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Capturing Context-Related Change in Emotional Dynamics via Fixed Moderated Time Series Analysis

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ABSTRACT
Much of recent affect research relies on intensive longitudinal studies to assess daily emotional experiences. The resulting data are analyzed with dynamic models to capture regulatory processes involved in emotional functioning. Daily contexts, however, are commonly ignored. This may not only result in biased parameter estimates and wrong conclusions, but also ignores the opportunity to investigate contextual effects on emotional dynamics. With fixed moderated time series analysis, we present an approach that resolves this problem by estimating context-dependent change in dynamic parameters in single-subject time series models. The approach examines parameter changes of known shape and thus addresses the problem of *observed* intra-individual heterogeneity (e.g., changes in emotional dynamics due to observed changes in daily stress). In comparison to existing approaches to *unobserved* heterogeneity, model estimation is facilitated and different forms of change can readily be accommodated. We demonstrate the approach's viability given relatively short time series by means of a simulation study. In addition, we present an empirical application, targeting the joint dynamics of affect and stress and how these co-vary with daily events. We discuss potentials and limitations of the approach and close with an outlook on the broader implications for understanding emotional adaption and development.

KEYWORDS
Dynamic modeling; emotional functioning; moderation; nonstationary processes; time series analysis

Introduction

Daily diaries or experience sampling gives rise to intensive repeated measures of intra-individual experiences in daily life (Ebner-Priemer & Trull, 2009). Behavioral affect research increasingly relies on these intensive longitudinal assessment protocols to target emotion regulation processes. The underlying rationale is that such processes may be identifiable from the temporal patterns contained in the observed affect trajectories (Boker, 2002; Kuppens & Verduyn, 2015; Ram & Gerstorf, 2009). Formal dynamic models that parameterize temporal regularities in short-term changes are used to cast these *dynamics* of emotional functioning.

However, a single dynamic model with time-invariant parameters implies time-invariant emotional dynamics, which may not always be sufficient to describe how a person functions over time. In fact, one can think of many factors relating to change in emotional dynamics, for instance, variations in daily context as individuals switch between work and home, engage in different social interactions or activities, or encounter stressful events (e.g., Koval & Kuppens, 2011; Kuppens, Allen, & Sheeber, 2010; Zautra, Berkhof, & Nicolson, 2002).

In this paper, we propose a dynamic model suited to capture changes in the dynamics of emotional functioning via time-varying model parameters. The model is implemented as a time series model applicable to data from single individuals. Within a given person, the approach allows to freely estimate the amount of change in model parameters that follows a known shape over time. In other words, we consider a model with time-varying parameters, where the change in these parameters is fully explained by an observed variable. This permits testing hypotheses about whether and to what extent differences in observed context are related to differences in emotional dynamics or whether and to what extent there are deterministic time trends in emotional dynamics. In formal terms, the proposed model addresses the issue of *observed* intra-individual heterogeneity as opposed to *unobserved* intra-individual

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heterogeneity. While the sources of heterogeneity are known in the first case, they are unknown in the latter.

The outline of the paper is as follows: First, we lay out a rationale for applying dynamic time series analysis (TSA) to affective phenomena. We briefly review the general line of argument, according to which dynamic models might inform us about emotion regulation processes. Additionally, we provide specific arguments in favor of time-varying emotional dynamics and recapitulate existing modeling solutions to the corresponding formal problem of time-varying dynamic parameters (i.e., intra-individual heterogeneity) in the $N = 1$ case. Second, we introduce our approach, fixed moderated TSA, in terms of model structure as conveyed in equations, in terms of model behavior as illustrated by simulated data, and in terms of parameter estimation. Third, a comprehensive simulation study is presented, in which we investigate the proposed model's performance given relatively short time series (i.e., $T = (100, 150, 200)$). Fourth, we apply the approach to data from the COGITO study (Schmiedek, Bauer, Lövdén, Brose, & Lindenberger, 2010). Using self-report data from nine younger adults across an average of 101 measurement occasions and 132 days, we model the joint daily dynamics of negative affect (NA) and perceived stress (PS) per participant. Variation in model parameters is estimated as a deterministic function of variation in self-reported daily events. We end by discussing potentials and limitations of the proposed approach and by addressing implications in the broader context of the literature on affective functioning.

TSA applied to affect data

General rationale

In this paper, we are concerned with TSA (Hamilton, 1994; Lütkepohl, 2005). A popular class of *dynamic* models for time series formalize change in a variable of interest as driven by an unobserved stochastic process that is random over time (i.e., a set of random variables that are independent over time; Browne & Nesselroade, 2005). When being randomly perturbed, the variable of interest, or process variable, may show nonrandom change, that is, it may reveal its "intrinsic dynamics" (Boker, 2002, p. 415). For instance, change in the process variable could be structured in that one observes carryover effects between time points, meaning that perturbations take time to "die out" (Hamilton, 1994, p. 54). This is the defining characteristic of a stationary auto-regressive (AR) model, which is the univariate variant of the stationary vector auto-regressive (VAR) model we rely on in the following. The behavior of a stationary AR model is shown in Panel A of Figure 1 and described in more detail in section *Fixed Moderated TSA*, subsection *Model Behavior*. If multiple variables are modeled simultaneously, it is possible to get at the *joint dynamics* among them, that is, how multiple variables affect each other over time when being perturbed. Dynamic longitudinal models are sometimes contrasted with static longitudinal models. While dynamic models formalize change in the variable of interest in reference to its own past, static models, such as growth curve models, formalize change in the variable of interest as a function of time or some independent time-varying variable. Put differently, in static systems, the output only depends on the input at a given point in time, whereas in dynamic systems, the output depends also on the past. Static models thus provide descriptions of the overall trajectory of a variable, but cannot *directly* address questions of (joint) dynamics (Hertzog & Nesselroade, 2003; McArdle, 2009; Voelkle, 2016; Voelkle & Oud, 2015).

Dynamic models are increasingly employed in the analysis of intensive longitudinal measures of emotional phenomena (Hamaker, Ceulemans, Grasman, & Tuerlinckx, 2015; Montpetit, Bergeman, Deboeck, Tiberio, & Boker, 2010; Pe et al., 2015; Röcke & Brose, 2013). The aim of these applications is to get at the processes that structure change in daily affective experiences, among them emotion regulation (Boker, 2002; Kuppens, Oravecz, & Tuerlinckx, 2010; Kuppens & Verduyn, 2015; Ram & Gerstorf, 2009), and specific model parameters are interpreted as parameters of regulatory processes (e.g., auto-regressive strength as "emotional inertia"; Brose, Schmiedek, Koval, & Kuppens, 2015; Koval, Kuppens, Allen, & Sheeber, 2012; Kuppens et al., 2010; Suls, Green, & Hillis, 1998).

The term TSA has been used to emphasize that the corresponding models are applicable to data from single individuals (Hamaker & Dolan, 2009; Hamaker, Dolan, & Molenaar, 2005; Molenaar, Sinclair, Rovine, Ram, & Corneal, 2009). Single-subject analyses are a necessity in single-case studies, for instance in clinical settings (Roche, Pincus, Rebar, Conroy, & Ram, 2014). But also with repeated measures from many individuals, a "person-centered" or "ideographic" approach can be appropriate if one views it as enabling "informed aggregations of information across multiple participants" (Nesselroade, 2010, p. 211). In concrete terms, TSA solutions from different individuals are independent and subsequent comparisons between individuals thus maximally unconstrained. Such a bottom-up approach to the scientific goal of establishing generalities across individuals can be an "epistemological necessity" when psychological processes are subject to profound inter-individual differences (Molenaar, 2004, p. 204).

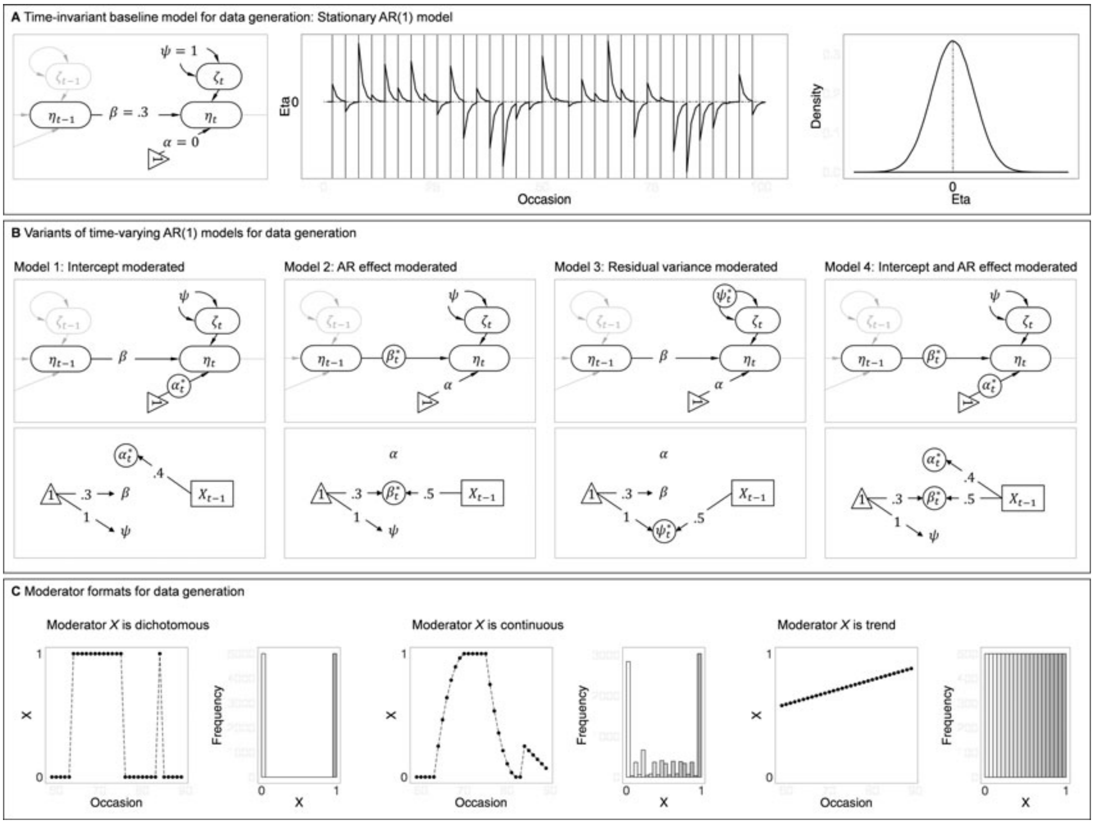

Figure 1. Ingredients for a demonstration of the model's behavior. In Panel A, the time-invariant baseline model is displayed in terms of a path diagram, a segment of a model-implied process trajectory, and the model-implied probability distribution. Data were generated based on four different time-varying model structures, depicted in Panel B, in combination with the three different moderator formats, shown in Panel C. Paths not drawn in path diagrams are zero.

Rationale for time-varying (emotional) dynamics

In many psychological applications, dynamic time series models formalize stationary processes. Stationary processes have stable characteristics over time, specifically, time-stable distributional moments (e.g., mean, variance, auto-covariance), and can thus be cast in terms of time-invariant parameters (Lütkepohl, 2005, p. 24). When thinking of psychological processes in general, and affective processes in particular, a stationary process seems to represent a relatively restrictive case, though. Complex and changing environments as well as ongoing developmental processes likely relate to variability and change in how we function and, thus, how psychological processes evolve over time (e.g., Hollenstein, Lichtwarck-Aschoff, & Potworowski, 2013; Molenaar, 2004; Nesselroade, 1991). For instance, an individual's emotional dynamics may fluctuate within a certain range (Chow, Zu, Shifren, & Zhang, 2011; Koval & Kuppens, 2011; Ram et al., 2014; Sliwinski, Almeida, Smyth, & Stawski, 2009; Zautra et al., 2002), possibly reflecting "state-dependent regulation" (De Haan-Rietdijk, Gottman, Bergeman, & Hamaker, 2016, p. 217). Furthermore, the possibility of temporal trends in the variability and predictability of affect has received interest in the context of forecasting major regime shifts such as transitions into depression (Scheffer et al., 2009; van de Leemput et al., 2014). Finally, major events or enduring changes in a person's environment may result in gradual long-term adjustments in the level around which an individual's emotional experiences fluctuate (Boker, 2015).

Modeling solutions to the problem of intra-individual heterogeneity

The processes portrayed above are all nonstationary in that more than one set of dynamic parameters (i.e., changing dynamic parameters) is required to characterize their behavior over time. We refer to this as a problem of *intra-individual heterogeneity* and thereby take Muthén's (1989) definition of *inter-individual heterogeneity* to the intra-individual level of analysis (Adolf, Schuurman, Borkenau, Borsboom, & Dolan, 2014; Dolan, 2009). Under intra-individual heterogeneity, variation can thus either pertain to a given set of dynamic parameters or to change in dynamic parameters. Clearly, an appropriate modeling approach to explicitly distinguish the two sources is required, if not out of substantive interest then for statistical reasons (e.g., De Haan-Rietdijk, Kuppens, & Hamaker, 2016; Dolan, Jansen, & van der Maas, 2004).

Traditionally, multivariate modeling solutions to the problem of heterogeneity have been proposed in the context of studying between-person differences, encompassing, for instance, multiple-group structural equation modeling (SEM) (e.g., Jöreskog, 1971), finite and infinite mixture SEM (Bauer, 2007; Dolan, 2009; Hessen & Dolan, 2009; Lubke & Muthén, 2005; D. Molenaar, 2015; Sterba, 2013), SEM with fixed moderators (e.g., Bauer & Hussong, 2009; Curran et al., 2014; D. Molenaar, Dolan, Wicherts, & van der Maas, 2010), SEM trees (Brandmaier, von Oertzen, McArdle, & Lindenberger, 2013), and locally weighted SEM (Hildebrandt, Lüdtke, Robitzsch, Sommer, & Wilhelm, 2016; Hülür, Wilhelm, & Robitzsch, 2011). With the rise of person-centered research methods (Hamaker, 2012; Molenaar & Campbell, 2009), problems of intra-individual heterogeneity and according modeling solutions receive increasing attention. In the following, we provide an overview over modeling solutions proposed for the $N = 1$ case. To organize this, we look at two criteria, the major one being whether models address heterogeneity as *observed* versus *unobserved* (cf. Lubke & Muthén, 2005), and the minor, orthogonal criterion being whether change in model parameters is cast in terms of abrupt switches versus smooth trajectories. The first criterion is chosen, because it best contrasts the approach suggested here to existing approaches. The second criterion may be of substantive interest, as the examples in the previous section showed. Obviously, neither is this overview exhaustive, nor are the criteria evoked to structure it.

Treating (sources of) intra-individual heterogeneity as *unobserved* is expedient if one neither knows when nor to what extent the parameters of a dynamic model change, or if the timing and extent of change are at least uncertain. Commonly used modeling solutions address this issue by setting up a parametric probability model for the change in parameters. That is, in addition to assuming an unobserved stochastic process leading to changes in the observed outcome variable, these models assume an unobserved stochastic process producing changes in the parameters of the unobserved stochastic process underlying the outcome variable. Fitting such a model involves not only estimating the unknown parameters underlying the observed *outcome* trajectory, but rather estimating the unknown parameters underlying the unobserved *parameter* trajectory underlying the observed outcome trajectory (Kim & Nelson, 1999).

Popular examples are so-called regime-switching models. (Chow & Zhang, 2013; De Haan-Rietdijk et al., 2016; Dolan et al., 2004; Hamaker & Grasman, 2012; Hamaker, Zhang, & van der Maas, 2009; Hamilton, 2010; Hunter, 2014a; Kim & Nelson, 1999). These evoke a small number of distinct parameter sets (i.e., dynamic regimes) between which the outcome process switches over time according to a discrete-valued parameter process, for instance a Markov chain. Regime-switching models thus allow investigating discontinuous and abrupt changes in model parameters. Other models, on the contrary, evoke a continuous-valued parameter process, for instance a Gaussian AR process, to account for gradual changes in process parameters over time (e.g., Boker, 2015; Chow, Ferrer, & Nesselroade, 2007; Chow et al., 2011; Molenaar, Beltz, Gates, & Wilson, 2016; Molenaar et al., 2009).

In both cases, discrete and continuous parameter processes, time-varying covariates may be incorporated into the model to reduce the uncertainty about parameter change. However, covariates are not *necessary* to identify unobserved heterogeneity. This is different in models for *observed* heterogeneity. Intra-individual heterogeneity may be treated as observed if the timing of changes in model parameters is assumed to be known. That is, if parameter changes can be coupled to a time-varying covariate or if a temporal pattern of a certain shape can be expected (e.g., a linear trend). In this case, there is no need to estimate the parameter trajectory from the data using a probability model. Instead, parameter change can directly and fully be accounted for by conditioning on the available information on the sources of change in parameters. Only the extent to which these fixed changes manifest, is then estimated from the data.

Including covariates as fixed moderators has received interest in cross-sectional modeling applications (Bauer & Hussong, 2009; Curran et al., 2014; Hildebrandt et al., 2016; D. Molenaar et al., 2010). Here, we propose to transfer the approach to the intra-individual level. We refer to this as *fixed moderated TSA*, which can be seen as an extension of VAR models including observed time-varying covariates as fixed or exogenous covariates (e.g., Lütkepohl, 2005, p. 387), which in the standard model only additively affect the outcome variable (but see Bringmann et al., 2013, who let selected parameters of a VAR model vary as a function of observed covariates). In fixed moderated TSA, moderators need not be observed, substantive covariates, though. As the data are analyzed in the time domain, model parameters can also vary as a function of time (cf. Bringmann et al., 2016). In fact, the flexibility of readily modeling various shapes of change is one advantage of this model. Other advantages concern an easy implementation and estimation, as we will show in the following sections. Although it seems unrealistic to have complete information about the timing of parameter changes, incorporating and exploring informed hypotheses can provide a pragmatic starting point for further research. The following sections present the model in more detail.

Fixed moderated TSA

Model structure

We employ a time-discrete VAR model of first order as the time-invariant baseline model. The process model is

$$\boldsymbol{\eta}_t = \boldsymbol{\alpha} + \mathbf{B}\boldsymbol{\eta}_{t-1} + \boldsymbol{\zeta}_t \tag{1}$$

with

$$\zeta_t \sim N(\mathbf{0}, \boldsymbol{\Psi})$$

where $\boldsymbol{\eta}_t$ is a $q \times 1$ vector of continuous process variables, $\boldsymbol{\alpha}$ is a $q \times 1$ vector of regression intercepts, $\mathbf{B}$ is a $q \times q$ matrix of auto- and cross-lagged regression weights, and ζ_t is a $q \times 1$ vector of stochastic process residuals. The process residuals at t are normally distributed with a $q \times 1$ zero mean vector and a $q \times q$ covariance matrix $\boldsymbol{\Psi}$, and are serially uncorrelated and uncorrelated with $\boldsymbol{\eta}_{t-1}$.

If the process variables are normally distributed at $t = 0$ and the process is stable (i.e., all eigenvalues of $\mathbf{B}$ have modulus less than 1, implying that the temporal dependencies are not too strong and perturbations die out on the long run), the process variables have a stationary long-run distribution. This is a normal distribution, with mean vector $\boldsymbol{\nu} = (\mathbf{I} - \mathbf{B})^{-1}\boldsymbol{\alpha}$ and covariance matrix, P, which can only be derived in vectorized form as $vec(\mathbf{P}) = (\mathbf{I} \otimes \mathbf{I} - \mathbf{B} \otimes \mathbf{B})^{-1} vec(\boldsymbol{\Psi})$, where $\mathbf{I}$ is a $q \times q$ identity matrix, $\otimes$ denotes the Kronecker product, and $vec(\boldsymbol{\Psi})$ is the vectorization of $\boldsymbol{\Psi}$ (Hamilton, 1994, p. 265; Lütkepohl, 2005, p. 13).

Since the process variables may be latent, we also specify a reflective measurement model of the form

$$\mathbf{y}_t = \boldsymbol{\tau} + \boldsymbol{\Lambda}\boldsymbol{\eta}_t + \boldsymbol{\varepsilon}_t \tag{2}$$

with

$$\boldsymbol{\varepsilon}_t \sim N(\mathbf{0}, \boldsymbol{\Theta})$$

where $\mathbf{y}_t$ is a $p \times 1$ vector of continuous observed indicators, $\boldsymbol{\tau}$ is a $p \times 1$ vector of measurement intercepts, $\boldsymbol{\Lambda}$ is a $p \times q$ matrix of factor loadings, and $\boldsymbol{\varepsilon}_t$ is a $p \times 1$ vector of measurement residuals with $p \times 1$ zero mean vector and $p \times p$ diagonal covariance matrix $\boldsymbol{\Theta}$. The measurement residuals are serially uncorrelated and uncorrelated with ζ_t and $\boldsymbol{\eta}_t$. The model implies that the indicators at time t are normally distributed with mean vector $\boldsymbol{\mu} = \boldsymbol{\tau} + \boldsymbol{\Lambda}\boldsymbol{\nu}$ and covariance matrix $\boldsymbol{\Sigma} = \boldsymbol{\Lambda}\mathbf{P}\boldsymbol{\Lambda}' + \boldsymbol{\Theta}$, where $\boldsymbol{\Lambda}'$ denotes the transpose of $\boldsymbol{\Lambda}$.

We now extend the structural model so that all model parameters can vary over time. The model is

$$\boldsymbol{\eta}_t = \boldsymbol{\alpha}_t^* + \mathbf{B}_t^*\boldsymbol{\eta}_{t-1} + \boldsymbol{\zeta}_t \tag{3}$$

with

$$\zeta_t \sim N\left(\mathbf{0}, \boldsymbol{\Psi}_t^*\right)$$

and

$$\begin{aligned} \boldsymbol{\alpha}_t^* &= g\left(X_{t-1}\right) \\ \mathbf{B}_t^* &= h\left(X_{t-1}\right) \\ \boldsymbol{\Psi}_t^* &= i\left(X_{t-1}\right) \end{aligned} \tag{4}$$

All model parameters can now vary as a function of a time-varying moderator X_t. This moderator enters the model in terms of fixed values and is assumed to completely determine variability in model parameters, that is, $g(\cdot)$, $h(\cdot)$, and $i(\cdot)$ are deterministic functions without stochastic residuals. The form of the functional relationships can be flexibly specified, and the extent of co-variation with the moderator is freely estimated per parameter. The functions $g(\cdot)$, $h(\cdot)$, and $i(\cdot)$ are themselves time-invariant.

Note that the moderator can be any time-varying variable, including time itself (c.f., Selig, Preacher, & Little, 2012). As the format of the moderator and the form of the functional relationship to the model parameters are (relatively) arbitrary, the model offers reasonable flexibility in testing for different forms of change in model parameters.

For illustrative purposes, we confine ourselves to the minimal example of a bivariate process model in the following, which is

$$\begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix}_t = \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix}_t^* + \begin{bmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{bmatrix}_t^* \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix}_{t-1} + \begin{bmatrix} \zeta_1 \\ \zeta_2 \end{bmatrix}_t \tag{5}$$

with

$$\begin{bmatrix} \zeta_1 \\ \zeta_2 \end{bmatrix}_t \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \psi_{11} & \psi_{12} \\ \psi_{21} & \psi_{22} \end{bmatrix}_t^*\right)$$

and a linear link function, such that

$$\begin{aligned} \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix}_t^* &= \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix}^{(0)} + X_{t-1}\begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix}^{(X)} \\ \begin{bmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{bmatrix}_t^* &= \begin{bmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{bmatrix}^{(0)} + X_{t-1}\begin{bmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{bmatrix}^{(X)} \\ \begin{bmatrix} \psi_{11} & \psi_{12} \\ \psi_{21} & \psi_{22} \end{bmatrix}_t^* &= \begin{bmatrix} \psi_{11} & \psi_{12} \\ \psi_{21} & \psi_{22} \end{bmatrix}^{(0)} + X_{t-1}\begin{bmatrix} \psi_{11} & \psi_{12} \\ \psi_{21} & \psi_{22} \end{bmatrix}^{(X)} \end{aligned} \tag{6}$$

As in an ordinary regression setup, all model parameters now decompose into an intercept or baseline component for $X_{t-1} = 0$, superscripted by (0), and a change component, associated with a one-unit change in X_{t-1} and superscripted by (X).

In case of a dummy-coded moderator $X_t = x_t \in \{0, 1\}$ the model can be re-parameterized as

$$\begin{aligned}
\begin{bmatrix}\alpha_1\\ \alpha_2\end{bmatrix}^*_t &= (1 - X_{t-1})\begin{bmatrix}\alpha_1\\ \alpha_2\end{bmatrix}^{(X=0)} + X_{t-1}\begin{bmatrix}\alpha_1\\ \alpha_2\end{bmatrix}^{(X=1)}\\
\begin{bmatrix}\beta_{11} & \beta_{12}\\ \beta_{21} & \beta_{22}\end{bmatrix}^*_t &= (1 - X_{t-1})\begin{bmatrix}\beta_{11} & \beta_{12}\\ \beta_{21} & \beta_{22}\end{bmatrix}^{(X=0)}\\
&\quad + X_{t-1}\begin{bmatrix}\beta_{11} & \beta_{12}\\ \beta_{21} & \beta_{22}\end{bmatrix}^{(X=1)}\\
\begin{bmatrix}\psi_{11} & \psi_{12}\\ \psi_{21} & \psi_{22}\end{bmatrix}^*_t &= (1 - X_{t-1})\begin{bmatrix}\psi_{11} & \psi_{12}\\ \psi_{21} & \psi_{22}\end{bmatrix}^{(X=0)}\\
&\quad + X_{t-1}\begin{bmatrix}\psi_{11} & \psi_{12}\\ \psi_{21} & \psi_{22}\end{bmatrix}^{(X=1)}
\end{aligned} \tag{7}$$

Parameters with superscript $(X = 0)$ still denote the baseline parameters given that $X_{t-1} = 0$. Parameters with superscript $(X = 1)$ now denote the process parameters given that $X_{t-1} = 1$. Instead of estimating change in parameters, the model switches between two distinct processes and estimates their parameters separately. Depending on one's research question, one may of course choose either of the two mathematically equivalent parameterizations, as long as the moderator is discrete and dummy-coded.

Two extensions of the model as presented in Equations (3) and (4), respectively, are possible. First, as in multiple regression, one may include multiple moderators. Second, the measurement model may be specified as time-varying in a similar way as the process model. Like in cross-sectional settings, this may then be used to address measurement theoretical questions (cf. Bauer & Hussong, 2009; Curran et al., 2014; Hildebrandt et al., 2016), specifically, it becomes possible to test for violations of factorial invariance over time (e.g., Adolf et al., 2014).

Model behavior

To gain a better understanding of the model's behavior, we present data generated from a univariate variant of the potentially latent process model in the following, thus ignoring the measurement part. Figure 1 illustrates the ingredients of this small, exemplary simulation, while Figure 2 displays the simulated data.

Panel A of Figure 1 shows on the left a path diagram of the time-invariant process model, which corresponds to the model as conveyed in Equation (1), but is reduced to one process variable. To generate the data, the parameter values are fixed to $\beta = .3$, $\alpha = 0$, and $\psi = 1$. The middle part of Panel A shows a realization of the model-implied process over time. The plotted trajectory indicates that a stationary first-order AR model implies some stable mean level (the black dash-dotted horizontal line) from which the process variable (e.g., a person's changing affective state; the bold black solid line) is continuously driven away by random shocks, which are realizations of the process residual variables, ζ_t. These may capture all kinds of situational influences (e.g., events; the occurrence of a shock or perturbation is marked by a black vertical line). Following such a perturbation to the process, we observe an exponentially shaped return to the mean, the rate of which is inversely related to the strength of the AR coefficient. That is, perturbations "die out" over time (Hamilton, 1994, p. 54), and they do so the faster the closer the AR coefficient is to zero. However, the closer the AR coefficient is to one, the longer the effects of perturbations remain in the system. In the context of dynamic models applied to affect data, the AR effect has therefore been interpreted as reflecting "emotional inertia" (e.g., Kuppens et al., 2010) or the tendency to experience "emotional residues" (Suls et al., 1998, p. 134). The final component on the right of Panel A shows the model-implied long-run probability distribution of the process variable. For the model under consideration, it is a univariate normal distribution with a mean that is a function of the intercept and the AR coefficient, and a variance that is a function of the AR coefficient and the processes residual variance. We include this probability distribution, as it demonstrates to what extent different states will be covered by the process on the long run. Hence, both the trajectory segment and the long-run probability distribution provide concrete and complementary descriptions of the kind of data we expect the model to fit well.

Panel B of Figure 1 displays four time-varying model structures that are based on Panel A's baseline model and differentially include the moderator. To draw the models, we adapted Curran and Bauer's (2007) scheme for path diagrams of hierarchical models. Encircled parameters are regressed not only on a constant of one (with the regression weight reflecting the baseline parameter value if the moderator is zero), but also on the moderator, and hence are time-varying. Note, however, that, unlike in Curran and Bauer's setup, there are no stochastic residuals pointing to the time-varying parameter variables, indicating that parameter change is not random, but fixed and determined by the moderator. The four models in Panel B entail the following scenarios: A one-unit increase in the moderator leads to a 0.4-unit increase in the intercept for Model 1, to a 0.5-unit increase in the AR effect for Model 2, to a 0.5-unit increase in the residual variance for Model 3, and to increases in both the intercept (0.4 units) and the AR effect (0.5-units) for Model 4.

For each of these four structures, we consider three formats of the moderator X as visualized via trajectory plots and corresponding long-run frequency distributions in Panel C of Figure 1. We incorporate the moderator as an equally distributed dichotomous variable (on the left), as a continuous variable (in the middle) and as a linear function of time (on the right). The continuous moderator is a smoothed version of the dichotomous moderator and is obtained by calculating a moving average with a window size of seven occasions (cf. Schilling & Diehl, 2014). We display the formula in the empirical illustration section of the paper (cf. Equation [12]). For now, it suffices to note that in contrast to the dummy code, the continuous moderator reflects the accumulation of moderator states over time. Note that the shape of the frequency distribution of this continuous moderator will depend on the temporal stability of the underlying dichotomous moderator. The depicted U-shaped distribution results from a rather stable dichotomous moderator (i.e., the moderator does not switch often between zero and one). Higher instability (frequent switching) would lead to a concentration of mass in the center of the distribution. The scale of the moderator is in all cases bounded between zero and one, and the associated color coding is retained in the following (white indicates zero).

By showing data generated from the combinations of model structures and moderator formats, Figure 2 demonstrates what happens when the different simulation ingredients come together. Panels A to C correspond to the different moderator formats and the rows within each Panel to the different model structures. The data are presented in terms of trajectories and long-run probability distributions, as was the case for the baseline model. We present trajectories for the moderated process (bold solid line) and the unmoderated counterfactual (dotted line). The process means conditional on the moderator are also included (dash-dotted line). The values the moderator takes on over time are indicated by background color. We distinguish the probability distribution of the process variable conditional on the moderator being zero (fine lines, filled white) and one (fine lines, filled gray) from the marginal distribution (in bold) over all values of the moderator. The marginal distribution is always a finite mixture of all conditional distributions, mixed according

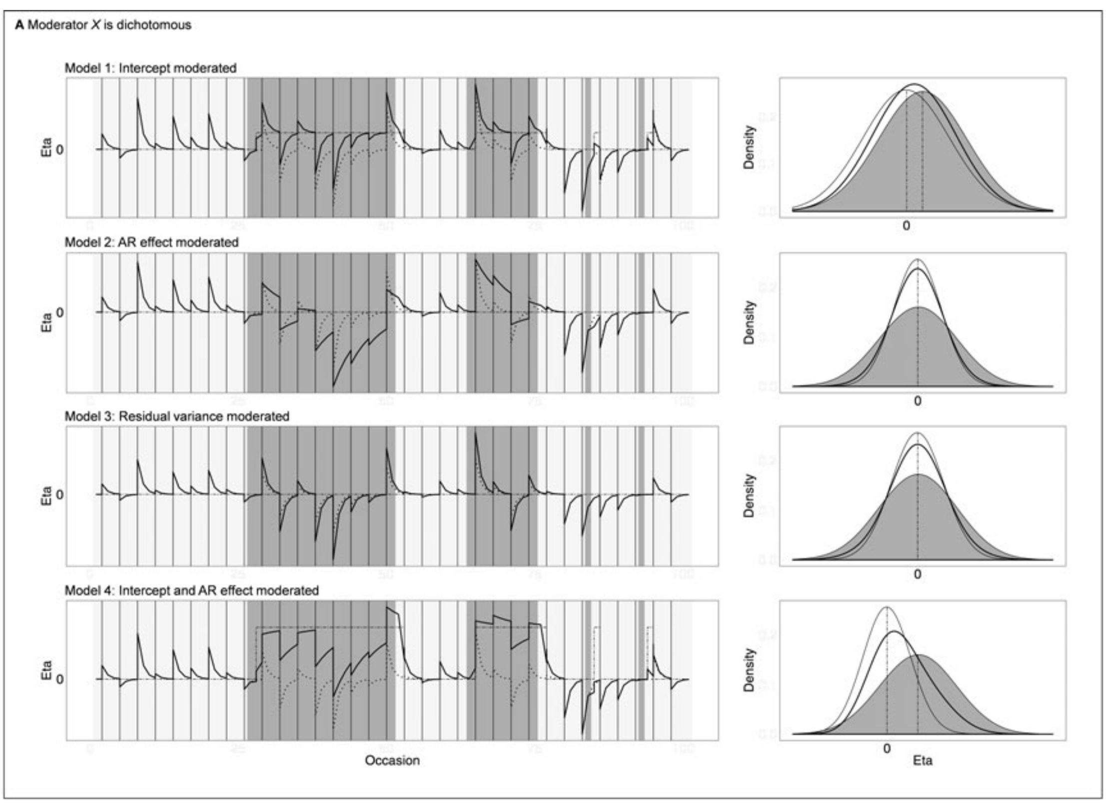

Figure 2. Model behavior under different constellations of time-invariant and time-varying parameters (Models 1–4) for different moderator formats (Panels A–C). The exact conditions under which the displayed data were generated are visualized in Figure 1. Implied process trajectories are shown on the left, with implied probability distributions on the right side of the panels. The trajectories of the moderated processes are represented by bold solid lines, the trajectories of the unmoderated counterfactual processes by dotted lines, and the mean trajectories of the moderated processes by dash-dotted lines. Moderator states over time are indicated by background color. The conditional probability distributions are printed with fine contours and in solid white for the moderator being zero and in solid gray for the moderator being one. The marginal probability distribution is depicted with bold contours.

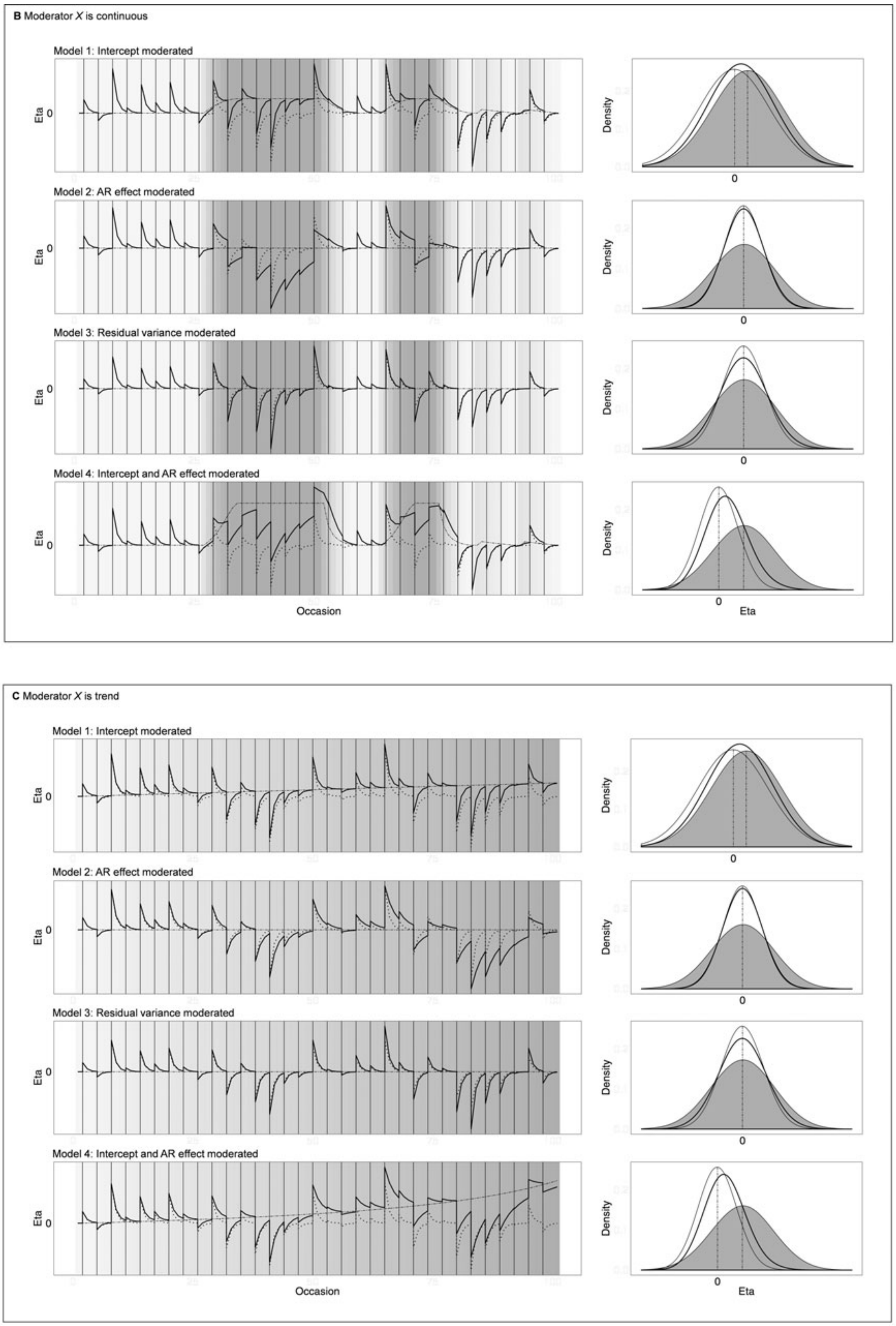

Figure 2. Continued

to the frequency distribution of the moderator, and hence no longer normal.

The trajectory plot in Row 1 of Panel A, Figure 2, reveals shifts in mean level due to the intercept shifting with the changing moderator. Note that this reflects a main effect of the earlier moderator state on the current process state after controlling for the effects of earlier process states. The two conditional probability distributions differ only in mean. In Row 2, the AR coefficient and thus the rate at which the process returns to its mean changes and we see how perturbations accumulate more when the moderator is "on" as compared to when it is "off." Remember that the AR coefficient also contributes to the mean. Here, however, we do not see an effect on the mean because the mean is 0 in this model. The conditional distributions differ in variance. Row 3 displays a situation in which the strength of a perturbation changes when the moderator is switched on, corresponding to a change in residual variance. Again, the conditional distributions differ in variance. In Row 4, we see the combination of altered mean level (due to changes in the intercept interacting with changes in the AR effect) and return rate (due to changes in the AR effect). The conditional distributions differ in mean and variance.

As the continuous moderator is derived from the dichotomous one, Panel B of Figure 2 recovers to some extent the just reported pattern of changes in dynamics across models. Differences lie in the smoothness of changes because the moderator changes are smoother. Effects thus build up over time and become more pronounced the longer the moderator is "on," before they fade out again.

In combination with a linearly trending moderator (Figure 2, Panel C), Model 1 (Row 1) leads to a linear mean change in the moderated process, Model 2 (Row 2) produces monotonically changing return rates, and Model 3 (Row 3) a monotonic change in the strength of perturbations. The combined case of Model 4 (Row 4) is particularly interesting, as the combined changes in the AR coefficient *and* intercept lead to a nonlinear mean trajectory. At the same time, the increase in the AR effect causes a decrease in the rate of return to the mean. The moderated process thus diverges from its mean over time.

Model estimation

We rely on the state-space modeling (SSM) framework and the Kalman filter to estimate the model (e.g., Chow, Ho, Hamaker, & Dolan, 2010; Durbin & Koopman, 2012; Harvey, 1989). SSM evokes a powerful multivariate modeling framework, which distinguishes latent process variables from measurement error. Implementing a dynamic model in SSM requires rewriting the process model in terms of its so-called state-space representation, that is, in terms of a first-order VAR process (which is possible for all VAR moving average models by extending the vector of latent process variables; e.g., Shumway & Stoffer, 2011). In our case, no reformulation of the model is needed. The Kalman filter is a recursive filtering procedure that capitalizes on this model structure to predict present process states from past ones (e.g., Durbin & Koopman, 2012; Hamilton, 1994; Harvey, 1989). Under the assumption that both the process errors at time t and the process variables at time $t = 0$ (i.e., $\boldsymbol{\eta}_0$) are normally distributed, the Kalman filter yields Maximum Likelihood (ML) estimates of the model parameters. Usually, for TSA, ML estimation via the Kalman filter is computationally more efficient than ML estimation in the context of SEM (Hamaker, Dolan, & Molenaar, 2003; Voelkle, Oud, von Oertzen, & Lindenberger, 2012).

For the present situation, the joint likelihood function can be written as (cf. Harvey, 1989, pp. 125–128)

$$L\left(\mathbf{y}; \boldsymbol{\Omega}\right) = \prod_{t=1}^{T} f\left(\mathbf{y}_t | \mathbf{Y}_{t-1}, X_{t-1}; \boldsymbol{\Omega}\right) \tag{8}$$

where

$$\begin{aligned} \boldsymbol{\Omega} &= \left(\boldsymbol{\tau}, \boldsymbol{\Lambda}, \boldsymbol{\Theta}, \boldsymbol{\alpha}_t^*, \mathbf{B}_t^*, \boldsymbol{\Psi}_t^*\right), \\ \mathbf{Y}_{t-1} &= \left(\mathbf{y}_{t-1}', \mathbf{y}_{t-2}', \ldots, \mathbf{y}_1'\right), \\ f\left(\mathbf{y}_t | \mathbf{Y}_{t-1}, X_{t-1}; \boldsymbol{\Omega}\right) &= N(\mathbf{y}_t | \mathbf{Y}_{t-1}, X_{t-1}; \boldsymbol{\tau} \\ &\quad + \boldsymbol{\Lambda}\boldsymbol{\eta}_{t|t-1}, \ \boldsymbol{\Lambda}\mathbf{P}_{t|t-1}\boldsymbol{\Lambda}' + \boldsymbol{\Theta}), \\ \boldsymbol{\eta}_{t|t-1} &= \boldsymbol{\alpha}_t^* + \mathbf{B}_t^*\boldsymbol{\eta}_{t-1|t-1}, \\ \mathbf{P}_{t|t-1} &= \mathbf{B}_t^*\mathbf{P}_{t-1|t-1}\mathbf{B}_t^{*'} + \boldsymbol{\Psi}_t^*. \end{aligned}$$

The vector $\boldsymbol{\Omega}$ subsumes all model parameters and $\mathbf{Y}_{t-1}$ contains all observations from time 1 up to time $t-1$. The function $f(\mathbf{y}_t | \mathbf{Y}_{t-1}, X_{t-1}; \boldsymbol{\Omega})$ is the distribution function of a multivariate normal with mean vector $\boldsymbol{\tau} + \boldsymbol{\Lambda}\boldsymbol{\eta}_{t|t-1}$ and covariance matrix $\boldsymbol{\Lambda}\mathbf{P}_{t|t-1}\boldsymbol{\Lambda}' + \boldsymbol{\Theta}$, which are determined by the model parameters and the process predictions $\boldsymbol{\eta}_{t-1|t-1}$, $\boldsymbol{\eta}_{t|t-1}$, $\mathbf{P}_{t-1|t-1}$, and $\mathbf{P}_{t|t-1}$, which are available from the Kalman filter recursions (for more details see Hamilton, 1994, pp. 377–381; Harvey, 1989, pp. 104–113). Conditioning on $\mathbf{Y}_{t-1}$ eliminates the potential temporal dependencies in the data, that is, it renders the individual likelihoods independent of each other and allows their multiplication to get the joint likelihood.

Conditioning on the moderator X addresses the problem of nonnormality due to nonstationarity or intra-individual heterogeneity introduced by X (see the mixture distributions displayed in Figure 2). As X is assumed to be observed and can directly be conditioned upon, the time-invariant Kalman filter can be employed. That is, although the extended model in Equations (3) and (4) formalizes a nonstationary process and implies nonnormally distributed data, conditioning on the moderator renders the process stationary and implies normally distributed data. In comparison, models with stochastically time-varying parameters pose more complex estimation problems in that estimation algorithms must be adapted to account for the uncertainty in model parameters (e.g., Kim & Nelson, 1999, pp. 99–102).

The Kalman filter recursions are started conditionally on $\boldsymbol{\eta}_{0|0}$, and $\mathbf{P}_{0|0}$. These two quantities are unknown as they concern the latent process state at $t = 0$, a time at which no data are available. They may either be fixed according to the moments of an (un)informative distribution or may, under stationarity, be equated to the earlier introduced long-run mean vector $\boldsymbol{\nu}$ and covariance matrix $\mathbf{P}$ of the latent process variables and may thus be set up as a function of the model parameters (cf. Harvey, 1989, p. 121). Note, however, that this, in our case, concerns the marginal distribution of the process variables over X. We derive the marginal distribution of the process variables for the case of a dichotomous, dummy-coded moderator in Appendix A. The moderator at $t = 0$ is assumed to be observed as for the other time points.

We estimate the model in the free and open source software OpenMx 2.2.4 (Boker et al., 2015; Neale et al., 2015) under R 3.2.1 (R Core Team, 2014). The function *mxExpectationStateSpace* automates the application of the Kalman filter for model estimation by deriving the Kalman filter–based model expectations from the user-provided parameter matrices (Hunter, 2014b). As the model expectations need to be derived conditional on the moderator, we capitalize on the software's flexibility in model specification and incorporate the moderator as a definition variable linking it to all model parameters via the specification of appropriate constraints. An annotated code example is provided in online supplemental material.

Simulation study

Purpose and study design

The main purpose of this simulation study is to investigate the performance of the model given relatively short time series (i.e., $T = 100$, $T = 150$, $T = 200$). For data generation, we use the bivariate process model presented in Equation (5) and include a dichotomous, dummy-coded moderator. We use two sets of parameter values. The parameters of the first set are

$$\begin{bmatrix}\alpha_1\\ \alpha_2\end{bmatrix}^*_t = \begin{bmatrix}2\\ 3\end{bmatrix}^{(0)} + X_{t-1}\begin{bmatrix}2\\ -1.5\end{bmatrix}^{(X)}$$
$$\begin{bmatrix}\beta_{11} & \beta_{12}\\ \beta_{21} & \beta_{22}\end{bmatrix}^*_t = \begin{bmatrix}.3 & -.15\\ -.15 & .3\end{bmatrix}^{(0)} + X_{t-1}\begin{bmatrix}.3 & -.1\\ -.1 & .3\end{bmatrix}^{(X)}$$
$$\begin{bmatrix}\psi_{11} & \psi_{12}\\ \psi_{21} & \psi_{22}\end{bmatrix}^*_t = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}^{(0)} + X_{t-1}\begin{bmatrix}.5 & 0\\ 0 & .5\end{bmatrix}^{(X)} \tag{9}$$

under the change parameterization (cf. Equation [6]), and

$$\begin{bmatrix}\alpha_1\\ \alpha_2\end{bmatrix}^*_t = (1 - X_{t-1})\begin{bmatrix}2\\ 3\end{bmatrix}^{(X=0)} + X_{t-1}\begin{bmatrix}4\\ 1.5\end{bmatrix}^{(X=1)}$$
$$\begin{bmatrix}\beta_{11} & \beta_{12}\\ \beta_{21} & \beta_{22}\end{bmatrix}^*_t = (1 - X_{t-1})\begin{bmatrix}.3 & -.15\\ -.15 & .3\end{bmatrix}^{(X=0)} + X_{t-1}\begin{bmatrix}.6 & -.25\\ -.25 & .6\end{bmatrix}^{(X=1)}$$
$$\begin{bmatrix}\psi_{11} & \psi_{12}\\ \psi_{21} & \psi_{22}\end{bmatrix}^*_t = (1 - X_{t-1})\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}^{(X=0)} + X_{t-1}\begin{bmatrix}1.5 & 0\\ 0 & 1.5\end{bmatrix}^{(X=1)} \tag{10}$$

under the switch parameterization (cf. Equation [7]). Here, it can be seen that we chose values such that the two conditional processes ($(X = 1)$ and $(X = 0)$) each have a mirror symmetric $\mathbf{B}$ matrix containing the AR and cross-regressive (CR) effects. Although this symmetry is neither necessary nor realistic for most practical situations, it is useful here as it enables an easier detection of finite sample biases in the following, which partly depend on the size of the estimated effects.

We additionally look at a second set of parameters within this simulation, which are

$$\begin{bmatrix}\alpha_1\\ \alpha_2\end{bmatrix}^*_t = (1 - X_{t-1})\begin{bmatrix}0\\ 5\end{bmatrix}^{(X=0)} + X_{t-1}\begin{bmatrix}3\\ 2\end{bmatrix}^{(X=1)}$$
$$\begin{bmatrix}\beta_{11} & \beta_{12}\\ \beta_{21} & \beta_{22}\end{bmatrix}^*_t = (1 - X_{t-1})\begin{bmatrix}.5 & .4\\ -.3 & .7\end{bmatrix}^{(X=0)} + X_{t-1}\begin{bmatrix}.3 & .5\\ 0 & .8\end{bmatrix}^{(X=1)}$$
$$\begin{bmatrix}\psi_{11} & \psi_{12}\\ \psi_{21} & \psi_{22}\end{bmatrix}^*_t = (1 - X_{t-1})\begin{bmatrix}1 & 0\\ 0 & 1.2\end{bmatrix}^{(X=0)} + X_{t-1}\begin{bmatrix}1.5 & 0\\ 0 & 1\end{bmatrix}^{(X=1)} \tag{11}$$

under the switch parameterization. These values are taken from a regime-switching model featuring in a recent paper (Hamaker & Grasman, 2012) that investigates model performance in small samples with missing data. We reuse the parameter values here to enable a performance comparison between the regime-switching model and our approach. The comparison grounds on the fact that the two models imply comparable parameter and outcome processes. The comparison is limited in that the models themselves approach heterogeneity in very different ways. As discussed earlier, the regime-switching model freely estimates the process of switching between dynamic regimes from the data and thus accommodates unobserved intra-individual heterogeneity. Our approach assumes that switching times between dynamic regimes are known, accommodating observed intra-individual heterogeneity.

We combine the process model variants displayed in Equations (9)–(11) with a minimal version of the measurement model, in which we specify a one-to-one relationship between manifest and latent variables by fixing $\boldsymbol{\Lambda}$ to an identity matrix and the measurement residual variances to 0.

The frequency distribution of the moderator is varied such that it is present (i.e., taking on the value 1) in 20, 30, ... , 80% of the occasions. The cutoffs of 20% and 80% imply that model estimation is supported by at least 20 observations per moderator state, which prevents us from larger numbers of nonconverging solutions. The first major part of the simulation is based on moderator occurrences that are random in time. In the final subsection *Temporal Patterns in the Moderator*, we complement this by looking at time-structured moderator changes and how this affects performance of the model.

We present the results in terms of biases of point estimates and coverage rates of the 95% confidence interval (CI) estimates. Relative parameter bias in % of the generating parameter value is reported if the generating parameter is different from 0, absolute bias otherwise. We rely on profile likelihood-based CI estimates as implemented in *OpenMx*. Unlike Wald-type CIs that only approximate the shape of the likelihood function around the maximum, likelihood-based interval estimates use the exact shape of the likelihood function, and are preferable with small sample sizes (Pek & Wu, 2015). If parameter correlations are reported, they are drawn from the averaged transformed Hessian matrix but can also be recovered empirically across replications.

Performance in small samples

Tables 1 and 2 show parameter biases and CI coverage rates per moderator frequency condition for the first parameter set under the switch parameterization (Equation [10]).

In general, we observe problems of finite sample bias that are more pronounced the shorter the overall time series. These biases are only specific to our model in the sense that, across moderator frequency conditions, we in principle compare models fitted to time series of varying length. Overall, bias is larger the shorter the conditional time series. That is, parameters with superscript $(X = 0)$ show increasing bias with increasing moderator frequency, whereas parameters with superscript $(X = 1)$ show decreasing bias with increasing moderator frequency. These trends are most obvious for the residual variances, but are also present for the AR and CR parameters (although this is sometimes obscured by compensating effects between the parameters), and for the intercepts. Here, the patterns of bias can best be seen for effects of bigger size, hence, the parameters with superscript $(X = 1)$. By plugging the mean estimates across replications into the model-implied expressions of the mean vector and covariance matrix (Appendix A), we learn that, per conditional process, the model-implied means are in relative consistence with the generating means. Thus, the biases in intercepts likely compensate for biases in the AR and CR effects as these highly correlated parameter sets both contribute to the means. Indeed, fixing all AR and CR effects to their true values removes the biases in intercepts. The model-implied variances and co-variances of the conditional processes, on the contrary, are too low in comparison to the generating ones. Underestimation of the variance given an unknown mean is a well-known problem in ML estimation (Bishop, 2006). In this model, the conditional variances and co-variances are a function of the AR and CR coefficients and the residual variances. Accordingly, fixing the intercepts, the AR, and CR effect (i.e., rendering the means known) removes the biases in residual variances. However, fixing the intercepts only (i.e., reducing the degrees of freedom in the estimation of the means) reduces bias in the AR and CR effects, but does not remove it. In fact, it has been shown that estimates of AR (and possibly CR) effects are biased toward 0 even with a known mean (Cheang & Reinsel, 2000; Marriott & Pope, 1954).

In sum, we observe three instantiations of finite sample bias. The first concerns underestimation of the variance given an unknown mean, the second concerns bias toward 0 in the AR (and CR) effects, and the third concerns compensatory bias in the intercepts. Note that as displayed in Table 3, the very same biases show up in a different pattern if we directly estimate change in model parameters via the change parameterization. The effects are again most pronounced for the residual variances, for which change is underestimated for low moderator frequencies and overestimated for high moderator frequencies. This is due to the way the above biases add up under this parameterization. Revisiting Table 2, we recall that we underestimate the baseline parameter much less than the alternative state parameter for low moderator frequencies under the switch parameterization. Because the alternative state parameter is the higher one in the generating model, we see an underestimation of the difference between the two processes. For low moderator frequencies, on the contrary, we underestimate the lower baseline

Table 1. Model performance (switch-parameterization) in terms of relative biases of point estimates in %.

Freq X	$\alpha_1^{(X=0)}$	$\alpha_2^{(X=0)}$	$\beta_{11}^{(X=0)}$	$\beta_{21}^{(X=0)}$	$\beta_{12}^{(X=0)}$	$\beta_{22}^{(X=0)}$	$\psi_{11}^{(X=0)}$	$\psi_{21}^{(X=0)}$	$\psi_{22}^{(X=0)}$	$\alpha_1^{(X=1)}$	$\alpha_2^{(X=1)}$	$\beta_{11}^{(X=1)}$	$\beta_{21}^{(X=1)}$	$\beta_{12}^{(X=1)}$	$\beta_{22}^{(X=1)}$	$\psi_{11}^{(X=1)}$	$\psi_{21}^{(X=1)}$	$\psi_{22}^{(X=1)}$
									$T = 200$									
.2	1.00	1.21	− 1.38	1.47	1.17	− 2.86	− 1.49	0.00	− 1.71	0.91	2.52	− 1.87	− 0.80	0.81	− 2.12	− 6.90	− 0.01	− 7.55
.3	1.13	0.78	− 1.21	0.02	1.59	− 2.64	− 1.55	0.00	− 1.99	0.70	4.04	− 1.11	1.24	1.31	− 2.60	− 4.73	0.00	− 5.31
.4	0.79	0.99	− 0.61	1.68	1.34	− 2.59	− 1.68	0.00	− 2.24	0.77	4.07	− 1.14	1.40	0.90	− 2.74	− 3.81	0.00	− 3.71
.5	0.44	0.74	− 0.09	1.21	0.78	− 1.89	− 1.94	0.00	− 2.56	0.86	3.95	− 0.99	1.62	1.60	− 2.63	− 2.87	0.00	− 2.65
.6	0.98	0.92	− 0.40	1.71	3.05	− 2.37	− 2.60	0.00	− 3.63	0.92	3.15	− 1.02	1.71	1.19	− 2.22	− 2.32	0.00	− 2.46
.7	0.30	0.73	0.12	1.91	1.17	− 1.91	− 3.77	0.00	− 4.29	1.01	3.55	− 1.04	2.25	1.08	− 2.38	− 2.08	0.00	− 2.05
.8	− 0.65	0.76	0.73	2.00	− 0.38	− 2.07	− 6.20	0.00	− 7.15	1.36	2.64	− 1.17	2.00	1.38	− 2.12	− 1.86	0.00	− 1.98
									$T = 150$									
.2	1.18	1.76	− 1.89	1.72	0.81	− 4.47	− 1.66	0.00	− 2.33	0.82	6.93	− 1.62	1.83	1.49	− 4.17	− 9.77	0.00	− 9.99
.3	1.22	1.59	− 1.49	1.73	1.16	− 3.91	− 1.92	0.00	− 2.94	0.49	5.65	− 1.11	1.02	− 0.09	− 4.12	− 6.44	0.00	− 7.33
.4	1.07	0.92	− 0.55	1.19	2.27	− 2.64	− 2.50	0.00	− 2.94	1.13	4.84	− 1.52	1.39	1.87	− 3.63	− 5.08	0.00	− 5.35
.5	0.56	1.01	− 0.30	1.63	1.29	− 2.98	− 3.07	0.00	− 3.75	0.73	4.54	− 1.00	1.95	0.75	− 3.13	− 3.61	0.00	− 4.07
.6	0.35	1.23	0.05	2.34	0.50	− 2.98	− 3.67	0.00	− 4.68	1.12	4.80	− 1.24	2.68	1.28	− 3.29	− 3.19	0.00	− 3.62
.7	0.02	0.79	0.28	1.89	1.44	− 1.60	− 5.29	0.00	− 6.12	1.14	5.05	− 1.22	3.40	0.80	− 3.36	− 2.55	0.00	− 3.04
.8	1.21	1.14	− 0.52	3.12	4.99	− 2.54	− 7.74	0.00	− 9.66	1.45	3.51	− 1.29	2.62	0.59	− 2.84	− 2.56	0.00	− 2.65
									$T = 100$									
.2	0.98	2.42	− 2.52	1.52	− 1.02	− 6.54	− 2.96	0.00	− 3.68	0.95	9.37	− 2.09	2.68	1.64	− 5.80	− 13.72	0.01	− 15.12
.3	1.34	2.72	− 1.83	3.97	1.28	− 6.55	− 3.28	0.00	− 4.40	1.58	8.95	− 2.24	2.33	3.41	− 5.98	− 9.55	0.00	− 9.95
.4	1.62	2.10	− 1.50	3.27	2.08	− 5.47	− 3.14	0.00	− 4.74	1.70	8.71	− 2.46	3.30	2.89	− 5.90	− 7.34	0.00	− 7.36
.5	1.32	1.82	− 0.61	2.79	3.30	− 4.70	− 4.52	0.00	− 6.08	1.56	7.06	− 1.95	3.00	2.56	− 5.03	− 5.56	0.01	− 6.14
.6	1.09	1.97	− 0.65	4.07	1.95	− 4.69	− 4.95	0.00	− 6.54	1.53	6.74	− 1.86	3.53	1.87	− 4.71	− 4.67	0.00	− 5.19
.7	0.98	1.64	− 0.13	3.93	2.95	− 3.95	− 7.39	0.00	− 9.31	2.01	6.20	− 2.01	4.05	2.15	− 4.67	− 4.17	0.00	− 4.39
.8	0.31	1.42	− 0.07	3.63	4.72	− 4.14	− 12.04	0.00	− 14.39	2.55	5.62	− 2.25	4.32	2.57	− 4.63	− 3.36	0.00	− 3.68

Note. The number of replications is 6,000 per line, Freq X is the frequency of X being "on" in relative numbers of occasions, Equation (10) contains the generating model. If generating parameter values are zero, absolute instead of relative biases are displayed.

Table 2. Model performance (switch-parameterization) in terms of coverage rates of 95 % interval estimates in %.

Freq X	$\alpha_1^{(X=0)}$	$\alpha_2^{(X=0)}$	$\beta_{11}^{(X=0)}$	$\beta_{21}^{(X=0)}$	$\beta_{12}^{(X=0)}$	$\beta_{22}^{(X=0)}$	$\psi_{11}^{(X=0)}$	$\psi_{21}^{(X=0)}$	$\psi_{22}^{(X=0)}$	$\alpha_1^{(X=1)}$	$\alpha_2^{(X=1)}$	$\beta_{11}^{(X=1)}$	$\beta_{21}^{(X=1)}$	$\beta_{12}^{(X=1)}$	$\beta_{22}^{(X=1)}$	$\psi_{11}^{(X=1)}$	$\psi_{21}^{(X=1)}$	$\psi_{22}^{(X=1)}$
									$T = 200$									
.2	94.92	94.97	94.25	94.93	94.75	94.25	94.12	94.70	94.17	93.53	94.13	93.27	93.77	93.52	93.52	92.50	93.60	92.77
.3	94.73	95.03	94.43	95.27	95.03	95.07	94.40	94.52	93.87	94.73	94.98	94.57	94.73	94.55	94.53	93.47	94.42	92.72
.4	94.48	94.55	94.45	94.65	94.38	94.60	94.23	94.47	94.28	94.50	94.50	94.40	94.57	94.92	94.78	93.77	94.55	94.27
.5	94.65	94.77	94.80	94.72	94.63	94.78	94.05	94.45	94.10	94.38	94.83	94.50	94.55	94.83	94.57	94.10	94.47	94.40
.6	94.35	94.50	93.98	94.13	94.35	94.15	94.00	94.38	94.07	94.55	94.70	94.33	94.78	94.57	94.37	94.57	95.13	94.77
.7	94.25	94.65	94.40	94.63	94.35	94.38	93.72	93.67	93.45	94.90	95.12	94.87	95.10	94.40	95.22	94.67	94.98	94.63
.8	93.95	94.10	93.63	94.25	93.80	94.13	92.48	93.57	93.00	94.62	95.00	94.80	94.58	94.80	94.68	94.68	94.32	94.87
									$T = 150$									
.2	94.72	95.02	94.87	94.55	94.90	94.52	94.30	94.40	94.30	93.33	93.05	93.77	93.67	93.48	93.60	92.42	93.18	91.58
.3	95.12	94.23	94.80	94.77	95.07	94.43	94.20	94.55	93.92	94.28	93.98	93.65	93.60	94.18	93.48	93.50	94.02	92.85
.4	94.58	94.47	94.62	94.67	94.83	93.88	94.43	93.77	94.25	94.02	94.68	94.08	94.45	94.38	94.33	94.07	93.93	93.42
.5	94.27	94.93	94.22	94.22	94.15	95.02	94.03	93.55	94.07	94.65	94.20	93.90	94.42	95.05	94.57	93.90	93.85	94.17
.6	94.30	94.13	94.38	93.92	94.75	94.27	93.15	94.37	94.30	94.83	94.28	94.42	94.63	94.53	94.13	94.55	94.40	93.20
.7	93.62	94.60	93.17	94.30	93.62	94.03	93.00	93.62	93.25	94.22	94.62	94.15	94.48	94.48	94.75	94.97	94.80	93.53
.8	93.35	93.88	93.28	93.80	93.73	93.58	91.63	92.70	90.95	94.63	94.10	94.87	94.30	94.58	94.68	94.43	94.95	94.27
									$T = 100$									
.2	95.05	94.23	94.85	94.13	94.73	94.55	93.87	94.33	94.23	92.58	92.65	92.43	92.27	92.85	92.78	82.13	92.12	82.02
.3	94.30	93.85	94.32	93.70	94.42	93.82	93.60	93.82	93.68	93.33	93.52	93.23	92.98	93.32	93.30	92.17	93.63	92.03
.4	94.62	93.85	94.67	93.97	94.10	93.93	93.97	94.38	93.82	94.23	93.72	93.60	93.58	94.20	94.15	92.80	93.72	93.13
.5	94.05	94.23	93.67	93.85	94.07	94.37	93.70	93.25	92.85	94.65	94.30	94.48	94.32	94.15	94.00	93.48	94.40	92.75
.6	93.43	93.92	93.52	94.18	94.32	94.40	93.47	93.23	92.33	94.22	94.18	94.25	93.97	94.38	94.27	93.62	94.43	93.95
.7	93.00	93.53	93.20	93.25	93.20	93.60	92.05	92.95	91.92	94.87	94.53	94.33	94.43	94.72	94.35	94.03	94.12	93.27
.8	92.53	92.80	92.28	92.68	92.98	93.20	89.52	91.02	90.52	93.95	94.53	94.07	94.57	94.73	93.68	94.12	94.33	94.10

Note. The number of replications is 6,000 per line, Freq X is the frequency of X being "on" in relative numbers of occasions, Equation (10) contains the generating model.

Table 3. Model performance (change-parameterization) in terms of relative biases of point estimates and coverage rates of 95% interval estimates in %.

	Relative biases									Coverage rates								
Freq X	$\alpha_1^{(X)}$	$\alpha_2^{(X)}$	$\beta_{11}^{(X)}$	$\beta_{21}^{(X)}$	$\beta_{12}^{(X)}$	$\beta_{22}^{(X)}$	$\psi_{11}^{(X)}$	$\psi_{21}^{(X)}$	$\psi_{22}^{(X)}$	$\alpha_1^{(X)}$	$\alpha_2^{(X)}$	$\beta_{11}^{(X)}$	$\beta_{21}^{(X)}$	$\beta_{12}^{(X)}$	$\beta_{22}^{(X)}$	$\psi_{11}^{(X)}$	$\psi_{21}^{(X)}$	$\psi_{22}^{(X)}$
									$T = 200$									
.2	0.72	−3.07	−1.35	1.43	2.78	−4.05	−18.07	0.00	−19.09	93.80	94.02	94.00	94.13	94.00	94.32	93.82	93.87	92.75
.3	0.06	−1.87	−1.01	1.19	−0.89	−2.49	−11.22	0.01	−11.92	94.78	94.53	94.43	94.25	94.23	95.00	93.95	94.28	93.62
.4	0.72	−1.41	−1.65	−0.95	1.99	−2.70	−6.26	0.01	−5.23	94.47	94.57	94.40	94.63	94.42	94.37	94.87	94.22	94.33
.5	1.33	−0.87	−2.21	−0.97	1.77	−2.40	−3.83	0.00	−4.32	94.25	95.15	94.05	95.05	94.67	94.65	94.87	94.80	94.75
.6	0.70	−1.68	−1.39	1.16	−2.94	−3.04	−1.08	0.00	−0.61	94.42	94.62	94.88	94.53	94.53	94.75	95.00	94.98	94.85
.7	2.00	−1.44	−2.33	2.37	0.59	−2.22	1.87	0.00	3.90	94.40	94.52	94.30	94.20	94.50	94.82	94.68	94.52	94.25
.8	3.19	−0.98	−2.98	2.23	2.39	−2.36	4.97	0.00	9.21	93.93	94.02	93.90	94.17	94.17	94.23	93.88	94.87	93.82
									$T = 150$									
.2	0.95	−2.63	−1.13	1.71	5.07	−2.88	−24.13	0.00	−24.82	93.52	93.98	93.78	94.08	92.93	94.27	92.33	93.42	92.80
.3	1.53	−1.98	−2.58	−2.56	3.73	−4.06	−17.11	0.00	−14.55	93.80	94.85	94.07	94.23	93.95	94.55	93.67	94.53	93.68
.4	0.34	−1.96	−1.50	0.59	−1.98	−3.01	−8.82	0.00	−8.58	94.33	94.60	94.03	94.52	93.95	94.77	93.97	94.52	94.43
.5	1.05	−2.55	−2.58	1.94	−0.48	−4.22	−6.11	0.00	−5.13	94.27	94.00	94.23	94.62	94.32	94.48	93.87	94.32	94.13
.6	0.94	−2.43	−1.66	3.30	−2.04	−3.65	−1.86	0.00	−1.00	94.33	94.47	94.13	94.38	94.35	94.25	94.60	94.55	94.37
.7	2.87	−2.32	−3.34	2.82	4.10	−4.17	3.28	0.00	3.08	94.63	94.38	94.22	94.10	94.55	94.00	93.68	93.70	94.25
.8	2.58	−1.28	−2.46	2.12	−4.13	−3.07	7.61	0.00	11.29	93.82	93.98	93.83	94.12	94.42	93.97	94.02	93.80	93.68
									$T = 100$									
.2	−0.79	−1.91	−0.56	0.68	−1.66	−3.43	−36.33	0.00	−39.38	93.12	92.75	92.80	92.75	92.88	93.05	83.32	91.92	83.25
.3	−0.33	−5.04	−1.74	1.74	−3.40	−7.54	−22.29	0.01	−22.73	93.38	93.33	93.32	93.68	93.83	93.15	92.47	93.60	92.17
.4	0.73	−4.04	−2.33	2.64	2.43	−5.95	−14.49	0.01	−15.23	94.27	94.37	94.25	93.93	94.27	94.55	93.90	93.93	93.62
.5	1.95	−3.89	−3.69	4.32	2.29	−5.36	−9.07	0.01	−6.42	94.10	94.42	93.65	94.12	94.15	94.53	93.47	94.08	94.25
.6	2.61	−4.09	−3.70	4.60	1.19	−5.89	−3.05	0.01	−0.93	93.72	94.25	93.97	94.42	93.77	94.62	93.90	94.37	94.88
.7	4.70	−0.86	−5.25	0.00	3.42	−4.58	2.53	0.00	6.42	93.85	94.05	93.60	93.58	93.97	93.97	94.20	94.17	94.28
.8	3.71	−0.85	−3.72	1.95	−3.25	−3.90	13.41	0.00	15.74	92.98	93.35	92.90	93.22	92.87	92.98	90.50	92.70	92.30

Note. The number of replications is 6,000 per line, Freq X is the frequency of X being "on" in relative numbers of occasions, Equation (9) contains the generating model. If generating parameter values are zero, absolute instead of relative biases are displayed.

parameter more than the higher alternative state parameter, resulting in an overestimation of the difference.

Our recommendation is to rely on CI estimates rather than point estimates in small samples. Although they mirror the pattern of biases described, the coverage rates of the profile likelihood-based CIs seem acceptable even for $T = 100$—at least if the frequency distribution of the moderator is relatively symmetric (i.e., with a mean of around .5).

Comparison to a regime-switching model

We now turn to the comparison between our and Hamaker and Grasman's model (2012). The properties of the Markov process underlying the regime switching in the latter model imply a long-run frequency of the moderator being "on" in around 60% of the occasions (verified by simulation). In Table 4, we therefore present results for data generated according to this moderator frequency condition without a temporal pattern in the moderator. The general conditions of this subsimulation are thus not qualitatively different from the conditions of the earlier simulation; we just look at a specific set of parameter values. In their 2012 paper, the authors do not report bias, but coverage rates that are substantially lower for $T = 100$ and get comparable with our results only at $T = 500$ (Hamaker & Grasman, 2012, p. 412), which suggests that the regime-switching model is more demanding in terms of required data. This is not surprising, given that the regime-switching model is more flexible in that it allows estimating switching between dynamic regimes from the data, whereas fixed moderated TSA incorporates switching times as known. Note that the regime-switching model is thus the more appropriate approach, if sources of intra-individual heterogeneity are unknown. However, if information about sources of heterogeneity is available, this comparison shows that they should (and how they could) be used with shorter time series.

Temporal patterns in the moderator

To explore whether temporal patterns in moderator occurrence have an effect on model performance, we compare two extreme temporal structures. A very slowly changing moderator was created by switching the moderator after the first half of the occasions, so that it is "on" during the first half, and "off" during the second half, or vice versa. A very fast changing moderator was created by switching the moderator at every occasion. The overall frequency of the moderator remains .5 for both the slow and the fast conditions. Additionally, we modified the generating model from Equation (10) such that there is no difference between the two conditional processes (i.e., the values superscripted by $(X = 0)$ also hold if the moderator is on). The motivation for doing this is similar to the one that let us simulate data from a symmetric $\mathbf{B}$ matrix, namely reducing complexity. Specifically, having conditional effects of identical size avoids differences in bias due to differences in size of true effects between the conditional processes. Note that this does not pose any identification problem to the model, as the amount of data available to estimate each conditional process remains the same. We should simply recover that the moderator has no effect and that there are no parameter differences between the two processes. A comparison of how the model in switch parameterization performs under the different temporal conditions, including the earlier setup in which moderator changes are just random over time, is presented in the upper part of Table 5.

Interestingly, the biases in the AR effects (and hence also the compensatory biases in the intercepts) are largest when the moderator changes slowly, and smallest when the moderator changes fast. Remember that AR (and CR) effect estimates are subject to finite sample bias in general, hence, also in time-invariant models (Cheang & Reinsel, 2000; Marriott & Pope, 1954). The lower part of Table 5 therefore shows biases in a time-invariant VAR model fitted to a time series of length $T = 50$. We used the parameter values superscripted by $(X = 0)$ from Equation (10) to generate the time-invariant data. In comparing the upper and the lower parts of Table 5, it seems that the biases occurring in the time-invariant TSA solution to a time series of length 50 represent the limiting case for the biases occurring in a fixed moderated TSA solution to a time series of unconditional length 100 and conditional length 50.

We conjecture that the beneficial effect in the fast condition arises in the following way. As mentioned earlier, conditioning on the moderator in fixed moderated TSA implies looking at time series that are subsets of the original time series and co-vary in length with moderator frequency. However, the situation is not exactly the same as looking at these time series subsets independently. Due to the recursive nature of the Kalman filter, we always make use of the *entire* time series, even when estimating the model parameters conditional on a moderator. Moreover, we make use of the entire series *optimally*, when the two conditional time series are most intermixed. The conditional time series are most intermixed if the moderator changes quickly, that is, in the extreme, it switches between being "on" and being "off" continuously. The conditional time series are least intermixed, if the moderator changes slowly. The estimation of each conditional parameter set relies on the process predictions available from the Kalman filter for the entire series. In the intermixed case, any process state prediction generated under

Table 4. Model performance (switch-parameterization) in terms of relative biases of point estimates and coverage rates of 95 % interval estimates in %.

Freq X	$\alpha_1^{(X=0)}$	$\alpha_2^{(X=0)}$	$\beta_{11}^{(X=0)}$	$\beta_{21}^{(X=0)}$	$\beta_{12}^{(X=0)}$	$\beta_{22}^{(X=0)}$	$\psi_{11}^{(X=0)}$	$\psi_{21}^{(X=0)}$	$\psi_{22}^{(X=0)}$	$\alpha_1^{(X=1)}$	$\alpha_2^{(X=1)}$	$\beta_{11}^{(X=1)}$	$\beta_{21}^{(X=1)}$	$\beta_{12}^{(X=1)}$	$\beta_{22}^{(X=1)}$	$\psi_{11}^{(X=1)}$	$\psi_{21}^{(X=1)}$	$\psi_{22}^{(X=1)}$
									Relative biases									
.6	0.00	3.93	− 1.71	0.00	2.90	− 3.27	− 7.19	0.00	− 7.46	1.04	11.97	− 4.87	0.00	2.34	− 2.97	− 4.91	0.00	− 4.80
									Coverage rates									
.6	94.15	93.80	93.92	93.87	94.52	93.72	92.33	93.50	92.58	94.00	93.82	94.43	94.13	94.40	93.97	93.53	94.35	93.68

Note. The number of replications is 6,000 per line, Freq X is the frequency of X being "on" in relative numbers of occasions, Equation (11) contains the generating model. If generating parameter values are zero, absolute instead of relative biases are displayed.

Table 5. Model performance (switch-parameterization) for different moderator change patterns and for a time-invariant reference model in terms of relative biases of point estimates in %

Change X	$\alpha_1^{(X=0)}$	$\alpha_2^{(X=0)}$	$\beta_{11}^{(X=0)}$	$\beta_{21}^{(X=0)}$	$\beta_{12}^{(X=0)}$	$\beta_{22}^{(X=0)}$	$\psi_{11}^{(X=0)}$	$\psi_{21}^{(X=0)}$	$\psi_{22}^{(X=0)}$	$\alpha_1^{(X=1)}$	$\alpha_2^{(X=1)}$	$\beta_{11}^{(X=1)}$	$\beta_{21}^{(X=1)}$	$\beta_{12}^{(X=1)}$	$\beta_{22}^{(X=1)}$	$\psi_{11}^{(X=1)}$	$\psi_{21}^{(X=1)}$	$\psi_{22}^{(X=1)}$
random	1.32	2.36	− 6.65	− 1.96	− 2.48	− 6.60	− 6.24	0.00	− 6.20	1.74	2.62	− 7.65	− 2.27	− 1.73	− 7.36	− 7.01	0.00	− 6.17
slow	4.08	5.42	− 14.68	− 2.85	− 1.92	− 14.65	− 5.97	0.01	− 5.96	3.04	4.98	− 15.23	− 4.10	− 5.29	− 14.10	− 5.94	0.00	− 5.94
fast	− 0.49	0.80	− 0.51	− 4.11	− 2.48	− 3.03	− 6.35	0.00	− 6.42	0.23	0.14	− 2.35	− 4.87	− 1.85	− 1.81	− 6.31	0.00	− 5.81
No X	α_1	α_2	β_{11}	β_{21}	β_{12}	β_{22}	ψ_{11}	ψ_{21}	ψ_{22}	—	—	—	—	—	—	—	—	—
$T = 50$	3.99	5.23	− 15.29	− 2.89	− 2.17	− 14.41	− 6.14	0.00	− 6.59	—	—	—	—	—	—	—	—	—

Note. The number of replications is 6,000 per line for the time-varying conditions including X, and 10,000 for the time-invariant condition excluding X. In the time-varying conditions, $T = 100$ and X is "on" in 50% of the occasions, slow change in X means X is only "on" during only the first or only the second half of the occasions, fast change means X alternates between occasions, and random change means no temporal structure in X. Under the time-invariant condition, we fitted a time-invariant VAR model to stationary data generated from this model. If generating parameter values are zero, absolute instead of relative biases are displayed.

one moderator value is still highly informed by earlier process predictions generated under that very same moderator value. This may then improve estimation accuracy for the AR (and CR) effects.

Application to data on daily emotional experiences from the COGITO study

Study design, participants, and measures of the COGITO study

The COGITO Study is a comprehensive longitudinal study conducted at the Max Planck Institute for Human Development, Berlin, during the years 2006 to 2007 and 2009 to 2010 (Cognition Ergodicity Study of the MPI for Human Development; Schmiedek et al., 2010). The heart of the study is an intensive longitudinal phase, which tracks cognitive and affective development of 101 younger and 103 older adults over 101 measurement occasions and 158 days, on average. To be measured, participants came to the laboratory every 1.5 days on average, so measurement occasions are irregularly spaced within persons. The daily laboratory sessions were mainly devoted to a broad range of cognitive tasks, but participants also reported affective experiences, self-regulation, motivation, stress experiences, and daily events.

The present analyses are based on the younger participants ($N = 101$; 51.5% women; age: 20–31, $M = 25.6$; daily sessions: 87–109, $M = 101$; days in study: 116–372, $M = 165$) and their reports on NA as assessed by the Positive and Negative Affect Schedule (Watson, Clark, & Tellegen, 1988), on PS as assessed by the Perceived Stress Scale (Cohen, Kamarck, & Mermelstein, 1983), and daily events assessed within seven different event domains (e.g., work, health, leisure, finances). For each domain, participants were asked to report whether they had experienced an event since the last session, whether an event was ongoing or whether they were expecting to experience an event later that day (e.g., "Has something happened at work or during professional training or studying?"; cf. Brose, Scheibe, & Schmiedek, 2013). They were then asked to rate the event in terms of valence (from "negative," to "neutral," to "positive") and personal relevance (from "doesn't move me" to "moves me a lot").

Model building

As an aspect of daily emotional life, we model the joint dynamics of NA and PS. For NA, we use the average score across the five most variable items (i.e., "distressed," "upset," "irritable," "nervous," and "jittery"), which were answered on an 8-point scale. For PS, we use the average score across two items related to perceived control (e.g., "To what extent do you today feel able to control the things in your life?"), also answered on an 8-point scale. Note that the scale is reversed, so lower values mean higher PS. By applying the time-invariant bivariate VAR model (Equation [1]; Model 1), we assume that an individual's current affect (corresponding to η_1) and stress levels (corresponding to η_2) depend on earlier affect and stress levels, but also on new, independent input at that particular occasion, such as situational influences or internal events unrelated to earlier affect or stress (i.e., the residuals). As before, we fix the measurement model so that there is a one-to-one relationship between observed and latent variables.

Extending the time-invariant baseline model, we allow all model parameters to vary intra-individually, conditional on daily events. We confine the analysis to events that were reported to have happened before the session or were still ongoing. We then collapse across event domains and look at whether events were reported as negative and as being somewhat relevant (at least "moves me a little"). The information on these negative events is incorporated as a dichotomous and a continuous moderator.

For the dichotomous moderator case (Model 2), we set up a dummy-coded variable, which indicates whether at least one relevant negative event occurred before a given occasion on the same day. Included as a moderator (cf. Equation [6]), this implies that parameters may be different on occasions preceded by relevant negative events as compared to occasions not preceded by those events (i.e., occasions preceded by positive, neutral, or irrelevant negative events). We set up the model according to the change parameterization.

For the continuous moderator case (Model 3), we calculate a linear weighted moving average of relevant negative events as an index of "stressor pile-up" (Schilling & Diehl, 2014) as

$$Z_t = \sum_{k=1}^{K} \left(X_{t-(k-1)} \left(K - (k-1) \right) \right) \left(\sum_{k=1}^{K} \left(K - (k-1) \right) \right)^{-1} \tag{12}$$

where Z_t is the continuous moderator, X_t is the dichotomous moderator, and K is the size of the window over which we average. This index reflects the accumulation of relevant negative events over K consecutive occasions up to and including the current one. In being accumulated, events get less weight the earlier they occurred. Distance from the current occasion is thereby taken into account linearly. In the present analyses, we set $K = 3$, so that $Z_t = (3X_t + 2X_{t-1} + X_{t-2})(3 + 2 + 1)^{-1}$. In the moderated model (cf. Equation [6]), Z_{t-1} then replaces X_{t-1}. If Z_{t-1} has a moderating effect, and Model 3 fits the data better than Model 2, it implies that relevant negative events have a stronger moderating impact if they follow up on each other (because Z_{t-1} takes on higher values) as compared to when they occur in isolation. The first two occasions of this moderator are imputed by the mean. As the continuous moderator was built from the dummy-coded moderator, it could not assume negative values and varied in the relatively restricted range between zero and one. We could therefore use simple linear functions to link it to the model parameters without the risk of pushing the parameter estimates pertaining to each conditional process to extreme values.

Model fitting

As described above, measurement intervals are irregularly spaced within individuals in this data set. Since participants could come to the laboratory on a daily basis, we can think of each day a person did not come in as a missing occasion. In discrete time TSA, this can be handled readily by the Kalman filter, which generates model-implied predictions of the processes' state also for missing occasions (done in case of Model 1; Durbin & Koopman, 2012). In discrete time moderated TSA, as employed here, the same strategy can be applied to missing values on the process variables. The moderator, however, enters the model as fixed. Hence, the model offers no account of how the moderator behaves over time and therefore has no implications for plausible values for occasions on which the moderator was not observed.

One way to deal with this problem would be multiple imputation (Buuren, 2012; Schafer & Graham, 2002). Here, we use a simple imputation method for the moderator: On missing occasions, we sample from the binomial distribution with the empirical probability of success set to the frequency of the dichotomous moderator observable in the available data (i.e., assuming stationarity of the moderator and no temporal structure). We generate 100 imputed data sets per person for the dichotomous moderator case and calculate the continuous moderator based on each of these. Using these imputed and recalculated data sets, we repeatedly fit Models 2 and 3. To a priori restrict the potential variability between modeling solutions within individuals, we confine this illustration to a subset of nine younger participants with no more than around 25% missing occasions.[1] An additional criterion guiding the selection of cases to analyze was variability in the moderator, leading us to leave out participants with relative event frequencies below .18, and above .82. Appendix B shows the trajectories of the selected cases for the sum scores of NA and PS with the dichotomous moderator visualized in the background. Per individual, we pool solutions across imputed data sets (i.e., point estimates and standard errors, calculation of within-imputation, between-imputation, and total variance) according to Rubin's rules as described in Schafer & Graham (2002).

We initialize the Kalman filter via the lag 0 moments of the observed time series. In fitting the models, we make use of (a modified version of) the OpenMx function *mxTryHard* that promotes and monitors iterative refitting and reuses previous parameter estimates as new starting values.

[1] Intra-class correlations (the amount of between-imputation variance over the amount of total variance) are <.10 for almost all parameters across individuals for Model 2 and <.14 for all parameters and cases except Cases 3 and 9 for Model 3. This indicates that the amount of variation between the 100 solutions based on the imputed data sets is relatively small.

Model comparisons

Within each person, we thus fit three different models, the time-invariant baseline Model 1, the dichotomously moderated Model 2, and the continuously moderated Model 3. For comparisons of Models 1 versus 2, and Models 1 versus 3, we rely on likelihood ratio testing (α= 0.05) since the models are nested. We pool χ^2-values as described in Buuren (2012). Additionally, we report the Akaike Information Criterion (AIC; Akaike, 1974) as averaged across solutions, whereby a lower average AIC implies better fit. We do not include both moderators simultaneously and can therefore not statistically test for an incremental effect of the continuous over the dichotomous moderator (note that the two are correlated since one is built from the other). Instead, we rely on the AIC and the actual modeling solutions.

Results

Figures 3 and 4 display the solutions of Models 2 and 3, respectively, in terms of 95% CI estimates across individuals. Table 6 summarizes the model comparisons.

Table 6. Model comparisons across individuals.

	Model 1	Model 2		Model 3	
Case	AIC	AIC	χ^2 (vs. Model 1)	AIC	χ^2 (vs. Model 1)
1	**<u>268.17</u>**	272.47	1.42	282.62	0.14
2	456.4	**<u>375.68</u>**	**10.96***	**393.01**	**8.55***
3	269.47	**<u>257.53</u>**	**3.15***	**266.21**	1.34
4	551.46	**<u>551.32</u>**	1.86	553.81	1.5
5	407.22	**<u>388.6</u>**	**3.8***	**395.81**	**2.7***
6	556.2	**<u>554.02</u>**	2.17	—	—
7	**<u>342.73</u>**	348.11	1.28	347.15	1.36
8	570.08	**<u>526.24</u>**	**6.71***	**549.95**	**3.65***
9	449.33	**<u>444.92</u>**	2.46	**449.2**	1.16

Note. * $p < (.05/m)$, $m = 9$ for Model 2 and $m = 8$ for Model 3. χ^2-values are pooled, the test statistic for the pooled χ^2-values is F-distributed under the null-hypothesis of equal model fit (Buuren, 2012). The best fitting model per comparison per case is printed in bold, and the best fitting model per case is printed in bold and underlined.

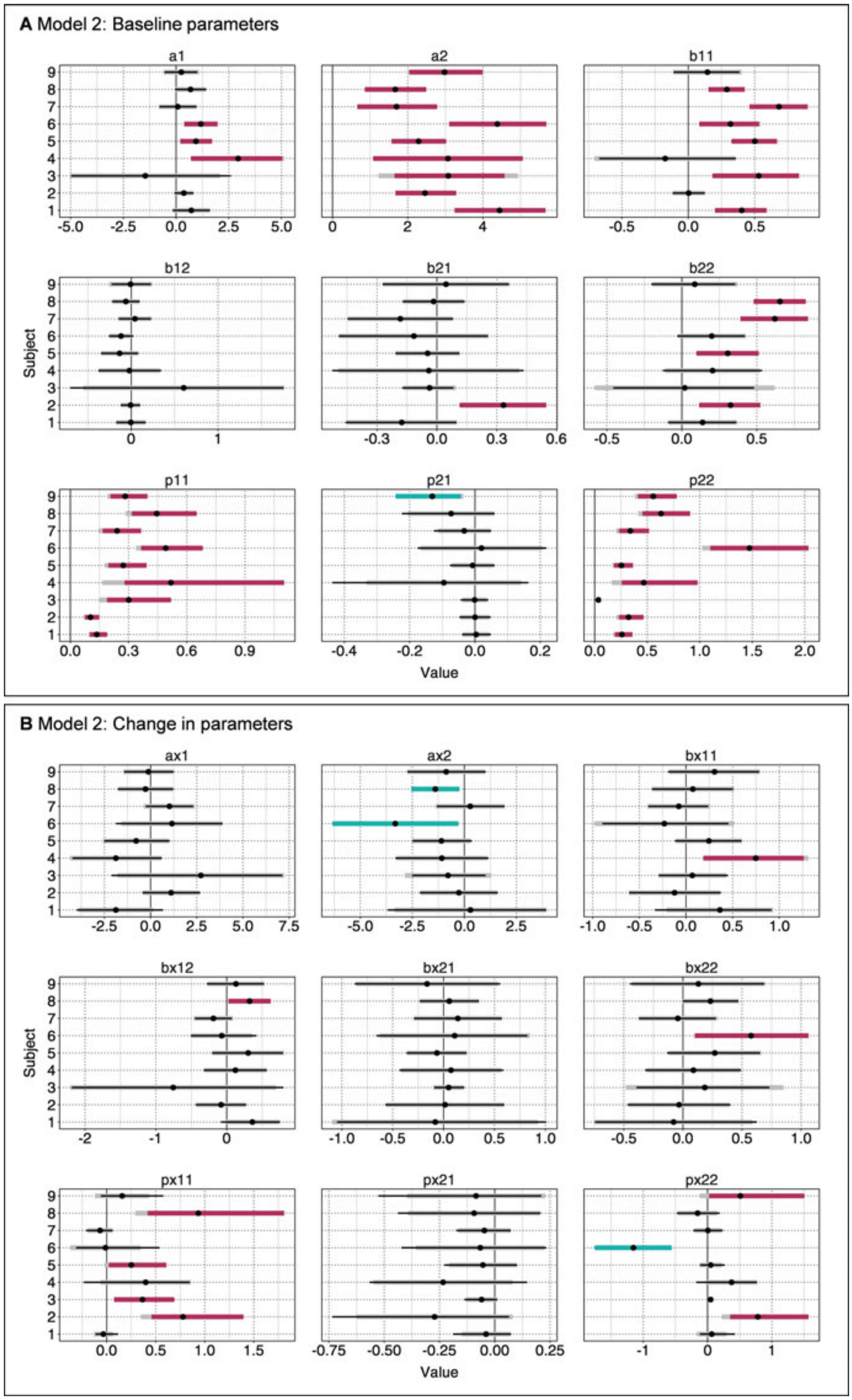

Figure 3. Individual solutions under Model 2 in terms of point and 95% CI estimates per parameter. Baseline parameters are displayed in Panel A, and change in parameters is displayed in Panel B. Profile likelihood-based CIs, averaged across solutions, are shown in black if including zero, in green if excluding zero and being negative, and in red if excluding zero and being positive. Wald-type CIs based on the pooled standard errors are shown in gray. The pooled point estimates are displayed as points.

We first consider Model 2. By incorporating the dichotomous moderator and thus discretely time-varying parameters into the time-invariant baseline model (Model 1), global model fit improves for seven individuals according to the AIC, and for four individuals according to likelihood ratio testing (Bonferroni corrected). Looking at the averaged profile likelihood-based CIs in Figure 3, and here especially at the parameters that estimate change (Panel B of Figure 3), we see different effects for those individuals for whom there is a decrease

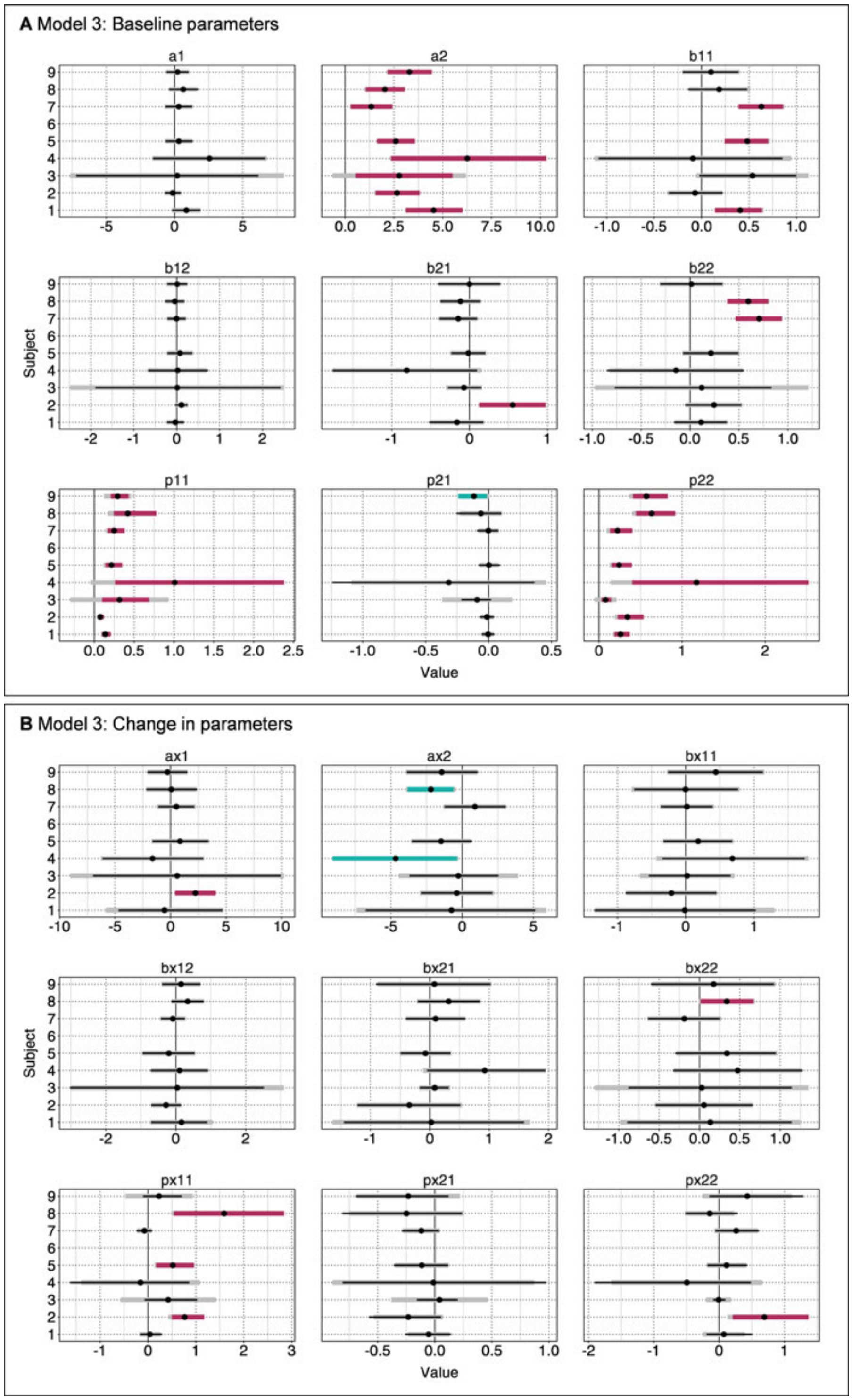

Figure 4. Individual solutions under Model 3 in terms of point and 95% CI estimates per parameter. Baseline parameters are displayed in Panel A, and change in parameters is displayed in Panel B. Profile likelihood-based CIs, averaged across solutions, are shown in black if including zero, in green if excluding zero and being negative, and in red if excluding zero and being positive. Wald-type CIs based on the pooled standard errors are shown in gray. The pooled point estimates are displayed as points.

in AIC from Model 1 to Model 2 (i.e., Cases 2 to 6, 8, and 9 as highlighted in Table 6). The residual variances of NA seem to be moderated most consistently, that is, for four individuals (i.e., "px11" in Panel B of Figure 3). These are also the cases for whom the inclusion of the moderator leads to a statistically significant increase in model fit (i.e., Cases 2, 3, 5, and 8 as highlighted in Table 6). Case 8 additionally has a negative main effect of the moderator on perceived control (i.e., "ax2" in Panel B of Figure 3). The moderated residual variances indicate increased variability in NA following negative events, after taking into account lagged effects of earlier affect and stress.

We now turn to Model 3 and the continuous moderator. Revisiting Table 6, we see that in comparison to Model 1, global model fit is improved for three individuals according to likelihood ratio testing (Bonferroni corrected). As for Model 2, those cases (now Cases 2, 5, and 8) are the ones showing moderated residual variances of NA (i.e., "px11" in Panel B of Figure 4). Case 8 additionally has a negative main effect on perceived control (i.e., "ax2" in Panel B of Figure 4), as under Model 1. As shown in Table 6, there is an AIC decrease as compared to Model 1 for five cases; however, the AIC favors Model 2 for all individuals. Note that there were some problems when fitting Model 3. That is, we did not get any converging solution for Case 6 (i.e., NPSOL status 6) who is therefore not included here. Also, across the 100 imputed data sets, 75 modeling solutions did not converge for Case 1, and 11 solutions were nonconvergent for Case 3, among them five conditionally unstable solutions (i.e., at least one eigenvalue of $\mathbf{B}^*$ and or $\mathbf{B}^{(0)}$ was equal to or larger than one in absolute value).[2]

Discussion of the empirical application

Looking at intra-individual emotional functioning within the young sample of the COGITO Study, there is some evidence for moderating effects of relevant negative events on the joint dynamics between NA and PS. The most prominent effect, present in about half of the subsample, was an increase in the variance of NA unexplained by earlier NA and PS after a relevant negative event had happened. We found no evidence for negative events having cumulative effects on the model parameters.

The observed effects on the process residual variances can be interpreted in different ways. It might be that some individuals are more (or more often) unstable in their NA levels following days with negative events. These effects might be hidden dynamic effects arising from temporal person-situation interactions that occur fast as compared to the temporal resolution of the current data and thus do not show up as lagged effects. Alternatively, the model might be reflecting a characteristic of the situation rather than the person, in that the days following negative events could have been more eventful. We can examine this post hoc to the extent events have been assessed. The zero-order intra-individual relationships between daily events of positive and negative valence, for instance, range from negative, to zero, to positive in the COGITO data. For a given individual, we may thus be contrasting days with negative events to days with positive events, days with both types of events to days without those events, or days with negative events to days with or without positive events. However, looking at the co-occurrence of relevant negative and positive events within the selected cases reveals that relationships are close to zero or negative (Spearman's rho ranges between .06 and −.22).

Some caution is advised, as individuals contribute conditional time series (given the moderator) of different lengths and thus of different degrees of bias and coverage rates, especially relevant for estimating (change in) the residual variances. Sorting solutions according to event frequency does however not reveal a clear trend (i.e., underestimation of change in residual variances for low moderator frequencies, overestimation for very high frequencies under the change-parameterization; from low to high event frequencies, the order of cases is 1, 6, 9, 2, 5, 8, 7, 3, 4). Also, in this application, we may well face a situation in which statistical power is sufficient to detect larger effects only. For Model 3, the imputation method is assumed to have added some noise too (as there was no temporal structure in the imputed dichotomous moderator data, which were used to calculate the continuous moderator). So, the absence of evidence for moderating effects is not necessarily to be taken as evidence for their nonexistence.

Since we use one indicator per construct, we stick to the constrained version of the measurement model in which we fix the factor loadings to one and the measurement residual variances to zero. With one indicator, the distinction of process and measurement residuals becomes relatively unstable unless the AR effect is very high or the time series is very long. Multiple indicator models are an alternative, which we have not considered in this illustration.

General discussion

In this paper, we propose fixed moderated TSA to study nonstationarities in affective processes due to time-varying process parameters. Such nonstationarities may for instance arise as a function of a person's interactions with specific situations in daily life. In the following discussion, we turn to potentials and limitations of the proposed modeling approach. This presentation is structured according to two main qualities of the model, namely, it being a model for observed heterogeneity and it being a time series model applicable to data from single individuals. We close with an outlook that addresses discussions in the current affect literature.

[2] These problematic instances seem to suffer from problems of empirical identification. That is, Cases 1 and 6 have the lowest relative moderator frequencies in the subsample selected (both around .18). Additionally, occurrences of the dichotomous moderator are unequally distributed across occasions, leading to little redundancy in values of the continuous moderator. Case 3 displays very little variation in PS over time.

Potentials

The model presented here is one for observed intra-individual heterogeneity as it treats the source of heterogeneity as known (i.e., the model features parameters that vary as a deterministic function of a fixed time-varying moderator). The model thus makes assumptions about the shape or pattern of temporal variability or change in model parameters while freely estimating the amount of change.

An advantage of this approach to intra-individual heterogeneity is its practicality. This concerns model implementation and estimation, in that incorporating intra-individual heterogeneity as observed yields a standard optimization problem for which solutions are widely implemented (e.g., the Kalman filter implementation in OpenMx). It also concerns feasibility in relatively small samples. As shown in the simulation, the model can be expected to be applicable given small to medium numbers of measurement occasions (e.g., $T = 100$–200) if there is sufficient variability in the moderator. With small sample sizes, however, problems of finite sample bias occur. The use of analytical or bootstrapped bias approximations for correction is possible (cf. Cheang & Reinsel, 2000). Here, we rely on profile likelihood-based 95% CI estimates that show acceptable coverage rates even with small T, as long as there is sufficient variability in the moderator. An interesting finding from the simulation study concerns the effects of temporal regularity in the moderator on model performance. In particular, it seems that more alternations between conditional processes are associated with less bias and higher coverage rates in the AR effects. We conjecture that in case of a more quickly changing moderator, parameter estimation via the Kalman filter makes better use of the entire time series data available.

Conceptually, the model incorporates and tests specific hypotheses about change in parameters. Despite its reliance on a priori knowledge about the potential sources of intra-individual heterogeneity, the model retains quite some flexibility. That is, time-varying moderators of various formats can be readily incorporated and thus parameter changes of very different shape can be tested within a single modeling framework. We consider this as another advantage of the presented approach. In the application to the COGITO data, we investigated discrete parameter changes as a function of negative events being present or absent as well as more continuous changes related to the accumulation of negative events over time. In an illustration of the model's behavior using simulated data, we also demonstrated that one is not restricted to substantive moderators or fluctuating parameter change. It is also possible to consider (e.g., gradual) changes as a function of time. Another flexibility regarding shapes of change concerns the functional relationship between the moderator and the model's parameters, which can be freely specified as long as it is deterministic. Also, hypotheses about change can be incorporated in a parameter-specific manner. Finally, models for unobserved heterogeneity have been criticized to run the risk of misrepresenting nonnormality not related to genuine heterogeneity as "spurious classes" (Bauer, 2007, p. 782). This is less likely to happen with a hypothesis-driven account to heterogeneity.

A distinguishing characteristic and important potential of the model is that it is a time series model applicable to data from single individuals. We argued that moderated TSA therefore provides the capacity to unconstrainedly examine the complex and profound inter-individual differences that intra-individual psychological phenomena are potentially subject to. Such a bottom-up approach seems appropriate in cases of extreme inter-individual differences, when aggregations across individuals must remain on a descriptive, informal level by virtue of the phenomenon under study. This is of course an extreme case and it has been argued that heterogeneity should be considered a gradual feature rather than an all-or-none situation (Brose, Voelkle, Lövdén, Lindenberger, & Schmiedek, 2015; Voelkle, Brose, Schmiedek, & Lindenberger, 2014). It may well be that individuals indeed share certain characteristics, hence, comprise a somewhat unified population. Based on this, one might then formulate an average intra-individual model structure and a formal between-subject model, which captures individual deviations from the average structure. The moderated VAR model presented here may be a candidate for an average within-subject model. If feasible, an integrated model affords simultaneous statistical inference intra- as well as inter-individually and can increase the precision of within-person estimates (von Oertzen & Boker, 2010). Note that, in these situations, single-subject modeling can still be a valid first step if a phenomenon is not well known and an explorative account is sought. Also, it can support model building (i.e., setting up appropriate priors in hierarchical Bayesian modeling; Schuurman, Grasman, & Hamaker, 2016).

Limitations

Obviously, the fact that heterogeneity needs to be observed may be considered an advantage of the approach but also a disadvantage. If no potential moderators are assessed or if hypotheses about their impact or about patterns of parameter change over time are lacking, the model is not applicable. That said, one need not have very strong hypotheses about parameter change in order to apply the model. A more explorative use seems warranted if backed up by subsequent effort to replicate and validate the captured instantiations of intra-individual heterogeneity and thus ensure their psychological substance. Of course, incorporating observed sources of heterogeneity does not preclude the possibility of unobserved heterogeneity in the data remaining unaccounted for.

Another limitation is that in the suggested fixed moderator approach, the moderator itself is not modeled. Hence, the model offers no account of how the moderator changes over time. If the moderator is a substantive covariate, this can pose the following problems. First, missing values on the moderator cannot be handled readily within the model. This also concerns attempts to take unequal measurement intervals into account by incorporating missing values or by taking a continuous time perspective (Driver, Oud, & Voelkle, 2017; Voelkle, Oud, Davidov, & Schmidt, 2012). Fortunately, as these are problems shared with a broad class of other modeling approaches that include fixed or exogenous variables (e.g., standard regression and multilevel modeling, VAR models with covariates), strategies to cope with this have been developed. Multiple imputation (Buuren, 2012; Schafer & Graham, 2002) is an acknowledged solution, for instance. Note that treating the moderator as fixed is also problematic if observed moderator scores are rather noisy, error-prone reflections of the *true* moderator scores. Distinguishing between these two sources of variation, noise and signal, is possible in the context of a measurement model. Including a measurement model for the moderator, however, requires shifting to models that incorporate interactions (i.e., nonadditive effects) among latent variables (e.g., Klein & Moosbrugger, 2000).

Finally, empirical identification can be an issue as we have seen in the illustration. Since the model decomposes (co-)variances, parameter estimation is complicated in the presence of limited variance in both the process variables and the moderator.

Substantive outlook

Surveying the recent affect literature including applied methodological work, there seems to be considerable enthusiasm about dynamic models holding potential for understanding emotional functioning. The main interest has thereby been in the identification of emotion regulation processes from intensive longitudinal data. In the present paper, we focused on the concern of flexibility of dynamic models. We argue that making models more flexible by allowing for change in dynamic parameters could address issues that are popular in the current affect literature.

First, it could immediately lead to more realistic and holistic descriptions of emotional functioning if models represent individuals as being flexible in their emotion regulation activities contingent upon internal or external context (e.g., Aldao, 2013; De Haan-Rietdijk et al., 2016; Sliwinski et al., 2009). Second, it can yield a central contribution to understanding how adaption to variable environments manifests in variable patterns of experiences and behaviors (cf. Gluck et al., 2012). Theories concerned with the adaptiveness of changes in emotional dynamics in reaction to contextual changes are currently getting established and might permit the derivation of concrete hypotheses (e.g., Bonanno & Burton, 2013; Hollenstein et al., 2013). Finally, looking at how (short-term) changes in specific emotional dynamics relate to (long-term) changes in global emotional outcomes can help to understand how daily emotional dynamics feature in the emergence and stabilization of inter-individual differences during emotional development (cf. Bos & De Jonge, 2014; Wichers, Wigman, & Myin-Germeys, 2015). We hope that taking a more flexible approach in modeling emotional dynamics by accounting for change in dynamics may help to develop more holistic, realistic, and mechanistic models of emotional development.

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Appendix A

In the following, we derive the marginal probability distribution of the process variables over a dichotomous, dummy-coded moderator. The structural model equals

$$\boldsymbol{\eta}_t = \boldsymbol{\alpha}^{(0)} + X_{t-1}\boldsymbol{\alpha}^{(X)} + (\mathbf{B}^{(0)} + X_{t-1}\mathbf{B}^{(X)})\boldsymbol{\eta}_{t-1} + \boldsymbol{\zeta}_t$$

with

$$\boldsymbol{\zeta}_t \sim N(0, \boldsymbol{\Psi}^{(0)} + X_{t-1}\boldsymbol{\Psi}^{(X)})$$

and

$$X_t = x_t \in (0, 1)$$

The conditional means are

$$\begin{aligned}
\boldsymbol{\nu}^{(0)} &= \mathrm{E}(\boldsymbol{\eta}_t | X_{t-1} = 0) \\
&= \mathrm{E}\big(\boldsymbol{\alpha}^{(0)} + \mathbf{B}^{(0)}(\boldsymbol{\eta}_{t-1}|X_{t-2} = 0\,) + \boldsymbol{\zeta}_t\big) = \boldsymbol{\alpha}^{(0)} + \mathbf{B}^{(0)}\boldsymbol{\nu}^{(0)} \\
\boldsymbol{\alpha}^{(0)} &= \boldsymbol{\nu}^{(0)} - \mathbf{B}^{(0)}\boldsymbol{\nu}^{(0)} \\
\boldsymbol{\alpha}^{(0)} &= (\mathbf{I} - \mathbf{B}^{(0)})\ \boldsymbol{\nu}^{(0)} \\
\boldsymbol{\nu}^{(0)} &= (\mathbf{I} - \mathbf{B}^{(0)})^{-1}\boldsymbol{\alpha}^{(0)}
\end{aligned}$$

and

$$\begin{aligned}
\boldsymbol{\nu}^{(X)} &= \mathrm{E}(\boldsymbol{\eta}_t | X_{t-1} = 1) \\
&= \mathrm{E}\big(\boldsymbol{\alpha}^{(0)} + \boldsymbol{\alpha}^{(X)} + (\mathbf{B}^{(0)} + \mathbf{B}^{(X)})(\boldsymbol{\eta}_{t-1}|X_{t-2} = 0) + \boldsymbol{\zeta}_t\big) \\
&= \boldsymbol{\alpha}^{(0)} + \boldsymbol{\alpha}^{(X)} + (\mathbf{B}^{(0)} + \mathbf{B}^{(X)})\boldsymbol{\nu}^{(X)} \\
\boldsymbol{\alpha}^{(0)} + \boldsymbol{\alpha}^{(X)} &= \boldsymbol{\nu}^{(X)} - (\mathbf{B}^{(0)} + \mathbf{B}^{(X)})\boldsymbol{\nu}^{(X)} \\
\boldsymbol{\alpha}^{(0)} + \boldsymbol{\alpha}^{(X)} &= (\mathbf{I} - (\mathbf{B}^{(0)} + \mathbf{B}^{(X)}))\ \boldsymbol{\nu}^{(X)} \\
\boldsymbol{\nu}^{(X)} &= (\mathbf{I} - (\mathbf{B}^{(0)} + \mathbf{B}^{(X)}))\ ^{-1}(\boldsymbol{\alpha}^{(0)} + \boldsymbol{\alpha}^{(X)})
\end{aligned}$$

given conditional stationarity, so, given,

$$\mathrm{E}(\boldsymbol{\eta}_t | X_{t-1} = 0) = \mathrm{E}(\boldsymbol{\eta}_{t-1} | X_{t-2} = 0)$$

and

$$\mathrm{E}(\boldsymbol{\eta}_t | X_{t-1} = 1) = \mathrm{E}(\boldsymbol{\eta}_{t-1} | X_{t-2} = 1)$$

The conditional variance-covariance matrices are

$$\begin{aligned}
\mathbf{P}^{(0)} &= \mathrm{E}((\boldsymbol{\eta}_t|X_{t-1} = 0)(\boldsymbol{\eta}_t|X_{t-1} = 0)') \\
&= \mathrm{E}((\mathbf{B}^{(0)}(\boldsymbol{\eta}_{t-1}|X_{t-2} = 0) + \boldsymbol{\zeta}_t)(\mathbf{B}^{(0)}(\boldsymbol{\eta}_{t-1}|X_{t-2} = 0) + \boldsymbol{\zeta}_t)') \\
&= \mathrm{E}((\mathbf{B}^{(0)}(\boldsymbol{\eta}_{t-1}|X_{t-2} = 0) + \boldsymbol{\zeta}_t)((\boldsymbol{\eta}_{t-1}|X_{t-2} = 0)'\mathbf{B}^{(0)'} + \boldsymbol{\zeta}_{t'})) \\
&= \mathrm{E}(\mathbf{B}^{(0)}(\boldsymbol{\eta}_{t-1}|X_{t-2} = 0)(\boldsymbol{\eta}_{t-1}|X_{t-2} = 0)'\mathbf{B}^{(0)\prime} + \boldsymbol{\zeta}_t\boldsymbol{\zeta}_t{}') \\
&= \mathbf{B}^{(0)}\mathrm{P}^{(0)}\mathbf{B}^{(0)'} + \boldsymbol{\Psi}^{(0)} \\
vec(\mathbf{P}^{(0)}) &= (\mathbf{B}^{(0)} \otimes \mathbf{B}^{(0)})vec(\mathbf{P}^{(0)}) + vec(\boldsymbol{\Psi}^{(0)}) \\
vec(\boldsymbol{\Psi}^{(0)}) &= vec(\mathbf{P}^{(0)}) - (\mathbf{B}^{(0)} \otimes \mathbf{B}^{(0)})vec(\mathbf{P}^{(0)}) \\
vec(\boldsymbol{\Psi}^{(0)}) &= (\mathbf{I} - (\mathbf{B}^{(0)} \otimes \mathbf{B}^{(0)}))vec(\mathbf{P}^{(0)}) \\
vec(\mathbf{P}^{(0)}) &= (\mathbf{I} - (\mathbf{B}^{(0)} \otimes \mathbf{B}^{(0)}))^{-1}vec(\boldsymbol{\Psi}^{(0)})
\end{aligned}$$

and

$$
\begin{aligned}
\mathbf{P}^{(X)} &= \mathrm{E}((\eta_t|X_{t-1}=1)(\eta_t|X_{t-1}=1)') \\
&= \mathrm{E}(((\mathbf{B}^{(0)}+\mathbf{B}^{(X)})(\boldsymbol{\eta}_{t-1}|X_{t-2}=1)+\boldsymbol{\zeta}_t) \\
&\quad \times ((\mathbf{B}^{(0)}+\mathbf{B}^{(X)})(\boldsymbol{\eta}_{t-1}|X_{t-2}=1)+\boldsymbol{\zeta}_t)') \\
&= \mathrm{E}(((\mathbf{B}^{(0)}+\mathbf{B}^{(X)})(\boldsymbol{\eta}_{t-1}|X_{t-2}=1)+\boldsymbol{\zeta}_t) \\
&\quad \times ((\boldsymbol{\eta}_{t-1}|X_{t-2}=1)'(\mathbf{B}^{0}+\mathbf{B}^{(X)})'+\boldsymbol{\zeta}_t')) \\
&= \mathrm{E}((\mathbf{B}^{(0)}+\mathbf{B}^{(X)})(\boldsymbol{\eta}_{t-1}|X_{t-2}=1) \\
&\quad \times (\boldsymbol{\eta}_{t-1}|X_{t-2}=1)'(\mathbf{B}^{(0)}+\mathbf{B}^{(X)})'+\boldsymbol{\zeta}_t\boldsymbol{\zeta}_t{}') \\
&= (\mathbf{B}^{(0)}+\mathbf{B}^{(X)})\mathbf{P}^{(0)}(\mathbf{B}^{(0)}+\mathbf{B}^{(X)})'+\boldsymbol{\Psi}^{(0)}+\boldsymbol{\Psi}^{(X)} \\
vec(\mathbf{P}^{(X)}) &= ((\mathbf{B}^{(0)}+\mathbf{B}^{(X)})\otimes(\mathbf{B}^{(0)}+\mathbf{B}^{(X)}))vec(\mathbf{P}^{(X)}) \\
&\quad +vec(\boldsymbol{\Psi}^{(0)}+\boldsymbol{\Psi}^{(X)})
\end{aligned}
$$

$$
\begin{aligned}
vec(\boldsymbol{\Psi}^{(0)}+\boldsymbol{\Psi}^{(X)}) &= vec(\mathbf{P}^{(X)})-((\mathbf{B}^{(0)}+\mathbf{B}^{(X)}) \\
&\qquad \otimes(\mathbf{B}^{(0)}+\mathbf{B}^{(X)}))vec(\mathbf{P}^{(X)}) \\
vec(\boldsymbol{\Psi}^{(0)}+\boldsymbol{\Psi}^{(X)}) &= (\mathbf{I}-((\mathbf{B}^{(0)}+\mathbf{B}^{(X)}) \\
&\qquad \otimes(\mathbf{B}^{(0)}+\mathbf{B}^{(X)})))vec(\mathbf{P}^{(X)}) \\
vec(\mathbf{P}^{(X)}) &= (\mathbf{I}-((\mathbf{B}^{(0)}+\mathbf{B}^{(X)}) \\
&\qquad \otimes(\mathbf{B}^{(0)}+\mathbf{B}^{(X)})))^{-1}(vec(\boldsymbol{\Psi}^{(0)}+\boldsymbol{\Psi}^{(X)})),
\end{aligned}
$$

given conditional stationarity, so that

$$
\begin{aligned}
&\mathrm{E}((\boldsymbol{\eta}_t|X_{t-1}=0)(\boldsymbol{\eta}_t|X_{t-1}=0)') \\
&\quad =\mathrm{E}((\boldsymbol{\eta}_{t-1}|X_{t-2}=0)(\boldsymbol{\eta}_{t-1}|X_{t-2}=0)')
\end{aligned}
$$

and

$$
\begin{aligned}
&\mathrm{E}((\boldsymbol{\eta}_t|X_{t-1}=1)(\boldsymbol{\eta}_t|X_{t-1}=1)') \\
&\quad =\mathrm{E}((\boldsymbol{\eta}_{t-1}|X_{t-2}=1)(\boldsymbol{\eta}_{t-1}|X_{t-2}=1)')
\end{aligned}
$$

In general, the moments of a finite mixture of multivariate normal distributions are

$$
\boldsymbol{\nu}_{mix}=\sum_{r=1}^{\mathrm{R}} w_r\boldsymbol{\nu}_{\mathrm{r}}
$$

and

$$
\mathbf{P}_{mix}=\sum_{r=1}^{\mathrm{R}} w_r\mathrm{P}_{\mathrm{r}}+\sum_{r=1}^{\mathrm{R}} w_r(\boldsymbol{\nu}_{\mathrm{r}}-\boldsymbol{\nu}_{mix})(\boldsymbol{\nu}_{\mathrm{r}}-\boldsymbol{\nu}_{mix})'
$$

with w_r being relative weights summing to one (Hardy, 2012). In our case of a dummy-coded dichotomous moderator, this is

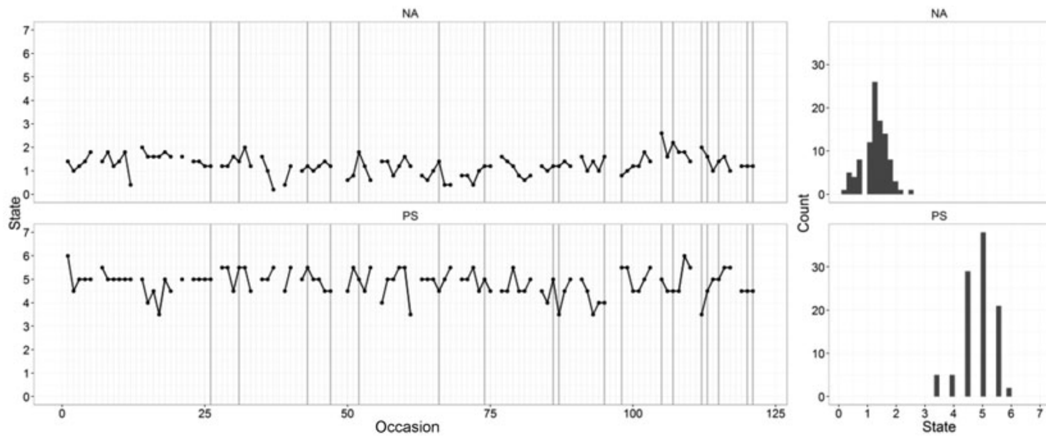
$$
\boldsymbol{\nu}_{mix}=(1-\bar{X})\boldsymbol{\nu}^{(0)}+\bar{X}\boldsymbol{\nu}^{(X)}
$$

and

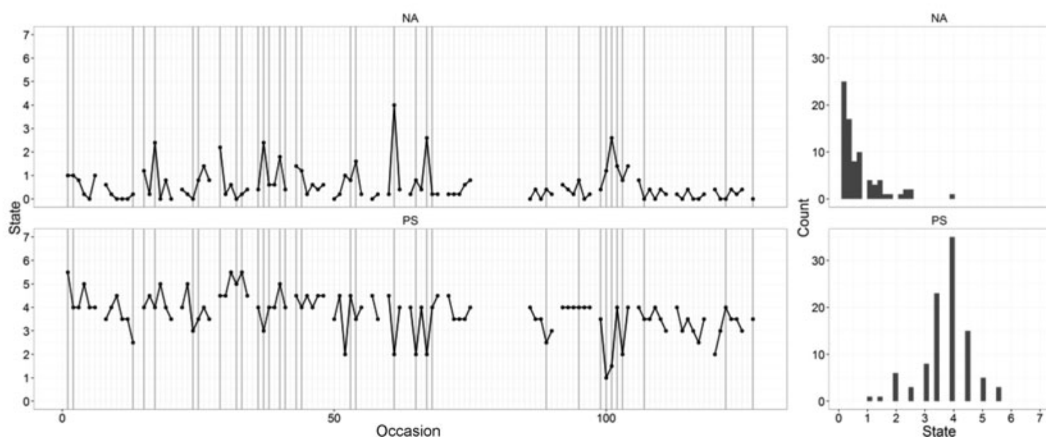
$$
\begin{aligned}
\mathbf{P}_{mix} &= (1-\bar{X})\mathrm{P}^{(0)}+\bar{X}\mathrm{P}^{(X)} \\
&\quad +(1-\bar{X})(\boldsymbol{\nu}^{(0)}-\boldsymbol{\nu}_{mix})(\boldsymbol{\nu}^{(0)}-\boldsymbol{\nu}_{mix})' \\
&\quad +\bar{X}(\boldsymbol{\nu}^{(X)}-\boldsymbol{\nu}_{mix})(\boldsymbol{\nu}^{(X)}-\boldsymbol{\nu}_{mix})'
\end{aligned}
$$

Appendix B

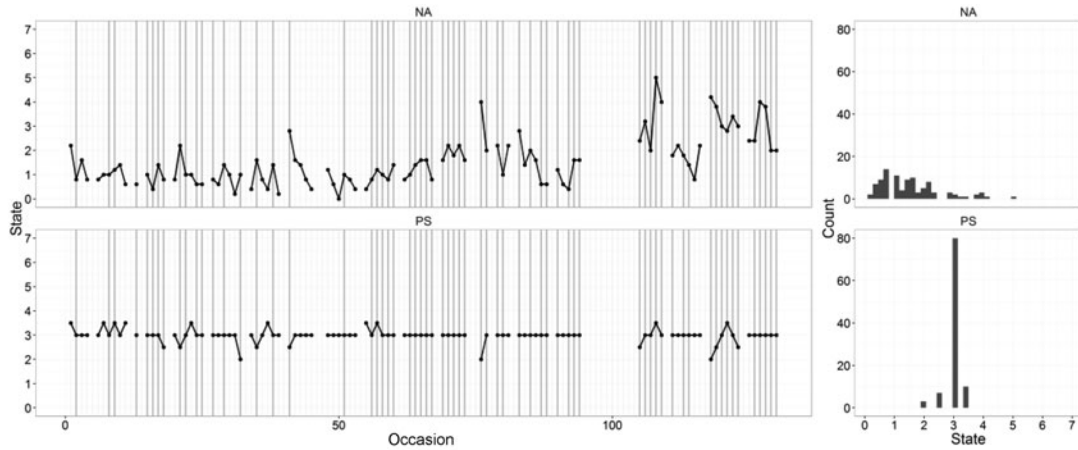
Case 1



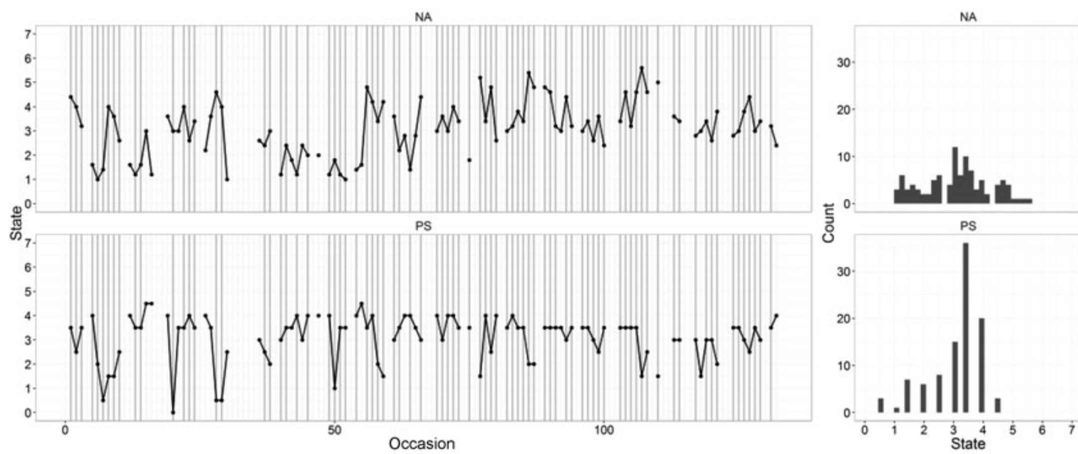
Case 2



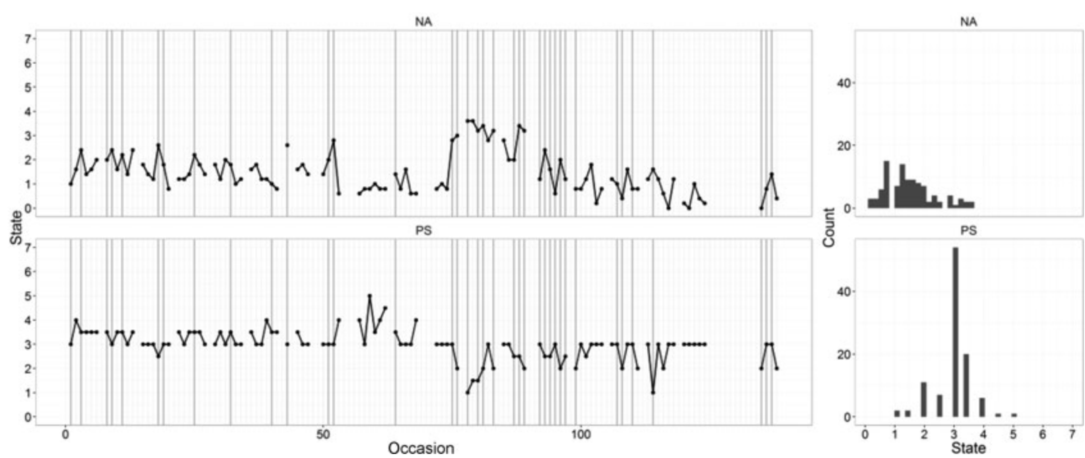
Case 3



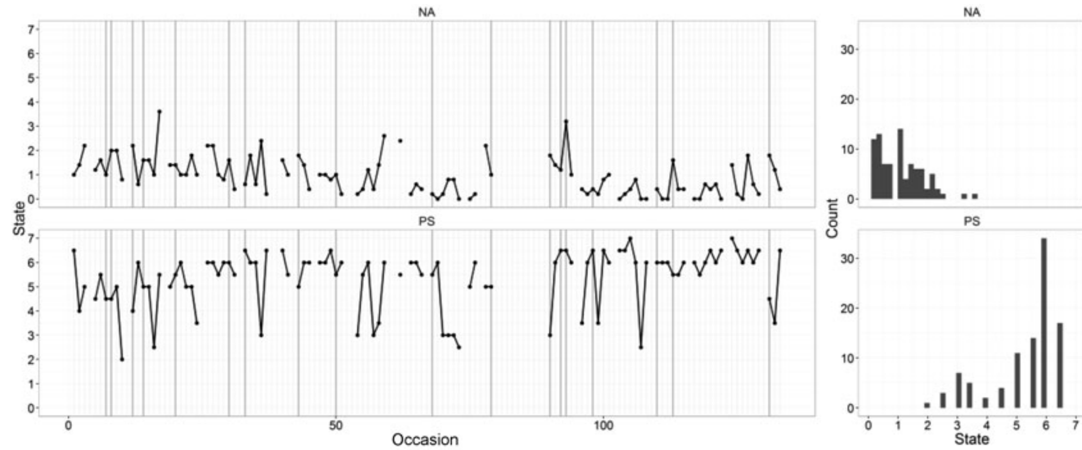
Case 4



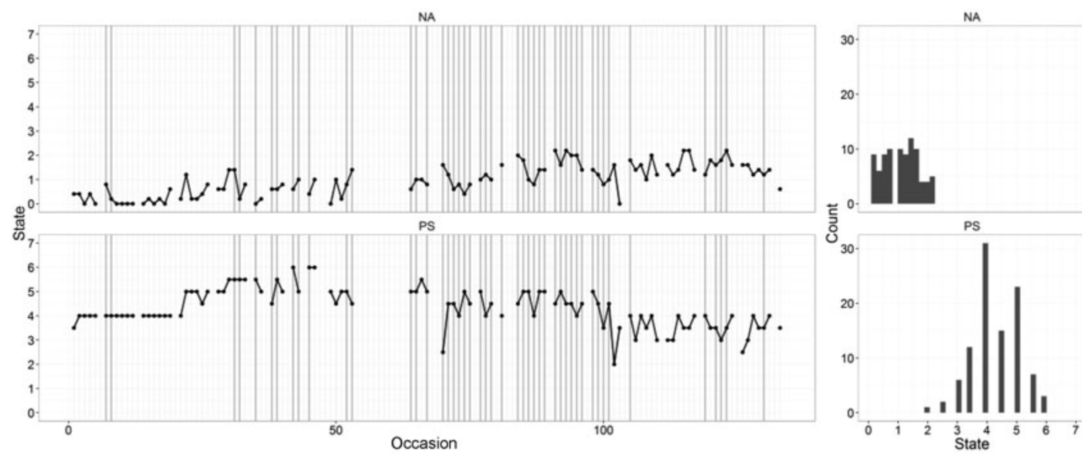
Case 5



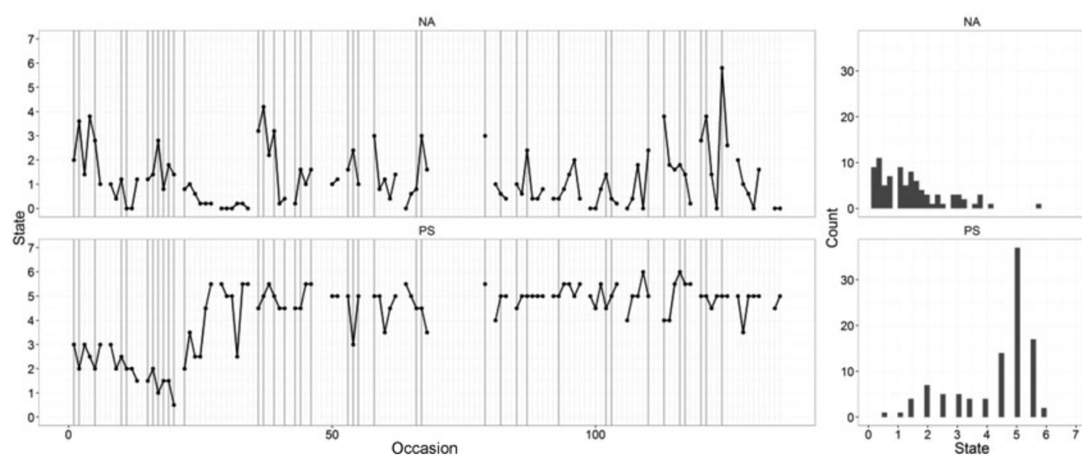
Case 6



Case 7



Case 8



Case 9