Self-consistent nonlinear kinetic modeling of runaway-electron dynamics

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Introduction Runaway electrons represent the greatest threat to the plasma-facing components of a tokamak when they are highly energetic and constitute a significant fraction of the electron population. This regime has not been previously accessible in modelling, since it requires a nonlinear relativistic treatment. To address this problem, we recently developed an efficient numerical tool called `NORSE` [1] for the study of runaway-electron momentum-space dynamics. The kinetic equation solved in `NORSE` includes a fully nonlinear relativistic collision operator [2, 3], making it possible to consider scenarios where the electric field is comparable to the Dreicer field (or larger), or the electron distribution function is otherwise far from a Maxwellian (which can be the case already in present-day runaway experiments). This capability makes `NORSE` unique in the field of runaway-electron studies.

Using `NORSE`, the transition to a regime where the entire electron population experiences continuous acceleration, so-called electron slide-away [4, 5], was recently investigated [1, 6]. It was also found that Ohmic heating and the rate of heat loss play an important role in determining the electron dynamics, with the latter affecting the average energy reached by the runaways by several orders of magnitude [6].

However, these studies were performed using a prescribed electric-field evolution, as is common when considering runaway-electron dynamics [7, 8]. Although this method may accurately describe scenarios where the runaways do not contribute significantly to the total plasma current, it is inadequate when the runaway current becomes substantial. This is because the electric field responsible for generating the runaways in the first place is in many cases determined by the evolution of the plasma current; if the runaways are able to affect the current, the problem becomes nonlinear and a self-consistent calculation of the evolution of the electric field and distribution function is necessary. As shown in Ref. [6], the runaway population quickly becomes substantial in a typical ITER disruption scenario, and a self-consistent treatment of the electric field is therefore a necessity in many `NORSE` simulations.

NORSE and a self-consistent calculation of the electric-field

The newly developed fully relativistic non-linear tool NORSE [1] is valid in spatially uniform plasmas, and solves the kinetic equation

$$\frac{\partial f}{\partial t} - \frac{e\mathbf{E}}{m_e c} \cdot \frac{\partial f}{\partial \mathbf{p}} + \frac{\partial}{\partial \mathbf{p}} \cdot (\mathbf{F}_s f) = C_{ee}\{f\} + C_{ei}\{f\} + S, \tag{1}$$

in 2D momentum space. Here f is the electron distribution function, t is the time, m_e, $-e$ and $\mathbf{p}$ are the electron rest mass, charge and momentum, $\mathbf{E}$ is the electric field, c is the speed of light, $\mathbf{F}_s$ is the synchrotron-radiation-reaction force [9], C_{ee} is the relativistic non-linear electron-electron collision operator of Braams & Karney [2, 3], C_{ei} is the electron-ion collision operator, and S represents heat and particle sources or sinks. For a detailed description of the various terms and operators, see Ref. [1].

In practice, the electric field E is related to the current density j through the self-inductance L of the plasma. In a torus, the inductance equation

$$E = E_a - \frac{LA}{2\pi R}\frac{\partial j}{\partial t} \tag{2}$$

can be derived from Maxwell's equations, assuming a toroidally symmetric radial distribution of current. Here E and E_a are the total and externally applied electric fields, A is the cross-sectional area (i.e. $jA = I$ with I the total plasma current), and R is the major radius. This simplified inductive model is adequate for our purposes, especially since the applicability of NORSE is limited to the vicinity of the magnetic axis due the 0D treatment of real space. Later, we will find it convenient to consider the normalized inductance $\hat{L} = L \cdot e^2 f_{M,0} A/(2\pi m_e R)$, where $f_{M,0}$ is the value of a Maxwellian at $p = 0$.

Equations (1) & (2) form a nonlinear coupled set, as the current density is a moment of the electron distribution function f. In each time step in NORSE, this set of equations is solved iteratively using Newton's method. The calculation converges in just a few iterations since the initial guess (the values from the previous time step) is of high quality. The method nevertheless requires the construction and solution of several linear systems for each time step, and is thus slower than a NORSE run with a prescribed $E(t)$.

Proof-of-principle scenario To demonstrate the implementation of the self-consistent electric-field calculation in NORSE, a proof-of-principle scenario is considered in Fig. 1. Initially, a constant electric field $E_a = 1\,\mathrm{V/m}$ was applied until the current density saturated. The parameters are $T = 20\,\mathrm{eV}$, $n_e = 10^{19}\,\mathrm{m}^{-3}$, $Z_{\mathrm{eff}} = 1$ and $\hat{L} = 10^{10}$. To increase the resistivity, starting at $t = 0$ the effective charge was drastically increased, as shown in Fig. 1a. Figure 1b shows that as the resistivity increases, the current starts to decay (albeit slowly on the time scale of the figure

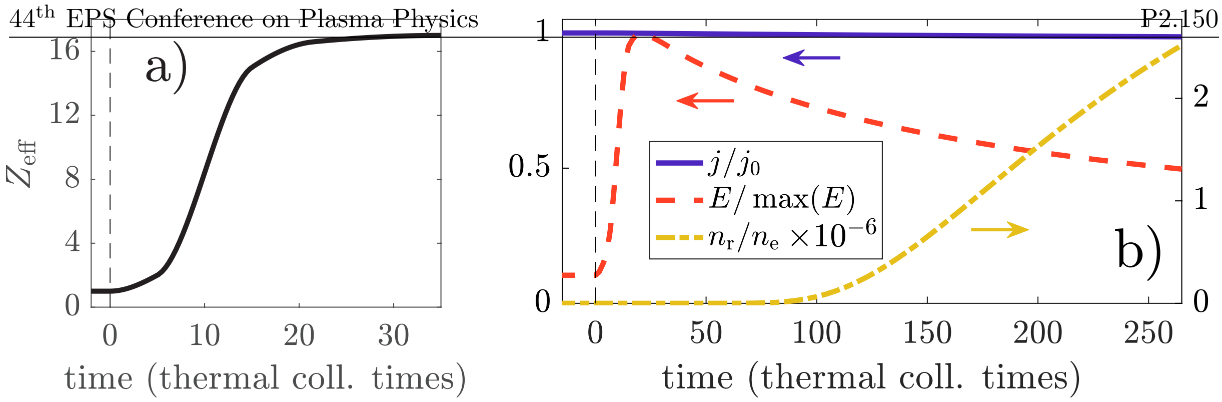

Figure 1: Proof-of-principle inductive scenario. a) Prescribed evolution of Z_{eff}, and b) the resulting change to the current density (blue, solid), electric field (red, dashed) and runaway fraction (yellow, dash-dotted). The current density is normalized to its initial value and the electric field to its maximum during the simulation. Arrows indicate which axis scale to refer to.

due to the large inductance). As a consequence, an electric field is induced in accordance with Eq. (2). The field reaches its peak on the time scale of a few tens of thermal collision times, before gradually decaying on a much longer time scale. The induced electric field in turn causes significant acceleration of runaway electrons. All these variations are in good agreement with the expected behavior in a similar scenario, thus demonstrating the succesful solution of the coupled system (1) & (2).

Analytical comparison In certain simplified cases, the governing equations can be solved analytically. Here we consider such a scenario in order to further validate the implementation in NORSE. In the absence of ions to collide with (i.e. $Z_{\mathrm{eff}} = 0$), an applied electric field will lead to indefinite acceleration of the electron population, and the change in current density becomes just $\partial j/\partial t = e^2 n_e E/m_e$. With this, Eq. (2) becomes

$$\frac{E}{E_a} = \frac{1}{1+n\hat{L}/f_{M,0}}; \tag{3}$$

the larger the self-inductance, the smaller the induced electric field (compared to the applied one). This scenario can be replicated in NORSE by starting a simulation with $E = E_a$, $Z_{\mathrm{eff}} = 0$ and $\hat{L} = 0$, and after some time switch to the desired inductance. After the system has relaxed, the value of E/E_a can be compared to the analytical prediction. This has been done in Fig. 2, which shows excellent agreement between NORSE and Eq. (3) for inductances varying over several orders of magnitude.

Conclusion and discussion In order to model the runaway-electron scenarios of greatest importance to the safety and long-term integrity of a tokamak, a nonlinear treatment of the electron distribution is necessary, as previously demonstrated. In such scenarios, a prescribed electric field is inadequate as the runaways make up a significant fraction of the electron population

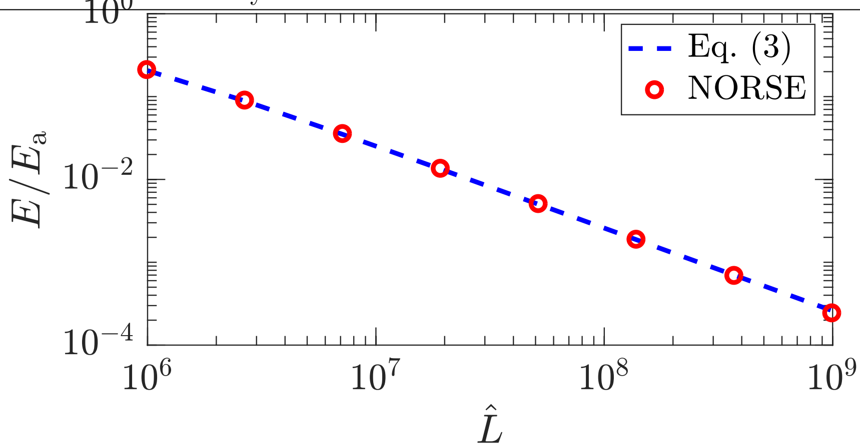

Figure 2: Comparison of the induced electric field between analytical (blue, dashed) and relaxed NORSE values (red markers) in a scenario without ions, for a number of values of the self-inductance $\hat{L}$.

and thus influence the current and electric field evolution. We have demonstrated the ability of the nonlinear relativistic tool NORSE to self-consistently calculate the inductive electric-field evolution together with the electron distribution function, paving the way for future study of disruptions and other dynamic scenarios.

Previously, fluid treatments have successfully been used to simulate the current and electric-field evolution during dynamic scenarios such as disruptions, using analytical formulae to estimate the runaway generation (see for instance [10–13]). Such formulas are derived as quasi-steady-state solutions to the kinetic equation and thus have limited applicability in highly dynamic situations. In addition, they only describe runaway generation. In contrast, a self-consistent NORSE calculation provides the full distribution function, and also captures runaway decay and relaxation phenomena. In addition, the impact of several other processes such as synchrotron radiation reaction, may be accurately modelled. NORSE thus provides the means to achieve improved understanding of runaway-electron behavior in dynamic scenarios; a topic of great importance to the magnetic-fusion community.

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