Function, Set-theoretical

In set-theory, a function is the assignment of a unique value taken from a set C, the 'Codomain,' to all the members of a set D of elements, the 'domain.' Both D and C may themselves consist of *n*-tuples of elements. D may be included in C and vice versa. It is common terminology to speak of 'a function from D to C,' or f: D → C, or to speak of D providing the *input*, and C the *output* of the function. A function is therefore a set of pairs ⟨x, y⟩ such that x is taken from a domain D and y is taken from a codomain C and for each x∈D there is precisely one y∈C.

The notion of function is fundamental to any form of calculus. In fact, a calculus is, in principle, a set of functions. Multiplication, for example, is a function from pairs of numbers to numbers, or *m*: ⟨x, y⟩ → z. Functions are also both frequent and important in ordinary life. It takes little effort to realize that we all live with functions, for example, from persons to dates and to places of birth, or, in monogamous societies, from husbands to wives and vice versa. There always is a function from rooms to temperatures, from objects to pairs of times and places, etc.

In *Formal Semantics* the notion of function is fundamental, as standard Formal Semantics is based on the principle that, given a model $\mathfrak{M}$ consisting of a particular situation ('world') W, a language L and an interpretation I, i.e., a specification of the extension of the terms and predicates of L in W, it must be possible to compute the truth-value in W of any well-formed sentence-structure ('tree'-structure) S of L by the application of a *functional calculus*. In a functional calculus one of the branches leading from a node N in the tree-structure S represents a function, while the one or more other (sister)-branches represent the elements whose n-tuples are input to the function. The value of the function is then passed on to the dominating node, and so on till the top node, which must represent a truth-value ('1' for truth and '0' for falsity). To give an extremely simple example, suppose L contains (generates) the tree structure $_S[_V[\text{laugh}]_{NP}[\text{J}]]$, where $_V[\text{laugh}]$ has a (characteristic) function associated with it, for example {⟨*Mary*,0⟩, ⟨*Sue*,1⟩, ⟨*John*,1⟩, . . . , ⟨*Carol*,0⟩}. Let $_{NP}[\text{J}]$ denote the element *John*. Then the value assigned by $_V[\text{laugh}]$ to *John* is 1. This value is passed on to the dominating node S. In other words, $_S[_V[\text{laugh}]_{NP}[\text{J}]]$ is true in this model. A language which is built and defined in such a way that it allows for the functional computation of truth-values for its sentences in any W, given some I, is called *compositional*. Formal Semantics is based on the assumption that natural languages are compositional.

See also: Bijection; Characteristic Function; Formal Semantics; Compositionality, Principle of.

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