Gauss-Bonnet supergravity in six dimensions

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The supersymmetrization of curvature squared terms is important in the study of the low energy limit of compactified superstrings where a distinguished role is played by the Gauss-Bonnet combination, which is ghost-free. In this letter, we construct its off-shell $\mathcal{N} = (1, 0)$ supersymmetrization in six dimensions for the first time. By studying this invariant together with the supersymmetric Einstein-Hilbert term we confirm and extend known results of the α'-corrected string theory compactified to six dimensions. Finally, we analyze the spectrum about the $\mathrm{AdS}_3 \times \mathrm{S}^3$ solution.

Introduction.—Six-dimensional (6D) supergravities are of considerable interest for numerous reasons that are rooted in their connection to superstring theory. This connection often guarantees improved quantum behavior for such theories. For instance, 6D anomaly-free matter-coupled models are known to arise from T^4 or K3 compactification of heterotic or type II string theories [1, 2], see also [3]. Another noteworthy model, the Salam-Sezgin model [4], which has been found to have an M/string theory origin [5], admits a unique supersymmetric $\mathrm{M}_4 \times \mathrm{S}^2$ vacuum [6] that may have interesting phenomenological applications [7–10].

Supergravity models in 6D also play an important role in the $\mathrm{AdS}_3/\mathrm{CFT}_2$ correspondence and BTZ black hole microstate counting since the ungauged theory accommodates a supersymmetric $\mathrm{AdS}_3 \times \mathrm{S}^3$ solution [11, 12]. However, precision tests of holography often require knowledge of supersymmetric higher order curvature invariants, motivating the need for their construction.

Higher order curvature terms are also important in string theory where the corrections take the form of an infinite series constrained by the on-shell supersymmetry order by order in the string tension α'. Upon reduction to six dimensions, where an off-shell formulation of supergravity is available, exact higher curvature invariants can be systematically constructed. The leading corrections come from curvature squared terms in which the Gauss-Bonnet (GB) combination $R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma} - 4R^{\mu\nu}R_{\mu\nu} + R^2 = 6R_{[\mu\nu}{}^{\mu\nu}R_{\rho\sigma]}{}^{\rho\sigma}$ is singled out as it is ghost free, and its equations of motion are second order in derivatives [13, 14]. These features facilitate the study of exact solutions and significantly simplifies the computation of physical quantities.

The purely gravitational higher curvature terms are related by supersymmetry to contributions depending on p-forms. These terms, that have not yet been systematically analyzed in the literature, play an important role in understanding the moduli in compactified string theory and the low energy description of string dualities, see, e. g., [15–17].

The construction of the GB supergravity invariant can be achieved using off-shell techniques for $\mathrm{D} \leq 6$. Their construction in 4D and 5D were presented in [18, 19] and [20–22], respectively, while for the 6D case only partial results were obtained thirty years ago [23–26]. In this work we complete the construction of the 6D off-shell $\mathcal{N} = (1, 0)$ GB term utilizing the techniques of [27–29].

A major advantage of an off-shell formulation is that the supersymmetry transformations do not receive higher order corrections. It also allows one to easily combine separate Lagrangians. In fact, by adding the off-shell supersymmetric Einstein-Hilbert term of [27] to our GB invariant we obtain an off–shell completion of the 6D Einstein-Gauss-Bonnet supergravity action originally derived in [17] by using the heterotic/IIA duality reduced to 6D $\mathcal{N} = (1, 1)$, see also [30], and by truncating to the NSNS $\mathcal{N} = (1, 0)$ sector. We conclude by analyzing the α'-corrected spectrum of fluctuations around the supersymmetric $\mathrm{AdS}_3 \times \mathrm{S}^3$ solution.

New supergravity invariants.—To begin with, we briefly summarize the field content of the standard Weyl multiplet of $\mathcal{N} = (1, 0)$ conformal supergravity in six dimensions [27]. This consists of $40 + 40$ bosonic and fermionic off-shell degrees of freedom. In the following we use the conventions of [31, 32] with the exception of a sign difference in the parity transformation. We denote spacetime indices by $\mu, \nu, \ldots$, tangent space indices by $a, b, \ldots$ and SU(2) indices by $i, j, \ldots$ Among the bosonic fields are the sechsbein $e_\mu{}^a$, SU(2) gauge fields $V_{\mu i}{}^j$ and a gauge field b_μ associated with dilatations. Furthermore, there are two composite bosonic gauge fields, namely the spin connection $\omega_\mu{}^{ab}$ and the gauge field $f_\mu{}^a$ associated with conformal boosts. In addition to the gauge connections, the bosonic content of the standard Weyl multiplet comprises of an anti-self-dual tensor T^-_{abc} and a real scalar D as covariant matter fields. The fermions consist of the gravitini, $\psi_{\mu i}$, that are the gauge fields of Q-supersymmetry, a composite gauge field $\phi_\mu{}^i$ associated with S-supersymmetry, and a chiral fermion χ^i. In what follows we will suppress the fermionic terms and we will refrain from discussing the superconformal transformations of the various fields in any detail, which are summarized in [31].

To describe curvature squared terms, we will make use of a variant 40+40 multiplet of conformal supergravity, known as the dilaton-Weyl multiplet [27]. This is obtained by coupling the standard Weyl multiplet to a tensor multiplet. The independent connections remain the same as in the standard Weyl multiplet but the covariant matter fields are exchanged with those of the tensor multiplet, which include a scalar field σ, a gauge 2-form $B_{\mu\nu}$ and a spinor ψ^i amongst its component fields. Upon doing so the bosonic covariant fields of the standard Weyl multiplet are then expressed as

$$T^-_{abc} = \frac{1}{2\sigma} H^-_{abc}, \tag{1a}$$
$$D = \frac{15}{4\sigma}\big(\mathscr{D}^a \mathscr{D}_a \sigma + \tfrac{1}{5} R\sigma + \tfrac{1}{3} T^-_{abc} H^{abc}\big) + \text{f.t.}, \tag{1b}$$

where "f.t." stands for neglected fermionic terms and H^-_{abc} denotes the anti-self-dual part of the three-form field strength $H_{abc} = 3 e_a{}^\mu e_b{}^\nu e_c{}^\rho \partial_{[\mu} B_{\nu\rho]}$. Here we have made use of the covariant derivative

$$\mathscr{D}_a = e_a{}^\mu \big(\partial_\mu - \tfrac{1}{2}\omega_\mu{}^{bc} M_{bc} - b_\mu \mathbb{D} - V_{\mu i}{}^j U_j{}^i\big), \tag{2}$$

where M_{ab}, $\mathbb{D}$ and $U_i{}^j$ are the Lorentz, dilatation and SU(2) generators, respectively. The Lorentz curvature is given by $R_{\mu\nu}{}^{cd} = R_{\mu\nu}{}^{cd}(\omega) := 2\partial_{[\mu}\omega_{\nu]}{}^{cd} + 2\omega_{[\mu}{}^{ce}\omega_{\nu]e}{}^d$, where $\omega_\mu{}^{cd} = \omega(e)_\mu{}^{cd} + 2e_\mu{}^{[c} b^{d]}$ is the Lorentz connection and $\omega(e)_\mu{}^{cd}$ is the usual torsion-free spin connection. The dilatation connection b_μ is pure gauge [27] and we will always consider it set to zero. Besides the Riemann tensor $R_{ab}{}^{cd} := R_{ab}{}^{cd}(\omega)$, we also use the tensors $R_a{}^b = R_{ac}{}^{bc}$ and $R = R_{ab}{}^{ab}$, which coincide with the Ricci and scalar curvature tensors, respectively. Note that by using the mapping (1), every invariant involving a coupling to the standard Weyl multiplet can be directly converted to one in terms of the dilaton-Weyl multiplet.

In three, four and five dimensions with eight supercharges, a special role is played by the linear multiplet action principle coupled to supergravity [33–35]. It schematically represents an action based on the product of a linear and a vector multiplet and describes supersymmetric extensions of $B_{d-2} \wedge F_2$ invariants in d-dimensions with F_2 a closed two form and B_{d-2} an unconstrained $(d-2)$-form. Although this action principle also exists for $(1,0)$ superconformal symmetry in six dimensions, and in fact was the main building block for superconformal invariants in [27], there is also another possibility. As briefly noted in [28], one can generate an action principle that is schematically the product of the tensor multiplet and a four-form multiplet, which is the supersymmetric extension of $B_2 \wedge H_4$. In components the associated Lagrangian takes the form

$$e^{-1}\mathcal{L}_{B_2\wedge H_4} = \tfrac{1}{4} B_{ab} C^{ab} - \tfrac{1}{4}\sigma C + \text{f.t.} \tag{3}$$

Here $C^{ab} = -\frac{1}{4!}\varepsilon^{abcdef} H_{cdef}$ is the Hodge dual of a closed four-form, $\mathrm{d}H_4 = 0$, and C is a real scalar field. The dimension 4 fields $C_{ab} = \frac{1}{12}\bar{Q}_i \gamma_{[a} Q_j B^{ij}_{b]}$ and $C = \frac{1}{12}\bar{Q}_i \gamma^a Q_j B^{ij}_a$ are descendant components of the four-form multiplet [37] which are defined in terms of a primary dimension 3 field $B^{ij}_a = B^{(ij)}_a$, see also [28, 29], and Q^i are the Q-supercharges. The density formula (3), which extends the one first introduced to describe the rigid supersymmetric Yang-Mills action [38], is the building block for constructing the curvature squared invariants in this letter.

Recently, a particular composite four-form multiplet defined solely using the fields of the standard Weyl multiplet has been constructed in [28]. It was used to describe an $\mathcal{N} = (1,0)$ conformal supergravity action [29]. The primary dimension 3 field of this multiplet has the form $B^{ij}_a = \frac{1}{4} T^-_{abc} F^{bc\,ij} + \text{f.t.}$, where $F_{\mu\nu}{}^{kl} := 2\partial_{[\mu} V_{\nu]}{}^{kl} - 2V_{[\mu}{}^{p(k} V_{\nu]p}{}^{l)}$ denotes the SU(2) curvature. The C and C^{ab} components of this composite multiplet were worked out in [29]. Plugging these results into (3) gives the new invariant

$$\begin{aligned} e^{-1}\mathcal{L}_{\text{new}} = \tfrac{1}{32}\Big\{ & \sigma C_{ab}{}^{cd} C_{cd}{}^{ab} - 3\sigma F_{ab}{}^{ij} F^{ab}{}_{ij} + \tfrac{4}{15}\sigma D^2 - 8\sigma T^{-dab}\big(\mathscr{D}_d \mathscr{D}^c T^-_{abc} + \tfrac{1}{2} R_d{}^c T^-_{abc} - \tfrac{1}{20} R T^-_{dab}\big) \\ & + 4\sigma(\mathscr{D}_c T^{-abc})\mathscr{D}^d T^-_{abd} + 4\sigma T^{-abc} T^-_{ab}{}^d T^{-ef}{}_c T^-_{efd} - \tfrac{8}{45} H_{abc} T^{-abc} D + 2H_{abc} C^{ab}{}_{de} T^{-cde} \\ & + 4H_{abc} T^{-ab}_d \mathscr{D}_e T^{-cde} - \tfrac{4}{3} H_{abc} T^{-dea} T^{-bcf} T^-_{def} - \tfrac{1}{4}\varepsilon^{abcdef} B_{ab}\big(C_{cd}{}^{gh} C_{efgh} - F_{cd}{}^{ij} F_{efij}\big)\Big\} + \text{f.t.} \end{aligned} \tag{4}$$

Here $C_{ab}{}^{cd}$ is the Weyl tensor and (1) holds in (4). The fermionic extension of (4) will appear in [36].

A supersymmetric extension of the Riemann curvature squared term was constructed in [23–26]. This was based on the action for a Yang-Mills multiplet coupled to conformal supergravity [27] and exploiting the feature that in the gauge

$$\sigma = 1, \quad b_\mu = 0, \quad \psi^i = 0, \tag{5}$$

the dilaton-Weyl multiplet can be mapped to a Yang-Mills vector multiplet taking values in the 6D Lorentz algebra [23]. It turns out that both the Yang-Mills and the supersymmetric Riemann squared invariants can be constructed by using the $B_2 \wedge H_4$ density formula together with appropriately chosen composite four-form multiplets. In superspace these were described in section 6 of [29] from which their component actions can be readily obtained. For the purpose of this letter, it is enough to present the bosonic part of the Riemann squared invariants in the gauge (5), which takes the form

$$e^{-1}\mathcal{L}_{\rm Riem^2} = -4F^{ab}{}_{ij}F_{ab}{}^{ij} + R^{abcd}(\omega_-)R_{ab\,cd}(\omega_-) - \tfrac{1}{4}\varepsilon^{abcdef}B_{ab}\,R_{cd}{}^{gh}(\omega_-)R_{ef\,gh}(\omega_-) + \text{f.t.} \quad (6)$$

Here $R_{ab}{}^{cd}(\omega_-)$ is the torsionful Lorentz curvature defined in terms of the modified connection

$$\omega_{\pm\mu}{}^{cd} := \omega_\mu{}^{cd} \pm \tfrac{1}{2}e_\mu{}^a H_a{}^{cd} \quad (7)$$

and it is such that

$$R_{ab}{}^{cd}(\omega_\pm) = R_{ab}{}^{cd} \pm \mathscr{D}_{[a}H_{b]}{}^{cd} - \tfrac{1}{2}H_{e[a}{}^{[c}H_{b]}{}^{d]e}. \quad (8)$$

As noted in [23], it is straightforward to restore a general gauge and undo the condition (5).

Now that we have described the new curvature squared invariant, we are ready to describe an off-shell extension of the 6D Gauss-Bonnet combination. It suffices to take the following combination of the Riemann squared and the new invariant

$$\mathcal{L}_{\rm GB} = -3\mathcal{L}_{\rm Riem^2} + 128\mathcal{L}_{\rm new}. \quad (9)$$

For the applications in this letter we will use the Gauss-Bonnet invariant in the gauge (5). Making use of (1) and (5) together with (8), (9) takes the form

$$\begin{aligned} e^{-1}\mathcal{L}_{\rm GB} &= 6R_{[\mu\nu}{}^{\mu\nu}(\omega_+)R_{\rho\sigma]}{}^{\rho\sigma}(\omega_+) + \tfrac{2}{3}R(\omega_+)H^2 \\ &- 4R^{\mu\nu}(\omega_+)H^2_{\mu\nu} + 4R_{\mu\nu\rho\sigma}(\omega_+)H^{\mu\rho\alpha}H^{\nu\sigma}{}_\alpha \\ &+ \tfrac{1}{9}(H^2)^2 - \tfrac{2}{3}H^4 + \epsilon^{\mu\nu\rho\sigma\lambda\tau}B_{\mu\nu}F_{\rho\sigma}{}^{ij}F_{\lambda\tau\,ij} \\ &+ \tfrac{1}{4}\epsilon^{\mu\nu\rho\sigma\lambda\tau}B_{\mu\nu}R_{\rho\sigma}{}^\alpha{}_\beta(\omega_+)R_{\lambda\tau}{}^\beta{}_\alpha(\omega_+) + \text{f.t.}, \end{aligned} \quad (10)$$

where $H^4 := H_{\mu\nu\sigma}H_{\rho\lambda}{}^\sigma H^{\mu\rho\delta}H^{\nu\lambda}{}_\delta$, $H^2_{\mu\nu} := H_\mu{}^{\rho\sigma}H_{\nu\rho\sigma}$ and $H^2 := H_{\mu\nu\rho}H^{\mu\nu\rho}$.

It is important to note that the B-field dependence of the supersymmetric GB invariant cannot be solely captured by a torsionful connection. This explains the previous unsuccessful attempts at the supersymmetrization of the 6D GB action [23, 24] where only the first and last two terms in (10) appeared in the bosonic part of the invariant. It is worth remarking that the off-shell GB action does not contain higher order kinetic terms for the massless supermultiplet of the two-derivative theory and the kinetic term for the SU(2) gauge fields $V_\mu{}^{ij}$ drops out.

On-Shell Einstein-Gauss-Bonnet Supergravity.—Now let us study a certain linear combination of the Einstein-Hilbert and GB invariants, which we refer to as Einstein-Gauss-Bonnet supergravity. In contrast, to the GB invariant which is based solely on the dilaton-Weyl multiplet, the Einstein-Hilbert invariant requires a compensating multiplet, which we choose to be the linear multiplet [27, 31]. It consists of an SU(2) triplet of scalars L_{ij}, a constrained vector field E_a, and an SU(2) Majorana spinor φ^i. Adopting the gauge (5) together with the SU(2) $\to$ U(1) gauge fixing conditions

$$L_{ij} = \tfrac{1}{\sqrt{2}}\delta_{ij}L, \quad V_\mu^{ij} = V'_\mu + \tfrac{1}{2}\delta^{ij}V_\mu, \quad (11)$$

where $L^2 = L_{ij}L^{ij}$, the EH action takes the form [32]

$$\begin{aligned} e^{-1}\mathcal{L}_{\rm EH} &= LR + L^{-1}\partial_\mu L\partial^\mu L - \tfrac{1}{12}LH_{\mu\nu\rho}H^{\mu\nu\rho} \\ &+ 2LV'^{ij}_\mu V'^\mu_{ij} - \tfrac{1}{2}L^{-1}E^\mu E_\mu + \sqrt{2}E^\mu V_\mu. \end{aligned} \quad (12)$$

The off-shell Einstein-Gauss-Bonnet supergravity is defined by the Lagrangian

$$2\kappa^2\mathcal{L} = \mathcal{L}_{\rm EH} + \tfrac{1}{16}\alpha'\mathcal{L}_{\rm GB}. \quad (13)$$

The solution $V_\mu{}^{ij} = E_\mu = 0$ is consistent with the equations of motion for the fields $(V_\mu{}^{ij}, E_\mu)$. Setting $L = e^{-2\upsilon}$ and pulling out most of the dependence on the three-form H_3, we obtain the Lagrangian for the on-shell Einstein-Gauss-Bonnet supergravity

$$\begin{aligned} 2\kappa^2 e^{-1}\mathcal{L} &= e^{-2\upsilon}[R + 4\partial_\mu\upsilon\partial^\mu\upsilon - \tfrac{1}{12}H_{\mu\nu\rho}H^{\mu\nu\rho}] \\ &+ \tfrac{1}{16}\alpha'\Big[6R_{[\mu\nu}{}^{\mu\nu}R_{\rho\sigma]}{}^{\rho\sigma} + \tfrac{1}{6}RH^2 - R^{\mu\nu}H^2_{\mu\nu} + \tfrac{5}{24}H^4 \\ &+ \tfrac{1}{2}R_{\mu\nu\rho\sigma}H^{\mu\nu\lambda}H^{\rho\sigma}{}_\lambda + \tfrac{1}{144}(H^2)^2 - \tfrac{1}{8}(H^2_{\mu\nu})^2 \\ &+ \tfrac{1}{4}\epsilon^{\mu\nu\rho\sigma\lambda\tau}B_{\mu\nu}R_{\rho\sigma}{}^\alpha{}_\beta(\omega_+)R_{\lambda\tau}{}^\beta{}_\alpha(\omega_+)\Big] + \text{f.t.} \end{aligned} \quad (14)$$

This result has a remarkable feature. In [17], it was conjectured that for the type II string, the B-field dependence in R^4 corrections is nearly completely captured in terms of the torsionful Riemann tensor (8) (except for the CP-odd sector). The claim was further studied by fixing the one-loop four-derivative corrections in six dimensions by means of a K3 reduction of type IIA and requiring that the dyonic string remains a solution, as well as the duality of this model to heterotic strings compactified on T^4. Here we provided an alternative derivation of the four-derivative corrections by exact supersymmetrization of the curvature squared invariants. Our result for the on-shell Einstein-Gauss-Bonnet supergravity precisely matches [17], thereby providing strong evidence for the conjecture put forward there.

$AdS_3\times S^3$ solution and spectrum.—It is known that the 6D two-derivative supergravities admit a supersymmetric $\mathrm{AdS}_3\times \mathrm{S}^3$ solution where the non-vanishing fields are given by

$$\begin{aligned} ds_6^2 &= \varrho^2(ds^2_{\rm AdS_3} + ds^2_{\rm S^3}), \\ H_3 &= 2\varrho^{-1}(\Omega_{\rm AdS_3} - \Omega_{\rm S^3}), \qquad L = 1, \end{aligned} \quad (15)$$

where $ds^2_{\rm AdS_3}$ and $ds^2_{\rm S^3}$ refer to the metrics on the unit radius of AdS_3 and S^3, respectively, $\Omega_{\rm AdS_3}$ and $\Omega_{\rm S^3}$ denote their volume forms, and ϱ is the constant curvature radius. This solution arises as the near horizon limit of dyonic strings. Similar to the $\mathrm{AdS}_5\times\mathrm{S}^5$ solution in type IIB [39], the $\mathrm{AdS}_3\times\mathrm{S}^3$ metric has vanishing Weyl tensor and scalar curvature [40]. One can show that the

$AdS_3 \times S^3$ solution in eq. (15) is also a solution of the Einstein-Gauss-Bonnet supergravity. The spectrum of the two-derivative theory (12) has been studied before in various works [11, 41, 42]. It contains only the short multiplets of $SU(1,1|2)$ dressed by the irreducible representations of the extra $SL(2,\mathbb{R}) \times SU(2)$, since the total isometry group associated with the supersymmetric $AdS_3 \times S^3$ vacuum is $SU(1,1|2) \times SL(2,\mathbb{R}) \times SU(2)$. A short multiplet of $SU(1,1|2)$ has the structure

$$(h,j) \oplus 2\times(h+\tfrac{1}{2}, j-\tfrac{1}{2}) \oplus (h+1, j-1), \quad h=j, \quad (16)$$

where h and j label the representations of the $SL(2,\mathbb{R}) \times SU(2)$ bosonic subgroup inside $SU(1,1|2)$. The irreducible representations of the extra $SL(2,\mathbb{R}) \times SU(2)$ group are labeled by $(\bar{h},\bar{j})$. All together, we use $DS^{(\bar{h},\bar{j})}(h,j)_S$ to denote a short multiplet of $SU(1,1|2) \times SL(2,\mathbb{R}) \times SU(2)$. The spectrum of the two-derivative theory (12) consists of eight infinite towers of short multiplets, each of which is labeled by an integer $\ell \geq 0$

$$\begin{aligned} &DS^{(\frac{\ell+3}{2},\frac{\ell+3}{2})}\Big(\tfrac{\ell+1}{2},\tfrac{\ell+1}{2}\Big)_S,\ 2\times DS^{(\frac{\ell}{2}+2,\frac{\ell}{2})}\Big(\tfrac{\ell+2}{2},\tfrac{\ell+2}{2}\Big)_S,\\ &DS^{(\frac{\ell+1}{2},\frac{\ell+1}{2})}\Big(\tfrac{\ell+3}{2},\tfrac{\ell+3}{2}\Big)_S,\ DS^{(\frac{\ell}{2}+2,\frac{\ell}{2})}\Big(\tfrac{\ell+4}{2},\tfrac{\ell+4}{2}\Big)_S,\\ &2\times DS^{(\frac{\ell}{2}+1,\frac{\ell}{2}+1)}\Big(\tfrac{\ell+2}{2},\tfrac{\ell+2}{2}\Big)_S,\ DS^{(\frac{\ell}{2}+2,\frac{\ell}{2})}\Big(\tfrac{\ell}{2},\tfrac{\ell}{2}\Big)_S. \quad (17)\end{aligned}$$

At each level ℓ, the spectrum contains 16+16 degrees of freedom which is the same as the spectrum around supersymmetric $Minkowski_6$.

Generic $SU(1,1|2)$ long multiplets have the form [11]

$$\begin{aligned}&(h,j)\oplus 2\times(h+\tfrac{1}{2},j-\tfrac{1}{2})\oplus(h+1,j-1)\oplus\\ &(h+\tfrac{1}{2},j+\tfrac{1}{2})\oplus 2\times(h+1,j)\oplus(h+\tfrac{3}{2},j-\tfrac{1}{2}). \quad (18)\end{aligned}$$

Our analysis shows that the linearized spectrum around $AdS_3 \times S^3$ of the action (13) contains, in addition to the eight infinite towers of short multiplets (17), four long multiplets whose AdS energies are independent of the Kaluza-Klein level ℓ, with $\ell \geq 0$,

$$\begin{aligned}&DS^{(\frac{\varrho^2}{\alpha'},\frac{\ell}{2})}\Big(\tfrac{\varrho^2}{\alpha'}+\tfrac{1}{2},\tfrac{\ell+1}{2}\Big)_L,\ DS^{(1-\frac{\varrho^2}{\alpha'},\frac{\ell}{2})}\Big(-\tfrac{\varrho^2}{\alpha'}-1,\tfrac{\ell}{2}\Big)_L,\\ &DS^{(1-\frac{\varrho^2}{\alpha'},\frac{\ell}{2})}\Big(-\tfrac{\varrho^2}{\alpha'}-\tfrac{1}{2},\tfrac{\ell+1}{2}\Big)_L,\ DS^{(\frac{\varrho^2}{\alpha'},\frac{\ell}{2})}\Big(\tfrac{\varrho^2}{\alpha'},\tfrac{\ell}{2}\Big)_L, \quad (19)\end{aligned}$$

where we have set $\kappa^2 = 1$ in (13). It should be emphasized that the auxiliary vector V_μ^{ij} has an important role in the analysis of the spectrum around $AdS_3 \times S^3$. Its KK massive states get reorganized into different multiplets. This would be impossible if one were considering the on-shell Gauss-Bonnet invariant (14) where $V_\mu^{ij} = 0$. The fact that all the states in our previous analysis fits into multiplets of $SU(1,1|2) \times SL(2,\mathbb{R}) \times SU(2)$ provides a further consistency check of the supersymmetric invariance of the off-shell GB action (10).

Unitarity requires $\bar{h} > 0$ and $h > j \geq 0$, thus $\alpha' > 0$ implies only the first and fourth series of the long multiplets can be unitary, with the restriction $\ell \leq \frac{\varrho^2}{\alpha'}$. However, unitarity of the representations carried by the linearized modes only means these modes are not tachyonic. Unitarity of the modes requires the Hamiltonian to be bounded from below. In fact, the $B\wedge R\wedge R$ and $B\wedge F\wedge F$ terms in the GB action induce Chern-Simons couplings in 3D via compactification on S^3 and an Ostrogradsky type analysis à la [43] shows that the Hamiltonian of the massive modes carrying unitary representations are in fact unbounded from below. These non-tachyonic ghost-like modes may encode the information of the massive string states propagating in the supersymmetric $AdS_3\times S^3\times K3(T^4)$ target space. The remaining modes carrying non-unitary representations are tachyonic and non-renormalizable. They may be removed from the spectrum by imposing proper boundary conditions.

Outlook.—The new GB invariant (10), together with the Riemann [23–26] and scalar curvature squared [44] combinations, allow one to construct all off-shell $\mathcal{N} = (1,0)$ curvature squared invariants in six dimensions. These results give the opportunity to extend known supergravity-matter models by including four-derivative invariants. By having full control of the off–shell supersymmetry transformations, one could determine whether BPS solutions of the two-derivative theory are solutions of the four-derivative ones.

In this letter, we studied the $AdS_3 \times S^3$ solution arising from the near horizon limit of the dyonic string. However, we have not checked whether the full dyonic string, interpolating between the $AdS_3 \times S^3$ and $Miknowski_6$, is unmodified by the GB invariant. It will be interesting to explore the full solution in the Einstein-Gauss-Bonnet supergravity, from which one can also extract the central charge of the dual 2D SCFT.

Compactifications to 4D of the GB invariant are also of interest. For instance, the string-string-string duality observed in the 4D STU model can be extended to include the higher derivative corrections by reducing the the 6D $\mathcal{N} = (1,0)$ Einstein-Gauss-Bonnet supergravity on a 2-torus [45].

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