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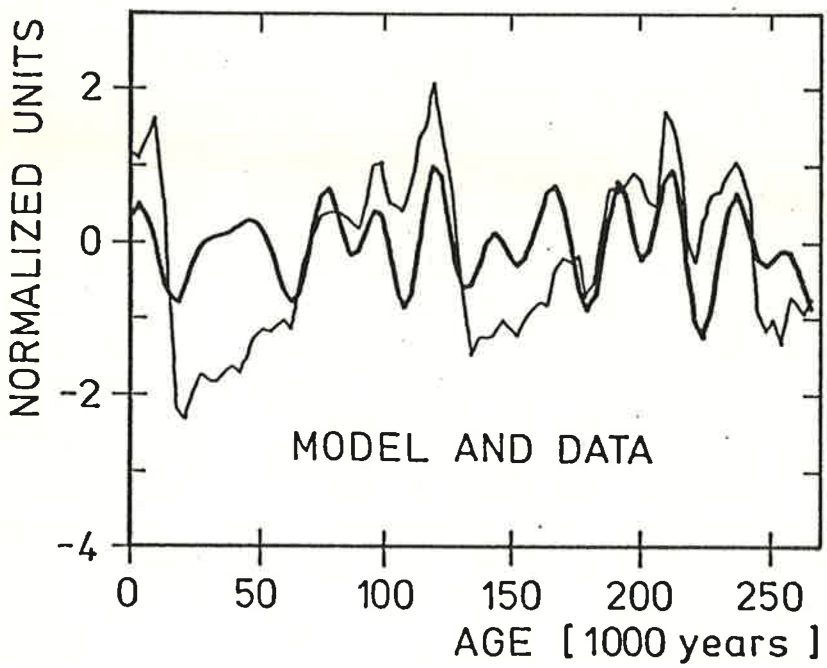

EXTRACTING THE PARAMETERS OF SIMPLE CLIMATE MODELS BY INVERSE MODELING OF THE DEEP-SEA CORE CLIMATIC RECORD

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Extracting the parameters of simple climate models by inverse modeling of the deep-sea core climatic record

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Abstract

A variational technique is presented to extract the parameters of simple climate models relating the variations of solar insolation and the deep-sea core climatic record. In the optimization process, a two-parameter linear model with solar insolation as input is fitted to the core data and the age-depth relation of the core is corrected simultanously. Preliminary results are shown for two 150 ky long time series consisting of the solar insolation July, 65°N and the planktonic $\delta^{18}O$ record taken from Meteor-core M-13519, which is related to changes in the global ice volume. The optimal model has a time constant of 22 500 years and explains 38% of the data variance. The optimized age-depth relation implies that the mean sedimentation rate varied between 1.65 cm / 1000 years during ice decay and 1.25 cm / 1000 years during ice build-up.

1. Introduction

The information contained in the sedimentary record of deep-sea cores has greatly improved our knowledge in climate dynamics on time scales from 10^3 to 10^6 years. In trying to fit climate models to these data, two extreme approaches have been pursued, aiming at quite different aspects in climate modeling.

Starting with Gates (1976), complicated general circulation models of the atmosphere, driven with boundary conditions set by deep-sea core data (CLIMAP-project, 1976) have been used to simulate atmospheric conditions during the last ice-age maximum. Recently, Kutzbach and Guetter (1986) extended this kind of modeling to other climate conditions during the last deglaciation period.

On the other hand, first order information on climate dynamics may already be obtained by fitting simple linear or non-linear models (containing a few free parameters) to the deep-sea core record. Using this approach Imbrie and Imbrie (1980) derived a mean time constant of 17 ± 3 ky involved in the interaction between the solar insolation July, 65°N and a planktonic $\delta^{18}O$ record related to changes of the global ice volume. By applying such inverse techniques to various climate variables from different locations, a global pattern of phase relationships may be derived.

A crucial point in calculating phase relations between climate variables derived from the deep-sea records, is the knowledge of the age-depth relation of the core data. There have been a number of attempts to date the deep-sea record, being a function of depth initally. In constructing an age-depth relation, all authors started with an initial time scale using radiometric dating at some depths and assuming a constant sedimentation rate in between. Further improvement was achieved by either using additional knowledge on sedimentation rates (Shackleton, 1977) or by postulating a linear relationship between the deep-sea record and Earth's orbital parameters (Imbrie et al., 1984), producing the SPECMAP time scale. Herterich and Sarnthein (1984) combined both concepts deriving the CARTUNE time scale, which matches the SPECMAP time scale within the (partly large) errors of the available radiometric dates.

To obtain information on time constants involved in the climate system, the above dating techniques are not best suited. The SPECMAP dates were derived by matching the phases of the $\delta^{18}O$ record, averaged over 5 cores, to the phases of a linear model with a fixed time constant of 17 ky and with Earth's orbital parameters as model input. On the other hand the CARTUNE time scale was developed such to obtain maximum coherence between the $\delta^{18}O$ record and solar insolation (July, 65°N), without using the information contained in the phases as a further criterion to evaluate the dating.

In this work we try to extract both the phase relations and the age-depth curve simultanously by fitting a linear model to the data and optimizing the age-depth relation at the same time, subject to some side conditions. In section 2 we describe the variational method used to solve this coupled optimization problem. The derived climatological model parameters (time constant and explained data variance) and the geological implications (sedimentation rates and ages of isotopic stages) are presented in section 3 and discussed in section 4.

2. Optimization methode

2.1 Formalism

The general method is desribed in Hasselmann and Herterich (1984). In the following, the procedure actually used in this work is outlined. Given any two time series, whose climatological relationship is going to be studied, the optimization technique extracts the parameters of some linear model relating the two data sets. In addition the dating of one time series (deep-sea core data) is optimized. The dating of the other time series (orbital parameters or solar insolation) is assumed to be correct.

We try to fit the deep-sea core record (in the following denoted by y) by a linear model $\hat{y}$ with the solar insolation curve x as the input. The model used here is defined by a first order differential equation with two free parameters a, λ, which control the strength of the forcing and the time constant of the linear model $T = 1/\lambda$, respectively:

$$\frac{d\hat{y}(t)}{dt} = -\lambda\hat{y}(t) + a\,x(t). \qquad (1)$$

A fit of the model $\hat{y}$ to the data y can be achieved by minimizing the error:

$$\varepsilon_1 = \int_0^{-\tau} (y - \hat{y})^2 dt\,, \qquad (2)$$

where the integration extends over the length of the time series from present at $t = 0$ down to some depth of the deep-sea core taken at $t = -\tau$. The minimum of ε_1 can either be found by numerical minimizing techniques or by solving the set of differential equations:

$$\frac{\partial\varepsilon_1}{\partial a} = 0,\ \frac{\partial\varepsilon_1}{\partial\lambda} = 0, \qquad (3)$$

thus defining a, λ of the optimal model fit.

Some modification, however, is nescessary because the dating of the deep-sea core record is not accurately known. Assuming that we already have an approximate age-depth relation $z_0(t)$, we will allow for small shifts $z' = z - z_0$ in the age-depth curve to obtain a better fit. The shifts z' (t) are uniquely defined if certain side conditions are introduced. We chose the following two: The new time scale should not be too far away from the initial age-depth relation and should not be too noisy. These conditions can be incorporated by defining two more errors:

$$\varepsilon_2 = \int_0^{-\tau} (z - z_o)^2 \, dt, \tag{4}$$

$$\varepsilon_3 = \int_o^{-\tau} \left(\frac{d^2 z'}{dt^2}\right)^2 dt \, , \tag{5}$$

and requiring now that the sum ε_0 of the three errors ε_1, ε_2, ε_3,

$$\varepsilon_o = \sum_{i=1}^{3} g_i \varepsilon_i \, , \tag{6}$$

should be minimized, where the coefficients g_i are weights to be chosen later.

The correction z' (t) can be found by variational calculus. It is the solution of the corresponding Euler equation:

$$g_3 \frac{d^4 z'}{dt^4} + g_2 z' = - g_1 \frac{dy}{dz} \left(y(z(t)) - \hat{y}(t)\right), \tag{7}$$

with $\hat{y}$ given by (1) for fixed parameter values a, λ of the linear model.

Equation (7) is a forth-order differential equation, however non-linear, since the inhomogeneity on the right hand side of eq. (7) is a non-linear function of the solution z'. Eq. (7) is solved iteratively by a Newton technique. The optimal parameters of the linear model can be defined by minimizing the error ε_1 (eq. 2) with respect to a and λ, where for each a, λ the correction z' to the initially chosen approximate age-depth relation $z_0(t)$ is determined by eq. (7).

2.2. Choice of weights g_i

For convenience, the weights g_i of eq. (7) are written as a product of a non-dimensional number g_i' and a quantity $\overline{g}_i$ such that for $g_i' = 1$, the correspon-ding expected errors ε_i (eqs. (2), (4), (5)) become non-dimensional and of order 1.

Dividing eq. (7) by g_1 does not change the solution z'. Thus z' actually depends only on the two ratios:

$$r_2 = \frac{g_2}{g_1} = \frac{g_2'\overline{g}_2}{g_1'\overline{g}_1} ,$$

$$r_3 = \frac{g_3}{g_1} = \frac{g_3'\overline{g}_3}{g_1'\overline{g}_1} .$$

As $\overline{g}_1$, $\overline{g}_2$, $\overline{g}_3$ are fixed values and g_1' a common factor in both ratios r_2, r_3, the whole space of possible solutions z' can be constructed by taking $g_1' = 1$ and varying only the normalized weights g_2', g_3'.

The right hand side of eq. (7) can be interpreted as a forcing term of the solution z', while the terms on the left hand side will damp the solution. Thus the weights g_2', g_3' should be small enough to allow for all nescessary changes in the age-depth relation z (t) to get a good fit between the model and the data. However, for very small values of g_2', g_3', the numerical technique to find the solution of the non-linear eq. (7) does no longer converge. In this case the basic assumption for convergence, namely that the initially chosen age-depth relation $z_o(t)$ is not far away from the optimized relation $z = z_o + z'$, is probably no longer valid.

There is some ambiguity how to choose the weights. One way to find some prefered set of weights is to investigate the range of results (parameters of the model and age-depth curves) for various values of the weights g_2', g_3' and then use other independent criteria favouring a specific choice of the weights. As will be shown in section 3.1 both the mean coherence achieved between solar insolation and the deep-sea core data, and the amount of the data variance explained by the model can serve as a criterion to choose the weights g_2', g_3'.

2.3. Choice of model and data

The optimization method is applied to a specific choice of time series. The deep-sea core data (Fig. 1) are planktonic $\delta^{18}O$ values from Meteor-core M-13519 (Sarnthein et al. 1984). These data are generally taken as a measure of changes in the global ice volume. We postulate that the ice volume is somehow linearly related to solar insolation. To be specific, we choose the insolation curve July, 65°N (Fig. 2). Guided by climate theory, we expect that variations in the ice volume are largely controlled by summer melting, while ice accumulation in winter is less affected. The latitude 65°N is selected, since this latitude marks the mean position of the Northern-Hemisphere-Ice-Sheets.

The variations of solar insolation (Berger, 1977) were calculated with a time step of Δt = 3 ky. This defines the model via eq. (1) taking the solar insolation July, 65°N for the input function x. With some starting linear age-depth relation (see section 3.1), the $\delta^{18}O$ data were interpolated to define the data at discrete times also with a time step of $\Delta t = 3$ ky. Both time series were normalized to zero mean and unit variance.

In this work, we concentrate on the last 150 ky, with only some preliminary experiments modeling the data back to isotopic stage 10.5 (Prell et al., 1986) at about 330 ky. Within the shorter time span (150 ky), the correlation between solar insolation and global ice volume is evident already from visual inspection (compare model and data in Fig. 3). This helps to evaluate the results of the optimization prior to applying this technique to the deeper parts of this core, where the correlation between solar insolation and data becomes progressively unclear. However, the data already cover a full glacial cycle such that the influence of different glacial conditions may show up in the resulting optimized age-depth relation.

3. Results

3.1. Choosing an initial age-depth relation

The initial age-depth relation from which optimization is started is taken to be a linear function of depth z. The top of the core at $z = 0$ is at the age $t = 0$ (present). To fix the slope of the age-depth relation, we assign an age of 118 ky (radiometric age given by Harmon et al. 1979) to the isotopic stage 5.5 (as defined by Prell et al. 1986), which is located at a depth of 170 cm in Meteor-core M-13519 used for this study. Isotopic stage 5.5 is related to an interglacial state with a corresponding global ice volume less than that reached during the last interglacial at about 6 ky B.P. (isotopic stage 1.1, see Fig. 1).

If no corrections z' in the age-depth relation are allowed ($g_2' = g_3' = 10^5$, sufficiently large to keep z' as the solution of (7) near zero), the model fits an infinitely high time constant ($\lambda = 0$) to the data, with 18% of the data variance explained by the model. In trying to match the data, the model produces the largest possible time delay.

This obvious failure of the model fit could have been anticipated already by correlating both time series visually. For a good fit between model and data, the isotopic stage 5.5 should be related to the solar insolation maximum at 126 ky. With an assumed age of 118 ky for isotopic stage 5.5, the necessary time delay the model has to produce is 8 ky. The corresponding phase shift would be 125° for the 23 ky precessional period, which dominates the spectrum of the solar insolation curve. This however, is outside the range of phase shifts which can be produced by a linear (first order) model (90° maximum).

Assigning higher ages to isotopic stage 5.5, the model fits finite time constants ($\lambda > 0$) and allows also higher variances explained by the model. A maximum explained variance of 22% is achieved for an age of 120 ky. In this case, the fitted time constant is 17.5 ky. In the following, this linear age-depth relation will be taken as the initial time scale from which optimization is started (see section 3.2).

The experiments above (with a linear age-depth relation of variable slope) show that the fitted time constant is very sensitive to the dating. Fitted time constants are obtained in the range from zero to infinity by varying the age of isotopic stage 5.5 only within the small intervall from 118 to 126 ky. The sensitivity would decrease for a longer time series with more dates available. In conclusion, meaningful time constants can be extracted from the data only, provided the initial dating is precise enough, or can be corrected by some optimization.

3.2. Optimized model parameters

Lowering the weights $g_2'$and g_3' will allow for deviations in the age-depth relation relative to the linear reference age-depth relation as defined above (section 3.1). The resulting corrections in the age-depth relation enables the model to improve the fit, yielding now higher values in the explained variance. An optimum is obtained for $g_2' = 1$ and $g_3' = 0.05$, in the sense that the explained variance is largest reaching 38% compared to 22% without the possibility to make corrections in the age-depth relation. The fitted time constant is now 22.5 ky.

In Fig. 3, both the data and the model fit (for $g_2' = 1$, $g_3' = 0.05$) are shown. The model follows the small amplitude changes of the data (ice volume) quite nicely. However, the characteristic transition between glacial and interglacial conditions with a period of 100 ky is clearly missing in the model. This is not surprising, since both solar insolation and the model do not contain a 100 ky cycle. Even if the 100 ky cycle would be added to the model, by taking eccentricity as input, the linear model would not be able to produce the asymmetric behaviour in the build-up and decay of the global ice volume.

The fitted time constant of 22.5 ky is comparable to the time constant of 17.5 Ky as derived by (Imbrie and Imbrie (1980), and lies within the range of physically plausible time constants suggested by climate dynamics. If the build-up of ice sheets is mainly controlled by snow fall, the time needed to produce an ice sheet of 4 km thickness will be 40 000 years, assuming a snowfall rate of 10 cm per year. On the other hand, a complete decay of an ice-sheet can occur within 10 000 years. The relatively small value of the variance (38%) of the data which is explained by the linear model is consistent with the fact that the ice volume changes are dominated by a 100 ky glacial / interglacial cycle, at least during the last 500 000 years, while the linear model only contains the band of periods from 20 000 to 40 000 years where solar insolation varies.

3.3. Optimized age-depth relation

In Fig. 4a, the corrections to the initially chosen linear age-depth relation as introduced by the optimization are shown. Ages have been shifted up to 4.3 ky. In Table 1, the optimized ages of the isotopic stages, as defined by Prell et al. (1986), are listed. With some exceptions, the ages match those given by Imbrie et al. (1984) using the SPECMAP time scale within the errors of radiometric dating. The ages for isotopic stage 1.1 (last climatic optimum at 6 ky B.P.) and isotopic stage 2.2. (last glacial maximum at 18 ky B.P), estimated as 9 ky and 20.3 ky, respectively in this work, are definitely too high. This is due to the boundary condition z' (t = o) = 0, although corrections are in a direction to improve these dates. The model, however, places these stages at nearly correct positions (see Fig. 3). This problem can be solved by either using an initial age-depth relation, which incorporates these (well known) dates including the possibility of $t < 0$ at the core top (z = O) or trying to increase the fine structure of the optimized age-depth relation such that these local changes in the sedimentation rate can be resolved. At older stages, deviations are partly due to the uncertainties in defining the isotopic stages in individual cores. To avoid these difficulties, Imbrie et al. (1984) used a stacked data set, where $\delta^{18}O$ values have been averaged over 5 deep sea cores.

Although absolute corrections in the dating are small, the changes in the sedimentation rate $s = dz / dt$ are significant. In Fig. 4b, the sedimentation rate derived from the optimized age-depth relation is shown. The sedimentation rate varies between 1.65 and 1.25 cm / 1000 y, with higher values around the top of interglacials and lower values at climates intermediate between interglacial and full glacial conditions.

The question arises, how real these variations in the sedimentation rate are. Experiments with different parameters g_i' show that generally the explained variance and the mean coherence between data and solar insolation increases as more structure in the age-depth relation is allowed. This gives some confidence to the derived sedimentation rate, since the "real" age-depth relation should also produce the highest coherence among all age-depth relations deviating from it.

3.4. Extending the fit downcore

The same steps in the optimization process as described in sections 3.1, 3.2 and 3.3. have been applied to the data set extended further downcore, reaching beyond isotopic stage 7.5, initially taken at an age of 270 ky. The optimal fit(see Fig. 5) yielded a time constant of 18.1 ky and an explained variance of 24% with the optimization parameters set at $g_2' = 1$ and $g_3' = 0.05$ as before.

The fitted time constant is about the same, showing that this parameter is also not very much dependent on the length of the data time series. However, the error of the estimated time constant will be reduced as the length of the time series increases. The smaller explained variance (24 % compared to 38% before) possibly means that there is degradation in the data, or that the contribution of non-linear effects is increasing in the past. The pattern of corrections in the age depth relation and the resulting changes in the sedimentation rates (Fig. 6a, b) basically repeat in the second glacial cycle ranging from isotopic stages 5.5. to 7.5.

The seemingly good fit between the model and the data (Fig. 5) also shows up in the calculated coherences and phases. In Fig. 7, the squared coherences between solar insolation and the data are plotted as a function of frequency, with high values in the band of periods between 20 ky and 40 ky, where most of the variance of the solar insolation input is concentrated. The higher coherences outside this band are not meaningful, since the solar insolation input has almost zero power there. Fig. 8 shows the calculated phases between solar insolation and data (thin line) as compared to the phases between solar insolation and model (thick line). Again, within the 20 ky to 40 ky band, model and data phases are consistent.

Preliminary experiments with the data set extended beyond isotopic stage 10.5 (at an age of 327 ky), yield similar time constants (around 20 ky) and the data variance explained by the model is reduced further (below 20%).

4. Summary and Discussion

In this work we presented a variational technique to extract the parameters of simple climate models from the analysis of the linear relation between solar insolation and climate variables, derived from deep-sea core data. This was achieved by fitting a linear model to the deep-sea core record and optimizing the age-depth relation of the core simultanously. The model input was some solar insolation curve and the model parameters were the time constant involved and the amplitude of the input forcing.

For the 150 ky long data time series, the estimated time constant is 22.5 ky and the fitted amplitude means that 38% of the variance of the data is explained by the model. From the optimized age-depth relation, the sedimentation rate of the specific core and the ages of isotopic stages could be derived.

For climate modeling purposes, the most interesting parameter is the time constant which is involved in the linear relationship between solar insolation (July, 65°N) and the deep-sea core data (global ice volume). The estimated value of 22.5 ky is well within the physically plausible range. A number of tests show that this estimated time constant is not an artificial result produced by some choice of the setting in the optimization. The derived time constant is rather independent on the choice of the weights g_i' and it does also not dependent on the initial age-depth relation starting the optimization.

The linear model explains about 38% of the variance of the data. The variance of the data is dominated by the glacial /interglacial cycle around 100 ky. This period is not contained in the spectrum of the solar insolation curve July, 65°N and therefore also not contained in the model.

The asymmetric shape of the ice-age cycle, which is clearly visible from the data (Fig. 1), suggests that the changes of the global ice volume cannot be explained by linear forcing alone. The same conclusion follows from an experiment performed by Imbrie (1984), trying to estimate the phase at the 100 ky period between the variations of eccentricity and global ice volume. The estimated phase at the 100 ky period was not consistent with a linear model which has been fitted to the higher frequency components of the Earth's orbital parameters in the 40 ky to 20 ky band.

The small local corrections in the age-depth relation, introduced by the optimization did not influence the fitted time constant very much. However, they are of geological significance, implying quite considerable changes of the local sedimentation rate. Since more fine structure in the age-depth relation and its derivative the sedimentation rate generally increases the coherence between solar insolation and deep-sea core data, the details in the sedimentation rate may be real.

It is planned to apply this optimization technique also to other input and output time series. The length of the time series to be analyzed will depend on the success of the optimization. If the optimization fails, the derived model parameters are probably not real. The input is not necessarily restricted to solar insolation curves. We would like to analyse also the dynamics between different climate parameters, derived from deep-sea cores from different locations. However, as long as the dating is not certain, we also need a chronometer, usually some solar insolation curve at some season and latitude, or the variations of the orbital parameters.

From such an approach, a global pattern of linear phase relationships between different climate variables may be derived to obtain a coarse picture of the basic dynamics of the climate system during the Pleistocene. This is also the scope of the SPECMAP group using the dates of isotopic stages as given by the SPECMAP time scale (Imbrie et al. 1984). This time scale uses an apriori given time constant of 17.5 K years involved in the changes of global ice volume forced by Earth's orbital parameters. It might be interesting to see, how the SPECMAP phase patterns compare to those derived by the technique suggested in this paper, where the time constant is not fixed but is itself part of the optimization process.

At a later stage it is also planned to fit simple non-linear models to the data, as for instance the model suggested by Imbrie and Imbrie (1980), containing two time constants, a larger one controlling the ice build-up, and a lower one involved in ice decay.

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References

Berger, A.L. (1977): Long term variations of the Earth's orbital elements. Celestial Mechanics 15, 53-74.

CLIMAP project (1976): The surface of the ice-age earth. Science, 191, 1131-1136.

Gates, W.L. (1979): The numerical simulation of the ice-age climate with a global general circulation model. J. Atmos. Sci., 33, 1844-1873.

Harmon, R.S. et al. (1979): Limits of U-series analysis: Phase 1: Results of the Uranium-Series Intercomparison Project. Geology, 7, 405-409.

Hasselmann, K. and K. Herterich (1983): Application of inverse modeling techniques to paleoclimatic data. In Paleoclimatic Research and Models, A. Ghazi (ed), D. Reidel Publ., Dordrecht, 52-68.

Herterich, K. and M. Sarnthein (1984): Brunhes time scale: Tuning by rates of calcium carbonate dissolution and cross spectral analysis with solar insolation. In Milankowitch and Climate. A.L. Berger et al. (eds.), 447-466.

Imbrie, J., and J.Z. Imbrie (1980): Modeling the climatic response to orbital variations. Science, 207, 943-953.

Imbrie et al. (1984): The orbital theory of Pleistocene climate: Support from a revised chronology of the marine $\delta^{18}O$ record. In Milankovitch and Climate. A.L. Berger et al. (eds.), 269-305.

Kutzbach, J.E. and P.J. Guetter (1986): The influence of changing orbital parameters and surface boundary conditions on climate simulations for the past 18 000 years. J. Atmos. Sci., 43, No 16, 1726-1759.

Prell, W.L. et al. (1986): Graphic correlation of oxygen isotope stratigraphy application to the late Quaternary. Paleoceanography, 1, 137-162.

Sarnthein, M. et al. (1984): Stable-isotope stratigraphy for the last 750 000 years: "Meteor" core 13519 from the eastern equatorial Atlantic. Meteor Forsch.-Ergebnisse, Reihe C, No. 38, 9-24.

Shackleton, N.J. and R.K. Matthews (1977): Oxygen isotope stratigraphy of late Pleistocene coral terrraces in Barbados. Nature 268, 618-620.

Table 1

isotopic stage	depth [cm]	age (ky)	
		this work	SPECMAP (Imbrie et al. 1984)
0	0	0	0
1.1	14.0	9.0	6.0
2.0	26.0	16.4	12.0
2.2	32.0	20.3	19.0
3.0	36.4	22.9	24.0
3.1	42.8	27.0	28.0
3.3	80.0	52.2	53.0
4.0	90.7	60.2	59.0
5.0	96.3	64.5	71.0
5.1	115.0	79.5	80.0
5.2	128.4	90.4	87.0
5.3	138	98.0	99.0
5.4	145.3	103.5	107.0
5.5	168.6	120.0	122.0
6.0	184.0	129.7	128.0

Table 1 Ages of isotopic stages (defined by Prell, 1986) as calculated in this work and compared to those ages determined from the SPECMAP time scale (Imbrie et al. 1984).

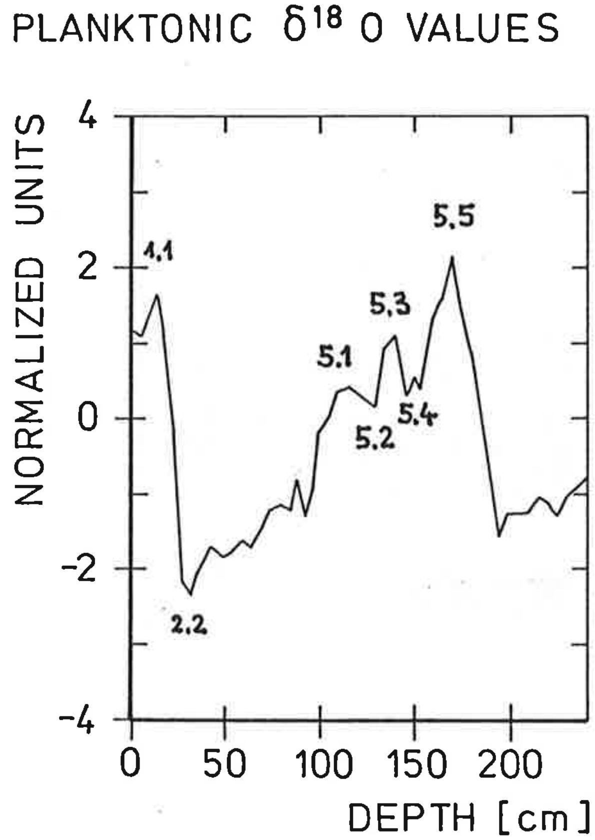

Fig. 1 Normalized planktonic $\delta^{18}O$ values (zero mean and unit variance) as a function of depth from Meteor deep-sea core 13519 (Sarnthein et al. 1984), taken as a measure of changes in the global ice volume, with high values indicating ice volume minima and low values corresponding to ice volume maxima. The numbers at the position of ice volume extrema denote isotopic stages as defined by Prell et al. (1986).

INSOLATION AND MODEL

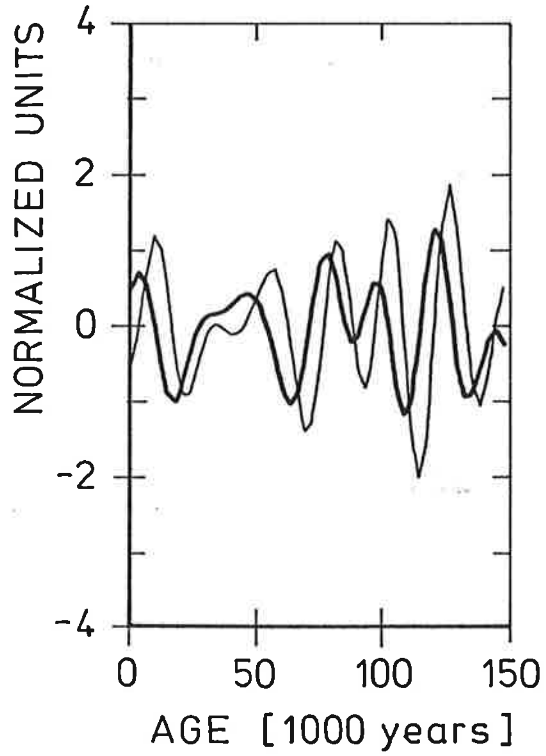

Fig. 2 Normalized solar insolation curve for July at 65°N (thin line) as a function of time (after Berger, 1982) and the output (thick line) of a linear model (eq. (1)) with this solar insolation curve as input.

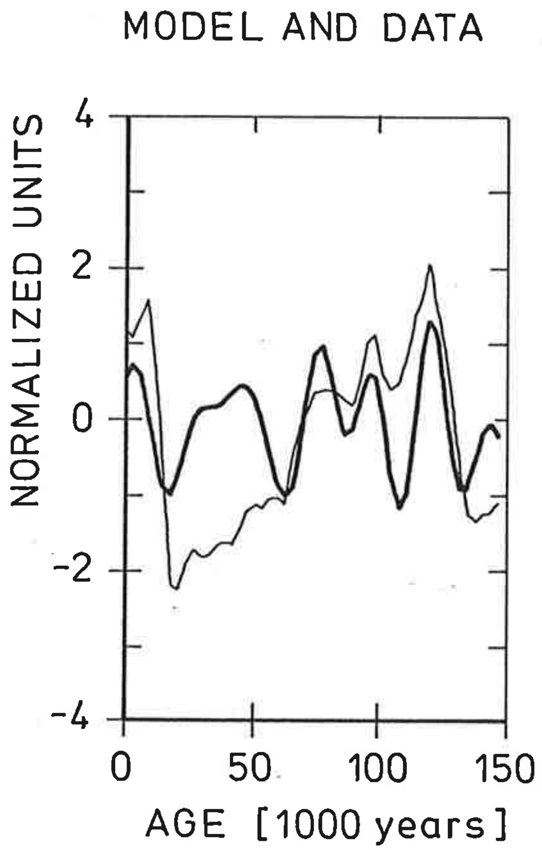

Fig. 3 Normalized δ180 record (thin line) and fitted model (thick line) as a function of time.

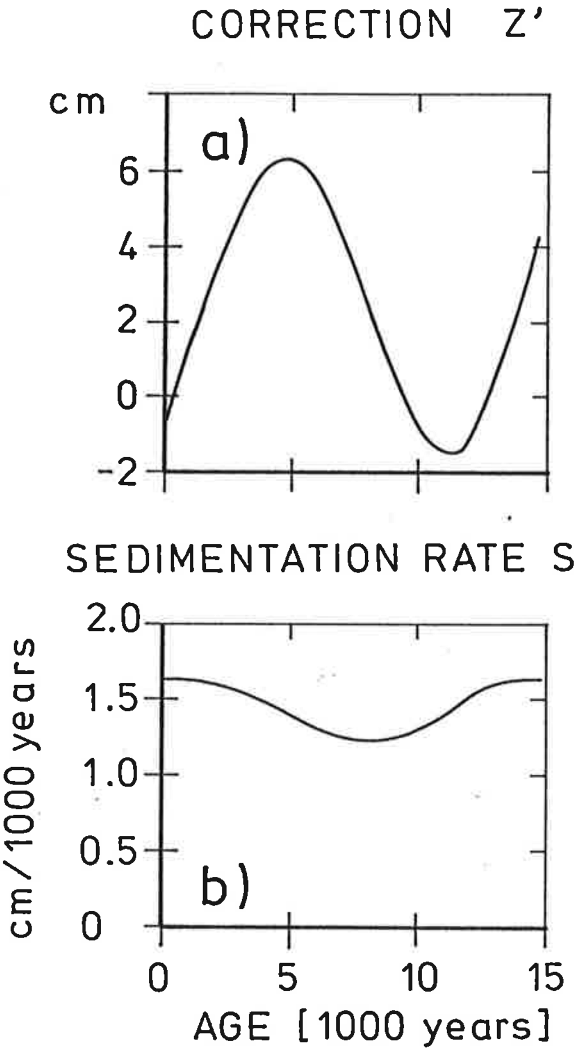

Fig. 4 a) Corrections z'[cm] to the initially chosen age-depth relation, introduced by the optimization; b) derived sedimentation rates [cm/1000 y].

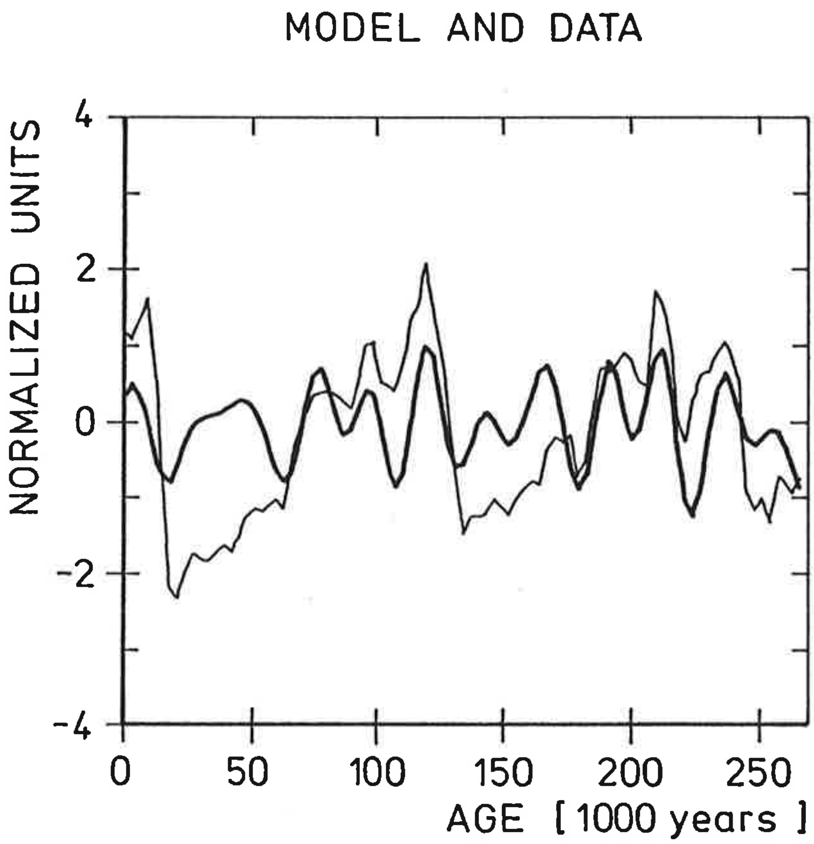

Fig. 5 Normalized (270 Ky long) δ180 record (thin line) and fitted model (thick line) as a function of time.

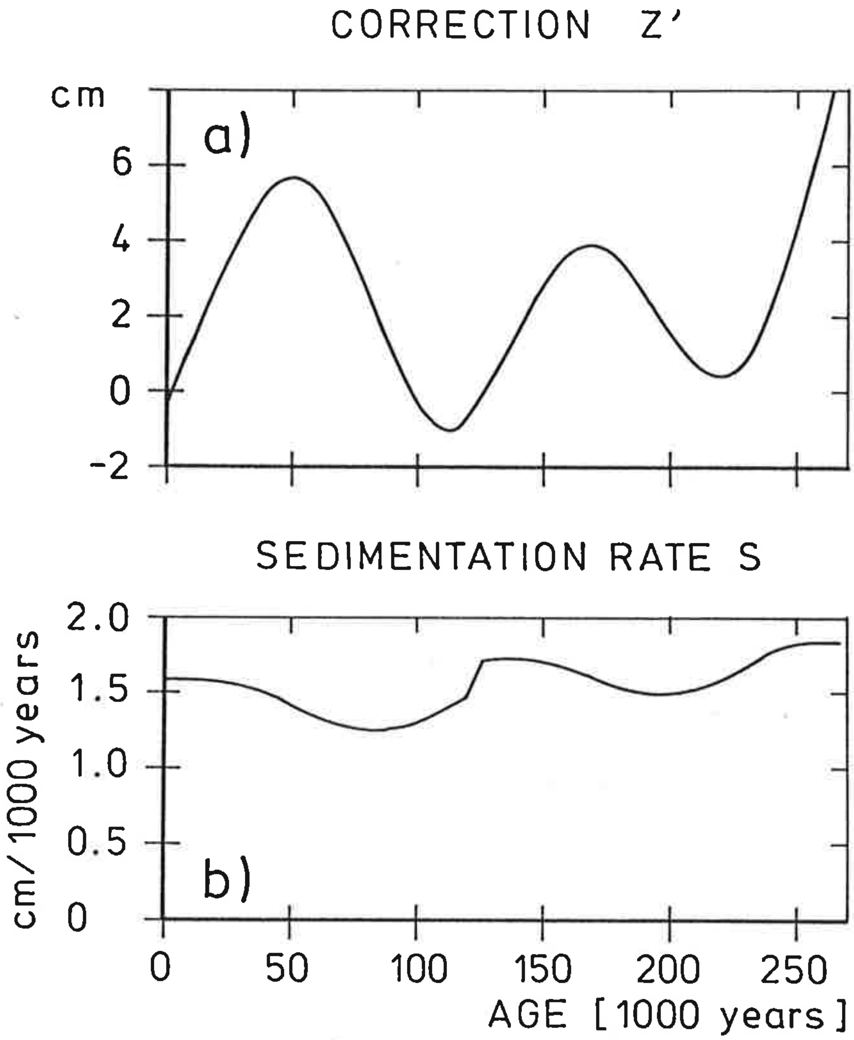

Fig. 6 a) Corrections z'[cm] to the initially chosen age-depth relation (last 270 ky) introduced by the optimization; b) derived sedimentation rates [cm / 1000 y].

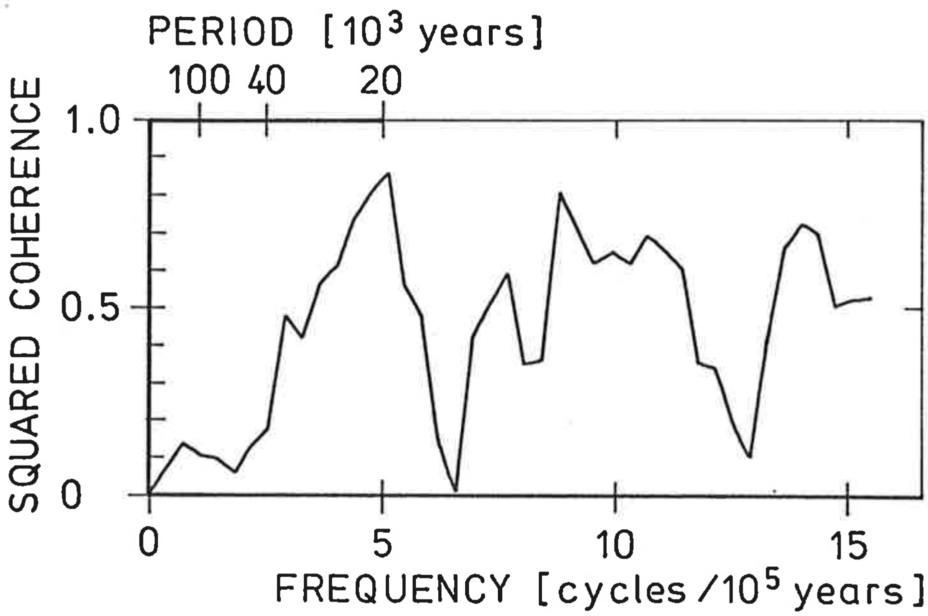

Fig. 7 Squared coherences between solar insolation (July, 65°N) and δ18O record (last 270 ky) as a function of frequency.

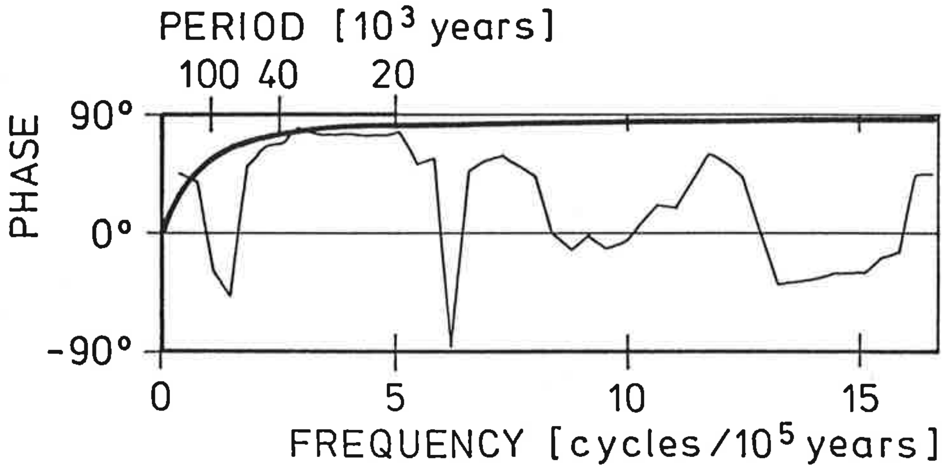

Fig. 8 Phases between solar insolation (July, 65°N) and δ180 record for the lasst 270 Ky (thin line) and theoretical phases between solar insolation and model (thick line) as a function of frequency.