STRATEGIES

FOR FUTURE CLIMATE RESEARCH

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ICE SHEET MODELLING

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1. Introduction

Within the last few years a number of ice sheet models of the General Circulation Model (GCM) type have been developed to simulate the present, past, and future states of ice sheets. This recent development is now a combined effort of glaciologists and climatologists, who previously viewed the ice from quite different viewpoints. Traditionally, glaciologists studied the physics of ice in some detail in field and laboratory experiments and used simple theoretical models to describe the observations of velocity and temperature distribution within ice sheets. On the other hand, scientists interested in climate did not even touch the mechanisms of temperature-dependent ice sheet flow until recently. The reason for neglecting ice flow on shorter time scale climatic modelling (the last 1000 years) is that the ice sheet shape does not change appreciably and thus may be treated as a fixed boundary condition for the atmosphere and the ocean. Even on longer time scales (ice ages), ice sheet buildup and decay were initially modelled as the results of snow accumulation and ablation, assuming perfect plastic flow within the ice sheet. This assumption uniquely relates the ice sheet shape to its volume. In this approach, the ice sheet is treated as a slave of the boundary conditions set by the atmosphere. Climate modellers interested in these longer time scales now view the ice sheets as an integral part of the climate system, with at least part of their dynamics being independent of the atmospheric boundary conditions.

The construction of Ice Sheet General Circulation Models (ISGCM) followed in many ways the concepts used in developing General Circulation Models for the atmosphere (AGCM) and the ocean (OGCM). Model equations are derived from the so-called primitive equations (conservation laws) by introducing appropriate approximations and parameterizations for those time and space scales not resolved in the model. The numerical methods are those developed for atmospheric and oceanic models. Model inter-comparisons which have been running for many years for atmospheric models are now also being initiated for ice sheet models (EISMINT, 1991). The idea is, first to sort out the influence of using different numerical techniques to solve the equations, and later to compare the models with respect to the physics contained in them.

In the following, only 3-d ice sheet models will be described. However, lower-order models (0-d, 1-d, and 2-d) are still valuable diagnostic tools. They can be integrated much faster and the specification of the upper boundary conditions of the ice sheet is much simpler. Although the 3-d models to be discussed are all based on some set of conservation laws (see section 2), the approximations introduced depend very much on the intended model application (section 3). Thus model intercomparison will certainly help to evaluate the applicability of certain model formulations. From the modelling results presented up to now (section 3) a preliminary conclusion may already be drawn (section 4), namely, that the ice sheet models are quite capable of simulating the observed ice sheets. However, the realistic definition of the boundary conditions is still an open question.

2. Model Equations

Ice sheets do not occupy a rather fixed domain as is the case with the atmosphere and the ocean. Ice sheets build up their volume during glacial periods and can disappear completely during interglacials. As they spread over the continents they can occasionally extend into the ocean and start to float. The floating part of an ice sheet is called an ice shelf. In the vicinity of the grounding line which marks the transition zone between ice sheet and ice shelf, ice flow is rather complicated. After describing the basic equations, approximations will be introduced for different ice sheet regimes and techniques to couple those regimes will be presented.

2.1 The prognostic equations

The model equations for an ice sheet (as for the atmosphere and the ocean) are based on the laws of conservation for mass energy and momentum.

The rate of change of ice sheet thickness h follows from mass conservation:

$$\frac{\partial h}{\partial t} = - \nabla \cdot q + b, \tag{2.1}$$

where $q = \int_{h_B}^{h_s} u_H \, dz$ is the vertically integrated horizontal ice velocity u_H from $z = h_B$, the height of the ice sheet bottom to $z = h_s$, the height of the ice sheet surface. The ice sheet is forced by the mass balance b at the surface. Ice loss at the bottom due to melting can be neglected.

The presentation of the model equations follows a more detailed description in Herterich (1991). An introduction to the physics of ice sheets is given by Paterson (1981). The theory of ice mechanics is discussed in depth by Hutter (1983).

The height h_B of the ice sheet bottom is not constant in time. The elastic lithosphere is suppressed into the astenosphere below (treated as a viscous fluid) by the weight of the ice sheet. The theory of bedrock dynamics has not yet provided a generally accepted set of equations for ice sheet modellers. If h_B is a function of time according to by some theory, the height of the ice sheet surface is obtained by $h_s = h_B + h$ (Fig. 1), with h from (2.1).

The calculation of ice flow which enters equation (2.1) is less straightforward and will be discussed in section 2.2 below. Ice flow is temperature dependent. Temperature T can be calculated from the energy conservation:

$$\frac{\partial T}{\partial t} = u \cdot \nabla T + k\frac{\partial^2 T}{\partial z^2} + d, \tag{2.2}$$

where u is the ice velocity, k the (molecular) diffusivity and d the rate of change of temperature due to internal frictional heat production. Since the ice sheet can be regarded as a medium with a small aspect ratio, horizontal diffusion can be neglected.

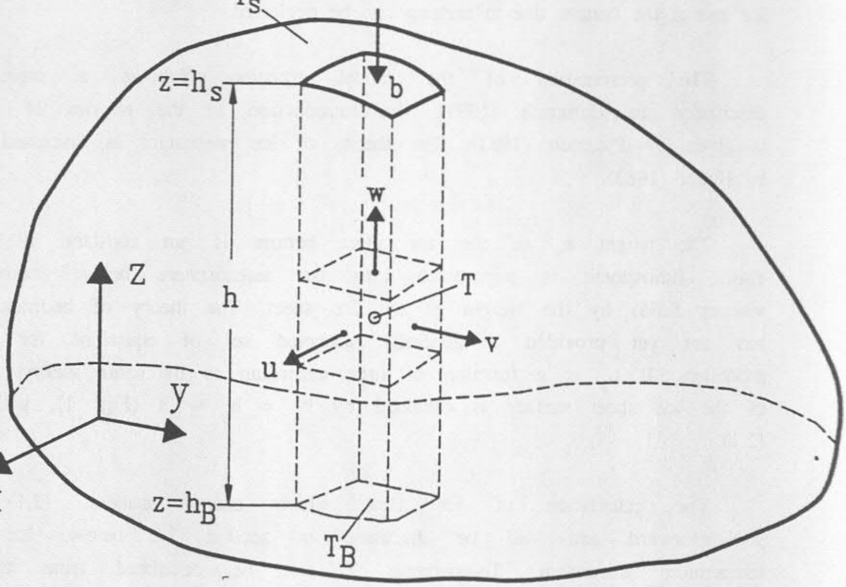

Fig. 1: Definition of model variables in the equations of the conservation of mass (2.1) and energy (2.2).

2.2 The diagnostic equations

Since the acceleration of ice particles can be neglected the conservation of momentum reduces to the balance of forces:

$$\nabla \cdot \sigma + \rho g = 0, \tag{2.3}$$

where σ is the symmetric stress tensor, ρ the density of ice taken to be a constant in most of the models and g the gravitational acceleration.

In a Cartesian coordinate system with the z-axes directed vertically upwards (2.3) becomes

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z} = 0, \tag{2.4}$$

$$\frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{yz}}{\partial z} = 0, \tag{2.5}$$

$$\frac{\partial \sigma_{xz}}{\partial x} + \frac{\partial \sigma_{yz}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} = -\rho g \tag{2.6}$$

The system (2.4), (2.5) and (2.6) is well suited for discussion of the major approximations which can be made for different ice sheet regimes. With regard to the vertical component, shear stresses can be assumed small compared to the normal pressure. Thus $\sigma_{xz}, \sigma_{yz} \ll \sigma_{zz}$ in eq. (2.6), yielding:

$$\sigma_{zz} = -\rho g(h_s - z). \tag{2.7}$$

For the grounded part of the ice sheet with a small aspect ratio, the horizontal gradient of σ_{xy} can be neglected in (2.4) and (2.5) and the normal pressure is assumed to be isotropic ($\sigma_{xx} = \sigma_{yy} = \sigma_{zz}$). Integration from the ice sheet surface $z = h_s$ to some depth z within the ice sheet then yields:

$$\sigma_{xz} = -\rho g \frac{\partial h_s}{\partial x}(h_s - z), \tag{2.8}$$

$$\sigma_{yz} = -\rho g \frac{\partial h_s}{\partial y}(h_s - z), \tag{2.9}$$

The so-called flow law provides a relation between the stress tensor σ and the tensor of the rate of change of deformation ε which, for small velocity gradients $\partial u_i/\partial x_k$ is defined by:

$$\varepsilon_{ik} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i}\right) \tag{2.10}$$

The flow law is a semi-empirical law. In the form suggested by Nye (1957) it reads:

$$\dot{\varepsilon}_{ik} = A(T')\sigma^{n-1}\sigma'_{ik}, \tag{2.11}$$

where the σ'_{ik} are the components of the stress deviator tensor defined by:

$$\sigma'_{ik} = \sigma_{ik} - \delta_{ik}p\ , \tag{2.12}$$

with p the isotropic normal pressure $p = \frac{1}{3}(\sigma_{xx} + \sigma_{yy} + \sigma_{zz})$ and δ_{ik} the Kronecker symbol. σ is the second invariant of the stress deviator tensor. The flow law (2.11) is invariant with respect to a rotation of the coordinate system and is independent of the isotropic normal pressure p. The exponent n ($n \approx 3$) and the factor of proportionality $A(T')$ are empirical numbers, the latter depending strongly on ice temperature T' measured with respect to pressure-corrected melting point. From the definition (2.12) and the flow law (2.11) it follows that

$$\dot{\varepsilon}_{xx} + \dot{\varepsilon}_{yy} + \dot{\varepsilon}_{zz} = 0,$$

which, with (2.10), is a statement of the incompressibility condition $\nabla \cdot u = 0$.

The flow law (2.11) and the force balance (2.3) together with the definitions (2.10) and (2.12) form a closed set of equations from which the ice velocity can be determined diagnostically.

In the case of the grounded part of the ice sheet, this leads to an analytical expression for the ice velocity (see section 2.3). For the floating part of the ice sheet, namely the ice shelves, a set of 2 coupled partial differential equations of elliptic type for the 2 components of the horizontal velocity result, and these can be solved iteratively. The ice velocity within the transition zone between the ice sheet and the ice shelf can also be calculated from a single elliptical equation, in this case for the 2 velocity components in a vertical plane perpendicular to the grounding line. However, presently existing ice sheet models all use a much reduced description for the transition zone.

2.3 The coupled system: ice-sheet, transition-zone, ice-shelf

The velocity field in the grounded part of the ice sheet can be determined from local characteristics, the ice thickness, the surface slope, and the vertical distribution of temperature (Herterich, 1988):

$$u_H(z) = u_H(z=h_B) + c \begin{pmatrix} \partial h_s/\partial x \\ \partial h_s/\partial y \end{pmatrix},$$

with

$$c = -2(\rho g)^n \left[\left(\frac{\partial h_s}{\partial x}\right)^2 + \left(\frac{\partial h_s}{\partial y}\right)^2 \right]^{\frac{n-1}{2}} \int_{h_b}^{z} A(T'(z'))(h_s - z')^n \, dz.$$

In the transition zone between the ice sheet and the ice shelf, the ice flow field is the solution of a boundary value problem in the vertical plane. The boundary conditions that have to be prescribed are the input flow from the ice sheet, the horizontal gradient of the output flow into the ice shelf, and the vertical velocity gradient at the bottom, upstream from the grounding line. From theoretical considerations, one might expect that the width of the transition zone in the flow direction would be small compared to the horizontal extensions of both the ice sheet and the ice shelf. This makes possible a much simpler treatment of the transition zone in coupled ice sheet/ice shelf models compared to the solving an elliptical partial differential equation as described in section 2.2. In a low resolution numerical model, the transition zone may enter only at grid points along the grounding line (Huybrechts, 1990) or may even be treated as a sub-grid scale process (Böhmer and Herterich, 1990).

For the ice shelves, the boundary conditions are the ice thickness and the ice velocities at the grounding line which are basically determined by the state of the ice sheet modified by the transition zone.

In coupled runs on longer time scales (> 1000 years), both the transition zone and the ice shelves can be treated as quasi-stationary subject to slowly changing the boundary conditions set by the ice sheet. Most of the modeling results discussed in section 3 have been gained with models without the treatment of ice shelves and the transition zone.

3. Modelling results

At present, only two large scale ice sheets exist, the Greenland and the Antarctic ice sheets. They are located in the polar regimes of both hemispheres and their coupling via the global atmosphere may be neglected in studies of their present state. Consequently, all 3-d modeling attempts to describe these ice sheets as they exist today have considered these ice sheets as separate units. Some attempts have been made, however, to model the 3 major ice-age ice sheets of the Northern Hemisphere, namely the Greenland ice sheet and the 2 ice sheets in North America and Eurasia, and their interaction.

3.1 The Antarctic ice sheet

Although the Antarctic ice sheet may not be in a stationary state today, modelling the stationary state is still of theoretical interest. With fixed ice thickness distribution and surface temperature, as observed today, the model has been integrated to a stationary state (Herterich, 1988). If the Antarctic ice sheet were in a stationary state today, the resulting field of flow and temperature in the model should resemble present observations. While the modelled ice flux is of the right order of magnitude, the divergence of the vertically integrated horizontal ice flow $\nabla \cdot q$ is not. In the stationary state we should have $\nabla \cdot q = b$ (the mass balance, see equation 2.1). This condition is only met in central regions of the Antarctic. Near the margin of the Antarctic and in areas of mass flux convergence, the model produces negative mass balances, contrary to observations which indicate a positive mass balance everywhere on the ice sheet surface. The reason for this failure may be the neglect of basal sliding ($u_H(h_B)=o$) in the model. In fact, the regions where the mass balance of the model is negative all lie in areas where the model predicts basal melting which is a condition that favors basal sliding (Fig. 2). The predicted bottom temperature is also consistent with the location of observed sub-ice lakes all situated in regions where the model predicts basal melting.

Instead of fixing the presently observed thickness of the Antarctic ice sheet, one may use presently observed accumulation rates and integrate the mass conservation equation (2.1) further. Such an integration was performed with a fully coupled ice sheet/ice shelf model (Böhmer and Herterich, 1990). In this model the width of the transition zone between the ice sheet and the ice shelf was assumed to be small compared to the horizontal resolution of the model (100 km). With this assumption it was possible to relate the slope of the ice sheet at the grounding line to the bottom friction coefficient, thus providing a coupling condition between ice sheet and ice shelf. With a constant value for this coefficient (0,05), the presently observed position of the grounding line could be reproduced in the model. Nevertheless the model responds quite sensitively to the assumed value of the bottom friction coefficient.

The most complete treatment of the Antarctic ice sheet was performed by Huybrechts (1990). The model contains an ice sheet, ice shelves and a transition zone in between and includes bedrock dynamics. The mass balance is parameterized in terms of the surface temperature. When the integration is started 160.000 years back in time and forced by observed atmospheric temperatures and sea level changes, the evolving Antarctic ice sheet in the model resembles the present distribution of ice sheet and ice shelves quite well.

3.2 The Greenland ice sheet

The very first modelling attempt using a fully thermomechanically coupled ice sheet model was performed by Jenssen (1977). Starting with the presently observed initial state, the model was integrated several thousand years. As the model ice sheet rapidly developed into a state quite different from the

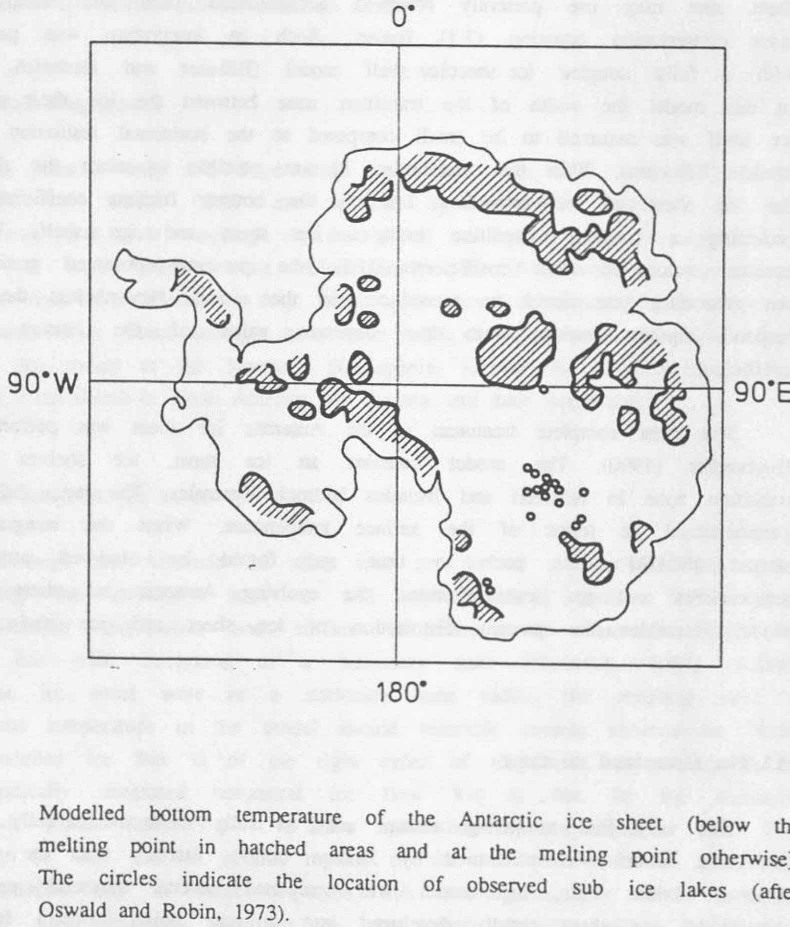

Fig. 2: Modelled bottom temperature of the Antarctic ice sheet (below the melting point in hatched areas and at the melting point otherwise). The circles indicate the location of observed sub ice lakes (after Oswald and Robin, 1973).

present state, it was concluded that either the Greenland ice sheet is not in a stationary state today or that some model assumptions about the physics and/or boundary conditions were wrong. With a new generation of super computers, more systematic modelling experiments are now possible.

Rather than trying to simulate the present state of the Greenland ice sheet, Huybrechts et al. (1991) modelled the build-up of the Greenland ice sheet starting from zero ice volume, assuming a number of different boundary conditions and forcing functions. These experiments showed that ice sheet evolution is quite sensitive to the assumed boundary conditions.

3.3 Ice-age ice sheets

During the last million years, the North American continent and Northern Europe have been covered by huge ice sheets most of the time (Fig. 3). During maximum glaciation, the ice volume of these two ice sheets was more than twice the volume of the Antarctic ice sheet. Modelling the build-up and decay of these ice sheets with 3-d models is far more complicated than simulating the presently existing ice sheets. Unlike the situation of the Antarctic and Greenland ice sheets, there are no data for defining the upper boundary conditions for the ice-age ice sheets. Furthermore, these ice sheets may have always been far from an equilibrium state.

The major problem is to define the mass balance at the ice sheet surface for the Northern Hemisphere during at least one ice age cycle (about 100.000 years). With presently available atmospheric models, this mass balance cannot be provided. General Circulation Models need too much computer time to cover an ice-age cycle and simpler models (energy balance models) without circulation can only badly model the hydrological cycle. Nevertheless, interesting results have been obtained using simplifying assumptions about the variation of the mass balance in space and time forcing 3-d ice sheets.

Andrews and Mahaffy (1976), using a model developed by Mahaffy (1975) without considering temperature dependence of ice flow, concentrated on the initial build-up phase of the North American ice sheet. With a variety of assumed mass balances and the feedback from the growing ice sheet, they were

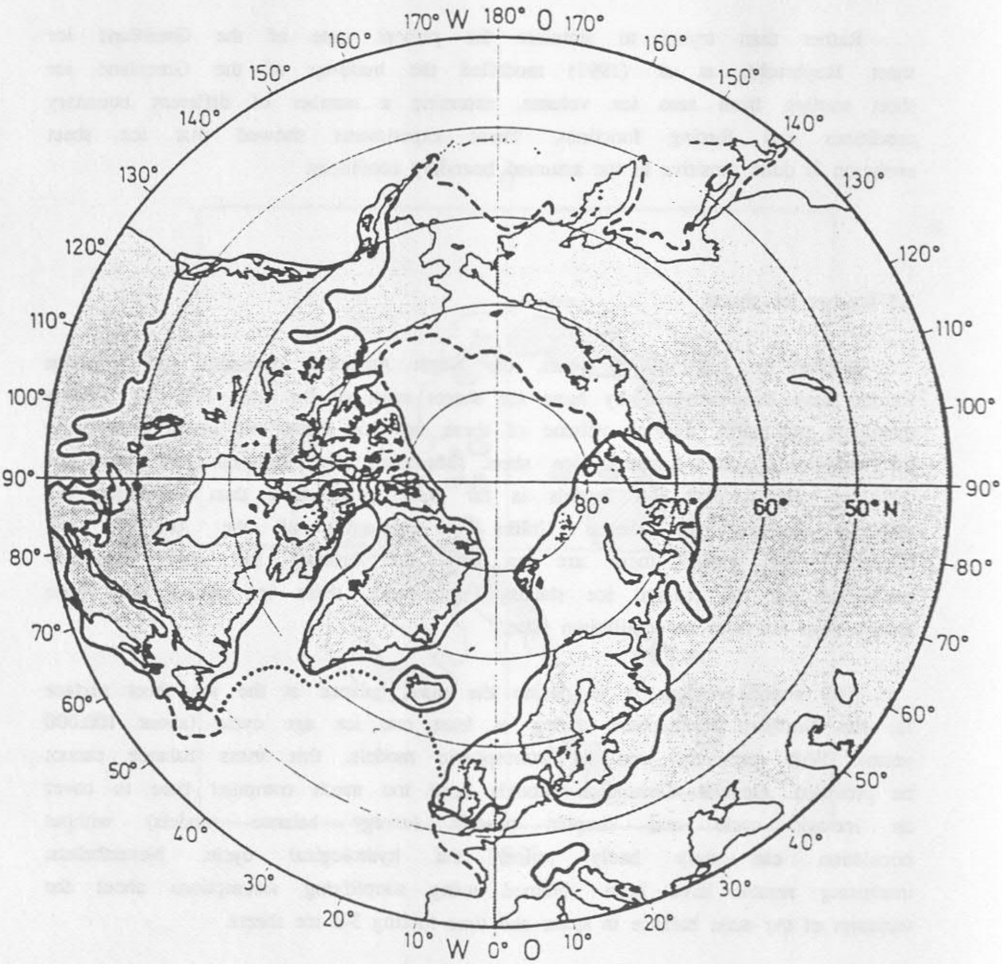

Fig. 3: Margin of Northern Hemisphere ice sheets (thick line), coast line (dashed line) and sea ice extent in summer (dotted line), 18.000 years before present.

able to match estimates of sea level changes caused by ice sheet build-up.

Starting from presently observed accumulation rates and ablation rates, Budd and Smith (1979) inspected changes of these fields with insolation. By applying these derived concepts to force the ice-age ice sheet covering North America, they were able to build up an ice sheet starting 120.000 years ago and to simulate its decay after 18.000 years B.P.

Oerlemans (1981) defined the mass balance by specifying the height of the snow line as a function of longitude and latitude for the area of the Eurasian ice sheet. The height of the snow line varied with summer insolation. He demonstrated that effects from the atmospheric circulation are important in ensuring that the Eurasian ice sheet in the model is at the observed position. The westerly winds lead to higher accumulation rates on the western side of the ice sheet and lower rates at the eastern margin. Thus the model ice sheet tended to grow into the upwind direction yielding a better fit with observed ice sheet margins.

Concerning the question of the possible existence of a Tibetan ice sheet during the ice ages put forward by Kuhle (1988), an inverse modeling experiment has been performed (Kuhle et al., 1989). Given the maximum possible extension of this still hypothetical ice sheet, the mass balance was defined in such a way as to keep the model ice sheet within these limits. The mass balance was defined in terms of three parameters: the height of the snow line (defined as the height where the yearly net mass balance is zero) an upper bound of maximum snowfall b_{max}, and the height difference Δh above the snow line where this maximum snowfall is reached. In between, a linear dependence of the mass balance on the height relative to the snowline is assumed. Since the height of the ice-age snow line can be reconstructed from the extension of fossil glaciers (Kuhle, 1988), and the parameter Δh may be related to the seasonal amplitude of temperature, the only free parameter is basically the maximum snowfall rate b_{max}. In a series of experiments, the reconstructed maximum snowfall rate resembled that observed in the monsoonal circulation pattern.

Lindstrom and Mactyeal (1989) forced their ice sheet model for the Northern Hemisphere with the observed variations in atmospheric CO_2 partial pressure as documented in the Vostock ice core (Barnola et al., 1989). To transform CO_2 variations into changes of mass balance, sensitivity experiments with Atmospheric Circulation Model experiments were used. They succeeded in building up the ice-age ice sheets as well as simulating ice sheet decay at the end of the last ice-age cycle.

In a model experiment of Deblonde and Peltier (1990), the mass balance was calculated from an energy balance model formulated for the whole globe and including a seasonal cycle. Accumulation and ablation were parameterized in terms of temperature and radiation. Starting the integration 122.000 B.P., the Northern Hemisphere ice sheets built up but did not decay in the model at the end of the last ice age cycle.

Recently, modelling experiments have been started with coupled global ice-sheet/atmosphere/ocean models. Schlesinger and Verbitzky (1991) used the 2-level atmospheric model coupled with a mixed layer ocean and an ice sheet model, applying an a-synchronous coupling. The inclusion of additional climate components and their coupling does not necessarily make the task of modelling ice-age ice sheets easier. Although, in the fully coupled mode, it is no longer necessary to define boundary conditions which are badly known, their realistic generation in the coupled model is also not guaranteed. However, if circulation effects from both the atmosphere and the ocean are to be studied, the fully coupled approach using circulation type models for atmosphere, ocean and ice sheets is the appropriate way to do this.

At MPI modelling experiments have also been started along these lines. The global ice sheet model is a vertically integrated version of the ice sheet model developed for the separate ice sheets. The ocean model used will be the planetary geostrophic model of Maier-Reimer et al. (1991). Finally, the atmospheric model is a two-level model developed by Lautenschlager (personal communication), based on the primitive equations and which does not yet contain a hydrological cycle. In initial experiments it will be parameterized in terms of variables defined in the dry atmosphere.

4 Conclusions

The model experiments listed in section 3 have shown that ice sheets react quite sensitively to changing mass balances. Unfortunately the mass balance is one of those climatic variables that cannot be modelled realistically at present. This situation holds not only for past climates but even for the present climate. Although we know that the mass balance on the Antarctic ice sheet is positive everywhere, we do not know its magnitude precisely enough to calculate the overall balance of the ice sheet, and determine whether Antarctic ice volume is increasing or decreasing. The situation is even worse for the Greenland ice sheet where the mass balance is positive at higher altitudes and negative in areas near the coast.

With such an ill-defined mass balance it is difficult to discriminate between different ice sheet models. In effect, all models described in section 3 were able to produce reasonable results independent of the model physics. Nevertheless, these experiments demonstrated the usefulness of 3-d models to answer questions beyond those treated with 2-d models. Assuming reasonable mass balance changes the ice-age ice sheets could be reproduced at observed locations. The variations in insolation seem to be sufficient to produce changes in the mass balance which lead to ice sheet growth as well as decay in at least some of the models. At present several models exist which are quite capable of matching ice volume changes qualitatively, although additional data sets will have to be incorporated in the models in order to test the validity of the model physics. These data could be atmospheric and oceanic variables obtained as the output of a coupled global ice sheet/atmosphere/ocean model as well as variables which have a direct impact on the land surface covered by these ice sheets.

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