A Call for New Physics : The Muon Anomalous Magnetic Moment and Lepton Flavor Violation

Manfred Lindner[1], Moritz Platscher[2], Farinaldo S. Queiroz[3]

Max-Planck-Institut für Kernphysik
Saupfercheckweg 1, 69117 Heidelberg, Germany

Abstract

We review how the muon anomalous magnetic moment $(g-2)$ and the quest for lepton flavor violation are intimately correlated. Indeed the decay $\mu \to e\gamma$ is induced by the same amplitude for different choices of in- and outgoing leptons. In this work, we try to address some intriguing questions such as: *Which hierarchy in the charged lepton sector one should have in order to reconcile possible signals coming simultaneously from $g-2$ and LFV? What can we learn if the $g-2$ anomaly is confirmed by the upcoming flagship experiments at FERMILAB and J-PARC, and no signal is seen in the decay $\mu \to e\gamma$ in the foreseeable future? On the other hand, if the $\mu \to e\gamma$ decay is seen in the upcoming years, do we need to necessarily observe a signal also in $g-2$?* In this attempt, we generally study the correlation between the two phenomena in a detailed analysis of simplified models. We derive master integrals and fully analytical and exact expressions for both phenomena. We investigate under which conditions the observations can be made compatible and discuss their implications. Lastly, we discuss in this context several extensions of the SM, such as the Minimal Supersymmetric Standard Model, Left-Right symmetric model, $B-L$ model, scotogenic model, two Higgs doublet model, Zee-Babu model, 3-3-1 model, and $L_\mu - L_\tau$ models.

[1]lindner@mpi-hd.mpg.de
[2]moritz.platscher@mpi-hd.mpg.de
[3]farinaldo.queiroz@mpi-hd.mpg.de

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1. Introduction

The muon anomalous magnetic moment ($g-2$) is a prime example of the success of quantum field theory [1]. Its precise measurement is paramount to understanding the effects of higher order corrections arising in perturbation theory. Furthermore, it potentially indicates the existence of new physics since there is a long standing deviation between the Standard Model (SM) prediction and the measurement, which raised much interest in the past [2]. On the other hand, lepton flavor violation (LFV) has been observed via neutrino oscillations since the late 90's [3, 4], but has thus far not been detected among charged leptons. Typically, new physics models that accommodate $g-2$ advocated the existence of new particles with masses around or below the TeV scale and face serious problems when confronted with constraints from LFV, which tend to force these particle to be rather heavy as illustrated in Fig. 1. Possibly, the ongoing $g-2$ experiments may reach a 5σ deviation from the SM in the foreseeable future, constituting an augury for new physics. Can such a signal be reconciled with limits from LFV? Is there room for signals in both observables? A new era in particle physics may be ahead of us and this review plans to pave the way. We outline what sort of models can accommodate signals and constraints once both data sets are accounted for in a systematic way. To do so, we structured the manuscript as follows:

(i) in Sec. 2 we review the theoretical aspects of the $g-2$;

(ii) in Sec. 3 we discuss the experimental apparatus for $g-2$;

(iii) in Sec. 4 we review the call for new physics in the $g-2$;

(iv) in Sec. 5 we give a brief introduction to LFV;

(v) in Sec. 6 we provide the experimental status;

(vi) in Sec. 7 we describe the foundation of the relation between $g-2$ and $\mu \to e\gamma$;

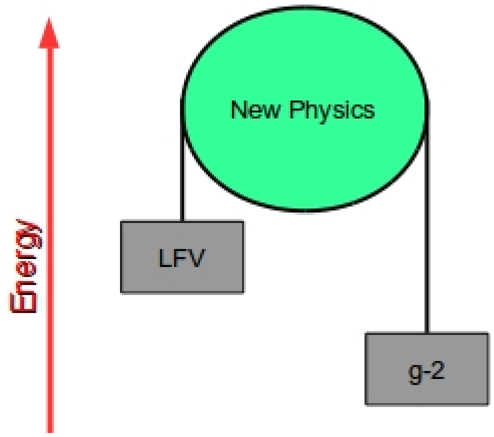


Figure 1: Muon anomalous magnetic moment points to new physics at relatively low scale, whereas lepton flavor violating processes typically push new physics to high scales. How are they connected? Can signals in both observables be reconciled? In this review we address these questions.

(vii) in Sec. 9, we investigate how one can reconcile possible signal seen in $g-2$ and $\mu \to e\gamma$ and the implications in several simplified models guarding $SU(2)_L$ invariance;

(viii) in Sec. 10 we discuss the correlation between $g-2$ and $\mu \to e\gamma$ in several UV complete models: the MSSM, $U(1)_{B-L}$, Left-Right symmetry, two Higgs doublets, Zee-Babu, scotogenic, the 3-3-1 model and $L_\mu - L_\tau$ models, to show that our findings are applicable to a multitude of popular particle physics models;

(ix) in Sec. 11 we finally draw our conclusions.

2. Muon Anomalous Magnetic Moment

In quantum mechanics we have learned that any charged particle has a magnetic dipole moment ($\overrightarrow{\mu}$) which is aligned with its spin ($\overrightarrow{s}$) and linked through the equation,

$$\overrightarrow{\mu} = g \left(\frac{q}{2m} \right) \overrightarrow{s}, \tag{1}$$

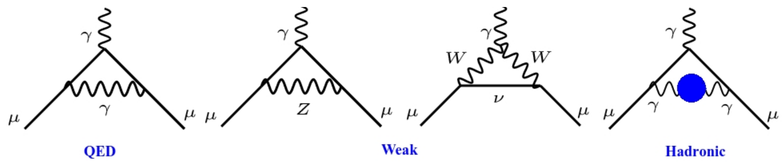


Figure 2: Lowest-order SM corrections to to a_μ. From left to right: QED , weak and hadronic.

where g is the gyromagnetic ratio, $q = \pm e$ is the electric charge of a given charged particle, and m its mass. In classical quantum mechanics, $g = 2$. However, loop corrections calculable in quantum field theories such as the SM yield small corrections to this number, as shown in Fig. 2. These corrections are parametrized in terms of $a_\mu = (g_\mu - 2)/2$, referred as the anomalous magnetic moment which has been calculated since the 1950s [1]. Ever since, a great deal of effort has been put forth to determine the SM prediction including higher orders of perturbation theory [5–13]. Considering SM contributions up to three orders in the electromagnetic constant, one finds:

$$\begin{aligned} a_\mu^{SM} &= 116591802(2)(42)(26) \times 10^{-11} \text{ [2]} \\ a_\mu^{SM} &= 116591828(2)(43)(26) \times 10^{-11} \text{ [14].} \end{aligned} \tag{2}$$

The difference central values are due to different results found for the hadronic vacuum polarization contributions. The three errors in parenthesis account for electroweak, lowest-order hadronic, and higher-order hadronic contributions, respectively [15].

Moreover, there is a great effort ongoing to reduce the theoretical errors [16–52]; however, calculating the SM contribution to a_μ is still a burden with large uncertainties arising [53, 54] most prominently from hadronic light-light corrections [47, 55–60].

Since the SM prediction to the muon anomalous magnetic moment is proportional to the electromagnetic constant, it is also important to measure the latter to a high precision. Up to now the electromagnetic constant is obtained through measurements of the electron magnetic moment [61–69]. In other words, the electron magnetic moment serves as input to determine the muon anomalous magnetic moment in the SM. Ideally, one could independently measure the electromagnetic constant and then compare with the measured electron magnetic moment. Albeit, the electron magnetic moment is not as sensitive to new physics effects due to the small electron mass. Indeed, its relative sensitivity to new physics is reduced by a factor of $m_\mu^2/m_e^2 \sim 40000$. Nevertheless, there are specific cases in which the electron magnetic moment plays a complementary probe to new physics [70, 71]. As for the tau magnetic moment [72, 73], which would in principle be an excellent probe for new physics, it is measured with a very poor precision, $-0.052 < a_\tau^{exp} < 0.013$ at 95% C.L. (also quoted as $a_\tau^{exp} = -0.018(17)$ [74]) due to its very short lifetime ($\sim 10^{-13}$ s). The quoted limit is the one adopted by the PDG [74], but there are many competing bounds in the literature [75–80]. Some are actually more stringent than the PDG one, lying in the range of $-0.007 < a_\tau^{exp} < 0.005$ [81, 82], using data on tau lepton production at LEP1, SLC, and LEP2. The SM prediction is 0.0117721(5) [72, 83], showing that even using this more restrictive limit we are still several factors away from probing new physics using the tau magnetic moment.

Now we have understood what is the muon anomalous magnetic moment, discussed the SM prediction to a_μ and the theoretical uncertainties, let's have a look at the measurement procedure.

3. Measuring the Muon Anomalous Magnetic Moment

A multitude of experiments have measured the muon anomalous magnetic moment through the principle of Larmor precession, whose frequency is proportional to the magnetic field which the charged particle is subject to. The measurement of the muon anomalous magnetic moment is illustrated in Fig. 3. After protons hit a target, charged pions are produced which then decay into polarized muons which are used by an injector which injects muons into the storage ring to which a uniform magnetic field ($\overrightarrow{B}$) perpendicular to muon spin

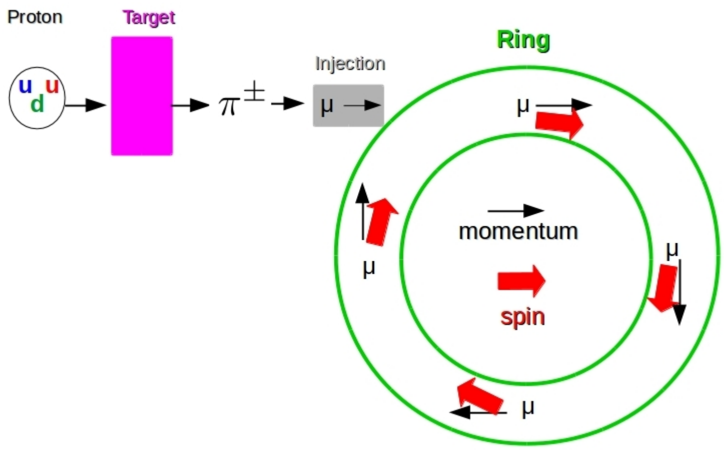


Figure 3: Illustrative figure showing how experiments measure the muon anomalous magnetic moment, using a beam of polarized muons and Larmor precession physics.

and orbit plane is applied. Using a vertically focused quadrupole electric field $\overrightarrow{E}$, one can find the frequency difference between the spin precession ($\overrightarrow{w_{a_\mu}}$) and the the cyclotron motion [100],

$$\overrightarrow{w_{a_\mu}} = \frac{e}{m_\mu}\left[a_\mu \overrightarrow{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1}\right)\overrightarrow{v} \times \overrightarrow{E}\right], \tag{3}$$

where $\gamma = (1 - v^2)^{-1/2}$, with v being the muon velocity. The fundamental idea concerning the measurement of a_μ consist of tuning the muon velocity such that $\gamma = 29.3$, removing the dependence on the electric field. This particular value is known as "magic γ" [101]. Next, one needs to measure the frequency $\overrightarrow{w_{a_\mu}}$ with high precision and extract a_μ. In Tab. 1 we present a comprehensive historic perspective of a_μ measurements going back to the first measurement in 1957. Interestingly, two Nobel prize winners (Leon Lederman, 1988 and Georges Charpak, 1992) were at some point involved in the measurement of the muon anomalous magnetic moment. In Fig. 4 one can easily see how the sensitivity has improved with time.

The most recent measurement comes from BNL (2006) data which found $a_\mu^{exp} = (116592089 \pm 63) \times 10^{-11}$, i.e. $\delta a_\mu^{exp} = 63 \times 10^{-11}$, reaching unprecedented

Determination	Beam	a_μ
SM [2]		0.00116 591 803(1)(42)(26)
Columbia-Nevis(1957) [84]	μ^+	0.00 ± 0.05
Columbia-Nevis(1959) [85]	μ^+	$0.001\,13^{+(16)}_{-(12)}$
CERN (1961) [86, 87]	μ^+	0.001 145(22)
CERN (1962) [88]	μ^+	0.001 162(5)
CERN (1968) [89]	$\mu^\pm$	0.001 166 16(31)
CERN (1975) [90]	$\mu^\pm$	0.001 165 895(27)
CERN (1979) [91]	$\mu^\pm$	0.001 165 911(11)
BNL E821 (2000) [92]	μ^+	0.001 165 919 1(59)
BNL E821 (2001) [93]	μ^+	0.001 165 920 2(16)
BNL E821 (2002) [94]	μ^+	0.001 165 920 3(8)
BNL E821 (2004) [95]	μ^-	0.001 165 921 4(8)(3)
BNL E821 (2006) [96]	$\mu^\pm$	0.001 165 920 89(63)
Current Discrepancy [97]		$\Delta a_\mu = 287(80) \times 10^{-11}$
		$\Delta a_\mu = 261(78) \times 10^{-11}$
Future sensitivity [98]		$\Delta a_\mu = 288(34) \cdot 10^{-11}$

Table 1: Historic overview of the measurements of the muon anomalous magnetic moment a_μ, along with current and future sensitivity of $\Delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{SM}}$. The errors in the SM values are due to electroweak corrections, leading and next-to-leading order hadronic corrections, respectively. Throughout this work we will adopt $\Delta a_\mu = 287(80) \times 10^{-11}$ as used by the PDG group [99]. For recent theoretical reviews see e.g. [97, 100].

sensitivity. The Muon $g-2$ Experiment at Fermilab (FNAL) aims to improve the statistical error by a factor of four, reaching a precision of ± 0.14 ppm, which translates into $\delta a_\mu^{exp} = 16 \times 10^{-11}$ [102–105]. The FNAL functioning is similar to that illustrated in Fig. 3 since the FNAL experiment uses the BNL ring, which was brought to FERMILAB. In other words, FNAL is a more sophisticated version of the BNL experiment.

If everything goes smoothly,the first results are expected to be announced around the beginning of 2019 [106], which will be followed by two other publications, in the course of a few years, aiming to reduce the systematic uncertainties by a factor of three and possibly achieve a ± 0.1 ppm statistical precision [106]. An important cross-check will be performed by the J-PARC experiment, located in Tokai, Japan, which initially plans to reach a statistical precision of 0.37 ppm, and should start taking data around 2020-2022. Its final goal is similar to FNAL, i.e. to reduce the statistical uncertainty down to a 0.1 ppm precision, as well as the systematics by a factor of three. We highlight that J-PARC is a very different experiment though [107–111], because it uses incident muons with much lower energies compared to FNAL [107, 112], a stronger magnetic field, and it does not adopt the "magic γ" approach, rather it will run with with zero electric field. Consequently, its systematic errors are also distinct [113–115]. Anyway, in the foreseeable these two flagship experiments will play an important role in particle physics regardless which direction their measurement will point to, but if indeed the central value remains roughly the same, the significance of the anomaly will be around or over 5σ, constituting a strong call for new physics [2], which is the focus of the following section.

4. An Augury for New Physics: Muon Anomalous Magnetic Moment

Comparing the SM prediction with the recent measurement from Brookhaven National Lab we find two values for the discrepancies depending on the value used for the hadronic vacuum polarization [2],

$$\begin{aligned} \Delta a_\mu &= a_\mu^{exp} - a_\mu^{SM} = (287 \pm 80) \times 10^{-11}\,(3.6\sigma), \\ \Delta a_\mu &= a_\mu^{exp} - a_\mu^{SM} = (261 \pm 78) \times 10^{-11}\,(3.3\sigma), \end{aligned} \tag{4}$$

which stands for the 3.6σ and 3.3σ deviations from the SM predictions, respectively. The significance of the excess can be dwindled with the use of τ data in the hadronic contributions to 2.4σ however [99]. Conversely, using recent results on the lowest-order hadronic evaluation, the difference between the SM prediction and the experimental value becomes larger yielding a significance of over 4σ [116, 117]. Unfortunately, the hadronic corrections, which are the main source of error in the SM prediction, result in uncertainties that mask the impact of the deviation.

Anyhow, the muon anomalous magnetic moment has triggered various interpretations in terms of new physics effects. Fortunately, we are currently at a very special moment because both experiment, FNAL and J-PARC are expected to reach unprecedented sensitivity and report results in the upcoming years. If the current anomaly is confirmed, the beginning of a new era might be ahead of us. Therefore, it is timely sensitive to discuss models which could possibly accommodate the deviation and their implications in a broad sense. (see [2, 71, 118–130] for reviews on $g-2$ with different focus). However, in the attempt to address the muon anomalous magnetic moment in the context of new physics, there are often stones encountered on the way, namely constraints stemming from LFV probes. Therefore, in the next section, we put the muon anomalous magnetic moment into perspective with LFV.

5. Lepton Flavor Violation

Perhaps more importantly than reviewing the muon anomalous magnetic moment by itself is to review its connection to the quest for LFV, for concreteness the decay $\mu \to e\gamma$ [166–177]. In the SM, lepton flavor is a conserved quantity since neutrinos are massless. However, we know that neutrinos do have masses and that they experience flavor oscillations [3, 4]. Though we have experimental confirmation of LFV from neutrino oscillations, we have not yet observed such violation from processes involving charged leptons. [4] Nevertheless, if LFV occurs among neutrino flavor, it is arguably natural to expect that

[4]Notice that LFV does not imply that neutrinos oscillate, since there are models where LFV occurs but neutrinos remain massless [178].

Complete overview of constraints on the $\tau \to X\gamma$ decay, $X = e, \mu$.

Determination	Bound
MARK-II (1982) [131, 132]	$\mathrm{BR}(\tau \to \mu\gamma) < 5.5 \times 10^{-4}$
MARK-II (1982) [131, 132]	$\mathrm{BR}(\tau \to \mathrm{e}\gamma) < 6.4 \times 10^{-4}$
CRYSTAL BALL (1988) [133]	$\mathrm{BR}(\tau \to \mathrm{e}\gamma) < 2 \times 10^{-4}$
ARGUS (1992) [134]	$\mathrm{BR}(\tau \to \mu\gamma) < 3.4 \times 10^{-5}$
ARGUS (1992) [134]	$\mathrm{BR}(\tau \to \mathrm{e}\gamma) < 1.2 \times 10^{-4}$
CLEO-II (1993)[135]	$\mathrm{BR}(\tau \to \mu\gamma) < 4.2 \times 10^{-6}$
DELPHI (1995)[136]	$\mathrm{BR}(\tau \to \mu\gamma) < 6.2 \times 10^{-5}$
DELPHI (1995)[136]	$\mathrm{BR}(\tau \to \mathrm{e}\gamma) < 1.1 \times 10^{-4}$
CLEO-II (1997)[137]	$\mathrm{BR}(\tau \to \mu\gamma) < 3 \times 10^{-6}$
CLEO-II (1997)[137]	$\mathrm{BR}(\tau \to \mathrm{e}\gamma) < 2.7 \times 10^{-6}$
CLEO-II (2000)[138]	$\mathrm{BR}(\tau \to \mathrm{e}\gamma) < 1.1 \times 10^{-6}$
BaBar (2002) [139]	$\mathrm{BR}(\tau \to \mu\gamma) < 2 \times 10^{-6}$
Belle (2002) [140]	$\mathrm{BR}(\tau \to \mu\gamma) < 6 \times 10^{-7}$
Belle (2004) [141]	$\mathrm{BR}(\tau \to \mu\gamma) < 3.1 \times 10^{-7}$
Belle (2005) [142]	$\mathrm{BR}(\tau \to \mathrm{e}\gamma) < 3.9 \times 10^{-7}$
BaBar (2005) [143]	$\mathrm{BR}(\tau \to \mu\gamma) < 6.8 \times 10^{-8}$
BaBar (2006) [144]	$\mathrm{BR}(\tau \to \mathrm{e}\gamma) < 1.1 \times 10^{-7}$
Belle (2008) [145]	$\mathrm{BR}(\tau \to \mu\gamma) < 4.5 \times 10^{-8}$
Belle (2008) [145]	$\mathrm{BR}(\tau \to \mathrm{e}\gamma) < 1.2 \times 10^{-7}$
BaBar (2010) [146]	$\mathrm{BR}(\tau \to \mu\gamma) < 4.4 \times 10^{-8}$
BaBar (2010) [146]	$\mathrm{BR}(\tau \to \mathrm{e}\gamma) < 3.3 \times 10^{-8}$
Current Bound [146]	$\mathrm{BR}(\tau \to \mu\gamma) < 4.4 \times 10^{-8}$
	$\mathrm{BR}(\tau \to \mathrm{e}\gamma) < 3.3 \times 10^{-8}$
Projected Sensitivity [147]	$\mathrm{BR}(\tau \to \mu\gamma) < 4.4 \times 10^{-9}$
	$\mathrm{BR}(\tau \to \mathrm{e}\gamma) < 3.3 \times 10^{-9}$

Table 2: Comprehensive overview of constraints on the decay $\tau \to e\gamma$. The projected sensitivity comes from superKEKB/Belle II which is expected to improve existing limits by a factor of 10 [147].

Historic overview of constraints on $\mu \to e\gamma$.

Determination	Beam	BR($\mu \to$ eγ)
SM [148]		$< 10^{-55}$
AFFLM (1959) [149]	μ^+	1.2×10^{-6}
PAR (1964) [150]	μ^+	2.2×10^{-8}
NaI (1977) [151]	μ^+	3.6×10^{-9}
SIN (1977) [152]	μ^+	1.1×10^{-9}
Clinton Anderson (1979) [153]	μ^+	1.9×10^{-10}
SIN (1980) [154]	μ^+	1×10^{-9}
NaI (1982) [155]	μ^+	1.7×10^{-10}
TRIUMF (1983) [156]	μ^+	1×10^{-9}
NaI-2 (1986) [157]	μ^+	4.9×10^{-11}
Crystal Box (1988) [158]	μ^+	4.9×10^{-11}
MEGA (1999) [159]	μ^+	1.2×10^{-11}
MEG (2008) [160]	μ^+	2.8×10^{-11}
MEG (2009) [161]	μ^+	1.5×10^{-11}
MEG (2011) [162]	μ^+	2.4×10^{-12}
MEG (2013) [163]	μ^+	5.7×10^{-13}
MEG (2016) [164]	μ^+	4.2×10^{-13}
Current Bound [164]		BR($\mu \to$ eγ) $< 4.2 \times 10^{-13}$
Projected Sensitivity [165]		BR($\mu \to$ eγ) $< 4 \times 10^{-14}$

Table 3: Comprehensive overview of constraints on the decay $\mu \to e\gamma$.

this violation also happens among charged leptons. From now on LFV will be regarded in the context of charged leptons.

In Fig. 1 we illustrate in a nutshell the effects that LFV and $g-2$ observables bring to new physics. While the non-observation of LFV typically pushes new physics to high energy scales, as indicated in Fig. 1, the $g-2$ anomaly favors lower energy scales. Thus, one may wonder whether the same new physics can still plausibly accommodate signals in both fronts.

Keep in mind though that, converse to the muon magnetic moment, an observation of LFV would be undoubtedly confirm the existence of new physics with tremendous implications. Many new physics models can accommodate LFV processes, especially $\mu \to e\gamma$. Several LFV processes have been experimentally searched for, such as $\tau \to e\gamma$, $\tau \to \mu\gamma$, $\mu \to eee$ and $\tau \to \mu\mu\mu$, and since no signal was observed stringent limits were derived [146] (See a sensitivity perspective in Fig. 4). In particular the τ decays, which are intimately connected to our reasoning, yield much weaker limits. For completeness, we provide in Tab. 2 a complete overview of all existing limits on this LFV decay mode. Eventually, we will discuss $SU(2)_L$ invariant simplified models, and when we do we will have the limits the BR($\tau \to$ Xγ) in mind but it is clear that will impose no further constraints on the model in the light of the loose constraints stemming from this decay. Therefore, our discussion regarding LFV will be focused on $\mu \to e\gamma$.

6. Observing the $\mu \to e\gamma$ conversion

Several searches have been performed for LFV, mostly based on the positively charged muon decay since it provides a unique opportunity to search for LFV compared with other channels. [5] This is due to the large number of muons available for experimental searches [169]. A positively charged muon decays at rest producing collinearly a $e^+\gamma$ pair with energies equal to half of the muon mass. The signal is clean, and since no excess events have been observed thus far, stringent limits were placed on the the branching ratio for the decay $\mu \to e\gamma$ (BR($\mu \to$ eγ)). See Tab. 3 for a comprehensive overview of all existing bounds. Nevertheless, we know that in the SM with massless neutrinos this flavor violating decay is non-existent. Moreover, this decay is still extremely

[5] A negative muon cannot be used because it is captured by a nucleus when it is stopped in material [169].

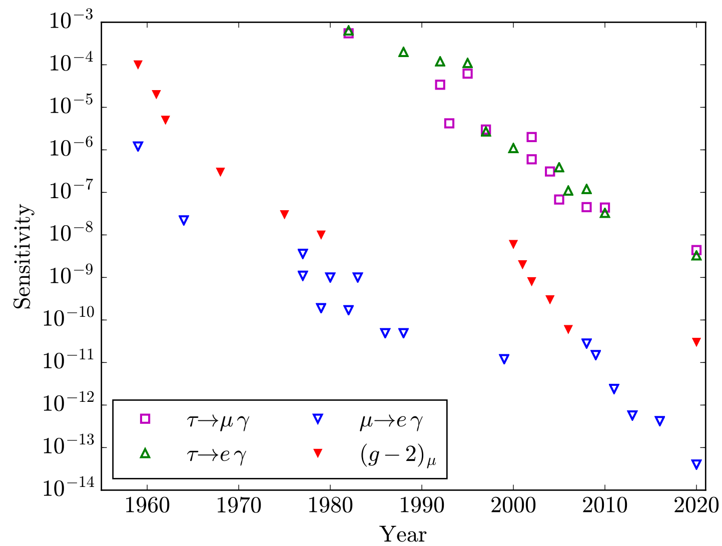


Figure 4: Historic development of the experimental sensitivities.

suppressed with the neutrino masses and mixing angles in agreement with atmospheric and solar-neutrino data [179]. Even in the popular canonical type I see-saw mechanism with heavy right-handed (RH) neutrinos, the branching ratio of the decay $\mu \to e\gamma$ is many orders of magnitude below current sensitivity limits [180]. Hence, the observation of LFV in the foreseeable future would be a paramount event since it conclusively confirms the presence of new physics, beyond the type I see-saw mechanism or any other SM extension describing massive neutrinos.

That said, there is a multitude of models perfectly capable of generating a sizable BR($\mu \to$ eγ), within current and projected sensitivity (see Table 3). For this reason we will glance at several simplified models that can be embedded in a plethora of particle physics models. Furthermore, we discuss several ultra violet (UV) complete models exploiting the complementarity between $g-2$ and $\mu \to e\gamma$. Previous works have been been done in this direction [71, 128–130, 181–196], but almost all of them have focused on a specific SM extension.

In particular, in [129] the authors discuss some of the models studied in our work, namely vector gauge boson and scalar, in a similar vein to [130] that encompasses many more possibilities. The former investigated how to reconcile $g-2$ and $\mu \to e\gamma$ in the context of scalar leptoquarks, whereas the latter was focused on providing a public code for computing $g-2$ only. Another interesting work that overlaps with ours was performed in [128] again with a strong focus on $g-2$ and collider constraints and again not as broad as this review.

We will try to keep our reasoning as general as possible and for this reason we will remain agnostic to the existence of electroweak and collider limits. We leave this work for the reader, after applying our findings to their favorite model. One clear example that justifies our means, is the case of a neutral vector mediator. Naively, one might think that it cannot accommodate $g-2$ due to collider bounds. Although, one may dress the Z' gauge boson with some sort of $L_\mu - L_\tau$ gauge symmetry to circumvent LEP and LHC limits. Thus, to avoid going much into the model dependence, we indeed set all existing bounds aside. However, at some point we will discuss both phenomena in UV complete models, and in this case it makes sense to consider existing limits. In summary one is able to systematically find in this work,

(i) New physics contributions to $g-2$ and $\mu \to e\gamma$ stemming from one and multiple fields via fully general master integrals, as well as compact expressions to facilitate their use.

(ii) Correlations between $g-2$ and $\mu \to e\gamma$ in several models keeping $SU(2)_L$ invariance as a guiding principle.

(iii) How possible signals seen both in $g-2$ and $\mu \to e\gamma$ observables can be made compatible.

(iv) The implications in case a signal in $g-2$ is observed in the presence of current and projected null results from $\mu \to e\gamma$ measurements, and vice-versa.

(v) We discuss the correlation between $g-2$ and $\mu \to e\gamma$ in several UV complete models, namely the MSSM, $U(1)_{B-L}$, Left-Right symmetry, two Higgs doublet models, the Zee-Babu model, and also the scotogenic, 3-3-1 and $L_\mu - L_\tau$ models.

We shall now describe the foundation of the complementarity between the $g-2$ and $\mu \to e\gamma$ observables.

7. General Framework

It is instructive to first take an effective field theory (EFT) viewpoint since it highlights the intimate relation of LFV decays and the anomalous magnetic moments of the leptons.

Assuming only conservation of charge and Lorentz invariance, the relevant effective operators are

$$\mathcal{L}_{\text{eff}} = \frac{\mu_{ij}^M}{2} \overline{\ell_i} \sigma^{\mu\nu} \ell_j F_{\mu\nu} + \frac{\mu_{ij}^E}{2} \overline{\ell_i} i\gamma^5 \sigma^{\mu\nu} \ell_j F_{\mu\nu}, \tag{5}$$

where the diagonal elements in the transition magnetic moment μ^M generate the anomalous magnetic dipole moments $\Delta a = \frac{1}{2}(g-2)$ of the leptons. Similarly, the flavor-diagonal part of μ^E gives contributions to the electric dipole moments, which we disregard in the present work. The off-diagonal elements, on the other hand, contribute to LFV decays such as $\mu \to e\gamma$. If $m_i \gg m_j$, it is convenient to define the dipole form factors A_M and A_E such that, neglecting contributions proportional to m_j, one may write $\mu_{ij}^{M/E} \equiv e m_i A^{M/E}/2$. With this definition, one obtains the following expressions for the anomalous magnetic moment and the branching ratio of the LFV decay:

$$\Delta a_{\ell_i} = A_{ii}^M m_i^2 \text{ (no sum)}, \tag{6a}$$

$$\text{BR}\left(\ell_i \to \ell_j \gamma\right) = \frac{3(4\pi)^3 \alpha_{\text{em}}}{4G_F^2} \left(\left|A_{ji}^M\right|^2 + \left|A_{ji}^E\right|^2 \right) \underbrace{\text{BR}\left(\ell_i \to \ell_j \nu_i \overline{\nu_j}\right)}_{\approx 1 \text{ for } \mu \to e\gamma}. \tag{6b}$$

Here, G_F is Fermi's constant of weak interactions and α_{em} is the electromagnetic fine-structure constant.

Naturally, any flavor non-diagonal coupling will activate LFV decays and at the same time yield contributions to the anomalous magnetic moments of the leptons. This interesting observation is the foundation of our present work in which we give fully general and yet compact expressions for the evaluation of LFV decays as well as the contributions to $g-2$. This should enable the readership to easily apply our results to their favorite model. We point out that this approach is complementary to the effective field theory approach of Ref. [177]. Furthermore, using the simplified model description guarding $SU(2)_L$ invariance, we believe that the results become more intuitive, motivating our work from a practical point of view.

The reader should be aware that there is a multitude of collider and electroweak precision limits that can be applied to many of the models we describe here. Since they are rather model dependent and our aim is to describe the correlation between the $g-2$ and $\mu \to e\gamma$ in a simplified framework, we have decided to leave those out of our discussion. Rather, we emphasize that, depending on the context in which the simplified models are embedded, the region of parameter space where one can accommodate $g-2$ and/or $\mu \to e\gamma$ might be actually ruled out. See [128] for a recent discussion in reference to LEP bounds.

8. New Physics Contributions

In this section we derive the new physics contributions to $g-2$ and $\mu \to e\gamma$. We focus for now on individual and multiple field corrections without imposing $SU(2)_L$ invariance to keep the results general and model independent, such that one can apply the results to a broader context. Only in Sec. 9 we will put them into perspective preserving $SU(2)_L$ invariance in the context of simplified SM extensions.

For now, we consider the most generic couplings that are allowed by the conservation of electric charge and Lorentz invariance. We start by addressing scalar mediators.

8.1. Scalar mediators

8.1.1. Neutral scalar

If additional electrically neutral scalar fields are present in a model, they will induce a shift in the leptonic magnetic moments and mediate LFV decays via the following interactions:

$$\mathcal{L}_{\rm int} = g_{s\,1}^{ij}\phi\,\overline{\ell_i}\,\ell_j + i g_{p\,1}^{ij}\phi\,\overline{\ell_i}\gamma^5\ell_j, \tag{7}$$

where both terms are manifestly hermitian. Notice that this sort of Lagrangian arises in many models. For instance, in $U(1)_X$ extensions of the SM, a scalar is usually needed to break the symmetry spontaneously. If SM particles are charged under $U(1)_X$, interactions between the SM fermions and the new scalar arise, including the one above. Moreover, in models where there is an inert scalar or the new neutral scalar mixes with the SM Higgs, this interaction Lagrangian also appears. In the latter case, however, the interaction strength is proportional to the (small) ratio m_f/v, where v is the vacuum expectation value (VEV) of the Higgs and m_f the fermion mass.

Decomposing the amplitude depicted in Fig. 5 as

$$-i\mathcal{M} = \overline{u_j}(p_2)\,(-ie\Gamma^\mu)\,u_i(p_1)\varepsilon_\mu(k), \tag{8}$$

we obtain for the dipole part of the vertex function[6]

$$\Gamma_1^\mu = \frac{i\sigma^{\mu\nu}k_\nu}{8\pi^2}\frac{m_i}{2}\sum_f \Big[\underbrace{g_{s\,1}^{fj}g_{s\,1}^{fi}I_{f,\,1}^{+\,+} + g_{p\,1}^{fj}g_{p\,1}^{fi}I_{f,\,1}^{+\,-}}_{\equiv(4\pi)^2A_{ji}^M} + i\gamma^5\underbrace{\left(g_{p\,1}^{fj}g_{s\,1}^{fi}I_{f,\,1}^{-\,+} - g_{s\,1}^{fj}g_{p\,1}^{fi}I_{f,\,1}^{-\,-}\right)}_{\equiv(4\pi)^2A_{ji}^E}\Big]. \tag{9}$$

Note that we have identified the form factors A_{ji}^M and A_{ji}^E in Eq. (9), where

$$\begin{aligned} A_{ji}^M &= \frac{1}{(4\pi)^2}\left(g_{s\,1}^{fj}g_{s\,1}^{fi}I_{f,\,1}^{+\,+} + g_{p\,1}^{fj}g_{p\,1}^{fi}I_{f,\,1}^{+\,-}\right)\\ A_{ji}^E &= \frac{1}{(4\pi)^2}\left(g_{p\,1}^{fj}g_{s\,1}^{fi}I_{f,\,1}^{-\,+} - g_{s\,1}^{fj}g_{p\,1}^{fi}I_{f,\,1}^{-\,-}\right). \end{aligned} \tag{10}$$

[6]This corresponds to the finite part of the amplitude proportional to $\sigma^{\mu\nu}k_\nu$. Note that all other contributions, especially those where the photon line is attached to the external leptons, yields non-dipole contributions ($\propto \gamma^\mu$). These are irrelevant for both LVF decays and $g-2$.

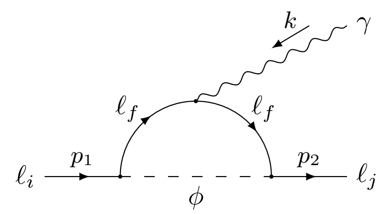


Figure 5: Diagrammatic representation of amplitude for the process $\ell_i \to \ell_j\gamma$ in the presence of a new neutral scalar ϕ.

The exact loop integral $I^{\pm\pm}_{f,1}$ is given in the appendix and for the special case of $\mu \to e\gamma$ it may be approximated as

$$\begin{aligned} I^{(\pm)_1\,(\pm)_2}_{f,1} &\simeq \int_0^1 \mathrm{d}x \int_0^{1-x} \mathrm{d}z \frac{xz + (\pm)_2(1-x)\frac{m_f}{m_\mu}}{-xzm_i^2 + xm_\phi^2 + (1-x)m_f^2} \\ &= \frac{1}{m_\phi^2}\int_0^1 \mathrm{d}x \int_0^1 \mathrm{d}y\, x^2 \frac{(1-x)y + (\pm)_2\epsilon_f}{(1-x)(1-xy\lambda^2) + x\epsilon_f^2\lambda^2} \\ &= \frac{1}{m_\phi^2}\left[\frac{1}{6} + (\mp)_2\epsilon_f\left(\frac{3}{2} + \log(\epsilon_f^2\lambda^2)\right)\right], \text{ for } m_\phi \to \infty. \end{aligned} \tag{11}$$

Here, we work to leading order in $m_j/m_i \ll 1$ and simplify the integration by defining $z \equiv (1-x)y$. Finally, we have used that the resulting integral is invariant under $x \to (1-x)$, and defined $\epsilon_f \equiv \frac{m_f}{m_\mu}$ and $\lambda \equiv \frac{m_\mu}{m_\phi}$.

With the form factors A^M_{ji} and A^E_{ji}, we can use Eq. (6) to obtain the corrections to $g-2$ $(i=j)$ and and $\mu \to e\gamma$ $(i \neq j)$.

$$\frac{1}{2}(g-2) \equiv \Delta a_\mu(\phi) = \frac{1}{8\pi^2}\frac{m_\mu^2}{m_\phi^2}\int_0^1 \mathrm{d}x \sum_f \frac{\left(g^{f\mu}_{s\,1}\right)^2 P_1^+(x) + \left(g^{f\mu}_{p\,1}\right)^2 P_1^-(x)}{(1-x)(1-x\lambda^2) + x\,\epsilon_f^2\lambda^2} \tag{12a}$$

where

$$P_1^\pm(x) = x^2\left(1 - x \pm \epsilon_f\right). \tag{12b}$$

Note that Eq. (12b) can be obtained from the second line of Eq. (11) by setting $y=1$ in the integrand and omitting the y-integration. For the case of a heavy mediator, $m_\phi \gg m_\mu, m_f$, the expression for Δa_μ can be approximated as

$$\begin{aligned}\Delta a_\mu(\phi) \simeq \frac{1}{4\pi^2}\frac{m_\mu^2}{m_\phi^2}\sum_f \Bigg[\left(g_{s\,1}^{f\mu}\right)^2\left(\frac{1}{6}-\epsilon_f\left(\frac{3}{4}+\log(\epsilon_f\lambda)\right)\right) + \\ + \left(g_{p\,1}^{f\mu}\right)^2\left(\frac{1}{6}+\epsilon_f\left(\frac{3}{4}+\log(\epsilon_f\lambda)\right)\right)\Bigg].\end{aligned} \tag{13}$$

Our results agree with those found in Refs. [128, 130]. For the LFV decay we find

$$\mathrm{BR}(\mu\to e\gamma) \approx \frac{3(4\pi)^3\alpha_{\rm em}}{4G_F^2}\left(|A_{e\mu}^M|^2+|A_{e\mu}^E|^2\right), \tag{14}$$

with $A_{e\mu}^M$ and $A_{e\mu}^E$ given in Eq. (10). In the limit where $m_\phi \gg m_\mu, m_f$, we can approximate this expression using the last line in Eq. (11):

$$\begin{aligned}A_{e\mu}^M = \frac{1}{16\pi^2 m_\phi^2}\sum_f \Bigg\{g_{s\,1}^{fe}g_{s\,1}^{f\mu}\left[\frac{1}{6}-\epsilon_f\left(\frac{3}{2}+\log(\epsilon_f^2\lambda^2)\right)\right] + \\ + g_{p\,1}^{fe}g_{p\,1}^{f\mu}\left[\frac{1}{6}+\epsilon_f\left(\frac{3}{2}+\log(\epsilon_f^2\lambda^2)\right)\right]\Bigg\},\end{aligned} \tag{15a}$$

$$\begin{aligned}A_{e\mu}^E = \frac{1}{16\pi^2 m_\phi^2}\sum_f \Bigg\{g_{p\,1}^{fe}g_{s\,1}^{f\mu}\left[\frac{1}{6}-\epsilon_f\left(\frac{3}{2}+\log(\epsilon_f^2\lambda^2)\right)\right] - \\ - g_{s\,1}^{fe}g_{p\,1}^{f\mu}\left[\frac{1}{6}+\epsilon_f\left(\frac{3}{2}+\log(\epsilon_f^2\lambda^2)\right)\right]\Bigg\}.\end{aligned} \tag{15b}$$

8.1.2. Singly charged scalar

Singly charged scalars are a clear signature of additional scalar doublets. They appear in the Zee-Babu model as well as in models with much richer scalar sectors such as multi-Higgs doublet models or scalar triplet models. For example, Left-Right models usually feature such triplet scalars which encompass singly charged scalars. The relevant interaction terms for the contribution of a scalar with unit charge to the amplitude $\mathcal{M}$ are given by

$$\mathcal{L}_{\rm int} = g_{s\,2}^{ij}\phi^+\,\overline{\nu_i}\,\ell_j + g_{p\,2}^{ij}\phi^+\,\overline{\nu_i}\gamma^5\ell_j + \text{ h.c.} \tag{16}$$

In what follows the ν_f^i do not have to be the SM neutrinos, but they can be any sort of exotic neutral leptons with arbitrary mass. Since we will provide a fully general result the reader is welcome to use it. Conversely, we will also give

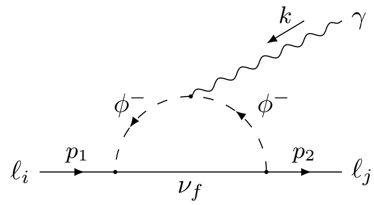


Figure 6: Process $\ell_i \to \ell_j\gamma$ mediated by a charged scalar.

simplified results taking the scalar to be much heavier than all other particles involved in the processes. It is worth emphasizing that if lepton number is explicitly violated in the given model, one might also find operators of the form $\phi^+\overline{\nu^c}\ell$. For the present analysis this will not change the calculation, hence we do not consider these operators explicitly.

From Fig. 6 the vertex function can be computed as

$$\begin{aligned}\Gamma_2^\mu = -\frac{i\sigma^{\mu\nu}k_\nu}{8\pi^2}\frac{m_i}{2}\sum_f \Big[& \underbrace{g_{s\,2}^{fj\,*} g_{p\,2}^{fi} I_{f,\,2}^{+\,+} + g_{p\,2}^{fj\,*} g_{p\,2}^{fi} I_{f,\,2}^{+\,-}}_{\equiv -(4\pi)^2 A_{ji}^M} - \\ & - \gamma^5 \underbrace{\left(g_{p\,2}^{fj\,*} g_{s\,2}^{fi} I_{f,\,2}^{-\,+} + g_{s\,2}^{fj\,*} g_{p\,2}^{fi} I_{f,\,2}^{-\,-}\right)}_{\equiv i(4\pi)^2 A_{ji}^E} \Big].\end{aligned} \tag{17}$$

From the full expression for the integral $I_{f,\,2}$ given in Eq. (A-2), we obtain for Δa_μ:

$$\Delta a_\mu\left(\phi^+\right) = -\frac{1}{8\pi^2}\frac{m_\mu^2}{m_{\phi^+}^2}\int_0^1 \mathrm{d}x \sum_f \frac{\left|g_{s\,2}^{f\mu}\right|^2 P_2^+(x) + \left|g_{p\,2}^{f\mu}\right|^2 P_2^-(x)}{\epsilon_f^2\lambda^2(1-x)\left(1-\epsilon_f^{-2}x\right)+x}, \tag{18a}$$

where

$$P_2^\pm(x) = x(1-x)\left(x \pm \epsilon_f\right), \quad \epsilon_f \equiv \frac{m_{\nu_f}}{m_\mu}, \quad \lambda \equiv \frac{m_\mu}{m_{\phi^+}}. \tag{18b}$$

In the limit of a heavy scalar mediator $\lambda \to 0$, this reduces to

$$\Delta a_\mu(\phi^+) \simeq \frac{-1}{4\pi^2}\frac{m_\mu^2}{m_{\phi^+}^2}\sum_f \left[\left|g_{s\,2}^{f\mu}\right|^2\left(\frac{1}{12}+\frac{\epsilon_f}{4}\right) + \left|g_{p\,2}^{f\mu}\right|^2\left(\frac{1}{12}-\frac{\epsilon_f}{4}\right)\right]. \tag{19}$$

Having identified A_{ji}^M and A_{ji}^E in Eq. (17), we can substitute them in Eq. (6) to find $\mathrm{BR}(\mu \to e\gamma) \approx \frac{3(4\pi)^3\alpha_{\rm em}}{4G_F^2}\left(|A^M_{e\mu}|^2 + |A^E_{e\mu}|^2\right)$, where

$$A^M_{e\mu} = \frac{-1}{(4\pi)^2}\sum_f \left(g_{s\,2}^{fe\,*} g_{s\,2}^{f\mu} I_{f,\,2}^{+\,+} + g_{p\,2}^{fe\,*} g_{p\,2}^{f\mu} I_{f,\,2}^{+\,-}\right) \tag{20a}$$

$$A^E_{e\mu} = \frac{-i}{(4\pi)^2}\sum_f \left(g_{p\,2}^{fe\,*} g_{s\,2}^{f\mu} I_{f,\,2}^{-\,+} + g_{s\,2}^{fe\,*} g_{p\,2}^{f\mu} I_{f,\,2}^{-\,-}\right) \tag{20b}$$

which in the case of a heavy scalar ($m_{\phi^+} \gg m_\mu, m_{\nu_f}$), simplify to

$$A^M_{e\mu} \simeq \frac{-1}{16\pi^2 m_{\phi^+}^2}\sum_f \left(g_{s\,2}^{fe\,*} g_{s\,2}^{f\mu}\left[\frac{1}{12} + \frac{\epsilon_f}{2}\right] + g_{p\,2}^{fe\,*} g_{p\,2}^{f\mu}\left[\frac{1}{12} - \frac{\epsilon_f}{2}\right]\right), \tag{21a}$$

$$A^E_{e\mu} \simeq \frac{-i}{16\pi^2 m_{\phi^+}^2}\sum_f \left(g_{p\,2}^{fe\,*} g_{s\,2}^{f\mu}\left[\frac{1}{12} + \frac{\epsilon_f}{2}\right] + g_{s\,2}^{fe\,*} g_{p\,2}^{f\mu}\left[\frac{1}{12} - \frac{\epsilon_f}{2}\right]\right), \tag{21b}$$

where we have used the approximate expression for $I_{f,\,2}$ found in the Appendix.

8.1.3. Doubly charged scalar

Doubly charged scalars are key features of the Type II see-saw model [197], and they are also predicted in models based on the $SU(3)_C \times SU(3)_L \times U(1)_N$ (3-3-1) gauge group. Such particles were vastly used to enhance the signal $H \to \gamma\gamma$, when a mild excess in the diphoton channel surfaced in the Higgs discovery [198–201]. Typically, such doubly charged scalars are accompanied by a singly charged one, however for now we will be restricted to the doubly charged scalar contribution only. In the case of a doubly charged scalar field, that might e.g. be a component of an $SU(2)_L$ triplet, the possible interactions take the form

$$\mathcal{L}_{\rm int} = g_{s\,3}^{ij}\phi^{++}\overline{\ell_i^{\mathcal{C}}}\ell_j + g_{p\,3}^{ij}\phi^{++}\overline{\ell_i^{\mathcal{C}}}\gamma^5\ell_j + \text{h.c.}, \tag{22}$$

which violate the SM lepton number symmetry explicitly. Note that $g_{s\,3}$ and $g_{p\,3}$ are forced to be symmetric matrices in flavor space.

Since all fields in the relevant interactions are electrically charged, there will be two contributions as shown in Fig. 7. The corresponding expressions can be obtained from the previous subsections with appropriate changes in the

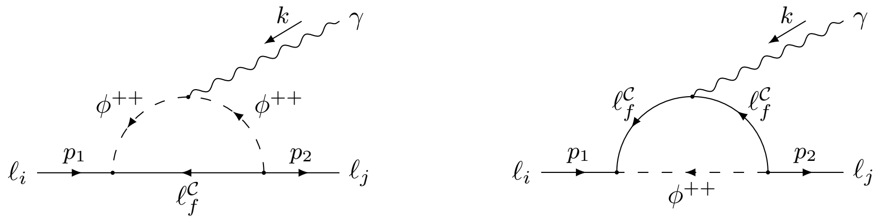


Figure 7: Process $\ell_i \to \ell_j \gamma$ mediated by a doubly charged scalar ϕ^{++}, where two diagrams contribute.

parameters. One obtains for the sum of the two diagrams

$$\begin{aligned}\Gamma_3^\mu = &- 8 \frac{i\sigma^{\mu\nu}k_\nu}{8\pi^2}\frac{m_i}{2}\sum_f \Big[g_{s\,3}^{fj\,*} g_{s\,3}^{fi} I_{f,\,2}^{+\,+} + g_{p\,3}^{fj\,*} g_{p\,3}^{fi} I_{f,\,2}^{+\,-} - \\ &\qquad - \gamma^5 \left(g_{p\,3}^{fj\,*} g_{s\,3}^{fi} I_{f,\,2}^{-\,+} + g_{s\,3}^{fj\,*} g_{p\,3}^{fi} I_{f,\,2}^{-\,-} \right) \Big] \\ &- 4 \frac{i\sigma^{\mu\nu}k_\nu}{8\pi^2}\frac{m_i}{2}\sum_f \Big[g_{s\,3}^{fj\,*} g_{s\,3}^{fi} I_{f,\,1}^{+\,+} + g_{p\,3}^{fj\,*} g_{p\,3}^{fi} I_{f,\,1}^{+\,-} - \\ &\qquad - \gamma^5 \left(g_{p\,3}^{fj\,*} g_{s\,3}^{fi} I_{f,\,1}^{-\,+} + g_{s\,3}^{fj\,*} g_{p\,3}^{fi} I_{f,\,1}^{-\,-} \right) \Big] ,\end{aligned} \tag{23}$$

where it is understood that the replacements $m_{\nu_f} \to m_f$, $m_{\phi,\phi^+} \to m_{\phi^{++}}$ are made in the loop functions $I_{f,\,1/2}^{\pm\,\pm}$. The multiple factors of 2 are due to the double unit charge of the scalar field and symmetry factors.

Again, we can extract the contributions to $g-2$, which read – in agreement with Ref. [130]:

$$\begin{aligned}\Delta a_\mu\left(\phi^{++}\right) = &- \frac{8}{8\pi^2}\frac{m_\mu^2}{m_{\phi^{++}}^2}\int_0^1 \mathrm{d}x \sum_f \frac{\left|g_{s\,3}^{f\mu}\right|^2 P_2^+(x) + \left|g_{p\,3}^{f\mu}\right|^2 P_2^-(x)}{\epsilon_f^2\lambda^2(1-x)(1-\epsilon_f^{-2}x)+x} - \\ &- \frac{4}{8\pi^2}\frac{m_\mu^2}{m_{\phi^{++}}^2}\int_0^1 \mathrm{d}x \sum_f \frac{\left|g_{s\,3}^{f\mu}\right|^2 P_1^+(x) + \left|g_{p\,3}^{f\mu}\right|^2 P_1^-(x)}{(1-x)(1-\lambda^2)+x\,\epsilon_f^2\lambda^2} .\end{aligned} \tag{24}$$

The functions $P_{1/2}^\pm$ are defined in Eqs. (12b) and (18b), however for $\epsilon_f \equiv \frac{m_f}{m_\mu}$ and $\lambda \equiv \frac{m_\mu}{m_{\phi^{++}}}$ in both functions. Note the relative sign between the second line in Eq. (24) and Eq. (12), which is due to the appearance of a *charge conjugate* lepton coupling to the photon.

For the case of a heavy mediator, one finds the simple expression [130]

$$\Delta a_\mu\left(\phi^{++}\right) = -\frac{1}{4\pi^2}\frac{m_\mu^2}{m_{\phi^{++}}^2}\sum_f\left[\left|g_{s\,3}^{f\mu}\right|^2\left(\frac{4}{3}-\epsilon_f\right)+\left|g_{p\,3}^{f\mu}\right|^2\left(\frac{4}{3}+\epsilon_f\right)\right]. \quad (25)$$

We have seen that the correction to $g-2$ arising from a doubly charged scalar is basically a combination of the neutral and charged scalar contributions with some minor changes. Notice that considering only flavor diagonal couplings, i.e. $\epsilon_f = 1$ and omitting the sum, the doubly charged scalar contribution to $g-2$ is negative. However, if the τ-lepton contributes significantly then the overall contribution might be positive, i.e. when $(g_s^{\tau\mu})^2 m_\tau/m_\mu > 4/3$, but this only true if $g_s^{\tau\mu} \neq g_p^{\tau\mu}$, otherwise the ϵ_f terms in Eq. (25) cancel.

As for $\mu \to e\gamma$ a similar treatment can be applied to find,

$$\mathrm{BR}(\mu\to e\gamma) \simeq \frac{3(4\pi)^3\alpha_{\rm em}}{4G_F^2}\left(|A_{e\mu}^M|^2+|A_{e\mu}^E|^2\right), \quad (26)$$

where

$$A_{e\mu}^M = \frac{-1}{(4\pi)^2}\sum_f\left(g_{s\,3}^{fe\,*}g_{s\,3}^{f\mu}I_{f,\,21}^{+\,+}+g_{p\,3}^{fe\,*}g_{p\,3}^{f\mu}I_{f,\,21}^{+\,-}\right) \quad (27a)$$

$$A_{e\mu}^E = \frac{-i}{(4\pi)^2}\sum_f\left(g_{p\,3}^{fe\,*}g_{s\,3}^{f\mu}I_{f,\,21}^{-\,+}+g_{s\,3}^{fe\,*}g_{p\,3}^{f\mu}I_{f,\,21}^{-\,-}\right) \quad (27b)$$

in agreement with [202], where with $I_{f,\,21} = 4(2I_{f,\,2}+I_{f,\,1})$.

In the limit $m_{\phi^{++}} \gg m_f$ we get,

$$\begin{aligned}
A_{e\mu}^M &\simeq \frac{-1}{8\pi^2 m_{\phi^{++}}^2}\sum_f\left(g_{s\,3}^{fe\,*}g_{s\,3}^{f\mu}\left[\frac{2}{3}-\epsilon_f\right]+g_{p\,3}^{fe\,*}g_{p\,3}^{f\mu}\left[\frac{2}{3}+\epsilon_f\right]\right),\\
A_{e\mu}^E &\simeq \frac{-i}{8\pi^2 m_{\phi^{++}}^2}\sum_f\left(g_{p\,3}^{fe\,*}g_{s\,3}^{f\mu}\left[\frac{2}{3}-\epsilon_f\right]+g_{s\,3}^{fe\,*}g_{p\,3}^{f\mu}\left[\frac{2}{3}+\epsilon_f\right]\right)
\end{aligned} \quad (28)$$

For the case where we have $y/\sqrt{2} = g_{s\,3} = \pm g_{p\,3}$, this expression reduces to,

$$\mathrm{BR}(\mu\to e\gamma) \simeq \frac{\alpha_{\rm em}\left|(y^\dagger y)_{e\mu}\right|^2}{3\pi G_F^2 m_{\phi^{++}}^4}. \quad (29)$$

We emphasize that Eq. (29) accounts only for the doubly charged scalar contribution to BR($\mu \to$ eγ). However, as mentioned above, doubly charged scalars usually arise in the context of Higgs triplets, so that a singly charged scalars will also contribute. Combining our results for the singly and doubly charged scalars we may obtain,

$$\mathrm{BR}(\mu \to e\gamma) \simeq \frac{27\alpha_{\rm em}\left|(y^\dagger y)_{e\mu}\right|^2}{64\pi G_F^2 m_{\phi^{++}}^4}, \tag{30}$$

where we assumed $m_{\phi^+} = m_{\phi^{++}}$. This result matches the well-known result for the Higgs triplet contribution derived in [203] and quoted in [204–206]. Thus, if one needs a more general assessment of the doubly charged contribution to $g-2$ and $\mu \to e\gamma$ Eq. (25) and Eq. (29) should be used respectively.

8.2. Gauge boson mediator

In this section we will discuss fully general interactions of neutral and charged gauge bosons which arise in many models that augment the SM with a new abelian symmetry or extended electroweak gauge sectors such as $SU(2)_R$, $SU(3)_L$ etc. Whether we consider new gauge bosons or fermion fields, in many cases the amplitude will involve a gauge boson propagator. In this subsection we list the relevant expressions for $\mathcal{M}$. We start with the charged gauge boson.

8.2.1. Neutral fermion – charged gauge boson

The introduction of several neutral fermions N_i opens the following vector and axial-vector interaction channels:

$$\mathcal{L}_{\rm int} = g_{v\,1}^{ij} W'^{+}_{\mu} \overline{N}_i \gamma^\mu \ell_j + g_{a\,1}^{ij} W'^{+}_{\mu} \overline{N}_i \gamma^\mu \gamma^5 \ell_j + \text{h.c.} \tag{31}$$

This expression may root in several high-energy models. For example, we could have that the W' is actually the SM $SU(2)_L$ gauge boson. In that case we would find that the coupling strength is proportional to some mixing of the N_i with the active neutrinos of the SM. Conversely, we could have that the W' is due to some extended gauge sector, e.g. $SU(2)_R$ of the Left-Right symmetric model. Since we are not worried about $SU(2)_L$ invariance at this point, we need to make further assumptions to carry out the relevant calculations. We have done so by performing all calculations in unitary gauge and taking the respective propagators for the internal W's.

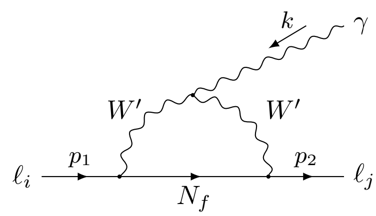


Figure 8: Process $\ell_i \to \ell_j\gamma$ mediated by a neutral fermion N and a W' boson.

For the diagram in Fig. 8, we obtain with the aid of the loop function $I_{f,3}^{\pm\,\pm}$ defined in Appendix in Eq. (A-5)

$$\begin{aligned}\Gamma_4^\mu = \frac{i\sigma^{\mu\nu}k_\nu}{8\pi^2}\frac{m_i}{2}\sum_f \Big[& g_{v\,1}^{fj\,*} g_{v\,1}^{fi} I_{f,3}^{+\,+} + g_{a\,1}^{fj\,*} g_{a\,1}^{fi} I_{f,3}^{+\,-} + \\ & + \gamma^5 \left(g_{a\,1}^{fj\,*} g_{v\,1}^{fi} I_{f,3}^{-\,+} + g_{v\,1}^{fj\,*} g_{a\,1}^{fi} I_{f,3}^{-\,-} \right) \Big] .\end{aligned} \tag{32}$$

From that we extract the relevant expressions for the $g-2$ give contributions of the form

$$\Delta a_\mu\left(N, W'\right) = \frac{-1}{8\pi^2}\frac{m_\mu^2}{m_{W'}^2}\int_0^1 \mathrm{d}x \sum_f \frac{\left|g_{v\,1}^{f\mu}\right|^2 P_3^+(x) + \left|g_{a\,1}^{f\mu}\right|^2 P_3^-(x)}{\epsilon_f^2\lambda^2(1-x)\left(1-\epsilon_f^{-2}x\right)+x}, \tag{33a}$$

with

$$P_3^\pm = -2x^2(1+x\mp 2\epsilon_f) + \lambda^2 x(1-x)(1\mp\epsilon_f)^2\left(x\pm\epsilon_f\right) \tag{33b}$$

and $\epsilon_f \equiv \frac{m_{N_f}}{m_\mu}$, $\lambda \equiv \frac{m_\mu}{m_{W'}}$, in agreement with Ref. [207].

It is interesting to see that the rather lengthy expression for $I_{f,3}$ reduces to a much simpler one if the internal boson is assumed to be decoupled, i.e. $m_{W'} \gg m_N, m_f$:

$$I_{f,3}^{(\pm)_1,(\pm)_2} \simeq \frac{1}{m_{W'}^2}\left[\frac{5}{6}\left(1+(\pm)_1 m_j/m_i\right) - 2(\pm)_2 m_{N_f}/m_i\right]. \tag{34}$$

Thus, if the intermediate W' is heavy, i.e. $\lambda \to 0$, we find the useful approximation

$$\Delta a_\mu(N, W') \simeq \frac{1}{4\pi^2}\frac{m_\mu^2}{m_{W'}^2}\sum_f \left[\left|g_{v\,1}^{f\mu}\right|^2\left(\frac{5}{6}-\epsilon_f\right) + \left|g_{a\,1}^{f\mu}\right|^2\left(\frac{5}{6}+\epsilon_f\right)\right]. \tag{35}$$

Similarly, taking both $m_N \simeq m_{W'} \gg m_f$ such that $\epsilon_f \lambda \simeq 1$, we find that the contribution is independent of ϵ_f if we allow the vector and axial-vector contributions to cancel, i.e. $|g_v| = |g_a| = |g|$. We obtain

$$\Delta a_\mu(N, W') \simeq \frac{17}{48\pi^2} \frac{m_\mu^2}{m_{W'}^2} \sum_f \left| g^{f\mu} \right|^2 . \tag{36}$$

As we will see below, this matches a well-known result in the Left-Right model for $\mu \to e\gamma$ where $g = g_R$, with g_R being the gauge coupling from the $SU(2)_R$ group. We emphasize that it is applicable only when $m_N \simeq m_{W'}$ both being much heavier than any charged lepton involved.

As for $\mu \to e\gamma$, we find,

$$A^M_{e\mu} = \frac{-1}{(4\pi)^2} \sum_f \left(g^{fe\,*}_{v\,1} g^{f\mu}_{v\,1} I^{+\,+}_{f,\,3} + g^{fe\,*}_{a\,1} g^{f\mu}_{a\,1} I^{+\,-}_{f,\,3} \right), \tag{37a}$$

$$A^E_{e\mu} = \frac{i}{(4\pi)^2} \sum_f \left(g^{fe\,*}_{a\,1} g^{f\mu}_{v\,1} I^{-\,+}_{f,\,3} + g^{fe\,*}_{v\,1} g^{f\mu}_{a\,1} I^{-\,-}_{f,\,3} \right), \tag{37b}$$

with $I^{\pm\,\pm}_{f,\,3}$ given in Eq. (A-5) in the Appendix.

Again taking the limit $m_{W'} \gg m_N, m_f$, we get

$$A^M_{e\mu} \simeq \frac{-1}{16\pi^2 m_{W'}^2} \sum_f \left(g^{fe\,*}_{v\,1} g^{f\mu}_{v\,1} \left[\frac{5}{6} - 2\epsilon_f \right] + g^{fe\,*}_{a\,1} g^{f\mu}_{a\,1} \left[\frac{5}{6} + 2\epsilon_f \right] \right), \tag{38a}$$

$$A^E_{e\mu} \simeq \frac{i}{16\pi^2 m_{W'}^2} \sum_f \left(g^{fe\,*}_{a\,1} g^{f\mu}_{v\,1} \left[\frac{5}{6} - 2\epsilon_f \right] + g^{fe\,*}_{v\,1} g^{f\mu}_{a\,1} \left[\frac{5}{6} + 2\epsilon_f \right] \right), \tag{38b}$$

However, for the regime in which $m_{N_f} = m_{W'} \gg m_f$, this results in ($|g_v| = |g_a| = |g|$)

$$A^M_{e\mu} = i A^E_{e\mu} \simeq \frac{-1}{16\pi^2 m_{W'}^2} \frac{17}{12} \sum_f g^{fe^*} g^{f\mu}. \tag{39}$$

and thus

$$\mathrm{BR}(\mu \to e\gamma) \simeq 6.43 \times 10^{-6} \left(\frac{1\,\mathrm{TeV}}{m_{W'}} \right)^4 \sum_f |g^{fe^*} g^{f\mu}|^2. \tag{40}$$

8.2.2. Singly charged fermion – neutral gauge boson

Exotic singly charged fermions commonly arise in 3-3-1 models [208, 209], in two Higgs doublet models [210], and many other theories, cf. Refs. [211–213].

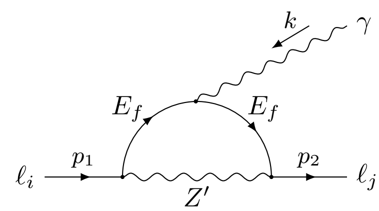


Figure 9: Process $\ell_i \to \ell_j \gamma$ mediated by a charged fermion E and a Z' boson.

Charged fermions could interact with the SM leptons via both a scalar and/or a neutral vector boson (Z'), i.e. via the Lagrangian

$$\mathcal{L}_{\text{int}} = g_{v\,2}^{ij} Z'_\mu \overline{E_i} \gamma^\mu \ell_j + g_{a\,2}^{ij} Z'_\mu \overline{E_i} \gamma^\mu \gamma^5 \ell_j + g_{s\,4}^{ij} h\, \overline{E_i}\, \ell_j + g_{p\,4}^{ij} h\, \overline{E_i} \gamma^5 \ell_j + \text{h.c.} \quad (41)$$

In the case where the new charged leptons E_i couple only to vector bosons, we set $g_{s/p\,5}^{ij} = 0$. As a result we obtain the following expression (cf. Fig. 9):

$$\begin{aligned}\Gamma^\mu_{5,v} = -\frac{i\sigma^{\mu\nu} k_\nu}{8\pi^2} \frac{m_i}{2} \sum_f \Big[& g_{v\,2}^{fj\,*} g_{v\,2}^{fi} I_{f,\,4}^{+\,+} + g_{a\,2}^{fj\,*} g_{a\,2}^{fi} I_{f,\,4}^{+\,-} + \\ & + \gamma^5 \left(g_{a\,2}^{fj\,*} g_{v\,2}^{fi} I_{f,\,4}^{-\,+} + g_{v\,2}^{fj\,*} g_{a\,2}^{fi} I_{f,\,4}^{-\,-} \right) \Big] ,\end{aligned} \quad (42)$$

with $I_{f,\,4}$ given exactly in Eq. (A-6).

We can obtain from the exact expression, for the case $m_i = m_j = m_\mu$, the result found in the literature [120, 207]:

$$\Delta a_\mu\,(E, Z') = \frac{1}{8\pi^2} \frac{m_\mu^2}{m_{Z'}^2} \int_0^1 \mathrm{d}x \sum_f \frac{\left| g_{v\,2}^{f\mu} \right|^2 P_4^+(x) + \left| g_{a\,2}^{f\mu} \right|^2 P_4^-(x)}{(1-x)\,(1-\lambda^2 x) + \epsilon_f^2 \lambda^2 x}, \quad (43a)$$

with

$$P_4^\pm = 2x(1-x)(x - 2 \pm 2\epsilon_f) + \lambda^2 x^2 (1 \mp \epsilon_f)^2 (1 - x \pm \epsilon_f) \quad (43b)$$

and $\epsilon_f \equiv \frac{m_{E_f}}{m_\mu}$, $\lambda \equiv \frac{m_\mu}{m_{Z'}}$.

Interestingly in the limit of a heavy Z', i.e. when $m_{Z'} \gg m_E, m_\mu$, $I_{f,\,4}$ simplifies to

$$I_{f,\,4}^{(\pm)_1,(\pm)_2} \simeq \frac{1}{m_{Z'}^2} \frac{2}{3} \left[1 + (\pm)_1 m_j/m_i - 3(\pm)_2 m_E/m_i \right], \quad (44)$$

which gives us an idea of the asymptotic behavior of $g-2$ and $\mu \to e\gamma$. Indeed, to leading order for a heavy gauge boson, one finds

$$\Delta a_\mu\left(E, Z'\right) \simeq \frac{-1}{4\pi^2}\frac{m_\mu^2}{m_{Z'}^2}\sum_f\left[\left|g_{v\,2}^{f\mu}\right|^2\left(\frac{2}{3}-\epsilon_f\right)+\left|g_{a\,2}^{f\mu}\right|^2\left(\frac{2}{3}+\epsilon_f\right)\right]. \tag{45}$$

As for $\mathrm{BR}(\mu \to e\gamma) \simeq \frac{3(4\pi)^3\alpha_{\mathrm{em}}}{4G_F^2}\left(|A_{e\mu}^M|^2+|A_{e\mu}^E|^2\right)$ one obtains:

$$A_{e\mu}^M = \frac{-1}{(4\pi)^2}\sum_f\left(g_{v\,2}^{f e\,*}g_{v\,2}^{f\mu}I_{f,\,4}^{+\,+}+g_{a\,2}^{f e\,*}g_{a\,2}^{f\mu}I_{f,\,4}^{+\,-}\right) \tag{46a}$$

$$\simeq \frac{-1}{24\pi^2 m_{Z'}^2}\sum_f\left(g_{v\,2}^{f e\,*}g_{v\,2}^{f\mu}\left(1-3\epsilon_f\right)+g_{a\,2}^{f e\,*}g_{a\,2}^{f\mu}\left(1+3\epsilon_f\right)\right), \tag{46b}$$

$$A_{e\mu}^E = \frac{i}{(4\pi)^2}\sum_f\left(g_{a\,2}^{f e\,*}g_{v\,2}^{f\mu}I_{f,\,4}^{-\,+}+g_{v\,2}^{f e\,*}g_{a\,2}^{f\mu}I_{f,\,4}^{-\,-}\right) \tag{46c}$$

$$\simeq \frac{i}{24\pi^2 m_{Z'}^2}\sum_f\left(g_{a\,2}^{f e\,*}g_{v\,2}^{f\mu}\left(1-3\epsilon_f\right)+g_{v\,2}^{f e\,*}g_{a\,2}^{f\mu}\left(1+3\epsilon_f\right)\right), \tag{46d}$$

where we have used the approximation $m_i \ll m_j \ll m_{Z'}$ and $m_E \ll m_{Z'}$.

The case where the charged fermions E_i couple to the SM leptons via a scalar only, the result is identical to the case of a neutral scalar. One can readily obtain:

$$\begin{aligned}\Gamma_{5,s}^\mu = \frac{i\sigma^{\mu\nu}k_\nu}{8\pi^2}\frac{m_i}{2}\sum_f\Big[&g_{s\,4}^{fj\,*}g_{s\,4}^{fi}I_{f,\,5s}^{+\,+}+g_{p\,4}^{fj\,*}g_{p\,4}^{fi}I_{f,\,5s}^{+\,-}- \\ &-\gamma^5\left(g_{p\,4}^{fj\,*}g_{s\,4}^{fi}I_{f,\,5s}^{-\,+}+g_{s\,4}^{fj\,*}g_{p\,4}^{fi}I_{f,\,5s}^{-\,-}\right)\Big].\end{aligned} \tag{47}$$

with $I_{f,\,5s}^{(\pm)_1,(\pm)_2} \equiv I_{f,\,1}(m_i,(\pm)_1 m_j,(\pm)_2 m_{E_f},m_h)$ as defined in Eq. (A-1). Similarly, the $g-2$ is easily generalized from Eq. (12):

$$\Delta a_\mu(E,\phi) = \frac{1}{8\pi^2}\frac{m_\mu^2}{m_\phi^2}\int_0^1 \mathrm{d}x\sum_f\frac{\left|g_{s\,4}^{f\mu}\right|^2 P_1^+(x)+\left|g_{p\,4}^{f\mu}\right|^2 P_1^-(x)}{(1-x)(1-\lambda^2)+x\,\epsilon_f^2\lambda^2}, \tag{48}$$

with $P_1^\pm$ defined above and $\epsilon_f \equiv \frac{m_{E_f}}{m_\mu}$, $\lambda \equiv \frac{m_\mu}{m_\phi}$. Similarly, the case of a heavy scalar mediator yields Eq. (13).

The corresponding $\mathrm{BR}(\mu \to e\gamma)$ can be found by combining the A_{ji}^M and A_{ji}^E functions defined in Eq. (6) and yield an expression identical to Eqs. (14) and (15).

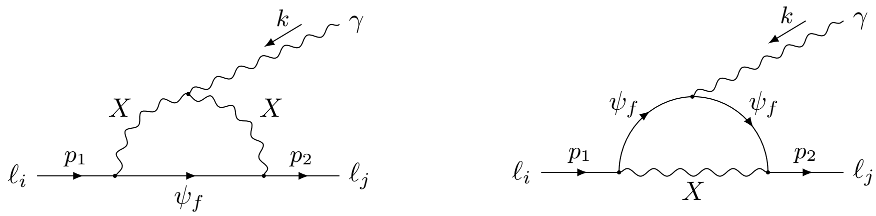


Figure 10: Process $\ell_i \to \ell_j \gamma$ mediated by a doubly charged fermion ψ and a X boson. Since both particles are charged, there are two separate contributions.

8.2.3. Doubly charged fermion – charged gauge boson

Doubly charged fermions, which appear as components of fermionic $SU(2)_L$ triplet fields [214], exotic doublets [215] as well in extensions of the SM with hypercharge $Y = 2$, as well as in composite Higgs models with extended isospin multiplets [216–219], will interact with SM leptons as follows:

$$\mathcal{L}_{\rm int} = g_{v\,3}^{ij} X_\mu^+ \overline{\ell_i} \gamma^\mu \psi_j + g_{a\,3}^{ij} X_\mu^+ \overline{\ell_i} \gamma^\mu \gamma^5 \psi_j + \text{h.c.}, \tag{49}$$

where the fermion ψ has electric charge $Q_\psi = -2e$. As for the case of a doubly charged scalar, the doubly charged fermion will contribute with two diagrams to the dipole form factors as shown in Fig. 10. Both contributions are related to the previous fermionic contributions as follows:

$$\begin{aligned}\Gamma_6^\mu = & -\frac{i\sigma^{\mu\nu}k_\nu}{8\pi^2}\frac{m_i}{2}\sum_f \Big[g_{v\,3}^{fj\,*} g_{v\,3}^{fi} I_{f,\,3}^{+\,+} + g_{a\,3}^{fj\,*} g_{a\,3}^{fi} I_{f,\,3}^{+\,-} + \\ & \qquad + \gamma^5 \left(g_{a\,3}^{fj\,*} g_{v\,3}^{fi} I_{f,\,3}^{-\,+} + g_{v\,3}^{fj\,*} g_{a\,3}^{fi} I_{f,\,3}^{-\,-} \right) \Big] \\ & - \frac{i\sigma^{\mu\nu}k_\nu}{8\pi^2} m_i \sum_f \Big[g_{v\,3}^{fj\,*} g_{v\,3}^{fi} I_{f,\,4}^{+\,+} + g_{a\,3}^{fj\,*} g_{a\,3}^{fi} I_{f,\,4}^{+\,-} + \\ & \qquad + \gamma^5 \left(g_{a\,3}^{fj\,*} g_{v\,3}^{fi} I_{f,\,4}^{-\,+} + g_{v\,3}^{fj\,*} g_{a\,3}^{fi} I_{f,\,4}^{-\,-} \right) \Big],\end{aligned} \tag{50}$$

with the masses m_E and m_N replaced with m_ψ in both loop functions $I_{f,\,3/4}^{\pm\,\pm}$ and the Z mass replaced with m_X in $I_{f,\,4}^{\pm\,\pm}$. This yields

$$\Delta a_\mu\,(\psi, X) = \frac{1}{8\pi^2}\frac{m_\mu^2}{m_X^2}\int_0^1 \mathrm{d}x \sum_f \frac{\left|g_{v\,3}^{f\mu}\right|^2 P_3^+(x) + \left|g_{a\,3}^{f\mu}\right|^2 P_3^-(x)}{\epsilon_f^2 \lambda^2 (1-x)\left(1 - \epsilon_f^{-2} x\right) + x} +$$

$$+\frac{2}{8\pi^2}\frac{m_\mu^2}{m_X^2}\int_0^1 \mathrm{d}x \sum_f \frac{\left|g_{v\,3}^{f\mu}\right|^2 P_4^+(x) + \left|g_{a\,3}^{f\mu}\right|^2 P_4^-(x)}{(1-x)\,(1-\lambda^2 x) + \epsilon_f^2\lambda^2 x}, \qquad (51)$$

with $\epsilon_f \equiv \frac{m_\psi}{m_\mu}$ and $\lambda \equiv \frac{m_\mu}{m_X}$; $P_{3/4}^{\pm}$ are defined in Eqs. (33b) and (43b). For a heavy mediator ($\lambda \to 0$) this yields

$$\Delta a_\mu\,(\psi, X) \simeq \frac{1}{4\pi^2}\frac{m_\mu^2}{m_X^2}\sum_f \left[\left|g_{v\,1}^{f\mu}\right|^2\left(3\epsilon_f - \frac{13}{6}\right) - \left|g_{a\,1}^{f\mu}\right|^2\left(3\epsilon_f + \frac{13}{6}\right)\right]. \qquad (52)$$

Notice that if the vector and axial-vector couplings are identical, the contribution to $g-2$ is positive.

Regarding $\mu \to e\gamma$, similarly to the previous subsections we find,

$$\mathrm{BR}(\mu \to e\gamma) \approx \frac{3(4\pi)^3\alpha_{\mathrm{em}}}{4G_F^2}\left(|A_{e\mu}^M|^2 + |A_{e\mu}^E|^2\right), \qquad (53)$$

where one should insert $A_{e\mu}^M = \frac{1}{(4\pi)^2}\sum_f \left(g_{v\,3}^{fe\,*} g_{v\,3}^{f\mu} I_{f,\,34}^{+\,+} + g_{a\,3}^{fe\,*} g_{a\,3}^{f\mu} I_{f,\,34}^{+\,-}\right)$ and $A_{e\mu}^E = \frac{-i}{(4\pi)^2}\sum_f \left(g_{a\,3}^{fe\,*} g_{v\,3}^{f\mu} I_{f,\,34}^{-\,+} + g_{v\,3}^{fe\,*} g_{a\,3}^{f\mu} I_{f,\,34}^{-\,-}\right)$, according to Eq. (50), and the short-hand notation $I_{f,\,34} = I_{f,\,3} + 2I_{f,\,4}$ has been used. In the usual approximation $m_\mu \ll m_X$, we obtain

$$A_{e\mu}^M \simeq \frac{1}{16\pi^2 m_{W'}^2}\sum_f \left(g_{v\,3}^{fe\,*} g_{v\,3}^{f\mu}\left(\frac{13}{6} - 6\epsilon_f\right) + g_{a\,3}^{fe\,*} g_{a\,3}^{f\mu}\left(\frac{13}{6} + 6\epsilon_f\right)\right), \qquad (54a)$$

$$A_{e\mu}^E \simeq \frac{-i}{16\pi^2 m_{W'}^2}\sum_f \left(g_{a\,3}^{fe\,*} g_{v\,3}^{f\mu}\left(\frac{13}{6} - 6\epsilon_f\right) + g_{v\,3}^{fe\,*} g_{a\,3}^{f\mu}\left(\frac{13}{6} + 6\epsilon_f\right)\right). \qquad (54b)$$

8.2.4. Charged Fermion – doubly charged vector boson

Finally, we discuss the case of a doubly charged vector boson, which again contributes through two independent diagrams. Doubly charged gauge bosons are a typical signature of the minimal 3-3-1 model [220]. There, the $SU(2)_L$ gauge group is extended to a $SU(3)_L$, with the generations in the fundamental representation of $SU(3)$. The third component in the fermion triplet has a charged lepton with opposite electric charged, e.g. e^c, and consequently in the covariant derivative a doubly charged gauge boson arises, with a charged current as shown below,

$$\mathcal{L}_{\mathrm{int}} = g_{v\,4}^{ij} U_\mu^{++}\,\overline{\ell_i^{\mathcal{C}}}\gamma^\mu\,\ell_j + g_{a\,4}^{ij} U_\mu^{++}\,\overline{\ell_i^{\mathcal{C}}}\gamma^\mu\gamma^5\ell_j + \mathrm{h.c.} \qquad (55)$$

Note that, while $g_{a\,4}$ is symmetric in flavor space, $g_{v\,4}$ is anti-symmetric and contains no diagonal entries. [7] Consequently, there will be symmetry factors in the vertex rules due to the appearance of identical fields. The two diagrams that contribute have topologies identical to the ones shown in Fig. 10 and give

$$\begin{aligned}\Gamma_9^\mu = &-8\frac{i\sigma^{\mu\nu}k_\nu}{8\pi^2}\frac{m_i}{2}\sum_f\left[g_{v\,4}^{fj\,*}g_{v\,4}^{fi}I_{f,3}^{+\,+} + g_{a\,4}^{fj\,*}g_{a\,4}^{fi}I_{f,3}^{+\,-} + \right.\\ &\left. +\gamma^5\left(g_{a\,4}^{fj\,*}g_{v\,4}^{fi}I_{f,3}^{-\,+} + g_{v\,4}^{fj\,*}g_{a\,4}^{fi}I_{f,3}^{-\,-}\right)\right] + \\ &+4\frac{i\sigma^{\mu\nu}k_\nu}{8\pi^2}\frac{m_i}{2}\sum_f\left[g_{v\,4}^{fj\,*}g_{v\,4}^{fi}I_{f,4}^{+\,+} + g_{a\,4}^{fj\,*}g_{a\,4}^{fi}I_{f,4}^{+\,-} + \right.\\ &\left. +\gamma^5\left(g_{a\,4}^{fj\,*}g_{v\,4}^{fi}I_{f,4}^{-\,+} + g_{v\,4}^{fj\,*}g_{a\,4}^{fi}I_{f,4}^{-\,-}\right)\right],\end{aligned} \tag{56}$$

such that

$$\begin{aligned}\Delta a_\mu\left(U^{++}\right) = &\frac{8}{8\pi^2}\frac{m_\mu^2}{m_U^2}\int_0^1 \mathrm{d}x\sum_f\frac{\left|g_{v\,4}^{f\mu}\right|^2 P_3^+(x) + \left|g_{a\,4}^{f\mu}\right|^2 P_3^-(x)}{\epsilon_f^2\lambda^2(1-x)\left(1-\epsilon_f^{-2}x\right)+x} - \\ &-\frac{4}{8\pi^2}\frac{m_\mu^2}{m_W^2}\int_0^1 \mathrm{d}x\sum_f\frac{\left|g_{v\,4}^{f\mu}\right|^2 P_4^+(x) + \left|g_{a\,4}^{f\mu}\right|^2 P_4^-(x)}{(1-x)\left(1-\lambda^2 x\right)+\epsilon_f^2\lambda^2 x},\end{aligned} \tag{57}$$

with $\epsilon_f \equiv \frac{m_f}{m_\mu}$ and $\lambda \equiv \frac{m_\mu}{m_U}$. Note the relative sign between the second term in Eq (57) and Eq. (51) due to the charge conjugate lepton coupling to the photon. In the heavy mediator limit ($m_{U^{++}} \gg m_f, m_\mu$), one obtains

$$\Delta a_\mu\left(U^{++}\right) \simeq \frac{1}{\pi^2}\frac{m_\mu^2}{m_U^2}\sum_f\left(\left|g_{v\,4}^{f\mu}\right|^2\left[-1+\epsilon_f\right] - \left|g_{a\,4}^{f\mu}\right|^2\left[1+\epsilon_f\right]\right), \tag{58}$$

which vanishes for $\epsilon_f = 1$ in agreement with the above argument.

Thus, if one considers only the muon in the loop ($\epsilon_f = 1$), the doubly charged gauge boson gives rise to a *negative* contribution to $g-2$, yielding tight constraints on the scale of symmetry breaking in the minimal 3-3-1 model [221].

---

[7] If the vector current is $j^\mu = \overline{\psi_i^c}\gamma^\mu\psi_j = -\psi_i^T C^{-1}\gamma^\mu C C^{-1}\psi_j = \psi_i^T\gamma^{\mu\,T}C^T\psi_j = \psi_i^T\gamma^{\mu\,T}(\psi_j^T C)^T = -\psi_i^T\gamma^{\mu\,T}(\psi_j^T C^{-1})^T = (\psi_j^T C^{-1}\gamma^\mu\psi_i)^T = -\overline{\psi_j^c}\gamma^\mu\psi_i$, so if $i = j$ the vector current must vanish, where we used $\psi^c = C\bar{\psi}^T$ and $\bar{\psi}^c = -\psi^T C^{-1}$, with C being the charged conjugation matrix.

Finally, we report the expression for $\mathrm{BR}(\mu \to e\gamma) \approx \frac{3(4\pi)^3 \alpha_{\rm em}}{4G_F^2} \left(|A^M_{e\mu}|^2 + |A^E_{e\mu}|^2\right)$ with

$$A^M_{e\mu} = \frac{-1}{(4\pi)^2} \sum_f \left(g_{v\,4}^{fe\,*} g_{v\,4}^{f\mu} I^{+\,+}_{f,\,3-4} + g_{a\,4}^{fe\,*} g_{a\,4}^{f\mu} I^{+\,-}_{f,\,3-4} \right),$$
$$A^E_{e\mu} = \frac{i}{(4\pi)^2} \sum_f \left(g_{a\,4}^{fe\,*} g_{v\,4}^{f\mu} I^{-\,+}_{f,\,3-4} + g_{v\,4}^{fe\,*} g_{a\,4}^{f\mu} I^{-\,-}_{f,\,3-4} \right), \tag{59}$$

and $I_{f,\,3-4} = -4(2I_{f,\,3} - I_{f,\,4}) \simeq -\left(1 \mp 2\epsilon_f\right)/m_U^2$ for a heavy mediator and $m_j \ll m_i$.

9. SU(2) Invariant Simplified Models

In this section, we will illustrate our results by numerically analyzing some specific simplified BSM scenarios which can be embedded in several extensions of the SM. To this end, we choose to restore $SU(2)_L$ invariance by combining the results of the previous section. For definiteness, we also restrict our attention to coupling matrices with a fixed flavor structure matrix Λ multiplied by a universal coupling g. Departures from this assumption will be addressed in the following section where we study UV complete models. In any case, the flavor structures we consider are labeled either as *strong* hierarchy, in which case we have

$$\Lambda = \begin{pmatrix} 1 & 10^{-5} & 10^{-8} \\ 10^{-5} & 1 & 10^{-5} \\ 10^{-8} & 10^{-5} & 1 \end{pmatrix}. \tag{60}$$

For the cases referred to as *mildly* hierarchical, we set

$$\Lambda = \begin{pmatrix} 1 & 10^{-3} & 10^{-6} \\ 10^{-3} & 1 & 10^{-3} \\ 10^{-6} & 10^{-3} & 1 \end{pmatrix}. \tag{61}$$

One might question the choices made for these hierarchies, since we know that both CKM and PMNS matrices present much weaker hierarchies. In case of weak hierarchies we know from current observations that any new physics giving rise to sizable contribution to $\mu \to e\gamma$ should usually come from scales much above 1 TeV. Although, new physics contributions do not have to follow either CKM or PMNS patterns. So one might wonder:

> *Which hierarchy in the charged lepton sector should one have in order to reconcile possible signals coming from $g-2$ and LFV? What can we learn if the $g-2$ anomaly is confirmed by the upcoming $g-2$ experiments, and no signal is seen in the $\mu \to e\gamma$ decay in the foreseeable future? If the $\mu \to e\gamma$ decay is seen in the upcoming years, do we need to necessarily observe a signal also in $g-2$?*

Those questions motivated the choice for the hierarchies above, in the sense that we choose the hierarchies that give rise to scenarios where one can reconcile possible signals in $g-2$ and $\mu \to e\gamma$ and/or a signal in one of either observable can be probed within current or future sensitivity of the experiments. In summary, the purpose of this approach is to illustrate three different scenarios:

(i) New physics signal in a_μ:

We will see that null results from LFV places stringent limits on new physics models capable of explaining $g-2$. In order to make both comparable, a strong hierarchy in the coupling is needed.

(ii) New physics signal in $\mu \to e\gamma$:

Assuming a signal is seen in $\mu \to e\gamma$ in the next generation of experiments and the $g-2$ anomaly is otherwise resolved, using the 1σ error bar on $g-2$, we can set strong limits on new physics interpretations to $\mu \to e\gamma$. We will see that most possible scenarios are already ruled out by $g-2$.

(iii) New physics signal seen in both a_μ and $\mu \to e\gamma$:

Depending on the hierarchy used, some models offer regions of parameter space where both signals can be simultaneously accommodated.

We discuss those three scenarios for several simplified models that preserve $SU(2)_L$ invariance below.

9.1. Scalar contributions

We begin the discussion of the results with simplified models describing additional scalar fields restoring the previously disregarded $SU(2)_L$ invariance.

9.1.1. Scalar doublet

Scalar doublets, as they occur in two Higgs doublet models, may also show up in the broken phase of a 3-3-1 symmetric model with a scalar $SU(3)_L$ sextet. We consider first the case of a scalar doublet ϕ with hypercharge $Y = 1/2$, which couples to the SM leptons as the Higgs doublet

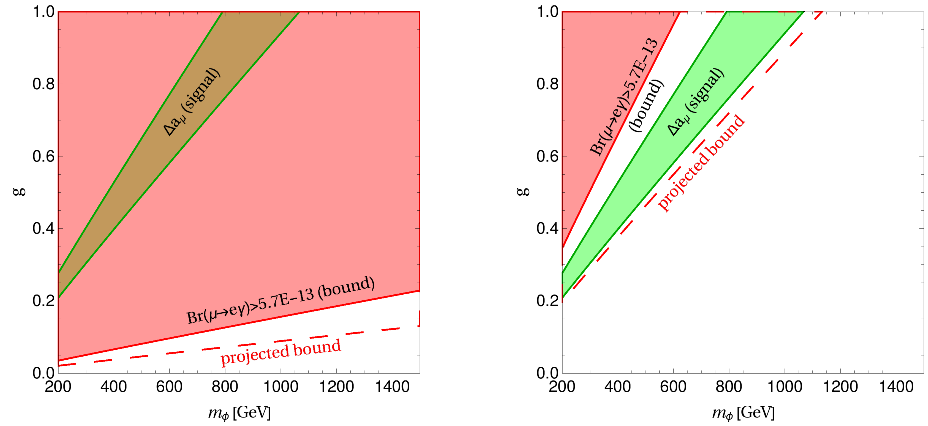
$$\mathcal{L}_{\rm int} = g_{L\,ij} \overline{e^i_R}\, \phi^\dagger \cdot \ell^j_L + \text{h.c.}\,, \tag{62}$$

where '$\cdot$' denotes the $SU(2)_L$ invariant product. In this case the imposed $SU(2)_L$ invariance dictates that $g_R = g_L{}^\dagger$ such that assuming real and symmetric couplings, we find that $g_p = -\text{Im}\, g_L = 0$ and $g_s = g_L$ for the electrically neutral component of ϕ.[8] In contrast, for the charged field, we have $g_p = -g_s = -g_L/2$. Since the doublet ϕ consists of a neutral and a charged component, the result is a sum of Eqs. (9) and (17), with SM leptons and neutrinos running in the loops, respectively.

Since we have obtained these results already in Eqs. (9) and and (17), there is no need to overdo. One can simply solve the integrals numerically to find the results shown in Fig. 11 and 12.

In Fig. 11 we assume that the $g-2$ deviation persists and check whether this is consistent with the current and project limits on $\text{BR}(\mu \to \text{e}\gamma)$ for the two aforementioned hierarchies. In both figures the green region represents the parameter space that in the plane g vs. m_ϕ which could explain the $g-2$ deviation assuming the central value to be the same, whereas the shaded red region (dashed line) accounts for the current (projected) limit stemming from $\mu \to e\gamma$.

[8] For the purpose of illustration, we consider here only the CP even part of ϕ^0. The CP odd scalar would have $g_s = 0$ and $g_p = g_L$.



(a) mild hierarchy **(b)** strong hierarchy

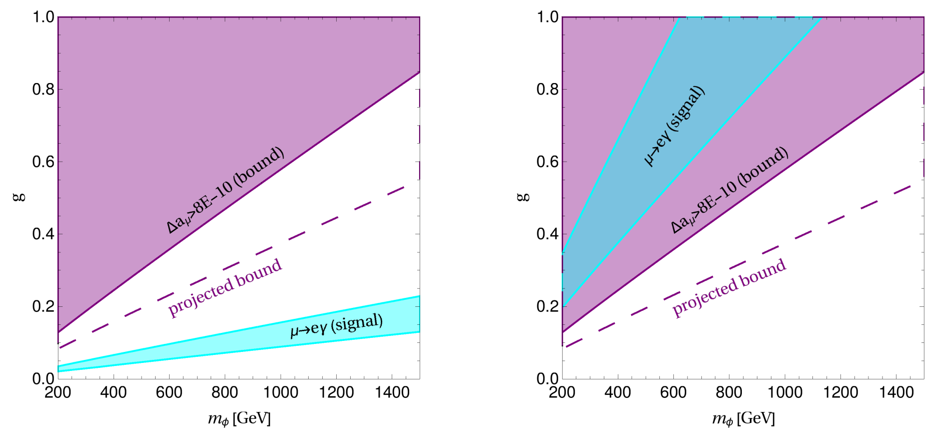
Figure 11: Results for a scalar doublet with hypercharge $Y = 1/2$. The green area corresponds to the signal region for Δa_μ, while the red region is excluded by $\mu \to e\gamma$ at 1σ. The projected bound is shown as a dashed line.

In the left panel the result is exhibited assuming mild hierarchy and one can see that the region of parameter space which accommodates a signal in $g-2$ is quite excluded by the current limits on BR($\mu \to$ eγ). Interestingly, for a strong hierarchy (right panel), the region which explains the $g-2$ deviation falls within the projected sensitivity on the $\mu \to e\gamma$ decay. It should be noted that the charged scalar contribution must be sub-leading because its contribution to Δa_μ is negative.

Panels 12(a) and 12(b) show the results with an orthogonal view. Assuming that Δa_μ is *not* due to BSM physics, what are the constraints for $\mu \to e\gamma$?

In both panels the blue area shows the region of parameter space in the plane g vs m_ϕ which could explain a signal in $\mu \to e\gamma$ with BR($\mu \to$ eγ) $= 4.2 \times 10^{-13} - 4 \times 10^{-14}$ which delimits the current and projected sensitivity. The shaded purple region (dashed line) delimits the current (projected) exclusion from the $g-2$ which is assumed to be otherwise resolved.

In Fig. 12(a) we adopt a mild hierarchy. As one might have anticipated, the limits from $g-2$ are very weak, even considering projected sensitivity. However,



(a) mild hierarchy

(b) strong hierarchy

Figure 12: Reversing the argument. The light blue region is the signal for $\mu \to e\gamma$ between the current and the projected bound and the violet region is excluded by Δa_μ assuming the anomaly is resolved. Again, the projected bound for $g-2$ is shown as a dashed line.

looking at Fig. 12(b) we may conclude that for a strong hierarchy a signal on $\mu \to e\gamma$ has already been ruled out by the $g-2$ constraint. The reason for this behavior is that Δa_μ grows with g^2, whereas $\mathrm{BR}(\mu \to e\gamma)$ grows with g^4. had we taken the signal in $\mu \to e\gamma$ to occur with a different $\mathrm{BR}(\mu \to \mathrm{e}\gamma)$, the signal region for $\mu \to e\gamma$ would shift. In particular, if one takes a signal in $\mu \to e\gamma$ to happen at one order of magnitude below the projected sensitivity, i.e. $\mathrm{BR}(\mu \to \mathrm{e}\gamma) = 4 \times 10^{-15}$, it means that the coupling in Figs. 12(a)-12(b) would have to be smaller roughly by a factor of two, since $\mathrm{BR}(\mu \to \mathrm{e}\gamma)$ goes with g^4, moving the signal region downwards.

Furthermore, we may observe from the results in Figs. 11 and 12 that the signal region/bounds of Δa_μ are rather insensitive to the chosen hierarchy, while for $\mu \to e\gamma$ the hierarchy is very decisive, illustrating that the Δa_μ is mostly sensitive to the flavor-diagonal couplings, while $\mu \to e\gamma$ probes the non-diagonal entries in Λ. Varying the hierarchies, we may naturally interpolate between both scenarios; however, a high degree of fine-tuning in the hierarchies would be needed to incorporate both viable signals $\mu \to e\gamma$ and Δa_μ in such a model. An example where the signal region for $g-2$ would have a significant change is when the $\tau-$lepton contribution becomes relevant for some reason.

In summary, the $g-2$ anomaly favors the large coupling and low mass regions, which are often disfavored by LFV searches cf. Fig. 1. Precisely for this reason, mild or strong hierarchies may yield signals in either $g-2$, or $\mu \to e\gamma$.

As a side note, keep in mind that a scalar doublet ϕ could also be of hypercharge $Y = -1/2$ such that it may be coupled to RH neutrinos in the following way

$$\mathcal{L}_{\text{int}} = g_{L\,ij} \overline{N_R^i}\, \phi^\dagger \cdot \ell_L^j + \text{h.c.} \tag{63}$$

However, glancing at Eq. (18a), one can see that $\Delta a_\mu > 0$ is possible only if we have a sizable RH neutrino mass m_N and dominant pseudo-scalar coupling. However, in an $SU(2)_L$ invariant framework this will not be possible to obtain, and hence a hypercharge $-1/2$ scalar doublet cannot explain the $g-2$ anomaly.

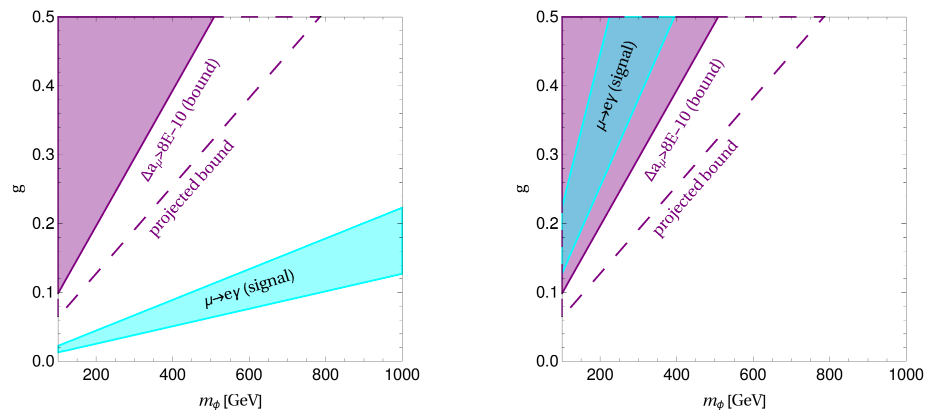
9.1.2. Scalar triplet

We conclude the discussion of scalar contributions to $\ell_i \to \ell_j\gamma$ with a model involving a $Y = 1$ scalar triplet Δ. Such a field contains a neutral, a singly and a doubly charged scalar component field with

$$\Delta = \begin{pmatrix} \Delta^+/\sqrt{2} & \Delta^{++} \\ \Delta^0 & -\Delta^+/\sqrt{2} \end{pmatrix}. \tag{64}$$

Such scalar triplets are arguably a signature of a type II see-saw mechanism for generating neutrino masses. They can naturally appear in Left-Right models. However, they might appear as result of a broken scalar sextet in 3-3-1 models for instance. The result is in principle a sum of *four* diagrams, one for the neutral and charged component each, and the two diagrams shown in Fig. 7 for the doubly charged field. Suppressing $SU(2)_L$ indices, the interaction term reads:

$$\mathcal{L}_{\text{int}} = g_L \overline{\ell_L^{\mathcal{C}}}^i i\sigma^2 \Delta \ell_L^j + \text{h.c.} \tag{65}$$



(a) mild hierarchy

(b) strong hierarchy

Figure 13: Results for possible LFV signals induced by a scalar triplet Δ coupling to SM leptons.

At this point, one should remark that due to electric charge conservation, the neutral component of Δ only couples to neutrinos, and hence has no effect on the charged leptons. On the other hand, we know from Sec. 8 that both the singly and doubly charged scalars tend to yield a negative Δa_μ, which cannot explain the observed excess.

However, assuming there is a signal in $\mu \to e\gamma$ with $\mathrm{BR}(\mu \to \mathrm{e}\gamma) = 4.2 \times 10^{-13} - 4 \times 10^{-14}$ which is exactly where the current limit lies, we can draw the regions where such a signal is expected for a mild (strong) hierarchy as shown in the left (right) panel of Fig. 13. We observe from Fig. 13 that there is a window for future observation of $\mu \to e\gamma$ if the Yukawa matrix has a mild hierarchy. In both panels the blue area shows the region of parameter space in the plane g vs. m_ϕ which could explain the signal in $\mu \to e\gamma$ and the shaded purple region (dashed line) represent the current (projected) exclusion from the $g-2$ which is assumed to be otherwise resolved. We emphasize that if we have adopted the signal in $\mu \to e\gamma$ to occur at $\mathrm{BR}(\mu \to \mathrm{e}\gamma) = 4 \times 10^{-15}$, the signal region would shift by a factor of two downwards, and in this case, even for the strong hierarchy case, a signal in $\mu \to e\gamma$ would be consistent with the current $g-2$

limit. However, one should keep in mind that collider searches for such scenarios restrict the mass of a doubly charged scalar to be larger than 400 GeV [222], already excluding a large region of the parameter space.

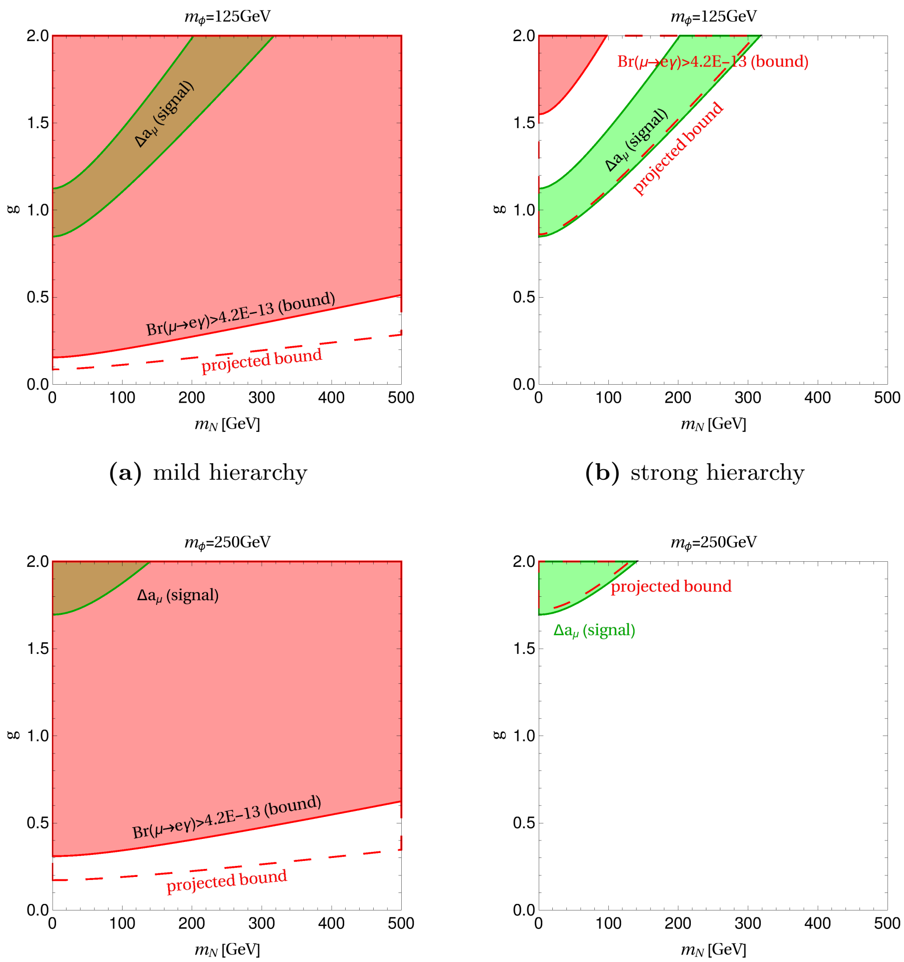
9.2. Fermion singlet contributions

Here, we will discuss the case in which fermionic $SU(2)_L$ singlet fields, with the electric charge equal to the field's hypercharge, contribute to $g-2$. Such fermions can be neutral, singly charged and even doubly charged as we explore below.

9.2.1. Neutral fermion singlet

Neutral fermions are present in many phenomenological studies of $g-2$ and in connection to dark matter [127]. For a new neutral fermion, we can have two potential interactions one of which is the scalar field which is identical to the result already given in Eq. (63). Repeating the previous analysis but now varying the fermion mass and fixing different scalar masses yields Figs. 14 and 15. In the upper panels of Fig. 14 we set $m_\phi = 125$ GeV with the left graph for mild hierarchy and the right for strong hierarchy. In the bottom panels we fix $m_\phi = 250$ GeV. In all these panels the green regions represent the parameter space in the plane g vs. m_N which could explain the $g-2$ deviation assuming the same central value, whereas the shaded red region (dashed line) accounts for the current (projected) limit stemming from $\mu \to e\gamma$.

Looking at Fig. 14, one may conclude that the $g-2$ anomaly favors light mediators (both scalar and fermion need to be light in this case), or large couplings of $\mathcal{O}(1)$. For a mild hierarchy with $m_\phi = (125, 250)$ GeV the deviation in $g-2$ is already ruled out by the $\mu \to e\gamma$ limit, whereas for a strong hierarchy signals in $g-2$ lie within current and future sensitivity for the $\mu \to e\gamma$ decay. One can take an orthogonal view to these findings as shown in Fig. 15. Again assuming a signal in $\mu \to e\gamma$ with $\mathrm{BR}(\mu \to \mathrm{e}\gamma) = 4.2 \times 10^{-13} - 4 \times 10^{-14}$, we learn that for LFV decays the hierarchy in flavor space is more decisive than the masses of the particles themselves. In the upper panels of Fig. 15 again we set



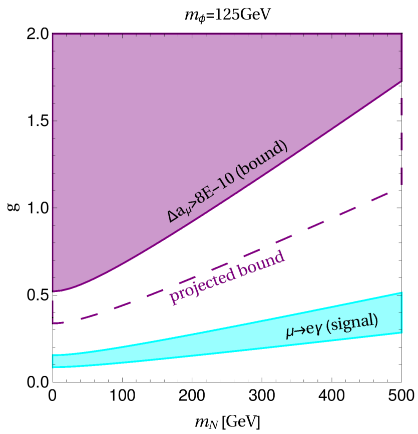
(a) mild hierarchy

(b) strong hierarchy

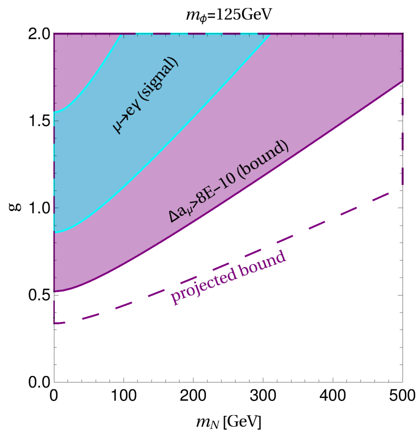
(c) mild hierarchy

(d) strong hierarchy

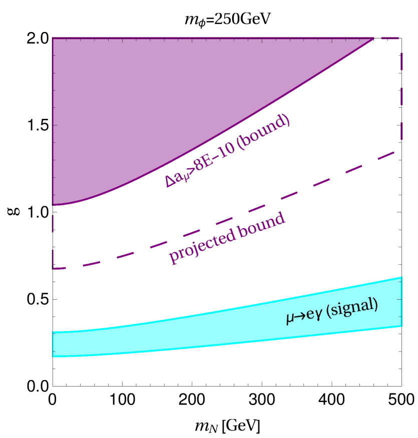
Figure 14: $g-2$ signal region for a neutral fermion N coupling to the SM leptons via a scalar ϕ for different scalar masses $m_\phi = (125, 250)$ GeV.



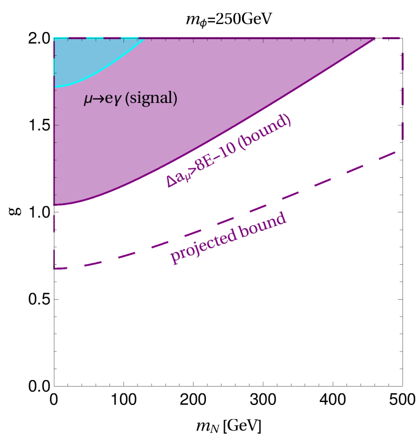
(a) mild hierarchy



(b) strong hierarchy



(c) mild hierarchy



(d) strong hierarchy

Figure 15: Signal region for $\mu \to e\gamma$ for a neutral fermion singlet.

$m_\phi = 125$ GeV, and in the bottom panels we keep $m_\phi = 250$ GeV. As expected a signal in $\mu \to e\gamma$ has an opposite effect compared to $g-2$, where the mild hierarchy was excluded. This time for a mild hierarchy a signal in $\mu \to e\gamma$ is perfectly consistent with $g-2$ physics, whereas for the strong hierarchy case the scenario is widely excluded by the $g-2$ limit.

The more interesting scenario is obtained if the neutral fermion couples to a spin-1 field and the charged leptons in a way similar to the SM W boson interactions:

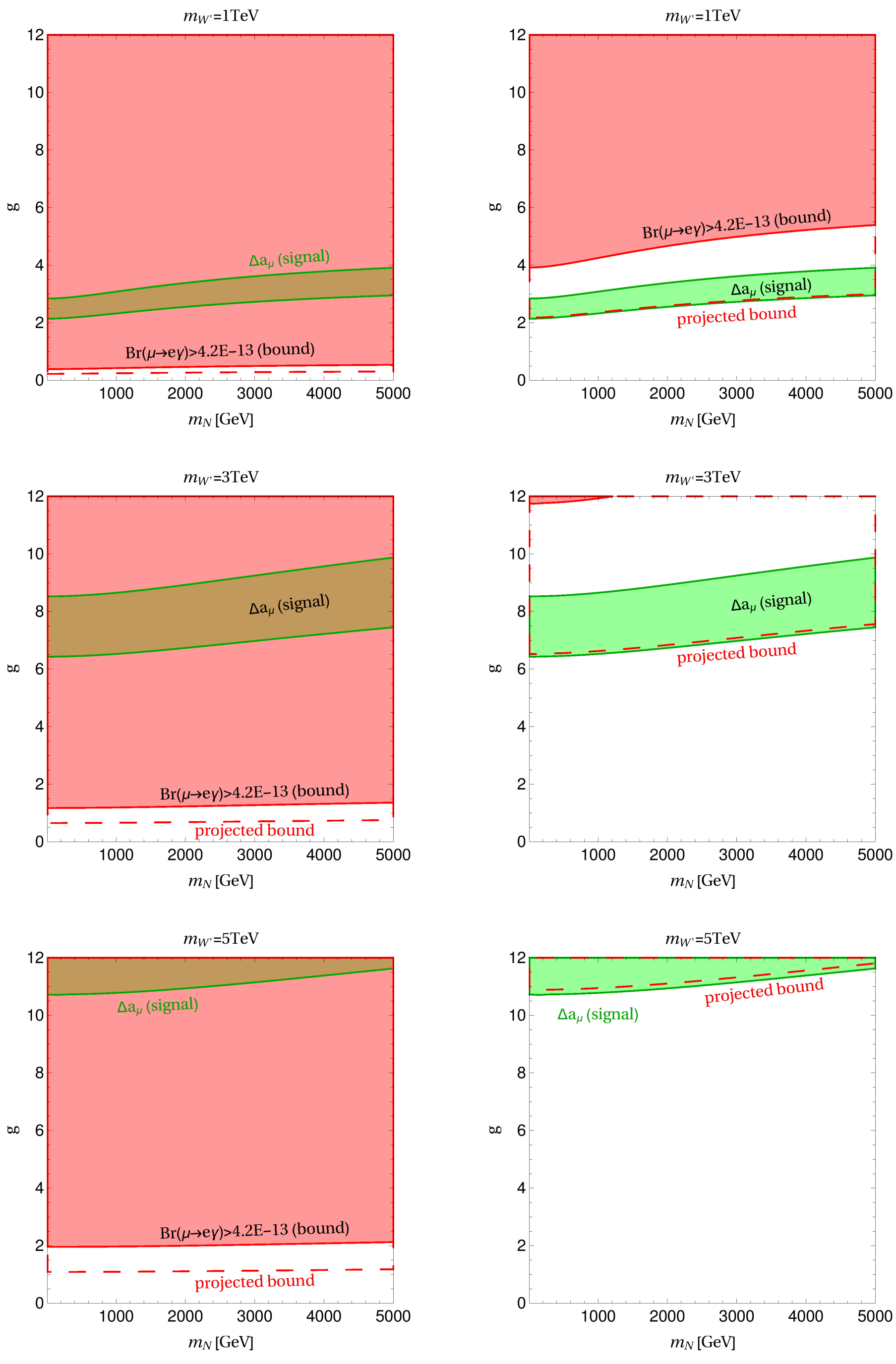
$$\mathcal{L}_{\rm int} = g_{Ri}\overline{N_R}\gamma^\mu e_R^i W'_\mu + \text{h.c.}, \tag{66}$$

as it occurs in Left-Right models.

The fully general result has been obtained in Eq. (33b), and limiting cases are given in Eq. (35) and Eq. (36). In what follows we simply solve Eq. (33b) numerically and display the results in the plane g vs m_N in Fig. 16, assuming the deviation in $g-2$ remains with the same central value, and overlay the limits from $\mu \to e\gamma$ using the same color scheme of the previous sections. In the left (right) graphs a mild (strong) hierarchy in the charged leptonic sector is adopted. In the upper panels of Fig. 16 we set $m_{W'} = 1$ TeV, in the middle ones $m_{W'} = 3$ TeV, and in the bottom $m_{W'} = 5$ TeV. For $m_{W'} = 1$ TeV we notice that the $g-2$ signal is consistent with $\mu \to e\gamma$ using strong hierarchies, and similarly for heavier masses but larger couplings are required. All cases with a mild hierarchy are excluded.

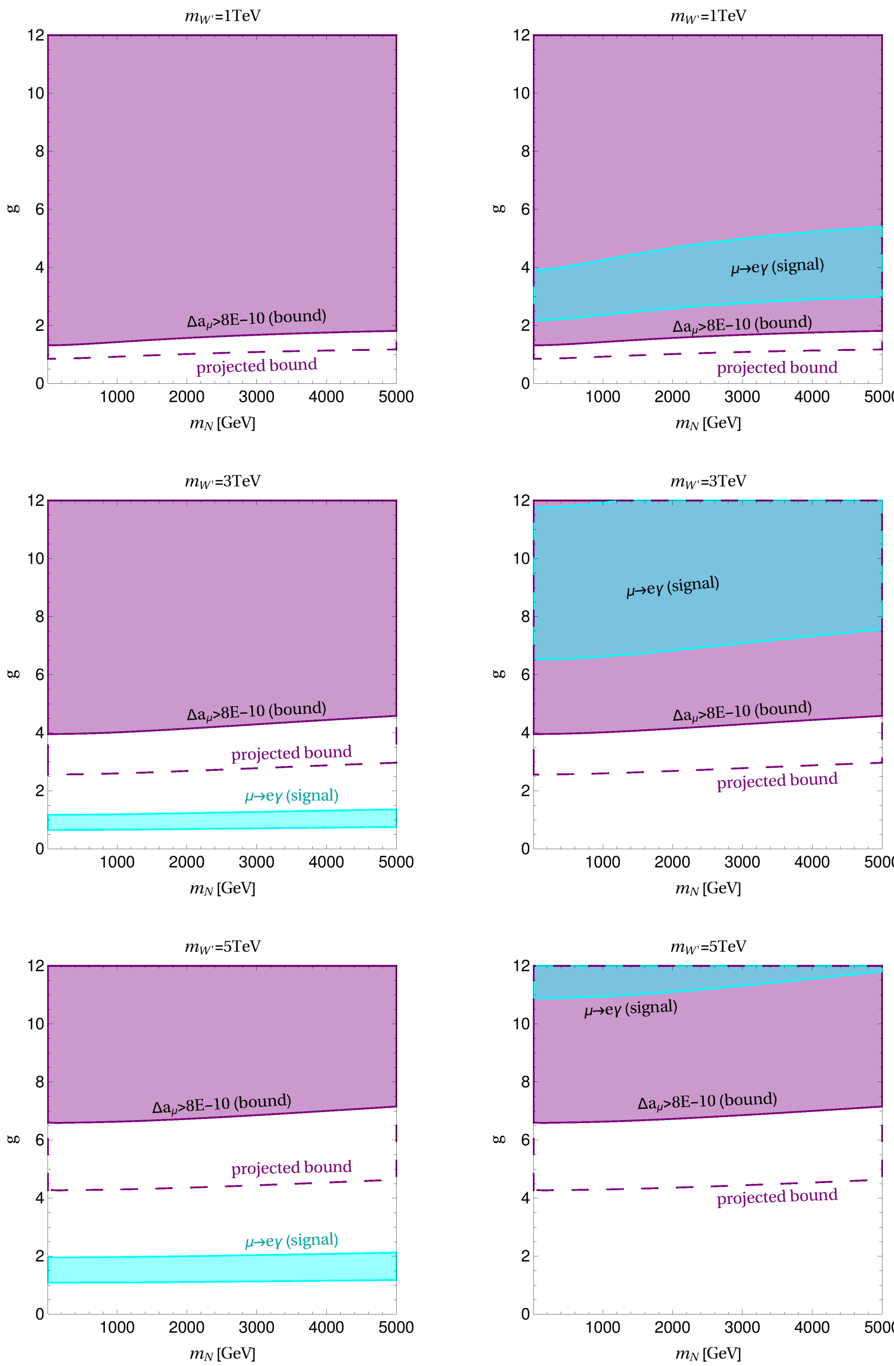
We emphasize that we are not taking into account the existence of collider bounds. Currently they exclude m'_W masses up to $3-4$ TeV [223] with a projected sensitivity of up to 6 TeV [224], assuming $g = e/s_W$. This is rather sensitive to the m_N though, and it weakens significantly when $m'_W \simeq m_N$ or when $m'_W \gg m_N$. So our findings have to be used with care.

In the converse approach, i.e. assuming now a signal in $\mu \to e\gamma$ with $\text{BR}(\mu \to \text{e}\gamma) = 4.2\times 10^{-13} - 4\times 10^{-14}$ we obtain Fig. 17, where the signal regions are delimited in blue, and the $g-2$ limits in violet as before. It is visible that a mild hierarchy is needed in order to reconcile signals in $\mu \to e\gamma$ and $g-2$.



(a) mild hierarchy **(b)** strong hierarchy

Figure 16: Contribution to the $g-2$ for a neutral fermion N coupling to a W' boson and the SM leptons as described in Eq. (66).



(a) mild hierarchy

(b) strong hierarchy

Figure 17: Potential $\mu \to e\gamma$ signal induced by a neutral fermion N coupling to a W' boson and the SM leptons as dictated by Eq. (66).

9.2.2. Charged fermion singlet

As we discussed previously in Eq. (41), a charged fermion might interact with the muon via both scalar and vector mediators. In the former case, we consider

$$\mathcal{L}_{\rm int} = g_{L\,i}\overline{E_R}\,\phi^\dagger \cdot \ell^i_L + \text{h.c.} \tag{67}$$

which resembles Eq. (62).

Similar interactions appear in 3-3-1 models [208, 209] and more exotic two Higgs doublet models [210]. We have treated this case before, so here we exhibit only the results summarized in Figs. 18 and 19.

In Fig. 18 we assume the deviation in $g-2$ is confirmed with the same central value and display in the left (right) panels the findings for a mild (strong) hierarchy in the charged lepton sector. Setting $m_\phi = (125, 250)$ GeV we see that the $g-2$ signal region has been excluded by $\mu \to e\gamma$, whereas for a strong hierarchy the $g-2$ signal region falls within current and projected sensitivity on the $\mu \to e\gamma$ decay. One may argue that we chose relatively light scalar masses, since for larger scalar masses the bounds from $\mu \to e\gamma$ weakens, but keep in mind that heavier scalars require larger couplings to still accommodate a signal in $g-2$. So generally heavier scalars are problematic.

As usual, the picture is reserved if one is interested in addressing a potential signal in $\mu \to e\gamma$ as one can see in Fig. 19, since only for a mild hierarchy a signal in $\mu \to e\gamma$ is in agreement with $g-2$ measurements.

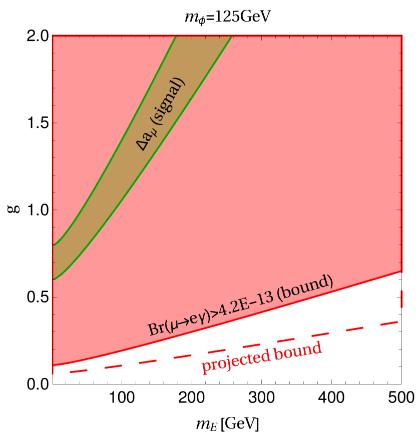
For a vector Z' mediator, one obtains results which are almost identical to the results for a neutral fermion coupling via a W' boson, and for this we skip this case.

9.3. Fermion multiplet contributions

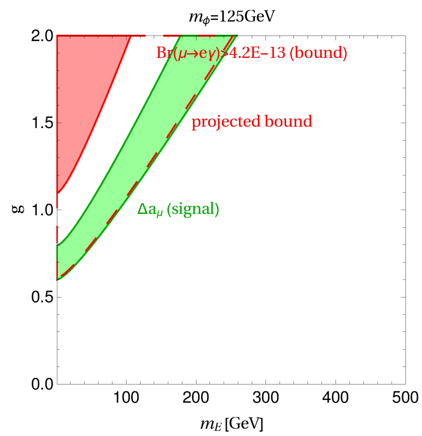
9.3.1. Fermion doublet

Let us consider the case of a new $SU(2)_L$ fermion doublet ψ_D which interacts with the SM charged leptons as

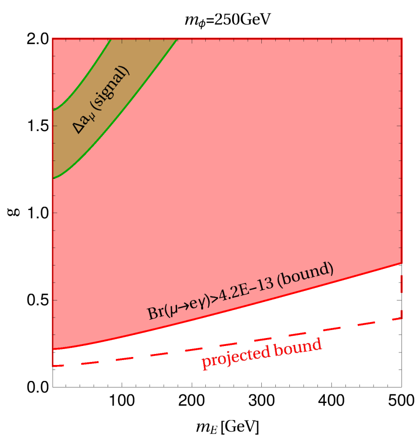
$$\mathcal{L}_{\rm int} = g_{L\,i}\,\overline{e^i_R}\,\phi^\dagger \cdot \psi_D. \tag{68}$$



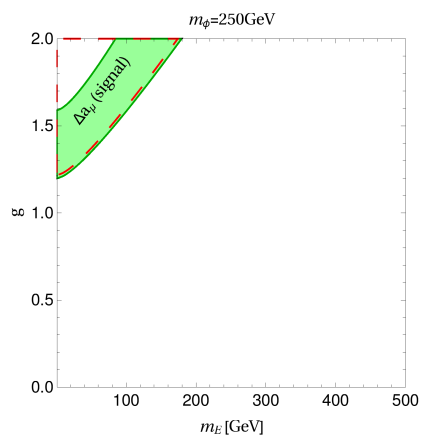
(a) mild hierarchy



(b) strong hierarchy

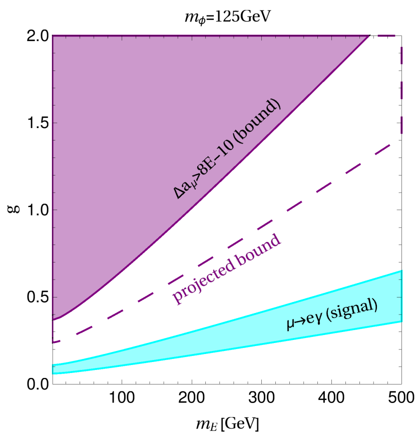


(c) mild hierarchy

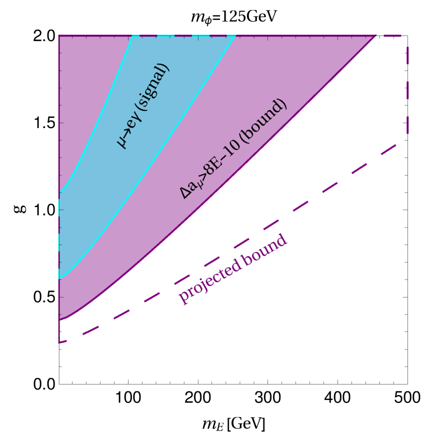


(d) strong hierarchy

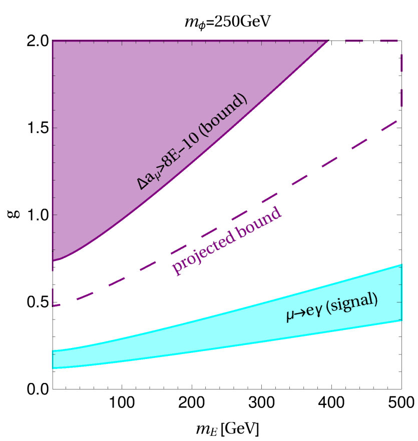
Figure 18: Δa_μ for a fermion with unit electric charge coupling to the SM leptons via a scalar doublet ϕ according to Eq. (67).



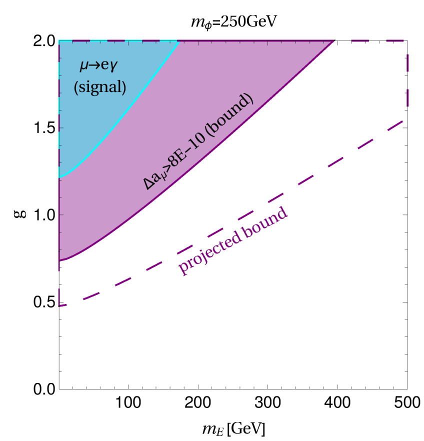
(a) mild hierarchy



(b) strong hierarchy

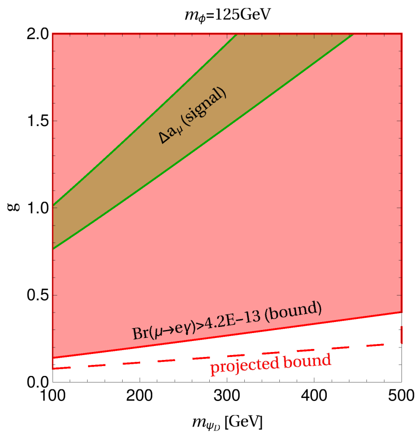


(c) mild hierarchy

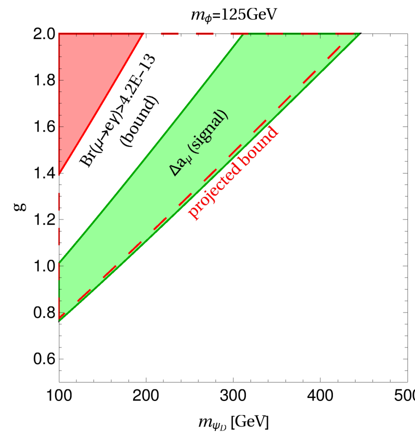


(d) strong hierarchy

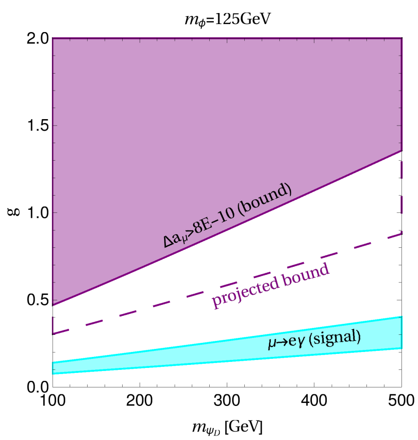
Figure 19: Signal region for $\mu \to e\gamma$ for a fermion with unit electric charge coupling to the SM leptons via a scalar doublet ϕ governed by Eq. (67).



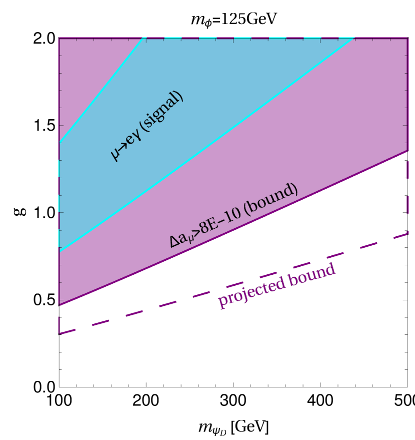
(a) mild hierarchy



(b) strong hierarchy



(c) mild hierarchy



(d) strong hierarchy

Figure 20: Results for a fermion doublet ψ_D.

The parameter scan for this model is shown in Fig. 20. Once more, a viable explanation for the excess Δa_μ can only be achieved by a light, Higgs-like scalar. The signal for $\mu \to e\gamma$, on the other hand, is highly sensitive to the hierarchy as the bottom panels highlight. We wish to highlight that Fig. 20(b) shows a scenario in which both the excess for $g-2$ and a future signal in $\mu \to e\gamma$ can be accommodated. Conversely, if no signal for $\mu \to e\gamma$ is seen in the next-generation experiments, there must be an even stronger hierarchy at work, if the excess is to be explained by this scenario.

9.3.2. Fermion triplet

We have not considered the case of a fermion field of double unit charge. The reason is the such a field always induces $\Delta a_\mu < 0$ [130]. However, such fields naturally arise in the context of $SU(2)_L$ triplets with hypercharge $Y = -1$. Such a triplet field could have Yukawa interactions of the form

$$\mathcal{L}_{\rm int} = g_{L i} \phi^\dagger \overline{\psi_T} \ell_L^i + \text{h.c.}, \tag{69}$$

where the triplet ψ_T may be written in component form as

$$\psi_T = \begin{pmatrix} \psi_T^-/\sqrt{2} & \psi_T^0 \\ \psi_T^{--} & -\psi_T^-/\sqrt{2} \end{pmatrix}. \tag{70}$$

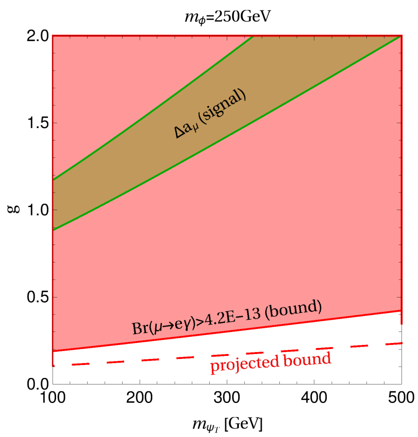
As before, the $g-2$ anomaly favors a light scalar mediator as we may conclude from Fig. 21. As in the previous cases, the hierarchy is irrelevant for the size of this contribution, but highly decisive for the viability of LFV decays.

9.4. Vector contributions

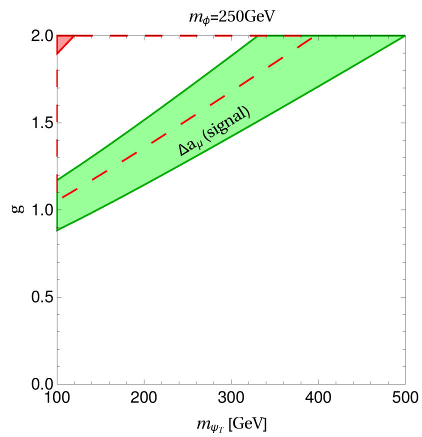
Finally, let us consider new vector fields with electrical charges $Q = 0, 1,$ and 2 as mediators.

9.4.1. Neutral vector boson

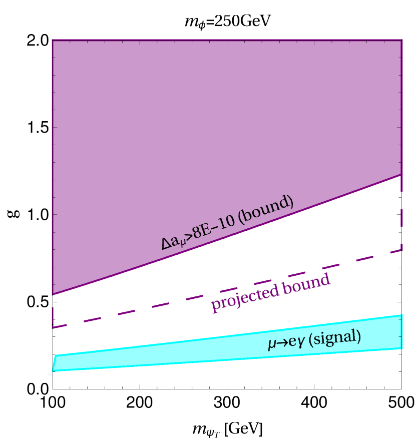
Heavy neutral gauge bosons (Z') arise in many popular models such as $B-L$, $L_\mu - L_\tau$, Left-Right models, and $3-3-1$ models. Their masses can be generated via spontaneous symmetry breaking due to a scalar field charged



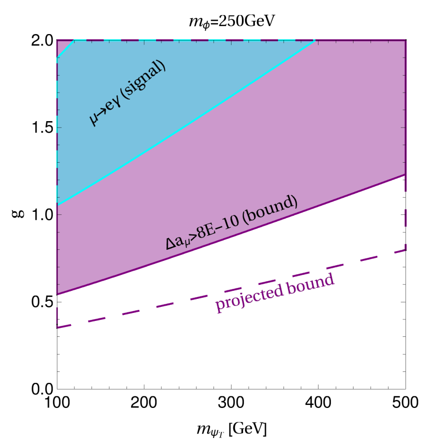
(a) mild hierarchy



(b) strong hierarchy

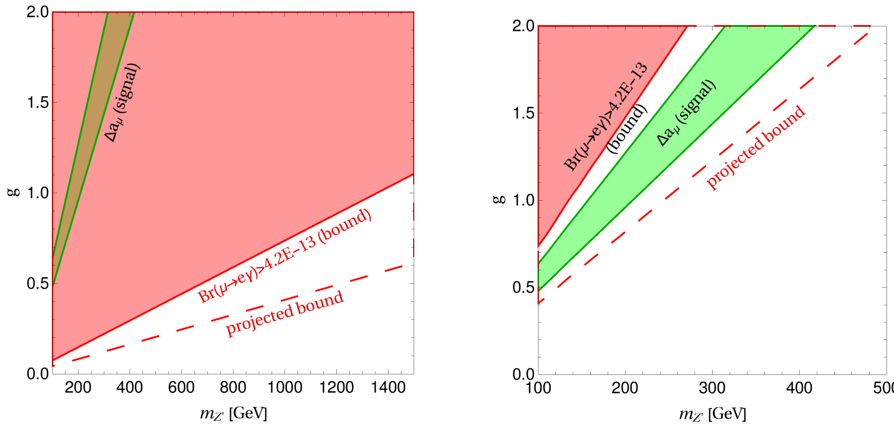


(c) mild hierarchy



(d) strong hierarchy

Figure 21: Results for a fermion triplet ψ_T of unit hypercharge.



(a) mild hierarchy

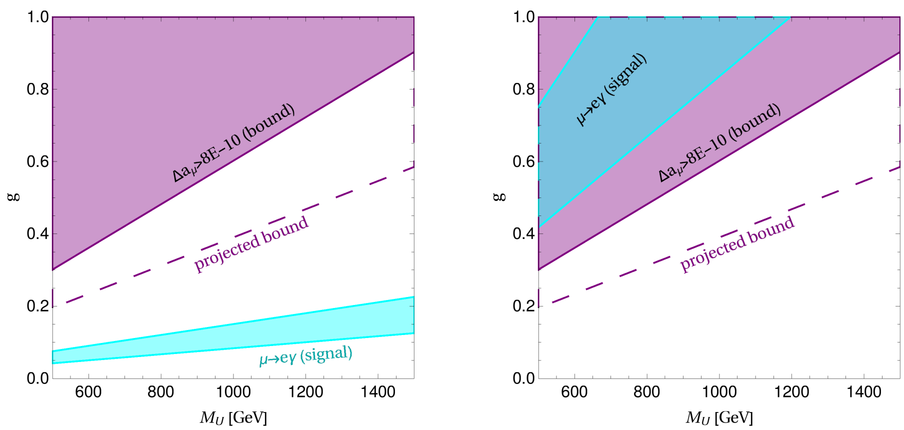
(b) strong hierarchy

Figure 22: A neutral Z' vector boson as mediator.

under the new gauge symmetry, or via the Stueckelberg mechanism [225–228], where the existence of such scalar field is not needed. In the former case, in addition to the Lagrangian we will describe below, interactions involving a scalar field would also show up, but we have already determined the $g-2$ and $\mu \to e\gamma$ contributions stemming from a scalar field. Thus, incorporating such interactions is straightforward by means of Eq. (7). That said, we will restrict our attention to the Lagrangian which contains the Z' boson. Moreover, such vector boson may have either vector couplings such as in the case of the $B-L$ model, [229, 230] axial-vector in some $U(1)_X$ models [231], or more often both. Thus, we will keep our reasoning as general as possible by writing the Z' interactions as follows,

$$\mathcal{L}_{\text{int}} = \left(g_{L ij} \overline{\ell_L}^i \gamma^\mu \ell_L^j + g_{R ij} \overline{e_R}^i \gamma^\mu e_R^j \right) Z'_\mu + \text{h.c.} \tag{71}$$

However, note that these interactions can always be made family-diagonal in case of universal couplings and consequently there cannot be any LFV involved. Therefore, only a signal in $g-2$ will be considered. Adopting the same procedure as before, in Fig. 22 we exhibit the region of parameter space which can explain the $g-2$ anomaly (using $g_L = g_R$). We find that this would require very



(a) mild hierarchy

(b) strong hierarchy

Figure 23: Results for a doubly charged vector boson U according to Eq. (73).

light boson masses $\ll 1\,\text{TeV}$ which are ruled out by collider searches if they have sizable couplings to either electrons or quarks [97]. We point out that in scenarios of non-universal couplings, LFV can be present via a Z' as discussed in [232]. For such case one can easily solve our fully generic result in Eq. (43b).

The previous caveat is avoided for a charged vector boson, conventionally called W'. In this case we cannot diagonalize the interaction in family space. We assume that this new vector boson couples only to left- or RH leptons, however, we remark that in the former case there exist other constraints from collider searches and electroweak precision tests (EWPT) [233, 234] which we disregard here. The interaction under consideration reads

$$\mathcal{L}_{\text{int}} = g_{Ri}\,\overline{\nu_R}\gamma^\mu e^i_R W'_\mu + g_{Li}\,\overline{\nu_L}\gamma^\mu e^i_L W'_\mu + \text{h.c.} \tag{72}$$

The term proportional to g_L is not relevant for our purposes because in this case it has to be accompanied by some small mixing, which would require very large couplings in order to produce a signal in $g-2$ or $\mu \to e\gamma$. The term proportional to g_R has already been considered in Eq. (66).

9.4.2. Doubly charged vector boson

The final case studied is that of a doubly charged vector boson U with interactions

$$\mathcal{L}_{\rm int} = \left(g_{L\,ij} \overline{\ell^{\mathcal{C}}_L}^i \gamma^\mu \ell^j_L + g_{R\,ij} \overline{e^{\mathcal{C}}_R}^i \gamma^\mu e^j_R \right) U^{++}_\mu + \text{h.c.} \tag{73}$$

Note however, that by virtue of the relation $\overline{\chi^{\mathcal{C}}}\gamma^\mu\psi = -\overline{\psi^{\mathcal{C}}}\gamma^\mu\psi$, the vector coupling matrix g_v is anti-symmetric and therefore contains no diagonal entries. Since its contribution to $g-2$ is negative as shown in Eq. (58), we show only the results for BR($\mu \to e\gamma$) in Fig. 23.

One concludes that only with a mild hierarchy in the charged leptonic sector one can reconcile a signal in $\mu \to e\gamma$ with constraints from $g-2$. It is also visible that even TeV scale masses offer a sizable contribution to $\mu \to e\gamma$ with gauge couplings of order of $\mathcal{O}(0.1)$.

10. UV Complete Models

Our goal in this section is to revisit existing results in the literature in the context of concrete models and show that one can apply our findings to well known extensions of the SM. We will discuss the Minimal Supersymmetric SM (MSSM), the Left-Right Symmetric model, as well as two classes of $B-L$ models. Furthermore, we discuss the scotogenic model, two Higgs doublet models, and the $L_\mu - L_\tau$, Zee-Babu and 3-3-1 models. Finally, we consider a model for a dark photon. We start our discussion with the MSSM.

10.1. Minimal Supersymmetric Standard Model

Supersymmetry (SUSY) is one of the most compelling extensions of the SM since it uniquely relates fermions and bosons in a relativistic quantum field theory [235]. Moreover, it cancels the quadratic divergences associated with the Higgs boson mass, stabilizing the weak scale against quantum corrections arising from high-energy scales [236]. Furthermore, it naturally leads to grand unification of the gauge couplings at high scales [237, 238]. We have not observed

leptons	quarks	Higgs	gauge bosons
sleptons	squarks	Higgsinos	gauginos
$\begin{pmatrix}\nu_e \\ e\end{pmatrix}_L, e_R, \ldots$	$\begin{pmatrix}u \\ d\end{pmatrix}_L, u_R, d_R, \ldots$	$\mathcal{H}_1, \mathcal{H}_2$	$B^\mu, W^{a\mu};\quad G^{a\mu}$
		$\gamma, Z, W^\pm, G^{0,\pm}, h^0, H^0, A^0, H^\pm$	
$\begin{pmatrix}\tilde{\nu}_e \\ \tilde{e}\end{pmatrix}_L, \tilde{e}_R, \ldots$	$\begin{pmatrix}\tilde{u}_L \\ \tilde{d}_L\end{pmatrix}, \tilde{u}_R, \tilde{d}_R, \ldots$	$\tilde{H}_1, \tilde{H}_2$	$\tilde{B}, \tilde{W}^a;\quad \tilde{g}^a$
		$\chi^0_{1,2,3,4}, \chi^\pm_{1,2}$	

Table 4: Particle content of the MSSM. Only the first generation is explicitly shown.

any supersymmetric particle yet at high-energy colliders [239–241], however, we can probe SUSY models by using precision measurements in low energy experiments specially those related to the muon anomalous magnetic moment and LFV [122, 242–255].

In this section we will discuss the correlation between $g-2$ and $\mu \to e\gamma$ using the MSSM. In order to follow our reasoning it is important to briefly describe the key features of the model. In Tab. 4 we show the particle content. One can see that the MSSM features two Higgs doublets $\mathcal{H}_{1,2}$ along with its supersymmetric partners called Higgsinos that are fermions, scalar SUSY partners of each chiral SM fermion called sfermions $\tilde{f}_{L,R}$, and fermion SUSY partners of the $U(1)_Y$, $SU(2)_L$ and $SU(3)_c$ SM gauge bosons, known respectively as bino ($\tilde{B}$), winos ($\tilde{W}^{\pm,3}$) and gluinos ($\tilde{g}$).

As far $g-2$ and $\mu \to e\gamma$ are concerned, two parameters play a key role in the MSSM. The first is the ratio of the vacuum expectation values of the two Higgs doublets ($\mathcal{H}_{1,2}$), defined as,

$$\tan\beta \quad = \quad \frac{v_2}{v_1}. \tag{74}$$

Since the muon Yukawa coupling y_μ is given by,

$$y_\mu = \frac{\mu_{mu}}{v_1} = \frac{m_\mu g_2}{\sqrt{2} M_W \cos_\beta}, \tag{75}$$

where $g_2 = e/s_W$, and the chirality-flips relevant for $g-2$ and $\mu \to e\gamma$ are

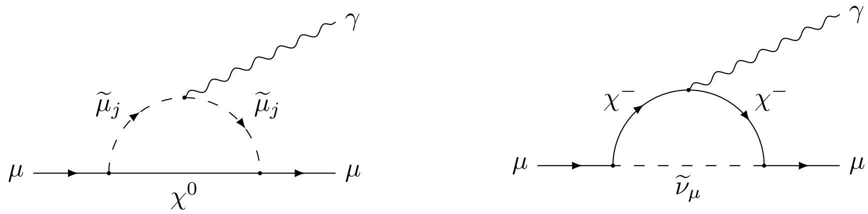


Figure 24

proportional to the muon mass, an enhancement to these observables might occur for large $\tan\beta \sim 1/\cos\beta$.

The second is the μ term which determines the Higgsino mass term and gives rise to the sfermions' interactions with $\mathcal{H}_{1,2}$ through the Lagrangian,

$$\mu\tilde{H}_1\tilde{H}_2 - \mu F_{H_1}\mathcal{H}_2 - \mu F_{H_2}\mathcal{H}_1 + \text{h.c.}, \tag{76}$$

where $F_{H_{1,2}}$ are auxiliary fields. The auxiliary fields are eliminated to generate Yukawa interactions of the form $\mathcal{H}_2^0\tilde{\mu}_L\tilde{\mu}_R^\dagger$ for instance. As we will see later on, the μ term and its sign are relevant to determine the neutralino and chargino contributions to $g-2$ and $\mu \to e\gamma$ since it appears in the mass matrices of both fields.

In order to provide an insight into the MSSM contributions to $g-2$ and $\mu \to e\gamma$, we will first work out the $g-2$ correction under some simplifying assumptions, revisiting well known results in certain regimes and then move to a very general approach and see whether signals in both observables can be accommodated.

As far as $g-2$ is concerned there are basically two sorts of diagrams (Fig. 24) contributing to $g-2$: (i) muon-neutralino-smuon; (ii) muon-chargino-sneutrino, which are found to be, respectively [256–272]

$$\Delta a_\mu^{\chi^0} = \frac{m_\mu}{16\pi^2}\sum_{i,m}\left\{-\frac{m_\mu}{12m^2_{\tilde{\mu}_m}}(|n^L_{im}|^2+|n^R_{im}|^2)F_1^N(x_{im}) + \frac{m_{\chi^0_i}}{3m^2_{\tilde{\mu}_m}}\mathrm{Re}[n^L_{im}n^R_{im}]F_2^N(x_{im})\right\}, \quad (77)$$

$$\Delta a_\mu^{\chi^\pm} = \frac{m_\mu}{16\pi^2}\sum_{k}\left\{\frac{m_\mu}{12m^2_{\tilde{\nu}_\mu}}(|c^L_k|^2+|c^R_k|^2)F_1^C(x_k) + \frac{2m_{\chi^\pm_k}}{3m^2_{\tilde{\nu}_\mu}}\mathrm{Re}[c^L_k c^R_k]F_2^C(x_k)\right\}, \quad (78)$$

with $i = 1,2,3,4$, and $k = 1,2$ denoting the neutralino and chargino mass eigenstate indices, $m = 1,2$ the smuon one, and the couplings given by,

$$\begin{aligned} n^R_{im} &= \sqrt{2}g_1 N_{i1}X_{m2} + y_\mu N_{i3}X_{m1}, \\ n^L_{im} &= \frac{1}{\sqrt{2}}\left(g_2 N_{i2} + g_1 N_{i1}\right)X^*_{m1} - y_\mu N_{i3}X^*_{m2}, \\ c^R_k &= y_\mu U_{k2}, \\ c^L_k &= -g_2 V_{k1}, \end{aligned} \quad (79)$$

where $g_1 = e/c_W$, and y_μ is the muon Yukawa coupling defined in Eq. (75). The kinematic loop functions, which are normalized to unity for $x = 1$, depend on the variables $x_{im} = m^2_{\chi^0_i}/m^2_{\tilde{\mu}_m}$, $x_k = m^2_{\chi^\pm_k}/m^2_{\tilde{\nu}_\mu}$ and are found to be,

$$F_1^N(x) = \frac{2}{(1-x)^4}\left[1-6x+3x^2+2x^3-6x^2\ln x\right], \quad (80)$$

$$F_2^N(x) = \frac{3}{(1-x)^3}\left[1-x^2+2x\ln x\right], \quad (81)$$

$$F_1^C(x) = \frac{2}{(1-x)^4}\left[2+3x-6x^2+x^3+6x\ln x\right], \quad (82)$$

$$F_2^C(x) = -\frac{3}{2(1-x)^3}\left[3-4x+x^2+2\ln x\right]. \quad (83)$$

With Eq. (78) one can compute the MSSM contribution to $g-2$ knowing the neutralino (χ^0)and chargino ($\chi^\pm$) and smuon ($\tilde{\mu}$) mass matrices which are given by, [273, 274]

$$M_{\chi^0} = \begin{pmatrix} M_1 & 0 & -\cos\beta\,\sin W\,M_Z & \sin\beta\,\sin W\,M_Z \\ 0 & M_2 & \cos\beta\,\cos W\,M_Z & -\sin\beta\,\cos W\,M_Z \\ -\cos\beta\,\sin_W\;M_Z & \cos\beta\,\cos W\,M_Z & 0 & -\mu \\ \sin\beta\,\sin_W\;M_Z & -\sin\beta\,\cos_W\;M_Z & -\mu & 0 \end{pmatrix}, \tag{84}$$

,

$$M_{\chi^\pm} = \begin{pmatrix} ccM_2 & \sqrt{2}\sin\beta\,M_W \\ \sqrt{2}\cos\beta\,M_W & \mu \end{pmatrix}, \tag{85}$$

and

$$M^2_{\tilde{\mu}} = \begin{pmatrix} m_L^2 + (s_W^2 - \frac{1}{2})m_Z^2\cos 2\beta & m_\mu(A^*_{\tilde{\mu}} - \mu\tan\beta) \\ m_\mu(A_{\tilde{\mu}} - \mu^*\tan\beta) & m_R^2 - s_W^2\,m_Z^2\cos 2\beta \end{pmatrix}, \tag{86}$$

where $A_{\tilde{\mu}}$ is the soft SUSY breaking parameter of the trilinear interaction $\tilde{\mu}_L - \tilde{\mu}_R$-Higgs, with the muon sneutrino mass being connected to the left-handed smuon mass parameter via,

$$m^2_{\tilde{\nu}} = m_L^2 + \frac{1}{2}M_Z^2\cos 2\beta. \tag{87}$$

These matrices are diagonalized using four matrices N, U, V and X which define the entries in Eqs. (79), and are determined as follows,

$$N^* M_{\chi^0} N^\dagger = \mathrm{diag}(m_{\chi_1^0}, m_{\chi_2^0}, m_{\chi_3^0}, m_{\chi_4^0}), \tag{88}$$

$$U^* M_{\chi^\pm} V^\dagger = \mathrm{diag}(m_{\chi_1^\pm}, m_{\chi_2^\pm}), \tag{89}$$

$$X M^2_{\tilde{\mu}} X^\dagger = \mathrm{diag}\,(m^2_{\tilde{\mu}_1}, m^2_{\tilde{\mu}_2}). \tag{90}$$

We have now gathered all relevant ingredients to compute the MSSM contribution to $g-2$ and have a grasp of the underlying physics.

10.1.1. Simplified Results

Similar SUSY masses

The simplest analytic result to obtain from Supersymmetry is to assume that all superpartners have the same mass $M_{\rm SUSY}$, which leads to [275],

$$\begin{aligned} \Delta a_\mu^{\chi^0} &= \frac{\tan\beta}{192\pi^2}\frac{m_\mu^2}{M_{\rm SUSY}^2}(g_1^2-g_2^2), \\ \Delta a_\mu^{\chi^\pm} &= \frac{\tan\beta}{32\pi^2}\frac{m_\mu^2}{M_{\rm SUSY}^2}g_2^2, \end{aligned} \tag{91}$$

resulting in,

$$\Delta a_\mu^{\rm SUSY} = \frac{\tan\beta}{192\pi^2}\frac{m_\mu^2}{M_{\rm SUSY}^2}(5g_2^2+g_1^2) = 14\tan\beta\left(\frac{100\ {\rm GeV}}{M_{\rm SUSY}}\right)^2 10^{-10}. \tag{92}$$

In this regime the chargino contribution dominates [275], and the effect of a large $\tan\beta$ is explicitly seen in Eq. (92). In Fig. 25 we display this dependence, where one can clearly see that relatively low masses are needed to account for the observed $g-2$ discrepancy.

This result holds true for one-loop corrections only. Albeit, two-loop effects which are negative, lead to effects of the order of 10% with, [125, 276–278]

$$\Delta a_{\mu\,2-{\rm loop}}^{SUSY} = \Delta a_{\mu\,1-{\rm loop}}^{SUSY}\left(1-\frac{4\alpha_{em}}{\pi}\ln\frac{M_{SUSY}}{m_\mu}\right), \tag{93}$$

thus not changing much the overall picture.

Large μ

Another important regime is for the large μ, i.e. $\mu, M_2 \gg M_1$, which implies that the diagrams with light bino and smuon are dominant. In this limit one finds, [279]

$$a_\mu^{{\rm Large}\,\mu} = \frac{g_1^2}{48\pi^2}\frac{m_\mu^2 M_1 {\rm Re}[\mu\tan\beta - A_\mu^*]}{m_{\tilde\mu_2}^2 - m_{\tilde\mu_1}^2}\left[\frac{F_2^N(x_{11})}{m_{\tilde\mu_1}^2} - \frac{F_2^N(x_{12})}{m_{\tilde\mu_2}^2}\right], \tag{94}$$

which for $m_{\tilde\mu_1} \approx m_{\tilde\mu_2} = 2.0 M_1$ reduces to,

$$a_\mu^{\rm light\ bino} = 18\tan\beta\left(\frac{100\ {\rm GeV}}{m_{\tilde\mu}}\right)^3\left(\frac{\mu - A_\mu\cot\beta}{1000\ {\rm GeV}}\right)10^{-10}, \tag{95}$$

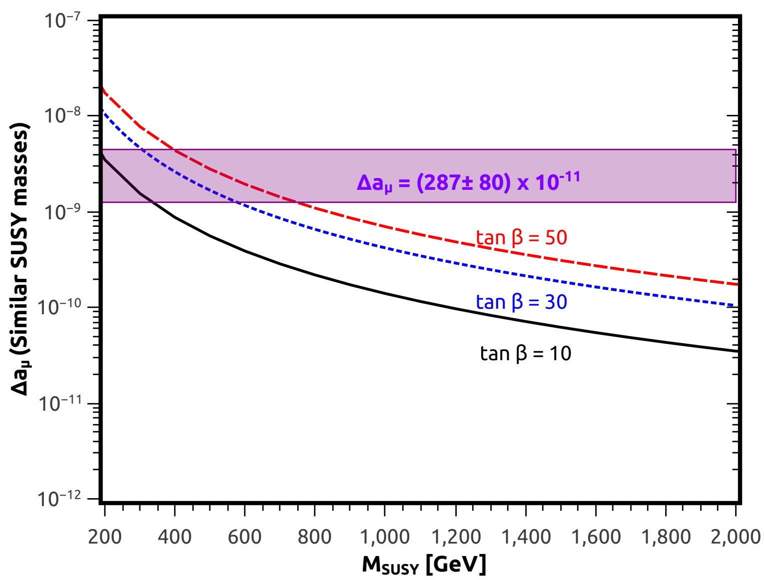


Figure 25: MSSM contribution to $g-2$ for different values of $\tan\beta$ assuming all supersymmetric particles have masses equal to M_{SUSY} according to Eq. (92).

where $x_{1m} = M_1^2/m_{\tilde{\mu}_m}^2$. First, notice that both the value of the μ term and its sign are relevant to determine whether the MSSM gives rise to a negative or positive contribution to $g-2$, along with the value for $\tan\beta$. Anyway, it is easy to see that there is plenty of room to accommodate the $g-2$ for several values of $\tan\beta, A_\mu$, and $m_{\tilde{\mu}}$. See Refs. [250, 279] for details.

10.1.2. Connecting $g-2$ and $\mu \to e\gamma$

The correlation between $g-2$ and $\mu \to e\gamma$ has been investigated before in the context of the MSSM [280, 281]. In order to connect $g-2$ and $\mu \to e\gamma$ we need to keep the discussion more general, leaving explicit the sneutrino mixings in the chargino contribution and the slepton mixings in the neutralino one [195]. We start with the chargino contribution.

Chargino contribution

To do so, we first start by assuming the mixing of the third generation to be decoupled from the first two. One can find that the sneutrino mass eigenvalues $m_{\tilde{\nu}_i}$ and the mixing angle ($\theta_{\tilde{\nu}}$) are determined through the diagonalization of the sneutrino mass matrix,

$$\begin{pmatrix} \cos\theta_{\tilde{\nu}} & \sin\theta_{\tilde{\nu}} \\ -\sin\theta_{\tilde{\nu}} & \cos\theta_{\tilde{\nu}} \end{pmatrix} \begin{pmatrix} m^2_{\tilde{L}_{11}} + \mathcal{D}^{\nu}_L & m^2_{\tilde{L}_{12}} \\ m^2_{\tilde{L}_{12}} & m^2_{\tilde{L}_{22}} + \mathcal{D}^{\nu}_L \end{pmatrix} \begin{pmatrix} \cos\theta_{\tilde{\nu}} & -\sin\theta_{\tilde{\nu}} \\ \sin\theta_{\tilde{\nu}} & \cos\theta_{\tilde{\nu}} \end{pmatrix} = \mathrm{diag}(m^2_{\tilde{\nu}_1}, m^2_{\tilde{\nu}_2}) \tag{96}$$

which leads to,

$$\tan 2\theta_{\tilde{\nu}} = \frac{2m^2_{\tilde{L}_{12}}}{m^2_{\tilde{L}_{11}} - m^2_{\tilde{L}_{22}}}, \tag{97}$$

where $m^2_{\tilde{L}_{11}}$, $m^2_{\tilde{L}_{22}}$, $m^2_{\tilde{L}_{12}}$ are the flavor-diagonal and off-diagonal soft mass parameters with $\mathcal{D}^{\nu}_L$ being the D-term contributions to their masses.

Now we can re-obtain the chargino contribution to $g-2$ leaving the dependence on $\theta_{\tilde{\nu}}$ explicit as follows,

$$a^{\tilde{\chi}^{\pm}_k}_{\mu} = \underbrace{\frac{m^2_{\mu}}{192\pi^2 m^2_{\tilde{\chi}^{\pm}_k}} \left(g_2^2|V_{k1}|^2 + y_{\mu}^2|U_{k2}|^2\right)\left[\sin^2\theta_{\tilde{\nu}}\, x_{k1}F^C_1(x_{k1}) + \cos^2\theta_{\tilde{\nu}}\, x_{k2}F^C_1(x_{k2})\right] +}_{A}$$
$$\underbrace{-\frac{2m_{\mu}}{48\pi^2 m_{\tilde{\chi}^{\pm}_k}}\, g_2 y_{\mu} Re[V_{k1}U_{k2}]\left[\sin^2\theta_{\tilde{\nu}}\, x_{k1}F^C_2(x_{k1}) + \sin^2\theta_{\tilde{\nu}}\, x_{k2}F^C_2(x_{k2})\right]}_{B}. \tag{98}$$

In the case of chargino dominance we find BR($\mu \to \mathrm{e}\gamma$) to be,

$$\begin{aligned} \mathrm{BR}(\mu \to \mathrm{e}\gamma) \quad = \quad & \frac{12\pi^3\alpha_{em}}{G_F m^4_{\mu}}\Bigg|\left(\frac{m^2_{\tilde{L}_{12}}}{m^2_{\tilde{\nu}_1} - m^2_{\tilde{\nu}_2}}\right)\Bigg[\frac{A\,\Delta_1}{x_{k_2}F^C_I(x_{k2}) + \sin^2\theta_{\tilde{\nu}}\Delta_1} \\ & + \frac{B\,\Delta_2}{x_{k_2}F^C_I(x_{k2}) + \sin^2\theta_{\tilde{\nu}}\Delta_2}\Bigg]\Bigg|^2. \end{aligned} \tag{99}$$

where $\Delta_i \equiv x_{k1} F_I^C(x_{k1}) - x_{k2} F_I^C(x_{k2})$, with $i = 1, 2$, and the A and B identified in Eq. (98). All coupling constants and functions have been defined previously.

Neutralino contribution

To compute the neutralino-slepton contributions to $g-2$ and $\mu \to e\gamma$ we need to work on a more general basis for the neutralino and slepton mixings. The diagonalization procedure for the neutralino was described earlier in Eq. (90). As for the sleptons, we follow the recipe of Ref. [195], where the full mixing structure was considered, and the mass mixing matrix K with $K\mathcal{M}^2K^\dagger = \mathrm{diag}(m^2_{\tilde{\ell}_1}, \ldots, m^2_{\tilde{\ell}_6})$ was derived. At the end one finds,

$$a_\mu^{\chi^0} = \frac{m_\mu}{16\pi^2} \sum_{i,m} \left\{ -\frac{m_\mu}{12 m^2_{\tilde{\mu}_m}} (|n^L_{im}|^2 + |n^R_{im}|^2) F_1^N(x_{im}) + \frac{m_{\chi_i^0}}{3 m^2_{\tilde{\mu}_m}} \mathrm{Re}[n^L_{im} n^R_{im}] F_2^N(x_{im}) \right\} \quad (100)$$

and,

$$\mathrm{BR}(\mu \to \mathrm{e}\gamma) = \frac{12\pi^3 \alpha_{em}}{G_F m_\mu^4} \left(|A|^2 + |B|^2 \right) \quad (101)$$

for the case of neutralino-slepton dominance, where

$$A = \frac{m_\mu}{16\pi^2} \sum_m \left\{ -\frac{m_\mu}{12 m^2_{\tilde{\chi}_i^0}} n^R_{\mu im} n^{R*}_{eim} x_{im} F_1^N(x_{im}) + \frac{1}{3 m_{\tilde{\chi}_i^0}} n^{L*}_{\mu im} n^{R*}_{eim} x_{im} F_2^N(x_{im}) \right\} \quad (102)$$

$$B = \frac{m_\mu}{16\pi^2} \sum_m \left\{ -\frac{m_\mu}{12 m^2_{\tilde{\chi}_i^0}} n^L_{\mu * im} n^L_{eim} x_{im} F_1^N(x_{im}) + \frac{1}{3 m_{\tilde{\chi}_i^0}} n^R_{\mu im} n^L_{eim} x_{im} F_2^N(x_{im}) \right\} \quad (103)$$

where $F_1^N(x)$ and $F_2^N(x)$ are defined in Eq. (83), $x_{im} = m^2_{\tilde{\chi}_i^0}/m^2_{\tilde{\ell}_m}$ and,

$$n^L_{\ell im} = \frac{1}{\sqrt{2}} \left(g_1 N_{i1} + g_2 N_{i2} \right) K^*_{m,\ell} - y_\ell N_{i3} K^*_{m,\ell+3},$$
$$n^R_{\ell im} = \sqrt{2} g_1 N_{i1} K_{m,\ell+3} + y_\ell N_{i3} K_{m,\ell}. \quad (104)$$

Combining Eqs. (98,99) and Eqs. (100,101), we can explore the correlation between $g-2$ and $\mu \to e\gamma$ in the MSSM for different regimes, namely *similar*

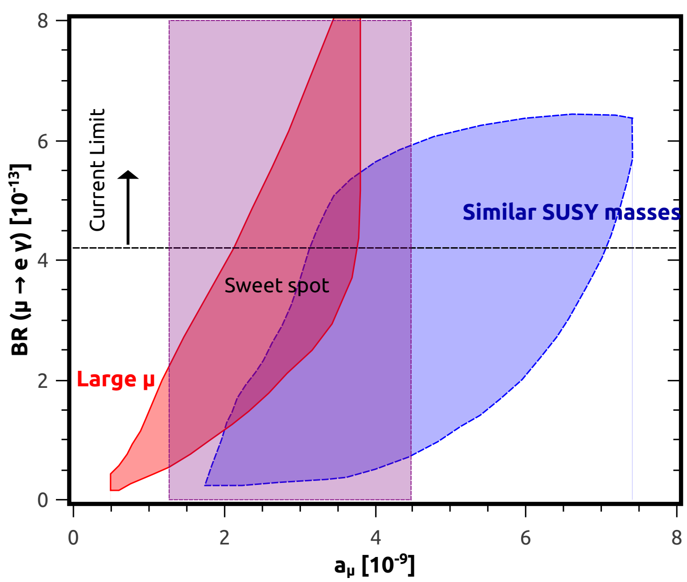


Figure 26: Correlation between $g-2$ and $\mu \to e\gamma$ in the MSSM for two different regimes: (i) Similar masses; (ii) Large μ regime. We overlay the 2σ band for $g-2$ and current limits on $\mu \to e\gamma$. Notice that there is a sweet spot lying below the current limit on $\mu \to e\gamma$ and in the 2σ region of $g-2$, where a signal in both observables are compatible with each other.

SUSY masses and *large μ term* as follows. We will adopt that squarks and gluinos are much heavier than the sleptons, charginos and neutralinos, with masses sufficiently large (TeV scale) to avoid LHC bounds at 13 TeV [240].

Similar SUSY masses

We will assume that supersymmetric particles have the same mass, varying from 300 GeV up to 800 GeV, keeping $\tan\beta = 50, A_\mu = 0$ and the hierarchies $m_{\tilde{L}_{12}}/m_{\tilde{L}_{11}} = 2 \times 10^{-5}$ and $m_{\tilde{R}_{12}}/m_{\tilde{R}_{11}} = 2 \times 10^{-5}$. This is similar to [195], except that there the authors scanned up to masses of 600 GeV. The result is shown in Fig. 26.

Large μ Regime

In this regime the most relevant parameters are the $\mu, \tan\beta, M_1, m^2_{\tilde{L}_{22}}$ and $m^2_{\tilde{R}_{22}}$. If $\tan\beta$ is relatively large, say $\tan\beta = 50$, the neutralino contribution starts dominating and growing with μ. Keeping the hierarchy $\mu > M_2 > M_1$, and varying $M_1, m^2_{\tilde{L}_{22}}$ and $m^2_{\tilde{R}_{22}}$ between $300 - 600$ GeV we find the result in Fig. 26, which agrees well with [195].

Looking at Fig. 26 we can see that there is some degree of correlation between $g-2$ and $\mu \to e\gamma$ in the scan performed, which was converted into regions using a interpolation function. A large region of the parameter space in the *similar SUSY masses* regime induces large contributions to a_μ incompatible with the data, whereas the *Large* μ typically yields corrections to a_μ in agreement with data. Interestingly, in both cases one can find a sweet spot within the 2σ band for $g-2$ shown in purple and below current limit on $\mu \to e\gamma$ where signals in observables can be made compatible with each other. The 2σ band for $g-2$ yields $\Delta a_\mu = (287 \pm 160) \times 10^{-11}$.

10.2. Left-Right Symmetry

Left-Right symmetric models are based on the gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ which under the addition of Left-Right parity means that the $SU(2)_L$ and $SU(2)_R$ couplings are identical, i.e. $g_L = g_R = g$, where $g_L = e/s_W$. These models may successfully be embedded in GUT theories, provide a natural environment for the see-saw mechanism, [282–286] and directly address parity violation at the weak scale [287, 288]. The fermion and scalar content of the model is,

$$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, Q_R = \begin{pmatrix} u_R \\ d_R \end{pmatrix}, \tag{105}$$

$$l_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}, l_R = \begin{pmatrix} N_R \\ e_R \end{pmatrix}, \tag{106}$$

$$\phi = \begin{pmatrix} \phi_1^0 & \phi_1^+ \\ \phi_2^- & \phi_2^0 \end{pmatrix}, \Delta_{L,R} = \begin{pmatrix} \delta^+_{L,R}/\sqrt{2} & \delta^{++}_{L,R} \\ \delta^0_{L,R} & -\delta^+_{L,R}/\sqrt{2} \end{pmatrix}, \tag{107}$$

with the fields transforming under parity and charge conjugation as follows: P: $Q_L \leftrightarrow Q_R, \phi \leftrightarrow \phi^\dagger, \Delta_L \leftrightarrow \Delta_R$; and C: $Q_L \leftrightarrow Q_R^c, \phi \leftrightarrow \phi^T, \Delta_{L,R} \leftrightarrow \Delta^*_{R,L}$. Here ϕ is a bi-doublet scalar not charged under B-L, whereas $\Delta_{L,R}$ are scalar triplets with $B-L=2$ [282, 285, 289]. The scalar sector of the model can take different forms, but with little impact on our reasoning. The vacuum expectations values follow the pattern below,

$$\langle\phi\rangle = \begin{pmatrix} \kappa_1/\sqrt{2} & 0 \\ 0 & \kappa_2/\sqrt{2} \end{pmatrix}, \ \langle\Delta_{L,R}\rangle = \begin{pmatrix} 0 & 0 \\ v_{L,R} & 0 \end{pmatrix}, \tag{108}$$

There are two important scales relevant for our purposes, one is the electroweak scale, with $\kappa \sim \kappa_1 \sim \kappa_2 \sim 246$ GeV, and the scale v_R at which the symmetries $SU(2)_R$ and $U(1)_{B-L}$ are spontaneously broken. After spontaneous symmetry breaking one finds,

$$\frac{M_{Z_R}}{M_{W_R}} = \frac{\sqrt{2} g_R/g_L}{\sqrt{(g_R/g_L)^2 - \tan^2\theta_W}}\,, \tag{109}$$

with $M_{W_R} = g_R v_R$. Remember that we will be assuming $g_L = g_R$ throughout, which implies that $M_{Z_R} \simeq 1.7 M_{W_R}$, unless stated otherwise.

The existence of new gauge bosons is a consequence of the extended gauge symmetry. They lead to the neutral current involving the Z' gauge boson,

$$\frac{g_L}{\sqrt{1-\delta\tan^2\theta_W}} \overline{f}\,\gamma_\mu \left(g_V^f - g_A^f\gamma^5\right)\, f\; Z'^\mu\,, \tag{110}$$

with the couplings determined by

$$g_V^f = \frac{1}{2}\left[\left\{\delta\tan^2\theta_W\left(T_{3L}^f - \mathrm{Q}^f\right)\right\} + \left\{T_{3R}^f - \delta\tan^2\theta_W \mathrm{Q}^f\right\}\right],$$
$$g_A^f = \frac{1}{2}\left[\left\{\delta\tan^2\theta_W\left(T_{3L}^f - \mathrm{Q}^f\right)\right\} - \left\{T_{3R}^f - \delta\tan^2\theta_W \mathrm{Q}^f\right\}\right],$$

where $T^f_{3L,3R} = \pm 1/2$ for $^{\text{up}}_{\text{down}}$-fermions, $\delta = g_L^2/g_R^2$, and Q^f being the corresponding electric charges. Moreover, the charged current is found to be,

$$\mathcal{L} = \frac{g_L}{\sqrt{2}}\left(\bar{l}_L U_L^\dagger \not{W}_L l'_L + \bar{Q}_L V_L^\dagger \not{W}_L Q'_L\right) + \text{h.c.} +$$
$$\frac{g_R}{\sqrt{2}}\left(\bar{l}_R U_R^\dagger \not{W}_R l'_R + \bar{Q}_R V_R^\dagger \not{W}_R Q'_R\right) + \text{h.c.},$$

where $U_{L/R}$ represent the PMNS mixing matrix for the left-handed and RH leptons and $V_{L/R}$ is the Cabibbo-Kobayashi-Maskawa matrix for the left and RH quarks.

Now that we have reviewed the model, we compute the Left-Right contribution to the observables of interest.

10.2.1. Results in the Left-Right Model

We will focus our discussion on the RH charged current, simply because the Z' contribution is dwindled, the scalar corrections are relatively small compared to the W_R mediated one [270], and on top of that are sensitive to the scalar content of the model which can vary. Thus in order to draw general conclusions we compute the one-loop processes that involve the W_R gauge boson and heavy RH neutrinos (N_R). We compute their contribution for two different regimes as follows,

(i) $M_{W_R} \gg M_{N_R}$

The charged current of the Left-Right model is identical to our simplified model with a gauge boson and a neutral fermion discussed in Sec. 9.2.1. Thus, we only need to adapt our findings knowing that g_v and g_a in Sec. 9.2.1, are now related as $g_v = g_a = g_R/\sqrt{2}\, U_R$, where U_R is the PMNS matrix for the RH leptons. Thus, in this regime we find,

$$\Delta a_\mu(N, W_R) = 2.2 \times 10^{-11} \left(\frac{g_R}{g_L}\right)^2 \left(\frac{1\,\text{TeV}}{M_{W_R}}\right)^2 \sum_N |U_{R\mu N}|^2, \tag{111}$$

and

$$\text{BR}(\mu \to e\gamma) \simeq 5 \times 10^{-8} \left(\frac{g_R}{g_L}\right)^4 \left(\frac{1\,\text{TeV}}{M_{W_R}}\right)^4 \times \sum_N |U_{ReN} U_{R\mu N}|^2, \tag{112}$$

where the sum in N runs over the three RH neutrino species.

It is clear that the $g-2$ contribution is rather small for TeV scale W_R masses, and one cannot push down the W_R masses to arbitrary low values due to the existence of collider and flavor constraints [223, 224, 290–296, 296–303]. Notice that the current limit on $\text{BR}(\mu \to e\gamma)$ enforces the product $|U_{ReN} U_{R\mu N}|$ to be below 5×10^{-3} for M_{W_R} masses at the TeV scale. Moreover, we conclude that one cannot reconcile possible signals in $g-2$ and $\mu \to e\gamma$.

(ii) $M_{W_R} \simeq M_{N_R}$

In this limit the results in a more general setting were derived in Eq. (36) and Eq. (39). After computing the coupling constants and matching the vector and axial-vector couplings to the Left-Right charged current, as done above, we obtain

$$\Delta a_\mu(N, W_R) \simeq 2.1 \times 10^{-11} \left(\frac{g_R}{g_L}\right)^2 \left(\frac{1\,\text{TeV}}{M_{W_R}}\right)^2 \sum_N |U_{R\mu N}|^2, \tag{113}$$

and

$$\text{BR}(\mu \to e\gamma) \simeq 2 \times 10^{-7} \left(\frac{g_R}{g_L}\right)^4 \left(\frac{1\,\text{TeV}}{M_{W_R}}\right)^4 \times \sum_N |U_{ReN} U_{R\mu N}|^2, \tag{114}$$

which agrees well with the result in [270].

The conclusion is similar to the previous regime, however $\text{BR}(\mu \to \text{e}\gamma)$ is about one order of magnitude larger, yielding tighter constraints on the W_R mass.

10.3. Two Higgs Doublet Model

The addition of a scalar doublet is perfectly possible since it does not disturb the parameter ρ determined in EWPT. In an $SU(2)_L \times U(1)_Y$ gauge theory with N scalar multiplets ϕ_i, the ρ parameter at tree-level is found to be [304],

$$\rho = \frac{\sum_{i=1}^{n} \left[I_i\,(I_i+1) - \frac{1}{4}\,Y_i^2\right] v_i}{\sum_{i=1}^{n} \frac{1}{2}\,Y_i^2 v_i}, \tag{115}$$

where I_i is the weak isospin, Y_i the weak hypercharge, and v_i are the VEVs of the neural fields. Since ρ is measured to be nearly one [305], we conclude that $SU(2)_L$ doublets with $Y = \pm 1$ along with singlets with $Y = 0$ do not alter the value of ρ, knowing that $I\,(I+1) = \frac{3}{4}\,Y^2$.

Thus, enlarging the SM with a scalar doublet is a natural framework, which is known as Two Higgs Doublet Model (2HDM) [306]. The 2HDM has a rich phenomenology and possesses several nice features such as links to Supersymmetry [273], axion models [307–310], baryogenesis [311–322] and even furnishes an environment for dark matter [323–327]. In what follows, we will be restricted to the non-supersymmetric 2HDM, see [328] for an extensive discussion. Generally speaking, there are several types of 2HDMs (see [121] for an excellent review). However, here we will focus on the type-III since there is a window for LFV. In what follows, we will be assuming the Higgs potential to be CP invariant. That said, the two Higgs doublets are,

$$\Phi_1 = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}\left(v + H_1^0 + iG^0\right) \end{pmatrix}, \qquad \Phi_2 = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}\left(H_2^0 + iA\right) \end{pmatrix}, \tag{116}$$

where the fields $G^{\pm}, G^0$ are Goldstone bosons, A is a CP-odd scalar, $H^{\pm}$ charged scalars, and finally we have $v = 246$ GeV. These doublets lead to a scalar potential which is found to be [329, 330],

$$\begin{aligned} V &= M_{11}^2\Phi_1^\dagger\Phi_1 + M^2\Phi_2^\dagger\Phi_2 - [M_{12}^2\Phi_1^\dagger\Phi_2 + \text{h.c.}] \\ &\quad +1/2\Lambda_1(\Phi_1^\dagger\Phi_1)^2 + 1/2\Lambda_2(\Phi_2^\dagger\Phi_2)^2 + \Lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \Lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) \\ &\quad + \left\{1/2\Lambda_5(\Phi_1^\dagger\Phi_2)^2 + \left[\Lambda_6(\Phi_1^\dagger\Phi_1) + \Lambda_7(\Phi_2^\dagger\Phi_2)\right]\Phi_1^\dagger\Phi_2 + \text{h.c.}\right\}. \end{aligned} \tag{117}$$

This scalar potential, with the symmetry breaking pattern of the two scalar doublets in Eq. (116), leads to a mixing between the neutral components which reads,

$$\begin{aligned} h &= H_1^0\ s_{\beta-\alpha} + H_2^0\, c_{\beta-\alpha}\,, \\ H &= H_1^0\ c_{\beta-\alpha} - H_2^0\, s_{\beta-\alpha}\,, \end{aligned} \tag{118}$$

with $\cos_{\beta-\alpha} \equiv \cos[(\beta-\alpha)]$ and $\sin_{\beta-\alpha} \equiv \sin[(\beta-\alpha)]$ where [329],

$$\sin_{2(\beta-\alpha)} = \frac{-2\Lambda_6 v^2}{m_H^2 - m_h^2}. \tag{119}$$

Moreover, the scalar potential gives rise to the scalar masses,

$$m_{H^\pm}^2 = M^2 + \frac{v^2}{2}\Lambda_3, \tag{120a}$$

$$m_A^2 - m_{H^\pm}^2 = -\frac{v^2}{2}(\Lambda_5 - \Lambda_4), \tag{120b}$$

$$m_H^2 + m_h^2 - m_A^2 = +v^2(\Lambda_1 + \Lambda_5), \tag{120c}$$

$$(m_H^2 - m_h^2)^2 = [m_A^2 + (\Lambda_5 - \Lambda_1)v^2]^2 + 4\Lambda_6^2 v^4. \tag{120d}$$

The Yuwaka Lagrangian of the 2HDM hosts the key information for the $g-2$ and $\mu \to e\gamma$ observables and it is found to be,

$$\begin{aligned}\mathcal{L}_Y = & \overline{Q}_j \widetilde{\Phi}_1 K_{ij}^* y_i^u u_{Ri} + \overline{Q}_i \Phi_1 y_i^d d_{Ri} + \overline{L}_i \Phi_1 y_i^e e_{Ri} \\ & + \overline{Q}_i \widetilde{\Phi}_2 [K^\dagger w^u]_{ij} u_{Rj} + \overline{Q}_i \Phi_2 [w^d]_{ij} d_{Rj} + \overline{L}_i \Phi_2 [w^e]_{ij} e_{Rj} + \text{h.c.}\,,\end{aligned} \tag{121}$$

where $\widetilde{\Phi}_i = i\sigma_2 \Phi_i^*$, Q, L are the quark and lepton $SU(2)_L$ doublets, K is the CKM matrix, y and w are the Yukawa couplings, and $i, j = 1, 2, 3$ run through the fermion generations. As in the SM, y is flavor conserving with

$$y_{ij} = \sqrt{2} m_f / v \delta_{ij}, \tag{122}$$

where m_f is the fermion mass, but w can have non-zero flavor changing entries relevant for $\mu \to e\gamma$. From Eq. (121) we get,

$$\begin{aligned}\mathcal{L}_Y \supset & \bar{e}\frac{1}{\sqrt{2}}\left[y^e(P_R + P_L)s_{\beta-\alpha} + (w^e P_R + w^{e\dagger} P_L)c_{\beta-\alpha}\right] e\, h \\ & + \bar{e}\frac{1}{\sqrt{2}}\left[y^e(P_R + P_L)c_{\beta-\alpha} - (w^e P_R + w^{e\dagger} P_L)s_{\beta-\alpha}\right] e\, H \\ & + \frac{i}{\sqrt{2}}\bar{e}\left(w^e P_R - w^{e^\dagger} P_L\right) e\, A \\ & + \bar{\nu}\,(w^e P_R)\, e\, H^+ + \text{h.c.},\end{aligned} \tag{123}$$

Eq. (123) gathers all the information needed to compute Δa_μ and $\mu \to e\gamma$ in the 2HDM type-III at the one-loop level.

10.3.1. Results

The Feynman diagrams that lead to corrections to $g-2$ and BR($\mu \to$ eγ) are displayed in Fig. 27. The $g-2$ contribution has already been obtained in

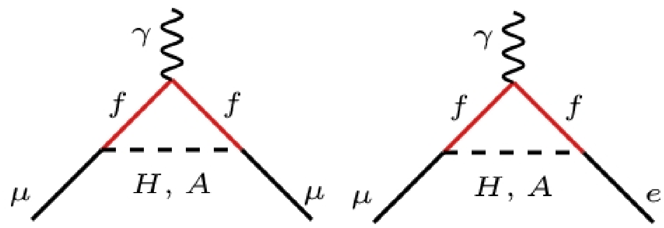


Figure 27: Feynman diagrams contributing to $g-2$ and $\mu \to e\gamma$ at one-loop level.

Eq. (13), and with a straightforward replacement of the (real) scalar couplings one finds,

$$\Delta a_\mu(\mathrm{H}+\mathrm{A}) \simeq \frac{1}{4\pi^2}\frac{m_\mu^2}{m_\phi^2}\sum_i \left[\left(y_{2i}^H\right)^2 \left(\frac{1}{6} - \epsilon_f\left(\frac{3}{4} + \log(\epsilon_f \lambda_H)\right)\right) + \right. \\ \left. + \, |y_{2i}^A|^2 \left(\frac{1}{6} + \epsilon_f\left(\frac{3}{4} + \log(\epsilon_i \lambda_A)\right)\right)\right]. \tag{124}$$

where $i = 1, 2, 3$ generally runs through all generations of charged leptons of mass m_f if one consider non-vanishing flavor violating mixings. Here, $\epsilon_f = m_f/m_\mu$, $\lambda_\phi = m_\mu/m_\phi$ ($\phi = H, A$) and,

$$y_{ij}^H \equiv \frac{1}{\sqrt{2}}\left(y_{ij}^e \cos_{\beta-\alpha} - w_{ij}^e \sin_{\beta-\alpha}\right), \\ y_{ij}^A \equiv i\frac{w_{ij}^e}{\sqrt{2}}, \tag{125}$$

which are easily identified from Eq. (123). Keeping in mind that in our notation y_{ij}^e is the Yukawa coupling appearing in front of the $\bar{e}_i e_j\, H$ interaction, whereas w_{ij}^e refers to the Yukawas of the interactions $\bar{e}_i e_j\, A$. Eq. (124) summarizes the one-loop contributions of neutral scalars in the 2HDM type-III. Since the singly charged scalar correction is negative and rather suppressed, Eq. (124) represents basically the overall prediction of the model. Moreover, the Higgs may also correct $g-2$ differently than in the SM. Such a correction can easily be extracted from the first term in Eq. (123), with $y_{ij}^h \equiv 1/\sqrt{2}(y_{ij}^e \sin_{\beta-\alpha} + w_{ij}^e \cos_{\beta-\alpha})$, when plugged in Eq. (124) along with the y_{ij}^H term. However, notice that in the decoupling limit, i.e. $\sin_{\beta-\alpha} \sim 1$ and $\cos_{\beta-\alpha} \sim -\Lambda_6 v^2/M^2 + O(v^4/M^4)$, the Higgs contribution is negligible since it will scale with the Yukawa coupling y_{22}^e,

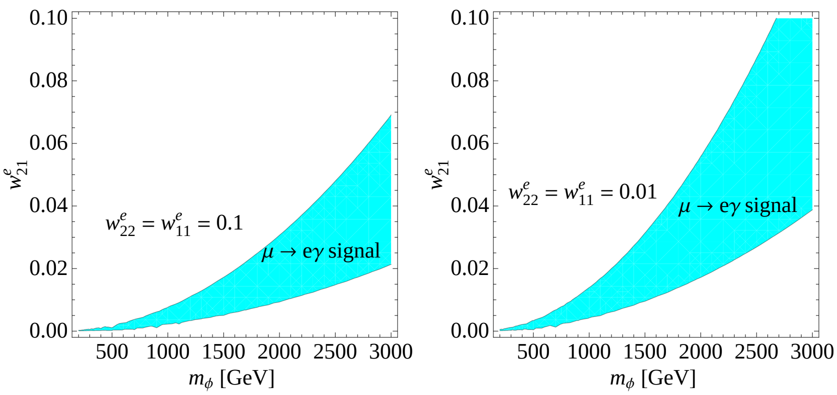


Figure 28: Region of parameter space in which the 2HDM type-III could address a signal in $\mu \to e\gamma$ with $\mathrm{BR}(\mu \to \mathrm{e}\gamma) = 4.2 \times 10^{-13} - 4 \times 10^{-14}$ for a specific set of couplings with $w^e_{22} = w^e_{11} = 0.1$ in the left panel, and $w^e_{22} = w^e_{11} = 0.01$.

which is proportional to m_μ/v. Thus, the leading corrections stem from H and A. That said in the decoupling limit one gets,

$$\Delta a_\mu(H + A) = 4.7 \times 10^{-11} \left(\frac{w^e_{2j}}{0.1}\right)^2 \left(\frac{100GeV}{m^2_\phi}\right)^2 . \tag{126}$$

As for the $\mathrm{BR}(\mu \to \mathrm{e}\gamma)$ we obtain,

$$\mathrm{Br}(\mu \to e\gamma) \approx \frac{3(4\pi)^3 \alpha_{\mathrm{em}}}{4G_F^2} \left(|A^M_{e\mu}|^2 + |A^E_{e\mu}|^2\right) \tag{127}$$

where,

$$\begin{aligned} A^M_{e\mu} = \frac{1}{16\pi^2 m^2_\phi} \sum_i \Bigg\{ & y^H_{2i} y^H_{1i} \left[\frac{1}{6} - \epsilon_i \left(\frac{3}{2} + \log(\epsilon^2_f \lambda^2)\right)\right] \\ & + y^A_{2i} y^A_{1i} \left[\frac{1}{6} + \epsilon_f \left(\frac{3}{2} + \log(\epsilon^2_i \lambda^2)\right)\right] \Bigg\}, \end{aligned} \tag{128}$$

and

$$\begin{aligned} A^E_{e\mu} = \frac{1}{16\pi^2 m^2_\phi} \sum_f \Bigg\{ & y^H_{2i} y^A_{1i} \left[\frac{1}{6} - \epsilon_i \left(\frac{3}{2} + \log(\epsilon^2_f \lambda^2)\right)\right] \\ & - y^H_{1i} y^A_{2i} \left[\frac{1}{6} + \epsilon_f \left(\frac{3}{2} + \log(\epsilon^2_i \lambda^2)\right)\right] \Bigg\}. \end{aligned} \tag{129}$$

In summary, Eqs. (127-129) represent the exact results for the $\mu \to e\gamma$ contribution in the 2HDM type-III at the one-loop level. Our results are more general than those presented in [178] which focused on the decoupling limit, i.e. when $\sin_{\beta-\alpha} \to 1$.

It is clear that the degree of complementarity between $g-2$ and $\mu \to e\gamma$ is rather arbitrary since they depend on the values used for the Yukawa couplings. In Fig. 28 we show that the 2HDM type-III can accommodate a signal in the $\mu \to e\gamma$ decay, delimited by the blue region, while avoiding $g-2$ constraints for two different choices of couplings. One can easily see that for these choices Δa_μ is very small using Eq. (126). For discussions concerning the scalar content of the model, which might be relevant to constrain the set up we just described, see [331–339]. However, for the heavy masses and small couplings used the parameter space is consistent with existing limits. Moreover, there are two-loop diagrams which also yield sizable contributions for some regions of the parameter space involving quarks and gauge bosons, as already pointed out in [340–342] and discussed further in [178, 343–345] which may affect our results.

10.4. Scotogenic Model

The scotogenic model is a scenario proposed in [346, 347], in which neutrinos acquire masses via their interactions with dark matter at the one-loop level. One simply extends the SM by a number of singlet fermions, conventionally dubbed RH neutrinos N_R^i, and a second scalar $SU(2)_L$ doublet η. In addition, a discrete $\mathbb{Z}_2$ symmetry is imposed under which the SM fields are even and both types of new fields are odd. This symmetry guarantees a number of important facts to hold true in the model: First, the new doublet does not acquire a VEV and consequently no tree-level neutrino masses arise. Second, there is no mixing between the new scalar particles and the SM Higgs. Lastly, the lightest particles charged under $\mathbb{Z}_2$ is either a fermion or a neutral scalar, making it stable. [9] The Lagrangian of the model is given by,

$$\mathcal{L} \supset -\frac{1}{2} M_i \overline{N_R^c}^i N_R^i - y_{ij} \overline{N_R^i} \widetilde{\eta}^\dagger L_L^j + \text{h.c.} - V(\phi, \eta), \tag{130}$$

[9] See [348–352] for some recent studies.

where the scalar potential is,

$$\begin{aligned} V(\phi,\eta) =& m_\phi^2\phi^\dagger\phi + m_\eta^2\eta^\dagger\eta + \frac{\lambda_1}{2}\left(\phi^\dagger\phi\right)^2 + \frac{\lambda_2}{2}\left(\eta^\dagger\eta\right)^2 + \lambda_3\left(\phi^\dagger\phi\right)\left(\eta^\dagger\eta\right) \\ &+ \lambda_4\left(\phi^\dagger\eta\right)\left(\eta^\dagger\phi\right) + \frac{\lambda_5}{2}\left[\left(\phi^\dagger\eta\right)^2 + \left(\eta^\dagger\phi\right)^2\right]. \end{aligned} \tag{131}$$

Indeed, we observe that unless η develops a VEV, neutrinos are massless at the tree-level. Upon electroweak symmetry breaking, the scalar sector contains, besides a Higgs boson shifted by its VEV v, four scalar degrees of freedom with masses,

$$m_{\eta^\pm}^2 = m_\eta^2 + v^2\lambda_3, \tag{132a}$$

$$m_{\eta_R}^2 = m_\eta^2 + v^2(\lambda_3+\lambda_4+\lambda_5), \tag{132b}$$

$$m_{\eta_I}^2 = m_\eta^2 + v^2(\lambda_3+\lambda_4-\lambda_5). \tag{132c}$$

One may calculate the one-loop correction to the neutrino masses, which amounts to [346, 353]

$$\mathcal{M}_{ij}^{(\nu)} = \frac{y_{ki}y_{kj}M_k}{32\pi^2}\left[\frac{m_R^2}{m_R^2-M_k^2}\log\left(\frac{m_R^2}{M_k^2}\right) - \frac{m_R^2}{m_I^2-M_k^2}\log\left(\frac{m_I^2}{M_k^2}\right)\right], \tag{133}$$

where a sum over repeated indices is implied. Note that in order to have at least two massive light neutrinos, we need at least two neutral fermions N_R^i.

The Yukawa interaction in Eq. (130) comprises an interaction of the form $y_{ij}\overline{\ell_L^i}\eta^- N_R^j$, very similar to our Eq. (16). Thus, the model gives rise to both LFV decays and a correction to $g-2$, the latter is however negative, as discussed above. The decay $\mu\to e\gamma$ is the most constraining one and proceeds with a branching ratio conventionally found in the literature as,

$$\mathrm{BR}\left(\ell_i\to\ell_j\gamma\right) = \frac{3(4\pi)^3\alpha_{\mathrm{em}}}{4G_F^2}|A_{ji}|^2\mathrm{Br}\left(\ell_\alpha\to\ell_\beta\nu_\alpha\overline{\nu_\beta}\right), \tag{134}$$

with the amplitude,

$$A_{ji} = \sum_{i=1}^{3}\frac{y_{kj}^* y_{ki}}{2(4\pi)^2}\frac{1}{m_{\eta^\pm}^2}F_2(x_i), \tag{135}$$

where $x_i = M_{N_i}^2/m_{\eta^\pm}^2$.

Starting from our exact expression (20) for the charged scalar, using $g_p = -g_s$, and approximating $m_e \ll m_\mu \ll m_{\eta^+/N}$, one gets,

$$F_2(x) = \frac{1 - 6x + 3x^2 + 2x^3 - 6x^2 \log x}{6(1-x)^4}, \tag{136}$$

in agreement with Refs. [350, 354]. For the contribution to $g-2$, we may consult Eq. (19) to observe that the contribution is negative.

Note that the results for the scotogenic model are simply those of the scalar $SU(2)_L$ doublet with hypercharge $Y = -1/2$ discussed in Sec. 9.1.1. The results are shown in Fig. 29. To illustrate the usefulness of our results, we have adapted the procedure to the one described in Sec. 9, and use $y_{ij} = g\Lambda_{ij}$ for the hierarchies described there. Clearly, we see that the signal region of $\mu \to e\gamma$ is very sensitive to the chosen hierarchy and naturally interpolates between scenarios already ruled out by current experiments, and such that can be probed only in the far future.

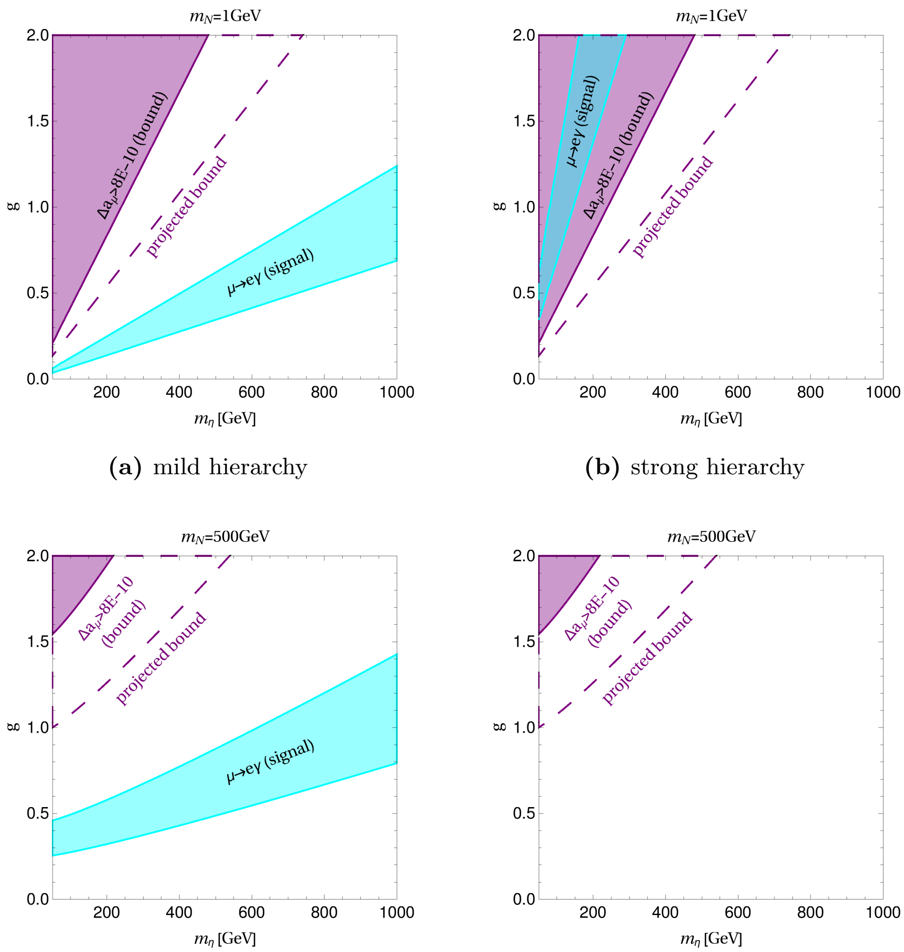
Note that in practice one should ensure that the couplings y_{ij} are also in agreement with neutrino oscillation data, DM constraints etc. We have not taken this into account and refer the reader to Refs. [350–352, 355–357] for more detailed studies along this path.

10.5. Zee-Babu Model

The Zee-Babu model [358–360] is yet another scenario which realizes neutrino masses at the loop-level. However, in this particular scenario, neutrinos remain massless up to two-loop order, where the singly and doubly charged scalars added to the SM field content induce a small Majorana mass. The Lagrangian of the model reads,

$$\mathcal{L} \supset f_{ij} \overline{\ell^c_{La}}^i \epsilon^{ab} \ell^{c\,j}_{Lb} h^+ + g_{ij} \overline{e^c_R}^i e_R{}^j k^{++} + \text{h.c.}\,, \tag{137}$$

where g is symmetric and f is anti-symmetric under the exchange of $i \leftrightarrow j$, and ψ^c denotes the charge conjugate spinor. While the above interaction can be made invariant under lepton number transformations if one assigns lepton numbers $L(h^+, k^{++}) = 2$, the scalar potential contains a coupling $\mu\, h^+ h^+ k^{--}$,



(a) mild hierarchy

(b) strong hierarchy

(c) mild hierarchy

(d) strong hierarchy

Figure 29: Signal region for $\mu \to e\gamma$ in the scotogenic model and constraints from $g-2$ for one light (top) and heavy (bottom) RH neutrinos.

which explicitly violates this symmetry by two units and induces Majorana neutrino masses. The expression for the neutrino mass matrix is,

$$\mathcal{M}_{ij}^{(\nu)} = 16\, \mu\, f_{ik}\, m_k\, g^*_{kl}\, I_{ln}\, m_n\, f_{nj}\,, \text{ where } I_{ln} \simeq \frac{\delta_{ln}}{(16\pi)^2 M^2} \frac{\pi^2}{3} \times \mathcal{O}(1) \tag{138}$$

and the factor of $\mathcal{O}(1)$ is due to a two-loop integral which must be evaluated numerically [361]. Furthermore, m_i are charged lepton masses and $M = \max(m_h, m_k)$.

Combining our results in Eqs. (13), (19), and (24), we may obtain an expression for the Zee-Babu fields' contribution the the $g-2$, as well as $\mu \to e\gamma$:[10]

$$\Delta a_\mu(h^+, k^{++}) = -\frac{m_\mu^2}{24\pi^2}\left(\frac{(f^\dagger f)_{\mu\mu}}{m_h^2} + 4\frac{(g^\dagger g)_{\mu\mu}}{m_k^2}\right), \tag{139}$$

$$\text{BR}(\mu \to e\gamma) = \frac{\alpha_{\text{em}}}{48\pi G_F^2}\left(\left|\frac{(f^\dagger f)_{e\mu}}{m_h^2}\right|^2 + 16\left|\frac{(g^\dagger g)_{e\mu}}{m_k^2}\right|^2\right), \tag{140}$$

valid in the limit $m_{h,k} \gg m_{\mu,e}$. This is in agreement with the results found in the literature [362–373]. Note that the Zee-Babu setting provides no explanation for the $g-2$ anomaly since the contribution is negative, nevertheless we may use it to derive constraints on the the parameter space by enforcing its contribution to be below the current and projected 1σ error bars in $g-2$ which read 80×10^{-11} and 34×10^{-11} respectively.

10.6. B-L Model

Since both baryon and lepton numbers are global symmetries in the SM, a natural and well motivated extension of the SM is the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$, which requires the addition of three RH neutrinos to cancel the triangle anomalies.[11]

In the $B - L$ model, the Z' possesses purely vectorial couplings to the

[10]Note that there is an extra factor of 2 coming with each insertion of f and a factor of 1/2 coming with each projector $P_{L/R}$.

[11]In fact, only the linear combination $B - L$ is anomaly-free, while $B + L$ is broken non-perturbatively. [374–378]

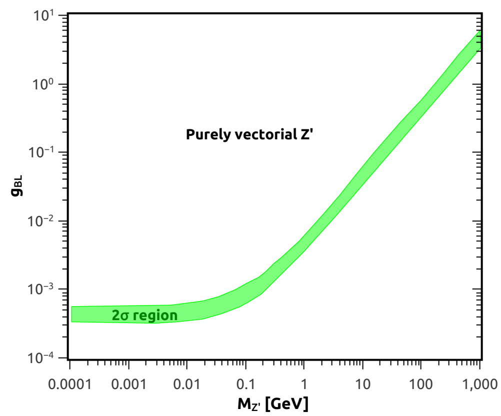


Figure 30: 2σ favored region for $g-2$ for a neutral vector boson with purely vectorial couplings to leptons.

fermions with,

$$\mathcal{L} \supset g_{BL} \sum_{i=1}^{3} \left(\bar{l}_i \gamma^\mu l_i + \bar{\nu}_i \gamma^\mu \nu_i \right) Z'_\mu. \tag{141}$$

The Z' gauge bosons gain mass either through a Stueckelberg mechanism [226] or a spontaneous symmetry breaking governed by a singlet scalar charged under B-L [379–382]. In the former case the $B-L$ symmetry remains unbroken. Either way, there is no $Z-Z'$ mass mixing at tree-level, and one can set the kinetic mixing to zero. That said, such vectorial interactions with charged leptons yield a contribution to $g-2$ but none to $\mu \to e\gamma$. The $g-2$ correction has been generally determined in Eq. (43). Since there is no mixing among lepton flavors, one can straightforwardly solve Eq. (43) to find the 2σ region for $g-2$ as drawn in Fig. 30. Notice that we have scanned over several orders of magnitude in the Z' mass and g_{BL} coupling, reaching 10 MeV mass. We point out that this result is applicable to any purely vector Z' model since the term in the Lagrangian Eq. (141) is rather general. Although, for sub-GeV masses one-loop corrections may induce a $Z-Z'$ kinetic mixing which could shift the favored $g-2$ region upwards. Anyway, existing collider limits on the Z' prohibit such a Z' to address the $g-2$ anomaly. Nevertheless one can extend this minimal $B-L$ scenario to accommodate a signal in $\mu \to e\gamma$ as we describe now.

particle	Q	u_R	d_R	L	e_R	ν_R	χ_1	χ_2	ϕ	s	Z'
spin	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0	0	1
Y_{B-L}	1/3	1/3	1/3	-1	-1	-1	-2	$+2$	0	-1	0

Table 5: Particle content and quantum numbers under $B-L$ symmetry.

10.7. B-L Model with Inverse See-Saw

In the previous $B-L$ model a signal in $g-2$ could be addressed in the light of existing constraints on the Z' mass. However, the canonical see-saw type-I with heavy RH neutrinos, which is naturally incorporated in the minimal $B-L$ model, gives rise to marginal contributions to $g-2$, and none to $\mu \to e\gamma$. Albeit, there is an alternative solution to accommodate a possible signal in $\mu \to e\gamma$ via a different type of see-saw mechanism known as inverse or low-scale see-saw where the mixing between the RH and active neutrinos is not so small. Thus, larger corrections to $\mu \to e\gamma$ are possible [193]. For some recent studies of a supersymmetric version of this model see also [383].

A possible realization of the inverse see-saw within the $B-L$ symmetry occurs by adding two singlet fermions $\chi_{1,2}$ per generation. The $B-L$ symmetry is broken spontaneously by introducing an $SU(2)_L$ singlet scalar with hypercharge $Y_{B-L} = -1$, which generates a mass for the new gauge boson, Z', associated with the $U(1)_{B-L}$ gauge group. The particle content is summarized in Tab. 5 for clarity. In addition to the $B-L$ symmetry, ones needs to impose upon the three singlet fermions represented by χ_1 a discrete $\mathbb{Z}_2$ symmetry to avoid mass terms such as $m\overline{\chi}_1\chi_2$.

The relevant part of the Lagrangian in this model is given by

$$\begin{aligned} \mathcal{L}_{B-L} \supset -\Big(\lambda_e \overline{L_L}\phi e_R + \lambda_\nu \overline{L_L}\tilde{\phi}\nu_R + \lambda_\chi \overline{\nu}^c_R s \chi_2 + \text{h.c.}\Big) - \\ - \frac{1}{\Lambda^3}\overline{\chi^c_1} {s^\dagger}^4 \chi_1 - \frac{1}{\Lambda^3}\overline{\chi^c_2} s^4 \chi_2 - V(\phi, s), \end{aligned} \tag{142}$$

with

$$V(\phi, s) = m_1^2 \phi^\dagger \phi + m_2^2 s^\dagger s + \lambda_1 (\phi^\dagger \phi)^2 + \lambda_2 (s^\dagger s)^2 + \lambda_3 (s^\dagger s)(\phi^\dagger \phi), \tag{143}$$

and $F'_{\mu\nu} = \partial_\mu Z'_\nu - \partial_\nu Z'_\mu$ being the field strength of the $U(1)_{B-L}$ gauge boson, which is minimally coupled via the covariant derivative

$$D_\mu = \partial_\mu - ig_s T^a G^a_\mu - ig\frac{\tau^i}{2} W^i_\mu - ig' Y B_\mu - ig_{BL} Y_{BL} Z'_\mu. \tag{144}$$

The last two terms in $\mathcal{L}_{B-L}$ are non-renormalizable terms, which are allowed by the symmetries and relevant for generating a TeV-scale mass for χ_1 and χ_2, as well as being required for the inverse see-saw mechanism. The scale Λ in these terms is a cut-off for the validity of the $B-L$ model.

As for neutrino masses, they are generated after spontaneous symmetry breaking via the Lagrangian,

$$\mathcal{L}^\nu_m = \mu_s \overline{\chi^c_2} \chi_2 + \left(m_D \bar{\nu}_L \nu_R + M_N \bar{\nu}^c_R \chi_2 + \text{h.c.} \right), \tag{145}$$

where $m_D = \frac{1}{\sqrt{2}} \lambda_\nu v$ and $M_N = \frac{1}{\sqrt{2}} \lambda_S v_s$, with $\mu_s = \frac{v_s^4}{4\Lambda^3}$.

Writing $\psi = (\nu^c_L, \nu_R, \chi_2)$, we can recast the Lagrangian above as $\overline{\psi^c} \mathcal{M}_\nu \psi$ with,

$$\mathcal{M}^{(\nu)} = \begin{pmatrix} 0 & m_D & 0 \\ m_D^T & 0 & M_N \\ 0 & M_N^T & \mu_s \end{pmatrix}. \tag{146}$$

Diagonalizing, this gives approximate masses

$$m_{\nu,\,\text{light}} \approx m_D \, M_N^{-1} \mu_s \, {M_N^{-1}}^T {m_D}^T \quad \text{and} \quad m_{\nu,\,\text{heavy}} \approx M_N \pm \mu_s. \tag{147}$$

For a cut-off Λ around 10^7 GeV, neutrino masses of the order eV can be achieved with Yukawa couplings of order one and $\mu_s \sim 10^{-9}$ GeV and TeV scale M_N.[12] Therefore, the Yukawa coupling λ_ν is no longer required to be tiny, which would preclude any experimental test of the scenario.

Note that, while the full 9×9 mass matrix in Eq. (146) is diagonalized by a unitary matrix U according to $\mathcal{M}^{(\nu)}_{\text{diag}} = U^\dagger \mathcal{M}^{(\nu)} U^*$, the light neutrino mass matrix in Eq. (147) is only a 3×3 sub-matrix that is not necessarily diagonalized

[12] Even without such a large cut-off, one may argue that μ_s is naturally small in 't Hooft's sense [384], since in the limit $\mu_s \to 0$ a global $U(1)$ lepton number symmetry can be defined.

in that way [385]. Thus, the model predicts that in general the leptonic mixing matrix $U_{\rm PMNS}$ is non-unitary. Most generally, we have

$$U = \begin{pmatrix} V_{3\times3} & V_{3\times6} \\ V_{6\times3} & V_{6\times6} \end{pmatrix}, \tag{148}$$

where one conventionally parametrizes $V_{3\times3} = \left(\mathbf{1} - \frac{1}{2}FF^\dagger\right) U_{\rm PMNS}$ such that the non-unitarity is measured exclusively by $F \equiv m_D M_N^{-1}$ instead of $U_{\rm PMNS}$ itself [386, 387]. Furthermore, the mixing of light and heavy states is given by $V_{3\times6} \simeq (0.I_{3\times3}, F)V_{6\times6}$. Finally, $V_{6\times6}$ is the matrix that diagonalizes the (ν_R, χ_2) subspace. As for the scalar sector, the Higgs boson h becomes a linear combination of ϕ and s [388].

We have assembled the basic building blocks of the model to compute $g-2$ and $\mu \to e\gamma$ in the model. The $g-2$ contribution stems from: (i) the W exchange via $\nu - \nu_R$ mixing; (ii) the $B-L$ gauge boson, Z'; (iii) the heavy Higgs, the latter of which is suppressed. The Z' contribution has been computed before, and the W exchange has been generally given in Eq. (35). One simply needs to plug in the vector and axial-vector coupling to find that the contribution is around $3 - 4 \times 10^{-9}$, thus being able to address the $g-2$ deviation at the 2σ level.

As for $\mu \to e\gamma$ the contribution to this decay is via W exchange, induced by the $\nu - \nu_R$ mixing. Again this calculation has been performed in Eq. (38), and after adapting the couplings to this specific model, one gets,

$$|\sum_N V_{\mu N} V_{eN}| \lesssim 10^{-5}, \tag{149}$$

where $N = \nu_R^1, \nu_R^2, \nu_R^3$.

10.8. 3-3-1 Model

This class of models represent electroweak extensions of the SM based on the gauge group $SU(3)_c \times SU(3)_L \times U(1)_X$, shortly referred to as 3-3-1 models. Due to the enlarged gauge symmetry, the fermionic generations are accommodated in the fundamental representation of $SU(3)_L$, i.e triplets. Since the SM spectrum should be reproduced, the triplet must contain the SM doublet, but the arbitrariness of the third component leads to a multitude of models based on this gauge symmetry [198, 220, 221, 389–408].

Generally speaking, 3-3-1 models send the appealing message of solving the puzzle of why there are three generations of fermions in Nature. The models are only self-consistent if there exist exactly three generations of fermions as a result of the triangle gauge anomalies and QCD asymptotic freedom [220, 389, 409]. Moreover, they can host a dark matter candidate [221, 399, 405–407, 410–421], generate neutrino masses [282–286, 422–424], among other things [425–434].

Anyways, since we are focused on $g-2$ and LFV we will adopt model known as 3-3-1 model with right handed neutrinos, $331\nu_R$ for short [392], where the third component of the fermion triplet is a RH neutrino as follows,

$$f_L^a = \begin{pmatrix} \nu_l^a \\ e_l^a \\ (\nu_R^c)^a \end{pmatrix} \sim (1,3,-1/3),\ e_R^a \sim (1,1,-1), \tag{150}$$

where $a = 1, 2, 3$.

We will set the hadronic sector aside since it is irrelevant for our purposes (see [392] for a more detailed discussion). In order to successfully generate masses for the fermions, one needs to invoke the presence of three scalar triplets and a sextet as follow,

$$\chi = \begin{pmatrix} \chi_1^0 \\ \chi_2^- \\ \chi_3^0 \end{pmatrix},\ \eta = \begin{pmatrix} \eta_1^0 \\ \eta_2^- \\ \eta_3^0 \end{pmatrix},\ \rho = \begin{pmatrix} \rho_1^+ \\ \rho_2^0 \\ \rho_3^+ \end{pmatrix},\ S = \begin{pmatrix} S_{11}^0 & S_{12}^- & S_{13}^0 \\ S_{12}^- & S_{22}^{--} & S_{23}^- \\ S_{13}^0 & S_{23}^- & S_{33}^0 \end{pmatrix}. \tag{151}$$

They have the following quantum numbers under the gauge group: $\chi \sim (1,3,-1/3)$, $\eta \sim (1,3,-1/3)$, $\rho \sim (1,3,2/3)$, $S \sim (1,6,-2/3)$. The spontaneous symmetry breaking pattern of the model proceeds via the triplet χ developing a non-trivial VEV, breaking $\mathrm{SU}(3)_L \times \mathrm{U}(1)_X \to \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$. This is followed by the breaking $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \to U(1)_{QED}$ via the VEVs of ρ and η, as indicated below.

$$\langle\chi\rangle = \begin{pmatrix} u'/\sqrt{2} \\ 0 \\ w/\sqrt{2} \end{pmatrix}, \langle\eta\rangle = \begin{pmatrix} u/\sqrt{2} \\ 0 \\ w'/\sqrt{2} \end{pmatrix}, \langle\rho\rangle = \begin{pmatrix} 0 \\ v\sqrt{2} \\ 0 \end{pmatrix}, \tag{152}$$

and

$$\langle S\rangle = \begin{pmatrix} v_{s1} & 0 & v_{s3} \\ 0 & 0 & 0 \\ v_{s3} & 0 & \Lambda \end{pmatrix}. \tag{153}$$

The role of the sextet is to give masses to the neutrinos, and in order to keep the symmetry breaking consistent, some conditions have to be satisfied, namely $\Lambda, w \gg v_{s3}, v, u \gg v_{s1}u'$, along with $w \gg w'$. In this way the SM gauge boson masses are correctly obtained, the ρ parameter remains close to unity, and the fermions acquire masses through the Yukawa Lagrangian which is divided into pieces, one where lepton flavor is conserved (LFC) and other where it is violated (LFV) as follows,

$$\mathcal{L}_{\rm LNC} \supset h^l_{ab}\bar{\psi}_{aL}\rho l_{bR} + h^\nu_{ab}\bar{\psi}^c_{aL}\psi_{bL}\rho + \text{h.c.}, \tag{154}$$

$$\mathcal{L}_{\rm LNV} \supset f^\nu_{ab}(\bar{\psi}^c_{aL})_m(\psi_{bL})_n(S^*)_{mn} + \text{h.c.} \tag{155}$$

where $a, b = 1, 2, 3$ account for the three generations and $m, n = 1, 2, 3$ indicate the entries of the sextet, and f_{ab} is symmetric. The last term in Eq. (154) gives rise to LFV interactions,

$$f_{12}\overline{(e_L)^c}\,\mu_L\,S^{++} = \frac{f_{12}}{2}\overline{e^c}\,\mu\,S^{++} - \frac{f_{12}}{2}\overline{e^c}\,\gamma_5\mu\,S^{++}, \tag{156}$$

which contributes to $\mu \to e\gamma$ via the presence of the doubly charged scalar in the sextet in Eq. (151) with $S^{++} \equiv S^{++}_{22}$. One can construct a similar term but proportional to f_{22} correcting $g-2$. Keep in mind that other charged leptons might run in the loop for $g-2$ and $\mu \to e\gamma$, and therefore the overall corrections in the 3-3-1 model have to be summed over all charged lepton flavors in general. We have computed both observables already in Eq. (24). Applying the results to the 3-3-1 model under study we get,

$$\Delta a_\mu\left(S^{++}\right) = -\frac{1}{4\pi^2}\frac{m_\mu^2}{m_{S^{++}}^2}\sum_b\left[|f_{2b}/2|^2\left(\frac{4}{3}-\epsilon_b\right)+|f_{2b}/2|^2\left(\frac{4}{3}+\epsilon_b\right)\right]. \tag{157}$$

where $\epsilon_b \equiv \frac{m_b}{m_\mu}$, and the sum is over all charged leptons of mass m_b in the loop, remembering that b is a fermion generation index. If one considers no mixing between the charged leptons then $m_b \equiv m_\mu$ and $\epsilon = 1$. In general, however, there might be a mixing with other charged leptons, and in that case one needs to sum over all fermion masses. This sum is only relevant in case there is mixing with the τ lepton.

As for the $\mu \to e\gamma$ we obtain,

$$\mathrm{BR}(\mu \to e\gamma) \simeq \frac{\alpha_{\mathrm{em}}\left|(f_{1a}^\dagger f_{2a})_{e\mu}\right|^2}{3\pi G_F^2 m_{S^{++}}^4}. \tag{158}$$

Considering only the non-diagonal term f_{12} non-vanishing with $f_{12} = f_{21} = 0.01$ and $f_{11} = 0.1$ we find $\mathrm{BR}(\mu \to \mathrm{e}\gamma) = 3.7\times 10^{-13}$ for $m_S^{++} = 2$ TeV, which is on the verge of being excluded. Since, generally speaking the $331\nu_R$ induces dwindled contributions to $g-2$ [221], apparently there is no room to accommodate possible signals in both observables. However, the $331\nu_R$ interestingly can give rise to a signal in $\mu \to e\gamma$ without getting in conflict with $g-2$. For other discussions of lepton flavor violation in 3-3-1 models see [435–437].

10.9. $L_\mu - L_\tau$

Lepton number is an accidental global symmetry of the SM, which is however broken by quantum corrections. It has been noted, however, that gauging any *difference* between two lepton family numbers with an abelian group leads to an anomaly free theory [438, 439, 439, 440]. $L_\mu - L_\tau$ is an explicit example which has been investigated in detail in [441–451]. The gauge group $SU(3)_c\times SU(2)_L\times U(1)_Y\times U(1)_{L_\mu - L_\tau}$ implies that only the second and third lepton generations are charged under the new abelian gauge symmetry and additionally, under which they carry opposite charges. As usual, the new abelian gauge group leads to the existence of a new massive gauge bosons, Z', which can acquire a mass either via spontaneous symmetry breaking governed by a new scalar field, or through the Stueckelberg mechanism [225, 226]. Either way, the new boson couples to the SM lepton doublets (L) via the term $\bar{L}\gamma^{\alpha} D_{\alpha} L$, where the covariant derivative is $D_{\alpha} = \partial_{\alpha} + ig'\, q\, Z'_{\alpha}$, with g' being the new gauge coupling of the $U(1)_{L_{\mu}-L_{\tau}}$ symmetry and q the corresponding charge ($q_{\mu,\nu_{\mu}} = 1$,$q_{\tau,\nu_{\tau}} = -1$). Writing down explicitly this term we get,

$$\mathcal{L}_{\text{fermions}} \supset g' \left(\bar{\mu}\gamma_{\alpha}\mu - \bar{\tau}\gamma_{\alpha}\tau + \bar{\nu_{\mu}}\gamma_{\alpha} P_L \nu_{\mu} - \bar{\nu_{\tau}}\gamma_{\alpha} P_L \nu_{\tau} \right) Z'^{\alpha} \,. \tag{159}$$

The very term in Eq. (159) gives rise to a contribution to $g-2$, which we find to be [cf. Eq. (43)],

$$\begin{aligned} &\Delta a_{\mu}\,(Z') = \frac{g'^{\,2}}{8\pi^2} \frac{m_{\mu}^2}{m_{Z'}^2} \int_0^1 \mathrm{d}x \frac{P_4^+(x)}{(1-x)\,(1-\lambda^2 x) + \epsilon_f^2 \lambda^2 x}, \\ &\text{where } P_4^+ = 2x^2(1-x), \text{ with } \lambda \equiv \frac{m_{\mu}}{m_{Z'}}, \end{aligned} \tag{160}$$

in agreement with [120, 207].

Approximating to leading order for a Z' much heavier than the muon, one finds,

$$\Delta a_{\mu}\,(Z') = \frac{g'^{\,2}}{12\pi^2} \frac{m_{\mu}^2}{m_{Z'}^2} \simeq 4 \times 10^{-9} \left(\frac{304\,\text{GeV}}{m_{Z'}^2} \right)^2 \left(\frac{g'}{0.5} \right)^2 , \tag{161}$$

which naturally addresses the $g-2$ measurement within 2σ. We emphasize that the result in Eq. (160) is completely general and applicable to any model with a gauge boson with purely vectorial couplings to muons. A more general general including both vector and axial-vector couplings and possible charged lepton mixings was obtained in Eq. (43b).

10.10. Dark Photon

Dark photon models refer to an abelian extension ($U(1)_X$) of the SM where the kinetic mixing dictates the observables [452–457]. In the minimal setup the model only contains a new vector boson which interacts with the SM particles through a kinetic mixing as follows,

$$\mathcal{L} = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} F'_{\mu\nu} F'^{\mu\nu} + \frac{\epsilon}{2} F'_{\mu\nu} B^{\mu\nu} + \frac{1}{2} m_{A'}^2 A'_{\mu} A'^{\mu}, \tag{162}$$

where $F'_{\mu\nu}$ and $B_{\mu\nu}$ are the field strengths of the $U(1)_X$ and $U(1)_Y$ gauge groups, respectively, ϵ the kinetic mixing parameter. Note that the mass term breaks $U(1)_X$ explicitly.

The kinetic mixing is the key input of the model and it governs the strength of the dark photon coupling with the SM particles. Such kinetic mixing is often taken to be zero, but it is generated via loops if there are new particles charged under $U(1)_X$. Changing to the basis of mass eigenstates, the $U(1)_X$ gauge boson, Z', is much lighter than the SM Z boson. For $\epsilon \ll 1$ we get,

$$\mathcal{L} = \epsilon' \, e \, Q \bar{f} \gamma^\mu f Z', \tag{163}$$

where $\epsilon' = \epsilon / c_W$.

We have already faced purely vectorial couplings to charged leptons in this article, more specifically in the context of the $B-L$ model. Therefore, the same results apply here, however, with an important difference: the dark photon can be much lighter than the Z boson and for this reason the integration of Eq. 43 is better handled numerically.

11. Summary and Outlook

In this article we have reviewed two key observables of modern particle physics, namely the muon anomalous magnetic moment and the lepton flavor violating decay $\ell_i \to \ell_j \gamma$. While recent measurements of the former observable may point towards new physics being around the corner, the latter gives rise to strong constraints on models beyond the Standard Model. We have reviewed the current experimental status of these observables in the light of the upcoming flagship experiments which will hopefully set a new direction in particle physics.

In the subsequent discussion, we have derived fully general expressions that allow the reader to compute the contribution of new physics to the rate of the process $\ell_i \to \ell_j \gamma$ as well as the anomalous magnetic dipole moment of a given lepton. We have studied these expressions extensively in the context of simplified, $SU(2)_L$ invariant extensions of the Standard Model. For definiteness, we have focused on the decay $\mu \to e\gamma$ and the anomalous magnetic moment of the muon. We have discovered that one may accommodate a signal in either observable while circumventing constrains from the other. In certain scenarios it is even feasible to address signals in both phenomena in the next generation of experiments.

In a final step, we have applied our findings to well-known UV-completions of the Standard Model to illustrate the broad applicability of our results to inspire any inclined reader to use those results for phenomenological studies in their favorite model.

We hope that with this review we have paved the way to new model building endeavors and motivated the interest in $g-2$ and $\mu \to e\gamma$ physics.

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A. Master integrals

In this appendix, we list the fully analytical loop integrals for the amplitude $\ell_i \to \ell_j\gamma$ for the different new particle contributions. All results have been numerically cross-checked with the Mathematica Package-X [458]. We begin with the neutral scalar integral represented by the graph in Fig. 5. It is given by

$$
\begin{aligned}
I_{f,1}^{(\pm)_1\,(\pm)_2} &\equiv I_{f,1}\left[m_i, (\pm)_1 m_j, (\pm)_2 m_f, m_\phi\right] \\
&= \int \underbrace{\mathrm{d}x\mathrm{d}y\mathrm{d}z\,\delta(1-x-y-z)}_{\equiv \mathrm{d}^3\mathbf{X}} \frac{x\left(y + (\pm)_1 z\,\frac{m_j}{m_i}\right) + (\pm)_2(1-x)\frac{m_f}{m_i}}{-xy\,m_i^2 - xz\,m_j^2 + x\,m_\phi^2 + (1-x)m_f^2}.
\end{aligned}
\tag{A-1}
$$

The equivalent diagram involving a charged internal scalar yields

$$I_{f,2}^{(\pm)_1\,(\pm)_2} = \int \mathrm{d}^3\mathbf{X} \frac{x\left(y + (\pm)_1 z\,\frac{m_j}{m_i} + (\pm)_1 \frac{m_{\nu_f}}{m_i}\right)}{-xy\, m_i^2 - xz\, m_j^2 + (1-x)m_{\phi^+}^2 + x\, m_{\nu_f}^2}. \tag{A-2}$$

This auxiliary function may be approximated for $m_j \ll m_i$ as

$$I_{f,2}^{(\pm)_1\,(\pm)_2} \simeq \frac{1}{m_{\phi^+}^2} \int_0^1 \mathrm{d}x \int_0^1 \mathrm{d}y\, x(1-x) \frac{xy + (\pm)_2 \epsilon_f}{\epsilon_f^2 \lambda^2 (1-x)\left(1 - \epsilon_f^{-2} xy\right) + x}. \tag{A-3}$$

Assuming in addition a very heavy mediator, i.e. $\lambda \to 0$, we find that

$$I_{f,2}^{(\pm)_1\,(\pm)_2} \simeq \frac{1}{m_{\phi^+}^2} \left[\frac{1}{12} + (\pm)_2 \frac{\epsilon_f}{2}\right]. \tag{A-4}$$

When there is a charged vector boson propagating in the loop, the occurrence of two gauge boson propagators complicates the calculation significantly. The result is rather lengthy and reads:

$$\begin{aligned}
I_{f,3}^{(\pm)_1\,(\pm)_2} &\equiv I_{f,3}\left[m_i, (\pm)_1 m_j, (\pm)_2 m_{N_f}, m_W\right] \\
&= \int \mathrm{d}^3\mathbf{X} \Bigg[\left[-xz\, m_i^2 - xy\, m_j^2 + (1-x) m_W^2 + x\, m_{N_f}^2\right]^{-1} \times \\
&\quad \times \Bigg\{ -(\pm)_2 3(1-x) \frac{m_{N_f}}{m_i} + (y + 2z(1-x)) + (\pm)_1 \frac{m_j}{m_i}(z + 2y(1-x)) \\
&\quad + \frac{m_i^2}{m_W^2} \Bigg[x\left((\pm)_1 (1-y)\frac{m_j}{m_i} - z\right)\left(z + (\pm)_1 y\,\frac{m_j}{m_i} + (\pm)_2 \frac{m_{N_f}}{m_i}\right)\left((\pm)_1 y\,\frac{m_j}{m_i} - (1-z)\right) \\
&\quad + xy\left(1 - (\pm)_2 \frac{m_{N_f}}{m_i}\right)\left(\frac{m_j^2}{m_i^2}(1-y) - z\right) \\
&\quad + xz\left(+(\pm)_1 \frac{m_j}{m_i} - (\pm)_2 \frac{m_{N_f}}{m_i}\right)\left((1-z) - y\,\frac{m_j^2}{m_i^2}\right)\Bigg]\Bigg\} \\
&\quad + m_W^{-2}\left[x(1-z)(\pm)_1 x(1-y)\frac{m_j}{m_i} - (\pm)_2 x \frac{m_{N_f}}{m_i} + y\left(1 - (\pm)_2 \frac{m_{N_f}}{m_i}\right) + \right. \\
&\quad \left. + z\left((\pm)_1 \frac{m_j}{m_i} - (\pm)_2 \frac{m_{N_f}}{m_i}\right)\right] \\
&\quad - m_W^{-2}\Bigg[(1-3x)\left((\pm)_2 \frac{m_{N_f}}{m_i} - (1-z) - (\pm)_1 (1-y)\frac{m_j}{m_i}\right) - xz - (\pm)_1 xy \frac{m_j}{m_i} \\
&\quad + \left((\pm)_2 \frac{m_{N_f}}{m_i} - 1\right)(1-3y)\left((\pm)_2 \frac{m_{N_f}}{m_i} - (\pm)_1 \frac{m_j}{m_i}\right)(1+3z)\Bigg] \times \\
&\quad \times \log\left(\frac{m_W^2}{-xz\, m_i^2 - xy\, m_j^2 + (1-x) m_W^2 + x\, m_{N_f}^2}\right)\Bigg]
\end{aligned} \tag{A-5}$$

The expression involving a neutral vector boson in the loop is similarly complicated:

$$\begin{aligned}
I_{f,4}^{(\pm)_1\,(\pm)_2} &\equiv I_{f,4}[m_i, (\pm)_1 m_j, (\pm)_2 m_{E_f}, m_Z] \\
&= \int \mathrm{d}^3\mathbf{X}\Bigg(\Big[-xz\,m_i^2 - xy\,m_j^2 + (1-x)m_{E_f}^2 + x\,m_Z^2\Big]^{-1} \times \\
&\quad \times \Bigg\{ 2x\left((1-z) + (\pm)_1(1-y)\frac{m_j}{m_i} - (\pm)_2 2\frac{m_{E_f}}{m_i}\right) \\
&\quad + \frac{m_i^2}{m_Z^2}\Bigg[(x-1)\left((\pm)_1\frac{m_j}{m_i} - (\pm)_2\frac{m_{E_f}}{m_i}\right)\left(z + (\pm)_1 y\,\frac{m_j}{m_i}\right)\left(1 - (\pm)_2\frac{m_{E_f}}{m_i}\right) \\
&\quad - z\left((\pm)_1\frac{m_j}{m_i} - (\pm)_2\frac{m_{E_f}}{m_i}\right)\left(xy\,\frac{m_j^2}{m_i^2} + (1-x+xz)\right) \\
&\quad - y(1-(\pm)_2\frac{m_{E_f}}{m_i})\left(xz + (1-x+xy)\frac{m_j^2}{m_i^2}\right)\Bigg]\Bigg\} \\
&\quad + m_Z^{-2}\left((\pm)_1\frac{m_j}{m_i} - (\pm)_2\frac{m_{E_f}}{m_i}\right) \times \\
&\qquad \times \left[z + (1-3z)\log\left(\frac{m_Z^2}{-xz\,m_i^2 - xy\,m_j^2 + (1-x)m_{E_f}^2 + x\,m_Z^2}\right)\right] \\
&\quad + m_Z^{-2}\left(1 - (\pm)_2\frac{m_{E_f}}{m_i}\right) \times \\
&\qquad \times \left[y + (1-3y)\log\left(\frac{m_Z^2}{-xz\,m_i^2 - xy\,m_j^2 + (1-x)m_{E_f}^2 + x\,m_Z^2}\right)\right]\Bigg).
\end{aligned} \tag{A-6}$$

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