Reply

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7 August 1986 and 6 October 1986

McIntosh (1986) raises an important question about the interpretation of the results of the optimization procedure employed by us in Schröter and Wunsch (1986). To state his objection in slightly different terms, it is that the curl of the wind stress, found to produce an extreme property of the ocean circulation, may be nonsmooth. If sufficiently nonsmooth, the finite difference representation of the stream function field will not be an accurate representation of the solution to the underlying partial differential equation.

If the windstress curl found to generate the extreme does not satisfy appropriate mathematical conditions, then clearly, as McIntosh notes, no finite difference representation, no matter how refined the grid, will ever adequately describe the solution to the partial differential equation. We agree that such behavior is indeed a *potential* source of trouble with the procedure we used and it is useful to have the point raised explicitly. However, we do not believe that it is a problem *in practice.*

The grid for the circulation problem was deliberately chosen to be quite coarse, but in conjunction with the frictional parameters, was sufficiently dense to specify all the length scales of interest. Curl of the wind stress values are only defined on the grid points. The correction, e, to the initial estimate of the wind-stress curl was also defined only on the grid points, and specified to lie within certain bounds—bounds which thereby prevent the gradients of the windstress curl from ever exceeding a given, rather modest, value.

Application of a finite difference approximation to the differential equation diminishes the set of possible solutions to those that can be represented by the eigensolutions of the discretized differential operator. For a uniform grid, the eigenfunctions of the problem are Fourier components up to order $2\Delta x$, where Δx is the grid spacing. These components and their numerical derivatives are well behaved, but they cannot describe smaller scale features. Thus, solutions with unbounded derivatives are excluded by the finite difference formulation.

It seems to us unreasonable to speculate about the hypothetical behavior of the windstress curl on spatial scales smaller than those resolved by the numerical grid, scales which play no role in the optimization. But suppose the grid were refined to arbitrarily small scales. One might redo the calculation permitting variations on spatial scales which were previously deliberately filtered out; then the behavior McIntosh notes could become an issue. Alternatively, and probably more reasonably if the grid were being refined, one would seek values at the new, denser, grid points which were instead a smooth interpolation of the previous coarse grid calculation. For ever-finer grids, we can prevent nonsmooth behavior in a number of ways; for example, the simple expedient of including an additional constraint, one limiting a norm of e or of its derivatives to some specified upper bound would suffice. Similarly, if the oscillation that McIntosh notes, between maximum and minimum values at neighboring grid points is deemed undesirable, it can be controlled with additional constraints.

There are several other ways one can obtain guaranteed smoothness: a simple one is to represent the wind field (or other unknowns) in a smooth set of basis functions (like the Wahba and Wendelberger, 1980, splines). Alternatively, the demand for an extreme can be applied to a suitably filtered form of the unknowns and the difference operators applied only to those averages.

The general optimization procedure we employ has the advantage that the relationship between the constraining model, written in finite difference form, and the objective function we seek to drive to an extreme, is explicit and transparent. The variational procedure described by Sasaki (1971), Wahba and Wendelberger (1980), Bennett and McIntosh (1982), and others, has the advantage, as McIntosh points out, of providing comparatively easy control of solution smoothness; it is a very elegant method. But it has the disadvantage of generating as an intermediary an Euler–Lagrange partial differential equation whose solution may be difficult to obtain, and whose interpretation may not be obvious. In any case, the availability of more than one procedure for combining dynamics with noisy observations can only be regarded as beneficial because in specific cases, one or the other may prove to be more convenient.

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