Parametrisation of optimal RMP coil phase on ASDEX Upgrade

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Edge Localised Modes (ELMs) are a repetitive instability, driven by the high currents and pressure gradients found at the edge of high confinement mode tokamak plasmas[2], which could potentially cause damage to the plasma facing components of the ITER tokamak [3]. ELMs can be mitigated by the application of Resonant Magnetic Perturbations (RMPs), using dedicated magnetic coils, now installed on many tokamaks in two sets, one toroidal ring of coils above the midplane, and another below. Typically the currents in these coils are approximately sinusoidal in the toroidal direction, with toroidal mode number n. The poloidal spectrum of the applied magnetic field can be tuned by introducing a toroidal phase offset between the upper and lower coil currents, defined here as $\Delta\Phi = \Phi_{upper} - \Phi_{lower}$, the effect of which is discussed in previous works [4,5]. The component of the applied field normal to the flux surfaces and aligned with the magnetic equilibrium, b^r_{res}, is a commonly used figure of merit for interpreting ELM mitigation experiments[6]. In previous works [6,7] a correlation was found between the mitigated ELM frequency and b^r_{res}, computed including the plasma response, which was varied by scanning $\Delta\Phi$. $\Delta\Phi_{opt}$ is thus defined here as

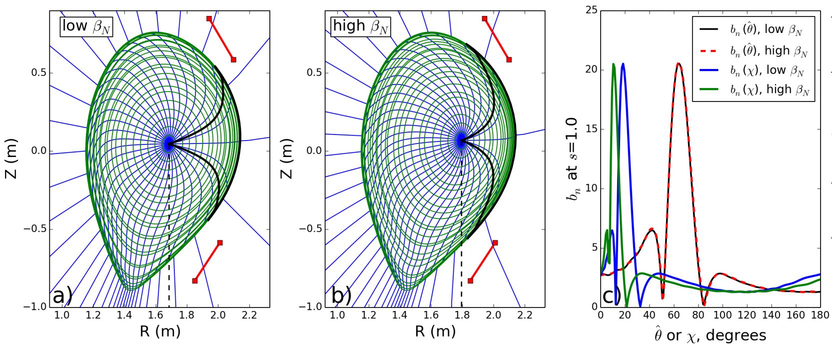

Figure 1. a,b) Grids showing the equilibrium based straight field line coordinate systems for low and high β_N cases, with the lines of $\chi = \pm 15$ highlighted (χ generalised poloidal angle). In the high β_N case, $\chi = \pm 15$ encompasses a larger section of the plasma boundary. c) The normal component of the applied vacuum perturbation at the plasma boundary. In real space (geometric coordinate θ) both applied fields are the same. However in magnetic geometry the high and low β_N fields differ, due to the redistribution of χ with changing β_N. In the high β_N case, the main peaks of the applied field (which naturally occur near the RMP coils) are compressed into a narrower range of χ, causing the poloidal spectrum to be shifted towards higher m. This is the cause of the dependence of alignment on β_N.

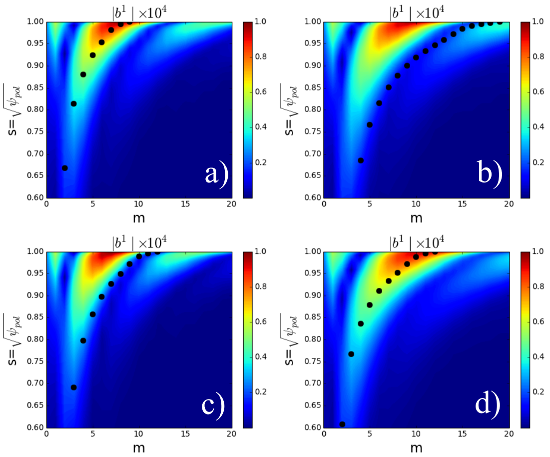

Figure 2. Changing q_{95} or n moves the $nq(s) = m$ line (black points) relative to the spectrum of the applied perturbation. a) and b) shows the $nq(s) = m$ line relative to the vacuum spectrum for a) a low q_{95} and b) a high q_{95} case. Increasing plasma pressure β_N distorts the equilibrium based magnetic geometry, redefining poloidal angle χ such that the vacuum spectrum is shifted to higher m. c) and d) shows the $nq(s) = m$ line relative to the vacuum spectrum for a low (c) and high (d) β_N case, with identical q_{95}.

$\Delta\Phi$ at which b^r_{res} is maximised. Since choosing $\Delta\Phi \sim \Delta\Phi_{opt}$ can optimize ELM mitigation for given plasma equilibrium, experimentalists are motivated to know $\Delta\Phi_{opt}$ ahead of experiments. However, computing it using MARS-F requires as input the tokamak equilibrium, plasma boundary, and kinetic profiles, which are not usually known before the experiment. A simpler method to estimate $\Delta\Phi_{opt}$ would therefore be desirable. The parameters of the equilibrium which most strongly modify the vacuum field for fixed $\Delta\Phi$, and hence $\Delta\Phi_{opt}$, are the plasma boundary shape (investigated in [4]), the normalised plasma pressure β_N, and the plasma safety factor at the 95% magnetic flux surface q_{95}, as explained in figures 1 and 2.

In this work, the dependence of $\Delta\Phi_{opt}$ on β_N and q_{95}, is studied. For this purpose a reference plasma equilibrium from the ASDEX Upgrade tokamak is scaled in pressure and current, to create a set of equilibria spanning a wide parameter space in (β_N, q_{95}), but with fixed plasma boundary shape and kinetic profiles. At each point in the parameter space, $\Delta\Phi_{opt}$ is computed using the code MARS-F[2], both in the vacuum approximation and including the plasma response. For given toroidal mode number n and constant plasma boundary shape, $\Delta\Phi_{opt}$ is found to vary smoothly with (β_N, q_{95}). A simple 2D quadratic function is proposed to parametrize the dependence of $\Delta\Phi_{opt}$ on (β_N, q_{95}), whose coefficients are computed by linear regression, and included here for researchers to use as a guide for future ASDEX Upgrade experiments. The accuracy of the 2D quadratic is assessed by comparing it's predictions with a set of validation points, each consisting of a $\Delta\Phi_{opt}$ computation using MARS-F, for distinct and widely varying experimental equilibria, plasma boundary shapes, and sets of kinetic profiles. The 2D quadratic is shown to be accurate to within 26 degrees for n=1 RMPs, and within 21 degrees for n=2 RMPs.

Using the CHEASE fixed boundary equilibrium solver, a reference equilibrium from ASDEX Upgrade shot 30835 at 3200ms, was scaled in 2 dimensions by scaling the plasma

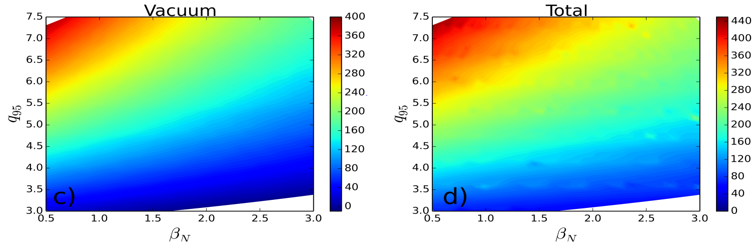

Figure 3. $\Delta\Phi_{opt}$ as a function of (β_N, q_{95}), computed for a set of scaled equilibria for constant kinetic profiles and plasma boundary shape.

pressure and plasma current profiles, to produce a dense set of equilibria which span a wide range of β_N and q_{95}. For each equilibrium point, the vacuum field and plasma response due to the applied RMP field were computed, by solving the linearised equations of resistive MHD using the MARS-F code.

$$\Delta\Phi_{opt} = \pm arccos\left(\frac{b^l_{res} \cdot b^u_{res}}{|b^l_{res}||b^u_{res}|}\right)$$

Equation [1]

$\Delta\Phi_{opt}$ was then computed for each point using Equation 1, where b^l_{res} and b^u_{res} are the complex valued outermost resonant components due to the lower and upper coils respectively, which are extracted from the MARS-F results. The sign uncertainty in Equation [1] is resolved by choosing the sign of $\Delta\Phi_{opt}$ which maximises b^r_{res}. Figure 3 shows $\Delta\Phi_{opt}$ for the scan of (β_N, q_{95}), for $n=2$, in the vacuum approximation and including the plasma response. $\Delta\Phi_{opt}$ increases with increasing q_{95}, and decreases with in increasing β_N.

$$\Delta\Phi_{opt,quad} = a(x^2y^2) + b(x^2y) + c(x^2) + d(xy^2) + e(xy) + f(x) + g(y^2) + h(y) + i$$

Equation [2]

To remove unphysical discontinuities, the phase wraps are removed from the results. The scan was also performed for $n=1,3$, and 4, which found the same general behaviour. The figure shows that for given n, $\Delta\Phi_{opt}$ is a smoothly varying function of (β_N, q_{95}). This allows the results to be parametrised with a simple analytic function, to allow researchers to estimate $\Delta\Phi_{opt}$ prior to experiments. A 2D quadratic function in β_N and q_{95} is chosen for ease of use, and because it closely fits the scan results. Equation 2 describes the form of the function, and table 1 lists the coefficient values of the 2D quadratic, found by linear regression. In Equation 2, let $\Delta\Phi_{opt,quad}$ be $\Delta\Phi_{opt}$ predicted by the 2D quadratic parametrisation, $x=\beta_N$ and $y=q_{95}$. To quantify the error of the 2D quadratic, which requires only β_N, q_{95} and n as input, the function was validated against MARS-F $\Delta\Phi_{opt}$ computation for a set of validation points, from several distinct ASDEX Upgrade experiments. Each validation point consists of a free boundary CLISTE equilibrium reconstruction based on magnetic measurements, radial profiles of electron density, electron temperature, ion temperature and bulk plasma toroidal rotation velocity, fitted to spatially resolved data from multiple diagnostics, and the experimentally applied RMP coil currents.

$$RMSE_{validation} = \left(\sum_i^n (\Delta\Phi^i_{opt,validation} - \Delta\Phi^i_{opt,quad})^2/n\right)^{\frac{1}{2}}$$

Equation [3]

For each validation point, the vacuum field and plasma response were computed using MARS-F, and then $\Delta\Phi_{opt}$ computed using Equation 1. To quantify the agreement between the 2D quadratic and the validation

coeff	a	b	c	d	e	f	g	h	i
n=1 vacuum	0.14	0.16	-1.67	-0.52	-7.69	18.74	-1.26	65.15	-312.19
n=1 total	0.43	-5.70	17.10	-2.74	29.94	-99.27	-0.46	49.97	-210.18
n=2 vacuum	0.15	1.71	-6.39	-0.25	-23.72	56.21	-3.15	127.83	-327.38
n=2 total	0.14	1.77	-8.53	-0.34	-22.03	63.89	-3.18	129.07	-286.34
n=3 vacuum	0.28	1.65	-6.71	-0.34	-34.04	76.08	-4.61	180.18	-676.44
n=3 total	0.22	1.81	-7.14	-0.56	-28.50	67.55	-3.96	171.31	-604.86
n=4 vacuum	0.36	2.05	-8.19	-0.51	-42.16	91.01	-5.15	219.78	-646.41
n=4 total	0.51	0.78	-6.19	-1.13	-35.52	85.06	-4.17	208.44	-572.30

Table 1: Coefficients of the quadratic parametrisation for $\Delta\Phi_{opt}$

points, a modified RMSE is defined in Equation 3. Figure 4 shows the values of $\Delta\Phi_{opt}$ computed by MARS-F for the validation points, compared with the values predicted by the 2D quadratic parametisation, and the RMSE between them. The RMSE may be improved by using a reference equilibrium which better represents an average ASDEX Upgrade discharge. With improvement, the 2D quadratic may become sufficiently accurate to estimate $\Delta\Phi_{opt}$. This work has shown that optimal coil phase $\Delta\Phi_{opt}$ on ASDEX Upgrade, defined as $\Delta\Phi$ which maximises the pitch aligned component of the applied RMP field, is a smoothly varying function of q_{95} and β_N, for given n and using model equilibria with fixed plasma shape and kinetic profiles. A 2D quadratic function is proposed to parametrise $\Delta\Phi_{opt}(\beta_N, q_{95}, n)$, and this function is compared quantitatively to the predictions of MARS-F including variations of plasma equilibrium, plasma boundary shape, and kinetic profiles. The 2D quadratic is found to match these predictions to within 26.5 degrees, improving the agreement, and quantifying the agreement for $n=3,4$, is left for future work.

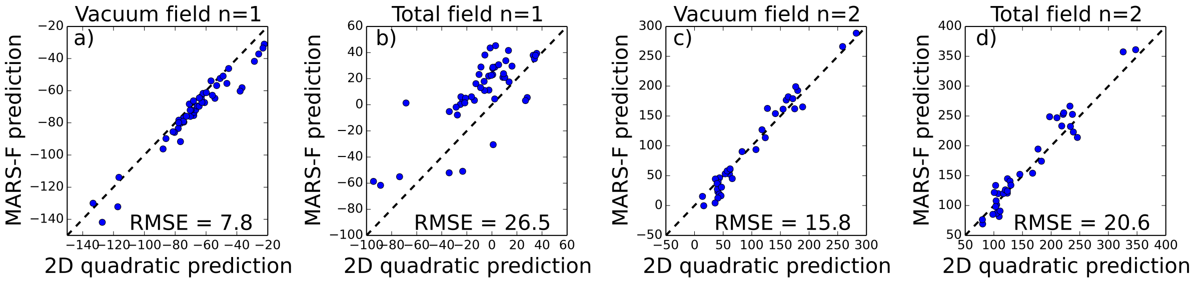

Figure 4. $\Delta\Phi_{opt}$ as computed by MARS-F, using the equilibrium, plasma boundary shape and kinetic profiles constructed from measurements, compared with $\Delta\Phi_{opt}$ predicted with the 2d quadratic parametrisation, which requires only β_N, q_{95} and n as input.

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