Helicity influence on Alfven wave heating in stellarators

I.O.Girka[1], R. Schneider[2]
[1] *Kharkiv National University, Svobody Sq.4, Kharkiv, 61077, Ukraine.*
[2] *Max Planck Institute fuer Plasmaphysik, Teilinstitut Greifswald, Wendelsteinstr., 1, Greifswald, D-17491, Germany.*

1. Introduction. Local Alfven resonance (AR) in the case of straight steady magnetic field $\vec{B}_0$ was intensively studied since Dolgopolov and Stepanov (1965), Uberoi (1972), Chen and Hasegawa (1974), Tataronis and Grossman (1973) etc. mainly due to application of this phenomenon to efficient plasma production and heating in fusion devices. In the cold approach, fields of RF waves are known to have infinite disruptions within AR region. Even weak effects (thermal motion of particles, finite inertia of electrons, weak nonlinearity and dissipation) can significantly affect on the conversion of these waves into small scale oscillations and provide for their efficacious absorption in this region. Singular behaviour of the radial wave electric field is shown in figures 1-3 with dashed lines.

AR is known to displace to plasma periphery while plasma density grows. This leads to undesirable plasma periphery heating, enhancement of plasma – wall interaction and further plasma contamination. Complex three-dimensional inhomogeneity of resonance regions and small scale of converted waves make a solution of the problem about conversion and absorption of RF waves in stellarators even with the help of PC very complicated so far. These reasons stimulate and justify attempts of an analytic solution of such problem.

Confining magnetic field of stellarators is characterised by a helical inhomogeneity. We restrict our consideration to the following representation of the confining magnetic field $\vec{B}_0(r,\vartheta,z)=\vec{e}_r B_{0r}+\vec{e}_\vartheta B_{0\vartheta}+\vec{e}_z B_{0z}$ in cylindrical coordinates:

$$B_{0r}=\delta\, sin\, l\theta\, B_0,\quad B_{0\vartheta}=\in^{(1)}{}_h\big/(\alpha r)cos\, l\theta\, B_0+\iota\, r/(R)B_0,\quad B_{0z}=B_0\left(1-\in^{(l)}{}_h\cos l\theta\right). \qquad (1)$$

Designations here are commonly used (see, e.g., Girka et al (2003)), $\delta=\left(1/k_s\right)d\in^{(1)}{}_h\big/dr$.

2. Derivation of the basic equation. We consider here the electromagnetic waves with a frequency $\omega<<\left|\omega_{ce}\right|$, ω_{pe} in the low-β plasma. In this plasma, the equilibrium density $n(r,\vartheta,z)$ can be introduced as a function of a single variable that is the magnetic surface, $n(r,\vartheta, z)=n(r_0)$, where

$$r_0=r-0.5\delta/\left(\alpha B_0\right)cos\left(l\theta\right)+O\left(\delta^2\right). \qquad (2)$$

The assumption of small current in helical winding ($\delta<<1$) allows us to write down the expressions for the components $\varepsilon_{1,2}$ of the dielectric permittivity tensor of the low-temperature plasma in the form of Fourier series with keeping the terms of the first order in δ only:

$$\varepsilon_{1,2}(r,\theta)=\varepsilon^{(0)}{}_{1,2}(r)+\varepsilon^{(1)}{}_{1,2}(r)\cos(l\theta)+O\left(\delta^2\right), \qquad (3)$$

where $\left|\varepsilon^{(1)}_{1,2}\right| \sim \left|\delta\varepsilon^{(0)}_{1,2}\right|$, $\varepsilon_{1,2}{}^{(0)}$ are zero-order terms and $\varepsilon_{1,2}{}^{(1)}$ are the first-order corrections.

Electron inertia is known to be negligibly weak for Alfven and fast magnetosonic branches of MHD oscillations. That is why the longitudinal component of the wave electric field is equal to zero everywhere in the plasma: $\left(\vec{E},\vec{B}_0\right)\equiv E_3=(B_{0r}E_r+B_{0\vartheta}E_\vartheta+B_{0z}E_z)\rightarrow 0$. This relation between the components of the RF electric field enables us to simplify the set of Maxwell equations. Following Dolgopolov and Stepanov (1965) and others we utilize also the approach of «narrow slab».

Judging by the symmetry of the problem we are looking for the solution of Maxwell equations in the following form,

$$E_r=\left[E_r^{(0)}(r)+E_r^{(+1)}(r)e^{il\theta}+E_r^{(-1)}(r)e^{-il\theta}\right]\exp i\left(k_z z+m\vartheta-\omega t\right). \qquad (4)$$

The representation (4) corresponds to the wave envelope that contains fundamental harmonic and two the nearest satellite harmonics. The expansions in Fourier series for the remaining components of the magnetic and electric fields of the wave are similar to (4).

Let's substitute expressions (4) for the wave fields and (3) for the components $\varepsilon_{1,2}$ into Maxwell equations and single out terms $\propto exp\{i[k_z z+m\theta-\omega t]\}$ and $\propto exp\{i[(k_z\mp k_s)z+(m\pm l)\theta-\omega t]\}$. Since the equation for the radial component of electric field is the most convenient for studying the structure of AR then we show here only the following closed set of the three equations for the amplitudes of the fundamental and satellite harmonics that is derived from Maxwell equations,

$$\left[\left(\varepsilon_1^{(0)}-N_z^2-2N_zN_\vartheta\iota\, r/R\right)E_r^{(0)}-A\right]+\frac{c^2\delta^2}{2\omega^2}\frac{d^2E_r^{(0)}}{dr^2}+0.5\varepsilon_1^{(1)}E_r^{(+1)}-\frac{c^2k_s\delta}{2\omega^2}\frac{dE_r^{(+1)}}{dr}$$

$$+\frac{c^2}{\omega^2}\delta\left(k_z+k_s\right)\frac{dE_r^{(+1)}}{dr}-\frac{c^2k_s\delta}{2\omega^2}\frac{dE_r^{(-1)}}{dr}+0.5\varepsilon_1^{(1)}E_r^{(-1)}-\frac{c^2}{\omega^2}\delta\left(k_z-k_s\right)\frac{dE_r^{(-1)}}{dr}=0, \qquad (5)$$

$$\frac{c^2}{2\omega^2}\delta\left(\mp 2k_z+k_s\right)\frac{dE_r^{(0)}}{dr}+\left(N_z\mp N_s\right)^2E_r^{(\pm 1)}$$

$$-\frac{c^2\delta^2}{2\omega^2}\frac{d^2E_r^{(\pm1)}}{dr^2}-N_z^2E_r^{(\pm1)}+\frac{c^2\delta^2}{4\omega^2}\frac{dE_r^{(\mp1)}}{dr}=0. \qquad (6)$$

The combination $\left(i\varepsilon_2^{(0)}E_\vartheta^{(0)}+N_\vartheta B_z^{(0)}\right)\equiv A$ in the left hand side of (5) appears to vary slowly in the vicinity of AR, although both fields $E_\vartheta^{(0)}$ and $B_z^{(0)}$ have a logarithmic singularity within the AR region in the cold approach. That is why it is natural to consider this combination as the constant that is in fact associated with a pumping wave driven by an antenna. Effect of the helical inhomogeneity of the confining magnetic field on the amplitude of the main harmonic outside the AR region is found only in the second approximation in the respect to the small parameter δ. Amplitudes of the satellite harmonics are less than that of the main harmonic outside the AR region, by the order of magnitude (see, e.g., Girka and Kovtun (2000)

$$E_r^{(\pm 1)} \sim \left|\delta E_r^{(0)}\right| . \tag{7}$$

3. AR fine structure. The set of equations (5), (6) consists of three differential equations of the second order. This set can be written down as the equation of the sixth order for the amplitude $E_r^{(0)}$ of the main harmonic. Analysis of the relation between the order of smallness and the order of the corresponding derivative allows us to simplify this sixth order equation with the accuracy of the order of $\sim\delta^{2/5}$ as follows:

$$-\frac{1}{8}\frac{c^4}{\omega^4}\delta^4\left(N_s^2+4N_z^2\right)\frac{d^4}{dr^4}E_r^{(0)}+N_s^2\left(N_s^2-4N_z^2\right)\left[\left(\varepsilon_1^{(0)}-N_z^2-2N_zN_\vartheta\iota\, r/R\right)E_r^{(0)}+A\right]=0. \tag{8}$$

If radial plasma density profile is linear one within the AR region, then the analytical solution of (8) can be obtained by the Laplace method in the following form:

$$E_r^{(0)}=\left(a^*k_1/N_s^2\right)Au_0\left[k_1(r-r_A)\right],\quad u_0(\xi)=\int_0^\infty \exp\left[i\left(t\xi+t^5/5\right)\right]dt\,, \tag{9}$$

$$k_1=\left(\frac{c^4\delta^4a^*}{8\omega^4N_s^2N_z^2}\frac{N_s^2+4N_z^2}{N_s^2-4N_z^2}\right)^{-1/5}\sim[k_z^2k_s^2/(\delta^4a^*)]^{1/5}\propto\delta^{-4/5}\,. \tag{10}$$

Here $a^*=\left|d\ln\varepsilon_1^{(0)}/dr\right|_{r_A}^{-1}$ is characteristic radial scale at which plasma density varies within the AR region. This solution satisfies the following boundary conditions: it is finite one; it describes the conversion of electromagnetic wave into the small scale wave which brings the energy away from the AR region; it damps under the account for a weak dissipation in $\varepsilon_1^{(0)}$.

Characteristic value of the amplitude $E_r^{(0)}$ of the fundamental harmonic of the radial component of the wave electric field within AR region can be evaluated from (9) and (10) by the order of magnitude as follows,

$$E_r^{(0)}\sim ak_1A/N_z^2\sim\left(\omega^4a^4N_s^2/\left(\delta^4N_z^8c^4\right)\right)^{\frac{1}{5}}A\,. \tag{11}$$

We can evaluate the characteristic value of $E_r^{(\pm1)}$, $E_r^{(\pm2)}$ … within the AR region by the order of magnitude using the eq. (8) as follows,

$$E_r^{(\pm1)}=-(\delta/2k_s)dE_r^{(0)}/dr\,,\ E_r^{(\pm2)}=-(\delta/4k_s)dE_r^{(\pm1)}/dr\,... \tag{12}$$

Analysis of (12) indicates that although the amplitudes $E_r^{(\pm 1)}$ of satellite harmonics grow within AR region even more rapidly than the amplitude $E_r^{(0)}$ of the fundamental harmonic - the amplitudes $E_r^{(\pm 1)}$ remain less than $E_r^{(0)}$ within AR region. Comparison of the values (7) and (12) shows that the difference in order of magnitude between $E_r^{(0)}$ and $E_r^{(\pm 1)}$ is not so pronounced within AR region as outside of it.

Studying the structure of the coefficients in eq. (8) and expression (10) for k_1 one can easily find out that the basic equation (8) becomes meaningless in the case $k_s=\pm 2k_z$. This condition is treated as follows: axial wave lenght of the fundamental harmonic of the wave is twice as large as the pitch length. Plasma Alfven heating in this resonant case, in which both fundamental harmonic of the wave envelope coupled due to plasma helical inhomogeneity have their AR in the same place, was studied by Girka et al (2003) earlier.

4. On the applicability of the obtained results. Account for electron inertia, ion Larmor radius and collisions in (8) can be easily carried out by the aid of the following replacement:

$$\varepsilon_1^{(0)} - N_s^2 - 2N_s N_\vartheta \iota\, r/R \rightarrow$$
$$\varepsilon_1^{(0)} - N_s^2 - 2N_s N_\vartheta \iota\, r/R + i\varepsilon_1^{(c)} + \left(\varepsilon_T + N_s^2/\varepsilon_3\right) c^2/\omega^2\, \partial^2/\partial r^2 \,. \qquad (13)$$

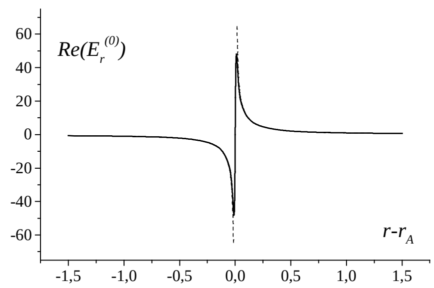

Figure 1: AR structure under account for the collisions (solid line).

The term ($i\varepsilon_1^{(c)}$) accounts for the collisions between plasma particles (see Akhiezer, Lapshin and Stepanov (1976)). AR structure under account for the collisions only is shown with solid line in the fig. 1. Effect of the collisions on the AR structure is usually weaker than that of finite ion Larmor radius ρ_{Li} in fusion devices. That is why the amplitude of the fundamental harmonic of the wave electric field ($E_r^{(0)} \sim A\omega/(N_z^2 \nu_{ab})$) is shown as the largest and the width of AR ($\Delta r_{coll} \sim a^* \nu_{ab}/\omega$) – as the narrowest as compared with the fig. 2 and 3. Here ν_{ab} is the frequency of collisions between the particles of species a and b.

The quantity of the factor ε_T in (13) accounts for ρ_{Li} (see Akhiezer et al (1975)). The finite electron inertia is also taken into account in (13) via the ε_3 component of the permittivity tensor. AR structure governed by the effect of ρ_{Li} and finite electron inertia is shown with solid line in the fig. 2. Effect of these two phenomena on the AR structure is assumed to be larger than that of the collisions. That is why the amplitude of the fundamental harmonic of the wave electric field ($E_r^{(0)} \sim (A/N_z^2)(a^*/\rho_{Li})^{2/3}$) is less one and the width of AR ($\Delta r_T \sim a^*(\rho_{Li}/a^*)^{2/3}$) – is the wider one as compared with the fig. 1.

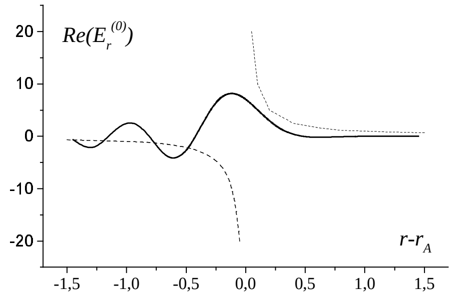

Figure 2: AR structure under account for the effect of ρ_{Li} and finite electron inertia (solid line).

Analyzing the basic eq. (8) with account for the replacement (13) one can find that effect of $\vec{B}_0$ weak helical inhomogeneity on the AR structure is more significant than those of finite ion Larmor radius ρ_{Li} and electron inertia if the following inequality is valid,

$$\delta^{12/5} >> (\rho_{Li}/a)^2 (k_z k_s a^{*2})^{6/5}. \qquad (14)$$

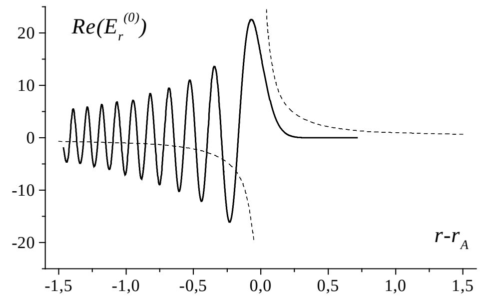

Figure 3. AR structure under account for the helical plasma inhomogenoity (solid line).

Obtained condition can be fulfilled in the peripheral plasma where the helical inhomogeneity of $\vec{B}_0$ is the most pronounced and plasma is colder than in the core. The condition (14) can be also treated as follows. Radial deviation r-r_A of the magnetic surface (2) from the cylinder with average radius is greater than that characteristic width $(\rho_{Li}^2 a^*)^{1/3}$ of AR region that is well known for the case of the straight magnetic field.

Behaviour of the amplitude of the main harmonic of the radial electric field of the wave is shown in the fig. 3. with the solid line under the condition (14), in which the influence of the plasma helical nonuniformity on the AR structure is the most pronounced as compared with that of collisions, ρ_{Li} and finite electron inertia. That is why the amplitude of the fundamental harmonic of the wave electric field is shown as the smallest one and the width of AR – as the widest one as compared with the fig. 1 and 2.

5. RF power absorption. The power absorbed by a unit length of the plasma torus in the vicinity of the AR region consists of the work done by the field of the fundamental harmonic over the radial RF currents, $P_r = 0.5\,\mathrm{Re}\left\{\int j_r^* E_r\, 2\pi r dr\right\}$ and the work done over the axial RF currents, $P_z = 0.5\,\mathrm{Re}\left\{\int j_z^* E_z\, 2\pi r dr\right\}$. Analytical expressions for $P_{r,z}$,

$$P_r = \frac{r\omega}{4}\left|\frac{d\varepsilon_1}{dr}\right|^{-1} |A|^2_{|r=r_A} \int_{-\infty}^{+\infty} \mathrm{Im}(u_0(x))dx\,, \qquad P_z = \frac{r\omega k_1^3 a^2}{4|\varepsilon_3|^2\alpha^2}\mathrm{Im}(\varepsilon_3)_{|r=r_A}|A|^2\int_{-\infty}^{+\infty}|u_0'(x)|^2 dx\,, \quad (15)$$

coincide with those in the case when AR structure is determined by finite ρ_{Li} or finite electron inertia at least by the order of magnitude.

6. Conclusions. Distribution of electromagnetic fields within AR region is determined and analysed under the condition (14) that means that the helical inhomogeneity of the confining magnetic field is dominant as compared with effect of electron inertia or finite ion Larmor radius. Characteristic width of AR region is greater in this case than in the straight magnetic field under the same other conditions. Then the evaluation of the AR width that is well-known for the case when AR structure is governed by the finite ion Larmor radius would be replaced by that derived here,

$$(\rho^2_{Li}a^*)^{1/3} \rightarrow [\delta^4 a^*/(k^2_z k^2_s)]^{1/5}. \qquad (16)$$

Characteristic value (12) of the amplitudes of the fundamental harmonic of the radial electric field of the electromagnetic wave is less than that in the case of straight magnetic field under the condition (14).

Amplitudes of satellite harmonics grow when coming to the AR region more rapidly than amplitudes of fundamental harmonics (see (12)). That is why discontinuity of the solutions to Maxwel equations for electromagnetic waves fields known to take place in the cold approach in the straight magnetic field is removed in the helical magnetic field.

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