Logarithmic corrections to black hole entropy from Kerr/CFT

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Abstract

It has been shown by A. Sen that logarithmic corrections to the black hole area-entropy law are entirely determined macroscopically from the massless particle spectrum. They therefore serve as powerful consistency checks on any proposed enumeration of quantum black hole microstates. Sen's results include a macroscopic computation of the logarithmic corrections for a five-dimensional near extremal Kerr-Newman black hole. Here we compute these corrections microscopically using a stringy embedding of the Kerr/CFT correspondence and find perfect agreement.

Contents

1 Introduction

The Bekenstein-Hawking area-entropy law universally applies to any self-consistent quantum theory of gravity. Efforts to understand how the former constrains the latter have led to a wealth of insights.

Five years ago Sen et al. [1–6] pointed out that the leading corrections to this law, which are of order $\log A$ (where A is the area) are also universal in that they depend only on the massless spectrum of particles and are insensitive to the UV completion of the theory. The basic reason for this is that the effects of a particle of mass m can be accounted for by integrating it out, which generates local higher derivative terms in the effective action. These lead to corrections to the entropy which are suppressed by inverse powers of $m^2 A$ and cannot give $\log A$ terms.

The macroscopically computed logarithms serve as a litmus test for any proposed enumeration of quantum black hole microstates which is more refined than the test provided by the area law. Sen extensively analyzed a number of stringy examples all of which passed the test with flying colors [7], and further noted that the macroscopic computation does not match the loop gravity result. He also posed a matching of the logarithms as a challenge for Kerr/CFT [8–11].

In this paper we show that the logarithms indeed match for one microscopic realization [12,13] of Kerr/CFT obtained by embedding a certain near-extremal five-dimensional Kerr-Newman black hole into a string compactification.[1] The microscopic dual is a two-dimensional field theory defined as the IR fixed point of the worldvolume field theory on a certain brane configuration with scaled fluxes. This IR limit is certainly nontrivial but is not a conventional 2D CFT. Its properties are incompletely understood and have been studied in a variety of approaches: see *e.g.* [14–20].

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[1]The near-extremal regime sidesteps subtleties with logarithmic corrections at extremality.

The example of Kerr/CFT we chose is the simplest possible case. In the simplest examples considered by Sen, the macro-micro match has a somewhat trivial flavor: all logs vanish in a certain thermodynamic ensemble, with logs generated on both sides of the match by Legendre transforms to a different ensemble. However, later examples become quite intricate and provide compelling tests of a variety of stringy constructions. In the five-dimensional Kerr/CFT match herein, all logs vanish in a certain thermodynamic ensemble, and the character of the match is as in the simplest of Sen's examples. Non-trivial aspects of our match reside in its reliance on the values of the Kerr/CFT central charges and Kac-Moody levels which enter the microscopic computation. In particular, we find that a necessary non-vanishing level of the current dual to the electric field is provided by a Chern-Simons term which is crucially present in the effective action obtained from string theory. While it is reassuring that the simplest case works, perhaps more challenging matches such as the extremal 4D case may eventually provide more refined and compelling tests of Kerr/CFT.

This paper is organized as follows. In section 2 we describe the black hole solution, take its near horizon limit, and determine the corresponding quantum mixed state in the CFT. In section 3 we begin by stating the result of [6] for the logarithmic corrections to the Bekenstein-Hawking entropy and then proceed to compute them microscopically in the dual theory for two different enhancements of the global symmetries. We match the Bekenstein-Hawking entropy of the near extremal black hole to first order in the Hawking temperature with the Cardy formula and, using the result of Appendix A for the gauge Kac-Moody level, we show that the logarithmic corrections also agree.

Previous work on logarithmic corrections to black hole entropy includes [21–31].

2   The five-dimensional Kerr-Newman black hole

We consider a charged and rotating black hole solution of five-dimensional Einstein gravity minimally coupled to a gauge field. The dynamics of the latter is specified by the Yang-Mills-Chern-Simons Lagrangian, so that the complete action is,[2]

$$S_5 = \frac{1}{4\pi^2} \int d^5x \left(\sqrt{-g} \left(R - \frac{3}{4}F^2 \right) + \frac{1}{4}\epsilon^{abcde} A_a F_{bc} F_{de} \right). \tag{2.1}$$

Specifically, we are interested in the following Kerr-Newman black hole solution to (2.1) considered in [12],

$$ds_5^2 = -\frac{(a^2 + \hat{r}^2)(a^2 + \hat{r}^2 - M_0)}{\Sigma^2} d\hat{t}^2 + \Sigma \left(\frac{\hat{r}^2 d\hat{r}^2}{f^2 - M_0\hat{r}^2} + \frac{d\theta^2}{4} \right) - \frac{M_0 F}{\Sigma^2} (d\hat{\psi} + \cos\theta \, d\hat{\phi}) \, d\hat{t}$$

$$+\frac{\Sigma}{4}(d\hat{\psi}^2 + d\hat{\phi}^2 + 2\cos\theta \, d\hat{\psi} \, d\hat{\phi}) + \frac{a^2 M_0 B}{4\Sigma^2} (d\hat{\psi} + \cos\theta \, d\hat{\phi})^2, \tag{2.2}$$

$$A = \frac{M_0 \sinh 2\delta}{2\Sigma} \left(d\hat{t} - \frac{1}{2}ae^\delta(d\hat{\psi} + \cos\theta \, d\hat{\phi}) \right), \tag{2.3}$$

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[2]This coincides with the bosonic sector of minimal supergravity in five dimensions.

where we have defined the quantities

$$B = a^2 + \hat{r}^2 - 2M_0 s^3 c^3 - M_0 s^4 (2s^2 + 3) , \qquad F = a(\hat{r}^2 + a^2)(c^3 + s^3) - aM_0 s^3 ,$$

$$\Sigma = \hat{r}^2 + a^2 + M_0 s^2 , \qquad f = \hat{r}^2 + a^2 , \tag{2.4}$$

and $s \equiv \sinh \delta , c \equiv \cosh \delta$. The geometry depends on three independent parameters (a, M_0, δ) and the physical quantities of the black hole, *i.e.*, its mass, angular momentum and electric charge, are given in terms of those parameters by

$$M = \frac{3M_0}{2} \cosh 2\delta , \qquad J_L = aM_0 (c^3 + s^3) , \qquad Q = M_0 sc . \tag{2.5}$$

In five dimensions, it is possible to have a second angular momentum, J_R, but we set $J_R = 0$. Note that the $SU(2)_L$ angle is identified $\hat{\psi} \sim \hat{\psi} + 4\pi$.

This black hole displays inner and outer horizons located at

$$r_\pm^2 = \tfrac{1}{2}(M_0 - 2a^2) \pm \tfrac{1}{2}\sqrt{M_0(M_0 - 4a^2)} . \tag{2.6}$$

At the (outer) horizon, the angular velocities are

$$\Omega_L \equiv \Omega_{\hat{\psi}} = \frac{4a}{M_0} \frac{1}{(c^3 - s^3) + (c^3 + s^3)\sqrt{1 - 4a^2/M_0}} , \qquad \Omega_R \equiv \Omega_{\hat{\phi}} = 0 , \tag{2.7}$$

and the electric potential is

$$\Phi = \frac{c^2 s - s^2 c + (c^2 s + s^2 c)\sqrt{1 - 4a^2/M_0}}{c^3 - s^3 + (c^3 + s^3)\sqrt{1 - 4a^2/M_0}} . \tag{2.8}$$

Finally, the Hawking temperature is given by

$$T_H = \frac{1}{\pi\sqrt{M_0}} \frac{\sqrt{1 - 4a^2/M_0}}{c^3 - s^3 + (c^3 + s^3)\sqrt{1 - 4a^2/M_0}} , \tag{2.9}$$

and the Bekenstein-Hawking entropy is

$$S_{BH} = \pi\sqrt{2} \, M_0 \sqrt{(c^6 + s^6)M_0 - 2(c^3 + s^3)^2 a^2 + (c^4 + c^2 s^2 + s^4)\sqrt{M_0(M_0 - 4a^2)}} . \tag{2.10}$$

The black hole approaches extremality in the limit $M_0 \to 4a^2$. In this limit, the two horizons (2.6) coalesce at $r_+ = a$ and the Hawking temperature (2.9) vanishes. The charges (2.5) become

$$M_{\text{ext}} = 6a^2 \cosh 2\delta , \qquad J_{L\,\text{ext}} = 4a^3 (c^3 + s^3) , \qquad Q_{\text{ext}} = 4a^2 sc , \tag{2.11}$$

and the angular velocity (2.7) and electric potential (2.8) become

$$\Omega_{L\,\text{ext}} = \frac{1}{a(c^3 - s^3)} , \qquad \Phi_{\text{ext}} = \frac{c^2 s - s^2 c}{c^3 - s^3} . \tag{2.12}$$

At extremality, the Bekenstein-Hawking entropy (2.10) reduces to

$$S_{BH\,\text{ext}} = 8\pi a^3 (c^3 - s^3) . \tag{2.13}$$

In this paper we are interested in the near-extreme case so we introduce a small parameter $\hat{\kappa}$ that measures the deviation from extremality and write $M_0 = 4a^2 + a^2\hat{\kappa}^2$. Substituting this into (2.10) and keeping terms up to linear order in $\hat{\kappa}$, the near extremal entropy is

$$
\begin{aligned}
S_{BH\,\text{near ext}} &= 8\pi a^3(c^3 - s^3) + 4\pi a^3(c^3 + s^3)\,\hat{\kappa}^2 + \mathcal{O}(\hat{\kappa}^2) \\
&= \frac{\pi^2}{3}\,(6J_{L\,\text{ext}})\left(\frac{1}{\pi}\frac{c^3 - s^3}{c^3 + s^3} + \frac{\hat{\kappa}}{2\pi}\right) + \mathcal{O}(\hat{\kappa}^2)\,.
\end{aligned}
\tag{2.14}
$$

2.1 Near horizon, near extremal limit

Consider the coordinate transformation

$$
t = \tfrac{1}{2}\,\epsilon\,\Omega_{L\,\text{ext}}\,\hat{t}\,, \qquad r = \frac{\hat{r}^2 - r_+^2}{\epsilon\,r_+^2}\,, \qquad \psi = \hat{\psi} - \Omega_{L\,\text{ext}}\hat{t}\,, \qquad \phi = \hat{\phi}\,.
\tag{2.15}
$$

Here, r_+ is the location of the outer horizon given in (2.6) and $\Omega_{L\,\text{ext}}$ is the extremal angular velocity (2.12). Making this coordinate transformation in the five-dimensional geometry (2.2), (2.3), with M_0 fixed to its extremal value, $M_0 = 4a^2$, and letting $\epsilon \to 0$, one obtains the extremal near horizon geometry given in [12].

Here, we are interested in reaching the near horizon geometry of the black hole close, but not exactly at, extremality. This is the analog of the so-called near-NHEK limit for 4D Kerr considered in [9]. In order to do this, we still make the coordinate transformation (2.15), but now parametrize deviations from extremality with a parameter κ defined by

$$
M_0 = 4a^2 + a^2\epsilon^2\kappa^2\,.
\tag{2.16}
$$

Then the metric (2.2) gives rise to

$$
\begin{aligned}
ds_5^2 &= \frac{M_{\text{ext}}}{12}\left[-r(r + 2\kappa)dt^2 + \frac{dr^2}{r(r + 2\kappa)} + d\theta^2 + \sin^2\theta d\phi^2 \right. \\
&\qquad \left. + \frac{27J_{L\,\text{ext}}^2}{M_{\text{ext}}^3}\Big(\pi T_L(d\psi + \cos\theta\,d\phi) + (r + \kappa)dt\Big)^2\right]
\end{aligned}
\tag{2.17}
$$

in the $\epsilon \to 0$ limit. Here, we have defined

$$
T_L \equiv \frac{1}{\pi}\frac{c^3 - s^3}{c^3 + s^3}\,.
\tag{2.18}
$$

This notation will be clarified in the next subsection. The location of the horizon in (2.17) is at $r = 0$ and the associated surface gravity is κ. We denote the corresponding Hawking temperature by

$$
T_R \equiv \frac{\kappa}{2\pi}\,.
\tag{2.19}
$$

When we identify κ with the parameter $\hat{\kappa}$ introduced in (2.14), the metric (2.17) corresponds to the near horizon geometry of the black hole (2.2) close to extremality in the following complementary sense as well. Making the coordinate transformation (2.15) with $\epsilon = 1$ and expanding the metric components in (2.2) to leading order in $r \sim \hat{\kappa} \ll 1$ we obtain (2.17) with $\kappa = \hat{\kappa}$. In the rest of the paper we make this identification throughout.

The gauge field corresponding to the near horizon, near extremal geometry is obtained by accompanying the coordinate transformation (2.15) with the gauge transformation

$$A \to A - d\Lambda \,, \qquad \text{with} \quad \Lambda \equiv \Phi_{\text{ext}} \hat{t} \,. \tag{2.20}$$

Then the gauge field (2.20) becomes

$$A = -\frac{1}{2} a e^{\delta} \tanh 2\delta \left(d\psi + \cos\theta d\phi + e^{-2\delta}(r + \kappa) dt \right) \tag{2.21}$$

in the $\epsilon \to 0$ limit.

2.2 Frolov-Thorne temperatures

We now move on to compute the Frolov-Thorne temperatures corresponding to the near-extremal Kerr-Newman black hole, by adapting the strategy of [9] to our present context. Consider a scalar field

$$\varphi = e^{-i\omega\hat{t} + im\hat{\psi}} \, \hat{R}(\hat{r}) \, S(\theta) \, T(\hat{\phi}) \tag{2.22}$$

on the the black hole background (2.2), with charge q under the gauge field (2.3). Zooming into the near horizon region requires performing the coordinate transformation (2.15) combined with the gauge transformation (2.20). The charged scalar (2.22) thus becomes

$$\varphi = e^{iq\Lambda} e^{-in_R t + in_L \psi} \, R(r) \, S(\theta) \, T(\phi) \tag{2.23}$$

with

$$\Lambda = \frac{2\Phi_{\text{ext}}}{\epsilon \, \Omega_{L\,\text{ext}}} t \,, \quad m = n_L \,, \quad \omega = \frac{1}{2} \epsilon \, \Omega_{L\,\text{ext}} \left(n_R + \frac{2}{\epsilon} \, n_L - \frac{2q\Phi_{\text{ext}}}{\epsilon \, \Omega_{L\,\text{ext}}} \right) \,. \tag{2.24}$$

Now, the scalar field is in a mixed quantum state whose density matrix has eigenvalues given by the Boltzmann factor $e^{-\frac{1}{T_H}(\omega - m\Omega_L + q\Phi)}$, where T_H is the Hawking temperature (2.9). Identifying

$$e^{-\frac{1}{T_H}(\omega - m\Omega_L + q\Phi)} = e^{-\frac{2n_L}{T_L} - \frac{n_R}{T_R} - \frac{q}{T_Q}} \tag{2.25}$$

and using (2.24) we find the following Frolov-Thorne temperatures:

$$T_R = \frac{2}{\epsilon \, \Omega_{L\text{ext}}} T_H = \frac{2a}{\epsilon \pi \sqrt{M_0}} \frac{(c^3 - s^3)\sqrt{1 - 4a^2/M_0}}{c^3 - s^3 + (c^3 + s^3)\sqrt{1 - 4a^2/M_0}} \,, \tag{2.26}$$

$$T_L = -\frac{2}{\Omega_L - \Omega_{L\text{ext}}} T_H = \frac{2a}{\pi \sqrt{M_0}} \frac{(c^3 - s^3)}{c^3 + s^3 + (c^3 - s^3)\sqrt{1 - 4a^2/M_0}} \,, \tag{2.27}$$

$$T_Q = \frac{1}{\Phi - \Phi_{\text{ext}}} T_H = \frac{1}{2\pi \sqrt{M_0}} \frac{c^3 - s^3}{s^2 c^2} \,. \tag{2.28}$$

Near extremality, M_0 is given by (2.16) and (2.26)–(2.28) become, in the $\epsilon \to 0$ limit,

$$T_R = \frac{\kappa}{2\pi} \,, \qquad T_L = \frac{1}{\pi} \frac{c^3 - s^3}{c^3 + s^3} \,, \qquad T_Q = \frac{1}{4\pi a} \frac{c^3 - s^3}{s^2 c^2} \,. \tag{2.29}$$

Recall that both T_R and T_L have already appeared in our discussion: the former as the Hawking temperature (2.19) of the near-horizon, near-extremal metric (2.17) and the latter as a parameter, (2.18), in that metric. The present analysis elucidates the names given previously to those quantities.

3 Logarithmic correction to entropy

The logarithmic correction to the microcanonical entropy of a non-extremal, rotating charged black hole in general spacetime dimension D has been computed by Sen in [6]. His result applies to the near extremal black hole considered in this paper, which has a small but non zero Hawking temperature. Equation (1.1) in [6] for the correction to the microcanonical entropy reads

$$S_{mc}\left(M, \vec{J}, \vec{Q}\right) = S_{BH}\left(M, \vec{J}, \vec{Q}\right) + \log a \left(C_{local} - \frac{D-4}{2} - \frac{D-2}{2}N_C - \frac{D-4}{2}n_V\right) \quad (3.1)$$

where $N_C = \left[\frac{D-1}{2}\right]$ is the number of Cartan generators of the spatial rotation group and n_V is the number of vector fields in the theory. C_{local} arises from one loop determinants of massless fields fluctuating in the black hole background and vanishes in odd dimensions. The remaining contribution in (3.1) comes from zero modes and Legendre transforms. Plugging $D = 5$, $N_C = 2$ and $n_V = 1$ into (3.1) we have

$$S_{mc} = S_{BH} - 4\log a , \quad (3.2)$$

with, for the case at hand, S_{BH} given by (2.10).

3.1 Microscopic computation

We now change gears and compute the logarithmic correction to the entropy of the microscopic theory dual to the Kerr-Newman black hole. In [12] this solution was embedded into string theory and the microscopic dual thereby shown to be the infrared fixed point of a 1+1 field theory living on the brane intersection. This fixed point is a possibly non-local deformation of an ordinary 1+1 conformal field theory which preserves at least one infinite-dimensional conformal symmetry. While the string theoretic construction implies the existence of the fixed point theory, it exhibits a new kind of $1+1$ D critical behavior and is only partially understood. The near horizon geometry (2.17) has a $SL(2,R)_R \times U(1)_L$ isometry subgroup coming from the isometries of the AdS_2 submanifold and the unbroken $U(1)_L \subset SU(2)_L$ rotation isometry respectively. Various infinite-dimensional enhancements of this global isometry, involving different boundary conditions, have been extensively considered in the literature, and may be relevant in different circumstances or for different computations. See [20] for a recent discussion. We consider two of them which turn out to both give the same log corrections.[3]

3.1.1 $Vir_R \times Vir_L$

In this subsection we consider a CFT in which the global symmetries are enhanced as

$$SL(2,R)_R \times U(1)_L \to Vir_R \times Vir_L , \quad (3.3)$$

where Vir_L and Vir_R are left and right moving Virasoro algebras with generators L_n and $\bar{L}_n$ respectively. L_0 generates ψ rotations and $\bar{L}_0$ generates AdS_2 time translations.

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[3]Had they been different, the matching of logarithmic corrections would have singled one out.

We put the CFT on a circle along $\psi - t$ and consider the ensemble

$$Z(\tau, \bar{\tau}) = \text{Tr}\, e^{2\pi i \tau L_0 - 2\pi i \bar{\tau} \bar{L}_0} \,. \tag{3.4}$$

We assume that

$$4\pi\tau = \beta_L - \beta_R + i(\beta_L + \beta_R) \tag{3.5}$$

and $4\pi\bar{\tau} = \beta_L - \beta_R - i(\beta_L + \beta_R)$. Standard modular invariance of this partition function is $Z(\tau, \bar{\tau}) = Z(-1/\tau, -1/\bar{\tau})$. The microscopic dual to the Kerr-Newman black hole we are considering in this paper has an additional $SU(2) \times U(1)$ global symmetry, corresponding to the $SU(2)$ rotation isometry and the $U(1)$ gauge symmetry. Turning on the associated chemical potentials, the partition function becomes

$$Z(\tau, \bar{\tau}, \vec{\mu}) = \text{Tr}\, e^{2\pi i \tau L_0 - 2\pi i \bar{\tau} \bar{L}_0 + 2\pi i \mu_i P^i} \tag{3.6}$$

and it obeys the modular transformation rule

$$Z(\tau, \bar{\tau}, \vec{\mu}) = e^{-\frac{2\pi i \mu^2}{\tau}} Z\left(-\frac{1}{\tau}, -\frac{1}{\bar{\tau}}, \frac{\vec{\mu}}{\tau}\right) \,. \tag{3.7}$$

Here μ_i are left chemical potentials associated with the left moving conserved charges P^i and $\mu^2 \equiv \mu_i \mu_j k^{ij}$ with k^{ij} the matrix of Kac-Moody levels of the left moving currents. In our case i, j run from 1 to 2 but, for the sake of generality, we temporarily assume they run from 1 to n. This partition function is related to the density of states, ρ, at high temperatures by

$$Z(\tau, \bar{\tau}, \vec{\mu}) = \int dE_L \, dE_R \, d^n p \, \rho(E_L, E_R, \vec{p}) \, e^{2\pi i \tau E_L - 2\pi i \bar{\tau} E_R + 2\pi i \mu_i p^i} \,, \tag{3.8}$$

where E_L, E_R, p^i are the eigenvalues of $L_0, \bar{L}_0, P^i$ respectively. For small τ, (3.7) implies that

$$Z(\tau, \bar{\tau}, \vec{\mu}) \approx e^{-\frac{2\pi i \mu^2}{\tau}} e^{-\frac{2\pi i E_L^v}{\tau} + \frac{2\pi i E_R^v}{\bar{\tau}} + \frac{2\pi i \mu_i p_v^i}{\tau}} \,. \tag{3.9}$$

Then, inverting (3.8), we obtain the following expression for the density of states:

$$\rho(E_L, E_R, \vec{p}) \simeq \int d\tau d\bar{\tau} \, d^n \mu \, e^{2\pi i \left(-\frac{\mu^2}{\tau} - \frac{E_L^v}{\tau} + \frac{E_R^v}{\bar{\tau}} - E_L \tau + E_R \bar{\tau} - \mu_i p^i\right)} \,, \tag{3.10}$$

where we have assumed that the vacuum is electrically neutral, $p_v^i = 0$. This integral may be evaluated by saddle point methods. The integrand reaches an extremum at

$$\tau_0 = \sqrt{\frac{4 E_L^v}{4 E_L - \mathcal{P}^2}} \,, \qquad \bar{\tau}_0 = -\sqrt{\frac{E_R^v}{E_R}} \,, \qquad \mu_{0i} = -k_{ij} p^j \sqrt{\frac{E_L^v}{4 E_L - \mathcal{P}^2}} \,, \tag{3.11}$$

where the matrix k_{ij} is the inverse of k^{ij} and $\mathcal{P}^2 \equiv p^i p^j k_{ij}$. The leading contribution to the entropy is obtained by evaluating (3.10) at the saddle (3.11). This gives

$$S = \log \rho_0 = 2\pi \sqrt{-E_L^v (4 E_L - \mathcal{P}^2)} + 2\pi \sqrt{-E_R^v (4 E_R)} \,. \tag{3.12}$$

Putting

$$E_L^v = E_R^v = -\frac{c}{24}, \quad E_L - \frac{\mathcal{P}^2}{4} = \frac{\pi^2}{6}c\,T_L^2, \quad E_R = \frac{\pi^2}{6}c\,T_R^2, \tag{3.13}$$

we have

$$S = \frac{\pi^2}{3}c\,T_L + \frac{\pi^2}{3}c\,T_R. \tag{3.14}$$

The analysis of [12,20] yields $c = 6J_{L\,\text{ext}}$ and using the values for T_L, T_R obtained in (2.29), we see that (3.14) matches the near-extremal Bekenstein-Hawking entropy (2.14) to linear order in κ. This extends the match of [12] from the extremal to the near-extremal regime.

The logarithmic correction ΔS to the leading entropy (3.12) is generated by Gaussian fluctuations of the density of states (3.10) about the saddle (3.11):

$$\Delta S = -\frac{1}{2}\log\frac{\det\mathcal{A}}{(2\pi)^{n+2}}, \tag{3.15}$$

where $\mathcal{A}$ is the determinant of the matrix of second derivatives of the exponent in the integrand of (3.10) with respect to τ, μ_i, $\bar{\tau}$. We find

$$\det\mathcal{A} = \frac{(2\pi)^{n+2}}{16}\left(-E_L^v\right)^{-\frac{n+1}{2}}\left(4E_L - \mathcal{P}^2\right)^{\frac{n+3}{2}}\left(-E_R^v\right)^{-\frac{1}{2}}\left(4E_R\right)^{\frac{3}{2}}\det k^{ij}. \tag{3.16}$$

We now fix $n = 2$ for the left moving $SU(2) \times U(1)$ current algebra corresponding to $SU(2)$ rotations and the gauge field. The $SU(2) \times U(1)$ charges are $p^1 = 0$ (because $J_R = 0$) and $p^2 \propto Q_{\text{ext}}$. The $U(1)$ Kac-Moody level $k^{22} \equiv k_Q$ is given in Appendix A, the $SU(2)$ level is $k^{11} \equiv k_J \propto c$ [32], and $k^{12} = k^{21} = 0$. Taking into account (2.11), we thus have the following scalings,

$$E_L^v, E_R^v, E_L - \mathcal{P}^2/4, E_R \sim a^3, \quad k_Q \sim a, \quad k_J \sim a^3. \tag{3.17}$$

Bringing (3.17) to (3.15, 3.16), we obtain

$$\Delta S = -5\log a. \tag{3.18}$$

3.1.2 $\widehat{U(1)}_R \times Vir_L$

In this subsection we consider a warped CFT, in which the global symmetries are enhanced as

$$SL(2,R)_R \times U(1)_L \to \widehat{U(1)}_R \times Vir_L. \tag{3.19}$$

Here $\widehat{U(1)}_R$ is a left moving Kac-Moody algebra whose zero mode $\tilde{R}_0$ generates the right sector time translations in AdS_2 and Vir_L is a left moving Virasoro algebra whose zero mode $\tilde{L}_0$ generates the left sector $U(1)_L$ rotational isometry. The symmetry algebra of our warped CFT is

$$\left[\tilde{L}_m, \tilde{L}_n\right] = (m-n)\tilde{L}_{m+n} + \frac{c}{12}(m^3 - m)\delta_{m+n},$$

$$\left[\tilde{R}_m, \tilde{R}_n\right] = \frac{k_R}{2}m\delta_{m+n}, \quad \left[\tilde{L}_m, \tilde{R}_n\right] = -n\tilde{R}_{m+n},$$

where $\tilde{L}_m$ and $\tilde{R}_m$ are the Virasoro and Kac-Moody generators respectively. Putting the theory on a circle along ψ, the partition function at inverse temperature β and angular

potential θ is given by $Z(\beta,\theta) = \mathrm{Tr}\, e^{-\beta\tilde{R}_0 + i\theta\tilde{L}_0}$. On the other hand, in [18] it was shown that by redefining the charges as

$$L_n = \tilde{L}_n - \frac{2}{k_R}\tilde{R}_0\tilde{R}_n + \frac{1}{k_R}\tilde{R}_0^2\delta_n\,, \quad R_n = \frac{2}{k_R}\tilde{R}_0\tilde{R}_n - \frac{1}{k_R}\tilde{R}_0^2\delta_n\,, \tag{3.20}$$

and putting the theory on the same circle but in the different ensemble[4]

$$Z(\tau,\bar{\tau}) = \mathrm{Tr}\, e^{2\pi i\tau L_0 - 2\pi i\bar{\tau}R_0}\,, \tag{3.21}$$

the partition function obeys the usual CFT modular invariance:

$$Z(\tau,\bar{\tau}) = Z(-1/\tau, -1/\bar{\tau})\,. \tag{3.22}$$

Assuming

$$4\pi\tau = \beta_L - \beta_R + i(\beta_L + \beta_R) \tag{3.23}$$

and $4\pi\bar{\tau} = \beta_L - \beta_R - i(\beta_L + \beta_R)$ we may then proceed as in the previous section replacing $\bar{L}_0$ with R_0 everywhere starting from equation (3.6) onwards. We thus arrive at the same results for the leading entropy and its logarithmic correction.

It should be noted that the enhancement (3.19) is somewhat unusual in the context of warped AdS$_3$ [17]. A third more natural enhancement $SL(2,R)_R \times U(1)_L \to Vir_R \times \widehat{U(1)}_L$ in that context is also possible [20]. However, this case may not be treated as (3.19) above because the arguments of [18] do not apply to the case when the identification in the bulk (along ψ) is precisely anti-aligned with the action of L_0 (along t). It is an important outstanding problem in Kerr/CFT to generalize the arguments of [18] to accomodate this case.

3.2 Match of the macroscopic and microscopic computations

We have already exhibited the match, in the near-extremal regime, of the bulk and microscopic results for the leading term of the entropy of the five-dimensional Kerr-Newman black hole under consideration: the Cardy formula (3.14) reproduces the near-extremal Bekenstein-Hawking entropy (2.14).

We will now show that the logarithmic corrections also agree. In order to furnish a sensible comparison, one must ensure that both results are given in the same ensemble. This is not the case for the macroscopic, (3.2), and microscopic, (3.18), results given above. The former assumes the entropy to be a function of the energy $Q[\partial_{\hat{t}}]$ conjugate to the asymptotic time which features in the full black hole solution (2.2), while the latter is instead a function of the energy $Q[\partial_t]$ conjugate to the near horizon time which appears in (2.17). The transformation between the macroscopic and microscopic density of states requires a Jacobian factor (Appendix B),

$$\rho_{\mathrm{bulk}} = \frac{\delta Q[\partial_t]}{\delta Q[\partial_{\hat{t}}]}\,\rho\,. \tag{3.24}$$

---

[4]A change of ensemble may result in different logarithmic corrections to the entropy. However, as explained in Appendix B, the change of ensemble corresponding to the charge redefinitions (3.20) here does not imply any change in the logarithmic correction to the entropy.

Now, from the change of coordinates (2.15) and the expression for the extremal angular velocity in (2.12), we see that this Jacobian scales like

$$\frac{\delta Q[\partial_t]}{\delta Q[\partial_{\hat{t}}]} \sim a \,. \tag{3.25}$$

Thus

$$\Delta S_{\text{bulk}} = \Delta S + \log a \,, \tag{3.26}$$

which indeed is satisfied by (3.2) and (3.18).

Acknowledgements

A   Computation of k_Q

In this appendix we compute the level k_Q of the $U(1)$ Kac-Moody algebra associated with the gauge field A_μ. We do not perform a full asymptotic symmetry group analysis here. We expect that with appropriate boundary conditions on the gauge field this Kac-Moody is consistent with the rest of the asymptotic symmetries used in Section 3. Here we are particularly interested in deriving the scaling of the level k_Q with a.

Thus we assume the $U(1)$ current algebra is generated by

$$\Lambda_\eta = \eta(\tilde{y}) \,, \tag{A.1}$$

where $\tilde{y} = \pi T_L \, \psi$. In modes, the generators

$$p_n = -2\pi T_L \, e^{-in\tilde{y}/(2\pi T_L)} \,, \tag{A.2}$$

satisfy the algebra

$$[p_m, p_n] = 0 \,. \tag{A.3}$$

Using the formulas in [33], one can compute the central extension in the corresponding Dirac bracket algebra. We find:

$$\{Q_{p_m}, Q_{p_n}\} \quad = \quad -im \, 24\pi^2 T_L^2 a e^\delta \tanh 2\delta \, \delta_{m+n} \,. \tag{A.4}$$

The central extension comes entirely from the Chern-Simons term in the action (2.1). Passing to the commutators $\{\,,\} \to -i[\,,]$ we obtain the current algebra,

$$[P_m, P_n] = \frac{k_Q}{2} m \delta_{m+n} \,, \tag{A.5}$$

with level given by

$$k_Q = 12 \, (2\pi T_L)^2 \, a e^\delta \tanh 2\delta \,. \tag{A.6}$$

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B Change of ensemble

Under a charge redefinition, $\vec{q} = \vec{q}(\vec{q}')$, the density of states, $\rho(\vec{q})$, transforms with the appropriate Jacobian factor as

$$\rho'(\vec{q}') = \frac{\partial(q_1, q_2, \ldots)}{\partial(q_1', q_2', \ldots)}\rho(\vec{q})\,. \tag{B.1}$$

The leading piece of the entropy $S = \log\rho$ typically scales like a^{D-2} for large $q \sim a$ and is therefore independent of the change of ensemble. However, the logarithmic correction, which scales like $\log a$, often picks up contributions from the Jacobian factor above. We have seen this explicitly in section 3.2 where the Jacobian (3.25) scales with a.

Another instance of a change of ensemble was mentioned in relation to the charge redefinitions in (3.20). In this case the Jacobian is

$$\frac{\partial(L_0, R_0)}{\partial(\tilde{L}_0, \tilde{R}_0)} = 2\frac{\tilde{R}_0}{k_R} = 2\sqrt{\frac{R_0}{k_R}}\,. \tag{B.2}$$

However, $k_R \propto c \sim a^3$ [20] and $R_0 \sim a^3$ so in this instance the Jacobian does not scale with a and therefore the logarithmic correction to the entropy is left intact by this particular change of ensemble.

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