for the proposition expressed by B to be true. (From here on, when necessary, a reference to a sentence is to be taken as a reference to the proposition expressed by it in every context of use.) Thus, A is an entailment of B (B \neq A). Since entailments are the business of logic, this implies that presupposition is in any case relevant in the logical analysis of natural language. Presuppositional entailments distinguish themselves, however, from other, 'classical,' entailments in that in an orderly presentation, transfer, and storage of information, that is, in a coherent discourse, they are, in some sense, prior to their carrier sentences. They restrict the domain within which their carrier sentence is interpretable. This, in turn, implies that presupposition is relevant in the analysis of the cognitive processes involved in the linguistic transfer of information. Such properties are commonly called 'discourse-related properties' of language.

The following examples illustrate the difference between classical and presuppositional entailments. In (1a, b), the first sentence classically entails (\models_c) the second; in (2a-d) the first sentence presupposes (\gg) the second:

The king has been assassinated \models_c The king is dead	(la)
Nob works hard ⊧ _c Nob works	(1b)
Nob lives in Manchester » There exists someone called 'Nob'	(2a)
Sue has forgotten that Nob was her student » Nob was Sue's student	(2b)
Nob has come back » Nob went away	(2c)
Nob still lives in Manchester » Nob lived in Manchester before	(2d)

In (2a) one finds an example of so-called 'existential presuppositions' (see Existential Presupposition). These were the main starting point for presupposition theory in philosophy. Number (2b) exemplifies 'factive presuppositions' (Kiparsky and Kiparsky 1971; see Factivity); the truth of the that-clause is presupposed. In (2c) we have a case of 'categorial presupposition' (see Categorial Presupposition); these are directly derived from the lexical meaning of the main predicate (come back). And (2d) belongs to a 'remainder category'; the presupposition being due to the adverb still.

The distinction between classical and presuppositional entailments gives rise to the question of whether the distinction is purely logical, or partly logical and partly to do with the orderly transfer of information—i.e., discourse-related, or entirely discourse-related, and hence irrelevant to logic. Answers to this question will be heavily theory-dependent and bound up with the question of how the disciplines concerned—mainly logic, semantics, and pragmatics—are to divide the labor. A decision will involve a whole theoretical paradigm, and only a wide variety of data, analyses, and other kinds of considerations will be able to tip the balance.

For some, discourse-related properties are *pragmatic*. The tendency here is to equate the logic and the semantics of language, the logic being classical and thus bivalent. Anything falling outside classical logic is taken to be pragmatic, including all discourse-bound aspects. In this view, presupposition is nonlogical and purely pragmatic, and the entire burden of explanation is thus placed on a pragmatic theory still largely to be developed.

Presupposition

A presupposition is a property of a sentence, making that sentence fit for use in certain contexts and unfit for use in other contexts. Most natural language sentences carry one or more presuppositions. If a sentence B carries a presupposition $A(B\gg A)$, then A must be true for B to be true, or more precisely, the proposition expressed by A must be true

Others take presupposition to be a *semantic* property. They make a primary distinction between what is part of the linguistic system, that is, at 'type'-level, and what results from the interaction of the linguistic system with any contingent state of affairs in the actual or any imagined world, that is, at 'token'-level (see Type/Token Distinction). In this view, all systematic linguistic aspects of the machinery, whereby speakers' cognitive contents (mental propositions) representing possible states of affairs are signified by uttered sentences and hence transferred to listeners, are considered to be semantic, whereas aspects to do with conditions of use are called pragmatic. Typically, in this view, semantics is taken to comprise a great deal more than what is provided by logic, and the logic to be adopted may well, if it incorporates the notion of presupposition, turn out to deviate from classical bivalent logic. In this semantic view, presupposition is at least partly, and for some entirely, a logical phenomenon. The terminological difference thus reflects different attitudes regarding the status of logic vis-à-vis semantics and the autonomy of the linguistic system, that is, the grammar and the semantics, of a natural language.

Finally, there is a diminishing school that looks upon presupposition as a purely *logical* phenomenon, requiring a nonclassical logic.

1. Operational Criteria

Whichever position one takes, it is clear that presuppositions are systematic properties of sentences, detectable ('observable') irrespective of actual conditions of use, though, apparently, their rationale is to restrict the usability of sentences to certain classes of context (discourse). This appears from the fact that, like classical entailments, presuppositions can be read off isolated sentences, such as those in (2), given without any special context. Yet these sentences evoke a certain context or class of contexts. Example (2a) evokes a context in which there is someone called 'Nob'; (2b) requires it to be given in the context, and thus evokes such a context, that Nob was Sue's student; (2c) requires it to be given that Nob was away; and (2d) that he lived in Manchester before. This, together with the logico-semantic property of entailment, provides a set of operational criteria to distinguish and recognize actual presuppositions of sentences.

First, if $B \gg A$ then in any case $B \models A$. This can be operationally tested (not defined) as follows. If the conjunction of sentences 'maybe not A, yet B' is recognized as per se incoherent then $B \models A$. Clearly, in all cases of (1) and (2) above, this test yields a positive result. For example, sentence (1a') is clearly per se incoherent (signaled by '!'):

! Maybe the king is not dead, yet he has been assassinated (1a')

But it yields a negative result when applied to (3a, b), since (3c) is coherent. Therefore, (3a) does not entail (3b) (i.e., $(3a) \not\models (3b)$). The relation between these two sentences is of a different kind:

Lady Fortune neighs (3a)

Lady Fortune is a horse (3b)

Lady Fortune may not be a horse, yet she neighs (3c)

The customary heuristic criterion for the entailment relation in $B \models A$ is the incoherence of 'not A, yet B.' This, however, is too strong, since it would incorrectly make (4a)

entail (4b), given the incoherence of (4c). Example (4d), on the other hand, is still coherent:

The king may have been assassinated (4a)

The king is dead (4b)

!The king is not dead, yet he may have been assassinated (4c)

The king may not be dead, yet he may have been assassinated (4d) (and thus be dead)

The difference is caused by the fact that natural language operators of epistemic possibility, such as English may, require compatibility of what is said in their scope with what is laid down as being the case in adjacent discourse (or in any knowledge store operational during the discourse). But they do not bring along the entailment of everything that is compatible with what is in their scope. Generally, if $B \models A$ (and A is not logically necessary), then $Possibly(B) \not\models A$. Yet not-A but Possibly(B) is incoherent for reasons to do with discourse construction: B (the scope of *Possibly*) is incompatible with the negation of its entailment A. But since B is in the scope of the entailment-canceling operator Possibly, no conclusion can be drawn with respect to the entailment properties of B. In the configuration Possibly(not-A), yet B, there is again discourse incoherence if B + A, and for the same reason. But now B does not stand under any entailment-canceling operator, and it is thus legitimate to draw a conclusion with respect to the entailment properties of B.

The 'entailment criterion,' that is, the incoherence of 'maybe not A, yet B,' yields identical results for all entailments, whether classical or presuppositional, and thus does not distinguish between the two categories. There is, however, a corollary which does make the distinction. If B>>A and B is the scope of an entailment-canceling operator, A will survive not as an entailment but as a more or less strongly invited inference. Generally, $O(B_A) > A$, where 'BA' stands for 'B presupposing A,' 'O' stands for any entailment-canceling imbedding operator, and '>' stands for invited inference. In standard terminology it is said that the presupposition of B is 'projected' through the imbedding operator. The conditions under which presuppositions of imbedded clauses are projected through imbedding operator structures constitute the well-known 'projection problem' of presupposition (see Projection Problem).

Projection is typical of presuppositions, not of classical entailments, of imbedded clauses. Thus, (5a) > (5b), but (6a) > (6b), precisely because (5b) is a presupposition of the imbedded clause *Nob lives in Manchester*, whereas (6b) is a classical entailment of *The king has been assassinated*.

Sue believes that Nob lives in Manchester (5a)

There exists someone called 'Nob' (5b)

Sue believes that the king has been assassinated (6a)

The king is dead (6b)

The projection criterion is most commonly used with the negation as the entailment-canceling operator. Strawson (1950; 1952) observed that presupposition is preserved as entailment under negation. In his view, a sentence like:

The present king of France is not bald

still presupposes, and thus entails, that there exists a king of France, who therefore, if (7) is true, must have hair on

his head. Strawson's observation was perhaps made without due consideration of the complications involved, since in many but not all cases presupposition is weakened to invited inference under negation. In any case, it was highly influential, and the so-called 'negation test' became the standard test for presupposition in much of the literature. Provided the notion of entailment is replaced by that of invited inference, this test is sound.

A further criterion to separate classical from presuppositional entailments is the 'discourse criterion.' Any bit of discourse 'A and/but B_A ' (taking into account anaphoric processes) will be felt to be orderly and well-planned. We shall use the term 'sequential' to refer to the typical quality of presuppositionally well-ordered texts, without implying that texts that are not, or not fully, sequential are therefore unacceptable in some sense. The concept of sequentiality is used only to characterize stretches of acceptable texts that have their presuppositions spelled out. Fully sequential texts will tend to be dull, but well-ordered. This is demonstrated by the following bits of discourse (' \sqrt ' signals sequentiality):

$$\sqrt{\text{Nob was Sue's student}}$$
, but she has forgotten that he was (8b)

$$\sqrt{\text{Nob was away, but he has come back}}$$
 (8c)

$$\sqrt{\text{Nob lived in Manchester before, and he still lives there}}$$
 (8d)

Classical entailments generally lack this property. When a classical entailment or an inductive inference precedes its carrier sentence the result may still be acceptable, yet there is a clear qualitative difference with sequential texts, as is shown in (9a, b), where a colon after the first conjunct is more natural ('#' signals nonsequential discourse):

The discourse criterion still applies when a presupposition is weakened to an invited inference. A discourse 'A and/but $O(B_A)$ ' will again be sequential:

$$\sqrt{\text{Nob was Sue's student}}$$
, but she has probably forgotten (10b) that he was

$$\sqrt{\text{Nob}}$$
 went away, and he has not come back (10c)

In practice, the combination of these tests will reliably set off presuppositions from classical entailments.

2. The Logical Problem

2.1 The Threat to Bivalence

The first to signal the fact that presuppositions are a threat to standard logic was Aristotle's contemporary, Eubulides of Miletus (Kneale and Kneale 1962: 113–17). He is known for his arguments against the Aristotelian logical axiom of 'strict bivalence,' also called the 'principle of the excluded third' (PET). This principle consists of two independent subprinciples: (a) all sentences always have precisely one truthvalue (hence no truth-value gaps), and (b) there are precisely two truth-values, 'true' and 'false' (weak bivalence). Eubulides formulated a few so-called 'paradoxes' (the most

famous of which is the liar paradox), including the 'Paradox of the horned man' (Kneale and Kneale 1962: 114): 'What you have not lost you still have. But you have not lost horns. So you still have horns.'

This paradox rests on presuppositional phenomena. Let B be You have lost your horns, and A You had horns. Now $B \gg A$, but $not(B) \not \gg A$, though not(B) > A. Eubulides, like Strawson, wanted presupposition to hold for the carrier sentence both with and without the negation. Then there would be both $B \models A$ and $not(B) \models A$, which, under PET, would mean that $not(A) \models not(B)$ and $not(A) \models B$. In other words, not(A) would have contradictory entailments, and thus be a necessary falsehood. A would thereby be a necessary truth, which, of course, is absurd for such a typically contingent sentence as You had horns. To avoid this, PET would have to be dropped. Although Aristotle himself was unable to show Eubulides wrong, there is a flaw in the paradox of the horned man. It lies in the first premiss What you have not lost you still have. For it is possible not to have lost something precisely because one never had it.

In the early 1950s, Eubulides' point was taken up by Strawson, who also posited the preservation of presupposition under negation. In Strawson's view, nonfulfillment of a presupposition leads to the carrier sentence A, and its negation, lacking a truth-value altogether. In allowing for truth-value gaps he thus denied subprinciple (a) of PET.

From a different angle, Frege (1892) had come to the same conclusion, at least for existential presuppositions. A sentence like:

is analyzed by Frege in the traditional way as 'Bald(the present king of France),' i.e., as a predicate with its subject argument term. The predicate bald extends over all bald individuals in this or any world. Now, to decide whether (11) is true, or false an individual i referred to by the definite description the present king of France is needed. If i is a member of the set of bald individuals the sentence is true; if not it is false. In the absence of any present king of France there is thus no way, in Frege's analysis, of computing the truth-value of either that sentence or its negation (i.e., (7)). Both will, therefore, fall into a truth-value gap.

Frege's argument posed a profound problem for standard logic. If sentence (11) is analyzed in the Fregean way then, in any strictly bivalent logic, the sentence *There is a king of France* must be considered a necessary truth, which is absurd. Put differently, the applicability of standard logic to English would have to be made dependent on the contingent condition of there being a king of France—a restriction no true logician will accept. Subsequently, two traditions developed in the effort to solve this problem, the Russell tradition and the Frege–Strawson tradition. In their present form, the two have begun to converge, yet they remain stuck in certain stubborn inadequacies. A third solution is beginning to present itself.

2.2 The Russell Tradition

It was this problem of empty reference that stirred the young Bertrand Russell into action. Having devised his solution to the problem of universal quantification over empty sets (*All square circles are in London*: true or false?), which had beset traditional predicate calculus ever since its

Aristotelian beginnings, he now proceeded to solving the problem of definite descriptions without a reference object. In 1905, Russell published his famous article *On referring*, where he proposed that a sentence like (11) should not be analyzed in the traditional (Fregean) way. Putting the new theory of quantification to use, he argued that (11) should be analyzed as follows:

$$\exists x [KoF(x) \land Bald(x) \land \forall y [KoF(y) \rightarrow x = y]]$$
 (12)

or: 'There is an individual x such that x is now king of France and x is bald, and for all individuals y, if y is now king of France, y is identical with x.' In other words: 'There is now precisely one king of France, and he is bald.' In order to save bivalence, Russell thus rejected the time-honored subject-predicate analysis used in logic as well as in grammar, replacing it by an analysis in terms of existential and universal quantification. The definite description the present king of France thus no longer forms a structural constituent of the logically analyzed sentence. It is dissolved into quantifiers and propositional functions.

The negation of (11), that is, (7), should be analyzed logically as (12) preceded by the negation operator, i.e., as (13a). However, for reasons best known to themselves, speakers often prefer to interpret (7) as (13b), with the negation restricted to the propositional function 'Bald(x)':

$$\neg [\exists x [KoF(x) \land Bald(x) \land \forall y [KoF(y) \rightarrow x = y]]]$$
 (13a)

$$\exists x [KoF(x) \land \neg [Bald(x)] \land \forall y [KoF(y) \rightarrow x = y]]$$
 (13b)

In practice, therefore, a sentence like (7) is ambiguous.

This proposal, known as Russell's *Theory of Descriptions*, quickly became standard among logicians and philosophers of language, precisely because it saved classical logic, with its cherished PET, from Frege's problem. At the same time, however, it brought about a deep rift between logic and grammar, since the Russellian way of analyzing sentences ran counter to any accepted notion of linguistic structure. From 1900 onward, grammarians (linguists) preached the irrelevance of logic to the study of language, and not until the 1970s did a rapprochement come about.

Although Russell's *Theory of Descriptions* saves classical logic, it fails to save the facts of natural language. Those who recognized this, modified Russell's analysis in various ways, without, however, giving up the original idea. There thus came about a 'Russellian tradition' in the analysis of definite descriptions, and presuppositions in general.

The first, and most obvious, objection concerns the so-called 'uniqueness clause' in (12)— $\forall y[KoF(y) \rightarrow x = y]$ —which is meant to ensure that only one king of France is said to exist and thus to account for the uniqueness expressed by the definite article. It is clear, however, that the use of the definite article involves no claim to uniqueness of existence, but only to discourse-bound uniqueness of reference. The uniqueness clause was thus quietly dropped early on in the piece.

Another objection is that this theory is limited to definite descriptions and thus in principle is unable to account for other than existential presuppositions. Factive and categorial presuppositions, as well as those derived from words like *all*, *still*, or *only*, fall in principle outside its scope. Yet analogous problems arise. For example, (14a)»(14c), yet likewise (for reasons to be discussed below) (14b)»(14c),

and (14b) is, to the best of our analytical powers, the logical negation of (14a):

Likewise $(15a)\gg(15c)$, and $(15b)\gg(15c)$ even though (15b) is the negation of (15a):

The presupposition structurally associated with cleft and pseudocleft sentences behaves in the same manner, as is seen from (16) and (17), exemplifying clefts and pseudoclefts, respectively:

Both (16a) and its negation (16b) presuppose (16c), and likewise for (17).

These are cases, overlooked by Eubulides, Strawson, and others, where presupposition is indeed fully preserved under negation. Consequently, in classical bivalent logic the presuppositions of sentences like (14a, b), (15a, b), (16a, b), or (17a, b) would be necessary truths.

Presupposition theorists see the same problem in (18), where both (18a) and its negation (18b) presuppose (18c):

In Russellian predicate calculus, however, (18a) does not entail (18c), and thus cannot presuppose it, whereas (18b) \(\pop(18c)\). Yet presupposition theorists will maintain that (18a) does entail (18c)—in fact, There may not be any men, yet all men are mortal is grossly incoherent—and that (18b) does not classically entail but presuppose (18c): There exist men and/but not all men are mortal is an acceptable discourse. Russellian predicate calculus thus seems to fit the presuppositional facts badly.

In order to generalize the *Theory of Descriptions* to other than existential presuppositions, some logicians have proposed to modify Russell's analysis as given in (12) to:

$$\exists x [KoF(x)] \land Bald(he)$$
 (19)

or 'There is now a king of France, and he is bald.' The bracketing structure is changed: The subject he of 'Bald' is no longer a bound variable, but an anaphoric expression. If a mechanism for this kind of anaphora can be provided, the analysis can be generalized to all kinds of presupposition. A sentence A_B is now analyzed as 'B and A_B ,' and A_B can be said to be normally analyzable as 'B and A_B ,' with small scope for not, though discourse conditions may force the analysis ' \Box [B and A_B],' with large scope for the negation. This analysis, which saves PET, is known as the

'conjunction analysis for presupposition.' Kamp (1981) and Groenendijk and Stokhof (1991), each with a specific anaphora mechanism, defend this analysis for existential presuppositions.

The introduction of an anaphora mechanism is necessary anyway, since the original Russellian analysis as given in (12) fails for cases like (20), where classical quantifier binding is impossible for the anaphoric expression it (the dog), which is in the scope of I hope whereas I hope is not in the scope of I know:

Geach argued (1972: 115-27) that a sentence like:

should be analyzed not with an anaphoric it, analogous to (19), but as $\exists x[Dog(x) \land White(x)]$, on the grounds that this is fully compatible with $\exists x[Dog(x) \land \neg White(x)]$, just as (21) is fully compatible with:

In the conjunction analysis, however, there is incompatibility between $\exists x[Dog(x)] \land White(it)$ on the one hand, and $\exists x[Dog(x)] \land \neg White(it)$ on the other, since $A \land B$ and $A \land \neg B$ are incompatible (contrary). Cases like (20), however, show that the bound variable analysis favored by Geach lacks generality (see Seuren 1977; 1985: 319–20).

Even so, the incompatibility problem remains for the conjunction analysis, which is unable to account for the fact that (23a) is coherent but (23b) is not:

Clearly, in (23a) there are two dogs, due to the repetition of there was a dog, but in (23b) there is only one. Yet the conjunction analysis cannot make that difference, since the repetition of there was a dog makes no semantic difference for it. Recently, attempts have been made to incorporate this difference into the logic (e.g., Kamp 1981; Heim 1982; Groenendijk and Stokhof 1991). The usual procedure is to attach a memory store to the model theory which keeps track of the elements that have so far been introduced existentially, i.e., some form of discourse-based semantics. Now, the second occurrence of there was a dog in (23a) represents a different proposition from the first, so that the propositional analysis is no longer $[a \wedge b] \wedge [a \wedge \neg b]$ but $[a \wedge b] \wedge [c \wedge \neg d]$, which shows no incompatibility.

The common motivation in this Russellian tradition of analyzing definite descriptions was always the wish to do justice to the facts of language without giving up PET as a logical axiom. In its latest forms, the conjunction analysis deviates in certain ways from Russell's predicate calculus, yet it leaves PET unaffected. Not all philosophers of language, however, were so attached to PET. Some felt that both the theory and the facts are better served without it.

Even in its most up-to-date versions, the conjunction analysis still has to cope with a number of problems. Thus, without ad hoc provisions it still seems necessary to postulate existence for term referents that are explicitly said not to exist:

The imaginary conspiracy was widely publicized

Clearly, analyses like 'there exists a monster of Loch Ness and/but it does not exist' or 'there existed an imaginary conspiracy and/but it was widely publicized' do injustice to both the logic and the semantics of such sentences. Moreover, the conjunction analysis forces one to say that the negation, in, for example, John did not buy a car is sentence negation, but in John did not buy the car it normally only negates the second conjunct and thus does not function as sentence negation. And if, in the latter case, the negation is indeed full sentence negation and thus cancels presuppositions, that is, as '\[\begin{array}{c} \be

marked and pragmatically plausible only in contexts where

a previously uttered or suggested A_B is radically denied

2.3 The Frege-Strawson Tradition

because of presupposition failure.

Strawson (1950; 1952) was the first to oppose the Russell tradition. Rejecting the *Theory of Descriptions*, he reverted to the traditional subject–predicate analysis for sentences with definite descriptions as their subject. He discussed existential presuppositions only, and only under extensional predicates, excluding cases like those in (24). He moreover neglected the Russellian wide-scope reading of negation, considering only the presupposition-preserving reading, interpreting that as the normal logical sentence negation. For Strawson, if $B \gg A$ then also $not(B) \gg A$, and when A fails to be true (presupposition failure), both B and not(B) lack a truth-value. The definition of presupposition is strictly logical: $B \gg A =_{Def} B \models A$ and $not(B) \models A$ and nontruth of A necessarily goes with both B and not(B) lacking a truth-value.

The logic of this system is bivalent with gaps, i.e., sentences without a truth-value in models where B is not true. Since lack of truth-value is hardly a valid input for a truth-function, Strawson's 'gapped bivalent propositional calculus' (GBC) is best reconstructed as shown in Fig. 1 (where

			^ B		∨ B			
~A	Α	1	2	*	1	2	*	
2 1 *	1 2 *	1 2 *	2 2 *	* *	1 1 *	1 2 *	*	

Figure 1. Gapped bivalent propositional calculus (GBC)

the symbol '~' stands for presupposition-preserving negation, '1' for truth, '2' for falsity, and '*' for lack of truth-value).

Insofar as truth-values are assigned, this calculus preserves the classical tables. Moreover, * ('unvalued') is 'infectious': wherever it appears in the input to a truth-function, the output is unvalued. GBC has the remarkable property of limiting the applicability of (bivalent) logic to cases where the, mostly contingent, presuppositions of the sentences involved are fulfilled (true). If U is the set of all possible states of affairs, then GBC operates in a different U for different sets of sentences. GBC is subject to a flexible, or 'dynamic' Us, defined for any specific set of sentences

S, and it can express propositions about states of affairs outside U_S only by the addition of existentially quantified sentences without presuppositions.

This analysis of presupposition was, partly successfully, criticized by Wilson (1975) and Boër and Lycan (1976). These authors side with Russell and show that in a sentence like (7) the negation is not presupposition-preserving since entailment does not hold. A sentence like (25) is coherent, though it requires emphatic, discourse-correcting accent on not

The present king of France is NOT bald. There is no king of France! (25)

Wilson, in particular, gives many examples of presuppositions of negated carrier sentences where presuppositional entailments are canceled under emphatic negation. The projection of the presupposition through negation, as well as the satisfaction of the discourse criterion are to be explained by a separate pragmatic theory. Logically speaking, presuppositions are simply entailments, though they have their own pragmatic properties. Logic has no place for them. Hence, these authors say, there is nothing amiss with classical bivalent logic as an analytic tool for language. This analysis may be called the 'entailment analysis' of presupposition.

If presuppositional entailments were always canceled by negation, little could be said against the entailment analysis (but for the failure of any pragmatic theory to account in anything like a satisfactory way for the projection and discourse properties of presuppositions of negated carrier sentences). But this is not so. Under certain definable conditions, natural language not is clearly presupposition-preserving (Seuren 1985: 228–33). Thus, in English, when sentence-negating not occurs in any other than the canonical position of negation, that is, in construction with the finite verb, it is per se presupposition-preserving. Examples (14b) and (18b) above, with fronted not, preserve their presuppositions. And in (26), not is in construction with the infinitive to realize, and therefore also preserves the factive presupposition induced by this verb:

Furthermore, as illustrated in (15) above, factive thatclauses in fronted position cause the negation over the factive main predicate to preserve the factive presupposition. Then, cleft and pseudocleft presuppositions are always saved under negation, as is seen in (16) and (17). In fact, the kind of discourse-correcting highly marked 'echo' negation found in (25) and similar examples is impossible for all the cases in (27):

!Not only Nob laughed. He didn't laugh!	(27a)
!Nor all men are mortal. There don't exist any men!	(27b)
! Nob seems NOT to realize that he is in trouble. He isn't in trouble!	(27c)
!That Nob laughed did NOT surprise Sue. He didn't laugh!	(27d)
!It was NOT Nob who laughed. Nóbody laughed!	(27e)

The same holds for the negation that is required with 'negative polarity items' (NPIs) in simple assertive main clauses (the NPIs are italicized):

(27f)

!Who laughed was not Nob. Nóbody laughed!

	(204)
! Nob does not mind that he is in trouble. He isn't in trouble!	(28b)
! Nob does not live in Kentucky any longer. He never lived there!	(28c)
! Nob has not come back yet. He never went away!	(28d)
!Sue has NOT seen Nob in weeks. She doesn't exist!	(28e)

(280)

! Nob did Not laugh at all. He doesn't exist!

This test to show the canceling of presuppositional entailment, applied in (25), (27), and (28) and used by Wilson as well as Boër and Lycan, is none other than the customary entailment criterion 'not A, yet B' mentioned earlier. It involves, moreover, the typical marked discourse-correcting emphatic *not* with 'echo'-effect. Application of the more refined test 'maybe not A, yet B' yields identical results for the cases at hand:

! Maybe Nob didn't laugh, yet not only Nob laughed	(29a)
! Maybe there exist no men, yet not all men are mortal	(29b)
! Maybe Nob is not in trouble, yet he seems not to realize that he is	(29c)
!Maybe Nob didn't laugh, yet that he laughed did not surprise Sue	(29d)
! Maybe no one laughed, yet it wasn't Nob who laughed	(29e)
! Maybe no one laughed, yet who laughed wasn't Nob	(29f)
! Maybe Nob isn't in trouble, yet he doesn't mind that he is	(29g)
! Maybe Nob never went away, yet he hasn't come back yet	(29h)
! Maybe Nob never lived in Kentucky, yet he doesn't live there any longer	(29i)

Curiously, however, application of the more refined test yields positive results for *all* presuppositions, not only those under presupposition-preserving negation:

!There may not be a king of France, yet he is not bald	(30a)
! Maybe Nob wasn't Sue's student, yet she has not	(30b)
forgotten that he was	

! Maybe Nob didn't go away, yet he hasn't come back (30c)

This would mean that negation does, after all, preserve presuppositions as full entailments. One notes, however, that when the more refined test is applied, as in (29) and (30), there is no way one can assign emphatic accent to not, because there is no discourse correction and hence no 'echo'-effect. The use of not in (29) and (30) is thus seen to be different from (25), and in general all similar cases presented in Wilson (1975) and elsewhere. And this use of not does preserve presuppositions.

This observation, together with the fact that under certain structurally definable conditions negation does preserve presuppositional entailments, renews the threat to classical bivalent logic. One proposal to solve this logical problem is to say that language conforms to a three-valued (trivalent) logic, which is identical to classical bivalent logic but for a distinction made between two kinds of falsity, each turned into truth (designated) by a separate negation operator. 'Minimal falsity' ('2') results when all presuppositions are true but not all classical entailments. 'Radical falsity' ('3') results when one or more presuppositions fail to be true. Correspondingly, 'minimal negation' (~) turns minimal falsity into truth, and truth into minimal falsity, leaving radical falsity unaffected, while 'radical negation' (~) turns radical falsity into truth, and both truth and

			∧ B			∨ B		
~A	Α	1	2	3	1	2	3	
2 1 3	1 2 3	1 2 3	2 2 2	3 2 3	1 1 1	1 2 3	1 3 3	
		ı						

Figure 2. Trivalent generalized calculus 1 (TGC1)

minimal falsity into minimal falsity. The radical negation enables one to utter a proposition about states of affairs falling outside the subuniverse for the discourse at hand, something which Strawson's GBC does not allow for.

From this point on there are two known ways to generalize 'classical bivalent propositional calculus' (CBC) to more values. The first is Kleene's (1938) 'trivalent generalized calculus' (TGC¹). It aims at preserving all theorems of CBC with bivalent ¬ replacing trivalent ~. This is what the truth-tables of Fig. 2 do. The generalization is that \land yields '2' whenever either conjunct is valued '2,' '1' only if both conjuncts are valued '1,' and '3' otherwise. Analogously, \lor yields '1' whenever either conjunct is valued '1,' '2' only if both conjuncts are valued '2,' and '3' otherwise.

This system is widely used by presuppositional logicians (e.g., Blau 1978; Blamey 1986). One notes that TGC¹ lacks the radical negation (≈), but the system will come to no harm if it is added, so that the two negations can formally distinguish between minimal and radical falsity.

The other generalization of CBC to more values is found in Seuren (1985; 1988). The operators \wedge and \vee select, respectively, the highest and the lowest of the component values. This results in truth-tables as shown in Fig. 3. Note that classical negation (\neg), which has been added for good measure, is the union of \sim and \simeq , as is likewise the case in TGC¹. In Seuren's view (as expressed in Seuren 1985; 1988), \neg does not occur in natural language, which has only \sim and \simeq .

It has been shown (Weijters 1985) that TGC^2 is equivalent with classical bivalent logic if only the operators \neg , \wedge , and \vee are used. Thus, closed under $\{\neg$, \wedge , \vee classical bivalent logic is independent of the number of truth-values (tv) employed, though any tv>2 will be vacuous.

Moreover, in both generalizations with n truth-values $(n \ge 2)$, there is, for any tv $i \ge 2$, a 'specific negation' N^i turning only that tv into truth, lower values into '2,' and leaving higher values unaffected. Thus, in TGC^2 , as in Fig. 3, N^2 is \sim and N^3 is \simeq . Classical bivalent \neg is the union of all specific negations. Consequently, in CBC, \neg is both the one specific negation allowed for and the union of all specific negations admitted. CBC is thus the most economical variety possible of a generalized calculus of either type, with just one kind of falsity.

				∧ B				∨ B			
$\neg A$	≃A	~A	Α	1	2	3	l	2	3		
2 1 1	2 2 1	2 1 3	1 2 3	1 2 3	2 2 3	3 3 3	1 1 1	1 2 2	1 2 3		
							1		- 1		

Figure 3. Trivalent generalized calculus 2 (TGC²)

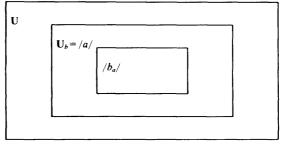


Figure 4. Valuation space construction of b_a and \mathbf{U}_b in \mathbf{U}

The distinction between the two kinds of falsity is best demonstrated by considering valuation spaces. Let U (the 'universe') be again the set of all possible states of affairs (valuations), and /A/ (the 'valuation space' of A) the set of all possible valuations in which A is true (A, B, ... being metavariables over sentences a, b, c, d, \ldots and their compositions in the language L). We now define U_A (the 'subuniverse' of A) as the set of all possible valuations in which the conjunction of all presuppositions of A is true. Since $A \models A$, /A/ is the valuation space of the conjunction of all entailments of A. And since $A \gg A$, $/A/\subset U_A$. $/\sim A/$ is the complement of /A/ in U_A , whereas $/\simeq A/$ is the complement of U_A in U. Clearly, $/\simeq A/\cup/\sim A/=/\neg A/$. If A has no presuppositions, then $/\sim A/=/\simeq A/=/\supset A/$. Conjunction and disjunction denote, as standard, intersection and union, respectively, of valuation spaces. For any valuation v_n , if $v_n \in A/$, $v_n(A) = 1$. If $v_n \in U_A$, $v_n(A) = 2$ and $v_n(\sim A) = 1$. If $v_n \in U - U_A$, $v_n(A) = 3$ and $v_n(\simeq A) = 1$ (for more details, see Seuren 1988).

The normal negation in language is the minimal negation (\sim) , denoting the complement of a sentence's valuation space within its subuniverse. And the normal truth-values speakers reckon with in undisturbed discourse are '1' and '2.' The function of the subuniverse of a sentence A, that is, U_A, is, typically, to limit the set of states of affairs (valuations) in which A can be uttered while being true or minimally false. Since presuppositions are type-level properties of sentences and thus structurally derivable from them, competent speakers immediately reconstruct UA on hearing A (this fact underlies the phenomenon of 'accommodation' or 'post hoc suppletion'; see below). And since they proceed on the default assumption of normal undisturbed discourse, the default use of negation will be that of \sim , \simeq being strongly marked in that it provokes an 'echo' of the nonnegated sentence (which, one feels, has been either uttered or anyway 'in the air' in immediately preceding discourse), and calls for a correction of preceding discourse.

The question of whether TGC² or TGC¹ is preferable for the description and analysis of presuppositional phenomena is hard to decide if presupposition is defined as follows (varying on Strawson's definition mentioned above):

$$B \gg A =_{Def} B \models A \text{ and } \sim B \models A \text{ and } \sim A \models \simeq B \text{ and } \simeq A \models \simeq B$$
 (31)

According to this purely logical definition in trivalent terms, if nontruth of A necessarily leads to radical falsity of B, then $B\gg A$. Extensive testing shows that, on this definition, both TGC^2 and TGC^1 suffer from empirical inadequacies.

 T_{GC}^2 is at a disadvantage for conjunctions of the form $A \wedge B_A$, since it predicts that $A \wedge B_A \gg A$ (nontruth of A)

gives radical falsity of B_A and thus of $A \wedge B_A$). A sentence like:

should thus presuppose that Nob's dog has died, which is not what the operational criteria tell us. TGC^1 fares better for this type of conjunction, since here, if $v_n(A) = 2$, $v_n(B_A) = 3$, and therefore, $v_n(A \wedge B_A) = 2$.

Both TGC¹ and TGC² make incorrect predictions for disjunctions (and hence conditionals). In both systems, the following theorem holds:

$$(\sim A \vee B_A) \wedge (\sim B_A \vee C_A) \gg A \tag{33}$$

Thus, a sentence like:

is said to presuppose that Nob's dog has died. And a sentence like:

presupposes that Nob's dog has died in TGC^1 , while it is entailed in TGC^2 . Both are thus seen to make incorrect predictions if and is made to translate \wedge , and 'if A then B' stands for $(\sim A \vee B)$.

3. The Discourse Approach

The problems raised by a purely logical definition of presupposition are compounded by the fact that a *logical* definition of presupposition, such as (31), inevitably entails that any arbitrary sentence will presuppose any necessary truth, which would take away all empirical content from the notion of presupposition. Attempts have therefore been made (for example, Gazdar 1979; Heim 1982; Seuren 1985) at viewing a presupposition A of a sentence B_A as restricting the interpretable use of B to contexts that admit of, or already contain, the information carried by A. If one sticks to trivalence, (31) may be weakened from a definition to a mere logical property:

If
$$B \gg A$$
 then $B \models A$ and $\sim B \models A$ and $\sim A \models \simeq B$ and $\simeq A \models \simeq B$ (36)

One advantage of this approach is that it leaves room for an account of the discourse-correcting 'echo' function of radical Not. Horn (1985; 1989) says, no doubt correctly (though his generalization to other metalinguistic uses of negation is less certain), that Not is 'metalinguistic,' in that it says something about the sentence in its scope. Neither TGC² nor TGC¹ accounts for this metalinguistic property. What Not('B_A') says about the sentence 'B_A' is that it suffers from presupposition failure, and thus cannot be coherently used in a discourse where A has been denied truth. Not is interpreted as the complex predicate 'belongs to the non-language for the discourse at hand.' The notion of 'non-language' is defined in terms of 'sequential discourse incrementation.'

The 'sequential incrementation' of a (monologue) discourse D is a process restricting D to specified valuation spaces. Intuitively, it locates the situation to be described in a progressively narrower section of U. Incrementation involves the assignment of the value '1' or '2' to sentences of the language L. The result of the sequential incrementation of A, or i(A), is a (further) restriction of the D under

construction to the intersection of /D/ and /A/. D+A is thus equivalent to $D \wedge A$: D can be considered the conjunction of all its sentences. The initial valuation space is U.

The 'sequentiality criterion' requires that:

- (a) if B>A then i(A) must precede i(B),
- (b) $i(A \wedge B)$ consists of i(A) followed by i(B),
- (c) for any D, $D/\supset D+A/$,

(d) no i(A) may result in the empty valuation space. Condition (a) requires that if A has not already been incremented prior to i(B_A) it is quietly 'slipped in.' This process is called 'accommodation' or 'post hoc suppletion.' A text requiring accommodation is not fully sequential. Condition (b) splits up a conjunction into separate subsequent incrementations of its conjuncts. Condition (c) is the 'informativeness condition.' It requires that every subsequent incrementation restricts the valuable space of D, thus specifying further the situation to be described. Condition (d) prevents logical inconsistency in any D. Again, the sequentiality criterion does not imply that a discourse or text not satisfying it is unacceptable in some sense. It only sets out the prototypical conditions of a possibly unexciting but well-ordered discourse.

If A is valued '2,' then B_A is excluded from D since now both B_A and $\sim B_A$ are valued '3,' which is not allowed in D: neither B nor $\sim B$ can be processed in D. Thus, at each stage Q in the development of D there is a set of sentences of L that are excluded from the further development of D, and also a set of sentences that can still be processed. The former is the 'nonlanguage' of D at Q, or $NL(D)^Q$; the latter is the 'presuppositional language' of D at Q, or $PL(D)^Q$. $PL(D)^Q$ is thus defined by the constraint that D contains no negation of any of the presuppositions of the sentences of $PL(D)^Q$.

For example, let D consist of the sentences a, b_a , and $\sim c_b$, in that order (b_a has no presuppositions beyond a, and analogously for c_b). Now $d_c \in \mathbf{NL}(\mathbf{D})^c$, since c is valued '2.' The sentence Not('d') is now true, as it says precisely this. In this interpretation, Not('d') is incremented the way sentences normally are: $/\mathbf{D}/$ will contain states of affairs involving sentences, which are objects like other objects. But the logical relation of Not('d') with respect to $\sim d$ and d cannot be expressed due to the metalevel shift. Not('d') is now understood as 'the truth-commitments entered into so far are satisfied and d suffers from presupposition failure; it therefore belongs to $\mathbf{NL}(\mathbf{D})^c$.' Both $\mathbf{i}(d)$ and $\mathbf{i}(\sim d)$ result in the empty valuation space, as $(/d/\cup/\sim d/)\cap/\sim c/=\varnothing$. This is illustrated in Fig. 5, where the area covered by vertical lines is $/\sim c/$.

Under this construal of the notion of sequential discourse, we are practically back at Strawson's GBC (Fig. 1), but with the extra provision of sentences (and other linguistic objects) as possible reference objects. It follows from this analysis that natural language negation is ambiguous between \sim and NoT, the latter being interpretable as 'belongs to $NL(D)^{now}$,' and thus providing a functional instrument for correcting any discourse whose sequential reconstruction (that is, with full post hoc suppletion of all implicit presuppositions) is inconsistent.

This may provoke fears of a resurrection of the 'liar paradox.' Yet this fear is unfounded. Following the medieval solution to this paradox in terms of *vacatio*, that is, lack of

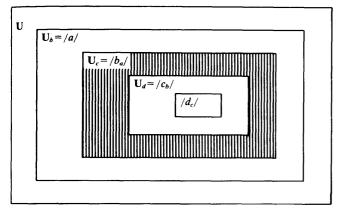


Figure 5. Valuation space construction of $/\sim c_{ba}/$ and $/d_c/$

reference (Bottin 1976; Seuren 1987), we say that the sentence *This proposition is false* fails to deliver a reference object for the subject term *this proposition*, so that the sentence fails to deliver a truth-value. This solution implies an infraction of PET, but so does the whole analysis.

A generalized trivalent logic is still useful in that it can express the logical properties of Not. If Not('d') (in the example of Fig. 5) is interpreted, after $\sim c_d$, as $\simeq d$, that is, as the intersection of $/\sim c/$ and $/\simeq d/$ (= $/\sim c/$ since $/\simeq d/$ = U-/c/), it is valued '1,' yet has no effect on the incrementation of D: it does not restrict further the valuation space of D and thus violates condition (c) of the sequentiality criterion. But now the metalinguistic character of Not is hignored and i($\simeq d$) is true but not informative.

From this point of view, TGC¹ and TGC² differ as follows. In TGC¹, as can be seen from Fig. 2, $U_{A \wedge B} = (U_A \cap U_B) \cup$ $/\sim A/\cup/\sim B/$. Thus, if A has no presuppositions, $U_{A \wedge B_A} =$ $(U \cap /A/) \cup (U-/A/) \cup (/A/-/B/) = U$. Consequently, $/\sim B_A/=/A/-/B/$, but $\sim (A \land B_A) \equiv \neg B_A$, even though $/B_A/=/A \wedge B_A/$. A fully sequential discourse will thus never be radically false in TGC¹, even when A is minimally false and all the remaining sentences are therefore radically false. But a discourse $[B_A + C_B + D_C + ...]$ (that is, with the initial presupposition kept implicit) will be radically false just in case A is not true. In TGC², on the other hand, the subuniverses for conjunction (and disjunction) run parallel: $U_{A \wedge B} = (U_A \cap U_B)$ (and $U_{A \vee B} = (U_A \cup U_B)$). Thus, $U_{[A+B_A+C_B+D_C]} = U_{[B_A+C_B+D_C]} = /C/$, as it makes no difference whether the presupposition is implicit or explicit. Since it makes a D radically false as soon as a radically false sentence is added, it allows for the rule that a minimally false D is made true simply by negating all its false sentences, without any sentence having to be rejected as belonging to NL(D), whereas a radically false D is made true by negating all its minimally false sentences and eliminating all sentences valued '3.' Such a trivalent logic requires that the notion of presupposition be limited to single increment units. In TGC², conjunctions (discourses) have subuniverses, but no presuppositions.

4. The Structural Source of Presuppositions

The structural source of three of the four types of presupposition that were distinguished at the outset of Sect. 1 can be identified uniformly: it lies in the lexical meaning conditions of the main predicate of the sentence (clause) in question. The lexical conditions of a predicate Pⁿ over individual

objects are the conditions that must be satisfied for any object, or n-tuple of objects, to be truthfully predicated by means of P^n . Thus, for the unary predicate bald the conditions must be specified under which any object can truthfully be called 'bald.' Or for the binary predicate wash it must be specified under what conditions it can truthfully be said of any pair of objects $\langle i, j \rangle$ that 'i washes j.' Analogously for predicates whose terms refer to things other than individual objects, such as sets of objects, or facts, or imbedded propositions.

In the light of presupposition theory, one can now, following Fillmore (1971), make a distinction between two kinds of lexical conditions (see *Lexical Conditions*), which we shall call the 'preconditions' and the 'satisfaction conditions.' The criterion distinguishing the two is that when any precondition is not fulfilled the sentence is radically false, whereas failure of a satisfaction condition results in minimal falsity. Fulfillment of all conditions results in truth.

The following notation makes the distinction formally clear. Let the extension of a predicate P^n be characterized by the function symbol σ . Then an n-ary predicate P^n over individuals will have the following schema for the specification of its lexical conditions:

$$\sigma(\mathbf{P}^{\mathbf{n}}) = \{ \langle i^{1}, i^{2}, \dots, i^{\mathbf{n}} \rangle : \dots \text{ (preconditions)} \dots \}$$
 (37) (satisfaction conditions) \dots \}

or: 'the extension of P^n is the set of all n-tuples of individuals $\langle i^1, i^2, \ldots, i^n \rangle$ such that ... (preconditions) ... and ... (satisfaction conditions) ... 'The preconditions and satisfaction conditions may affect any or all of the members of the n-tuple. The predicate *bald*, for example, can be considered to have a lexical specification of the following structure (without any pretension to lexicographical adequacy):

Categorial presuppositions are thus clearly derivable from lexical preconditions. The same holds for factive presuppositions, to be derived from a precondition associated with the factive predicate in question to the effect that the proposition expressed by the factive that-clause must be true. Discourse-semantically this means that the factive that-clause must be incremented in the truth-domain of D, or anyway in the same (sub)-D as the carrier sentence, and prior to it (see Discourse Semantics).

A similar treatment is now obvious for existential presuppositions. An existential presupposition is associated with a particular argument term of a given predicate Pⁿ, and derivable from the precondition that the reference object of that argument term must exist in the real world. It is then said that Pn is 'extensional with respect to that argument term.' The predicate talk about, for example, is extensional with respect to its subject term, but not with respect to its object term, since one can very well talk about things that do not exist. Most predicates are extensional with respect to all of their terms, so that one may consider extensignality to be the default case. From a notational point of view it is therefore preferable to mark the nonextensional arguments of a predicate, for example, by means of an asterisk. The lexical description of talk about will then be structured as in (39), where the asterisk on 'j' (that is, the reference object of the object term) indicates that this predicate is nonextensional with respect to its object term:

$$\sigma(\text{talk about}) = \{ \langle i, j^* \rangle : \dots (\text{preconditions}) \dots | \dots \rangle$$
(satisfaction conditions) \dots \}

The predicate *exist* is to be specified as nonextensional with respect to its subject term:

$$\sigma(\text{exist}) = \{i^* | i \text{ is in the real world}\}$$
 (40)

Discourse-semantically, this means that a definite subject of the verb *exist* must be represented somewhere in D, but not necessarily in the truth-domain of D. It may very well be located in some intensional subdomain, for example, the subdomain of things that Nob keeps talking about, as in:

The incremental effect of (41) is that the representation of the thing that is said to exist is moved up to the truth-domain of D (see *Discourse Semantics*).

The remainder category of presuppositions, induced by words like *only*, *no longer*, *still*, or by contrastive accent and (pseudo)cleft constructions, appears not to be derivable in this way. The choice here is either to derive them by adhoc rules, or to adopt a syntactic analysis in terms of which these words and constructions figure as (abstract) predicates at the level of representation that is taken as input to the incrementation procedure. On the whole, the literature is remarkably silent on this question. In general, the prefference is for adhoc derivations of presuppositions (e.g., Gazdar 1979).

See also: Accommodation and Presupposition; Discourse Semantics; Projection Problem.

Bibliography

Blamey S 1986 Partial logic. In: Gabbay D, Guenthner F (eds.) Handbook of Philosophical Logic, vol. 3. Reidel, Dordrecht

Blau U 1978 Die dreiwertige Logik der Sprache. Ihre Syntax, Semantik und Anwendung in der Sprachanalyse. De Gruyter, Berlin

Boër S E, Lycan W G 1976 The myth of semantic presupposition. Working Papers in Linguistics 21: 2-90

Bottin F 1976 Le antinomie semantiche nella logica medievale. Antenore, Padua

Fillmore C J 1971 Types of lexical information. In: Steinberg D D, Jakobovits L A (eds.) 1971

Frege G 1892 Ueber Sinn und Bedeutung. Zeitschrift für Philosophie und philosophische Kritik 100: 25-50

Gazdar G 1979 Pragmatics: Implicature, Presupposition, and Logical Form. Academic Press, New York

Geach P T 1972 Logic Matters. Blackwell, Oxford

Groenendijk J. Stokhof M 1991 Dynamic predicate logic. *La Ph* 14: 39–100

Heim I 1982 The semantics of definite and indefinite noun phrases.

(Doctoral dissertation, University of Massachusetts at Amherst)

Horn L R 1985 Metalinguistic negation and pragmatic ambiguity. *Lg* 61: 121-74

Horn L R 1989 A Natural History of Negation. University of Chicago Press, Chicago, IL

Kamp H 1981 A theory of truth and semantic representation. In: Groenendijk J, Janssen T, Stokhof M (eds.) Formal Methods in the Study of Language I. Mathematisch Centrum, Amsterdam

Kempson R M 1975 Presupposition and the Delimitation of Semantics. Cambridge University Press, Cambridge Kiparsky P, Kiparsky C 1971 Fact. In: Steinberg D D, Jakobovits L A (eds.) 1971

Kleene S 1938 On notation for ordinal numbers. *Journal of Symbolic Logic* 3: 150-55

Kneale W C, Kneale M 1962 The Development of Logic. Clarendon Press, Oxford

Russell B 1905 On denoting. Mind 14: 479-93

Seuren P A M 1977 Forme logique et forme sémantique: Un argument contre M. Geach. Logique et Analyse 20: 338-47

Seuren P A M 1985 Discourse Semantics. Blackwell, Oxford

Seuren P A M 1987 Les paradoxes et le langage. Logique et Analyse 30: 365-83

Seuren P A M 1988 Presupposition and negation. *Journal of Semantics* 6: 175-226

Steinberg D D, Jakobovits L A (eds.) 1971 Semantics. An Interdisciplinary Reader in Philosophy, Linguistics, and Psychology. Cambridge University Press, Cambridge

Strawson P F 1950 On referring. Mind 59: 320-44

Strawson P F 1952 Introduction to Logical Theory. Methuen, London

Weijters A 1985 Presuppositional propositional calculi. Appendix to Seuren P A M 1985

Wilson D 1975 Presuppositions and Non-Truth-Conditional Semantics. Academic Press, London

P. A. M. Seuren