On the distribution of stellar-sized black hole spins

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Abstract. Black hole spin will have a large impact on searches for gravitational waves with advanced detectors. While only a few stellar mass black hole spins have been measured using X-ray techniques, gravitational wave detectors have the capacity to greatly increase the statistics of black hole spin measurements. We show what we might learn from these measurements and how the black hole spin values are influenced by their formation channels.

1. Introduction

Gravitational wave observations will be sensitive to the spins of black holes [3]. Simulated search pipelines indicate that accounting for black hole spin can substantially improve the sensitivity of gravitational wave searches for binaries containing one [2] or two [15] black holes. However, this improvement comes at a cost of increased computational resources and is only realised if the spins of black holes are indeed significant. It is therefore necessary to balance the increased cost with the expectation that black holes will indeed have large spins. Here we discuss that evidence.

Truly isolated black holes are expected to satisfy the Kerr bound, $a_* \equiv cJ/GM^2 < 1$, where J and M denote the angular momentum and mass of the black hole and c and G the speed of light and Newton's gravitational constant respectively. This bound is manifested in the formula for the spatial coordinate of the event horizon in the Kerr solution, $r = M + M\sqrt{1 - a_*^2}$.

The value of a_* can formally be computed for any object that has a mass and angular momentum and it is worth recalling that the Kerr bound is only a bound on black holes. Many non-compact objects such as the Earth, the Sun and extremely rapidly rotating massive stars like VFTS 102[4] do not satisfy this bound, whilst compact objects like the rapidly rotating millisecond pulsar PSR J1748-2446ad [6] and the near-extremal black hole candidate Cygnus X-1[5] do. This is described in Table 1 where the spin values for the solid objects assume they are constant density spheres.

While the exterior spacetimes of these objects are all approximately vacuum, axisymmetric and stationary as required by the Kerr solution, except for Cygnus X-1 they do not describe black holes (do not contain horizons) and so can have a different multipole structure to the Kerr solution and are not constrained by the Kerr bound.

Things are even more extreme for elementary particles. The quantum spin of an elementary particle can be related to an asymptotic classical angular momentum in the sense of the Einstein-de Haas effect. In this way we can simply calculate values for the mass, specific angular momentum and charge of elementary particles. An electron's mass is 2.3×10^{-66}secs and its specific angular momentum is 6.4×10^{-22} so its a_* value is a whopping 2.8×10^{44}. (In fact things are even more extreme in the context of the Kerr-Newman solution as the electron has a
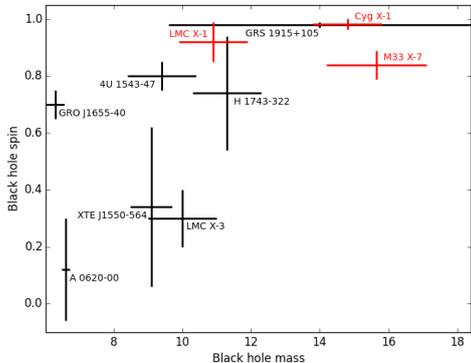
Figure 1. Masses and spins for 10 black holes with approximate error bars. The three high-mass-X-ray-binary systems, LMC X-1, Cygnus X-1 and M33 X-7 are indicated by names above the line and in red online.
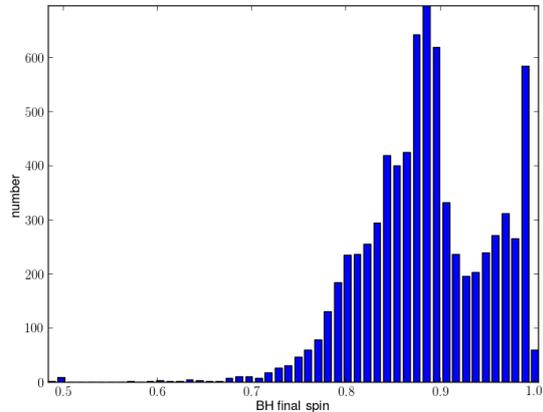
Figure 2. Distribution (upper bound) of spins for black holes in NSBH binaries, with masses greater than $2M_\odot$, assuming NS natal spins of 0, BH natal spins of 0.5 and merging in 15 billion years. Adapted from [14].

square root charge of 4.6×10^{-45} seconds). The reason for the violation of the Kerr bound in these cases is that elementary particles typically have Planckian values of spin, but not of mass. Of the Standard Model elementary particles, only the Higg's boson satisfies the Kerr bound because its charge and spin are thought to be zero.

Object	Mass [s]	J/M [s]	a_*
Earth	1.5×10^{-11}	1.3×10^{-8}	895
Sun	4.9×10^{-6}	6.1×10^{-6}	1.2
VFTS 102	1.2×10^{-4}	9.3×10^{-3}	75
PSR J1748-2446ad	6.9×10^{-6}	2.9×10^{-6}	0.4
Cygnus X-1	7.30×10^{-5}	7.23×10^{-5}	0.99

Table 1. Approximate values of mass and specific angular momentum for the Earth, Sun, a rapidly spinning massive star VFTS 102, a rapidly spinning neutron star PSR J1748-2446ad and a rapidly spinning black hole Cygnus X-1. For ease of comparison, both the mass and specific angular momentum values are given in seconds.

2. X-ray observations

X-ray observations of accretion disks have been able to measure the spins of around 10 stellar mass black holes [11]. These are displayed in Fig 1.

Of these most are Low Mass X-ray Binary (LMXB) sources and are unlikely to form the double compact object systems necessary to be seen by the current generation of ground-based gravitational wave detectors. Three of the systems (LMC X-3, M33 X-7 and Cygnus X-1) are High Mass X-ray Binaries (HMXB) where the companion to the black hole is a massive star, with mass greater than $\sim 10M_\odot$ and these systems do have a chance to form either neutron star-black hole binaries or binary black holes systems. These systems in themselves are not targets for current ground-based GW observatories as they are many millions of years away from merging but it is interesting to note that all three of these HMXBs have black holes with large spins $a* > 0.85$ and this suggests that the population of black holes in compact binary systems might be dominated by black holes with large spins. The probability of obtaining three values all above 0.7 from a flat distribution is only 3% .

On the other hand, observations of neutron star pulsars suggest that the spins of neutron stars in compact binaries are very low $a* < 0.05$. If this is indeed the case it implies that the formation channels of these objects are very different in terms of their angular momentum evolution. Neutron stars may be born with low spins or they may spin down rapidly after birth. The observation of young slowly spinning pulsars, such as the Crab pulsar rotating at only ~ 30Hz with a spin down rate of ~ 0.01Hz per year, 1000 years after its birth, constrains models of rapid spin-down for neutron stars.

3. Before collapse - tidal locking

To form a compact binary system requires two large heavy stars in a binary. Single massive stars are typically slowly rotating at the end of their lives dues to stellar wind losses and not differentially rotating due to angular momentum redistribution by magnetic torques [1]. But stars in binaries can spin up, increasing their rotational angular momentum by mass transfer [10]. The spin rate of both stars can be affected by tides and mass transfer [10]. Tidal interactions will tend to circularize the orbits of massive stars[7]. Massive stars in binaries can have rotational velocities ~ 200km/s [10].

Close binary systems are expected to be tidally locked [8]. The tidal locking timescale is between 100 and 10,000 years for helium core stars before they turn into black holes [12]. The tidal locking is more efficient on the larger object the closer to equal mass the bodies, perhaps giving an explanation why BHs in LMXBs have a wide range of spins, but predominantly high spins in HMXBs. It has been shown that there is a correlation between the black hole mass and orbital period in X-ray transients that may be partly explained by tidal locking [9]. For simplicity here we will assume that the moment of inertia for each star is given by kmr^2 where k is a constant (equal to $2/5$ for solid balls of uniform density). The relationship between radius and mass is often assumed to be $r \sim m^\alpha$ where typical values of α for massive stars are 0.5 to 0.8. Then we find that the dimensionless spin parameter a_* for an object with mass m_1 is given by

$$a_* = k\sqrt{\frac{c^2 r_\odot}{Gm_\odot}} \left(\frac{r_\odot}{a}\right)^{3/2} \left(\frac{m_1}{m_\odot}\right)^{2\alpha-1} \sqrt{\frac{m_1}{m_\odot} + \frac{m_2}{m_\odot}}. \tag{1}$$

where a denotes the semi-major axis of the orbit. Note that the three HMXB systems would have $a_* > 1$ if they were tidally locked stars at their current mass and separation values.

Observed rotation rates of massive stars in binaries tend to be significantly higher than the tidal locking rate [10], since a $10 - 10$ binary would need a separation of $10r_\odot$ to be tidally locked if the rotational velocity is ~ 300km/s, corresponding to $a_* \sim 100$. Stars that are further apart will have a lower rotational velocity. A separation of 1000 solar radii would have a tidally locked velocity of ~ 300m/s, since $a_* \sim a^{-3/2}$.

4. During collapse - core collapse and supernovae

Different scenarios have been proposed for how collapse proceeds for massive stars. These distinguish between failed core collapse (collapsar) with no explosion and successful core collapse (supernova) where the outer envelope is partially ejected [17]. Direct collapse may be favoured for heavy systems as the energy flux from the core is not enough to expel the outer shell. If a star collapses directly to a black hole, without shedding any matter and conserving its mass and its angular momentum, then its a_* value would be conserved.

For failed core collapse models, O'Connor and Ott [13] find a linear relationship between initial specific angular momentum of the star and final black hole spin parameter (their fig 11). The relation is approximately $j = 2.75a_*$. The specific angular momentum before collapse, j,

can be calculated using the tidal lock model above, to obtain a final a_* after collapse of,

$$a_* = 155k \left(\frac{r_\odot}{a}\right)^{3/2} \left(\frac{m}{m_\odot}\right)^{2\alpha+0.5}.$$ (2)

It should be noted that for rapidly spinning black holes, centrifugal support will tend to prevent the black hole from accreting large amounts of matter during a collapse scenario. If an initial, near extremal black hole forms, subsequent shells with $a_* > 1$ will not be accreted but expelled [9]. In this case, as in the successful core collapse, detailed modeling is required to derive final mass and spin values for the remnant.

5. After collapse - common envelope phase
The distribution of upper limits on the final BH spins, assuming natal spins of zero for neutron stars and 0.5 for black holes, for merging NSBH systems in [14] is shown in Fig. 2. This shows that if accretion in the common envelope phase does behave as a thin disk, then the final spins of BHs in NSBH binaries may be quite large, with most values falling in the range 0.8 to 1.0.

The distribution in Fig. 2 actually contains two distinct populations with different formation channels. Most of the high spin systems are actually light black holes that formed initially after collapse as neutron stars and subsequently accreted sufficient material during common envelope to form black holes, where the threshold for black hole formation is set at $2M_\odot$. For these light systems, the accretion of a small amount of matter is enough to spin them up considerably.

The other distribution consists of objects that collapsed to black holes before the common envelope phase. These systems are typically more massive and hence do not spin up as much during common envelope.

6. Conclusion
The limited X-ray evidence suggests that black holes in compact binaries may have high spins. We have discussed several mechanisms that could lead to these high spins, including tidal locking of massive progenitor stars, rapid collapse to black holes and common envelope accretion. There are still large uncertainties of all these processes, although they tend to reinforce one another. Measuring the spins of a large number of black holes with gravitational wave detectors will help to constrain some of these model uncertainties.

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