

Quasilinear particle transport from gyrokinetic instabilities in general magnetic geometry

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The quasilinear particle flux arising from gyrokinetic instabilities is calculated in the electrostatic and collisionless approximation, keeping the geometry of the magnetic field arbitrary. In particular, the flux of electrons and heavy impurity ions is studied in the limit where the former move quickly, and the latter slowly, along the field compared with the mode frequency. Conclusions are drawn about how the particle fluxes of these species depend on the magnetic-field geometry, mode structure and frequency of the instability. Under some conditions, such as everywhere favourable or unfavourable magnetic curvature and modest temperature gradients, it is possible to make general statements about the fluxes independently of the details of the instability. In quasi-isodynamic stellarators with favourable bounce-averaged curvature for most particles, the particle flux is always outward if the temperature gradient is not too large, suggesting that it might be difficult to fuel such devices with gas puffing from the wall. In devices with predominantly unfavourable magnetic curvature, the particle flux can be inward, resulting in spontaneous density peaking in the centre of the plasma. An estimate for the level of transport caused by magnetic-field fluctuations arising from ion-temperature-gradient instabilities is also given and is shown to be small compared with the electrostatic component.

1 Introduction

Turbulence is ubiquitous in both tokamaks and stellarators, and generally degrades the energy confinement, which is therefore maximised if there is as little turbulence as possible in the plasma. Such a state is however undesirable because heavy impurity ions then tend to accumulate in the plasma core under the influence of neoclassical transport [1, 2, 3, 4, 5, 6]. This is a particular problem in stellarators, where neoclassical transport is much larger than in tokamaks. Moreover, in stellarators this transport is such as to cause a hollow electron density profile in the absence of turbulence, because the thermodiffusion – i.e. particle transport driven by the temperature gradient – is nearly always outward for the electrons. In the absence of a central particle source it thus becomes impossible to maintain a steady-state density profile [7]. Turbulence is however usually unavoidable, and it is therefore of interest to try to predict its effect on the particle transport. In tokamaks, this has been done with considerable success, using quasilinear theory of gyrokinetic instabilities [8, 9, 10, 11], but no similar study of stellarator plasmas has been published, except for the study of impurity transport by Mikkelsen et al. [12].

This is the aim of the present article, where the gyrokinetic turbulent particle transport is calculated in the quasilinear approximation. As always in gyrokinetics, this transport is intrinsically ambipolar [13], so in a pure plasma the calculation can be done either for the ions or for the electrons; the two results always coincide. Of course, the gyrokinetic equation for both species needs to be solved to obtain the mode structure and frequency, but valuable information about the particle transport can be extracted from the electron equation alone. In contrast, little can be said using only the ion equation, for if the electrons for instance are taken to be adiabatic (Boltzmann-distributed), the particle flux vanishes identically.

In an impure plasma, it is of great interest to calculate the transport of the impurity ions, which tend to spoil the energy confinement by causing excessive radiation losses. This transport can be calculated relatively easily for impurities that are sufficiently heavy that their motion along the magnetic field can be neglected in the gyrokinetic equation.

The outcome of these calculations is a particle transport law for each species a ,

$$\Gamma_a = -n_a \left(D_{a1} \frac{d \ln n_a}{d\psi} + D_{a2} \frac{d \ln T_a}{d\psi} + C_a \right), \quad (1)$$

relating the cross-field particle flux Γ_a to the density and temperature gradients as well as a term, here denoted by C_a , related to the curvature of the magnetic field. In tokamaks, this term usually describes inward-directed transport and is referred to as the ‘‘curvature pinch’’.

2 Linear gyrokinetics

Our analysis proceeds from the linearised gyrokinetic equation in the electrostatic and collisionless approximation. The notation and the analysis in this section follow Ref. [14], but are reproduced here for convenience. The distribution function of each species a is written as

$$f_a = f_{a0} \left(1 - \frac{e_a \phi(\mathbf{r}, t)}{T_a} \right) f_{a0} + g_a(\mathbf{R}, v, \lambda, t),$$

where \mathbf{r} denotes the particle position, \mathbf{R} the guiding-centre position, ϕ the electrostatic potential, e_a the charge, v the speed, $\lambda = v_\perp^2 / (v^2 B)$ the ratio between the magnetic moment $\mu = m_a v_\perp^2 / (2B)$ and the kinetic energy $m_a v^2 / 2 = x^2 T_a$, and

$$f_{a0} = n_a \left(\frac{m_a}{2\pi T_a} \right)^{3/2} e^{-x^2}$$

the Maxwellian with number density $n_a(\psi)$ and temperature $T_a(\psi)$. The magnetic field is written in Clebsch form, $\mathbf{B} = \nabla\psi \times \nabla\alpha$, the wave vector in ballooning space as $\mathbf{k}_\perp = k_\psi \nabla\psi + k_\alpha \nabla\alpha$, and the diamagnetic and drift frequencies, respectively, are defined by

$$\begin{aligned} \omega_{*a} &= \frac{k_\alpha T_a}{e_a} \frac{d \ln n_a}{d\psi}, \\ \omega_{*a}^T &= \omega_{*a} \left[1 + \eta_a \left(x^2 - \frac{3}{2} \right) \right], \\ \omega_{da} &= \mathbf{k}_\perp \cdot \mathbf{v}_{da}, \end{aligned}$$

where $\eta_a = d \ln T_a / d \ln n_a$ and \mathbf{v}_{da} denotes the drift velocity. In this notation, the equation for g_a becomes

$$i v_\parallel \nabla_\parallel g_a + (\omega - \omega_{da}) g_a = \frac{e_a \phi}{T_a} J_0 \left(\frac{k_\perp v_\perp}{\Omega_a} \right) (\omega - \omega_{*a}^T) f_{a0}, \quad (2)$$

where J_0 is a Bessel function, $\Omega_a = e_a B / m_a$ the gyrofrequency and the derivatives are taken at fixed energy and magnetic moment.

The gyrokinetic equation (2) usually needs to be solved numerically. Indeed, great efforts have gone into the construction of computer codes for this purpose. As is well known, it is however a simple matter to solve the equation analytically in the limit of fast and slow-moving particles.

For gyrokinetic instabilities other than the electron-temperature-gradient mode, the frequency tends to lie well below the electron bounce/transit frequency, ω_b . The electron distribution function can then be expanded in the small parameter $\omega / \omega_b \ll 1$, $g_e = g_{e0} + g_{e1} + \dots$, so that

$$v_{\parallel} \nabla g_{e0} = 0,$$

implying that g_{e0} vanishes for untrapped particles, since it must do so at infinity in ballooning space, whereas for trapped ones

$$i v_{\parallel} \nabla g_{e1} + (\omega - \omega_{de}) g_{e0} = -\frac{e\phi}{T_e} J_0 (\omega - \omega_{*e}^T) f_{e0}. \quad (3)$$

In each magnetic well, labelled by an integer j and defined by the condition $\lambda B < 1$, the first term in this equation is annihilated by a bounce average,

$$\bar{\phi}_j(\lambda) = \frac{1}{\tau_j} \int \frac{\phi(l) dl}{\sqrt{1 - \lambda B(l)}},$$

where the integral is carried out between the two bounce points of the j :th well and

$$\tau_j(\lambda) = \int \frac{dl}{\sqrt{1 - \lambda B(l)}} \quad (4)$$

denotes the normalised bounce time. The application of this bounce average to Eq. (3) gives

$$g_{e0}^{\text{tr}} = -\frac{\omega - \omega_{*e}^T}{\omega - \bar{\omega}_{de}} J_0 \frac{e\bar{\phi}_j}{T_e} f_{e0} \quad (5)$$

in each trapping region.

The opposite limit, where the mode frequency exceeds the bounce frequency, applies to heavy ions, particularly impurities with charge numbers $Z \gg 1$. In this limit the solution of the gyrokinetic equation (2) becomes to leading order

$$g_Z = \frac{\omega - \omega_{*Z}^T}{\omega - \omega_{dZ}} J_0 \frac{Ze\phi}{T_Z} f_{Z0}. \quad (6)$$

3 Quasilinear particle flux

The cross-field $\mathbf{E} \times \mathbf{B}$ velocity is

$$\mathbf{v}_E = \frac{\mathbf{b} \times J_0 \nabla \phi}{B} \cdot \nabla \psi = -ik_\alpha J_0 \phi,$$

where J_0 again denotes the gyro-averaging operator, and the quasilinear turbulent flux of any species a is thus given by

$$\Gamma_a = \Re \left\langle \int f_a \mathbf{v}_E^* \cdot \nabla \psi d^3 v \right\rangle = -k_\alpha \Im \left\langle \phi^* \int g_a J_0 d^3 v \right\rangle,$$

where an asterisk indicates complex conjugation and angular brackets the flux-surface average [15]

$$\langle \dots \rangle = \lim_{L \rightarrow \infty} \int_{-L}^L (\dots) \frac{dl}{B} \Big/ \int_{-L}^L \frac{dl}{B}.$$

3.1 Electron flux

Using Eq. (5) we thus obtain the electron flux as

$$\Gamma_e = k_\alpha \Im \left\langle \phi^* \int_{\text{tr.}} \frac{\omega - \omega_{*e}^T}{\omega - \bar{\omega}_{de}} J_0^2 \frac{e \bar{\phi}_j}{T_e} f_{e0} d^3 v \right\rangle.$$

Here, the integral is only taken over the trapped part of velocity space, and can be simplified by using

$$\int_{-\infty}^{\infty} \phi^*(l) dl \int_{1/B_{\max}}^{1/B_{\min}} \frac{\bar{\phi}_j d\lambda}{\sqrt{1 - \lambda B}} = \int_{1/B_{\max}}^{1/B_{\min}} d\lambda \sum_j \tau_j |\bar{\phi}_j|^2,$$

where the sum is taken over all relevant magnetic wells (regions with magnetic field strength $B < 1/\lambda$). The electron flux can thus be written as

$$\Gamma_e = \frac{ek_\alpha}{T_e} \Im \int_0^\infty f_{e0} 2\pi v^2 dv \int_{\text{tr.}} \sum_j \frac{\omega - \omega_{*e}^T}{\omega - \bar{\omega}_{de}} \tau_j |\bar{\phi}_j|^2 J_0^2 d\lambda \Big/ \int \frac{dl}{B},$$

where the λ -integral is taken over all trapped particles, i.e.

$$\frac{1}{B_{\max}} < \lambda < \frac{1}{B_{\min}},$$

with B_{\min} and B_{\max} denoting the minimum and maximum field strength on the flux surface in question. This expression for the flux can be simplified by using Eq. (4) and interchanging the integrations in l and λ , giving

$$\Gamma_e = \frac{ek_\alpha}{T_e} \Im \left\langle \int_{\text{tr.}} \frac{\omega - \omega_{*e}^T}{\omega - \bar{\omega}_{de}} f_{e0} |\bar{\phi}|^2 J_0^2 d^3 v \right\rangle.$$

When interpreting this expression, it must be remembered that the quantities $\bar{\omega}_{de}$ and $\bar{\phi}$ depend both on the velocity space variable λ (through the bounce average) and, discretely, on the coordinate along the field line l , which selects the appropriate trapping well over which the bounce average is taken.

Finally, if we write $\omega = \omega_r + i\gamma$ and for $\gamma > 0$ define the function

$$\Delta_\gamma(x) = \frac{\gamma/\pi}{\gamma^2 + x^2},$$

which approaches a Dirac delta function in the limit $\gamma \rightarrow 0+$, then we can write our result as

$$\begin{aligned} \Gamma_e \frac{d \ln n_e}{d\psi} &= -\pi\omega_{*e} \left\langle \int_{\text{tr.}} \left| \frac{e\bar{\phi}}{T_e} \right|^2 \Delta_\gamma(\omega_r - \bar{\omega}_{de}) (\omega_{*e}^T - \bar{\omega}_{de}) J_0^2 f_{e0} d^3v \right\rangle \\ &= -\frac{dn_e}{d\psi} \left(D_{e1} \frac{d \ln n_e}{d\psi} + D_{e2} \frac{d \ln T_e}{d\psi} + C_e \right), \end{aligned} \quad (7)$$

where we have recalled Eq. (1). We thus see that the particle flux is a sum of three terms, proportional to the density and temperature gradients in ω_{*e}^T and to the magnetic curvature in $\bar{\omega}_{de}$, respectively. Since γ must be positive for quasilinear theory to apply, the function Δ_γ is also positive, and it follows that the diffusion coefficient multiplying the density gradient is always positive, $D_{e1} > 0$, as required by the condition of positive entropy production. (Quasilinear transport satisfies both an H-theorem and Onsager symmetry.) The term containing the temperature gradient $dT_e/d\psi$ is called thermodiffusion and can have either sign. The term C_e from the magnetic curvature is usually called the ‘curvature pinch’ on the grounds that it usually describes inward transport in tokamaks, but as we shall see it can be directed outward in stellarators.

3.2 Impurity ion flux

Similarly, but somewhat more straightforwardly, the quasilinear flux of heavy impurity ions can be obtained from Eq. (6),

$$\Gamma_Z = -\frac{Zek_\alpha}{T_Z} \Im \left\langle |\phi|^2 \int \frac{\omega - \omega_{*Z}^T}{\omega - \omega_{dZ}} J_0^2 f_{Z0} d^3v \right\rangle,$$

and can thus be written as

$$\Gamma_Z \frac{d \ln n_Z}{d\psi} = -\pi\omega_{*Z} \left\langle \left| \frac{Ze\phi}{T_Z} \right|^2 \int \Delta_\gamma(\omega_r - \omega_{dZ}) (\omega_{*Z}^T - \omega_{dZ}) J_0^2 f_{Z0} d^3v \right\rangle. \quad (8)$$

$$= -\frac{dn_Z}{d\psi} \left(D_{Z1} \frac{d \ln n_Z}{d\psi} + D_{Z2} \frac{d \ln T_Z}{d\psi} + C_Z \right).$$

Note that, in contrast to the electrons, not only trapped ions but also circulating ones contribute to the transport. Again, D_{Z1} is positive whereas D_{Z2} and C_Z can have either sign.

4 Important special cases

The expressions (7) and (8) are valid within the assumptions of the model, but require knowledge about the eigenfunction $\phi(l)$ and the eigenvalue ω of the linear stability problem. Without this knowledge, little can in general be said about the magnitude and directions of the fluxes. However, in certain special cases valuable conclusions can be drawn without knowing much about the linear instability.

4.1 Good average curvature

One such case for the electron transport occurs when the bounce-averaged magnetic curvature is favourable (or ‘good’) for all orbits, i.e. $\omega_{*e} \bar{\omega}_{de} < 0$, and, additionally, η_e lies between 0 and 2/3. When these two conditions are satisfied, then $\omega_{*e} \bar{\omega}_{de}^T$ is negative throughout velocity space and according to Eq. (7)

$$\Gamma_e \frac{d \ln n_e}{d\psi} < 0,$$

so that the electron particle flux is in the direction of lower density. These conditions, which can be satisfied in quasi-isodynamic stellarators [16], also imply that collisionless trapped-electron modes are stable, both linearly [17] and nonlinearly [18].

Nothing can be concluded about the impurity flux in this case, however, since the *local* (rather than the bounce-averaged) drift frequency appears in Eq. (8). Even if the bounce-average curvature is favourable, the local curvature will usually be unfavourable somewhere, particularly in the region where $|\phi|$ peaks.

4.2 Bad magnetic curvature

If the magnetic curvature is everywhere unfavourable, $\omega_{*a} \omega_{da} > 0$, and the temperature gradient is neither too large nor too small, definite statements can be made about the

direction of both the electron and impurity ion fluxes, as can be seen from the expression

$$\omega_{*a} (\omega_{*a}^T - \omega_{da}) = \omega_{*a}^2 \left[1 - \frac{3\eta_a}{2} + x^2 \left(\eta_a - \frac{\tilde{\omega}_{da}}{\omega_{*a}} \right) \right], \quad (9)$$

where we have written $\omega_{da} = \tilde{\omega}_{da}(\lambda)x^2$, noting that the drift velocity is proportional to energy. If now

$$\frac{2}{3} < \eta < \min_{\lambda} \frac{\tilde{\omega}_{da}(\lambda)}{\omega_{*a}}, \quad (10)$$

the expression (9) is negative definite and it follows from Eqs. (7) and (8) that

$$\Gamma_e \frac{d \ln n_e}{d\psi} > 0,$$

$$\Gamma_Z \frac{d \ln n_Z}{d\psi} > 0,$$

so both the electron and impurity fluxes are in the direction of the respective density gradient, i.e. usually into the plasma. The resulting density peaking in the centre of the plasma is generally desirable for the electrons but undesirable for the impurities. Note that the condition (10) depends on the density gradient for the particle species in question and therefore gets progressively more difficult to satisfy when the density profile becomes steeper. The peaking of this profile will stop when the particle flux balances the sources within the plasma.

The condition that the magnetic curvature be unfavourable everywhere applies in a Z -pinch, a screw-pinch, a reversed-field pinch and in a dipole magnetic field. In a typical tokamak or stellarator, however, the magnetic curvature is good in some places on each magnetic surface and bad in others, but the fluctuation amplitude $|\phi|$ usually peaks in the bad-curvature region on the outboard side of the torus. If this peaking is sufficiently strong, the conclusions of this subsection then apply. In particular, if $\eta > 2/3$ and the transport caused by the temperature gradient can then be so strong that it exceeds that from the density gradient, resulting in a density profile that peaks in the core of the plasma, regardless of other properties of the turbulence.

4.3 Thermodiffusion

We now specifically consider the effect of the temperature gradient on the particle transport. From Eq. (7) the electron thermodiffusion coefficient is equal to

$$D_{e2} = \frac{\gamma}{n_e} \left(\frac{k_{\alpha} T_e}{e} \right)^2 \left\langle \int_{\text{tr.}} |\bar{\phi}|^2 \frac{x^2 - 3/2}{\gamma^2 + (\omega_r - \bar{\omega}_{de})^2} f_{e0} d^3v \right\rangle, \quad (11)$$

where we have set the Bessel function equal to unity for simplicity. As Angioni et al. [9] have pointed out, the integrand is positive for $x^2 > 3/2$ and negative for $x^2 < 3/2$, and were it not for the energy-dependence of $\bar{\omega}_{de}$ in the denominator, the integral would vanish. Ion-temperature-gradient (ITG) modes in tokamaks usually propagate in the ion diamagnetic direction, which we take to be positive, $\omega > 0$, and since $|\phi|$ peaks on the outboard side of the torus, $\bar{\omega}_{de}$ is negative, making $(\omega_r - \bar{\omega}_{de})^2$ an increasing function of energy. The expression (11) then becomes negative, implying inward thermodiffusion of electrons. If, on the other hand, ω and $\bar{\omega}_{de}$ have the same sign and $\omega/\bar{\omega}_{de} \gg 1$, as expected for trapped-electron modes (TEMs), then the integral in Eq. (11) becomes negative, implying outward thermodiffusion. This is thought to explain ‘density pump-out’ in tokamaks with electron cyclotron resonance heating [10].

A similar argument can now be made for thermodiffusion of impurity ions. According to Eq. (8), the thermodiffusion coefficient is

$$D_{Z2} = \frac{\gamma}{n_Z} \left(\frac{k_\alpha T_Z}{Ze} \right)^2 \left\langle |\phi|^2 \int \frac{x^2 - 3/2}{\gamma^2 + (\omega_r - \omega_{dZ})^2} f_{Z0} d^3v \right\rangle,$$

where we have ignored the Bessel function on the grounds that its argument is much smaller for heavy ions than for light ones. If it is of order unity for the bulk ions, it should thus be permissible to neglect finite-gyroradius effects for heavy impurities. For ITG modes in tokamaks, and for other modes propagating in the same direction of the impurity drift, $\omega\omega_{dZ} > 0$, we expect outward thermodiffusion, $D_{Z2} > 0$.

4.4 Curvature pinch

The presence of a ‘curvature-pinch’ term in Eqs. (1), (7) and (8) is fundamentally interesting since it implies that the state of zero particle flux is not necessarily one where the density and temperature gradients vanish. Even if they do, there is in general a finite particle flux, which reflects a tendency of the plasma to ‘spontaneously’ develop a density gradient [11, 19]. The underlying reason is that the lowest accessible energy state is generally not homogeneous if the plasma dynamics is constrained to conserve adiabatic invariants [18].

In the present context, the curvature pinch can be read off from Eqs. (7) and (8):

$$C_e = \frac{\pi e k_\alpha}{n T_e} \left\langle \int_{\text{tr.}} \bar{\omega}_{de} |\bar{\phi}|^2 \Delta_\gamma (\omega_r - \bar{\omega}_{de}) J_0^2 f_{e0} d^3v \right\rangle,$$

$$C_Z = \frac{\pi Z e k_\alpha}{n_Z T_Z} \left\langle |\bar{\phi}|^2 \int \omega_{dZ} \Delta_\gamma (\omega_r - \omega_{dZ}) J_0^2 f_{Z0} d^3 v \right\rangle.$$

Hence it is clear that the curvature pinch is indeed caused by magnetic curvature through the drift frequency ω_d . Its direction is most clearly understood from a comparison with ordinary density-gradient-driven diffusion,

$$\frac{C_e}{D_{e1} \frac{d \ln n}{d \psi}} = - \left\langle \int_{\text{tr.}} \frac{\bar{\omega}_{de}}{\omega_{*e}} |\bar{\phi}|^2 \Delta_\gamma J_0^2 f_{e0} d^3 v \right\rangle / \left\langle \int_{\text{tr.}} |\bar{\phi}|^2 \Delta_\gamma J_0^2 f_{e0} d^3 v \right\rangle.$$

Hence it is clear that the curvature pinch is indeed a ‘pinch’, i.e. it transports electrons up the density gradient, if the bounce-averaged magnetic curvature is unfavourable, $\bar{\omega}_{de} \omega_{*e} > 0$, for most relevant orbits. This tends to be the case in tokamaks [11, 19] but not in quasi-isodynamic stellarators [14, 17], where the curvature pinch is thus outward [20].

Such devices thus suffer from a curvature ‘anti-pinch’, potentially leading to hollow density profiles. The corresponding impurity flux,

$$\frac{C_Z}{D_{Z1} \frac{d \ln n_Z}{d \psi}} = - \left\langle \int \frac{\omega_{dZ}}{\omega_{*Z}} |\bar{\phi}|^2 \Delta_\gamma J_0^2 f_{Z0} d^3 v \right\rangle / \left\langle \int |\bar{\phi}|^2 \Delta_\gamma J_0^2 f_{Z0} d^3 v \right\rangle,$$

is however beneficial, driving the impurities in the same direction as ordinary diffusion, if the fluctuations peak in regions of favourable local magnetic curvature, $\omega_{dZ} \omega_{*Z} < 0$.

4.5 Near-marginality

Just above the linear stability threshold, $\gamma \rightarrow 0+$, the function Δ_γ approaches a Dirac delta function, which simplifies the calculation of the velocity-space integral in Eqs. (7) and (8). One of the two velocity-space integrals can then be carried out, but the resulting expression does not yield much more information than already discussed above.

For instance, the electron flux (7) becomes in this limit

$$\begin{aligned} \Gamma_e = & -k_\alpha^2 \frac{dn}{d\psi} \int_{1/B_{\max}}^{1/B_{\min}} \sum_j \frac{\tau_j |\bar{\phi}|^2}{|\tilde{\omega}_{de}|} \sqrt{\frac{\pi \omega}{\tilde{\omega}_{de}}} \Theta(\omega \tilde{\omega}_{de}) e^{-\omega/\tilde{\omega}_{de}} \\ & \times \left[1 + \eta_e \left(\frac{\omega}{\tilde{\omega}_{de}} - \frac{3}{2} \right) - \frac{\omega}{\omega_{*e}} \right] d\lambda / \int \frac{dl}{B}, \end{aligned}$$

where Θ denotes the Heaviside function, which in this expression ensures that the frequencies ω and $\tilde{\omega}_{de}(\lambda)$ have the same sign, so that only resonant orbits contribute to the flux. From this result, we again see that the flux is in the direction of increasing density if the condition (10) is satisfied. We also note that thermodiffusion is outward if $\omega/\tilde{\omega}_{de} > 3/2$, but otherwise it is difficult to draw any general conclusions.

5 Extension to electromagnetic instabilities

The two main limitations of our results are the neglect of collisions and electromagnetic effects, which arise at finite plasma beta and perturb the equilibrium magnetic field. The electromagnetic fluctuation components are of two types: one associated with magnetic compressibility, δB_{\parallel} , and the other with “magnetic flutter”, $\delta \mathbf{B}_{\perp}$ arising from perturbed electric currents parallel to \mathbf{B} . Magnetic compressibility affects ion-temperature-gradient modes if the electron pressure is of the order of $\beta_e \sim L_p/R$, where L_p denotes the pressure gradient length scale and R the curvature radius of the magnetic field [22]. As is well known in the gyrokinetic simulation community, the destabilising effect of δB_{\parallel} is largely cancelled by a reduction in the equilibrium ∇B -drift (at constant magnetic curvature), so that little net effect is seen if one is careful to keep the magnetic drifts consistent with the magnetic equilibrium [22]. This is true even when β is large enough to excite kinetic ballooning modes [23].

Perpendicular magnetic fluctuations, coming from a term containing the magnetic potential A_{\parallel} in the gyrokinetic equation, arise already at smaller pressures, of order [22]

$$\beta \sim \left(\frac{k_{\perp} \rho_i k_{\parallel} c_s}{\omega} \right)^2,$$

where ρ_i denotes the ion gyroradius and c_s the sound speed. These fluctuations cause extra particle transport by electrons streaming along perturbed magnetic field lines, but only if these reconnect and form overlapping magnetic islands, so that the field becomes chaotic. A quasilinear estimate of the island width, and of this transport, can be given if the electrostatic perturbation driven by ion-scale turbulence generates a magnetic perturbation at the electron scale, $\delta < \rho_i$, where magnetic flux surfaces are not preserved and reconnection can indeed occur. This is an intricate calculation that, to our knowledge, has only been performed in the case of the slab ion-temperature-gradient mode [24]. In a more general setting, it is possible to highlight some features of the quasilinear stochastic transport without going into too many details. First of all, the amplitude of the magnetic perturbation that generates a magnetic island must be related to the amplitude of the electrostatic potential, ϕ . The amplitude of electromagnetic fluctuations, A_{\parallel} , can be considered the unknown of the ion-scale problem if it is assumed that the solution of the leading-order electrostatic problem at the char-

acteristic scale of the instability, $k_\psi \nabla \psi \sim k_\alpha/a$, is known, where $a \sim |\nabla \alpha|^{-1}$ denotes a normalising length. This means that the eigenfunction ϕ and the eigenvalue ω of the electrostatic mode (typically an ITG or TEM instability) must be given for a specific wave number k_α/a . Then, A_\parallel is determined by multiple asymptotic matching between the ion-region solution, $(k_\psi \nabla \psi) \rho_i \sim 1$, and the leading-order electrostatic solution. At the ion scale, A_\parallel is a β -correction to the electrostatic potential associated with the driving turbulence so that $A_\parallel \propto \beta \phi$, [see Eq. (11) of [24]]. Not surprisingly, this implies that the addition of an electromagnetic component to Eq. (7) would only contribute significantly to the transport if β is large enough. We note that the explicit form of A_\parallel depends on the details of the leading order electrostatic problem. Under the assumption $\delta \ll \rho_i$, the matching of A_\parallel found from by perturbing the electrostatic problem to the value $A_\parallel|_\delta$, evaluated from Ampere's law at the electron scale δ , gives the amplitude $A_\parallel|_\delta$ as a function of ϕ . The specific form of the electron response is thus eventually required. In this regard, the analysis of Connor et al. was performed for semicollisional electrons. However, a simple adaptation of their analysis to the collisionless case is straightforward and gives the magnetic perturbation at the rational surface

$$v_{thi} A_\parallel(0) = \beta_i b_0^2 \frac{\delta L_s}{\rho_i L_T} F\left(\frac{\omega_0}{\eta_e \omega_{*e}}, \beta_i \frac{\rho_i L_s^2}{\delta L_T^2}, b_0, I_e\right) \phi, \quad (12)$$

where the function F takes into account of the details of the driving instability at the ion scale [i.e. it is a function of the Δ^* quantity introduced by the authors], and given by the electrostatic problem, I_e is an $\mathcal{O}(1)$ number resulting from the integration of a function of the collisionless electron conductivity [25, 26, 27], $b_0 = k_\alpha^2 (\rho_i/a)^2/2$, $L_T^{-1} = d \log T / d \hat{\psi}$, L_s denotes the shear length, $\hat{\psi} = \psi / (a^2 B_0)$, and B_0 a reference magnetic field. In Eq. (12), the explicit dependence on b_0 and L_s/L_T are specific of the slab ITG mode, however the δ/ρ_i and β_i -dependencies are general. By using $\delta B_r \sim (k_\alpha \nabla \alpha) A_\parallel(0)$, a quasilinear estimate for the collisionless Rechester-Rosenbluth transport coefficient gives [28]

$$\chi_e = v_{the} L \left(\frac{\delta B_\perp}{B} \right)^2 \sim \left(\frac{v_{the}}{k_\alpha |\nabla \alpha|} \right) \frac{\beta_i^2 b_0^2}{T_e^2 / T_i^2} \frac{\delta L_s^3}{\rho_i^2 L_T^2} F^2 \frac{k_\alpha^2 |\nabla \alpha|^2 \phi^2}{v_{thi}^2 B_0^2}, \quad (13)$$

where $L = L_s / (k_\alpha |\nabla \alpha| \delta)$ is the length that a particle needs to travel along the magnetic field to encounter the electron scale δ . Let us now consider the particle transport due to the effective diffusion caused by magnetic perturbations: $\partial_t n = \chi_e \nabla^2 n$. This will

compete with the radial $\mathbf{E} \times \mathbf{B}$ transport of Eq. (7): $\Gamma_e d \ln n / d\psi = \omega_{*e} n I_p$, where I_p is an $\mathcal{O}(1)$ quantity defined by the right-hand side of Eq. (7). We can now compare the ratio of the particle diffusion due to the stochastic magnetic field, and the one given by the radial component of the $\mathbf{E} \times \mathbf{B}$ drift

$$\begin{aligned} \frac{\chi_e L_T^{-2} n}{\Gamma_e \frac{d}{d\psi} \ln n} &= 2a \nabla \alpha \frac{\beta_i^2 b_0^4}{(T_e/T_i)^{5/2}} \sqrt{\frac{m_i}{m_e}} \frac{\delta^3 L_s^3}{\rho_i^3 L_T^3} \frac{F^2 k_\alpha^2}{I_p} \frac{|\nabla \alpha|^2 \phi^2}{v_{thi}^2 B_0^2} \\ &\sim \beta_i^2 b_0^4 \sqrt{\frac{T_i m_i}{T_e m_e}} \frac{\delta}{\rho_i} \left(\frac{L_s}{L_T} \right)^3 \ll 1. \end{aligned} \quad (14)$$

Since $\delta < \rho_i$ and in accordance with the result of Connor et al. [24], this is a very small number whose smallness is dictated by the electron-ion scale separation, by β , and by b_0 .

6 Conclusions

Our conclusions can be summarised as follows. Not all of them are novel, in particular those relating to tokamaks, but we nevertheless list all significant findings here for convenience:

- Most of the electron transport is carried by trapped particles.
- In tokamaks, most trapped particles experience unfavourable magnetic curvature on an orbit-average. Instabilities propagating in the ion diamagnetic direction therefore tend to produce inward thermodiffusion, and those propagating in the electron direction cause outward thermodiffusion, which is thought to explain ‘density pump-out’ during electron-cyclotron-resonance heating. The curvature pinch is directed inward and tends, in combination with ITG-driven thermodiffusion, to cause slightly peaked density profiles.
- In so-called maximum- J devices [21], such as quasi-isodynamic stellarators [17], where most trapped electrons experience favourable magnetic curvature on a time average, the net particle transport is always in the direction of decreasing density if $0 < \eta_e < 2/3$, and the curvature pinch is outward. This result suggests that it could be relatively difficult to achieve efficient fuelling of such stellarators using gas puffing at the plasma edge. Such difficulties have indeed been observed in the first operational phase of Wendelstein 7-X.

- In magnetic configurations with unfavourable magnetic curvature everywhere, such as magnetic dipoles, reversed-field pinches, Z-pinches and screw pinches, both the electron and the impurity fluxes are in the direction of increasing density, i.e., usually into the plasma, if the condition (10) is satisfied. Spontaneous density peaking should then occur at least to the point where Eq. (10) is no longer satisfied.
- In turbulence driven by the ion-temperature-gradient instability, the contribution to transport from parallel streaming along chaotic magnetic field lines is relatively small.

The main limitation of our calculation is the neglect of collision, which in particular for heavy impurities restricts its validity to high-temperature plasmas.

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