PRIMARY MENTAL ABILITIES

By L. L. THURSTONE
PROFESSOR OF PSYCHOLOGY
THE UNIVERSITY OF CHICAGO

THE UNIVERSITY OF CHICAGO PRESS
CHICAGO · ILLINOIS

PREFACE

THIS is the first major application of the theory of factor analysis that was described in *The Vectors of Mind*.[1] The study has been in progress for several years. When this study was planned, we postulated a number of tentative psychological categories or factors which served merely to insure that a wide variety of tests of the paper-pencil sort were included. The primary factors that appeared have a general relation to the tentative categories with which we started, but they are not identical with the tentative categories. We had postulated a verbal factor, but we found two distinct verbal factors in the analysis. We found that the number factor is highly restricted. We had postulated different reasoning factors for verbal, numerical, and spatial material; but this tentative classification was not sustained. The reasoning tests revealed two factors that we have called "induction" and "deduction," the latter being less clearly indicated than the inductive factor. These reasoning factors seem to transcend the immediate character of the material of the tests. We had separate tentative categories for visualizing in flat space and in solid space, but our analysis did not reveal such a division. These tests collapsed into a single visual space factor. From the methodological standpoint these findings give strength to the factorial methods in that they do not merely reproduce the classifications that we had in mind. The factorial methods have so far indicated their effectiveness in testing psychological hypotheses. It is in this function that the factorial methods will justify themselves in experimental and theoretical psychology.

Since the completion of this experiment, we have completed another experimental study of the perceptual factor which was not one of the tentative categories in the original test material. This factor needs much further experimental study before its psychological nature will be known. Another experiment is now in progress on the inductive factor which promises to be of fundamental psychological interest. These subsequent studies have had the advantage of some orientation

[1] L. L. Thurstone, *The Vectors of Mind* (Chicago: University of Chicago Press, 1935). A preliminary report on the primary factors was published in a paper, "The Factorial Isolation of Primary Abilities," *Psychometrika*, Vol. I, No. 3.

about the first seven primary factors as landmarks. Further experimental studies of the factors will be much more refined and crucial in character in that the experimental tests can be constructed more precisely to test specific psychological questions. In the exploratory study that we are reporting in this monograph we did not have the advantage of orientation about any known landmarks. Consequently, the tests in the present study were often more complex as to factorial composition than we had anticipated. The tests that have been constructed for the subsequent studies are more nearly pure in that some of them could be designed so as to feature one factor with little admixture of the others. This process will continue for some time until we shall be able to prepare psychological tests that involve only one or two factors instead of three, four, or five, as is the case with most of the tests in current use.

Experimental work in this field is hampered by the variance of unknown nature that is necessarily involved in paper-and-pencil tests for groups of subjects. Eventually we shall probably reduce the amount of testing in groups with paper-pencil tests and turn to individual tests with more refined procedures. At the present state of knowledge it is probably better to find the principal landmarks in the cognitive and conative primary traits by means of group procedures. When the general nature of the most important primary traits is known, it will be possible to design individual tests that are more crucial and incisive than can be made at present. The next stage in these developments will probably be individual testing with material specially designed to feature the primary factors that have been found by the group methods. It is our belief that the appraisal of the cognitive and conative primary traits will eventually be accomplished in terms of discriminatory and other rather simple perceptual tasks with individuals in the laboratory and that the tests will look entirely different, superficially, from the traits that they may be found to signify. It may even happen that such abilities as Number, Induction, and Memory may be appraised by tachistoscopically presented discriminatory tasks that do not contain any numbers, that do not call for memorizing in the usual sense, and which do not involve inductive or deductive thinking in explicit verbalized form. It seems probable that the various tests for personality traits will entirely disappear from the paper-pencil lists. The temperamental traits will probably be appraised by the individual differences in conative characteristics that may be tested by individual methods. This may be one of the most fertile fields for experimental psychological study.

So far in our work we have not found the general factor of Spearman, but our methods do not preclude it. The presence of a general factor could be indicated by a large part of the communality of each test that remains unaccounted for by the common factors that can be identified in a simple structure. So far we have not found any conclusive evidence for a general common factor in Spearman's sense, but some situations may be found in which such an interpretation is justifiable. As far as we can determine at present, the tests that have been supposed to be saturated with the general common factor divide their variance among primary factors that are not present in all the tests. We cannot report any general common factor in the battery of fifty-six tests that have have been analyzed in the present study.

This study has revealed more problems than it has answered, but the new psychological questions that we ask about the primary factors are more specific than we could have asked before the primaries were discovered. The resulting experiments can be planned so as to be more crucial and determinate than the exploratory experiment that is here reported.

The computational labor on this study has been much greater than in comparable studies that are now being made. The reason is largely in the fact that new analytical methods have been tried on the data. These efforts have resulted in improved analytical and computational methods which are now being used to advantage in subsequent studies.

In this monograph we have described each of the fifty-six psychological tests that were used in the investigation and we have given a few sample items from each test to show its nature. The tests have not been reproduced in full. For the benefit of those who may want to study the tests in further detail we have assembled a limited number of sets of tests which are available as a supplement to this monograph. The supplement contains full-size copies of all the tests in the battery. The supplement may be useful to those who are assembling new test batteries for factorial studies in this field.

I wish to acknowledge financial assistance from the Social Science Research Committee of the University of Chicago. A grant for this study was made by the American Council on Education, and I am also grateful for the financial assistance that has been given by the Carnegie Corporation of New York. We were fortunate also to have some assistance from the Civil Works Education Service. Special mention should be made of the loyalty and interest of my research assistants, namely, Miss Leone Chesire, Mr. Ledyard Tucker, Mr. Herbert

Landahl, Mr. Robert Blakey, Mr. Thomas Jeffrey, and Mr. Willis Schaeffer. Several of them have discovered new theorems and invented new laborsaving computational methods in factorial analysis. These are being published in separate papers. I must mention also the enthusiasm with which two hundred and forty university students gave fifteen hours of time and their sustained interest and effort in taking the fifty-six psychological tests. I appreciate the editorial assistance of the University of Chicago Press and, in particular, the editorial advice of Miss Mary D. Alexander.

In making an acknowledgment to my wife, Thelma Gwinn Thurstone, I mention not only the customary author's indebtedness for the proofreading. In the entire theoretical development and in the preparation of appropriate psychological tests I have depended on her for ideas and for psychological judgment which have in large part determined the present investigation.

L. L. THURSTONE

November 1937

TABLE OF CONTENTS

CHAPTER I

THE TEST EXPERIMENT

INTRODUCTION TO THE MULTIPLE-FACTOR PROBLEM

THE multiple-factor methods have been developed primarily for the solution of some fundamental psychological problems, although these analytical methods are so general that they will probably be found useful as well in solving other problems in the biological and in the social sciences. They will be described here in the psychological setting in which they were developed. A detailed exposition of the factorial methods involves a mathematical treatment of the postulates and theorems and of the analysis of experimental data by which meaningful factors are isolated. This has been presented in a previous volume.[1] The purpose here is to present in a relatively nontechnical form the nature of the factorial methods and the assumptions that are involved. The present attempt is to describe the fundamental ideas of factor analysis in terms of only the simplest mathematical ideas in the belief that some readers who are not interested in the mathematical proofs of the previous volume may, nevertheless, have some interest to know the essential nature of these methods.

One of the oldest psychological problems is to describe and to account for individual differences in human abilities. How are these abilities and the great variations in human abilities to be comprehended? And just what is an ability? For centuries philosophers have been free to set up arbitrary classifications of personality types and lists of abilities, and there have been almost as many classifications as there have been writers. The factorial methods have for their object to isolate the primary abilities by objective experimental procedures so that it may be a question of fact how many abilities are represented in a set of tasks, and whether a particular objective performance represents an ability that is in some fundamental sense primary.

This problem can be illustrated by numerous examples. One of these is to explain the ability to visualize. Some individuals are described as visualizers in their problem-solving, while others are said to be poor visualizers. This particular problem was investigated by Sir Francis Galton over fifty years ago. The psychological problem is to ascertain

[1] L. L. Thurstone, *The Vectors of Mind* (Chicago: University of Chicago Press, 1935).

whether the ability to visualize flat form, as in memorizing the detail of a design, is the same as the ability to foresee readily how the pieces of a jigsaw puzzle are going to fit together before they have been picked up. Or does the latter involve some additional kinaesthetic ability? And is this sort of visualizing the same ability as that which is required to imagine solid objects as they would appear from different sides and the ways in which they might fit together? And is this visualizing the same ability as that which is required to imagine the movement of solid objects as in machine design? These are examples of problems of fundamental interest to psychological science, and if they can be solved the results will also be of considerable practical significance.

Reasoning ability involves similar problems. Is reasoning a simple fundamental ability that can operate, as it were, on an indefinite variety of things, or is it so highly specific that there is a separate reasoning ability for every possible thing that we may reason about? The truth is probably between these extremes, but just how specific are the abilities to reason? How many reasoning abilities are there, and just what is each one of them like? Perhaps there is no primary reasoning ability at all, so that what we ordinarily call "reasoning" is a complex of more elementary powers no one of which can be called "reasoning." These are but two examples of the types of psychological problems for which the multiple-factor methods have been developed.

These problems will eventually be resolved so that human abilities can be comprehended in neurological, physiological, psychological, and genetic terms. If one of these sciences should isolate an ability as being in some sense primary, it should eventually be possible to identify the same ability in terms of the other sciences because there is probably no fundamentally sharp line of separation between them. It is quite likely that the primary mental abilities will be fairly well isolated by the factorial methods before they are verified by the methods of neurology or genetics. Eventually the results of the several methods of investigating the same phenomena must agree.

The first simplifying assumption of the factorial methods is that, if the objective performance of a task requires a certain number of fundamental or primary abilities, then the performance can be expressed, in first approximation, as a linear function of the primaries. The interpretation of a linear function in this context can be illustrated by a hypothetical example. Consider an arithmetical task which requires two hypothetical abilities that may be called "number speed" and "numerical reasoning." One subject may be superior in setting up the problems, but he may be slow in the numerical calculation. Another subject may be slow in reasoning out the problem, but he may be fast in the calculation. Other subjects may be fast in both, or slow in both. In the factorial analysis an objective performance is stated as a weighted sum of two contributions, one for each of the two abilities that are involved in the task. If the task involves very little calculating but mostly reasoning, then the weight for the first factor is smaller than for the second factor. The assumption that the performance can be regarded as a sum of the contributions of the two abilities is represented in the expression

$$s = a_1x_1 + a_2x_2 \, , \tag{1}$$

in which s represents the standard score of an individual in the arithmetical task, x_1 and x_2 represent the standard scores of the individual in the two fundamental abilities, and a_1 and a_2 are the weights of the two abilities in the arithmetical task. If we know to what extent each of the two abilities are required in the task, the weights a_1 and a_2, and if we know how a person scores in each of the two fundamental abilities that are involved in the task, x_1 and x_2, then the objective performance s can be predicted.

It is conceivable that two individuals with different abilities will obtain the same objective performance s. This can be illustrated in terms of the present example. If the two weights a_1 and a_2 are comparable so that the task involves both of the hypothetical abilities, numerical speed and numerical reasoning, then one of the individuals may obtain an acceptable performance if he rates high in x_1 and low in x_2, while another individual may obtain the same score on the task by the opposite configuration of abilities, namely, a low rating x_1 and a high rating x_2. The best possible performance is to be expected from a person who rates high in both of the abilities that are involved in the task. If a task requires only the first ability, then the weight a_1 will be high while the weight a_2 will be zero. In that case an individual who possesses the second ability with a high rating x_2 will not profit in this particular task because the product a_2x_2 will still be zero. In other words, the fact that a person has a high rating in a particular ability does not help him to superior performance in a task unless the task involves the ability in question.

It may very well happen that the several abilities that are involved in a complex task do not enter into the performance in a simple summative way. Perhaps the equation (1) should be very much more complex. Even if that should eventually be found to be the case, it is fortunate, nevertheless, that most complex functions can be represented in first approximation by a linear expression of the form (1). That is a circumstance on which the factorial methods have been built with the full realization that the solutions that we obtain will be in the nature of first approximations. The factorial methods should enable us to ascertain the nature of the principal landmarks in mental ability. Eventually the simple factorial methods will be superseded by more involved functions which cannot be written at the present state of knowledge. It must be remembered that psychology does not even know if there is such a thing as reasoning ability, or visualizing ability, or mental speed, or number ability. Some of these abilities have been surmised but not objectively proved to be unique and primary mental powers. Since none of the primary mental abilities has been objectively isolated, we have not been able to appraise individuals as regards their several talents or disabilities.

The weights a_1 and a_2 in the foregoing equation describe the tests, while the coefficients x_1 and x_2 describe the individuals. If there are n tests and r abilities, the tests may be described factorially in a table with n rows and r columns. The rows refer to the tests and the columns to the fundamental abilities. Such a table is called a *factorial matrix*.

The individual subjects may be described also in a table of coefficients in a similar manner. If there are N individuals and if a particular experimental study involves r abilities, then the individuals may be described in a table with N rows and r columns. Each row describes one of the subjects.

Factorial analysis begins with the record of objective performances of individuals, and the problem is to isolate the fundamental abilities, to describe their nature, to ascertain the loading of each fundamental ability in each test, and to describe each subject as regards each of the fundamental abilities. An attempt will be made here to describe the logical difficulties involved in solving this problem and the way in which these difficulties are overcome.

If we start with a set of records of the objective performances of N individuals in n tests, it is evident that these performances may be broken up into parts quite arbitrarily in an infinite number of ways. With N individuals and n tests there are nN scores to be acounted for. In order that a factorial analysis shall be scientifically significant, several restrictions must be placed on the solution. The first requirement is that a scientific explanation must be simpler than the number of things that are to be covered by the explanation. This requirement is that the number of degrees of freedom of a scientific hypothesis must be smaller than the number of degrees of freedom of the phenomena that are to be explained. The principle can be treated mathematically in general form, and it is involved explicitly in statistical theory. Most scientists do not deal with this principle explicitly because a good scientist uses the principle intuitively. It is almost second nature for him in his work. However, in some problems the complexities are such that intuitive judgment becomes uncertain, and then an explicit treatment of the concept becomes helpful. An absurd example of the violation of this principle is the postulating of a separate instinct or mental faculty for everything that people do. A similar case is that in which, when a hypothesis fails to be verified in successive experiments, it is amended with additional parameters or complexities until it becomes so unwieldy that we are left unconvinced and dissatisfied. This is an intuitive application of the principle. In the factorial problem this principle demands that the scores of the N individuals in n tests must be explained with r factors and that the number of factors must be smaller than the number of tests. If we must postulate a new ability or factor for every new test that is added to the experiments, then we have accomplished nothing. It can be proved that such a resolution can always be made routinely. This requirement, $r<n$, means that the number of factors must be much smaller than the number of tests.

A reduction of the scores is obtained in terms of the relations between the scores of the same individual on different tests and of the relations between different individuals on the same tests. These relations give a leverage on the problem. The relation between the scores of the same individuals on two tests is given by the correlation coefficient. This coefficient is a pure number that expresses the degree to which a person with high performance in one test tends to make a superior performance in a second test. If the correlation is high, then whatever the abilities may be in the tests, they are similarly loaded in the two tests. If the correlation is zero, then the abilities required in the two tests are so different that, if a man's performance is known in one of the tests, one cannot predict how he would perform in the second test. The raw data in terms of which a factorial analysis begins are a square table of correlation coefficients which shows the degree of similarity of every test with every other test in an experiment. Such a square table is called a *correlational matrix.*

The number of factors that are involved in the battery of n tests is the rank of the correlational matrix. This is a property of the matrix which can be determined merely from the coefficients. When the rank of the matrix has been determined, it constitutes a partial answer to the factor problem, since we know then how many factors or abilities are involved in the experiment, but it does not give any information as to what the factors or abilities are. The application of this theorem, that the number of abilities involved in a set of tests is the rank of the correlational matrix, meets with a practical difficulty. In any experimental situation the correlation coefficients are affected by sampling errors so that, if the experiment were repeated, the correlation coefficients would not be exactly the same. The corresponding coefficients in two similar experiments would differ slightly on account of the variable errors. The scientific problem is therefore to find a square matrix of coefficients of lowest possible rank that duplicates the experimentally observed coefficients within the known sampling errors of the experimental coefficients. This problem can be solved in several ways. One of the simplest solutions is the centroid method of factoring a correlational matrix.[2]

The first object of a factorial analysis is to produce a factorial matrix. If there are n tests and r factors in an experiment, the factorial matrix will have n rows and r columns. Each row describes a test in that the entries a_1, a_2, a_3, ... , show the extent to which each of the factors is required by each one of the tests. At this point an interesting indeterminacy appears in the factor problem. While the rank r of the matrix of correlation coefficients gives the number of factors in terms of which the test scores and the correlations can be comprehended, there is an infinite number of ways in which a set of r factors may be chosen to explain the tests. A simple analogy to this problem is the situation that would arise if we knew the differences between the elevations of ten mountains. Even though we had a table showing the difference in elevation between every mountain and every other mountain in a particular list, the data would give no information as to the absolute height of any one of them.

This indeterminacy constitutes a crucial problem in factor analysis. The nature of this difficulty can be shown further in terms of a simple geometrical representation of the problem. *Figure 1* is drawn to represent two tests, (1) and (2), and their intercorrelation. In this diagram the correlation between the two tests is given by the product of

[2] *Ibid.*, chaps. ii and iii and Appen. I.

the lengths of the two vectors *1* and *2* and the cosine of their angular separation. A vector is a directed quantity. An example of a vector quantity is a force. The strength of the force is represented by the length of an arrow, and the direction of the force is represented by the direction in which the arrow is pointing. The velocity of an object can also be so represented. The speed of the object is shown by the length of the vector, while the direction of the motion is shown by the direction in which the velocity vector is pointing. Such quantities are distinguished from scalar quantities which are not characterized by any particular direction. Examples of scalar quantities are price, volume, and temperature, which are nondirectional in character.

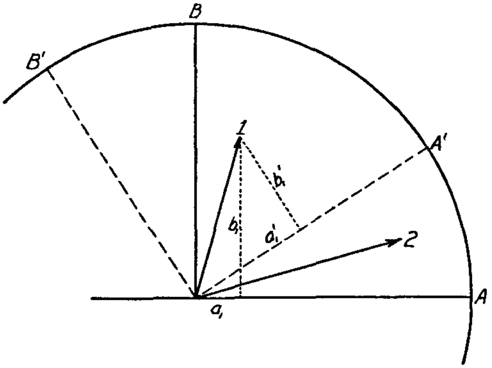


FIGURE 1

In the same diagram two fundamental abilities A and B are also represented. They are shown at right angles to represent that they are uncorrelated fundamental abilities. The test vector (1) may be described as a_1 of A plus b_1 of B in a manner similar to the familiar resolution of forces in a parallelogram. The second test vector may be described in the same way, namely, as a_2 of A and b_2 of B. The indeterminacy appears here in the fact that the two axes A and B may be rotated to a new position such as A' and B'. In this situation the first test is described in terms of a_1' of A' and b_1' of B'. These descriptions will be entirely different numerically, and the fundamental factors or abilities would be different in the two cases, but both of these arrangements account equally well for the intercorrelations of the experimentally observed values in the correlational matrix. This becomes apparent also when it is recalled that in this geometrical representation the experimentally observed correlation coefficient is equal to the product of the lengths of the two test vectors and the cosine of the angular separation. But this relation is entirely unaffected by the location of the co-ordinate axes A and B or A' and B'. The indeterminacy concerns then the choice of a reference frame which is so far entirely arbitrary.

If there are three factors or abilities involved in the tests of an experiment, then each test may be represented as an old-fashioned hatpin that is stuck into a cork. These hatpins must be so arranged in the cork that the experimentally observed correlations between the tests are equal to the scalar products of the pairs of test vectors, i.e., the product of the lengths of each pair of hatpins and the cosine of their angular separation. Such a model can always be constructed if there are only three fundamental abilities involved in the experiment. But no numerical description can be given for the tests in terms of fundamental abilities until a reference frame has been chosen, i.e., the x, y, and z axes. So far as the mathematical problem is concerned, these axes can be located arbitrarily, but the scientific problem demands that the fundamental abilities which are represented by the co-ordinate axes shall be uniquely determined and that they shall be meaningful.

This indeterminacy has been resolved by discovering another restriction on the configuration that must be found in order that the reference frame shall be scientifically meaningful. The factorial matrix describes each of the n tests in terms of r fundamental abilities. For example, if there are six fundamental abilities involved in an experiment with fifty tests, then it is not likely that every test requires all six fundamental abilities. The factorial matrix would have fifty rows and six columns. If the abilities were known, it would probably be very difficult to construct a set of fifty tests each one of which required all the abilities. It is much more likely that each test will require one or more abilities but that these will differ from one test to another. For example, if one of the tests calls for the ability to visualize, it is not likely that all the verbal tests will contain this ability. If one of the tests involves a number factor, it is not likely that all the tests require this factor. It is to be expected, therefore, that even for a random battery of tests there will be a large number of zero entries in the factorial matrix which describes the tests in terms of the fundamental abilities. There will be one or more zeros in every row. This circumstance can be capitalized in finding a unique solution for the factor problem in which the fundamental abilities shall be simple and meaningful.

This situation is shown in *Figure 2*, which represents a three-dimensional configuration of test vectors. Let *ABC* represent a spherical triangle and let *O* be the origin at the center of the sphere. In the model the cork would be at the center, and the test vectors would be represented by hatpins. If a test vector lies in the plane *AB*, the interpretation is that the test can be described in terms of the fundamental ability vectors *A* and *B* and that the ability *C* is not involved in the test. All the tests in the plane *AB* are characterized by the absence of ability *C*. All the tests in the plane *BC* are characterized by the absence of the ability *A*. A test which contains only one ability such as *B*

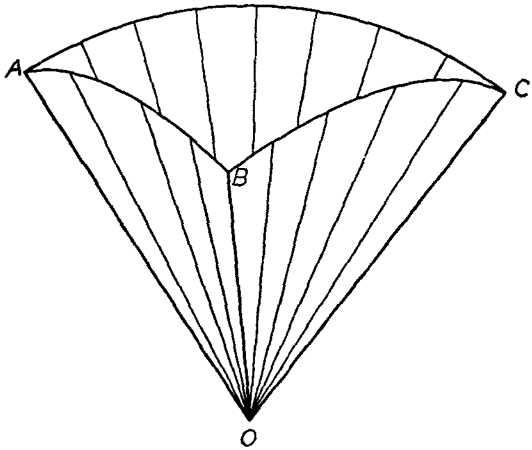


FIGURE 2

would lie in the axis *OB*. If the tests are such that none of them calls for all three of the fundamental abilities, then the test vectors lie in the three planes *AB*, *AC*, and *BC*, and none of them lies inside the cone. Such a model can always be constructed from the observed data if only three factors are present in the battery and if none of the tests calls for all three of the primary abilities. In this case the three corners of the configuration can be seen by inspection, and the reference axes are then placed at *A*, *B*, and *C*. The factorial description is the simplest when none of the tests requires all three of the fundamental abilities and when the number of zero entries in the factorial matrix is maximized. Furthermore, if such a configuration is discovered, it necessarily means that there is an underlying order in the abilities that are involved because, if a correlational matrix with rank 3 were written at random, the resulting model would show the test vectors scattered over the sphere without any defined pattern, and there would then be a limited number of zeros that could be introduced into the factorial matrix. When a simple pattern is discovered by which none of the test vectors lies inside the configuration and by which a large number of zero entries appear in the factorial matrix, the inference must be made that an underlying simple order has been discovered in the abilities. This type of configuration has been called a *simple structure* or *simple configuration.* In actual experimental work with tests it is not often that the number of fundamental abilities is so small as three, but analytical methods have been developed for dealing with the same problem in any number of dimensions.

The corners of a configuration as in *Figure 2* define the fundamental abilities which have been called "primary abilities." It can be shown that the cosine of the angular separation between any two of the primary abilities is equal to the correlation between the two primary abilities in the experimental population for which the correlations were computed.

When the primary axes have been discovered, it is a problem of considerable psychological interest to ascertain what each primary ability is like. This is done by inspecting the tests which are then known to require the primary and the tests in which the primary is known to be absent. In case there is uncertainty about the exact nature of a primary factor, or in case two rival interpretations are made, it is possible by the factorial methods to resolve the question on an experimental basis instead of by argument about rival classifications of personality as in the past.

PLAN OF THE TEST BATTERY

In a previous volume the theory of multiple-factor analysis was developed especially for the purpose of isolating primary traits. Several smaller studies have been made in order to test the factorial methods, but the present experiment constitutes the first major application of the methods for isolating primary traits. The experiment was planned to include a fairly wide variety of tests covering verbal, numerical, and visual tasks in the hope that some of the primary traits involved in current psychological tests would appear.

The present investigation was made with a battery of fifty-six psychological tests that were given to a group of 240 volunteers. A list of the names of the tests in the battery with code numbers and time limits is given in *Table 1.* In preparing this battery, the tests were assembled so as to represent a fairly wide range of the mental activities that are typical in current psychological tests, with special emphasis on those tests which are used as measures of general intelligence. It was decided to conduct each of the tests in several sections: (1) the instructions to the subject with a few examples, (2) a fore-exercise in which the subject was given the opportunity to try the task under short time-limit conditions, and (3) the test proper. The fore-exercise was ignored in scoring since it was regarded as a part of the instructions. Its purpose was solely to give the subject a clear idea of what was expected in each of the tests. Since current tests are seldom constructed in this manner, it was found desirable to construct all the tests specially for this study. However, ideas were drawn from current psychological tests, and in some cases current tests were adapted to the present test program.

The tests in the battery were grouped roughly according to the abilities which are called for, but this classification is, of course, only tentative since it is the purpose of this investigation to isolate some of the primary mental abilities. The tentative groups are given in *Table 1*, but it should be kept in mind that the purpose of this tentative classification was merely to insure that each of these descriptive categories was represented by several tests.

In assembling the battery, it was considered essential that various types of abstraction be included. Abstraction is represented in verbal tests, in numerical tests, and in geometrical and spatial tests. The verbal tests were constructed so as to include word associations of restricted character as well as the freer forms of verbal and logical thinking. The verbal material is quite extensive so that primary verbal abilities should have good opportunity to reveal themselves.

The geometrical and spatial material was introduced into the tests with several problems in mind. It is a psychological question of fact whether separate abilities are involved in thinking about flat forms which are to be compared as to detail, flat forms which are to be fitted as in a form board, solid forms which involve three dimensions, and the movement of solid forms in space. It is conceivable that visualizing abilities involve all these activities, and it is also conceivable that separate kinaesthetic abilities are introduced by the space tests and by the tests in which the subject must think about the movement of solid objects in space. Some of the space tests involve reasoning, and it is another psychological question whether reasoning involves a distinct mental ability which transcends the detailed form on which it is exercised. Reasoning might appear as several distinct abilities, depending on whether it is exercised on verbal, numerical, or spatial material. A

Table 1

The Test Battery

Code Number	Name of Test	Time Limits (in Minutes)		Scoring Formula
		Fore-exercise	Test Proper	
	Abstraction:			
4	Reading I	4	8	R
5	Reading II	....	8	R
6	Verbal Classification	5	15	R
7	Word Grouping	2	8	R
8	Figure Classification	3	10	R
	Verbal Tests:			
9	Controlled Association	5	16	R (Total number of words)
10	Inventive Opposites	2	6	R (Total number of words)
11	Completion	2	5	R
12	Disarranged Words	1	8	R
13	First and Last Letter	5	3, 3	R (Number of words accepted)
14	Disarranged Sentences	2	4	R–W
15	Anagrams	4	10	R (Number of words accepted)
16	Inventive Synonyms	2	6	R
	Space:			
17	Block Counting	7	7	R
18	Cubes	7	5	R–W
19	Lozenges A	5	4	R
20	Flags	2	2	R–W
21	Form Board	3	8	R
22	Lozenges B	4	5	R–W
23	Surface Development	4	6	R–W
24	Punched Holes	3	3	R
25	Mechanical Movements	3	20	R–W
	Form:			
26	Identical Forms	3	5	R
27	Pursuit	3	3	R
28	Copying	4	3	R–W
29	Areas	5	6	R
	Number:			
30	Number Code	5	8	R
31	Addition	....	3	R
32	Subtraction	....	3	R
33	Multiplication	....	3	R
34	Division	....	3	R
35	Tabular Completion	....	10	R
	Numerical Reasoning:			
36	Estimating	7	15	....
37	Number Series	4	10	R
38	Numerical Judgment	3	10	R–W
39	Arithmetical Reasoning	1	20	R

Table 1—Continued

Code Number	Name of Test	Time Limits (in Minutes)		Scoring Formula
		Fore-exercise	Test Proper	
	Verbal Reasoning:			
40.	Reasoning	3	6	R–W
41.	Verbal Analogies	3	6	R
42.	False Premises	3	8	R–W
	Space Reasoning:			
43.	Code Words	4	20	R
44.	Pattern Analogies	3	6	R
45.	Syllogisms	3	6	R–W
	Rote Learning:			
46.	Word-Number			R
	Page 1	2	....	
	Pages 2 and 3	3	....	
	Page 4	1	....	
	Page 7	....	5	
	Page 8	....	3	
47.	Initials			R
	Page 1	3	....	
	Page 2	1½	....	
	Page 3	....	6	
	Page 4	....	3	
48.	Number-Number			R
	Page 1	4	....	
	Page 2	1	....	
	Page 3	....	5	
	Page 4	....	3	
49.	Word Recognition			R–W
	Page 1	3	....	
	Page 2	1½	....	
	Page 5	....	5	
	Page 6	....	4	
50.	Figure Recognition			R–W
	Page 1	3	....	
	Page 2	1	....	
	Page 5	....	2	
	Page 6	....	4	
51.	Picture Recall	2	5	R
	Unclassified:			
52.	Theme	....	20	....
53.	Hands	4	3	R–W
54.	Rhythm	3	6	R
55.	Sound Grouping	3	9	R
56.	Spelling	....	7	R–W
57.	Grammar	....	15	R
58.	Vocabulary (Chicago)	....	15	R
59.	Word Count	....	....	R (Total number of words)
60.	Vocabulary	....	10	R

considerable variety of combinations of these tentative factors was introduced into the battery.

The memory tests were designed within the limitations of paper and pencil tests for this program. There was some uncertainty as to whether these tests should be named "memory," or "memorizing," or "attention." It is a psychological question of fact whether good memory in general is a primary ability, whether it is identical with the ability to memorize intentionally, and whether this is also the same as the ability to give sustained attention to a task or problem. There is also the psychological problem as to whether immediate memory involves the same primary ability as the ability to remember past experience even when the subject does not exercise any special intention to retain the experience for future recall. These problems are not adequately represented in the present battery since they probably call for a special factorial study of some magnitude.

THE SUBJECTS

When the test battery was being assembled, the question arose of how to obtain a fairly large group of subjects who would be willing to spend about fifteen hours on the psychological tests. Various proposals were considered, such as payment for student service at the usual hourly rate and the possibility of obtaining a large group of subjects in various types of institutions where people are available with plenty of time and not much to do. It was decided to try asking for student volunteers with the promise of an individual report on the findings for each student. The present battery of fifty-six psychological tests is probably one of the most comprehensive of the test batteries that have so far been constructed. It is customary to give vocational and educational counseling on test data which are meager in comparison with the individual results that are available for each student who takes the present battery of tests. Hence it seemed amply justifiable to offer a free service in the appraisal of several mental abilities by the present battery as well as it can be done at the present state of knowledge. It did not seem fair to ask the subjects to wait until the factorial analysis and the extraction of the primary factors had been completed. For this reason the subjects were promised a preliminary individual report in which the tests were grouped in psychological categories which could be justified in terms of current psychological knowledge and practice.

An announcement was posted on several bulletin boards, and several offices were provided with application blanks for the service. The subjects were told that the records of their performances would be used in an attempt to isolate the primary mental abilities and that they would each be given a preliminary report of the findings as well as a final report in a year or two when the analysis of all the records had been completed. In the application blank the students were asked to give information about themselves on the following points: amount of general schooling, name of high school and previous college attendance, the college subject of principal interest, fluency in speaking English (for the purpose of excluding foreign-born subjects who might be handicapped in taking speed tests in English), native country and state, expected occupational choice, occupational ambitions, preferred avocations, willingness to devote six three-hour sessions to the present program of psychological tests, preferred hours, address, and telephone number. They were told that they would be notified by mail if their application was accepted.

It was discovered that about one hundred of the applicants were willing to devote the mornings of a spring vacation week between quarters to the psychological tests, and consequently the tests were given in the lecture-room of Eckhart Hall. Other sessions for about a hundred students were held in the evenings. Several students missed a few sessions, and they were given the opportunity to complete the whole program of tests in the following week. Announcement was made that no student's tests would be scored unless he had completed the whole program of fifty-six tests. The interest of the students in the tests was remarkable. Instead of regarding the tests as a disagreeable chore, the subjects seemed to enjoy them, and comments to this effect were frequent. The examiners had the opportunity during an intermission at each session to talk with the students and to discover their reactions to the tests. The subjects worked with intense interest and effort during the full time limit for each test. This was a fortunate circumstance because it implied that the score of each man could be taken safely to represent his performance under considerable pressure of motivation and interest. The subjects finished the fifteen hours of testing with applause. At the end of the last session the writer acknowledged again the assistance that the students had given toward the solution of a problem of fundamental importance to science as well as to practical affairs. They were promised preliminary individual reports in line with the best knowledge then available and individual final reports when the factorial analysis was complete. The success of the undertaking so far shows clearly that studies of this kind can be made with volunteers and that the original problem of how to get subjects willing to be tested for fifteen hours had been satisfactorily resolved.

The total experimental group consisted of 240 volunteer subjects, and 218 (91 per cent) finished all five sessions. This group includes 40 students at the Central Y.M.C.A. College who volunteered as subjects. The experimental group was divided as follows: 132 Freshmen, 36 Sophomores, 30 Juniors, 12 Seniors, and 14 college graduates. The age distribution is shown in *Table 2*. The modal age was 18.

Table 2

Age Distribution of Subjects

Age	16	17	18	19	20	21	22	23	24	25	Above 25
f	9	27	77	44	20	19	8	8	7	5	16

Table 3

Psychological Examination of the American Council on Education

Gross Score	University of Chicago Freshmen (1933) $N=646$		National Norms (1933) 203 Colleges $N=40{,}229$	
	P.R.	x_2	P.R.	x_1
118	.05	−1.64	.28	− .58
143	.10	−1.28	.42	− .20
157	.15	−1.04	.51	.03
169	.20	− .84	.58	.20
176	.25	− .67	.63	.33
184	.30	− .52	.67	.44
192	.35	− .39	.71	.55
201	.40	− .25	.76	.71
209	.45	− .13	.79	.81
218	.50	.00	.83	.95
225	.55	.13	.86	1.08
232	.60	.25	.88	1.17
239	.65	.39	.90	1.28
248	.70	.52	.92	1.41
256	.75	.67	.94	1.55
264	.80	.84	.95	1.64
275	.85	1.04	.97	1.88
290	.90	1.28	.98	....
309	.95	1.64	.99	....

The subjects in the present study were a highly selected group. The fact that they volunteered for the experiments probably accounts in part for the fact that their average mental endowment was exceptionally high. It is of theoretical interest to ascertain whether they constituted a normal distribution of test intelligence in terms of standards that are available for larger populations. It was found that 113 of the 240 subjects had previously taken the psychological examination of the American Council on Education. On the assumption that these 113 students were representative of the entire group of 240, it is possible to compare the subjects in this investigation with the Freshman

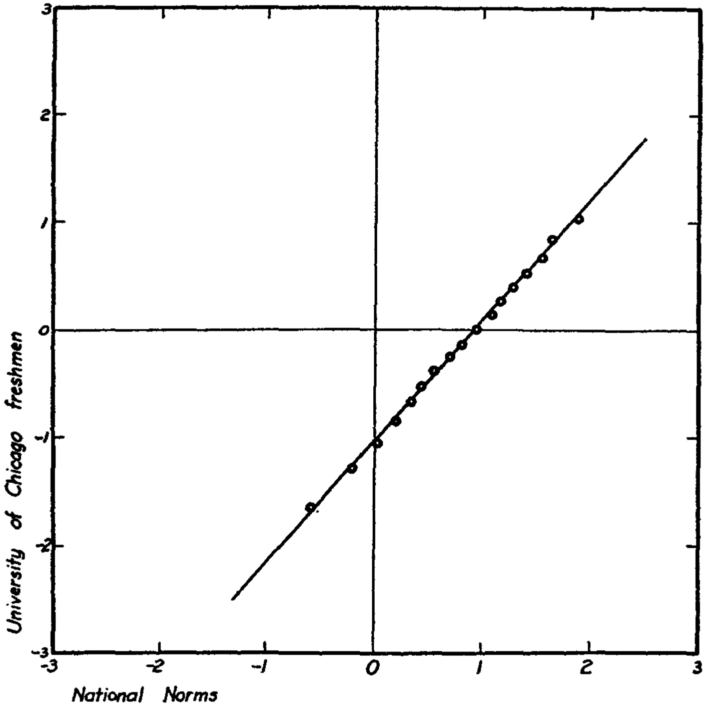


FIGURE 3

classes at the University of Chicago and with the national norms on the same examination for 203 American colleges.

In *Table 3* are recorded the data for comparing the Chicago Freshmen with the national norms.[3] The raw scores in the table were selected so as to correspond with percentile ranks in intervals of .05 for the Chicago Freshmen. The corresponding x-values are the deviations of a normal distribution in terms of its standard deviation. A similar tabulation is made for the national norms for the same raw scores. *Figure 3* shows the plot of the two x-values for the same raw scores.

[3] L. L. Thurstone and Thelma Gwinn Thurstone, "The 1933 Psychological Examination," *Educational Record,* XV, No. 2 (April, 1934), 173.

The linearity of the plot proves that the two distributions may be regarded as normal distributions on the same absolute scale of test ability.[4]

The fundamental scaling equation is

$$M_2 + x_2\sigma_2 = M_1 + x_1\sigma_1 \,, \tag{2}$$

in which M_1 and M_2 are the absolute means and σ_1 and σ_2 are the absolute dispersions. The equation may be written in the form

$$x_2 = \frac{\sigma_1}{\sigma_2}\, x_1 + \frac{M_1 - M_2}{\sigma_2} \,. \tag{3}$$

The equation of the linear plot is

$$x_2 = 1.12x_1 - 1.04 \,, \tag{4}$$

and hence the slope is

$$\frac{\sigma_1}{\sigma_2} = 1.12 \,, \tag{5}$$

and the x_2-intercept is

$$\frac{M_1 - M_2}{\sigma_2} = -1.04 \,. \tag{6}$$

The national norms show the greater variability, and the difference between the two means is 1.04 σ_2 on the Chicago distribution.

A similar comparison was made between the national norms and the group of 113 subjects in the present study for whom American Council test records were available. *Table 4* shows the numerical comparisons, and these are represented in *Figure 4*. Here the fundamental scaling equation is

$$M_3 + x_3\sigma_3 = M_1 + x_1\sigma_1 \,, \tag{7}$$

which may be written in the form

$$x_3 = x_1\, \frac{\sigma_1}{\sigma_3} + \frac{M_1 - M_3}{\sigma_3} \,. \tag{8}$$

The equation of the linear plot is

$$x_3 = 1.19x_1 - 1.63 \,. \tag{9}$$

[4] L. L. Thurstone, "A Method of Scaling Psychological and Educational Tests," *Journal of Educational Psychology*, XVI, No. 7 (October, 1925), 433–51.

The slope, 1.19, is the ratio of the dispersions of the two groups. The national norms show the greater dispersion. The additive term, 1.63, is the difference between the two means in terms of the standard deviation of the experimental group. The linearity of the diagram proves that these two distributions may also be regarded as normal distributions on the same absolute scale of test ability.

Table 4

Psychological Examination of the American Council on Education

Interval	National Norms (1933) $N=40{,}229$ 203 Colleges		Test Subjects Having A.C.E. Records $N=113$		
	P.R. Upper Boundary	x_1	N_1	P_3 Upper Boundary	x_3
.35–.40	.40	−.25	1	.01	....
.40–.45	.45	−.13	2	.02	....
.45–.50	.50	.00	4	.04	−1.75
.50–.55	.55	.13	6	.05	−1.64
.55–.60	.60	.25	10	.09	−1.34
.60–.65	.65	.39	16	.14	−1.08
.65–.70	.70	.52	18	.16	−.99
.70–.75	.75	.67	22	.19	−.88
.75–.80	.80	.84	32	.28	−.58
.80–.85	.85	1.04	38	.34	−.41
.85–.90	.90	1.28	48	.42	−.20
.90–.95	.95	1.64	73	.65	.39
.95–.96	.96	1.75	76	.67	.44
.96–.97	.97	1.88	80	.71	.55
.97–.98	.98	....	83	.73	.61
.98–.99	.99	....	89	.79	.81
.99–.999	.999	....	112	.99	....
.999	....	....	113	1.00	....

In *Figure 5* the three distributions have been superimposed on the same absolute scale. The three curves show the relation between the national norms which represent 203 American colleges, the Freshman class at the University of Chicago in 1933, and 113 of the subjects in the present study. The linearity of the relations in *Figures 3* and *4* is conspicuous, and it is especially convincing for the applicability of the absolute scaling method. This analysis supports the conclusion that the group of subjects in this study constituted a normal distribution of test intelligence on the same absolute scale on which the distribution of 40,229 Freshmen in 203 American colleges is normal. The relation between these two distributions is shown in the comparison between curves *1* and *3* in *Figure 5*.

ADMINISTRATION OF THE TESTS

It was at first planned to give the tests in six sessions, but it was found possible to complete the entire test battery in five sessions of about three hours each. An intermission of about ten minutes was

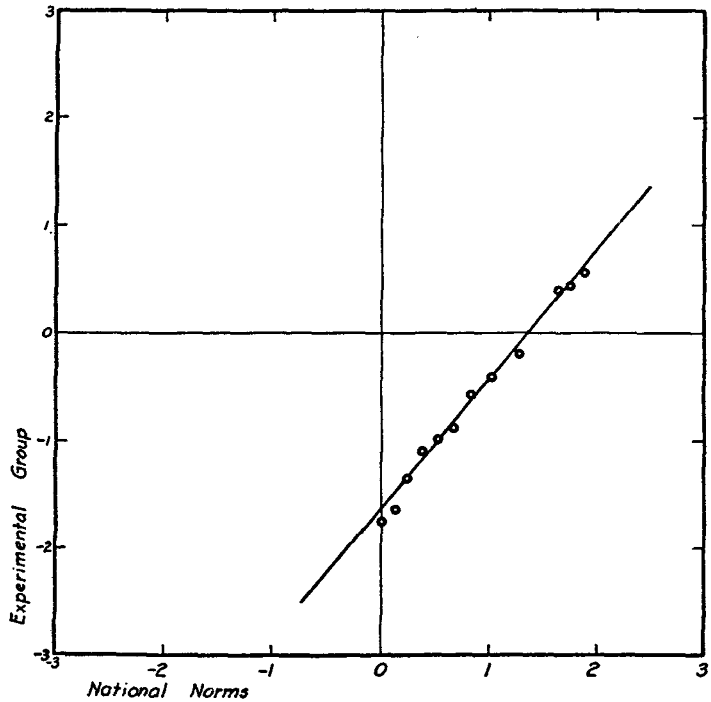


FIGURE 4

given in the middle of each session. The tests were given in the following order in the several sessions:

Session	
1.	Tests 26, 4, 5, 17, 6, 27, 30, 46, 9, 25
2.	Tests 28, 10, 36, 11, 53, 12, 18, 43, 19, 47, 20
3.	Tests 29, 13, 37, 21, 14, 40, 22, 41, 23, 48, 7
4.	Tests 15, 8, 42, 16, 24, 49, 54, 55, 38, 44
5.	Tests 51, 31, 32, 33, 34, 56, 50, 39, 45, 58, 59, 60, 35, 57, 52

In arranging the order of the tests, an attempt was made to vary the nature of the material as regards verbal, numerical, and visual content so as to minimize fatigue or boredom with repetition of the same type of mental activity. The tests were administered to three separate groups of not more than one hundred students in each group. Each test with fore-exercise was bound in a separate pamphlet. The pamphlets were distributed and collected separately for each test by ten or twelve examiners. This interruption between tests serves the useful function of avoiding the continued strain which would result if a long

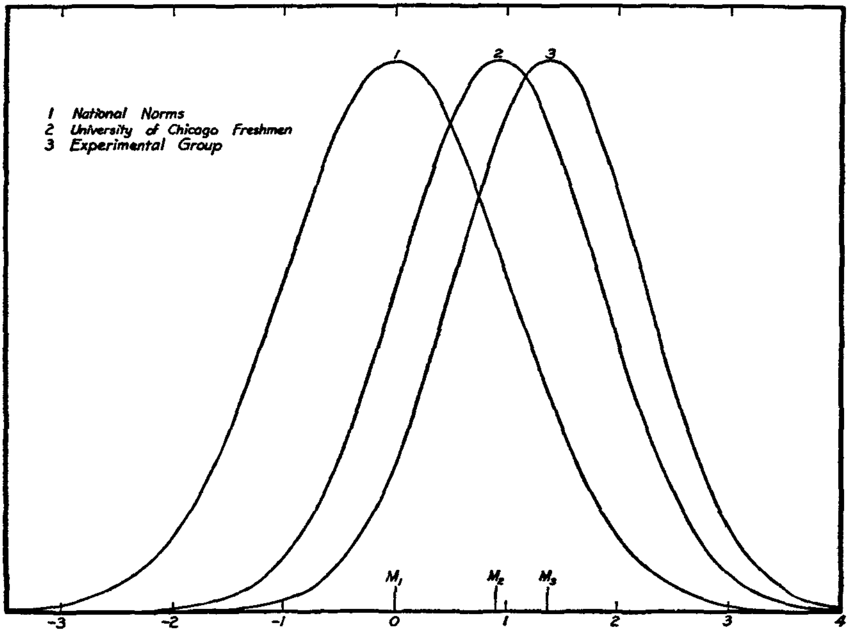


FIGURE 5

series of tests were bound together in the same booklets and if the termination of one test were followed immediately by the beginning of the next one. To handle the tests in separate pamphlets takes more time and more examiners, but it is probably a better procedure. It introduces an interruption between successive tests in the form of a natural rest of a few minutes between the time limits of the tests.

CHAPTER II

DESCRIPTION OF THE TESTS

THE tests will be described in the order of their code numbers listed in *Table 1–i*. Each test is illustrated by a few sample items. In many cases the sample items are taken from the instructions and the fore-exercise. The complete set of fifty-six lithographed tests has been published as a supplement to this monograph.[1]

Reading I (4) *and Reading II* (5).—These tests were prepared by the author, for use at Carnegie Institute of Technology in 1919, with the assistance of Miss Helen Davis. Both of these tests were used in the present battery in their original form. They have been used by Professor E. L. Thorndike as a part of the tests known as the "CAVD" tests. Both of these tests were planned as tests of reading in which the subject should return a fairly large number of objective responses in a limited time. Since the tests call for the interpretation of proverbs and of quotations with paraphrased statements, they have the advantage that the subject does not merely repeat the statements in the tests. He must interpret their meaning in order to make correct responses. There is a short fore-exercise which is followed by the test proper. Since the second reading test uses the same plan with quotations instead of proverbs, no special instructions or fore-exercises are needed for Reading II.

SAMPLES FROM READING I (24 ITEMS)

This is a test of your ability to understand what you read. Read proverb *A*.

A. Sail when the wind blows.

Two and only two of the following sentences have nearly the same meaning as proverb *A*. Find these two sentences.

__✓__ Strike when the iron is hot.
_____ One must howl with the wolves.
__✓__ Make hay while the sun shines.
_____ Make not your sail too large for the ship.

The first and third statements have been checked because they have nearly the same meaning as proverb *A*.

Now check the two sentences in the group below which have nearly the same meaning as proverb *B*.

[1] L. L. Thurstone, *Psychological Tests for a Study of Mental Abilities* (Chicago: University of Chicago Press, 1937).

B. Tall oaks from little acorns grow.

______ No grass grows on a beaten road.
______ Large streams from little fountains flow.
______ The exception proves the rule.
______ Great ends from little beginnings.

SAMPLES FROM READING II (25 ITEMS)

1. "In calamity any rumor is considered worth listening to."—PUBLIUS SYRUS.
   ______ A drowning man grasps at a straw.
   ______ Rumors are seldom truthful.
   ______ Thy neighbor is thy teacher.
   ______ Any port is welcome in a storm.
2. "No great genius was ever without some mixture of madness, nor can anything grand or superior to the voice of common mortals be spoken except by the agitated soul."—ARISTOTLE.
   ______ Genius is essentially hard work and persistence.
   ______ Contented and serene characters are the ones that produce the work of genius.
   ______ Genius and insanity have certain elements in common.
   ______ Strokes of genius are likely to come after times of great disturbance or stress for the individual.

Verbal Classification (6) was prepared by Thelma Gwinn Thurstone. The intention was to devise a verbal form of test which called for the same type of classification as the geometrical forms in Spearman's test (8) for "*g*." This test was prepared to parallel Spearman's classification test with verbal material. In preparing several tests for the same ability, it is quite likely that variation in difficulty will cause the test to require different abilities. The present Verbal Classification test is probably easier than the Figure Classification test of Spearman. The tasks are similar in kind except for the substitution of words for figures.

SAMPLES FROM VERBAL CLASSIFICATION (70 ITEMS)

Column 1, below, is a list of *animals*. Column 2 is a list of *furniture*. Column 3 has some words about animals and some words about furniture. *Desk* is furniture, and 2 is written after it. *Sheep* is an animal, and 1 is written after it. The rest of the words under 3 have been marked in the same way.

1	2	3	
cow	table	desk	2
horse	chair	sheep	1
bird	bookcase	rocker	2
dog	lamp	dresser	2
		cat	1
		donkey	1

Below is another problem of the same kind. Column 1 is a list of *colors*, and column 2 is a list of *sports*. The first few words under 3 have been marked for you. You mark the others.

1	2	3	
blue	golf	red	1
green	tennis	pink	1
violet	football	baseball	2
yellow	swimming	diving	____
		orange	____
		purple	____

In the following problems, mark each word under 3.

1	2	3	
mountain	profound	upward	____
tower	submerged	depression	____
mast	pit	surmount	____
aloof	dive	soar	____
		depth	____
		sunk	____
simple	barnstorm	charming	____
tasteful	gaudy	roar	____
celestial	torn	panic	____
chaste	driven	classic	____
		crazy	____
		pure	____
lacerate	suffer	wince	____
torture	ache	crucify	____
bite	twinge	crush	____
pinch	writhe	smart	____
		moan	____
		cut	____

Word-grouping (7) was prepared by Thelma Gwinn Thurstone. It is similar to a form of test in current use and is intended to be suitable for college students. Since the task is quite simple to comprehend, the instructions and the fore-exercise are combined in the first page, which is followed by the test proper.

SAMPLES FROM WORD-GROUPING (71 ITEMS)

In the line below, notice that the four words, *dog*, *lion*, *cat*, and *giraffe*, can be grouped together because they are all names of animals. The word *chair* does not belong with the others because it is not the name of an animal. Since *chair* is the second word, 2 is written in the blank at the right.

1—dog 2—chair 3—cat 4—lion 5—giraffe 2

Similarly, four of the words in the line below can be grouped together because they are alike in some way, while one of the words does not belong because it is different. Write the number of that word in the blank at the right.

1—carrot 2—radish 3—beet 4—book 5—turnip ____

Book does not belong because the other four are all vegetables. You should have written 4 in the blank.

In each of the lines below, find the four words which can be *grouped together* because they are alike in some way. Then write the number of the extra word in the blank at the right.

1—aunt	2—uncle	3—barn	4—mother	5—father	_____
1—paper	2—chisel	3—axe	4—knife	5—saw	_____
1—lake	2—ocean	3—sea	4—gulf	5—field	_____
1—youth	2—boy	3—man	4—maiden	5—girl	_____
1—run	2—swim	3—walk	4—march	5—hike	_____

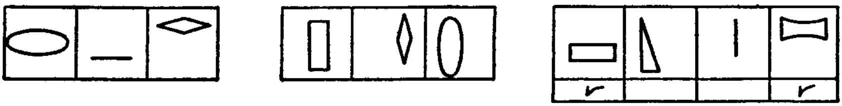
Figure Classification (8) is part of a test used by Professor Charles Spearman and has been reproduced in this battery with his permission. It was included because, according to Professor Spearman, it is one of the best tests for his central intellective factor "*g*," and it was of course desirable to have that factor well represented in this battery. The test was arranged with one page of instructions and examples, which was followed by a two-page fore-exercise. Then followed the test proper in the next eight pages.

SAMPLES FROM FIGURE CLASSIFICATION (28 ITEMS)

In each line below, there is a rule by which the symbols in Group I differ from those in Group II. *There is a new rule for each line. Your problem is to discover the rule in each line.* Some sample problems are worked for you below.

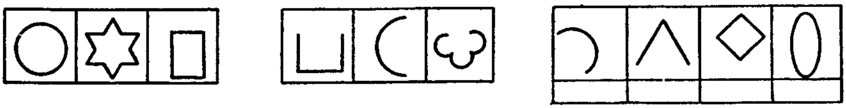
In the first line below the rule is that the symbols in Group I are *horizontal* while those in Group II are *vertical.* Each of the *test symbols* at the right belongs either to Group I or to Group II. The test symbols that belong to Group I have been checked.

GROUP I GROUP II TEST SYMBOLS



The rule in the problem below is that the figures in Group I are *closed* while those in Group II are *open.* Now check the test symbols that belong to Group I.

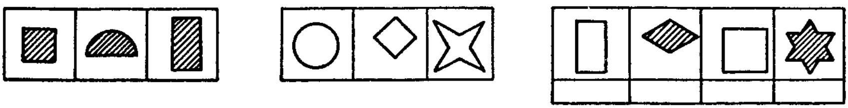
GROUP I GROUP II TEST SYMBOLS



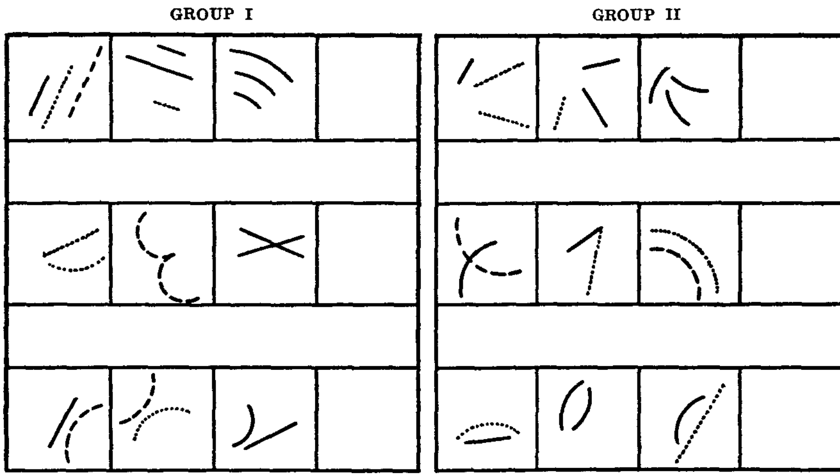
You should have checked the third and fourth symbols. They are closed figures.

Now find the rule in the following line by which Group I differs from Group II. Then check the test symbols that belong to Group I.

GROUP I GROUP II TEST SYMBOLS



You should have checked the second and fourth symbols. They are shaded.



Controlled Association (9) was prepared as a test of word fluency. It is intended to appraise the ability to produce a large number of relevant word associations. For this reason it was not written in the objective form. There may be a difference in the facility with which a subject can check relevant items already given and his facility in producing relevant associations. The instructions and fore-exercise are combined in the first page of the test.

SAMPLES FROM CONTROLLED ASSOCIATION (8 SECTIONS)

This is a test of your ability to think of words. Several words have been written in the blank space after the word "GOOD." Read them.

GOOD:

right	*virtuous*	*deserving*
excellent	*noble*	*pleasing*
superior	*worthy*	*creditable*
commendable	*pious*	*admirable*

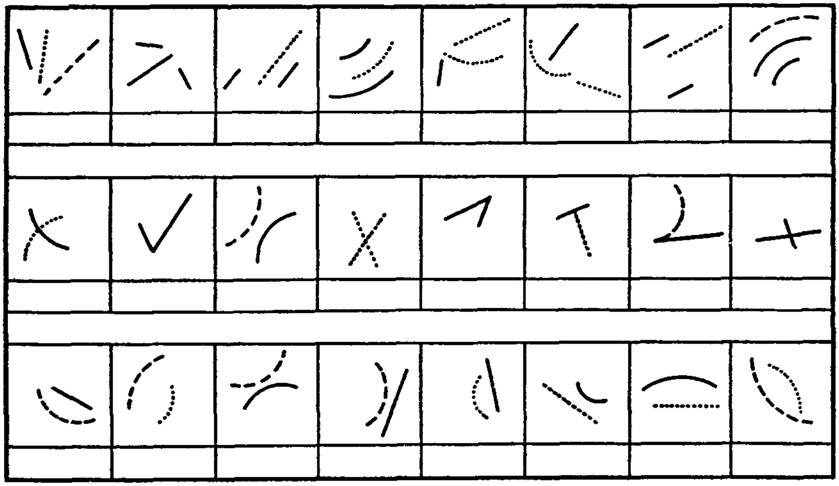
Notice that all the words *written* above are somewhat like the word "GOOD" in meaning.

In the blank spaces below write as many words as you can which have somewhat the same meaning as the given word. You may use slang or foreign words. Go right ahead.

Write words similar
in meaning to
HAPPY:

Write words similar
in meaning to
LARGE:

TEST SYMBOLS



Inventive Opposites (10) was prepared on a plan similar to that of Controlled Association. In the opposites test the subject is asked to produce two opposites for every given stimulus word. The initial letter of the response word is given. The opposites test in this form is likely to feature verbal fluency more than the opposites test which involves only checking one of several given response words. However, this is a question of fact, and it is the object of the present study to determine whether verbal fluency is a primary ability distinct from other abilities.

SAMPLES FROM INVENTIVE OPPOSITES (30 ITEMS)

This is a test of ability to think of words. Think of two different words opposite in meaning to the word *narrow* below. One word should begin with *b*. The other should begin with *w*. The words are *broad* and *wide*. These words have been written in the blanks.

narrow	b *broad*	w *wide*

Now think of two words opposite in meaning to the word *large*. The first should begin with *l*; the second with *s*.

large	l ________	s ________

The words are *little* and *small*. Write *little* in the first blank. Write *small* in the second.

strong	f ________	w ________
wrong	r ________	c ________
dark	b ________	l ________

Completion (11) was adapted from the Completion tests that have been used in the Psychological Examination of the American Council on

Education. It is one form of vocabulary test in that a definition is given and the subject is asked to supply the word. He is given the initial letter of the response word. It is likely that this test combines vocabulary and verbal fluency, if these are distinct abilities.

SAMPLES FROM COMPLETION (39 ITEMS)

There is one word missing in each of the following sentences. The first letter of the missing word is shown in the parentheses.

A contest of speed is a..................... (r) race

In order to complete the sentence we have written the word *race* in the blank space. The two sentences below are completed correctly.

A fluid used in writing is.................. (i) ink

A short sleep or doze is a.................. (n) nap

Complete the following sentences in the same way. The word you use must begin with the letter shown in parantheses.

A person licensed to practice medicine is a..... (d) ______

A cap used in sewing to protect the finger when pushing the needle is a.................. (t) ______

The red fluid that circulates in the veins and arteries of man is....................... (b) ______

The part of the day between noon and evening is.................................... (a) ______

A head covering is called a................. (h) ______

Disarranged Words (12) is similar to a form of test in current use.

SAMPLES FROM DISSARRANGED WORDS (72 ITEMS)

Rearrange the letters on each of the following lines to spell the name of an *animal.* In the first line, the letters (ebar) can be arranged to spell *bear,* which is written in the blank space. In the next line, the letters (odg) spell *dog,* which is written in the blank space. In the same way the letters (atc) spell *cat.*

ANIMALS

ebar bear
odg dog
atc cat

Rearrange the letters on each of the following lines to spell the name of a *boy.* The first two names have already been written for you. Write the third.

BOYS' NAMES

lpau Paul
rcla Carl
honj ______

Rearrange the letters on each of the following lines to spell the name of a *bird.*

BIRDS

uckd ______
cowr ______
wahk ______

First and Last Letter (13) was devised by Thelma Gwinn Thurstone as another test of the facility with which words come to mind. The subject is given the first and last letter and asked to write as many words as possible that have the given initial and terminal letters. There are marked individual differences in this ability.

SAMPLES FROM FIRST AND LAST LETTER

In the blanks below, write as many different words as you can that begin with *S*, and end with *L*. The words may be long or short. You may write the names of persons, or places or foreign words. Errors in spelling will not be counted against you.

As examples, the first three lines have already been filled in for you. Write as many other words as you can.

Write words that
begin with *S* and end with *L*.

1. sell
2. Saul
3. spell
4. __________
5. __________

Disarranged Sentences (14) was assembled from several forms of the Army Alpha tests.

SAMPLES FROM DISARRANGED SENTENCES (81 ITEMS)

In the sentence below the words are mixed up. When the words are put in the right order, they read, "Chicago is a large city." This statement is true. Therefore a plus sign (+) is written in the parentheses.

(+) Chicago large a city is

If the words in the next sentence are put in the right order, they read, "The dog is a rare animal." This statement is false. Therefore a minus sign (−) is written in the parentheses.

(−) animal a is the rare dog

Below are three more sentences which have been marked correctly.

(+) eyes some brown are
(−) blue human is usually hair
(−) of is green made moon cheese the

Mark the following sentences yourself. Put a plus sign (+) in the parentheses if the sentence is true. Put a minus sign (−) if it is false. Do *not* take time to write out the sentence in the correct order.

( ) white blackboard the is
( ) is falling rain water
( ) fly some can birds
( ) blue is sky the sometimes
( ) an never dies evergreen
( ) large is an beast ant a

Anagrams (15) is similar to current tests of the same name.

SAMPLES FROM ANAGRAMS

Make as many different words as you can, using only the letters in the word G-E-N-E-R-A-T-I-O-N-S. You may use long or short words and may include the names of persons, places, or foreign words. In any one word do not use a letter more times than it appears in G-E-N-E-R-A-T-I-O-N-S.

Sample words have been written in the first few lines. Continue writing as many words as you can using only the letters given.

G-E-N-E-R-A-T-I-O-N-S

1. art ____________
2. era ____________
3. snore ____________
4. ____________
5. ____________

Inventive Synonyms (16) is arranged in a manner similar to that of Inventive Opposites in that the subject is asked to supply two synonyms for each stimulus word and is given the initial of each of the response words.

SAMPLES FROM INVENTIVE SYNONYMS (29 ITEMS)

This is a test of ability to think of words. Think of two different words that mean the same as the word *tiny* below. One word should begin with *s*. The other should begin with *l*. The words are *small* and *little*. These words have been written in the blanks.

tiny	s small ________	l little ________

Now think of two words that mean the same as *aid*. The first should begin with *h;* the second with *a*.

aid	h ________	a ________

The words are *help* and *assist*. Write *help* in the first blank. Write *assist* in the second.

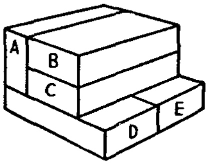
Go ahead with the exercises on this page. For each word that is given write *two* other words of *similar* meaning. The words *must* begin with the letters indicated.

huge	b ________	l ________
possess	h ________	o ________
infant	c ________	b ________

Block-counting (17) was adapted from one of the tests of Professor T. W. MacQuarrie. The present form has been made longer, but the items are of the same character. It should be a good test of visualizing in solid space, and it gives a large number of independent responses in a relatively short time. The instructions and fore-exercise were combined on the first page of the test.

SAMPLES FROM BLOCK-COUNTING (16 PROBLEMS)

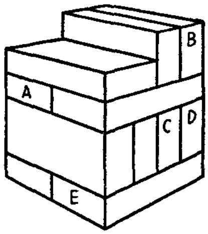
Figure 1 represents a pile of blocks. All of them are the same size and shape. The block *A* touches four other blocks (*B*, *C*, *D*, and *E*). Therefore we have written 4 after *A* in the column to the right. Block *B* touches two other blocks. Therefore 2 is written after *B* in the column to the right. Block *C* touches 4 other blocks and 4 is written after *C*. The rest of the column is marked in the same way.



A	4
B	2
C	4
D	3
E	3

FIG. 1

Fill in the blank spaces to show how many blocks touch each of the lettered blocks. The blocks are all of the same size and shape.



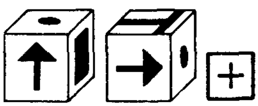
A	
B	
C	
D	
E	

Cubes (18) is an adaptation of a more difficult form of test that has been used by Professor Carl C. Brigham in experimental tests for the College Entrance Examination Board and is described in *A Study of Error*.[2] This is probably the most difficult of the space tests in the present battery. The instructions and fore-exercise are given on separate pages with separate time limits in order that the subject may be carefully introduced to the exact nature of the task.

SAMPLES FROM CUBES (32 PROBLEMS)

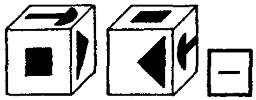
The drawings in this test represents cubes. There is a different design on each face of the cube. A cube has *six* faces.

Notice that both of the drawings below can represent the *same* cube. *Be sure you see that the first and second drawings represent the same cube turned into two different positions.* Since both drawings can represent the same cube, a plus sign (+) has been placed in the blank square at the right.

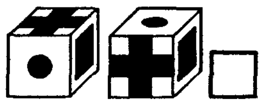


[2] Carl C. Brigham, *A Study of Error* (New York: College Entrance Examination Board, 1932).

Notice that the two drawings below represent two *different* cubes and that a minus sign (−) has been placed in the blank square at the right. *Be sure that you see that it would be impossible to turn the cube shown in the first drawing so that it would look EXACTLY like the cube shown in the second drawing.* Unless you see this clearly, you cannot solve the test items. There is a different design on each face of the cube.

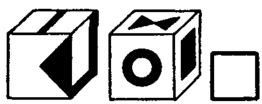


If the two drawings below *can* represent the same cube turned into different positions, put a plus sign (+) in the square at the right. If the two drawings below *cannot* represent the same cube, put a minus sign (−) in the blank at the right. *Remember that there is a different design on each face of the cube.*



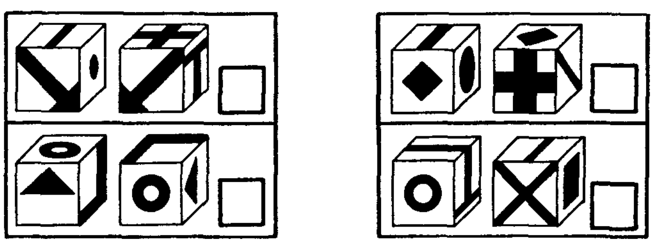
You should have marked the above problem minus (−). Study it carefully to be *sure* you see that this is true.

Now mark the problem below.



You should have marked the above problem plus (+).

In the following practice problems put a plus sign (+) if the two drawings can both represent the same cube. Put a minus sign (−) if they cannot both represent the same cube. Remember that there is a different design on each of the six faces of a cube.



Lozenges A (19) was prepared by the author at Carnegie Institute of Technology. The purpose of this test was to introduce kinaesthetic imagery into a paper-pencil test without involving mechanical devices. It is quite likely that kinaesthetic imagery is involved in this test for most people and that it also involves large loadings of one or more visualizing factors.

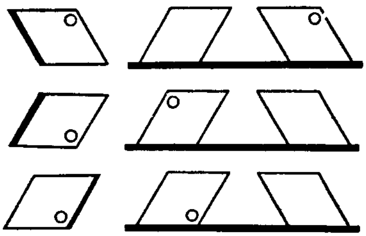
SAMPLES OF LOZENGES A (48 ITEMS)

Figure 1 represents a lozenge-shaped card. It has a hole in one corner. It is painted black on one edge. Imagine that it is picked up, turned over, and placed *face down* so that the black edge touches the long black line in Figure 2. Decide which of the two diagrams it would fit. Where would the hole be? It is shown in Figure 2.

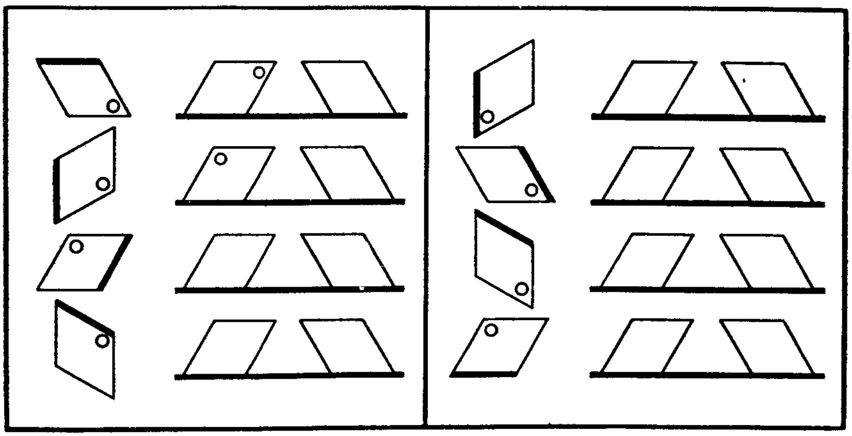
FIG. 1

FIG. 2

In each of the other three sample problems imagine that the card is picked up, turned over, and placed *face down* so that the black edge touches the long black line. Decide which of the two diagrams it would fit and where the hole would be. In each of these examples a small circle has been drawn to show where the hole would be.



For each of the following problems decide which of the two diagrams the card would fit when it has been turned over and draw a small circle to show where the hole would be. The first two problems below have been marked for you.



Flags (20) was prepared by the author at Carnegie Institute of Technology in studies of visualizing. This is one of the simplest of the visualizing or space tests in this battery and probably does not involve so much kinaesthetic imagery as the Lozenges test.

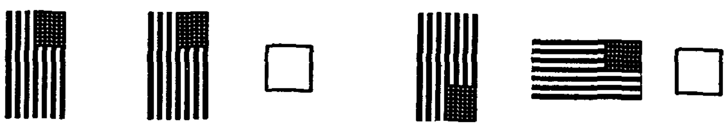
SAMPLES FROM FLAGS (48 ITEMS)

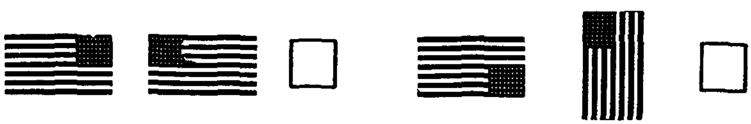
Look at the two flag pictures below. They represent opposite faces of the same flag. A minus sign (−) has been placed in the square to indicate that the pictures show opposite faces of the flag.

Now compare the next two pictures. If they represent the same face of the flag, mark them plus (+). If they represent opposite faces of the same flag, mark them minus (−).

The two pictures show the same face of the flag, so a plus sign (+) should be written in the square.

Mark in this way each of the following pairs of flag pictures. If the two pictures shows the same face of the flag, write a plus sign (+) in the square. If the two pictures show opposite faces of the flag, write a minus sign (−) in the square. Go right ahead. Do not wait for any signal.

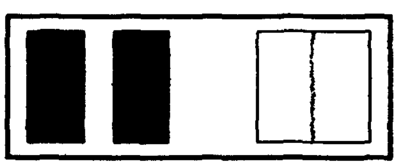




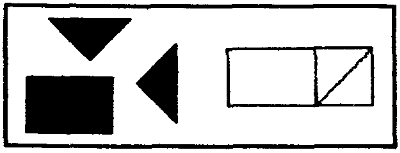
Form Board (21) is identical with the Minnesota Form Board test except for the addition of printed instructions and a fore-exercise. These were combined in the first page of the test.

SAMPLES FROM FORM BOARD (42 ITEMS)

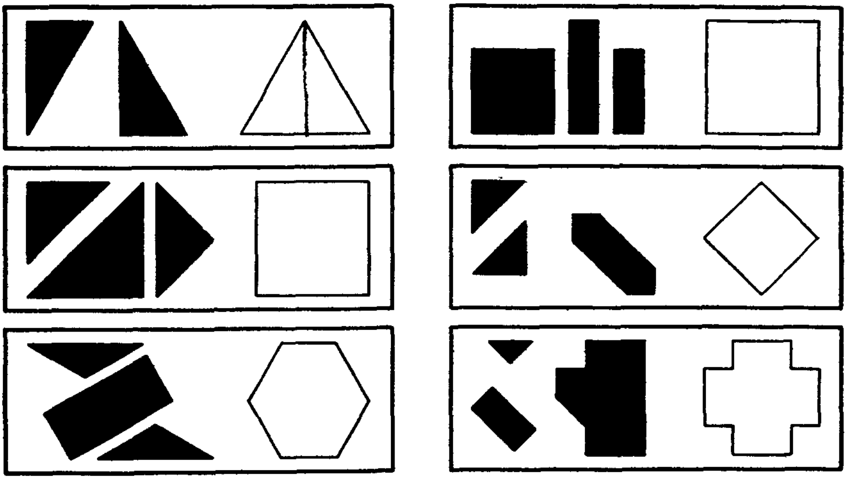
The two black pieces can be placed together to form the outline at the right, a square. A pencil line has been drawn to show how the two pieces can be placed so as to *fit exactly* on the square. A horizontal line through the center of the square would also be a correct answer.



The three black pieces can be placed so as to make the rectangular outline. Pencil lines have been drawn to show one possible arrangement of the black pieces.



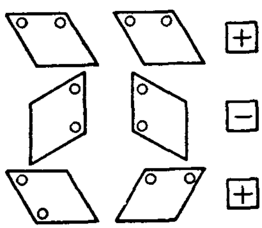
In the following practice problems the first one has already been marked for you. Now draw pencil lines in the white outlines to show how the black pieces can be placed so as to *fit* the outlines. Do not waste time over extreme accuracy. Merely indicate by lines how the black pieces may be placed so as to cover the given outlines.



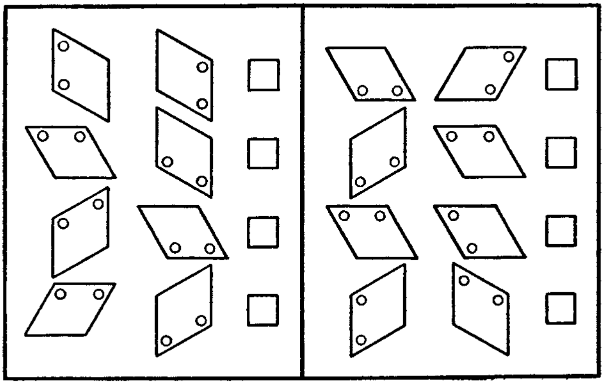
Lozenges B (22) was designed by the author at Carnegie Institute of Technology in a study of visualizing.

SAMPLES FROM LOZENGES B (78 ITEMS)

Each pair of diagrams represents a lozenge-shaped card. If the two diagrams show the same side of the card, a plus sign is written in the square. If they show opposite sides of the card, a minus sign is written in the square.



Now work the following practice problems. Write plus if the diagrams show the same side of the card; write minus if they show the opposite sides. Go ahead.



Surface Development (23) was prepared by the author. This is a new test of visualizing and space thinking.

SAMPLES FROM SURFACE DEVELOPMENT (6 PROBLEMS)

In this test you will be shown pictures of some simple objects that can be made from cardboard or sheet metal. With each picture there is also a diagram showing how a piece of cardboard might be cut and folded so as to make the object. The dotted lines show where the cardboard or metal should be folded.

Look at the *picture* and the *diagram* below. The diagram shows a cardboard cut so that it might be folded into the shape of a box. Notice the side *A* in the picture and also the side *A* in the diagram. Notice that the edges *e, d, f, h,* in the picture are also marked in the diagram.

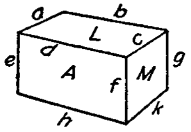
In the *schedule* there are indicated certain parts of the diagram which correspond to certain parts of the picture. You are to fill in the blank spaces in the schedule.

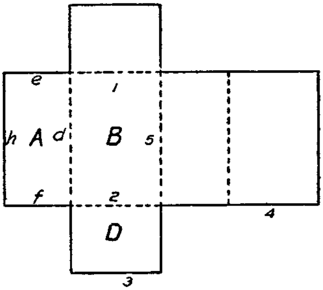
In the first line of the schedule we are *to find the part of the picture* that corresponds to the side *B* in the diagram. Look at *B* in the diagram and you will see that it corresponds to *L* in the picture. Therefore *L* has been written in the blank space opposite *B* in the schedule.

In the next line of the schedule we are *to find the part of the picture* that corresponds to *1* in the diagram. Look at the edge *1* in the diagram and you will see that it corresponds to the edge *a* in the picture. Therefore the letter *a* has been written in the schedule.

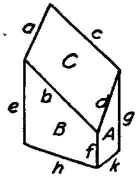
Fill in the other blanks in the same manner.

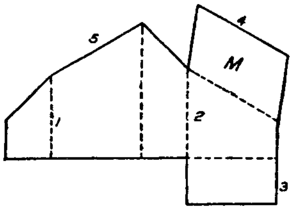
PICTURE	DIAGRAM	SCHEDULE





Part of the picture	Part of the diagram
L	B
a	1
	2
	3
	4
	5





Part of the picture	Part of the diagram
	M
	1
	2
	3
	4
	5

Punched Holes (24) is an adaptation and extension of a form of test in current use.

SAMPLES FROM PUNCHED HOLES (10 ITEMS)

Figure 1 represents a square sheet of paper. It is folded on the dashed line and then it looks like Figure 2. A hole is punched through the folded paper as shown in Figure 2. Show by small circles *in Figure 1*, where the holes would be when the paper is unfolded. The small circles have been drawn for you in this example.

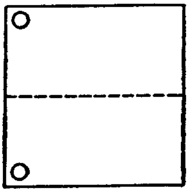


FIG. 1

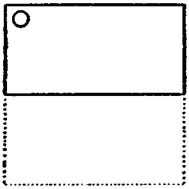


FIG. 2

Figure 3 is another sheet of paper. When it is folded along the dashed line, it looks like Figure 4. A hole is punched as shown in Figure 4. Show *in Figure 3* where the holes would be when the paper is unfolded.

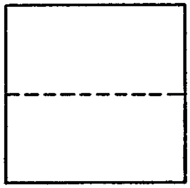


FIG. 3

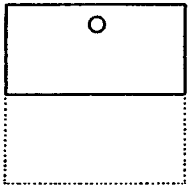


FIG. 4

Figure 5 is a sheet of paper which is folded along the dashed line. Then it looks like Figure 6. It is folded once more and then it looks like Figure 7. A hole is then punched through it. Show *in Figure 5* where the holes will be when the paper is unfolded. The holes have been marked for you in this example.

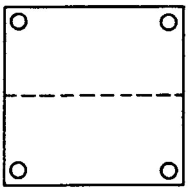


FIG. 5

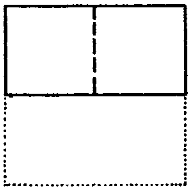


FIG. 6

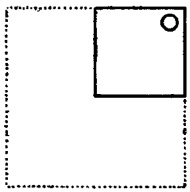
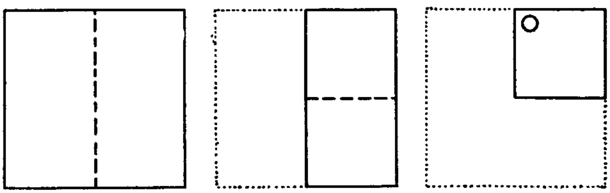


FIG. 7

These diagrams represent in the same manner a sheet of paper that is folded twice. A hole is then punched through it. Show by small circles in the first square where the holes will be when the paper is unfolded.



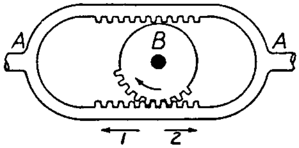
Mechanical Movements (25) is an extension of a test prepared by the author at Carnegie Institute of Technology. It was planned to be a test of kinaesthetic imagery.

SAMPLES FROM MECHANICAL MOVEMENTS (22 PROBLEMS)

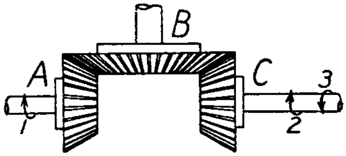
In this test you will be shown pictures of mechanical movements. You will be asked questions about them.

In each picture the part that makes the others move is called the *driver*. The solid black circles represent axles which can turn but cannot move from where they are shown.

Now answer the questions after each of the pictures below. Go right ahead.

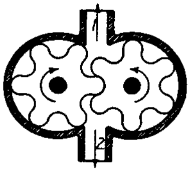


1. If *B* starts moving in the direction shown, which way will *A* move, *1* or *2?* . . . . ______
2. In which direction will *A* be moving when *B* has turned half way around from where it is now? . . . . ______



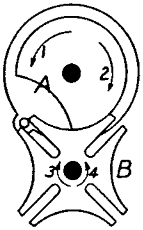
A, *B*, and *C* are three beveled gears.

1. If *A* is turning in the direction shown, which way is *C* turning, *2* or *3?* . . . . ______
2. Can *B* be the driver? . . . . Yes No
(Draw a ring around the correct answer.)



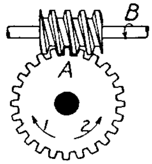
The gears of this rotary oil pump fit very close to the sides and end of the case and against each other.

1. If the gears turn in the direction shown, which way will the oil be forced? . . . . ______

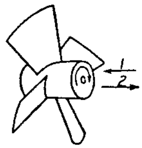


1. Which part moves only part of the time? . . . . ______
2. Which part is the driver? . . . . ______
3. Can this mechanism operate when *A* moves in direction *1?* . . . . . . Yes No
4. Can this mechanism operate when *A* moves in direction *2?* . . . . . Yes No

5. If B moves in the direction 4, which way is A moving? . . . .
6. How many times does A turn around while B is turning around once? . . . .



1. If B turns in the direction shown, which will A turn? . . . .
2. Can A be the driver? . . . . Yes No
3. If the worm were taken off, reversed, and replaced on the same shaft, which way would A turn? . . . .

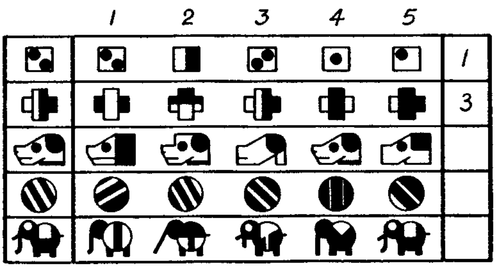


1. If the fan is turning in the direction shown, which way does it blow the air? . . . .
2. If this were a propeller on a boat, which way would the boat move? . . . .
3. If this were a propeller on an airplane, which way would the plane move? . . . .

Identical Forms (26) was adapted from a current test with the assistance of Mr. Alfred Sterges. This test was included in the battery in the hope of discovering by factor analysis whether the quickness in perceiving details in flat form is distinct from the ability to visualize the movement of objects in flat space and in solid space.

SAMPLES FROM IDENTICAL FORMS (60 ITEMS)

The first figure in each line below is exactly the same as one of the five numbered figures following. In the blank space at the right of each line, write the number of the figure which is exactly the same as the first figure in the line. The first two blank spaces have been filled in correctly. You fill in the other three. Go right ahead.



Pursuit (27) was adapted and extended from a short test of similar form by Professor T. W. MacQuarrie. Several of his tests have been reproduced in the present battery with his permission.

SAMPLES FROM PURSUIT (8 DIAGRAMS)

Figure 1 consists of ten lines which run across the diagram from left to right. The beginning of each line is numbered. Notice that line *1* begins in the upper left corner and ends near the bottom at the right. The figure *1* has been written in the space where it ends. Line *2* ends at the bottom space on the right, and *2* is written in that space.

Now start with line *3* on the left. Follow it across to the right and write the figure *3* in the space at which you end.

The figure *3* should be written in the fourth space from the top on the right side.

Follow each of the remaining lines from its beginning at the left to the space where it ends at the right. Write its number in the space where it ends.

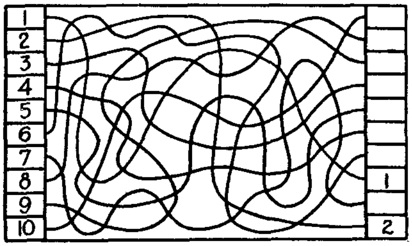


FIG. 1

Copying (28) was adapted from a short test of the same kind by Professor T. W. MacQuarrie. It involves not only visualizing a pattern in flat space but the score is also influenced by the subject's motor coordination in reproducing the pattern. It is a psychological question whether the muscular component in a test of this sort is the same as the kinaesthetic imagery which is almost certainly called for in some of the other tests.

SAMPLES FROM COPYING (36 DIAGRAMS)

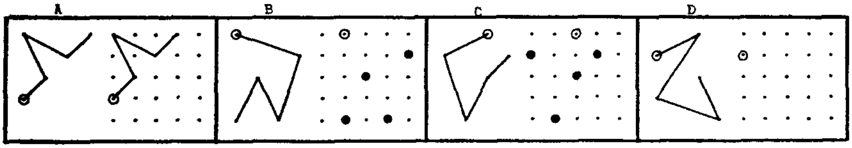
In this test you are to copy each of the figures in the dotted space to the right of it. The little circles show you where to begin. There is a dot for every corner.

Figure A has been copied as an example in the dotted space to the right of it.

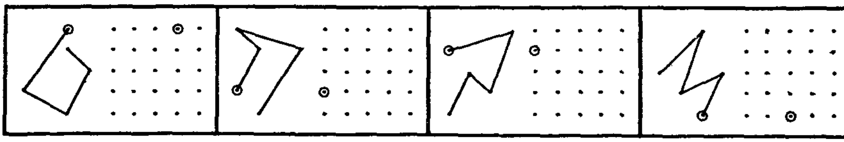
Copy Figure B in the dotted space. Begin at the small circle and use the heavy dots as guides.

Copy Figure C in the dotted space in the same manner.

Copy Figure D in the dotted space.



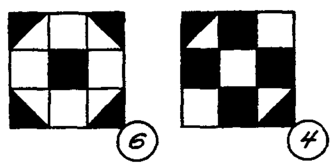
Copy each of the following figures in the same way. Begin at the small circles. Your lines do not have to be straight, but they should begin and end on dots. Make corrections, if you wish, but do not waste time erasing.



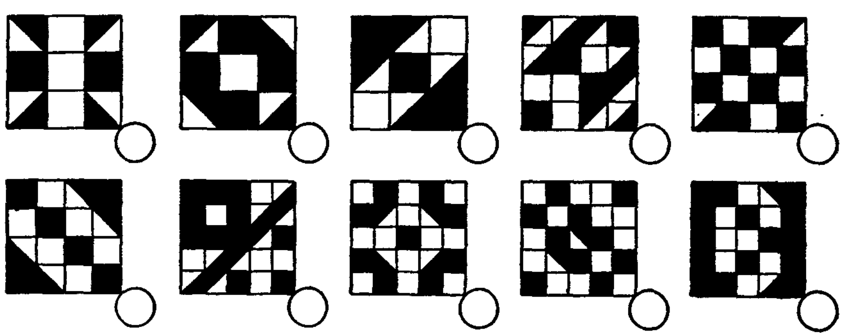
Areas (29) was adapted from one of the tests of Professor Carl C. Brigham.[3]

SAMPLES FROM AREAS (35 DIAGRAMS)

In each of the following figures, determine the total white area in terms of squares. The answers are shown in the attached circles.



Do the same with the following figures. Give your answers in terms of squares but disregard the size of the squares. Use fractions where necessary.



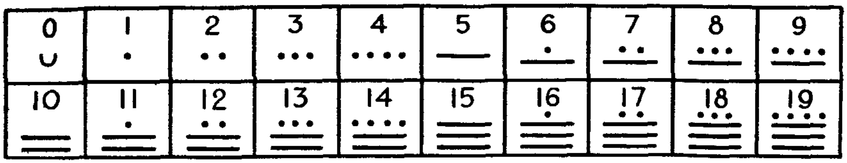
Number Code (30) was designed by Thelma Gwinn Thurstone. It makes use of the Mayan number code. This test involves some number facility, and it probably calls for one or more of the visualizing factors. It may also be complicated by a learning factor. It was introduced in

[3] *Ibid.*

an effort to diversify the number tests. Most number tests seem to be limited to simple numerical calculation and to arithmetical reasoning problems.

SAMPLES FROM NUMBER CODE (40)

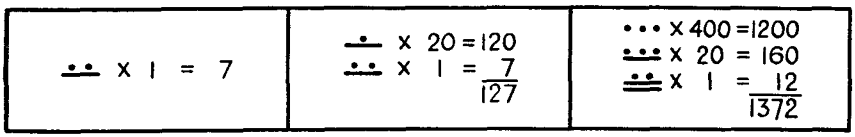
In this test you will be asked to use a number code based on twenty symbols instead of the ten digits to which we are accustomed. There is a symbol for each of the numbers from 0 to 19, as shown below. Notice that a bar means 5 and that a dot means 1. For example, the number 9 is represented by a symbol consisting of a bar and four dots. Zero is represented by a U-shaped symbol as shown.



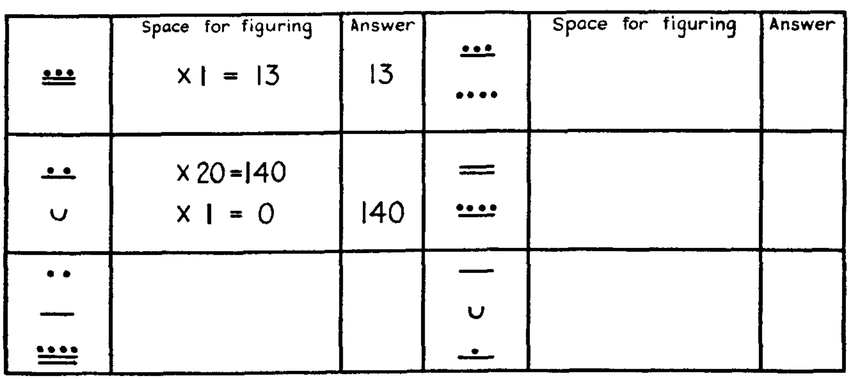
For numbers larger than nineteen, the symbols are combined, one above the other. This is shown in Example 2 below. When there are two symbols, one above the other, the upper one is to be multiplied by 20 and the bottom symbol is to be multiplied by 1. The answer is the sum. Study Example 2.

For numbers larger than 399, three symbols are used, one above the other. The uppermost symbol is multiplied by 400, the next by 20, and the bottom symbol by 1. The answer is the sum. Study Example 3.

EXAMPLE 1 EXAMPLE 2 EXAMPLE 3



Now solve the six practice problems below. The first two have already been solved for you.



Addition (31) is a simple test of ordinary addition of seven two-digit numbers.

Subtraction (32) is a simple test of subtraction involving four-digit numbers.

Multiplication (33) is a simple test involving the multiplication of six-digit numbers by a single-digit number.

Division (34) is a simple test involving the division of seven-digit numbers by a single-digit number.

Tabular Completion (35) was designed by the author as a part of a clerical examination. The subject is asked to fill in the missing numerical entries in a table by examining the column headings. It involves very simple reasoning and very simple calculation.

SAMPLES FROM TABULAR COMPLETION (4 TABLES)

In the following table there are blank spaces. Fill in the correct numbers in these spaces. Get the necessary information from the rest of the table. Use the space below each table and on the opposite page for figuring.

Year	Marriages		Divorces			
	Number	Increase over Preceding Year	Number	Increase over Preceding Year	Number Granted to Husband	Number Granted to Wife
1894	566,161	12,512	37,568	100	12,551	25,017
1895	598,855	32,694	40,387	2,819		26,931
1896	613,873	15,018	42,937		14,448	
1897	622,350			1,762	14,765	29,934
Total	2,401,239	No total	165,591	No total	55,220	110,371

Estimating (36) is an extension of a test prepared by the author in the 1924 edition of the Psychological Examination of the American Council on Education. It is a numerical reasoning test in which the subject is asked to write his own premises. Casual inspection of this test frequently brings the comment that it is a test of encyclopedic information, but this is entirely erroneous. It is a test of numerical reasoning. In the original form the subject was asked to write the answer in a blank space. It was scored by considering the ratio of the subject's answer to the correct answer. The frequency distribution of the logarithms of the answers should be Gaussian. There are striking sex differences in this test in that the majority of women and an appreciable number of men dislike the task. In the present form the test was prepared with four alternative answers, and the subject is asked to check the one answer which seems to him to be most reasonable. The best scores are obtained by those subjects who start with some generally known facts as premises and who make deductions about the expected order of magnitude of the answer. The test was scored by giving two points for each right answer and one point for each deviation of not more than one step.

SAMPLES FROM ESTIMATING (18 ITEMS)

This is a test of your skill in estimating facts by reasonsing.

In the illustration below you are asked to estimate the number of children in the United States under five years of age. Four answers are given, *one* of which is *correct*.

Estimate the number of children in the United States in 1930 under five years of age.	5,100,000 ______
	11,500,000 __√__
10 ⌊ 120,000,000	58,400,000 ______
12,000,000	115,700,000 ______

There are many ways of finding out which of the above answers is correct. For example, if you happen to know that there are about 120,000,000 people in the United States, you might estimate that the average life-span is fifty years (roughly) which means that (very roughly) one-tenth of the population would be under five. This makes about 12,000,000. Knowing that *one* of the above answers is *correct* you would check 11,500,000 because it is the answer nearest to your estimate. This is the correct answer.

Below is another illustration.

Estimate the population per square mile in the United States in 1930.	8.5 ______
	41.3 __√__
$\frac{3{,}000 \times 1{,}000}{3{,}000{,}000}$ $\quad \frac{120{,}000{,}000}{3{,}000{,}000} = 40$	206.3 ______
	1032.5 ______

In solving this problem, you might estimate that the size of the United States is (roughly) 1,000 miles by 3,000 miles. This makes a total of 3,000,000 square miles. Divide the population of 120,000,000 by 3,000,000. This gives about 40 persons per square mile. You would then check 41.3 which is the correct answer.

Answer the sample problems given below in the same way. Do not "just guess." *Reason* out each answer in some way, as illustrated above. You may use the extra space for figuring. Indicate the correct answer by a check mark.

Estimate the number of automobiles registered in the United States in 1930.	5,000 ______
	26,000,000 ______
	120,000,000 ______
	509,000,000 ______
Estimate the number of paving bricks necessary to pave a section of street, 20′ by 100′.	1,000 ______
	6,000 ______
	30,000 ______
	100,000 ______
Estimate the number of people that can be seated in a space 30′ by 40′ using ordinary folding chairs. Do not allow for aisles.	123 ______
	415 ______
	1248 ______
	3572 ______

Number Series (37) is similar to the number-series tests in current use.

SAMPLES FROM NUMBER SERIES (22 SERIES)

The numbers in each row of this test follow one another according to some rule. You are to find the rule and fill in the blanks to fit the rule.

In the example below each number can be obtained from the one before it by the rule *add 2.* The blanks have been filled in accordingly.

2	4	6	8	10	12	14

Find the rule in the series below and fill in the blanks. You may use addition, subtraction, multiplication, division, or any combination of these.

10	8	11	____	12	____	13

The above series goes by alternate steps of subtracting 2 and adding 3. You should have written 9 and 10 in the blanks.

Find the rule in each series below and write the numbers in the blanks accordingly. There is a different rule for each line. Go right ahead. Do not wait for any signal.

19	18	17	____	15	14	____
8	11	14	____	20	____	
27	____	23	23	19	19	____

Numerical Judgment (38) was prepared by the author for this test battery. It is intended to test numerical judgment rather than speed and exactness of calculation. It is so arranged that the subject makes a judgment about the order of magnitude of the expected answer. Exact answers are not required.

SAMPLES OF NUMERICAL JUDGMENT (24 ITEMS)

In this test you are shown some arithmetical problems that have already been worked out. Four answers are given for each problem. One of these is always the right answer. You are asked merely to check the right answer. You may use the space on the page for figuring but do not waste time working out the exact answer.

In the first problem below you can readily see that the first number is nearly 4 and the second number is nearly 7. Since $4 \times 7 = 28$, look for the answer that is nearest 28. This is the third answer and it is checked.

	7.563327	____
$4.12395 \times 6.82187 =$	14.012468	____
$4 \times 7 = 28$	28.133051	✓
	56.103378	____

In the problem below you see that the numerator is nearly 30 and the denominator is nearly 6. Since $30 \div 6 = 5$, we check the second answer which is the one nearest 5.

$\frac{29.6718}{5.7261} =$	4.4278	____
	5.1819	✓
6)30 / 5	6.9271	____
	8.4293	____

Since you *know* that one answer is *correct*, there are many other tricks for finding out *which* answer that is. For example, in the problem below you see that $30 \times 30 = 900$. Therefore 29×29 must be *less* than 900. You can also see that $9 \times 9 = 81$, so that the right-hand figure of the answer will be 1. Hence 841 is the only possible correct answer.

$(29)^2 =$	755.	______
30	841.	______
30	865.	______
900	901.	______

In the problems below use any tricks or short cuts to find out which answer is correct and check that answer. Do not waste time checking exact answers because one of the given answers is the correct one.

	2.621	______
$3.01224 \times 4.86537 =$	6.782	______
	14.656	______
	21.387	______
	6.5643	______
$\frac{53.29736}{5.01258} =$	10.6327	______
	91.7136	______
	134.6973	______
	7698.	______
1351 + 8271 + 72 + 3 + 51 +	9875.	______
2 + 1 + 13 + 9 + 4 + 23 +	13561.	______
8 + 19 + 22 + 4 + 6 + 16 =	20679.	______
	11569.	______
$(197)^2 =$	23417.	______
	38809.	______
	62187.	______

Arithmetical Reasoning (39) contains nineteen problems and is similar to current tests of this type.

Reasoning (40) is a syllogism test in which the subject is asked to judge whether an inference follows from the given premises. This test was prepared with the assistance of Mr. Merrill E. Roff.

SAMPLES FROM REASONING (33 SYLLOGISMS)

This is a test of your ability to tell the difference between good and bad reasoning. The first argument below is *good* reasoning and is marked plus (+). The second argument *appears* similar but is *bad* reasoning and is marked minus (−).

All sports are dangerous, and football is a sport.
Therefore, football is dangerous. ___+___

Some sports are dangerous, and football is a sport.
Therefore, football is dangerous. ___−___

Now mark the two arguments below in the same way.

All wealthy men pay taxes. Mr. White pays taxes.
Therefore, Mr. White is wealthy. ______

All wealthy men pay taxes. Mr. White is wealthy.
Therefore, Mr. White pays taxes. ______

The first argument above is *bad* and should have been marked minus (−). The second should have been marked plus (+).

Mark the following arguments in the same way. Put a plus sign (+) for good reasoning and a minus sign (−) for bad reasoning.

Knavery and folly always go together; so, knowing him to be a fool, I distrusted him. ______

Since all metals are elements, the most rare of all metals must be the most rare of all elements. ______

None of her dresses are blue. Therefore this blue dress is not hers. ______

Haste makes waste, and waste makes want. Therefore a man never loses by delay. ______

Verbal Analogies (41) is similar to current tests of this type. It was prepared with the assistance of Mrs. Dorothy Slesinger.

SAMPLES FROM VERBAL ANALOGIES (56 ITEMS)

Read the following row of words:

1—foot:2—shoe 3—hand: 4—thumb 5—head 6—glove 7—finger 8—clasp _6_

The first two words, *foot* and *shoe,* are united by a certain relation, the shoe is worn on the foot. The next word is *hand.* Which of the five words following can be combined with *hand* in the way given by the *foot-shoe* relation? The answer is *glove* because the glove is worn on the hand. Therefore a *6* is written in the blank at the right.

In the following two exercises find the word at the right which is related to the third word in the same way that the second word is related to the first. Write the corresponding numbers in the blanks at the right.

1—fish:2—water 3—bird: 4—blue 5—robin 6—ocean 7—sky 8—high ______

1—mayor:2—city 3—captain: 4—ship 5—private 6—general 7—store 8—lieutenant ______

False Premises (42) was designed as a test of syllogistic reasoning in which the subject does not have the advantage of dealing with meaningful sentences. The premises are absurd and so are the conclusions. The subject is asked merely to judge whether the conclusions follow from the given premises, even though the premises are known to be either false or meaningless in the ordinary interpretation of the words involved. This test was prepared with the assistance of Mr. Merrill E. Roff.

SAMPLES FROM FALSE PREMISES (25 SYLLOGISMS)

This is a test of your ability to tell the difference between good and bad reasoning. *You must judge only the reasoning* in the following arguments because every statement is false or even absurd.

The first argument below is *good* reasoning and is marked plus (+). The second argument *appears* similar but is *bad* reasoning and is marked minus (−).

All haystacks are catfish. All catfish are typewriters. Therefore all haystacks are typewriters. _+_

All haystacks are typewriters. All catfish are typewriters.
Therefore all haystacks are catfish. _−_

Notice that the first argument below is bad reasoning, while the second is good reasoning. Mark them accordingly.

Some lagoons are hilltops, and all hilltops are hungry; therefore all lagoons are hungry. ______

All lagoons are hilltops, and all hilltops are hungry; therefore all lagoons are hungry. ______

Mark the following practice problems in the same way. If the *reasoning is good*, mark the argument plus (+). If the *reasoning is bad*, mark the argument minus (−). Go right ahead. Do not wait for any signal.

All pigs can fly, and all elephants are pigs; therefore all elephants can fly. ______

Some radishes are rumble-seats, and some rumble-seats sing soprano; therefore some radishes sing soprano. ______

No two people look just alike. Jimmie and Johnnie look just alike; therefore Jimmie and Johnnie are not two people. ______

All street lamps have adenoids, and some street lamps are telephones; therefore some telephones have adenoids. ______

Since all thimbles are geraniums, the most ferocious thimble must be the most ferocious geranium. ______

Chimpanzees are more lurid than buggies, for buggies are not lurid at all, and chimpanzees cannot be buggies. ______

If limousines were more valuable than fleas, I should be the King of Spades; but I am not the King of Spades, therefore limousines are not more valuable than fleas. ______

All violins that are made of natural gas taste bluish-pink. This violin does not taste bluish-pink; therefore it is not made of natural gas. ______

Code Words (43) was adapted from a test of Godfrey Thomson. It is a type of reasoning test involving the deciphering of code. The items in this test were prepared with the assistance of Mr. Harold O. Gulliksen.

SAMPLES FROM CODE WORDS (21 PROBLEMS)

This is a test of your ability to translate code words. Each symbol represents a letter in the English alphabet. A different code is used in each set of code words.

The words written in code below mean *no*, *to*, and *on*, but they are not arranged in that order.

WORDS	CODE WORDS	TRANSLATION
no	[>	t o
to	> ∫	o n
on	∫ >	n o

The letter *t* occurs only once in the words *no*, *to*, *on*. The symbol that occurs only once is [. Therefore [must represent *t* and the first code word is *to*. *To* has been written in the correct place. Since [> is *to*, > is *o*. Then the second code word is *on*, and the third is *no*.

Reason out and answer the examples below in the same way.

WORDS	CODE WORDS	TRANSLATION
call	δ ω η η	— — — —
miss	θ φ γ γ	— — — —
make	δ φ β λ	— — — —
assent	ξ ρ ρ π ✓ α	— — — — — —
allots	ξ θ θ β ω π	— — — — — —
accent	ξ φ φ ε α ρ	— — — — — —
adduce	ξ ω ω π ✓ α	— — — — — —

Pattern Analogies (44) was adapted from a similar test in the Psychological Examination of the American Council on Education.

SAMPLES FROM PATTERN ANALOGIES (23 ITEMS)

Each problem in this test consists of eight figures. The first three are called *A*, *B*, and *C*. The next five are called *1*, *2*, *3*, *4*, and *5*.

In Sample I below figure *A* is a large circle. Figure *B* is a small circle. By what rule is figure *A* changed in making figure *B?* The rule is "making it smaller." Apply this rule to figure *C* which is a large square. The result is a small square. Find the small square in the row of five figures at the right. It is figure *2*. Therefore *2* is written in the blank at the right.

Sample I

A B C 1 2 3 4 5

In Sample II below the rule is "Figure *A* is turned upside down to make figure *B*." If this rule is applied to figure *C*, the result is figure *4*. Therefore *4* is written in the blank at the right.

Sample II

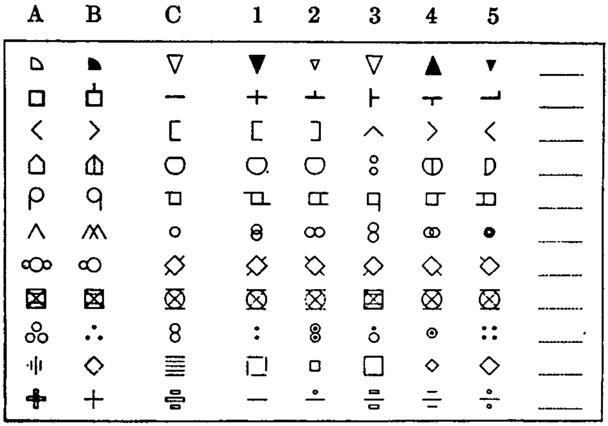
A B C 1 2 3 4 5

In Sample III below the rule has two parts, "Make figure *B* larger than figure *A* and of the opposite color." If this rule is applied to figure *C*, the result is figure *1*. Write *1* in the blank at the right.

Sample III

A B C 1 2 3 4 5

First, decide what rule is used to make figure *B* from figure *A*.
Second, apply this rule to figure *C* and find the resulting figure among the figures *1–5*.
Third, write the number of this figure in the blank at the end of the row.



Syllogisms (45) was prepared by the author at Carnegie Institute of Technology. It was intended as a test in syllogistic form with extreme monotony of material. The three syllogism tests were included in order to ascertain whether the factors would be considerably affected by monotony or extreme variation in subject matter or by making the premises simple, meaningful, or absurd.

SAMPLES FROM SYLLOGISM TEST (32 SYLLOGISMS)

Read carefully: You will be shown a list of arguments, each followed by a conclusion. Some of the conclusions are right, others are wrong.

On the line after each argument make a plus sign (+) if the conclusion is right; make a minus sign (−) if the conclusion is wrong.

The three following examples are marked correctly.

Jones is younger than Brown.
Brown is younger than Smith.
Therefore Smith is younger than Jones. __−__

Smith is older than Jones.
Jones is older than Brown.
Therefore Brown is younger than Smith. __+__

Smith is older than Jones.
Smith is younger than Brown.
Therefore Brown is older than Jones. __+__

Answer the following practice problems in the same way.

Brown is older than Jones.
Jones is older than Smith.
Therefore Brown is older than Smith. _____

Smith is younger than Jones.
Brown is older than Jones.
Therefore Smith is older than Brown. _____

Jones is younger than Brown.
Smith is younger than Jones.
Therefore Smith is older than Brown. _____

Smith is older than Jones.
Smith is younger than Brown.
Therefore Brown is younger than Jones. _____

Brown is older than Smith.
Jones is younger than Smith.
Therefore Jones is older than Brown. _____

Brown is younger than Smith.
Brown is older than Jones.
Therefore Jones is younger than Smith. _____

Word-Number (46) was prepared as a test involving memorizing. The subject memorizes a set of paired associates. Each stimulus word is to be associated with a response number. In the recall the subject is given the stimulus word, and he is asked to write the corresponding response number. The test is arranged with instructions and a fore-exercise followed by a recall. A second fore-exercise, which is longer, is then given. It is followed by a recall. The test proper with twenty words and associated numbers is then presented. This is followed by the recall. The test really consists of three sections with a presentation and recall in each section. This was done in order to make sure that the subjects understood the nature of the task.

It is of interest to ascertain whether a good memory as ordinarily interpreted and the ability to memorize intentionally involve the same abilities. It should be possible to investigate this problem by the factorial methods. Another psychological problem is to ascertain whether immediate memory in memorizing test items and memory over several months' or a year's time involve the same abilities. Still another psychological question concerns the relation of attention to memory and memorizing and learning ability. It would be of psychological interest to ascertain whether the ability to memorize for immediate recall involves the same primary factors as the ability to give sustained attention to the details of a task. The present battery was not designed to answer these questions, but we have included six tests under the general category of retentivity with varying material. Three of these tests are in the form of learning of paired associates.

Initials (47) is a paired-associates test in two sections. Each section is divided into two parts, namely, presentation and recall. The subject is first given a list of names with initials. He is then asked to recall the initials. Then follows the test proper with twenty-five names and initials which he is asked to memorize. Then comes the recall in which the twenty-five names are given, and the subject is asked to supply the initials.

Number-Number (48) is a paired-associates test in the same form as the two previous tests in which the stimulus consists of a two-digit number, and the response is another two-digit number. This test is also given in two sections with two parts for each section. Together with the instructions the subject is given five paired numbers to associate. He is then asked to recall the five response numbers when the stimulus numbers are presented. He is given the opportunity to write the numbers if he wants to learn them in that manner. Then follow the memorizing of the twenty pairs of numbers and the recall in which the twenty stimulus numbers are presented in random order. The subject is asked to fill in the response numbers.

Word Recognition (49) is another test of retention. In this test the subject is given a set of words to examine carefully so that he will be able to recall which words occur in the list. In the recall he is given a longer list of words, and the task is to check those words which were presented in the first list. The test is given in two sections with two parts in each section. In the instructions the subject is given three words, and he is then shown a list of nine words in random order. He is asked to check the three words which occurred in the first list. He is then asked to study a second list of six words. In the recognition he is given eighteen words, and he is asked to check the six words in this list which were previously presented. Then follows the test proper, in which the subject is asked to study carefully a list of fifty words. In the recall he is given a list of one hundred and fifty words which includes the stimulus list of fifty words. The subject is asked to check the stimulus words which occur in the longer list. In preparing the final recognition list, the distractors were written by a rather definite plan. Besides the correct word there is also a synonym and a word which sounds like the stimulus word or which is similarly spelled. This plan for the distractors makes the test more difficult than if the recognition list were any random list of words. Fractional scoring could be used in order to investigate individual differences in types of errors. It is conceivable that some subjects would more frequently err by checking incorrect words that sound like the stimulus word and that others would check words that look like the stimulus word or whose meaning is similar to the stimulus word.

Figure Recognition (50) was designed to parallel in form the Word Recognition test. This test is given in three sections with two parts for each section. The subject is first given several figures to examine. He is then given a longer list of figures and is asked to check the figures which occur in the first list. The subject is then given a second list of figures to examine. In the longer recognition list he is asked to check the figures previously seen. Then follows the test proper with twenty figures which the subject is asked to examine. The final recognition test has sixty figures including the twenty stimulus figures. The subject is asked to check those which he has previously seen.

Picture Recall (51).—This is another test involving retention in a form which has been used in previous studies. The subject is shown a picture which he is asked to examine for several minutes. After removing the picture, he is given a list of questions concerning the details of the picture.

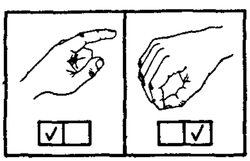
Theme (52) is a simple test in which the subject is asked merely to describe a person whom he knows quite well. It was scored by English instructors as a short theme.

Hands (53) was designed by the author as a part of a battery of tests for the study of visualizing. The original purpose of this test was to involve visualizing and kinaesthesis in nongeometrical and nonmechanical form. The subject is asked to identify each of a set of pictures of hands as to whether it represents a right hand or a left hand. There are conspicuous individual differences in this test.

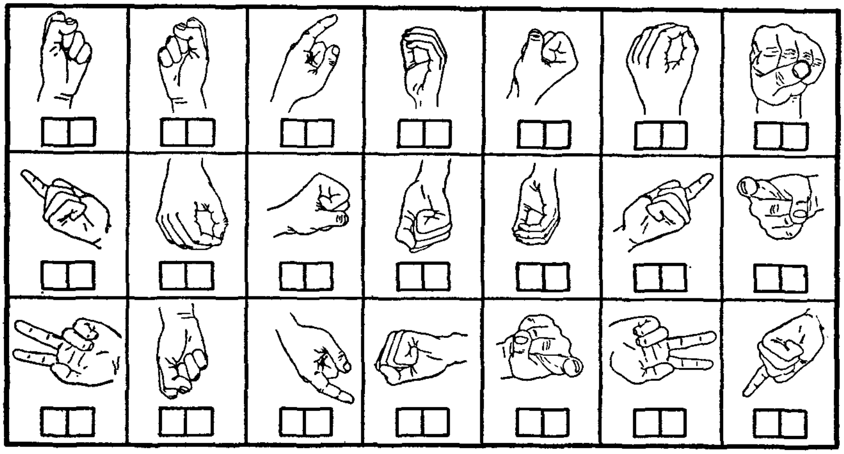
SAMPLES FROM HANDS (49 ITEMS)

In this test you will be shown a series of pictures of hands. Some of these pictures represent right hands, others represent left hands. Below each picture you will find two small squares.

If the picture represents a right hand, put a check mark in the right square; if it represents a left hand, put a check mark in the left square, as shown in the following samples, which are correctly marked.



Now mark the samples below in the same way. Go right ahead. Do not wait for any signal.



Rhythm (54) was designed by Thelma Gwinn Thurstone. In this test the subject is asked to examine sets of four lines and to check the one line whose rhythm is different from that of the other three. This test should involve some auditory and kinaesthetic factors in addition to possible verbal factors. It is also an attempt to introduce some material of auditory significance in paper-pencil form. But it is likely that auditory factors will be better isolated by tests that are more conspicuously auditory in character.

SAMPLES FROM RHYTHM (26 ITEMS)

Three of the lines below are alike in rhythm, but one is different. Notice that the *rhythm* of the third line is different from the others. Therefore a check mark has been placed in the blank before the third line.

______ Mary, Mary, quite contrary

______ Peter, Peter, pumpkin eater

__√__ Hey diddle diddle, the cat and the fiddle

______ Simple Simon met a pie man

Read the four lines below and decide which line differs in rhythm from the others. Place a check mark in the blank before that line.

Consider only the *rhythm*. Length of line is not to be considered.

______ The mountains look on Marathon

______ It was night in the lonesome October

______ And when she fell in feeble health, ye blessed her that she died

______ We are but pebbles on this rocky shore of life

You should have placed a check mark before the second line.

In each exercise below three lines are alike in rhythm and one is different. Check the line which is different in each group.

______ By the flow of the inland river.

______ Yet the cards they were stacked.

______ I do not own an inch of land.

______ Then I strove to go down to the sea.

______ If wishes were horses, then beggars would ride

______ To market, to market, to buy a fat pig

______ There was an old woman who lived in a shoe

______ There were two birds sat on a stone

Sound-grouping (55) was designed by Thelma Gwinn Thurstone as another attempt to introduce auditory factors in paper-pencil form. In this test the subject is given sets of four words, one of which sounds different from the other three.

SAMPLES FROM SOUND-GROUPING (87 ITEMS)

In the line below, notice that the three words, *comb*, *foam*, and *home*, can be grouped together because they sound somewhat alike. The word *come* does not belong with the other three because it sounds different. Since *come* is the fourth word, *4* has been written in the blank at the right.

1—comb 2—foam 3—home 4—come........ 4

Three of the words in the line below belong together because they sound alike, while one of the words does not belong because it sound different. Write the number of that word in the blank at the right.

1—praise 2—chase 3—maize 4—phase........ ______

The word *chase* does not belong with the other three. You should have written *2* in the blank.

In each of the lines below, think how the words would sound if you pronounced them aloud. Find the three words which can be grouped together because they sound alike in some way. Then write the number of the other word in the blank space at the right. Go right ahead. Do not wait for any signal.

1—toil 2—spoil 3—spool 4—coil.......... ______

1—mate 2—male 3—late 4—prate........ ______

1—altar 2—halter 3—psalter 4—barter........ ______

1—gone 2—cone 3—groan 4—known....... ______

1—good 2—hood 3—mood 4—wood........ ______

Spelling (56) is an objective true-false test of one hundred words.

Grammar (57) is an objective test of grammar in which the subject is asked to correct each wrong sentence by changing a single word. This test was assembled by Mr. John M. Stalnaker. The test contains fifty sentences.

Vocabulary (58) is an objective test of one hundred items which was assembled by Mr. John M. Stalnaker. It is more difficult than (60).

Free Writing (59) is merely the total word count of the theme written in test 52.

Word Knowledge (60) is the Thorndike Vocabulary Test.

In addition to the fifty-six measures in the present battery, each of the two hundred and forty subjects was asked to fill in a vocational interest schedule for which a separate analysis has been made.

CHAPTER III

THE FACTORIAL ANALYSIS

THE INTERCORRELATIONS

AFTER the tests had been given, they were scored according to the scoring formulas that are given in *Table 1–i*. The multiple-factor analysis required first the intercorrelations of the fifty-six tests. This computing involved nearly sixteen hundred correlation coefficients, and it was decided to reduce the computational labor by using tetrachoric correlation coefficients instead of product-moment coefficients.

The use of tetrachoric coefficients involves an assumption that has a fundamental relation to the psychological postulates on which the analysis is based. The factorial methods assume that each test performance can be expressed in first approximation as a linear function of the several factors. If one or both of the tests whose distributions enter into a correlation table are skewed, then the product-moment coefficient will deviate from the corresponding tetrachoric coefficient for the same table if the dichotomies are placed at or near the medians. There is no reason to suppose that the particular shape of the distribution of raw scores has fundamental psychological significance, since its shape can be altered by arbitrarily introducing different proportions of items of various degrees of difficulty. There was no previous standardization to introduce in these tests either a rectangular or any other form of distribution of difficulty. Consequently the distribution of raw scores may be regarded as essentially arbitrary. Most of the distributions are unimodal, as was to be expected. The simplest psychological assumption that can be made is that each of the primary abilities is distributed normally in the experimental population. If that assumption is true, then any linear function of the standard scores in the primary abilities will also be normally distributed in the experimental population. Hence the distributions of scores in the tests should be normal.

The most complete procedure would therefore seem to be to normalize each of the distributions of raw scores and then to compute the product-moment coefficients. But this assumption that the given distributions of raw scores require normalizing before entering the correlation tables is also just the assumption that is implied by the tetrachoric coefficient. Hence, in using tetrachoric coefficients, we are estimating the product-moment coefficient for the normalized distributions of scores. In using the tetrachoric coefficient, we are sacrificing some accuracy but we are not introducing any new assumptions into the factorial analysis. If the raw scores were allowed to enter directly into the correlation coefficients, we should have incorrect values in case the raw distributions deviate from normality, since we assume that these distributions are normal.

The factorial analysis could be made with the explicit assumption that the raw distributions are of the correct form, but such an assumption would require defense in that the forms of the raw distributions are known to be essentially arbitrary. A situation might arise in which the latter procedure might be psychologically defensible. If it should be discovered that some of the raw distributions are bimodal, then our present procedure, which implies the normalizing of the distributions, would be incorrect. Bimodal distributions are seldom if ever found for psychological tests, so that the safest procedure at the present time seems to be one in which the underlying theoretical distributions are assumed to be normal. The fact that bimodal distributions are not found in psychological tests may be due to the complexity of the tests. If relatively pure measures of the primary abilities can be developed, it would be of psychological importance to discover truly bimodal distributions that can be attributed to discontinuities in the fundamental measures of a primary ability. The factorial analysis of this case has been previously discussed.[1] For the present it seems best not to assume any discontinuities in the primary abilities and to proceed with the simpler and more generally acceptable case, namely, that the underlying theoretical distributions of test abilities are normal even if slight deviations from normality are found in the arbitrary distributions of raw scores. This is the assumption on which the tetrachoric correlation coefficient is based.

The computation of the tetrachoric coefficients was made by means of facilitating tables by which each coefficient can be determined in a few minutes.[2] The tetrachoric coefficients are given in *Table 2* of the *Appendix*. The correlations are positive or zero with few exceptions. The distribution of correlation coefficients is given in *Table 1*. The

[1] L. L. Thurstone, *The Vectors of Mind* (Chicago: University of Chicago Press, 1935), chap. viii.

[2] Leone Chesire, Milton Saffir, and L. L. Thurstone, *Computing Diagrams for the Tetrachoric Correlation Coefficient* (Chicago: University of Chicago Bookstore, 1933).

probable error of a tetrachoric coefficient whose true value is zero is about .07 for a population of two hundred and forty subjects when the dichotomic lines are at the medians.[3] It is therefore to be expected that a few coefficients will have negative values which should not be lower than about −.30 on the assumption that the true values of some of the coefficients should be as low as zero. This is what we find in *Table 1*. The present results support the general observation that psychological tests have correlations that are positive or zero. The tests whose correlations are zero are usually of restricted range of content.

Table 1

Frequency Distribution of the Intercorrelations of the Fifty-seven Tests

Class Interval		*f*
From	To	
−.25	−.15	2
−.15	−.05	13
−.05	.05	52
.05	.15	177
.15	.25	304
.25	.35	394
.35	.45	293
.45	.55	208
.55	.65	117
.65	.75	28
.75	.85	8

The tests in common use in intelligence tests have positive intercorrelations, since their complexities involve common elements.

The distributions of raw scores are given in *Table 1* of the *Appendix*. It will be seen that most of them are unimodal and that the indications of bimodality are not sufficiently clear to justify the assumption of bimodal discontinuities. There is some suspicion of discontinuity in the visualizing tests which may later be the subject of special study.

The reliabilities of the tests have been estimated by the tetrachoric correlations of odd and even items. These reliability coefficients have been recorded in the last column of *Table 3* in the *Appendix*. It is to be expected that the reliabilities will exceed or equal the communalities, and this is found to be true for all tests except four, namely, Arithmetical Reasoning (39), False Premises (42), Pattern Analogies (44), and Vocabulary (60). The inversion for the first two of these tests can per-

[3] Truman L. Kelley, *Statistical Method* (New York: Macmillan Co., 1923), p. 257, eq. 211.

haps be explained by the relatively small number of items, but no explanation seems evident for the other two inversions.

THE CENTROID MATRIX

The factorial analysis is made in two stages: (1) the factoring of the correlational matrix and (2) the rotation of the co-ordinate axes to the primary axes. It is not to be expected that the co-ordinate axes will be meaningful until they have been rotated to the primary axes of a simple configuration. The present correlational matrix (*Appendix, Table 2*) was factored by the centroid method, which is probably the simplest of several available methods that do not raise the rank of the matrix with communalities in the diagonals.

The residuals were studied after each factor to ascertain whether additional factors should be extracted. If the number of factors is equal to the number of tests, the residuals vanish identically without adjustment in the diagonal cells,[4] but such a result is of no scientific interest. The rotation of the factorial matrix can be accomplished even if more factors have been extracted from the correlational matrix than can be given a clear interpretation. However, in order that a factor shall be meaningful, it seems to be essential that several different tests have significant projections on the factor. If several tests have only slight projections on an additional factor, then their magnitudes may be more or less comparable with the projections from chance variations, and the interpretation may then be obscured. The present data were analyzed to twelve factors. The twelfth-factor residuals were small enough so that their contributions to the correlation coefficients were ignored in the subsequent rotation.

The residuals have been tabulated for each factor and are summarized in *Tables 2a* and *2b*. There were 1,596 residuals for each factor. The table shows the mean and the standard deviation of the distribution of residuals for each factor. The algebraic sum of the residuals is identically zero for each factor. The standard deviation was determined from the truncated distribution of residuals, all taken with positive sign, and with the point of truncation at the algebraic mean which is zero.

The modal correlation coefficient in *Table 2* of the *Appendix*, summarized in *Table 1* of this chapter, is about .35. The probable error of a tetrachoric coefficient of .35 for a population of 240 and with both of the dichotomic lines at the medians is .062. The standard error would therefore be about .09. The higher coefficients have smaller probable

[4] *The Vectors of Mind*, chap. iii.

errors. It will be seen in *Tables 2a* and *2b* that the standard deviations of the residuals are much smaller than would be expected by this calculation.

With each successive factor it is to be expected that the dispersion of the residuals will be reduced. This effect can be seen by inspection of

Table 2a

Frequency Distribution of Residuals

Residuals, Disregarding Sign	Frequencies for Each Centroid Factor							
	I	II	III	IV	V	VI	VII	VIII
.000–.020	199	260	281	313	344	380	397	411
.020–.040	203	255	293	291	317	317	329	365
.040–.060	187	240	242	272	276	279	283	279
.060–.080	191	197	227	229	216	216	224	225
.080–.100	163	166	169	164	174	163	154	125
.100–.120	153	150	139	123	109	100	86	87
.120–.140	111	115	96	88	68	64	61	50
.140–.160	88	72	64	44	35	37	36	29
.160–.180	62	43	28	33	33	28	19	17
.180–.200	52	29	18	14	12	7	6	6
.200–.220	50	27	17	12	8	3	0	1
.220–.240	41	18	7	6	2	2	1	1
.240–.260	23	7	9	4	2			
.260–.280	20	4	4	1				
.280–.300	17	6	0	1				
.300–.320	9	0	1	0				
.320–.340	7	2	0	0				
.340–.360	4	2	0	0				
.360–.380	5	0	1	1				
.380–.400	6	1						
.400–.420	0	0						
.420–.440	2	0						
.440–.460	2	1						
.460–.480	1	1						
Sum	1,596	1,596	1,596	1,596	1,596	1,596	1,596	1,596
Mean deviation	.0998	.0771	.0686	.0639	.0590	.0558	.0530	.0507
Standard deviation	.1269	.0977	.0859	.0793	.0738	.0700	.0667	.0640

Table 2. In *Figure 1* the standard deviation of residuals has been plotted against the number of factors. The dispersion of the residuals is reduced as the number of factors is increased. The break in the continuity of the curve at the tenth factor is caused by a change in the method of computation for the last four factors in *Figure 1*. The centroid method was used with sign-changing until each column of the residual matrix had a majority of positive signs. At the tenth factor a refinement was introduced in that the sign-changing was continued until the algebraic sum of each column of residuals was positive or zero. This required

slightly more labor and a few more sign changes for each factor. However, it improves the solution in that it increases the amount of the total variance of the tests that is accounted for by each new factor. Hence the dispersion of the residuals is depressed more rapidly than when this final adjustment in sign-changing is omitted. The maximum slope of

Table 2b

Frequency Distribution of Residuals

Residuals, Disregarding Sign	Frequencies for Each Centroid Factor				
	IX	X	XI	XII	XIII
.000–.010	215	245	260	262	273
.010–.020	192	227	228	250	252
.020–.030	203	201	204	219	243
.030–.040	171	150	200	193	177
.040–.050	163	164	176	167	181
.050–.060	148	136	114	135	112
.060–.070	118	112	94	96	105
.070–.080	85	87	105	89	92
.080–.090	69	77	79	65	50
.090–.100	70	65	40	43	47
.100–.110	45	38	30	32	22
.110–.120	31	27	22	15	17
.120–.130	25	26	13	13	16
.130–.140	22	13	12	5	3
.140–.150	10	14	10	7	0
.150–.160	14	4	4	1	3
.160–.170	5	3	0	0	0
.170–.180	4	1	1	2	3
.180–.190	4	4	2	1	
.190–.200	2	1	2	0	
.200–.210		1		1	
Sum	1,596	1,596	1,596	1,596	1,596
Mean deviation	.0487	.0456	.0423	.0401	.0387
Standard deviation	.0612	.0579	.0535	.0507	.0489

the curve of *Figure 1* would be obtained by determining the principal axes, but this would be more laborious and would necessitate an estimate of the communality of each test.

The intercorrelations in the present study were factored first to nine factors. Considerable work was done on the rotation of the first nine columns of the centroid matrix. It was later decided to continue the factoring. This was done to twelve factors after which the rotations gave psychologically meaningful factors. Finally, an additional factor, the thirteenth, was computed, but it did not show sufficiently large entries in the centroid matrix to justify rotation. The centroid matrix to twelve factors is shown in *Table 3* of the *Appendix*. The last two col-

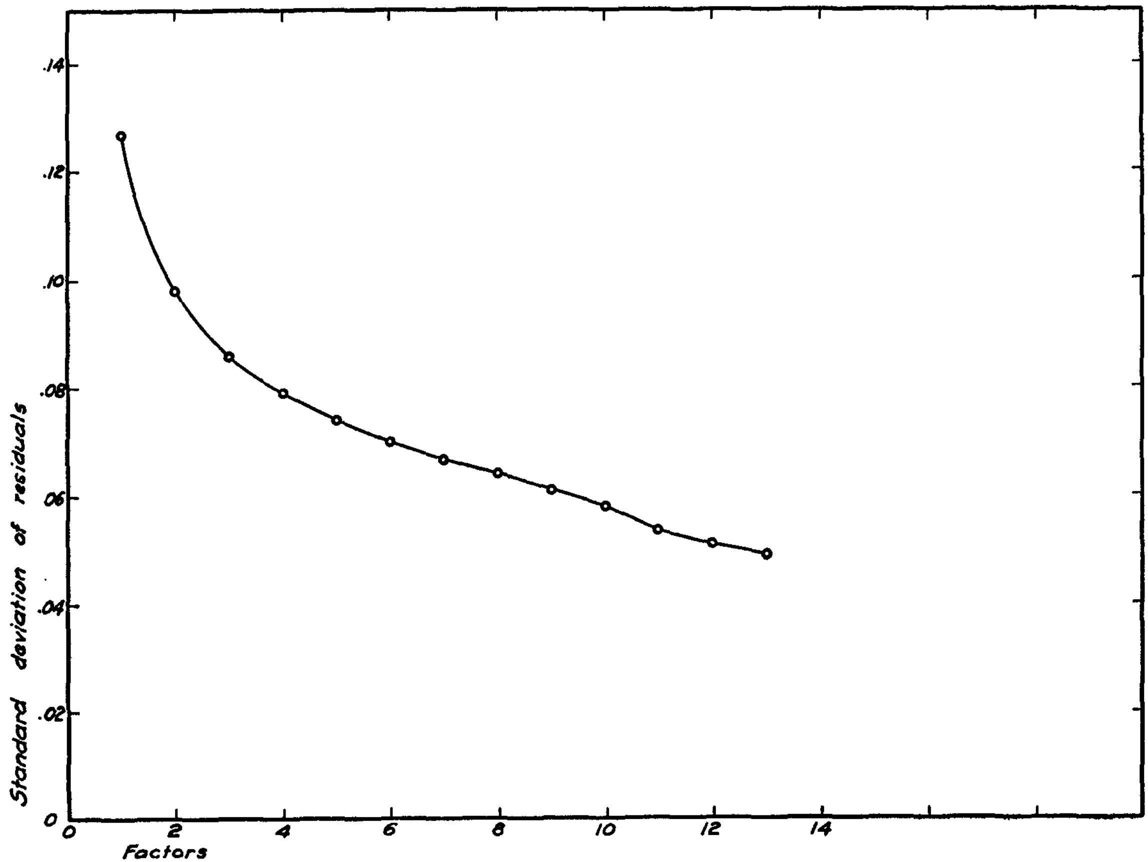

Standard deviation of residuals
.14
.12
.10
.08
.06
.04
.02
0
0
2
4
6
8
10
12
14
Factors

FIGURE 1

umns of this table give the communalities and the reliabilities. This is the basic table for the rotational solution. It will be noted that the first column has all positive values and that each of the subsequent columns contains both positive and negative entries, as was to be expected.

THE NUMBER OF COMMON FACTORS

A recurring problem in factor analysis is to determine how many factors to extract from the correlational matrix. Several properties of the factorial matrix can be used as aids in making the judgment as to how many factors to extract. In addition to these properties, an empirical rule has been found to be useful.

It is, of course, always possible to continue the process of extracting factors until one can ascertain by mere inspection of the residuals that more than enough factors have been extracted. The subsequent rotations may then reveal meaningful interpretation of several factors and some hints about the interpretation of minor factors which merge with the variable errors. This process is not very satisfactory in practice because of the great amount of labor in the computation of superfluous factors.

One simple procedure is to compare the dispersion of the residuals with the probable errors of the given coefficients, but this is an uncertain criterion because the dispersion decreases only slightly for each factor after the first two or three factors have been extracted. This comparison has been shown in *Table 2* and in *Figure 1*.

Another interpretation of the centroid matrix is to consider the contributions that each new column makes toward the correlation coefficients. If the two largest absolute values in a new column are, say, .20 and .30, then the maximum contribution of the new factor is .06 to one of the given correlations or to one of the given residuals. If there are only a few such values in the new column, it seems reasonable to suppose that the factor is at least not of major significance in the system.

A variant of this interpretation is to examine the largest positive and the largest negative entries in a new column of the factorial matrix. If these are, say, +.20 and −.30, then the thickness of the configuration does not exceed .50 in the new dimension, and this should be the maximum projection that can be obtained by rotation for the minor dimension of the system. These two methods of inspection are not definite as criteria for the number of factors, but they aid in making a decision as to when a sufficient number of factors has been drawn from the correlational matrix.

Another method of determining the proper number of factors would be to consider the obtained dispersions of the residuals for two successive factors in relation to the dispersions that would be expected if the variance in the tables were all due to chance or variable errors. A criterion of this type would involve, for each factor, the number of degrees of freedom in the factorial matrix after determining each factor. A satisfactory criterion along these lines has not yet been developed.

In order to study this problem, a set of five fictitious correlational matrices was prepared. The first was constructed for a fictitious set of twenty variables with a factorial matrix of rank one. The corresponding correlation coefficients were computed. To these theoretical coefficients were added variable errors at random in the table but with due regard for the fact that the standard error of a correlation coefficient is a function of the coefficient itself. The variable errors were in magnitude those which would be expected with a population of two hundred individuals. The result was a correlational matrix that was known to contain one significant factor and variable chance errors.

The correlational matrix so constructed was then reduced by extracting several centroid factors. For each factor there were tabulated various indices that might be used to determine the number of significant factors. An empirical rule has been found that has been successful, so far, in indicating the number of significant factors. As yet there is not available any rational proof for this criterion. Since it seems to be successful, it is likely either that it is susceptible of proof or that it is an approximation of a rational criterion.

The empirical rule is simply that, when all the significant factors have been extracted so that only chance variation remains in the residuals, then an empirical criterion ϕ takes a limiting value. The rule is given by the relation

$$\phi = \sqrt{\frac{\Sigma\rho_{s+1}}{\Sigma\rho_s}} = \frac{n-1}{n}, \tag{1}$$

where n is the number of variables in the correlational matrix, $\Sigma\rho_s$ is the sum of the absolute values of the residuals after s factors, and $\Sigma\rho_{s+1}$ is the sum of the absolute values of the residuals after $(s + 1)$ factors. If the relation of equation (1) obtains, then there are s significant factors in the correlational matrix. The summations include the diagonal terms. The summation $\Sigma\rho_s$ includes the adjusted diagonal terms of the correlation matrix just before the $(s + 1)$th factor is extracted. The summation $\Sigma\rho_{s+1}$ includes the diagonal elements in the residual matrix without adjustment in the diagonal cells after $(s + 1)$ factors have been drawn. This empirical criterion was discovered by Mr. Ledyard Tucker in studying records of several fictitious examples that were constructed for this problem.

Fortunately the criterion is independent of the standard errors of the given correlation coefficients. It seems to reach the limiting value when the residual matrix of order $n \times n$ represents a configuration that is of approximately equal extension in n dimensions.

In *Table 3* we have a record of the application of the empirical criterion to five sets of fictitious correlational matrices in each of which the number of significant factors was known. In set *c* of *Table 3* are recorded the results with a fictitious correlational matrix of twenty variables and of rank one with variable errors. The ratio $(n - 1)/n$ is here $19/20 = .950$. After one factor had been drawn, the criterion ϕ was .165, which is obviously too low. After the second factor had been drawn, the criterion rose to .949, which is practically identical with the limiting value of .950. The interpretation is then that the second factor for which equation (1) became satisfied was superfluous.

In set *d* of *Table 3* are the records for a fictitious correlational matrix with twenty variables, and which was constructed so as to represent two significant common factors plus chance errors. The chance errors were in each of these problems so chosen as to represent a population of $N = 200$. The sixth column shows the numerical values of the empirical criterion ϕ for each of four successive factors. The limiting value of ϕ for twenty random factors of equal significance is .950. After the first factor had been drawn, the value of ϕ was .593, and hence more factors should be tried. After the second factor had been drawn, ϕ was .426, and hence another factor should be tried. After the third factor the criterion ϕ jumped to .935, which is almost equal to the limiting value of .950. An additional factor gave $\phi = .964$, which slightly exceeds the limiting value. In this case the third factor was judged to be the first superfluous one. It may happen that the criterion .935 is significantly lower than .950, in which case three factors should be retained for rotational purposes. After completing the rotations, the third factor would drop out as insignificant and meaningless. This situation can be brought about by the fact that the centroid method adds new dimensions as far as possible in the common-factor space, and as far as possible, outside the error space. It may happen that the successive centroid axes are not entirely in the common-factor space and that a third factor is called for in this problem in order to give rotational freedom for eliminating the error space from the first two factors.

Table 3

Determination of the Number of Factors

Number of Significant Factors	Factors Extracted	$\Sigma\rho_s$	$\Sigma\rho_{s+1}$	ϕ^2	ϕ	$\frac{n-1}{n}$
	a) Eight variables. Random coefficients. No common factors. $n=8$. (First set)					
0	1	14.660	11.352	.7744	.880	.875
	b) Eight variables. Random coefficients. No common factor. $n=8$. (Second set)					
0	1	14.800	10.982	.7420	.861	.875
	c) Twenty variables. One common factor and residual errors. $n=20$					
1	1	325.25	8.85	.0272	.165	.950
1	2	8.49	7.65	.9011	.949	.950
	d) Twenty variables. Two common factors and residual errors. $n=20$					
2	1	224.23	78.73	.3511	.593	.950
2	2	79.81	14.49	.1816	.426	.950
2	3	14.31	12.51	.8742	.935	.950
2	4	12.80	11.89	.9289	.964	.950
	e) Twenty variables. Four common factors and residual errors. $n=20$					
4	1	145.24	98.05	.6751	.822	.950
4	2	99.21	64.76	.6528	.808	.950
4	3	66.13	47.18	.7134	.845	.950
4	4	47.82	16.20	.3388	.582	.950
4	5	15.73	13.32	.8468	.920	.950
4	6	13.54	13.22	.9764	.988	.950

In set *e* we have the record for a similar fictitious matrix of twenty variables. But this one was constructed with four common factors plus chance errors in the correlation coefficients. This section of *Table 3* shows a sudden rise in the criterion ϕ at the fifth factor, which is the first superfluous factor. At the sixth factor the empirical criterion exceeds the limiting value of .950.

One correlational matrix was constructed with eight variables in which the coefficients were all randomly distributed in the lower half of the matrix. The upper half was made symmetrical with the lower half to simulate the symmetric correlational matrix. The notation $\Sigma\rho_s$ in set *a* represents the sum of these random coefficients. After the first factor was drawn, the criterion ϕ was .880, which was practically equal to the limiting value for eight variables, namely, $7/8 = .875$. The inference would therefore be made that the configuration extends equally

Table 4

Fourteen Tests at West Point (Brigham)

Factors	$\Sigma\rho_s$	$\Sigma\rho_{s+1}$	ϕ^2	ϕ	$\frac{n-1}{n}$
1	113.131	21.324	.188	.434	.929
2	21.468	12.643	.589	.767	.929
3	12.674	7.797	.615	.784	.929
4	7.864	2.953	.376	.613	.929
5	2.947	2.585	.877	.937	.929

in all of its dimensions, which include the variable errors, and that therefore no meaningful common factor can be expected.

A second correlational matrix of order 8×8 was set up in the same manner with coefficients which were drawn at random from a previously constructed normal distribution. The two matrices differed only in being two random samples. In set *b* the criterion ϕ was .861 after the first factor was drawn, and this value approached closely to the limiting value of $7/8 = .875$. For each of these two matrices of order 8×8 a complete set of eight factors was drawn. The criterion values hovered about the limiting value. In these sixteen values of ϕ the maximum was .908 and the minimum was .790.

Table 4 is a record of the empirical criterion for the factorization of the intercorrelations of fourteen tests that were given at West Point. The data were supplied by Professor C. C. Brigham, of Princeton University. Previous study of these intercorrelations has revealed four clear factors, after which the residuals practically vanish. There seems little doubt that four common factors are sufficient for these data. The limiting value of the criterion is here $13/14 = .929$. The criterion for the fifth factor was .937, which slightly exceeds the limiting value. The next previous value was .613, which is evidently too low. Hence the fifth factor would be judged here to be the first superfluous factor. This agrees with previous study of these data according to which they contain only four significant common factors.

Table 5

The Values of ϕ for the Experimental Data

$\Sigma\rho_s$	Factor	$\Sigma\rho_{s+1}$	ϕ^2	ϕ
1088.840	1	336.184	.3088	.556
336.954	2	259.920	.7714	.878
260.956	3	230.990	.8852	.941
231.873	4	214.667	.9258	.962
215.566	5	198.040	.9187	.959
198.715	6	187.112	.9416	.970
187.928	7	178.172	.9481	.974
179.098	8	169.872	.9485	.974
170.960	9	163.433	.9560	.978
164.207	10	153.309	.9336	.966
154.146	11	141.984	.9211	.960
142.688	12	134.819	.9449	.972
135.428	13	130.069	.9604	.980

In *Table 5* we have a record of the application of the empirical criterion to the fifty-seven tests in the present experiment. For each of the successive factors the value of ϕ was computed as shown. The limiting value here was 56/57 = .982. The criterion rose to this value, namely, .980, after thirteen factors had been drawn. This finding supports our analysis of the first twelve factors as representative of the significant common-factor variance in the tests.

CHAPTER IV

ROTATION OF THE ORTHOGONAL REFERENCE FRAME

SIMPLE CONFIGURATION

THE centroid method was used to obtain an arbitrary orthogonal reference frame for the test vectors. The projections on these arbitrary axes are shown in *Table 3* of the *Appendix*. The next problem was to rotate this frame with regard to the configuration of fifty-seven test vectors in twelve dimensions so that the co-ordinate axes became meaningful

By psychological considerations the hypothesis can be entertained that the primary factors act positively unless they are absent from a performance. If this assumption is correct, then the projections of the test vectors on the co-ordinate axes should be either positive or zero. In rotating the axes, an attempt has been made so to place them that the test vectors have no significant negative projections, while the axes have been retained in their mutually orthogonal relation.

Consideration has also been given to the desirability of maximizing the number of projections that are zero or near zero. It has been found that the configuration of the present battery of tests is quite well defined in a positive orthogonal manifold.

One of the criteria that have been used in locating a unique orthogonal reference frame for the fifty-seven tests is the maximizing of the number of entries in the factorial matrix that are zero or near zero. The psychological plausibility of this criterion can be seen by considering the interpretation of the factors. Let us suppose that there exist distinct factors, mental faculties or powers, such as facility with numbers, facility with words, inductive resourcefulness, ability to think in visual terms, quickness of perception of detail, and retentiveness. Let there be a list of eight or ten such faculties or possibly fifty or sixty of them.

Now consider a matrix of order $n \times r$ which represents the normal or average factorial composition of n tasks and r factors. Each row of such a factorial matrix then represents one of the tasks. List at random such tasks. It is almost certain that no one of the tasks will involve all the abilities or faculties. In fact, it would be difficult, if not impossible, to invent a unified task or problem or test question which required the exercise of all the factors just enumerated. The best way to convince one's self about the psychological plausibility of this assumption is to attempt the assembly of a list of tests so designed that each one of them demands the exercise of each of the enumerated mental powers. It is almost certain that any collection of tests that we can assemble will have a factorial description with a very large number of vanishing entries. For example, a test in which the subject writes the opposites of given words as quickly as he can think of them is almost certain to make little or no demand on number facility or the perception of detail. A numerical task is likely to make a negligible demand on word fluency or memory. It is by these psychological considerations that we have confidence in being able to find, by rotation of the axes, such a reference frame that a unique set of vanishing entries appears in the factorial matrix.

Situations can be found in which no unique solution is attainable. Such a case is that in which a small number of tests are of such complexity that no unique configuration exists. Such a case is also that in which some of the minor common factors are present in the tests to so small an extent that their variances are comparable with the chance-error variance. Then a unique rotational solution is obscured by the errors. The usual case which is illustrated by the present study is that some of the factors are determinable. The remaining ones are obscured by being of minor significance in the experimental tests and by being comparable in variance to the chance errors.

In the present problem it was encouraging to find that, when the rotations had been effected which maximized the number of nearly vanishing entries in the factorial matrix, all the significantly negative entries disappeared. The result was a factorial matrix, shown in *Table 4* of the *Appendix*, in which the entries are positive or zero and in which no significant negative entries appear. This finding gives added confidence to the psychological interpretation of the factors even though we realize that the factorial analysis is based on the underlying assumption that each test performance is, in first approximation, a linear function of the primary factors.

Many different methods have been used in these successive rotations, and it would be impossible to describe them all. Nor would it be profitable for the reader to follow in detail the many methods that we have tried and improved, or discarded, in the course of our work. The graphical method of rotating in one plane at a time is still probably the best single method. It was the first one that the writer used, and it is still more effective than any other method that has been found so far. But the graphical method is not ideal. It will, no doubt, be replaced by better methods eventually. While the principle of the graphical rotations was described several years ago[1] and later in *The Vectors of Mind*, there has not been published an adequate description of the method as applied to an actual problem. Perhaps the best way to describe it is in its application to a short fictitious problem, where it can be illustrated more economically for the reader than in the present twelve-dimensional case.

Some of the rotations were determined by the single hyperplane method which has been described elsewhere.[2] It is applicable only to the oblique case where each hyperplane is determined separately. The data of this problem were first analyzed to produce a set of oblique primary axes, but it was found that most of them were nearly orthogonal. The data were then analyzed again with graphical rotations which maintained the reference axes in their mutually orthogonal relations. The resulting factorial matrix is nearly the same as that of the oblique methods. Unless there is good evidence to the contrary, it is probably psychologically preferable to have a set of orthogonal primary factors.

GRAPHICAL METHOD OF ROTATION

The application of this method assumes that the correlational matrix has been factored into a factorial matrix. In the present problem this was done in reducing the correlational matrix of *Table 2* to the centroid matrix of *Table 3* in the *Appendix*.

Each column of the factorial matrix F is plotted against every other column. If there are r columns in F, there will be $\frac{1}{2}r(r-1)$ diagrams. Each of the plots is examined to determine whether an orthogonal rotation of a pair of axes can be made so as (1) to increase the number of nearly vanishing entries in the factorial matrix or (2) to decrease the number of negative projections (or their absolute magnitudes) on the axes.

Let *Figure 1* represent such a plot for the orthogonal axes I and III. In this figure it is apparent that a rotation of the axes through the positive angle ϕ_1 will place the axes at the positions I' and III' relative to

[1] L. L. Thurstone, *The Theory of Multiple Factors* (Ann Arbor, Mich.: Edwards Bros., 1932).

[2] Thurstone, "The Bounding Hyperplanes of a Configuration of Traits," *Psychometrika*, I, No. 1 (1936), 61–68.

the configuration of test vectors, and that the new location of the axes will reduce the number and the magnitudes of the negative projections on I, and increase the number of nearly vanishing projections on both of the axes.

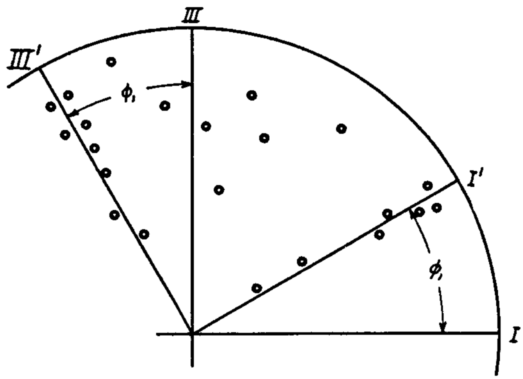


FIGURE 1

Let another plot be illustrated by *Figure 2* which shows the projection of the configuration on the plane II–V. A rotation of these axes through the negative angle ϕ_2 will locate them at II' and V'. Here the significant negative projections on V are eliminated, and the number of nearly vanishing projections is increased for both axes.

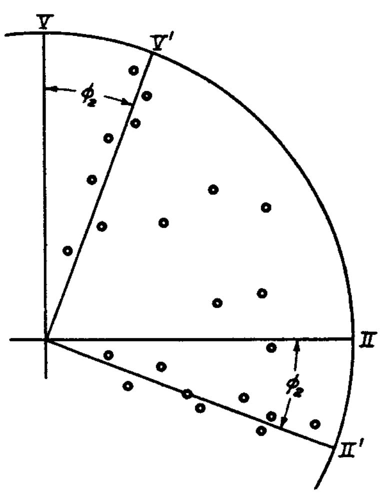


FIGURE 2

After each diagram has been inspected, a set of rotations is selected which involves each axis not more than once. For example, if the matrix F has six columns, it is possible by the graphical method, in its simpler form, to combine the rotations of as many as three pairs of axes in the same orthogonal transformation. It may be that profitable rotations are found only for one pair of axes, or for two pairs of axes, in which case the remaining axes are left unaltered in the transformation. They may be moved in subsequent transformations.

Table 1 shows the manner in which the orthogonal transformation is designed to represent the rotations that were determined in *Figure 1* and in *Figure 2*. The given factorial matrix may be denoted F_0. An orthogonal transformation T_1 is to be found that will rotate the axes as determined in the diagrams. This orthogonal rotation of the axes is represented by the matrix equation

$$F_0 T_1 = F_1 \,, \tag{1}$$

where F_1 is the new factorial matrix after one rotation. The transformation matrix T_1 is shown in *Table 1*.

Table 1

Orthogonal Transformation T_1

	I'	II'	III'	IV'	V'	VI'
I	$\cos \phi_1$ (+)		$-\sin \phi_1$ (−)			
II		$\cos \phi_2$ (+)			$-\sin \phi_2$ (+)	
III	$\sin \phi_1$ (+)		$\cos \phi_1$ (+)			
IV				1		0
V		$\sin \phi_2$ (−)			$\cos \phi_2$ (+)	
VI				0		1

By *Figure 1* the axes I and III are to be rotated through the angle ϕ_1. The orthogonal transformation for this rotation is

$$\begin{array}{c} \\ I \\ III \end{array} \begin{array}{cc} I' & III' \\ \left\| \begin{array}{cc} \cos \phi_1 & -\sin \phi_1 \\ \sin \phi_1 & \cos \phi_1 \end{array} \right\| \end{array} ,$$

and this matrix is represented in *Table 1* by the four intersections of rows I and III and the two corresponding columns. By *Figure 2* the axes II and V are to be rotated through the angle ϕ_2. The corresponding orthogonal transformation is

$$\begin{array}{c} \\ II \\ V \end{array} \begin{array}{cc} II' & V' \\ \left\| \begin{array}{cc} \cos \phi_2 & -\sin \phi_2 \\ \sin \phi_2 & \cos \phi_2 \end{array} \right\| \end{array} ,$$

and this matrix is represented in *Table 1* by the four intersections of rows *II* and *V* and the two corresponding columns. If there is no rotation indicated in the diagram for *IV* and *VI*, then these two axes remain unaltered. This is represented in *Table 1* by the identity matrix

$$\begin{array}{c} \\ IV \\ VI \end{array} \begin{array}{cc} IV' & VI' \\ \left\| \begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array} \right\| \end{array},$$

by which the new axes IV' and VI' are identical with the given axes *IV* and *VI*.

The angle ϕ_1 is taken positive in *Figure 1*, and the angle ϕ_2 is negative in *Figure 2*. The sign of each numerical entry in *Table 1* is shown in parentheses in each cell. The geometric interpretation of each column in that table can be illustrated by column V'. The two entries in that column mean that the new unit vector V' is the vectorial sum of (-sin ϕ_2) of the unit vector *II* and (cos ϕ_2) of the unit vector *V*. The resultant vector V' is also a unit vector since the transformation is orthogonal. The coefficients (-sin ϕ_2) and (cos ϕ_2) are both numerically positive. This interpretation can be seen in *Figure 2*, where the unit vector V' is the resultant of a fraction of the vector *II* plus a fraction of the vector *V*.

The transformation matrix T_1 of *Table 1* is orthogonal as may be seen by inspection. The entries in each column of the transformation matrix T_1 show the direction cosines of a new unit reference vector. Applying this transformation, we have the new factorial matrix F_1 in equation (1).

The columns of the new matrix F_1 are plotted against each other in a set of $\frac{1}{2}r(r - 1)$ diagrams with orthogonal axes. New rotations are determined graphically for a rotation

$$F_1T_2 = F_2 , \tag{2}$$

and this process is continued until after s rotations the entries in F_s satisfy the criteria that have been imposed for a simple configuration or a positive orthogonal manifold.

If the centroid matrix be denoted F, its successive rotated forms may be denoted F_s, where s is the number of rotations, each determined by a set of diagrams. The successive orthogonal rotations may be denoted T_s so that

$$F_{s-1}T_s = F_s\,. \tag{3}$$

The several transformations T_s may be combined into a single orthogonal transformation T_c, where

$$T_c = \prod_{s=1}^{t} F_s \tag{4}$$

for t successive rotations. Then

$$FT_c = F_s\,, \tag{5}$$

where T_c is the orthogonal transformation which carries the centroid matrix F to its final rotated form F_s in which the axes may be expected to be meaningful if the rotational criteria can be satisfied. This transformation can also be regarded as the matrix of the direction cosines of the new reference axes in terms of the orthogonal centroid reference frame. This matrix is shown in *Table 2*, where the columns have been given letter designations for the factors. This is also matrix T_c of equation (5), but it will be referred to as the matrix Λ_{mp} whose columns show the r direction cosines λ_{mp} for each of the r factorial axes.

Table 2

Orthogonal Transformation λ_{mp} *for Rotating Centroid Matrix* F_{jm} *to the Primary Factorial Matrix* F_{jp}

	S	*P*	*N*	*V*	*M*	*W*	*I*	*R*	*D*	*10*	*11*	*12*
I	.304	.288	.280	.372	.282	.264	.248	.368	.298	.279	.154	.279
II	−.339	.227	−.406	.477	.067	.374	−.189	−.059	−.035	−.027	−.498	.088
III	−.498	−.518	.557	.161	.129	.085	.214	.040	.064	.044	−.234	−.139
IV	−.434	.144	−.341	−.330	.440	−.319	.349	.293	.161	−.071	−.009	.178
V	−.190	.616	.415	.136	−.152	−.347	.004	.170	−.394	−.147	−.099	−.173
VI	−.314	−.211	−.190	.424	−.458	−.206	−.047	.262	−.003	.000	.493	.268
VII	.156	.046	.098	.193	−.199	−.368	.201	−.353	.513	−.400	−.291	.279
VIII	−.072	−.052	.278	−.231	.125	−.090	−.680	.139	.020	.013	−.135	.581
IX	.203	−.213	−.075	.449	.621	−.377	−.184	−.135	−.242	−.181	.155	−.105
X	.070	−.069	.058	−.077	.038	.426	.083	.250	−.153	−.824	.139	.094
XI	−.377	.301	.159	−.032	.183	.223	−.091	−.589	.168	−.044	.519	.039
XII	.053	−.075	.016	−.029	−.015	.044	.425	−.318	−.593	.145	−.067	.569

When the transformation of *Table 2* is applied to the centroid matrix, we have

$$F_{jm}\lambda_{mp} = F_{jp}\,, \tag{6}$$

where

F_{jm} is the given centroid matrix, *Table 3* of the *Appendix*
λ_{mp} is the orthogonal transformation of *Table 2*
F_{jp} is the rotated factorial matrix, *Table 4* of the *Appendix*

The rotated matrix F_{jp} is the matrix in which we may hope to discover psychologically meaningful factors. It should be borne in mind that there is nothing in the factorial methods to guarantee that a simple configuration exists in any given table of intercorrelations. The configuration may not even be uniquely determined by the correlations. A unique configuration may not reveal a simple structure. Even with a simple structure it may not be possible to identify psychologically more than a few of the factors.

Inspection of *Table 4* in the *Appendix* shows that in all but the last three residual columns the projections of the test vectors on the new orthogonal reference axes are positive or near zero and that there are no significant negative projections. A projection is regarded as nearly zero if it is in the range plus or minus .20. A projection of .20 signifies a contribution of 4 per cent of the total variance of a test. In this table of nine factors there is no negative projection longer than −.14, which represents less than 2 per cent of the total variance of a test. Such projections are regarded as negligible as far as psychological interpretation is concerned. The last three columns are residual factors which have not been given any psychological interpretation.

CHAPTER V

INTERPRETATION OF THE FACTORS

THE most interesting part of a factor analysis is the psychological interpretation of the factors. It is encouraging to discover that some of the primary factors that we have found in these data correspond closely to group factors that have been previously identified.

The rotated factorial matrix is *Table 4* of the *Appendix*. It will be examined by columns. The first column has thirty-six projections in the range of ±.20, and the lowest value is −.092. A projection or factor loading of .20 accounts for only 4 per cent of the total variance of a test. We have not regarded a projection as significant in naming a factor unless it is as large as .40. The naming of a factor cannot be made with confidence unless the projections are as large as .50 or .60 so that the factor accounts for a fourth or a third of the variance of a test. Confidence in naming a factor is also determined by the number of tests that have significant projections of .40 or higher on the factor. The thirty-six projections that are nearly vanishing in the first column represent more than half of the tests in the battery. This factor does not enter negatively in any of the tests.

Considering only those tests which have projections of about .40 or higher, we have the following:

(6) Verbal Classification	.411	(22) Lozenges B	.633
(8) Figure Classification	.393	(23) Surface Development	.551
(17) Block-counting	.413	(27) Pursuit	.584
(18) Cubes	.626	(45) Syllogisms	.430
(19) Lozenges A	.448	(53) Hands	.455
(20) Flags	.636	(55) Sound-grouping	.412
(21) Form Board	.415		

It requires very little inspection of these tests to bring out the fact that their common element is visual or spatial in character. The factor has been denoted *S*. A question concerning the general character of this common factor might be raised for Syllogisms (45) and Sound-grouping (55). The essentially visual character of the syllogism test is readily seen by inspection. The premises are concerned about the relative ages of three individuals, and the simplest way to keep them in mind is to represent them by lines or relative elevations. This is the way most people solve these syllogisms, and hence facility in visualizing augments the score. The appearance of a visual factor in Sound-grouping is puzzling and calls for separate experimentation with visual and auditory material to ascertain why the factor *S* should appear in Sound-grouping. The Verbal Classification test involves many categories that can be readily visualized, and it is not unlikely that this type of imagery facilitates the task.

An important check on the visual character of the factor *S* is to inspect the thirty-six tests in which this factor is absent. One might question why Mechanical Movements (25) does not have a large saturation of the factor *S*. The explanation may be that this test calls for some kinaesthetic factor which is not demanded by the other tests. This hypothesis can be checked by separate experimentation with a combination of visual and kinaesthetic material in the same battery.

Until further experimental evidence is available on the saturation of this factor in auditory and in kinaesthetic material, we seem to be justified in characterizing this factor as facility in spatial and visual imagery. It is probably the same factor that has appeared previously as a spatial or visual group factor in the experiments of Kelley.[1] Several of the factors in the present study are probably the same as those which Kelley has listed.

The second column of *Table 4* in the *Appendix* has thirty-seven entries in the nearly vanishing range of ±.20 with a minimum value of —.072. The negligible entries represent two-thirds of the entries in this column. The significant entries above .40 are as follows:

(6) Verbal Classification	.537	(41) Verbal Analogies	.417
(7) Word-grouping	.573	(44) Pattern Analogies	.435
(11) Completion	.422	(51) Picture Recall	.545
(14) Disarranged Sentences	.461	(60) Vocabulary (Thorndike)	.412
(26) Identical Forms	.603		

In this list Identical Forms has the highest saturation of .603, which represents more than one-third of the variance of the test. Our problem is to identify, if possible, the psychological trait which is common to these tests and which is absent from the thirty-seven tests with negligible saturations for this factor. A hypothesis which agrees with introspective study of the mental operations essential in these tests is that the factor is essentially perceptual in character. Strictly speaking, all the tests in the battery involve perception, and vision in particular.

[1] T. L. Kelley, *Crossroads in the Mind of Man* (Stanford University, Calif.: Stanford University Press, 1928).

But we are here concerned with those traits which are responsible for the individual differences that are revealed in the correlation coefficients of the battery. The perceptual function here seems to be a facility in perceiving detail that is imbedded in irrelevant material. The simplest expression of this function would be a task in which the subject is asked to identify some particular detail that is buried in distracting material. Given the task to find a particular word in a page of print, some people seem to be able to locate it by a dispersed attention to the page as a whole, while others require systematic search through each successive line of print.

Unfortunately we had no such hypothesis in mind when the test battery was planned so that this type of factor is not conspicuous in the battery except in Identical Forms (26). However, the factor is not lost in the specific of Test 26 because it is significantly present in some of the other tests, though not so strongly. In Picture Recall (51) the subject studies a picture for a short time in order to fix in mind the detail that may be asked for in a recall test. The study of the picture for this purpose may be facilitated by the present perceptual trait. In Verbal Classification (6) and in Word-grouping (7) there is also a large perceptual element, since each contains many easy items. The score is therefore determined largely by the speed of perceiving superficially apparent relations. The same is true for Disarranged Sentences, in which the best subjects sense the meaning of each sentence without explicit rearrangement of the word order. The high saturation of the factor P in the Thorndike Vocabulary test (60) is of interest when compared with the low saturation of the same factor in the Vocabulary Test (58). The explanation is probably in the great difference in difficulty of these two tests. The Thorndike Vocabulary test was easy for the subjects in this experiment. Hence their scores were in part due to speed of reading, which may be largely a perceptual factor.

A new set of ten tests has been devised for an experimental study of the factor P in order to determine whether it is conspicuous in perceptual tests of the type of Identical Forms when the nature of the perceptual detail is subjected to considerable variation. If our hypothesis is correct that the factor P involves facility in finding or in recognizing particular items in a perceptual field, then the saturation of this factor should be large in a variety of tests which have this characteristic in common even though they differ markedly in other respects.

It might be suggested here that the various tests for reading readiness of young children are probably good examples of the factor P.

If this should be verified, it would be psychologically interesting to determine whether slow and fast readers can be differentiated by the factor *P* under similar conditions of practice in reading. It will also be of interest to determine to what extent this factor is involved in what is sometimes called "quick intelligence" as distinguished from its more analytical and reflective aspects.

The third column in *Table 4* of the *Appendix* represents a factor that is easily identified in the tests of the present battery. This column contains thirty-eight entries in the range ±.20 that we consider to be negligible. This represents two-thirds of the tests in the entire battery. The tests which have saturations of .40 in this factor are as follows:

(30) Number Code	.625	(34) Division	.619
(31) Addition	.755	(35) Tabular Completion	.392
(32) Subtraction	.670	(38) Numerical Judgment	.432
(33) Multiplication	.812	(39) Arithmetical Reasoning	.383

The common characteristic of all these tests is their numerical character, but they do not include all the tests with numerical content. The exceptional case is Estimating (36), which has a projection of only +.020 on this factor. In that test the subject is certainly thinking about numerical quantities, and yet it separates sharply from the rest of the number tests. This fact was anticipated in discussions about the nature of the test. Many people dislike the estimating test. It is essentially logical in character, as can be seen from its factorial composition. It demands very little proficiency in rapid calculation. All the number tests in the foregoing list demand considerable facility in numerical calculation. The only tests in this list that are restricted to numerical calculation are Addition, Subtraction, Multiplication, and Division. The others all require other factors as well. The tests of Tabular Completion, Numerical Judgment, and Arithmetical Reasoning all require verbal and logical factors in addition to the numerical, and hence their variances are divided among more factors than the four simple tests of rapid calculation.

Since this factor is so clearly limited to the numerical tests, we can name it with some confidence as far as the present battery of tests is concerned. It has been denoted *N*. This factor has appeared in many previous studies of the mental abilities. It has appeared as a disturber of Spearman's tetrad difference criterion for a single factor. It has been treated as a group factor by several investigators, and it was isolated as a separate factor by Kelley. There can be little doubt that this factor is the same as the one which has been previously identified as numerical in character.

The insistence of the numerical factor makes it almost certain that it represents a unique ability, but one is puzzled about its psychological or genetic character in view of the fact that calculation is more naturally thought of as a cultural rather than as a biological category. There seems to be some evidence for the genetic interpretation of number facility as an inherited trait. Occasionally this ability is found to be extremely conspicuous even at an early age, and it seems then to be more or less independent of other abilities. Some number freaks are well endowed in other abilities, but some of them are otherwise mediocre. The fact that occasional feeble-minded individuals possess some degree of number ability indicates further that it constitutes a more or less independent mental ability. Of interest for this problem is the frequent occurrence of considerable mental ability in the verbal factors combined with what seems to be a blind spot for numerical and logical relations. Literary people are not infrequently of this type. In studying this problem, we should keep in mind the possibility that "number" as such may not adequately describe this factor psychologically or genetically, that the factor N may be more basic and general than number, and that the number tests constitute good examples of it. This is a possibility for all the factors. The question of whether they extend to broader categories than those which are indicated in the test batteries in which they are discovered can be answered only by discovering further tests of wider range in which the same factor is unmistakably present. Thus, if we should find some nonnumerical tests with high saturation of the factor N, then it would be necessary to extend our comprehension of the factor to a category broader than "number" but which would include numerical tasks.

It has been customary to group psychological tests in three large categories, namely, verbal, numerical, and spatial. The spatial tests are here included in what we have called the factor S. The tests of simple numerical calculation are represented by the factor N. The verbal tests divide themselves in our factorial matrix into two factors. These are the factors V and W whose differentiation is of considerable psychological interest.

In the column V we find thirty tests with projections in the range $\pm .20$ that we call negligible. This is over half of the tests in the present battery. The strongest negative projection is $-.065$, which shows that the hyperplane is essentially positive.[2] The tests with projections greater than .40 on this reference axis are as follows:

(4) Reading I (proverbs)	.552	(41) Verbal Analogies	.597
(5) Reading II (quotations)	.506	(42) False Premises	.424
(7) Word-grouping	.456	(55) Sound-grouping	.453
(9) Controlled Association	.450	(57) Grammar	.498
(10) Inventive Opposites	.635	(58) Vocabulary (Chicago)	.395
(16) Inventive Synonyms	.495	(60) Vocabulary (Thorndike)	.385
(40) Reasoning	.420		

The tests in this list are evidently logical in character. In all these tests the subject must deal with ideas, and the factor is evidently characterized primarily by its reference to ideas and the meanings of words. It can be called "verbal relations," with the reservation that still another verbal factor involves verbal material in a psychologically different manner.

The factor *W* has thirty-nine test projections that are in the range of $\pm .20$ and with a minimum projection of $-.134$. This hyperplane is also essentially positive, and more than half of the tests do not demand this factor. Listing the tests with significant projections on this axis, we have:

(12) Disarranged Words	.512	(56) Spelling	.508
(13) First and Last Letter	.388	(57) Grammar	.530
(15) Anagrams	.534	(60) Vocabulary (Thorndike)	.413

In studying these two lists of verbal tests, it becomes apparent that the second list is concerned with tests in which the subject deals with single and isolated words. This is true without exception in the second list. The test with highest saturation in the factor *W* is Anagrams (15). In this test the subject is asked merely to make words that are composed of certain restricted letters. Nothing is asked about the meanings of these words. The First and Last Letter demands the association of a word which has the given initial and terminal letters. In Disarranged Words the letters of a word are given in pied order, and the subject is asked to write the word as it would be spelled when the letters are written in correct order. The tests Spelling and Grammar were both in the form in which single words were to be found correct or incorrect. The grammar test specified that, if a sentence was wrong, it could be corrected by changing one, and only one, word. That word was to be marked.

[2] L. L. Thurstone, *The Vectors of Mind* (Chicago: University of Chicago Press, 1935), chap. vi, p. 166.

In the interpretation of the factor *W* we seem to have been fortunate in the inclusion of so many word tests in a variety of forms. With a shorter list of verbal tests it might happen that the two verbal factors would not be so clearly separated as they seem to be in the present battery. The factor *W* seems to have as its principal characteristic a fluency in dealing with words. This factor seems to be separate from the verbal factor *V*, which is concerned with ideas and meanings. It is to be expected that some of the verbal tests have appreciable saturation in both factors such as the opposites in which the subject deals with words and also with ideas. The factor *W* does not extend to other forms of fluency because in that case it would have been found in some of the nonverbal speed tests in which the *W* factor is conspicuously absent.

The factor *V* should be investigated with further experimental study of tests that involve the manipulation of ideas in verbal and in essentially nonverbal form. It is quite likely, as far as one can judge from the present data, that the factor *V* will be identified largely in terms of the verbal manipulation of ideas as they occur in sustained verbal discourse.

The factor *M* is represented by one of the columns in the factorial matrix with thirty-seven projections in the range $\pm.20$ and with a minimum value of $-.080$. The tests with significant projections on this factor are as follows:

(46) Word-Number .... .529
(47) Initials .... .487
(48) Number-Number .... .664
(49) Word Recognition .... .381
(50) Figure Recognition .... .420

All these tests were designed as memory tests. The first three tests in this list were in the paired-associates form, and the others were in the recognition form. The paired-associates recall form has the larger saturation of the memory factor. There seems to be no doubt that this factor is concerned with memory, and it has been denoted *M*.

The problems of memory are especially interesting for the experimental opportunities that they offer in relation to factorial methods. The memory factor was listed as a separate factor by Kelley, and it has appeared as a group factor in many experimental studies. It is clear also in the present study. Because of its significance for education and because of its theoretical importance in psychology, it should be exhaustively studied. A few of the problems in this field may be mentioned here with reference to the possible use of factorial methods in their solution.

One of the problems in memory is the possible differentiation between incidental memory and memorizing. Another problem in memory is to ascertain whether and to what extent the memory function is differentiated for the several modalities. There are at least two possibilities. There may be a separate memory factor for each modality. There may be a general memory factor and a separate imagery factor for each modality. The person who remembers faces but not names may have good memory but poor verbal fluency. Another individual with equally good memory may be deficient in visual imagery. For him it should be easier to remember names than faces. These and similar questions could be answered by the factorial analysis of appropriate tests.

The question whether there is a factorial distinction between rote memory and memory for ideas can be answered in a similar way. Here it is essential again that the experimental battery be sufficiently broad so as to represent verbal and logical factors which may be responsible for the differentiation. The memory tests should be sufficiently diversified so that several memory factors have the opportunity to reveal themselves. Another memory problem is to determine to what extent inductive ingenuity is involved in memorizing. This relation can be experimentally ascertained by arranging some memory tests with the inductive aids already added to the material and other tests in which these aids are absent. The problem is related to the possible distinction between rote memory and what is sometimes called "logical memory." "Logical memory" is a misnomer because it refers to memory of ideas which can be reproduced in paraphrased form as distinguished from verbatim reproduction. The factorial methods should lend themselves well to the solution of these psychological problems.

One of the most interesting factors is that which has been denoted *I*. There were thirty-six tests, more than half of the battery, with small projections in the range $\pm .20$, and the minimum projection was $-.110$. The tests with significant saturations in this factor were as follows:

(8) Figure Classification	.405	(37) Number Series	.503
(29) Areas	.477	(44) Pattern Analogies	.392
(35) Tabular Completion	.479		

The characteristic that these tests have in common is that they all demand the subject to find a rule or principle for each item in the test. This is conspicuous in several of the tests such as the well-known Number Series test. Here the subject makes his responses merely to show that he has discovered the rule for each item. The same is true in

Spearman's Figure Classification. The Pattern Analogies is also in this class in that the analogies are not obvious. The subject usually tries several hypotheses before hitting upon the correct rule. A simple test of analogies would be visual and perceptual in its principal factors. In Tabular Completion we have the same element in that the missing entries of the table can be filled in after the subject has discovered the principle by which the table was drawn.

Verbal Classification (6) was designed to be a verbal parallel to Spearman's Figure Classification (8). The Verbal Classification test has a projection of .306 in the inductive factor I, but its saturation is higher in the perceptual factor P. This relation and several other similar cases illustrate the principle that the factorial constitution of a test may be materially altered by merely changing its difficulty and without changing the content or the form. Another way in which the factorial components of a test may be altered is to give the same test at different age levels. A very common example is the case of a simple arithmetical problem, say, $3 \times 1\frac{1}{4}$. To answer a score of such items quickly at the age of fifteen is indicative of the factor N, but a four-year-old who solves such items rationally may reveal logical faculties, perhaps inductive, rather than superiority in the factor N. This is an example of what might be taking place. Verbal Classifications may be a test of logical faculties for a young child, whereas for the educated adult it may be so easy as to represent little more than perceptual speed. These relations have not been adequately recognized in recent studies of the changes in mental organization with age. That problem may also be further complicated by the possibility that the several mental abilities may have entirely different growth functions. For example, the perceptual factor may mature much earlier than the verbal or inductive factors. These considerations make it advisable to isolate the mental abilities separately at each age level and to move with caution in extrapolating the factorial interpretation of a test for widely different ages.

If it should be established that the intercorrelations of psychological tests tend to decrease with age, the effect can be interpreted in terms of a rather simple hypothesis. If we assume that the mental abilities of the young child are not clearly differentiated, he will use a wider spread of abilities in solving a problem than later when he can restrict his efforts to those mental abilities that are most appropriate for the problem. This effect is readily seen in the muscular co-ordination of children in which larger muscle groups are involved than when the same co-ordination is effected by an older child or by an adult. Examples are early efforts in writing, at the piano, and in typewriting. If the mental abilities become more and more differentiated with exercise and maturity, it should follow that psychological tests become less and less correlated with age.

There remain two factors whose interpretation is tentative and not so clear as the first seven factors in *Table 4*. The last four columns have been regarded as residual factors, and they have not been rotated into a simple configuration. The two factors that are only tentatively identified have been denoted *R* and *D*. The column *R* has twenty-eight entries in the range ±.20 with a minimum of −.101. The factor *R*, therefore, enters positively only into the test performances. The tests which have significant projections on this factor are as follows:

(11) Completion	.481	(56) Spelling	.410
(25) Mechanical Movements	.414	(58) Vocabulary (Chicago)	.457
(38) Numerical Judgment	.534	(60) Vocabulary (Thorndike)	.545
(39) Arithmetical Reasoning	.583		

The characteristic that is common to these tests, and to a lesser degree in the tests with projections between .30 and .40, is not easy to determine. The conventional categories of verbal, numerical, and spatial are clearly not applicable here. A close study of all these tests, together with the twenty-eight tests in which this factor is absent, has enabled us to make a tentative psychological category for the tests of the present battery. The common characteristic seems to be the successful completion of a task that involves some form of restriction in the solution. The test with strongest saturation in this factor is Arithmetical Reasoning. Further study of arithmetical and other restrictive tasks will reveal the nature of this factor.

Column *D* has thirty-three entries in the range ±.20 with a minimum value of −.097. We conclude, therefore, that the factor *D* enters positively into the tests. The tests with significant projections are as follows:

(8) Figure Classification	.398	(40) Reasoning	.525
(25) Mechanical Movements	.403	(42) False Premises	.578

This list of tests is not sufficiently extensive to make the identification with certainty. Its obvious common feature is the deductive nature of the four tests. In Figure Classification the subject must first find the rule and must then apply it to the given examples. In Mechanical Movements the principle is apparent in each figure, and the movement must be traced in the successive parts. This factor can be investigated experimentally with a variety of test material in which the deductive feature is emphasized. The psychological identification of the factor can be made with confidence only in case it remains conspicuously present in special test batteries in which other features are varied while the deductive nature of the tests is retained. If the present tentative identification of the factor D is not correct, then such a special subgroup in a test battery will reveal a diversity of factors instead of a single common factor.

The factorial methods are still imperfect but can be developed to become more powerful analytical tools. Among the theoretically challenging possibilities for their further development we might consider the analysis of a score matrix by pairs of columns instead of by pairs of rows. The latter case gives the conventional correlations or cross-products of test scores that are summed over individuals. The former case gives cross-products or correlations of individuals, summed over tests. The development of this case leads to the description of an individual as a linear combination of several reference individuals which may be regarded as types. This kind of factorial analysis has been discussed by Eckart and Young[3] and by Stephenson.[4] It has interesting possibilities for the psychological analysis of traits.

Another possibility is to transcend the restrictions of linearity in the fundamental assumptions of factor methods in their present form. The more developed a science becomes, the fewer are the linear equations in its theoretical structure. It is possible that the residual factors are due largely to the warping that is introduced by this assumption of linearity. Fortunately a limited range of a curved surface can be represented by a plane as a first approximation. This approximation is implied every time that we write a Pearson correlation coefficient or a regression equation. The critics of the assumption of linearity in factor analysis should recall that the same assumption underlies most of their current statistical procedures. While working with this simplifying assumption, we can expect to find the principal landmarks or dimensions of mind. Refinement in mental measurement will eventually involve nonlinear functions and phenomena of discreteness. In our present position, confronting the psychological chaos of speculation about individual differences in ability and personality, it is more profitable to

[3] Carl Eckart and Gale Young, "The Approximation of One Matrix by Another of Lower Rank," *Psychometrika*, I, No. 3 (1936), 211–18.

[4] William Stephenson, "The Foundations of Psychometry," *ibid.*, pp. 195–209.

attempt the analytical extraction of meaningful categories with objective criteria for their uniqueness than to join the perfectionists who can show what we already know, namely, that our assumptions oversimplify the phenomena that we are trying to comprehend. Even with the imperfections of the factorial methods in their present form, we can resolve many psychologically interesting problems objectively.

The identification of the factors cannot be accepted unless the factors make psychological sense. The interpretation of the factors in this study has been made with this requirement in mind. Factorial analysis aids in discovering the principal dimensions in the experimental observations, and it is our further task to discover the psychological meaning of each factor. Only to the extent that we succeed in isolating meaningful categories in these studies are we advancing psychological science.

In the current psychological literature there is appearing a large number of factorial studies. Unfortunately, most of these studies are not taking advantage of the technical advances that are already available, and consequently the results are not so conclusive as they could be even within present limitations of the factor methods. It may be of some assistance to list here a few of the deficiencies in many current studies which could be remedied even in the present state of knowledge about factor analysis.

One of the most common characteristics of factor studies is the computation of a factorial matrix and the attempt to make a psychological interpretation without rotating the reference frame. It does not matter whether the factorial matrix has been produced by the centroid method or by the principal axes or components. In either case the configuration must be rotated before the factors can be even expected to have psychological meaning. One cannot expect to find several meaningful factors by any of the current methods of factoring a correlational matrix without rotation of the reference frame.

Ideally, we should not be satisfied until the factorial description of a test remains invariant when it is moved from one test battery to another. If a test demands the functioning of several factors in certain proportions, then, surely, it does not make sense if these factorial components are completely altered merely by asking the same subjects to take additional tests next week. By so doing, we add new tests to the battery and we alter the principal axes and the principal components of the tests. These axes are determined by the battery as a whole. This is an absurdity. We cannot, then, make any claim that the factors are unique and psychologically meaningful traits of the individual subjects. My own first efforts in factor analysis involved this absence of psychological uniqueness, and I therefore discarded the principal component description of the tests even though the solution was mathematically unique within each test battery. The solution which I have found for this problem is to rotate the reference frame into a simple structure if it exists in the battery. The factorial description of each test is then independent of the fact that the tests are assembled in a particular battery. To increase the number of verbal tests or to withdraw some spatial tests, for example, has no effect on the reference frame for a simple structure. This type of invariance gives the possibility of identifying the reference axes in terms of unique traits of the individual subjects.

CHAPTER VI

INDIVIDUAL MENTAL PROFILES

THE REDUCED TEST BATTERY

THE identification of separate and unique mental abilities has its principal interest in the possibility of describing the mental endowment of each individual in each of the abilities. This problem was the subject of chapter x in *The Vectors of Mind,* where the appropriate regression equations were derived for two cases, namely, the prediction of primary abilities from test scores and the prediction of test scores from primary abilities. Both of these cases are of practical educational significance.

In building a battery of tests for practical use in estimating primary abilities, we start with the battery of fifty-seven experimental tests by selecting a few tests to represent each primary. In making this selection, we have kept in mind several considerations. The experimental tests in the present investigation were relatively short so that the battery would cover the range of factors in current psychological tests and so that each factor would be overdetermined with a large number of tests. Hence the reliabilities are not so high as they will be when the tests are lengthened for subsequent editions of practical test batteries. Now that several of the factors are somewhat better understood, it should also be expected that tests will be improved by increasing the saturation of the factor that each test is expected to measure and by decreasing the saturations of other factors that are measured by other tests. What we have called the "complexity" of each test should be reduced.[1] It seems likely that such improvements in psychological tests will mark a new line of development. Instead of improving a composite test by raising its correlation with some equally complex practical criterion, such as academic scholarship, the tests will be improved by making them relatively pure measures of the primary abilities. In general, this will make the tests look simpler, and in some cases the tests will appear to be remote from the practical activities that psychological tests are sometimes made to simulate. The simplification of the tests

[1] L. L. Thurstone, *The Vectors of Mind* (Chicago: University of Chicago Press, 1935), chap. vi, p. 155.

toward relatively pure measures of primary factors will also enable us to use them over a wider age range, but the factorial composition of a test may alter with extreme variation in the age of the subjects.

In general, the selection of tests has been made so as to reduce complexity. A shorter battery has been selected also with the consideration that, if it is successful, there will be a practical demand for longer tests and for parallel forms. Some types of material lend themselves better for these purposes than others. Two tests were selected to represent each primary factor with the exception that one factor, N, is represented by three short tests and one of the tentative factors, D, is represented by only one test in the reduced battery. The total testing time for the reduced battery is three hours, but this time will probably be increased when the separate tests are made longer and hence more reliable.

Table 1

Tests	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	Specifics for 20	Specifics for 27	h^2	L^2
20	.49	−.53	−.30	−.30	−.13	.08	.02	.18	.06	.11	−.06	−.10	.46	.....	.7864	.....
27	.47	−.42	−.07	−.31	−.01	−.13	.09	−.11	−.03	−.13	−.20	.09	.....	.63	.6014	...
Sum	.96	−.95	−.37	−.61	−.14	−.05	.11	.07	.03	−.02	−.26	−.01	.46	.63	....	3.0497
Normal	.55	−.54	−.21	−.35	−.08	−.03	.06	.04	.02	−.01	−.15	−.01	.26	.36	.80	.99

The method of combining several tests into a single composite test for each primary factor will be illustrated with the two tests that were selected for measuring the space factor. These two tests were Flags (20) and Pursuit (27). In *Table 1* these two tests are represented by their projections on the centroid axes as shown in *Table 3* of the *Appendix*. The first two rows of this table show the centroid co-ordinates of each test. For each test there is added an extra column for its specific factor. The two specifics are assumed to be uncorrelated. In other words, it is here assumed that the common factors account for the correlation between the two tests. This assumption is nearly correct, since the residual correlations are neglected.

In the total-factor space these two test vectors are summed as shown in the third row of the table, where L is the length of the vectorial sum. In the last row of the table are shown the projections of a unit vector colinear with the vector sum. It represents a new test whose raw score is the sum of the standard scores in tests (20) and (27), and which has been reduced to standard score. A similar procedure was used for combining the other tests into a composite test for each primary factor.

In *Table 2* are summarized the results of computation like *Table 1* for each primary factor. Each row of *Table 2* shows the projection of a composite test C on the orthogonal centroid co-ordinate axes. The subscripts refer to primaries so that, for example, C_S is the composite test of (20) and (27) for the primary S. In the left column of *Table 2* are shown the tests selected for each primary factor, including the two tentative factors D and R. This matrix shows the composite tests C in terms of the centroid axes and is denoted F_C.

In selecting the two tests, Flags (20) and Pursuit (27), consideration was given to the fact that other tests involving space, such as Cubes (18), are more complex. The two tests selected are simple and can easily be reproduced in parallel forms.

Table 2

Matrix F_c *of the Composite Test Vectors* **C** *in Terms of the Orthogonal Centroid Axes*

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	h^2
C_S (20, 27)....	.55	−.54	−.21	−.35	−.08	−.03	.06	.04	.02	−.01	−.15	−.01	.7963
C_P (7, 26).....	.65	.20	−.38	.03	.30	−.10	.15	.04	−.09	−.12	.02	−.16	.7804
C_N (31, 33, 34)	.52	−.29	.55	−.24	.25	−.11	.02	.15	−.18	.09	.12	−.05	.8695
C_V (10, 41) ...	.81	.28	−.07	−.14	.16	.24	−.07	−.09	.18	−.12	.02	.04	.9040
C_M (46, 47)...	.54	.28	.16	.21	−.22	−.16	−.08	.11	.25	−.23	.15	−.05	.6726
C_W (12, 15)...	.61	.22	.08	−.14	−.28	−.11	−.20	−.06	−.25	−.04	.20	−.01	.6848
C_I (29, 35)....	.69	−.31	.24	.22	−.07	.03	−.01	−.18	−.04	.01	.01	.16	.7439
C_R (11, 39)....	.83	.03	.04	.13	.14	.08	−.34	−.06	−.01	−.06	−.19	−.09	.9014
C_D (40)	.66	.13	.14	.22	−.28	.36	.17	−.03	.03	.12	.03	−.19	.8106

The perceptual factor P is represented by Word-grouping (7) and Identical Forms (26). Both of these tests are quite simple and can be reproduced in parallel forms. Care must be taken in constructing a word-grouping test for young subjects because for them it is likely to become a test of induction unless it is made perceptually simple.

The factor N is naturally represented by Addition (31), Multiplication (33), and Division (34). The test Subtraction (32) was not used, partly because it is not needed in a short battery and partly because it seems to reveal stronger verbal components than the three other number tests.

Verbal relations in factor V is represented by the well-known Opposites (10) and Verbal Analogies (41). In the experimental battery the Opposites test was constructed in the inventive form, where the subject writes his responses. The original intention was to allow verbal recall or fluency to operate independently of any other factor that may characterize "checking right answers" in a recognition form. It is not yet known whether the simpler and more objective form can be used here, but this question will be investigated in the near future in a special study of the verbal factors. As far as possible the tests should be objective, but in the present investigation the purpose was to identify factors and only secondarily to introduce convenience in scoring. There is also a possibility that this factor will be interpreted to be more general than its present verbal connotation would indicate. The factor might turn out to be characterized in terms of symbolic representation and meaning in a form that is more general than its present restriction to verbal thinking. This question can be answered in terms of additional experimental study by the factorial methods.

The memory factor M is represented in the reduced battery by Word-Number (46) and Initials (47). The Number-Number test (48) is probably a better test of memory, but the test is more difficult.

Disarranged Words (12) and Anagrams (15) are used in the reduced battery for measuring the word factor W.

The inductive factor I is measured by Areas (29) and Tabular Completion (35). The Figure Classification test (8) would seem to be an appropriate test for this factor, but it has high saturation on both the factor I and the tentative factor D. The latter factor needs further experimental study. The two tests chosen are simple and can be reproduced readily in any number of parallel forms for future editions.

The two tentative factors R and D are measured by Completion (11) and Arithmetical Reasoning (39) for R and Reasoning (40) for D. There were two syllogism tests with saturation on D, namely, Reasoning (40) and False Premises (42). The latter test is strange in appearance and is probably not adapted for general use without some special explanation by the examiner.

The nine composite tests can be expressed in terms of the orthogonal primary axes. This can be accomplished by postmultiplying the matrix F_C by the orthogonal transformation Λ. We have then

$$R_{cp} = F_{cm}\lambda_{mp} , \tag{1}$$

which is shown in *Table 3*. The last three factors, *10*, *11*, *12*, have not been given psychological interpretation and are arbitrary.

THE REGRESSION x ON s

The estimate of standard scores of an individual on each primary ability involves the regression x_{pi} on S_{ji} where x_{pi} is the standard score of individual i on the primary factor p, and S_{ji} is the standard score of individual i on test j. The regression equation[2] is

$$y_{pi} = \sum_j W_{pj} S_{ji} \, , \tag{2}$$

where y_{pi} is the predicted value of x_{pi}, and

$$W_{pj} = R_{pk} R_{jk}^{-1} \tag{3}$$

Writing the equation in matrix form,

$$y_{pi} = R_{pk} R_{jk}^{-1} S_{ji} \, . \tag{4}$$

In the present case the composite tests may be denoted c and d to dis-

Table 3

Matrix R_{cp} *of the Composite Test Vectors* C *in Terms of the Orthogonal Primary Vectors* T_p

Axis	*S*	*P*	*N*	*V*	*M*	*W*	*I*	*R*	*D*	*10*	*11*	*12*
C_S	.697	.004	.339	.022	−.058	.000	.062	.173	.154	.182	.301	.104
C_P	.258	.690	.049	.257	.082	.042	−.002	.220	.265	.145	−.048	.151
C_N	−.012	.007	.857	.089	.034	.198	.134	.180	.156	.097	.043	.024
C_V	.159	.335	.094	.689	.155	.173	.093	.261	.058	.300	.157	.194
C_M	−.054	.067	.008	.279	.595	.162	.004	.051	.273	.331	−.056	.157
C_W	.063	.126	.090	.166	.141	.615	.103	.069	.270	.366	.067	.106
C_I	.112	−.006	.300	.094	.253	.083	.552	.272	.201	.230	.241	.180
C_R	.126	.257	.157	.297	.259	.190	.199	.627	.072	.384	.133	.056
C_D	−.047	−.053	−.035	.420	.182	.166	.202	.356	.525	.011	.212	.291

tinguish them from the separate experimental tests j and k. We have then

$$y_{pi} = R_{pc} R_{dc}^{-1} S_{di} \, . \tag{5}$$

The matrix R_{pc} and R_{dc} are square and of the order 9, since we have nine primary factors and as many composite tests. The transpose of the matrix R_{pc} is shown in *Table 3*. The matrix R_{cd} can be obtained either from R_{cp} in *Table 3* or from the matrix $R_{cm} = F_c$ in *Table 2* except that the diagonals of R_{cd} are unity. Then

$$R_{cd} = R_{cp} R'_{cp} + D \, , \tag{6}$$

[2] This regression equation is derived in *ibid.*, p. 226.

where D is the diagonal matrix which makes the diagonal entries of R_{cd} unity. *Table 4* shows the matrix R_{cd}, and *Table 5* shows the inverse R_{cd}^{-1}. *Table 6* shows the matrix R_{cp}, and *Table 7* shows the weights W_{pd}

Table 4

The Matrix R_{cd} *(Equation 6)*

	c_S	c_P	c_N	c_V	c_M	c_W	c_I	c_R	c_D
C_S..........	1.00	.31	.38	.33	.05	.23	.41	.38	.20
C_P..........	.31	1.00	.17	.61	.31	.33	.25	.53	.33
C_N..........	.38	.17	1.00	.29	.17	.33	.48	.40	.24
C_V..........	.33	.61	.29	1.00	.47	.48	.44	.73	.55
C_M..........	.05	.31	.17	.47	1.00	.44	.34	.45	.44
C_W..........	.23	.33	.33	.48	.44	1.00	.38	.49	.41
C_I..........	.41	.25	.48	.44	.34	.38	1.00	.59	.50
C_R..........	.38	.53	.40	.73	.45	.49	.59	1.00	.52
C_D..........	.20	.33	.24	.55	.44	.41	.50	.52	1.00

Table 5

The Inverse of the Matrix R_{cd} *Is the Matrix* R_{cd}^{-1}

	c_S	c_P	c_N	c_V	c_M	c_W	c_I	c_R	c_D
C_S..........	1.405	−.248	−.280	−.138	.281	−.040	−.368	−.122	.084
C_P..........	−.248	1.700	.094	−.765	−.065	−.051	.221	−.373	.019
C_N..........	−.280	.094	1.433	.017	.048	−.226	−.446	−.217	.079
C_V..........	−.138	−.765	.017	2.843	−.279	−.212	.131	−1.204	−.515
C_M..........	.281	−.065	.048	−.279	1.528	−.347	−.153	−.183	−.251
C_W..........	−.040	−.051	−.226	−.212	−.347	1.532	−.032	−.209	−.155
C_I..........	−.368	.221	−.446	.131	−.153	−.032	2.003	−.726	−.508
C_R..........	−.122	−.373	−.217	−1.204	−.183	−.209	−.726	2.871	−.103
C_D..........	.084	.019	.079	−.515	−.251	−.155	−.508	−.103	1.721

Table 6

The Projections of the Composite Test Vectors C *on the Orthogonal Primary Vectors Constitute the Matrix* R_{cp}

	S	P	N	V	M	W	I	R	D
C_S..........	.697	.004	.339	.022	−.058	.000	.062	.173	.154
C_P..........	.258	.690	.049	.257	.082	.042	−.002	.220	.265
C_N..........	−.012	.007	.857	.089	.034	.198	.134	.180	.156
C_V..........	.159	.335	.094	.689	.155	.173	.093	.261	.058
C_M..........	−.054	.067	.008	.279	.595	.162	.004	.051	.273
C_W..........	.063	.126	.090	.166	.141	.615	.103	.069	.270
C_I..........	.112	−.006	.300	.094	.253	.083	.552	.272	.201
C_R..........	.126	.257	.157	.297	.259	.190	.199	.627	.072
C_D..........	−.047	−.053	−.035	.420	.182	.166	.202	.356	.525

which are determined by the multiplication

$$(7) \qquad W_{pd} = R_{pc}R_{dc}^{-1}$$

so that

$$(8) \qquad y_{pi} = W_{pd}S_{di} ,$$

by which the standard scores x_{pi} may be estimated.

Table 7

The Weights W_{pd}

	c_S	c_P	c_N	c_V	c_M	c_W	c_I	c_R	c_D
S..........	.82	.12	−.28	.05	.00	.02	−.01	−.09	−.17
P..........	−.23	.81	.00	.10	−.11	.02	.01	.05	−.31
N..........	.08	.01	.95	.04	.02	−.13	.02	−.14	−.20
V..........	−.12	−.20	.06	1.09	.00	−.15	−.19	−.29	.21
M..........	−.08	−.04	−.08	−.18	.67	−.12	.18	.19	−.09
W..........	−.11	−.13	.09	−.02	−.10	.73	−.16	.04	.02
I..........	−.18	−.03	−.12	−.03	−.20	.01	.78	−.02	.00
R..........	−.03	−.03	−.01	−.38	−.22	−.22	−.16	1.09	.28
D..........	.13	.38	.09	−.52	.17	.15	−.03	−.32	.68

THE INTERCORRELATIONS OF THE PRIMARY SCORES

Ideally, the scores x_{pi} and x_{qi} on any pair of primary abilities should be zero, since the primary axes are orthogonal. In practice the correlations will not be exactly zero. The correlations of the predicted standard scores can be estimated as follows.

The moment matrix for the primaries p and q may be written

$$(9) \qquad M_{pq} = \sum_i y_{pi}y_{iq}$$

or, in matrix form,

$$(10) \qquad M_{pq} = y_{pi}y_{iq} ,$$

where y_{iq} is the transpose of y_{pi}. Substituting (8) in (10),

$$(11) \qquad \frac{1}{N} M_{pq} = \frac{1}{N} W_{pd}S_{di}S_{ic}W_{cq} .$$

Let

$$m_{pq} \equiv \frac{1}{N} M_{pq} .$$

Then

(12) $$m_{pq} = W_{pd} R_{dc} W_{cq} .$$

By (7),

(13) $$m_{pq} = R_{pc}(R_{dc}^{-1} R_{dc}) R_{dc}^{-1} R_{dq} ,$$

which can be simplified to

(14) $$m_{pq} = R_{pc} R_{dc}^{-1} R_{dq} ,$$

and by (7),

(15) $$m_{pq} = W_{pd} R_{dq} .$$

The predicted values of x_{pi} are denoted y_{pi}. The standard deviation of x_{pi} is unity by definition, and hence the standard deviations of y_{pi} will be less than unity. Let the standard deviation of y_{pi} be σ_p. If we let $\sigma = \sigma_p = \sigma_q$ denote a diagonal matrix with entries σ_p, then σ_p^{-1} is the inverse, namely, a diagonal matrix with entries $1/\sigma_p$. We have then

(16) $$R_{pq} = \sigma_p^{-1} m_{pq} \sigma_q^{-1} ,$$

whose diagonal elements are unity. Then

(17) $$R_{pq} = \sigma_p^{-1} W_{pd} R_{dq} \sigma_q^{-1} ,$$

and by (7)

(18) $$R_{pq} = \sigma_p^{-1} R_{pc} R_{dc}^{-1} R_{dq} \sigma_q^{-1}$$

In order to solve for R_{pq}, it is necessary to compute the values in the matrix $R_{pc} R_{dc}^{-1} R_{dq}$. The diagonal values of this matrix are equal to σ_p^2. From these may be computed the values $1/\sigma_p$, which are the entries in the diagonal matrix σ_p^{-1}. By (18) the intercorrelations of the estimated primary scores y_{pi} may be determined. They should agree approximately with the correlations that may be actually computed for the scores

y_{pi} and y_{qi}. The agreement will not be exact because the common-factor variance of each test is usually an underestimation. This is due to the fact that in any practical problem the common factors of minor significance are ignored.

In *Table 8* we have the estimated correlations between the standard scores y_{pi} in the primary factors. It will be seen that they are low. The two highest coefficients, namely, .22 and .24, involve the tentative factor *D*. These intercorrelations were determined by equation (18).

Table 8

Predicted Correlations R_{pq}

	S	*P*	*N*	*V*	*M*	*W*	*I*	*R*	*D*
S....	(1.00)	.15	.06	− .06	− .18	− .15	− .08	.02	.05
P....	.15	(1.00)	− .01	.20	− .01	.00	− .11	.13	.00
N....	.06	− .01	(1.00)	− .03	− .06	.10	.11	.05	.04
V....	− .06	.20	− .03	(1.00)	.11	11	− .06	.09	.02
M....	− .18	− .01	− .06	.11	(1.00)	.04	.18	.15	.22
W....	− .15	.00	.10	.11	.04	(1.00)	.01	.00	.24
I....	− .08	− .11	.11	− .06	.18	.01	(1.00)	.25	.10
R....	.02	.13	.05	.09	.15	.00	.25	(1.00)	.07
D....	.05	.00	.04	.02	.22	.24	.10	.07	(1.00)

THE MENTAL PROFILES AND VOCATIONAL INTERESTS

It is of psychological interest to compare the mental profiles in the seven factors and in the two additional tentative factors with the vocational preferences of the subjects. Each subject filled in a vocational-interest schedule.

In the interpretation of the mental profiles it has been found best not to confine one's attention to the absolute level of each ability on the scale determined by the experimental group. The best interpretation of the mental profile is made by considering the relations of the several abilities to one another and to the mean level of the individual himself. All the primary abilities of an individual may be above the mean for a particular group, but, if one or two of his abilities are much higher than the other abilities, his vocational preferences are likely to show some correspondence to his mental profile.

A few of the individual subjects will be described here as regards the relation between the mental profile and the occupational preferences. These cases are selected from the two hundred and forty subjects to represent various combinations. Most of the subjects do not show marked atypicalities in their mental profiles so that for them the relation of the profile to the occupational preferences is not so clear.

One student who rates high in both of the verbal factors V and W and average or below in all the others likes advertising and writing. It must be recalled that since this experimental group was extremely highly selected, an average score here represents a good performance in relation to the American college population. This student has a lower performance in the number factor. Another student has his own highest performances in the two verbal factors V and W and in the memory factor M. He wants advertising and selling. Another student who has the ambition to be a poet and writer is characterized by a conspicuously low score on the number factor, and his highest score is made on the memory factor.

A profile with the two highest scores in verbal relations V and in the perceptual factor P and a conspicuously low score in the problem-solving factor R is for a student who would like to be an actor. He is interested in entertaining as a career. Another student who has a conspicuously low score in the number factor with the other factors roughly equal wants to be a psychiatrist, a philosopher, and a teacher.

A profile which is highest in the deductive factor and relatively high in space, number, and induction is that of a student who wants to study engineering. Another profile is high in the problem-solving factor R. It is for a student who wants to be a mathematician.

One profile which is noticeable because of the conspicuously high score in the visual or space factor belongs to a student who wants to be a chemist or radio engineer. Another similar profile, characterized by a high score on the space factor S, belongs to a student who wants to be a geologist.

Some of the profiles are characterized by an extremely low score in some one factor. One such profile in which verbal relations V is conspicuously low, and the space factor S somewhat higher than the rest, is for a student who wants to be a scientist and a mechanical engineer.

Another profile has the two lowest scores, relative to itself, in verbal relations and memory. The two highest scores on this profile are number and the tentative problem-solving factor R. This student wants to be a research chemist. A student whose highest factor is memory and who has a conspicuously low score in the factor R wants to teach history. Some of these relations between mental profiles and occupational choices are amusing in that some of them seem to be determined by a conspicuously high factor, and others seem to be determined by a dislike that is perhaps associated with a conspicuously low factor. One such profile which has a conspicuously low score in the number factor *N*, and a relatively high score in the word factor *W*, is for a student who wants to be a journalist and advertiser. Two profiles with highest relative scores in verbal relations *V* belong to students who want to become teachers. Another profile with highest relative score in the factor *R* is for a student who wants to become a physicist and mathematician.

Further experimental study of the psychological nature of these factors together with other information about the subjects such as their likes and dislikes, their occupational preferences, and their actual performances will eventually give us more confidence in the interpretation of mental profiles.

APPENDIX

Table 1

Distribution of Raw Scores

4		5		6		7		8		9		10		11	
Reading I		Reading II		Verbal Classification		Word-grouping		Figure Classification		Controlled Associations		Inventive Opposites		Completion	
Score	*f*	Score	*f*	Score	*f*	Score	*f*	Score	*f*	Score	*f*	Score	*f*	Score	*f*
5	1	1, 2	1	2– 6	1	21–24	1	0, 1	36	15– 20	1	4– 7	1	17	2
6	0	3, 4	2	6–10	1	24–27	2	2, 3	20	20– 25	3	7–10	1	18	1
7	0	5, 6	3	10–14	5	27–30	3	4, 5	8	25– 30	4	10–13	3	19	2
8	2	7, 8	2	14–18	3	30–33	4	6, 7	5	30– 35	14	13–16	0	20	2
9	2	9, 10	8	18–22	6	33–36	6	8, 9	8	35– 40	18	16–19	1	21	1
10	4	11, 12	12	22–26	4	36–39	9	10, 11	6	40– 45	25	19–22	2	22	2
11	6	13, 14	12	26–30	10	39–42	18	12, 13	24	45– 50	24	22–25	4	23	5
12	10	15, 16	17	30–34	17	42–45	21	14, 15	24	50– 55	25	25–28	7	24	5
13	9	17, 18	26	34–38	33	45–48	20	16, 17	23	55– 60	37	28–31	9	25	9
14	12	19, 20	20	38–42	37	48–51	25	18, 19	17	60– 65	18	31–34	21	26	7
15	11	21, 22	42	42–46	29	51–54	24	20, 21	25	65– 70	9	34–37	24	27	6
16	11	23, 24	48	46–50	42	54–57	24	22, 23	17	70– 75	18	37–40	32	28	11
17	9	25, 26	41	50–54	26	57–60	34	24, 25	11	75– 80	12	40–43	24	29	9
18	15			54–58	10	60–63	17			80– 85	10	43–46	42	30	11
19	28			58–62	7	63–66	11			85– 90	3	46–49	26	31	12
20	24			62–66	3	66–69	3			90– 95	2	49–52	16	32	14
21	36									95–100	2	52–55	6	33	9
22	28									100–105	5	55–58	4	34	25
23	17									105–110	1	58–61	1	35	19
24	9													36	23
25	0													37	16
														38	19
														39	14
	234		234		234		222		224		231		224		224

Table 1—Continued

12		13		14		15		16		17		18		19	
Disarranged Words		First and Last Letters		Disarranged Sentences		Anagrams		Inventive Synonyms		Block-counting		Cubes		Lozenges A	
Score	*f*	Score	*f*	Score	*f*	Score	*f*	Score	*f*	Score	*f*	Score	*f*	Score	*f*
								12–15	1	0– 5	1	0, 1	6	−1– 2	1
20–23	1	5– 8	5	27–31	1	6– 9	2	15–18	4	5–10	1	2, 3	5	2– 5	8
23–26	0	8–11	9	31–35	5	9–12	12	18–21	7	10–15	2	4, 5	7	5– 8	15
26–29	6	11–14	32	35–39	7	12–15	20	21–24	10	15–20	5	6, 7	11	8–11	19
29–32	4	14–17	34	39–43	12	15–18	31	24–27	5	20–25	11	8, 9	18	11–14	18
32–35	7	17–20	39	43–47	19	18–21	44	27–30	14	25–30	14	10, 11	22	14–17	17
35–38	17	20–23	41	47–51	31	21–24	43	30–33	24	30–35	21	12, 13	27	17–20	23
38–41	18	23–26	28	51–55	25	24–27	32	33–36	33	35–40	43	14, 15	25	20–23	17
41–44	35	26–29	12	55–59	31	27–30	18	36–39	22	40–45	43	16, 17	24	23–26	10
44–47	19	29–32	10	59–63	33	30–33	12	39–42	36	45–50	32	18, 19	13	26–29	17
47–50	29	32–35	4	63–67	16	33–36	6	42–45	26	50–55	30	20, 21	18	29–32	13
50–53	26	35–38	4	67–71	24	36–39	2	45–48	24	55–60	16	22, 23	12	32–35	9
53–56	17	38–41	2	71–75	9	39–42	2	48–51	9	60–65	11	24, 25	10	35–38	7
56–59	22	41–44	2	75–79	7	42–45	0	51–54	6	65–70	2	26, 27	7	38–41	12
59–62	15	44–47	1	79–83	2	45–48	1	54–57	1	70–75	1	28, 29	13	41–44	9
62–65	5							57–60	2	75–80	1	30, 31	4	44–47	8
65–68	2											32, 33	2	47–50	15
68–71	1														
	224		223		222		225		224		234		224		218

Table 1—Continued

20		21		22		23		24		25		26		27	
FLAGS		FORM BOARD		LOZENGES B		SURFACE DEVELOPMENT		PUNCHED HOLES		MECHANICAL MOVEMENTS		IDENTICAL FORMS		PURSUIT	
Score	*f*	Score	*f*	Score	*f*	Score	*f*	Score	*f*	Score	*f*	Score	*f*	Score	*f*
−1– 3	5	4, 5	1	−10– −5	1	− 9– −6	2	0	4	−13– −8	3	21–24	2	25–28	4
3– 7	8	6, 7	1	− 5– 0	1	− 6– −3	2	1	4	− 8– −3	3	24–27	1	28–31	5
7–11	8	8, 9	1	0– 5	2	− 3– 0	3	2	4	− 3– 2	12	27–30	4	31–34	4
11–15	9	10, 11	2	5– 10	12	0– 3	6	3	10	2– 7	25	30–33	0	34–37	14
15–19	15	12, 13	10	10– 15	5	3– 6	12	4	7	7– 12	29	33–36	6	37–40	12
19–23	23	14, 15	10	15– 20	14	6– 9	9	5	19	12– 17	28	36–39	19	40–43	34
23–27	30	16, 17	13	20– 25	14	9– 12	10	6	46	17– 22	29	39–42	19	43–46	28
27–31	23	18, 19	18	25– 30	12	12– 15	14	7	37	22– 27	30	42–45	27	46–49	22
31–35	26	20, 21	19	30– 35	18	15– 18	17	8	22	27– 32	24	45–48	31	49–52	30
35–39	27	22, 23	21	35– 40	13	18– 21	28	9	40	32– 37	16	48–51	35	52–55	26
39–43	24	24, 25	11	40– 45	18	21– 24	33	10	31	37– 42	11	51–54	23	55–58	16
43–47	19	26, 27	14	45– 50	8	24– 27	32			42– 47	7	54–57	27	58–61	17
47–51	7	28, 29	17	50– 55	4	27– 30	17			47– 52	6	57–60	33	61–64	15
		30, 31	26	55– 60	13	30– 33	14			52– 57	9	60–63	6	64–67	5
		32, 33	25	60– 65	21	33– 36	14			57– 62	2			67–70	0
		34, 35	12	65– 70	17	36 39	8							70–73	2
		36, 37	11	70– 75	30										
		38, 39	6	75– 80	19										
		40, 41	4												
	224		222		222		221		224		234		233		234

Table 1—Continued

28		29		30		31		32		33		34		35	
Copying		Areas		Number Code		Addition		Subtraction		Multiplication		Division		Tabular Completion	
Score	*f*	Score	*f*	Score	*f*	Score	*f*	Score	*f*	Score	*f*	Score	*f*	Score	*f*
−12, −11	1	4, 5	2	1, 2	6	2	2	1, 2	1	1, 2	4	0	1	4, 5	1
−10, − 9	1	6, 7	4	3, 4	3	3	5	3, 4	0	3, 4	21	1	3	6, 7	3
− 8, − 7	1	8, 9	5	5, 6	8	4	7	5, 6	0	5, 6	38	2	7	8, 9	2
− 6, − 5	2	10, 11	10	7, 8	1	5	5	7, 8	0	7, 8	63	3	15	10, 11	6
− 4, − 3	4	12, 13	15	9, 10	1	6	11	9, 10	4	9, 10	25	4	27	12, 13	5
− 2, − 1	5	14, 15	21	11, 12	3	7	14	11, 12	4	11, 12	27	5	28	14, 15	4
0, 1	12	16, 17	25	13, 14	4	8	20	13, 14	10	13, 14	21	6	34	16, 17	6
2, 3	12	18, 19	25	15, 16	5	9	23	15, 16	19	15, 16	7	7	29	18, 19	12
4, 5	19	20, 21	20	17, 18	8	10	28	17, 18	20	17, 18	5	8	20	20, 21	10
6, 7	31	22, 23	31	19, 20	13	11	25	19, 20	39	19, 20	4	9	16	22, 23	23
8, 9	21	24, 25	29	21, 22	12	12	17	21, 22	26	21, 22	1	10	14	24, 25	31
10, 11	24	26, 27	20	23, 24	30	13	14	23, 24	23	23, 24	0	11	8	26, 27	40
12, 13	35	28, 29	11	25, 26	31	14	13	25, 26	16	25, 26	1	12	4	28, 29	39
14, 15	20	30, 31	4	27, 28	21	15	12	27, 28	14	27, 28	1	13	3	30, 31	28
16, 17	16			29, 30	18	16	9	29, 30	15			14	5	32, 33	6
18, 19	11			31, 32	20	17	3	31, 32	8			15	1		
20, 21	6			33, 34	13	18	3	33, 34	7			16	2		
22, 23	0			35, 36	14	19	1	35, 36	12			17	0		
24, 25	1			37, 38	11	20	1					18	1		
26, 27	1			39, 40	12	21	1								
28, 29	1					22	1								
						23	2								
						24	1								
	224		222		234		218		218		218		218		216

Table 1—Continued

36		37		38		39		40		41		42		43	
Estimating		Number Series		Numerical Judgment		Arithmetic		Reasoning		Verbal Analogies		False Premises		Code Words	
Score	*f*	Score	*f*	Score	*f*	Score	*f*	Score	*f*	Score	*f*	Score	*f*	Score	*f*
3, 4	2	1	1	− 6, − 5	8	0	1	− 7, − 6	2	7–10	1	− 5, − 4	1	0	2
5, 6	1	2	0	− 4, − 3	8	1	3	− 5, − 4	0	10–13	2	− 3, − 2	1	1	5
7, 8	4	3	1	− 2, − 1	17	2	10	− 3, − 2	1	13–16	1	− 1, 0	2	2	3
9, 10	7	4	2	0, 1	27	3	8	− 1, 0	4	16–19	4	1, 2	6	3	5
11, 12	6	5	4	2, 3	19	4	17	1, 2	5	19–22	13	3, 4	9	4	1
13, 14	11	6	11	4, 5	24	5	15	3, 4	7	22–25	12	5, 6	14	5	7
15, 16	10	7	10	6, 7	26	6	21	5, 6	15	25–28	21	7, 8	17	6	8
17, 18	20	8	6	8, 9	18	7	29	7, 8	14	28–31	19	9, 10	27	7	6
19, 20	28	9	10	10, 11	24	8	21	9, 10	17	31–34	36	11, 12	31	8	8
21, 22	35	10	16	12, 13	13	9	23	11, 12	15	34–37	26	13, 14	29	9	15
23, 24	39	11	25	14, 15	8	10	17	13, 14	26	37–40	26	15, 16	31	10	17
25, 26	37	12	16	16, 17	12	11	15	15, 16	15	40–43	20	17, 18	20	11	25
27, 28	25	13	30	18, 19	14	12	8	17, 18	17	43–46	16	19, 20	19	12	21
		14	26	20, 21	4	13	10	19, 20	15	46–49	13	21, 22	11	13	23
		15	23	22, 23	1	14	7	21, 22	19	49–52	7	23, 24	7	14	35
		16	13			15	8	23, 24	15	52–55	5			15	20
		17	9			16	2	25, 26	9					16	12
		18	8			17	3	27, 28	10					17	8
		19	5					29, 30	7					18	2
		20	3					31, 32	6						
		21	2					33, 34	3						
	225		221		223		218		222		222		225		223

Table 1—Continued

44		45		46		47		48		49		50		51	
PATTERN ANALOGIES		SYLLOGISMS		MEMORY I		MEMORY III		MEMORY II		WORD RECOGNITION		FIGURE RECOGNITION		PICTURE RECALL	
Score	f	Score	f	Score	f	Score	f	Score	f	Score	f	Score	f	Score	f
5	1	− 2, − 1	2	0	7	− 1, 0	1	0	1	−31− −27	1	− 2	1	13, 14	1
6	1	0, 1	2	1	16	1, 2	16	1	4	−27− −23	0	− 1	1	15, 16	0
7	0	2, 3	3	2	23	3, 4	32	2	11	−23− −19	2	0	0	17, 18	1
8	1	4, 5	4	3	18	5, 6	32	3	13	−19− −15	3	1	0	19, 20	10
9	1	6, 7	6	4	24	7, 8	34	4	25	−15− −11	5	2	2	21, 22	8
10	3	8, 9	10	5	24	9, 10	38	5	21	−11− − 7	3	3	1	23, 24	11
11	9	10, 11	4	6	19	11, 12	23	6	30	− 7− − 3	7	4	4	25, 26	24
12	12	12, 13	13	7	22	13, 14	15	7	30	− 3− 1	12	5	8	27, 28	26
13	16	14, 15	9	8	10	15, 16	12	8	20	1− 5	12	6	3	29, 30	28
14	17	16, 17	8	9	8	17, 18	8	9	18	5− 9	11	7	4	31, 32	33
15	26	18, 19	15	10	9	19, 20	4	10	12	9− 13	18	8	6	33, 34	30
16	24	20, 21	14	11	14	21, 22	4	11	14	13− 17	13	9	2	35, 36	28
17	35	22, 23	12	12	7	23, 24	4	12	8	17− 21	27	10	8	37, 38	10
18	27	24, 25	17	13	8	25, 26	1	13	7	21− 25	21	11	5	39, 40	8
19	29	26, 27	10	14	5			14	4	25− 29	30	12	4		
20	15	28, 29	13	15	7			15	3	29− 33	12	13	13		
21	1	30, 31	17	16	4			16	0	33− 37	12	14	13		
22	4	32, 33	59	17	3			17	1	37− 41	18	15	7		
				18	2					41− 45	12	16	11		
				19	1					45− 49	6	17	30		
				20	2							18	16		
												19	6		
												20	73		
	222		218		233		224		222		225		218		218

Table 1—Continued

52		53		54		55		56		57		58		59		60	
THEME		HANDS		RHYTHM		SOUND-GROUPING		SPELLING		GRAMMAR		VOCABULARY		FREE-WRITING		WORD KNOWLEDGE	
Score	*f*	Score	*f*	Score	*f*	Score	*f*	Score	*f*	Score	*f*	Score	*f*	Score	*f*	Score	*f*
0.0	3	−10– –6	2	1, 2	5	32–35	1	2– 7	2	7–10	1	33–37	1	6– 9	1	40, 41	1
0.5	1	– 6– –2	3	3, 4	21	35–38	1	7–12	3	10–13	1	37–41	2	9–12	1	42, 43	0
1.0	4	– 2– 2	7	5, 6	34	38–41	4	12–17	2	13–16	1	41–45	4	12–15	5	44, 45	1
1.5	6	2– 6	8	7, 8	46	41–44	8	17–22	4	16–19	2	45–49	1	15–18	10	46, 47	1
2.0	20	6– 10	20	9, 10	33	44–47	6	22–27	7	19–22	1	49–53	2	18–21	18	48, 49	1
2.5	10	10– 14	19	11, 12	23	47–50	7	27–32	5	22–25	6	53–57	2	21–24	25	50, 51	1
3.0	23	14– 18	13	13, 14	19	50–53	9	32–37	11	25–28	8	57–61	5	24–27	30	52, 53	2
3.5	22	18– 22	20	15, 16	8	53–56	10	37–42	11	28–31	13	61–65	5	27–30	34	54, 55	1
4.0	20	22– 26	18	17, 18	11	56–59	15	42–47	19	31–34	23	65–69	11	30–33	31	56, 57	0
4.5	22	26– 30	15	19, 20	8	59–62	12	47–52	13	34–37	24	69–73	6	33–36	21	58, 59	6
5.0	29	30– 34	16	21, 22	2	62–65	21	52–57	23	37–40	43	73–77	7	36–39	11	60, 61	1
5.5	13	34– 38	18	23, 24	7	65–68	22	57–62	16	40–43	44	77–81	10	39–42	10	62, 63	3
6.0	19	38– 42	10	25, 26	8	68–71	22	62–67	26	43–46	36	81–85	11	42–45	11	64, 65	6
6.5	10	42– 46	27			71–74	22	67–72	18	46–49	15	85–89	16	45–48	4	66, 67	3
7.0	4	46– 50	28			74–77	22	72–77	28			89–93	15	48–51	2	68, 69	11
7.5	4					77–80	21	77–82	10			93–97	5	51–54	0	70, 71	7
8.0	3					80–83	15	82–87	8			97–101	1	54–57	1	72, 73	15
8.5	2					83–86	3	87–92	5					57–60	2	74, 75	23
9.0	1					86–89	3	92–97	7							76, 77	19
9.5	0															78, 79	10
10.0	1															80, 81	4
	217		224		225		224		218		218		104		217		116

*Table 2**

Tetrachoric Correlations of 57 Psychological Tests for 240 Subjects

Test	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
4		+83	+54	+49	+19	+47	+47	+58	+35	+27	+37	+14	+46	+13	+10	+28	00	+32	+20	+22
5	+83		+63	+61	+15	+42	+61	+64	+42	+34	+42	+28	+54	+15	+25	+46	+12	+46	+34	+27
6	+54	+63		+76	+46	+37	+52	+55	+29	+31	+64	+24	+48	+36	+48	+38	+37	+58	+56	+42
7	+49	+61	+76		+31	+37	+57	+62	+31	+42	+59	+22	+48	+34	+34	+25	+21	+46	+31	+30
8	+19	+15	+46	+31		+10	+05	+15	+30	+31	+33	+22	+19	+31	+59	+40	+42	+51	+54	+42
9	+47	+42	+37	+37	+10		+49	+43	+16	+37	+28	+37	+37	+06	−10	−06	−16	+21	−07	+15
10	+47	+61	+52	+57	+05	+49		+73	+51	+52	+60	+37	+72	+26	+26	+28	+08	+43	+22	+32
11	+58	+64	+55	+62	+15	+43	+73		+57	+46	+60	+35	+70	+32	+43	+33	+20	+62	+29	+32
12	+35	+42	+29	+31	+30	+16	+51	+57		+48	+37	+46	+59	+16	+40	+37	+22	+43	+31	+35
13	+27	+34	+31	+42	+31	+37	+52	+46	+48		+28	+56	+46	+21	+28	+25	+21	+37	+30	+23
14	+37	+42	+64	+59	+33	+28	+60	+60	+37	+28		+22	+46	+25	+34	+31	+28	+37	+40	+31
15	+14	+28	+24	+22	+22	+37	+37	+35	+46	+56	+22		+48	+06	+19	+07	+12	+28	+15	+22
16	+46	+54	+48	+48	+19	+37	+72	+70	+59	+46	+46	+48		+09	+18	+25	+06	+37	+18	+20
17	+13	+15	+36	+34	+31	+06	+26	+32	+16	+21	+25	+06	+09		+61	+51	+48	+61	+40	+42
18	+10	+25	+48	+34	+59	−10	+26	+43	+40	+28	+34	+19	+18	+61		+47	+68	+61	+56	+56
19	+28	+46	+38	+25	+40	−06	+28	+33	+37	+25	+31	+07	+25	+51	+47		+62	+67	+64	+54
20	00	+12	+37	+21	+42	−16	+08	+20	+22	+21	+28	+12	+06	+48	+68	+62		+59	+61	+63
21	+32	+46	+58	+46	+51	+21	+43	+62	+43	+37	+37	+28	+37	+61	+61	+67	+59		+58	+58
22	+20	+34	+56	+31	+54	−07	+22	+29	+31	+30	+40	+15	+18	+40	+56	+64	+61	+58		+57
23	+22	+27	+42	+30	+42	+15	+32	+32	+35	+23	+31	+22	+20	+42	+56	+54	+63	+58	+57	
24	+28	+40	+47	+33	+51	+05	+23	+33	+37	+33	+15	+15	+20	+50	+42	+63	+53	+74	+62	+55
25	+28	+34	+44	+30	+40	+21	+32	+47	+37	+40	+18	+18	+34	+45	+45	+43	+33	+61	+42	+36
26	+25	+33	+71	+42	+22	+15	+22	+46	+25	+12	+40	+10	+37	+40	+45	+29	+21	+40	+25	+43
27	+17	+09	+33	+25	+40	+09	+13	+22	+06	+25	+37	+25	00	+46	+61	+38	+57	+49	+49	+43
28	+21	+35	+58	+40	+53	+12	+35	+38	+31	+15	+40	+22	+24	+45	+47	+43	+34	+70	+42	+50
29	+43	+32	+44	+20	+31	+05	+30	+30	+34	+18	+28	+19	+18	+40	+42	+35	+33	+55	+42	+41
30	+19	+27	+45	+43	+40	+07	+42	+30	+30	+25	+40	+28	+46	+58	+48	+39	+58	+48	+48	+45
31	+01	+11	+18	+18	+19	+06	+10	−06	+12	+31	+21	+16	+12	+27	+21	+17	+28	+06	+19	+14
32	+26	+25	+25	+28	+10	+18	+40	+07	+28	+28	+37	+25	+25	+12	+14	+17	+20	+24	+21	+16
33	−03	+08	+28	+17	+10	+17	+18	+15	+35	+22	+32	+26	+22	+26	+26	+15	+23	+23	+22	+11
34	+18	+16	+40	+15	+12	+06	+29	+26	+25	+25	+28	+16	+30	+22	+37	+22	+27	+24	+30	+27
35	+40	+43	+41	+27	+37	+11	+30	+30	+29	+27	+25	+22	+37	+18	+36	+36	+20	+40	+33	+30
36	+26	+31	+20	+24	−03	+06	+30	+30	+11	+06	+34	−10	+28	+40	+26	+33	+27	+30	+33	+11
37	+40	+38	+45	+43	+52	+10	+37	+18	+29	+26	+44	+26	+35	+26	+36	+42	+17	+33	+41	+33
38	+32	+34	+31	+33	+16	+09	+20	+35	+22	00	+25	+16	+18	+28	+27	+22	+23	+30	+18	+34
39	+34	+31	+50	+37	+30	+16	+42	+42	+34	+16	+35	+31	+29	+50	+39	+34	+34	+57	+52	+42
40	+59	+66	+42	+48	+28	+21	+43	+50	+34	+40	+42	+30	+56	+11	+27	+46	+30	+42	+30	+33
41	+57	+64	+64	+64	+46	+40	+60	+60	+40	+46	+59	+28	+61	+46	+46	+49	+42	+54	+40	+46
42	+58	+54	+48	+43	+43	+22	+43	+42	+31	+40	+38	+22	+52	+13	+29	+29	+16	+26	+29	+32
43	+52	+60	+68	+58	+53	+18	+46	+50	+53	+34	+46	+28	+54	+45	+40	+55	+50	+56	+53	+50
44	+51	+59	+60	+59	+51	+21	+48	+62	+45	+37	+46	+31	+47	+37	+52	+49	+28	+62	+28	+43
45	+52	+51	+65	+48	+52	+28	+44	+39	+35	+43	+31	+19	+40	+37	+51	+47	+48	+50	+64	+52
46	+43	+25	+30	+18	−02	+28	+22	+32	+29	+18	+28	+19	+27	+10	+12	+10	−04	+32	+33	+30
47	+43	+53	+31	+31	+06	+42	+57	+40	+29	+48	+31	+45	+54	+03	−03	+07	+03	+37	+19	+30
48	+25	+38	+32	+10	+21	+13	+28	+21	+07	+13	+29	+22	+17	+04	+20	+12	+16	+20	+32	+27
49	+27	+48	+40	+30	+19	+11	+32	+35	+38	+24	+30	+31	+40	+18	+15	+31	+05	+42	+33	+18
50	+22	+28	+33	+31	+22	+08	+30	+36	+25	+19	+21	+15	+21	+08	+20	+32	+15	+42	+23	+20
51	+07	+12	+29	+28	+02	+16	+25	+28	+29	+03	+48	+04	+21	+25	+26	+03	+06	+19	+10	+12
52	+51	+62	+51	+45	+15	+43	+69	+46	+25	+31	+37	+16	+49	+03	+16	+34	−04	+37	+25	+25
53	−05	+02	+25	+09	+21	00	+13	+13	+21	+21	+34	+22	+09	+48	+42	+34	+59	+37	+53	+40
54	+35	+31	+36	+09	+25	+21	+35	+23	+25	+21	+31	+34	+30	+05	+31	+32	+11	+23	+20	+30
55	+40	+46	+61	+59	+51	+25	+49	+40	+38	+37	+51	+31	+51	+18	+36	+49	+37	+37	+42	+33
56	+40	+42	+48	+45	+16	+31	+57	+43	+46	+40	+34	+37	+46	+08	+18	+16	+02	+33	+18	+15
57	+42	+53	+48	+46	+30	+28	+69	+49	+46	+48	+45	+53	+61	+11	+29	+37	+21	+33	+28	+28
58	+57	+75	+38	+46	00	+50	+66	+76	+35	+31	+30	+20	+46	+11	+05	+35	+12	+42	+03	+29
59	+15	+26	+38	+29	+12	+29	+19	+25	+22	+10	+31	+07	+34	+07	+04	+10	00	−06	+10	−02
60	+72	+76	+67	+67	+28	+25	+61	+77	+62	+48	+43	+40	+68	+28	+35	+37	+27	+46	+22	+21

* In this table the decimal point preceding the entries has been omitted.

Table 2—Continued

Test	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43
4	+28	+28	+25	+17	+21	+43	+19	+01	+26	−03	+18	+40	+26	+40	+32	+34	+59	+57	+58	+52
5	+40	+34	+33	+09	+35	+32	+27	+11	+25	+08	+16	+43	+31	+38	+34	+31	+66	+64	+54	+60
6	+47	+44	+71	+33	+58	+44	+45	+18	+25	+28	+40	+41	+20	+45	+31	+50	+42	+64	+48	+68
7	+33	+30	+42	+25	+40	+20	+43	+18	+28	+17	+15	+27	+24	+43	+33	+37	+48	+64	+43	+58
8	+51	+40	+22	+40	+53	+31	+40	+19	+10	+10	+12	+37	−03	+52	+16	+30	+28	+46	+43	+53
9	+05	+21	+15	+09	+12	+05	+07	+06	+18	+17	+06	+11	+06	+10	+09	+16	+21	+40	+22	+18
10	+23	+32	+22	+13	+35	+30	+42	+10	+40	+18	+29	+30	+30	+37	+20	+42	+43	+60	+43	+46
11	+33	+47	+46	+22	+38	+30	+30	−06	+07	+15	+26	+30	+30	+18	+35	+42	+50	+60	+42	+50
12	+37	+37	+25	+06	+31	+34	+30	+12	+28	+35	+25	+29	+11	+29	+22	+34	+34	+40	+31	+53
13	+33	+40	+12	+25	+15	+18	+25	+31	+28	+22	+25	+27	+06	+26	00	+16	+40	+46	+40	+34
14	+15	+18	+40	+37	+40	+28	+40	+21	+37	+32	+28	+25	+34	+44	+25	+35	+42	+59	+38	+46
15	+15	+18	+10	+25	+22	+19	+28	+16	+25	+26	+16	+22	−10	+26	+16	+31	+30	+28	+22	+28
16	+20	+34	+37	00	+24	+18	+46	+12	+25	+22	+30	+37	+28	+35	+18	+29	+56	+61	+52	+54
17	+50	+45	+40	+46	+45	+40	+58	+27	+12	+26	+22	+18	+40	+26	+28	+50	+11	+46	+13	+45
18	+42	+45	+45	+61	+47	+42	+48	+21	+14	+26	+37	+36	+26	+36	+27	+39	+27	+46	+29	+40
19	+63	+43	+29	+38	+43	+35	+39	+17	+17	+15	+22	+36	+33	+42	+22	+34	+46	+49	+29	+55
20	+53	+33	+21	+57	+34	+33	+58	+28	+20	+23	+27	+20	+27	+17	+23	+34	+30	+42	+16	+50
21	+74	+61	+40	+49	+70	+55	+48	+06	+24	+23	+24	+40	+30	+33	+30	+57	+42	+54	+26	+56
22	+62	+42	+25	+49	+42	+42	+48	+19	+21	+22	+30	+33	+33	+41	+18	+52	+30	+40	+29	+53
23	+55	+36	+43	+43	+50	+41	+45	+14	+16	+11	+27	+30	+11	+33	+34	+42	+33	+46	+32	+50
24		+61	+30	+40	+42	+52	+47	+11	+15	+21	+24	+35	+20	+48	+30	+53	+48	+49	+39	+72
25	+61		+21	+24	+48	+42	+37	+11	+07	+04	+15	+42	+17	+30	+28	+52	+51	+46	+35	+48
26	+30	+21		+22	+42	00	+22	00	+02	+03	+19	+16	00	+16	+10	+16	+10	+37	+22	+40
27	+40	+24	+22		+42	+43	+40	+34	+21	+29	+40	+22	+20	+34	+28	+25	+22	+29	+16	+54
28	+42	+48	+42	+42		+49	+40	+30	+19	+26	+28	+43	+18	+50	+30	+55	+33	+52	+20	+49
29	+52	+42	00	+43	+49		+45	+18	+19	+29	+42	+52	+30	+42	+43	+56	+39	+43	+41	+47
30	+47	+37	+22	+40	+40	+45		+64	+57	+62	+51	+52	+36	+40	+43	+66	+40	+60	+39	+66
31	+11	+11	00	+34	+30	+18	+64		+62	+62	+54	+40	+06	+35	+28	+29	+21	+25	+18	+30
32	+15	+07	+02	+21	+19	+19	+57	+62		+67	+53	+31	+15	+38	+37	+38	+33	+34	+22	+28
33	+21	+04	+03	+29	+26	+29	+62	+62	+67		+62	+47	+14	+30	+52	+48	+10	+16	+11	+33
34	+24	+15	+19	+40	+28	+42	+51	+54	+53	+62		+56	+11	+41	+57	+62	+27	+28	+29	+48
35	+35	+42	+16	+22	+43	+52	+52	+40	+31	+47	+56		+13	+68	+48	+58	+45	+40	+36	+36
36	+20	+17	00	+20	+18	+30	+36	+06	+15	+14	+11	+13		+23	+09	+30	+28	+40	+20	+33
37	+48	+30	+16	+34	+50	+42	+40	+35	+38	+30	+41	+68	+23		+43	+72	+55	+54	+42	+58
38	+30	+28	+10	+28	+30	+43	+43	+28	+37	+52	+57	+48	+09	+43		+64	+37	+16	+32	+42
39	+53	+52	+16	+25	+55	+56	+66	+29	+38	+48	+62	+58	+30	+72	+64		+50	+48	+46	+62
40	+48	+51	+10	+22	+33	+39	+40	+21	+33	+10	+27	+45	+28	+55	+37	+50		+54	+78	+48
41	+49	+46	+37	+29	+52	+43	+60	+25	+34	+16	+28	+40	+40	+54	+16	+48	+54		+40	+72
42	+39	+35	+22	+16	+20	+41	+39	+18	+22	+11	+29	+36	+20	+42	+32	+46	+78	+40		+47
43	+72	+48	+40	+34	+49	+47	+66	+30	+28	+33	+48	+36	+33	+58	+42	+62	+48	+72	+47	
44	+50	+57	+34	+46	+57	+57	+47	+16	+16	+10	+28	+26	+31	+43	+46	+54	+49	+67	+48	+69
45	+55	+40	+35	+43	+51	+54	+50	+28	+29	+38	+45	+60	+27	+46	+33	+50	+42	+50	+51	+66
46	+20	+20	+21	+02	+24	+20	+26	+14	+12	+11	+14	+30	+04	+24	+06	+32	+23	+19	+32	+17
47	+33	+28	+28	+16	+21	+28	+35	+18	+21	+20	+19	+23	+12	+30	+10	+22	+50	+31	+43	+48
48	+20	+10	+10	+10	+39	+36	+41	+38	+07	+30	+26	+39	+04	+46	+40	+37	+20	+16	+37	+44
49	+36	+30	+16	+03	+28	+42	+30	+05	+12	+04	+09	+23	+08	+20	+18	+23	+33	+25	+38	+27
50	+29	+23	+20	−10	+23	+35	+17	−10	+09	+02	−01	+13	+11	+18	+15	+17	+33	+31	+23	+42
51	+10	+16	+28	+12	+21	+10	+32	+16	+10	+16	+10	+10	+19	+10	+22	+19	−07	+25	+06	+28
52	+25	+19	+25	+12	+29	+22	+17	+12	+40	+29	+28	+22	+06	+34	+37	+34	+46	+43	+48	+40
53	+33	+15	+03	+40	+33	+42	+50	+24	+22	+30	+30	+27	+19	+17	+22	+29	+21	+28	+07	+37
54	+35	+02	+06	+29	+12	+29	+05	+05	+24	+04	+20	+32	+04	+27	+18	+13	+23	+34	+13	+23
55	+33	+15	+25	+46	+28	+33	+49	+22	+40	+22	+40	+39	+11	+38	+28	+29	+33	+59	+41	+62
56	+05	+24	+19	+07	+18	+11	+18	+09	+37	+26	+30	+23	+20	+39	+40	+46	+45	+28	+52	+24
57	+28	+28	+12	+16	+31	+32	+32	+19	+42	+26	+43	+33	+28	+39	+37	+40	+62	+46	+64	+43
58	+30	+15	+20	+10	+10	+09	+16	+14	+19	+12	+13	+16	−03	+18	+30	+38	+27	+49	+24	+31
59	+08	+10	+22	00	+10	+17	+17	+26	+10	+24	+22	+17	−06	+21	+25	+06	+07	+16	+16	+20
60	+37	+60	+40	+11	+46	+40	+35	+12	+12	+10	+30	+37	+24	+31	+46	+52	+58	+62	+62	+62

Table 2—Continued

Test	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
4	+51	+52	+43	+43	+25	+27	+22	+07	+51	−05	+35	+40	+40	+42	+57	+15	+72
5	+59	+51	+25	+53	+38	+48	+28	+12	+62	+02	+31	+46	+42	+53	+75	+26	+76
6	+60	+65	+30	+31	+32	+40	+33	+29	+51	+25	+36	+61	+48	+48	+38	+38	+67
7	+59	+48	+18	+31	+10	+30	+31	+28	+45	+09	+09	+59	+45	+46	+46	+29	+67
8	+51	+52	−02	+06	+21	+19	+22	+02	+15	+21	+25	+51	+16	+30	00	+12	+28
9	+21	+28	+28	+42	+13	+11	+08	+16	+43	00	+21	+25	+31	+28	+50	+29	+25
10	+48	+44	+22	+57	+28	+32	+30	+25	+69	+13	+35	+49	+57	+69	+66	+19	+61
11	+62	+39	+32	+40	+21	+35	+36	+28	+46	+13	+23	+40	+43	+49	+76	+25	+77
12	+45	+35	+29	+29	+07	+38	+25	+29	+25	+21	+25	+38	+46	+46	+35	+22	+62
13	+37	+43	+18	+48	+13	+24	+19	+03	+31	+21	+21	+37	+40	+48	+31	+10	+48
14	+46	+31	+28	+31	+29	+30	+21	+48	+37	+34	+31	+51	+34	+45	+30	+31	+43
15	+31	+19	+19	+45	+22	+31	+15	+04	+16	+22	+34	+31	+37	+53	+20	+07	+40
16	+47	+40	+27	+54	+17	+40	+21	+21	+49	+09	+30	+51	+46	+61	+46	+34	+68
17	+37	+37	+10	+03	+04	+18	+08	+25	+03	+48	+05	+18	+08	+11	+11	+07	+28
18	+52	+51	+12	−03	+20	+15	+20	+26	+16	+42	+31	+36	+18	+29	+05	+04	+35
19	+49	+47	+10	+07	+12	+31	+32	+03	+34	+34	+32	+49	+16	+37	+35	+10	+37
20	+28	+48	−04	+03	+16	+05	+15	+06	−04	+59	+11	+37	+02	+21	+12	00	+27
21	+62	+50	+32	+37	+20	+42	+42	+19	+37	+37	+23	+37	+33	+33	+42	−06	+46
22	+28	+64	+33	+19	+32	+33	+23	+10	+25	+53	+20	+42	+18	+28	+03	+10	+22
23	+43	+52	+30	+30	+27	+18	+20	+12	+25	+40	+30	+33	+15	+28	+29	−02	+21
24	+50	+55	+20	+33	+20	+36	+29	+10	+25	+33	+35	+33	+05	+28	+30	+08	+37
25	+57	+40	+20	+28	+10	+30	+23	+16	+19	+15	+02	+15	+24	+28	+15	+10	+60
26	+34	+35	+21	+28	+10	+16	+20	+28	+25	+03	+06	+25	+19	+12	+20	+22	+40
27	+46	+43	+02	+16	+10	+03	−10	+12	+12	+40	+29	+46	+07	+16	+10	00	+11
28	+57	+51	+24	+21	+39	+28	+23	+21	+29	+33	+12	+28	+18	+31	+10	+10	+46
29	+57	+54	+20	+28	+36	+42	+35	+10	+22	+42	+29	+33	+11	+32	+09	+17	+40
30	+47	+50	+26	+35	+41	+30	+17	+32	+17	+50	+05	+49	+18	+32	+16	+17	+35
31	+16	+28	+14	+18	+38	+05	−01	+16	+12	+24	+05	+22	+09	+19	+14	+26	+12
32	+16	+29	+12	+21	+07	+12	+09	+10	+40	+22	+24	+40	+37	+42	+19	+10	+12
33	+10	+38	+11	+20	+30	+04	+02	+16	+29	+30	+04	+22	+26	+26	+12	+24	+10
34	+28	+45	+14	+19	+26	+09	−01	+10	+28	+30	+20	+40	+30	+43	+13	+22	+30
35	+26	+60	+30	+23	+39	+23	+13	+10	+22	+27	+32	+39	+23	+33	+16	+17	+37
36	+31	+27	+04	+12	+04	+08	+11	+19	+06	+19	+04	+11	+20	+28	−03	−06	+24
37	+43	+46	+24	+30	+46	+20	+18	+10	+34	+17	+27	+38	+39	+39	+18	+21	+31
38	+46	+33	+06	+10	+40	+18	+15	+22	+37	+22	+18	+28	+40	+37	+30	+25	+46
39	+54	+50	+32	+22	+37	+23	+17	+19	+34	+29	+13	+29	+46	+40	+38	+06	+52
40	+49	+42	+23	+50	+20	+33	+33	−07	+46	+21	+23	+33	+45	+62	+27	+07	+58
41	+67	+50	+19	+31	+16	+25	+31	+25	+43	+28	+34	+59	+28	+46	+49	+16	+62
42	+48	+51	+32	+43	+37	+38	+23	+06	+48	+07	+13	+41	+52	+64	+24	+16	+62
43	+69	+66	+17	+48	+44	+27	+42	+28	+40	+37	+23	+62	+24	+43	+31	+20	+62
44		+43	+18	+31	+22	+31	+29	+34	+34	+16	+28	+59	+31	+53	+22	+19	+63
45	+43		+23	+37	+20	+30	+13	+03	+38	+39	+33	+54	+33	+43	+33	+11	+43
46	+18	+23		+43	+42	+36	+26	+13	+19	+15	+07	+12	+27	+18	+18	+13	+28
47	+31	+37	+43		+43	+48	+42	+19	+48	+03	+28	+25	+33	+43	+32	+03	+45
48	+22	+20	+42	+43		+20	+36	+25	+34	+26	+08	+13	+13	+25	−10	+16	+38
49	+31	+30	+36	+48	+20		+40	+19	+38	+20	+20	+40	+33	+28	+28	+13	+37
50	+29	+13	+26	+42	+36	+40		+19	+25	+24	+06	+23	+14	+28	+19	−12	+37
51	+34	+03	+13	+19	+25	+19	+19		+31	+12	+12	+28	+10	00	+12	+46	+37
52	+34	+38	+19	+48	+34	+38	+25	+31		−03	+34	+43	+46	+46	+56	+31	+57
53	+16	+39	+15	+03	+26	+20	+24	+12	−03		+21	+37	−08	+28	−22	−10	+09
54	+28	+33	+07	+28	+08	+20	+06	+12	+34	+21		+56	+30	+37	+38	+06	+26
55	+59	+54	+12	+25	+13	+40	+23	+28	+43	+37	+56		+46	+61	+38	+10	+59
56	+31	+33	+27	+33	+13	+33	+14	+10	+46	−08	+30	+46		+63	+60	+17	+62
57	+53	+43	+18	+43	+25	+28	+28	00	+46	+28	+37	+61	+63		+36	+07	+62
58	+22	+33	+18	+32	−10	+28	+19	+12	+56	−22	+38	+38	+60	+36		+32	+82
59	+19	+11	+13	+03	+16	+13	−12	+46	+31	−10	+06	+10	+17	+07	+32		+10
60	+63	+43	+28	+45	+38	+37	+37	+37	+57	+09	+26	+59	+62	+62	+82	+10	

Table 3

The Centroid Matrix. Projections of the Test Vectors on Twelve Arbitrary Orthogonal Axes, Determined by the Centroid Method

Test	*I*	*II*	*III*	*IV*	*V*	*VI*	*VII*	*VIII*	*IX*	*X*	*XI*	*XII*	h_j^2	Rel.
4	.61	.37	.03	.26	.04	.23	.18	−.09	.15	−.02	−.21	.12	.75	.77
5	.71	.39	−.05	.24	.08	.20	.12	.09	.11	.03	−.13	.19	.85	.95
6	.79	.11	−.24	.05	.20	−.20	.22	−.16	−.03	.04	−.12	−.04	.87	.96
7	.67	.26	−.23	−.05	.30	.14	.08	−.03	−.05	−.04	.09	−.08	.71	.93
8	.51	−.29	−.21	−.04	−.16	−.02	.31	−.28	−.15	−.06	.15	.05	.64	.90
9	.36	.44	.13	−.09	.06	−.08	−.03	−.05	.10	−.44	−.10	.19	.61	.90
10	.69	.43	.07	−.15	.08	.12	−.24	−.06	.17	−.07	−.09	.04	.81	.91
11	.71	.36	−.22	.05	.09	.04	−.32	.00	−.08	−.15	−.17	−.07	.86	.84
12	.59	.17	−.05	−.04	−.17	−.02	−.18	.05	−.26	.07	.15	−.07	.55	.95
13	.53	.21	.03	−.30	−.30	.11	−.03	−.05	−.14	−.25	.14	−.07	.63	.68
14	.65	.19	−.13	−.18	.23	−.23	−.02	−.07	.20	.14	.13	−.11	.71	.97
15	.44	.21	.18	−.20	−.30	−.16	−.15	−.15	−.17	−.13	.19	.05	.56	
16	.65	.46	.05	−.03	.02	.09	−.01	−.02	−.07	−.06	.15	−.16	.70	.89
17	.50	−.44	−.26	−.09	.16	.07	−.25	.07	.03	−.19	−.13	−.07	.68	.97
18	.60	−.40	−.31	−.20	−.06	−.13	−.01	−.06	−.15	.05	−.13	−.18	.75	.86
19	.60	−.27	−.28	−.01	−.12	.21	.15	.29	.02	.18	−.13	.09	.73	.98
20	.49	−.53	−.30	−.30	−.13	.08	.02	.18	.06	.11	−.06	−.10	.79	.96
21	.74	−.23	−.32	.08	−.17	.00	−.24	.15	−.11	−.16	−.07	.19	.90	.97
22	.61	−.38	−.23	−.07	−.24	−.14	.14	.07	.24	.12	−.04	−.09	.76	.98
23	.59	−.29	−.15	−.05	−.20	−.16	.06	.14	.17	−.17	−.18	−.01	.64	.94
24	.64	−.33	−.18	.20	−.26	.15	.14	.22	−.05	−.07	−.07	.12	.78	.94
25	.56	−.16	−.17	.29	−.15	.20	−.12	−.03	−.24	−.22	−.03	−.19	.67	.91
26	.44	.08	−.41	.10	.21	−.31	.18	.10	−.11	−.17	−.06	−.20	.65	.98
27	.47	−.42	−.07	−.31	−.01	−.13	.09	−.11	−.03	−.13	−.20	.09	.60	.98
28	.63	−.29	−.16	.16	.10	−.14	−.10	−.15	−.08	−.15	.08	.12	.64	.88
29	.60	−.31	.06	.22	−.13	.02	−.12	−.18	.04	.05	−.07	.19	.62	.94
30	.70	−.35	.16	−.15	.22	.11	−.08	.18	.09	−.07	.30	−.15	.89	.95
31	.38	−.26	.45	−.25	.22	−.04	.12	.19	−.04	−.06	.29	−.10	.68	.85
32	.46	−.01	.46	−.38	.13	.05	.06	.24	−.04	.17	.13	.06	.70	.95

Table 3—Continued

Test	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	h_j^2	Rel.
33	.43	−.25	.50	−.26	.24	−.19	−.14	.21	−.19	.11	.13	.07	.79	.73
34	.52	−.24	.45	−.10	.18	−.06	.06	−.03	−.22	.18	−.11	−.09	.68	.86
35	.60	−.22	.36	.16	.00	.03	.10	−.13	−.11	−.03	.08	.09	.62	.96
36	.33	−.11	−.11	−.02	.14	.32	−.19	−.05	.29	.17	−.08	−.18	.45	.72
37	.64	−.18	.28	.14	.05	.02	.21	−.24	.09	.00	.14	.06	.68	.80
38	.52	−.19	.32	.18	.22	−.06	−.10	.02	−.18	.26	−.22	.04	.65	.70
39	.70	−.31	.29	.17	.14	.10	−.26	−.10	.07	.04	−.15	−.09	.84	.69
40	.66	.13	.14	.22	−.28	.36	.17	−.03	.03	.12	.03	−.19	.81	.82
41	.77	.07	−.20	−.10	.21	.31	.11	−.10	.15	−.15	.13	.03	.87	.97
42	.63	.19	.17	.20	−.23	.15	.20	−.12	.08	.10	−.06	−.35	.77	.50
43	.81	−.14	−.14	.10	.15	.13	.15	.01	.08	.05	.15	.06	.80	.90
44	.73	−.01	−.24	.16	.06	.11	−.01	−.25	−.24	−.02	.00	.00	.75	.58
45	.73	−.18	.02	−.09	−.08	.03	.23	−.10	.06	−.10	−.16	.07	.69	.97
46	.38	.13	.11	.21	−.13	−.22	−.12	.08	.23	−.18	.08	−.15	.42	.91
47	.53	.34	.17	.15	−.25	−.05	−.02	.11	.20	−.21	.18	.06	.65	.89
48	.42	−.10	.26	.36	.02	−.31	−.04	−.06	.29	.09	.20	.03	.62	.68
49	.47	.19	−.08	.20	−.27	−.10	−.09	.17	.00	.10	.16	.09	.47	.88
50	.38	.10	−.17	.26	−.19	.00	−.21	.09	.12	.18	.29	.18	.50	.66
51	.32	.09	−.16	.04	.40	−.35	−.19	.08	.03	.08	.13	−.11	.50	.77
52	.58	.39	.14	.13	.07	−.11	.10	.13	.08	.13	−.18	.22	.67	
53	.40	−.42	−.09	−.28	−.15	−.10	−.17	−.01	.23	.17	.19	.04	.60	.98
54	.41	.13	.02	−.19	−.17	−.12	.14	−.13	.00	.08	−.15	.38	.47	.88
55	.68	.13	−.12	−.32	.00	−.01	.28	−.18	−.03	.24	.08	.19	.81	.95
56	.54	.40	.21	−.12	−.07	.00	−.15	−.08	−.12	.19	−.21	−.14	.66	.90
57	.67	.26	.18	−.18	−.20	.13	−.04	−.24	−.05	.23	−.05	−.04	.76	.88
58	.51	.51	−.02	−.02	.12	.21	.03	.36	−.16	−.10	−.42	.23	.97	.96
59	.27	.17	.12	.08	.33	−.28	.18	.15	−.12	−.04	−.03	−.17	.41	
60	.77	.38	−.11	.19	.07	.20	−.21	−.14	−.18	.14	−.05	−.08	.96	.84

Table 4

Rotated Orthogonal Factorial Matrix

Test	*S*	*P*	*N*	*V*	*M*	*W*	*I*	*R*	*D*	*10*	*11*	*12*	*13*
4	.002	.169	−.130	.552	.208	−.007	.306	.330	.148	.081	−.126	.340	−.124
5	.023	.283	−.061	.506	.253	.082	.204	.351	.099	.100	−.089	.502	−.146
6	.411	.537	.109	.301	.181	.119	.306	.269	.260	.058	−.098	.133	−.035
7	.115	.573	.083	.456	.011	.128	.045	.231	.171	.125	.087	.183	.055
8	.393	.196	.080	.054	−.052	.058	.405	−.096	.398	.118	.269	.130	.122
9	−.016	.142	.003	.450	.160	.072	.070	−.047	−.041	.480	−.334	.087	.069
10	.073	.194	.048	.635	.225	.327	.029	.308	−.054	.316	−.025	.105	.069
11	.136	.422	−.119	.333	.165	.291	.001	.481	.048	.428	−.029	.124	−.159
12	.063	.219	.058	.102	.101	.512	.030	.216	.255	.232	.122	.183	−.092
13	.124	.030	.037	.366	−.039	.388	−.023	−.028	.342	.436	.093	.110	−.059
14	.301	.461	.224	.395	.321	.279	.005	.112	.066	−.015	.041	−.028	.204
15	.042	−.002	.092	.182	.127	.534	.140	−.101	.204	.378	−.011	.003	−.070
16	−.087	.332	.041	.495	.147	.365	.007	.190	.308	.209	−.023	.133	−.051
17	.413	.208	.218	−.014	.029	−.134	−.020	.364	−.015	.371	.354	.017	.106
18	.626	.211	.199	−.065	−.034	.145	.099	.291	.272	.158	.247	−.009	−.036
19	.448	.067	.120	.083	.034	−.001	.020	.335	.191	−.006	.281	.530	.103
20	.636	−.002	.261	−.018	−.035	.014	−.110	.216	.179	.061	.434	.178	−.192
21	.415	.214	.034	−.057	.190	.156	.129	.329	.095	.501	.281	.397	.121
22	.633	.018	.162	.052	.316	.024	.067	.150	.307	.004	.281	.174	−.027
23	.551	.002	.157	.053	.282	−.059	.041	.180	.241	.294	.086	.195	−.066
24	.336	.020	.074	−.028	.145	−.067	.184	.275	.340	.241	.270	.527	.098
25	.069	.188	−.095	−.028	.044	.020	.167	.414	.403	.402	.286	.142	.111
26	.324	.603	.005	−.017	.125	−.057	−.049	.147	.291	.131	−.175	.074	−.187
27	.584	.011	.332	.052	−.073	−.018	.222	.083	.088	.260	.094	.011	.082
28	.270	.359	.179	−.045	.217	.027	.368	.173	.103	.341	.248	.098	.107
29	.223	−.024	.125	.030	.276	.077	.477	.295	.071	.209	.278	.167	.031
30	.109	.205	.625	.228	.194	.025	−.035	.200	.211	.230	.448	.100	−.047
31	−.063	.029	.755	.144	.049	.040	−.016	−.039	.218	.099	.126	.024	−.033
32	−.010	−.072	.670	.323	.001	.298	−.035	.075	.060	.005	.014	.203	.149

Table 4—Continued

Test	*S*	*P*	*N*	*V*	*M*	*W*	*I*	*R*	*D*	*10*	*11*	*12*	*13*
33	−.021	.009	.812	−.016	.068	.284	.071	.140	−.026	.137	.020	.049	−.095
34	.060	−.016	.619	.094	−.020	.179	.299	.352	.196	.010	−.026	−.019	−.190
35	−.027	.008	.392	.132	.166	.073	.479	.180	.275	.189	.134	.150	−.149
36	.140	.053	.020	.314	.101	−.060	−.055	.393	−.065	−.070	.377	−.047	.129
37	.059	.087	.348	.296	.258	.003	.503	.091	.287	.078	.181	.095	.205
38	.012	.061	.432	−.021	.128	.136	.358	.534	.017	−.007	−.034	.103	−.025
39	.077	.009	.383	.173	.281	.027	.331	.583	.080	.234	.262	−.030	.050
40	−.046	−.053	−.035	.420	.182	.166	.202	.356	.525	.011	.212	.292	.111
41	.215	.417	.122	.597	.056	−.013	.138	.161	.159	.232	.313	.242	.052
42	.018	−.007	−.027	.424	.261	.159	.190	.358	.578	−.027	.035	.071	.075
43	.241	.380	.248	.304	.214	.009	.251	.232	.233	.083	.326	.344	−.170
44	.197	.435	−.029	.179	.007	.182	.392	.341	.254	.236	.217	.160	.204
45	.430	.048	.241	.324	.101	.004	.325	.192	.290	.224	.076	.215	−.203
46	−.009	.069	.024	.117	.529	.066	−.030	.096	.217	.242	−.043	−.006	−.128
47	−.092	.036	−.009	.350	.487	.206	.033	−.014	.250	.321	−.046	.270	−.086
48	−.040	.085	.243	.024	.664	.032	.300	.060	.107	−.014	.076	.007	−.262
49	.040	.126	−.080	.035	.381	.339	.035	.102	.182	.110	.067	.355	.109
50	−.012	.162	−.142	.015	.420	.280	.069	.058	−.004	.036	.296	.330	−.072
51	.085	.545	.208	−.020	.288	.135	−.090	.147	−.097	.004	−.037	−.096	.176
52	.050	.169	.110	.357	.320	.214	.187	.255	.016	.019	−.323	.385	.172
53	.455	−.059	.289	.019	.235	.196	.009	−.020	−.019	.046	.467	.005	−.116
54	.336	−.036	.055	.252	.051	.281	.319	−.053	−.039	.090	−.163	.268	.037
55	.412	.266	.191	.453	−.026	.375	.285	−.027	.149	−.077	.066	.268	.035
56	.032	.036	.065	.386	.081	.508	.082	.410	.138	.083	−.195	.002	.113
57	.135	−.023	.074	.498	.069	.530	.257	.272	.218	.060	.063	.074	.015
58	.013	.178	−.018	.395	−.080	.158	−.084	.457	−.079	.292	−.391	.545	−.115
59	−.052	.360	.295	.056	.121	−.004	−.014	.136	.192	−.017	−.344	.005	.135
60	−.034	.412	−.121	.385	.120	.413	.220	.545	.163	.168	.096	.184	−.274

INDEX

INDEX

PRINTED IN U.S.A.