabla^\gamma_K W)N_{PQ} - \frac{1}{4}(\gamma_a)_{\beta\gamma}\varepsilon^{JKLP}(\nabla^\gamma_L\nabla^\delta_P W)S_{\delta K} - \frac{\mathrm{i}}{24}(\gamma_a)_{\beta\gamma}(\gamma^c)_{\delta\rho}\varepsilon^{JKLP}(\nabla^\gamma_K\nabla^\delta_L\nabla^\rho_P W)K_c\,, \tag{A.10b}$$

JHEP03(2017)109

$$[\nabla_a, \nabla_b] = \frac{1}{8}\varepsilon_{abc}(\gamma^c)_{\alpha\beta}\varepsilon^{PQIJ}\Big(\mathrm{i}(\nabla_I^\alpha \nabla_J^\beta W)N_{PQ} + \frac{\mathrm{i}}{3}\varepsilon^{LIJK}(\nabla_I^\alpha \nabla_J^\beta \nabla_K^\gamma W)S_{\gamma L} + \frac{1}{24}(\gamma^d)_{\gamma\delta}\varepsilon^{IJKL}(\nabla_I^\alpha \nabla_J^\beta \nabla_K^\gamma \nabla_L^\delta W)K_d\Big)\,, \tag{A.10c}$$

where the super Cotton scalar W satisfies the following dimension 2 Bianchi identity

$$\nabla^{\alpha I}\nabla_\alpha^J W = \frac{1}{4}\delta^{IJ}\nabla_P^\alpha \nabla_\alpha^P W\ . \tag{A.11}$$

For each SO(4) vector V_I we can associate a second-rank isospinor $V_{i\bar{i}}$

$$V_I \leftrightarrow V_{i\bar{i}} := (\tau^I)_{i\bar{i}}V_{i\bar{i}}\,, \quad (V_{i\bar{i}})^* = V^{i\bar{i}}\ . \tag{A.12}$$

The original SO(4) connection turns into a sum of two SU(2) connections

$$\Phi_A = (\Phi_{\mathrm{L}})_A + (\Phi_{\mathrm{R}})_A\,, \quad (\Phi_{\mathrm{L}})_A = \Phi_A{}^{kl}L_{kl}\,, \quad (\Phi_{\mathrm{R}})_A = \Phi_A{}^{\bar{k}\bar{l}}R_{kl}\ . \tag{A.13}$$

Here L_{kl} is the $\mathrm{SU}(2)_{\mathrm{L}}$ generator and $R_{\bar{k}\bar{l}}$ is the $\mathrm{SU}(2)_{\mathrm{R}}$ generator. They are related to the SO(4) generators N_{KL} as

$$N_{KL} \ \to\ N_{k\bar{k}l\bar{l}} = \varepsilon_{\bar{k}\bar{l}}L_{kl} + \varepsilon_{kl}R_{\bar{k}\bar{l}}\ . \tag{A.14}$$

The left and right operators act on the covariant derivatives as

$$[L^{kl}, \nabla_\alpha^{i\bar{i}}] = \varepsilon^{i(k}\nabla_\alpha^{l)\bar{i}}\,, \quad [R^{kl}, \nabla_\alpha^{i\bar{i}}] = \varepsilon^{\bar{i}(\bar{k}}\nabla_\alpha^{i\bar{l})}\ . \tag{A.15}$$

In the isospinor notation, the Bianchi identity on W becomes

$$\nabla^{\alpha i\bar{i}}\nabla_\alpha^{j\bar{j}}W = \frac{1}{4}\varepsilon^{ij}\varepsilon^{\bar{i}\bar{j}}\nabla^\alpha_{k\bar{k}}\nabla_\alpha^{k\bar{k}}W\ . \tag{A.16}$$

The algebra of spinor covariant derivatives becomes

$$\begin{aligned}\{\nabla_\alpha^{i\bar{i}}, \nabla_\beta^{j\bar{j}}\} = {}& 2\mathrm{i}\varepsilon^{ij}\varepsilon^{\bar{i}\bar{j}}\nabla_{\alpha\beta} + 2\mathrm{i}\varepsilon_{\alpha\beta}\varepsilon^{\bar{i}\bar{j}}WL^{ij} - 2\mathrm{i}\varepsilon_{\alpha\beta}\varepsilon^{ij}WR^{\bar{i}\bar{j}} \\ & - \mathrm{i}\varepsilon_{\alpha\beta}\varepsilon^{ij}\nabla^\gamma{}_k{}^{\bar{i}}WS_\gamma^{k\bar{j}} + \mathrm{i}\varepsilon_{\alpha\beta}\varepsilon^{\bar{i}\bar{j}}\nabla^{\gamma i}{}_{\bar{k}}WS_\gamma^{j\bar{k}} \\ & + \frac{1}{4}\varepsilon_{\alpha\beta}\Big(\varepsilon^{ij}\nabla_\gamma{}_k{}^{\bar{i}}\nabla_\delta^{k\bar{j}}W - \varepsilon^{\bar{i}\bar{j}}\nabla_\gamma{}^j{}_{\bar{k}}\nabla_\delta^{i\bar{k}}W\Big)K^{\gamma\delta}\end{aligned} \tag{A.17}$$

and the action of the S-supersymmetry generator on $\nabla_\alpha^{i\bar{i}}$ is

$$\{S_\alpha^{i\bar{i}}, \nabla_\beta^{j\bar{j}}\} = 2\varepsilon_{\alpha\beta}\varepsilon^{ij}\varepsilon^{\bar{i}\bar{j}}\mathbb{D} - 2\varepsilon^{ij}\varepsilon^{\bar{i}\bar{j}}M_{\alpha\beta} + 2\varepsilon_{\alpha\beta}\varepsilon^{\bar{i}\bar{j}}L^{ij} + 2\varepsilon_{\alpha\beta}\varepsilon^{ij}R^{\bar{i}\bar{j}}\ . \tag{A.18}$$

B The geometry of SO(4) superspace

For many applications it is useful to work with a superspace formulation with a smaller structure group than that of conformal superspace. The superspace formulation of [14, 27], known as SO(4) superspace, provides such a formulation and may be obtained from

JHEP03(2017)109

conformal superspace via a degauging procedure [18]. For the $\mathcal{N}=4$ case one chooses the structure group to be SO(4). The SO(4) superspace formulation for $\mathcal{N}=4$ conformal supergravity has been used to construct general off-shell supergravity-matter couplings [14].

The covariant derivatives have the form:

$$\mathcal{D}_A = E_A - \Omega_A - \Phi_A \ . \tag{B.1}$$

Here $E_A = E_A{}^M(z)\partial_M$ is the supervielbein, with $\partial_M = \partial/\partial z^M$, Ω_A is the Lorentz connection, and $\Phi_A = \frac{1}{2}\Phi_A{}^{KL}N_{KL}$ is the SO(4)-connection. The supergravity gauge group is generated by local transformations of the form

$$\delta_K \mathcal{D}_A = [K, \mathcal{D}_A]\,, \qquad K = K^C(z)\mathcal{D}_C + \frac{1}{2}K^{cd}(z)M_{cd} + \frac{1}{2}K^{PQ}(z)N_{PQ}\,, \tag{B.2}$$

with all the gauge parameters obeying natural reality conditions.

The covariant derivatives satisfy the (anti)commutation relations

$$[\mathcal{D}_A, \mathcal{D}_B\} = -T_{AB}{}^C\mathcal{D}_C - \frac{1}{2}R_{AB}{}^{KL}N_{KL} - \frac{1}{2}R_{AB}{}^{cd}M_{cd}\,, \tag{B.3}$$

with $T_{AB}{}^C$ the torsion, $R_{AB}{}^{cd}$ the Lorentz curvature and $R_{AB}{}^{KL}$ the SO(4) curvature. The algebra of covariant derivatives must be constrained to describe conformal supergravity. The appropriate constraints [27] lead to the following anti-commutation relation [14]:

$$\begin{aligned}\{\mathcal{D}^I_\alpha, \mathcal{D}^J_\beta\} &= 2\mathrm{i}\delta^{IJ}(\gamma^c)_{\alpha\beta}\mathcal{D}_c - 2\mathrm{i}\varepsilon_{\alpha\beta}C^{\gamma\delta IJ}M_{\gamma\delta} - 4\mathrm{i}S^{IJ}M_{\alpha\beta} \\ &\quad + \Big(\mathrm{i}\varepsilon_{\alpha\beta}W^{IJKL} - 4\mathrm{i}\varepsilon_{\alpha\beta}S^{K[I}\delta^{J]L} + \mathrm{i}C_{\alpha\beta}{}^{KL}\delta^{IJ} - 4\mathrm{i}C_{\alpha\beta}{}^{K(I}\delta^{J)L}\Big)N_{KL}\ .\end{aligned} \tag{B.4a}$$

Here the dimension-1 components are real and satisfy the symmetry properties

$$W^{IJKL} = W^{[IJKL]} = \varepsilon^{IJKL}W\,, \qquad S^{IJ} = S^{(IJ)}\,, \qquad C_a{}^{IJ} = C_a{}^{[IJ]}\ . \tag{B.5}$$

It is useful to decompose the torsion superfield S^{IJ} into its trace ($\mathcal{S}$) and traceless ($\mathcal{S}^{IJ}$) parts as

$$S^{IJ} = \mathcal{S}\delta^{IJ} + \mathcal{S}^{IJ}\,, \quad \mathcal{S} = \frac{1}{\mathcal{N}}\delta_{IJ}S^{IJ}\,, \quad \delta_{IJ}\mathcal{S}^{IJ} = 0\ . \tag{B.6}$$

The torsion superfields satisfy the Bianchi identities

$$\mathcal{D}^I_\alpha \mathcal{S}^{JK} = 2\mathcal{T}_\alpha{}^{I(JK)} + \mathcal{S}_\alpha{}^{(J}\delta^{K)I} - \frac{1}{\mathcal{N}}\mathcal{S}_\alpha{}^I\delta^{JK}\,, \tag{B.7a}$$

$$\begin{aligned}\mathcal{D}^I_\alpha C_{\beta\gamma}{}^{JK} &= \frac{2}{3}\varepsilon_{\alpha(\beta}\Big(C_{\gamma)}{}^{IJK} + 3\mathcal{T}_{\gamma)}{}^{JKI} + 4(\mathcal{D}^{[J}_{\gamma)}\mathcal{S})\delta^{K]I} + \frac{(\mathcal{N}-4)}{\mathcal{N}}\mathcal{S}_{\gamma)}{}^{[J}\delta^{K]I}\Big) \\ &\quad + C_{\alpha\beta\gamma}{}^{IJK} - 2C_{\alpha\beta\gamma}{}^{[J}\delta^{K]I}\,,\end{aligned} \tag{B.7b}$$

$$0 = \Big(\mathcal{D}^{\gamma(I}\mathcal{D}^{J)}_\gamma - \frac{1}{4}\delta^{IJ}\mathcal{D}^{\gamma K}\mathcal{D}_{\gamma K} - 4\mathrm{i}\mathcal{S}^{IJ}\Big)W\ . \tag{B.7c}$$

It is often useful to make use of the isomorphism $\mathrm{SO}(4) \cong \big(\mathrm{SU}(2)_\mathrm{L} \times \mathrm{SU}(2)_\mathrm{R}\big)/\mathbb{Z}_2$ and make use of isospinor notation, $\mathcal{D}^I_\alpha \to \mathcal{D}^{i\bar{i}}_\alpha$, by replacing each SO(4) vector index by a pair of isospinor ones. For our notation and conventions we refer the reader to [14].

JHEP03(2017)109

After introducing isospinor notation, the covariant derivatives are

$$\mathcal{D}_A = (\mathcal{D}_a, \mathcal{D}_\alpha^{i\bar{i}}) = E_A - \Omega_A - \Phi_A \,, \tag{B.8}$$

where the original SO(4) connection Φ_A now turns into a sum of two SU(2) connections

$$\Phi_A = (\Phi_{\rm L})_A + (\Phi_{\rm R})_A \,, \quad (\Phi_{\rm L})_A = \Phi_A{}^{kl} L_{kl} \,, \quad (\Phi_{\rm R})_A = \Phi_A{}^{\bar{k}\bar{l}} R_{\bar{k}\bar{l}} \,. \tag{B.9}$$

The two SU(2) generators act on the spinor covariant derivatives $\mathcal{D}_\alpha^{i\bar{i}} := \mathcal{D}_\alpha^I (\tau_I)^{i\bar{i}}$ as follows:

$$\big[L^{kl}, \mathcal{D}_\alpha^{i\bar{i}}\big] = \varepsilon^{i(k} \mathcal{D}_\alpha^{l)\bar{i}} \,, \quad \big[R^{\bar{k}\bar{l}}, \mathcal{D}_\alpha^{i\bar{i}}\big] = \varepsilon^{\bar{i}(\bar{k}} \mathcal{D}_\alpha^{i\bar{l})} \,. \tag{B.10}$$

The algebra of spinor covariant derivatives is

$$\begin{aligned}
\{\mathcal{D}_\alpha^{i\bar{i}}, \mathcal{D}_\beta^{j\bar{j}}\} = {} & 2{\rm i}\varepsilon^{ij}\varepsilon^{\bar{i}\bar{j}} (\gamma^c)_{\alpha\beta} \mathcal{D}_c + 2{\rm i}\varepsilon_{\alpha\beta}\varepsilon^{\bar{i}\bar{j}} (2\mathcal{S} + X) L^{ij} - 2{\rm i}\varepsilon_{\alpha\beta}\varepsilon^{ij} \mathcal{S}^{kl\bar{i}\bar{j}} L_{kl} + 4{\rm i} C_{\alpha\beta}{}^{\bar{i}\bar{j}} L^{ij} \\
& + 2{\rm i}\varepsilon_{\alpha\beta}\varepsilon^{ij} (2\mathcal{S} - X) R^{\bar{i}\bar{j}} - 2{\rm i}\varepsilon_{\alpha\beta}\varepsilon^{\bar{i}\bar{j}} \mathcal{S}^{ij\bar{k}\bar{l}} R_{\bar{k}\bar{l}} + 4{\rm i} C_{\alpha\beta}{}^{ij} R^{\bar{i}\bar{j}} \\
& + 2{\rm i}\varepsilon_{\alpha\beta} (\varepsilon^{\bar{i}\bar{j}} C^{\gamma\delta ij} + \varepsilon^{ij} C^{\gamma\delta\bar{i}\bar{j}}) M_{\gamma\delta} - 4{\rm i} (\mathcal{S}^{ij\bar{i}\bar{j}} + \varepsilon^{ij}\varepsilon^{\bar{i}\bar{j}} \mathcal{S}) M_{\alpha\beta} \,,
\end{aligned} \tag{B.11}$$

where the torsion components satisfy certain Bianchi identities given in [14].[18]

C Super-Weyl gauge conditions

In this appendix we show how one can use the super-Weyl freedom to impose certain gauge conditions in SO(4) superspace. In particular, within the SO(4) superspace formulation we will show that one can impose either

$$C_a{}^{\bar{i}\bar{j}} = 0 \,, \quad 2S + W = 0 \tag{C.1}$$

or

$$C_a{}^{ij} = 0 \,, \quad 2S - W = 0 \,. \tag{C.2}$$

We begin by introducing, within the SO(4) superspace geometry, an off-shell self-dual vector multiplet $G^{\bar{i}\bar{j}}$ and an anti-self-dual vector multiplet G^{ij}. They are constrained by the differential constraints for $\mathcal{O}(2)$ multiplets

$$\mathcal{D}_\alpha^{i(\bar{i}} G^{\bar{j}\bar{k})} = 0 \,, \quad \mathcal{D}_\alpha^{(i\bar{i}} G^{jk)} = 0 \,. \tag{C.3}$$

Using these constraints it is possible to build some of the components of the torsion in terms of these multiplets. In particular, one finds

$$2S - W = \frac{{\rm i} G_+}{8} \mathcal{D}^{\gamma i\bar{i}} \mathcal{D}_{\gamma i\bar{i}} G_+^{-1} \,, \tag{C.4a}$$

$$2S + W = \frac{{\rm i} G_-}{8} \mathcal{D}^{\gamma i\bar{i}} \mathcal{D}_{\gamma i\bar{i}} G_-^{-1} \,, \tag{C.4b}$$

$$C_{\alpha\beta}{}^{ij} = -\frac{\rm i}{4} G_+ \mathcal{D}_\alpha^{(i\bar{k}} \mathcal{D}_\beta{}^{j)}{}_{\bar{k}} G_+^{-1} \,, \tag{C.4c}$$

[18] As compared to [14], we have relabelled the superfield $B_{\alpha\beta}{}^{ij}$ by $C_{\alpha\beta}{}^{ij}$.

JHEP03(2017)109

$$C_{\alpha\beta}{}^{\bar{i}\bar{j}} = -\frac{\mathrm{i}}{4} G_- \mathcal{D}_\alpha{}^{k(\bar{i}} \mathcal{D}_{\beta k}{}^{\bar{j})} G_-^{-1} \,, \tag{C.4d}$$

$$S^{(k}{}_p{}^{\bar{i}\bar{j}} G^{l)p} = -\frac{\mathrm{i}}{16} \{\mathcal{D}^{\gamma p(\bar{i}}, \mathcal{D}_{\gamma p}{}^{\bar{j})}\} G^{kl} \,, \tag{C.4e}$$

$$S^{ij}{}_{\bar{p}}{}^{(\bar{k}} G^{\bar{l})\bar{p}} = -\frac{\mathrm{i}}{16} \{\mathcal{D}^{\gamma(i\bar{p}}, \mathcal{D}_\gamma{}^{j)}{}_{\bar{p}}\} G^{\bar{k}\bar{l}} \,, \tag{C.4f}$$

where $G_+^2 = G^{\bar{i}\bar{j}} G_{\bar{i}\bar{j}}$ and $G_-^2 = G^{ij} G_{ij}$.

The vector multiplets transform homogeneously under super-Weyl transformations

$$G^{\bar{i}\bar{j}} \to \mathrm{e}^{\sigma} G^{\bar{i}\bar{j}} \,, \quad G^{ij} \to \mathrm{e}^{\sigma} G^{ij} \,, \tag{C.5}$$

which tells us that the super-Weyl freedom can be completely fixed by imposing the gauge condition $G_+ = 1$ or $G_- = 1$. If we impose $G_+ = 1$ we find the conditions (C.1), while if we impose $G_- = 1$ we find the conditions (C.2). Therefore, these conditions can always be imposed by an appropriate super-Weyl transformation.

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JHEP03(2017)109