

Visuospatial Reasoning

Barbara Tversky

Visuospatial reasoning is not simply a matter of running to retrieve a fly ball or wending a way through a crowd or plotting a path to a destination or stacking suitcases in a car trunk. It is a matter of determining whether gears will mesh (Schwartz & Black, 1996a), understanding how a car brake works (Heiser & Tversky, 2002), discovering how to destroy a tumor without destroying healthy tissue (Duncker, 1945; Gick & Holyoak, 1980, 1983), and designing a museum (Suwa & Tversky, 1997). Perhaps more surprising, it is also a matter of deciding whether a giraffe is more intelligent than a tiger (Banks & Flora, 1977; Paivio, 1978), whether one event is later than another (Boroditsky, 2000), and whether a conclusion follows logically from its' premises (Barwise & Etchemendy, 1995; Johnson-Laird, 1983). All these abstract inferences, and more, appear to be based on spatial reasoning. Why is that? People begin to acquire knowledge about space and the things in it probably before they enter the world. Indeed, spatial knowledge is critical to survival and spatial inference critical to effective survival. Perhaps because of the (literal) ubiq-

uity of spatial reasoning, perhaps because of the naturalness of mapping abstract elements and relations to spatial ones, spatial reasoning serves as a basis for abstract knowledge and inference. The prevalence of spatial figures of speech in everyday talk attests to that: We feel close to some people and remote from others; we try to keep our spirits up, to perform at the peak of our powers, to avoid falling into depressions, pits, or quagmires; we enter fields that are wide open, struggling to stay on top of things and not get out of depth. Right now, in this section, we establish fuzzy boundaries for the current field of inquiry.

Reasoning

Before the research, a few words about the words. The core of *reasoning* seems to be, as Bruner put it years ago, going beyond the information given (Bruner, 1973). Of course, nearly every human activity requires going beyond the information given. The simplest recognition or generalization task, as well as the simplest action, require going beyond

the information given, as according to a far more ancient saying, you never step into the same river twice. Yet many of these tasks and actions do not feel cognitive, do not feel like reasoning. However, the border between perceptual and cognitive processes may be harder to establish than the borders between countries in conflict. Fortunately, psychology is typically free of territorial politics, so establishing boundaries between perception and cognition is not essential. There seems to be a tacit understanding as to what counts as perceptual and what as cognitive, although for these categories just as for simpler ones, such as chairs and cups, the centers of the category enjoy more consensus than the borders. Invoking principles or requirements for the boundaries between perception and cognition – consciousness, for example – seems to entail more controversy than the separation into territories.

How do we go beyond the information given? Going beyond the information given does not necessarily mean adding information. One way to go beyond the information given is to transform the information given. This is the concern of the earlier part of the manuscript. Going beyond the information given can also mean transforming the given information, sometimes according rules, as in deductive reasoning. Another way to go beyond the information given is to make inferences or judgments from it. Inference and judgment are the concerns of the later part of the manuscript. Now some more distinctions regarding the *visuospatial* portion of the title.

Representations and Transformations

Truths are hard to come by in science, but useful fictions, approximate truths, abound. One of these is the distinction between representations and transformations, between information and processes, between data and the operations performed on data. Representations place limits on transformations as they select and structure the information captured from the world or the mind. Distinguishing representations and transformations, even under direct observation of

the brain, is another distinction fraught with complexity and controversy. Evidence brought to bear for one can frequently be reinterpreted as evidence for the other (e.g., Anderson, 1978). Both representations and transformations themselves can each be decomposed into representations and transformations. Despite these complications, the distinction has been a productive way to think about psychological processes. In fact, it is a distinction that runs deep in human cognition, captured in language as subject and predicate and in behavior as agent/object and action. The distinction will prove useful here, more than as a way of organizing the literature (for related discussion, see Doumas & Hummel, Chap. 4).

It has been argued that the very establishment of representations entails inferential operations. A significant example are the Gestalt principles of perceptual organization – grouping by similarity, proximity, common fate, and good continuity – that contribute to scene segmentation and representation. These are surely a form of visuospatial inference. Representations are internal translations of external stimuli (or internal data); as such, they not only eliminate information from the external world, they also add to it and distort it in the service of interpretation or behavior. Thus, if inference is to be understood in terms of operating on or manipulating information to draw new conclusions, then it begins in the periphery of the sensory systems with leveling and sharpening and feature detection and organization. Nevertheless, the field has accepted a level of description of representations and transformations, one higher than the levels of sensory and perceptual processing; that level is reflected here.

Visuospatial

What makes visuospatial representations visuospatial? Visuospatial transformations visuospatial? First and foremost, visuospatial representations capture visuospatial properties of the world. They do this in a way

that preserves, at least in part, the spatial structural relations of that information (see Johnson-Laird, 1983; Pierce in Houser & Kloesel, 1992). This means that visuospatial properties that are close or above or below in the world preserve those relations in the representations. Visual includes static properties of objects, such as shape, texture, and color, or between objects and reference frames, such as distance and direction. It also includes dynamic properties of objects such as direction, path, and manner of movement. By this account, visuospatial transformations are those that change or use visuospatial information. Many of these properties of static and dynamic objects and of spatial relations between objects are available from modalities other than vision. This may explain why well-adapted visually impaired individuals are not disadvantaged at many spatial tasks (e.g., Klatzky, Golledge, Cicinelli, & Pellegrino, 1995). Visuospatial representations are regarded as contrasting with other forms of representation, notably linguistic. The similarities (e.g., Talmy, 1983, 2001) and differences between visuospatial and linguistic representations provide insights into both.

Demonstrating properties of internal representations and transformations is tricky for another reason; representations are many steps from either (controlled) input or (observed) output. For these reasons, the study of internal representations and processes was eschewed not only by behaviorists, but also by experimentalists. It was one of the first areas to flourish after the so-called Cognitive Revolution of the 1960s, with a flurry of innovative techniques to demonstrate form and content of internal representations and the transformations performed on them. It is that research that we now turn.

Representations and Transformations

Visuospatial reasoning can be approached bottom-up by studying the elementary representations and processes that presumably form the building blocks for more complex reasoning. It can also be approached

top-down by studying complex reasoning that has a visuospatial basis. Both approaches have been productive. We begin with elements.

Imagery as Internalized Perception

The major research tradition studying visuospatial reasoning from a bottom-up perspective has been the *imagery* program, pioneered by Shepard (see Finke & Shepard, 1986; Shepard & Cooper, 1982; Shepard & Podgorny, 1978, for overviews) and Kosslyn (1980, 1994b), which has aimed to demonstrate parallels between visual perception and visual imagery. There are two basic tenets of the approach, one regarding representations and the other regarding operations on representations: that mental images resemble percepts, and that mental transformations on images resemble observable changes in things in the world, as in mental rotation, or perceptual processes performed on things in the world, as in mental scanning. Kosslyn (1994b) has persisted in these aims, more recently demonstrating that many of the same neural structures are used for both. Not the demonstrations per se, but the interpretations of them have met with controversy (e.g., Pylyshyn, 1978, 1981). In attempting to demonstrate the similarities between imagery and perception, the imagery program has focused both on properties of objects and on characteristics of transformations on objects – the former, representations, and the latter, operations or transformations. The thrust of the research programs has been to demonstrate that images are like internalized perceptions and transformations of images like transformations of things in the world.

REPRESENTATIONS

In the service of demonstrating that images preserve characteristics of perceptions, Shepard and his colleagues brought evidence from similarity judgments as support. They demonstrated “second-order isomorphisms,” similarity spaces for perceived and imagined stimuli that have the same structure, that is, are fit by the

same underlying multidimensional space (Shepard & Chipman, 1970). For example, similarity judgments of shapes of cutouts of states conform to the same multidimensional space as similarity judgments of imagined shapes of states. The same logic was used to show that color is preserved in images, as well as configurations of faces (see Gordon & Hayward, 1973; Shepard, 1975). Similar reasoning was used to demonstrate qualitative differences between pictorial and verbal representations in a task requiring sequential same–different judgments on pairs of schematic faces and names (Tversky, 1969). The pictorial and verbal similarity of the set of faces was orthogonal so the “different” responses were a clue to the underlying representation; times to respond “different” are faster when more features between the pairs differ. These times indicated that when participants expected the target (second) stimulus would be a picture, they encoded the first stimulus pictorially, whether it had been a picture of a face or its name. The converse also held: When the target stimulus was expected to be a name, participants coded the first stimulus verbally irrespective of its presented modality.

To demonstrate that mental images preserve properties of percepts, Kosslyn and his colleagues presented evidence from studies of reaction times to detect features of imagined objects. One aim is to show that properties that take longer to verify in percepts take longer to identify in images. For example, when participants were instructed to construct images of named animals in order to judge whether the animal had a particular part, they verified large parts of animals, such as the back of a rabbit, faster than small but highly associated ones, such as the whiskers of a rat. When participants were not instructed to use imagery to make judgments, they verified small associated parts faster than large ones. When not instructed to use imagery, participants used their general world knowledge to make judgments (Kosslyn, 1976). Importantly, when the participants explicitly used imagery, they took longer to verify parts, large or small, than when they relied on world knowledge.

Additional support for the claim that images preserve properties of percepts comes from tasks requiring construction of images. Constructing images takes longer when there are more parts to the image, even when the same figure can be constructed from more or fewer parts (Kosslyn, 1980).

The imagery-as-internalized-perception has proved to be too narrow a view of the variety of visuospatial representations. In accounting for syllogistic reasoning, Johnson-Laird (1983) proposed that people form mental models of the situations described by the propositions (see Johnson-Laird, Chap. 9). Mental models contrast with classic images in that they are more schematic than classical images. Entities are represented as tokens, not as likenesses, and spatial relations are approximate, almost qualitative. A similar view was developed to account for understanding text and discourse, that listeners and readers construct schematic models of the situations described (e.g., Kintsch & van Dijk, 1983; Zwaan & Radvansky, 1998). As is seen, visuospatial mental representations of environments, devices, and processes are often schematic, even distorted, rather than detailed and accurate internalized perceptions.

TRANSFORMATIONS

Here, the logic is the same for most research programs, and in the spirit of Shepard's notion of second-order isomorphisms: to demonstrate that the times to make particular visuospatial judgments in memory increase with the times to observe or perform the transformations in the world. The dramatic first demonstration was mental rotation (Shepard & Metzler, 1971): time to judge whether two figures in different orientations (Figure 10.1) are the same or mirror images correlates linearly with the angular distance between the orientations of the figures. The linearity of the relationship – 12 points on a straight line – suggests smooth continuous mental transformation. Although linear functions have been obtained for the original stimuli, strings of 10 cubes with two bends, monotonic, but

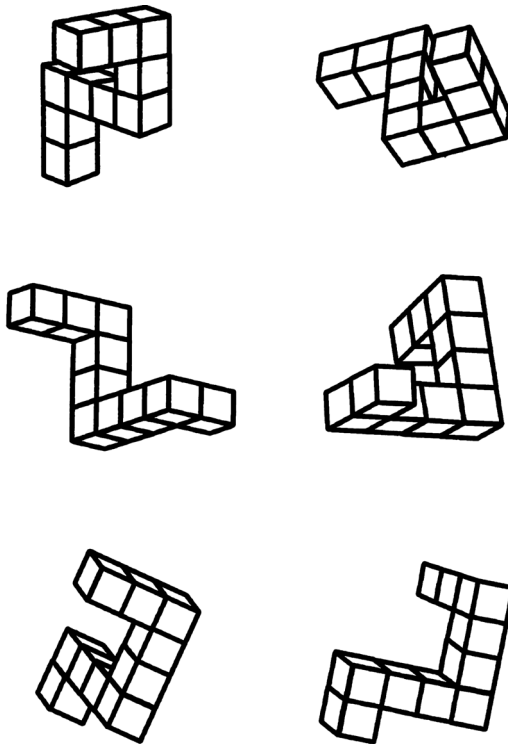


Figure 10.1. Mental rotation task of Shepard and Metzler (1971). Participants determine whether members of each pair can be rotated into congruence.

not linear functions are obtained for other stimuli, such as letters (Shepard & Cooper, 1982). There are myriad possible mental transformations, only a few of which have been studied in detail. They may be classified into mental transformations on other objects and individuals, and mental transformations on one's self. In both cases, the transformations may be global, wholistic, of the entire entity, or the transformations may be operations on parts of entities.

Mental Transformations on Objects. Rotation is not the only transformation that objects in the world undergo. They can undergo changes of size, shape, color, internal features, position, combination, and more. Mental performance of some of these transformations has been examined. The time to mentally compare the shapes of two rectangles differing in size increases as the actual size difference between them in-

creases (Bundesen, Larsen, & Farrell, 1981; Moyer, 1973). New objects can be constructed in imagery, a skill presumably related to design and creativity (e.g., Finke, 1990, 1993). In a well-known example, Finke, Pinker, and Farah (1989) asked students to imagine a capital letter J centered under an upside-down grapefruit half. Students reported "seeing" an umbrella. Even without instructions to image, certain tasks spontaneously encourage formation of visual images. For example, when participants are asked whether a described spatial array, such as *star above plus*, matches a depicted one, response times indicate that they transform the description into a depiction when given sufficient time to mentally construct the situation (Glushko & Cooper, 1978; Tversky, 1975).

In the cases of mental rotation, mental movement, and mental size transformations, objects or object parts undergo imagined transformations. There is also evidence that objects can be mentally scanned in a continuous manner. In a popular task introduced by Kosslyn and his colleagues, participants memorize a map of an island with several landmarks, such as a well and a cave. Participants are then asked to conjure an image of the map and to imagine looking first at the well, and then mentally scanning from the well to the cave. The general finding is that mental scanning between two imagined landmarks increases linearly as the distance between them increases (Denis & Kosslyn, 1999; Kosslyn, Ball, & Rieser, 1978; Figure 10.2). The phenomenon holds for spatial arrays established by description rather than depiction, again, under instructions to form and use images (Denis, 1996). Mental scanning occurs for arrays in depth and for flat perspectives on 3D arrays (Pinker, 1980). In the previous studies, participants were trained to mentally scan, and directed to do so, leaving open the question of whether it occurs spontaneously. It seems to be in a task requiring direction judgments on remembered arrays. Participants first saw an array of dots. After the dots disappeared, an arrow appeared on the screen. The task was to say whether the arrow pointed to

the previous location of a dot. Reaction times increased with distance of the arrow to the likely dot, suggesting that participants mentally scan from the arrow to answer the question (Finke & Pinker, 1982, 1983). Mental scanning may be part of catching or hitting the ball in baseball, tennis, and other sports.

Applying Several Mental Transformations.

Other mental transformations on objects are possible, for example, altering the internal configuration of an object. To solve some problems, such as geometric analogies, people need to apply more than one mental transformation to a figure to obtain the answer. In most cases, the order of applying the transformations is optional; that is, first rotating and then moving a figure yields the same answer as first moving and then rotating. Nevertheless, people have a preferred order for performing a sequence of mental transformations, and when this order is violated, both errors and performance time increase (Novick & Tversky, 1987). What accounts for the preferred order? Although the mental transformations are performed in working memory, the determinants of order do not seem to be related to working memory demands. *Move* is one of the least demanding transformations, and it is typically performed first, whereas *rotate* is one of the most difficult transformations and is performed second. Then transformations of intermediate difficulty are performed. What correlated with the order of applying successive mental transformations is the order of drawing. *Move* determines where the pencil is to be put on the paper, the first act of drawing. *Rotate* determines the direction in which the first stroke should be taken, and it is the next transformation. The next transformations to be applied are those that determine the size of the figure and its internal details (*remove, add part, change size, change shading, add part*). Although the mental transformations have been tied to perceptual processes, the ordering of performing them appears to be tied to a motor process, the act of drawing or constructing a figure. This finding presaged later work showing that complex

visuospatial reasoning has not only perceptual, but also motor, foundations.

Mental Transformations of Self. That mental imagery is both perceptual and motor follows from broadening the basic tenets of the classical account for imagery. According to that account, mental processes are internalizations of external or externally driven processes, perceptual ones according to the classic view (e.g., in the chapter title of Shepard & Podgorny, 1978, "Cognitive processes that resemble perceptual processes"). The acts of drawing a figure or constructing an object entail both perceptual and motor processes working in concert, as do many other activities performed in both real and virtual worlds, from shaking hands to wayfinding.

Evidence for mental transformations of self, or motor imagery, rather than or in addition to visual imagery has come from a variety of tasks. The time taken to judge whether a depicted hand is right or left correlates with the time taken to move the hand into the depicted orientation, as if participants were mentally moving their hands in order to make the right/left decision (Parsons, 1987b; Sekiyama, 1982). Mental reorientation of one's body has been used to account for reaction times to judge whether a left or right arm is extended in pictures of bodies in varying orientations from upright (Parsons, 1987a). In those studies, reaction times depend on the angle of rotation and the degree of rotation. For some orientations, notably the picture plane, the degree of rotation from upright has no effect. This allows dissociating mental transformations of other, in this case, mental rotation, from mental transformations of self, in this case, perspective transformations, as the latter do yield increases in reaction times with degree of rotation from upright (Zacks, Mires, Tversky, & Hazeltine, 2000; Zacks & Tversky, in press). Imagining one's self interacting with a familiar object such as a ball or a razor, selectively activates left inferior parietal and sensorymotor cortex, whereas imagining another interacting with the same objects selectively activates right inferior parietal,



Figure 10.2. Mental scanning. Participants memorize map and report time to mentally scan from one feature to another (after Kosslyn, Ball, & Rieser, 1978).

precuneus, posterior cingulate, and frontopolar cortex (Ruby & Decety, 2001).

There have been claims that visual and motor imagery, or as we have put it, mental transformations of object and of self, share the same underlying mechanisms (Wexler, Kosslyn, & Berthoz, 1998; Wolschlagel & Wolschlagel, 1998). For example, performing clockwise physical rotations facilitates performing clockwise mental rotations, but interferes with performing counterclockwise mental rotations. However, this may be because planning, performing, and monitoring the physical rotation requires both perceptual and motor imagery. The work of Zacks and collaborators (Zacks et al., 2000; Zacks & Tversky, in press) and Ruby and Decety (2001) suggests that these two classes of mental transformations are dissociable. Other studies directly comparing the two systems supports their dissociability: The consequences of using one can be different from the consequences of using the other (Schwartz, 1999; Schwartz & Black, 1999; Schwartz & Holton, 2000). When people imagine wide and narrow glasses filled to the

same level, and are asked which would spill first when tilted, they are typically incorrect from visual imagery. However, if they close their eyes and imagine tilting each glass until it spills, they correctly tilt a wide glass less than a narrow one (Schwartz & Black, 1999). Think of turning a car versus turning a boat. To imagine making a car turn right, you must imagine rotating the steering wheel to the right; however, to imagine making a boat turn right, you must imagine moving the rudder lever left. In mental rotation of left and right hands, the shortest motor path accounts for the reaction times better than the shortest visual path (Parsons, 1987b). Mental enactment also facilitates memory, even for actions described verbally (Englekamp, 1998). Imagined motor transformations presumably underlie mental practice of athletic and musical routines, techniques known to benefit performance (e.g., Richardson, 1967).

The reasonable conclusion, then, is that both internalized perceptual transformations and internalized motor transformations can serve as bases for transformations in mental imagery. Perceptual and motor imagery can work in concert in imagery, just as perceptual and motor processes work in concert in conducting the activities of life.

ELEMENTARY TRANSFORMATIONS

The imagery-as-internalized-perception approach has provided evidence for myriad mental transformations. We have reviewed evidence for a number of mental perceptual transformations: scanning, change orientation, location, size, shape, color, construct from parts, and rearrange parts. Then we have motor transformations: motions of bodies, wholes, or parts. This approach has the potential to provide a catalog of elementary mental transformations that are simple inferences and that can combine to enable complex inferences.

The work on inference, judgment, and problem solving will suggest transformations that have yet to be explored in detail. Here, we propose a partial catalog of candidates for elementary properties of representations

and transformations, expanding from the research reviewed:

- *Determining static properties of entities:* figure/ground, symmetry, shape, internal configuration, size, color, texture, and more
- *Determining relations between static entities:*
 - With respect to a frame of reference: location, direction, distance, and more
 - With respect to other entities, comparing size, color, shape, texture, location, orientation, similarity, and other attributes
- *Determining relations of dynamic and static entities:*
 - With respect to other entities or to a reference frame: direction, speed, acceleration, manner, intersection/collision
- *Performing transformations on entities:* change location (scanning), change perspective, orientation, size, shape; moving wholes, reconfiguring parts, zooming, enacting
- *Performing transformations on self:* change of perspective, change of location, change of size, shape, reconfiguring parts, enacting

INDIVIDUAL DIFFERENCES

Yes, people vary in spatial ability. However, spatial ability does not contrast with verbal ability; in other words, someone can be good or poor at both, as well as good in one and poor in the other. In addition, spatial ability (like verbal ability) is not a single, unitary ability. Some of the separate spatial abilities differ qualitatively; that is, they map well onto the kinds of mental transformations they require. A meta-analysis of a number of factor analyses of spatial abilities yielded three recurring factors (Linn & Peterson, 1986): spatial perception, spatial visualization, and mental rotation. Rod-and-frame and water-level tasks load high on spatial perception; this factor seems to reflect choice of frame of reference, within an object or extrinsic. Performance on embedded

figures, finding simple figures in more complex ones, loads high on spatial visualization, and performance on mental rotation tasks naturally loads high on the mental rotation factor. As frequently as they are found, these three abilities do not span the range of spatial competencies. Yet another partially independent visuospatial ability is visuospatial memory, remembering the layout of display (e.g., Betrancourt & Tversky, in press). The number of distinct spatial abilities as well as their distinctness remain controversial (e.g., Carroll, 1993; Hegarty & Waller, in press).

More recent work explores the relations of spatial abilities to the kinds of mental transformations that have been distinguished, for example, imagining an object rotate versus imagining changing one's own orientation. The mental transformations, in turn, are often associated with different brain regions (e.g., Zacks, Mires, Tversky, & Hazeltine, 2000; Zacks, Ollinger, Sheridan, & Tversky, 2002; Zacks & Tversky, in press). Kozhevnikov, Kosslyn, and Shepard (in press) proposed that spatial visualization and mental rotation correspond respectively to the two major visual pathways in the brain – the ventral “what” pathway underlying object recognition and the dorsal “where” pathway underlying spatial location. Interestingly, scientists and engineers score relatively high on mental rotation and artists score relatively high on spatial visualization. Similarly, architects and designers score higher than average on embedded figure tasks but not on mental rotation (Suwa & Tversky, 2003). Associating spatial ability measures to mental transformations and brain regions are promising directions toward a systematic account of spatial abilities.

Inferences

Inferences from Observing Motion in Space

To ensure effective survival, in addition to perceiving the world as it is we need to also anticipate the world that will be. This

entails inference, inferences from visuospatial information. Some common inferences, such as determining where to intersect a flying object, in particular, a fly ball (e.g., McBeath, Shaffer, & Kaiser, 1995), or what moving parts belong to the same object (e.g., Spelke, Vishton, & von Hofsten, 1995) are beyond the scope of the chapter. From simple, abstract motions of geometric figures, people, even babies, infer causal impact and basic ontological categories, notably, inanimate and animate. A striking demonstration of perception of causality comes from the work of Michotte (1946/1963; see Buehner & Cheng, Chap. 7). Participants watch films of a moving object, A, coming into contact with a stationary object, B. When object B moves immediately, continuing the direction of motion suggested by object A, people perceive A as launching B, A as causing B to move. When A stops so both A and B are stationary before B begins to move, the perception of a causal connection between A's motion and B's is lost; their movements are seen as independent events. This is a forceful demonstration of immediate perception of causality from highly abstract actions, as well as of the conditions for perception of causality. What seems to underlie the perception of causality is the perception that object A *acts on* object B. Actions on objects turn out to be the basis for segmenting events into parts (Zacks, Tversky, & Iyer, 2001).

In Michotte's (1946/1963) demonstrations, the timing of the contact between the initially moving object and the stationary object that begins to move later is critical. If A stops moving considerably before B begins to move, then B's motion is perceived to be independent of A's. B's movement in this case is seen as self-propelled. Self-propelled movement is possible only for animate agents, or, more recently in the history of humanity, for machines. Possible paths and trajectories of animate motion differ from those for inanimate motion. Preschool children can infer which motion paths are appropriate for animate and inanimate motion, and even for abstract stimuli; they also offer sensible explanations for their inferences (Gelman, Durgin, & Kaufman, 1995).

From abstract motion paths, adults can make further inferences about what generated the motion. In point-light films, the only thing visible is the movement of lights placed at motion junctures of, for example, at the joints of people walking or along branches of bushes swaying. From point-light films, people can determine whether the motion is walking, running, or dancing, of men or of women, of friends (Cutting & Kozlowski, 1977; Johansson, 1973; Kozlowski & Cutting, 1977), of bushes or trees (Cutting, 1986). Surprisingly, from point-light displays of action, people are better at recognizing their own movements than those of friends, suggesting that motor experience contributes to perception of motion (Prasad, Loula, & Shiffrar, 2003). Even abstract films of movements of geometric figures in sparse environments can be interpreted as complex social interactions, such as chasing and bullying, when they are especially designed for that (Heider & Simmel, 1944; Martin & Tversky, 2003; Oatley & Yuill, 1985) or playing hide-and-seek, but interpreting these as intentional actions is not immediate; rather, it requires repeated exposure and possibly instructions to interpret the actions (Martin & Tversky, 2003).

Altogether, simply from abstract motion paths or animated point-light displays, people can infer several basic ontological categories: causal action, animate versus inanimate motion, human motion, motion of males or females and familiar individuals, and social interactions.

Mental Spatial Inferences

INFERENCES IN REAL ENVIRONMENTS

Every kid who has figured out a short-cut, and who has not, has performed a spatial inference (for a more recent overview of kids, see Newcombe & Huttenlocher, 2000). Some of these inferences turn out to be easier than others, often surprisingly. For example, in real environments, inferences about where objects will be in relationship to one's self after imagined movement in the environment turn out to be relatively accurate when the imagined movement is a

translation, that is, movement forward or backward aligned with the body. However, if the imagined movement is rotational, a change in orientation, updating is far less accurate (e.g., Presson & Montello, 1994; Reiser, 1989). When asked to imagine walking forward a certain distance, turning, walking forward another distance, and then pointing back to the starting point, participants invariably err by not taking into account the turn in their pointing (Klatzky, Loomis, Beall, Chance, & Golledge, 1998). If they actually move forward, turn, and continue forward, but blindfolded, they point correctly. Spatial updating in real environments is more accurate after translation than after rotation, and updating after rotation is selectively facilitated by physical rotation. This suggests a deep point about spatial inferences and possibly other inferences, that in inference, mental acts interact with physical acts.

GESTURE

Interaction of mind and body in inference is also revealed in gesture. When people describe space but are asked to sit on their hands to prevent gesturing, their speech falters (Rauscher, Krauss, & Chen, 1996), suggesting that the acts of gesturing promote spatial reasoning. Even blind children gesture as they describe spatial layouts (Iverson & Goldin-Meadow, 1997).

The nature of spontaneous gestures suggests how this happens. When describing continuous processes, people make smooth, continuous gestures; when describing discrete ones, people make jagged, discontinuous ones (Alibali, Bassok, Solomon, Syc, & Goldin-Meadow, 1999). For space, people tend to describe environments as if they were traveling through them or as if they were viewing them from above. The plane of their gestures differs in each case, in correspondence with the linguistic perspective they adopt (Emmorey, Tversky, & Taylor, 2000). Earlier, mental transformations that appear to be internalized physical transformations, such as those underlying handedness judgments, were described. Here, we

also see that actual motor actions affect and reflect the character of mental ones.

INFERENCES IN MENTAL ENVIRONMENTS

The section on inference opened with spatial inferences made in real environments. Often, people make inferences about environments they are not currently in, for example, when they tell a friend where to how to get to their house and where to find the key when they arrive. For familiar environments, people are quite competent at these sorts of spatial inferences. The mental representations and processes underlying these inferences have been studied for several kinds of environments, notably the immediately surrounding visible or tangible environment and the environment too large to be seen at a glance. These two situations, the space around the body, and the space the body navigates, seem to function differently in our lives, and consequently, to be conceptualized differently (Tversky, 1998).

Spatial updating for the space around the body was first studied using language alone to establish the environments (Franklin & Tversky, 1990). It is significant that language alone, with no specific instructions to form images, was sufficient to establish mental environments that people could update easily and without error. In the prototypical *spatial framework* task, participants read a narrative that describes themselves in a 3D spatial scene, such as a museum or hotel lobby (Franklin & Tversky, 1990; Figure 10.3). The narrative locates and describes objects appropriate to the scene beyond the observer's head, feet, front, back, left, and right (locations chosen randomly). After participants have learned the scenes described by the narratives, they turn to a computer that describes them as turning in the environment so they are now facing a different object. The computer then cues them with direction terms, *front*, *back*, *head*, and so on, to which the participants respond with the name of the object now in that direction. Of interest are the times to respond, depending on the direction from the body. The classical imagery account would predict that participants will imagine themselves in

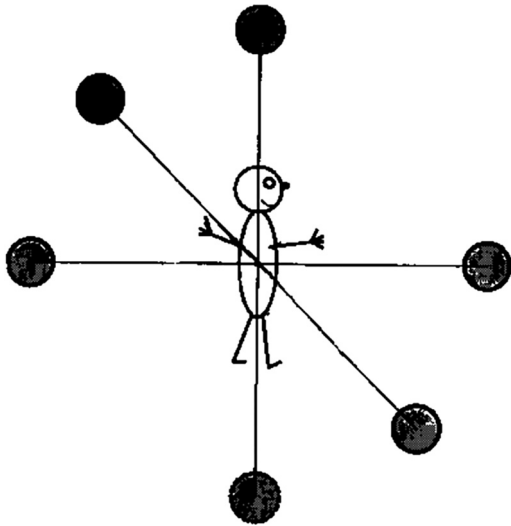


Figure 10.3. Spatial framework situation. Participants read a narrative describing objects around an observer (after Bryant, Tversky, & Franklin, 1992).

the environment facing the selected object, and then imagine themselves turning to face each cued object in order to retrieve the object in the cued direction. The imagery account predicts that reaction times should be fastest to the object in front, then to the objects 90 degrees away from front, that is, left, right, head, and feet, and slowest to objects 180 degrees from front, that is, objects to the back. Data from dozens of experiments fail to support that account.

Instead, the data conform to the *spatial framework theory* according to which participants construct a mental spatial framework from extensions of three axes of the body, head/feet, front/back, and left/right. Times to access objects depend on the asymmetries of the body axes, as well as the asymmetries of the axes of the world. The front/back and head/feet axes have important perceptual and behavioral asymmetries that are lacking in the left/right axis. The world also has three axes, only one of which is asymmetric, the axis conferred by gravity. For the upright observer, the head/feet axis coincides with the axis of gravity, so responses to head and feet should be fastest, and they are. According to the spatial framework account, times should be next fastest to the front/back axis

and slowest to the left/right axis, the pattern obtained for the prototypical situation. When narratives describe observers as reclining in the scenes, turning from back to side to front, then no axis of the body is correlated with gravity so times depend on the asymmetries of the body, and the pattern changes. Times to retrieve objects in front and back are then fastest because the perceptual and behavioral asymmetries of the front/back axis are most important. This is the axis that separates the world that can be seen and manipulated from the world that cannot be seen or manipulated.

By now, dozens of experiments have examined patterns of response times to systematic changes in the described spatial environment (e.g., Bryant, Tversky, & Franklin, 1992; Franklin, Tversky, & Coon, 1992). In one variant, narratives described participants at an oblique angle outside the environment looking onto a character (or two!) inside the environment; in that case, none of the axes of the observer's body is correlated with axes of the characters in the narrative, and the reaction times to all directions are equal (Franklin et al., 1992). In another variant, narratives described the scene, a special space house constructed by NASA, as rotating around the observer instead of the observer turning in the scene (Tversky, Kim, & Cohen, 1999). That condition proved difficult for participants. They took twice as long to update the environment when the environment moved than when the observer moved, a case problematic for pure propositional accounts of mental spatial transformations. Once they had updated the environment, retrieval times corresponded to the spatial framework pattern.

Yet other experiments have varied the way the environment was conveyed, comparing description, diagram, 3D model, and life (Bryant & Tversky, 1999; Bryant, Tversky, & Lanca, 2001). When the scene is conveyed by narrative, life, or a 3D model, the standard spatial framework pattern obtains. However, when the scene is conveyed by a diagram, participants spontaneously adopt an external perspective on the environment.

Their response times are consonant with performing a mental rotation of the entire environment rather than performing a mental change of their own perspective with respect to a surrounding environment (Bryant & Tversky, 1999). Which viewpoint participants adopt, and consequently which mental transformation they perform, can be altered by instructions. When instructed to do so, participants will adopt the internal perspective embedded in the environment in which the observer turns from a diagram or the external perspective from a model in which the entire environment is rotated, with the predicted changes in patterns of retrieval times. Similar findings have been reported by Huttenlocher and Presson (1979), Wraga, Creem, and Proffitt (2000), and Zacks et al. (in press).

ROUTE AND SURVEY PERSPECTIVES

When people are asked to describe environments that are too large to be seen at a glance, they do so from one of two perspectives (Taylor & Tversky, 1992a, 1996). In a *route* perspective, people address the listener as “you,” and take “you” on a tour of the environment, describing landmarks relative to your current position in terms of your *front*, *back*, *left*, and *right*. In a *survey* perspective, people take a bird’s eye view of the environment and describe locations of landmarks relative to one another in terms of *north*, *south*, *east*, and *west*. Speakers (and writers) often mix perspectives, contrary to linguists who argue that a consistent perspective is needed both for coherent construction of a message and for coherent comprehension (Taylor & Tversky, 1992, 1996; Tversky, Lee, & Mainwaring, 1999). In fact, construction of a mental model is faster when perspective is consistent, but the effect is small and disappears quickly during retrieval from memory (Lee & Tversky, in press). In memory for locations and directions of landmarks, route and survey statements are verified equally quickly and accurately irrespective of the perspective of learning, provided the statements are not taken verbatim from the text (Taylor & Tversky, 1992b). For

route perspectives, the mental transformation needed to understand the location information is a transformation of self, an egocentric transformation of one’s viewpoint in an environment. For survey perspectives, the mental transformation needed to understand the location information is a transformation of other, a kind of mental scanning of an object.

The prevalence of these two perspectives in imagery, the external perspective viewing an object or something that can be represented as an object and the internal perspective viewing an environment from within, is undoubtedly associated with their prevalence in the experience of living. In life, we observe changes in the orientation, size, and configuration of objects in the world, and scan them for those changes. In life, we move around in environments, updating our position relative to the locations of other objects in the environment. We are adept at performing the mental equivalents of these actual transformations. There is a natural correspondence between the internal and external perspectives and the mental transformations of self and other, but the human mind is flexible enough to apply either transformation to either perspective. Although we are biased to take an external perspective on objects and mentally transform them and biased to take an internal perspective on environments and mentally transform our bodies with respect to them, we can take internal perspectives on objects and external perspectives on events. The mental world allows perspectives and transformations, whereas the physical world does not. Indeed, conceptualizing a 3D environment that surrounds us and is too large to be seen at once as a small flat object before the eyes, something people, even children, have done for eons whenever they produce a map, is a remarkable feat of the human mind (cf. Tversky, 2000a).

EFFECTS OF LANGUAGE ON SPATIAL THINKING

Speakers of Dutch and other Western languages use both route and survey perspectives. Put differently, they can use either a

relative spatial reference system or an absolute (extrinsic) spatial reference system to describe locations of objects in space. Relative systems use the spatial relations “left,” “right,” “front,” and “back” to locate objects; absolute or extrinsic systems use terms equivalent to “north,” “south,” “east,” and “west.” A smattering of languages dispersed around the world do not describe locations using “left” and “right” (Levinson, 2003). Instead, they rely on an absolute system, so a speaker of those languages would refer to your coffee cup as the “north” cup rather than the one on “your right.” Talk apparently affects thought. Years of talking about space using an absolute spatial reference system has fascinating consequences for thinking about space. For example, speakers of absolute languages reconstruct a shuffled array of objects relative to extrinsic directions in contrast to speakers of Dutch, who reconstruct the array relative to their own bodies. What’s more, when speakers of languages with only extrinsic reference systems are asked to point home after being driven hither and thither, they point with impressive accuracy, in contrast to Dutch speakers, who point at random. The view that the way people talk affects how they think has naturally aroused controversy (see Gleitman & Papafragou, Chap. 26), but is receiving increasing support from a variety of tasks and languages (e.g., Boroditsky, 2001; Boroditsky, Ham, & Ramscar, 2002). Taking a broader perspective, the finding that language affects thought is not as startling. Language is a tool, such as measuring instruments or arithmetic or writing; learning to use these tools also has consequences for thinking.

Judgments

Complex visuospatial thinking is fundamental to a broad range of human activity, from providing directions to the post office and understanding how to operate the latest electronic device to predicting the consequences of chemical bonding or designing a

shopping center. Indeed, visuospatial thinking is fundamental to the reasoning processes described in other chapters in this handbook, as discussed in the chapters on similarity (see Goldstone & Son, Chap. 2), categorization (see Medin & Rips, Chap. 3), induction (see Sloman & Lagnado, Chap. 5), analogical reasoning (see Holyoak, Chap. 6), causality (see Buehner & Cheng, Chap. 7), deductive reasoning (see Evans, Chap. 8), mental models (see Johnson-Laird, Chap. 9), and problem solving (see Novick & Bassok, Chap. 14). Fortunately for both reader and author, there is no need to repeat those discussions here.

Distortions as Clues to Reasoning

Another approach to revealing visuospatial reasoning has been to demonstrate the ways that visuospatial representations differ systematically from situations in the world. This approach, which can be called the *distortions* program, contrasts with the classical imagery approach. The aim of the distortions approach is to elucidate the processes involved in constructing and using mental representations by showing their consequences. The distortions approach has focused more on relations between objects and relations between objects and reference frames, as these visuospatial properties seem to require more constructive processes than those for establishing representations of objects. Some systematic distortions have also been demonstrated in representations of objects.

REPRESENTATIONS

Early on, the Gestalt psychologists attempted to demonstrate that memory for figures got distorted in the direction of good figures (see Riley, 1962). This claim was contested and countered by increasingly sophisticated empirical demonstrations. The dispute faded in a resolution: visual stimuli are interpreted, sometimes as good figures; memory tends toward the interpretations. So if $o - o$ is interpreted as “eyeglasses,” participants later draw the connection curved, whereas if it is interpreted as “barbells,” they do not (Carmichael, Hogan, & Walter,

1932). Little noticed is that the effect does not appear in recognition memory (Prentice, 1954). Since then, and relying on the sophisticated methods developed, there has been more evidence for shape distortion in representations. Shapes that are nearly symmetric are remembered or judged as more symmetric than they actually are, as if people code nearly symmetric objects as symmetric (Freyd & Tversky, 1984; McBeath, Schiano, & Tversky, 1997; Tversky & Schiano, 1989). Given that many of the objects and beings that we encounter are symmetric, but are typically viewed at an oblique angle, symmetry may be a reasonable assumption, although one that is wrong on occasion. Size is compressed in memory (Kerst & Howard, 1978). When portions of objects are truncated by picture frames, the objects are remembered as more complete than they actually were (Intraub, Bender, & Mangels, 1992).

REPRESENTATIONS AND TRANSFORMATIONS: SPATIAL CONFIGURATIONS AND COGNITIVE MAPS

The Gestalt psychologists also produced striking demonstrations that people organize the visual world in principled ways, even when that world is a meaningless array (see Hochberg, 1978). Entities in space, especially ones devoid of meaning, are difficult to understand in isolation, easier to grasp in context. People group elements in an array by proximity or similarity or good continuation. One inevitable consequence of perceptual organizing principles is distorted representations.

Many of the distortions reviewed here have been instantiated in memory for perceptual arrays that do not stand for anything. They have also been illustrated in memory for cognitive maps and for environments. As such, they have implications for how people reason in navigating the world, a visuospatial reasoning task that people of all ages and parts of the world need to solve. Even more intriguing, many of these phenomena have analogs in abstract thought.

For the myriad spatial distortions described here (and analyzed more fully in

Tversky, 1992, 2000b, 2000c), it is difficult to clearly attribute error to either representations or processes. Rather the errors seem to be consequences of both, of schematized, hence distorted, representations constructed ad hoc in order to enable specific judgments, such as the direction or distance between pairs of cities. When answering such questions, it is unlikely that people consult a library of "cognitive maps." Rather, it seems that they draw on whatever information they have that seems relevant, organizing it for the question at hand. The reliability of the errors under varying judgments makes it reasonable to assume erroneous representations are reliably constructed. Some of the organizing principles that yield systematic errors are reviewed in the next section.

Hierarchical Organization. Dots that are grouped together by good continuation, for example, parts of the same square outlined in dots, are judged to be closer than dots that are actually closer but parts of separate groups (Coren & Girgus, 1980). An analogous phenomenon occurs in judgments of distance between buildings (Hirtle & Jonides, 1985): Residents of Ann Arbor think that pairs of university (or town) buildings are closer than actually closer pairs of buildings that belong to different groups, one to the university and the other to the town. Hierarchical organization of essentially flat spatial information also affects accuracy and time to make judgments of direction. People incorrectly report that San Diego is west of Reno. Presumably this error occurs because people know the states to which the cities belong and use the overall directions of the states to infer the directions between cities in the states (Stevens & Coupe, 1978). People are faster to judge whether one city is east or north of another when the cities belong to separate geographic entities than when they are actually farther, but part of the same geographic entity (Maki, 1981; Wilton, 1979).

A variant of hierarchical organization occurs in locating entities belonging to a bounded region. When asked to remember the location of a dot in a quadrant, people

place it closer to the center of the quadrant, as if they were using general information about the area to locate entity contained in it (Huttenlocher, Hedges, & Duncan, 1991; Newcombe & Huttenlocher, 2000).

Amount of Information. That representations are constructed on the fly in the service of particular judgments seems to be the case for other distance estimates. Distances between A and B, say two locations within a town, are greater when there are more cross streets or more buildings or more obstacles or more turns on the route (Newcombe & Liben, 1982; Sadalla & Magel, 1980; Sadalla & Staplin, 1980a, 1980b; Thorndyke, 1981), as if people mentally construct a representation of a path from A to B from that information and use the amount of information as a surrogate for the missing exact distance information. There is an analogous visual illusion: A line appears longer if bisected, and longer still with more tick marks (at some point of clutter, the illusion ceases or reverses).

Perspective. Steinberg regaled generations of readers of the *New Yorker* and denizens of dormitory rooms with his maps of views of the world. In the each view, the immediate surroundings are stretched and the rest of the world shrunk. The psychological reality of this genre of visual joke was demonstrated by Holyoak and Mah (1982). They asked students in Ann Arbor to imagine themselves on either coast and to estimate the distances between pairs of cities distributed more or less equally on an east-west axis across the states. Regardless of imagined perspective, students overestimated the near distances relative to the far ones.

Landmarks. Distance judgments are also distorted by landmarks. People judge the distance of an undistinguished place to be closer to a landmark than vice versa (McNamara & Diwadkar, 1997; Sadalla, Burroughs, & Staplin, 1980). Landmark asymmetries violate elementary metric assumptions, assumptions that are more or less realized in real space.

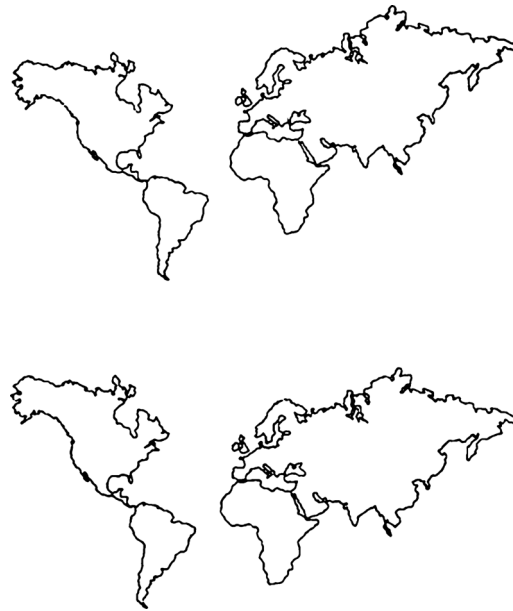


Figure 10.4. Alignment. A significant majority of participants think the incorrect lower map is correct. The map has been altered so the United States and Europe and South American and Africa are more aligned (after Tversky, 1981).

Alignment. Hierarchical, perspective, and landmark effects can all be regarded as consequences of the Gestalt principle of grouping. Even groups of two equivalent entities can yield distortion. When people are asked to judge which of two maps is correct, a map of North and South America in which South America has been moved westward to overlap more with North America, or the actual map, in which the two continents barely overlap, the majority of respondents prefer the former (Tversky, 1981; Figure 10.4). A majority of observers also prefer an incorrect map of the Americas and Europe/Africa/Asia in which the Americas are moved northward so the United States and Europe and South America and Africa are more directly east-west. This phenomenon has been called *alignment*; it occurs when people group two spatial entities and then remember them more in correspondence than they actually are. It appears not only in judgments of maps of the world, but also in judgments of directions between cities, in

memory for artificial maps, and in memory for visual blobs.

Spatial entities cannot be localized in isolation; they can be localized with respect to other entities or to frames of reference. When they are coded with respect to another entity, alignment errors are likely. When entities are coded with respect to a frame of reference, rotation errors, described in the next section, are likely.

Rotation. When people are asked to place a cutout of South America in a north-south east-west frame, they upright it. A large spatial object, such as South America, induces its own coordinates along an axis of elongation and an axis parallel to that one. The actual axis of elongation of South America is tilted with respect to north-south, and people upright it in memory. Similarly, people incorrectly report that Berkeley is east of Stanford, when it is actually slightly west. Presumably this occurs because they upright the Bay Area, which actually runs at an angle with respect to north-south. This error has been called *rotation*; it occurs when people code a spatial entity with respect to a frame of reference (Tversky, 1981; Figure 10.5). As for rotation, it appears in memory for artificial maps and uninterpreted blobs, as well as in memory for real environments. Others have replicated this error in remembered directions and in navigation (e.g., Glicksohn, 1994; Lloyd & Heivly, 1987; Montello, 1991; Presson & Montello, 1994).

Are Spatial Representations Incoherent? This brief review has brought evidence for distortions in memory and judgment for shapes of objects, configurations of objects, and distances and directions between objects that are a consequence of the organization of the visuospatial information. These are not errors of lack of knowledge; even experienced taxi drivers make them (Chase & Chi, 1981). Moreover, many of these biases have parallels in abstract domains, such as judgments about members of one's own social or political groups relative to judgments about members of other groups (e.g., Quattrone, 1986).

What might a representation that captures all these distortions look like? Nothing that can be sketched on a sheet of paper, that is, coherent in two dimensions. Landmark asymmetries alone disallow that. It does not seem likely that people make these judgments by retrieving a coherent prestored mental representation, a "cognitive map," and reading the direction or distance from it. Rather, it seems that people construct representations on the fly, incorporating only the information needed for that judgment, the relevant region, the specific entities within it. Some of the information may be visuospatial, from experience or from maps, some may be linguistic. For these reasons, "cognitive collage" seems a more apt metaphor than "cognitive map" for whatever representations underlie spatial judgment and memory (Tversky, 1993). Such representations are schematic, they leave out much information and simplify others. Schematization occurs for at least two reasons. More exact information may not be known and therefore cannot be represented. More exact information may not even be needed as the situation on the ground may fill it in. More information may overload working memory, which is notoriously limited. Not only must the representation be constructed in working memory, but also a judgment made on the representation. Schematization may hide incoherence, or it may not be noticed. Schematization necessarily entails systematic error.

Why do Errors Persist? It is reasonable to wonder why so many systematic errors persist. Some reasons for the persistence of error have already been discussed, that there may be correctives on the ground, that some errors are a consequence of the schematization processes that are an inherent part of memory and information processing. Yet another reason is that the correctives are specific – now I know that Rome is north of Philadelphia – and do not affect or even make contact with the general information organizing principle that generated the error, and that serves us well in many situations (e.g., Tversky, 2003a).

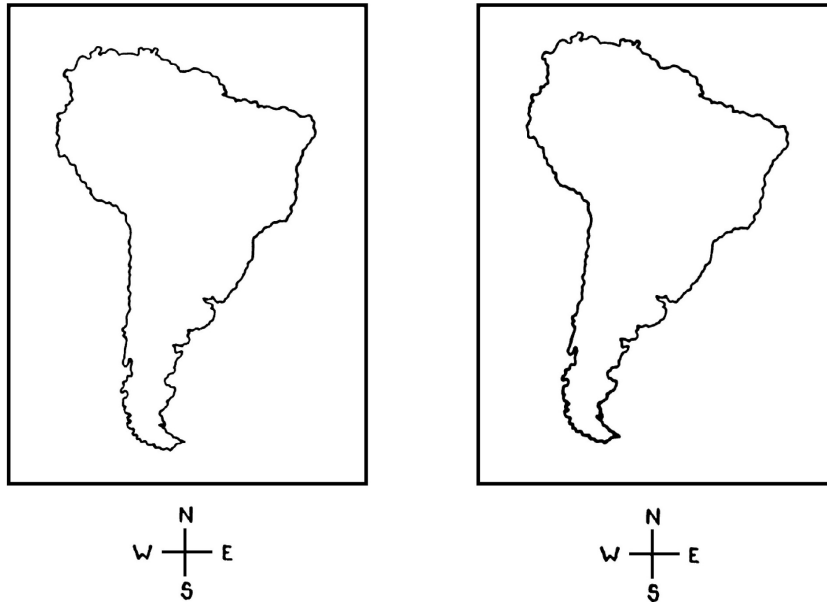


Figure 10.5. Rotation. When asked to place a cutout of South American in a NSEW framework, most participants upright it, as in the left example (after Tversky, 1981).

From Spatial to Abstract Reasoning

Visuospatial reasoning does not only entail visuospatial transformations on visuospatial information. Visuospatial reasoning also includes making inferences from visuospatial information, whether that information is in the mind or in the world. An early demonstration was the *symbolic distance effect* (e.g., Banks & Flora, 1977; Moyer, 1973; Paivio, 1978). The time to judge which of two animals is more intelligent or pleasant is faster when the entities are farther on the dimension than when they are closer, as if people were imagining the entities arrayed on a line corresponding to the abstract dimension. It is easier, hence faster, to discriminate larger distances than smaller ones. Note that a subjective experience of creating and using an image does not necessarily accompany making these and other spatial and abstract judgments. Spatial thinking can occur regardless of whether thinkers have the sensation of using an image. So many abstract concepts have spatial analogs (for related discussion, see Holyoak, Chap. 6).

Indeed, spatial reasoning is often studied in the context of graphics, maps, diagrams, graphs, and charts. External representations bear similarities to internal representations, if only because they are creations of the human mind, cognitive tools to increase the power of the human mind. They also bear formal similarities, in that both internal and external representations are mappings between elements and relations. External representations are constrained by a medium and unconstrained by working memory; for this reason, inconsistencies, ambiguities, and incompleteness may be reduced in external representations.

Graphics: Elements

The readiness with which people map abstract information onto spatial information is part of the reason for the widespread use of diagrams to represent and convey abstract information, from the sublime, the harmonies of the spheres, rampant in religions spanning the globe, to the mundane corporate charts and statistical graphs.

Graphics, such as these, consist of elements and spatial relations among the elements. In contrast to written (alphabetic) languages, both elements and use of space in graphics can convey meaning rather directly (e.g., Bertin, 1967/1983; Pinker, 1994; Tversky, 1995, 2001; Winn, 1989). Elements may consist of likenesses, such as road signs depicting picnic tables, falling rocks, or deer. Elements may also be figures of depiction, similar to figures of speech: synecdoche, where a part represents a whole, common in ideographic writing, for example, using a ram's horns to represent a ram; or metonymy, where an association represents an entity or action, common in computer menus, such as scissors to denote *cut* text or a trashcan to allow deletion of files.

Graphics: Relations

Relations among entities preserve different levels of information. The information preserved is reflected in the mapping to space. In some cases, the information preserved is simply categorical; space is used to separate entities belonging to different categories. The spaces between words, for example, indicate that one set of letters belongs to one meaning and another set to another meaning. Space can also be used to represent ordinal information, for example, listing historic events in their order of occurrence, groceries by the order of encountering them in the supermarket, and companies by their profits. Space can be used to represent interval or ratio information, as in many statistical graphs, where the spatial distances among entities reflect their distances on some other dimension.

SPONTANEOUS USE OF SPACE TO REPRESENT ABSTRACT RELATIONS

Even preschool children spontaneously use diagrammatic space to represent abstract information (e.g., diSessa, Hammer, Sherin, & Kolpakowski, 1991; Tversky, Kugelmass, & Winter, 1991). In one set of studies (Tversky et al., 1991), children from three language communities were asked to place stickers on paper to represent spatial, tem-

poral, quantitative, and preference information, for example, to place stickers for TV shows they loved, liked, or disliked. Almost all the preschoolers put the stickers on a line, preserving ordinal information. Children in the middle school years were able to represent interval information, but representing more than ordinal information was unusual for younger children, despite strong manipulations to encourage them. Not only did children (and adults) spontaneously use spatial relations to represent abstract relations, but children also showed preferences for the direction of increases in abstract dimensions. Increases were represented from right to left or left to right (irrespective of direction of writing for quantity and preference) or down to up. Representing increasing time or quantity from up to down was avoided. Representing increases as upward is especially robust; it affects people's ability to make inferences about second-order phenomena such as rate, which is spontaneously mapped to slope, from graphs (Gattis, 2002; Gattis & Holyoak, 1996). The correspondence of upward to more, better, and stronger appears in language – on top of the world, rising to higher levels of platitude – and in gesture – thumbs up, high five – as well as in graphics. These spontaneous and widespread correspondences between spatial and abstract relations suggest they are cognitively natural (e.g., Tversky, 1995a, 2001).

The demonstrations of spontaneous use of spatial language and diagrammatic space to represent abstract relations suggests that spatial reasoning forms a foundation for more abstract reasoning. In fact, children used diagrammatic space to represent abstract relations earlier for temporal relations than for quantitative ones, and earlier for quantitative relations than for preference relations (Tversky et al., 1991). Corroborative evidence comes from simple spatial and temporal reasoning tasks, judging whether one object or person is before another. In many languages, words for spatial and temporal relations, such as *before*, *after*, and *in between*, are shared. That spatial terms are the foundation for the temporal comes from research showing priming of temporal

perspective from spatial perspective, but not vice versa (Boroditsky, 2000). More support for the primacy of spatial thinking for abstract thought comes from studies of problem solving (Carroll, Thomas, & Mulhota, 1980). One group of participants was asked to solve a spatial problem under constraints, arranging offices to facilitate communication among key people. Another group was asked to solve a temporal analog, arranging processes to facilitate production. The solutions to the spatial analog were superior to those to the temporal analog. When experimenters suggested using a diagram to yet another group solving the temporal analog, their success equaled that of the spatial analog group.

DIAGRAMS FACILITATE REASONING

Demonstrating that using a spatial diagram facilitates temporal problem solving also illustrates the efficacy of diagrams in thinking, a finding amply supported, even for inferences entailing complex logic, such as double disjunctions, although to succeed, diagrams have to be designed with attention to the ways that space and spatial entities are used to make inferences (Bauer & Johnson-Laird, 1993). Middle school children studying science were asked to put reminders on paper. Those children who sketched diagrams learned the material better than those who did not (Rode & Stern, in press).

DIAGRAMS FOR COMMUNICATING

Many maps, charts, diagrams, and graphs are meant to communicate clearly for travelers, students, and scholars, whether they are professionals or amateurs. To that end, they are designed to be clear and easy to comprehend, and they meet with varying success. Good design takes account of human perceptual and cognitive skills, biases, and propensities. Even ancient Greek vases take account of how they will be seen. Because they are curved round structures, creating a veridical appearance requires artistry. The vase "Achilles and Ajax playing a game" by the Kleophrades Painter in the Museum of Metropolitan Art in New York City (Art.

65.11.12, ca. 500–480 B.C.) depicts a spear that appears in one piece from the desired viewing angle, but in three pieces when viewed straight on (J. P. Small, personal communication, May 27, 2003).

The perceptual and cognitive processes and biases that people bring to graphics include the catalog of mental representations and transformations that was begun earlier. In that spirit, several researchers have developed models for graph understanding, notably Pinker (1990), Kosslyn (1989, 1994a), and Carpenter and Shah (1998) (see Shah 2003/2004, for an overview). These models take account of the particular perceptual or imaginal processes that need to be applied to particular kinds of graphs to yield the right inferences. Others have taken account of perceptual and cognitive processing in the construction of guidelines for design of (e.g., Carswell & Wickens, 1990; Cleveland, 1985; Kosslyn, 1994a; Tufte, 1983, 1990, 1997; Wainer, 1984, 1997). In some cases the design principles are informed by research, but in most they are informed by the authors' educated sensibilities and/or rules of thumb from graphic design.

Inferences from Diagrams: Structural and Functional. The existence of spontaneous mapping of abstract information onto spatial does not mean that the meanings of diagrams are transparent and can be automatically and easily extracted (e.g., Scaife & Rogers, 1995). Diagrams can support many different classes of inferences, notably, structural and functional (e.g., Mayer & Gallini, 1990). Structural inferences, or inferences about qualities of parts and the relations among them, can be readily made from inspection of a diagram. Distance, direction, size, and other spatial qualities and properties can be "read off" a diagram (Larkin & Simon, 1987), at least with some degree of accuracy. "Reading off" entails using the sort of mental transformations discussed earlier, mental scanning, mental distance, size, shape, or direction judgments or comparisons. Functional inferences, or inferences about the behavior of entities, cannot be readily made from inspection of a diagram in the absence of

additional knowledge or assumptions, often a consequence of expertise. Spatial information may provide clues to functional information, but it is not sufficient for concepts such as force, mass, and friction. Making functional inferences requires linking perceptual information to conceptual information; it entails both knowing how to “read” a diagram, that is, what visuospatial features and relations to inspect or transform, and knowing how to interpret that visuospatial information.

Structural and functional inferences respectively correspond to two senses of *mental model* prevalent in the field. In both cases, *mental model* contrasts with *image*. In one sense, a mental model contrasts with an image in being more skeletal or abstract. This is the sense used by Johnson-Laird in his book, *Mental Models* (1983), in his explication of how people solve syllogisms (see Johnson-Laird, Chap. 9, and Evans, Chap. 8). Here, a mental model captures the structural relations among the parts of a system. In the other sense, a mental model contrast with an image in having moving parts, in being “runnable” to derive functional or causal inferences (for related discussion on causality, see Buehner and Cheng, Chap. 7, and on problem solving, see Chi and Ohlsson, Chap. 16). This is the sense used in another book also titled *Mental Models* (Gentner & Stevens, 1983). One goal of diagrams is to instill mental models in the minds of their users. To that end, diagrams abstract the essential elements and relations of the system they are meant to convey. As is seen, conveying structure is more straightforward than conveying function.

What does it mean to say that a mental model is “runnable?” One example comes from research on pulley systems (Hegarty, 1992). Participants were timed to make two kinds of judgments from diagrams of three-pulley systems. For true-false judgments of structural questions, such as “The upper left pulley is attached to the ceiling,” response times did not depend on which pulley in the system was queried. For judgments of functional questions, such as “The upper left pulley goes clockwise,” response times did

depend on the order of that pulley in the mechanics of the system. To answer functional questions, it is as if participants mentally animate the pulley system in order to generate an answer. Mental animation, however, does not seem to be a continuous process in the same way as physical animation. Rather, mental animation seems to be a sequence of discrete steps, for example, the first pulley goes clockwise, and the rope goes under the next pulley to the left of it, so it must go counterclockwise. That continuous events are comprehended as sequences of steps is corroborated by research on segmentation and interpretation of everyday events, such as making a bed (Zacks, Tversky, & Iyer, 2001).

It has long been known that domain experts are more adept at functional inferences from diagrams than novices. Experts can “see” sequences of organized chess moves in a midgame display (Chase & Simon, 1973; De Groot, 1965). Similarly, experts in Go (Reitman, 1976), electricity (Egan & Schwartz, 1979), weather (Lowe, 1989), architecture (Suwa & Tversky, 1997), and more make functional inferences with ease from diagrams in their domain. Novices are no different from experts in structural inferences.

Inferences from Diagrams of Systems. The distinction between structural and functional inferences is illustrated by work on production and comprehension of diagrams for mechanical systems, such as a car brake, a bicycle pump, or a pulley system (Heiser & Tversky, 2002; Figure 10.6). Participants were asked to interpret a diagram of one of the systems. On the whole, their interpretations were structural, that is, they described the relations among the parts of the system. Another set of participants was given the same diagrams, enriched by arrows indicating the sequence of action in the systems. Those participants gave functional descriptions; that is, they described the step-by-step operation of the system. Reversing the tasks, other groups of participants read structural or functional descriptions of the systems and produced diagrams of them. Those who

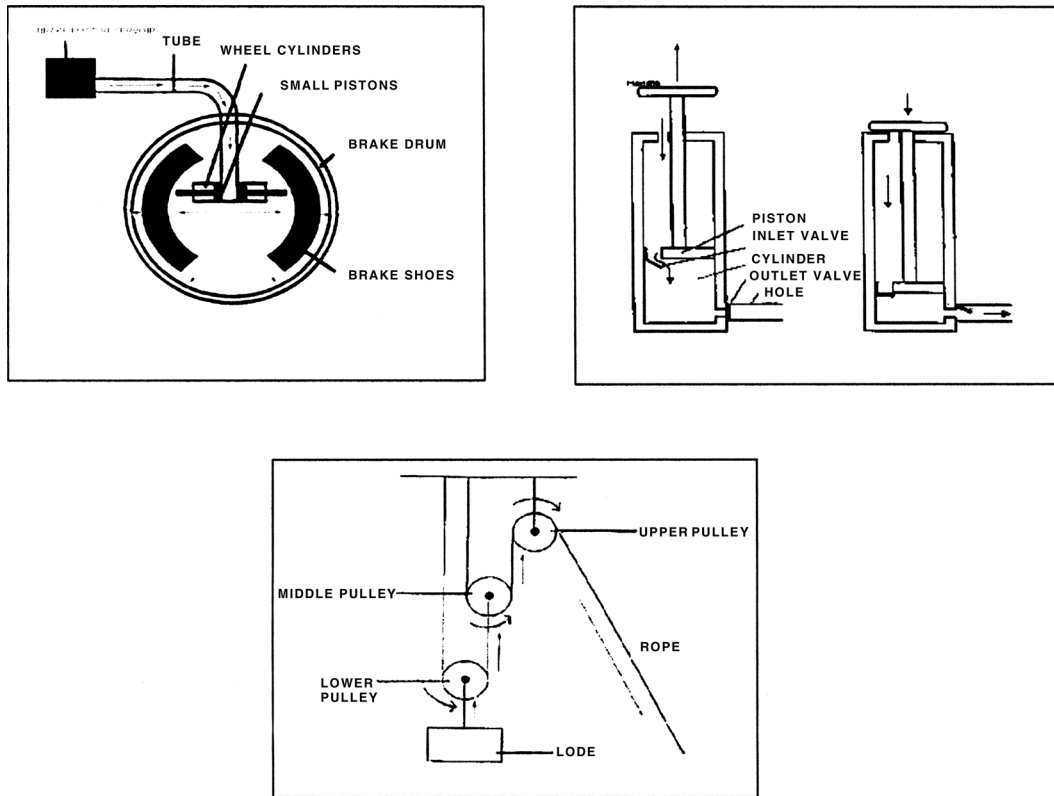


Figure 10.6. Diagrams of a car brake and a bicycle pump (both after Mayer & Gallini, 1990), and a pulley system (after Hegarty, 1992). Diagrams without arrows encouraged structural descriptions and diagrams with arrows yielded functional descriptions (Heiser and Tversky, in press).

read functional descriptions used arrows in their diagrams far more than those who read structural descriptions. Arrows are an extrapictorial device that have many meanings and functions in diagrams, for example, pointing, indicating temporal sequence, causal sequence, and path and manner of motion (Tversky, 2001).

Expertise came into play in a study of learning rather than interpretation. Participants learned one of the mechanical systems from a diagram with or without arrows or from structural or functional text. They were later tested on both structural and functional information. Participants high in expertise/ability (self-assessed) were able to infer both structural and functional information from either diagram. In contrast, participants low in expertise/ability could derive structural but not functional information from the diagrams. Those participants

were able to infer functional information from functional text. This finding suggests that people with high expertise/ability can form unitary diagrammatic mental models of mechanical systems that allow spatial and functional inferences with relative ease, but people with low expertise/ability have and use diagrammatic mental models for structural information, but rely on propositional representations for functional information.

Enriching Diagrams to Facilitate Functional Inferences. As noted, conveying spatial or structural information is relatively straightforward in diagrams. Diagrams can use space to represent space in direct ways that are readily interpreted, as in maps and architectural sketches. Conveying information that is not strictly spatial, such as change over time, forces, and kinematics, is less straightforward. Some visual conventions for

conveying information about dynamics or forces have been developed in comics and in diagrams (e.g., Horn, 1998; Kunzle, 1990; McCloud, 1994), and many of these conventions are cognitively compelling. Arrows are a good example. As lines, arrows indicate a relationship, a link. As asymmetric lines, they indicate an asymmetric relationship. The arrowhead is compelling as an indicator of the direction of the asymmetry because of its correspondence to arrowheads common as weapons in the world or its correspondence to Vs created by paths of downward moving water. A survey of diagrams in science and engineering texts shows wide use of extrapictorial diagrammatic devices, such as arrows, lines, brackets, and insets, although not always consistently (Tversky, Heiser, Lozano, MacKenzie, & Morrison, in press). As a consequence, these devices are not always correctly interpreted. Some diagrams of paradigmatic processes, such as the nitrogen cycle in biology or the rock cycle in geology, contain the same device, typically an arrow, with multiple senses, pointing or labeling, indicating movement path or manner, suggesting forces or sequence, in the same diagram. Of course, there is ambiguity in many words that appear commonly in scientific and other prose, words that parallel these graphic devices, such as *line* and *relationship*. Nevertheless, the confusion caused by multiple senses of diagrammatic devices in interpreting diagrams suggests that greater care in design is worthwhile.

An intuitive way to visualize change over time is by animations. After all, an animation uses change over time to convey change over time, a cognitively compelling correspondence. Despite the intuitive appeal, a survey of dozens of studies that have compared animated graphics to informationally comparable static graphics in teaching a wide variety of concepts, physical, mechanical, and abstract, did not find a single example of superior learning by animations (Tversky, Morrison, & Betrancourt, 2002). Animations may be superior for purposes other than learning, for example, in maintaining perspective or in calling attention to a solution

in problem solving. For example, a diagram containing many arrows moving toward the center of a display was superior to a diagram with static arrows in suggesting the solution to the Duncker radiation problem, how to destroy a tumor without destroying healthy tissue (Pedone, Hummel, & Holyoak, 2001; see Holyoak, Chap. 6, Figure 6.4). The failure of animations to improve learning itself becomes intuitive on further reflection. For one thing, animations are often complex, so it is difficult for a viewer to know where to look and to make sense of the timing of many moving components. However, even simple animations, such as the path of a single moving circle, are not superior to static graphics (Morrison & Tversky, in press). The second reason for the lack of success of animations is one reviewed earlier. If people think of dynamic events as sequences of steps rather than continuous animations, then presenting change over time as sequences of steps may make the changes easier to comprehend.

Diagrams for Insight

Maps for highways and subways, diagrams for assembly and biology, graphs for economics and statistics, and plans for electricians and plumbers are designed to be concise and unambiguous, although they may not always succeed. Their inventors want to communicate clearly and without error. In contrast are graphics created to be ambiguous, to allow reinterpretation and discovery. Art falls into both those categories. Early design sketches are meant to be ambiguous, to commit the designer to only those aspects of the design that are likely not to change, and to leave open other aspects. One reason for this is fixation; it is hard to “think out of the box.” Visual displays express, suggest, more than what they display. That expression in fact, came from solution attempts to the famous nine-dot problem (see Novick & Bassok, Chap. 14, Fig. 14.4). Connect all nine dots in a 3×3 array using four straight lines without lifting the pen from the paper. The solution that is hard to see is to extend the lines beyond the “box” suggested



Figure 10.7. A sketch by an architect designing a museum. Upon reinspection, he made an unintentional discovery (Suwa, Tversky, Gero, & Purcell, 2001).

by the 3×3 array. The Gestalt psychologists made us aware of the visual inferences the mind makes without reflection, grouping by proximity, similarity, good continuation, and common fate.

INFERENCES FROM SKETCHES

Initial design sketches are meant to be ambiguous for several reasons. In early stages of design, designers often do not want to commit to the details of a solution, only the general outline, leaving open many possibilities; gradually, they will fill in the details. Perhaps more important, skilled designers are able to get new ideas by reexamining their own sketches, by having a conversation with their sketches, bouncing ideas off them (e.g., Goldschmidt, 1994; Schon, 1983; Suwa & Tversky, 1997; Suwa, Tversky, Gero, & Purcell, 2001). They may construct sketches with one set of ideas in mind, but on later reexamination they see new configurations and relations that generate new design ideas. The productive cycle between reexamining and reinterpreting is revealed in the protocol of one expert architect. When he saw a new

configuration in his own design, he was more likely to invent a new design idea; similarly, when he invented a new design idea, he was more likely to see a new configuration in his sketch (Suwa et al., 2001; Figure 10.7).

Underlying these unintended discoveries in sketches is a cognitive skill termed *constructive perception*, which consists of two independent processes: a perceptual one, mentally reorganizing the sketch, and a conceptual one, relating the new organization to some design purpose (Suwa & Tversky, 2003). Participants adept at generating multiple interpretations of ambiguous sketches excelled at the perceptual ability of finding hidden figures and at the cognitive ability of finding remote meaningful associations, yet these two abilities were uncorrelated.

Expertise affects the kinds of inferences designers are able to make from their sketches. Novice designers are adept at perceptual inferences, such as seeing proximity and similarity relations. Expert designers are also adept at functional inferences, such as “seeing” the flow of traffic or the changes in light from sketches (Suwa & Tversky, 1997).

Conclusions and Future Directions

Starting with the elements of visuospatial representations in the mind, we end with visuospatial representations created by the mind. Like language, graphics serve to express and clarify individual spatial and abstract concepts. Graphics have an advantage over language in expressiveness (Stenning & Oberlander, 1995); graphics use elements and relations in graphic space to convey elements and relations in real or metaphoric space. As such, they allow inference based on the visuospatial processing that people have become expert in as a part of their everyday interactions with space (Larkin & Simon, 1997). As cognitive tools, graphics facilitate reasoning, both by externalizing, thus offloading memory and processing, and by mapping abstract reasoning onto spatial comparisons and transformations. Graphics organize and schematize spatial and abstract information to highlight and focus the essential information. Like language, graphics serve to convey spatial and abstract concepts to others. They make private thoughts public to a community that can then use and revise those concepts collaboratively.

Of course, graphics and physical and mental transformations on them are not identical to visuospatial representations and reasoning, they are an expression of it. Talk about space and actions in it were probably among the first uses of language, telling others how to find their way and what to look for when they get there. Cognitive tools to promote visuospatial reasoning were among the first to be invented from tokens for property counts, believed to be the precursor of written language (Schmandt-Besserat, 1992), to trail markers to maps in the sand. Spatial thought, spatial language, and spatial graphics reflect the importance and prevalence of visuospatial reasoning in our lives, from knowing how to get home to knowing how to design a house, from explaining how to find the freeway to explaining how the judicial system works, from understanding basic science to inventing new conceptions of the origins of the universe. Where do we go from here? Onward and upward!

Acknowledgments

I am grateful to Phil Johnson-Laird and Jeff Zacks for insightful suggestions on a previous draft. Preparation of this chapter and some of the research reported were supported by Office of Naval Research, Grant Numbers N00014-PP-1-O649, N000140110717, and N000140210534 to Stanford University.

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