Logic and Truth-values in Language

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1. TRUTH-VALUE GAPS

The principle of the excluded third (PET) has done a great deal of harm in the semantics of natural language. The principle expresses two independent properties of languages. First, a language following PET is *fully valued*: all its sentences always have a truth-value. Secondly, there are no more than two truth-values. The result of these two conditions taken together is that a sentence of a language that follows PET has no choice but to be true or not-true (false). I shall put forward the claim that no natural language follows PET because neither of the two conditions is fulfilled. (In the argument I shall not make use of the well-known phenomena of vagueness in natural language. These phenomena result mainly from unclarities in the truth-testing procedures for atomic sentences, either because some predicates have, or seem to have, fuzzy boundary conditions for their satisfaction conditions, or because there is a 'reference group' for comparison which is not explicitly mentioned, or both. Thus, when I say that bees are clever, the truth of what I say depends on the comparison class of cleverness. This class will be different when I say that *this* bee is clever, referring to a particular bee. In any case, in our argument it will be assumed that the truth-testing procedures for atomic sentences are clear and unambiguous.)

The claim that natural languages are fully valued is easily dismantled. Let English be defined as the set of well-formed English sentences, where the grammar of English, G, defines the conditions of well-formedness of and assigns grammatical structure to the sentences of English. There can be no doubt that:

(1) The quick brown fox jumps over the lazy dog

is a well-formed sentence of English, complete with a grammatical structure. Yet here and now, between you and me, this sentence has no truth-value. It is possible, therefore, for a sentence of English to be without a truth-value.

Some philosophers have tried to escape from this conclusion by proposing analyses for sentences whereby a truth-value is always assigned. But we can say *a priori* that such analyses will fail to capture the meaning of the sentences whose lacking truth-value is thus reasoned away. Russell and Quine are the main proponents of this programme. According to Russell, sentence (1) should be analysed more or less as:

(2) There is one and only one x which is a fox, brown and quick, and one and only one y which is a dog and lazy, and x jumps over y

If taken literally, this sentence will have to be false as long as the world contains more than one quick brown fox or more than one lazy dog, and the chances of this sentence being true are therefore very slim indeed, much slimmer than speakers of English are willing to admit.

It might be objected, in Russell's defence, that an analysis like (2) should be taken to apply to a limited verification domain or situation, call it V, and that (2) should be tested against V, and not against the whole universe, for its truth-value. This objection is no doubt in principle correct, and the many questions it evokes should be given serious consideration. Yet it undermines the claim that English follows PET, since a truth-value is now no longer a function of pairs consisting of sentences and the world, but of pairs consisting of a sentence and some given verification domain. And whereas the world is always there, there is not always a V for a sentence. Hence the function assigning truth-values is sometimes undefined, so that no truth-value results.

The fact that it is not sentences of a language taken by themselves that have truth-values, but sentences as they are used in certain contexts, was driven home by Strawson (1950), as he attacked Russell on this score. But although Strawson's contributions in the early '50s provoked lively debates, this particular point was not taken until quite recently. In what is known as formal pragmatics, formal semantic treatments now aim at providing accounts for the fact that sentences are not interpreted, and are not assigned truth-values, in isolation but in their context of use. In standard semantic calculus (possible world semantics) Russell's analysis of definite descriptions is followed in principle but the variables are pragmatically linked up with entities in limited domains. In this area of semantic study PET has thus effectively been given up. But this is achieved, in principle, by extra apparatus attached to a formal semantics for a fully valued bivalent language. PET is fully restored when this extra pragmatic apparatus is removed.

Strawson, as is well-known, had a further point to make. He maintained that a natural language sentence A may have an entailment B and that the negation of A, $\neg$A, may also entail B without B being necessarily (always) true. Such entailments have become known as *presuppositions.* Although

Strawson sometimes speaks of a "logical relation", he effectively leaves the question unanswered whether presupposition should be considered a logical or a semantic phenomenon. In any case, if a language L follows PET, and if a *valuation for L* is a truth-value assignment for the sentences of L (i.e., a word description), then if A $\models$ B, for all valuations v where v(A) = +("A is true"), v(B) = +, but not necessarily vice versa. And if $\neg$A $\models$ B, then for all v such that v($\neg$A) = +, v(B) = +. Since *tertium non datur,* v(B) = + for all possible valuations. Strawson needs the truth-value gap for unvalued sentences to escape from the consequence that presuppositions are necessary truths. Now it is possible for both A and not-A to entail B, and yet for B to be false or undefined in some valuations. But then, if B is not true, v(A) as well as v($\neg$A) are undefined ("U").

At the same time Strawson wished to keep classical logic unchanged, which is impossible if holes are allowed to fall in arbitrary places in the field of valuations. Then it would be possible, for example, for both A and B to be marked true in some valuation v, and for A $\wedge$ B to be marked U. This embarrassment cannot be avoided simply by allowing truth-value gaps to occur only for logically atomic sentences, and letting the logically complex sentences be undefined only when at least one of the component sentences is undefined. This would be nothing but a straightforward application of the notion of truth-function. Yet it would invalidate the argument form known as "addition", by which, given the truth of A, the truth is guaranteed of A $\vee$ B for any arbitrary B, since there is no guarantee that B is not marked U. The question therefore arises if and how holes can be allowed in the field of valuations without a loss of logic.

If truth-value gaps are allowed to occur arbitrarily for logically atomic sentences, a great deal of classical logic is preserved. Only 'additive' entailment schemata (which involve the introduction of arbitrary sentences) will be victimized. One might well ask if this would not be a welcome loss, given the well-known fact that additive entailments do not suit linguistic intuitions very well. Van Fraassen (1966; 1971), whose point of view was purely logical and not linguistic, developed a way of regimenting truth-value gaps in valuation fields without any loss of classical logic. This system is known as the system of *supervaluations.*

This system, which has been acclaimed as extremely elegant, starts from a classical logical language L which is fully valued and has two truth-values. Let the atomic sentences of L, L_a, be valued in an arbitrary way (these valuations may be constrained by relations of semantic entailment, but this is not relevant now). The logically complex sentences are valued according to the truth-functions. Supervaluations are now generated in the following manner. Take an arbitrary finite set of classical valuations C; for each sentence S in L such that v(S) = + for all v $\in$ C, s(S) = + (i.e., the value for S in the supervaluation being generated is +); for each sentence S in L such that v(S) = - for all v ∊ **C**, s(S) = -; for all remaining sentences in L there is no uniform truth-value in the valuations of **C**, and for these sentences s(S) = U (i.e. "undefined"). We thus associate with each supervaluation s a set of sentences X such that for each S ∊ X, s(S) = + or s(S) = -, and for all remaining sentences of L, s(S) = U. Moreover, for each Y ⊆ L_a there is a set of supervaluations **S** such that each s ∊ **S** has a classical value for precisely the sentences in Y, and U for the remaining sentences in L_a. Clearly, if Y = L_a, **S**(Y) is identical with the field of classical valuations for L.

Consider the classical language L_a = {A,B,C} with the usual syntax for the truth-functional operators. Let v_1 - v_8 be the classical valuations in the classical valuation space **V**, and s_1 - s_{10} be supervaluations in the supervaluation space **SV**, where s_1 - s_{10} are defined as follows:

C (s_1) = { v_1, v_3, v_5, v_7}	**C** (s_6) = {v_5, v_6, v_7, v_8}
C (s_2) = { v_1, v_2, v_5, v_6}	**C** (s_7) = { v_1, v_5 }
C (s_3) = { v_1, v_2, v_3, v_4 }	**C** (s_8) = { v_1, v_3 }
C (s_4) = { v_2, v_4, v_6, v_8}	**C** (s_9) = { $v_1, v_2, v_3, v_5, v_6, v_7$ }
C (s_5) = { v_3, v_4, v_7, v_8}	**C** (s_{10}) = { $v_1, v_2, v_3, v_4, v_5, v_7$}

Table 1

V:	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8	**SV:** s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9	s_{10}
A	+	-	+	-	+	-	+	-	+	U	U	-	U	U	+	+	U	U
B	+	+	-	-	+	+	-	-	U	+	U	U	-	U	+	U	U	U
C	+	+	+	+	-	-	-	-	U	U	+	U	U	-	U	+	U	U
¬A	-	+	-	+	-	+	-	+	-	U	U	+	U	U	-	-	U	U
¬B	-	-	+	+	-	-	+	+	U	-	U	U	+	U	-	U	U	U
¬C	-	-	-	-	+	+	+	+	U	U	-	U	U	+	U	-	U	U
A ∧ B	+	-	-	-	+	-	-	-	U	U	U	-	-	U	+	U	U	U
A ∧ C	+	-	+	-	-	-	-	-	U	U	U	-	U	-	U	+	U	U
A ∨ B	+	+	+	-	+	+	+	-	+	+	U	U	U	U	+	+	+	U
A ∨ C	+	+	+	+	+	-	+	-	+	U	+	U	U	U	+	+	U	+

It is clear that all entailments in the classical calculus are maintained in **SV**. A ⊨ B in the classical calculus is defined in terms of valuations as follows: A ⊨ B iff for all valuations v where v(A) = +, v(B) = +. Let T(S), for any sentence S ∊ L, be the set of valuations v where v(S) = +, then if some supervaluation s_n gives truth for S, **C** (s_n) ⊆ T(S). If A ⊨ B, then if for some s_n, s_n(A) = +, **C** (s_n) ⊆ T(A) ⊆ T(B).

The truth-functional operators, however, do not remain entirely truth-functional. Truth-functionality is preserved throughout, except for conjunction and disjunction, which are not truth-functional for the values <U,U>. As Table 2 illustrates, the followint three-valued truth-tables can now be set up, with the values " + ", " - ", and "U":

Table 2

			∧ B			v B	
¬A	A	+	-	U	+	-	U
-	+	+	-	U	+	+	+
+	-	-	-	-	+	-	U
U	U	U	-	-/U	+	U	+/U

(Note that Table 2 is really three-valued, since U is not 'infectious' in the sense that a truth-value gap for a constituent sentence does not automatically make for a truth-value gap for the complex expression, as in Table 4 below.) As is illustrated by s_9 and s_{10}, disjunctions are not truth-functional for < U,U >: although A, B, and C are all U in s_9 and s_{10}, s_9(A v B) = +, but s_{10}(A v B) = U: likewise s_9(A v C) = U but s_{10}(A v C) = +. It can be shown similarly that conjunction gives either U or - for <U,U>.

Let a pair < L,V > of a classical language L and a classical valuation space V be called a *system.* Since entailment relations restrict the admissible valuations in V (when A ⊨ B, no valuation v is admitted where v(A) = + and v(B) = -), a system reflects a logical calculus, in fact the classical calculus. (We may allow for further non-logical entailment relations between (sets of) atomic sentences in L_a - such as *Jack has been murdered* ⊨ *Jack is dead* -, so that V is further restricted by such semantic entailments; but this is not relevant here.) For any system < L,V > a new system < L,SV > can be generated by defining all possible supervaluations for the classical system < L,V >. Now not only are all logical (and semantic) entailment relations in < L,V > preserved in < L,SV >, it is also the case that V ⊂ SV, since all classical valuations are also supervaluations: iff $C(s_n) = \{ v_m \}$, then $s_n = v_m$.

We can now make a system < L,SV > reflect presupposition by further restricting the admissible supervaluations in the following way. We have seen that for any set of sentences $X \subseteq L_a$ there is a set of supervaluations S(X) such that for every $s \in S(X)$, s(S) = + or s(S) = - if $S \in X$, and s(S) = U if $S \in L_a$ and $S \notin X$. We say that X *generates* S(X) by selecting all and only those valuations in **V** where every S ϵ X has a uniform value. (The cardinality of **S**(X) is thus 2^n, where n is the number of sentences in X.) We now add a further condition to any X $\subseteq L_a$ generating **S**(X): if any S ϵ X presupposes an atomic sentence A, then A ϵ X and for all s ϵ **S**(X), s(A) = +. (Note that this does not *define* presupposition: the notion of presupposition is presupposed.) It follows that in a *presuppositional* system < L,**SV** > it is no longer so that **V** $\subset$ **SV**: those classical valuations where a presupposed sentence is valued "false" are inadmissible as super-valuations. The result is that, if S presupposes A, in any *super*valuation where S has a classical value, A is valued "true".

The point of a presuppositional supervaluation system is that it allows for sub-languages X $\subseteq L_a$ such that some sentence or set of sentences Y $\subseteq$ X is invariably valued "true" in all s ϵ **S**(X). Y may then be said, with some justification, to be presupposed by X.

For this to be an account of presuppositional phenomena in natural language it is necessary to assume that natural languages fit semantically into systems of the type <L,**SV**>, and this assumption may strike some as being perhaps a little far-fetched and at least in need of independent confirmation. Although the supervaluational account of presupposition shows that Strawson's position (which does not differ essentially from the position taken by Frege (1892)) is logically coherent, or at least open to a logically coherent interpretation, one may well wonder if a simpler solution would not do. (In fact, as we shall see below, a system defined by the logical operators $\wedge$, v, and $\neg$ - see Table 5 - also preserves the whole of classical logic independently of the number of truth-values. Presuppositions can then be defined non-trivially by the introduction of choice (= minimal) negation and radical negation, whose union equals the classical exclusion negation $\neg$.) What Strawson himself had in mind was clearly not a system of supervaluations, but rather a system with two truth-values (i.e., bivalent), but with truth-value gaps for atomic sentences, resulting in truth-value gaps for logically complex sentences with one or more constituent sentences suffering from a truth-value gap. We then have a partially valued but bivalent language, with a valuation field as in Table 3:

Table 3

V:	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
A	+	–	U	+	–	U	+	–	U	+	–	U	+	–	U	+	–	U	+	–	U	+	–	U	+	–	U
B	+	+	+	–	–	–	U	U	U	+	+	+	–	–	–	U	U	U	+	+	+	–	–	–	U	U	U
C	+	+	+	+	+	+	+	+	+	–	–	–	–	–	–	–	–	–	U	U	U	U	U	U	U	U	U
¬A	–	+	U	–	+	U	–	+	U	–	+	U	–	+	U	–	+	U	–	+	U	–	+	U	–	+	U
¬B	–	–	–	+	+	+	U	U	U	–	–	–	+	+	+	U	U	U	–	–	–	+	+	+	U	U	U
¬B	–	–	–	–	–	–	–	–	–	+	+	+	+	+	+	+	+	+	U	U	U	U	U	U	U	U	U
A ∧ B	+	–	U	–	–	U	U	U	U	+	–	U	–	–	U	U	U	U	+	–	U	–	–	U	U	U	U
A ∧ C	+	–	U	+	–	U	+	–	U	–	–	U	–	–	U	–	–	U	U	U	U	U	U	U	U	U	U
A ∨ B	+	+	U	+	–	U	U	U	U	+	+	U	+	–	U	U	U	U	+	+	U	+	–	U	U	U	U
A ∨ C	+	+	U	+	+	U	+	+	U	+	–	U	+	–	U	+	–	U	U	U	U	U	U	U	U	U	U

The concomitant truth-tables are as follows:

Table 4

		∧ B			∨ B		
¬A	A	+	-	U	+	-	U
-	+	+	-	U	+	+	U
+	-	-	-	U	+	-	U
U	U	U	U	U	U	U	U

These tables are strictly bivalent, since "U" is nothing but the absence of a value. If one wishes to keep the whole of classical logic intact, it suffices to stipulate that the logic is limited to valued sentences only, so that unvalued sentences simply play no part in the logic. This, it seems, corresponds most closely to Strawson's intentions. But it is equally possible to let the logic apply to all sentences, whether valued or not. In that case the 'additive' entailments, which involve the introduction of arbitrary new sentences (such as the entailment of addition), are lost. One might well argue, however, that far from being a disadvantage, this is precisely an advantage since these entailments strike the intuitive observer as unnatural.

If we now assume a relation of semantic entailment " $>>$ " such that if A $>>$ B, then A $\models$ B and ¬A $\models$ B, we see that the valuations 4, 5, 7, 8, 13, 14, 16, 17, 22, 23, 25 and 26 are inadmissible. In the remaining admissible valuations, however, B is not always valued positively, but sometimes also negatively, and sometimes as U. Hence, B need not be a so-called necessary truth. Note that if, for some v, v(B) = - or v(B) = U, v(A) = U whenever A $>>$ B. Analogously, in Van Fraassen's system, if, for some s, s(B) = - or s(B) = U, s(A) = U, as we have seen.

Note, moreover, that the so-called necessary truths and necessary falsehoods behave differently in a supervaluational system and in a system with truth-value gaps for atomic sentences. A ∨ ¬A, for example, remains true in all classical valuations and in all supervaluations, but it is either true or undefined in a system of simple truth-value gaps. Likewise, A ∧ ¬A is false in all (super)valuations à la Van Fraassen, but either false or undefined in a simple truth-value gap system.

2. TWO NEGATIONS AND THREE TRUTH-VALUES

It is thus clear that there are various ways in which a logical language can be partially valued without logical disaster, and there is, therefore, no *a priori* argument that a logical analysis of natural languages must be based on the assumption that natural languages are fully valued. Given our very clear intuition that most sentences of no matter which natural language, when taken by themselves, are without a truth-value (as we illustrated earlier with sentence (1)), there seems to be no reason why it should be assumed that natural languages are fully valued. This in itself invalidates PET as a principle for the logical analysis of natural languages.

There remains, however, the second question of the number of truth-values. In order to answer this question we must look again at Strawson's relation of presupposition. Is it true that there is a relation in natural language which corresponds to Strawson's notion of presupposition? The answer is, it seems, that there is indeed something like that notion, but it cannot be expressed in a valuation space such as in Table 1 or Table 3. What is needed is a three-valued language with truth-value gaps.

The stumbling block in the Frege-Strawson-Van Fraassen account is that non-truth of a presupposition requires the lack of a truth-value for the carrier sentence. But this is not so for those cases where we wish to speak of presupposition. The classical case of presupposition is, of course, Russell's celebrated sentence:

(3) The present king of France is bald

which is said to presuppose that there is a king of France at the moment of speaking. Strawson maintained that this sentence is without a truth-value given that there is at present no monarchy in France. Yet he did admit (1954) that we are inclined to speak of falsity, and not of a truth-value gap, when some bragger says:

(4) The king of France asked me for a ride

Or, analogously, take the case of a mother who is putting her young child to bed and tells the child a bedtime story. Unfortunately, however, the story is about the Abominable Snowman, and the child, instead of going to sleep, starts to cry and refuses to sleep because it is frightened. Now the mother understands her mistake, and in order to put things right she says:

(5) The Abominable Snowman won't come tonight: he doesn't exist

If the Frege-Strawson-Van Fraassen account of presupposition were correct, the first sentence should lack a truth-value on account of the truth of the second. And the mother would forever be unable to soothe

the child. It is, therefore, necessary that a truth-value should be assigned to sentences even when their presuppositions fail to be true. This value cannot be 'true', because the sentence entails its presupposition. Nor can it be 'false' since the negated sentence entails the presupposition as well. There must, therefore, be a third truth-value, besides the value 'undefined', for sentences whose presuppositions are not fulfilled. We shall speak of two kinds of falsity, *minimal* and *radical* falsity. PET then stands no chance at all.

This argument would suffice, were it not that it has been observed, correctly, that it is not so that in those cases where we wish to speak of presupposition both the presupposing sentence and its negation *entail* the presupposition. That the sentence itself entails the presupposition can be accepted (where it should be clear that such entailments are not *logical* in the current sense of the word, but *semantic* in that they derive from the semantic properties of individual lexical items). But it cannot be accepted that the negated sentences also entail the presuppositions. These at most *suggest* the truth of the presupposition, but they do not necessitate it. That this is so appears from the fact that one can say without contradiction, e.g.:

(6) The present king of France is NOT bald: there ís no king of France!

It may be granted that a special emphatic intonation is required to make (6) acceptable, but the fact remains that (6) is not contradictory. Why under normal intonation a sentence like *The present king of France is not bald* should *suggest* that there is a king of France, so that the sentence effectively says that this person has a hairy scalp, is considered, by those who insist on this observation, to be a fact to be accounted for by some pragmatic theory, based on tacitly or implicitly accepted principles of cooperative communicative behaviour.

This observation, put forward with a wealth of examples by authors such as Kempson (1975), Wilson (1975), Boër and Lycan (1976), has led to what is known as the "entailment analysis" of presupposition. In this view, presupposition falls entirely outside the realm of logical analysis and is a purely pragmatic phenomenon. The logic remains classical and bivalent (the question of partial valuations is not raised in these discussions).

If the entailment analysis of presupposition is correct, there is indeed no case for a third truth-value, and all we have to debunk PET is the argument of partial valuations. An account will still have to be provided for the fact that presuppositions tend to be *suggested* when they are not entailed, and the theory of pragmatics as it exists today does not seem to offer much promise in this respect. But the conclusion that natural language is three-valued is not justified if the entailment analysis is correct. It appears, however, that the entailment analysis is, after all, not correct, so that presupposition cannot be explained (away) as a merely pragmatic phenomenon. It must be reinstated as a semantic phenomenon.

Although the entailment analysis sprang from improved observations compared with the Frege-Strawson analysis, this does not mean that the new analysis will carry the day. In fact, as we step up the level of observation the entailment analysis perishes without delay. It perishes because there are fairly numerous cases where the negation of a presupposition does indeed bring about a contradiction with the negated carrier sentence. Moreover, there are also cases where a negated presupposition combines quite easily with the negated carrier sentence; in these cases the suggestion of the presupposition is even hardly there. Without pretending completeness, we can mention the following classes of cases (the exclamation mark signifies contradiction; the check sign means compatibility):

A. *Morphologically incorporated negations* (except when incorporated into an existential quantifier: *nobody, never*). These preserve presuppositional entailments:

(7) a. Harry is co-operative } ⊨ Harry exists
b. Harry is unco-operative }
c. ! Harry is UNco-operative: he doesn't exist
d. √ Harry is NOT co-operative: he doesn't exist

B. *Negations in non-canonical positions* (for English: not constructed with the finite verb). They likewise preserve presuppositional entailments (unless incorporated into an existential quantifier):

(8) a. All the doors were shut } ⊨ there were doors
b. Not all the doors were shut }
c. ! NOT all the doors were shut: there wére no doors
d. √ Tim did NOT shut all the doors: there wére no doors

C. *Non-extraposed factive subject-clauses.* They preserve presupposition under negation:

(9) a. That Bill was guilty surprised her } ⊨ Bill was guilty
b. That Bill was guilty did not surprise her }
c. ! That Bill was guilty did NOT surprise her: he wásn't guilty
d. √ It did NOT surprise her that Bill was guilty: he wásn't guilty
(Note that (9c) is not contradictory when there is contrastive stress on the subject-clause and parenthesis intonation on *he wasn't guilty*. It is, however, clearly contradictory with accents on *not* and *wasn't*.)

D. *Cleft and pseudocleft constructions.* These preserve presupposition under negation:

(10) a. What he said was "aargh!"
b. What he said was not "aargh!" } ⊨ he said something
c. ! What he said was NOT "aargh!": he didn't say anything at all
d. √ It is not so that what he said was "aargh!": he didn't say anything at all

E. *Negative polarity items* (NPI). These are lexical items or grammatical constructions which (for reasons as yet unknown) require a negation (some only require a negative word which may be weaker than full negation, e.g. *hardly, difficult*) in simple declarative sentences on pain of ungrammaticality. English NPI's include:

can possibly	*mind that*	*any - whatsoever*	*ever*	*as much as* + VP
can help	*give a damn that*	*at all*	*budge*	*need/dare* + infin.
can seem to	*bat and eyelid*	*all that* + adj.	*any more*	*so* - adj. - *as*
matter that	*lift a finger*	*in the least*	*care to*	*half* + gradable adj.

(Note that their behaviour in questions and embedded clauses differs considerably. They differ, moreover, in that some NPI's allow emphatic or contrastive accent to substitute for the negation: *I DO mind that...* The verbs *mind that, matter that, interest that,* moreover, are besides NPI's also factive and thus presuppose the truth of the factive *that*-clause.)

NPI's preserve presupposition under negation:

(11) a. Joe DOES mind that the boss is an alcoholic
b. Joe doesn't mind that the boss is an alcoholic } ⊨ the boss is an alcoholic
c. ! Joe does NOT mind that the boss is an alcoholic: the man is a teetotaller

We thus have pairs of sentences and their negations, both of which share an entailment, called "presupposition". This takes us back to the Frege-Strawson position, although the class of cases where the position holds turns out much more restricted than it was thought to be.

Notice, moreover, that besides the NPI's just illustrated, there are also *positive polarity items* (PPI). They are also very numerous in all languages of the world. Their behaviour, however, seems, at first sight, not to be the exact opposite of NPI's. It is not so that they cannot stand under negation on pain of ungrammaticality. On the contrary, they do occur under negation, but when they do, there is what is known in the linguistic literature as the 'echo-effect'. Moreover, a fact not known in the literature, a negated sentence with a PPI requires that one or more presuppositions of the non-negated sentence be dropped. Examples of PPI are:

relatively		*still*	*perhaps*	*bristle with*	*few*
reasonably		*plenty of*	*certainly*	*teem with*	*forever + be*
rather	+adj.	*splendid*	*surely*	*several*	*most*
pretty		*decent (metaph.)*	*awful*	*some*	*the whole bloody lot*
far from		*terrific*	*swarm with*	*not*	
				hardly	*yet have to*
					already

Their effect under negation can be seen in the following pair:

(12) a. ! Harry no longer lives in Paris. He has never lived there
b. √ Harry does NOT still live in Paris. He has never lived there

The expression *no longer* in (12a) is an NPI and is responsible for the presupposition that Harry has lived in Paris in the past. The sentence asserts that this state of affairs has been discontinued. Due to the fact that *no longer* is an NPI the presupposition cannot be cancelled. In (12b), however, there is the PPI *still*, with the same presupposition as *no longer*, though with the opposite assertive force: *Harry still lives in Paris* asserts that the presupposed state of affairs still continues. When this sentence is negated, as in (12b), there is no problem at all in cancelling the presupposition.

These observations force us to accept at least that there are two distinct uses of the word *not* in English (in other languages the situation is entirely analogous). Let us speak of a *minimal* and a *radical* use of *not*, the former protecting all presuppositions of the non-negated sentence, and the latter requiring that at least one of them be dropped. It will be argued in a moment that there are not only two uses of *not*, but that there are in effect two negations *not* in English, a minimal and a radical *not*, which differ truth-functionally. But we shall not prejudice the issue, and be content for the moment with speaking of no more than different uses of *not*.

Having made this distinction we notice that NPI's and PPI's do behave symmetrically with respect to each other: NPI's require the minimal (use of) *not* and do not, therefore, affect presuppositions; PPI's require the absence of the minimal (use of) *not* and the only negation they allow of is the radical (use of) *not*, so that presuppositions are not safe under negated PPI's.

Before we proceed, let us have a look at one further peculiarity of the radical (use of) *not*. It is to be observed that radical *not* seems to have a preference for the position of full sentential operator. It definitely dislikes being placed in the scope of another operator or embedded in a subordinate clause. Whereas (13a) clearly falls within the boundaries of acceptable speech, (13b) and (13c) border on the unacceptable:

(13) a. John's child is NOT asleep: he hás no children
b. ! Some of John's children are NOT asleep: he hás no children
c. ! Ed hopes that John's child is NOT asleep: John hás no children

Yet radical *not* does allow for higher operators, and for certain kinds of embedding, as in:

(14) a. In Ed's world of fancy, John's child is NOT asleep, because Ed thinks John has no children
b. If indeed John's child is NOT asleep because he has no children, all is well
c. If John has no children, then indeed his child is NOT asleep

But in these cases one has to do with reports of what someone believes or has said.

Most of the observations given above regarding the minimal and the radical (uses of) *not* are as yet without any proper linguistic explanation: all we can do here is accept the observations and see what follows from them, even if we do not know why the facts should be as they are. We have seen that the minimal conclusion is that there are two uses of *not*. The question now arises which of the two theories is to be preferred: (a) *not* occurs in different structural positions, and in some positions it preserves presuppositions, whereas in other positions it does not; (b) there are two truth-functionally distinct negations, both constrained in the structural positions they may occur in, but they have at least one position in common, the position of full sentential operator, in which the one preserves presuppositions and the other does not. Proponents of theory (a) will face the task of specifying the positions in which presuppositions are preserved under *not* and those where they are not preserved. They must, moreover, make sure that their structural analyses do not depart too wildly from the sentences they are analysing. They must, in some principled way, conform to Russell's principle of "parity of form" (1905: 483), by which it is required that what presents itself in sentences as being of a particular form or structure should be analysed, as much as possible, uniformly. Distinctions are allowed only if nothing else helps. Proponents of theory (b), on the other hand, will probably face a simpler task of designing logical analytic structures, but they carry the full burden of the distinction between two negations: they must show, by the principle of parity of form, that nothing else helps, or at least, if this cannot be shown, they must claim that nothing else helps and wait and hope that the claim will not be disproved.

Let us consider theory (a). Its earliest proponent was Bertrand Russell, who proposed (1905) to analyse the sentence *The king of France is not* *bald* as either (15a) or (15b) (whereby we neglect the uniqueness clause):

(15) a. $\neg \exists x(KF(x) \wedge B(x))$
b. $\exists x(KF(x) \wedge \neg B(x))$

This analysis is quickly shown to be inadequate. One reason why it fails is that it leads to scope problems when generalized to other cases. Take, e.g.:

(16) Ed thought that there was a doctor, and he hoped that *the doctor* would help

If we dissolve the definite description *the doctor* into a quantifier and its variable, as Russell did, then the quantifier will have to come either in the scope of *he hoped*, which is not what the sentence means, or outside *he hoped*, which is again not what the sentence means. A second reason is that only existential presuppositions would fit this analysis. Other types of presupposition cannot be handled this way, as is easily seen.

An improvement would consist in analysing presupposition-carrying sentences always as a conjunction, where the first conjunct expresses the presupposition and the second conjunct expresses the assertion. The negation can then be made to take the whole conjunction in its scope, or only the second conjunct. This is an improvement since the analysis is no longer restricted to just existential presuppositions. Thus, (9b) above would be analysed as either (17a) or (17b):

(17) a. Bill was guilty and $\neg$ (that surprised her)
b. $\neg$ (Bill was guilty and that surprised her)

Yet scope problems keep rearing their head. In the second conjunct of (16), for example, the presupposition would either have to precede *he hoped* ("There was a doctor, and Ed hoped that he would help") or be in the scope of *he hoped* ("he hoped that there was a doctor and that the doctor would help"). In neither case, however, do we get the right meaning. Moreover, this type of analysis fails to explain why (18a) is contradictory, but (18b) is not:

(18) a. ! There was a doctor, and he helped, and he did not help
b. √ There was a doctor and he helped, and there was a doctor and he did not help

Finally, there is a lack of structural regularity between external and internal negations in the analyses, on the one hand, and the corresponding positions of the word *not* in the actual sentences. In Sentence (8b), for example, the negation is presupposition-preserving and should therefore be internal. Yet it occurs in the most external position possible in the English sentence.

This last obstacle also enables us to give short shrift to the theory that distinguishes between sentence negation and verb-phrase negation, where the former does not preserve presuppositions but the latter does. Again, if the *not* in (8b) is verb-phrase negation, why is it playing around with *all* at the beginning of the sentence?

Gabbay & Moravcsik (1978) gets around this particular obstacle by defining a "constituent negation" along with classical sentential negation. The constituent negation, or "denial", is used (p. 251) "not only to pose a contradictory to some proposition, but to claim that something is wrong with a proposition, and to indicate – insofar as possible - which is the objectionable item". For example:

(19) N_{John} (John eats fish)

should be read, with "denial" on the constituent *John*, as "it is not John who eats fish". Truth is brought about for (19) if there is an element other than John, to be taken from a pre-defined complement class, eating fish. The negation of denial should then be presupposition-preserving, whereas classical sentential negation would not be.

While such a proposal might bring some relief for cases as illustrated in (7b), (8b), and perhaps, under some suitable analysis, for (10b), it fails to explain (9b) as well as the cases mentioned under E, the NPI-cases. Moreover, if the concept of "constituent negation" is viable at all, constituents with built-in negation must be prime cases. Yet there often are morphologically incorporated negations which do not "deny" the constituent they form a structural part of. For example:

(20) Nobody left because the play was boring. (Those who left did so because they had a train to catch)

Here the constituent "denied", in the sense of Gabbay & Moravcsik, is the subordinate clause *because the play was boring*. A paraphrase is:

(21) It is not because the play was boring that some people left

In the same way, in (22) *not* should "deny" *until yesterday*, but it clearly does not do so:

(22) Not until yesterday did he post the letter

Note, moreover, that the Gabbay & Moravcsik proposal in fact concedes the point that negation in human language is pluriform: there is no longer one truth-functionally identical negation occurring in different positions.

Other proposals can no doubt be devised in order to circumvent the conclusion that human language has two truth-functionally distinct negations. But as long as no convincing alternative is provided, we are justified in accepting this conclusion. It keeps semantic analyses relatively simple and straightforward from a structural point of view, or at least it avoids the specific structural complications imported by variant-position analyses. At the moment it seems sensible to claim that nothing helps short of a distinction between two truth-functionally distinct propositional negation operators, the *minimal* negation which preserves presuppositions, and the *radical* negation which eliminates them or at least makes their status insecure. If this claim proves untenable, it will be in virtue of an alternative analysis which is shown to be superior or preferable, for some valid reason. So far, however, such an alternative analysis has not come forth.

What has come forth is an objection of a principled kind against the claim that *not* in English is ambiguous. This objection is made in Gazdar (1979: 65), and it is based on the consideration that if *not* is ambiguous in English, one would expect its translations into other languages, in general, not to be ambiguous. Ordinary homonymous lexical items, such as *stock, ball,* or *bank,* are disambiguated in other languages. Lexical homonymy is almost by definition idiosyncratic for the language at hand, as every foreign language learner knows. Among homonyms, however, *not* is idiosyncratic in that it is, apparently, homonymous in all languages in the world. Many languages distinguish between various kinds of negation: emphatic versus weak negation; indicative versus subjunctive negation; indicative versus imperative negation, etc. But no language has been spotted so far where a lexical distinction is made between minimal and radical negation. Does this not weaken our position? Does it not speak against us that a distinction which is, apparently, so prominent in semantic analysis is so weakly represented on the morphological level in the languages of the world?

In answering this question it should be taken into consideration that *not*, or its counterparts in other languages, is an idiosyncratic case among homonyms not only because it is, apparently, never lexically disambiguated between a minimal and a radical negation, but also for another good reason. *Not* is not just an ordinary lexical item, nor is the ambiguity of the arbitrary kind customary in lexical homonyms. On the contrary, *not* obviously occupies a very special place in the lexicon, and the ambiguity is between two readings which are closely connected and, as we shall see in a moment, complementary in a truth-functional three-valued system.

This in itself should make uncertain the expectation that *not*, as a possibly ambiguous item, will behave just like other lexical homonyms.

But there is another consideration as well, to do with the discourse properties of presupposition. It is a general rule that when A >> B ("A presupposes B"), not only A ⊨ B and ~ A ⊨ B ("minimally negated A entails B"), but B is also required in preceding discourse for A or ~ A to be interpretable. If B is not already present in the discourse that has been built up when A or ~ A is uttered, it is inserted *post hoc* as long as there are no logical, semantic or cognitive factors preventing such an insertion. The crucial fact is that whenever a presupposition B is excluded from a discourse, either because it has been explicitly negated or because B clashes semantically or cognitively with the discourse, then A, or ~ A, is uninterpretable and unacceptable in that discourse. For example, when the discourse has been about a room, a subsequent utterance of a sentence may presuppose that it has windows, doors, a TV-set, furniture, but not, in general, a steering wheel, - simply because in our culture the notion of a room with a steering wheel has not so far gained currency. A sentence like:

(23) The steering wheel of the room was out of order

will then be uninterpretable and hence unacceptable. (See Seuren (forthcoming) for a fuller discussion of the discourse properties of presupposition.)

In the light of this discourse principle, it is clear that there is a self-regulating mechanism selecting the minimal or the radical meaning of *not*. As long as the discourse either already contains the presupposition B or allows for it to be added *post hoc*, the minimal reading of *not* is forced, due to the requirement that presuppositions must be present or must be inserted *post hoc* in the discourse, ánd the fact that radical negation requires the falsity of at least one presupposition of the carrier sentence. If, however, the discourse contains the explicit negation of the presupposition or blocks the insertion of the presupposition for semantic or cognitive reasons, the only interpretable reading for *not*-A is the reading with the radical negation: ≃ A. It is therefore not necessary to have two distinct lexical representations, one for minimal and one for radical negation: the principles of discourse construction sort the difference out automatically. For all practical purposes, therefore, the classical bivalent negation, ¬A, will do: the conditions of use do the rest.

A reply to this might be that this argument shows that no distinction is needed between truth-functionally distinct negations, since whatever distinction is needed is made by the discourse on every occasion of use. Negation may remain bivalent: we can do with three truth-values and only one negation. Whether it lets the presuppositions through or blocks them is a matter of discourse. This reply is not unreasonable, but it should be realized that the price paid for this solution is that negation is no longer truth-functional. For in a discourse where a sentence A is entirely acceptable and interpretable so that all its presuppositions are either already in the discourse or can be supplied without problems, yet A is radically false because in fact one of its presuppositions is not fulfilled, the sentence *not*-A will be interpreted with minimal negation, the discourse being what it is. Yet *not*-A is still not true, even though A was radically false. It seems that loss of truth-functionality is a higher price than those who would propose this solution would be prepared to pay.

Let us sum up the situation. Our position implies not only that natural languages are partially valued, but also that the logic at work in natural language arguments is three-valued, with the values "true", "minimally false", and "radically false", and that there are two negations in language, the minimal and the radical negation. This makes for quite an unusual picture from a logician's point of view. Should this worry us? The answer is "no", as long as the logic is sound. In fact, one would expect that the logic of language is not quite the same as the logic of logic or the logic of mathematics. Language serves so many different, mainly mundane, purposes in life, and it does so by depending so heavily on factors of discourse and context, that it would indeed be highly surprising if it turned out to be describable semantically in terms of a fully valued bivalent logical language. Every natural language is the product of natural growth (both in the biological and in the social sense of "natural"), and one should expect nature to find her own, usually unexpectedly clever and functional, means to achieve her end. (Heraclitus said: "Nature likes to hide herself.")

Let us, finally, have a look at the three-valued propositional calculus that would do the job of accounting for logical and semantic entailment in arguments conducted with the help of natural language sentences. We adopt the following truth-tables, where the value "1" stands for truth, "2" for minimal falsity, and "3" for radical falsity. Besides the minimal negation ($\sim$) and the radical negation ($\simeq$), we still define the classical bivalent negation ($\neg$) separately, although it is defined as the union of the minimal and the radical negations. (For a full discussion of this logic, see the Appendix by A. Weyters in Seuren (forthcoming).)

Table 5

				∧ B			∨ B		
¬A	≃A	~A	A	1	2	3	1	2	3
2	2	2	1	1	2	3	1	1	1
1	2	1	2	2	2	3	1	2	2
1	1	3	3	3	3	3	1	2	3

(Since the language is partially valued, any complex expression where at least one constituent sentence is unvalued, will be unvalued.)

This logic has the property of deviating minimally from classical bivalent logic. The conjunctive operator " ∧ " is definable for any n-valued logic ($n > 1$) as a binary operator selecting the highest of the component values; the disjunction " ∨ " is defined as a binary operator selecting the lowest of the component values; the bivalent negation "¬ " is defined for any n-valued logic as a unary operator converting truth into minimal falsity and all other values into truth. Any formula expressed in terms of these three operators which is classically valid is valid within this system, no matter how many truth-values are defined in the logic. In other words, the logical system (∧, ∨, ¬) is independent of the number of values. The bivalent system is simply the system (∧, ∨, ¬) with the lowest possible number of values.

Moreover, for any n-valued system (∧, ∨, ¬), n–1 negations can be defined in such a way that each negation yields truth for exactly one value of its argument proposition, and if $\sim_1, \sim_2, \ldots, \sim_{n-1}$ are the negations in question, then $\sim_1 A \vee \sim_2 A \vee \ldots \vee \sim_{n-1} A \equiv \neg A$. Table 5 contains the negations " ~ " (= $\sim_1$) and " ≃ " (= $\sim_2$), where the former yields truth for minimal falsity of A only, and the latter for radical falsity of A only. Table 5, in fact, represents the three-valued member of a family of systems characterized by the properties just given. (For fuller details, see Section 6.3 of the Appendix by A. Weyters in Seuren (forthcoming).) It is clear that any n-valued system in this hierarchy contains the classical system (∧, ∨, ¬) as a proper subpart, since the union of all n–1 negations in an n-valued system equals the classical bivalent negation. The logic we are using is thus less unorthodox than might appear at first sight. The ulterior question of why nature should prefer the three-valued member of the family, with its two specific negations, for the purposes of natural language, is a question which would require more space than is afforded here to find a proper answer.

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