STRATEGIES FOR FUTURE CLIMATE RESEARCH

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DATA ASSIMILATION IN ATMOSPHERIC MODELS

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1. Introduction

The prediction of weather and climate and the monitoring of the earth atmosphere require an accurate determination of the state of the atmosphere. This will include the basic parameters: horizontal wind, temperature and moisture through the whole depth of the atmosphere and surface pressure. Additional observations are required from the surface of the earth; temperature, soil moisture, snow and ice as well as relevant data of the biosphere.

In order to provide initial data for the latest generation of numerical forecasting models having as much as 6 million variables (ECMWF, 1991) with a suitable accuracy (say, $v \approx 1\text{-}3$ m/s, $T \approx 1^{\circ}$C and $r \approx 5\text{-}10\%$), we would have to face an enormous observation problem.

Such a daunting undertaking is presently not feasible, neither for economical nor for technical reasons. In fact, it is only during the last decade or so that we have had at our disposal an observing system, which for the first time has made it possible to observe the large scale aspects of the global atmosphere on scales which by and large are an order of magnitude coarser than the resolution of the most advanced models.

Fortunately, as we will be able to demonstrate in section 3, this extremely demanding observational requirement can be relaxed significantly due to the strong dynamical and physical coupling between meteorological observations in time and space.

Although data assimilation, that is the interaction or the integration of an analysis and a forecasting system, was recognized in the early days of numerical weather prediction (e.g. Bergthorsson and Döös, 1955), it was not until the 1970s, in relation to the preparation for the Global Weather Experiment, that the efforts to build comprehensive data assimilation systems started.

As will be demonstrated in section 4 below, the remarkable improvement in the quality of numerical weather prediction, e.g. Bengtsson (1991), to a considerable degree is related to the quality and consistency of the data assimilation procedures. A very special by-product of the data assimilation has been the possibility to detect systematic observation biases and through that improve the quality of the observing system itself.

Most of the operational data assimilation systems developed so far are not fully four-dimensional but instead arrange data in time intervals, ranging from a few hours up to 12 hours. At each time slot, available data are analysed using a previous short-range forecast as a first guess or an estimate. The new observations are merged with the first guess and an initial state is calculated based upon a statistically optimized procedure.

During recent years several attempts have been undertaken to develop consistent four-dimensional assimilation procedures, implying a minimization of a suitable cost function in time and space (e.g. Courtier and Talagrand, 1990). Although these methods are presently excessively time-consuming and still at an early state of development, they deserve nevertheless serious attention. In section 5 we will report of some recent results.

2. The observations

The global observing system is composed of a number of sub- systems providing measurements either from in situ or movable platforms, or measurements by remote sensing, such as satellite temperature soundings. A summary of present observing systems is given below including coverage, typical accuracies and frequencies of observations (figure 1).

Three-dimensional coverage and accuracies are key factors. Observational errors consist of both instrument errors and errors of representativeness. Estimated observation errors are given in Bengtsson (1975) and Shaw et al. (1987).

2.1. Surface stations

Surface stations are distributed irregularly over land and along well travelled shipping lanes. The surface pressure, temperature, wind components, relative humidity and different weather data are normally observed at least every three hours. The surface pressure is reduced to mean sea level pressure

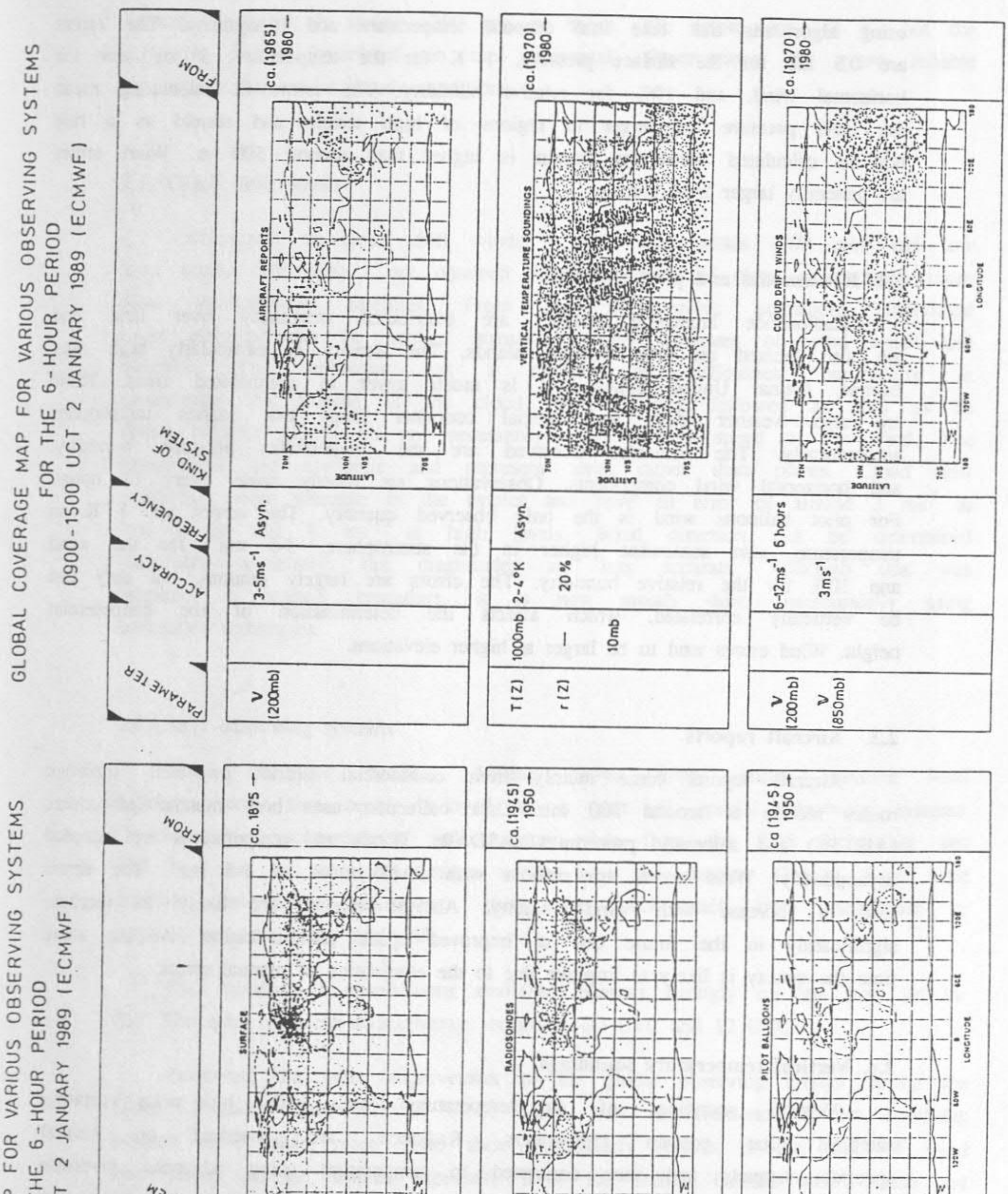


Fig. 1 Global coverage map for various observing systems for the 6-hour period from 0900-1500 UCT 2 January 1989 (ECMWF).

using algorithms that take into account temperature and topography. The errors are 0.5 mb for the surface pressure, 1 K for the temperature, 3 ms^{-1} for the horizontal wind, and 10% for relative humidity. The errors in calculating mean sea level pressure are larger in regions of high terrain and should as a rule not be calculated when the terrain is higher than around 500 m. Wind errors are generally larger from ships.

2.2. Radiosondes and pilot balloons

Radiosonde launching stations are distributed irregularly over land and are also located on some remote islands. The density is particularly high over Europe, China, United States, but is much lower in uninhabited areas. There are also weather ships and special container ships that launch radiosondes automatically. The variables observed are the temperature, pressure, humidity, and horizontal wind components. Observations are mostly made every 12 hours. For pilot balloons wind is the only observed quantity. The errors are 1 K in temperature, also somewhat higher in the stratosphere, 3-5 ms^{-1} for the wind and 10% for the relative humidity. The errors are largely random, but they can be vertically correlated, which affects the determination of the geopotential height. Wind errors tend to be larger at higher elevations.

2.3. Aircraft reports

Aircraft reports come mainly from commercial aircraft on well- travelled routes mostly at around 200 mb. Data collection uses both manual procedures (AIREPS) and automatic procedures (ASDAR). Winds and temperatures are sampled asynoptically. Wind errors are random with RMS errors of 3-5 ms^{-1}. The errors tend to increase with wind velocity. Aircraft reports are likely to increase significantly in the future due to improved space communication. At the same time the quality is likely to improve due to the elimination of manual errors.

2.4. Vertical temperature soundings

Vertical soundings of air temperatures are made by polar orbiting satellites that contain radiometers. Radiances are observed on several spectral channels and then converted to temperature using advanced inversion algorithms. The coverage is global but asynoptic. Figure 1 shows data from two orbiting satellites. It is important to note that the "observations" represent areas rather than points and have a horizontal resolution of about 250 km. The vertical resolution is coarse, 200 mb or more. The errors are 2-3 K, but the error is highly correlated in the horizontal. Observations are more reliable in clear air than under cloudy conditions.

2.5. Cloud drift winds

Observation of cloud drift winds - high level winds (200 mb) and low level winds (850 mb) - are obtained from sequences of cloud photographs taken from geostationary satellites. From the photographic sequences, individual cloud elements can be tracked manually, giving estimates of wind speed and direction. The technique has a number of difficulties, particularly in determining the altitude of the cloud element being followed as well as its speed, because clouds do not necessarily move at the speed of the wind. The observations are asynoptic and represent areas rather than points. Cloud drift winds are most valuable in the tropics and have an error of around 3 ms^{-1} at low levels and 8 ms^{-1} at high levels. Wind direction can be determined reasonably accurately, but magnitudes are less accurate. Although this was originally a manual procedure, it is now mostly done automatically using correlation techniques.

2.6 Other observing systems

Other observing systems, drifting buoys, drop sondes, constant level balloons, have also been deployed as a part of special observing systems. Orbiting active scatterometers will generate data from the ERS I satellite and will be used to measure surface winds over the oceans. Ground base wind profilers (microwave instruments using the back-scatter from turbulence or aerosols) are now being deployed.

The number of observations available depends strongly on the time of the day. The most complete data coverage occurs at 00 UTC and 12 UTC.

Following the great improvement of the global observing system during the Global Weather Experiment in 1979, hardly any improvements in the observing systems have taken place since then. Particularly serious is the lack of wind observations in the tropics especially over the Indian Ocean, where there are presently hardly any usable wind observations.

Another problem is the lack of quality of the observations from the space-based systems. Satellite temperature soundings contribute only marginally to the observing system of the Northern Hemisphere. These observations have in fact recently been excluded north of 20° S and below 100 mb in the present forecasting system at ECMWF (ECMWF, 1991). Improved retrieval methods, presently being developed, may change this situation.

There is no question that major improvements in the coverage and quality of atmospheric observation must be of highest priority for the future.

3. The data-assimilation problem

The concepts of data assimilation was clearly recognized already by Vilhelm Bjerknes. The following is a quote from his article "Das Problem der Wettervorhersage betrachtet vom Standpunkt der Mechanik und der Physik", in "Meteorologische Zeitschrift", 1904:

"The major task of observational meteorology will be to obtain regular simultaneous observations of all parts of the atmosphere at the surface of the earth and aloft, over land and over sea.

The first task of the theoretical meteorologist will be to work out, on the basis of these observations, the best possible overall picture of the physical and dynamical state of the atmosphere at the time of these observations. And this representation must have such a form that will enable it to serve as a starting point for weather prediction by rational dynamical physical methods.

Even this first preliminary task is a sizeable one. For it is of course much more difficult to represent the state of the atmosphere at all elevations than only at sea level, as it is now done. In addition, our direct observations of the higher layers of air will always be very limited. **One must therefore use each observation from the high levels to the utmost**. From the directly observable quantities one has to **compute** to the greatest extent all accessible data about the **non-observable ones**. In doing this, one has to utilize the physical relationship between the quantities. Even to construct the coherent picture of the total state of the atmosphere out of scattered observations one has **to use, to a large extent, dynamical/physical methods**."

Of fundamental importance for data-assimilation is the relative importance of the different atmospheric parameters. We will illustrate this by the following simple example.

For a barotropic rotating fluid having a free surface (see figure 2) the equations of motion and the continuity equation in a linearized form can be written as follows:

$$\frac{\partial u}{\partial t} = fv - g\frac{\partial h}{\partial x} \tag{3.1}$$

$$\frac{\partial v}{\partial t} = fu - g\frac{\partial h}{\partial y} \tag{3.2}$$

$$\frac{\partial h}{\partial t} = H\left(\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}\right) \tag{3.3}$$

u und v are the horizontal wind components, $f=2\Omega \sin \psi$ the vertical component of the Coriolis force, h the variation of the height of the free surface and H the averaged height.

The equations (3.1)-(3.2) can be expressed in a rotational part and a non-rotational part by making use of the relation

$$\mathbf{v} = \underset{\text{rot. part}}{\mathbf{k} \times \nabla \Psi} \quad + \quad \underset{\substack{\text{nonrot.}\\ \text{part}}}{\nabla \chi} \tag{3.4}$$

At the same time we will write

$$\Psi = \Psi_0 e^{i(kx+ly-ct)}$$

$$\chi = \chi_0 e^{i(kx+ly-ct)} \tag{3.5}$$

where k and l are the zonal and the meridional wave number and c the phase speed of the wave.

For a given value of k and l the equation (3.1)-(3.3) can now be written:

$$\frac{\partial \Psi}{\partial t} = -f\chi \tag{3.6}$$

$$\frac{\partial \chi}{\partial t} = f\Psi - gh \tag{3.7}$$

$$\frac{\partial h}{\partial t} = H(k^2+l^2)\chi$$

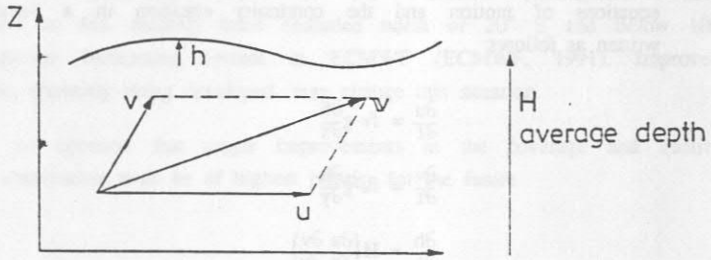


Fig. 2 Shallow water equations integrated for a fluid with constant density and a free surface.

The geostrophic wind correction

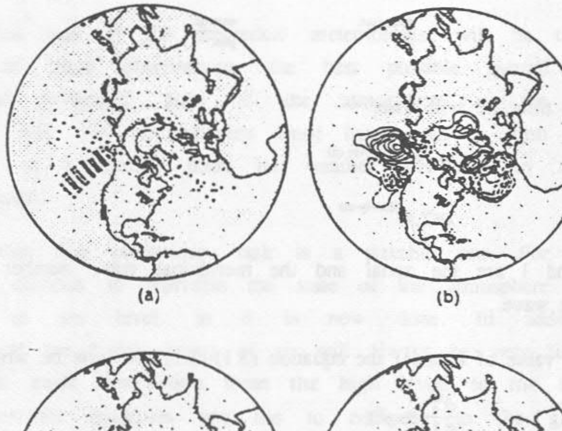


Fig. 3 Illustration of the rejection phenomenon: (a) location of height insertions, (b) difference between height observations and model prediction (meters), (c): projection of the difference on the fast manifold (gravity waves), (d) projection on the slow manifold (Rossby waves). After Daley and Puri (1980).

This system has three eigensolutions. One is a geostrophically balanced mode and two others are high frequency modes corresponding to propagating gravity waves along the forward and backward direction, having eigenvalues:

$$\lambda_2,\ \lambda_3 = \pm\ \{gH(k^2+l^2+f^2)\}^{1/2} \tag{3.9}$$

In the stationary case: $\psi_t = \chi_t = h_t = 0$, we have

$$\chi_s = 0$$
$$h_s = \frac{f}{g}\psi_s \tag{3.10}$$

showing that the stationary case is in geostrophic balance. It follows from equation (3.6) and (3.8) that the expression $H(k^2+l^2)\Psi + fh$ is invariant (constant in time).

For an initial field (subscript i) we now have the relation

$$H(k^2+l^2)\Psi_s + fh_s = H(k^2+l^2)\Psi_i + fh_i \tag{3.11}$$

By making use of (3.10) we can now write (3.11) as

$$\Psi_s = \alpha\Psi_i + (1-\alpha)\Psi_i' \tag{3.12}$$

where

$$\Psi_i' = \frac{f}{g}\, h_i \qquad \text{(geostrophic balance)}$$

and

$$\alpha = \frac{gH(k^2+l^2)}{gH(k^2+l^2)+f^2} \tag{3.13}$$

$\alpha \to 1 \to h_s \approx \frac{f}{g}\,\Psi_i$ Mass field adjustment to initial wind field (large wave numbers or small f)

$\alpha \to 0 \to \Psi_s \approx \frac{g}{f}\, h_i$ Wind field adjustment to initial mass field (small wave numbers and large f)

Relation (3.13) means specifically that for small scale systems or for small scale increments in a data assimilation procedure as well as for low latitudes the windfield in general constitutes the primary information. Height information should therefore not be inserted into a model on its own, but must be at least accompanied with the associated geostrophic wind, else the windfield of the first guess will suppress the inserted new information. Alternatively for the larger scales the mass field information is crucial.

The rejection phenomenon can be graphically illustrated by figure 3, taken from Daley and Puri (1980). This experiment was performed with a global non-linear shallow water model using real observations. Figure 3a shows the position of the height observations that were inserted in the model at a given time. The partial satellite tracks are clearly visible. Figure 3b shows the difference (in meters) between the new height observations and the model new height at the insertion time. The contour interval is 20 meters (this field could be likened to the observation increment used in statistical interpolation). Figure 3d shows the projection of the new height difference field in (b), onto the slow manifold of the model, and figure 3c shows the projection onto the high- frequency inertia-gravity waves. As can be seen from figure 3b, the direct insertion of the new height information causes inertia-gravity wave shock and is essentially rejected by the model.

When inserted temperatures, say, are rejected by the model, the model windfield is not substantially altered from what it would have been in the absence of data insertion. Thus, the model temperature and windfields after a period of data insertion do not strongly reflect the inserted data. Consequently, atmospheric analyses obtained after a period of data assimilation tend to reflect too strongly the model state that would have been obtained without data insertion.

The effect of geostrophic wind correction can be seen in figure 4. This figure, which has the same format as figure 3, shows (a) the projection on the slow manifold and (b) the inertia-gravity waves for the observed minus forecast new height field shown in figure 3b. In figure 4, a procedure analogous to the geostrophic wind correction has been used (Puri, 1981). Clearly, most of the height information has been forced onto the slow manifold of the assimilating model, and there is little energy in the inertia- gravity waves. In other words, the geostrophic wind correction procedure has prevented the new height information shown in figure 3b, being rejected by the model.

In the same way as the wind and the mass field can reconstitute each other through the geostrophic relation, comprehensive numerical models are

Fig. 4 Effect of geostrophic wind correction in the same format as in figure 3. (a) projection on the fast manifold, (b) projection on the slow manifold. After Daley and Puri (1980).

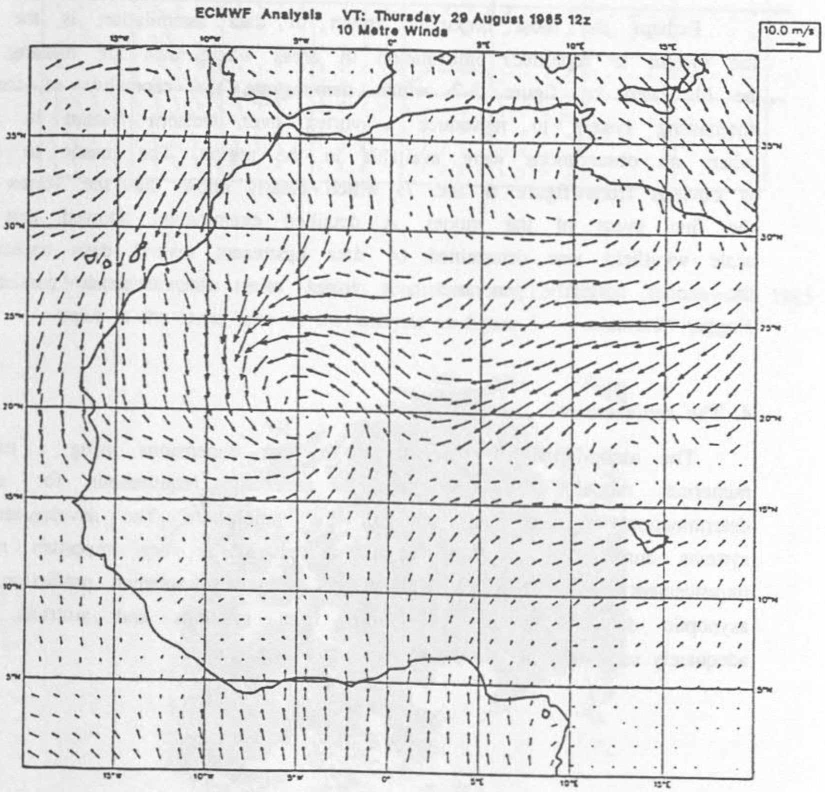


Fig. 5 ECMWF analysis for 12 UTC, 29 August 1985.

also capable of reconstituting other kinds of missing information. This was for example pointed out by Smagorinsky (1969) who demonstrated that a dynamical model, for example, was able to reconstitute the humidity field in most of the details within a day or two in an experiment in which humidity initially was described as a zonal mean. It should be perhaps stressed that this is strongly latitude dependant and plays a role mainly in the extratropics where the dynamical forcing is dominating. In the tropics, humidity observations often play a crucial role, and as has been demonstrated by Krishnamurti et al. (1983) are required for an accurate prediction of tropical disturbancies.

Numerical experiments (e.g. Arpe et al., 1985, Hollingsworth et al., 1985) have shown that an accurate specification of the coupled wind and mass fields in the baroclinic zones is crucial for forecast quality, while a detailed analysis of the boundary layer appears to be less important. The explanation is again that a numerical model is able to generate missing information indicating a considerable degree of redundancy among the large scale atmospheric variables.

Perhaps the most important aspect of data assimilation is the ability of the system to reproduce information in areas where data are missing. This can be illustrated by figure 5-7 which demonstrate the capability of the ECMWF forecasting system to reproduce a vortex over western Sahara in a situation where no observations were available in the region. The reason to the success is obvious from figure 6 and 7, which clearly show that the vortex existed in the first guess of the model. A detailled examination showed that the large scale windfield was determined by data upstreams several days earlier and that the model correctly generated a vortex when the airstream passed over the Hoggar Mountain.

4. The achievement

The assimilation of observations in four dimensions using a comprehensive numerical model as a tool is a necessary requirement for an accurate determination of the initial state of the atmosphere. The development of such systems during the past 10 years has played a very important role in the improvement which has taken place in numerical weather prediction. Moreover, asynoptic data, such as observations from satellites and aircrafts, cannot be adequately used without four-dimensional data simulation.

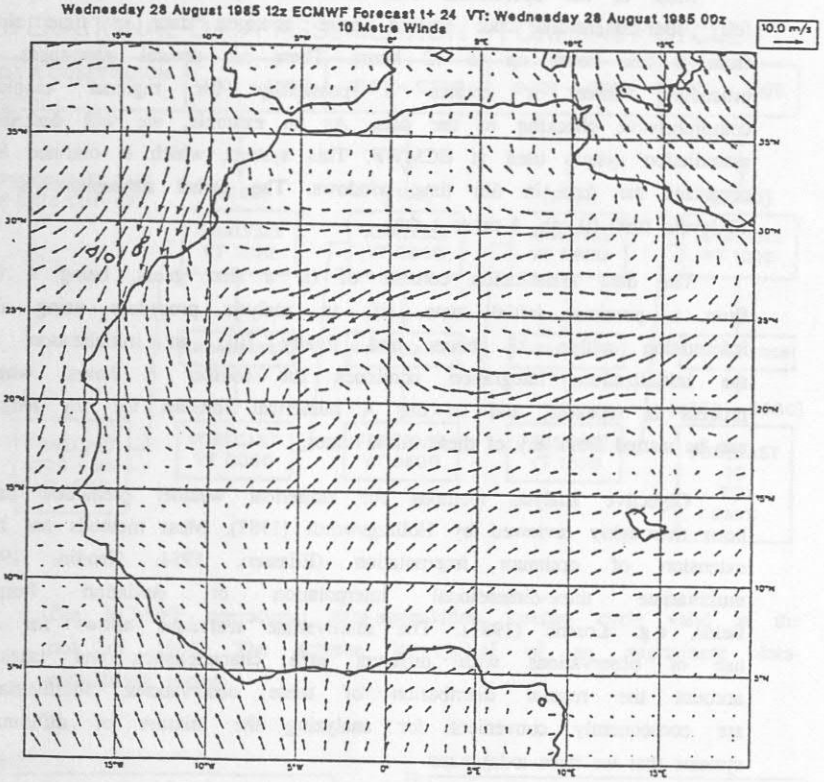


Fig. 6 24-hour forecast from initial condition on 12 UTC, 28 August 1985 valid at the same time as the analysis in figure 5.

Fig. 7 Meteosat IR image of a dust vortex (arrow) at 1155 UTC 29 August 1985.

Most of the operational data simulation systems developed so far are not fully four-dimensional but instead have arranged data in time slots ranging from a few hours up to 12 hours. There are several advantages in such a procedure, since it creates a possibility for regional monitoring and comprehensive checking of the data. As an example, we will describe the data assimilation system used at ECMWF. This system, which is outlined in figure 8, organizes the data in 6hr time windows. The global atmosphere is analysed at 17 levels 1000-10 mb, 4 times a day.

The data assimilation consists of (i) a first guess, being a 6hr forecast from a previous initial state, (ii) an analysis procedure using all available information within ∓3 hours and finally (iii) an initialization step. After the initialization, integration continues for another 6 hours, whereafter the process is repeated, and so on. A numerical forecast for any length of time can be started from any of these initial states.

Objective analysis methods for numerical weather prediction have recently been thoroughly reviewed by Hollingsworth (1987). Most methods are based on an extension of optimum interpolation (Eliassen, 1954, Gandin, 1963) to a multivariate three-dimensional interpolation or deviation from forecast fields, e.g. Lorenc (1981). The multivariate technique allows for a consistent use of observations with different area characteristics and takes into full account the regular distribution of these observations. Multivariate methods are consequently convenient for analyzing the mixture of different observing systems that we have today.

The quality of numerical forecasts have undergone significant improvements during the 80s. This can be seen from figure 9, which presents the standard deviation of the monthly average forecast area for 72 hours forecast for the height fields at 500 and 1000 mb. These data have been compiled under a WMO project (Bengtsson and Lange, 1982, and Lange and Hellsten, 1983) and summarize the results of daily operational forecasts from several centres for a period of 9 years.

It can be noted that the best available model in 1987 has a standard deviation error of only 55% of the best model available in 1979. Other studies indicate (e.g. Bengtsson, 1991) that the skill of useful forecasts has been extended by about three days in the 1980s. Since the observing systems have been virtually unchanged during this period, the improvement is clearly due to model developments and to a better use of data due to improved data assimilation procedures.

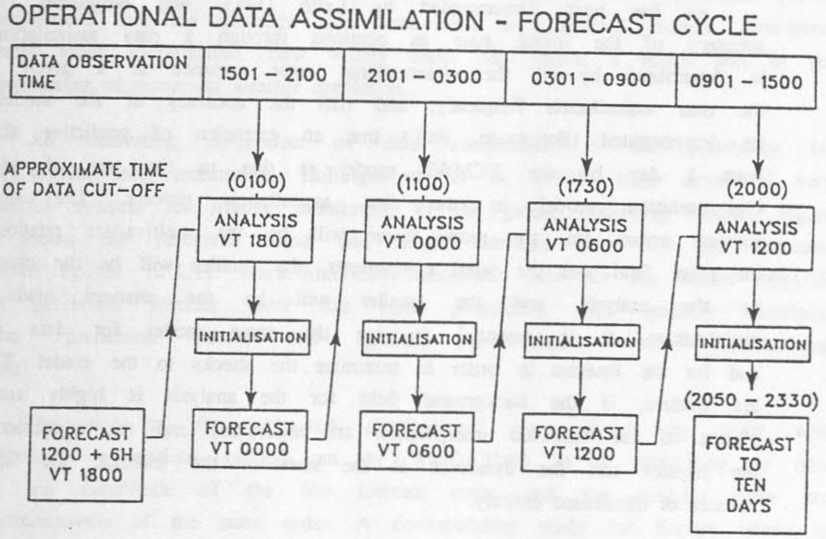


Fig. 8 The ECMWF operational data-assimilation forecast cycle valid at the end of 1988. The scheme is typical of an intermittent data-assimilation system.

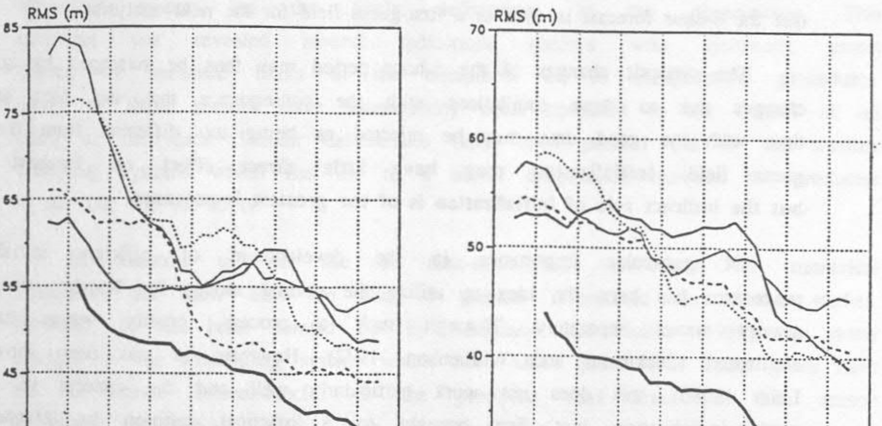


Fig. 9 Standard deviation scores for 72-hour operational forecasts for the 500 hPa (top) and 1000 hPa (bottom) height fields for a period of nine years. The scores have been collected and calculated under the WMO/CAS Forecast Intercomparison Project. Thin full curve, Federal Republic of Germany; dotted curve, France; dashed curve, United Kingdom; heavy curve, NMC, Washington, and thick curve, ECMWF.

As has been demonstrated by Leith (1983) and Bengtsson (1989) the accuracy of the initial state as obtained through a data assimilation process is determined by (i) the observational error variance at a given time, (ii) the data assimilation frequency, and (iii) the accuracy of the model. It can be demonstrated (Bengtsson, ibid.) that an extension of predictive skill of at least 1 day by the ECMWF model is due to the use of an accurate high-resolution model in the data assimilation process. The smaller the forecast errors are, the more linear will be the multivariate relation between the mass field and the wind components, the smaller will be the changes made by the analysis, and the smaller will be the changes made by the initialization. It is essential to use the same model for data assimilation and for the forecast in order to minimize the shocks to the model. The reasons are evident; if the background field for the analysis is highly accurate, the effects of the foreward interpolation are minimized and the adjustment between the physics and the dynamics at the start of the forecast, the "spin-up", is reduced or eliminated entirely.

If analysed data are used directly as the initial condition for a forecast, imbalance between the mass and windfield will cause the forecast to be contaminated by spurious, high-frequency gravity wave oscillations on much larger amplitudes than are observed in the real atmosphere. Although these oscillations tend to die away slowly due to various dissipation mechanisms in the model, they may be quite detrimental to the data-assimilation cycle in that the 6-hour forecast is used as a first-guess field for the next analysis.

The synoptic changes of the 6-hour period may thus be swamped by spurious changes due to these oscillations with the consequence that the next analysis time with the good data may be rejected as being too different from the first guess field. **Initialization may have little direct effect on forecast skill, but the indirect role of initialization is of the greatest importance.**

Of particular importance in the development of efficient initialization procedures has been the idea to utilize the normal modes for Rossby waves and gravity waves separately. Through such a process, gravity waves can be eliminated (Dickinson and Williamson, 1972). However, as has been shown by Leith (1980), this does not work particularly well and the concept of normal mode initialization was first brought to a practical solution by Machenhauer (1977) and Baer and Tribbia (1977) who independantly incorporated the effect of non-linear terms. The physical meaning of this non-linear normal mode initialization was to put the initial tendency of the gravity wave modes equal to zero. Andersson (1977), Daley (1979) and Temperton and Williamson (1979) demonstrated that the methods also work well in multilevel models. Non-linear normal mode initialization, now widely used, has played a major part in the improvement of numerical weather prediction.

An interesting by-product of data assimilation is the possibility for data control and monitoring. Hollingsworth et al. (1986) have developed very powerful systems for routine monitoring of the global observing system. Figure 10 shows the perceived wind and height forecast error for 6hr forecasts verified against 12 UTC North-American radiosonde data for the winter 1984/85. The perceived forecast error has been partitioned into spatially correlated errors (prediction errors) and spatially uncorrelated errors (observational errors).

The size of the estimated observational error agrees by large with independent evaluations carried out by WMO/CIMO. It is interesting to note that the magnitude of the 6hr forecast error and the analysis error are approximatively of the same order. A corresponding study for Europe shows in fact that the observation error is somewhat larger than the forecast error, presumably due to the many different types of radiosonde equipment in Europe compared to North America.

As a consequence of the high performance of the data simulation system, ECMWF has started on a systematic basis to inform operators of the global observing system on the likely deficiencies in the observations. This excercise has revealed several radiosonde stations with systematic errors caused by technical faults in the equipment or to inappropriate procedures. Many of these stations have subsequently been corrected to the benefit of all users of the data. Similar deficiencies have been pointed out in the satellite observing systems which has led to a useful cooperation between the producers of the data and the users.

In addition to the use of data-assimilated information for numerical prediction, the same data are also of greatest importance for climate studies. However, as has been pointed out by Bengtsson and Shukla (1989), the many changes in the forecasting system which have taken place over many years have led to minor inconsistencies in the produced data sets. For this reason Bengtsson and Shukla (ibid.) have suggested that it would be of fundamental importance for climate research that the data over a period of some 15 years were to be reassimilated by an up-to- date frozen data assimilation system.

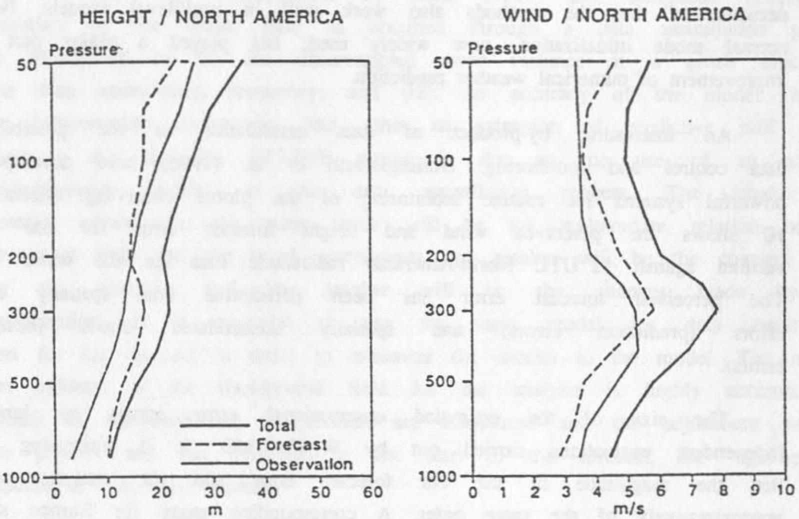


Fig. 10 The total perceived height and wind forecast error over North America. The perceived error has been divided into a correlated part (forecast error) and an uncorrelated part (observation error) as indicated in the legend. The calculations are based on 6-hour forecasts for the first quarter of 1984 with the ECMWF system; the unit is m (height) and ms-1 (wind). Note that the sizes of the prediction error and the observational error are comparable.

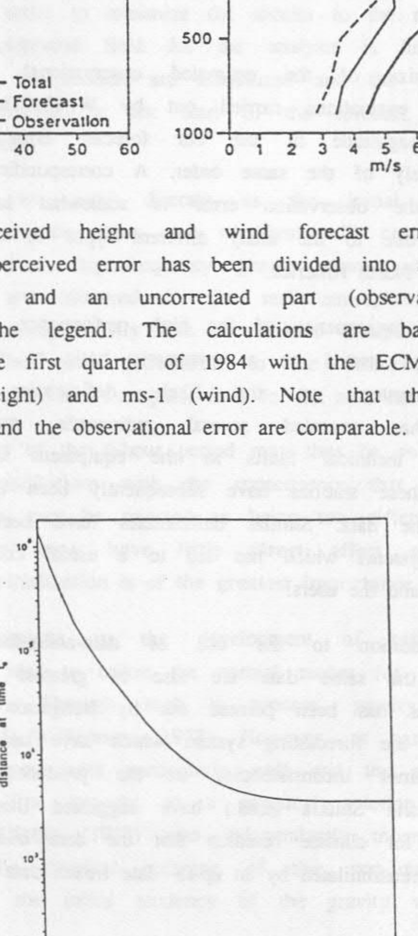


Fig. 11 Variation of the distance between the reference and the model state at the initial time of the assimilation period as a function of the number of iterations (Thépaut and Courtier, 1991)

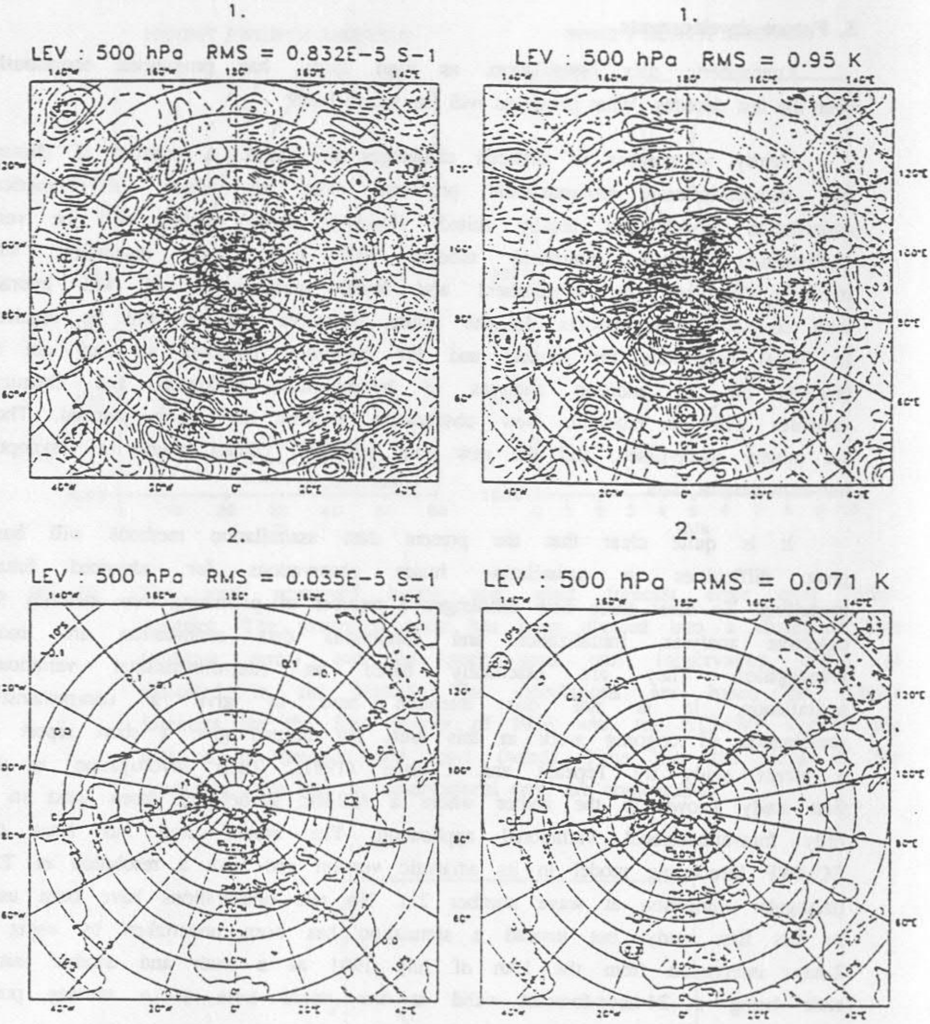
5. Future developments

Atmospheric data assimilation, as used today, has progressed enormously over the last decades. What prospects will the future hold?

Clearly, a number of difficult challenges lie ahead. A number of climate and interdisciplinary environmental problems exist for which the numerical assimilation models are ideally suited. However, these models will be very sophisticated. Future numerical models will have high resolution, will routinely simulate the stratosphere and lower mesosphere and will interact with coupled ocean models. In the future, objective analyses will be required as initial conditions for models and for diagnostic purposes as they are at present; but in addition analyses of hydrological, biological and chemical variables will be required. New observation systems are being planned. They are mostly space-based, and the new data sets will consist mainly of asynoptic non-conventional data.

It is quite clear that the present data assimilation methods will have great difficulties in assimilating future observations for advanced future modelling. To meet these new challenges a number of promising new methods for objective analyses, initialization and continuous data assimilation are under investigation. They are essentially based on four-dimensional variational assimilation. It is not our intention here to give a comprehensive presentation of ongoing work in this area, but instead give a short report of a recent study by Tépeaut and Courtier (1991). Their investigation is the first study known to the author where a realistic model has been used in a fully four-dimensional variational application. The model used has been the ECMWF forecasting model in its adiabatic version and with a resolution of T21 (triangular truncation at wave number 21). No real observations have been used in this first study, but instead a simulation has been undertaken by using a 2-day- integration from the 12th of July 1991 as a truth and another initial state being a 24-hour-forecast valid at the initial time as a starting point for the forecast experiment.

The cost function used in the experiment has been the difference in the total energy between the reference state and the experimental state. In the basic experiment "observations" have been inserted at +6 hrs and the initial state has been reconstituted through a successive forward-backward integration using the adjoint of the model for the backward integration. After 30 iterations, the cost function is reduced by 6 orders of magnitude and the



1.

LEV : 500 hPa RMS = 0.832E-5 S-1

1.

LEV : 500 hPa RMS = 0.95 K

2.

LEV : 500 hPa RMS = 0.035E-5 S-1

2.

LEV : 500 hPa RMS = 0.071 K

Fig. 12 Vorticity (left) and temperature (right) difference fields at 500 hPa.

1. Difference between the starting field for the minimisation and the reference.
2. Difference between the final point after 30 iterations and the reference. (The contour interval is $0.4x10^{-5}s^{-1}$ for the vorticity field and 0.4K for the temperature field). (Thépaut and Courtier, ibid.).

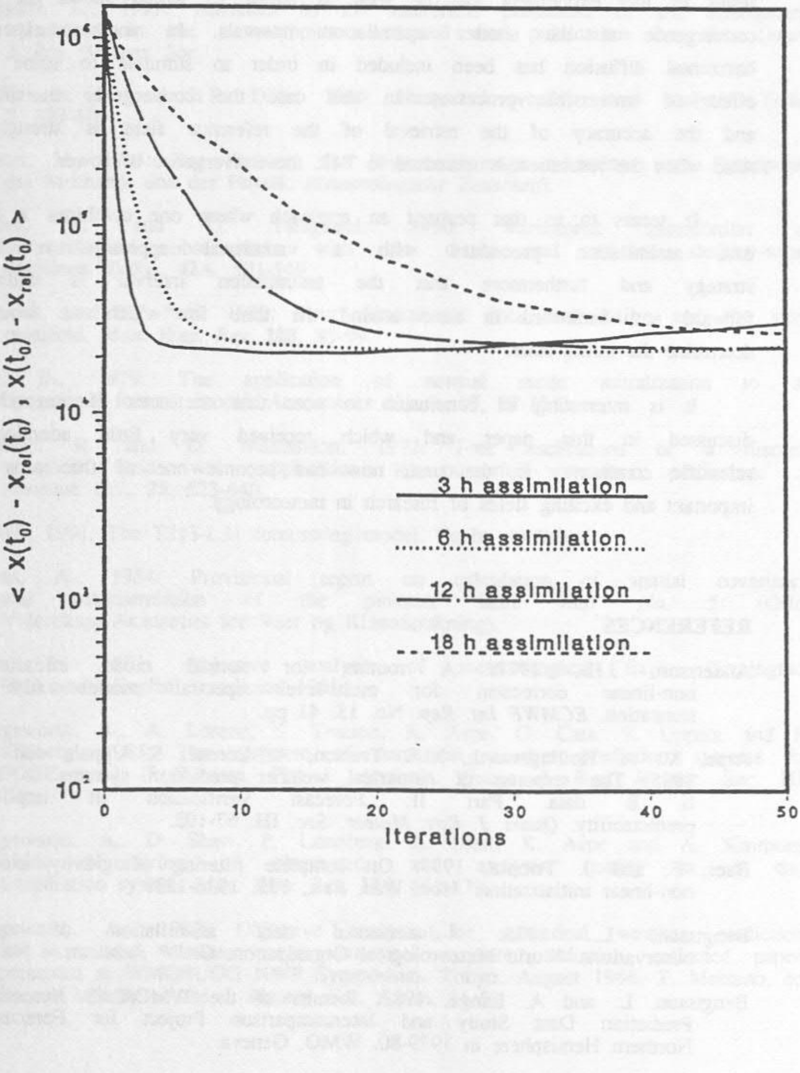


Fig. 13 The same as Fig. 11 but for different time periods of assimilation (Thépaut and Courtier, ibid.).

difference at the initial time between the two states has been reduced by some three orders of magnitude (see figure 11). Figure 12 shows the difference for vorticity and temperature at the initial time and at the 30th iteration.

A number of different experiments have been carried out, one of them addressing the impact of the length of the assimilation time interval. The result of this experiment can be seen in figure 13 which shows the very rapid convergence at the shorter assimilation intervals. In another experiment the horizontal diffusion has been included in order to simulate to some extent the effect of irreversible processes. In this case the convergence is much slower and the accuracy of the retrieval of the reference field is strongly affected. Also when the resolution is increased to T42, the convergence is slower.

It seems to us that perhaps an approach where one combines a conventional data assimilation procedure with a variational approach is a realistic strategy and furthermore that the assimilation interval is extended both forward and backward in time around the time for which we would like to determine the initial state.

It is interesting in conclusion to note that the area of research we have discussed in this paper and which received very little attention by the scientific community in the past, now has become one of the most interesting, important and exciting fields of research in meteorology.

REFERENCES

Andersson, J.H., 1977: A routine for normal mode initialization with non-linear correction for multi-level spectral model with triangular truncation. *ECMWF Int. Rep.* No. **15**, 41 pp.

Arpe, K., A. Hollingsworth, M.S. Tracton, A. Lorenc, S. Uppala and P Kållberg, 1985: The response of numerical weather prediction systems to FGGE level II B data. Part II: Forecast verification in implication for predictability. *Quart. J. Roy. Meteor. Soc.* **III**, 67-102.

Baer, F. and J. Tribbia, 1977: On complete filtering of gravity modes through non-linear initialization. *Mon. Wea. Rev.*, **105**, 1536-1539.

Bengtsson, L., 1975: 4-dimensional data assimilation of meteorological observations. World Meteorological Organization, *GARP Publication* No. **15**.

Bengtsson, L. and A. Lange, 1982: Results of the WMO/CAS Numerical Weather Prediction Data Study and Intercomparison Project for Forecasts for the Northern Hemisphere in 1979-80. WMO, Geneva.

Bengtsson, L. and J. Shukla, 1988: Integration of space and in situ observations to study global climate changes. *Bull. Amer. Meteor. Soc.* **69**, 1130-1143.

Bengtsson, L., 1989: On the growth of errors in data assimilation systems: ECMWF seminar on data-assimilation and the use of satellite data. ECMWF 5-9 September 1988, 3-16.

Bengtsson, L., 1991: Advances in the numerical prediction of the atmospheric circulation in the extra-tropics. Accepted for publication by the *Quart. J. Roy. Meteor. Soc.*

Bergthorsson, P. and B.R. Döös, 1955: Numerical weather map analysis. *Tellus* 7, 329-40

Bjerknes, V., 1904: Das Problem der Wettervorhersage betrachtet vom Standpunkt der Mechanik und der Physik. *Meteorologsche Zeitschrift*.

Courtier, P. and O. Talagrand, 1990: Variational assimilation of meteorological observations with the direct and adjoint shallow-water equations. *Tellus*, **42A**, 531-549

Daley, R. and K. Puri, 1980: Four dimensional data assimilation and the slow manifold. *Mon. Wea. Rev.* **108**, 85-99.

Daley, R., 1979: The application of normal mode initialization to an operational forecast model. *Atmosphere Ocean*, **17**, 97-124.

Dickinson, R. and D. Williamson, 1972: Free oscillations of a discreet stratified fluid with application to numerical weather prediction. *J. Atmosph. Sci.*, **29**, 623-640.

ECMWF, 1991: The T213-L31 forecasting model. To be published.

Eliassen, A., 1954: Provisional report on calculation of spatial covariance and autocorrelation of the pressure field. *Rep. No.* 5 (Oslo, Videnskaps-Akademiet for Vaer og Klimaforskning).

Gandin, L. 1963: Objective analysis of meteorological fields (Leningrad, Gridromet) English translation 1965.

Hollingsworth, A., A. Lorenc, S. Tracton, K. Arpe, G. Cats, S. Uppala and P. Kållberg, 1985: The response of numerical weather prediction systems for FGGE level II B data. Part I: Analysis. *Quart. J. Roy. Meteor. Soc.* **III**, 1-66.

Hollingsworth, A., D. Shaw, P. Lönnberg, L. Illari, K. Arpe and A. Simmons, 1986: Monitoring of observation and analysis quality in a data assimilation system. *Mon. Wea. Rev.* **114**, 861-879.

Hollingsworth, A., 1987: Objective analysis for numerical weather prediction. In *Short and medium range numerical weather prediction*, collected papers presented at WMO/IUGG NWP Symposium, Tokyo, August 1986, T. Matsuno, ed. Special volume of the *J. Meteor. Soc. Japan*, 11-59.

Krishnamurti, T.M., K. Ingles, S. Cocke, T. Kitade and R. Pasch, 1983: Details of low latitude medium-range weather prediction using a global spectral model. II Effects of orography and physical initialization. *Florida State Univ. Rep.* 83-11. 1-206.

Lange, A. and E. Hellsten, 1983: Results of the WMO/CAS NWP Data Study and Intercomparison Project for Forecasts for the Northern Hemisphere in 1981-82. *WMO Short-Medium-Range Weather Prediction Res. Publ. Ser.* No. **2**. WMO, Geneva.

Leith, C., 1980: Non-linear normal mode initialization and quasi-geostrophic theory, *J. Atmos. Sci.*, **37**, 958-968.

Leith, L.E., 1983: Predictability in theory and practice. *Large-Scale Dynamical Processor in the Atmosphere* (B.J. Hoskins and R.P. Pearce, eds.) Academic Press, 365-383.

Lorenc, A. 1981: A global three-dimensional multivariate statistical interpolation scheme. *Mon. Wea. Rev.* **109**, 701-721.

Machenhauer, B., 1977: On the dynamics of gravity oscillations in a shallow water model with application to normal mode initialization. *Contrib. Atmos. Phys.*, **50**, 253-271.

Puri, K. (1981) Local geostrophic wind correction in the assimilation of height data and its relationship to the slow manifold. *Mon. Wea. Rev.* **109**, 52-55.

Shaw, D., P. Lönnberg, A. Hollingsworth and P. Undén, 1987: Data assimilation: The 1984/85 revision of the ECMWF mass and wind analysis. *Quart. J. Roy. Meteor. Soc.* **113**, 533-66.

Smagorinsky, J., 1969: Problems and promises of deterministic extended range forecasting. *Bull. Am. Meteorol. Soc.* **50**, 286-311.

Temperton, C. and D. Williamson, 1981: Normal mode initialization for a multi-level gridpoint model. Part I: Linear aspects. *Mon. Wea. Rev.*, **109**, 729-743.

Thépaut, J.-N., and P. Courtier, 1991: Four-dimensional variational data assimilation using the adjoint of a multilevel primitive equation model. Submitted to *Quart. J. Roy. Meteor. Soc.*