

# Energy momentum tensor for translation invariant renormalizable noncommutative field theory

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## Abstract

In this paper, we derive the energy momentum tensor for the translation invariant noncommutative Tanasa scalar field model. The Wilson regularization procedure is used to improve this tensor and the local conservation property is recovered. The same question is addressed in the case where the Moyal star product is deformed including the tetrad fields. It provides with an extension of the recent work [J. Phys. A: Math. Theor. 43 (2010) 155202], regarding the computation and properties of the Noether currents to the renormalizable models.

Pacs numbers: 02.40.Gh, 11.10.Nx

Key words: Energy-Momentum-Tensor, translation invariant, renormalizable model, noncommutative field theory.

## 1 Introduction

Noncommutative (NC) geometry and its applications to quantum field theory (QFT) namely NC-QFT receives an increasing attention this two decades due to the advent of the class of renormalizable actions [1]-[10]. The NCQFT arises as a scenario for the Planck scale behavior of physical theories, at which the non-locality of interactions has to appear and break down the notion of continuous spacetime [11]-[12]. It is most often performed over a Moyal space  $\mathbb{R}_\theta^d$ . This space is the deformation of  $d$ -dimension Euclidean space  $\mathbb{R}^d$  endowed with a constant Moyal product of functions:

$$(f \star g)(x) = \mathbf{m} \left\{ e^{i\frac{\theta^{\rho\sigma}}{2} \partial_\rho \otimes \partial_\sigma} f(x) \otimes g(x) \right\}, \quad x \in \mathbb{R}^d, \quad f, g \in C^\infty(\mathbb{R}^d). \quad (1)$$

where  $\mathbf{m}(f \otimes g) = f \cdot g$ , and such that the coordinates functions  $x^\sigma$  and  $x^\rho$  satisfied the commutation relation

$$[x^\rho, x^\sigma]_\star = i\theta^{\rho\sigma}. \quad (2)$$

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$\theta^{\rho\sigma}$  is a skew symmetric constant tensor and elements have the dimension of length square. It is possible to construct the NCQFT in a nontrivial background metric, generally by imposing the non-constant deformation matrix  $\theta^{\rho\sigma} = \theta^{\rho\sigma}(x)$ , which naturally results in the difficulty of finding a suitable explicit closed Moyal-type formula [13]-[17]. In the context of a dynamical NC field theory this can be realized by replacing the vector field  $\partial_\mu$  on the tangent space  $T_x\mathbb{R}_\theta^d$  by  $X_a = e_a^\mu(x)\partial_\mu$ , where the tetrad  $e_a^\mu(x)$  is a tensor depending on the coordinate functions. The generalized Moyal star-product becomes

$$(f \star g)(x) = \mathbf{m} \left\{ e^{i\frac{\theta^{ab}}{2} X_a \otimes X_b} f(x) \otimes g(x) \right\}, \quad x \in \mathbb{R}^d, \quad f, g \in C^\infty(\mathbb{R}^d). \quad (3)$$

In general case the vector field  $X_a$  are noncommutative, respect to the Lie bracket “[ $\cdot$ ,  $\cdot$ ]”. The particular condition  $[X_a, X_b] = 0$  results in the constraints on the tetrad  $e_a^\mu$  and leads to the definition of one new field  $\varphi^a$  such that the inverse  $e_\nu^a$  of  $e_a^\nu$  is proportional to  $\partial_\nu \varphi^a$ . Since  $X_a \varphi^b = \delta_a^b$ , the field  $\varphi^b$  can be viewed as new coordinates along the  $X_a$  directions and therefore will be taking into account in the redefinition of the functional action [15]. The Moyal space  $\mathbb{R}_\theta^d$  of this type becomes curve with the background metric  $g_{\mu\nu} = e_\mu^a e_\nu^b \delta_{ab}$ . Let us mention that the commuting vector field  $X_a$  ensures the associativity of the star product (3). But the loss of the associativity propertie becomes evident in the general case where  $[X_a, X_b] \neq 0$ . Nevertheless, this propertie maybe satisfy in a space with a nearly Euclidean metric in which it is natural to choose a tetrad field  $e_a^\mu(x)$  that lies nearly along the coordinate axes  $e_a^\mu(x) = \delta_a^\mu + \omega_a^\mu(x)$  where  $\omega_a^\mu(x)$  is a coordinate dependent small quantity to be determined [18]-[19].

The basis problem, which has accompanied the development of NC field theory is the UV-IR mixing in the perturbation computation. This pathology maybe solved by introducing in the scalar field action, i.e. the  $\varphi_\star^4$ -model, the so called Grosse-Wulkenhaar (GW) harmonic term [3]-[5]. The GW model breaks the  $U(N)$  symmetry invariance in the IR regime but is asymptotically safe in the UV regime. The model is also non-invariant under the translation and rotation of spacetime. The only know invariance satisfied by the model is the so called Langman-Szabo duality [20]. The study of the symmetry consequence such as the Noether current are addressed for the GW model by imposing a constraint on the Euler-Lagrange (EL) equations of motion [21]-[23]. In [24] the same question is addressed in the case of twisted star product definition in the field theory.

Using the same idea of the perturbative computation of the renormalization procedure, other theoretical model have been proved renormalizable. The theoretical ingredient to perform this issue is the so call multiscale analysis, developed by Rivasseau [26]. One of these models which we will focus in this work is the translation invariant renormalizable scalar model discovered by Tanasa et al [6]-[10]. The Tanasa model comes from the NC  $\varphi^4$  model by adding a new contribution  $\frac{a}{\theta^2 p^2}$  on the propagator in the momentum space, and on which the problem of UV/IR mixing is solved. At any order in perturbation theory, the  $\beta$  functions of the model are given [25]. Despite all these interesting results, the corresponding current derived from the symmetry properties of the Tanasa model is not yet be given in the litteratures. Our purpose in this paper is to investigate the computation of the energy momentum tensor (EMT) of the Tanasa model and study its regularization in both ordinary Moyal space and twisted Moyal space. The Wilson regularization procedure is used to recover the local conservation as given in [27]-[36].

The paper is organized as follows. In section (2) we compute the EMT for translation invariant Tanasa model in the ordinary Moyal space. The regularization of this tensor is also given. In the section (3) the same computation is performed in the case of the twisted Moyal plane. Our conclusion and remarks are given in section (4).

## 2 EMT for renormalizable Tanasa model in Moyal space

In this section we derive the EL equation of motion and the EMT for the translation invariant nonlocal functional action. Let us consider the scalar field theoretic model in which we begin with the Lagrange density, which is a function of the field  $\varphi$ , its first partial spacetime derivatives  $\partial_\mu\varphi$ , and the inverse derivatives  $\partial_\mu^{-1}\varphi$  :

$$S_\star[\varphi] = \int d^d x \mathcal{L}_\star(\varphi, \partial_\mu\varphi, \partial_\mu^{-1}\varphi), \quad \partial_\mu^{-1}\varphi(x) = \int d^d x' \varphi(x'), \quad (4)$$

where  $\mathcal{L}_\star$  mean that the ordinary product of function in the action  $S$  is replaced by the Moyal product i.e.:

$$\mathcal{L}_\star(\varphi, \partial_\mu\varphi, \partial_\mu^{-1}\varphi) = \mathcal{L}(\varphi, \partial_\mu\varphi, \partial_\mu\partial_\nu\varphi, \partial_\mu\partial_\nu\partial_\sigma\varphi, \dots, \partial_\mu^{-1}\varphi). \quad (5)$$

Under the translation group which transform coordinates as:  $x^\mu \rightarrow x^\mu + a^\mu$  ( $a^\mu$  is a constant vector), the field  $\varphi$  is then transform as  $\varphi(x) \rightarrow \varphi(x) + a^\mu\partial_\mu\varphi(x)$ , the variation of the action (4) gives

$$S_\star[\varphi] \rightarrow S_\star[\varphi] + \delta S_\star[\varphi] = S_\star[\varphi] + a^\mu\partial_\mu S_\star[\varphi], \quad (6)$$

where we have assumed that the field  $\varphi$  vanishes when  $|x|$  approaches infinity. The translation invariant of the action  $\delta S_\star[\varphi] = 0$ , implies the existence of current densities  $J_\nu$  such that:

$$\delta\varphi \star \frac{\delta S_\star}{\delta\varphi} + \partial^\nu J_\nu = 0. \quad (7)$$

Thus, the EL equations for the Lagrangian density  $\mathcal{L}_\star$  becomes

$$E_\varphi = \frac{\partial\mathcal{L}_\star}{\partial\varphi} - \partial_\mu\left(\frac{\partial\mathcal{L}_\star}{\partial\partial_\mu\varphi}\right) - \partial_\mu^{-1}\left(\frac{\partial\mathcal{L}_\star}{\partial\partial_\mu^{-1}\varphi}\right) = 0, \quad (8)$$

and the conserved EMT can be derived by replacing in the relation (7):  $\delta\varphi$  by  $-a^\mu\partial_\mu\varphi$ , such that

$$\int d^d x \left( -a^\rho \partial^\mu T_{\mu\rho} + E_\varphi \right) = 0, \quad (9)$$

where

$$T_{\mu\rho} = \frac{1}{2} \left\{ \frac{\partial\mathcal{L}_\star}{\partial\partial_\mu\varphi}, \partial_\rho\varphi \right\}_\star + \frac{1}{2} \left\{ \partial_\mu^{-1} \left( \frac{\partial\mathcal{L}_\star}{\partial\partial_\mu^{-1}\varphi} \right), \partial_\mu^{-1} \partial_\rho\varphi \right\}_\star - g_{\mu\rho} \mathcal{L}_\star. \quad (10)$$

Consider as an example the translation invariant noncommutative field theory [8], defined by

$$S_\star[\varphi] = \int d^d x \left[ \frac{1}{2} \partial_\mu\varphi \star \partial^\mu\varphi + \frac{m^2}{2} \varphi \star \varphi + \frac{a}{2\theta^2} \partial_\mu^{-1}\varphi \star \partial_\mu^{-1}\varphi + \frac{\lambda}{4!} \varphi_\star^4 \right], \quad (11)$$

where

$$\partial_\mu^{-1}\varphi(x) := \int_{-\infty}^x d^d x' \varphi(x'), \quad \int_{-\infty}^x \frac{\delta\varphi(x')}{\delta\varphi(y)} dx' = \Theta(x - y). \quad (12)$$

$\Theta(x)$  is the Heaviside function. The variation principle gives the EL equations of motion

$$\frac{\delta S_\star}{\delta\varphi} = 0 \Leftrightarrow -\partial_\mu\partial_\mu\varphi + m^2\varphi + \frac{\lambda}{3!}\varphi_\star^3 - \frac{a}{\theta^2}\partial_\mu^{-1}\partial_\mu^{-1}\varphi = 0 \quad (13)$$

in which the Einstein summation holds, and the EMT becomes,

$$T_{\mu\rho} = \frac{1}{2} \left\{ \partial_\mu \varphi, \partial_\rho \varphi \right\}_* + \frac{a}{2\theta^2} \left\{ \partial_\mu^{-1} \partial_\mu^{-1} \varphi, \partial_\mu^{-1} \partial_\rho \varphi \right\}_* - g_{\mu\rho} \mathcal{L}_* \quad (14)$$

The tensor  $T_{\mu\rho}$  is nonsymmetric and nonlocally conserved. Let  $T_{\mu\rho}^s$  be the symmetric tensor associated to  $T_{\mu\rho}$  i.e.  $T_{\mu\rho}^s = (T_{\mu\rho} + T_{\rho\mu})/2$ , we get

$$T_{\mu\rho}^s = \frac{1}{2} \left\{ \partial_\mu \varphi, \partial_\rho \varphi \right\}_* + \frac{a}{4\theta^2} \left\{ \partial_\mu^{-1} \partial_\mu^{-1} \varphi, \partial_\mu^{-1} \partial_\rho \varphi \right\}_* + \frac{a}{4\theta^2} \left\{ \partial_\rho^{-1} \partial_\rho^{-1} \varphi, \partial_\rho^{-1} \partial_\mu \varphi \right\}_* - g_{\mu\rho} \mathcal{L}_* \quad (15)$$

Note that the procedure of regularization of the EMT performed by Gerhold et al [27] can be used. Consider the star product  $\star'$  given by

$$f \star' g = \mathbf{m} \left\{ \frac{\sin \left( \frac{1}{2} \theta^{\mu\nu} \partial_\mu \otimes \partial_\nu \right)}{\frac{1}{2} \theta^{\mu\nu} \partial_\mu \otimes \partial_\nu} (f \otimes g) \right\}, \quad (16)$$

which satisfy the following identity:  $\theta^{\mu\nu} \partial_\mu f \star' \partial_\nu g = -i[f, g]_\star$ . Then after few computation we get

$$\partial^\rho T_{\rho\mu}^s = \frac{\lambda}{4!} \left[ [\partial_\mu \varphi, \varphi]_\star, \varphi \star \varphi \right]_* = i \frac{\lambda}{4!} \theta^{\alpha\beta} \partial_\alpha \left( [\partial_\mu \varphi, \varphi]_\star \star' \partial_\beta (\varphi \star \varphi) \right). \quad (17)$$

The locally conserved EMT becomes

$$\begin{aligned} T_{\rho\mu}^{s,r} &= \frac{1}{2} \left\{ \partial_\mu \varphi, \partial_\rho \varphi \right\}_* + \frac{a}{4\theta^2} \left\{ \partial_\mu^{-1} \partial_\mu^{-1} \varphi, \partial_\mu^{-1} \partial_\rho \varphi \right\}_* + \frac{a}{4\theta^2} \left\{ \partial_\rho^{-1} \partial_\rho^{-1} \varphi, \partial_\rho^{-1} \partial_\mu \varphi \right\}_* \\ &\quad - i \frac{\lambda}{4!} \theta^{\alpha\beta} g_{\alpha\rho} \left( [\partial_\mu \varphi, \varphi]_\star \star' \partial_\beta (\varphi \star \varphi) \right) - g_{\mu\rho} \mathcal{L}_*. \end{aligned} \quad (18)$$

Note that the limite  $a \rightarrow 0$  gives the EMT for the scalar field theory on Moyal space derived in [27] and [28] from which the Belifante PDE can be given. Also by adding the quantity  $\frac{1}{6}(g_{\mu\rho} \square - \partial_\mu \partial_\rho)(\varphi \star \varphi)$  in the expression (15) and by setting  $m = 0$ , we obtain the traceless EMT.

### 3 The EMT for the Tanasa model in the twisted Moyal space

This section is devoted to the computation of the EMT of the twisted Tanasa model. Before defined this model we give some definitions and identities satisfied by the star product (3). These will be used to calculate the  $\varphi$  and the  $\varphi^a$  variation of the functional action (for more explanation see [15]). Expanding the dynamical  $\star$ -product (3) as follows

$$f \star g \equiv e^\Delta(f, g) = \sum_{n=0}^{\infty} \frac{\Delta^n}{n!} (f, g), \quad \Delta(f, g) = \frac{i}{2} \theta^{ab} (X_a f)(X_b g), \quad (19)$$

allows us to defined the four operators:

$$\begin{aligned} T(\Delta) &= \frac{e^\Delta - 1}{\Delta} & S(\Delta) &= \frac{\sinh(\Delta)}{\Delta} \\ R(\Delta) &= \frac{\cosh(\Delta) - 1}{\Delta} \text{ and } \tilde{X}^a &= \frac{i}{2} \theta^{ab} X_b, \end{aligned} \quad (20)$$

such that the following identities hold:

$$f \star g = fg + X_a T(\Delta)(f, \tilde{X}^a g) \quad (21)$$

$$f \star g - g \star f = [f, g]_\star = 2X_a S(\Delta)(f, \tilde{X}^a g) \quad (22)$$

$$f \star g + g \star f = \{f, g\}_\star = 2fg + 2X_a R(\Delta)(f, \tilde{X}^a g). \quad (23)$$

$S(\Delta)(\cdot, \tilde{X}\cdot)$  is a bilinear antisymmetric operator such that

$$T(\Delta)(f, \tilde{X}^a g) - T(\Delta)(g, \tilde{X}^a f) = 2S(\Delta)(f, \tilde{X}^a g). \quad (24)$$

The integral of the form  $\int d^d x (f \star g)$  is not cyclic; even with suitable boundary conditions at infinity, i.e.

$$\int d^d x (f \star g) \neq \int d^d x (g \star f). \quad (25)$$

Using now the measure  $e d^d x$  where  $e = \det(e_\mu^a)$ , a cyclic integral can be defined so that, up to boundary terms:

$$\int e d^d x (f \star g) = \int e d^d x (fg) = \int e d^d x (g \star f). \quad (26)$$

From now the peculiar Euler Lagrange equations of motion can be readily derived by direct application of the variational principle and the use of formulas of derivatives and variations given in [15] by:

$$\delta_{\varphi^c} e = e X_a(\delta\varphi^a), \quad \delta_{\varphi^c} e^{-1} = -e^{-1} X_a(\delta\varphi^a), \quad e X_a(f) = \partial_\mu(e e_\mu^a f). \quad (27)$$

To compute  $\delta_{\varphi^c}$  variations, consider the functions  $f$  and  $g$ , which do not depend on  $\varphi^c$ . It turns out that the following identity is useful:

$$\delta_{\varphi^c}(f \star g) = -(\delta\varphi^c X_c f) \star g - f \star (\delta\varphi^c X_c g) + \delta\varphi^c X_c(f \star g). \quad (28)$$

In view of these considerations, the following NC scalar invariant model, so called the twisted Tanasa model is described by the functional action

$$\begin{aligned} S_\star[\varphi] &= \int e d^d x \left\{ \frac{1}{2} \partial_\mu \varphi \star \partial^\mu \varphi + \frac{a}{2\theta^2} \partial_\mu^{-1} \varphi \star \partial_\mu^{-1} \varphi + \frac{m^2}{2} \varphi \star \varphi + \frac{\lambda}{4!} \varphi \star \varphi \star \varphi \star \varphi \right. \\ &\quad \left. + \frac{1}{2} \partial_\mu \varphi_c \star \partial^\mu \varphi^c + \frac{a}{2\theta^2} \partial_\mu^{-1} \varphi_c \star \partial_\mu^{-1} \varphi^c \right\} \star e^{-1} \\ &= \int e d^d x (\mathcal{L}_\star \star e^{-1}). \end{aligned} \quad (29)$$

Before start the investigation of the EMT, let us recall that the case where  $a = 0$  in (29) is reduced to the well know scalar field theory in the litterature (see [15] for more details). Then we will focus our attention on to the  $\varphi$  and  $\varphi^c$  variation of the quantity

$$S_\partial = \frac{a}{2\theta^2} \int e d^d x \left[ \partial_\mu^{-1} \varphi \star \partial_\mu^{-1} \varphi + \partial_\mu^{-1} \varphi_c \star \partial_\mu^{-1} \varphi^c \right] \star e^{-1}. \quad (30)$$

Recall that the field  $\varphi_c$  do not depend for  $\varphi$ . We get

$$\begin{aligned} \delta_\varphi S_\partial &= \frac{a}{2\theta^2} \int e d^d x \left[ \partial_\mu^{-1} \delta\varphi \star \partial_\mu^{-1} \varphi \star e^{-1} + \partial_\mu^{-1} \varphi \star \partial_\mu^{-1} \delta\varphi \star e^{-1} \right] \\ &= \frac{a}{2\theta^2} \int e d^d x \left[ \partial_\mu^{-1} \delta\varphi \star \{ \partial_\mu^{-1} \varphi, e^{-1} \}_\star + 2X_a S(\Delta)(\partial_\mu^{-1} \varphi, \tilde{X}^a(\partial_\mu^{-1} \delta\varphi \star e^{-1})) \right] \\ &= \frac{a}{2\theta^2} \int e d^d x \left[ (\partial_\mu^{-1} \delta\varphi) \{ \partial_\mu^{-1} \varphi, e^{-1} \}_\star + X_a T(\Delta)(\partial_\mu^{-1} \delta\varphi, \tilde{X}^a(\{ \partial_\mu^{-1} \varphi, e^{-1} \}_\star)) \right. \\ &\quad \left. + 2X_a S(\Delta)(\partial_\mu^{-1} \varphi, \tilde{X}^a(\partial_\mu^{-1} \delta\varphi \star e^{-1})) \right], \end{aligned} \quad (31)$$

where we have used the identities

$$\partial_\mu^{-1}\varphi \star \partial_\mu^{-1}\delta\varphi \star e^{-1} = \partial_\mu^{-1}\delta\varphi \star e^{-1} \star \partial_\mu^{-1}\varphi + 2X_a S(\Delta)(\partial_\mu^{-1}\varphi, \tilde{X}^a(\partial_\mu^{-1}\delta\varphi \star e^{-1})) \quad (32)$$

$$(\partial_\mu^{-1}\delta\varphi) \star \{\partial_\mu^{-1}\varphi, e^{-1}\}_\star = (\partial_\mu^{-1}\delta\varphi)\{\partial_\mu^{-1}\varphi, e^{-1}\}_\star + X_a T(\Delta)(\partial_\mu^{-1}\delta\varphi, \tilde{X}^a(\{\partial_\mu^{-1}\varphi, e^{-1}\}_\star)). \quad (33)$$

Consider the following relation in which the index  $\mu$  is suppose to satisfied the Einstein summation:

$$e(\partial_\mu^{-1}\delta\varphi)\{\partial_\mu^{-1}\varphi, e^{-1}\}_\star = \partial_\mu \left[ (\partial_\mu^{-1}\delta\varphi)\partial_\mu^{-1}(e\{\partial_\mu^{-1}\varphi, e^{-1}\}_\star) \right] - \delta\varphi \partial_\mu^{-1}(e\{\partial_\mu^{-1}\varphi, e^{-1}\}_\star). \quad (34)$$

Therefore we get the  $\varphi$  variation of  $S_\partial$  as

$$\delta_\varphi S_\partial = \int d^d x \left[ \delta\varphi E_\partial^\varphi + \partial_\sigma \mathcal{K}_\partial^\sigma \right], \quad (35)$$

where  $E_\partial$  contribute to the EL equations of motion and  $\mathcal{K}_\partial^\mu$  to the current:

$$E_\partial^\varphi = -\frac{a}{2\theta^2} \partial_\mu^{-1}(e\{\partial_\mu^{-1}\varphi, e^{-1}\}_\star), \quad (36)$$

$$\begin{aligned} \mathcal{K}_\partial^\sigma &= \frac{a}{2\theta^2} \left[ (\partial_\sigma^{-1}\delta\varphi)\partial_\sigma^{-1}(e\{\partial_\sigma^{-1}\varphi, e^{-1}\}_\star) + ee_b^\sigma T(\Delta)(\partial_\mu^{-1}\delta\varphi, \tilde{X}^b(\{\partial_\mu^{-1}\varphi, e^{-1}\}_\star)) \right. \\ &\quad \left. + 2ee_b^\sigma S(\Delta)(\partial_\mu^{-1}\varphi, \tilde{X}^b(\partial_\mu^{-1}\delta\varphi \star e^{-1})) \right]. \end{aligned} \quad (37)$$

Using the same technical computation to the remain expression of the functional action (29) the EL equations of motion of the field  $\varphi$  become

$$\begin{aligned} E_\varphi &= -\frac{1}{2} \partial_\mu (e\{\partial^\mu\varphi, e^{-1}\}_\star) - \frac{a}{2\theta^2} \partial_\mu^{-1}(e\{\partial_\mu^{-1}\varphi, e^{-1}\}_\star) + \frac{m^2}{2} e\{\varphi, e^{-1}\}_\star \\ &\quad + \frac{\lambda}{4!} e\{\varphi \star \varphi, \{\varphi, e^{-1}\}_\star\}_\star = 0, \end{aligned} \quad (38)$$

which is reduced to (13) in the limit where  $X_a \rightarrow \partial_a$ . Hence, the corresponding current is

$$\begin{aligned} \mathcal{K}^\sigma &= \frac{a}{2\theta^2} \left[ (\partial_\sigma^{-1}\delta\varphi)\partial_\sigma^{-1}(e\{\partial_\sigma^{-1}\varphi, e^{-1}\}_\star) + ee_b^\sigma T(\Delta)(\partial_\mu^{-1}\delta\varphi, \tilde{X}^b(\{\partial_\mu^{-1}\varphi, e^{-1}\}_\star)) \right. \\ &\quad \left. + 2ee_b^\sigma S(\Delta)(\partial_\mu^{-1}\varphi, \tilde{X}^b(\partial_\mu^{-1}\delta\varphi \star e^{-1})) \right] + \frac{e\delta\varphi}{2} \cdot \{\partial^\sigma\varphi, e^{-1}\}_\star \\ &\quad + ee_b^\sigma \left[ T(\Delta) \left( \delta\partial_\mu\varphi, \frac{\tilde{X}^b}{2} \{\partial^\mu\varphi, e^{-1}\}_\star \right) + S(\Delta) \left( \partial_\mu\varphi, \tilde{X}^b(\partial^\mu\delta\varphi \star e^{-1}) \right) \right] \\ &\quad + ee_b^\sigma \left[ \frac{m^2}{2} T(\Delta) \left( \delta\varphi, \tilde{X}^b \{\varphi, e^{-1}\}_\star \right) + m^2 S(\Delta) \left( \varphi, \tilde{X}^b(\delta\varphi \star e^{-1}) \right) \right] \\ &\quad + ee_b^\sigma \left[ \frac{\lambda}{4!} T(\Delta) \left( \delta\varphi, \tilde{X}^b \{\varphi \star \varphi, \{\varphi, e^{-1}\}_\star\}_\star \right) + \frac{\lambda}{12} S(\Delta) \left( \varphi, \tilde{X}^b(\delta\varphi \star \varphi \star \varphi \star e^{-1}) \right) \right. \\ &\quad \left. + \frac{\lambda}{12} S(\Delta) \left( \varphi \star \varphi, \tilde{X}^b(\delta\varphi \star \varphi \star e^{-1}) \right) + \frac{\lambda}{12} S(\Delta) \left( \varphi \star \varphi \star \varphi, \tilde{X}^b(\delta\varphi \star e^{-1}) \right) \right]. \end{aligned} \quad (39)$$

In the other hand we are interested to the  $\varphi_c$  variation of (30). This variation is subdivided into two contributions namely  $A_\partial$  and  $B_\partial$  such that

$$\begin{aligned} \delta_{\varphi^c} S_\partial &= \frac{a}{2\theta^2} \delta_{\varphi^c} \left\{ \int ed^d x \partial_\mu^{-1}\varphi \star \partial_\mu^{-1}\varphi \star e^{-1} \right\} + \frac{a}{2\theta^2} \delta_{\varphi^c} \left\{ \int ed^d x \partial_\mu^{-1}\varphi_c \star \partial_\mu^{-1}\varphi^c \star e^{-1} \right\} \\ &= A_\partial + B_\partial \end{aligned} \quad (40)$$

where

$$\begin{aligned}
A_\partial &= \frac{a}{2\theta^2} \int d^d x (\delta_{\varphi^c} e) (\partial_\mu^{-1} \varphi \star \partial_\mu^{-1} \varphi \star e^{-1}) + \frac{a}{2\theta^2} \int ed^d x \delta_{\varphi^c} (\partial_\mu^{-1} \varphi \star \partial_\mu^{-1} \varphi \star e^{-1}) \\
&= \frac{a}{2\theta^2} \int d^d x \left\{ \partial_\sigma \left( ee_a^\sigma \delta \varphi^a \partial_\mu^{-1} \varphi \star \partial_\mu^{-1} \varphi \star e^{-1} \right) - ee_a^\sigma \delta \varphi^a \partial_\sigma (\partial_\mu^{-1} \varphi \star \partial_\mu^{-1} \varphi \star e^{-1}) \right\} \\
&+ \frac{a}{2\theta^2} \int ed^d x \left\{ -\delta \varphi^a X_a \partial_\mu^{-1} \varphi \{ \partial_\mu^{-1} \varphi, e^{-1} \}_\star - X_b T(\Delta) (\delta \varphi^a X_a \partial_\mu^{-1} \varphi, \tilde{X}^b \{ \partial_\mu^{-1} \varphi, e^{-1} \}_\star) \right. \\
&- 2X_b S(\Delta) (\partial_\mu^{-1} \varphi, \tilde{X}^b \delta \varphi^a X_a \partial_\mu^{-1} \varphi \star e^{-1}) - X_a (\partial_\mu^{-1} \varphi \star \partial_\mu^{-1} \varphi \star \delta \varphi^a e^{-1}) \\
&+ X_a (\partial_\mu^{-1} \varphi \star \partial_\mu^{-1} \varphi) \delta \varphi^a e^{-1} + X_b T(\Delta) (X_a (\partial_\mu^{-1} \varphi \star \partial_\mu^{-1} \varphi), \tilde{X}^b \delta \varphi^a e^{-1}) \\
&\left. + \delta \varphi^a X_a (\partial_\mu^{-1} \varphi \star \partial_\mu^{-1} \varphi \star e^{-1}) \right\} \tag{41}
\end{aligned}$$

and

$$\begin{aligned}
B_\partial &= \frac{a}{2\theta^2} \int d^d x (\delta_{\varphi^c} e) (\partial_\mu^{-1} \varphi_c \star \partial_\mu^{-1} \varphi^c \star e^{-1}) + \frac{a}{2\theta^2} \int ed^d x \delta_{\varphi^c} (\partial_\mu^{-1} \varphi_c \star \partial_\mu^{-1} \varphi^c \star e^{-1}) \\
&= \frac{a}{2\theta^2} \int d^d x \left\{ \partial_\sigma \left( (ee_a^\sigma \delta \varphi^a) \partial_\mu^{-1} \varphi_c \star \partial_\mu^{-1} \varphi^c \star e^{-1} \right) - ee_a^\sigma \delta \varphi^a \partial_\sigma (\partial_\mu^{-1} \varphi_c \star \partial_\mu^{-1} \varphi^c \star e^{-1}) \right\} \\
&+ \frac{a}{2\theta^2} \int d^d x \left\{ -\delta \varphi_c \partial_\mu^{-1} (e \{ \partial_\mu^{-1} \varphi^c, e^{-1} \}_\star) + \partial_\mu (\partial_\mu^{-1} \delta \varphi_c \partial_\mu^{-1} (e \{ \partial_\mu^{-1} \varphi, e^{-1} \}_\star)) \right. \\
&+ 2\partial_\sigma \left( ee_a^\sigma S(\Delta) (\partial_\mu^{-1} \varphi_c, \tilde{X}^a \delta \varphi^c \star e^{-1}) \right) + \partial_\sigma \left( ee_a^\sigma T(\Delta) (\partial_\mu^{-1} \delta \varphi_c, \tilde{X}^a \{ \partial_\mu^{-1} \varphi^c, e^{-1} \}_\star) \right) \left. \right\} \\
&+ \frac{a}{2\theta^2} \int ed^d x \left\{ -\delta \varphi^a X_a \partial_\mu^{-1} \varphi_c \{ \partial_\mu^{-1} \varphi^c, e^{-1} \}_\star - X_b T(\Delta) (\delta \varphi^a X_a \partial_\mu^{-1} \varphi_c, \tilde{X}^b \{ \partial_\mu^{-1} \varphi^c, e^{-1} \}_\star) \right. \\
&- 2X_b S(\Delta) (\partial_\mu^{-1} \varphi_c, \tilde{X}^b \delta \varphi^a X_a \partial_\mu^{-1} \varphi^c \star e^{-1}) - X_a (\partial_\mu^{-1} \varphi_c \star \partial_\mu^{-1} \varphi^c \star \delta \varphi^a e^{-1}) \\
&+ \delta \varphi^a e^{-1} X_a (\partial_\mu^{-1} \varphi_c \star \partial_\mu^{-1} \varphi^c) + X_b T(\Delta) (X_a (\partial_\mu^{-1} \varphi_c \star \partial_\mu^{-1} \varphi^c), \tilde{X}^b \delta \varphi^a e^{-1}) \\
&\left. + \delta \varphi^a X_a (\partial_\mu^{-1} \varphi_c \star \partial_\mu^{-1} \varphi^c \star e^{-1}) \right\}. \tag{42}
\end{aligned}$$

Taking into account all of these quantities, the contribution to the EL equations of motion is

$$\begin{aligned}
E_\partial^{\varphi\varphi^c} &= \frac{a}{2\theta^2} \left[ -e X_a \partial_\mu^{-1} \varphi \{ \partial_\mu^{-1} \varphi, e^{-1} \}_\star + X_a (\partial_\mu^{-1} \varphi \star \partial_\mu^{-1} \varphi) - e X_a \partial_\mu^{-1} \varphi_c \{ \partial_\mu^{-1} \varphi^c, e^{-1} \}_\star \right. \\
&\left. + X_a (\partial_\mu^{-1} \varphi_c \star \partial_\mu^{-1} \varphi^c) - \partial_\mu^{-1} (e \{ \partial_\mu^{-1} \varphi^c, e^{-1} \}_\star) \right]. \tag{43}
\end{aligned}$$

The contribution to the current  $J^\sigma$  denote by  $J_\partial^\sigma$  takes the form

$$\begin{aligned}
J_\partial^\sigma &= \frac{a}{2\theta^2} \left[ ee_b^\sigma \delta \varphi^b \partial_\mu^{-1} \varphi \star \partial_\mu^{-1} \varphi \star e^{-1} - ee_b^\sigma T(\Delta) (\delta \varphi^a X_a \partial_\mu^{-1} \varphi, \tilde{X}^b \{ \partial_\mu^{-1} \varphi, e^{-1} \}_\star) \right. \\
&- 2ee_b^\sigma S(\Delta) (\partial_\mu^{-1} \varphi, \tilde{X}^b \delta \varphi^a X_a \partial_\mu^{-1} \varphi \star e^{-1}) - ee_b^\sigma (\partial_\mu^{-1} \varphi \star \partial_\mu^{-1} \varphi \star \delta \varphi^b e^{-1}) \\
&+ ee_b^\sigma T(\Delta) (X_a (\partial_\mu^{-1} \varphi \star \partial_\mu^{-1} \varphi), \tilde{X}^b \delta \varphi^a e^{-1}) + ee_b^\sigma \delta \varphi^b (\partial_\mu^{-1} \varphi_c \star \partial_\mu^{-1} \varphi^c \star e^{-1}) \\
&+ (\partial_\sigma^{-1} \delta \varphi_c) \partial_\sigma^{-1} (e \{ \partial_\sigma^{-1} \varphi^c, e^{-1} \}_\star) + 2ee_b^\sigma S(\Delta) (\partial_\mu^{-1} \varphi_c, \tilde{X}^b \delta \varphi^c e^{-1}) \\
&+ ee_b^\sigma T(\Delta) (\partial_\mu^{-1} \delta \varphi_c, \tilde{X}^b \{ \partial_\mu^{-1} \varphi^c, e^{-1} \}_\star) - ee_b^\sigma T(\Delta) (\delta \varphi^a X_a \partial_\mu^{-1} \varphi_c, \tilde{X}^b \{ \partial_\mu^{-1} \varphi^c, e^{-1} \}_\star) \\
&- 2ee_b^\sigma S(\Delta) (\partial_\mu^{-1} \varphi_c, \tilde{X}^b \delta \varphi^a X_a \partial_\mu^{-1} \varphi^c \star e^{-1}) - ee_b^\sigma (\partial_\mu^{-1} \varphi_c \star \partial_\mu^{-1} \varphi^c \star \delta \varphi^a e^{-1}) \\
&\left. + ee_b^\sigma T(\Delta) (X_a (\partial_\mu^{-1} \varphi_c \star \partial_\mu^{-1} \varphi^c), \tilde{X}^b \delta \varphi^a e^{-1}) \right]. \tag{44}
\end{aligned}$$

By performing the same computation to the other terms in the action (29) we get the EL equations of motion

$$E^{\varphi\varphi^c} = E_\partial^{\varphi^c} - X_c \varphi \mathcal{E}_\varphi + X_c \mathcal{L}_\star^\Omega - \frac{1}{2} X_c \varphi \partial_\mu (e \{ \partial^\mu \varphi, e^{-1} \}_\star) - e \frac{\Omega^2}{2} \varphi X_c \tilde{x} \cdot \{ \tilde{x} \varphi, e^{-1} \}_\star$$

$$-\frac{e}{2}X_c\partial_\mu\varphi.\{\partial^\mu\varphi,e^{-1}\}_\star-\frac{e}{2}X_c\partial_\mu\varphi_a.\{\partial^\mu\varphi^a,e^{-1}\}_\star-\partial_\mu\left(\frac{e}{2}\{\partial^\mu\varphi_c,e^{-1}\}_\star\right)=0 \quad (45)$$

and the current

$$\begin{aligned} \mathcal{J}^\sigma &= \mathcal{J}_\partial^\sigma + \mathcal{K}^\sigma(\delta\varphi \rightarrow -\delta\varphi^c X_c\varphi) + \frac{e\delta\varphi^c}{2}X_c\varphi.\{\partial^\sigma\varphi,e^{-1}\}_\star + \frac{e\delta\varphi^c}{2}.\{\partial^\sigma\varphi_c,e^{-1}\}_\star \\ &+ ee_b^\sigma \left\{ -\mathcal{L}_\star \star (\delta\varphi^b e^{-1}) + \delta\varphi^b (\mathcal{L}_\star \star e^{-1}) + T(\Delta) \left( X_c(\mathcal{L}_\star), \tilde{X}^b(\delta\varphi^c e^{-1}) \right) \right. \\ &+ \left. \frac{1}{2}T(\Delta) \left( \partial_\mu(\delta\varphi^c e_c^\rho) \partial_\rho\varphi, \tilde{X}^b \{\partial^\mu\varphi, e^{-1}\}_\star \right) + S(\Delta) \left( \partial_\mu\varphi, \tilde{X}^b \left( (\partial_\mu(\delta\varphi^c e_c^\rho) \partial_\rho\varphi) \star e^{-1} \right) \right) \right\} \\ &+ \frac{1}{2}ee_b^\sigma \left\{ -T(\Delta) \left( \delta\varphi^c X_c \partial_\mu\varphi_a, \tilde{X}^b \{\partial^\mu\varphi^a, e^{-1}\}_\star \right) - 2S(\Delta) \left( \partial^\mu\varphi_a, \tilde{X}^b \left( (\delta\varphi^c X_c \partial_\mu\varphi^a) \star e^{-1} \right) \right) \right. \\ &+ \left. 2S(\Delta) \left( \partial_\mu\varphi_a, \tilde{X}^b (\partial^\mu\delta\varphi^a \star e^{-1}) \right) + T(\Delta) \left( \partial_\mu\delta\varphi_a, \tilde{X}^b \{\partial^\mu\varphi^a, e^{-1}\}_\star \right) \right\}, \end{aligned} \quad (46)$$

such that

$$\delta_{\varphi^c}\mathcal{S}_\star = \int d^d x \left( \delta\varphi^c E^{\varphi\varphi^c} + \partial_\sigma \mathcal{J}^\sigma \right). \quad (47)$$

Now using the results in the previous paragraph where we studied the general properties of the total variation of the Lagrangian, we discuss the translation invariant symmetry of the model and compute the conserve current namely the EMT. In general, a symmetry of the action involves a certain change of variables. Performing a functional variation of the fields and a coordinates transformations

$$\varphi'(x) = \varphi(x) + \delta\varphi(x), \quad \varphi'^c(x) = \varphi^c(x) + \delta\varphi^c(x), \quad x'^\mu = x^\mu + a^\mu, \quad (48)$$

and by using the identity  $d^D x' = [1 + \partial_\mu a^\mu + \mathcal{O}(\epsilon^2)]d^D x$ , leads to the following variation of the action, to first order in  $\delta\varphi(x)$ ,  $\delta\varphi^c(x)$  and  $a^\mu$ :

$$\begin{aligned} \delta\mathcal{S}_\star &= \int ed^D x \left\{ \left| \frac{\partial x'}{\partial x} \right| \star (\mathcal{L}'_\star \star e^{-1}) \right\} - \int ed^D x (\mathcal{L}_\star \star e^{-1}) \\ &= \int d^D x \left\{ \delta_\varphi \left( (\mathcal{L}_\star \star e^{-1}) e \right) + \delta_{\varphi^c} \left( (\mathcal{L}_\star \star e^{-1}) e \right) \right. \\ &\quad \left. + a^\mu \star \partial_\mu [(\mathcal{L}_\star \star e^{-1}) e] + \partial_\mu a^\mu \star (\mathcal{L}_\star \star e^{-1}) e \right\}. \end{aligned} \quad (49)$$

Now by integrating on a submanifold  $M \subset \mathbb{R}^D$  with fields non vanishing at the boundary (so that the total derivative terms do not disappear), we get:

$$\delta\mathcal{S}_\star = \int_M d^D x \partial_\sigma \left[ \mathcal{K}^\sigma + \mathcal{J}^\sigma + a^\sigma \star \left( (\mathcal{L}_\star^\Omega \star e^{-1}) e \right) \right] \quad (50)$$

coupled to the transformations  $\delta\varphi = -a^\nu \partial_\nu\varphi$ ,  $\delta\varphi^c = -a^\nu \partial_\nu\varphi^c$ ,  $a^\nu = \text{constant}$ , that we substitute into (50) and taking into account the identities  $\delta\varphi^c X_c \partial_\mu\varphi = \partial_\mu(\delta\varphi^c X_c\varphi) - \partial_\mu(\delta\varphi^c e_c^\rho) \partial_\rho\varphi$  such that  $\delta\varphi^c X_c \partial_\mu\varphi = \partial_\mu\delta\varphi = -a^\nu \partial_\nu \partial_\mu\varphi$  and the fact that  $e_\nu^a = \partial_\nu\varphi^a$ , we come from the relation

$$0 = \delta\mathcal{S}_\star = -a^\nu \int_M d^D x \partial_\sigma \mathcal{T}_\nu^\sigma, \quad (51)$$

where the EMT takes the form

$$\mathcal{T}_\nu^\sigma = \frac{e}{2} \left[ (\partial_\nu\varphi) \{\partial^\sigma\varphi, e^{-1}\}_\star + (\partial_\nu\varphi_c) \{\partial^\sigma\varphi^c, e^{-1}\}_\star \right]$$



$$\begin{aligned}
& + \frac{a}{2\theta^2} \left[ \partial_\sigma^{-1} (\partial_\nu \varphi) \partial_\sigma^{-1} \left( e \{ \partial_\sigma^{-1} \varphi, e^{-1} \}_* \right) + \partial_\sigma^{-1} (\partial_\nu \varphi_c) \partial_\sigma^{-1} \left( e \{ \partial_\sigma^{-1} \varphi^c, e^{-1} \}_* \right) \right] \\
& - e e_b^\sigma \left[ \mathcal{L}_* \star (e^{-1} \partial_\nu \varphi^b) + T(\Delta) \left( X_c \mathcal{L}_*, \tilde{X}^b (e^{-1} \partial_\nu \varphi^c) \right) \right]. \tag{52}
\end{aligned}$$

This tensor is neither symmetric and non locally conserved. Note that to recovered the EMT given in (14) we write  $T_{\nu\rho} = g_{\sigma\rho} \mathcal{T}_{\nu\sigma}$  and takes the limit  $e_a^\mu \rightarrow \delta_a^\mu$ . The expression (52) can be symmetrized as

$$\begin{aligned}
\mathcal{T}_{\nu\sigma}^s &= \frac{e}{4} \left[ (\partial_\nu \varphi) \{ \partial_\sigma \varphi, e^{-1} \}_* + (\partial_\nu \varphi_c) \{ \partial_\sigma \varphi^c, e^{-1} \}_* + (\partial_\sigma \varphi) \{ \partial_\nu \varphi, e^{-1} \}_* + (\partial_\sigma \varphi_c) \{ \partial_\nu \varphi^c, e^{-1} \}_* \right] \\
& + \frac{a}{4\theta^2} \left[ \partial_\sigma^{-1} (\partial_\nu \varphi) \partial_\sigma^{-1} \left( e \{ \partial_\sigma^{-1} \varphi, e^{-1} \}_* \right) + \partial_\sigma^{-1} (\partial_\nu \varphi_c) \partial_\sigma^{-1} \left( e \{ \partial_\sigma^{-1} \varphi^c, e^{-1} \}_* \right) \right. \\
& + \partial_\nu^{-1} (\partial_\sigma \varphi) \partial_\nu^{-1} \left( e \{ \partial_\nu^{-1} \varphi, e^{-1} \}_* \right) + \partial_\nu^{-1} (\partial_\sigma \varphi_c) \partial_\nu^{-1} \left( e \{ \partial_\nu^{-1} \varphi^c, e^{-1} \}_* \right) \left. \right] \\
& - \frac{e}{2} \left\{ g_{\rho\sigma} e_b^\rho \left[ \mathcal{L}_* \star (e^{-1} \partial_\nu \varphi^b) + T(\Delta) \left( X_c \mathcal{L}_*, \tilde{X}^b (e^{-1} \partial_\nu \varphi^c) \right) \right] \right. \\
& \left. + g_{\rho\nu} e_b^\rho \left[ \mathcal{L}_* \star (e^{-1} \partial_\sigma \varphi^b) + T(\Delta) \left( X_c \mathcal{L}_*, \tilde{X}^b (e^{-1} \partial_\sigma \varphi^c) \right) \right] \right\}. \tag{53}
\end{aligned}$$

Now we can regularize the EMT (52). Due to the very complex form of expression in the general case we focus our attention to the case where the coordinates base  $e_a^\mu(x)$  is to be  $e_a^\mu = \delta_a^\mu + \omega_{ab}^\mu x^b$  such that the tensor  $(\omega_{ab}^\mu)$  is symmetric between the index  $a$  and  $b$ , i.e.  $\omega_{ab}^\mu = \omega_{ba}^\mu$ . The commutation relation between the vectors fields  $X_a$  is:

$$[X_a, X_b] = (\omega_{ba}^\mu - \omega_{ab}^\mu) \partial_\mu = 0, \tag{54}$$

and therefore the twisted star product is associative. We adequately choose the elements of the matrix  $(\omega_{ab}^\mu)$  such that the matrix representation of  $(e_a^\mu)$  is given in dimension  $d = 4$  by

$$(e)_a^\mu = \begin{pmatrix} 1 + \omega_{11}^1 x^1 + \omega_{12}^1 x^2 & \omega_{11}^2 x^1 + \omega_{12}^2 x^2 & 0 & 0 \\ \omega_{12}^1 x^1 + \omega_{22}^1 x^2 & 1 + \omega_{12}^2 x^1 + \omega_{22}^2 x^2 & 0 & 0 \\ 0 & 0 & 1 + \omega_{33}^3 x^3 + \omega_{34}^3 x^4 & \omega_{33}^4 x^3 + \omega_{34}^4 x^4 \\ 0 & 0 & \omega_{34}^3 x^3 + \omega_{44}^3 x^4 & 1 + \omega_{34}^4 x^3 + \omega_{44}^4 x^4 \end{pmatrix}. \tag{55}$$

Then, the determinants  $e^{-1}$  and the inverse  $e$  becomes

$$e^{-1} = 1 + \omega_\mu x^\mu, \quad e = 1 - \omega_\mu x^\mu, \tag{56}$$

where the components of the vector  $\omega_\mu$  are

$$\omega_1 = \omega_{11}^1 + \omega_{12}^2, \quad \omega_2 = \omega_{22}^2 + \omega_{12}^1, \quad \omega_3 = \omega_{33}^3 + \omega_{34}^4, \quad \omega_4 = \omega_{44}^4 + \omega_{34}^3. \tag{57}$$

The noncommutative tensor takes the form  $\theta^{\mu\nu}(x) = \theta e^{-1} J^{\mu\nu}$  where  $(J)$  stands for the symplectic matrix in four dimensions. Besides, the inverse matrix  $e_\mu^a$  can be written as  $e_\mu^a = \delta_\mu^a + \omega_\mu^{ab} x_b$ , where  $\omega_\mu^{ab} = -\omega_\mu^{ba}$ , and the solution of the field equation  $e_\mu^a = \partial_\mu \phi^a$  is well given by

$$\phi^a = x^a + \frac{1}{2} \omega_\mu^{ab} x_b x^\mu. \tag{58}$$

Using all these considerations, after few algebraic computation we come to the relation

$$\begin{aligned}
\partial^\nu \mathcal{T}_{\nu\sigma}^s &= \frac{2e\lambda}{4!} X_a S(\Delta) ([\partial_\sigma \varphi, \varphi]_*, \tilde{X}^a (\varphi \star \varphi \star e^{-1})) \\
&= \frac{2\lambda}{4!} \partial_\gamma \left( e e_a^\gamma S(\Delta) ([\partial_\sigma \varphi, \varphi]_*, \tilde{X}^a (\varphi \star \varphi \star e^{-1})) \right), \tag{59}
\end{aligned}$$

where the followings identities are used

$$\{\partial_\sigma \varphi, e^{-1}\}_\star = 2e^{-1} \partial_\sigma \varphi, \quad (60)$$

$$e^{-1} \partial_\sigma \varphi^c = \delta_\sigma^c + \delta_\sigma^c \omega_\mu x^\mu + \omega_\sigma^{cd} x_d, \quad (61)$$

$$T(\Delta) \left( X_c \mathcal{L}_\star, \tilde{X}^b (e^{-1} \partial_\sigma \varphi^c) \right) = 0. \quad (62)$$

As the ordinary Moyal plane the EMT defined on the twisted Moyal space can be regularized. We get

$$\mathcal{T}_{\nu\sigma}^{s,r} = \mathcal{T}_{\nu\sigma}^s - \frac{2\lambda}{4!} g_{\gamma\nu} \left( e e_a^\gamma S(\Delta) ([\partial_\sigma \varphi, \varphi]_\star, \tilde{X}^a (\varphi \star \varphi \star e^{-1})) \right). \quad (63)$$

## 4 Conclusion and remarks

In conclusion, we summarize our results. We have developed the variational techniques for the determination of the EL equations of motion of a Lagrangian that depends on  $\partial_\mu^{-1} \varphi$ . We have computed the EMT for the Tanasa model, in ordinary and twisted Moyal spaces. The Wilson regularization procedure is also given to improve the corresponding tensors.

Let us remark that introducing  $x$ -dependence in the deformation matrix  $(\theta^{\mu\nu})$  of the star product leads to the definition of nontrivial background metric. Then the EMT associated to translation invariant field theory may provided from the core of the Einstein equation, when we assume that gravity can be incorporated in the noncommutativity. Also, the EMT given in (53) can be regularized without choosing the tetrad as  $e_a^\mu = \delta_a^\mu + \omega_{ab}^\mu x^b$ . Due to the very complex and lengthy form of the results this computation is not given in this paper, but maybe deduced, thanks to the example proposed in this paper.

## Acknowledgements

EB work is partially supported by the Abdus Salam International Centre for Theoretical Physics (ICTP, Trieste, Italy) through the Office of External Activities (OEA)-Prj-15. The ICMPA is in partnership with the Daniel Iagolnitzer Foundation (DIF), France. DOS research at Max-Planck Institute is supported by the Alexander von Humboldt foundation.

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