Bicoherence analysis of fast ion driven transient plasma waves

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Introduction Bicoherence analysis is a widely applied method for identifying quadratic nonlinearity of stationary processes [1]. The classic bicoherence, however, does not converge for signals exhibiting rapidly changing amplitudes or frequencies, and thus tends to give false positives: high bicoherence values even without phase coupling (type I error). In this paper we present a novel method to eliminate these false bicoherences and apply the technique to study the quadratic wave-wave coupling of Energetic particle driven Geodesic Acoustic Modes (EGAMs) [2, 3] and Toroidicity-induced Alfvén Eigenmodes (TAEs) [4]. Significant bicoherence between these modes indicates that direct mode coupling has to be considered when determining mode evolution and stability. The usual definition way to calculate bicoherence of frequency components f_1, f_2 of a signal $x(t)$ is:

$$b^2(f_1,f_2) = \frac{|\mathbf{A}\left[X\left(f_1\right)X\left(f_2\right)X^*\left(f_1+f_2\right)\right]|^2}{\mathbf{A}[|X\left(f_1\right)X\left(f_2\right)|^2]\mathbf{A}[|X^*\left(f_1+f_2\right)|^2]}, \text{ with } \mathbf{A}[Y] := \lim_{N\to\infty}\frac{1}{N}\sum_{i=1}^{N}Y_i, \tag{1}$$

with $X^*(f)$ denoting the complex conjugate of the $X(f)$ Fourier transform of signal $x(t)$. For stationary cases $\mathbf{A}$ equals the expected value. The value of b represents what is the fraction of the signal power of the f_1, f_2 components that appears in a phase coupled way ($\varphi_{f_1} + \varphi_{f_2} - \varphi_{f_1+f_2} = \text{const.}$) on the $f_1 + f_2$ sum frequency. The calculation of the bispectrum – the nominator of equation (1) – can be visualized as a random walk process on the complex plane (see figure 1a). In the case of stationary, uncoupled phenomena, the value of bicoherence will converge to 0 as the number of averages $N \to \infty$. In reality the length of the measured signal is finite, and the number of blocks is also limited by the desired frequency resolution, therefore $b > 0$ values can statistically appear. In the case of instationary phenomena – illustrated in figure 1b – the situation can be worse, as blocks with large amplitudes introduce a systemic error in the calculation of the statistical averages (eq. (1)).

To quantitatively estimate the bias introduced by both the finite number of averages and possible instationarities, we can numerically estimate the $\rho(b)$ probability distribution of b for

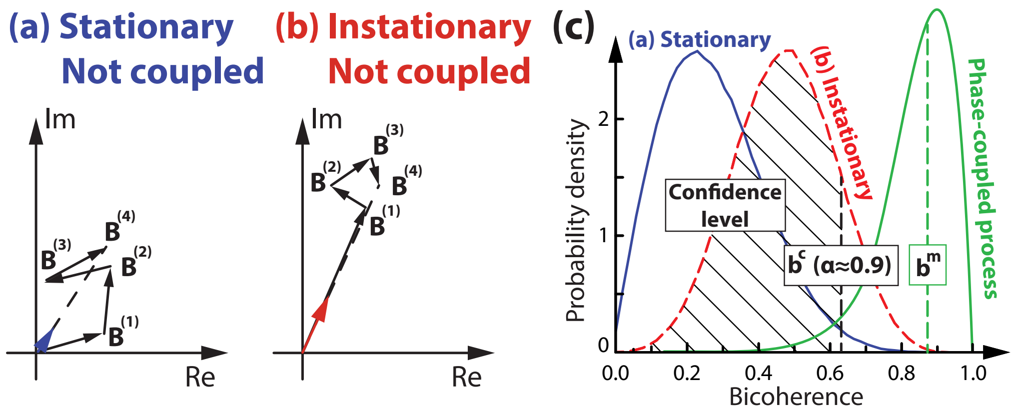

Figure 1: Bicoherence calculation visualized as a random walk on the complex plane for two not coupled processes with (a) stationary and (b) instationary amplitudes. (c) Bicoherence PDF calculated for stationary (blue) and instationary (red) random processes, as well as for a phase coupled case (green).

any uncoupled phenomenon with arbitrary amplitude evolution, where the "true" value of the bicoherence should be $\bar{b} = 0$. The Phase Randomized Bicoherence Probability Density Function ($\rho(b)$) characterizes the bicoherence distribution of an uncoupled process. It can be calculated for any given amplitude evolution measured by taking the absolute value of the $X_i(f)$ Fourier components from the measured signal, but assigning uniform random phases ($\varphi_i(f) \in [0, 2\pi[$) for each frequency in each block. With enough (e.g. $\mathscr{O}(10^3)$) realizations, $\rho(b)$ is estimated with reasonable accuracy. Then we can define an $\alpha = \int_0^{b^c} \rho(b) \mathrm{d}b$ confidence level, which assigns a critical b^c bicoherence value at each f_1, f_2 point. If the measured bicoherence is larger than b^c, then the probability of the bicoherence being false is $1 - \alpha$, otherwise it is accepted as a result of phase coupling between the components. Figure 1c shows $\rho(b)$ for a stationary, uncoupled (blue solid) and an instationary, uncoupled (red dashed) process with $N = 10$ averages calculated for $5 \cdot 10^5$ realizations. We can see that in both cases the expected value $\bar{b} > 0$, and is significantly higher for the instationary case. This can lead to high values of bicoherence (false positives or type I errors) even in the complete lack of phase coupling. The green solid line in figure 1c shows $\rho(b)$ for a hypothetical phase-coupled process. The black striped area of the instationary case illustrates the calculation of the b^c critical bicoherence value for a given α confidence level.

We have implemented the method based on the $\rho(b)$estimation into the NTI Wavelet Tools [7] signal processing software package. Numerical optimization allows fast calculation of $\rho(b)$ for typical plasma physics problems in a reasonable time. First, we validated the method and its implementation using simple synthetic models. Furthermore, we have analysed outputs of XHMGC [5] simulations which did not have wave-wave coupling enabled. As expected, mode interaction only through the fast particle phase space did not lead to significant bicoherence between the modes.

Experimental results Dedicated experiments have been carried out on ASDEX Upgrade to study the nonlinear physics of energetic particle (driven) modes (EPMs). It was aimed to maximize the ratio of fast particle pressure ("drive") and background density & temperature (ion Landau damping) to drive TAEs and EPMs unstable. Following an earlier recipe [3, 6], 93 keV off-axis NBI ($P_{NBI} = 2.5$ MW) was used in the ramp-up stage of low density ($\sim 2 \cdot 10^{19}$ m^{-3}) discharges with relatively high W concentration, further reducing background ion temperature via radiation. The latter is achieved by executing the shots just before boronization.

We have used data from the ballooning- and Mirnov coils of ASDEX Upgrade ($f_{sampling} =$ 2 MHz) [7], which measure the variation in the radial- and poloidal component of the magnetic field, respectively. Strongly driven modes are usually characterized by rapidly changing amplitudes and/or frequencies and/or radial structure(s) [3], which requires data processing methods suitable for fast transients. The Short Time Fourier Transform (STFT) spectrogram of shot #32388 is shown in figure 2a, which identifies three main frequency ranges of interest, two of which showing strongly chirping behaviour.

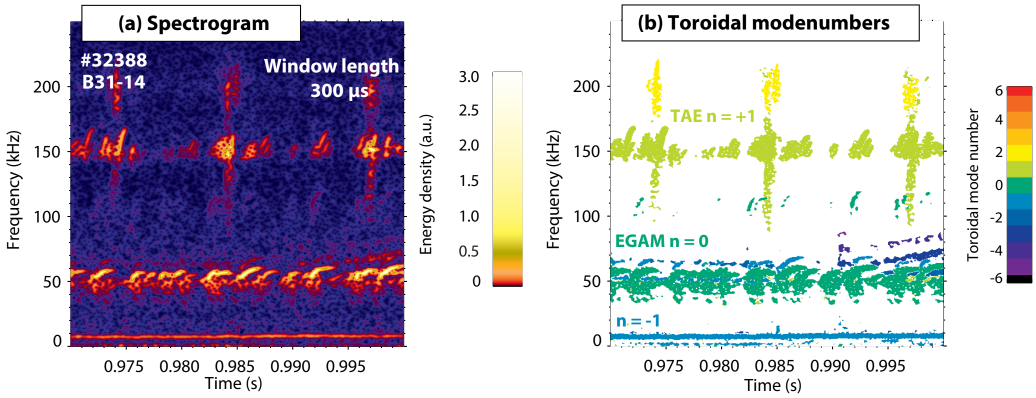

Figure 2: (a) Spectrogram and (b) n toroidal mode number spectrogram for shot #32388.

Modenumber spectrograms [7] calculated from coil arrays (shown for toroidal mode numbers in figure 2b) aid the identification of the modes. In the range of 40-60 kHz upwards chirping $(m,n) = (-2,0)$ EGAMs and in the range of 140-170 kHz chirping $(-3,+1)$ TAEs have been found. Due to a radially inverted energetic particle gradient and a strong anisotropy in pitch angle, both the TAE and EGAM propagate in the electron diamagnetic direction.

Figure 3a shows the "raw" bicoherence calculated for the time interval shown in figure 2a. The effect of broadband perturbations is visible in the frequency band $130 < f_1 + f_2 < 230$ kHz, where the average bicoherence is systematically high. High bicoherence around $(55,55)$ kHz indicates that the EGAMs are strongly nonlinear, which is further supported by the appearence of weak upper harmonics at ~ 110 kHz, shown in figures 2a-b. Further conclusions are hard to draw from the "raw" calculation, due to the possible widespread appearence of false positives.

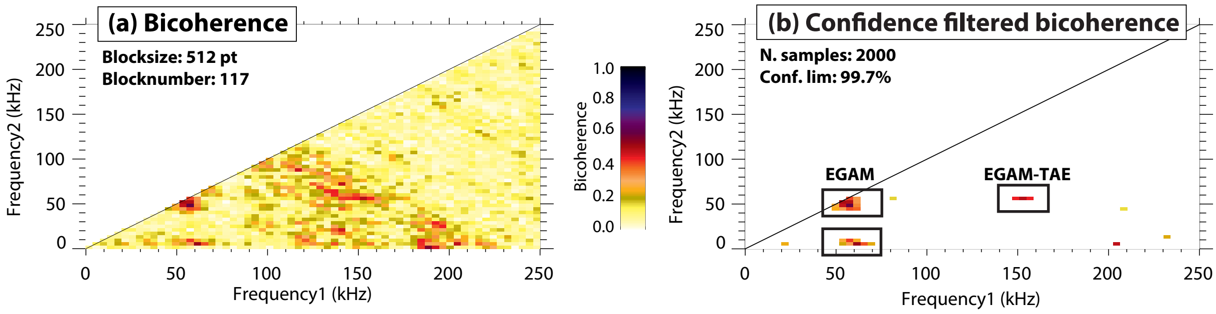

Figure 3: (a) "Raw" and (b) filtered bicoherence calculated for shot #32388 (see figure 2).

The filtering method described in the introduction was applied to the case discussed here, as shown in figure 3b. For the estimate of $\rho(b)$, 2000 realizations were calculated using the measured amplitudes but randomized phases, and a confidence level limit of $\alpha = 99.7\%$ was set. A few remaining false bicoherence points – roughly $1-\alpha$ times the total number of total bicoherence points – can still be observed due to the statistical behavior of the method. The filtering process highlights high bicoherence around $(150,55)$ kHz with a 99.7% confidence. **This indicates a significant nonlinear interaction between the EGAMs and the TAEs.** Similar behaviour is also observed in other shots, not described in detail here due to space limitations (see the poster for more details). We note that the frequency region of high bicoherence is slightly offset from the observed mode frequencies. As both the amplitude and the frequency increase during the chirp, this implies the existence of a mode amplitude threshold, above which nonlinearities become significant. A nonlinear wave-wave interaction may bear great importance in the case of ITER, and in burning plasmas in general. Wave nonlinearities may stabilise or destabilize marginally stable modes, thus influencing energetic particle transport. Further experimental studies and the analysis of XHMGC simulations with wave-wave coupling are planned for the future. The ultimate goal is to determine the coupling coefficients using the measured signal to be incorporated into more advanced theoretical and numerical models.

Acknowledgements This work has been carried out within the framework of the EUROfusion Consortium and has received funding from the Euratom research and training programme 2014-2018 under grant agreement No 633053. The views and opinions expressed herein do not necessarily reflect those of the European Commission.

References

[1] Young C. Kim, *et al.*, IEEE Transactions on Plasma Science **7** 120 (1979)
[2] G.Y. Fu *et al.*, Phys. Rev. Lett., **101** 185002 (2008)
[3] L. Horváth *et al.*, Nuclear Fusion, **56** 112003 (2016)
[4] M. Schneller *et al.*, Plasma Phys. Control. Fusion **58** 014019 (2015)
[5] S. Briguglio *et al.*, Physics of Plasmas, **2** 3711 (1995)
[6] Ph. Lauber *et al.*, Nuclear Fusion, **52** 094007 (2012)
[7] L. Horváth *et al.*, Plasma Phys. Control. Fusion **57** 125005 (2015)