NOTES AND CORRESPONDENCE

Disqualifying Two Candidates for the Energy Balance of Oceanic Internal Waves

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ABSTRACT

It is shown that spreading of tidal energy across the wave continuum and bottom scattering play no role in the spectral energetics of ocean internal waves.

1. Introduction

At present the explanation of the oceanic internal wave spectrum in terms of generation, internal transfer and dissipation of wave energy is mainly hampered by two facts. First, except for some observed correspondences between wind fluctuations and the wave field in the upper ocean (Käse and Olbers, 1979; Davis *et al.*, 1981) the extensive search in field data failed to reveal any obvious relation of the wave spectrum to external forcing (e.g., Wunsch and Webb, 1979). The deep-ocean wave field appears to be in a universal equilibrium (see, e.g., Garrett and Munk, 1979). Theoretical investigations, on the other hand, though demonstrating the importance of at least half a dozen processes in the spectral balance (see, e.g., Olbers, 1981) did not yet yield a unified picture of the spectral balance on the basis of these processes. The particular role of wave-triad interactions in shaping of the spectrum, however, seems to be obvious (McComas and Müller, 1981). Surprisingly, almost all processes which have been studied within a spectral framework affect the spectrum at a rate of $\sim 10^{-3}$ W m^{-2} implying a time scale of roughly 10 days for the observed energy of 10^3 J m^{-2} in the wave field.

The purpose of this note is to narrow down the list of further possible candidates (see, e.g., Müller and Olbers, 1975) which have been proposed to play a role in the spectral energetics. We show that spreading of tidal energy across the internal wave continuum by resonant coupling and internal spectral transfer by scattering of waves at irregularities of the ocean floor are irrelevant for the energy balance.

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2. Spreading of tidal energy across the wave continuum

It has been suggested (e.g., Bell, 1975a) that internal tides generated by interaction of the barotropic tidal current with abyssal topography may spread their energy over the internal wave spectrum by weakly nonlinear coupling of the tidal wave components with waves in the continuum. The rate of change of tidal energy E_T in the mode λ at wavevector $\mathbf{k}$ by this process is given by (Olbers, 1974)

$$\frac{dE_T}{dt} = -\delta_D E_T, \tag{1}$$

where the decay rate at mode number λ, wave vector $\mathbf{k}$ with frequency $\omega^\lambda = \omega^\lambda(\mathbf{k})$

$$\begin{aligned}\delta_D = \delta_D(\mathbf{k}) = \sum_{\mu,\nu} \int d^2k' \int d^2k'' \\ \times \{T^+\delta(\mathbf{k} - \mathbf{k}' - \mathbf{k}'')\delta(\omega^\lambda - \omega^\mu - \omega^\nu) \\ \times [A^\mu(\mathbf{k}') + A^\nu(\mathbf{k}'')] + 2T^-\delta(\mathbf{k} + \mathbf{k}' - \mathbf{k}'') \\ \times \delta(\omega^\lambda + \omega^\mu - \omega^\nu)[A^\mu(\mathbf{k}') - A^\nu(\mathbf{k}'')]\}\end{aligned} \tag{2}$$

is an integral of the ambient wave action spectrum $A^\lambda(\mathbf{k})$, weighted by positive cross sections T^+ and T^-. The first term of δ_D corresponding to sum interactions is always positive but contributes only if $\omega > 2f$. For semidiurnal tide, $\omega = M_2$, this is north of 28.7° latitude. The second term arising from difference interactions may attain negative values and contributes everywhere. The energy lost (or gained) by the tide must appear in (or taken from) the internal wave continuum. The expression (2) has been derived on the basis of the weak interaction approximation which has fallen into some discredit with respect to internal wave interactions (e.g., Holloway, 1980). The main objection against the approach is that for a given spectral shape the decay rates may be comparable or even less than the frequencies of the interacting wave triads. This is not the case for the interactions considered here.

The decay rate δ_D has been evaluated by Olbers (1974) for an early version of the Garrett and Munk model spectrum and recently by Pomphrey *et al.* (1980) for the GM76 model (with high-wave-number slope −2). Fig. 1 is an enlargement of the low-frequency and low mode-number region of their Fig. 4 which displays the results for an exponential stability frequency and a Coriolis parameter $f = 7.3 \times 10^{-5}$ s^{-1} corresponding to a latitude of 30° where $M_2/f = 1.93$. The baroclinic tidal energy is predominantly in the lowest few modes with a typical energy density of $\sim 10^2$ J m^{-2} (Wunsch, 1975). As shown in Fig. 1, decay times $1/\delta_D$ range from 20 to 200 days so that the tidal loss dE_T/dt amounts to $\sim 5 \times 10^{-6}$ to 5×10^{-5} W m^{-2}. This result cannot immediately be extrapolated to the global problem since it strictly applies to the latitude of 30°. Here the spreading of tidal energy over the wave continuum is far too ineffective in the energetics of the wave field comparing dE_T/dt to other wave generation processes. Approaching the equator this statement must slightly be weakened. The behavior of the decay may be inferred from a parameterization given by Pomphrey *et al.* (1980) according to which

$$\delta_D/f = \delta_D(\omega/f, 30°)/f(30°) \quad (3)$$

for frequencies $\omega > 3f$. Thus, for example, at 5.7° latitude, where $M_2/f = 10$ and $f = 1.4 \times 10^{-5}$ s^{-1}, the scaling (3) yields a decay time of about 10 days and an energy flux of about 10^{-4} W m^{-2} which is still an order of magnitude too small. At high latitudes where $M_2/f < 2$, the decay is not necessarily positive (the spectrum at λ and $\mathbf{k}$ may grow on account of the continuum wave spectrum) as indicated in the results of Pomphrey *et al.* (1980). This feature constitutes an a priori objection against the possibility that the internal wave-tidal coupling plays a uni-

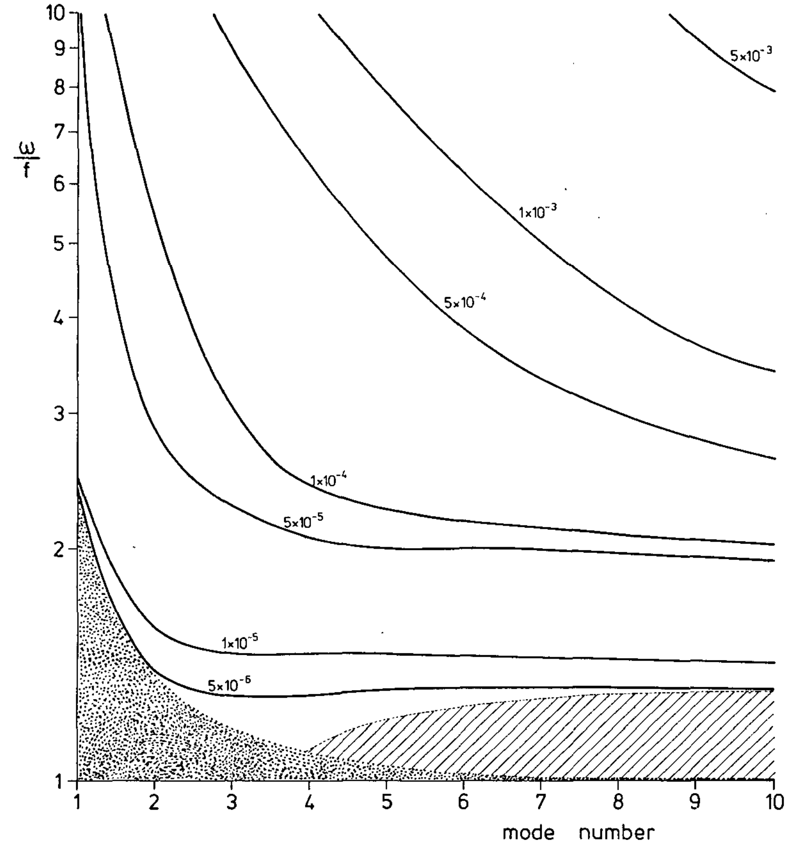

FIG. 1. Decay rate δ_D of a line in an ambient GM76 model spectrum, scaled by $N = 5.2 \times 10^{-3}$ s^{-1} versus frequency ω/f and mode number. In the hatched region δ_D is negative corresponding to a growth of the line. In the speckled region $\delta_D = 0$ because there are no triads in the spectrum satisfying the resonance condition.

versal role in the energy balance of the internal wave field.

3. Bottom scattering

The effect of random bottom inhomogeneities on internal wave modes was studied by Cox and Sandstrom (1962) in an attempt to explain the conversion of the surface tide into internal tides. The source term at the horizontal wave vector **k** and the (local) vertical wavenumber β for a random wave field is of the form (Müller and Olbers, 1975)

$$S_{bs}(\mathbf{k}, \beta) = \int d^2k' \int_0^\infty d\beta' T^{bs} F^{bs}(\mathbf{k} - \mathbf{k}') \times \{v_3(\mathbf{k}, \beta)A(\mathbf{k}', \beta') + v_3(\mathbf{k}', \beta')A(\mathbf{k}, \beta)\}, \quad (4)$$

with a cross section T^{bs}, a bottom spectrum $F^{bs}(\mathbf{k})$ and vertical group velocity $v_3(\mathbf{k}, \beta)$. This flux has not yet been adequately evaluated. The first term represents the flux to the three-dimensional wave vector $(\mathbf{k}, \beta)$ arising from the scattering of the downward propagating spectral component, the second term describes the extraction of energy at $(\mathbf{k}, \beta)$ due to scattering of this component into other upward propagating waves. An order of magnitude of the source terms follows from (4)

$$\int d^2k \int d\beta |S_{bs}| \Big/ \int d^2k \int d\beta v_3 A \approx \frac{\langle s^2 \rangle}{\pi}. \quad (5)$$

Typical slopes of abyssal hill topography s are in the range 10^{-2} to 10^{-1} (Bell, 1975b) so that roughly a fraction 10^{-3} to 10^{-2} of the downward propagating energy flux will be redistributed in wavenumber space. Since the downward flux is at most some 10^{-3} W m^{-2} (Olbers, 1981) we may conclude that, compared to the redistribution by wave-wave interaction, the bottom scattering is negligible.

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Observations of Quasi-Two-Dimensional Turbulence in Tidal Currents

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ABSTRACT

Observational evidence for the existence of quasi-two-dimensional turbulence in tidal currents is derived from the auto- and cross-correlation spectra of vertically separated current meters. The observed quasi-two-dimensional turbulence seems to exhibit a k^{-1} wavenumber spectrum over an eddy length-scale interval from 600 to 60 m, which is thought to be produced by topographic vortex stretching over the irregular sea bed in a tidal area.

1. Introduction

Tidal currents in the vicinity of the coast belong to the most vigorous flow phenomena in the sea, characterized by Reynolds numbers of the order of 10^7–10^8 (Grant *et al.*, 1962). Nonlinear instabilities therefore give rise to a large variety of "secondary currents" ranging from the smallest three-dimensional eddies in the dissipation range (Grant *et al.*, 1962)—with a scale of $\sim 10^{-3}$ m, carried along by

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