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Component reduction and the superconformal gravity invariants[1]

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Abstract.
     We present the component structure of the superconformal gravity invariants in six dimensions, which was recently elaborated in arXiv:1701.08163.

1. Introduction

Conformal gravity invariants appear in the study of conformal field theories on an arbitrary curved background in even spacetime dimensions, where they correspond to certain conformal anomalies. The conformal anomalies express the tracelessness of the energy-momentum tensor in correlation functions and have been classified by Deser and Schwimmer long ago [1]. It is known that there are two main types: type A and type B. There is always one type A anomaly corresponding to the topological Euler invariant but an increasing number of type B anomalies in increasing spacetime dimension. For instance, there is one type B anomaly in four dimensions and three type B anomalies in six dimensions (6D). The type B anomalies correspond to the conformal gravity invariants, being Weyl invariant expressions constructed from the curvature tensors and its covariant derivatives.

     In six dimensions, the conformal gravity invariants may be written as spacetime integrals of the form

$$I_{\mathrm{i}} = \int \mathrm{d}^6 x \, e \, \mathcal{L}_{\mathrm{i}} \,, \quad \mathrm{i} = 1, 2, 3 \,, \tag{1.1}$$

where

$$\mathcal{L}_1 = C_{abcd} C^{aefd} C_e{}^{bc}{}_f \,, \tag{1.2a}$$

$$\mathcal{L}_2 = C_{abcd} C^{cdef} C_{ef}{}^{ab} \,, \tag{1.2b}$$

$$\mathcal{L}_3 = C_{abcd}(\delta^a_e \Box - 4R_e{}^a + \frac{6}{5}\delta^a_e R)C^{ebcd} + \text{total derivative} \,, \quad \Box := \mathcal{D}^a \mathcal{D}_a \,. \tag{1.2c}$$

Here $\mathcal{D}_a$ is the covariant derivative, C_{abcd} is the Weyl tensor satisfying the properties

$$C_{abcd} = C_{[ab][cd]} \,, \quad C_{[abc]d} = 0 \,, \quad \mathcal{D}_{[a}C_{bc]}{}^{de} = -\frac{2}{3}\mathcal{D}_f C_{[ab}{}^{f[d}\delta^{e]}_{c]} \,, \tag{1.3}$$

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[1] Based on the talk presented by JN at ISQS25 (Prague, Czech Republic, 6 – 10 June, 2017).

and R_{ab} and R are the Ricci curvature and scalar curvature, respectively. The Lagrangian $\mathcal{L}_3$ can be completed by adding a total derivative (which we suppress for simplicity) in such a way that it transforms homogeneously under Weyl transformations as $\mathcal{L}_1$ and $\mathcal{L}_2$. The properties of the curvature tensor implies that all conformal gravity invariants are described by the linear combination

$$I = c_1 I_1 + c_2 I_2 + c_3 I_3 \ , \tag{1.4}$$

where c_i are arbitrary coefficients.

In the context of superconformal field theories, which exhibit invariance under the supersymmetric extension of conformal symmetry, conformal field theories in six dimensions become more intriguing. Remarkably, the structure of the superconformal algebra implies that 6D is the highest spacetime dimension in which superconformal field theories can exist [2]. On the other hand, the only known non-trivial unitary conformal field theories in 6D are supersymmetric and arise in string theory, realizing either $\mathcal{N} = (1,0)$ or $\mathcal{N} = (2,0)$ superconformal symmetry. These properties place 6D superconformal field theories in a special class in the space of conformal field theories. Their type B anomalies should be supersymmetric and correspond to superconformal gravity invariants. Therefore, the construction of superconformal gravity invariants helps one elaborate on the anomaly structure of the 6D superconformal field theories.

Supersymmetry imposes constraints on the structure of the superconformal invariants, which now involves additional fields such as the gravitini. Thus one typically expects the number of independent superconformal invariants to reduce with more supersymmetry. The corresponding superconformal algebras exist for $\mathcal{N} = (n,0)$ but are usually limited to the $n \leq 2$ cases to ensure that superconformal gravity does not contain higher spin fields. In regards to the supersymmetric type B anomalies, there are two obvious points that should be addressed for the corresponding invariants. These include the number of independent conformal gravity invariants that one can extend to the supersymmetric case and the other is their full supersymmetric forms when coupled to the rest of the Weyl multiplet of superconformal gravity whose structure in components was first described in [3]. In principle, these points can be addressed indirectly by computing the conformal anomaly of various $(1,0)$ or $(2,0)$ matter multiplets coupled to (super)gravity. Such computations have been limited to only the purely gravitational parts, see eg. [4, 5, 6], from which one can deduce the following constraints on the coefficients c_i for the $(1,0)$ and $(2,0)$ cases, respectively:

$$\begin{aligned} c_1 - 2c_2 - 6c_3 &= 0 & \mathcal{N} = (1,0) \ , \tag{1.5a} \\ c_2 - 3c_3 &= 0 & \mathcal{N} = (2,0) \ . \tag{1.5b} \end{aligned}$$

The above constraints tell us that we expect two independent invariants in the $(1,0)$ case and only one independent invariant in the $(2,0)$ case.[2] Their purely gravitational form is given by eq. (1.4) with coefficients restricted by the constraints (1.5). Despite this information, very little is known from the computation of the purely gravitational parts of these invariants about their supersymmetric completions.

In this paper we present the results of [7] in which the full component structure of 6D conformal supergravity invariants were given. Our goals will be the following: (i) to present the direct path of [7] to constructing the component structure of 6D $\mathcal{N} = (1,0)$ superconformal gravity from superspace; (ii) to use the invariants to deduce information about the structure of the $(2,0)$ invariant; and (iii) to verify the known constraints on the coefficients c_i.

This paper is organized as follows. In section 2 we give a brief review of the superspace formulation of 6D superconformal gravity in [8] and present the procedure of [7] in extracting component expressions. In section 3 we present the $\mathcal{N} = (1,0)$ superconformal gravity invariants

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[2] The $(1,0)$ constraint also holds in the $(2,0)$ case.

in component form. Remarkably, as will be shown in section 4, the component bosonic action of the $\mathcal{N} = (2,0)$ superconformal gravity invariant can be almost completely recovered by considering its truncation onto a $\mathcal{N} = (1,0)$ invariant. Finally, conclusions are presented in section 5.

2. From superspace to components

The $\mathcal{N} = (1,0)$ superconformal tensor calculus was developed over thirty years ago in [3] for the construction of general supergravity-matter systems. The formulation met with some success in describing certain higher-derivative invariants, such as the supersymmetric Riemann curvature squared term [9]. However, it has given little insight into the construction of further higher derivative invariants, in particular the conformal supergravity invariants, which is the focus of our presentation.

Apart from superconformal tensor calculus, there is another approach to $\mathcal{N}$-extended conformal supergravity for dimension $D \leq 6$ based on using a curved $\mathcal{N}$-extended superspace $\mathcal{M}^{D|\delta}$, where δ is the number of fermionic dimensions. Within the superspace setting, one can choose to gauge only part of the superconformal algebra and rely on super-Weyl transformations to realise the conformal structure, which leads to the conventional superspace approach. It was developed for the 4D $\mathcal{N} = 1$ and $\mathcal{N} = 2$ cases in [10, 11] and presented for the 5D $\mathcal{N} = 1$ case in [12] which was extended to the 6D $\mathcal{N} = (1,0)$ case in [13]. However, for applications involving component reduction it is technically easier to make use of another superspace formulation, known as conformal superspace [14, 15, 16, 17, 8]. It is based on gauging the entire superconformal algebra and consequently makes more direct the connection with the component formulation, superconformal tensor calculus.[3] Conformal superspace has already proved useful in the construction of higher derivative invariants, e.g. [17, 18, 19, 20], and for this reason it provides an ideal formulation for the description of the higher derivative superconformal gravity invariants.

In this section we briefly present the 6D $\mathcal{N} = (1,0)$ conformal superspace developed in [8] and demonstrate how the component fields within superconformal tensor calculus originate within it following [7]. For the sake of brevity we refer the reader to [8, 7] for our conventions and the superconformal algebra, which contains the supertranslations $P_A = (P_a, Q_\alpha^i)$, the Lorentz generators M_{ab}, the dilatations $\mathbb{D}$, the SU$(2)_{\rm R}$ transformations J_{ij} and the special conformal transformations $K^A = (K^a, S_i^\alpha)$.

2.1. Gauging the superconformal algebra in superspace

To gauge the superconformal algebra one takes a $\mathcal{N} = (1,0)$ curved superspace $\mathcal{M}^{6|8}$ locally parametrised by coordinates $z^M = (x^m, \theta_{\tt i}^\mu)$, where $m = 0, 1, 2, 3, 4, 5$, $\mu = 1, 2, 3, 4$ and ${\tt i} = \underline{1}, \underline{2}$. One now associates with each generator $X_{\underline{a}} = (M_{ab}, J_{ij}, \mathbb{D}, S_k^\gamma, K^c)$ a connection one-form $\omega^{\underline{a}} = (\Omega^{ab}, \Phi^{ij}, B, \mathfrak{F}_\gamma^k, \mathfrak{F}_c) = {\rm d}z^M \omega_M{}^{\underline{a}}$ and with P_A the vielbein $E^A = (E_i^\alpha, E^a)$. They may be used to construct the covariant derivatives

$$\nabla_A = E_A{}^M \partial_M - \frac{1}{2}\Omega_A{}^{ab}M_{ab} - \Phi_A{}^{ij}J_{ij} - B_A\mathbb{D} - \mathfrak{F}_{A\beta}{}^j S_j^\beta - \mathfrak{F}_{Ab}K^b \ . \tag{2.1}$$

It is important to note that the connections can be defined such that the generators appearing in the structure group act on the covariant derivatives in the same way as they do on P_A except with P_A replaced by ∇_A.

The supergravity gauge transformations of the covariant derivatives may be summarised as

$$\delta_\mathcal{K} \nabla_A = [\mathcal{K}, \nabla_A] \ , \quad \mathcal{K} := \xi^A \nabla_A + \frac{1}{2}\Lambda^{bc}M_{bc} + \Lambda^{ij}J_{ij} + \sigma\mathbb{D} + \Lambda_\alpha^i S_i^\alpha + \Lambda_a K^a \ , \tag{2.2}$$

---

[3] The conventional formulations can be obtained by partially fixing the gauge freedom.

where the gauge parameters associated with $\mathcal{K}$ satisfy natural reality properties. It should be mentioned that the supergravity transformations act on a tensor superfield U as $\delta_{\mathcal{K}}U = \mathcal{K}U$. The superfield U is said to be *primary* and of dimension Δ if $K_A U = 0$ and $\mathbb{D}U = \Delta U$.

The above geometric setup contains too many fields to describe conformal supergravity alone. In order to describe conformal supergravity it is necessary to impose constraints on the covariant derivative algebra. These are algebraic constraints on the superspace curvatures and their corresponding consequences due to the Bianchi identities. As in lower dimensions, see [14, 15, 16, 17], it is natural to constrain the covariant derivative algebra to resemble those of supersymmetric Yang-Mills. However, this will lead to the corresponding composite component connections having non-trivial dependence on the covariant fields of the Weyl multiplet, which unnecessarily complicates the corresponding component results. To remedy this, one can redefine the vector covariant derivative $\nabla_a \to \hat{\nabla}_a$ by modifying the Lorentz and K_A connections to remove such dependence and result in the "traceless frame" of [7]. We present the resulting covariant derivative algebra below.

The torsion and curvatures appear in the the (anti)commutation relations

$$[\hat{\nabla}_A, \hat{\nabla}_B\} \equiv -\mathcal{F}_{AB} = -\hat{T}_{AB}{}^C \hat{\nabla}_C - \frac{1}{2}\hat{R}(M)_{AB}{}^{cd}M_{cd} - \hat{R}(J)_{AB}{}^{kl}J_{kl}$$
$$- \hat{R}(\mathbb{D})_{AB}\mathbb{D} - \hat{R}(S)_{AB}{}^k_\gamma S^\gamma_k - \hat{R}(K)_{ABc}K^c \, , \tag{2.3}$$

where $\hat{\nabla}_A = (\hat{\nabla}_a, \nabla^i_\alpha)$, and the torsion and curvatures are constrained to satisfy the Bianchi identities $[\hat{\nabla}_{[A}, \mathcal{F}_{BC}\} = 0$. The covariant derivative algebra is constrained in such a way that all curvatures are expressed in terms of the super-Weyl tensor $W^{\alpha\beta}$ and its covariant derivatives. The super-Weyl tensor [13] is a symmetric primary superfield of dimension 1,

$$W^{\alpha\beta} = W^{\beta\alpha} \, , \quad K^A W^{\beta\gamma} = 0 \, , \quad \mathbb{D}W^{\alpha\beta} = W^{\alpha\beta} \, , \tag{2.4}$$

and satisfies the following superspace differential constraints[4]

$$\nabla^{(i}_\alpha \nabla^{j)}_\beta W^{\gamma\delta} = -\delta^{(\gamma}_{[\alpha}\nabla^{(i}_{\beta]}\nabla^{j)}_\rho W^{\delta)\rho} \, , \tag{2.5a}$$

$$\nabla^k_\alpha \nabla_{\gamma k} W^{\beta\gamma} - \frac{1}{4}\delta^\beta_\alpha \nabla^k_\gamma \nabla_{\delta k} W^{\gamma\delta} = 8\mathrm{i}\nabla_{\alpha\gamma}W^{\gamma\beta} \, . \tag{2.5b}$$

The covariant derivative algebra is given by

$$\{\nabla^i_\alpha, \nabla^j_\beta\} = -2\mathrm{i}\varepsilon^{ij}(\gamma^c)_{\alpha\beta}\hat{\nabla}_c - 2\mathrm{i}\varepsilon^{ij}(\gamma_a)_{\alpha\beta}W^{abc}M_{bc} - \frac{3}{2}\varepsilon^{ij}\varepsilon_{\alpha\beta\gamma\delta}X^{\gamma k}S^\delta_k$$
$$-\mathrm{i}\varepsilon^{ij}(\gamma^a)_{\alpha\beta}\left(\frac{1}{4}\eta_{ac}Y - \hat{\nabla}^b W_{abc} + W_a{}^{ef}W_{cef}\right)K^c \, . \tag{2.6}$$

The nontrivial torsion and curvature components in the commutator $[\hat{\nabla}_a, \hat{\nabla}^j_\beta]$ are given by

$$\hat{T}_{a\beta k}^{j\gamma} = -\frac{1}{2}(\gamma_a)_{\beta\delta}W^{\delta\gamma}\delta^j_k \, , \tag{2.7a}$$

$$\hat{R}(\mathbb{D})_{a\beta}^{j} = -\frac{\mathrm{i}}{2}(\gamma_a)_{\beta\gamma}X^{\gamma j} \, , \tag{2.7b}$$

$$\hat{R}(M)_{a\beta}^{j\,cd} = \mathrm{i}\delta^{[c}_a(\gamma^{d]})_{\beta\gamma}X^{\gamma j} - \mathrm{i}(\gamma_a{}^{cd})_{\gamma\delta}X^{j\gamma\delta}_\beta + 2\mathrm{i}(\gamma_a)_{\beta\gamma}(\gamma^{cd})_\delta{}^\rho X^{j\gamma\delta}_\rho \, , \tag{2.7c}$$

$$\hat{R}(J)_{a\beta}^{j\,kl} = 2\mathrm{i}(\gamma_a)_{\beta\gamma}X^{\gamma(k}\varepsilon^{l)j} \, , \tag{2.7d}$$

---

[4] The superfield $W^{\alpha\beta}$ is equivalent to an anti-self-dual superfield $W_{abc} = \frac{1}{8}(\gamma_{abc})_{\alpha\beta}W^{\alpha\beta}$.

$$
\begin{aligned}
\hat{R}(S)_{a\ \beta\gamma}^{\ jk} &= -\frac{\mathrm{i}}{4}(\gamma_a)_{\beta\delta}\, Y_\gamma^{\ \delta jk} + \frac{3\mathrm{i}}{20}(\gamma_a)_{\gamma\delta} Y_\beta^{\ \delta jk} \\
&\quad -\frac{\mathrm{i}}{8}(\gamma_a)_{\beta\delta}\hat{\nabla}_{\gamma\rho} W^{\delta\rho}\varepsilon^{jk} + \frac{\mathrm{i}}{40}(\gamma_a)_{\gamma\delta}\hat{\nabla}_{\beta\rho} W^{\delta\rho}\varepsilon^{jk} \\
&\quad -\frac{\mathrm{i}}{8}(\gamma_a)_{\delta\epsilon}\,\varepsilon_{\beta\rho\tau\gamma}\, W^{\delta\rho}W^{\epsilon\tau}\varepsilon^{jk}\ ,
\end{aligned} \tag{2.7e}
$$

$$
\begin{aligned}
\hat{R}(K)_{a\ \beta c}^{\ j} &= \frac{\mathrm{i}}{4}(\gamma_c)_{\beta\gamma}\hat{\nabla}_a X^{\gamma j} - \frac{\mathrm{i}}{4}(\gamma_{acd})_{\gamma\delta}\hat{\nabla}^d X_\beta^{j\gamma\delta} + \frac{\mathrm{i}}{3}(\gamma_a)_{\beta\delta}(\gamma_{cd})_\rho^{\ \gamma}\hat{\nabla}^d X_\gamma^{j\delta\rho} \\
&\quad -\frac{\mathrm{i}}{8}(\gamma_a)_{\beta\gamma}(\gamma_c)_{\delta\rho} W^{\gamma\delta} X^{\rho j} + \frac{5\mathrm{i}}{12}(\gamma_a)_{\beta\rho}(\gamma_c)_{\gamma\epsilon} W^{\gamma\delta} X_\delta^{j\rho\epsilon} \\
&\quad +\frac{\mathrm{i}}{4}(\gamma_a)_{\gamma\rho}(\gamma_c)_{\beta\epsilon} W^{\gamma\delta} X_\delta^{j\rho\epsilon} - \frac{\mathrm{i}}{2}(\gamma_a)_{\gamma\rho}(\gamma_c)_{\delta\epsilon} W^{\gamma\delta} X_\beta^{j\rho\epsilon}\ .
\end{aligned} \tag{2.7f}
$$

Here we have introduced the dimension 3/2 superfields

$$
X_\gamma^{k\alpha\beta} = -\frac{\mathrm{i}}{4}\nabla_\gamma^k W^{\alpha\beta} - \delta_\gamma^{(\alpha} X^{\beta)k}\ , \quad X^{\alpha i} := -\frac{\mathrm{i}}{10}\nabla_\beta^i W^{\alpha\beta} \tag{2.8}
$$

together with the dimension 2 descendant superfields:

$$
Y_\alpha^{\ \beta ij} := -\frac{5}{2}\Big(\nabla_\alpha^{(i} X^{\beta j)} - \frac{1}{4}\delta_\alpha^\beta \nabla_\gamma^{(i} X^{\gamma j)}\Big) = -\frac{5}{2}\nabla_\alpha^{(i} X^{\beta j)}\ , \tag{2.9a}
$$

$$
Y := \frac{1}{4}\nabla_\gamma^k X_k^\gamma\ , \tag{2.9b}
$$

$$
Y_{\alpha\beta}^{\ \ \gamma\delta} := \nabla_{(\alpha}^k X_{\beta)k}^{\ \ \gamma\delta} - \frac{1}{6}\delta_\beta^{(\gamma}\nabla_\rho^k X_{\alpha k}^{\ \ \delta)\rho} - \frac{1}{6}\delta_\alpha^{(\gamma}\nabla_\rho^k X_{\beta k}^{\ \ \delta)\rho}\ . \tag{2.9c}
$$

The action of the spinor covariant derivatives, as well as the S-supersymmetry generators, on the defined superfields may be found in [8]. The remaining commutator $[\hat{\nabla}_a, \hat{\nabla}_b]$ in the covariant derivative algebra follows from the superspace Bianchi identities for the covariant derivatives. Finally, the action of the generators $X_{\underline{a}}$ on the covariant derivatives is the same as their action on P_A with P_A replaced with $\hat{\nabla}_A$ with one exception. The exception is $[S_i^\alpha, \hat{\nabla}_a]$ which now obtains explicit dependence on the super-Weyl tensor,

$$
\begin{aligned}
[S_i^\alpha, \hat{\nabla}_b] &= -\mathrm{i}(\tilde{\gamma}_b)^{\alpha\beta}\nabla_{\beta i} + \frac{1}{10}W_{bcd}(\tilde{\gamma}^{cd})^\alpha_{\ \gamma} S_i^\gamma - \frac{1}{4}X_i^\alpha K_b \\
&\quad +\Big[\frac{1}{4}(\tilde{\gamma}_{bc})^\alpha_{\ \beta} X_i^\beta + \frac{1}{2}(\gamma_{bc})_\beta^{\ \gamma} X_{\gamma i}^{\ \ \beta\alpha}\Big] K^c\ .
\end{aligned} \tag{2.10}
$$

It is important to stress that the entire covariant derivative algebra is expressed directly in terms of the super-Weyl tensor and its descendents. In particular, it is trivial to see that when $W^{\alpha\beta}$ vanishes the supergeometry is superconformally flat. Furthermore, the standard Weyl multiplet of $6D$ $\mathcal{N} = (1,0)$ conformal supergravity is encoded in the superspace geometry. The component fields can be readily identified as certain $\theta = 0$ parts of the superspace gauge one-forms and descendents of $W^{\alpha\beta}$, which we now turn to demonstrating.

2.2. Identifying the component structure

In contrast with conformal superspace, the superconformal tensor calculus framework [3] involves gauging the superconformal algebra in spacetime. Associated respectively with local translations, Q-supersymmetry, SU(2) R-symmetry, and dilatations are the vielbein $e_m^{\ a}$, the gravitino $\psi_{mi}^{\ \ \alpha}$, the SU(2) gauge field $\mathcal{V}_m^{\ ij}$, and a dilatation gauge field b_m. The remaining gauge symmetries are gauged by the spin connection $\omega_m^{\ cd}$, the S-supersymmetry connection $\phi_{m\alpha}^{\ \ i}$, and

the special conformal connection $\mathfrak{f}_{ma}$. These are composite fields, meaning that they are not independent and are expressed in terms of other fields of the Weyl multiplet. To ensure that the last three connections are composite, one imposes conventional constraints on the vielbein curvature $R(P)_{mn}{}^a$, the gravitino curvature $R(Q)_{mn_i}{}^\alpha$, and the conformal Lorentz curvature $R(M)_{mn}{}^{ab}$. Upon doing this it is necessary to introduce additional degrees of freedom since the bosonic and fermionic degrees of freedom do not match. An off-shell representation is achieved by introducing three covariant fields: a real anti-self-dual tensor T_{abc}^-, a chiral fermion $\chi^{\alpha i}$, and a real scalar field D which modify the supersymmetry algebra, the curvatures and the constraints imposed on the curvatures in a consistent way [3]. The component fields of the 6D $\mathcal{N} = (1,0)$ Weyl multiplet [3] can be identified within the geometry of conformal superspace, which begins with a manifest off-shell gauging of the superconformal algebra from the very beginning. We will elaborate further on this point below.

The identification of the component one-forms is straightforward. We start with the vielbein $(e_m{}^a)$ and gravitini $(\psi_{m_i}{}^\alpha)$. These appear as the $\theta = 0$ projections of the coefficients of $\mathrm{d}x^m$ in the supervielbein E^A,

$$e^a = \mathrm{d}x^m e_m{}^a = E^a \| \,, \qquad \psi_i^\alpha = \mathrm{d}x^m \psi_{m_i}^\alpha = 2E_i^\alpha \| \,, \qquad (2.11)$$

where the double bar denotes setting $\theta = \mathrm{d}\theta = 0$.[5] The remaining fundamental and composite one-forms correspond to double-bar projections of superspace one-forms,

$$\mathcal{V}^{kl} := \Phi^{kl} \| \,, \quad b := B \| \,, \quad \hat{\omega}^{cd} := \hat{\Omega}^{cd} \| \,, \quad \hat{\phi}_\gamma^k := 2\hat{\tilde{\mathfrak{F}}}_\gamma^k \| \,, \quad \hat{\mathfrak{f}}_c := \hat{\tilde{\mathfrak{F}}}_c \| \,. \qquad (2.12)$$

The covariant matter fields are contained within the super-Weyl tensor W_{abc} and its independent descendants as follows:

$$T_{abc}^- \quad := \quad -2W_{abc}| \,, \qquad\qquad\qquad\qquad (2.13a)$$

$$\chi^{\alpha i} \quad := \quad \frac{15}{2}X^{\alpha i}| = -\frac{3\mathrm{i}}{4}\nabla_\beta^i W^{\alpha\beta}| \,, \qquad (2.13b)$$

$$D \quad := \quad \frac{15}{2}Y| = -\frac{3\mathrm{i}}{16}\nabla_\alpha^k \nabla_{\beta k} W^{\alpha\beta}| \,. \qquad (2.13c)$$

The differential constraints on the superfield $W^{\alpha\beta}$ do not allow for any other independent descendents apart from those that correspond to curvatures but these are composites of the already defined component fields.

By taking the double bar projection of $\hat{\nabla}$, we define the component vector covariant derivative $\hat{\nabla}_a$ to coincide with the projection of the superspace derivative $\hat{\nabla}_a|$,

$$e_m{}^a \hat{\nabla}_a = \partial_m - \frac{1}{2}\psi_{m_i}^\alpha \nabla_\alpha^i| - \frac{1}{2}\hat{\omega}_m^{cd} M_{cd} - \hat{b}_m \mathbb{D} - \hat{\mathcal{V}}_m{}^{kl} J_{kl} - \frac{1}{2}\hat{\phi}_{m\alpha}{}^i S_i^\alpha - \hat{\mathfrak{f}}_{ma} K^a \,. \qquad (2.14)$$

The projected spinor covariant derivative $\nabla_\alpha^i|$ corresponds to the generator of Q-supersymmetry, and is defined so that if $\mathcal{U} = U|$, then $\hat{Q}_\alpha^i \mathcal{U} = \nabla_\alpha^i|\mathcal{U} := (\nabla_\alpha^i U)|$. Note that there is no ambiguity for the other generators as e.g. $M_{cd}\mathcal{U} = (M_{cd}U)|$, and so local diffeomorphisms, Q-supersymmetry transformations, and so forth descend naturally from their corresponding rule in superspace.

The component supercovariant curvature tensors are defined by the expressions $\hat{R}(P)_{ab}{}^c = \hat{T}_{ab}{}^c|$ and $\hat{R}(Q)_{ab}{}_k^\gamma = \hat{T}_{ab}{}_k^\gamma|$, and with $\hat{R}(M)_{ab}{}^{cd}$, $\hat{R}(J)_{ab}{}^{kl}$, $\hat{R}(\mathbb{D})_{ab}$, $\hat{R}(S)_{ab}{}_\gamma^k$ and $\hat{R}(K)_{abc}$ coinciding with the lowest components of the corresponding superspace curvatures. The constraints on the superspace curvatures determine how to supercovariantize a given component

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[5] In what follows a single line next to a superfield denotes setting $\theta = 0$.

curvature. One first need to simply take the double bar projection of the superspace torsion and each of the superspace curvatures. Upon doing so one finds [7]

$$\hat{R}(P)_{ab}{}^{c} = 0 \; , \tag{2.15a}$$

$$\hat{R}(Q)_{abk} = \frac{1}{2}\hat{\Psi}_{abk} + i\tilde{\gamma}_{[a}\hat{\phi}_{b]k} + \frac{1}{24}T^{-}_{cde}\tilde{\gamma}^{cde}\gamma_{[a}\psi_{b]k} \; , \tag{2.15b}$$

$$\hat{R}(\mathbb{D})_{ab} = 2e_{a}{}^{m}e_{b}{}^{n}\partial_{[m}b_{n]} + 4\hat{\mathfrak{f}}_{[ab]} + \psi_{[a}{}^{i}\hat{\phi}_{b]i} + \frac{i}{15}\psi_{[a}{}^{j}\gamma_{b]}\chi_{j} \; , \tag{2.15c}$$

$$\hat{R}(M)_{ab}{}^{cd} = \mathcal{R}_{ab}{}^{cd}(\hat{\omega}) + 8\delta^{[c}_{[a}\hat{\mathfrak{f}}_{b]}{}^{d]} + i\psi_{[aj}\gamma_{b]}\hat{R}(Q)^{cdj} + 2i\psi_{[aj}\gamma^{[c}\hat{R}(Q)_{b]}{}^{d]j}$$
$$- \psi_{[aj}\gamma^{cd}\hat{\phi}_{b]}{}^{j} - \frac{2i}{15}\delta^{[c}_{[a}\psi_{b]j}\gamma^{d]}\chi^{j} + \frac{i}{2}\psi_{[a}{}^{j}\gamma^{e}\psi_{b]j}T_{e}^{-cd} \; , \tag{2.15d}$$

$$\hat{R}(J)_{ab}{}^{kl} = \mathcal{R}_{ab}{}^{kl}(\mathcal{V}) + 4\psi_{[a}{}^{(k}\hat{\phi}_{b]}{}^{l)} + \frac{4i}{15}\psi_{[a}{}^{(k}\gamma_{b]}\chi^{l)} \; , \tag{2.15e}$$

where we have introduced the derivatives

$$\hat{\mathcal{D}}_{m} = \partial_{m} - \frac{1}{2}\hat{\omega}_{m}{}^{bc}M_{bc} - b_{m}\mathbb{D} - \mathcal{V}_{m}{}^{ij}J_{ij} \; , \quad \hat{\mathcal{D}}_{a} = e_{a}{}^{m}\hat{\mathcal{D}}_{m} \; , \tag{2.16}$$

together with

$$\hat{\Psi}_{abk}{}^{\gamma} := 2e_{a}{}^{m}e_{b}{}^{n}\hat{\mathcal{D}}_{[m}\psi_{n]k}{}^{\gamma} \; , \tag{2.17a}$$

$$\hat{\mathcal{R}}_{ab}{}^{cd} := \hat{\mathcal{R}}_{ab}{}^{cd}(\omega) = e_{a}{}^{m}e_{b}{}^{n}\left(2\partial_{[m}\omega_{n]}{}^{cd} - 2\omega_{[m}{}^{ce}\omega_{n]e}{}^{d}\right) \; , \tag{2.17b}$$

$$\hat{\mathcal{R}}_{ab}{}^{kl} := \hat{\mathcal{R}}_{ab}{}^{kl}(\mathcal{V}) = e_{a}{}^{m}e_{b}{}^{n}\left(2\partial_{[m}\mathcal{V}_{n]}{}^{kl} + 2\mathcal{V}_{[m}{}^{p(k}\mathcal{V}_{n]p}{}^{l)}\right) \; . \tag{2.17c}$$

On the other hand, the superspace curvatures are constrained to imply

$$\hat{R}(P)_{ab}{}^{c} = 0 \; , \tag{2.18a}$$

$$\gamma^{b}\hat{R}(Q)_{abk} = 0 \; , \tag{2.18b}$$

$$\hat{R}(M)_{ac}{}^{bc} = 0 \; , \tag{2.18c}$$

which explains why we called the choice of vector covariant derivatives the "traceless" frame since these conditions are modified using a different choice of vector covariant derivative. The conditions (2.18) allow one to solve for the composite connections as follows:

$$\hat{\omega}_{abc} = \omega(e)_{abc} - 2\eta_{a[b}b_{c]} - \frac{i}{4}\psi_{b}{}^{k}\gamma_{a}\psi_{ck} - \frac{i}{2}\psi_{a}{}^{k}\gamma_{[b}\psi_{c]k} \; , \tag{2.19a}$$

$$\hat{\phi}_{m}{}^{k} = \frac{i}{16}\left(\gamma^{bc}\gamma_{m} - \frac{3}{5}\gamma_{m}\tilde{\gamma}^{bc}\right)\left(\hat{\Psi}_{bc}{}^{k} + \frac{1}{12}T^{-}_{def}\tilde{\gamma}^{def}\gamma_{[b}\psi_{c]}{}^{k}\right) \; , \tag{2.19b}$$

$$\hat{\mathfrak{f}}_{a}{}^{b} = -\frac{1}{8}\mathcal{R}_{a}{}^{b}(\hat{\omega}) + \frac{1}{80}\delta_{a}^{b}\mathcal{R}(\hat{\omega}) + \frac{1}{8}\psi_{[aj}\gamma^{bc}\hat{\phi}_{c]}{}^{j}$$
$$- \frac{1}{80}\delta_{a}^{b}\psi_{cj}\gamma^{cd}\hat{\phi}_{d}{}^{j} + \frac{i}{16}\psi_{cj}\gamma_{a}\hat{R}(Q)^{bcj} + \frac{i}{8}\psi_{cj}\gamma^{[b}\hat{R}(Q)_{a}{}^{c]j} + \frac{i}{60}\psi_{aj}\gamma^{b}\chi^{j}$$
$$+ \frac{i}{16}\psi_{a}{}^{j}\gamma_{c}\psi_{dj}T^{-bcd} - \frac{i}{160}\delta_{a}^{b}\psi_{c}{}^{j}\gamma_{d}\psi_{ej}T^{-cde} \; . \tag{2.19c}$$

One could now plug the above results into eqs. (2.15) to arrive at expressions for the curvatures. Here we will simply refer the reader to [7] for the details. We also refer the reader there for supersymmetry transformations of the independent component fields and expressions for the curvatures $\hat{R}(S)$ and $\hat{R}(K)$. The above results represent the main results necessary for the component reduction of the conformal supergravity invariants from their superspace description.

3. The $\mathcal{N} = (1,0)$ superconformal gravity invariants

It turns out there are exactly two independent $\mathcal{N} = (1,0)$ superconformal gravity invariants that were constructed in superspace in [8]. One of these invariants contains a certain combination of C^3 terms at the component level, while the other contains a $C\Box C$ term at the component level. Their construction is based on the use of the superform approach to constructing supersymmetric invariants [22, 23, 24].

The superform approach to constructing supersymmetric invariants is a general approach based on the idea that a closed super D-form automatically leads to a supersymmetric invariant in D dimensions. In six-dimensions, the approach starts with a six-form $J = \frac{1}{6!}\mathrm{d}z^{M_6} \wedge \cdots \wedge \mathrm{d}z^{M_1} J_{M_1\cdots M_6}$ that is required to be closed $\mathrm{d}J = 0$. The supersymmetric invariant is given by

$$S = \int_{\mathcal{M}^6} i^* J = \int \mathrm{d}^6 x \, e \, {}^*J|_{\theta=0} \, , \qquad {}^*J := \frac{1}{6!}\varepsilon^{mnpqrs}J_{mnpqrs} \, , \tag{3.1}$$

where $i : \mathcal{M}^6 \to \mathcal{M}^{6|8}$ is the inclusion map and i^* is its pullback, the effect of which is to project $\theta_i^\mu = \mathrm{d}\theta_i^\mu = 0$. The closure of J, $\mathrm{d}J = 0$, ensures that the action (3.1) is invariant under super-diffeomorphisms. It is also required that J transform by (at most) an exact form under all gauge transformations. The component action may then be obtained as an expansion in higher products of the gravitino fields by expressing the action in terms of the tangent frame[6]

$$\begin{aligned}
S &= \int \mathrm{d}^6 x \, \frac{1}{6!}\varepsilon^{m_1\cdots m_6} E_{m_6}{}^{A_6} \cdots E_{m_1}{}^{A_1} J_{A_1\cdots A_6}|_{\theta=0} \, , \\
&= \frac{1}{6!}\int \mathrm{d}^6 x \, e \, \varepsilon^{a_1\cdots a_6} \Big[J_{a_1\cdots a_6} + 3\psi_{a_1 i}{}^\alpha J^i_{\alpha a_2\cdots a_6} + \frac{15}{4}\psi_{a_2 j}{}^\beta \psi_{a_1 i}{}^\alpha J^{i\,j}_{\alpha\beta a_3\cdots a_6} \\
&\quad + \frac{5}{2}\psi_{a_3 k}{}^\gamma \psi_{a_2 j}{}^\beta \psi_{a_1 i}{}^\alpha J^{i\,j\,k}_{\alpha\beta\gamma a_4 a_5 a_6} + \mathcal{O}(\psi^4)\Big]|_{\theta=0} \, .
\end{aligned} \tag{3.2}$$

Appropriate closed superforms describing the supersymmetric C^3 and $C\Box C$ invariants were given in [8]. They were entirely built out of primary composite superfields that were in turn built out the the super-Weyl tensor and their covariant derivatives. The correspondence outlined in the previous section from superspace to components as well as eq. (3.2) allows one to convert these superforms into their corresponding component actions. We do not provide the tedious details of this procedure here and instead simply present a large sector of the bosonic terms. We refer the reader to [7] for the details, including the full component actions.

3.1. The supersymmetric C^3 invariant

Below we present the supersymmetric C^3 invariant. For the sake of brevity we only give the bosonic sector and suppress terms involving the covariant field T^-_{abc}. The component Lagrangian is given by

$$\begin{aligned}
\mathcal{L}_{C^3} &= C_{abcd}C^{abef}C^{cd}{}_{ef} + 2\,C_{abcd}C^{aefc}C^b{}_e{}^d{}_f - \frac{3}{4}C_{abcd}\mathcal{R}^{ab\,ij}\mathcal{R}^{cd}{}_{ij} + \frac{3}{2}\mathcal{R}_{ab}{}^{ij}\mathcal{R}^{ac}{}_i{}^k\mathcal{R}^b{}_{c\,jk} \\
&\quad + \frac{3}{16}\varepsilon^{abcdef}\mathcal{R}_{abi}{}^j\mathcal{R}_{cdj}{}^k\mathcal{R}_{efk}{}^i - \frac{4}{75}D^3 - \frac{1}{10}DC_{abcd}C^{abcd} + \frac{8}{5}D\mathcal{R}_{ab}{}^{ij}\mathcal{R}^{ab}{}_{ij} \\
&\quad + \mathcal{O}(T^-) + \text{fermion terms} \, ,
\end{aligned} \tag{3.3}$$

where $\mathcal{O}(T^-)$ denotes the terms involving the covariant field T^-_{abc}. We have included the SU(2) Chern-Simons form as the first term on line 2 since in this form the Lagrangian is completely superconformally invariant.

---

[6] The expansion should be slightly modified if J is not invariant under additional gauge transformations. We refer the reader to [8] for the details.

3.2. The supersymmetric $C\Box C$ invariant

The $C\Box C$ invariant can also be given and it is described by the following component Lagrangian:

$$
\begin{aligned}
\mathcal{L}_{C\Box C} &= C_{abcd}(\delta^a_e\Box - 4\mathcal{R}_e{}^a + \frac{6}{5}\delta^a_e\mathcal{R})C^{ebcd} + 4C_{abcd}C^{aefc}C^b{}_e{}^d{}_f - C_{ab}{}^{cd}C_{cd}{}^{ef}C_{ef}{}^{ab} \\
&\quad -3\mathcal{R}_{abij}\Box\mathcal{R}^{abij} + 3\mathcal{R}_a{}^b\mathcal{R}^{acij}\mathcal{R}_{bcij} - \frac{3}{2}\mathcal{R}\mathcal{R}^{abij}\mathcal{R}_{abij} + 6\,\mathcal{R}_a{}^b{}_i{}^j\mathcal{R}_b{}^c{}_j{}^k\mathcal{R}_c{}^a{}_k{}^i \\
&\quad +6\,C^{abcd}\,\mathcal{R}_{ab}{}^{ij}\mathcal{R}_{cd\,ij} + \frac{4}{15}\,D\Box D + \frac{4}{75}\mathcal{R}D^2 + \frac{8}{75}\,D^3 + \frac{2}{5}D\,C_{abcd}C^{abcd} \\
&\quad -\frac{14}{5}D\,\mathcal{R}_{ab}{}^{ij}\mathcal{R}^{ab}{}_{ij} + \mathcal{O}(T^-) + \text{fermion terms} + \text{total derivative}\,, \qquad (3.4)
\end{aligned}
$$

where we have introduced the definition $\Box := \hat{\mathcal{D}}^a\hat{\mathcal{D}}_a$ and for simplicity we have suppressed a total derivative.

4. The $\mathcal{N} = (2, 0)$ superconformal gravity invariant

The $\mathcal{N} = (2, 0)$ Weyl multiplet of conformal supergravity was constructed in [21]. Many of the formulae there could actually be fixed by considering their truncations from $(2, 0)$ to $(1, 0)$. In a similar way, almost all of the bosonic terms of a $(2, 0)$ conformal supergravity invariant can be obtained. In this section, we show that there is at most one $(2, 0)$ conformal supergravity invariant and present a significant portion of its bosonic terms following [7].

We begin by recalling the component structure of the Weyl multiplet of $\mathcal{N} = (2, 0)$ conformal supergravity [21]. In the off-shell gauging of the $(2, 0)$ superconformal algebra, one associates the following independent fields with the local translations, Q-supersymmetry, $USp(4)$ R-symmetry, and the dilatations: the vielbein $e_m{}^a$, the gravitino $\psi_m{}^i$, the $USp(4)$ gauge field $\mathcal{V}_m{}^{ij}$, and the dilatation gauge field b_m. The remaining gauge symmetries are associated with composite connections, which include the spin connection $\omega_m{}^{cd}$, the S-supersymmetry connection $\phi_m{}^i$ and the special conformal connection $\mathfrak{f}_{ma}$. An off-shell representation of the conformal supersymmetry algebra is achieved by introducing three covariant matter fields: $T_{abc}{}^{ij} = T_{[abc]}{}^{[ij]}$, $\chi^{i,jk} = \chi^{i,[jk]}$ and $D^{ij}{}_{kl} = D^{[ij]}{}_{[kl]} = D_{kl}{}^{ij}$. Here $T_{abc}{}^{ij}$ is anti-self-dual with respect to its Lorentz vector indices and all covariant matter fields of the Weyl multiplet are traceless with respect to the invariant antisymmetric tensor Ω^{ij} of $USp(4)$. These covariant fields are used to build the full covariant curvatures given in [21].

We now turn to outlining how to perform the truncation from $(2, 0)$ to $(1, 0)$. The $(2, 0)$ Weyl multiplet decomposes into a number of $(1, 0)$ multiplets, including a Weyl multiplet on which half of the supersymmetry is manifest, two gravitini multiplets associated with the extra spin-$3/2$ gauge fields, and a Yang-Mills multiplet associated with the extra R-symmetry connections. To obtain only the fields of the $(1, 0)$ Weyl multiplet we truncate away the additional gravitino multiplets and Yang-Mills multiplet as in [21]. We split the $USp(4)$ indices $i = 1, \cdots, 4$ to $(i = 1, 2, i' = 1, 2)$ and switch off the third and fourth gravitini $\psi_m{}^{i'} = 0$. We also identify $V_m{}^{ij} = \hat{\mathcal{V}}_m{}^{ij}$ and switch off the other components of the $USp(4)$ connection $V_m{}^{i'j'} = V_m{}^{ij'} = 0$. These constraints induce restrictions on the covariant fields of the Weyl multiplet so that the Q- and S-supersymmetry transformations are consistent. These constraints are given by

$$
T_{abc}{}^{ij} = \varepsilon^{ij}T^-_{abc}\,, \qquad T_{abc}{}^{i'j'} = -\varepsilon^{i'j'}T^-_{abc}\,, \qquad (4.1a)
$$

$$
\chi_i{}^{jk} = \varepsilon^{jk}\chi_i\,, \quad \chi_{i'}{}^{j'k'} = -\varepsilon^{j'k'}\chi_i\,, \quad \chi_{i'}{}^{j'k} = \frac{1}{2}\delta^{j'}_{i'}\chi^k\,, \qquad (4.1b)
$$

$$
D^{ij}{}_{kl} = -\varepsilon^{ij}\varepsilon_{kl}D\,, \quad D^{ij}{}_{k'l'} = \varepsilon^{ij}\varepsilon_{k'l'}D\,, \quad D^{i'j'}{}_{k'l'} = -\varepsilon^{i'j'}\varepsilon_{k'l'}D\,, \qquad (4.1c)
$$

$$
D^{ij'}{}_{kl'} = -\frac{1}{2}\delta^i_k\delta^{j'}_{l'}D\,. \qquad (4.1d)
$$

Now we are in a position to consider the $(2,0)$ uplift of our $(1,0)$ conformal supergravity invariants. However, it becomes immediately apparent that each of the two $(1,0)$ invariants can not be uplifted due to the presence of the term $DC^{abcd}C_{abcd}$ in their Lagrangians. The problem is that there is no scalar built out of the fields of the $(2,0)$ Weyl multiplet which will truncate to this $(1,0)$ term. However, there is precisely one linear combination for which all such problematic terms cancel and it corresponds to

$$I_{C\Box C} + 4I_{C^3} \ , \tag{4.2}$$

which tells us that the $(2,0)$ combination must be unique. The $(2,0)$ uplift of the invariant fixes almost all the bosonic terms except for a few involving the covariant field $T_{abc}{}^{ij}$. For our purposes here, we will ignore all terms explicitly involving this covariant field. The result of the uplift gives the following bosonic terms for the $(2,0)$ invariant:

$$\begin{aligned}
\mathcal{L}_{(2,0)} &= C_{abcd}(\delta^a_e \Box - 4\mathcal{R}_e{}^a + \frac{6}{5}\delta^a_e \mathcal{R})C^{ebcd} + 12C_{abcd}C^{aefc}C^b{}_e{}^d{}_f + 3C_{ab}{}^{cd}C_{cd}{}^{ef}C_{ef}{}^{ab} \\
&\quad -3\mathcal{R}_{ab}{}^{ij}\Box\mathcal{R}^{ab}{}_{ij} + 3\mathcal{R}_a{}^b \mathcal{R}^{acij}\mathcal{R}_{bcij} - \frac{3}{2}\mathcal{R}\mathcal{R}^{abij}\mathcal{R}_{abij} + 6\,\mathcal{R}_a{}^b{}_i{}^j\mathcal{R}_b{}^c{}_j{}^k\mathcal{R}_c{}^a{}_k{}^i \\
&\quad +3\,C^{abcd}\,\mathcal{R}_{ab}{}^{ij}\mathcal{R}_{cd\,ij} + \frac{1}{75}D^{ij}{}_{kl}\Box D^{kl}{}_{ij} + \frac{1}{375}\mathcal{R}D^2 - \frac{2}{1125}D^{ij}{}_{kl}D^{kl}{}_{pq}D^{pq}{}_{ij} \\
&\quad -\frac{2}{5}D^{ij}{}_{kl}R(V)_{abi}{}^k R(V)^{ab}{}_j{}^l + \mathcal{O}(T) + \text{fermion terms} + \text{total derivative} \ , \tag{4.3}
\end{aligned}$$

where $\mathcal{O}(T)$ represents terms involving $T_{abc}{}^{ij}$.

5. Conclusion

In conclusion, we presented the 6D $\mathcal{N} = (1,0)$ conformal superspace formulation of conformal supergravity constructed in [8] and showed how it encodes the component fields of the Weyl multiplet. We presented the linearly independent component invariants for $\mathcal{N} = (1,0)$ conformal supergravity constructed in [7], which to date are probably the most complicated supergravity actions in six dimensions. Remarkably, these results allowed us to obtain a great deal of information about the component action of the $\mathcal{N} = (2,0)$ conformal supergravity invariant for which one can recover almost all its bosonic terms.

Using the results presented here, we can provide an independent derivation to the ones in the literature of the relations (1.5). The argument goes as follows. A general conformal gravity invariant may be written in terms of the purely gravitational parts of the $(1,0)$ and $(2,0)$ invariants as

$$c_1\mathcal{L}_1 + c_2\mathcal{L}_2 + c_3\mathcal{L}_3 = (c_1 - 2c_2 - 6c_3)\mathcal{L}_1 + (c_2 - 3c_3)\mathcal{L}_{C^3} + c_3\mathcal{L}_{(2,0)} \ . \tag{5.1}$$

For the $\mathcal{N} = (1,0)$ case, the first term on the right hand side must vanish since such a combination cannot be supersymmetrised, which requires the first condition of (1.5). For the $\mathcal{N} = (2,0)$ case, the second term on the right hand side must vanish since $\mathcal{L}_{C^3}$ cannot be completed alone into a $(2, 0)$ invariant, which gives the second constraint of (1.5). Thus the constraints (1.5) follow directly from a completely supersymmetric argument.

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