Hysteresis and Fast Timescale in Transport Relation of Toroidal Plasmas

K. Itoh[1,2], S.-I. Itoh[2,3], K. Ida[1,2], S. Inagaki[2,3], Y. Kamada[4], K. Kamiya[4], J. Q. Dong[5], C. Hidalgo[6], T. Evans[7], W. H. Ko[8], H. Park[8,9], T. Tokuzawa[1], S. Kubo[1], T. Kobayashi[1], Y. Kosuga[2,3], G. S. Yun[10], S. D. Song[5], N. Kasuya[2,3], Y. Nagashima[2,3], C. Moon[1], M. Yoshinuma[1], R. Makino[1], T. Tsujimura[1], H. Tsuchiya[1], U. Stroth[11]

[1]National Institute for Fusion Science, Toki 509-5292, Japan

[2]Research Center for Plasma Turbulence, Kyushu University, Kasuga, 816-8580 Japan

[3]Research Institute for Applied Mechanics, Kyushu University, Kasuga, 816-8580, Japan

[4]Japan Atomic Energy Agency, Naka, Ibaraki, 311-0193, Japan

[5]Southwestern Institute of Physics, PO Box 432, Chengdu, People's Republic of China

[6] Laboratorio Nacional de Fusión. Asociación EURATOM-CIEMAT, Madrid, Spain

[7]General Atomics, San Diego, USA

[8]National Fusion Research Institute, Korea

[9]Ulsan National Institute of Science and Technology, Ulsan 689-798, Republic of Korea

[10]Pohang University of Science and Technology, Pohang, Gyeongbuk 790-784, Republic of Korea

[11]Institut für Plasmaphysik Garching, Germany

E-mail contact of main author: itoh@nifs.ac.jp

Abstract This article assesses the understanding of and impacts by the hysteresis of transport relation. The rapid changes of fluxes compared to slow changes of plasma parameters are overviewed for both core and edge plasmas. In this article, the modulation ECH experiment is explained, in which the heating power repeats on-and-off periodically, revealed the hysteresis and fast changes in gradient-flux relation. The decisive progress is that both the hystereses in the *gradient-flux* and *gradient-fluctuation* relations were observed simultaneously. Hysteresis with rapid timescale exists in the channels of energy, electron and impurity densities, and plausibly in momentum. Advanced methods of data analysis are explained. The transport hysteresis can be studied by observing the higher harmonics of temperature perturbation δT_{m} in heating modulation experiments. The hysteresis introduces

the term δT_{m}, which depends on the harmonic number m in algebraic manner (not exponential decay). Next, causes of hysteresis and fast timescale are discussed. The nonlocal-in-space coupling works here, but does not suffice. One mechanism for 'the heating heats turbulence' is that the external source S in phase space for heating has its fluctuation in turbulent plasma. This coupling can induce the direct input of heating power into fluctuations. The height of the jump in transport hysteresis is smaller for heavier hydrogen isotope, and could be one of origins of isotope effect on confinement. Finally, the impacts of transport hysteresis on the control system are assessed. The control system must be designed so as to protect the system from sudden plasma loss.

1. Introduction and Background of the Problem

In the history of study of confinement in toroidal plasmas, the approach to analyze the response to periodic modulation of heating power has been applied routinely. (See a review, e.g., [1].) The 'heat pulse' thermal diffusivity, χ_{HP}, has been deduced, but the discrepancy between this and the thermal diffusivity in the stationary power balance, χ_{PB}, has long been recognized as a mystery. The understanding of the difference between χ_{HP} and χ_{PB} has been advanced recently. There have been many experimental reports (e.g., on W7-AS [2], on D-III D [3]) that the gradient-flux relation has a hysteresis (depending on switching on/off of ECH heating), and the heat flux may directly depend on the heating power (Fig.1 [3, 4]). This view shows a contrast to the conventional view of the transport, in which the heat flux is expressed in terms of the local plasma parameters and their gradients. If the hystreresis in the gradient-flux relation exists, it is not surprising the difference between χ_{HP} and χ_{PB} is quite large. An essential question to the pioneering work (like [1, 2]) was whether the profile of absorbed power by modulated heating was correctly predicted by theory or not. The measurement on the fluctuation intensity in the modulated heating has been performed on LHD [5]. The new, independent data of fluctuation intensity was found to change against the local temperature gradient with hysteresis on LHD [5, 6]. Thus, the hysteresis is not an artifact due to the error in evaluating the power absorption profile. The development in identifying transport hysteresis has been assessed in literatures [4, 7].

If the hysteresis really exists and the heat flux directly depends on the heating power, the discrepancy between 'power-balance' and 'heat-pulse' conductivities [1] arises. In addition, this hysteresis can have profound impact on our evaluating the dynamical response of burning plasmas. The understanding of these observations is necessary for the accurate prediction of burning plasma and to design the temporal control system against the dynamic change of core plasma.

2. Assessment of Experimental Observations

2.1 Modulation ECH (MECH) experiment

The key experimental discovery is the identification of the difference between the time scale of response of heat flux and that of global parameters after the sudden change of heating power. The modulation ECH (MECH) experiment, in which the heating power repeats on-and-off following the step function,

$$H(t) = \sum_{m=1}^{\infty} \frac{1}{2m-1} \sin\left((2m-1)\Omega\, t\right) , \tag{1}$$

has revealed the fast changes in gradient-flux relation. The switching time in the modulational EC heating is shorter than a mili-second. The global plasma parameters change in the order of energy confinement time (a few 10 to few 100 ms). The change of the heat flux was found to occur with the former short time, not with those for global parameters [5, 6]. Thus, the heat flux cannot be expressed by a function of local global plasma parameters. The decisive progress is that both the hystereses in the *gradient-flux* and *gradient-fluctuation* relations were observed simultaneously as is illustrated in Fig.2 [7]. The hysteresis in gradient-flux relation indicates the influence of heating on transport is considered to appear. This conclusion is based on the comparison of time scales.

Analyses of observations that can be interpreted as the hysteresis have been undertaken on various experiments [8, 9]. Discussions were made on observations on LHD, DIII-D, HL-2A, JFT-2M, JT-60U, KSTAR, TJ-II, and W7-AS.

2.2 Advanced methods of data analysis

The discrimination of two time scales in changes of heat flux and of plasma temperature becomes clear by improving the statistical accuracy in observing dynamical changes. By introducing the convolution analysis, in which the time is segmented by time periods $t_\mathrm{j} < t < t_\mathrm{j} + 2\pi/\Omega$ ($t_\mathrm{j} = 2\pi \mathrm{j}/\Omega$) and data in this segment, $X(t)$ at $t = t_\mathrm{j} + s$, is relabeled as $X(s;\ \mathrm{j})$ so that an average is taken over N-periods as

$$X(s) = N^{-1} \sum_{j=1}^{N} X(s;\ j) \tag{2}$$

By this convolution method, the periodic change of temporal evolution of $T(r,\ t)$ at the on- and off- of heating power is measured accurately. In the case of [5], the time resolution of $T(r,\ t)$ at on-off of heating reaches sub-miliseconds, and the discrimination between two times scales in flux and gradient of temperature became unambiguous. In terms of Fourier components, the higher harmonics in $T(r,\ t)$, e.g., 9th or higher harmonics, can be observed accurate enough compared to the noise level [4]. This has become available by the introduction of the convolution method in the analysis.

The transport hysteresis is searched for most simply, by observing the higher harmonics of temperature perturbation in the MECH experiment. We study the case where the gradient-flux relation (in the radial region where the heating power is absent) has the 'jump' in the hysteresis, q_jump, in addition to conventional diffusive terms,

$$q_r = -n\chi\,\nabla T + n\,VT + q_{\;jump} \tag{3}$$

where q_r is the radial heat flux, n is the plasma density and χ and V are conventional transport coefficients. Let us write the contribution of the jump to the energy balance equation as $-\nabla\cdot q_{\;jump} = n\,Q\,H(t)$. Its dependence on $H(t)$ models the fact that the jump in the hysteresis occurs in the short time in comparison with the period of heating modulation. One has the equation for the perturbed temperature δT as

$$\frac{\partial}{\partial t}\delta T - \chi_{HP}\nabla^2\delta T + V_{HP}\nabla\delta T = Q\,H(t)\,. \tag{4}$$

We assume that Q is a smooth function in radius for the analytic transparency, so that $|\nabla Q/Q|$ is much smaller than the wavenumber of the temperature perturbation. (This condition is satisfied in a wide region of the plasma column in the example of [5].) The special solution, which varies slowly in radius following Q, is given as [10]

$$T_{2m-1} \propto \frac{-Q}{(2m-1)^2}\cos\bigl((2m-1)\Omega t\bigr)\,. \tag{5}$$

The homogeneous solution, which is used in the conventional analysis with the diffusion model, obeys the dispersion relation

$$\omega = -i\chi_{HP}k^2 + V_{HP}\,k\,, \tag{6a}$$

for the waveform

$$\delta T \sim \exp\left(-i\omega t + ikx\right) \tag{6b}$$

with $k = k_r + ik_i$, where x is the distance from the reference radius. In using Eq.(3), the geometrical effects in cylinder or torus are often neglected. (The range of validity of this geometrical simplification is discussed in [9].) The wave number is predicted from Eq.(6a) as a function of the frequency ω as

$$k = \frac{1}{2\chi_{HP}}\left(-iV_{HP} \pm \sqrt{-V_{HP}^{2} + 4i\omega\chi_{HP}}\right) \tag{6c}$$

For the components with higher frequencies, $(2m-1) >> V_{HP}^{2}/4\chi_{HP}\Omega$, one has from Eq.(6c) as

$$k_{2m-1} \sim \frac{1+i}{\sqrt{2}}\sqrt{\frac{2m-1}{\chi_{HP}}\Omega}\ , \tag{7a}$$

for the (2m-1)th harmonics. That is, for the homogeneous solution (the case where the heat flux is given by diffusion model), the profile of temperature perturbation of the *n*-th harmonics $\delta T_{\mathrm{2m-1}}$ under the MECH Eq.(1) satisfies the relation

$$\delta T_{2m-1}(r) \ \sim\ \delta T_{1}(r)^{\alpha}\text{, and }\ \alpha \propto \sqrt{m-1/2}\ . \tag{7b}$$

One comes to the conclusion that, while the diffusion model of flux predicts the exponential decay of perturbation amplitude at higher harmonics, the jump in the gradient-flux relation with hysteresis induces the algebraic decay with respect to the harmonic number.

Comparing Eqs.(5) and (7b), one sees that the radial profile of the higher harmonic is a key to investigate the property of the gradient-flux relation. The dependence of amplitude on the harmonic number has been tested on LHD and TJ-II (Fig.3(a) and (b)). It has been shown that the amplitude of higher harmonics decays in radius with the similar decay index with the fundamental mode. The exponential dependence on the harmonic number, which should appear if the system obeys the diffusive model, is not observed. The similar analysis has been performed on the MECH experiment on KSTAR and DIII-D [9]. Figures 3 (c) and 3(d) are the cases of KSTAR and DIII-D, respectively. In the propagation region of 0/3 < r/a < 0.5, the radial shapes of amplitudes of 3rd and 5th harmonics are similar to the one for fundamental mode.

In the conventional analysis, the slow radial decay of perturbation amplitudes of higher harmonics leads to the fitting to larger values of transport coefficients for higher harmonics. In the conventional analysis, in which the hysteresis is neglected, one uses the homogeneous solution to obtain the interpretation from Eq.(6a) as

$$\chi_{HP} = \frac{1}{k_r^{2} + k_i^{2}}\frac{k_i}{k_r}\omega\ , \qquad V_{HP} = \frac{k_r^{2} - k_i^{2}}{k_r^{2} + k_i^{2}}\frac{1}{k_r}\omega \tag{8}$$

Under the condition that the dependence of amplitude on radius is common to higher harmonics (as is in, e.g., Fig.3(a)), '*k*' in Eq.(8) has only weak dependence on harmonic number. Therefore, the conventional analysis comes to the fitting that the deduced transport coefficient is in proportion to the harmonic number, in the limit of higher harmonics, when the hysteresis exists in the gradient-flux relation.

The result that the fitted transport coefficient is larger for higher harmonics (as in [9]) indicates the violation of the local diffusion model. If the diffusion model is correct, the transport coefficient must be recovered as an independent value for harmonic number. Here we conclude that, if the hysteresis like Figs.1 or 2 appears in the plasma transport property, the conventional diffusion model gives the result that the fitted transport coefficient is in proportion to the harmonic number. It is thus concluded, that if the conventional analysis deduces the result such that the fitted transport coefficient is larger for higher harmonics, then it is highly plausible that the hysteresis exists in transport relation.

In the case of NBI heating, if the source is modulated like Eq.(1), δT_{2m-1} is predicted to be proportional to $Q\ (2m-1)^{-3}$ in the large-m limit. Thus, the evidence of the transport hysteresis will be found in much wider circumstances.

2.3 Possible hysteresis in other transport channels

Hysteresis with rapid timescale exists in core plasma transport in tokamaks and helical systems, in the channels of energy, electron and impurity drndities, and plausibly in momentum transport. The mechanisms that induces nonlocal-in-space coupling work here as have been assessed in [7], but do not suffice. The very fast time scale in change of flux indicates that the heating can directly enhance the turbulence and transport: i.e., 'the heating heats turbulence'.

The rapid response of particle flux at the onset of heating [11], sudden jump in gradient-flux relation for carbon impurity flux [12], and dynamic hysteresis loop in velocities [13] have been observed. Interference between the particle flux and heat flux in the MECH has been investigated [14].

3. Physics Models for Hysteresis in Transport Relation and Hydrogen Isotope Effect

Theories predict that '*the heating heats turbulence*'. One possibility is that the external source S in phase space for heating has its fluctuation in turbulent plasma: $\mathrm{S}[f] = \mathrm{S}[f_0] + (\mathrm{dS}/\mathrm{d}f)\ \delta f$, where δf is the perturbation of distribution function. The fluctuation intensity I is given as

$$I = I_0/(1 - \Gamma_h), \tag{9}$$

where I_0 is the intensity without heating effect. The heating effect is represented by the parameter $\Gamma_h = (dP/dp)/\gamma_{damp}$, where $\gamma_{damp} = \chi_N k^2_\perp$ is the damping rate without heating effect, k is the relevant wavenumber, P is the heating power and p is the plasma pressure, and is the diffusion coefficient by ambient microscopic turbulence χ_N [15]. The difference $I - I_0$ at the onset of heating may correspond to the jump in hysteresis. When P is stronger, Γ_h is larger and $I - I_0$ becomes larger. This explains the observation in [5].

The 'isotope effect' on the direct heating effect [16] is also discussed. The local turbulent diffusivity χ_N is assumed to be an increasing function of mass number A of hydrogen isotope. Then Γ_h is a decreasing function of A, if other parameters are common. For instance, $\Gamma_h \sim A^{-0.5}$, if $\chi_N \sim \chi_{gB} \sim A^{0.5}$ and k independent of ρ_i. Equation (9) shows that the height of the jump in transport hysteresis is smaller for larger A. The change of the temperature gradient, $\chi \Delta \left| \nabla T \right| = (1 - \alpha) \Delta q_r/n$ against the change of power per particle $\Delta q_r/n$ is calculated in Fig.4, where χ is the local thermal diffusivity. Here the parameter α represents the relative contribution of the jump in the hysteresis, and is related with Γ_h as $\alpha \sim \Gamma_h/(1-\Gamma_h)$. Comparing the cases of heavy (2) and light (1) hydrogen isotopes, one has

$$\frac{\left(\Delta \left| \nabla T \right|\right)_2}{\left(\Delta \left| \nabla T \right|\right)_1} = \frac{A_1^{0.5}}{A_2^{0.5}} \frac{1 - \alpha_2}{1 - \alpha_1} \tag{10}$$

where the local gyro-Bohm relation is assumed for χ_N. The hysteresis can offset the isotope-dependence of local diffusion; If the condition $\alpha_2 < \left(A_2/A_1\right)^{0.5} \alpha_1 + 1 - \left(A_2/A_1\right)^{0.5}$ is satisfied, the heaviour hydrogen isotope can show a reduced transport (better confinement) in comparison with the light hydrogen isotope.

4. Impacts on Control System in Fusion Device

The hysteresis in the core transport has a strong impact on the control system against transient change of plasmas. For instance, the short interval of heating has strong influence on the edge plasma conditions. A short pulse-like change of heating in the core is smoothed, if the propagation of variation $X(x, t)$ is a diffusive process (where the diffusivity is denoted by χ):

$$X(x, t) \sim (\chi t)^{-1/2} \exp(-x^2/\chi t) \qquad (11)$$

At the distance of the order of plasma radius a, the half width of a peak in time is of the order of a^2/χ. By this broadening, the peak height of the impact becomes very small when the pulse arrives at the edge. In contrast, when the hysteresis like Fig.2 exists in the gradient-flux relation, the sudden change propagates rapidly, without being smoothed by diffusive process. The transport hysteresis can cause a large change in heat flux at edge, so as to cause the rapid H-L back transition and sudden heat load at the divertor. Observations in Ref. [17] indicate that the edge plasma parameter starts to change very shortly when the core heating is turned off. The transition from the H-mode with small ELMs to the ELM-free H-mode happens (almost immediately), when the core heating power is turned off. Considering the fact that the abrupt change of transport occurs when the core heating is turned on or off in W7-AS plasmas [17], it is highly plausible that the abrupt change of heat flux propagates rapidly (as was observed on D III-D and LHD) and that the rapid transmission of the change of heat flux to the edge happens.

This problem of rapid transition of change might have an important influence on the dynamic property of burning plasmas in experimental reactor. The possibility that the transport hysteresis causes the giant ELMs, burn through of detached plasmas, etc. will be a serious issue of the edge and scrape-off layer. The influence is more serious as the plasma becomes larger. The dynamics of plasma response includes much faster time scale than the diffusive change; the control system to protect the system from sudden plasma loss must be designed accordingly.

Summary

In short, (1) hysteresis in the gradient-flux relation can exist unambiguously in core plasma transport, for channels of energy, electron and impurity densities, and others. (2) The theoretical modelling is in progress, and (3) the knowledge and understanding of transport hysteresis is critical for control systems of fusion devices. This might also give a clue to understanding the hydrogen-isotope effect in plasma confinement.

Acknowledgements:

This study is partly supported by a Grant-in-Aid for Scientific Research of JSPS, Japan (15H02155, 15K14282, 15H02335 and 16H02442) and by the collaboration programs of NIFS (NIFS13KOCT001) of the RIAM of Kyushu University, and of QST-universities, and by Asada Science Foundation.

References

[1] N. J. Lopes Cardozo: Plasma Phys. Control. Fusion **37**, 799 (1995).

[2] U. Stroth, L. Giannone, H.-J. Hartfuss, the ECH Group and theW7-AS Team: *Plasma Phys. Control. Fusion* **38** (1996) 611

[3] K. W. Gentle, M. E. Austin, J. C. DeBoo, T. C. Luce and C. C. Petty: *Phys. Plasmas* **13** (2006) 012311

[4] S.-I. Itoh, et al.: Plasma Fusion Research **11** (2016) 2503086;

[5] S. Inagaki, et al.: Nucl. Fusion **53** (2013) 113006;

[6] S. Inagaki, et al.: Plasma and Fusion Research **8** (2013) 1202172

[7] K. Ida, et al.: Nucl. Fusion **55** (2015) 013022;

[8] S. Inagaki, et al.: Plasma and Fusion Research Rapid Commun. **9** (2014) 1202052;

[9] T. Kobayashi, et al.: this IAEA Conference (2016);

[10] K. Itoh, et al.: J. Phys. Soc. Jpn. **85** (2016) 014501;

[11] R. Makino, et al.: Plasma and Fusion Research **8** (2013) 2402115;

[12] M. Yoshinuma, et al.: Nucl. Fusion **55** (2015) 083017;

[13] K. Kamiya, et al.: submitted to Phys. Plasmas (2016);

[14] S. D. Song, et al.: Nucl. Fusion **52** (2012) 033006

[15] S.-I. Itoh, et al.: Nucl. Fusion **53** (2013) 073035;

[16] S.-I. Itoh, et al.: Nucl. Fusion **56** (2016) in press;

[17] M Hirsch, et al.: Plasma Phys. Control. Fusion **50** (2008) 053001

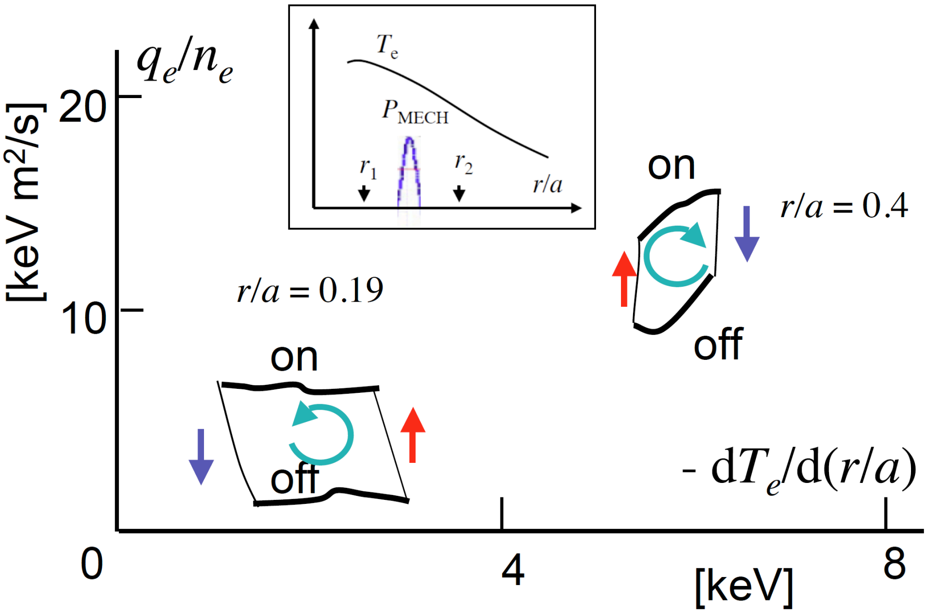

Fig.1: Observations of the gradient-flux relation via MECH experiment on D-III D (drawn based on [2], and reproduced from [4]) .

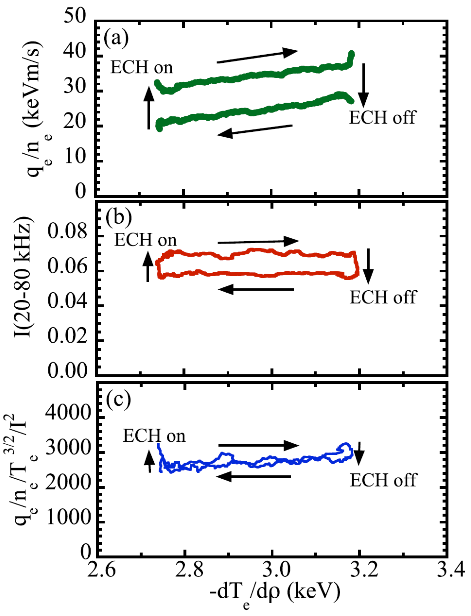

Fig.2: Measurement of gradient-flux relation on LHD. Heat flux per particle (top) and fluctuation intensity (middle) as functions of temperature gradient. Normalized heat flux is also shown (bottom). (Reproduced from [7].)

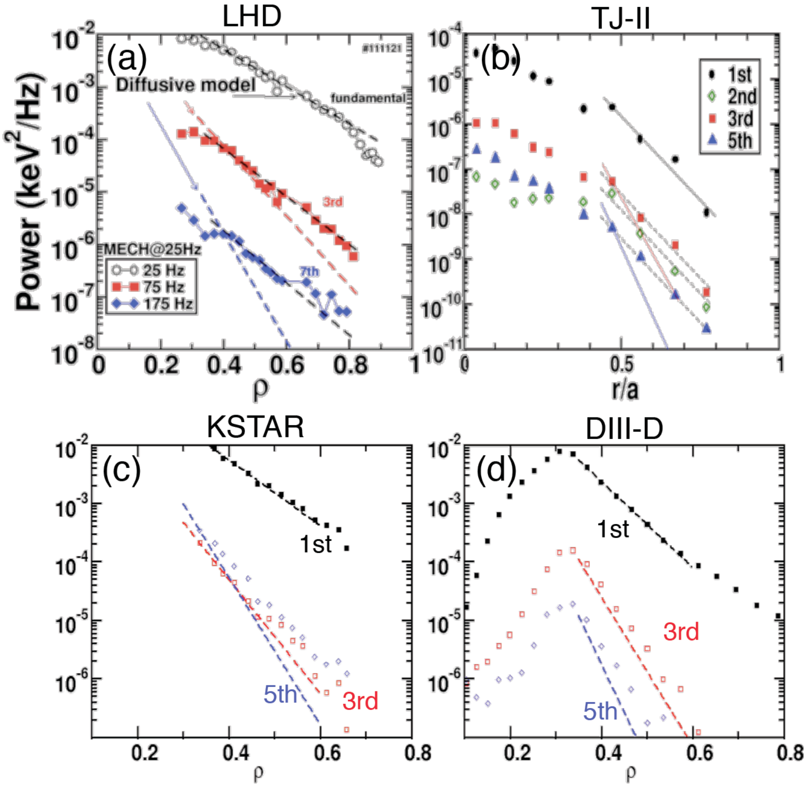

Fig. 3: Radial profiles of the electron temperature perturbation amplitude for LHD (a), TJ-II (b), KSTAR (c) and DIII-D (d).

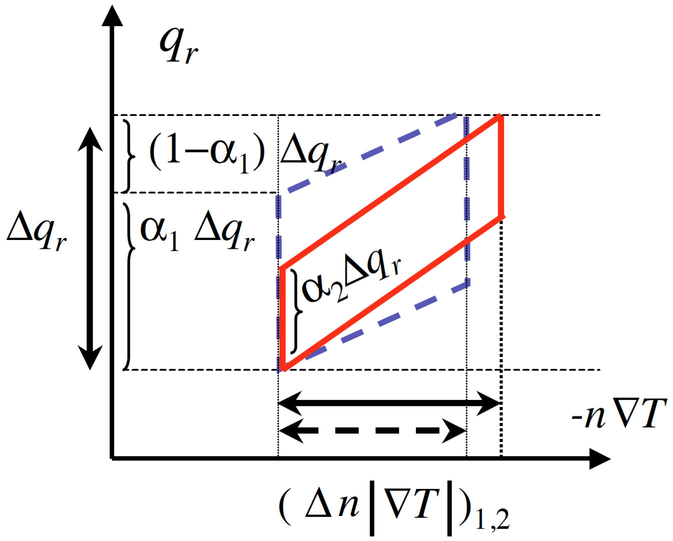

Fig.4: Illustration of the isotope effect in the presence of the hysteresis of the gradient flux relation. The cases for light isotope (thick dashed line, and suffix 1) and heavy isotope (solid line, and suffix 2) sre shown, respectively. Change of gradient against variation of the heat flux per particle $\Delta q_r/n$, $\chi\Delta|\nabla T| = (1-\alpha)\Delta q_r/n$, is shown by the thick arrow on the horizontal axis.