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Concrete Operations and Attentional Capacity

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In order to test predictions regarding the attentional capacity requirements of Piaget's stage of concrete operations, a battery of concrete operational tasks (class inclusion, transitivity of length and of weight, multiplication of classes, multiplication of relations) and two measures of attentional capacity (backward digit span and the Figural Intersection Test) were administered to 120 first, second, and third graders. With the exception of class inclusion, the results supported the hypothesis that three units of capacity would be necessary for solving each concrete operational task. Class inclusion was considerably easier than the other concrete operational tasks and was solved by nearly all children, a result which suggests that this version of the class inclusion task could possibly be solved without operational reasoning. The data also supported the prediction that transitivity of length and of weight, although differing in overall difficulty, would not differ in capacity demand. Instead, the horizontal decalage between length and weight was explained in terms of children's tendency to infer relative weight from relative size. © 1989 Academic Press, Inc

In Piaget's first published articles on cognitive development (Piaget, 1921, 1923), he argued that a relation of mutual dependence exists between children's forms of reasoning and the "breadth" of their attention, understood in terms of "the number of objects [of thought] capable of being simultaneously associated in the field of attention" (Piaget, 1923, pp. 170–171). In his words, the field of attention "conditions" the logical form of children's reasoning (Piaget, 1923, p. 172) even as the logical form "collaborates" in widening that field (Piaget, 1921, p. 480). Similar ideas may be found in his later writings, although the terminology varied. In his book on equilibration he suggested that limitations in children's

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reasoning coincide with a narrowness in their "field or 'span' of assimilation" (Piaget, 1975/1985, p. 114), and in *Experiments on Contradiction* he stated that overcoming contradictions involved a widening of "the system of reference" in terms of the number of concepts simultaneously coordinated (Piaget, 1974/1980, p. 242). Although Piaget called attention in these passages to a quantitiative dimension in children's reasoning, he did not conceive of it as a primary cause of cognitive development nor did he attempt to measure this dimension separately in order to predict performance on specific reasoning tasks.

Beginning with Pascual-Leone (1970), the prediction of children's reasoning from estimates of their attentional capacity has been a major goal of several neo-Piagetian theories of cognitive development (see also Case, 1985; Halford, 1982; McLaughlin, 1963; Pascual-Leone, 1980; reviewed by Chapman, 1987). A common feature of these theories is the proposition that the growth of attentional capacity is a *causal* factor in the development of children's reasoning, such that a certain level of capacity is necessary but not sufficient for a given level of reasoning. Although substantial evidence exists to support a general dependency of reasoning upon capacity (see reviews by Case, 1985; Halford, 1982), the above-mentioned theories differ with respect to (a) the nature of the capacity construct, (b) the ways in which units of capacity are defined and measured, and (c) the specific relations of correspondence between units of capacity and stages or levels of cognitive development.

The general purpose of the present study is to provide a test of one approach toward the resolution of these issues, based on Chapman's (1987) structural–functional model of children's reasoning. First, the basic assumptions of the model are stated, and then the predictions of the model regarding the relation between concrete operations and attentional capacity are described. The term "attentional capacity" is used generically throughout the article to refer to any quantitative dimension in the contents of immediate attention, regardless of how that dimension might be defined in the context of specific theories.

Basic Assumptions

One of the main assumptions of the proposed model is that the phenomenon to be investigated in studies of children's reasoning is the *form of reasoning* exhibited by children in justifying their judgments in problem-solving tasks. The term "form of reasoning" is used to refer to the type of inferential relation uniting children's judgments (as the conclusions of their inferences) with their explanations (the premises) of those inferences. The object of investigation is conceived in terms of children's ability to exhibit a given form of reasoning in their verbal argumentation, rather than in terms of a mental process acting as a causal or functional antecedent of children's verbalizations (cf. Brainerd, 1973, 1977; Reese & Schack, 1974). As an example of this approach, Chapman and Lindenberger (1988) distinguished between "functional" and "operational" forms of reasoning on a transitivity task, depending on whether children justified judgments of length as a function of spatial orientation ("This one is longer, because it's on the right") or as the product of an operational composition ("A is longer than C, because A is longer than B and B is longer than C").

A second assumption is that the structure of children's forms of reasoning can be represented in terms of the relations among *operatory variables*, understood as the aspects or dimensions of a task situation that the subject recognizes as potentially varying (as assuming a range of possible values), within the experiential context of the task. So defined, operatory variables can be distinguished from operatory *constants:* aspects or dimensions that the subject recognizes as invariant in the particular context. In a typical reasoning task, both the premises and the conclusion stated by the subject can be described structurally as expressing particular values of different operatory variables. For example, in the Chapman and Lindenberger (1988) study, children using functional reasoning recognized potential variation in both the *relative length* and the *spatial orientation* of each pair of sticks to be compared (one stick could be either longer or shorter and either to the right or to the left of the other). Therefore, two operatory variables, corresponding to relative length and spatial orientation, respectively, were involved.

The third basic assumption of the model is that relations of correspondence exist between the structural and the functional levels of description. In particular, we propose (a) that the structural act of assigning a value to an operatory variable corresponds to the functional consumption of a fixed amount of attentional capacity and (b) that this fixed amount of capacity can be considered a "unit." Thus, the total capacity requirement of a given form of reasoning is equal to the number of operatory variables that are assigned values simultaneously in employing that form of reasoning in a particular task. Operatory constants (aspects of the task situation that the subject regards as invariant) generally can be coded in long- or intermediate-term memory and accordingly do not require capacity in immediate attention.

Observe that this third assumption is consistent with both continuous and discontinuous models of attentional capacity; all that is asserted is that the structural process of value assignment corresponds to the functional consumption of a fixed amount of capacity. For present purposes, it is immaterial whether this fixed amount of capacity represents an indivisible quantum or an arbitrary division of a continuous quantity.

Task Analysis

The foregoing assumptions provide guidelines for the analysis of particular reasoning tasks. First, the forms of reasoning used by children in solving the problems in question are determined by classifying their responses (judgments and explanations) and ordering them developmentally by type. Second, the structures of each form of reasoning are identified in terms of the operatory variables involved and their interrelations. Third, the attentional capacity demand for each form of reasoning is determined from the number of operatory variables that are assigned values simultaneously in employing that form of reasoning.

Each of these three steps corresponds to a different phase in the Piagetian or neo-Piagetian research programs. The first step corresponds to the taxonomic aspect of Piaget's research in cognitive development: the classification of different forms of reasoning and the seriation of these forms into developmental sequences. (On the taxonomic character of Piaget's research, see Chapman, 1987, 1988.) The second step corresponds to Piaget's structural analyses of the thinking characteristic of different cognitive–developmental stages in terms of operatory logic (e.g., Piaget, 1949/1972). However, one cannot assume that Piaget's own structural analyses are adequate in every case. In particular, we suggest that structural analysis must closely conform to the verbal arguments actually used by children. In some cases, this approach will lead to results different from those reached by Piaget (cf. the discussion of class inclusion in the Discussion section). Finally, the third step of analysis corresponds to the neo-Piagetian goal of determining the functional information-processing requirements of different structurally defined developmental stages (Case, 1985; Chapman, 1987; Halford, 1982; Pascual-Leone, 1970, 1984). As previously mentioned, little agreement exists among different investigators on exactly how this project ought to be accomplished (Case, 1985; Chapman, 1987; Halford, 1982; Pascual-Leone, 1970, 1980, 1984).

In the following sections, we apply this method of analysis to the tasks exemplifying the four major groupings of concrete operations according to Inhelder and Piaget (1959/1964): class inclusion, the transitivity of length and of weight, the multiplication of classes, and the multiplication of relations. Two predictions are generated by this analysis. The first involves the attentional capacity requirements of concrete operations in general, and the second an explanation of the typical horizontal decalage between length and weight on the transitivity task.

Class inclusion. In the typical Piagetian class inclusion task (Piaget & Szeminska, 1941/1952), children are shown a collection of objects (e.g., wooden beads), most of which are of one color (e.g., red) and the rest of another color (e.g., white). Children are asked if there are more red beads or more wooden beads and are credited with class inclusion if they indicate that there are more wooden beads because the red beads are *included* in the total class of wooden beads. According to Piaget and Szeminska's structural analysis, class inclusion is characterized by an operation of class addition having the form $A + A' = B$, where A

represents a subclass of objects defined by a particular property (e.g., the *red* beads), B represents the supraordinate class defined by some other property (the *wooden* beads), and A' is the complement of A under B (the wooden beads that are *not* red). Understanding class inclusion thus implies understanding that a supraordinate class (B) is necessarily equivalent to the sum of its subclasses (A and A'). Applying this operation in a concrete task situation involves assigning particular values to the class variables A, A', and B. Children must view (a) all the red objects as constituting a single class, and similarly for (b) all the wooden beads and (c) the beads that are not red. In other words, class inclusion involves what Inhelder and Piaget (1959/1964) called the coordination of *intension* (the defining properties of a class) and *extension* (the objects belonging to that class) (see also Pascual-Leone & Smith, 1969). In our view, the formula $A + A' = B$ ought not to be construed as a computational routine by which a correct answer to the class inclusion problem is *obtained*, but as an expression of the logical relations among A, A', and B that *result* if intension and extension of these classes are correctly coordinated. With respect to the capacity requirements of class inclusion, the important thing is that all three class variables, A, A', and B, must be assigned values simultaneously. Under the assumption that each simultaneous value assignment requires a "unit" of capacity, the operation of class addition would require a minimum of 3 such units.

Transitivity of length and weight. In the transitivity of length task, children are first shown two sticks A and B identified by color, such that B is visibly longer than A (i.e., $A < B$). Then they are shown sticks B and C, such that $B < C$ is also visible. Finally, they are shown A and C together in such a way that they cannot see the difference in length and are asked which is longer, A or C? Children are credited with transitive reasoning if they conclude that $A < C$ because $A < B$ and $B < C$. According to Piaget's (1949/1972) operatory logic, transitive reasoning of this kind involves an operation called the "addition of asymmetrical relations," having the form $(A < B) + (B < C) = (A < C)$. This operation derives from the action of seriating objects according to a physical dimension such as length or weight. In terms of the present model, applying this operation in practice involves the cognitive construction of an ordered series $X \rightarrow Y \rightarrow Z$, where X, Y, and Z are the respective "positions" in the series, and the sign "$\rightarrow$" represents the relation of succession constituted by the operation of "placing" one object after another according to some physical dimension. (On the construction of an ordered series through operations of "placement," see Piaget, 1946/1970, pp. 280–285.) The series can be constructed from the pairwise physical comparisons $A < B$ and $B < C$, and the physical relation between A and C can then be determined by noting the relative positions of these two objects in that series. With respect to the capacity requirements of transitive reasoning, three relational variables ($X \rightarrow Y$, $Y \rightarrow Z$, and $X \rightarrow Z$) must be assigned values simultaneously ($A < B$, $B < C$, and $A < C$, respectively). Under the assumption that each value assignment requires 1 "unit" of capacity, the total capacity demand of the task is 3 units. Moreover, the same amount of capacity should be required regardless of the physical dimension (e.g., length or weight) upon which objects are compared.

However, the foregoing analysis is valid only under the condition that the values of the variables in question are simultaneously represented in children's immediate attention. The analysis does *not* apply if premise comparisons are presented repeatedly, as in Bryant and Trabasso's (1971) memory-for-premises training paradigm. Repeated presentation of the comparisons $A < B$ and $B < C$ allows the series $A < B < C$ to be schematized in long-term memory (i.e., the relations $X \rightarrow Y$ and $Y \rightarrow Z$ acquire the *constant* values $A < B$ and $B < C$, respectively). Given such a scheme, the relation between A and C can then be determined from the series. As indicated, the values of only two variables ($X \rightarrow Y$ and $Y \rightarrow Z$) need be represented simultaneously in children's immediate attention in order to schematize the series. Thus, the Bryant–Trabasso paradigm requires only 2 rather than 3 units of capacity. When long-term memory is engaged, the demands on children's immediate attention are reduced relative to the "standard" transitive reasoning task in which premise comparisons are not presented repeatedly. Under the assumption that 2 "units" of attentional capacity develop by 5–6 years on the average, the foregoing analysis explains (a) why the Bryant–Trabasso paradigm is easier than the "standard" paradigm, and (b) why children younger than 5 years of age are generally unable to succeed on the Bryant–Trabasso version (Halford, 1984).

The hypothesis that 3 units of attentional capacity are necessary for success on the standard transitive reasoning task applies regardless of the physical content involved. In other words, the typical horizontal decalage between the transitivity of length and the transitivity of weight (Piaget & Inhelder, 1941/1974) is not to be explained in terms of a difference in capacity demand (i.e., along the same lines as Pascual-Leone, 1980, explained the decalage between conservation of physical quantity and that of weight). Instead, this decalage is explained by children's tendency to infer weight as a function of size, even after they are capable of the transitivity of length. Thus, Chapman and Lindenberger (1988) found the length–weight decalage to occur only when both the size and the weight of comparison objects were varied in an uncorrelated manner, and not when the objects were all the same size.

Multiplication of classes and relations. In assessing the multiplication of classes and the multiplication of relations, children were shown 2×2 matrices in which three of the four cells were filled with objects. In each case, they were asked to fill in the missing cell with an object that completed the matrix. In the multiplication of classes task, the rows of the matrix were defined by classes of objects differing in shape, and the columns by classes differing in color. In the multiplication of relations task, the rows and columns were defined by relations of shape and size, respectively. In each case, children were credited with passing the task if they (a) filled the empty cell with an object having the properties or relations defined by the relevant rows and columns and (b) referred to those properties or relations in explaining their choices. Three operatory variables must be assigned values in solving these tasks: one variable representing the class (or relational term) corresponding to the relevant row, one variable representing the class (or relational term) corresponding to the relevant column, and one variable representing the class intersection (or compound relational term) corresponding to the cell to be filled.

Summary

The foregoing analyses demonstrate that 3 units of attentional capacity are required for each of the concrete operational tasks considered. This conclusion is consistent with Pascual-Leone's (1970, 1984) previous prediction that 3 units of "mental capacity" are necessary for concrete operations. In Pascual-Leone's theory, that prediction followed as a generalization from theoretical analyses of specific concrete operational tasks (Pascual-Leone, 1976, 1980; Pascual-Leone & Smith, 1969). According to the present model, the same prediction can be seen to follow as a deductive consequence from Piaget's (1949/1972) definition of concrete operations as compositions of at least two transformational "elements" leading to a third within a total system. In this model, each of the three elements is represented by an operatory variable, and a unit of capacity is required for assigning a value to each variable. Therefore, a minimum of 3 units should be required for any instance of concrete operational reasoning.

The specific goal of this study was to test the prediction (a) that 3 units of attentional capacity should be necessary but not sufficient for concrete operational reasoning on all tasks and (b) that these tasks should be solved by at least some children having only 3 units of capacity (i.e., that 4 units should *not* be necessary). A secondary purpose was to test the prediction that the transitivity of length and the transitivity of weight should have the same capacity demand, although they might differ in overall difficulty for other reasons. Thus, 3 units of capacity should be necessary for both tasks, and both should in fact be solved by some children with only 3 units.

METHOD

Subjects

A total of 120 first, second, and third graders, including 20 boys and 20 girls at each grade level, were studied in two elementary schools in a middle- to lower-middle-class neighborhood in West Berlin. Mean ages of first, second, and third graders were 7.06 (SD = .42), 7.91 (SD = .42), and 8.95 (SD = .36) years, respectively. All children were given a battery of concrete operational tasks, including class inclusion, transitivity of length, transitivity of weight, multiplication of classes, and multiplication of relations, in one 45-min sitting. In addition, they were given two measures of attentional capacity: backward digit span and Pascual-Leone's (undated) Figural Intersection Test. Backward digit span was presented to children at the end of the same session in which they were tested on the concrete operational tasks. The Figural Intersection Test was administered in another session along with several other measures unrelated to this study.

Procedure

Concrete operations. The procedures followed for the class inclusion, multiplication of classes, and multiplication of relations tasks were identical to those described by Smedslund (1964), with one exception: The multiplication of relations task was given in a 2 × 2 matrix form similar to that followed in the multiplication of classes task in order to equate the number of cells children would have to consider on each task. (In Smedslund's original multiplication of relations task, children were shown only the linear series formed by the diagonal elements of a 3 × 3 or a 4 × 4 matrix and not the matrix as a whole.) In all three tasks, small pieces of plastic varying in color and shape were used. In the class inclusion task, children were shown a group of red plastic figures, 10 of which were round and 3 of which were square. They were then asked whether there were more "red ones" or more "round ones" and why. Children were credited with operational class inclusion if they gave a correct judgment (more red ones), followed by one of Smedslund's criterial explanations ("Because they're all red"; "Because there are some square ones, too"; "Because the round ones and the square ones are more than the round ones alone."). In the multiplication tasks, children were shown 2 × 2 matrices in which three of the four cells were occupied, and they were asked to choose an appropriate object to fill the remaining cell. Again, they were asked to justify their answers. They were credited with operational multiplication of classes or relations if their choice of objects and explanations reflected the classes or relations defining the rows and columns of the matrices (e.g., "this one goes there, because it's both round [referring to the columns] and big [referring to the rows]"). Following

Smedslund's prcedures, each of the foregoing tasks was repeated using different objects (e.g., white rather than red plastic figures for class inclusion), once with the objects uncovered and once covered within each trial. Children were credited with operational reasoning if they gave a criterial response (judgment plus explanation) in one of the two trials.

Transitivity of length and of weight was assessed according to the procedures described by Chapman and Lindenberger (1988) for standard transitivity tasks. Children were shown three comparison objects identified by color (sticks of slightly different lengths or balls of different sizes and weights). Objects were presented two at a time with the third object hidden out of sight. Length comparisons were demonstrated by holding the respective sticks upright next to one another on a tabletop across from the children being tested. Weight comparisons were demonstrated by means of a balance scale after making sure that children understood that the heavier of the two objects would go down. First, they were shown that one object (*A*) was longer (or heavier) than a second object (*B*), then that *B* was longer (or heavier) than a third object (*C*). The positions of the longer (heavier) objects to children's right or left were counterbalanced. Next, children were asked about the relative length or weight of *A* and *C*, with object *B* hidden out of sight. They were considered to have demonstrated operational transitivity if they gave a correct judgment and an explanation involving composition of relations (e.g., "*A* is longer than *C*, because *A* was longer than *B* and *B* was longer than *C*"). The sticks used in the length task were 14.1, 14.4, and 14.7 cm in length. The balls used in the weight task were 25 g (diameter 7.5 cm), 180 g (5.5 cm), and 340 g (6.5 cm).

Capacity measures. The backward digit span task was administered as described by Wechsler (1974), and the Figural Intersection Test according to Pascual-Leone (undated). A short form of the Figural Intersection Test was used, with four items per level of capacity from level 2 (two overlapping figures) to level 7 (seven overlapping figures). The children's task in this test was to place a dot in the configuration of overlapping figures printed on the left of the page such that the dot was contained within each of the individual figures shown separately on the right of the page. On two of the four items of each level, all of the overlapping figures were "relevant"; that is, the dot was to be placed inside all of them. On the remaining two items of each level, one "irrelevant" figure, a figure that should not contain the dot, was included. This procedure was followed in order to prevent the use of a simple centering strategy (i.e., "place the dot toward the center of the configuration of overlapping figures") that could have resulted in correct answers by chance.

In the backward digit span task, units of capacity were estimated in terms of the number of digits children successfully reversed, and a criterion of two successes out of three trials at each level of capacity was used for categorizing a child at a given level. In the Figural Intersection Test, estimated units of capacity were equal to the number of overlapping figures in test items successfully passed, under the assumption that 1 unit of capacity was necessary to "keep in mind" each individual figure. Children were classified at the highest capacity level for which they solved at least three of four items successfully.

Data Analysis

The data were analyzed by means of prediction analysis (Hildebrand, Laing, & Rosenthal, 1977). This method allows for tests of theoretically based predictions that particular "error cells" in a cross-classification matrix should be empty, given perfect measurement. The empirical "success" of such predictions is evaluated by means of the DEL statistic, a measure of proportional error reduction. The value of DEL represents the extent to which the obtained frequency of errors deviates from the number expected by chance, based on the marginal distributions. More specifically, DEL is equal to 1 minus the ratio of obtained to expected errors. Thus, a DEL value of 1.0 means that no errors were observed; a value of .50 means that the number of errors is 50% less than that expected; a value of zero means that the number of errors is equal to that expected; and so on.

The statistical significance of DEL is tested by computing its estimated variance, from which a z score or confidence interval can be derived. In the present study, two statistical tests are of interest: (a) whether or not DEL is significantly greater than zero (i.e., whether or not the frequency of obtained errors is less than the number expected), and (b) whether or not DEL is significantly less than 1.0 (i.e., whether or not the frequency of obtained errors is greater than zero). In addition to the success of one's predictions as measured by DEL, Hildebrand et al. (1977) recommend consideration of the *precision* of those predictions as measured by U, the a priori probability of an error given the marginal distributions. Because U is equivalent to the expected number of errors divided by the total sample and the latter is constant (= 120) across all analyses reported in this paper, information regarding the relative precision of those analyses is provided in each case by the expected number of errors itself. Prediction analyses were computed with the aid of computer programs provided by von Eye and Krampen (1987).

RESULTS

Capacity Measures and Age

Although no specific hypotheses regarding the relations between age and mental capacity or between the two measures of capacity used in this study were made, an analysis of those relations provides a useful check on the validity of the measures. Under the assumptions that capacity

TABLE 1
PERCENTAGE OF TOTAL SAMPLE BY AGE AND CAPACITY MEASURE

Capacity measures and units of capacity (k)	Age in years 6	7	8	9	Overall percentage
Backward digit span					
$k = 2$	6.7	6.7	3.3	0.8	17.5
$k = 3$	10.8	22.5	14.2	8.3	55.8
$k = 4$	0.0	5.8	12.5	5.0	23.3
$k = 5$	0.0	0.0	1.7	1.7	3.4
Figural Intersection Test					
$k = 1$	0.8	0.8	0.0	0.8	2.5
$k = 2$	9.2	5.0	6.7	2.5	23.3
$k = 3$	5.8	20.0	10.8	5.0	41.7
$k = 4$	1.7	7.5	8.3	3.3	20.8
$k = 5$	0.0	0.8	3.3	0.8	5.0
$k = 6$	0.0	0.8	2.5	1.7	5.0
$k = 7$	0.0	0.0	0.0	1.7	1.7
Overall percentage	17.5	35.0	31.7	15.8	100.0

Note. $N = 120$.

increases with age and that both capacity measures tap the same underlying dimension, one would expect (a) that both measures would show age-related increases in capacity and (b) that the two measures would yield generally the same capacity estimates. Also of interest is the extent to which each measure yields estimates consistent with the age norms specified in previous capacity theories.

A cross-classification of age and capacity measures is presented in Table 1. Children in the present sample were from 6 to 9 years old, and performance on the capacity measures is given in terms of estimated units of capacity (k) from 2 to 4, the range of interest in this study. As indicated in the table, estimated capacity on the backward digit span ranged from 2 to 5 units, with the great majority of children falling between 2 and 4 units. These results are consistent with Pascual-Leone's (1970) age norms for the growth of M-capacity (3 units at age 7–8, 4 at age 9–10). On the Figural Intersection Test, however, capacity estimates ranged from 1 to 7 units, a result that deviates from those age norms at the upper end of the distribution. This result can perhaps be explained by the fact that this task can potentially be solved through a sequential strategy of finding the common area between two figures and then progressively reducing the common area by considering each additional figure one at a time. Children's use of such a strategy could result in inflated capacity estimates (see Pascual-Leone, undated, p. 13n). The fact that the Figural Intersection Test yields capacity estimates with too wide a range is noted in the manual (Pascual-Leone, undated, p. 9).

TABLE 2
PERCENTAGE OF TOTAL SAMPLE BY CAPACITY MEASURES

Figural Intersection Test	Backward digit span			
	$k = 2$	$k = 3$	$k = 4$	$k = 5$
$k = 1$	0.8	1.7	0.0	0.0
$k = 2$	10.8	9.2	3.3	0.0
$k = 3$	4.2	28.3	8.3	0.8
$k = 4$	0.8	12.5	6.7	0.8
$k = 5$	0.0	3.3	1.7	0.0
$k = 6$	0.8	0.8	2.5	0.8
$k = 7$	0.0	0.0	0.8	0.8

Note. $N = 120$.

However, the use of a short version of the test in this study may also have increased the scatter of capacity estimated relative to the original version.

Consistent with the age norms, most children did not have 3 units of capacity on the Figural Intersection Test until 7 years of age. In contrast, most 6-year-olds already had 3 units of capacity on the backward digit span, suggesting some difference in the error rates of the two measures at the lower end of the distribution. In fact, the distributions of the backward digit span and Figural Interaction Test scores did not differ significantly among 6-year-olds: six children had higher scores on the former than on the latter, and four children had the reverse (sign test nonsignificant). Both measures were significantly correlated with age. The Pearson correlation between backward digit span and age was $r(118) = .41$, $p < .001$; that between the Figural Intersection Test and age was $r(118) = .33$, $p < .001$.

The cross-classification of the two capacity measures is presented in Table 2. The Pearson correlation between them was $r(118) = .43$, $p < .001$. With age partialed out, the two measures were still significantly correlated: $r(118) = .34$, $p < .001$. Consistent with the observation that capacity estimates on the Figural Intersection Test may overstate true capacity above $k = 4$, much of the scatter in Table 2 results from these high estimates; if estimates for the Figural Intersection Test are collapsed for $k = 4$ and above, the frequencies of observations in the diagonal cells (i.e., those cells in which estimates from the two tests agree) are consistently greater than those in the off-diagonal cells. On the whole, the distributions of backward digit span and Figural Intersection Test scores did not differ significantly in the total sample: 30 children had higher scores on the former than on the latter, and 34 had the reverse (sign test nonsignificant). In short, the results are consistent with the assumption that the two tests measure the same underlying dimension

TABLE 3
PERCENTAGE OF TOTAL SAMPLE BY AGE AND CONCRETE OPERATIONAL TASK

Concrete operational task	Age in years 6	7	8	9	Overall percentage	Correlation between age and performance
Class inclusion						
Fail	2.5	1.7	0.0	0.8	5.0	
Pass	15.0	33.3	31.7	15.0	95.0	.15
Transitivity of length						
Fail	11.7	22.5	16.7	3.3	54.2	
Pass	5.8	12.5	15.0	12.5	45.8	.28*
Transitivity of weight						
Fail	16.7	25.8	20.0	9.2	71.7	
Pass	0.8	9.2	11.7	6.7	28.3	.26*
Multiplication of classes						
Fail	10.8	16.7	5.8	1.7	35.0	
Pass	6.7	18.3	25.8	14.2	65.0	.39*
Multiplication of relations						
Fail	8.3	19.2	6.7	3.3	37.5	
Pass	9.2	15.8	25.0	12.5	62.5	.26*

Note. $N = 120$.
* $p < .01$.

of capacity, at least for the range of interest in this study (from $k = 2$ to $k = 4$).

Concrete Operations and Age

The cross-classification of age and concrete operational performance is presented in Table 3. As shown in this table, pass rates increased with age on all tasks except class inclusion, on which performance was nearly perfect from the beginning. This age increase is reflected in the significant point-biserial correlations between age and performance given in the right-hand column of the table (again, with the exception of class inclusion). The decalage between length and weight is manifest in the fact that more children passed the transitivity of length task at all ages. In the total sample, 23 children passed the length task without passing the weight task, but only 2 children showed the reverse pattern (sign test, $z = 4.00$, $p < .001$).

Concrete Operations and Attentional Capacity

The major predictions of this study had to do with the relation between concrete operational performance and attentional capacity. Specifically, the possession of 3 units of capacity was predicted to be a necessary but not sufficient condition for passing the concrete operational tasks. In other words, children having only 2 units of capacity should not be able to pass any of the concrete operational tasks. A further prediction was that 4 units of capacity should *not* be necessary for concrete operational performance: Some children with only 3 units of capacity should indeed be able to solve the respective tasks.

In order to minimize false negative measurement error, children's scores on the two capacity measures were combined. The rationale behind this procedure was analogous to that underlying factor analysis: By combining variables, sources of error unique to each separate measure can be minimized. Thus, some children may have failed to reverse three digits not because they lacked 3 units of capacity, but because they lacked a sufficient facility with numbers. Similarly, some children may have failed to attain a given level of capacity on the Figural Intersection Test not because they lacked sufficient capacity, but because they lacked a familiarity with geometric figures. Such sources of measurement error were reduced by classifying children at a given capacity level if they had attained that level on *one or the other* of the two measures.

The cross-classification of concrete operational tasks and levels of capacity is presented in Table 4. With respect to concrete operations, children were divided into those passing and those failing each task, as indicated in the rows of the table. With respect to capacity levels, two separate analyses were conducted: In the left half of Table 4, children were divided into those having 2 or fewer units of capacity ($k \leq 2$) on both measures versus those having 3 or more units ($k \geq 3$) on one or the other measure. In the right half of the table, children were divided into those having 3 or fewer units ($k \leq 3$) on both measures versus those with 4 or more units ($k \geq 4$) on one or the other measure. Cross-classification of concrete operational tasks and capacity levels resulted in a series of 2×2 tables as indicated by the boxes in the body of Table 4. The frequencies of obtained errors (with expected errors in parentheses) are shown in italics within each 2×2 table, and the DEL values pertaining to those error cells are given immediately to the right of each 2×2 table.

According to the hypothesis that 3 units of capacity should be necessary for solving each concrete operation task, the frequency of obtained errors in the *left* half of Table 4 should be significantly less than those expected. In other words, the confidence intervals around DEL should not include *zero*. As shown in this table, this condition was met for all concrete operational tasks except class inclusion. In fact, the confidence interval around those DEL values both excluded zero and included 1.0. In other words, the frequency of obtained errors was significantly less than the number expected, but not significantly greater than zero. Such a result is precisely what would be expected under the assumption (a) that 3 units of capacity were necessary for solving the respective concrete operational tasks and (b) that capacity was measured with low error. As

TABLE 4

FREQUENCIES BY TASK AND CAPACITY LEVEL (CAPACITY MEASURES COMBINED)

Concrete operational task	Capacity levels and DEL values					
	$k \leq 2$ on both measures	$k \geq 3$ on one or both measures	DEL[a]	$k = 3$ on both measures	$k \geq 4$ on one or both measures	DEL[a]
Class inclusion						
Fail	2	4		5	1	
Pass	*12 (13.3)*	102	.10 ± .50	*61 (62.7)*	53	.03 ± .08
Transitivity of length						
Fail	13	52		47	18	
Pass	*1 (6.4)*	54	.84 ± .31	*19 (30.3)*	36	.37 ± .18
Transitivity of weight						
Fail	13	73		58	28	
Pass	*1 (4.0)*	33	.75 ± .49	*8 (18.7)*	26	.57 ± .25
Multiplication of classes						
Fail	12	30		32	10	
Pass	*2 (9.1)*	76	.78 ± .32	*34 (42.9)*	44	.21 ± .13
Multiplication of relations						
Fail	12	33		34	11	
Pass	*2 (8.8)*	73	.77 ± .33	*32 (41.3)*	43	.22 ± .13

Note. Error cells indicated by italics (with expected errors in parentheses).

[a] With 95% confidence interval.

indicated by the frequencies of expected errors (given in parentheses), the precision of the predictions in these analyses was relatively low. This result is attributable to the fact that few children had 2 or fewer units of capacity on both measures.

According to the further hypothesis that some children with only 3 units of capacity *should* be able to solve the concrete operational tasks, the frequency of obtained errors in the *right* half of Table 4 should be significantly greater than zero. In other words, the confidence intervals around DEL should not include *1.0*. As shown in the table, this condition was met for all concrete operational tasks, significantly including transitivity of weight. As indicated by the frequencies of expected errors, the precision of these analyses was high relative to that of the previous analyses, a result of the fact that a relatively high proportion of the sample had a capacity of $k \leq 3$ on both measures.

In order to ensure that the results shown in Table 4 were not attributable to just one of the two capacity measures, the analyses just described were redone for each measure separately. The results of these analyses are presented in Tables 5 and 6 for the Figural Intersection Test and backward digit span, respectively. The formats of these tables are identical with that of Table 4.

Consistent with the assumption that combining capacity measures would result in less measurement error, the DEL values shown in Tables 5 and 6 were smaller than those in Table 4. Nevertheless, the overall results were roughly the same for each measure as for both measures combined. For the Figural Intersection Test (Table 5), the confidence intervals around DEL in the left half of the table excluded zero for all concrete operational tasks except class inclusion, and the confidence intervals around DEL in the right half of the table excluded 1.0 for all tasks. For backward digit span (Table 6), the confidence intervals for DEL on the left side of the table excluded zero only for transitivity of length and multiplication of classes, but the confidence intervals for DEL on the right side of the table excluded 1.0 for all tasks.

DISCUSSION

With the exception of class inclusion, the results clearly supported the primary hypothesis of this study: that 3 units of attentional capacity (but not 4) should be necessary for passing the concrete operational tasks investigated. This conclusion followed from the findings that (a) the number of children passing concrete operational tasks with 2 or fewer units of attentional capacity on both capacity measures was significantly less than the number expected and (with capacity measures combined) not significantly greater than zero; (b) the number of children passing concrete operational tasks with 3 or fewer units of capacity was indeed significantly greater than zero. These findings are consistent with the

TABLE 5
FREQUENCIES BY TASK AND CAPACITY LEVEL (FIGURAL INTERSECTION TEST)

Concrete operational task	Capacity levels and DEL values					
	$k \leq 2$ on both measures	$k \geq 3$ on one or both measures	DEL[a]	$k \leq 3$ on both measures	$k \geq 4$ on one or both measures	DEL[a]
Class inclusion						
Fail	4	2		5	1	
Pass	*27* (*29.45*)	87	.08 ± .25	*76* (*76.95*)	38	.01 ± .05
Transitivity of length						
Fail	23	42		50	15	
Pass	*8* (*14.21*)	47	.44 ± .33	*31* (*37.13*)	24	.17 ± .16
Transitivity of weight						
Fail	27	59		65	21	
Pass	*4* (*8.78*)	30	.54 ± .39	*16* (*22.95*)	18	.30 ± .27
Multiplication of classes						
Fail	18	24		37	5	
Pass	*13* (*20.15*)	65	.35 ± .29	*44* (*52.65*)	34	.16 ± .09
Multiplication of relations						
Fail	20	25		40	5	
Pass	*11* (*19.38*)	64	.43 ± .28	*41* (*50.63*)	34	.19 ± .10

Note. Error cells indicated by italics (with expected errors in parentheses).
[a] With 95% confidence interval.

TABLE 6
FREQUENCIES BY TASK AND CAPACITY LEVEL (BACKWARD DIGIT SPAN)

Concrete operational task	Capacity levels and DEL values					
	$k \leq 2$ on both measures	$k \geq 3$ on one or both measures	DEL[a]	$k \leq 3$ on both measures	$k \geq 4$ on one or both measures	DEL[a]
Class inclusion						
Fail	2	4		5	1	
Pass	*19 (19.95)*	95	.05 ± .37	*83 (83.60)*	31	.01 ± .04
Transitivity of length						
Fail	17	48		58	7	
Pass	*4 (9.63)*	51	.58 ± .38	*30 (40.33)*	25	.26 ± .15
Transitivity of weight						
Fail	17	69		73	13	
Pass	*4 (5.95)*	30	.33 ± .58	*15 (24.93)*	19	.40 ± .25
Multiplication of classes						
Fail	13	29		36	6	
Pass	*8 (13.65)*	70	.41 ± .40	*52 (57.20)*	26	.09 ± .08
Multiplication of relations						
Fail	13	32		39	6	
Pass	*8 (13.13)*	67	.39 ± .41	*49 (55.00)*	26	.11 ± .09

Note. Error cells indicated by italics (with expected errors in parentheses).
[a] With 95% confidence interval.

general hypothesis of neo-Piagetian capacity theories that the level of children's reasoning is limited by their attentional capacity (Case, 1985; Halford, 1982; Pascual-Leone, 1970, 1980). More specifically, it is consistent with Pascual-Leone's (1970) prediction that M-capacity of $e + 3$ should be necessary for concrete operational reasoning, under the assumption that "$e + 3$" is the amount of capacity estimated by $k = 3$ on the capacity measures used in this study.

The results were also consistent with the secondary hypothesis of this study: that both the transitivity of length and the transitivity of weight should have the same capacity demand. Although the two tasks differed in overall difficulty, 3 (but not 4) units of capacity were necessary to solve them both. In other words, the horizontal decalage between these tasks cannot be explained in terms of a difference in capacity demand on the basis of this evidence.

Two aspects of these results require further explanation: (a) Why did class inclusion not require a minimum of 3 units of capacity as did the other concrete operational tasks? (b) How can the horizontal decalage between the transitivity of length and the transitivity of weight be explained, if not by a difference in capacity demand?

Class Inclusion

Perhaps the simplest explanation of the finding that class inclusion did not require 3 units of capacity (and was accordingly much easier than the other concrete operational tasks) is that the task used in this study did not require a true operational understanding of class inclusion. In Piagetian logic, this understanding is defined in terms of the operation of "class addition": the recognition that a supraordinate class B is necessarily equal to the sum of its subclasses A and A', where A is defined positively by a directly perceptible characteristic, and A' is defined negatively as all the members of B that are *not* members of A. The prediction that 3 units of capacity would be necessary was based on the assumption that the task would be solved with such an operation and that children would therefore have to assign values to the three variables A, A', and B. However, class inclusion performance is known to be affected by a variety of factors, and different versions of the task have been found to be solved by children at different ages (Winer, 1980). One is therefore justified in asking whether some versions do not actually assess what is intended (Smith, 1982).

For example, most researchers who have considered both judgments and explanations in their response criteria have accepted as valid an argument (judgment plus explanation) having the form, "There are more X ones, *because they are all X*" (where X is the property defining the supraordinate class). Such an explanation was accepted by Smedslund

(1964), whose criteria and procedures were followed in this study. In accepting such an explanation, researchers have assumed in effect that children implicitly intended the word "all" to subsume the two subclasses involved. However, the judgment that all the objects present have a certain property *X* (e.g., are "red") can be made without considering the properties that define the subclasses (e.g., whether they are round or square). Smedslund's procedures in fact ensure that children will make such a judgment as a consequence of his preparatory questions. Further, the way in which the test question is phrased ("Are there more red ones or more round ones?") encourages the assumption that there are only two possibilities to be considered: There are either *more red ones* or *more round ones*. Under this restricted condition (i.e., that the two classes are not coextensive), the judgment of "more" can be inferred directly from the judgment of "all": If all the objects are red, then there cannot be more round ones; therefore, there must be more red ones. If the possibility that the classes are coextensive were not eliminated from the beginning, however, then the possibility would remain that the red ones and the round ones were equal in number so that "more" could not be inferred directly from "all."

In short, Smedslund's procedures might allow children to infer a judgment of "more" from a judgment of "all" without having to consider the relation between the two subclasses and the supraordinate class which includes them both. The understanding of that relation, however, is what the class inclusion task is meant to assess. Our claim is not that all children giving the explanation "because they are all *x*" lack a true understanding of class inclusion, but only that this explanation is ambiguous. Some children who give this explanation may indeed intend the word "all" to refer to both subclasses, but other children answering in the same way might be inferring "more" directly from "all" in the manner just described. Because the latter form of reasoning involves only two operatory variables (corresponding to the supraordinate class and to the single subordinate class with which it is compared), it would have a capacity demand of only 2 units consistent with the results of this study.

Preliminary data from a pilot study by Drummond and Chapman (1988) conform to this interpretation. Children were given the Smedslund (1964) version of the class inclusion task, but explanations of the form "They are all *X*" were distinguished from explanations that contained a reference to both superordinate and subordinate classes (e.g., "because they're all red and only some are round"). The major result was that 2 units of capacity were sufficient for the first type of explanation, but 3 units were necessary for the second. Although this finding must be considered provisional, it suggests that one cannot assume the two types of answers to be semantically equivalent in all cases.

Length and Weight

The fact that 3 (but not 4) units of capacity were found to be necessary for both transitivity of length and transitivity of weight suggests that the horizontal decalage between these two tasks cannot be accounted for in terms of differential capacity demands. An alternative explanation is the following: (a) Young children tend to infer weight as a function of size (Piaget, 1971/1974, Ch. 10), and (b) some children continue to infer weight from size in the transitivity task even after they become capable of operational transitivity of length because they do not realize that weight is not always proportional to perceived size. The horizontal decalage between length and weight is the developmental time that elapses before children fully appreciate the implications of this fact for the transitivity problem. Under any other hypothesis, Chapman and Lindenberger's (1988) finding that the length–weight decalage does not occur when comparison objects are all the same size is difficult to explain.

Structural–Functional Theory

This study is an example of what Chapman (1987) called a *structural–functional* approach to cognitive development. Such an approach is characterized by an attempt (a) to identify children's forms of reasoning in terms of their structural properties, and (b) to specify functional conditions necessary for a particular form of reasoning. In these terms, this study has been an investigation of organismic conditions (a certain level of attentional capacity) that make concrete operational reasoning possible. The results are consistent with the conclusion that 3 units of capacity as defined in this article are necessary but not sufficient for concrete operations.

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