

Structure of the lightest tin isotopes

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We link the structure of nuclei around ^{100}Sn , the heaviest doubly magic nucleus with equal neutron and proton numbers ($N = Z = 50$), to nucleon-nucleon (NN) and three-nucleon (NNN) forces constrained by data of few-nucleon systems. Our results indicate that ^{100}Sn is doubly magic, and we predict its quadrupole collectivity. We present precise computations of ^{101}Sn based on three-particle–two-hole excitations of ^{100}Sn , and reproduce the small splitting between the lowest $J^\pi = 7/2^+$ and $5/2^+$ states. Our results are consistent with the sparse available data.

Introduction – ^{100}Sn is a nucleus of superlatives: It is the heaviest self-conjugate ($N = Z = 50$) nucleus [1], exhibits the largest strength in allowed β decay [2], is close to the proton dripline [3], and is the endpoint of a region of nuclei with enhanced α decays [4, 5]. While these properties make ^{100}Sn the cornerstone of a most interesting region of the nuclear chart, our understanding of this nucleus and its neighbors is still rather limited, see [6] for a review. No data exist regarding the spectrum of ^{100}Sn , and the spin assignments for low-lying states in ^{101}Sn are controversial [7, 8]. Likewise, the evolution of collective observables towards neutron number $N = 50$ is experimentally unclear at present [9–14]. On the other hand, with naively expected shell closures for both protons and neutrons, and the stabilizing effects of the Coulomb and centrifugal barriers, ^{100}Sn should be particularly suitable for a reliable theoretical treatment.

In this Letter, we calculate properties of ^{100}Sn and neighboring nuclei using realistic interactions between protons and neutrons. This is in contrast to large-scale shell-model (LSSM) calculations [15–19] in this region of the nuclear chart that employ ^{80}Zr or ^{88}Sr cores and phenomenologically adjusted interactions based on the G -matrix approach [20]. The strong nuclear force is rooted in the fundamental theory of strong interactions, quantum chromodynamics (QCD), and is manifested in dominant two-nucleon (NN) forces and weaker but pivotal three-nucleon (NNN) forces between protons and neutrons. Effective field theories (EFTs) of QCD provide us with a systematically improvable low-momentum expansion of these interactions [21–23]. So far, interactions derived from the EFT framework have been applied to light and medium-mass nuclei (see [24–27] for recent reviews).

The extension of *ab initio* computations from light [24, 25, 28] to heavier nuclei is based on the development and application of quantum many-body methods that exhibit

a polynomial scaling in mass number [29–39]. Medium-mass and heavy nuclei can technically be computed with these methods, but most interactions developed thus far considerably overbind heavier nuclei [40]. In the quest for nuclear interactions with more acceptable saturation properties [41–44], one interaction labeled 1.8/2.0(EM) has emerged that describes binding energies, two-neutron separation energies, and the first 2^+ excited state in nuclei up to neutron-rich nickel isotopes remarkably well, while charge radii are too small [45–50]. It is primarily this interaction from Ref. [41] that we will employ in the computation of ^{100}Sn and its neighbors.

This Letter is organized as follows. First, we briefly describe the Hamiltonian, the employed model spaces, and computational methods. Then, we validate the interactions in the tin region by computing known binding energies and level splittings, and address method uncertainties by employing the coupled-cluster method [26, 51] and the valence-space in-medium similarity-renormalization-group method (VS-IMSRG) [52] coupled with the importance-truncated large-scale shell model [53]. Finally, we present results for the structure of the lightest isotopes of tin.

Hamiltonian and model space – We employ the intrinsic Hamiltonian

$$H = \sum_{i < j} \left(\frac{(\mathbf{p}_i - \mathbf{p}_j)^2}{2mA} + V_{NN}^{(i,j)} \right) + \sum_{i < j < k} V_{NNN}^{(i,j,k)}, \quad (1)$$

where the NN and NNN potentials (V_{NN} and V_{NNN} , respectively) are the interactions 1.8/2.0(EM), 2.0/2.0(EM), 2.2/2.0(EM), and 2.0/2.0(PWA) from Ref. [41]. These interactions result from a similarity-renormalization-group (SRG) [54] evolution of the chiral NN interaction of [55] to cutoffs $\lambda = 1.8, 2.0$, and 2.2 fm^{-1} , respectively. The NNN potential is not evolved but rather taken as the leading NNN forces from chiral EFT [56–58] and has a cutoff $\Lambda_{NNN} = 2.0 \text{ fm}^{-1}$.

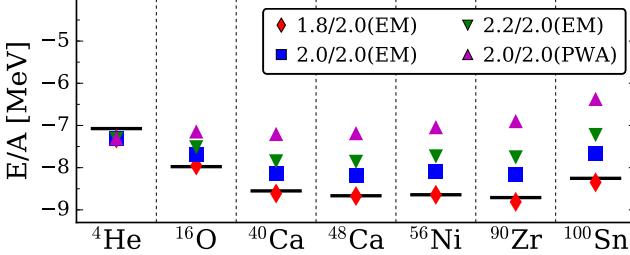


FIG. 1. (Color online) Ground-state energies per nucleon E/A for selected closed-shell nuclei computed with the closed-shell IMSRG [34] using the interactions of Ref. [41] in comparison with experiment (black horizontal lines).

The two low-energy constants of the short-range part of the NNN forces are adjusted to binding energy of the triton and the radius of the α particle, following Ref. [59]. These interactions are quite soft (due to the relatively small cutoffs), which allows us to achieve reasonably well converged binding energies and spectra in nuclei up to neutron-rich ${}^{78}\text{Ni}$ [47, 50], and in the neutron-deficient tin isotopes considered in this work.

Figure 1 shows the computed ground-state energies per nucleon for ${}^4\text{He}$, ${}^{16}\text{O}$, ${}^{40,48}\text{Ca}$, ${}^{56}\text{Ni}$, ${}^{90}\text{Zr}$, and ${}^{100}\text{Sn}$ with the single-reference IMSRG [34, 36]. The 1.8/2.0(EM) interaction consistently yields the best agreement with data. Presently, it is unclear what distinguishes this interaction from the other similarly obtained interactions; however this soft interaction puts us in a fortuitous situation to make theoretical predictions (albeit without rigorous uncertainty quantification) for binding energies and spectra in nuclei as heavy as ${}^{100}\text{Sn}$.

Coupled-cluster calculations use a Hartree-Fock basis constructed from a harmonic-oscillator basis of up to 15 major oscillator shells. For VS-IMSRG we use a similar basis, except that the Hartree-Fock reference is constructed with respect to an ensemble state above the ${}^{80}\text{Zr}$ core following Ref. [52]. All calculations are performed at oscillator frequencies in the range $\hbar\omega = 12 - 16$ MeV, which include the minimum in energy for the largest model space we consider. We use the normal-ordered two-body approximation [35, 40, 60] for the NNN interaction with an additional energy cut on three-body matrix elements $e_1 + e_2 + e_3 \leq E_{3\max}$. When $E_{3\max}$ is increased from 16 to 18, the binding energy of ${}^{100}\text{Sn}$ changes by 2% for the hardest interaction 2.0/2.0(PWA), while for the softest interaction, 1.8/2.0(EM), the change is less than 1%.

Method – The coupled-cluster method is an ideal tool to compute doubly magic nuclei and their neighbors [26, 29, 30, 32, 33, 38, 61–63]. This method computes the similarity transform $\overline{H} \equiv \exp(-T)H_N \exp(T)$ of the Hamiltonian H_N , obtained by normal ordering the free-space Hamiltonian (1) with respect to the closed-shell Hartree-Fock reference of ${}^{100}\text{Sn}$. The clus-

ter operator T includes particle-hole excitations and is truncated at the coupled-cluster singles-doubles (CCSD) level. Usually CCSD accounts for about 90% of the correlation energy (i.e., the energy beyond Hartree Fock) [51]. For a higher precision of the ground-state energy, we include triples excitations of the cluster operator T perturbatively within the Λ -CCSD(T) method [64]. Excited states in ${}^{100}\text{Sn}$ are computed with an equation-of-motion (EOM) method including $3p\text{-}3h$ corrections via a generalization of the ground state Λ -CCSD(T) approximations to excited states with EOM-CCSD(T) [65]. The neighboring nuclei ${}^{101,102}\text{Sn}$ are computed as one- and two-particle attached states [66–68] of the ${}^{100}\text{Sn}$ similarity transformed Hamiltonian \overline{H} . The two-particle attached states of ${}^{102}\text{Sn}$ are truncated at the $3p\text{-}1h$ level, while the particle-attached states of ${}^{101}\text{Sn}$ are computed at the $2p\text{-}1h$ level with perturbative $3p\text{-}2h$ corrections included (described below). Further details of the coupled-cluster approach to nuclei are presented in a recent review [26].

We briefly describe our new approach to include perturbative $3p\text{-}2h$ corrections to the particle-attached states of ${}^{101}\text{Sn}$. Generalizing the completely renormalized (CR) EOM-CCSD(T) approximation from quantum chemistry [69, 70] and nuclear physics [38, 62, 71] to particle-attached excited states yields the correction

$$\delta\omega_\nu^{3p-2h} = \sum_{i < j} \sum_{a < b < c} \mathcal{L}_{\nu,ij}^{abc} \mathcal{R}_{\nu,ij}^{abc} \mathcal{M}_{\nu,ij}^{abc}. \quad (2)$$

Here ν denotes the state of interest, i, j (and a, b, c) are occupied (and unoccupied) orbitals in the ${}^{100}\text{Sn}$ reference $|\Phi\rangle$, \mathcal{L}_ν and \mathcal{M}_ν represent the left and right $3p\text{-}2h$ moments

$$\begin{aligned} \mathcal{L}_{\nu,ij}^{abc} &= \langle \Phi | L_\nu^{2p-1h} \overline{H} | \Phi_{ij}^{abc} \rangle, \quad \mathcal{M}_{\nu,ij}^{abc} = \langle \Phi_{ij}^{abc} | \overline{H} R_\nu^{2p-1h} | \Phi \rangle, \\ |\Phi_{ij}^{abc}\rangle &\text{ are } 3p\text{-}2h \text{ excited states, and } \mathcal{R}_\nu \text{ is the resolvent} \end{aligned}$$

$$\mathcal{R}_{\nu,ij}^{abc} = \langle \Phi_{ij}^{abc} | (\omega_\nu^{2p-1h} - \overline{H})^{-1} | \Phi_{ij}^{abc} \rangle. \quad (3)$$

Here ω_ν^{2p-1h} is the $2p\text{-}1h$ energy corresponding to the states L_ν^{2p-1h} and R_ν^{2p-1h} of ${}^{101}\text{Sn}$. We draw the reader's attention to the similar structure between the bi-variational expression (2) and second-order perturbation theory. This method is the completely renormalized particle-attached equation-of-motion (CR-PA-EOM). In our results for ${}^{101}\text{Sn}$, we used three different approximations (labeled A,B,C) for the energy denominator in Eq. (3). Approximation A uses in place of \overline{H} the Hartree-Fock single-particle energies, approximation B uses the one-body part of \overline{H} , and approximation C uses both the one- and two-body parts of \overline{H} . Thus, approximation C is the most complete choice for the resolvent and most accurately approximates the full calculation [62].

The IMSRG and its VS-IMSRG variant are effective tools for computing doubly magic nuclei and for constructing valence-space interactions from NN and NNN

interactions that can be subsequently diagonalized using shell-model techniques [27, 34, 52, 72–74]. These methods also rely on similarity transformations $\bar{H} \equiv \exp(\Omega)H_N\exp(-\Omega)$ where H_N is the normal-ordered Hamiltonian with respect to the ensemble reference of each target nucleus. For nuclei in the ^{100}Sn region, the VS-IMSRG yields an anti-Hermitian Ω , truncated at the one- and two-body level, which decouples the major oscillator shell above ^{80}Zr . The ensuing large-scale eigenvalue problem is solved via the importance-truncated shell model [53].

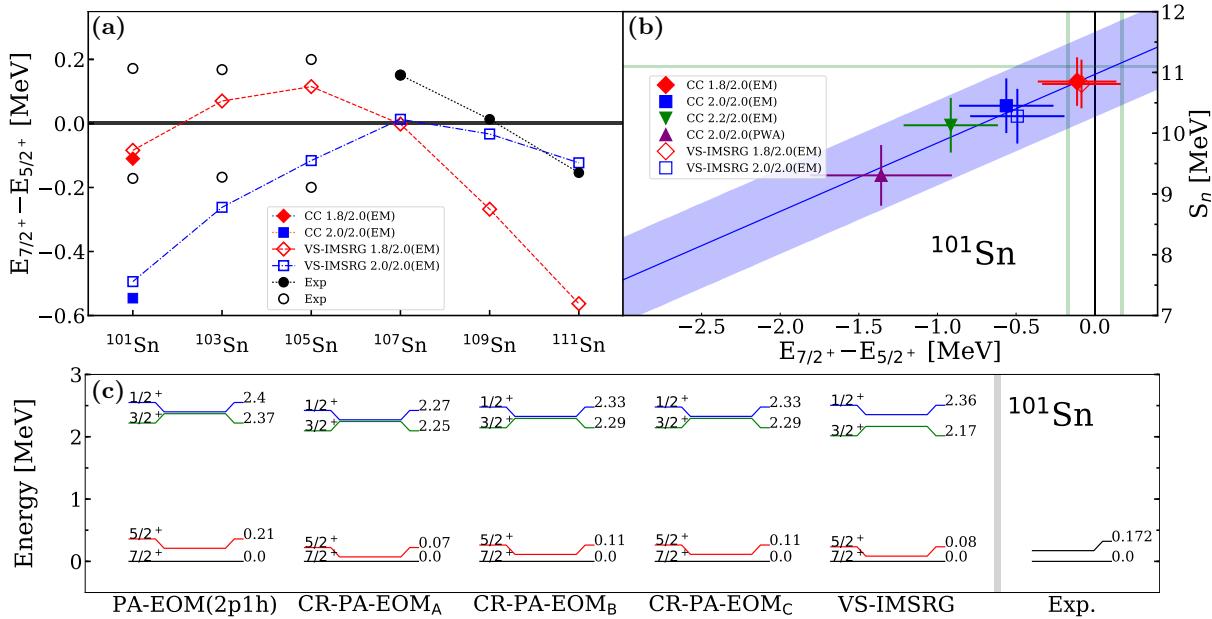
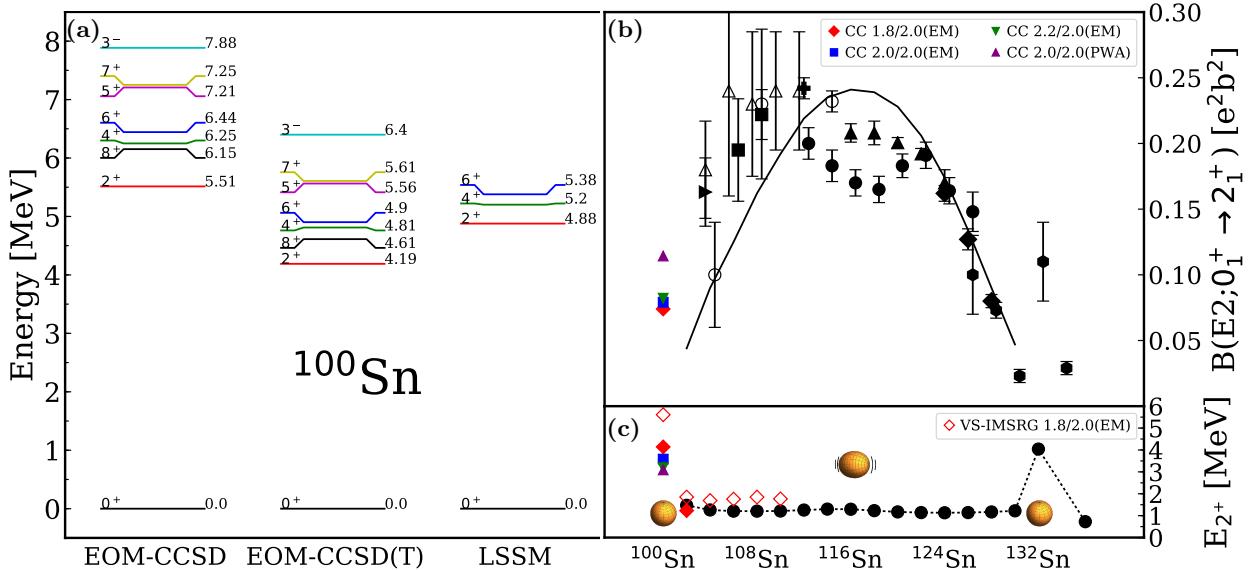
Results – Results for ^{100}Sn are shown in Fig. 2. Panel (a) shows the low-lying states in ^{100}Sn computed in the EOM-CCSD and EOM-CCSD(T) approximations with the 1.8/2.0(EM) interaction. We also show the phenomenological LSSM results of Ref. [2]. The relatively large excitation gap of about 4 MeV is consistent with ^{100}Sn being doubly magic, a finding which is—to our knowledge—qualitatively ubiquitous in all previous theoretical investigations. Panel (b) shows our EOM-CCSD predictions for the $B(E2)$ in ^{100}Sn for the 1.8/2.0(EM), 2.0/2.0(EM), 2.2/2.0(EM), and 2.0/2.0(PWA) interactions together with the experimental $B(E2)$ values for the isotopes $^{104-132}\text{Sn}$ [9–14]. Our computed $B(E2)$ values are similar in size to that of ^{132}Sn and consistent with ^{100}Sn being doubly magic. They also fall within expectations from phenomenological shell-model calculations [6] and from extrapolations of data in light tin isotopes. Panel (c) shows CC results for 1.8/2.0(EM), 2.0/2.0(EM), 2.2/2.0(EM), and 2.0/2.0(PWA) interactions and the VS-IMSRG result for the 1.8/2.0(EM) interaction for the energy of the first $J^\pi = 2_1^+$ state in light even isotopes $^{100-110}\text{Sn}$, and data for $^{102-132}\text{Sn}$. The $B(E2)$ values computed are consistent with the computed excitation energies of the first $J^\pi = 2_1^+$, in the sense that, for a given interaction, the larger the energy, the smaller the $B(E2)$ value. We note that despite the consistency with experiment, the 1.8/2.0(EM) interaction produces radii that are too small, and this would certainly affect the $B(E2)$. The systematic trend of known and computed 2_1^+ energies in the tin isotopes again suggests that ^{100}Sn is doubly magic. In ^{100}Sn , this energy is similar to that of the doubly magic nucleus ^{132}Sn [75, 76]. The VS-IMSRG result for the 2_1^+ state in ^{100}Sn is about 1.5 MeV higher than the EOM-CCSD(T) result, but close to the similarly approximated EOM-CCSD result shown in panel (a). Using the discrepancy between methods as an estimate of the uncertainty in the many-body method, our results for the energy of 2_1^+ state in $^{102-110}\text{Sn}$ are consistent with the data.

In ^{101}Sn , the two lowest states are separated by only 172 keV [7, 8]. Observation of ^{105}Te α decay in coincidence with the 172 keV γ line indicates that the dominant α decay of ^{105}Te is to the first excited state in ^{101}Sn , implying that these states have identical spins [8]. We recall that the lowest two states in the odd isotopes $^{105-113}\text{Te}$

and $^{101-105}\text{Sn}$ are only about 0.2 MeV apart and lack definite spin assignments. In tin, this near degeneracy between the $J^\pi = 5/2^+$ and $7/2^+$ states persists up to ^{111}Sn , and the ground-state spin changes from $J^\pi = 5/2^+$ in ^{107}Sn [77] and ^{109}Sn to $J^\pi = 7/2^+$ in ^{111}Sn . The level ordering in ^{101}Sn to ^{105}Sn between the $J^\pi = 5/2^+$ and $7/2^+$ states is not known. This is reflected in panel (a) of Fig. 3 which compares available data (full and open black points for definite and tentative spins assignments, respectively) with CC and VS-IMSRG predictions for the energy splitting in odd tin isotopes using the interactions 1.8/2.0(EM) and 2.0/2.0(EM). Both interactions yield a small splitting between the $J^\pi = 5/2^+$ and $7/2^+$ states, but they differ on its precise size and sign. Panel (b) of Fig. 3 plots the calculated energy splitting between the $J^\pi = 5/2^+$ and $7/2^+$ states versus the neutron separation energy of ^{101}Sn computed with the CR-PA-EOM using the 1.8/2.0(EM), 2.0/2.0(EM), 2.2/2.0(EM), and 2.0/2.0(PWA) interactions. Also shown are estimated uncertainties due to finite model-space sizes and the employed methods, and a blue (diagonal) band encompassing these uncertainties (see Ref. [45] for details). The horizontal and vertical green lines indicate experimental data. The intersection of the blue diagonal band with the precisely known neutron separation energy S_n yields one estimate of the systematic uncertainty for the energy splitting between the $J^\pi = 5/2^+$ and $7/2^+$ states in ^{101}Sn . Clearly, theory is not sufficiently precise to make a definite prediction for the ground-state spin of ^{101}Sn as the predicted range for the energy splitting can support either $J^\pi = 5/2^+$ or $7/2^+$ as the ground state. Again, the 1.8/2.0(EM) interaction is closest to data. Panel (c) of Fig. 3 shows the lowest states in ^{101}Sn , computed with the 2p-1h particle-attached EOM-CC method, the CR-PA-EOM developed in this work, and the VS-IMSRG for the 1.8/2.0(EM) interaction. We find that for this interaction, the different methods agree on the level ordering, and the energy splitting varies by at most 140 keV. While the upcoming measurements will yield definite spin assignments [78], getting theory to a level where such fine details can be unambiguously resolved will require more work.

Figure 4 shows the convergence of the $5/2^+$ and $7/2^+$ states in ^{101}Sn and ^{105}Te with the number of particle-hole excitations (T_{\max}) in the importance-truncated large-scale shell-model calculations using the 1.8/2.0(EM) and 2.0/2.0(EM) interactions. In both, ^{101}Sn and ^{105}Te , we obtain nearly degenerate $J^\pi = 5/2^+$ and $7/2^+$ states consistent with data.

Conclusions and Outlook – Our computations demonstrated that tin nuclei can be described by NN and NNN interactions constrained by few-body data. We found that ^{100}Sn is doubly magic and presented results and predictions for its structure and low-lying collectivity. For an increased precision of excited states in ^{101}Sn , we developed a method that includes three-particle-two-



hole excitations in our coupled-cluster calculations. One

interaction reproduced both binding energies and the

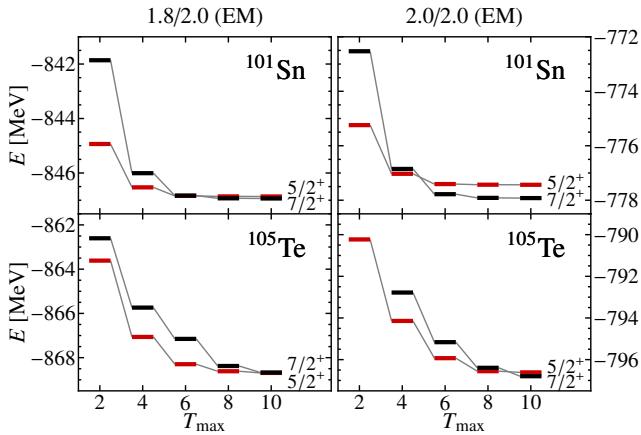


FIG. 4. (Color online) Ground- and first excited states in ^{101}Sn and ^{105}Te obtained in VS-IMSRG for the 1.8/2.0(EM) and 2.0/2.0(EM) interactions, with spins $J^\pi = 5/2^+$ (red) and $J^\pi = 7/2^+$ (black).

near degeneracy between the lowest $J^\pi = 5/2^+$ and $7/2^+$ states in the odd-mass isotopes $^{101-111}\text{Sn}$ and ^{105}Te . This work opens the avenue for reliable calculations of even heavier nuclei based on NN and NNN interactions.

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