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Economic efficiency of CO_2 reduction programs

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ABSTRACT: A highly simplified time-dependent low-dimensional system was designed to describe conceptually the interaction of climate and economy. Enhanced emission of carbon dioxide (CO_2) is understood as an agent that not only favors instantaneous consumption but also causes unfavorable climate changes at a later time. We consider the problem of balancing these 2 counterproductive effects of CO_2 emissions on a finite time horizon. The climate system is represented by just 2 parameters, namely globally averaged near-surface air temperature and globally averaged tropospheric CO_2 concentration. The costs of abating CO_2 emissions are monitored by a function which depends quadratically on the percentage reduction of emission compared to an 'uncontrolled emission' scenario. Parameters are fitted to historical climate data and to estimates from studies of CO_2 abatement costs. Two optimization approaches, which differ from earlier attempts to describe the interaction of economy and climate, are discussed. In the 'cost-oriented' strategy an optimal emission path is identified which balances the abatement costs and explicitly formulated damage costs. These damage costs, the estimates of which are very uncertain, are hypothesized to be a linear function of the time derivative of temperature. In the 'target-oriented' strategy an emission path is chosen so that the abatement costs are minimal while certain restrictions on the terminal temperature and concentration change are met.

KEY WORDS: Climate change · Economic efficiency · Optimization · Simplified models

(1) INTRODUCTION

There is a vast body of theoretical and empirical evidence that human-caused emissions of greenhouse gases significantly increase the atmospheric concentration of these gases (cf. IPCC 1990). This, in turn, may lead to an increase in the global mean temperature.[1] Because the temperature increase is predicted to be the fastest experienced over the last 10 000 yr, it may require costly adaptation of the world economy. This adaptation might partly be avoided by the initiation of a program to abate greenhouse gas emissions.

The choice between the options of abatement or of adaptation raises economic questions. A key question relates to the long time scales involved. The quantity of greenhouse gas in the atmosphere represents an accumulation of past emissions, and it will decline only slowly after a reduction in emissions. In addition, temperature changes lag changes in concentration. The implication is that the abatement costs are to be paid by earlier generations than those who have to adapt to temperature change. This means that the problem is to find time paths for emissions, concentration and temperature which balance the welfare of present and future generations. Another question, which is beyond the scope of the present paper, concerns the efficient and fair allocation of abatement costs between rich and poor economies.

The present paper focuses on the design of strategies to rationally balance the welfare of present and future generations. The tools available for this purpose are time-dependent[2] emission reduction programs. There are many different possibilities for formulating emission abatement programs which are dynamically effi-

[1]There is a consensus among atmospheric scientists that an increased concentration of airborne greenhouse gas concentrations will significantly increase the near-surface air temperature of the world. The remaining skepticism stems from the lack of proof that the presently observed warming is not due to internal climate variations (cf. Pennell et al. 1993)

[2]Economists use the words 'intertemporally efficient'

cient. We suggest 2 different possibilities for handling one essential problem, namely the formulation of the human adaptation costs due to climate change. Thus the main emphasis in this paper is on the theoretical problem of defining an appropriate dynamic optimization format for studying the properties of emission reduction programs. Our simplified calculations do *not* yield results which could be used for any political decisions on emission reduction programs.

To formulate these ideas more strictly, we assume that climate change is controlled only by anthropogenic greenhouse gas emissions. Further, we assume that the state of the climate system can be monitored by a (low-dimensional) vector of state variables. These could be, for example, the concentration of different greenhouse gases and the temperature in different regions. When, in addition, we assume that instantaneous human welfare depends on emission abatement and on the state of the climate system, an optimal control problem can be defined by maximizing human welfare within some planning horizon. This means that, given the initial state of the climate, the problem is to choose a time path for greenhouse gas emissions which brings the climate to some admissible terminal state within some time period so that human welfare, measured by some functional criteria, is as large as possible.

In our paper we intended to use analytical mathematical solutions and to operate with as few degrees of freedom as possible in order to minimize the problem of defining constants. Therefore, the models presented below contain many simplifications. The vector of state has only 2 components, namely the mean concentration of CO_2 and the globally averaged near-surface air temperature. No regional differences are specified. A simple abatement cost function is specified which disregards the fact that investments generally occur not instantaneously but over time.

We introduce a cost-oriented strategy and a target-oriented strategy.

(1) In the cost-oriented model we assume that the adaptation costs can be expressed as a function of the rate of climate change[3] and that these costs can be measured in terms of the same units as the costs of emission abatement. Many arguments support the assumption that the rate of climate change is more important for damages than the climate change itself.[4] In general, it is difficult to know whether a climate with a global mean near-surface air temperature 1.5 to 5 °C higher than the pre-industrial level is more or less desirable from the human point of view. It is, however, clear that, if the temperature increases rapidly, the costs of adaptation may be painfully high, examples being problems in heavily populated low-lying areas and changes in agriculture and forestry. A severe obstacle in the 'cost-oriented' formulation is the lack of reliable estimates of damage costs.

(2) In the target-oriented formulation we try to circumvent the damage-cost issue by assuming that it is possible to specify some upper limit to temperature change within the next 100 yr. In this specification we require that the temperature and greenhouse gas concentration be close to some equilibrium after 100 yr. The objective in this approach of time-dependent cost efficiency is to reach the specified targets with minimum abatement costs. This approach has close counterparts in the economic literature of pollution control models which disregard the accumulation of the pollutant (see e.g. Baumol & Oates 1988) and specify the optimal emission allocation over different emission sources for reaching an exogenously given upper bound for emissions at minimum costs. But most models which, like our model, do take into account the accumulation of the pollutant use the damage functions approach, even if the damage estimation is more difficult in this case. Our target-oriented strategy tries to specify a cost-efficient CO_2 abatement program without applying the damage function approach. Instead, the damage control is taken care of by imposing side conditions to limit the set of admissible emission paths.

The paper is organized as follows. In Section 2 we define our optimal control format, i.e. the welfare and cost functions, and the dynamics of the state variables. In Section 3 we specify the various constants needed for our simple climate-economy model. For that purpose we make use of historical climate data, climate model output and estimates published in the economic literature. In Section 4 we present the solution to the models, i.e. the optimal time paths for CO_2 emissions, with the associated time paths for CO_2 concentration, temperature, abatement costs and a tax on CO_2 emissions. The sensitivity of the optimal solutions with respect to changes in the adaptation cost assumption and in the target equilibrium temperature is discussed. In Section 5 we discuss 2 interesting economic questions, namely the cost of delaying the switch from the uncontrolled emissions policy to 'Optimal Temperature Stabilizing Programs' and the economic rationale of current proposals to maintain emissions at 1990 levels. Finally, in the concluding Section 6 we discuss the possibilities of overcoming the most serious oversimplifi-

[3] To clarify our somewhat imprecise wording: if the climate is monitored by a parameter T, with the present value $T(0)$, then 'climate change' at time t is the difference $T(t) - T(0)$. The rate of climate change is defined as the time derivative $\partial T(t)/\partial t$

[4] Note that Nordhaus (1992) and Peck & Teisberg (1991) formulate the adaptation costs as a function of temperature even though these authors point out that the rate of temperature change might be significant for damages

cations of our models. Mathematical details of the optimal control problem are presented in the 'Appendix'

(2) THE MODEL

(a) The welfare function and cost functions

We assume that the human objective is to maximize the present value of net production plus a 'bequest'. For that purpose we measure human welfare, **W**, by integrating a discounted output function, **U**:

$$\mathbf{W} = \int_0^{t_1} \mathbf{U}[E(t), T'(t), t]e^{-\delta t}dt + e^{-\delta t_1}[G_1 C(t_1) + G_2 T(t_1)] \quad (1)$$

where δ is the rate of discount; t is time in years; t_1 is the length of the planning horizon; $E(t)$ is annual anthropogenic CO_2 emission (Gt C)[5]; and $T(t)$ and $C(t)$ denote the differences between the prevailing and the pre-industrial global mean near-surface air-temperature (°C) and atmospheric CO_2 concentration (ppm) respectively. The prime ' represents a time derivative so that $T'(t)$ represents the time derivative of $T(t)$. $e^{-\delta t_1}$ [...] is a bequest function which measures the economic value of the environment at the end of the planning horizon; the coefficients G_1 and G_2 are described below.

The function **U**(...) in Eq. (1) specifies the annual output as a function of CO_2 concentration, of the rate of temperature increase and of time. The interpretation of the integral is that the aim is to maximize the present value of production net of 'environmental costs', i.e. net of CO_2 abatement costs and adaptation or damage costs due to a temperature increase.

In Eq. (1) we apply a finite planning horizon, t_1. The reason for this choice is that scenarios of abatement costs and uncontrolled emissions are normally based on a time horizon of t_1 = 100 yr or so. By using a linear bequest function in Eq. (1) we evaluate the state of the physical system — as given by the CO_2 concentration and the temperature — left for future generations after the planning horizon. The coefficients G_1 and G_2 are chosen such that the bequest function reflects an infinitely long planning horizon (for details see the last paragraph in the 'Appendix').

The instantaneous net output, $\mathbf{U}[E(t), T'(t), t]$, is assumed to be a function of emissions and temperature changes:

$$\mathbf{U}[E(t), T'(t), t] = \{\mathbf{U}_0 - \mathbf{A}[E(t), t] - \mathbf{D}[T'(t)]\}e^{rt} \quad (2)$$

[5]As mentioned before, we assume for the sake of simplicity that climate change may be monitored by using a single globally averaged near-surface temperature. Also, we assume that there is just 1 greenhouse gas, namely CO_2. The unit Gt C means gigatons carbon with giga = 10^9

with the abatement function $\mathbf{A} = \mathbf{A}[E(t), t]$ and damage or adaptation function $\mathbf{D} = \mathbf{D}[T'(t), t]$. $\mathbf{U}_0$ is the output at the initial time, r is the rate of growth of net output given constant abatement and adaptation costs. $\mathbf{A}e^{rt}$ and $\mathbf{D}e^{rt}$ are the abatement costs and the dam-age or adaptation costs.

To specify the abatement function **A** we assume that **A** is a quadratic function, $\mathbf{A}[E(t), t] = aR(t)^2$, of the rate $R(t) = 1 - E(t)/E_b(t)$ of abatement from the uncontrolled emission $E_b(t)$. The quadratic form is chosen because we want to solve the model analytically. Then

$$\mathbf{A}[E(t), t] = a\left(1 - \frac{E(t)}{E_b(t)}\right) \quad (3)$$

Note that ae^{rt} represents the costs of turning off all emissions at any time. In the following we assume

$$E_b(t) = E_0 e^{qt} \quad (4)$$

so that a constant level of $R(t)$ implies increasing abatement costs (at the rate of r) and exponentially increasing emissions (at the rate of q). Accordingly, if emissions are kept at the present level, $E_b(0)$, the abatement rate R and the abatement function **A** must increase: $R(t) = 1 - e^{-qt}$ and $\mathbf{A}[E_b(0), t] = a(1 - e^{-qt})^2$.

It should be noted that the abatement cost function gives long-run abatement costs. This means that emissions are assumed to respond instantaneously to any abatement. A more satisfactory approach would be to take into account the low rate of adjustment of the capital stocks in production and abatement and express the costs as functions of the rates of investment. However, we will use the long-run cost function because CO_2 abatement cost studies usually give estimates of long-run abatement costs only.

For the damage or adaptation function **D** we assume a linear relationship[6]

$$\mathbf{D}[T'(t)] = dT'(t) \quad (5)$$

With the assumptions of Eqs. (4) & (5) we find the output (Eq. 2) to be an increasing (concave) function of the rate of emissions E:

$$\partial \mathbf{U}(t)/\partial E(t) = \frac{2a}{E_0} e^{(r-q)t}[1 - E(t)/E_0 e^{qt}] > 0 \quad (6)$$

$$\text{if } E(t) < E_b(t)$$

and the level of output is lower the higher the rate of temperature change:

[6]We have chosen this linear relationship because it will allow us to solve the resulting optimization problem analytically. This choice is not ideal. A linear relationship implies for $T' < 0$ negative adaptation costs, which may not be reasonable. However, in the cases we are going to study the level of T' will be non-negative. A more satisfactory form of the adaptation cost function may be $d[T'(t)]^b$ with $b > 1$

$$\partial \mathbf{U}(t)/\partial T' = -de^{rt} < 0 \quad (7)$$

In addition it can be shown by taking the time derivative of net output with constant temperature that output is growing, given $\mathbf{U}_0 \geq a$ and $r \geq q$, although the level of emissions is kept at some constant positive level. In the following we will deal with the case where $\mathbf{U}_0 \geq a$ and $r \geq q$.

The constants $\mathbf{U}_0$, a, d, r, q, and δ are specified in Section 3a.

(b) The connection between emissions and climate change

The emission $E(t)$ and the temperature $T(t)$ are connected through the concentration of airborne CO and 2 highly simplified equations:

$$C'(t) = \beta E(t) - \sigma C(t) \quad (8)$$

$$T'(t) = \mu C(t) - \alpha T(t) \quad (9)$$

The constants α, μ and σ are empirically derived parameters (see Section 3b) and the conversion rate β, which relates emission to concentration, is set to 0.47 (ppm Gt^{-1} C yr^{-1}) (cf. Maier-Reimer & Hasselmann 1987). $\sigma C(t)$ and $\alpha T(t)$ are restoring (elasticity) terms which force the temperature and CO_2 concentration back to the pre-industrial levels (or, in more physical terms, memory terms which determine the rate of relaxation of the solutions to the equilibrium levels determined by the external forcing). Two source terms, $\beta E(t)$ and $\mu C(t)$, drive the system away from the pre-industrial levels.

The system (Eqs. 8 & 9) suffers from a series of severe simplifications. One is that there is only 1 greenhouse gas, namely CO_2. Other important trace gases such as methane, CFCs and N_2O cannot rigorously be collected into a single 'equivalent' CO_2 gas because they have different residence times in the atmosphere. Both CO_2 and CFCs are very long-lived gases whereas N_2O and methane are less persistent. Another severe simplification is the assumption of just 1 memory term per equation. For the temperature there are at least 2 significant agents with different memory times: the upper well-mixed ocean and the stably stratified deep ocean. The interaction of temperature and concentration takes place in more degrees of freedom than just 1. Last but not least, the internal dynamics of the climate system exhibit a number of significant nonlinearities. Nevertheless, we find the system (Eqs. 8 & 9) sufficient to describe first-order interactions.

The initial conditions are

$$T(0) = T_0 \quad (10)$$

$$C(0) = C_0 \quad (11)$$

Given an emission path $E(t)$, the concentration path $C(t)$ and the temperature path $T(t)$ are completely determined. Thus the emissions enter both the out-put function (Eq. 1) and the physical system (Eqs. 8 to 11).

Eqs. (1) to (5) and (8) to (11) define a 'nonautonomous 2 state variable optimal control model' (e.g. Seierstad & Sydsæter 1987). The analytical solution for this problem is presented in the 'Appendix'. The solution includes optimal time-dependent emission levels $E^*(t)$, concentration $C^*(t)$ and temperatures $T^*(t)$.

The stationary solution to Eqs. (8) and (9) is

$$\hat{C} = \frac{\alpha}{\mu}\hat{T} = \frac{\beta}{\sigma}\hat{E} \quad (12)$$

(3) SPECIFICATION OF CLIMATE AND ECONOMIC PARAMETERS

(a) Economic parameters

There are 6 economic constants which must be specified, namely the present day output $\mathbf{U}_0$, the constants a and d in the abatement function $\mathbf{A}$ (Eq. 3) and in the damage function $\mathbf{D}$ (Eq. 5), the unperturbed growth rate of global output r, the increase of uncontrolled emissions q and the discount rate δ. We derive numerical values for these constants from various point estimates presented by Manne & Richels (1991) and by Nordhaus (1991a, b). The high uncertainty, particularly with respect to the parameters a and δ, cannot be overemphasized [see the caveats by Nordhaus (1991b) and critique by Ayres & Walter (1991)].

Following Manne & Richels (1991) we set $E_0 = 6.3$, $r = 0.02$, $q = 0.017$,[7] and $\mathbf{U}_0 = 23 \times 10^{12}$ \$.[8] For the discount rate we choose $\delta = 0.03$ in the standard case.

To obtain an estimate for the parameter a in Eq. (3) we make use of Nordhaus' (1991b) survey of CO_2 abatement costs studies. These studies usually give discrete estimates of the relationship between CO_2 tax rates and the related percentage reduction in CO_2. Interpreting the results of different studies as single observations, Nordhaus derives CO_2 abatement costs as a function of the percentage reduction in CO_2 emissions. Approximating this cost curve by a quadratic function yields $a = 10^{12}$ \$.

Thus $r > q$ and $\mathbf{U}_0 > a$ so that, according to Eq. (2), economic growth is possible even if emissions are kept constant at some level.

[7] This value of q represents a growth of uncontrolled emissions which is faster than that envisaged by the IPCC (1990) as 'Scenario A'. Six global economic models, the predictions of which were summarized by Dean & Hoeller (1992), forecasted higher emissions than the IPCC; there is high uncertainty surrounding the issue of 'business as usual'

[8] \$ is defined as US dollars adjusted to 1990 values

We turn next to the question of the function **D**, which describes the damage and adaptation costs due to temperature rise. There are only a few studies which have considered this difficult question and the level of uncertainty is enormous. We have already introduced the highly simplifying assumption that the adaptation function $\mathbf{D}[T'] = dT'$ is linear. Now we must specify the slope d of this function. A recent study by Cline (1992) attempts to estimate adaptation costs as a percentage decline in gross world product (GWP) at 2040 if emissions are left uncontrolled. According to climate model simulations of the impact of continuously increased atmospheric greenhouse gas concentrations, the linearly approximated annual increase in temperature from 1985 through 2040 is of the order of 0.03 °C yr^{-1} (Cubasch et al. 1994). Thus we express the parameter d as a function of the percentage decrease due to a 0.03 °C yr^{-1} warming; in other words $d = \gamma \mathbf{U}_0/0.03$. $\mathbf{U}_0$ equals the GWP in 1990. The parameter γ, which represents the adaptation costs in terms of a percentage decline in annual GWP, is highly uncertain and will therefore vary between 0 and 4%.

(b) Climate parameters

In Eqs. (8) & (9) all constants except β are unknown and have to be fitted to data either from observations or from simulations. To find σ we minimize

$$< \|\Delta C(t)/\Delta t - \beta \cdot E(t) + \sigma \cdot C(t)\|^2 > \; = \; \min \qquad (13)$$

where <...> denotes expectation and $\Delta t = 1$ yr. For $C(t)$ we use the observed record of annual mean CO_2 concentrations from the pre-industrial state in 1860, where $C(t) = 0$, until 1987. This record was reconstructed by a spline interpolation between direct observation, data taken at Mauno Loa, Hawaii, USA, and ice core data from Siple station (Antarctica). For the emission, $E(t)$, we use the data from Marland (1989) for the same time interval. This gives $\sigma = 0.018$ yr^{-1}, indicating a memory of 55 yr for the concentration.

To fit the 2 constants α and μ in the temperature equations the records of annual mean CO_2 concentrations and temperature for the same period, 1860 to 1987, were used. The annually and globally averaged temperatures were taken from IPCC (1990). The temperature time series exhibits significant high-frequency variability so that it is necessary to filter it before fitting α and μ to the data. As a filter we used the 11 yr running mean filter

$$\tilde{T}(t) = \frac{1}{11}\sum_{\tau=-5}^{5} T(\tau)$$

This procedure results in $\alpha = 3.0 \times 10^{-2}$ and $\mu = 4.5 \times 10^{-4}$.

The 2 constants α and μ were also fitted to data simulated in a general circulation model (GCM) experiment. In this experiment (Cubasch et al. 1992), a dynamical model of the fully coupled ocean-atmosphere system was subjected to a prescribed increase in greenhouse gas concentrations. The greenhouse gas concentrations were specified in terms of 'equivalent CO_2 concentrations' according to Scenario A of the IPCC (i.e. no emission control), with 335 ppm for 1985 and 627 ppm for 2085. The resulting values, $\alpha = 2.6 \times 10^{-2}$ and $\mu = 3.9 \times 10^{-4}$, compare well with the numbers obtained from the observed concentrations and emissions.

The reconstructed concentrations and temperatures for the interval 1860 to 1985, as well as the observed emissions, concentrations and temperatures for that period, are shown in Fig. 1. The concentration response to the observed emission path is fitted satisfactorily by our linear model. The main feature of the noisy observed time series, i.e. the accelerating warming, is reproduced. A significant part of the observed temperature is left unexplained; the processes responsible for the unexplained variance are unknown at present. Possible agents that might account for this unexplained variance are natural (internal) variability of the climate system or the man-made injection of sulfate aerosols into the atmosphere.

In Fig. 2b, c the concentration and temperature responses of Eqs. (8) to (11) to the 4 emission scenarios A to D (Fig. 2a) proposed by the IPCC are given. The model reproduces the increases in temperature and CO_2 concentration that were anticipated by the IPCC using much more sophisticated models (Fig. 2).

We also ran the model (Eqs. 8 to 11) with the 2 constants α and μ as obtained from the GCM experiments. Responses to both historical emissions (Fig. 1) and emissions envisaged by the IPCC (not shown) are close to the time paths which we obtained when we used the constants derived from observed data.

(4) OPTIMAL SOLUTIONS AND SENSITIVITY ANALYSIS

(a) Definition of optimal strategies

For both the cost-oriented and the target-oriented strategy our aim is to find an optimal emission path $E(t) > 0$ so that the induced welfare **W** (Eq. 1) is maximized within a finite time horizon $[0,t_1]$.

In the cost-oriented strategy no side conditions to limit emissions are imposed, and both abatement costs and damage costs contribute to the output function. No restrictions on the range of atmospheric CO_2 concentrations or temperature changes are applied. However,

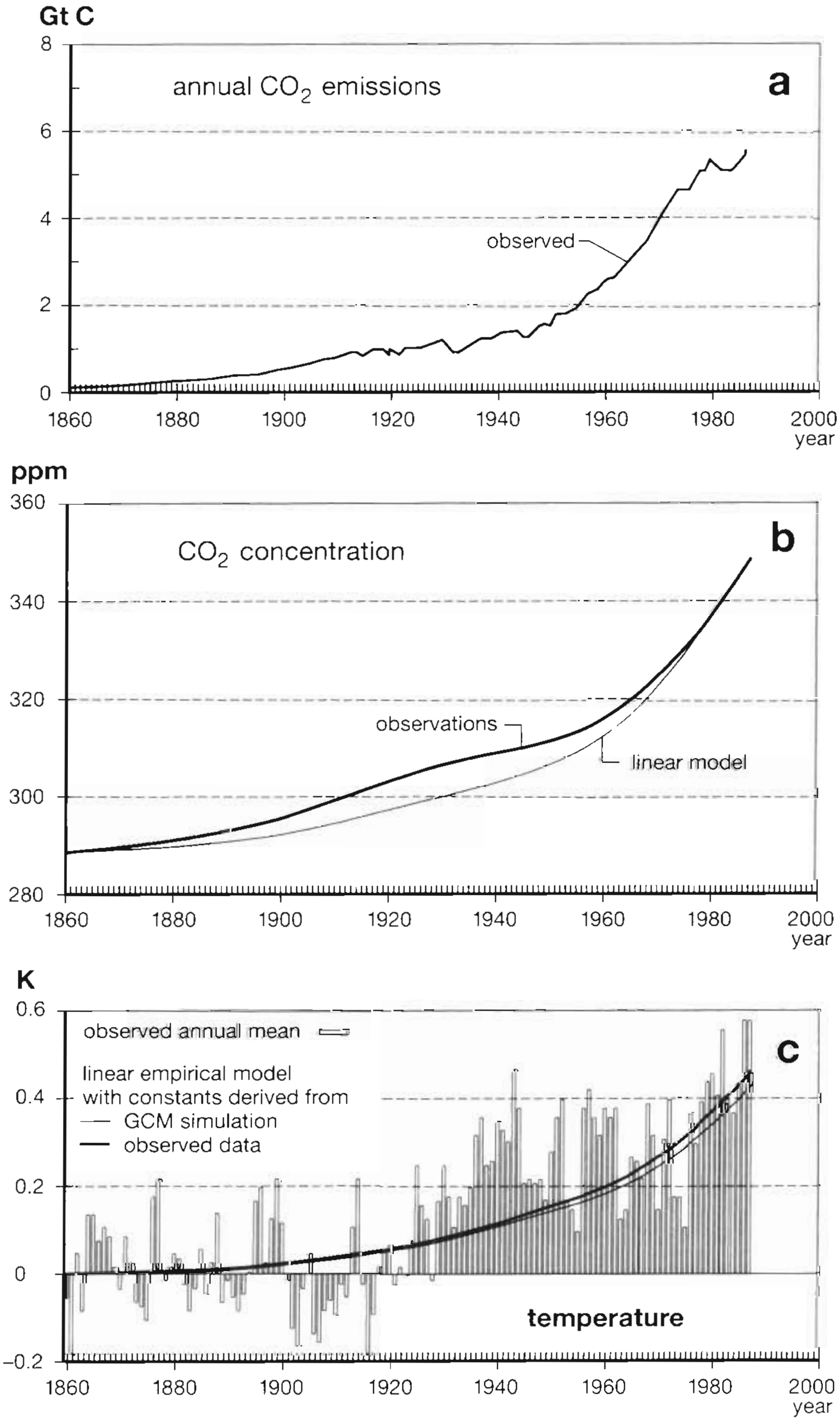

Fig. 1. Historical paths of CO_2 emission, CO_2 concentration and temperature from 1860 to 1985. (a) Observed annual CO_2 emissions (Gt C); (b) observed and modeled CO_2 concentration (ppm); (c) observed temperature variations (bars) from 1860 through 1987 and modeled temperature increase, as a response to the observed CO_2 emissions given in (a). Thick line: constants derived from the historical data; thin line: constants derived from the climate model output

the endpoint levels of carbon concentration and temperature enter the problem via the bequest functions. The 'Appendix' (last paragraph) shows that the coefficients G_1 and G_2 can be set to reflect an infinite planning horizon. The main obstacle with the cost-oriented strategy is the enormous uncertainty in estimating the damage or adaptation costs (see Ayres & Walter 1991, Nordhaus 1991a, Cline 1992, Schelling 1992).

Because of the uncertainty of the damage costs, the target-oriented strategy operates without explicitly specified damage costs and without bequest functions. Instead the parameter d in (Eq. 5) is set to zero so that the output function (Eq. 2) includes only the abatement costs $\mathbf{U}[E(t),T'(t),t] = \{\mathbf{U}_0 - \mathbf{A}[E(t),t]\}e^{rt}$. The maximum welfare $\mathbf{W}$ is obtained by $E(t) = E_b(t)$, but at the same time temperature would steadily rise and

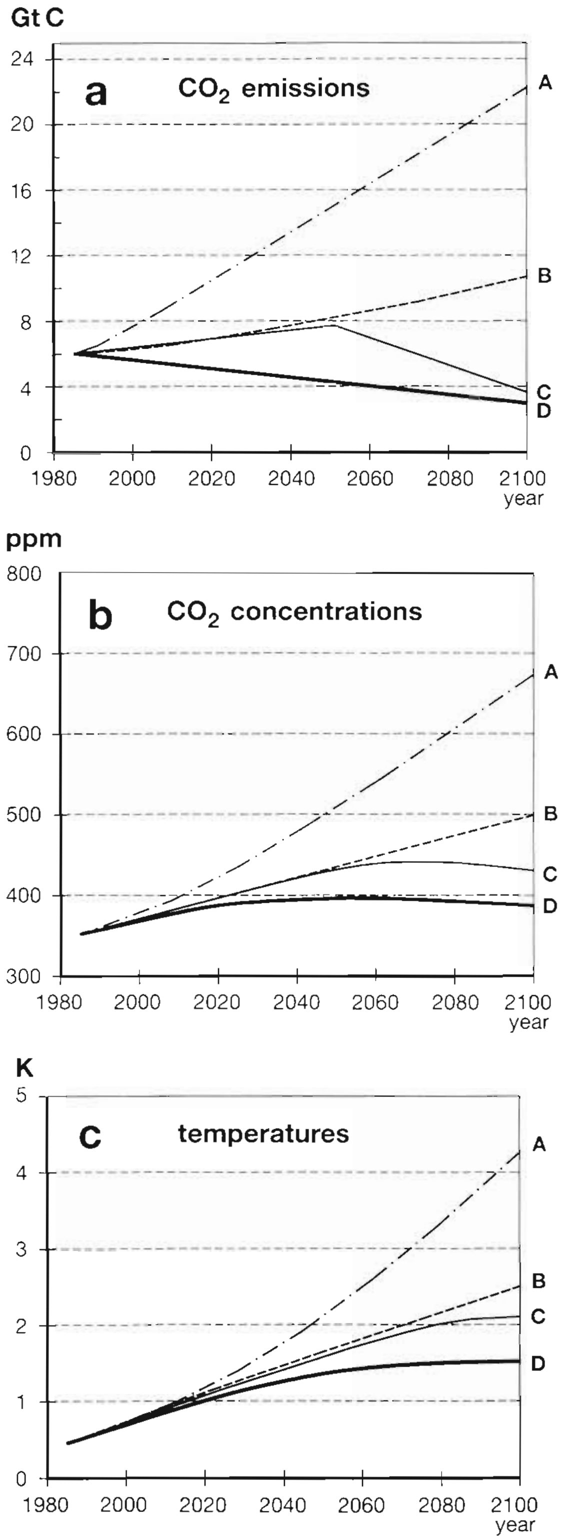

Fig. 2. (a) IPCC (1990) scenarios A to D of annual CO_2 emission (Gt C) for 1985 through 2100. (b) Response of the atmospheric CO_2 concentration, in ppm, to the emission scenarios A to D (see panel a) as derived from the simplified linear model (Eqs. 8 & 9). The concentration is given as the deviation from 1860 levels. (c) Same as (b) but for temperature

would cause unknown damage to the economy and to the earth's ecology. To limit these damages we introduce additional side conditions which will limit the set of admissible emission paths in the optimization problem.

An interesting property of the target-oriented strategy is its independence of the cost parameter a (>0). The quadratic form of the abatement function together with the other properties of the model is sufficient for determining the optimal solution.

The restrictions required for the target-oriented strategy can take various forms. Here, we use an approach which imposes upper-limit restrictions on the terminal temperature and concentration:

$$C(t_1) \le \hat{C} \tag{14}$$

$$T(t_1) \le \hat{T} \tag{15}$$

Since we want the climate system to be close to a new equilibrium at the end of the planning horizon, $\hat{T}$ and $\hat{C}$ are required to satisfy the stationarity condition (Eq. 12), $\hat{T} = \mu/\alpha\hat{C}$. Then the only parameter which is subjected to an upper-limit restriction is $\hat{T}$.

Even though $d = 0$, the damage costs are taken into account in this target-oriented strategy because of the side conditions imposed in Eqs. (14) & (15). Instead the damage cost function is a step function switching costs from zero to infinity at a certain threshold.

We will show that in most cases the optimal temperature and concentration will increase monotonically and that the optimal solutions which satisfy Eqs. (14) & (15) normally will stay below $\hat{T}$ and $\hat{C}$ for almost all $t \in [0, t_1]$. Also, the rate at which temperature increases is limited because the CO_2 concentration and temperature respond slowly to changes in emission.

Because of the stationarity condition (Eq. 12) 3 different cases, or regimes, can occur at the end of the planning horizon:

(I) $T(t_1) = \hat{T}$ and $C(t_1) = \hat{C}$: both the temperature and the CO_2 concentration are equal to the upper bounds. Then the temperature has been stabilized: $T'(t_1) = 0$.

(II) $T(t_1) < \hat{T}$ and $C(t_1) = \hat{C}$: the terminal temperature is below the upper bound while the CO_2 concentration is at the maximum admissible level. The temperature is rising at the end of the planning horizon: $T'(t_1) > 0$.

(III) $T(t_1) = \hat{T}$ and $C(t_1) < \hat{C}$: the final temperature equals the upper bound restriction but the concentration is below the maximum admissible level. The temperature is decreasing: $T'(t_1) < 0$.

The conditions defined in Eqs. (12), (14) & (15) thus exclude the possibility that the terminal temperature is rising when it equals the upper bound restriction. This case should be ruled out because there would be no assurance that the upper bound temperature could be maintained with a non-negative emission after the end of the planning horizon. In other words, the emission

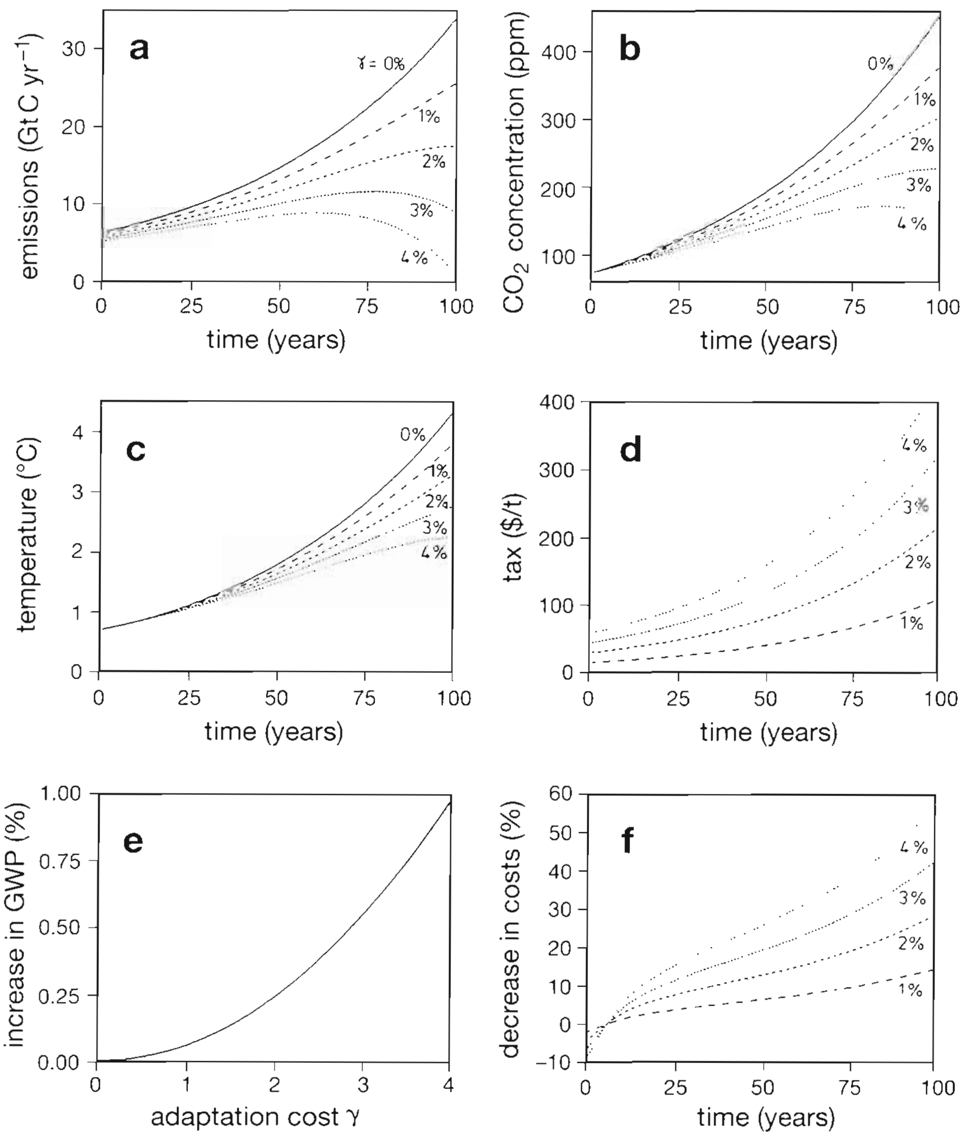

Fig. 3. Results obtained for the cost-oriented strategy (nonzero damage and bequest functions but no restrictions on the climate system) conditional on different values of γ (γ is the damage caused by an annual temperature increase of 0.03 °C, given in percentage of the 1990 GWP). (a) to (d) & (f) show optimal emissions E^{*} (in Gt C yr^{-1}), CO_2 concentrations C^{*} (in ppm, relative to the 1860 level), global air temperature T^{*} (in °C, relative to the 1860 level), the carbon tax (in \$ t^{-1}) and the reduction of costs earned by applying the optimal cost-oriented strategy (in % of damage costs incurred by a business-as-usual policy without control). (e) shows the percentage increase in discounted world production due to optimal emission control when γ varies between 0 and 4%

abatement program would have failed to have stabilized the climate system. In the 3 cases listed above it is possible to maintain the upper bound temperature with some feasible emission control policy after t_1. In this sense the scope of this condition is extended beyond the planning horizon.

An alternative set of restrictions would be to use Eqs. (14) & (15) with strict equalities. The set $S^{=}$ of admissible emission paths satisfying the equalities $C(t_1) = \hat{C}$ and $T(t_1) = \hat{T}$ is smaller than the set $S^{<}$ of admissible emission paths satisfying the inequalities in Eqs. (14) & (15). Therefore the optimal solution in $S^{=}$ is more expensive than the optimal solution in $S^{<}$. The enhanced costs of the $S^{=}$ solution are required to establish the prescribed CO_2 concentration $\hat{C}$ and the temperature $\hat{T}$ at the end of the planning horizon. We do not think that such an approach makes sense.

In the following we present the optimal solutions, labelled E^{*}, obtained from different strategies. The analytical solutions are given in the 'Appendix'.

(b) The cost-oriented strategy: nonzero damage and bequest functions but no physical restrictions

For the cost-oriented strategy we must specify γ (representing the adaptation costs, as percentage of the world annual output, that correspond to a temperature increase of 0.03 °C yr^{-1}). In Fig. 3a the optimal emission paths $E^{*}(t)$ are given for γ = 0, 1, 2, 3 and 4%. The optimal paths are strongly dependent on the choice of γ. The initial optimal emissions vary between $E^{*}(0) = 6.3$ Gt C for γ = 0% (note that $E^{*} = E_b$ if γ = 0%) and 5 Gt C for γ = 4%. When $t = 70$ the optimal emission varies between 20.5 Gt C ($\gamma = 0\%$) and 8 Gt C ($\gamma = 4\%$).

Fig. 3b, c shows the responses of concentration $C^{*}(t)$ and temperature $T^{*}(t)$ to the optimal emission paths (Fig. 3a) conditional on the choice of γ. Without emission control the temperature rises by 4.3 °C above the pre-industrial level within 100 yr. When t = 100 the temperature increase is 2 °C and its rate of change is approximately zero, for $\gamma = 4\%$.

The optimal policies connected with $\gamma < 3\%$ never yield a stabilization of the atmospheric CO_2 concentration. Only for $\gamma \geq 3\%$ is the concentration kept approximately constant for the last decades of the planning horizon.

Fig. 3d represents the CO_2 tax levels which are needed to induce the emission trajectories shown in Fig. 3a. The emission tax levels equal the derivative of abatement costs with respect to the emission along the optimal trajectories, i.e. $\partial \mathbf{A}[E^{*}(t)]/\partial E$. The time-dependent taxes depend strongly on the specification of the adaptation costs (represented by γ). The initial taxes vary between zero (no control) and 60 \$ t^{-1} C.

Fig. 3e compares the value of maximized welfare to the outcome when emissions are left uncontrolled. In other words the figure shows the maximum economic gain which would be achieved by controlling emissions optimally. The increase in welfare is given as a function of the adaptation cost estimate γ. Thus when γ = 4 the optimal policy increases the level of present value output by 1%. Note that the gain from optimal emission control also includes the higher level of bequest that is left for future generations after the planning horizon.

Another possible way of evaluating the potential gains which can be achieved by emission control is to estimate the decrease in environmental costs, which are defined as the sum of adaptation and abatement costs. Fig. 3f compares the optimized environmental costs with the outcome of uncontrolled emissions, conditional on the choice of γ. During the first 6 yr the optimized sum of abatement and adaptation costs exceeds the adaptation costs realized by the uncontrolled policy. However, after 6 yr the annual environmental costs are lower if emissions are optimally controlled compared to the outcome of uncontrolled emissions. When $\gamma = 4\%$ the optimal policy finally reduces environmental costs by 50% at the end of the planning horizon.

(c) The target-oriented strategy: restrictions on the terminal temperature and concentration

In this subsection we present optimal policies defined by the target-oriented strategy, in which welfare $\mathbf{W}$ takes into account only the discounted abatement costs $\mathbf{A}[E(t),t]e^{rt}$. The negative effects of climate change are no longer represented by a damage cost function (i.e. d = 0) or by the bequest functions (i.e. $G_1 = G_2 = 0$) but by restrictions on the terminal CO_2 concentration and on the global mean temperature. These upper bounds, $\hat{T}$ and $\hat{C}$, are connected through Eq. (12).

We consider the terminal upper bounds $\hat{T}$ = 6.8, 4, 3.5, 3, 2.5, or 2 °C and show in Fig. 4a the optimal emission trajectories related to the different $\hat{T}$ values. Note

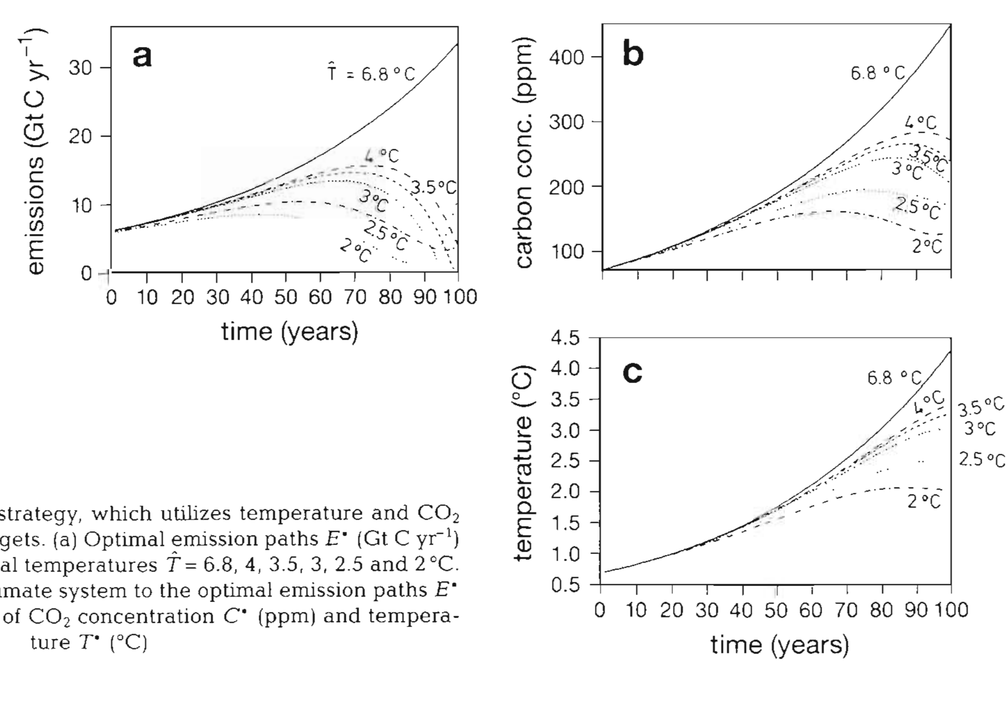

Fig. 4. The target-oriented strategy, which utilizes temperature and CO_2 concentration stabilizing targets. (a) Optimal emission paths E^{*} (Gt C yr^{-1}) conditional on target terminal temperatures $\hat{T}$ = 6.8, 4, 3.5, 3, 2.5 and 2 °C. (b, c) The response of the climate system to the optimal emission paths E^{*} shown in panel (a) in terms of CO_2 concentration C^{*} (ppm) and temperature T^{*} (°C)

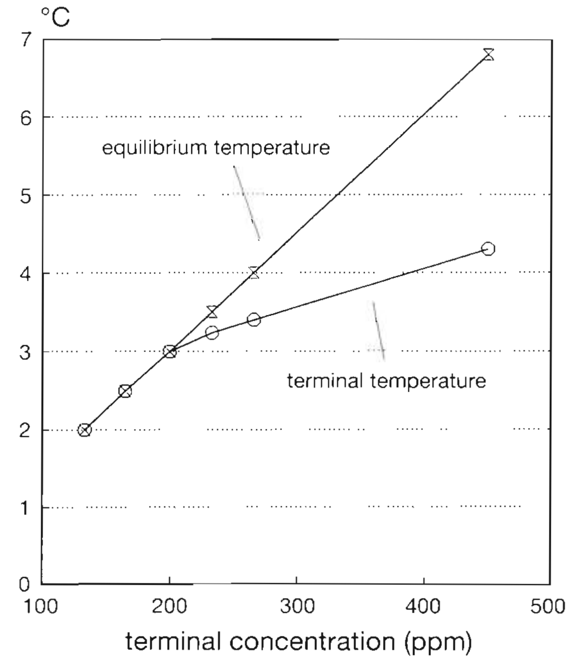

Fig. 5. The target-oriented strategy: terminal temperature $T(t_1)$ of different optimal paths conditional on a prescribed upper limit for the terminal concentration $\hat{C}$ or, equivalently, of temperature $\hat{T} = \alpha/\mu\hat{C}$. For $\hat{T} \leq 3$°C and $\hat{C} \leq 200$ ppm, the climate system ends in an equilibrium (Regime I) at the end of the planning horizon, whereas for larger upper limits the optimally controlled temperature stays well below the allowed upper limit at the end of the planning horizon but the temperature is rising at that time

that, if emissions are uncontrolled, the carbon concentration increases to a level of about $\hat{C} = 450$ ppm, which implies an equilibrium temperature of $\hat{T} = 6.8$ °C. When $\hat{T}$ is 3.5 or 3 °C the optimal emission level eventually becomes zero. When $\hat{T} < 3$ °C the endpoint restriction for the temperature (Eq. 15) becomes binding. The shape of the optimal emission trajectories with $\hat{T} <$ 2.5 °C is markedly different from those with $\hat{T} > 3$ °C.

The difference between the uncontrolled outcome and different optimal policies is small during the first years but becomes much larger after 50 yr. That low $\hat{T}$ levels are achieved even though the optimal emissions do not deviate much from the uncontrolled levels in the early years of the planning horizon has 4 explanations:

(1) The positive rate of discount implies that the present generation values later abatements as less costly than those made earlier.

(2) According to the time-dependent cost function $\mathbf{A}[E(t),t]$ the cost of reducing the first x units of emissions from the uncontrolled level decreases exponentially with time.

(3) The removal of carbon dioxide from the atmosphere was specified in Eq. (8) by $-\sigma C$, which is a linear and increasing function of the CO_2 concentration. Thus an emission control policy which allows for more emissions in the early years of the program efficiently utilizes the ability of the climate system to dispose of the additional CO_2.

(4) Similarly, a policy which permits initially increased carbon dioxide concentrations makes efficient use of the climate system's tendency to have temperatures return towards the pre-industrial levels.

In Fig. 4b, c the optimal paths for CO_2 concentration and temperature are shown for different choices of the maximum terminal temperature $\hat{T}$. The terminal concentration equals in all cases the 'equilibrium' concentration, i.e. the prescribed upper limit $\hat{C} = (\alpha/\mu)\hat{T}$. The optimally controlled temperature curves all become almost stationary at the end of the planning horizon of 100 yr.

To analyze the state of the controlled climate system at the end of the planning horizon, we have plotted in Fig. 5 the maximum permissible (equilibrium) terminal temperature $\hat{T}$ and the optimally controlled terminal temperature $T(t_1)$ as a function of the upper-limit terminal (equilibrium) concentration $\hat{C}$.

For terminal concentrations of ≤200 ppm and terminal temperatures of ≤3 °C, the terminal temperature equals the upper limit and the system is in Regime I [$T(t_1) = \hat{T}$, $C(t_1) = \hat{C}$ and $T'(t_1) = 0$; Section 4a], i.e. the climate will stay in equilibrium if, after t_1, an emission policy is chosen so that the concentration will be kept constant. For higher terminal concentrations, the optimally controlled emissions yield a temperature change less than the maximal allowed change of >3 °C. Thus, the climate system is in Regime II [$T(t_1) < \hat{T}$, $C(t_1) = \hat{C}$ and $T'(t_1) > 0$] in which the terminal temperature is rising. In these cases it is optimal to leave the terminal temperature below $\hat{T}$ but the concentration will equal $\hat{C}$. However, the temperature has not been stabilized and temperature will rise if concentrations are kept constant after t_1.

(5) DISCUSSION

(a) Costs of a delay of an optimal emission program

What are the costs of postponing an optimal emission abatement program? In Fig. 6 we show the costs that are incurred if the optimal program, given by the target-oriented strategy with $\hat{T} = 2.5$ °C or by the cost-oriented strategy with $\gamma = 3$ %, is delayed by 0 to 60 yr. All other parameters have been assigned our standard values, such as $t_1 = 100$ yr and $\delta = 3$ %.

In the framework of the target-oriented strategy an optimal solution with non-negative emissions does exist, even after a delay of 60 yr. The present value of

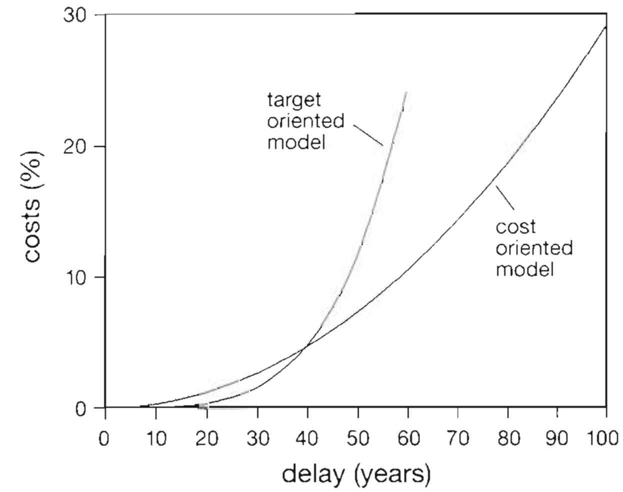

Fig. 6. Costs ($) of delaying the initiation of an optimal emission program given by the target-oriented strategy, with bounds on the terminal concentration and temperature [$T(t_1) \leq \hat{T} = 2.5\,°C$], and by the cost-oriented strategy, with explicitly given damage costs calculated with $\gamma = 3\%$

the postponed emission control is about 25% higher than that of an immediately launched program. Surprisingly, a delay of 40 yr raises the optimal costs by only ca 5%.

The other curve in Fig. 6 shows the increase in environmental costs if the optimal cost-oriented program is delayed. The curve shows that the costs of delaying less than 40 yr are higher than for the target-oriented strategy. However, delays longer than 40 yr increase the level of abatement costs in the target-oriented strategy more than they increase the sum of abatement and adaptation costs in the cost-oriented optimal program.

The costs of postponing emission control programs obtained by the 2 strategies are not directly comparable, however. In the cost-oriented strategy the damages are explicitly included in the costs, whereas in the target-oriented strategy the damage costs are free to vary as long as the incurred climate change does not exceed the imposed upper limit.

Following the studies by Nordhaus (1977, 1991a, b, 1993) and Peck & Teisberg (1991) we used an annual rate of discount of $\delta = 3\%$. How do the optimal trajectories depend on this choice? Tahvonen et al. (1994) show for the target-oriented strategy that a higher rate of discount implies higher emissions towards the beginning of the planning interval, and a lower rate implies increased emission towards the end. This result is intuitively meaningful — an decrease of future costs moves part of the abatement to future generations.

In the case of the cost-oriented strategy the dependence between rate of discount δ and optimal emission trajectories $E^*(t)$ is more complicated. In the 'Appendix' (Eq. A26) we compute the derivative $\partial E^*(t)/\partial\delta$, the sign of which is undetermined. A closer look at Eq. (A26) reveals that the dependence of the optimal emission level E^* on the rate of discount δ is U-shaped. If $r = \delta$ we have $\partial E^*(t)/\partial\delta < 0$ but when δ is large enough $\partial E^*(t)/\partial\delta$ becomes positive. This characteristic of the cost-oriented model is a complicated matter and is studied in a companion paper by Tahvonen et al. (1994). The optimal emission may decrease with an increase in the rate of discount. At first glance this behaviour is surprising, but it may be traced back to the fact that it is the rate of the temperature change which enters the damage costs. Future generations are better off if they inherit a stable and 'not too low' temperature which keeps adaptation costs low without too high abatement costs. Thus, when the welfare of future generations is taken to be more important (low δ), the optimal approach may be to increase the present emission level, with the consequence that generations in the near future, instead of generations in the far future, will face the required adaptation costs due to rising temperature.

(b) Evaluation of the emission stabilizing proposal

Perhaps the most common proposal for stabilizing temperature increase has been to keep annual emissions at the 1990 level of about 6 Gt C (see e.g. Manne & Richels 1991). We label this policy $E_s = E_b(0) \approx 6$ Gt C.

We may formally assess the usefulness of this proposal in the framework of our 'cost-oriented strategy'. From Fig. 3 we can see that the E_s policy approximates the optimal emission control policy E^* if the level of γ is between 3 and 4%.

Another way to evaluate this proposal is to ask if the value $\mathbf{W}_s$ of the E_s policy is higher than the value $\mathbf{W}_b$ of the 'business-as-usual' policy, E_b. The answer depends on the parameter γ. For $\gamma = 3.2\%$ we find the 2 values identical:

$$\mathbf{W}_s = \int_0^{t_1} [\mathbf{U}_0 + \mathbf{A}(E_s) + \mathbf{D}(T'_s)]e^{(r-\delta)t}\,dt + e^{-\delta t_1}[G_1 C_s(t_1) + G_2 T_s(t_1)]$$

$$= \int_0^{t_1} [\mathbf{U}_0 + \mathbf{D}(T'_b)]e^{(r-\delta)t}\,dt + e^{-\delta t_1}[G_1 C_b(t_1) + G_2 T_b(t_1)]$$

$$= \mathbf{W}_b \qquad (16)$$

This means that in terms of the functional criteria of the cost-oriented strategy, the emission stabilizing program E_s is superior to the business-as-usual policy E_b only if the annual adaptation costs due to a temperature increase of 0.03 °C yr^{-1} are such that world output will be reduced by at least 3.2 %.

According to our simple climate model (Eqs. 8 & 9) the E_s policy would raise the temperature by 2 °C and the concentration by 146 ppm (relative to pre-industrial levels) at the end of the planning horizon of 100 yr. To equilibrate the concentration of $\hat{C}$ = 146 ppm, a temperature $\hat{T}$ = 2.2 C would be required. The E_s policy thus leads the climate system into Regime II (see Section 4a) with increasing temperatures at the end of the planning horizon.

In Section 4b we already argued that 'intertemporal cost efficiency' does not imply constant emissions. Therefore we consider whether it is possible to find a time-dependent strategy E^*_s which is less expensive in terms of abatement costs than the E_s policy but which yields the same terminal conditions. We find E^*_s with the target-oriented strategy with strict equalities for the terminal conditions: $T(t_1)$ = 2 °C and $C(t_1)$ = 146 ppm. In terms of climate change the cost efficient policy E^*_s is superior to the emission stabilizing policy E_s since the T^*_s temperature is approximately stable during the last 15 yr (not shown) whereas $T'_s(t_1) > 0$.

The cost savings achieved by applying the optimal E^*_s policy instead of the emission-stabilizing program E_s, i.e. $\int[\mathbf{A}(E^*) - \mathbf{A}(E_s)]e^{(r-\delta)t}\,dt$, is 6 % and 10 % of the present value of the emission-stabilizing program if δ = 0.03 and 0.04 respectively.

(6) CONCLUSION

(a) Summary

We have tried to combine the key features of complex climate and CO_2 abatement cost models in an optimal control framework. Two different strategies were studied. The first, cost-oriented approach generates an optimal time-dependent (intertemporal) balance between adaptation and abatement costs. The second, target-oriented approach specifies emission trajectories which drive the temperature-CO_2 system to some equilibrium target state by minimizing the present value of abatement costs. Both of these optimization problems were simple enough to be solved analytically. The parameter values for the climate equations were estimated using empirical climate data. The economic parameters were derived by utilizing the CO_2 abatement cost studies of Nordhaus (1991b).

The purpose of the paper is *not* to propose realistic emission control policies. Instead our intent was to discuss concepts in the design of such policies by explicitly coupling together highly aggregated models of climate and economy. In that sense our concept represents the first realization of a Global-Environment-and-Men model (Hasselmann 1990). Our paper may also be seen as a reformulation of Nordhaus' (1991a) study, in which he defined adaptation costs as a function of temperature and considered only stationary climate conditions and constant emissions.[9]

In our study we made a number of broad assumptions, which prevent the results from being directly usable. Instead our analyses serve only to elucidate the general dynamics. Important assumptions are that climate change is controlled entirely by 1 greenhouse gas emission, the possibility to reduce the complex climate system to just 2 highly aggregated variables (namely near-surface temperature and CO_2 concentration), and that our human welfare function $\mathbf{W}$ in (Eq. 1) makes sense.

(b) Evaluation of the cost-oriented and target-oriented strategies

Although our system involves many simplifications, it is possible to evaluate the relative merits of the 2 strategies.

The strength of the cost-oriented approach is that the solution trajectories give the balance between abatement and adaptation costs directly. In this approach it is optimal to cut emissions more heavily in the early years of the planning horizon. Although this approach is economically appealing, its usefulness is limited because of the lack of reliable adaptation cost estimates. A drawback of the cost-oriented strategy is its failure to lead the climate system to equilibrium (Fig. 7). For damage levels $\gamma < 3$ %, the optimal temperature $T^*(t_1)$ at the end of the planning horizon of t_1 = 100 yr is below the temperature $\hat{T}$ which equilibrates the CO_2 concentration $C^*(t_1)$ at that time. The difference between the actual temperature and the temperature that is in equilibrium with the actual CO_2 concentration may be seen as a potential climate change. If, after 100 yr, the concentration is kept at the 100 yr level, the temperature will rise above the temperature at the end of the planning horizon of 100 yr. This potential accounts for 1.1 °C after 100 yr if γ = 2 %. Only with a damage level of γ = 4 % will the system be (almost) in equilibrium at t_1 and not warm up further after 100 yr.

[9]In an earlier paper, Nordhaus (1977) presented a model which aimed at a limitation of increases in atmospheric CO_2 concentration within a finite time horizon. His main conclusion was that, owing to discounting, the major reductions in an optimal program should be scheduled afer about 40 yr. This finding is consistent with our results

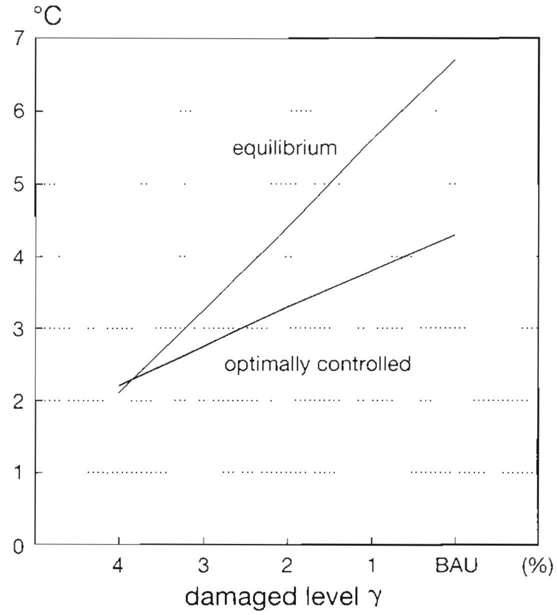

Fig. 7. The cost-oriented strategy: equilibrium temperatures and optimally controlled temperatures after 100 yr, conditional on different assumed damage levels γ. BAU (business-as-usual) refers to $\gamma = 0$ when no control is required

The target-oriented strategy is less appealing than the cost-oriented strategy since the former depends on an exogenous decision, namely on the maximum permissible terminal temperature or CO_2 concentration. However, actual pollution control decisions are seldom based on explicit knowledge of the damage costs, so that it is worth studying models which operate with exogenously determined targets. The main finding for the target-oriented strategy is that it yields almost stable climatic situations if the terminal upper temperature limit is not too high (Fig. 5). The largest reductions of emissions occur near the terminal date because of discounting, the temporarily increased efficiency of abatement actions and the ability of the climate system to dispose of CO_2 within the planning horizon if the CO_2 has been emitted early (see Section 4c for details). The obvious disadvantage with this strategy is that damage costs are accounted for only if temperature and CO_2 concentration pass the terminal threshold. All damage costs below that limit are allowed to vary freely in the exercise of allocating costs in an optimal manner.

Part of the damage costs can be measured quite accurately (e.g. sea level rise and its effect on safety and ground water) whereas other aspects can hardly be specified. This leads us to suggest that our 2 approaches be combined together, so that the terminal conditions are used to keep the climate system close to an equilibrium, whereas the abatement costs *plus the measurable part of the damage costs* enter the welfare function **W**.

(c) Simplicity of our model

For our system we have deliberately chosen a very simple framework, with a fully parameterized climate/economic model which has no internal dynamics and is always in equilibrium with the (exogenous) climate state. One reason for doing so was to keep the system analytically manageable. The other reason was to avoid taking the same approach as various other studies in the climate impact community (e.g. Jonas et al. 1992) which have tried to implement climate models of intermediate complexity. Such models, like IMAGE (e.g. Jonas et al. 1992), use energy balance models and rely on the climate dynamics described by highly simplified physics. We, on the other hand, were in the luxurious position of having output available from GCMs, which are based on the best presently available representation of our physical knowledge. Instead of using a physical model, which has been fitted to the full system by tuning physical parameters and by selecting the right elements from a plethora of potentially relevant physical processes, we simply fit a linear *statistical* model to the GCM output. A highly satisfactory result of our study is that the observed data as well as the GCM simulated data assign the same numbers to the parameters of our statistical model[10].

Our models can be extended in various directions. Some of the most obvious are:

- The climate model should include equations for each greenhouse gas, such as CFCs, N_2O and methane.
- The dynamics of different gases can be represented more accurately by using several memory terms.
- The model should also have spatial dimensions, since the temperature change will not be equally distributed.
- On the economic side, the functional forms used may be too restrictive. The model should be based on a richer growth model with abatement investment dynamics.
- According to our present assumptions, the only 2 options available to respond to the threat of man-made climate change are to reduce emissions or to adapt to the climate change. Keith & Dowlatabadi (1992) propose a third option, namely geoengineering. This label covers various technical activities either to reduce the concentration of CO_2 in the atmosphere (by direct

[10]This coincidence of parameters derived from observations and simulations suggests first that our models describe realistically the impact of enhanced CO_2 concentration, and second that the temperature increase observed in the last 100 yr is indeed man-made

Appendix. Solution to the optimal control problem

Our mathematical control problem is to maximize

$$\mathbf{W} = \int_0^{t_1} [\mathbf{U}_0 - a(1-E(t)/E_0 e^{qt})^2 - d[\mu C(t) - \alpha T(t)]\, e^{(r-\delta)t}\, dt + e^{-\delta t}\, [G_1 C(t_1) + G_2 T(t_1)] \quad (A1)$$

under the constraints

$$C'(t) = \beta E(t) - \sigma C(t) \quad (A2)$$
$$T'(t) = \mu C(t) - \alpha T(t) \quad (A3)$$
$$T(0) = T_0 \quad (A4)$$
$$C(0) = C_0 \quad (A5)$$
$$T(t_1) \le \hat{T} \quad (A6)$$
$$C(t_1) \le \alpha \hat{T}/\mu \quad (A7)$$
$$\text{and} \quad 0 \le E(t) \le E_0 e^{qt} \quad (A8)$$

The control variable is the emission $E(t)$ and the state variables are CO_2 concentration $C(t)$ and temperature $T(t)$.
To solve the control problem (Eqs. A1 to A8) we use Theorems 2 and 3 in Seierstad & Sydsæter (1987). The current value Hamiltonian of the problem is:

$$\mathcal{H}[C(t), T(t), E(t), \psi(t), \phi(t), t] = \{\mathbf{U}_0 - a[1-E(t)/E_0 e^{qt}]^2 - d[\mu C(t) - \alpha T(t)]\} e^{rt} + \psi(t)[\beta E(t) - \sigma C(t)] + \phi(t)[\mu C(t) - \alpha T(t)]$$

where $\psi(t)$ and $\phi(t)$ are the costates of $C(t)$ and $T(t)$. Let $E^*(t)$ be a piecewise continuous control function defined on $[0, t_1]$ which solves Eqs. (A1) to (A8). Then there exist continuous and piecewise continuously differentiable functions $\psi(t)$ and $\phi(t)$ such that for all $t \in [0, t_1]$:

$$E^*(t) \text{ maximizes } \mathcal{H}(C, T, \psi, \phi, t) \text{ for all } E(t) \in [0, E_0 e^{qt}] \quad (A9)$$
$$\psi'(t) = d\mu e^{rt} - \phi\mu + \psi(\delta+\sigma) \quad (A10)$$
$$\phi'(t) = -d\alpha e^{rt} + \phi(\delta+\alpha) \quad (A11)$$
$$C'(t) = \beta E(t) - \sigma C(t) \quad (A12)$$
$$T'(t) = \mu C(t) - \alpha T(t) \quad (A13)$$
$$C(0) = C_0 \quad (A14)$$
$$T(0) = T_0 \quad (A15)$$

In the cost-oriented problem, when d, G_1, $G_2 > 0$, we apply the transversality conditions

$$\psi(t_1) = G_1 \quad (A16)$$
$$\phi(t_1) = G_2 \quad (A17)$$

In the target-oriented strategy, when $d = G_1 = G_2 = 0$, the transversality conditions are

$$\psi(t_1) \le 0, \quad \psi(t_1) = 0 \quad \text{if } C(t_1) < \alpha/\mu\, \hat{T} \quad (A18)$$
$$\phi(t_1) \le 0, \quad \phi(t_1) = 0 \quad \text{if } T(t_1) < \alpha/\mu\, \hat{T} \quad (A19)$$

Eqs. (A10) to (A13) form 4 first-order ordinary linear differential equations for the 4 variables $T(t)$, $C(t)$, $\phi(t)$ and $\psi(t)$. The general solution for ψ and ϕ is given by

$$\phi(t) = A_1 e^{(\delta+\alpha)t} + \frac{d\alpha e^{rt}}{\alpha+\delta-r} \quad (A20)$$

$$\psi(t) = \frac{\mu A_1 e^{(\delta+\alpha)t}}{\sigma-\alpha} + A_2 e^{(\delta+\sigma)t} + \frac{(r-\delta) d\mu e^{rt}}{(\alpha+\delta-r)(\delta-r+\sigma)} \quad (A21)$$

The maximization of $\mathcal{H}$ with respect to $E(t)$ requires

$$\frac{\partial \mathcal{H}}{\partial E} = e^{(r-q)t}\, 2a\, [1 - E(t)/E_0 e^{qt}] + \psi(t)\beta = 0 \quad (A22)$$

with $E(t) \in [0, E_0 e^{qt}]$. If $\partial\mathcal{H}/\partial E(t)|_{E(t)=0} \le 0$ then $E^*(t) = 0$ and if $\partial\mathcal{H}/\partial E(t)|_{E(t)=E_0 e^{qt}} \ge 0$ then $E^*(t) = E_0 e^{qt}$. Eq. (A22) implies for interior solutions

$$E^*(t) = \frac{\beta}{2a} \psi(t) E_0 e^{(2q-r)t} + E_0 e^{qt} \quad (A23)$$

Now $E(t)$ can be substituted from Eq. (A12) which then yields

$$C(t) = \frac{-A_1 E_0^2 \beta^2 \mu e^{(\alpha+\delta+2q-r)t}}{2a(\alpha+\delta+2q-r+\sigma)(\alpha-\sigma)} + \frac{A_2 E_0^2 \beta^2 e^{(\delta+2q-r+\sigma)t}}{2a(\delta+2q-r+2\sigma)} - \frac{E_0^2 \beta^2 d\mu(\delta-r) e^{2qt}}{2a(\alpha+\delta-r)(\delta-r+\sigma)(2q+\sigma)} + \frac{E_0 \beta e^{qt}}{q+\sigma} + A_3 \beta e^{-\sigma t} \quad (A24)$$

Next, by using Eqs. (A23) & (A13) the time path for the temperature can be computed:

$$T(t) = \frac{-A_1 E_0^2 \beta^2 \mu^2 e^{(\alpha+\delta+2q-r)t}}{2a(2\alpha+\delta+2q-r)(\alpha+\delta+2q-r+\sigma)(\alpha-\sigma)} + \frac{A_2 E_0^2 \beta^2 \mu e^{(\delta+2q-r+\sigma)t}}{2a(\alpha+\delta+2q-r+\sigma)(\delta+2q-r+2\sigma)} - \frac{E_0^2 \beta^2 d\mu^2 (\delta-r) e^{2qt}}{2a(\alpha+\delta-r)(\delta-r+\sigma)(2q+\sigma)(\alpha+2q)} + \frac{E_0 \beta\mu e^{qt}}{(q+\sigma)(\alpha+q)} + \frac{A_3 \beta e^{-\sigma t}}{\alpha-\sigma} + A_4 e^{-\alpha t} \quad (A25)$$

where constants A_1, A_2, A_3 and A_4 must be specified so that the initial conditions (Eqs. A14 & A15) and the transversality conditions (Eqs. A16 & A17 or A18 & A19) are satisfied.
If we use Eqs. (A21) and (A23) we may compute the derivative of the optimal emission level $E^*(t)$ at any time t with respect to the discount parameter δ:

$$\frac{\partial E^*(t)}{\partial \delta} = -E_0 \beta d\mu e^{2qt} \frac{\alpha\sigma - \delta^2 + r(2\delta - r)}{2a(\alpha+\delta-r)^2(\delta-r+\sigma)^2} \quad (A26)$$

This derivative has no uniform sign.
Next, the coefficients G_1 and G_2 are determined such that during the first t_1 years the finite time solution exactly coincides with the infinite time solution. We label the solutions of the costates as $\psi_\infty(t)$ and $\phi_\infty(t)$ for the infinite time problem. If $r-\delta < 0$ then $\lim_{t\to\infty} \psi_\infty(t) = \lim_{t\to\infty} \phi_\infty(t) = 0$ and according to Theorem 16 in Seierstad & Sydsaeter (1987), $A_1 = A_2 = 0$. The equality of the finite and infinite time solutions during the first t years requires

$$\phi_\infty(t_1) = \frac{d\alpha}{\alpha+\delta-r} e^{rt_1} = G_1 \quad (A27)$$

$$\psi_\infty(t_1) = \frac{(r-\delta) d\mu}{(\alpha+\delta-r)(\delta-r+\sigma)} e^{rt_1} = G_2 \quad (A28)$$

In other words, the coefficients G_1 and G_2 of the bequest function must equal the costates of the infinite problem at time t_1. Thus, the finite time solution with the bequest function, which represents the present value of future damage up to infinity, can be obtained by setting $A_1 = A_2 = 0$. The maximum gain from the optimal program $E^*(t)$ can be computed by using the information in Eqs. (A27) and (A28).

ocean disposal of CO_2, ocean fertilization, reforestation) or to mitigate the effect of the enhanced CO_2 concentration (solar shields, increased aerosol concentration).

At the price of increased complexity of the mathematics we can implement these extensions in our framework by modifying the cost function and the physical model. Even if the resulting model were still highly simplified, the mathematical solution would require the use of numerical approximation techniques.

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