Radiation pressure calibration and test mass reflectivities for LISA Pathfinder

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Abstract. This paper describes a series of experiments which were carried out during the main operations of LISA Pathfinder. These experiments were performed by modulating the power of the measurement and reference beams. In one series of experiments the beams were sequentially switched on and off. In the other series of experiments the powers of the beams were modulated within 0.1% and 1% of the constant power. These experiments use recordings of the total power measured on the photodiodes to infer the properties of the Optical Metrology System (OMS), such as reflectivities of the test masses and change of the photodiode efficiencies with time. In the first case the powers are back propagated from the different photodiodes to the same place on the optical bench to express the unknown quantities in the measurement with the complimentary photodiode measurements. They are combined in the way that the only unknown left is the test mass reflectivities. The second experiment compared two estimates of the force applied to the test masses due to the radiation pressure that appears because of the beam modulations. One estimate of the force is inferred from the measurements of the powers on the photodiodes and propagation of this measurement to the test masses. The other estimation of the force is done by calculating it from the change in the main scientific output of the instrument – differential displacement of the two test masses.

1. Introduction

LISA Pathfinder (LPF) is a technology demonstration mission for the future gravitational wave observatory in space – Laser Interferometer Space Antenna (LISA). The Optical Metrology System (OMS) on board LPF measures relative position of two free falling test masses and the position of one of the test masses relative to the optical bench. The measurements are performed with interferometers that are bonded on the ultra-stable bench between the test masses. The OMS of LPF has four interferometers bonded on the optical bench:

- *X12* – interferometer that measures relative displacement of the two test masses;
- *X1* – interferometer that measures the displacement of the first test mass relative to the optical bench;
- *Frequency* – interferometer with the deliberate pathlength mismatch used for the frequency stabilisation of the laser;
- *Reference* – interferometer, which measurement is subtracted from the other interferometer measurements to get rid of the common mode noise.
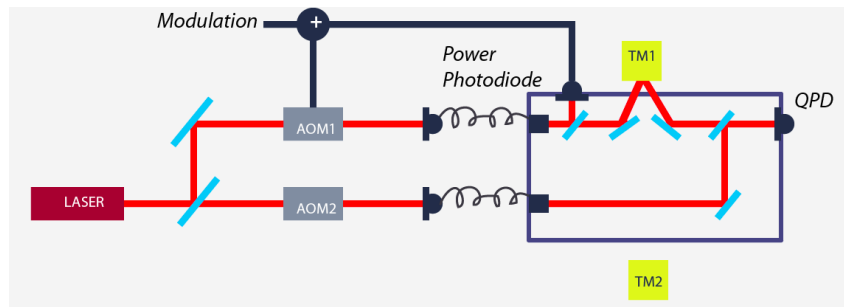
Figure 1. This is an outline of the experiment that presents how the modulation is done within the OMS (Optical Metrology System) which comprises of laser assembly, Acousto-Optical Modulators (AOMs), Test Masses (TMs), modulation bench and optical bench.

There are two quadrant photodiodes for each interferometer. In addition there are two single element photodiodes that measure the power transmitted to the optical bench. These measurements are used for the stabilisation of the power that goes to the optical bench. Other components on the optical bench are beam splitters and mirrors, the reflectivities and transmissivities for which we know within errors. The outline of the optical bench with the labels for its components is shown on the Figure 2.

2. Experiment setup
Two types of measurements were performed:

 (i) Sequence of the beam on/beam of measurements. We measure the sum of the power on the quadrants of the Quadrant Photodiodes (QPDs) and use the ratio of the measurements from two interferometers to estimate the test mass reflectivities.
 (ii) Modulation of the measurement beam, modulation of the reference beam and counter modulation of both beams. The modulations were done with 0.1% and 1% of the original power at frequencies around 2 mHz.

The schematic of the experiment setup is presented on the Figure 1. The power is modulated by adding signal to the error point of the power stabilisation loop.

3. Test mass reflectivities
To calculate test mass reflectivities we treat test mass as one of the components in the light path that goes from Fibre Injector Optical Subassemblies (FIOS) to the respective QPDs. To put it simply we need to divide the measured power by the injected power. However, we do not know the efficiencies of the photodiodes and values of the power injected on the optical bench well enough. Therefore we need to use a combination of the measurements to get rid of the unknown values. To eliminate QPD efficiencies we use the measurements from the same QPDs but for the experiments when either measurement of reference beams were on. To measure the powers we use the QPDs that do not have a test mass in the path of their beam such as the measurements from the *Reference* or from the *Frequency* interferometer. We propagate the power back through component from the QPDs to the place where it is injected on the optical bench. The powers propagated from either *X1* or *X12* interferometers we express through the powers propagated from either *Reference* or *Frequency*. We use *Reference* interferometer for the calculations, for the other one equations can be derived in the similar way and will give us redundant measurements. Moreover we will use only A photodiodes, for the B photodiodes calculations are done similarly and they also provide redundant measurements.

Let us derive the equation for the first test mass reflectivity only, for the second test mass it is done in a similar way.

First, let us define the convention for the naming that is going to be used throughout this section:

- E will stand for the efficiencies with the respective photodiode name in the subscript;
- Σ will stand for the total power on the photodiode with the respective photodiode name in the subscript;
- subscripts "m" or "r" for the total power measurements will define whether measurement or reference beam is on, respectively.

Let us propagate the power for the measurement P_{m} (beam 1) and reference P_{r} (beam 2) beams to the *Reference interferometer* photodiode PDRA:

$$P_{\mathrm{m}} \cdot L_{\mathrm{m}RA} = E_{RA} \cdot \Sigma_{\mathrm{m}RA}$$
$$P_{\mathrm{r}} \cdot L_{\mathrm{r}RA} = E_{RA} \cdot \Sigma_{\mathrm{r}RA} \tag{1}$$

where the optical paths are

$$L_{\mathrm{m}RA} = BS16t \cdot BS1t \cdot BS2r \cdot M11 \cdot M12 \cdot M14 \cdot BS5r$$
$$L_{\mathrm{r}RA} = BS11t \cdot BS4t \cdot BS9r \cdot BS5t. \tag{2}$$

Let us propagate the power for the measurement P_{m} (beam 1) and reference P_{r} (beam 2) beams to the *X1 interferometer* photodiode:

$$P_{\mathrm{m}} \cdot L_{\mathrm{m}1A} \cdot TM1r \cdot WIN1t^2 = E_{1A} \cdot \Sigma_{\mathrm{m}1A} \tag{3}$$
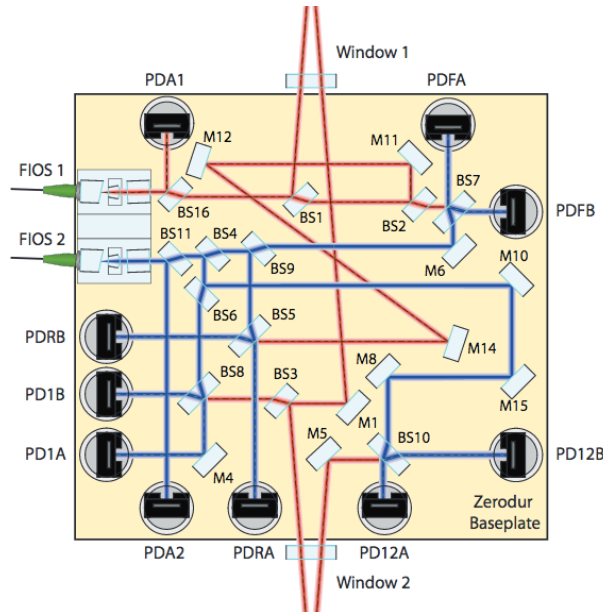$$P_{\mathrm{r}} \cdot L_{\mathrm{r}1A} = E_{1A} \cdot \Sigma_{\mathrm{r}1A}$$

Figure 2. Optical bench schematic with the names of the components and photodiodes. Image courtesy of Institute for Gravitational Research, Glasgow and Albert Einstein Institute, Hannover

where optical paths are

$$L_{\mathrm{m1}A} = BS16t \cdot BS1r \cdot M1 \cdot BS3t \cdot BS8r \cdot M4$$

$$L_{\mathrm{r1}A} = BS11t \cdot BS4r \cdot BS6t \cdot BS8t \cdot M4 \tag{4}$$

Then we take ratios of $P_{\mathrm{m}}/P_{\mathrm{r}}$ for both equations 1 and 3

$$\frac{P_{\mathrm{m}}}{P_{\mathrm{r}}} \cdot \frac{L_{\mathrm{m}RA}}{L_{\mathrm{r}RA}} = \frac{\Sigma_{\mathrm{m}RA}}{\Sigma_{\mathrm{r}RA}} \qquad\qquad \frac{P_{\mathrm{m}}}{P_{\mathrm{r}}} \cdot \frac{L_{\mathrm{m1}A}}{L_{\mathrm{r1}A}} \cdot TM1r \cdot WIN1t^2 = \frac{\Sigma_{\mathrm{m1}A}}{\Sigma_{\mathrm{r1}A}} \tag{5}$$

and equate them. This provides the way to express test mass one reflectivity in terms of powers measured with the QPDs of the *X1* and *Reference* interferometers and the optical components

$$TM1r \cdot WIN1t^2 = \frac{L_{\mathrm{r1}A}}{L_{\mathrm{m1}A}} \cdot \frac{\Sigma_{\mathrm{m1}A}}{\Sigma_{\mathrm{r1}A}} \cdot \frac{\Sigma_{\mathrm{r}RA}}{\Sigma_{\mathrm{m}RA}} \cdot \frac{L_{\mathrm{m}RA}}{L_{\mathrm{r}RA}}. \tag{6}$$

It should be noted that we cannot independently measure the properties of the optical window. Therefore we measure a combination of the test mass reflectivity multiplied by the square of the optical window transmissivity.

4. Photodiode calibration

Next set of experiments are performed by modulating the power of the measurement and reference beams with 0.1 and 1% around the constant power. It introduces additional force from the radiation pressure at the frequencies of the modulation. There are two independent ways to estimate this force. First, from the differential displacement measured by the interferometer. Differential acceleration can be inferred from the differential displacement in the following way

$$\Delta g(t) = \Delta \ddot{x}(t) - \Delta g_c + \omega_{12}^2 x_1(t) + \omega_2^2 \Delta x(t) - \delta_{\mathrm{ifo}} \ddot{x}_1(t), \tag{7}$$

where $x_1(t)$ is the measurement of the position of the first test mass to the optical bench. $\Delta x(t)$ is the relative position of the two test masses. ω_2^2 and ω_{12}^2 are the relative stiffness and stiffness for the second test mass which couple spacecraft motion into $\Delta g(t)$. $\Delta g_c(t)$ is the commanded force on the test mass two and δ_{ifo} is the cross coupling term from the interferometer readout [2].

Second way to calculate the force acting on the test masses from the radiation pressure, is by estimating the radiation pressure that laser beam applies on the test masses. This can be inferred from by propagating the power measured with the photodiodes to the test masses in the following way

$$a_{\mathrm{est}} = \frac{1 + TM1r}{Mc} P_{\mathrm{TM1}} + \frac{1 + TM2r}{Mc} P_{\mathrm{TM2}}. \tag{8}$$

Here c is the speed of light, M is the mass of the test masses, $TM1r$ is the reflectivity of the test mass one and $TM2r$ is the reflectivity of the test mass two. Variables P_{TM1} is the power propagated to the test mass one and P_{TM2} power propagated to the test mass two. For example, powers from the QPDs propagated to the test masses are

$$P(TM1) = \Sigma_1 / (M1 \cdot BS3t \cdot 0.5(BS8t + BS8r \cdot M4)), \tag{9}$$

$$P(TM2) = \Sigma_{12} / (M5 \cdot 0.5(BS10r + BS10t)). \tag{10}$$

The radiation pressure measurement is a completely independent check of the differential acceleration measurement. At the same time the residual differential acceleration measurement can be used to calibrate the measurement of the power at the photodiodes which involve unknown efficiency of the photodiode. Repeating this experiment several time during the mission allows to measure the decay of the photodiodes.

5. Acknowledgments

References

[1] Robertson D.I. *et al*, Construction and testing of the optical bench for LISA Pathfinder, CQG 2013
[2] Armano, M. *et al*, Sub-Femto-*g* Free Fall for Space-Based Gravitational Wave Observatories: LISA Pathfinder Results, PRL 2016