Curvature squared invariants in six-dimensional $\mathcal{N} = (1,0)$ supergravity

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ABSTRACT: We describe the supersymmetric completion of several curvature-squared invariants for $\mathcal{N} = (1,0)$ supergravity in six dimensions. The construction of the invariants is based on a close interplay between superconformal tensor calculus and recently developed superspace techniques to study general off-shell supergravity-matter couplings. In the case of minimal off-shell Poincaré supergravity based on the dilaton-Weyl multiplet coupled to a linear multiplet as a conformal compensator, we describe off-shell supersymmetric completions for all the three possible purely gravitational curvature-squared terms in six dimensions: Riemann, Ricci, and scalar curvature squared. A linear combination of these invariants describes the off-shell completion of the Gauss-Bonnet term, recently presented in arXiv:1706.09330. We study properties of the Einstein-Gauss-Bonnet supergravity, which plays a central role in the effective low-energy description of α'-corrected string theory compactified to six dimensions, including a detailed analysis of the spectrum about the $\mathrm{AdS}_3 \times \mathrm{S}^3$ solution. We also present a novel locally superconformal invariant based on a higher-derivative action for the linear multiplet. This invariant, which includes gravitational curvature-squared terms, can be defined both coupled to the standard-Weyl

or dilaton-Weyl multiplet for conformal supergravity. In the first case, we show how the addition of this invariant to the supersymmetric Einstein-Hilbert term leads to a dynamically generated cosmological constant and non-supersymmetric (A)dS$_6$ solutions. In the dilaton-Weyl multiplet, the new off-shell invariant includes Ricci and scalar curvature-squared terms and possesses a nontrivial dependence on the dilaton field.

Contents

1 Introduction

Over the years, six-dimensional (6D) $\mathcal{N} = (1,0)$ supergravity theories [1–7] have been a fertile ground of studies in various contexts due to their relationship with string theories and 10D supergravities, and the role they have played in various phenomenological scenarios. It was already realized in the 80s by Salam and Sezgin that a prototypical scenario in six-dimensional $\mathcal{N} = (1,0)$ gauged supergravity did not have Minkowski$_6$ (nor (A)dS$_6$) backgrounds as solutions and the equations of motion led to a spontaneous compactification to lower-dimensional spaces [8]. In the absence of a background three-form flux [9], smooth symmetric solutions of the Salam-Sezgin model take the form of a half-BPS Minkowski$_4\times$S^2 background [8, 10], which are phenomenologically relevant and can also be embedded in a String/M-theory framework [11]. In the context of warped braneworld scenarios, 6D supergravities have also been investigated in the past to propose possible scenarios to solve the cosmological constant problem and build models possessing dS$_4$ vacua, see, e.g., [12–14].

Being based on a 6D theory possessing chiral fermions, all the previously mentioned models are generically anomalous. As part of the effort to embed these theories in a consistent quantum theory of gravity, anomaly free 6D $\mathcal{N} = (1,0)$ supergravities have been constructed in the ungauged and gauged cases [15–27]. In the ungauged case various anomaly-free models were originally constructed by means of compactification of the heterotic string and Green-Schwarz anomaly cancellation. Recently, the classification of 6D $\mathcal{N} = (1,0)$ supergravity theories consistent with quantum gravity have also been systematically approached in the context of F-theory compactifications on elliptically fibered Calabi-Yau threefolds. We refer the reader to [28–39] and references therein for some of the literature on the subject.

The ungauged six-dimensional supergravity based on the dilaton-Weyl multiplet admits a unique supersymmetric $\text{AdS}_3 \times \text{S}^3$ solution. The solution is supported by a self-dual 3-form flux and possesses vanishing Weyl tensor reminiscent of the $\text{AdS}_5 \times \text{S}^5$ solution in IIB supergravity. Moreover, there have been arguments indicating that the supersymmetric $\text{AdS}_5 \times \text{S}^5$ is an exact solution in the full string theory based on its vanishing Weyl tensor. By analogy, it is tempting to conjecture that the supersymmetric $\text{AdS}_3 \times \text{S}^3$ solution is an exact solution in the six-dimensional compactified string theory. We have verified this up to the inclusion of the Gauss-Bonnet and Riemann squared super-invariants [40].[1] We recall that the $\text{AdS}_3 \times \text{S}^3$ geometry is locally $\text{BTZ} \times \text{S}^3$ and it is also the near horizon geometry of a black string. Thus, the 6D supergravity models can also be a useful arena for studying the black hole/string entropy correspondence. Although the supersymmetric $\text{AdS}_3 \times \text{S}^3$ solution is not affected by the curvature-squared corrections, α' corrections in the action do modify the macroscopic entropy via Wald's entropy formula and thus can be used to compare with future microscopic computations at the same order. Curvature-squared terms are particularly important for computing the entropy of small black holes as the "would-be" leading Bekenstein-Hawking piece vanishes and therefore the curvature-squared corrections serve as the first non-vanishing contribution. The inclusion of the curvature-squared invariants also modifies the spectrum of fluctuations around the supersymmetric $\text{AdS}_3 \times \text{S}^3$ solution. On top of the short multiplets of $\text{SU}(1,1|2)$, there are also infinite towers of long $\text{SU}(1,1|2)$ multiplets with mass proportional to the inverse of the α' parameter. Expanding on the results presented in [40], in this paper we give a detailed analysis of the spectrum around the supersymmetric $\text{AdS}_3 \times \text{S}^3$ solution. Note also that the spectrum of fluctuations provides logarithmic corrections to the effective action, which has been proposed as a new probe to the possible UV completion of a low-energy effective theory of quantum gravity [41].

It is worth stressing that there is a sharp difference between the spectrum of the short and long multiplets mentioned above. The spectrum of short multiplets is protected and can in principle be inferred by using group theoretical considerations along the analysis of [42]. Contrarily, the AdS energies of the long-multiplet states are not protected by shortening conditions and depend on the detail of the supergravity theory (in a low-energy approximation). Once the model is fixed, we can adopt the same method as [43] to compute the spectrum of long multiplets by solving the linearized equations for the supegravity fields about the supersymmetric AdS vacuum. In this regard, our results based on the new off-shell Einstein-Gauss-Bonnet supergravity presented in [40] and, in more detail, in the present paper are new and overcome the limitations of an algebraic approach.

Higher order curvature terms are of importance in string theory where the leading supergravity actions are necessarily corrected by an infinite series of quantum corrections parametrized by the string tension α' and the string coupling g_s. The purely gravitational higher-curvature terms are also related by supersymmetry to contributions depending on p-forms. These terms, that have not yet been systematically analyzed in the literature,

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[1]One can also show that it is true with the inclusion of the Ricci scalar squared invariant given in this paper.

play an important role in understanding the moduli in compactified string theory and the low-energy description of string dualities, see, e. g., [44–46].

Since multiplets of extended supersymmetry can be decomposed into $(1,0)$ multiplets, in principle, all supergravity models in 6D can be formulated in the $(1,0)$ framework. Among all the supergravity models in 6D, the most interesting ones are those coming from Calabi-Yau compactifications of string theory/F-theory which are anomaly free and possess the duality properties inherited from the parent theories. For instance, the two $(1,1)$ supergravities descending from the Heterotic string on T^4 and IIA-string on K3 are expected to be related to each other by the six-dimensional Heterotic/IIA duality transformation to all order in α', see the discussion in [46]. So far the 6D duality has been verified rigorously at the two-derivative level and connects the two $(1,1)$ supergravity theories introduced by Romans in [47]. Beyond the leading order, examining the duality becomes a difficult task as it requires the knowledge of the fully-fledged supersymmetric higher-derivative corrections involving delicate couplings between gravity and matter fields. Some recent progress was made to the first order in α' in [46]. Furthermore, 6D $\mathcal{N} = (1,0)$ supergravity admits an off-shell formulation which significantly facilitates the construction of higher-derivative super-invariants. Thus the off-shell $(1,0)$ supergravity should provide a useful framework for testing string duality at higher order. We also notice that under the proposed six-dimensional duality transformation [46], fields belonging to the dilaton-Weyl multiplet, which is one of the two variant multiplets of 6D $(1,0)$ conformal supergravity [48], form a closed structure. Consequently, testing the duality can be further simplified by focusing on the subsector consisting of only the dilaton-Weyl multiplet, which will be a main player in our paper.

In compactified string theory the leading corrections in the higher-derivative series come from curvature-squared terms given by the Gauss-Bonnet (GB) combination which has the form $\mathcal{R}^{abcd}\mathcal{R}_{abcd} - 4\mathcal{R}^{ab}\mathcal{R}_{ab} + \mathcal{R}^2 = 6\mathcal{R}_{[ab}{}^{ab}\mathcal{R}_{cd]}{}^{cd}$, where $\mathcal{R}_{abcd}$ is the Riemann tensor, $\mathcal{R}_{ab}$ is the Ricci tensor, and $\mathcal{R}$ is the Ricci scalar. This is the simplest example of Lovelock gravity theory and as such it is singled out since it is ghost free, and its equations of motion are second order in derivatives [49, 50]. In the case of string theory compactified to $\mathrm{D} \leq 6$, the construction of higher-derivative supergravity invariants can be simplified thanks to the fact that off-shell supersymmetric techniques can be efficiently used. In particular, the construction of the GB supergravity invariant by using off-shell techniques was achieved in 4D in [51–58], in 5D in [59–61], while for the 6D case only partial results were obtained more than thirty years ago [62–65]. Before continuing our discussion it is worth underlining what was the state of the art in the description of curvature squared invariants in off-shell 6D $\mathcal{N} = (1,0)$ supergravity and what is new in our paper. Note that in 6D, up to total derivatives, the general non-supersymmetric curvature squared Lagrangian has the form $\alpha \mathcal{R}^{abcd}\mathcal{R}_{abcd} + \beta \mathcal{R}^{ab}\mathcal{R}_{ab} + \gamma \mathcal{R}^2$.

A supersymmetric extension of the Riemann curvature squared term was constructed more than 30 years ago [62–65]. This invariant has been studied in some detail in the literature. For instance, it has been coupled to the gauged chiral supergravity in six dimensions extending the Salam-Sezgin model with curvature squared corrections [66] and the exact spectrum of this model around the half-BPS Minkowski$_4 \times \mathrm{S}^2$ background was

analyzed in [67]. A feature of this model that is worth underlining is that, compared to the 6D $\mathcal{N} = (1,0)$ Einstein-Hilbert Poincaré supergravity, the addition of the Riemann squared invariant modifies the spectrum of the theory. In fact, as we will also review in section 4 of our paper, off-shell $\mathcal{N} = (1,0)$ Poincaré supergravity (we focus for simplicity on the ungauged case) can be constructed by coupling the dilaton-Weyl multiplet of conformal supergravity [48] to a linear multiplet. The independent field content of the first off-shell multiplet comprises the vielbein and chiral gravitini ($e_m{}^a$, ψ_{mi}), the dilatation (b_m) and $\mathrm{SU}(2)_R$ ($\mathcal{V}_m{}^{kl}$) gauge connections together with a real scalar field σ, a chiral fermion ψ^i, and a gauge two form b_{mn}. The off-shell linear multiplet is described by an $\mathrm{SU}(2)_R$ triplet of scalar fields L^{ij} together with a gauge 4-form b_{mnpq} and a chiral fermion φ_α^i. The off-shell Einstein-Hilbert (EH) invariant that can be constructed with the previous field content [48] is such that $\{e_m{}^a, \psi_{mi}, b_{mn}, L = \sqrt{L_{ij}L^{ij}/2}, \varphi^i\}$ are the on-shell physical fields while $\{\sigma, b_m, b_{mnpq}, \mathcal{V}_m{}^{kl}, \psi^i\}$ are either pure gauge degrees of freedom or auxiliary fields. Once the Riemann tensor squared invariant is added to the supersymmetric EH term some of the latter fields start to propagate and are not auxiliary any longer [66, 67]. In principle, their elimination can be performed in a nontrivial expansion in α', see [66] for the analysis at first order. Moreover, the gravitational spectrum is also modified due to the higher-derivative term that in this case generate ghost modes. A similar pattern emerges when one consider a linear combination of the supersymmetric EH term and a supersymmetric completion of the scalar curvature squared which was constructed in 2013 in [68].

Recently, in [40] we presented a new independent curvature squared invariant completing the classification of the supersymmetric extension of the three curvature squared terms $\alpha \mathcal{R}^{abcd}\mathcal{R}_{abcd} + \beta \mathcal{R}^{ab}\mathcal{R}_{ab} + \gamma \mathcal{R}^2$. By taking a linear combination of the Riemann squared invariant and the new invariant, in [40] we described for the first time the full bosonic sector of the off-shell $\mathcal{N} = (1,0)$ Gauss-Bonnet term. Besides being free of ghosts in the gravitational sector, the supersymmetric Gauss-Bonnet invariant stands apart being the only off-shell curvature squared invariant that remarkably preserves the same structure of physical and auxiliary degrees of freedom of the EH term. It is exemplary to note that a general combination of Riemann squared, Ricci squared and scalar curvature squared, contains a dynamical terms $\mathcal{R}_{ab}{}^{kl}(\mathcal{V})\mathcal{R}^{ab}{}_{kl}(\mathcal{V})$ for the $\mathrm{SU}(2)_R$ connection $\mathcal{V}_m{}^{kl}$ which exactly cancels out only for the GB invariant. One of the main aims of this paper is to describe in detail the construction of the three curvature squared invariant with the techniques of [48, 69, 70]. Though these invariants were already known in the literature, some of the detail of their construction were missing. In particular, in [40] we presented the Gauss-Bonnet invariant but this paper gives a comprehensive description of its construction and the analysis of the spectrum for the Einstein-Gauss-Bonnet supergravity.

Another main purpose of this paper is to present two other new curvature-squared invariants. They both arise from a four-derivative action for a compensating linear multiplet coupled to off-shell $\mathcal{N} = (1,0)$ conformal supergravity. If the linear multiplet is coupled to the dilaton-Weyl multiplet, as for all the curvature squared invariants discussed above, one obtains a new invariant that contains a linear combination of Ricci tensor and scalar curvature squared terms multiplied by a prefactor for the dilaton field $L = \mathrm{e}^{-v}$ which also appears in higher-derivative self-interactions. Alternatively, by coupling the

new higher-derivative action for the linear multiplet to the standard-Weyl multiplet of off-shell conformal supergravity[2] one can obtain another supersymmetric extension of a linear combination of Ricci tensor and scalar curvature squared terms. This latest invariant differs considerably from the previous four since it is constructed by using a variant off-shell supergravity multiplet. Interestingly, as an application, we will show that the coupling of this latest invariant to the supersymmetric Einstein-Hilbert term in a standard-Weyl multiplet leads to a dynamically generated cosmological constant and non-supersymmetric (A)dS$_6$ solutions.

Before turning to the main technical sections of our paper, it is worth emphasizing some important features and recent developments of off-shell supergravity that might not be familiar to all the readers and will play an important role in our paper. A main feature of off-shell techniques is that the local supersymmetry transformations close without the use of the equations of motion. This feature makes off-shell formalisms extremely powerful to describe general supergravity-matter systems without having to worry about the dependence of the supersymmetry transformations upon specific models. In particular, this is a clear advantage if one is interested in constructing higher-derivatives interactions that on-shell introduce highly nontrivial modifications to the supersymmetry transformations. As such, a description of the supergravity effective action of superstrings where supersymmetry closes off-shell would eliminate the complexity of an infinite series of α' and g_s corrections that in an on-shell setting not only appear in the action but also in the supersymmetry transformations that are both corrected order by order. Furthermore, an off-shell description naturally solves the problem that higher-derivative terms in the on-shell string theory effective action possess ambiguities arising from curvature dependent field redefinitions of the metric, such as $g'_{mn} = g_{mn} + ag_{mn}\mathcal{R} + b\mathcal{R}_{mn} + \ldots$ with $\mathcal{R}_{mn}$ the Ricci tensor and $\mathcal{R}$ the scalar curvature. This conceptual issue, which plagues the organization of the effective action, simply disappears if supersymmetry is implemented off-shell since no such redefinition leaves the supersymmetry algebra invariant. It is important to stress that such an off-shell organization of the low-energy string effective action remains in general an open problem. One of its very nontrivial issues is to understand how the off-shell supergravity "auxiliary" fields get integrated out[3] and if and how they alter the low-energy spectrum. Though still a longstanding open problem, in this paper we will show how it is possible to deal with these issues in the restrictive truncation to six dimensional $\mathcal{N} = (1,0)$ and at first order in α'.

Formalisms to describe off-shell supergravity-matter systems make use of component field techniques within the superconformal tensor calculus or superspace approaches. The literature on these subjects is vast and we only refer to standard reviews for the 4D case,

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[2]As we will review in detail later, the standard-Weyl multiplet of $\mathcal{N} = (1,0)$ conformal supergravity includes $(e_m{}^a, \psi_{mi}, b_m, \mathcal{V}_m{}^{kl})$ independent gauge fields, as for the dilaton-Weyl multiplet, but comprises an anti-self-dual tensor T^-_{abc}, a real scalar field D, and a chiral fermion χ^i as covariant matter fields [48].

[3]We refer the reader to [71] for some references on the subject and a very interesting example of how non-trivial and counterintuitive the integration of auxiliary fields can be. In [71] it was shown how the infinite series of higher derivative interactions of DBI theories coupled to supergravity can remarkably arise by integrating out the auxiliary fields of 4D $\mathcal{N} = 2$ conformal supergravity in an off-shell $\mathcal{R}^4$ action.

respectively [72] and [73–75]. In the case of 6D $\mathcal{N} = (1, 0)$ supergravity, the superconformal tensor calculus was first applied in [48] and further developed in [66, 76] where the complete off-shell action for minimal Poincaré supergravity was presented [76] as well as that of the gauged minimal 6D supergravity [66]. For standard superspace techniques applied to supergravity in six dimensions see [65, 77–82] and [83]. Note that these references employ a "Wess-Zumino" superspace approach analogue to the one used to study 4D $\mathcal{N} = 1$ (see, e.g., [73–75]) and $\mathcal{N} = 2$ supergravity (see, e.g., [84–87]) where the structure group of the superspace geometry is chosen to be the Lorentz group times (a subgroup of) the R-symmetry group.

It turns out that the superconformal tensor calculus and the standard superspace approach to off-shell supergravity are naturally related through the so-called *conformal superspace*. In this formalism the entire superconformal algebra is manifestly gauged in superspace combining the main advantages of both approaches and providing a streamlined approach for component reduction of superspace results. As such, this approach is a bridge between the superconformal tensor calculus, where the superconformal group is gauged manifestly in standard space-time, and standard superspace approaches, where typically part of the superconformal transformations are non-linearly realized as super-Weyl transformations [73, 75, 88]. Conformal superspace was first introduced for 4D $\mathcal{N} = 1, 2$ supergravity in [89, 90] (see also the seminal work by Kugo and Uehara [91]) and it was developed and extended to 3D $\mathcal{N}$-extended supergravity [92], 5D $\mathcal{N} = 1$ supergravity [61], and recently to 6D $\mathcal{N} = (1, 0)$ supergravity [69], see also [70]. In the last few years, this approach has proven to be efficient to construct higher-derivative supergravity invariants. These include the construction of: the three-dimensional $3 \leq \mathcal{N} \leq 6$ conformal supergravity actions [93, 94]; the 4D $\mathcal{N} = 2$ Gauss-Bonnet invariant [58]; curvature squared invariants in 5D $\mathcal{N} = 1$ supergravity [61]; the 6D $\mathcal{N} = (1, 0)$ conformal supergravity actions related to the Type-B conformal anomalies [69, 70]; and recently the 6D $\mathcal{N} = (1, 0)$ Gauss-Bonnet invariant [40]. This paper will then show how this formalism can be efficiently employed to construct curvature-squared invariants for six-dimensional $(1, 0)$ supergravity.

This paper is organized as follows. In section 2 we describe the 6D $\mathcal{N} = (1, 0)$ locally superconformal multiplets that will be used in our paper. In particular, we will describe the standard-Weyl multiplet, the non-abelian vector multiplet, the linear multiplet, the gauge 3-form multiplet, the tensor multiplet and the dilaton-Weyl multiplet. We will describe the structure of each of the multiplets both in superspace and in components. In section 3, by using the superform approach to the construction of supersymmetric invariants [95–99], we describe how to construct various locally superconformal invariants that will play the role of action principles. In section 4 we review the construction of the minimal off-shell $\mathcal{N} = (1, 0)$ two-derivative Poincaré supergravity theory of [48] within our approach. The off-shell extension of the Einstein-Hilbert term arises by using a linear multiplet compensator coupled to a dilaton-Weyl conformal supergravity multiplet. Section 5 contains some of the main results of our paper: the locally $\mathcal{N} = (1, 0)$ supersymmetric extension of all the curvature squared terms for the minimal Poincaré supergravity of section 4 which is based on a dilaton-Weyl multiplet of conformal supergravity. In particular, two of these invariants, the Riemann squared and an invariant that was first constructed in [40], are

locally superconformal and do not need the coupling to a matter compensator in contrast to a supersymmetric extension of the scalar curvature squared term. We also present in detail the off-shell Gauss-Bonnet invariant which is relevant to describing α' corrections in string theory compactified to six dimensions. Section 6 is devoted to defining the 6D $\mathcal{N} = (1,0)$ Einstein-Gauss-Bonnet supergravity theory which arises by adding the off-shell Gauss-Bonnet to the Einstein-Hilbert invariant. By integrating out the auxiliary fields, whose on-shell values can be set to zero as in the two-derivative Poincaré theory of section 4, we derive the on-shell Einstein-Gauss-Bonnet supergravity which was first obtained in [46] by using string theory arguments. In section 7 we present new curvature squared invariants based on an higher-derivative action for the linear multiplet compensator which, depending on the choice of the dilaton-Weyl or the standard-Weyl as conformal supergravity multiplets, proves to describe two new curvature-squared terms compared to the ones of section 5. In section 8 we turn to an application of the results of section 6 and describe in detail the computation of the spectrum of the Einstein-Gauss-Bonnet supergravity theory around the supersymmetric $AdS_3 \times S^3$ vacuum. In section 9 we conclude by discussing our results and possible future lines of research based on our findings. The paper includes four appendices. Appendix A summarises useful results of [69, 70] regarding 6D $\mathcal{N} = (1,0)$ conformal superspace that are necessary for our paper. In appendix B we collect relevant descendant components for the composite gauge 3-form multiplet based on the primary (5.17), which are necessary to derive the complete result for the invariant (5.19). Appendix C includes the full bosonic part of the new locally superconformal invariant constructed in section 7. In appendix D we describe how to map our notation and conventions, based on [69, 70], to the ones of [48] and [76].

2 $\mathcal{N} = (1,0)$ superconformal multiplets

In this section, we describe several superconformal multiplets that will serve as building blocks for the various curvature squared invariants presented in this paper. We will first discuss the standard-Weyl multiplet of conformal supergravity before moving on to the description of various matter multiplets, including the non-abelian vector, linear, and gauge 3-form multiplets, and concluding with the description of the tensor and dilaton-Weyl multiplets. For each of the multiplets, we first elaborate on a superspace description and then provide their structure in terms of component fields. The readers only interested in the component results can direct their attention to the second half of each subsection. For 6D $\mathcal{N} = (1,0)$ superspace we make use of the formulation and results of [69, 70].

Before turning to the technical presentation of this section it is worth commenting about the fact that the non-abelian vector multiplet is described by a closed super 2-form, the tensor multiplet is described by a closed super 3-form, the gauge 3-form multiplet is described by a closed super 4-form, and the linear multiplet is described by a closed super 5-form. As was shown in [100], these multiplets are in fact part of a tensor hierarchy of superforms that also contains as top-form the closed super 6-form used in [69, 70]. Note that, despite the natural organization of these multiplet in a tensor hierarchy, we will organize this section by following a more traditional order, as for instance similar to the

one used in [48], and present these multiplets by increasing complexity of their structure and the physical role they will play in the applications studied in our paper. For instance, we leave the description of the tensor multiplet to the end of this section. Compared to the other matter multiplets, the tensor multiplet stands apart since in the flat limit it is on-shell and in the curved case it is directly linked to the description of the off-shell dilaton-Weyl multiplet of conformal supergravity.

2.1 The standard-Weyl multiplet

The standard-Weyl (or type I) multiplet of $\mathcal{N} = (1,0)$ conformal supergravity is associated with the local off-shell gauging of the superconformal group $\mathrm{OSp}(8^*|1)$ [48]. The multiplet contains $40 + 40$ physical components described by a set of independent gauge fields: the vielbein $e_m{}^a$ and a dilatation connection b_m; the gravitino $\psi_{mi}{}^\alpha$, associated with the gauging of Q-supersymmetry; and $\mathrm{SU}(2)_R$ gauge fields $\mathcal{V}_m{}^{ij}$. The other gauge fields associated with the remaining generators of $\mathrm{OSp}(8^*|1)$ are composite fields. In addition to the independent gauge connections, the standard-Weyl multiplet comprises a set of covariant matter fields: an anti-self-dual tensor T_{abc}^-; a real scalar field D; and a chiral fermion χ^i. We start by reviewing how to embed this in conformal superspace [69] and then, following [70], we will show how to derive the component structure of the multiplet.

2.1.1 The standard-Weyl multiplet in superspace

We begin with a curved six-dimensional $\mathcal{N} = (1,0)$ superspace $\mathcal{M}^{6|8}$ parametrized by local bosonic (x^m) and fermionic (θ_i) coordinates:

$$z^M = (x^m, \theta_i^\mu), \tag{2.1}$$

where $m = 0, 1, \cdots, 5$, $\mu = 1, \cdots, 4$ and $i = 1, 2$. By gauging the full 6D $\mathcal{N} = (1,0)$ superconformal algebra we introduce covariant derivatives $\nabla_A = (\nabla_a, \nabla_\alpha^i)$ that have the form

$$\nabla_A = E_A - \frac{1}{2}\Omega_A{}^{ab}M_{ab} - \Phi_A{}^{kl}J_{kl} - B_A\mathbb{D} - \mathfrak{F}_{AB}K^B. \tag{2.2}$$

Here $E_A = E_A{}^M\partial_M$ is the inverse super-vielbein, M_{ab} are the Lorentz generators, J^{ij} are generators of the $\mathrm{SU}(2)_R$ R-symmetry group, $\mathbb{D}$ is the dilatation generator and $K^A = (K^a, S_i^\alpha)$ are the special superconformal generators. The supervielbein one form is $E^A = \mathrm{d}z^M E_M{}^A$ with $E_M{}^A E_A{}^N = \delta_M^N$, $E_A{}^M E_M{}^B = \delta_A^B$. The Lorentz $\Omega_A{}^{ab} = -\Omega_A{}^{ba}$, $\mathrm{SU}(2)_R$ $\Phi_A{}^{kl} = \Phi_A{}^{lk}$, dilatation B_A and special conformal $\mathfrak{F}_{AB}$ connections are associated with their respective structure group generators $(M_{ab}, J^{ij}, \mathbb{D}, K^a, S_i^\alpha)$. The super one-form connections are $\Omega^{ab} := \mathrm{d}z^M\Omega_M{}^{ab} = E^A\Omega_A{}^{ab}$, $\Phi^{kl} = \mathrm{d}z^M\Phi_M{}^{kl} = E^A\Phi_A{}^{kl}$, $B := \mathrm{d}z^M B_M = E^A B_A$, and $\mathfrak{F}_B := \mathrm{d}z^M\mathfrak{F}_{MB} = E^A\mathfrak{F}_{AB}$.

To describe the standard 6D $(1,0)$ Weyl multiplet in conformal superspace, one constrains the algebra of covariant derivatives

$$\begin{aligned}
[\nabla_A, \nabla_B\} = &-T_{AB}{}^C\nabla_C - \frac{1}{2}R(M)_{AB}{}^{cd}M_{cd} - R(J)_{AB}{}^{kl}J_{kl} \\
&- R(\mathbb{D})_{AB}\mathbb{D} - R(S)_{AB}{}_\gamma^k S_k^\gamma - R(K)_{ABc}K^c,
\end{aligned} \tag{2.3}$$

to be completely determined in terms of the super-Weyl tensor superfield $W^{\alpha\beta}$ [69, 83, 101] satisfying

$$W^{\alpha\beta} = W^{\beta\alpha} , \quad K^A W^{\alpha\beta} = 0 , \quad \mathbb{D} W^{\alpha\beta} = W^{\alpha\beta} , \tag{2.4}$$

and the Bianchi identities

$$\nabla_\alpha^{(i} \nabla_\beta^{j)} W^{\gamma\delta} = -\delta_{[\alpha}^{(\gamma} \nabla_{\beta]}^{(i} \nabla_\rho^{j)} W^{\delta)\rho} , \tag{2.5a}$$

$$\nabla_\alpha^k \nabla_{\gamma k} W^{\beta\gamma} - \frac{1}{4} \delta_\alpha^\beta \nabla_\gamma^k \nabla_{\delta k} W^{\gamma\delta} = 8\mathrm{i} \nabla_{\alpha\gamma} W^{\gamma\beta} . \tag{2.5b}$$

Due to the relation $W^{\alpha\beta} = 1/6(\tilde{\gamma}^{abc})^{\alpha\beta} W_{abc}$, the super-Weyl tensor is equivalent to an anti-self-dual rank-3 tensor superfield W_{abc}. In (2.3) $T_{AB}{}^C$ is the torsion, and $R(M)_{AB}{}^{cd}$, $R(J)_{AB}{}^{kl}$, $R(\mathbb{D})_{AB}$, $R(S)_{AB\gamma}^K$ and $R(K)_{ABc}$ are the curvatures corresponding to the Lorentz, $\mathrm{SU}(2)_R$, dilatation, S-supersymmetry and special conformal boosts, respectively. Their expressions in terms of $W^{\alpha\beta}$ and its descendant superfields of dimension 3/2

$$X^{\alpha i} := -\frac{\mathrm{i}}{10} \nabla_\beta^i W^{\alpha\beta} , \quad X_\gamma^{k\alpha\beta} := -\frac{\mathrm{i}}{4} \nabla_\gamma^k W^{\alpha\beta} - \delta_\gamma^{(\alpha} X^{\beta)k} , \tag{2.6}$$

and of dimension 2

$$Y_\alpha^{\beta ij} := -\frac{5}{2} \left(\nabla_\alpha^{(i} X^{\beta j)} - \frac{1}{4} \delta_\alpha^\beta \nabla_\gamma^{(i} X^{\gamma j)} \right) = -\frac{5}{2} \nabla_\alpha^{(i} X^{\beta j)} , \tag{2.7a}$$

$$Y := \frac{1}{4} \nabla_\gamma^k X_k^\gamma , \tag{2.7b}$$

$$Y_{\alpha\beta}{}^{\gamma\delta} := \nabla_{(\alpha}^k X_{\beta)k}{}^{\gamma\delta} - \frac{1}{6} \delta_\beta^{(\gamma} \nabla_\rho^k X_{\alpha k}{}^{\delta)\rho} - \frac{1}{6} \delta_\alpha^{(\gamma} \nabla_\rho^k X_{\beta k}{}^{\delta)\rho} , \tag{2.7c}$$

are collected in appendix A. There we also collect the (anti-)commutators among the structure group generators and with the covariant derivatives. Note that, compared to [69], in this paper we will make use of conformal superspace with a redefined vector covariant derivative which corresponds to choosing the "traceless" frame conventional constraints employed for the first time in [70].[4] The superspace and component structures corresponding to this choice are summarized below and in appendix A.

The superfields $X^{\alpha i}$, $X_\gamma^{k\alpha\beta}$, $Y_\alpha^{\beta ij}$, Y and $Y_{\alpha\beta}{}^{\gamma\delta}$ satisfy the nontrivial Bianchi identities (A.7) [69] that are consequences of (2.5). These imply that the previous five superfields are the only independent descendants obtained by acting with spinor derivatives on $W^{\alpha\beta}$. At higher mass dimension all the descendants are vector derivatives of the previous five, see (A.5). See also (A.6) for the action of the S-generators on $X^{\alpha i}$, $X_\gamma^{k\alpha\beta}$, $Y_\alpha^{\beta ij}$, Y and $Y_{\alpha\beta}{}^{\gamma\delta}$ that prove to be all annihilated by K^a.

In conformal superspace, the gauge group of conformal supergravity, $\mathcal{G}$, is generated by *covariant general coordinate transformations*, δ_{cgct}, associated with a local superdiffeomorphism parameter ξ^A and *standard superconformal transformations*, $\delta_{\mathcal{H}}$, associated with

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[4]In [70] we denoted the covariant derivatives of [69] as $\nabla_A = (\nabla_a, \nabla_\alpha^i)$ while derivatives in different frames were denoted by $\hat{\nabla}_A = (\hat{\nabla}_a, \nabla_\alpha^i)$. Since in this paper we will always use the traceless frame of [70] we will remove everywhere the hats but the reader should not confuse $\nabla_A = (\nabla_a, \nabla_\alpha^i)$ with the ones of [69]. Equation (A.8) explains the relation between the two vector derivatives.

the following local superfield parameters: the dilatation σ, Lorentz $\Lambda^{ab} = -\Lambda^{ba}$, $\mathrm{SU(2)}_R$ $\Lambda^{ij} = \Lambda^{ji}$, and special conformal (bosonic and fermionic) transformations $\Lambda_A = (\Lambda_a, \Lambda_\alpha^i)$. The covariant derivatives transform as

$$\delta_\mathcal{G} \nabla_A = [\mathcal{K}, \nabla_A]\,, \tag{2.8}$$

where $\mathcal{K}$ denotes the first-order differential operator

$$\mathcal{K} = \xi^C \nabla_C + \frac{1}{2} \Lambda^{ab} M_{ab} + \Lambda^{ij} J_{ij} + \sigma \mathbb{D} + \Lambda_A K^A\,. \tag{2.9}$$

A covariant (or tensor) superfield U transforms as

$$\delta_\mathcal{G} U = (\delta_{\mathrm{cgct}} + \delta_\mathcal{H}) U = \mathcal{K} U\,. \tag{2.10}$$

The superfield U is said to be *primary* and of dimension Δ if $K_A U = 0$ and $\mathbb{D}U = \Delta U$. The super-Weyl tensor $W^{\alpha\beta}$ is a primary dimension 1 covariant superfield.

2.1.2 The standard-Weyl multiplet in components

Following the analysis of [70], let us now describe how to obtain the component description of the Weyl multiplet from the previous superspace geometry.

The vielbein $(e_m{}^a)$ and gravitini (ψ_{mi}^α) appear as the $\theta = 0$ projections of the coefficients of $\mathrm{d}x^m$ in the supervielbein E^A one-form,

$$e^a = \mathrm{d}x^m e_m{}^a = E^a \|\,, \qquad \psi_i^\alpha = \mathrm{d}x^m \psi_{mi}^\alpha = 2E_i^\alpha \|\,, \tag{2.11}$$

where the double bar denotes setting $\theta = \mathrm{d}\theta = 0$ [70, 102, 103]. The remaining fundamental and composite one-forms correspond to double-bar projections of superspace one-forms,

$$\mathcal{V}^{kl} := \Phi^{kl} \|\,, \quad b := B \|\,, \quad \omega^{cd} := \Omega^{cd} \|\,, \quad \phi_\gamma^k := 2\mathfrak{F}_\gamma^k \|\,, \quad \mathfrak{f}_c := \mathfrak{F}_c \|\,. \tag{2.12}$$

The covariant matter fields are contained within the super-Weyl tensor W_{abc} and its independent descendants as follows:

$$T_{abc}^- := -2W_{abc}|\,, \tag{2.13a}$$

$$\chi^{\alpha i} := \frac{15}{2} X^{\alpha i}| = -\frac{3\mathrm{i}}{4} \nabla_\beta^i W^{\alpha\beta}|\,, \tag{2.13b}$$

$$D := \frac{15}{2} Y| = -\frac{3\mathrm{i}}{16} \nabla_\alpha^k \nabla_{\beta k} W^{\alpha\beta}|\,, \tag{2.13c}$$

where a single line next to a superfield denotes setting $\theta = 0$. The lowest components of the other nontrivial descendants of $W^{\alpha\beta}$, specifically $X_\alpha^{i\,\beta\gamma}|$, $Y_\alpha{}^{\beta kl}|$ and $Y_{\alpha\beta}{}^{\gamma\delta}|$, prove to be directly related to component curvatures and then they are composite fields.

By taking the double bar projection of $\nabla = E^A \nabla_A$, the component vector covariant derivative ∇_a is defined to coincide with the projection of the superspace derivative $\nabla_a|$,[5]

$$e_m{}^a \nabla_a = \partial_m - \frac{1}{2} \psi_{mi}^\alpha \nabla_\alpha^i| - \frac{1}{2} \omega_m{}^{cd} M_{cd} - b_m \mathbb{D} - \mathcal{V}_m{}^{kl} J_{kl} - \frac{1}{2} \phi_{m\alpha}^i S_i^\alpha - \mathfrak{f}_{ma} K^a\,. \tag{2.14}$$

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[5]Depending on the context it should be clear to the reader whether ∇_a denotes the superspace or the component vector derivatives.

In this framework, the projected spinor covariant derivative $\nabla_\alpha^i|$ corresponds to the generator of Q-supersymmetry, and is defined so that if $\mathcal{U} = U|$, then $Q_\alpha^i \mathcal{U} := \nabla_\alpha^i|\mathcal{U} := (\nabla_\alpha^i U)|$. For the other generators, as e.g. $M_{cd}\mathcal{U} = (M_{cd}U)|$, there is no ambiguity in identifying the bar projection and then the local diffeomorphisms, Q-supersymmetry transformations, and so on descend naturally from their corresponding rule in superspace.

The component supercovariant curvature tensors, arising from the commutator of two ∇_a derivatives, are defined as $R(P)_{ab}{}^c = T_{ab}{}^c|$ and $R(Q)_{abk}{}^\gamma = T_{abk}{}^\gamma|$, and with $R(M)_{ab}{}^{cd}$, $R(J)_{ab}{}^{kl}$, $R(\mathbb{D})_{ab}$, $R(S)_{ab\gamma}{}^k$ and $R(K)_{abc}$ coinciding with the lowest components of the corresponding superspace curvatures that are given in appendix A. Note that (A.4a), (A.4c) and (A.4b) imply that $X_\alpha^{i\,\beta\gamma}|$, $Y_{\alpha\beta}{}^{\gamma\delta}|$ and $Y_\alpha{}^{\beta kl}|$ are identified with the $R(Q)_{abk}{}^\gamma$, $R(M)_{ab}{}^{cd}$ and $R(J)_{ab}{}^{kl}$ component curvatures, respectively.

The constraints on the superspace curvatures determine how to supercovariantize a given component curvature by simply taking the double bar projection of the superspace torsion and each of the superspace curvature two forms. Upon doing so one finds [70]

$$R(P)_{ab}{}^c = 0\,, \tag{2.15a}$$

$$R(Q)_{abk} = \frac{1}{2}\Psi_{abk} + \mathrm{i}\tilde{\gamma}_{[a}\phi_{b]k} + \frac{1}{24}T^-_{cde}\tilde{\gamma}^{cde}\gamma_{[a}\psi_{b]k}\,, \tag{2.15b}$$

$$R(\mathbb{D})_{ab} = 2e_a{}^m e_b{}^n \partial_{[m}b_{n]} + 4\mathfrak{f}_{[ab]} + \psi_{[a}{}^i\phi_{b]i} + \frac{\mathrm{i}}{15}\psi_{[a}{}^j\gamma_{b]}\chi_j\,, \tag{2.15c}$$

$$R(M)_{ab}{}^{cd} = \mathcal{R}_{ab}{}^{cd}(\omega) + 8\delta_{[a}^{[c}\mathfrak{f}_{b]}{}^{d]} + \mathrm{i}\psi_{[aj}\gamma_{b]}R(Q)^{cdj} + 2\mathrm{i}\psi_{[aj}\gamma^{[c}R(Q)_{b]}{}^{d]j}$$
$$\qquad - \psi_{[aj}\gamma^{cd}\phi_{b]}{}^j - \frac{2\mathrm{i}}{15}\delta_{[a}^{[c}\psi_{b]j}\gamma^{d]}\chi^j + \frac{\mathrm{i}}{2}\psi_{[a}{}^j\gamma^e\psi_{b]j}\,T_e{}^{-cd}\,, \tag{2.15d}$$

$$R(J)_{ab}{}^{kl} = \mathcal{R}_{ab}{}^{kl}(\mathcal{V}) + 4\psi_{[a}{}^{(k}\phi_{b]}{}^{l)} + \frac{4\mathrm{i}}{15}\psi_{[a}{}^{(k}\gamma_{b]}\chi^{l)}\,, \tag{2.15e}$$

where we have introduced the derivatives

$$\mathcal{D}_m = \partial_m - \frac{1}{2}\omega_m{}^{bc}M_{bc} - b_m\mathbb{D} - \mathcal{V}_m{}^{ij}J_{ij}\,, \quad \mathcal{D}_a = e_a{}^m\mathcal{D}_m\,, \tag{2.16}$$

together with the curvature and field strengths

$$\Psi_{abk}{}^\gamma := 2e_a{}^m e_b{}^n\mathcal{D}_{[m}\psi_{n]k}{}^\gamma\,, \tag{2.17a}$$

$$\mathcal{R}_{ab}{}^{cd} := \mathcal{R}_{ab}{}^{cd}(\omega) = e_a{}^m e_b{}^n\left(2\partial_{[m}\omega_{n]}{}^{cd} - 2\omega_{[m}{}^{ce}\omega_{n]e}{}^d\right)\,, \tag{2.17b}$$

$$\mathcal{R}_{ab}{}^{kl} := \mathcal{R}_{ab}{}^{kl}(\mathcal{V}) = e_a{}^m e_b{}^n\left(2\partial_{[m}\mathcal{V}_{n]}{}^{kl} + 2\mathcal{V}_{[m}{}^{p(k}\mathcal{V}_{n]p}{}^{l)}\right)\,. \tag{2.17c}$$

On the other hand, due to the superspace curvature constraints $T_{ab}{}^c = 0$, (A.4a) and (A.4b), the "traceless" conventional constraints for the component curvatures follow [70]

$$R(P)_{ab}{}^c = 0\,, \tag{2.18a}$$

$$\gamma^b R(Q)_{abk} = 0\,, \tag{2.18b}$$

$$R(M)_{ac}{}^{bc} = 0\,. \tag{2.18c}$$

The conditions (2.18) allow one to solve for the composite connections as follows:

$$\omega_{abc} = \omega(e)_{abc} - 2\eta_{a[b}b_{c]} - \frac{\mathrm{i}}{4}\psi_b{}^k\gamma_a\psi_{ck} - \frac{\mathrm{i}}{2}\psi_a{}^k\gamma_{[b}\psi_{c]k}\,, \tag{2.19a}$$

$$\phi_m{}^k = \frac{\mathrm{i}}{16}\left(\gamma^{bc}\gamma_m - \frac{3}{5}\gamma_m\tilde{\gamma}^{bc}\right)\left(\Psi_{bc}{}^k + \frac{1}{12}T^-_{def}\tilde{\gamma}^{def}\gamma_{[b}\psi_{c]}{}^k\right)\,, \tag{2.19b}$$

$$\begin{aligned}
\mathfrak{f}_a{}^b = &-\frac{1}{8}\mathcal{R}_a{}^b(\omega) + \frac{1}{80}\delta_a^b\mathcal{R}(\omega) + \frac{1}{8}\psi_{[aj}\gamma^{bc}\phi_{c]}{}^j - \frac{1}{80}\delta_a^b\psi_{cj}\gamma^{cd}\phi_d{}^j \\
&+ \frac{\mathrm{i}}{16}\psi_{cj}\gamma_a R(Q)^{bcj} + \frac{\mathrm{i}}{8}\psi_{cj}\gamma^{[b}R(Q)_a{}^{c]j} + \frac{\mathrm{i}}{60}\psi_{aj}\gamma^b\chi^j \\
&+ \frac{\mathrm{i}}{16}\psi_a{}^j\gamma_c\psi_{dj}\,T^{-bcd} - \frac{\mathrm{i}}{160}\delta_a^b\psi_c{}^j\gamma_d\psi_{ej}\,T^{-cde}\,,
\end{aligned} \tag{2.19c}$$

where $\omega(e)_{abc} = -\frac{1}{2}(\mathcal{C}_{abc} + \mathcal{C}_{cab} - \mathcal{C}_{bca})$ is the usual torsion-free spin connection given in terms of the anholonomy coefficient $\mathcal{C}_{mn}{}^a := 2\,\partial_{[m}e_{n]}{}^a$.

The supersymmetry transformations of the independent fields in the standard-Weyl multiplet may also be read off from the superspace results

$$\delta e_m{}^a = -\mathrm{i}\xi_k\gamma^a\psi_m{}^k\,, \tag{2.20a}$$

$$\delta\psi_{mi} = 2\mathcal{D}_m\xi_i + \frac{1}{12}T^{-abc}\tilde{\gamma}_{abc}\gamma_m\xi_i + 2\mathrm{i}\tilde{\gamma}_m\eta_i\,, \tag{2.20b}$$

$$\delta\mathcal{V}_m{}^{kl} = -4\xi^{(k}\phi_m{}^{l)} - \frac{4\mathrm{i}}{15}\xi^{(k}\gamma_m\chi^{l)} + 4\psi_m{}^{(k}\eta^{l)}\,, \tag{2.20c}$$

$$\delta b_m = \xi_i\phi_m{}^i + \frac{\mathrm{i}}{15}\xi_i\gamma_m\chi^i + \psi_m{}^i\eta_i - 2e_m{}^a\lambda_a\,, \tag{2.20d}$$

$$\delta T^-_{abc} = -\frac{\mathrm{i}}{8}\xi^k\gamma^{de}\gamma_{abc}R(Q)_{dek} - \frac{2\mathrm{i}}{15}\xi_k\gamma_{abc}\chi^k\,, \tag{2.20e}$$

$$\delta\chi^i = \frac{1}{2}D\xi^i - \frac{3}{4}R(J)_{ab}{}^{ij}\tilde{\gamma}^{ab}\xi_j + \frac{1}{4}\nabla_a T^-_{bcd}\tilde{\gamma}^{bcd}\gamma^a\xi^i - \mathrm{i}T^-_{abc}\tilde{\gamma}^{abc}\eta^i\,, \tag{2.20f}$$

$$\delta D = -2\mathrm{i}\,\xi_j\slashed{\nabla}\chi^j - 4\chi^k\eta_k\,. \tag{2.20g}$$

Here

$$\nabla_d T^-_{abc} = \mathcal{D}_d T^-_{abc} + \frac{\mathrm{i}}{15}(\gamma_{abc})_{\alpha\beta}\psi_{dk}^\alpha\chi^{\beta k} + \frac{\mathrm{i}}{2}(\gamma_{abc})_{\alpha\beta}\psi_{dk}^\gamma X_\gamma^{k\alpha\beta}\,, \tag{2.21a}$$

$$\slashed{\nabla}\chi^j = \slashed{D}\chi^j + \frac{\mathrm{i}}{15}T^-_{bcd}\tilde{\gamma}^{bcd}\gamma^a\phi_a{}^j\,, \tag{2.21b}$$

and we have restricted to the Q, S and K transformations, $\delta = \delta_Q + \delta_S + \delta_K$, whose local component parameter are given by ξ_i, η^i and λ_a respectively defined as the $\theta = 0$ components of the corresponding superfield parameters, $\xi_i^\alpha|$, $\eta_\alpha^i := \Lambda_\alpha^i|$ and $\lambda_a := \Lambda_a|$.

2.2 The non-abelian vector multiplet

By following the discussion and conventions of [69, 70], let us turn to the description of a non-abelian vector multiplet.

2.2.1 The non-abelian vector multiplet in superspace

To describe the non-abelian vector multiplet in superspace, we introduce the gauge-covariant derivatives

$$\boldsymbol{\nabla} = E^A\boldsymbol{\nabla}_A\,, \quad \boldsymbol{\nabla}_A := \nabla_A - \mathrm{i}\boldsymbol{\mathcal{V}}_A\,, \tag{2.22}$$

where the gauge connection one-form $\boldsymbol{\mathcal{V}}_1 = E^A \boldsymbol{\mathcal{V}}_A$ takes its values in the Lie algebra of the (unitary) Yang-Mills gauge group, G_{YM}, with its (Hermitian) generators commuting with all the generators of the superconformal algebra. The gauge-covariant derivatives satisfy the algebra

$$[\boldsymbol{\nabla}_A, \boldsymbol{\nabla}_B\} = -T_{AB}{}^C \boldsymbol{\nabla}_C - \frac{1}{2} R(M)_{AB}{}^{cd} M_{cd} - R(J)_{AB}{}^{kl} J_{kl} - R(\mathbb{D})_{AB} \mathbb{D}$$
$$- R(S)_{AB}{}^\gamma_k S^k_\gamma - R(K)_{AB}{}^c K_c - \text{i} \boldsymbol{\mathcal{F}}_{AB} \,, \tag{2.23}$$

where the torsion and curvatures are those of conformal superspace while $\boldsymbol{\mathcal{F}}_{AB}$ corresponds to the gauge covariant field strength two-form $\boldsymbol{\mathcal{F}}_2 = \boldsymbol{\nabla}\boldsymbol{\mathcal{V}}_1 = \frac{1}{2} E^B \wedge E^A \boldsymbol{\mathcal{F}}_{AB}$. The field strength $\boldsymbol{\mathcal{F}}_2$ satisfies the Bianchi identity

$$\boldsymbol{\nabla}\boldsymbol{\mathcal{F}}_2 = 0 \quad \Longleftrightarrow \quad \boldsymbol{\nabla}_{[A}\boldsymbol{\mathcal{F}}_{BC)} + T_{[AB}{}^D \boldsymbol{\mathcal{F}}_{|D|C)} = 0 \,. \tag{2.24}$$

The Yang-Mills gauge transformation acts on the gauge-covariant derivatives $\boldsymbol{\nabla}_A$ and a matter superfield U (transforming in some representation of the gauge group) as

$$\boldsymbol{\nabla}_A \;\rightarrow\; \text{e}^{\text{i}\boldsymbol{\tau}} \boldsymbol{\nabla}_A \text{e}^{-\text{i}\boldsymbol{\tau}}, \qquad U \;\rightarrow\; U' = \text{e}^{\text{i}\boldsymbol{\tau}} U \,, \qquad \boldsymbol{\tau}^\dagger = \boldsymbol{\tau} \,, \tag{2.25}$$

where the Hermitian gauge parameter $\boldsymbol{\tau}(z)$ takes its values in the Lie algebra of G_{YM}. This implies that the gauge one-form and the field strength transform as follows:

$$\boldsymbol{\mathcal{V}}_1 \;\rightarrow\; \text{e}^{\text{i}\boldsymbol{\tau}} \boldsymbol{\mathcal{V}}_1 \text{e}^{-\text{i}\boldsymbol{\tau}} + \text{i}\,\text{e}^{\text{i}\boldsymbol{\tau}} \text{d}\,\text{e}^{-\text{i}\boldsymbol{\tau}} \,, \qquad \boldsymbol{\mathcal{F}}_2 \;\rightarrow\; \text{e}^{\text{i}\boldsymbol{\tau}} \boldsymbol{\mathcal{F}}_2 \text{e}^{-\text{i}\boldsymbol{\tau}} \,. \tag{2.26}$$

Some components of the field strength have to be constrained in order to describe an irreducible multiplet. In conformal superspace the right constraints are [69, 83, 104]

$$\boldsymbol{\mathcal{F}}^{i\,j}_{\alpha\beta} = 0 \,, \quad \boldsymbol{\mathcal{F}}_{a\,\beta}{}^j = (\gamma_a)_{\alpha\beta} \boldsymbol{\Lambda}^{\beta i} \,, \tag{2.27a}$$

where $\boldsymbol{\Lambda}^{\alpha i}$ is a conformal primary of dimension 3/2, $S^\gamma_k \boldsymbol{\Lambda}^{\alpha i} = 0$ and $\mathbb{D}\boldsymbol{\Lambda}^{\alpha i} = \frac{3}{2}\boldsymbol{\Lambda}^{\alpha i}$. The Bianchi identity (2.24) together with the constraints (2.27a) fix the remaining component of the two-form field strength to be

$$\boldsymbol{\mathcal{F}}_{ab} = -\frac{\text{i}}{8}(\gamma_{ab})_\beta{}^\alpha \boldsymbol{\nabla}^k_\alpha \boldsymbol{\Lambda}^\beta_k \tag{2.27b}$$

and fix $\boldsymbol{\Lambda}^{\alpha i}$ to obey the differential constraints [69, 83, 104]:

$$\boldsymbol{\nabla}^k_\gamma \boldsymbol{\Lambda}^\gamma_k = 0 \,, \quad \boldsymbol{\nabla}^{(i}_\alpha \boldsymbol{\Lambda}^{\beta j)} = \frac{1}{4}\delta^\beta_\alpha \boldsymbol{\nabla}^{(i}_\gamma \boldsymbol{\Lambda}^{\gamma j)} \,. \tag{2.28}$$

It is useful to list some identities for spinor covariant derivatives acting on the primary $\boldsymbol{\Lambda}^{\alpha i}$ and the descendant superfields $\boldsymbol{\mathcal{F}}_\alpha{}^\beta = -\frac{1}{4}(\gamma^{ab})_\alpha{}^\beta \boldsymbol{\mathcal{F}}_{ab}$ and

$$\boldsymbol{X}^{ij} := \frac{\text{i}}{4}\boldsymbol{\nabla}^{(i}_\gamma \boldsymbol{\Lambda}^{\gamma j)} \,. \tag{2.29}$$

These are

$$\boldsymbol{\nabla}_\alpha^i \boldsymbol{\Lambda}^{\beta j} = -\mathrm{i}\delta_\alpha^\beta \boldsymbol{\Lambda}^{ij} - 2\mathrm{i}\varepsilon^{ij}\boldsymbol{\mathcal{F}}_\alpha{}^\beta \,, \tag{2.30a}$$

$$\boldsymbol{\nabla}_\alpha^i \boldsymbol{\mathcal{F}}_\beta{}^\gamma = -\boldsymbol{\nabla}_{\alpha\beta}\boldsymbol{\Lambda}^{\gamma i} - \delta_\alpha^\gamma \boldsymbol{\nabla}_{\beta\delta}\boldsymbol{\Lambda}^{\delta i} + \frac{1}{2}\delta_\beta^\gamma \boldsymbol{\nabla}_{\alpha\delta}\boldsymbol{\Lambda}^{\delta i} - \varepsilon_{\alpha\beta\rho\tau}W^{\gamma\rho}\boldsymbol{\Lambda}^{\tau i} \,, \tag{2.30b}$$

$$\boldsymbol{\nabla}_\alpha^i \boldsymbol{X}^{jk} = 2\varepsilon^{i(j}\boldsymbol{\nabla}_{\alpha\beta}\boldsymbol{\Lambda}^{\beta k)} \,. \tag{2.30c}$$

It is also useful to note that the S-supersymmetry generator acts on the descendants of the superfield $\boldsymbol{\Lambda}^{\alpha i}$ as follows:

$$S_k^\gamma \boldsymbol{\mathcal{F}}_\alpha{}^\beta = -4\mathrm{i}\delta_\alpha^\gamma \boldsymbol{\Lambda}_k^\beta + \mathrm{i}\delta_\alpha^\beta \boldsymbol{\Lambda}_k^\gamma \,, \qquad S_k^\gamma \boldsymbol{X}^{ij} = -4\mathrm{i}\delta_k^{(i}\boldsymbol{\Lambda}^{\gamma j)} \,. \tag{2.31}$$

Note that the two-form field strength is invariant under special superconformal transformations and is hence a primary superform, $K^C \boldsymbol{\mathcal{F}}_2 = 0$.

2.2.2 The non-abelian vector multiplet in components

The component structure of the vector multiplet can be readily extracted from the superfield description above, see [70] for more details. The gaugino of the vector multiplet is given by the projection $\boldsymbol{\Lambda}^{\alpha i}|$. The component one-form $\boldsymbol{v}_m$ and its field strength $\boldsymbol{f}_{mn}$ are given by $\boldsymbol{\mathcal{V}}_m|$ and $\boldsymbol{\mathcal{F}}_{mn}|$, respectively. The *supercovariant* field strength $\boldsymbol{F}_{ab}$ is simply given by $\boldsymbol{\mathcal{F}}_{ab}|$ and is related to $\boldsymbol{f}_{mn} = 2\partial_{[m}\boldsymbol{v}_{n]}$ as

$$\boldsymbol{F}_{ab} = e_a{}^m e_b{}^n \boldsymbol{f}_{mn} + \psi_{[a k}\gamma_{b]}\boldsymbol{\Lambda}^k \,. \tag{2.32}$$

The last physical field of the vector multiplet is simply the bar-projection of $\boldsymbol{X}^{ij}$.

In what follows, to avoid awkward notation, when the correct interpretation is clear from the context, we will associate the same symbol for the covariant component fields and the associated superfields. The superfields $\boldsymbol{\Lambda}^{\alpha i}$, $\boldsymbol{X}^{ij}$, together with $\boldsymbol{\mathcal{F}}_\alpha{}^\beta$, are all annihilated by K^a hence all their bar-projections are K-primary fields. Using this fact together with (2.30) and (2.31) the $\delta = \delta_Q + \delta_S + \delta_K$ transformations of the component fields follow:

$$\delta\boldsymbol{\Lambda}^j = -\mathrm{i}\,\xi_i \boldsymbol{X}^{ij} + \frac{\mathrm{i}}{2}\tilde{\gamma}^{ab}\xi^j \boldsymbol{\mathcal{F}}_{ab} \,, \tag{2.33a}$$

$$\delta\boldsymbol{X}^{ij} = -2\,\xi^{(i}\boldsymbol{\nabla\!\!\!/}\,\boldsymbol{\Lambda}^{j)} - 4\mathrm{i}\,\eta^{(i}\boldsymbol{\Lambda}^{j)} \,, \tag{2.33b}$$

$$\delta\boldsymbol{\mathcal{F}}_{ab} = 2\,\xi_i\gamma_{[a}\boldsymbol{\nabla}_{b]}\boldsymbol{\Lambda}^i + T_{abc}^-\xi_i\gamma^c\boldsymbol{\Lambda}^i + 2\mathrm{i}\,\eta^i\tilde{\gamma}_{ab}\boldsymbol{\Lambda}_i \,, \tag{2.33c}$$

where

$$\boldsymbol{\nabla}_a\boldsymbol{\Lambda}^i = \boldsymbol{\mathcal{D}}_a\boldsymbol{\Lambda}^i + \frac{\mathrm{i}}{2}\boldsymbol{X}^{ij}\psi_{aj} - \frac{\mathrm{i}}{4}\tilde{\gamma}^{bc}\psi_a{}^i \boldsymbol{\mathcal{F}}_{bc} \,, \tag{2.34a}$$

$$\boldsymbol{\mathcal{D}}_a := e_a{}^m \left(\partial_m - \frac{1}{2}\omega_m{}^{cd}M_{cd} - b_m\mathbb{D} - \boldsymbol{\mathcal{V}}_m{}^{kl}J_{kl} - \mathrm{i}\boldsymbol{v}_m \right) \,. \tag{2.34b}$$

The transformation rule of the component connection $\boldsymbol{v}_m$ can be computed by first noticing that $K^A \boldsymbol{\mathcal{F}}_2 = 0$ and then by taking the double bar projection of the supergravity gauge transformation of $\boldsymbol{\mathcal{V}}_1$, $\delta_{\mathcal{G}}\boldsymbol{\mathcal{V}}_1 = E^A\xi^B \boldsymbol{\mathcal{F}}_{BA}$, which leads to

$$\delta\boldsymbol{v}_m = e_m{}^a\xi_k^\gamma \boldsymbol{\mathcal{F}}_{\gamma a}^k| = -\xi_k\gamma_m\boldsymbol{\Lambda}^k \,. \tag{2.35}$$

2.3 The linear multiplet

Let us turn to describing the 6D off-shell linear multiplet coupled to conformal supergravity.

2.3.1 The linear multiplet in superspace

The linear multiplet, or $\mathcal{O}(2)$ multiplet, can be described using a 4-form gauge potential $B_4 = \frac{1}{4!}E^D \wedge E^C \wedge E^B \wedge E^A B_{ABCD}$ possessing the gauge transformation

$$\delta_{\rho_3} B_4 = \mathrm{d}\rho_3 \,, \qquad (2.36)$$

where the gauge parameter ρ_3 is an arbitrary super 3-form. The corresponding 5-form field strength is

$$E_5 = \mathrm{d}B_4 = \frac{1}{5!}E^E \wedge E^D \wedge E^C \wedge E^B \wedge E^A E_{ABCDE} \,, \qquad (2.37)$$

where

$$E_{ABCDE} = 5\nabla_{[A}B_{BCDE)} + 10T_{[AB}{}^F B_{|F|CDE)} \,. \qquad (2.38)$$

The field strength satisfies the Bianchi identity

$$\mathrm{d}E_5 = 0 \quad \Longleftrightarrow \quad \nabla_{[A}E_{BCDEF)} + \frac{5}{2}T_{[AB}{}^G E_{|G|CDEF)} = 0 \,. \qquad (2.39)$$

In order to describe the linear multiplet we need to impose covariant constraints on the field strength E_5

$$E_{abc}{}^{i\ j}_{\alpha\ \beta} = 4\mathrm{i}(\gamma_{abc})_{\alpha\beta}L^{ij} \,, \quad L^{ij} = L^{ji} \,, \quad S^\gamma_k L^{ij} = 0 \,, \quad \mathbb{D}L^{ij} = 4L^{ij} \,, \qquad (2.40\mathrm{a})$$

and require all lower mass-dimension components of E_{ABCDE} to vanish. The remaining components of E_5 are constrained by the Bianchi identities (2.37) to be defined in terms of the superfield L^{ij}, and its descendants as

$$E_{abcd}{}^i_\alpha = \frac{1}{3}\varepsilon_{abcdef}(\gamma^{ef})_\alpha{}^\beta \nabla_{\beta j}L^{ij} \,, \qquad (2.40\mathrm{b})$$

$$E_{abcde} = \frac{\mathrm{i}}{12}\varepsilon_{abcdef}(\tilde{\gamma}^f)^{\alpha\beta}\nabla^k_\alpha \nabla^l_\beta L_{kl} \equiv \varepsilon_{abcdef}\mathrm{E}^f \,, \qquad (2.40\mathrm{c})$$

where L^{ij} is constrained to satisfy the defining constraint for the linear multiplet[6]

$$\nabla^{(i}_\alpha L^{jk)} = 0 \,. \qquad (2.41)$$

The highest dimension component of the superform satisfies the Bianchi identity $\nabla_a \mathrm{E}^a = 0$ with $\mathrm{E}^a = \frac{1}{5!}\varepsilon^{abcdef}E_{bcdef}$.[7]

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[6]The linear multiplet in six dimensions was introduced in [105, 106] for the Minkowski case extending the 4D results of [104, 107–111]. See [48, 83, 112, 113] for other references about the coupling of the linear multiplet to 6D supergravity.

[7]The reader should not confuse E^a with the vector supervielbein.

In order to elaborate on the component structure of the superfield L^{ij}, the following identities prove useful:

$$\nabla^i_\alpha L^{jk} = -2\varepsilon^{i(j}\varphi^{k)}_\alpha\,, \tag{2.42a}$$

$$\nabla^i_\alpha \varphi^j_\beta = -\frac{\mathrm{i}}{2}\varepsilon^{ij}\mathrm{E}_{\alpha\beta} - \mathrm{i}\nabla_{\alpha\beta}L^{ij}\,, \tag{2.42b}$$

$$\nabla^k_\gamma \mathrm{E}_{\alpha\beta} = -8\nabla_{\gamma[\alpha}\varphi^k_{\beta]} - 2\nabla_{\alpha\beta}\varphi^k_\gamma + 2\varepsilon_{\alpha\beta\gamma\delta}W^{\delta\rho}\varphi^k_\rho\,, \tag{2.42c}$$

where we have defined the descendant superfields

$$\varphi^i_\alpha := -\frac{1}{3}\nabla_{\alpha j}L^{ij}\,, \qquad \mathrm{E}_a = -\frac{\mathrm{i}}{4}(\tilde{\gamma}_a)^{\alpha\beta}\nabla^k_\alpha\varphi_{\beta k}\,, \quad \mathrm{E}_{\alpha\beta} = (\gamma^a)_{\alpha\beta}\mathrm{E}_a\,. \tag{2.43}$$

These prove to be annihilated by K^a and to satisfy

$$S^\beta_j \varphi^i_\alpha = 8\delta^\beta_\alpha L^i{}_j\,, \qquad S^\gamma_k \mathrm{E}_{\alpha\beta} = -40\mathrm{i}\delta^\gamma_{[\alpha}\varphi_{\beta]k}\,. \tag{2.44}$$

These turn out to be equivalent to the condition that E_5 is a primary superform $K^C E_5 = 0$.

2.3.2 The linear multiplet in components

The covariant component fields of the linear multiplet can be identified by the component projections of L^{ij} and φ^i_α. The component gauge 4-form and its 5-form field strength can be identified by the component projections $b_{mnpq} := B_{mnpq}|$ and $h_{mnpqr} = 5\partial_{[m}b_{npqr]}$, respectively. In the paper we will also use the Hodge dual of the 4-form $\tilde{b}^{mn} = \frac{1}{4!}\varepsilon^{mnpqrs}b_{pqrs}$.[8] The supercovariant field strength is just the component projection of E_{abcde}, or equivalently E^a, the top component of the superform E_5. At the component level it holds

$$\mathrm{E}^a = h^a - \psi_{bi}\gamma^{ab}\varphi^i - \frac{\mathrm{i}}{2}\psi_{bi}\gamma^{abc}\psi_{cj}L^{ij}\,, \qquad h^a = \frac{1}{5!}\varepsilon^{amnpqr}h_{mnpqr}\,. \tag{2.45}$$

In the previous equation and in what follows we drop the component projection of the descendant fields when it is clear from context what we mean. The supersymmetry transformations of the covariant fields can be read off from (2.42) and (2.44) which give

$$\delta L^{ij} = 2\xi^{(i}\varphi^{j)}\,, \tag{2.46a}$$

$$\delta\varphi^i = \frac{\mathrm{i}}{2}\xi^i\gamma^a\mathrm{E}_a - \mathrm{i}\xi_j\slashed{\nabla}_a L^{ji} - 8\eta_j L^{ij}\,, \tag{2.46b}$$

$$\delta\mathrm{E}_a = 2\xi_i\gamma_{ab}\nabla^b\varphi^i + \frac{1}{12}\xi^i\gamma_a\tilde{\gamma}^{bcd}T^-_{bcd}\varphi_i - 10\mathrm{i}\eta^i\tilde{\gamma}_a\varphi_i\,, \tag{2.46c}$$

where

$$\nabla_a L^{ij} = \mathcal{D}_a L^{ij} - \psi_a{}^{(i}\varphi^{j)}\,, \tag{2.47a}$$

$$\nabla_a\varphi^i = \mathcal{D}_a\varphi^i - \frac{\mathrm{i}}{4}\psi_a{}^i\gamma_b\mathrm{E}^b - \frac{\mathrm{i}}{2}\psi_{aj}\slashed{\nabla}L^{ij} + 4\phi_{aj}L^{ij}\,. \tag{2.47b}$$

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[8]It is worth reminding that in our notation, see appendix A of [69], the Levi-Civita tensors with curved indices include e or e^{-1} factors since we use the following definitions $\varepsilon^{mnpqrs} := \varepsilon^{abcdef}e_a{}^m e_b{}^n e_c{}^p e_d{}^q e_e{}^r e_f{}^s$, and $\varepsilon_{mnpqrs} := e_m{}^a e_n{}^b e_p{}^c e_q{}^d e_r{}^e e_s{}^f \varepsilon_{abcdef}$, with ε_{abcdef} such that $\varepsilon_{012345} = -\varepsilon^{012345} = 1$.

The transformation of the component gauge field b_{mnpq} can be obtained by projecting to components the supergravity gauge transformation of B_4, $\delta_{\mathcal{G}} B_4 = \frac{1}{4!} E^{A_4} \wedge \cdots \wedge E^{A_1} \xi^B E_{BA_1\cdots A_4}$. This leads to

$$\delta b_{mnpq} = e_{[q}{}^d e_p{}^c e_n{}^b \Big(e_{m]}{}^a \xi_j^\beta E_{\beta abcd}^j | + 2\psi_{m]i}^\alpha \xi_j^\beta E_{\beta\alpha abc}^{j\,i} | \Big)$$
$$= -\varepsilon_{mnpqef}\, \xi_j \gamma^{ef} \varphi^j + 8\mathrm{i}\psi_{[mi}\gamma_{npq]}\xi_j L^{ij}\,. \tag{2.48}$$

The transformations of the components of the linear multiplet reproduce, up to the change of notation described in appendix D, the results of [48].

2.4 The gauge 3-form multiplet

In this section we introduce another off-shell multiplet that will play a central role in the construction of an invariant action principle in our paper. This was described in [100] and also in [69, 70]. It contains a closed 4-form field strength amongst its component fields, which can be solved in terms of a gauge 3-form. We will refer to it as the gauge 3-form multiplet.

2.4.1 The gauge 3-form multiplet in superspace

The gauge 3-form multiplet can naturally be described in superspace using a super 3-form $B_3 = \frac{1}{3!} E^C \wedge E^B \wedge E^A B_{ABC}$ possessing the gauge transformation

$$\delta_{\rho_2} B_3 = \mathrm{d}\rho_2\,, \tag{2.49}$$

where ρ_2 is a two-form gauge parameter. The corresponding field strength is

$$H_4 = \mathrm{d}B_3 = \frac{1}{4!} E^D \wedge E^C \wedge E^B \wedge E^A H_{ABCD}\,, \tag{2.50a}$$

$$H_{ABCD} = 4\nabla_{[A} B_{BCD)} + 6T_{[AB}{}^F B_{|F|C)}\,. \tag{2.50b}$$

The 4-form field strength satisfies the Bianchi identity

$$\mathrm{d}H_4 = 0 \implies \nabla_{[A} H_{BCDE)} + 2T_{[AB}{}^F H_{|F|CDE)} = 0\,. \tag{2.51a}$$

In order to describe an irreducible 3-form multiplet it is necessary to impose the following covariant constraints [100] on the field strength H_4:

$$H_{ab\gamma\delta}^{kl} = \mathrm{i}(\gamma_{abc})_{\gamma\delta} B^{ckl}\,, \quad S_k^\gamma B_a^{ij} = 0\,, \quad \mathbb{D} B_a^{ij} = 3B_a^{ij}\,, \tag{2.51b}$$

with all lower dimension components of the superform vanishing. The Bianchi identity (2.51a) fixes the remaining components as

$$H_{abc\delta}^l = -\frac{1}{12}\varepsilon_{abcdef}(\gamma^{de})_\delta{}^\rho \nabla_{\rho p} B^{flp}\,, \tag{2.51c}$$

$$H_{abcd} = \frac{\mathrm{i}}{48}\varepsilon_{abcdef}(\tilde{\gamma}^e)^{\alpha\beta} \nabla_{\alpha k} \nabla_{\beta l} B^{fkl}\,, \tag{2.51d}$$

and requires $B_a{}^{ij}$ to satisfy the constraints[9]

$$\nabla_\alpha^{(i} B^{\beta\gamma jk)} = -\frac{2}{3}\delta_\alpha^{[\beta}\nabla_\delta^{(i} B^{\gamma]\delta jk)}\,, \tag{2.52a}$$

$$[\nabla_\alpha^{(i}, \nabla_{\beta k}] B^{\alpha\beta j)k} = -8\mathrm{i}\,\nabla_{\alpha\beta} B^{\alpha\beta ij}\,. \tag{2.52b}$$

The superfield $B_a{}^{ij}$ has a large number of descendants, only some of which will appear in the field strength H_4 or in the action principle we will construct based upon it. The relevant ones are

$$\Lambda^{\alpha ijk} := \frac{\mathrm{i}}{3}\nabla_\beta^{(i} B^{\beta\alpha jk)}\,, \tag{2.53a}$$

$$\Lambda_{\alpha a}{}^i := \frac{2\mathrm{i}}{3}\nabla_{\alpha j} B_a{}^{ij}\,, \tag{2.53b}$$

$$C_{ab} := \frac{1}{8}(\tilde{\gamma}_a)^{\alpha\beta}\nabla_{\alpha k}\Lambda_{\beta b}{}^k\,, \tag{2.53c}$$

where the reader is cautioned that C_{ab} is a generic rank-two tensor containing both a symmetric and antisymmetric part. These descendants satisfy the following useful relations, which are consequences of the constraints (2.52)

$$\nabla_\alpha^i B_a{}^{jk} = -\frac{\mathrm{i}}{2}(\gamma_a)_{\alpha\beta}\Lambda^{\beta ijk} - \mathrm{i}\,\varepsilon^{i(j}\Lambda_{\alpha a}{}^{k)}\,, \tag{2.54a}$$

$$\begin{aligned}\nabla_\alpha^i \Lambda_{\beta b}{}^j = &-\frac{1}{2}(\gamma_b)_{\alpha\gamma}C_\beta{}^{\gamma ij} + \frac{1}{6}(\gamma_b)_{\beta\gamma}C_\alpha{}^{\gamma ij} + \varepsilon^{ij}\,(\gamma^a)_{\alpha\beta}C_{ab}\\ &- 2(\gamma^a)_{\alpha\beta}\nabla_a B_b{}^{ij} + 4\,(\gamma^c)_{\alpha\beta}W_{cba}B^{aij}\,,\end{aligned} \tag{2.54b}$$

$$\begin{aligned}\nabla_\alpha^i C_{ab} = &-\frac{\mathrm{i}}{8}\eta_{ab}\rho_{\alpha\beta}{}^{\beta i} - \frac{\mathrm{i}}{8}(\gamma_{ab})_\beta{}^\gamma\rho_{\alpha\gamma}{}^{\beta i} + \frac{\mathrm{i}}{8}(\gamma_b)_{\alpha\beta}(\tilde{\gamma}_a)^{\gamma\delta}\rho_{\gamma\delta}{}^{\beta i} + \mathrm{i}(\gamma_{ac})_\alpha{}^\beta\nabla^c\Lambda_{\beta b}{}^i\\ &+ \left(2\mathrm{i}\,\eta_{ab}(\gamma_c)_{\alpha\beta} - 2\mathrm{i}\,\eta_{ac}(\gamma_b)_{\alpha\beta} - \mathrm{i}(\gamma_{abc})_{\alpha\beta}\right)X_j^\beta B^{cij} - 4\mathrm{i}\,(\gamma_a)_{\alpha\delta}B^{cij}(\gamma_{bc})_\gamma{}^\beta X_{\beta j}{}^{\gamma\delta}\\ &-\frac{\mathrm{i}}{2}W_{acd}(\gamma^{cd})_\alpha{}^\beta\,\Lambda_{\beta b}{}^i + 2\mathrm{i}\,W_b{}^{cd}\,(\gamma_{ad})_\alpha{}^\beta\,\Lambda_{\beta c}{}^i\,,\end{aligned} \tag{2.54c}$$

and

$$S_k^\gamma \Lambda_{\alpha a}{}^i = -\frac{44\mathrm{i}}{3}\delta_\alpha^\gamma B_{ak}{}^i - \frac{4\mathrm{i}}{3}(\gamma_{ab})_\alpha{}^\gamma B^b{}_k{}^i\,, \tag{2.55a}$$

$$S_k^\gamma C_{ab} = -\frac{9}{2}(\tilde{\gamma}_a)^{\gamma\delta}\Lambda_{\delta bk} - \frac{1}{2}(\tilde{\gamma}_b)^{\gamma\delta}\Lambda_{\delta ak} + \frac{1}{2}\eta_{ab}(\tilde{\gamma}_c)^{\gamma\delta}\Lambda_\delta{}^c{}_k + \frac{1}{2}(\tilde{\gamma}_{abc})^{\gamma\delta}\Lambda_\delta{}^c{}_k\,, \tag{2.55b}$$

while each is annihilated by K^a. These conditions are equivalent to the condition that H_4 is primary, $K^C H_4 = 0$. Note in the supersymmetry transformations (2.54), additional component fields appear. These are defined by the descendants

$$C_\alpha{}^{\beta ij} := \frac{3}{4}\nabla_{\alpha k}\Lambda^{\beta ijk}\,, \qquad \rho_{\alpha\beta}{}^{\gamma i} := -\frac{2\mathrm{i}}{3}\nabla_{[\alpha j}C_{\beta]}{}^{\gamma ij}\,, \tag{2.56}$$

but we do not give their transformations here as they are unnecessary for what follows. A more complete discussion of this multiplet can be found in [69, 70].

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[9]It was shown in [69, 70] that a reducible multiplet described by a primary superfield $B_a{}^{ij}$ satisfying only (2.52a) but not (2.52b) can be used to construct a non-primary super 6-form. This plays an important role in, e.g., providing a density formula for the full superspace integral and to describe one of the two $\mathcal{N} = (1,0)$ conformal supergravity actions.

2.4.2 The gauge 3-form multiplet in components

The component structure of the 3-form multiplet follows from the superspace constraints (2.52). Besides the lowest components of the descendant superfields $\Lambda_{\alpha a}{}^i$ and C_{ab}, there are other component fields of $B_a{}^{ij}$ that we do not summarise here, see [70] for more details. In later sections, we will only make use of the component projections of $B_a{}^{ij}$, $\Lambda_{\alpha a}{}^i$, and C_{ab} whose $\delta = \delta_Q + \delta_S + \delta_K$ transformation, that follow from (2.54) and (2.55), prove to be

$$\delta B_a{}^{jk} = -\frac{\mathrm{i}}{2}\xi_i\gamma_a\Lambda^{\beta ijk} + \mathrm{i}\,\xi^{(j}\Lambda_a{}^{k)}\,, \tag{2.57a}$$

$$\delta\Lambda_{\beta b}{}^j = \frac{1}{2}C_\beta{}^{\gamma ij}(\gamma_b\xi_i)_\gamma + \frac{1}{6}\xi_i^\alpha C_\alpha{}^{\gamma ij}(\gamma_b)_{\beta\gamma} + (\gamma^a\xi^j)_\beta C_{ab} + 2(\gamma^a\xi_i)_\beta\nabla_a B_b{}^{ij}$$
$$+ 2\,(\gamma^c\xi_i)_\beta T^-_{cba}B^{aij} + \frac{44\mathrm{i}}{3}\eta_{\alpha j}B_a{}^{ij} + \frac{4\mathrm{i}}{3}(\gamma_{ab}\eta_j)_\alpha B^{bij}\,, \tag{2.57b}$$

$$\delta C_{ab} = -\frac{\mathrm{i}}{8}\eta_{ab}\,\xi_i\gamma^c\rho_c{}^i + \frac{\mathrm{i}}{8}\xi_i\gamma^c\tilde{\gamma}_{ab}\rho_c{}^i + \frac{\mathrm{i}}{2}\xi_i\gamma_b\rho_a{}^i + \mathrm{i}\,\xi_i\gamma_{ac}\nabla_c\Lambda_b{}^i$$
$$+ \left(\frac{4}{15}\mathrm{i}\,\eta_{ab}\,\xi_i\gamma_c\chi_j - \frac{4\mathrm{i}}{15}\,\eta_{ac}\,\xi_i\gamma_b\chi_j - \frac{2\mathrm{i}}{15}\xi_i\gamma_{abc}\chi_j - 4\mathrm{i}\,\xi_i\gamma_a R(Q)_{bcj}\right)B^{cij}$$
$$+ \frac{\mathrm{i}}{4}T^-_{acd}\,\xi_i\gamma^{cd}\Lambda_b{}^i - \mathrm{i}\,T_b^{-cd}\,\xi_i\gamma_{ad}\Lambda_c{}^i - \frac{9}{2}\eta^k\tilde{\gamma}_a\Lambda_{bk} - \frac{1}{2}\eta^k\tilde{\gamma}_b\Lambda_{ak}$$
$$+ \frac{1}{2}\eta_{ab}\,\eta^k\tilde{\gamma}_c\Lambda^c{}_k + \frac{1}{2}\,\eta^k\tilde{\gamma}_{abc}\Lambda^c{}_k\,. \tag{2.57c}$$

2.5 The tensor multiplet and the dilaton-Weyl multiplet

So far we have described various off-shell multiplets coupled to the standard-Weyl (or type I) multiplet of $\mathcal{N} = (1,0)$ conformal supergravity [48]. In this subsection we introduce a variant off-shell formulation in which the matter fields (T^-_{abc}, χ^i, D) of the standard-Weyl multiplet are replaced by a scalar σ, a gauge two-form tensor b_{mn}, and a chiral ψ_i matter fields. These fields belong to a 6D $(1,0)$ tensor multiplet [105, 114] which once coupled to the standard-Weyl multiplet, can be used to define a new multiplet of conformal supergravity: the dilaton-Weyl (or type II) multiplet [48]. This plays an important role in six-dimensional supergravity since, to date, it has proven to be the simplest formulation that can be consistently used to construct actions for general off-shell supergravity-matter systems. In flat superspace, the tensor multiplet has been constructed as a super gauge two-form [115]. Extending the curved superspace analysis of [83], we will first introduce the dilaton-Weyl multiplet by consistently describing the tensor multiplet gauge two-form in conformal superspace, see also [113]. Successively, we will reproduce the description in components of [48].

2.5.1 The tensor multiplet and the dilaton-Weyl multiplet in superspace

Consider a super two-form gauge potential $B_2 = \frac{1}{2}E^B \wedge E^A B_{AB}$ possessing the gauge transformation

$$\delta_{\rho_1}B_2 = \mathrm{d}\rho_1\,, \tag{2.58}$$

where ρ_1 is a one-form gauge parameter. The corresponding field strength is

$$H_3 = \mathrm{d}B_2 = \frac{1}{3!}E^C \wedge E^B \wedge E^A H_{ABC}\,, \tag{2.59}$$

where

$$H_{ABC} = 3\nabla_{[A}B_{BC)} + 3T_{[AB}{}^F B_{|F|C)} \,. \tag{2.60}$$

The field strength must satisfy the Bianchi identity

$$\mathrm{d}H_3 = 0 \implies \nabla_{[A}H_{BCD)} + \frac{3}{2}T_{[AB}{}^E H_{|E|CD)} = 0 \,. \tag{2.61}$$

In order to describe the tensor multiplet we need to impose some covariant constraints on the field strength H_3 [83, 115]

$$H^{i\,j\,k}_{\alpha\beta\gamma} = 0 \,, \quad H_a{}^{i\,j}_{\alpha\beta} = 2\mathrm{i}\varepsilon^{ij}(\gamma_a)_{\alpha\beta}\Phi \,, \quad S^\alpha_i \Phi = 0 \,, \quad \mathbb{D}\Phi = 2\Phi \,. \tag{2.62}$$

By using the Bianchi identities (2.61), the remaining components of the superform H_3 are uniquely fixed in terms of Φ and its descendants. The solution implies that Φ satisfies the differential constraint corresponding to the tensor multiplet

$$\nabla^{(i}_\alpha \nabla^{j)}_\beta \Phi = 0 \,, \tag{2.63}$$

and also determines the higher dimension super-form components to be

$$H_{ab}{}^i_\alpha = (\gamma_{ab})_\alpha{}^\beta \nabla^i_\beta \Phi \,, \tag{2.64a}$$

$$H_{abc} = -\frac{\mathrm{i}}{8}(\tilde{\gamma}_{abc})^{\gamma\delta}\nabla^k_\gamma \nabla_{\delta k}\Phi - 4W_{abc}\Phi \,. \tag{2.64b}$$

It is worth noting that the constraint (2.63) can be solved in terms of a constrained pre-potential $V^{\alpha i}$ as follows [83, 115, 116]

$$\Phi = \nabla^k_\gamma V^\gamma_k \,, \quad \nabla^{(i}_\alpha V^{\beta j)} - \frac{1}{4}\delta^\beta_\alpha \nabla^{(i}_\gamma V^{\gamma j)} = 0 \,. \tag{2.65}$$

It is in fact simple to prove that the following two-form

$$\mathcal{B}_2 = -8\mathrm{i}E^\beta_j \wedge E^a(\gamma_a)_{\beta\gamma}V^{\gamma j} - E^b \wedge E^a(\gamma_{ab})_\beta{}^\alpha \nabla^i_\alpha V^\beta_i \tag{2.66}$$

obeys $\mathrm{d}\mathcal{B}_2 = H_3$. The prepotential $V^{\alpha i}$ is defined up to a shift by a superfield describing an abelian vector multiplet, $V^{\alpha i} \to V^{\alpha i} + \Lambda^{\alpha i}$ which leaves H_3 of (2.62) and (2.64) invariant.

We introduce the following descendants of Φ

$$\psi^i_\alpha := \nabla^i_\alpha \Phi \,, \qquad H_{\alpha\beta} = \frac{1}{6}(\gamma^{abc})_{\alpha\beta}H^+_{abc} = -\mathrm{i}\nabla^k_{(\alpha}\psi_{\beta)k} \,, \tag{2.67}$$

where we used the decomposition of H_{abc} into self-dual and anti-self-dual parts $H_{abc} = H^+_{abc} + H^-_{abc}$. The superfields ψ^i_α and $H_{\alpha\beta}$ satisfy

$$\nabla^i_\alpha \psi^j_\beta = -\frac{\mathrm{i}}{2}\varepsilon^{ij}H_{\alpha\beta} - \mathrm{i}\varepsilon^{ij}\nabla_{\alpha\beta}\Phi \,, \tag{2.68a}$$

$$\nabla^k_\gamma H_{\alpha\beta} = -4\nabla_{\gamma(\alpha}\psi^k_{\beta)} \,, \tag{2.68b}$$

and

$$S^\beta_j \psi^i_\alpha = 4\delta^\beta_\alpha \delta^i_j \Phi \,, \qquad S^\gamma_k H_{\alpha\beta} = -24\mathrm{i}\delta^\gamma_{(\alpha}\psi_{\beta)k} \,. \tag{2.69}$$

Both ψ_α^i and $H_{\alpha\beta}$ are annihilated by K^a. Note also that (2.69) turn out to be equivalent to H_3 being a primary superform $K^C H_3 = 0$.

Assuming the tensor multiplet superfield Φ is everywhere nonvanishing, $\Phi \neq 0$, which is a standard requirement for a conformal compensator, it is straightforward to check that the constraint (2.63) implies the following relations

$$W_{abc} = -\frac{1}{4\Phi} H_{abc}^-, \tag{2.70a}$$

$$X^{\alpha i} = -\frac{\mathrm{i}}{4\Phi}\left(\nabla^{\alpha\beta}\psi_\beta^i - W^{\alpha\beta}\psi_\beta^i\right), \tag{2.70b}$$

$$Y = \frac{1}{2\Phi}\left(\nabla^a\nabla_a\Phi - 4X^{\alpha i}\psi_{\alpha i} - \frac{2}{3}W^{abc}H_{abc}\right). \tag{2.70c}$$

This shows that the covariant superfield of the standard-Weyl multiplet, W_{abc} and its descendants, are now composed of the tensor multiplet covariant superfields Φ, H_{abc} and their descendants. The result is a superspace description of the dilaton-Weyl (or type II) multiplet of conformal supergravity [48, 83].

2.5.2 The tensor multiplet and the dilaton-Weyl multiplet in components

The covariant component fields of the tensor multiplet can be identified by the component projections $\sigma := \Phi|$ and $\psi_\alpha^i|$. The supercovariant 3-form field strength is just the component projection of the superfield H_{abc}. By taking the double bar projection of (2.59) one can derive

$$H_{abc} = h_{abc} + \frac{3}{2}\psi_{[a}{}^i\gamma_{bc]}\psi_i + \frac{3\mathrm{i}}{2}\psi_{[a}{}^i\gamma_b\psi_{c]i}\sigma. \tag{2.71}$$

Using (2.68) and (2.69), and suppressing the component projection on the superfields ψ_α^i and H_{abc}^+, one can obtain the $\delta = \delta_Q + \delta_S + \delta_K$ transformations of the components:

$$\delta\sigma = \xi_i\psi^i, \tag{2.72a}$$

$$\delta\psi^i = \frac{\mathrm{i}}{12}\xi^i\gamma_{abc}H^{abc} + \mathrm{i}\xi^i\nabla\!\!\!\!/\,\sigma + 4\eta^i\sigma, \tag{2.72b}$$

$$\delta H_{abc}^+ = -\frac{1}{2}\xi_k\gamma^d\tilde{\gamma}_{abc}\nabla_d\psi^k - 3\mathrm{i}\eta^k\tilde{\gamma}_{abc}\psi_k, \tag{2.72c}$$

where

$$\nabla_a\sigma = \mathcal{D}_a\sigma - \frac{1}{2}\psi_{ai}\psi^i, \tag{2.73a}$$

$$\nabla_a\psi^i = \mathcal{D}_a\psi^i - \frac{\mathrm{i}}{24}\gamma^{bcd}\psi_a{}^i H_{bcd} - \frac{\mathrm{i}}{2}\psi_a{}^i\nabla\!\!\!\!/\,\sigma - 2\phi_a{}^i\sigma. \tag{2.73b}$$

The transformation of the component gauge two-form $b_{mn} = B_{mn}|$ can be obtained by using the fact that B_2 is a primary superform and by projecting to components the supergravity gauge transformation of B_2, $\delta_{\mathcal{G}}B_2 = \frac{1}{2}E^B \wedge E^A \xi^C H_{CAB}$, leading to

$$\delta b_{mn} = e_{[n}{}^b\left(e_{m]}{}^a\xi_k^\gamma H_{\gamma ab}^k| + \psi_{m]i}^\alpha\xi_k^\gamma H_{\gamma\alpha b}^{ki}|\right) = \xi_i\gamma_{mn}\psi^i + 2\mathrm{i}\xi_i\gamma_{[m}\psi_{n]}{}^i\sigma. \tag{2.74}$$

The relations (2.70) that define the dilaton-Weyl multiplet in superspace translate to the following at the component level

$$T_{abc}^- = \frac{1}{2\sigma} H_{abc}^- \,, \tag{2.75a}$$

$$\chi^i = -\frac{15\mathrm{i}}{8\sigma} \slashed{\nabla} \psi^i - \frac{5\mathrm{i}}{32\sigma} T_{abc}^- \tilde{\gamma}^{abc} \psi^i \,, \tag{2.75b}$$

$$D = \frac{15}{4\sigma} \nabla^a \nabla_a \sigma - 2\chi^i \psi_i + \frac{15}{12\sigma} T_{abc}^- H^{abc} \,, \tag{2.75c}$$

with

$$\nabla^a \nabla_a \sigma = \left(\mathcal{D}_a \nabla^a - 4\mathfrak{f}_a{}^a + \frac{\mathrm{i}}{2} \psi_{ai} \gamma^a \chi^i \right) \sigma - \frac{1}{2} \psi_{ai} \left(\nabla^a \psi^i + \frac{1}{4} T_{bcd}^- \gamma^a \tilde{\gamma}^{bcd} \psi^i \right)$$
$$+ \frac{\mathrm{i}}{2} \phi_a{}^i \tilde{\gamma}^a \psi_i \,, \tag{2.76}$$

where $\nabla_a \sigma$ and $\nabla_a \psi^i$ are respectively given in (2.73a) and (2.73b), while

$$\mathfrak{f}_a{}^a = -\frac{1}{20} \mathcal{R} - \frac{\mathrm{i}}{8} \psi^a{}_k \gamma^b R(Q)_{ab}{}^k + \frac{\mathrm{i}}{60} \psi_{ak} \gamma^a \chi^k + \frac{1}{20} \psi_{ak} \gamma^{ab} \phi_b{}^k + \frac{\mathrm{i}}{40} \psi_a{}^k \gamma_b \psi_{ck} T^{-abc} \,. \tag{2.77}$$

In all the previous equations the reader should keep in mind that on the right hand side T_{abc}^-, χ^i and D are built from the tensor multiplet covariant fields σ, ψ^i and H_{abc}. Equations (2.75) in fact show how in the dilaton-Weyl multiplet the covariant fields of the standard-Weyl multiplet are replaced with the fields of the tensor multiplet. The dilaton-Weyl multiplet will play a central role in the construction of curvature squared terms in later sections.

The supersymmetry transformations of the independent component connections of the dilaton-Weyl multiplet $(e_m{}^a, \psi_{mi}, \mathcal{V}_m{}^{kl}, b_m)$ can be read straightforwardly from the transformation rules of the same fields in the standard-Weyl multiplet (2.20a)–(2.20d) where now (2.75) should be used everywhere. The transformation rules of the independent matter fields (σ, ψ^i, b_{mn}) of the dilaton-Weyl multiplet are respectively given by (2.72a), (2.72b) and (2.74).

In this section we have described in detail various off-shell matter multiplets coupled to the standard-Weyl multiplet. For each of these multiplets, it is clear that the same analysis holds straightforwardly in the case of the coupling to the dilaton-Weyl multiplet. In fact, to switch from one off-shell conformal supergravity multiplet to the other, it is only necessary to use equations (2.75) everywhere or their superfield equivalents.

3 Locally superconformal action principles

In this paper we will make use of three action principles to construct various locally superconformal invariants. One of these involves the product of a linear multiplet with an Abelian vector multiplet and describes the supersymmetric extension of a $B_4 \wedge \mathcal{F}_2$ term. This action formula was the main building block for the supergravity invariants in [48]. In superspace it may be described by a full superspace integral

$$\int \mathrm{d}^6 x \, \mathrm{d}^8 \theta \, E \, U_{ij} \, L^{ij} \,, \tag{3.1}$$

where U_{ij} is the Mezincescu prepotential for the vector multiplet [117, 118], L^{ij} is a linear multiplet and $E = \mathrm{Ber}(E_M{}^A)$ is the Berezinian (or superdeterminant) of the supervielbein.

A second locally superconformal invariant is the so-called A action principle of [69, 70]. This is based on a primary dimension-9/2 superfield $A_\alpha{}^{ijk}$ whose differential constraint will be reviewed later in this section. This describes the bottom component of a covariant closed super 6-form that was originally constructed in [100].

Another action formula involves the product of a tensor multiplet with a gauge 3-form multiplet and it describes the supersymmetric extension of a $B_2 \wedge H_4$ term. Its existence was noted in [69] where it was described in terms of the A action principle with a superfield Lagrangian $A_\alpha{}^{ijk}$ chosen as

$$A_\alpha{}^{ijk} = \varepsilon_{\alpha\beta\gamma\delta} V^{\beta(i} B^{\gamma\delta jk)} , \tag{3.2}$$

where $V^{\alpha i}$ is a prepotential for the tensor multiplet (2.65) while $B^{\alpha\beta ij} = (\tilde{\gamma}^a)^{\alpha\beta} B_a{}^{ij}$ is the primary superfield describing a gauge 3-form supermultiplet as in section 2.4. The bosonic component structure of this new locally superconformal invariant was given in [40] while its complete structure is given for the first time here in our paper.

The A action principle was already studied in detail in [69, 70]. In this section we are going to present a construction of both the $B_4 \wedge \mathcal{F}_2$ and $B_2 \wedge H_4$ action principle by using the superform approach to construct supersymmetric invariants [95–99]. The advantage compared to using (3.1) and (3.2) is two-fold: firstly, we will see that no prepotential superfields, either for the vector or tensor multiplets, appear explicitly; secondly, it is straightforward to reduce the results to component fields making direct contact with the superconformal tensor calculus. Our approach follows the one of, e.g., [93, 119–121].

3.1 Superform construction of locally superconformal invariants

For a 6D $\mathcal{N} = (1,0)$ superspace, we introduce a closed 6-form J

$$J = \frac{1}{6!} \mathrm{d}z^{M_6} \wedge \cdots \wedge \mathrm{d}z^{M_1} J_{M_1\cdots M_6}(z) , \qquad \mathrm{d}J = 0 . \tag{3.3}$$

Such a closed superform leads to the supersymmetric action principle [95–99]

$$S = \int_{\mathcal{M}^6} i^* J = \int \mathrm{d}^6 x \, e \,{}^* J|_{\theta=0} , \qquad e := \det e_m{}^a , \tag{3.4a}$$

$${}^* J := \frac{1}{6!} \varepsilon^{m_1 m_2 m_3 m_4 m_5 m_6} J_{m_1 m_2 m_3 m_4 m_5 m_6} , \tag{3.4b}$$

where $i : \mathcal{M}^6 \to \mathcal{M}^{6|8}$ is the inclusion map and i^* is its pullback which effectively acts as the double bar projection, $\theta_i^\mu = \mathrm{d}\theta_i^\mu = 0$. Due to the transformation rule of a closed 6-form

$$\delta_\xi J = \mathcal{L}_\xi J \equiv i_\xi \mathrm{d}J + \mathrm{d}i_\xi J = \mathrm{d}i_\xi J , \tag{3.5}$$

up to boundary terms that we are going to neglect, the closure of J guarantees that the action is invariant under general coordinate transformations of superspace generated by a vector field $\xi = \xi^A E_A = \xi^M \partial_M$.

The action principle (3.4) is manifestly invariant under superdiffeomorphisms. In addition, the action must be invariant under all other gauge transformations. Among other

possible gauge transformations of matter multiplets in a specific model, for conformal supergravity we need to include the other superconformal transformations describing the structure group of conformal superspace, which form the subgroup $\mathcal{H}$. To ensure the invariance of (3.4) then J is demanded to transform.

A special case is when the closed 6-form is itself invariant, $\delta_{\mathcal{H}} J = 0$. This implies that if one instead decomposes J in the tangent frame,

$$J = \frac{1}{6!} E^{A_6} \wedge \cdots \wedge E^{A_1} J_{A_1 \cdots A_6} \,, \tag{3.6}$$

the components $J_{A_1 \cdots A_6}$ transform covariantly and obey the covariant constraints

$$\nabla_{[A_1} J_{A_2 \cdots A_7)} + 3 T_{[A_1 A_2}{}^B J_{|B|A_3 \cdots A_7)} = 0 \,. \tag{3.7}$$

In particular, their S and K transformations are given by

$$S_j^\beta J_{a_1 \cdots a_n \alpha_1}{}^{i_1} \cdots{}^{i_{6-n}}_{\alpha_{6-n}} = -\mathrm{i}\, n (\tilde{\gamma}_{[a_1})^{\beta\gamma} J_{\gamma j a_2 \cdots a_n]\alpha_1}{}^{i_1} \cdots{}^{i_{6-n}}_{\alpha_{6-n}} \,, \qquad K^b J_{A_1 \cdots A_6} = 0 \,. \tag{3.8}$$

These are equivalent to demanding J to be a primary six-form $K^A J = 0$.

Once constructed an appropriate invariant closed super 6-form J, it is straightforward to describe a gauge invariant action principle in components. In fact, by expressing the action (3.4) in terms of tangent frame indices one obtains

$$
\begin{aligned}
S &= \frac{1}{6!} \int \mathrm{d}^6 x \, e \, \varepsilon^{m_1 \cdots m_6} E_{m_6}{}^{A_6} \cdots E_{m_1}{}^{A_1} J_{A_1 \cdots A_6}|_{\theta=0} \,, \\
&= \frac{1}{6!} \int \mathrm{d}^6 x \, e \, \varepsilon^{a_1 \cdots a_6} \Bigg[J_{a_1 \cdots a_6} - 3 \psi_{a_1 i}{}^\alpha J_{a_2 \cdots a_6 \alpha}{}^i - \frac{15}{4} \psi_{a_1 i_2}{}^{\alpha_2} \psi_{a_2 i_1}{}^{\alpha_1} J_{a_3 \cdots a_6 \alpha_1 \alpha_2}{}^{i_1 i_2} \\
&\quad + \frac{5}{2} \psi_{a_1 i_3}{}^{\alpha_3} \psi_{a_2 i_2}{}^{\alpha_2} \psi_{a_3 i_1}{}^{\alpha_1} J_{a_4 a_5 a_6 \alpha_1 \alpha_2 \alpha_3}{}^{i_1 i_2 i_3} + \frac{15}{16} \psi_{a_1 i_1}{}^{\alpha_1} \cdots \psi_{a_4 i_4}{}^{\alpha_4} J_{a_5 a_6 \alpha_1 \cdots \alpha_4}{}^{i_1 \cdots i_4} \\
&\quad - \frac{3}{16} \psi_{a_1 i_1}{}^{\alpha_1} \cdots \psi_{a_5 i_5}{}^{\alpha_5} J_{a_6 \alpha_1 \cdots \alpha_5}{}^{i_1 \cdots i_5} - \frac{1}{64} \psi_{a_1 i_1}{}^{\alpha_1} \cdots \psi_{a_6 i_6}{}^{\alpha_6} J_{\alpha_1 \cdots \alpha_6}^{i_1 \cdots i_6} \Bigg]|_{\theta=0} \,, \tag{3.9}
\end{aligned}
$$

which is the standard expansion of supergravity actions as a supercovariant power series in the gravitini. Let us now turn to the examples relevant for our paper.

3.2 The $B_4 \wedge \mathcal{F}_2$ action principle

Consider the gauge 4-form multiplet associated to a linear multiplet, as in section 2.3, together with an Abelian vector multiplet, see section 2.2. By considering the gauge 4-form B_4 and the vector multiplet two-form field strength $\mathcal{F}_2$ we can construct the primary super 6-form $B_4 \wedge \mathcal{F}_2$. This is manifestly invariant under vector multiplet transformations, that leave invariant $\mathcal{F}_2$, while it transforms as an exact form under the gauge transformation of the 4-form (2.36), $\delta_{\rho_3}(B_4 \wedge \mathcal{F}_2) = \mathrm{d}(\rho_3 \wedge \mathcal{F}_2)$, so that its exterior derivative is gauge invariant

$$\mathrm{d}(B_4 \wedge \mathcal{F}_2) = E_5 \wedge \mathcal{F}_2 \,. \tag{3.10}$$

It turns out that the covariant constraints on $\mathcal{F}_2$ and E_5 mean that $E_5 \wedge \mathcal{F}_2$ is Weil trivial [122]. In other words, it turns out that there is a second super 6-form $\Sigma_{E_5 \wedge \mathcal{F}_2}$ obeying

$\mathrm{d}\Sigma_{E_5\wedge\mathcal{F}_2} = E_5 \wedge \mathcal{F}_2$ such that $\Sigma_{E_5\wedge\mathcal{F}_2}$ is constructed entirely in terms of the covariant superfields in $\mathcal{F}_2$ and E_5. This implies that $\Sigma_{E_5\wedge\mathcal{F}_2}$ is primary and gauge invariant. We call this super 6-form the curvature induced form. This means that by construction the primary super 6-form

$$J_{B_4\wedge\mathcal{F}_2} = B_4 \wedge \mathcal{F}_2 - \Sigma_{E_5\wedge\mathcal{F}_2} \tag{3.11}$$

is closed, $\mathrm{d}J_{B_4\wedge\mathcal{F}_2} = 0$, such that $\delta_{\rho_3}J_{B_4\wedge\mathcal{F}_2} = \mathrm{d}(\rho_3 \wedge \mathcal{F}_2)$ and then, according to the discussion in the previous section, it leads to a locally superconformal and gauge invariant action.

It is not difficult to solve explicitly for $\Sigma_{E_5\wedge\mathcal{F}_2}$. It obeys

$$\mathrm{d}\Sigma_{E_5\wedge\mathcal{F}_2} = E_5 \wedge \mathcal{F}_2 \iff \nabla_{[A_1}\Sigma_{A_2\cdots A_7)} + 3T_{[A_1A_2}{}^B\Sigma_{|B|A_3\cdots A_7)} = 3\mathcal{F}_{[A_1A_2}E_{A_3\cdots A_7)}, \tag{3.12}$$

with $\mathcal{F}_{AB}$ the components of the abelian vector multiplet super two-form $\mathcal{F}_2$, eqs. (2.27), and $E_{A_1\cdots A_5}$ the components of the linear multiplet super 5-form E_5, eq. (2.40). A covariant superform solution $\Sigma_{E_5\wedge\mathcal{F}_2}$ is given by

$$\Sigma_{abcde}{}^i_\alpha = 2\varepsilon_{abcdef}(\gamma^f)_{\alpha\beta}\Lambda^\beta_j L^{ij}, \tag{3.13a}$$

$$\Sigma_{abcdef} = -\varepsilon_{abcdef}\Big(X^{ij}L_{ij} - 2\mathrm{i}\Lambda^{\alpha i}\varphi_{\alpha i}\Big), \tag{3.13b}$$

with all components at lower dimension vanishing.

If we now plug the resulting closed 6-form (3.11) into (3.9) we obtain the component action principle

$$S_{B_4\wedge\mathcal{F}_2} = \int \mathrm{d}^6x \, e \left(-\frac{1}{2}f_{mn}\tilde{b}^{mn} + X^{ij}L_{ij} - 2\mathrm{i}\Lambda^i\varphi_i - \psi_{ai}\gamma^a\Lambda_j L^{ij}\right), \tag{3.14}$$

where we have suppressed spinor indices and we remind that we defined

$$\tilde{b}^{mn} := \frac{1}{4!}\varepsilon^{mnpqrs}b_{pqrs}. \tag{3.15}$$

The action (3.14), which we will refer to as $B_4 \wedge \mathcal{F}_2$ action principle, was obtained for the first time in [48].

3.3 The $B_2 \wedge H_4$ action principle

By starting from the gauge two-form multiplet associated to a tensor multiplet, as in section 2.5, together with a gauge 3-form multiplet of section 2.4, we can construct another gauge invariant action principle using the same logic as in the previous subsection.

Considering the gauge two-form B_2 and the closed 4-form field strength $H_4 = \mathrm{d}B_3$ we construct the primary super 6-form $B_2 \wedge H_4$. This is invariant under the 3-form gauge transformation, $\delta_{\rho_2}B_3 = \mathrm{d}\rho_2$, it transforms as an exact form under the two-form gauge transformation (2.58), $\delta_{\rho_1}(B_2 \wedge H_4) = \mathrm{d}(\rho_1 \wedge H_4)$, so that

$$\mathrm{d}(B_2 \wedge H_4) = H_3 \wedge H_4, \tag{3.16}$$

is gauge invariant. The covariant constraints on H_3, eqs. (2.62) and (2.64), and on H_4, eq. (2.51), are such that the super 7-form $H_3 \wedge H_4$ is again Weil trivial and then it is possible to construct a curvature induced super 6-form $\Sigma_{H_3 \wedge H_4}$, so that

$$J_{B_2 \wedge H_4} = \Sigma_{H_3 \wedge H_4} - B_2 \wedge H_4 \qquad (3.17)$$

is closed, $\mathrm{d}J_{B_2 \wedge H_4} = 0$, and such that $\delta_{\rho_1} J_{B_2 \wedge H_4} = \mathrm{d}(\rho_1 \wedge H_4)$ leading to a locally super-conformal and gauge invariant action.

The superform equation defining $\Sigma_{H_3 \wedge H_4}$ is

$$\mathrm{d}\Sigma_{H_3 \wedge H_4} = H_3 \wedge H_4 \iff \nabla_{[A}\Sigma_{BCDEFG)} + 3T_{[AB}{}^H \Sigma_{|H|CDEFG)} = 5H_{[ABC}H_{DEFG)}\,, \qquad (3.18)$$

with H_3 corresponding to a tensor multiplet, see section 2.5, and H_4 corresponding to the gauge 3-form multiplet, see section 2.3. A covariant solution of equation (3.18) is given by a super 6-form $\Sigma_{H_3 \wedge H_4}$ possessing the following nontrivial components

$$\Sigma_{abcd\rho\tau}{}^{pq} = \frac{\mathrm{i}}{2}\varepsilon_{abcdef}(\gamma^{efg})_{\rho\tau}\Phi B_g{}^{pq}\,, \qquad (3.19\mathrm{a})$$

$$\Sigma_{abcde\delta}{}^{l} = -\frac{1}{4}\varepsilon_{abcdef}\left[2\psi_{\delta k}B^{fkl} + \mathrm{i}\Phi\Lambda_\delta{}^{fl} + (\gamma^{fg})_\delta{}^\rho\left(2\psi_{\rho k}B_g{}^{kl} - \mathrm{i}\Phi\Lambda_{\rho g}{}^l\right)\right]\,, \qquad (3.19\mathrm{b})$$

$$\Sigma_{abcdef} = \frac{1}{4}\varepsilon_{abcdef}(\tilde\gamma^g)^{\alpha\beta}\psi_{\alpha k}\Lambda_{\beta g}{}^k + \frac{1}{4}\varepsilon_{abcdef}\Phi C\,, \qquad (3.19\mathrm{c})$$

with all lower dimension components vanishing and the descendant superfield $\Lambda_{\alpha a}{}^i$ defined in (2.53b) and C given by the trace of C_{ab}, defined in eq. (2.53c),

$$C := \eta^{ab}C_{ab} = \frac{\mathrm{i}}{12}(\tilde\gamma^a)^{\alpha\beta}\nabla_{\alpha k}\nabla_{\beta l}B_a{}^{kl} = \frac{\mathrm{i}}{12}\nabla_{\alpha k}\nabla_{\beta l}B^{\alpha\beta kl} = \frac{1}{8}(\tilde\gamma^a)^{\alpha\beta}\nabla_{\alpha k}\Lambda_{\beta a}{}^k\,. \qquad (3.20)$$

The antisymmetric part of C_{ab} is related to the supercovariant four-form field strength H_{abcd} via (2.51d), leading to

$$C_{[ab]} := -\frac{1}{12}\varepsilon_{abcdef}h^{cdef} - \frac{3i}{4}\psi^c{}_j\gamma_{[ab}\Lambda_{c]}{}^j - \frac{i}{8}\varepsilon_{abcdef}\psi^c{}_j\gamma^{deg}\psi^f{}_k\,B_g{}^{jk}\,, \qquad (3.21)$$

where $h_{mnpq} = 4\partial_{[m}b_{npq]}$. If we now plug the resulting closed 6-form (3.17) into (3.9) we obtain the component action principle

$$\begin{aligned}
S_{B_2 \wedge H_4} = \int \mathrm{d}^6 x\, e\, \Bigg\{ &-\frac{1}{4}\sigma C - \frac{1}{4}\psi_j\tilde\gamma^g\Lambda_g{}^j + \frac{1}{4}b_{ab}C^{ab} \\
&+ \frac{1}{8}\psi_{ai}\left[2\psi_j B^{aij} + \mathrm{i}\sigma\Lambda^{ai} + \gamma^{ab}\left(2\psi_j B_b{}^{ij} - \mathrm{i}\sigma\Lambda_b{}^i\right) + \frac{3\mathrm{i}}{2}b_{bc}\gamma^{[ab}\Lambda^{c]i}\right] \\
&+ \frac{\mathrm{i}}{4}\psi_{ai}\left[\frac{1}{2}\gamma^{abc}\sigma B_c{}^{ij} - b_{cd}\gamma^{[abc}B^{d]ij}\right]\psi_{bj}\Bigg\}\,,
\end{aligned} \qquad (3.22)$$

where spinor indices have been suppressed. Note that the couplings involving b_{ab} can be rewritten as $\varepsilon^{abcdef}b_{ab}h_{cdef}$. We will refer to the invariant (3.22) as the $B_2 \wedge H_4$ action principle. The bosonic part of the action (3.22) appeared for the first time in [40]. It is important to mention that although one can formally work in the standard-Weyl multiplet

in the construction of the above invariant, the tensor multiplet is on-shell. To have an off-shell description, one must identify the tensor multiplet as the one of the dilaton-Weyl multiplet. Note that by turning off the supergravity multiplet in (3.22) one obtains the action principle that was first used in [105] to describe the action for the rigid supersymmetric Yang-Mills multiplet. Let us see how (3.22) allows us to extend that result to the curved case.

3.3.1 Non-Abelian vector multiplet action

It is worth presenting a first nontrivial application of the $B_2 \wedge H_4$ action principle (3.22): a direct construction of the non-Abelian vector multiplet action in a general dilaton-Weyl multiplet background. Previously, this action had been constructed in [48] undertaking the following steps: (i) restrict to an Abelian vector multiplet; (ii) in a dilaton-Weyl background, construct a composite linear multiplet in terms of an Abelian vector multiplet;[10] (iii) plug the composite linear multiplet in the $B_4 \wedge \mathcal{F}_2$ action principle; (iv) realise that the resulting action is invariant also in the case of a non-Abelian vector multiplet.

In the case of a general non-Abelian vector multiplet, described by the superfield strength $\boldsymbol{\Lambda}^{\alpha i}$, as observed in [69] it is straightforward to construct a composite gauge 3-form multiplet in terms of the primary dimension three superfield

$$B^{\alpha\beta ij} = -4\mathrm{i}\,\mathrm{Tr}\left(\boldsymbol{\Lambda}^{\alpha(i}\boldsymbol{\Lambda}^{\beta j)}\right), \tag{3.23a}$$

which, due to (2.28), satisfies the constraints (2.52). The descendants of the composite $B^{\alpha\beta ij}$ are then

$$\Lambda_{\alpha a}{}^i = \frac{2}{3}(\gamma_a)_{\beta\gamma}\,\mathrm{Tr}\left[\mathrm{i}\delta_\alpha^\beta \boldsymbol{X}^{ij}\boldsymbol{\Lambda}_j^\gamma - 6\mathrm{i}\boldsymbol{\mathcal{F}}_\alpha{}^\beta\boldsymbol{\Lambda}^{\gamma i}\right], \tag{3.23b}$$

$$C = 2\,\mathrm{Tr}\left[\boldsymbol{\mathcal{F}}^{ab}\boldsymbol{\mathcal{F}}_{ab} - \boldsymbol{X}^{ij}\boldsymbol{X}_{ij} - 2\mathrm{i}\boldsymbol{\Lambda}^{\alpha k}\boldsymbol{\nabla}_{\alpha\beta}\boldsymbol{\Lambda}_k^\beta\right], \tag{3.23c}$$

$$C_{[ab]} = \frac{1}{2}\,\mathrm{Tr}\left[\varepsilon_{abcdef}\boldsymbol{\mathcal{F}}^{cd}\boldsymbol{\mathcal{F}}^{ef} - \mathrm{i}\boldsymbol{\nabla}^c\big((\gamma_{abc})_{\alpha\beta}\boldsymbol{\Lambda}^{\alpha k}\boldsymbol{\Lambda}_k^\beta\big)\right]. \tag{3.23d}$$

If we plug these results into the $B_2 \wedge H_4$ action principle (3.22) we find the action for a general non-Abelian vector multiplet

$$S_{\sigma F^2} = \int \mathrm{d}^6x\, e\,\mathrm{Tr}\left\{-\frac{1}{2}\sigma\boldsymbol{f}^{ab}\boldsymbol{f}_{ab} + \frac{1}{2}\sigma\boldsymbol{X}^{ij}\boldsymbol{X}_{ij} + \frac{1}{8}\varepsilon^{abcdef}b_{ab}\boldsymbol{f}_{cd}\boldsymbol{f}_{ef}\right\}$$
$$+ \text{fermions}, \tag{3.24}$$

where we neglected the fermionic terms. The action, up to a change of notations, coincides with the result of [48].

---

[10]In superspace, with $\Lambda^{\alpha i}$ the superfield strength of an Abelian vector multiplet, the composite linear multiplet is described by the superfield $\mathcal{F}^{ij} := \big(\mathrm{i}(\nabla_\alpha^{(i}\Phi)\Lambda^{\alpha j)} + \frac{\mathrm{i}}{4}\Phi\nabla_\alpha^{(i}\Lambda^{\alpha j)}\big) = \big(\mathrm{i}\psi_\alpha^{(i}\Lambda^{\alpha j)} + \Phi X^{ij}\big)$ which can be easily proven to be a dimension-4 primary such that $\nabla_\alpha^{(i}\mathcal{F}^{jk)} = 0$ [83].

3.4 The *A* action principle

Let us conclude this section by reviewing the salient results concerning the *A* action principle. The reader should refer to [69, 70] for a complete analysis.

The *A* action principle is based on a primary dimension 9/2 superfield $A_\alpha{}^{ijk} = A_\alpha{}^{(ijk)}$ obeying the reality condition $\overline{A_\alpha{}^{ijk}} = A_{\alpha\,ijk}$ and satisfying the differential constraint

$$\nabla_{(\alpha}^{(i} A_{\beta)}{}^{jkl)} = 0\,. \tag{3.25}$$

As already mentioned, this superfield arises as the bottom component of a covariant closed super 6-form [69, 100]. By using the superform approach for the construction of supersymmetric invariants it is possible to obtain the invariant [70]

$$S_A = \int \mathrm{d}^6 x\, e \bigg\{ F - \frac{\mathrm{i}}{4} \psi_{ai} \Omega'^{ai} - \frac{\mathrm{i}}{144} \psi_{di} \gamma^{de} \gamma^{abc} \psi_{ej} S^+_{abc}{}^{ij} - \frac{\mathrm{i}}{12} \psi_{ai} \gamma^{abc} \psi_{bj} E_c{}^{ij} \\ + \frac{1}{16} (\psi_{ai} \gamma^{abc} \psi_{bj}) (\psi_{ck} A^{ijk}) \bigg\}\,, \tag{3.26}$$

where the following descendant component fields of $A_\alpha{}^{ijk}$ have been introduced:

$$S^+_{abc}{}^{ij} := \frac{3}{32} (\tilde{\gamma}_{abc})^{\alpha\beta} \nabla_{\alpha k} A_\beta{}^{ijk}|\,, \qquad E_a{}^{ij} := \frac{3}{16} (\tilde{\gamma}_a)^{\alpha\beta} \nabla_{\alpha k} A_\beta{}^{ijk}|\,, \tag{3.27a}$$

$$\Omega'_{a\alpha}{}^i := \frac{\mathrm{i}}{32} (\tilde{\gamma}_a)^{\beta\gamma} (\nabla_{\beta j} \nabla_{\gamma k} A_\alpha{}^{ijk}| + \nabla_{\alpha j} \nabla_{\beta k} A_\gamma{}^{ijk}|)\,, \tag{3.27b}$$

$$F := \frac{\mathrm{i}}{2^4 4!} \varepsilon^{\alpha\beta\gamma\delta} \nabla_{\alpha i} \nabla_{\beta j} \nabla_{\gamma k} A_\delta{}^{ijk}|\,. \tag{3.27c}$$

The superconformal transformations of the previous multiplet are rather involved, see [70] for details. The main point is that (3.26) proves to be manifestly locally superconformal invariant. For the scope of this paper we will only need to know that the purely bosonic part of this action can be extracted from the F component (3.27c) of the $A_\alpha{}^{ijk}$ multiplet.

4 Einstein-Hilbert Poincaré supergravity

In this section we describe how to construct in conformal superspace a supersymmetric extension of the Einstein-Hilbert term and then describe the off-shell and on-shell two-derivative Poincaré supergravity theory reproducing the results of [48].

4.1 Linear multiplet action

An action for the linear multiplet can be constructed using the $B_4 \wedge \mathcal{F}_2$ action principle (3.14) with the vector multiplet built out of the linear multiplet [48]. The appropriate composite vector multiplet is described by the primary dimension-3/2 superfield strength

$$\Lambda^{\alpha i} = -\frac{1}{2L} \nabla^{\alpha\beta} \varphi_\beta^i - \frac{1}{2L} \big(W^{\alpha\beta} \varphi_\beta^i - 2\mathrm{i} X_j^\alpha L^{ji} \big) + \frac{1}{4L^3} L_{jk} (\nabla^{\alpha\beta} L^{ij}) \varphi_\beta^k \\ - \frac{1}{8L^3} L^{ij} \mathrm{E}^{\alpha\beta} \varphi_{\beta j} - \frac{\mathrm{i}}{8L^5} \varepsilon^{\alpha\beta\gamma\delta} \varphi_{\beta j} \varphi_{\gamma k} \varphi_{\delta l} L^{ij} L^{kl}\,, \tag{4.1}$$

where $L^2 := \frac{1}{2}L^{ij}L_{ij}$ is assumed to be nonvanishing, $L \neq 0$. The component fields of the composite vector multiplet can be computed straightforwardly. They include the $\theta = 0$ projection of $\Lambda^{\alpha i}$ together with the descendant components of the composite vector multiplet, $X^{ij} = \frac{\mathrm{i}}{4}\nabla^{(i}_\gamma \Lambda^{\gamma j)}|$ and $\mathcal{F}_{ab} = -\frac{\mathrm{i}}{8}(\gamma_{ab})_\beta{}^\alpha \nabla^k_\alpha \Lambda^\beta_k|$,[11]

$$
\begin{aligned}
X^{ij} = {}&-\frac{1}{2}L^{-1}\nabla^a\nabla_a L^{ij} - \frac{1}{15}L^{-1}L^{ij}D + \frac{1}{16}L^{-3}\mathrm{E}^a\mathrm{E}_a L^{ij} - \frac{1}{4}L^{-3}\mathrm{E}^a L^{k(i}\mathcal{D}_a L^{j)}{}_k \\
&+ \frac{1}{4}L^{-3}L_{kl}(\mathcal{D}^a L^{k(i})\mathcal{D}_a L^{j)l} + \text{fermions}\,,
\end{aligned} \tag{4.2a}
$$

$$
\mathcal{F}_{ab} = \frac{1}{2}\mathcal{D}_{[a}\big(L^{-1}\mathrm{E}_{b]}\big) + \frac{1}{2}L^{-1}\mathcal{R}_{ab}{}^{ij}L_{ij} - \frac{1}{4}L^{-3}L_{ij}(\mathcal{D}_{[a}L^{ki})\mathcal{D}_{b]}L^j{}_k + \text{fermions}\,, \tag{4.2b}
$$

where

$$
\nabla^a\nabla_a L^{ij} = \mathcal{D}^a\nabla_a L^{ij} - 8\mathfrak{f}_a{}^a L^{ij} + \text{fermions}\,, \tag{4.3}
$$

the expression for $\nabla_a L^{ij}$ is given in (2.47a), and $\mathfrak{f}_a{}^a$ is given in (2.77). The f_{mn} field strength (2.32) in the case of the composite vector multiplet proves to be

$$
f_{mn} = 2\partial_{[m}\Gamma_{n]} - \frac{1}{4}L^{-3}L_{ij}(\partial_{[m}L^{ki})\partial_{n]}L^j{}_k\,, \tag{4.4a}
$$

$$
\Gamma_m := \frac{1}{2}L^{-1}\left(\mathcal{V}_m{}^{ij}L_{ij} + \frac{1}{2}h_m\right) + \text{fermions}\,. \tag{4.4b}
$$

The previous composite vector multiplet coincides, up to change of notation, to the one originally constructed in [48] where the reader can also find the completion with the fermionic terms.

Using (3.14) and disregarding a total derivative, the action for the linear multiplet takes the form

$$
\begin{aligned}
S_{\text{EH}} = \int \mathrm{d}^6 x\, e\, \bigg\{ &-\frac{2}{5}\mathcal{R}L - \frac{2}{15}DL - \frac{1}{8L}\mathrm{E}^a\mathrm{E}_a - \frac{1}{2L}\mathrm{E}^a\mathcal{V}_a{}^{ij}L_{ij} - \mathcal{D}^a\mathcal{D}_a L \\
&+ \frac{1}{4L}(\mathcal{D}^a L^{ij})\mathcal{D}_a L_{ij} + \frac{1}{8L^3}\tilde{b}^{mn}L_{ij}(\partial_m L^{ki})\partial_n L^j{}_k \bigg\} + \text{fermions}\,.
\end{aligned} \tag{4.5}
$$

The previous action is by construction locally superconformal invariant independently of the conformal supergravity background chosen, either the standard- or dilaton-Weyl multiplets which we have not specified so far in this section. In the standard-Weyl multiplet background the DL coupling in (4.5) implies $L = 0$ on-shell. This is inconsistent with the requirement of L being a conformal compensator, $L \neq 0$, and makes the action (4.5) alone inconsistent in a standard-Weyl multiplet background. A standard resolution of this issue is to consider (4.5) in a dilaton-Weyl multiplet background where, thanks to (2.75c)

$$
D = \frac{15}{4\sigma}\mathcal{D}^a\mathcal{D}_a\sigma + \frac{3}{4}\mathcal{R} + \frac{5}{4\sigma}T^-_{abc}H^{+abc} + \text{fermions} \tag{4.6}
$$

---

[11]Here and in what follows, whenever we only explicitly give the bosonic sectors, any supercovariant field (which involves fermionic terms) can be replaced with its purely bosonic analogue, e.g. E^a with h^a, H_{abc} with h_{abc} and so on, since it will only change the suppressed fermionic terms.

and the action (4.5) becomes

$$S_{\text{EH}} = \int d^6 x\, e \left\{ -\frac{1}{2} L \mathcal{R} + \frac{1}{4L} (\partial^m L_{ij}) \partial_m L^{ij} + \frac{1}{L} \mathcal{V}^{m\,k}_{\ \ i} L_{jk} (\partial_m L^{ij}) + \frac{1}{L} \mathcal{V}_m^{\ (i}_{\ \ k} L^{j)k} \mathcal{V}^m_{\ il} L^l_{\ j} \right.$$
$$\left. - \frac{L}{2\sigma} \mathcal{D}^a \mathcal{D}_a \sigma - \frac{L}{6\sigma} T^-_{abc} H^{+abc} - \frac{1}{8L} \mathrm{E}^a \mathrm{E}_a - \frac{1}{2L} \mathrm{E}^a \mathcal{V}_a^{\ ij} L_{ij} - \frac{1}{8L^3} \tilde{b}^{mn} L_i^{\ j} (\partial_m L^{ki}) \partial_n L_{jk} \right\}$$
$$+ \text{fermions}. \tag{4.7}$$

Another option to solve the inconsistent dynamics of (4.5) in a standard-Weyl multiplet background would be to add to (4.5) other invariants such that the resulting dynamical system possesses equations of motions for D that are self consistent. We will show an example of this option later in section 7 when we will introduce a new curvature squared invariant in a standard-Weyl multiplet background that will contain a D^2 term in its action.

4.2 Off-shell Einstein-Hilbert Poincaré supergravity

The six dimensional $\mathcal{N} = (1,0)$ supersymmetric extension of the Einstein-Hilbert term and consequently the off-shell Poincaré supergravity action can be given by coupling a linear multiplet compensator to a dilaton-Weyl multiplet, eq. (4.7), followed by the gauge fixing of the redundant superconformal symmetries. The gauge fixing conditions we impose in superspace are

$$B_M = 0, \quad \Phi = 1, \tag{4.8a}$$

$$L_{ij} = \delta_{ij} L. \tag{4.8b}$$

At the component level these imply

$$b_m = 0, \quad \sigma = 1, \quad \psi^i = 0, \tag{4.9a}$$

$$L_{ij} = \delta_{ij} L. \tag{4.9b}$$

The gauge conditions (4.8a) and (4.9a) fix dilatations, conformal boosts and S-supersymmetry transformations while (4.8b) and (4.9b) breaks the $\text{SU}(2)_R$ down to a residual $\text{U}(1)_R$ gauge symmetry. After fixing the gauge, the remaining physical fields are

$$\{ e_m^{\ a}, \psi_m^{\ i}, b_{mn}, V_m, V_m'^{\ ij}, L, \varphi^i, b_{mnrs} \}, \tag{4.10}$$

which form the minimal off-shell Poincaré supergravity multiplet. Note that we have decomposed the gauge field $\mathcal{V}_m^{\ ij}$ of the $\text{SU}(2)_R$ symmetry into its trace and traceless parts as

$$\mathcal{V}_m^{\ ij} = V_m'^{\ ij} + \frac{1}{2} \delta^{ij} V_m, \qquad V_m'^{\ ij} \delta_{ij} = 0. \tag{4.11}$$

To preserve the gauge fixing conditions (4.9) under the residual Poincaré supergravity transformations the following decomposition rules for the dependent compensating gauge

parameters have to be used [66]

$$\eta^i = -\frac{\mathrm{i}}{96}\gamma^{abc}\xi^i H_{abc}\,, \tag{4.12a}$$

$$\lambda_m = -\frac{1}{2}\xi^i \phi_{mi} - \frac{\mathrm{i}}{30}\xi^i \gamma_m \chi_i + \frac{1}{2}\eta^i \psi_{mi}\,, \tag{4.12b}$$

$$\lambda'^{ij} = \frac{1}{L}S'_k{}^{(i}\delta^{j)k}\,, \qquad S'^{ij} = -\xi^{(i}\varphi^{j)} + \frac{1}{2}\delta^{ij}\delta_{kl}\xi^k\varphi^l\,. \tag{4.12c}$$

Note that, besides the Q-supersymmetry transformations parametrized by local $\xi_i^\alpha|$ parameter, the trace λ of the SU(2)$_R$ parameter, $\lambda^{ij} := \Lambda^{ij}| = \lambda'^{ij} + \frac{1}{2}\delta^{ij}\lambda$, generates a residual arbitrary U(1)$_R$ gauge transformation. The other residual gauge transformations are arbitrary covariant general coordinate and Lorentz transformations parametrized by $\xi^a|$ and $\lambda^{ab} := \Lambda^{ab}|$, respectively. As a consequence, an off-shell $\mathcal{N} = (1,0)$ Poincaré supergravity background with the field content given in (4.10) has, up to cubic fermions, the following supersymmetry transformation rules [66]

$$\delta e_m{}^a = -\mathrm{i}\xi_i\gamma^a\psi_m{}^i\,, \tag{4.13a}$$

$$\delta\psi_{mi} = 2\left(\partial_m + \frac{1}{4}\omega_m{}^{ab}\tilde{\gamma}_{ab}\right)\xi_i + 2\mathcal{V}_{mi}{}^j\xi_j - \frac{1}{8}H_{mbc}\tilde{\gamma}^{bc}\xi_i\,, \tag{4.13b}$$

$$\delta b_{mn} = 2\mathrm{i}\xi_i\gamma_{[m}\psi_{n]}{}^i\,, \tag{4.13c}$$

$$\delta\varphi^i = \mathrm{i}\delta^{ij}\gamma^m\xi_j\partial_m L - \frac{\mathrm{i}}{2}\gamma^a\xi^i \mathrm{E}_a - 2\mathrm{i}V_m'{}^{(i}{}_k\delta^{j)k}\gamma^m\xi_j L + \frac{\mathrm{i}}{6}L\delta^{ij}\gamma^{abc}\xi_j H_{abc}\,, \tag{4.13d}$$

$$\delta L = \xi^i\varphi^j\delta_{ij}\,,$$

$$\delta b_{mnpq} = -\varepsilon_{mnpqef}\,\xi_j\gamma^{ef}\varphi^j + 8\mathrm{i}\psi^i_{[m}\gamma_{npq]}\xi^j\delta_{ij}L\,, \tag{4.13e}$$

$$\delta V_m'{}^{ij} = -\mathrm{i}\xi^{(i}\gamma^n\widehat{R}_{mn}{}^{j)}(Q) + \frac{\mathrm{i}}{2}\xi^k\gamma^n\widehat{R}_{mn}{}^\ell(Q)\delta_{k\ell}\delta^{ij} + \frac{\mathrm{i}}{6}\xi^{(i}\gamma^{abc}\psi_m^{j)}H^{abc}$$
$$\quad - \frac{\mathrm{i}}{12}\xi^k\gamma^{abc}\psi_m^\ell\delta_{k\ell}\delta^{ij}H_{abc}\,, \tag{4.13f}$$

$$\delta V_m = -\mathrm{i}\xi^i\gamma^n\widehat{R}_{mn}{}^j(Q)\delta_{ij} + \frac{\mathrm{i}}{6}\xi^i\gamma^{abc}\psi_m^j\delta_{ij}H_{abc}\,, \tag{4.13g}$$

where

$$\widehat{R}_{mn}{}^i(Q) = 2D_{[m}\psi_{n]}{}^i + 2V'_{[m}{}^{ij}\psi_{n]j} + \frac{1}{4}\gamma^{ab}\psi_{[m}H_{n]ab}\,, \tag{4.14a}$$

$$D_m\psi_n^i = \left(\partial_m + \frac{1}{4}\omega_m{}^{cd}\tilde{\gamma}_{cd}\right)\psi_n{}^i + \frac{1}{2}V_m\delta^{ij}\psi_{nj}\,. \tag{4.14b}$$

By imposing the gauge fixing (4.9) in the action (4.7) it follows that the off-shell two-derivative Poincaré supergravity Lagrangian takes the form [66]

$$e^{-1}\mathcal{L}_{\mathrm{EH}} = -\frac{1}{2}L\mathcal{R} + \frac{1}{2}L^{-1}\partial_m L\partial^m L - \frac{1}{24}LH^{abc}H_{abc} + LV_m'{}^{ij}V'^m{}_{ij} - \frac{1}{8L}\mathrm{E}^a\mathrm{E}_a - \frac{1}{2}\mathrm{E}^m V_m$$
$$+ \text{fermions}\,. \tag{4.15}$$

The off-shell Poincaré supergravity action is invariant under the transformation rules (4.13).

4.3 On-shell Poincaré supergravity

When considering the Einstein-Hilbert supergravity (4.15), the equations of motion for the $V_m'^{ij}, V_m$ and b_{mnpq} fields simply imply [66]

$$V_m = V_m'^{ij} = b_{mnpq} = 0\,, \tag{4.16}$$

leaving the following on-shell Einstein-Hilbert supergravity Lagrangian

$$e^{-1}\mathcal{L} = -\frac{1}{2}e^{-2v}\left(\mathcal{R} - 4\partial^m v\,\partial_m v + \frac{1}{12}H_{mnp}H^{mnp}\right) + \text{fermions} \tag{4.17}$$

where we have set $L = e^{-2v}$ with v being the dilaton field. The massless supermultiplet of the two-derivative theory is therefore given by the following component fields

$$\{e_m{}^a\,,\psi_{mi}\,,b_{mn}\,,v\,,\varphi^i\}\,, \tag{4.18}$$

whose supersymmetry transformation rules, that follow from (4.13), up to cubic fermion terms are

$$\delta e_m{}^a = \mathrm{i}\xi^i\gamma^a\psi_{mi}\,, \tag{4.19a}$$

$$\delta\psi_m{}^i = 2\left(\partial_m\xi^i + \frac{1}{4}\omega_m{}^{ab}\tilde{\gamma}_{ab}\right)\xi^i - \frac{1}{4}H_{mab}\tilde{\gamma}^{ab}\xi^i\,, \tag{4.19b}$$

$$\delta b_{mn} = 2\mathrm{i}\xi_i\gamma_{[m}\psi_{n]}{}^i\,, \tag{4.19c}$$

$$\delta\varphi^i = -2\mathrm{i}e^{-2v}\delta^{ij}\gamma^m\xi_j\partial_m v + \frac{\mathrm{i}}{6}e^{-2v}\delta^{ij}\gamma^{abc}\xi_j H_{abc}\,, \tag{4.19d}$$

$$\delta v = -\frac{1}{2}e^{2v}\xi^i\varphi^j\delta_{ij}\,. \tag{4.19e}$$

Note that, since the fields $V_m'^{ij}, V_m$ and b_{mnpq} have been integrated out, the previous supersymmetry transformations close only on the mass-shell.

5 Curvature squared invariants in a dilaton-Weyl background

Here we turn to constructing curvature squared supergravity invariants. All the locally superconformal invariants described in this section are based on the $B_2 \wedge H_4$ action principle; hence they will all implicitly be in a dilaton-Weyl multiplet background. Within our framework, we will describe the off-shell locally $\mathcal{N} = (1,0)$ supersymmetric extensions of all the possible purely gravitational curvature squared terms that can be described as linear combinations of Riemann squared, Ricci tensor squared, and scalar curvature squared terms.

5.1 The Riemann curvature squared invariant

A supersymmetric extension of the Riemann curvature squared term was constructed in [62–66]. It has been coupled to the gauged chiral supergravity in six dimensions extending the Salam-Sezgin model with curvature squared corrections. The exact spectrum of the Riemann squared extended Salam-Sezgin model around the half-BPS Minkowski$_4\times$S^2

background was analyzed for the first time in [67]. The construction of the Riemann curvature squared invariant of [62–66] was based on the action for a Yang-Mills multiplet coupled to conformal supergravity [48] and the feature that in the gauge (4.9a) the dilaton-Weyl multiplet can be mapped to a Yang-Mills vector multiplet taking values in the 6D Lorentz algebra [62]. This observation is sometimes referred to as the "Yang-Mills trick". The superspace analogue of the "Yang-Mills trick" was elaborated in section 6 of [70]. Let us here further elaborate on this analysis.

In a dilaton-Weyl multiplet background, with tensor superfield $\Phi \neq 0$, following [69] one can introduce the spinor covariant derivative,

$$\mathscr{D}_\alpha^i = \Phi^{-\frac{1}{4}} \left(\nabla_\alpha^i + (\nabla_\beta^i \ln \Phi) M_\alpha{}^\beta - 2(\nabla_\alpha^j \ln \Phi) J_j{}^i - \frac{1}{2}(\nabla_\alpha^i \ln \Phi)\mathbb{D} \right). \tag{5.1}$$

This is chosen so that given a primary tensor superfield U of dimension Δ, the superfield $\mathscr{D}_\alpha^i U$ is also a primary superfield of dimension Δ. Moreover, Φ is annihilated by $\mathscr{D}_\alpha^i$, $\mathscr{D}_\alpha^i \Phi = 0$. When acting on a primary superfield, the algebra of covariant derivatives becomes[12]

$$\{\mathscr{D}_\alpha^i, \mathscr{D}_\beta^j\} = -2\mathrm{i}\varepsilon^{ij}\mathscr{D}_{\alpha\beta} - 4\mathrm{i}\varepsilon^{ij}\mathscr{W}^{abc}(\gamma_a)_{\alpha\beta}M_{bc} - 4\mathrm{i}\varepsilon^{ij}\mathscr{N}^{abc}(\gamma_a)_{\alpha\beta}M_{bc} - 16\mathrm{i}\mathscr{N}_{\alpha\beta}J^{ij}, \tag{5.2}$$

where

$$\mathscr{D}_{\alpha\beta} = -\frac{\mathrm{i}}{4}\{\mathscr{D}_\alpha^k, \mathscr{D}_{\beta k}\} - 2\mathscr{N}^{bcd}(\gamma_b)_{\alpha\beta}M_{cd} - 2\mathscr{W}^{bcd}(\gamma_b)_{\alpha\beta}M_{cd} \tag{5.3}$$

and the primary dimensionless superfields $\mathscr{N}_{\alpha\beta}$ and $\mathscr{W}^{\alpha\beta}$ are defined as

$$\mathscr{N}_{\alpha\beta} := \frac{1}{3!}(\gamma^{abc})_{\alpha\beta}\mathscr{N}_{abc} = -\frac{\mathrm{i}}{16}\Phi^{\frac{3}{2}}\nabla_{(\alpha}^k \nabla_{\beta)k}\Phi^{-2}, \qquad \mathscr{W}^{\alpha\beta} := \Phi^{-\frac{1}{2}}W^{\alpha\beta}. \tag{5.4}$$

By using the previous composite superfields, one can construct a primary dimension-3/2 superfield valued in the Lorentz algebra $\mathbf{\Lambda}^{\alpha i} := \mathbf{\Lambda}^{\alpha i}{}_\beta{}^\gamma M_\gamma{}^\beta$ [70]

$$\mathbf{\Lambda}^{\alpha i} = \Phi^{3/4} \left(\mathscr{D}_\beta^i \mathscr{W}^{\alpha\gamma} - \frac{2}{3}\varepsilon^{\alpha\gamma\delta\rho}\mathscr{D}_\delta^i \mathscr{N}_{\rho\beta} - \frac{1}{3}\delta_\beta^\alpha \mathscr{D}_\delta^i \mathscr{W}^{\gamma\delta} \right) M_\gamma{}^\beta. \tag{5.5}$$

This satisfies constraints that are formally the same as those of a non-Abelian vector multiplet:

$$\mathbf{\Lambda}^{\alpha i}{}_\beta{}^\beta = 0, \quad \mathscr{D}_\alpha^{(i}\mathbf{\Lambda}^{\beta j)}{}_\gamma{}^\delta - \frac{1}{4}\delta_\alpha^\beta \mathscr{D}_\rho^{(i}\mathbf{\Lambda}^{\rho j)}{}_\gamma{}^\delta = 0, \quad \mathscr{D}_{\alpha i}\mathbf{\Lambda}^{\alpha i}{}_\beta{}^\gamma = 0. \tag{5.6}$$

The primary $\mathbf{\Lambda}^{\alpha i}{}_\beta{}^\gamma$ can be equivalently rewritten as

$$\mathbf{\Lambda}^{\alpha i}{}_\beta{}^\gamma = 4\mathrm{i}X_\beta^{i\,\alpha\gamma} - \frac{4\mathrm{i}}{3}\delta_\beta^\alpha X^{\gamma i} + \frac{\mathrm{i}}{3}\delta_\beta^\gamma X^{\alpha i} - \Phi^{-1}\left(W^{\alpha\gamma}\psi_\beta^i - \frac{1}{3}\delta_\beta^\alpha W^{\gamma\delta}\psi_\delta^i + \frac{1}{3}\delta_\beta^\gamma W^{\alpha\delta}\psi_\delta^i \right)$$
$$+ \varepsilon^{\alpha\gamma\delta\rho}\Phi^{-2}\left[\frac{1}{2}(\nabla_{\delta(\rho}\Phi)\psi_{\beta)}^i - \frac{1}{3}\Phi\nabla_{\delta(\rho}\psi_{\beta)}^i - \frac{1}{8}H_{\rho\beta}\psi_\delta^i - \frac{\mathrm{i}}{4}\Phi^{-1}\psi_\delta^i\psi_{(\rho}^k\psi_{\beta)k} \right], \tag{5.7}$$

---

[12]The resulting superspace geometry for the covariant derivatives $\mathscr{D}_A = (\mathscr{D}_\alpha^i, \mathscr{D}_a)$ is equivalent to the SU(2) superspace formulation of conformal supergravity of [83] with the torsion superfield component $C_a{}^{ij}$ set to zero, $C_a{}^{ij} = 0$. This is also equivalent to the superspace geometry of [65]. Using in (5.1) instead a generic real unconstrained dimension-2 primary superfield X instead of Φ, as described in [69], one would obtain the general SU(2) superspace geometry of [83] with $C_a{}^{ij} \neq 0$.

where we have used the tensor multiplet relations

$$\psi_\alpha^i = \nabla_\alpha^i \Phi\,, \qquad H_{\alpha\beta} := -\mathrm{i}\nabla_{(\alpha}^k \psi_{\beta)k}\,, \qquad \nabla_\alpha^i \nabla_\beta^j \Phi = -\frac{\mathrm{i}}{2}\varepsilon^{ij} H_{\alpha\beta} - \mathrm{i}\varepsilon^{ij}\nabla_{\alpha\beta}\Phi\,, \tag{5.8}$$

and it should be kept in mind that in a dilaton-Weyl background $W^{\alpha\beta} = -\frac{1}{4}\Phi^{-1}H^{\alpha\beta}$. A remarkable property of the composite superfield (5.7) is that, in the gauge where $B_M = 0$ and $\Phi = 1$, equivalent to $b_m = 0$, $\sigma = 1$ and $\psi^i = 0$, the following relations for the descendants of $\mathbf{\Lambda}^{\alpha i}{}_\beta{}^\gamma$ hold:

$$\boldsymbol{\mathcal{F}}_{ab}{}^{cd} := -\frac{\mathrm{i}}{16}(\gamma_{ab})_\beta{}^\alpha (\gamma^{cd})_\delta{}^\gamma \nabla_\alpha^k \mathbf{\Lambda}_k{}^\beta{}_\gamma{}^\delta| = \mathcal{R}_{ab}{}^{cd}(\omega_+) + \text{fermions}\,, \tag{5.9a}$$

$$\boldsymbol{X}^{ijab} := \frac{\mathrm{i}}{8}(\gamma^{ab})_\gamma{}^\beta \nabla_\alpha^{(i}\mathbf{\Lambda}^{\alpha j)}{}_\beta{}^\gamma| = -2\mathcal{R}^{abij} + \text{fermions}\,. \tag{5.9b}$$

Here $\mathcal{R}_{ab}{}^{cd}(\omega_+)$ is the torsionful Riemann tensor which is defined through (2.17b) in terms of the shifted Lorentz connection

$$\omega_{\pm m}{}^{cd} := \omega_m{}^{cd} \pm \frac{1}{2}e_m{}^a H_a{}^{cd}\,, \tag{5.10}$$

such that

$$\mathcal{R}_{ab}{}^{cd}(\omega_\pm) = \mathcal{R}_{ab}{}^{cd}(\omega) \pm \mathcal{D}_{[a}H_{b]}{}^{cd} + \frac{1}{2}H_{e[a}{}^{[c}H_{b]}{}^{d]e}\,. \tag{5.11}$$

Considering the analogy of $\mathbf{\Lambda}^{\alpha i}{}_\beta{}^\gamma$ with a Yang-Mills multiplet and the known construction of the Riemann squared action from a vector multiplet one, it is natural to argue that a gauge 3-form multiplet is described by the following composite superfield

$$B^{\alpha\beta ij} = -16\mathrm{i}\,\text{Tr}\left[\mathbf{\Lambda}^{\alpha(i}\mathbf{\Lambda}^{\beta j)}\right] = -16\mathrm{i}\mathbf{\Lambda}^{\alpha(i}{}_\gamma{}^\delta \mathbf{\Lambda}^{\beta j)}{}_\delta{}^\gamma = 8\mathrm{i}\mathbf{\Lambda}^{\alpha(i}{}_{cd}\mathbf{\Lambda}^{\beta j)cd}\,. \tag{5.12}$$

Thanks to the fact that $\nabla_\gamma^k\,\text{Tr}\left[\mathbf{\Lambda}^{\alpha(i}\mathbf{\Lambda}^{\beta j)}\right] = \Phi^{\frac{1}{4}}\mathscr{D}_\gamma^k\,\text{Tr}\left[\mathbf{\Lambda}^{\alpha(i}\mathbf{\Lambda}^{\beta j)}\right]$, one can prove that the composite (5.12) satisfies the constraints (2.52). Then, by plugging (5.12) in the $B_2 \wedge H_4$ action principle, or equivalently by plugging (5.9) in (3.24), after imposing the gauge condition (4.9a), one obtains the action

$$S_{\text{Riem}^2} = \int \mathrm{d}^6 x\, e\bigg\{\mathcal{R}^{abcd}(\omega_+)\,\mathcal{R}_{abcd}(\omega_+) - \frac{1}{4}\varepsilon^{abcdef}b_{ab}\,\mathcal{R}_{cd}{}^{gh}(\omega_+)\,\mathcal{R}_{efgh}(\omega_+)$$
$$- 4\mathcal{R}^{abij}\mathcal{R}_{abij}\bigg\} + \text{fermions}\,. \tag{5.13}$$

Up to a change of notation, the previous result coincides with the known Riemann squared invariant [62–66].

5.2 The scalar curvature squared invariant

In section 4 we have shown how to construct the locally supersymmetric extension of the Einstein-Hilbert term by using the composite Abelian vector multiplet based on the superfield $\Lambda^{\alpha i}$ defined by eq. (4.1). It is clear that the composite

$$B^{\alpha\beta ij} := -4\mathrm{i}\Lambda^{(\alpha i}\Lambda^{\beta)j}\,, \tag{5.14}$$

satisfies the constraints (2.52). Moreover, it is straightforward to check that by plugging (5.14) in the $B_2 \wedge H_4$ action principle, in the gauge (4.8), one can find the supersymmetric extension of a scalar curvature squared that was first constructed in [68]. Let us here review the salient results for this invariant in components.

In the gauge (4.9), by using the components of the composite vector multiplet (4.2), it is straightforward to obtain the following useful expressions

$$f_{mn} = \partial_{[m}\left(V_{n]} + \frac{1}{2}L^{-1}\mathrm{E}_{n]}\right) + \text{fermions}\,, \tag{5.15a}$$

$$X_{ij}X^{ij} = \frac{1}{2}\Upsilon^2 - \frac{1}{2}|\Xi|^2 + \text{fermions}\,, \tag{5.15b}$$

where

$$\Upsilon = -\frac{1}{2}\mathcal{R} - \frac{1}{24}H_{mnp}H^{mnp} - L^{-1}\partial^m\partial_m L + \frac{1}{2}L^{-2}(\partial^m L)\partial_m L$$
$$- 2Z^m\bar{Z}_m + \frac{1}{8}L^{-2}\mathrm{E}^m\mathrm{E}_m\,, \tag{5.15c}$$

$$\Xi = -2L^{-1}\mathcal{D}^m(LZ_m) + \mathrm{i}L^{-1}\mathrm{E}^m Z_m\,, \tag{5.15d}$$

with the complex vector field Z_m defined as

$$Z_m = V'^{\,12}_m + \mathrm{i}V'^{\,11}_m\,. \tag{5.15e}$$

Using the action (3.24), we obtain (up to a numerical factor) the following Lagrangian for the Ricci scalar curvature squared invariant [68]

$$\begin{aligned}
e^{-1}\mathcal{L}_{R^2} = \frac{1}{16}\Bigg[&\mathcal{R} + \frac{1}{12}H_{mnr}H^{mnr} + 2L^{-1}\partial^m\partial_m L - L^{-2}(\partial_m L)\partial^m L \\
&+ 4Z^m\bar{Z}_m - \frac{1}{4}L^{-2}\mathrm{E}^m\mathrm{E}_m\Bigg]^2 \\
&- \frac{1}{4}\Big[2L^{-1}\mathcal{D}^m(LZ_m) - \mathrm{i}L^{-1}\mathrm{E}^m Z_m\Big]\Big[2L^{-1}\mathcal{D}^n(L\bar{Z}_n) + \mathrm{i}L^{-1}\mathrm{E}^n\bar{Z}_n\Big] \\
&+ \frac{1}{8}\varepsilon_{mnpqrs}b^{mn}\partial^p\left(V^q + \frac{1}{2}L^{-1}\mathrm{E}^q\right)\partial^r\left(V^s + \frac{1}{2}L^{-1}\mathrm{E}^s\right) \\
&- \frac{1}{2}\partial_{[m}\left(V_{n]} + \frac{1}{2}L^{-1}\mathrm{E}_{n]}\right)\partial^m\left(V^n + \frac{1}{2}L^{-1}\mathrm{E}^n\right) + \text{fermions}\,. \tag{5.16}
\end{aligned}$$

The resulting Ricci scalar curvature squared action is invariant under the off-shell supersymmetry transformation rules (4.13).

5.3 The new curvature squared invariant

Recently, a particular composite gauge 3-form multiplet defined solely using the standard-Weyl multiplet has been constructed in [69]. This was used to describe one of the $\mathcal{N} = (1,0)$ conformal supergravity actions [69, 70]. Such a composite gauge 3-form multiplet is defined in terms of the following primary dimension 3 superfield [69]

$$B^{\alpha\beta\,ij} = -4W^{\gamma[\alpha}Y_\gamma{}^{\beta]ij} - 32\mathrm{i}\,X_\gamma{}^{\alpha\delta(i}X_\delta{}^{\beta\gamma j)} + 10\mathrm{i}\,X^{\alpha(i}X^{\beta j)}\,, \tag{5.17}$$

satisfying the constraints (2.52). Its lowest component is

$$B_a{}^{ij} = T_{abc}^- R(J)^{bc\,ij} + \mathrm{i}\, R(Q)_{bc}{}^i \gamma_a R(Q)^{bcj} + \frac{2\mathrm{i}}{45}\chi^i \gamma_a \chi^j \,. \tag{5.18}$$

The structure of its descendants, including the $\Lambda_{\alpha a}{}^i$ and C_{ab} components appearing in the $B_2 \wedge H_4$ action principle (3.22), were worked out in [70] and are collected for convenience of the reader in appendix B. As we first described in [40], by plugging these results into (3.22) and performing some integrations by parts, one obtains the new curvature squared invariant

$$
\begin{aligned}
S_{\mathrm{new}} = \frac{1}{32} \int \mathrm{d}^6 x\, e\, \Big\{ & \sigma C_{ab}{}^{cd} C_{cd}{}^{ab} - 3\sigma \mathcal{R}_{ab}{}^{ij}\mathcal{R}^{ab}{}_{ij} + \frac{4}{15}\sigma D^2 + 4\sigma(\mathcal{D}_c T^{-abc})\mathcal{D}^d T^-_{abd} \\
& - 8\sigma T^{-dab}\Big(\mathcal{D}_d \mathcal{D}^c T^-_{abc} - \frac{1}{2}\mathcal{R}_d{}^c T^-_{abc}\Big) + 4\sigma T^{-abc}T^{-d}_{ab}T^{-ef}{}_c T^-_{efd} - 2H_{abc}C^{ab}{}_{de}T^{-cde} \\
& - \frac{8}{45}H_{abc}T^{-abc}D + 4H_{abc}T_d^{-ab}\mathcal{D}_e T^{-cde} - \frac{4}{3}H_{abc}T^{-dea}T^{-bcf}T^-_{def} \\
& - \frac{1}{4}\varepsilon^{abcdef}b_{ab}\Big(C_{cd}{}^{gh}C_{efgh} - \mathcal{R}_{cd}{}^{ij}\mathcal{R}_{efij}\Big)\Big\} + \text{fermions} \,,
\end{aligned}
\tag{5.19}
$$

where the reader should keep in mind that, in a dilaton-Weyl background, $T^-_{abc} = \frac{1}{2}H^-_{abc}$ and D satisfies (4.6). Note that in (5.19) we focused only on the bosonic part of the invariant. In principle, by plugging into (3.22) the expressions for $B_a{}^{ij}$, eq. (5.17), together with its descendants $\Lambda_{\alpha a}{}^i$ and C_{ab} given in eq. (B.1), the reader can obtain the full result including all the fermionic terms.

In the gauge (4.9a) the new invariant contains the following linear combination of Riemann squared, Ricci squared and scalar curvature squared terms

$$e^{-1}\mathcal{L}_{\mathrm{new}} = \frac{1}{32}\left(\mathcal{R}_{abcd}\mathcal{R}^{abcd} - \mathcal{R}_{ab}\mathcal{R}^{ab} + \frac{1}{4}\mathcal{R}^2\right) + \cdots \,. \tag{5.20}$$

This completes the construction of the three independent supersymmetric curvature squared invariants. Let us now turn to describing some physical applications.

6 Einstein-Gauss-Bonnet supergravity

In section 4 we reviewed the off-shell supersymmetrization of the Einstein-Hilbert action while in the previous section we have described three linearly independent curvature squared invariants of the six dimensional $\mathcal{N} = (1,0)$ supergravity. Due to the off-shell nature of these models, we can combine them to form a general curvature squared extended Poincaré supergravity Lagrangian

$$\mathcal{L} = \alpha \mathcal{L}_{\mathrm{Riem}^2} + \beta \mathcal{L}_{R^2} + \gamma \mathcal{L}_{\mathrm{new}} \,, \tag{6.1}$$

without having to modify the supersymmetry transformations. As described for the first time in [40], the off-shell Gauss-Bonnet combination corresponds to $\alpha = -3$, $\beta = 0$ and

$\gamma = 128$. For this particular choice of parameters, in the gauge (4.9a), if one uses the following identities

$$0 = H_{abc}H^{ade}\mathcal{D}^b H^c{}_{de} \,, \tag{6.2a}$$

$$0 = \varepsilon_{abcdef}\left(\mathcal{R}_{gh}{}^{ef}H^{gha}H^{bcd} + \mathcal{R}_g{}^f H^{gab}H^{cde}\right) \,, \tag{6.2b}$$

$$0 = \varepsilon^{abcdef}\left(H_{abc}H_{deg}\mathcal{D}^h H^g{}_{fh} - 2H_{abc}H_{dgh}\mathcal{D}_e H^{gh}{}_f\right) \,, \tag{6.2c}$$

and introduces the notation,

$$H^2_{ab} := H_a{}^{cd}H_{bcd} \,, \qquad H^2 := H_{abc}H^{abc} \,, \qquad H^4 := H_{abe}H_{cd}{}^e H^{acf}H^{bd}{}_f \,, \tag{6.3a}$$

up to fermionic terms the Lagrangian for the supersymmetric Gauss-Bonnet invariant (5.19) takes the following very compact form

$$\begin{aligned}
e^{-1}\mathcal{L}_{\text{GB}} = {}& \mathcal{R}_{abcd}\mathcal{R}^{abcd} - 4\mathcal{R}_{ab}\mathcal{R}^{ab} + \mathcal{R}^2 \\
& - \frac{1}{2}\mathcal{R}_{abcd}H^{abe}H^{cd}{}_e + \mathcal{R}^{ab}H^2_{ab} - \frac{1}{6}\mathcal{R}H^2 + \frac{5}{24}H^4 + \frac{1}{144}(H^2)^2 - \frac{1}{8}(H^2_{ab})^2 \\
& + \varepsilon^{abcdef}b_{ab}\mathcal{R}_{cd}{}^{ij}\mathcal{R}_{efij} - \frac{1}{4}\varepsilon^{abcdef}b_{ab}\mathcal{R}_{cd}{}^{gh}(\omega_-)\mathcal{R}_{efgh}(\omega_-) \\
& + \text{fermions} \,.
\end{aligned} \tag{6.4}$$

This can equivalently be written

$$\begin{aligned}
e^{-1}\mathcal{L}_{\text{GB}} = {}& 6\mathcal{R}_{[ab}{}^{ab}(\omega_-)\mathcal{R}_{cd]}{}^{cd}(\omega_-) - 4\mathcal{R}_{abcd}(\omega_-)H^{abe}H^{cd}{}_e + 4\mathcal{R}^{ab}(\omega_-)H^2_{ab} - \frac{2}{3}\mathcal{R}(\omega_-)H^2 \\
& - \frac{2}{3}H^4 + \frac{1}{9}(H^2)^2 + \varepsilon^{abcdef}b_{ab}\left[\mathcal{R}_{cd}{}^{ij}\mathcal{R}_{efij} - \frac{1}{4}\mathcal{R}_{cd}{}^{gh}(\omega_-)\mathcal{R}_{efgh}(\omega_-)\right] \\
& + \text{fermions} \,.
\end{aligned} \tag{6.5}$$

In this form it becomes evident that the b_2-field dependence of the supersymmetric Gauss-Bonnet invariant cannot be captured solely in terms of a Riemann tensor with a torsionful connection. This explains the unsuccessful attempts to construct the Gauss-Bonnet invariant in previous works where this was assumed as a working condition [62, 63].

By using the action (6.4), we can now easily construct the 6D $\mathcal{N} = (1,0)$ off-shell Einstein-Gauss-Bonnet (EGB) supergravity. In the gauge (4.9), this is given by the following combination of the supersymmetric completion of the Einstein-Hilbert term (4.15) and the Gauss-Bonnet term (6.4)

$$2\kappa^2\mathcal{L}_{\text{EGB}} = \mathcal{L}_{\text{EH}} + \frac{1}{16}\alpha'\mathcal{L}_{\text{GB}} \,. \tag{6.6}$$

We have introduced the Newton constant κ^2. The Einstein-Gauss-Bonnet supergravity (6.6) is invariant under the off-shell supersymmetry transformation rules (4.13).

6.1 On-shell Einstein-Gauss-Bonnet supergravity

In section 4 we have shown how the on-shell Einstein-Hilbert supergravity (4.17) arises from (4.15) by using the equations of motion $V'_m{}^{ij} = V_m = b_{mnpq} = 0$. A remarkable feature

of the supersymmetric Einstein-Gauss-Bonnet model is that the massless supermultiplet of the two-derivative theory does not acquire mass due to the inclusion of higher-derivative terms. Furthermore, the kinetic term $\mathcal{R}_{ab}{}^{ij}\mathcal{R}^{ab}{}_{ij}$ for the $\mathrm{SU}(2)_R$ gauge field $\mathcal{V}_m{}^{ij}$ that exists in both the Riemann-squared and the new invariants, cancels out in the Gauss-Bonnet combination. Therefore, there are no new dynamical degrees of freedom induced by the higher-derivative terms and the solution (4.16) remains consistent in the Einstein-Gauss-Bonnet supergravity, leading to the on-shell theory associated to the following Lagrangian

$$
\begin{aligned}
2\kappa^2 e^{-1}\mathcal{L} = &-e^{-2v}\left(\mathcal{R} - 4\partial^m v\,\partial_m v + \frac{1}{12}H_{mnp}H^{mnp}\right)\\
&+\frac{1}{16}\alpha'\bigg[6\mathcal{R}_{[mn}{}^{mn}\mathcal{R}_{rs]}{}^{rs} - \frac{1}{2}\mathcal{R}_{mnrs}H^{mnt}H^{rs}{}_t + \mathcal{R}^{mn}H_{mn}^2 - \frac{1}{6}\mathcal{R}H^2 + \frac{5}{24}H^4\\
&\qquad + \frac{1}{144}(H^2)^2 - \frac{1}{8}(H_{mn}^2)^2 - \frac{1}{4}\varepsilon^{mnpqrs}b_{mn}\mathcal{R}_{pq}{}^{ab}(\omega_-)\mathcal{R}_{rsab}(\omega_-)\bigg]\\
&+\text{fermions}\,.
\end{aligned}
\tag{6.7}
$$

As a consequence, up to cubic fermions, the on-shell $\mathcal{N} = (1,0)$ Einstein-Gauss-Bonnet supergravity has the same supersymmetry transformations as the on-shell Einstein-Hilbert supergravity given by (4.19).

Note that our on-shell Einstein-Gauss-Bonnet supergravity action precisely matches the result derived in [46]. In fact, we have actually fixed the proportionality constant of the α' term in (6.6) by comparing the on-shell Lagrangian (6.7) with [46].

To conclude this section, let us comment further on the relevance of our result in comparison with the analysis of Liu and Minasian in [46]. There, it was conjectured that for the type II-string, the NSNS b_2-field dependence in the R^4 corrections is nearly completely captured in terms of the torsionful Riemann tensor (5.11) (except for the CP-odd sector). The claim was further investigated in [46] by fixing the one-loop four-derivative corrections by means of a K3 reduction to six dimensions of type IIA and requiring that the dyonic string remains as a solution and that the model remain dual to the heterotic string compactified on T^4. In our work, we provided an alternative derivation of the four-derivative corrections by exact supersymmetrization of the curvature squared invariants. The fact that our results for the Einstein-Gauss-Bonnet supergravity match, and extend off-shell, the results of [46] thereby provides strong evidence for their conjecture.

7   New curvature squared invariants with a linear multiplet compensator

In a conformal supergravity background described by the dilaton-Weyl multiplet, we constructed three independent curvature squared supergravity invariants in section 5. These were all based on the $B_2 \wedge H_4$ action principle of subsection 3.3. One of these invariants was a locally superconformal extension of a four-derivative invariant for a linear multiplet. A natural question one could ask is whether this is the only four-derivative superconformal invariant for a linear multiplet. It turns out that this is not the case as one can simply write down another local four-derivative superconformal invariant using a full superspace

integral as follows

$$\int d^6x d^8\theta\, E\, L^{1/2}\,, \qquad E := \mathrm{Ber}[E_M{}^A]\,, \tag{7.1}$$

where $L^{1/2} = \sqrt{\frac{1}{2}L^{ij}L_{ij}}$ and E is the Berezinian (or superdeterminant) of the superviel-bein.[13] Moreover, (7.1) does not coincide with the four-derivative linear multiplet invariant of the previous sections since it is well-defined in the standard-Weyl multiplet (as it does not involve the tensor multiplet).

As of yet, we do not have a manifestly superconformal component (or superform) representation of the full superspace integral based on a primary density formula (or superform).[14] For this reason, we will endeavour to make use of the manifestly superconformal A action principle for the construction of a new local four-derivative invariant based on the linear multiplet.[15] In this section, we will then show that this new invariant leads to curvature squared terms such that: (i) once added to the Einstein-Hilbert term in a standard-Weyl multiplet background, the new invariant leads to a dynamical system with consistent equations of motion for the D field which dynamically generates a cosmological constant term that leads to (non-supersymmetric) (A)dS$_6$ solutions; (ii) in a dilaton-Weyl multiplet background the new invariant proves to have a nontrivial coupling to the dilaton and is distinct from the curvature squared invariants constructed in section 5.

7.1 A new locally superconformal higher-derivative action for the linear multiplet

In [112] it was recently proven that, given a so-called real $\mathcal{O}^*(4)$ multiplet, which is a real dimension -4 primary superfield T satisfying

$$\overline{T} = T\,, \qquad \nabla^k_{(\alpha}\nabla_{\beta)k}T = 0 \quad \Longrightarrow \quad \nabla^i_{(\alpha}\nabla^j_{\beta)}T = 0\,, \tag{7.2}$$

and a real $\mathcal{O}(4)$ multiplet which is a primary superfield T^{ijkl} of dimension 8 satisfying

$$T^{ijkl} = T^{(ijkl)}\,, \qquad \overline{(T^{ijkl})} = T_{ijkl}\,, \qquad \nabla^{(p}_{\alpha}T^{ijkl)} = 0\,, \tag{7.3}$$

it is possible to construct a manifestly locally superconformal action with the A action principle. In fact, for any pair of $\mathcal{O}(4)$ and $\mathcal{O}(4)^*$ multiplets, the following superfield

$$A_\alpha{}^{ijk} := -\mathrm{i}T\nabla_{\alpha l}T^{ijkl} - 5\mathrm{i}T^{ijkl}\nabla_{\alpha l}T\,, \tag{7.4}$$

proves to be a primary of dimension 9/2 satisfying the symmetry and reality conditions $\overline{A_\alpha{}^{ijk}} = A_{\alpha\,ijk}$ and $A_\alpha{}^{ijk} = A_\alpha{}^{(ijk)}$ together with the differential constraint $\nabla^{(i}_{(\alpha}A_{\beta)}{}^{jkl)} = 0$, (3.25). Then, given any pair of real $\mathcal{O}(4)$ and $\mathcal{O}^*(4)$ multiplets one can construct a manifestly locally superconformal action.

---

[13]The reader can look at [69, 83] for discussions about general properties of the full superspace integral under local superconformal transformations.

[14]See [69] for a non-primary superform description of the full superspace integral based on the so-called B action principle.

[15]This invariant presumably coincides with the one defined by the full superspace integral above.

Using a linear (real $\mathcal{O}(2)$) multiplet as in subsection 2.3 that is nowhere vanishing, $L^2 = \frac{1}{2} L^{ij} L_{ij} \neq 0$, it is possible to construct a pair of composite real $\mathcal{O}(4)$ and $\mathcal{O}^*(4)$ multiplets. The real $\mathcal{O}(4)$ multiplet superfield is

$$L^{ijkl} := -\frac{1}{96} \varepsilon^{\alpha\beta\gamma\delta} \nabla_\alpha^{(i} \nabla_\beta^j \nabla_\gamma^k \nabla_\delta^{l)} L^{3/2} \,, \tag{7.5}$$

and the real $\mathcal{O}^*(4)$ multiplet is L^{-1}, which due to (2.41) obeys $\nabla_{(\alpha}^k \nabla_{\beta)k} L^{-1} = 0$. This implies that we can straightforwardly construct a new higher-derivative action for the linear multiplet using the A action principle with

$$A_\alpha{}^{ijk} := -\mathrm{i} L^{-1} \nabla_{\alpha l} L^{ijkl} - 5\mathrm{i} L^{ijkl} \nabla_{\alpha l} L^{-1} \,. \tag{7.6}$$

To show that, it suffices to focus on the bosonic part of this invariant. With the assistance of the computer algebra program *Cadabra* [123, 124], we computed the bosonic part of the action. The full result of this calculation is presented appendix C. For the scope of this section, we can simplify the result (C.1) by going to a conformal and $\mathrm{SU}(2)_R \to \mathrm{U}(1)_R$ gauge where

$$b_m \equiv 0 \,, \qquad L^{ij} = \delta^{ij} L \,, \tag{7.7}$$

and neglect, besides the fermion fields, also the $\mathrm{SU}(2)_R$ connection and curvature together with the E_a and T_{abc}^- fields. This leads to

$$
\begin{aligned}
e^{-1} \mathcal{L}_{\mathrm{new-linear}} = {}& -\frac{1}{12} L^{\frac{1}{2}} \Box D - \frac{1}{20} L^{\frac{1}{2}} D^2 - \frac{1}{12} D L^{-\frac{1}{2}} \Box L + \frac{1}{24} L^{-\frac{3}{2}} D (\mathcal{D}^a L) \mathcal{D}_a L \\
& -\frac{5}{8} L^{-\frac{1}{2}} \Box^2 L + \frac{15}{8} L^{-\frac{3}{2}} (\mathcal{D}^a L) \nabla_a \Box L + \frac{15}{16} L^{-\frac{3}{2}} (\Box L) \Box L \\
& + \frac{5}{32} L^{-\frac{3}{2}} (\nabla^a \nabla^b L) \nabla_a \nabla_b L - \frac{295}{128} L^{-\frac{5}{2}} (\mathcal{D}^a L)(\mathcal{D}_a L) \Box L \\
& -\frac{85}{64} L^{-\frac{5}{2}} (\mathcal{D}^a L)(\mathcal{D}^b L) \nabla_a \nabla_b L + \frac{1355}{1024} L^{-\frac{7}{2}} (\mathcal{D}^a L)(\mathcal{D}_a L)(\mathcal{D}^b L) \mathcal{D}_b L \\
& + \cdots \,.
\end{aligned} \tag{7.8}
$$

Here we note that given a field ϕ constrained to be a Lorentz scalar conformal primary ($K^a \phi = 0$) of dimension Δ ($\mathbb{D}\phi = \Delta\phi$), as for instance the fields L and D, one finds

$$\nabla_a \nabla_b \phi = \mathcal{D}_a \mathcal{D}_b \phi - 2\Delta \mathfrak{f}_{ab} \phi \,, \tag{7.9a}$$

$$\Box \phi = \mathcal{D}^2 \phi - 2\Delta \mathfrak{f}_a{}^a \phi \,, \tag{7.9b}$$

$$\nabla_a \Box \phi = \mathcal{D}_a \mathcal{D}^2 \phi - 2\Delta \mathfrak{f}_b{}^b \mathcal{D}_a \phi + 4(2 - \Delta) \mathfrak{f}_a{}^b \mathcal{D}_b \phi - 2\Delta (\mathcal{D}_a \mathfrak{f}_b{}^b) \phi \,, \tag{7.9c}$$

$$
\begin{aligned}
\Box^2 \phi = {}& \mathcal{D}^4 \phi + 8(2 - \Delta) \mathfrak{f}^{ab} \mathcal{D}_a \mathcal{D}_b \phi - 4(\Delta + 1) \mathfrak{f}_a{}^a \mathcal{D}^2 \phi \\
& + 4 \Big((2 - \Delta)(\mathcal{D}^a \mathfrak{f}_a{}^b) - \Delta(\mathcal{D}^b \mathfrak{f}_a{}^a) \Big) \mathcal{D}_b \phi \\
& - 2\Delta \Big((\mathcal{D}^2 \mathfrak{f}_a{}^a) - 2(\Delta + 2)(\mathfrak{f}_a{}^a)^2 - 4(\Delta - 2) \mathfrak{f}^{ab} \mathfrak{f}_{ab} \Big) \phi \,, \tag{7.9d}
\end{aligned}
$$

with

$$\Box := \nabla^a \nabla_a \,, \qquad \mathcal{D}^2 := \mathcal{D}^a \mathcal{D}_a \,, \qquad \mathcal{D}^4 := (\mathcal{D}^2)^2 \,, \tag{7.10}$$

and

$$\mathfrak{f}_{ab} = -\frac{1}{8}\mathcal{R}_{ab} + \frac{1}{80}\eta_{ab}\mathcal{R}\,, \qquad \mathfrak{f}_a{}^a = -\frac{1}{20}\mathcal{R}\,. \tag{7.11}$$

The action $S_{\text{new-linear}}$ based on (C.1) is locally superconformal invariant both in the standard and dilaton-Weyl backgrounds for conformal supergravity. In the second case, the reader should keep in mind that the D and T_{abc}^- fields are composite fields that should be expressed in terms of σ and H_{abc} by using (2.75). By looking at the relations (7.9) it is clear that the new invariant $S_{\text{new-linear}}$ includes $\mathcal{R}^{ab}\mathcal{R}_{ab}$ and $\mathcal{R}^2$ curvature squared terms. The exact contributions depend on the conformal supergravity background used. We leave a detailed analysis and application of the new invariant for future work but we now describe some simple properties of $S_{\text{new-linear}}$ in a standard-Weyl and dilaton-Weyl background, respectively.

7.2 The invariant in a standard-Weyl multiplet background, dynamically generated cosmological constant, dS_6 and AdS_6 solutions

Given two real constant parameters A and B, let us consider a linear combination

$$S = AS_{\text{EH}} + BS_{\text{new-linear}}\,, \tag{7.12}$$

of the Einstein-Hilbert term described by the action S_{EH} (4.5) for a linear multiplet, and the new invariant $S_{\text{new-linear}}$ constructed in the previous subsection. In this subsection we are going to analyse properties of the action (7.12) in a standard-Weyl multiplet background of conformal supergravity coupled only to a linear multiplet. We fix dilatations, special conformal transformations and $\text{SU}(2)_R \to \text{U}(1)_R$ by imposing the following gauge conditions

$$B_M = 0\,, \qquad L^{ij} = \delta^{ij}\,, \qquad L = 1\,, \tag{7.13a}$$

which in components imply

$$b_m = 0\,, \qquad L^{ij} = \delta^{ij}\,, \qquad L = 1\,, \qquad \varphi^i = 0\,. \tag{7.13b}$$

In this section, besides the fermions, we neglect the $\text{SU}(2)_R$ fields together with the E_a and T_{abc}^- fields. Then, once imposed these restrictions and the gauge fixing (7.13) in (4.5) and (7.8), up to total derivatives the action (7.12) becomes

$$S = -\int \mathrm{d}^6 x\, e\left\{\frac{2A}{5}\Big(\mathcal{R}+\frac{1}{3}D\Big) + \frac{B}{20}(\mathcal{R}+D)D + \frac{15B}{32}\Big(\mathcal{R}^{ab}\mathcal{R}_{ab} - \frac{21}{150}\mathcal{R}^2\Big)\right\} + \cdots\,. \tag{7.14}$$

It is then clear that, thanks to the D^2 term in the new invariant (7.8), the previous action possesses a remarkable feature compared to the two-derivative linear multiplet action (4.5): provided the constant $B \neq 0$, the dynamics for the D field is completely consistent without the need of a second compensating multiplet. Note that this feature was noticed also in [125] for models of 4D $\mathcal{N} = 2$ supergravity including an off-shell scalar curvature squared invariant constructed from a linear multiplet. What is also interesting about (7.14) is that

a cosmological constant is generated dynamically by integrating out the D field. This is especially remarkable since, to the best of our knowledge, no supersymmetric $\mathcal{N} = (1,0)$ cosmological constant is known.

Let us integrate out the auxiliary field D from (7.14). Its equation of motion implies

$$D = -\frac{4A}{3B} - \frac{1}{2}\mathcal{R} + \cdots, \tag{7.15}$$

where the ellipses refer to terms including $\mathrm{SU}(2)_R$ gauge fields, E_a, and T_{abc}^- fields, together with fermions, that we have neglected in this section. Inserting this result into (7.14) we obtain

$$S = \frac{2A}{3} \int \mathrm{d}^6 x \, e \left\{ \frac{2A}{15B} - \frac{1}{2}\mathcal{R} - \frac{45B}{64A}\mathcal{R}^{ab}\mathcal{R}_{ab} + \frac{15B}{128A}\mathcal{R}^2 \right\} + \cdots. \tag{7.16}$$

After defining the cosmological constant as

$$\Lambda := -\frac{2A}{15B}, \tag{7.17}$$

and choosing for convenience $A = 3/2$,

$$S = -\int \mathrm{d}^6 x \, e \left\{ \Lambda + \frac{1}{2}\mathcal{R} - \frac{3}{32\Lambda}\left(\mathcal{R}^{ab}\mathcal{R}_{ab} - \frac{1}{6}\mathcal{R}^2 \right) \right\} + \cdots. \tag{7.18}$$

Note that the dynamically generated cosmological constant Λ can take both positive and negative values. It is straightforward to prove that the previous action possesses both dS_6 and AdS_6 solutions. In fact, the relevant part of the metric equations of motion of (7.18) reads

$$\mathcal{G}_{mn} - g_{mn}\Lambda - \frac{3}{16\Lambda}\left(2\mathcal{R}_m{}^r\mathcal{R}_{nr} - \frac{1}{2}g_{mn}\mathcal{R}^{rs}\mathcal{R}_{rs} - \frac{1}{3}\mathcal{R}\mathcal{R}_{mn} + \frac{1}{12}g_{mn}\mathcal{R}^2 \right) = 0, \tag{7.19}$$

with $\mathcal{G}_{mn}$ the Einstein tensor. The trace of (7.19) reads

$$\mathcal{R} + 3\Lambda - \frac{3}{32\Lambda}\left(\mathcal{R}^{mn}\mathcal{R}_{mn} - \frac{1}{6}\mathcal{R}^2 \right) = 0. \tag{7.20}$$

Let us consider an ansatz for an $(\mathrm{A})\mathrm{dS}_6$ metric

$$\mathcal{R}_{mnrs} = k\,(g_{mr}g_{ns} - g_{ms}g_{nr}) \quad \Longrightarrow \quad \mathcal{R} = 30\,k, \tag{7.21}$$

for a given real constant parameter k. Then we find that the trace equation (7.20) implies

$$k = -\frac{1}{10}\Lambda, \tag{7.22}$$

which solves (7.19). Given that $\Lambda \neq 0$, we can have both dS_6 or AdS_6 solutions depending on the sign of Λ.

It is also straightforward to show that these solutions cannot be supersymmetric. Note that the gauge fixing conditions (7.13) leave the χ^i chiral matter fermion of the standard-Weyl multiplet in the spectrum. Neglecting contributions from the $\mathrm{SU}(2)_R$ and T_{abc}^- fields, the residual gauge preserving supersymmetry transformation of χ^i follow from (2.20f)

$$\delta\chi^i = \frac{1}{2}D\xi^i + \cdots . \tag{7.23}$$

A necessary condition for the dS_6 or AdS_6 backgrounds to possess unbroken supersymmetry is then $0 \equiv \delta\chi^i = D$. The D equation of motion (7.15) for these solutions reads

$$D = -\frac{4A}{3B} - \frac{1}{2}\mathcal{R} = 10\Lambda - 15\,k\,, \tag{7.24}$$

which implies that in order for us to be able to set $D = 0$ we must have

$$k = \frac{2}{3}\Lambda\,. \tag{7.25}$$

This result clearly conflicts with (7.22) given that $\Lambda \neq 0$. Thus the (A)dS$_6$ solution (7.22) cannot preserve supersymmetry. This is expected since it is known that the supersymmetric extensions of the dS_6 and AdS_6 symmetry groups $\mathrm{SO}(1,6)$ and $\mathrm{SO}(2,5)$ describe two real forms of the exceptional supergroup $\mathrm{F}(4)$, respectively $\mathrm{F}^1(4)$ and $\mathrm{F}^2(4)$, that possess 16 fermionic generators, see [126–130]. Hence it is necessary to consider extended 6D supergravities if one is interested in supersymmetric (A)dS$_6$ backgrounds. Nevertheless, we find intriguing the existence of the non-supersymmetric (A)dS$_6$ solutions triggered by the non-trivial higher-derivative term described in this section.

7.3 The invariant in a dilaton-Weyl background

A natural question to be asked is whether the new invariant constructed in this section is independent of the ones described in section 5. It is simple to see that the answer is yes. To prove this, it is first necessary to remember that all the invariants in section 5 were defined in a dilaton-Weyl background for conformal supergravity. Hence, to compare the results, in the new invariant described by (7.8) we need to express the D and T_{abc}^- fields in terms of the fields σ and H_{abc} of the tensor multiplet by using (2.75). It then suffices to compare the results in the conformal and $\mathrm{SU}(2)_R$ gauge given by (4.9). Besides the gauge conditions mentioned before, for simplicity we neglect all fields in the supergravity multiplet except the vielbein and the dilaton $L = \mathrm{e}^{-2v}$. In the equations in this subsection the ellipses will indicate all the terms neglected as described above. It turns out that by

using the following relations that are consequences of (7.9)

$$D|_{\sigma=1} = \frac{3}{4}\mathcal{R} + \cdots, \tag{7.26a}$$

$$\Box D|_{\sigma=1} = \frac{3}{4}\mathcal{D}^2\mathcal{R} + \frac{9}{20}\mathcal{R}^2 + \cdots, \tag{7.26b}$$

$$\nabla_a\nabla_b L = \mathcal{D}_a\mathcal{D}_b L + \mathcal{R}_{ab}L - \frac{1}{10}\eta_{ab}\mathcal{R}L + \cdots, \tag{7.26c}$$

$$\Box L = \mathcal{D}^2 L + \frac{2}{5}\mathcal{R}L + \cdots, \tag{7.26d}$$

$$\nabla_a\Box L = \mathcal{D}_a\mathcal{D}^2 L + \mathcal{R}_{ab}\mathcal{D}^b L + \frac{3}{10}\mathcal{R}\mathcal{D}_a L + \frac{2}{5}L\mathcal{D}_a\mathcal{R} + \cdots, \tag{7.26e}$$

$$\Box^2 L = \mathcal{D}^4 L + 2\mathcal{R}^{ab}\mathcal{D}_a\mathcal{D}_b L + \frac{4}{5}\mathcal{R}\mathcal{D}^2 L + \frac{6}{5}(\mathcal{D}^a\mathcal{R})\mathcal{D}_a L$$
$$+ L\left(\frac{2}{5}\mathcal{D}^2\mathcal{R} - \mathcal{R}^{ab}\mathcal{R}_{ab} + \frac{19}{50}\mathcal{R}^2\right) + \cdots, \tag{7.26f}$$

one can obtain from (7.8) the following Lagrangian for the new invariant in a dilaton-Weyl background

$$
\begin{aligned}
e^{-1}\mathcal{L}_{\text{new-linear}} = \mathrm{e}^{-v}\bigg[& \frac{25}{32}\mathcal{R}^{ab}\mathcal{R}_{ab} - \frac{1}{5}\mathcal{R}^2 - \frac{5}{16}\mathcal{D}^2\mathcal{R} + \frac{15}{8}\mathcal{R}^{ab}\mathcal{D}_a\mathcal{D}_b v - \frac{5}{16}\mathcal{R}\mathcal{D}^2 v \\
& - \frac{25}{16}\mathcal{R}^{ab}(\mathcal{D}_a v)\mathcal{D}_b v - \frac{5}{32}\mathcal{R}(\mathcal{D}^a v)\mathcal{D}_a v + \frac{5}{4}\mathcal{D}^4 v + \frac{5}{2}(\mathcal{D}^a v)\mathcal{D}_a\mathcal{D}^2 v \\
& - 5(\mathcal{D}^a v)\mathcal{D}^2\mathcal{D}_a v - \frac{35}{8}(\mathcal{D}^a\mathcal{D}^b v)\mathcal{D}_a\mathcal{D}_b v + \frac{5}{4}(\mathcal{D}^2 v)^2 \\
& - \frac{15}{8}(\mathcal{D}^a v)(\mathcal{D}^b v)\mathcal{D}_a\mathcal{D}_b v - \frac{25}{16}(\mathcal{D}^a v)(\mathcal{D}_a v)\mathcal{D}^2 v \\
& + \frac{35}{64}(\mathcal{D}^a v)(\mathcal{D}_a v)(\mathcal{D}^b v)\mathcal{D}_b v\bigg] \\
& + \cdots
\end{aligned}
\tag{7.27}
$$

It is evident that it not possible to express the previous invariant as a linear combination of the three invariants constructed in section 5. The new invariant appears multiplied by a different factor of the dilaton and it includes higher-derivative terms for v. We leave the analysis of possible dynamical features of the new invariant studied in this section for future work and now come back to analysing in detail physical properties of the Einstein-Gauss-Bonnet supergravity of section 6.

8   Spectrum of Einstein-Gauss-Bonnet supergravity about AdS$_3$ × S^3

In this section, we elaborate on the derivation of the spectrum of the Einstein-Gauss-Bonnet supergravity about the supersymmetric AdS$_3$ × S^3 vacuum whose main results were originally presented in [40]. Although the spectrum of the short multiplets can be inferred from [42, 43], the spectrum of the long multiplets computed here comprises new results. As the AdS energies of the long-multiplet states are not protected by shortening conditions and depend on the detail of the theory, they cannot be obtained by the group

theory based approach of [42]. We therefore adopted the same method as [43] by solving the linearized equations for supegravity fields about the supersymmetric AdS vacuum.

Before turning to the main technical discussion, it is important to underline that, in order to precisely adhere to the results presented in [40], in this section we decided to use the same conventions as [40]. These differ compared to the conventions we use elsewhere in this paper. To avoid possible confusion, the reader should consider results presented in this section as self-contained.

In particular, as in [40], in this section we adopt the conventions of [66, 76] with the exception of a sign difference in the parity transformation and, consequently, the Levi-Civita tensor (which agrees with the one in our paper).[16] Other differences of conventions used in this section are: the six-dimensional vector indices here are labeled by $\mu, \nu, \cdots$; the Lorentz and $\mathrm{SU}(2)_R$ connections and curvatures, which here will be denoted as $(\omega_\mu{}^{ab}, R_{\mu\nu}{}^{ab})$ instead of $(\omega_m{}^{ab}, \mathcal{R}_{mn}{}^{ab})$ and $(V_\mu{}^{ij}, F_{\mu\nu}{}^{ij})$ instead of $(\mathcal{V}_m{}^{ij}, \mathcal{R}_{mn}{}^{ij})$, all have a sign flip compared to the same tensors in the rest of the paper; the vielbein $e_\mu{}^a$ (up to the nomenclature for the curved 6D indices) and the field σ are identical to the ones in the rest of the paper; the gauge 2-form and its field strength 3-form here will be denoted by $B_{\mu\nu}$ and $H_{\mu\nu\rho}$ but they are otherwise identical to b_{mn} and h_{mnp} of the rest of the paper; the scalar field L^{ij} of the linear multiplet is identical to the one of the rest of the paper with the difference that in this section we use the definition $L^2 := L^{ij} L_{ij}$; the derivatives ∇_μ used in this section (which should not be confused with the locally superconformally covariant derivatives of the rest of the paper) are solely defined in terms of the metric and are the standard general coordinate covariant derivatives possessing no other connections other than the Levi-Civita connection.

Before continuing our analysis, it is also worth giving the Einstein-Hilbert and Gauss-Bonnet supergravity invariants in these conventions. After assuming the gauge fixing conditions

$$\sigma = 1, \quad L_{ij} = \frac{1}{\sqrt{2}} \delta_{ij} L, \quad \psi^i = 0, \quad b_\mu = 0, \tag{8.1}$$

and applying the decomposition of the original $\mathrm{SU}(2)_R$ gauge field V_μ^{ij} under the residual $\mathrm{U}(1)_R$ symmetry

$$V_\mu^{ij} = V_\mu'^{ij} + \frac{1}{2} \delta^{ij} V_\mu, \tag{8.2}$$

up to fermionic terms, the Lagrangian of Einstein-Hilbert supergravity takes the form

$$e^{-1}\mathcal{L}_{\mathrm{EH}} = LR + L^{-1}\partial_\mu L \partial^\mu L - \frac{1}{12} L H_{\mu\nu\rho} H^{\mu\nu\rho} - 4L Z_\mu \bar{Z}^\mu - \frac{1}{2} L^{-1} \mathrm{E}^\mu \mathrm{E}_\mu + \sqrt{2} \mathrm{E}^\mu V_\mu$$
$$+ \text{fermions}, \tag{8.3}$$

where we have defined

$$Z_\mu = V_\mu'^{11} + \mathrm{i} V_\mu'^{12}. \tag{8.4}$$

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[16]See appendix D to compare the conventions of [76] with the ones of the rest of our paper.

The off-shell Gauss-Bonnet invariant (6.4) and in the conformal and $SU(2)_R \to U(1)_R$ gauge takes the form

$$e^{-1}\mathcal{L}_{\text{GB}} = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2 + \frac{1}{2}R_{\mu\nu\rho\sigma}H^{\mu\nu\lambda}H^{\rho\sigma}{}_\lambda - R^{\mu\nu}H^2_{\mu\nu} + \frac{1}{6}RH^2$$
$$+ \frac{1}{144}(H^2)^2 - \frac{1}{8}(H^2_{\mu\nu})^2 + \frac{5}{24}H^4 + \frac{1}{4}\epsilon^{\mu\nu\rho\sigma\lambda\tau}B_{\mu\nu}R_{\rho\sigma}{}^\alpha{}_\beta(\omega_+)R_{\lambda\tau}{}^\beta{}_\alpha(\omega_+)$$
$$- 2\epsilon^{\mu\nu\rho\sigma\lambda\tau}B_{\mu\nu}F_{\rho\sigma}(Z)\bar{F}_{\lambda\tau}(Z) + \frac{1}{2}\epsilon^{\mu\nu\rho\sigma\lambda\tau}B_{\mu\nu}F_{\rho\sigma}(V)F_{\lambda\tau}(V)$$
$$+ \text{fermions}\,, \tag{8.5}$$

where

$$F_{\mu\nu}(Z) = 2\partial_{[\mu}Z_{\nu]} - 2iV_{[\mu}Z_{\nu]}\,, \quad F_{\mu\nu}(V) = 2\partial_{[\mu}V_{\nu]} + 4iZ_{[\mu}\bar{Z}_{\nu]}\,. \tag{8.6}$$

Note that $R_{\mu\nu}{}^\alpha{}_\beta(\omega_+)$, which later will also be simply denoted by $R^+{}_{\mu\nu}{}^\alpha{}_\beta$, is the Riemann tensor defined with a torsionfull connection $\omega_{\pm\mu}{}^{cd} := \omega_\mu{}^{cd} \pm \frac{1}{2}e_\mu{}^a H_a{}^{cd}.$[17]

The total Lagrangian of the Einstein-Gauss-Bonnet supergravity is parametrized as (for convenience we set $\kappa^2 = 1$)

$$\mathcal{L}_{\text{tot}} = \mathcal{L}_{\text{EH}} + \frac{\alpha'}{16}\mathcal{L}_{\text{GB}}\,. \tag{8.7}$$

Accordingly, the equation of motion for the dilaton field L is given by

$$R + L^{-2}\partial_\mu L\partial^\mu L - 2L^{-1}\Box L - \frac{1}{12}H^2 - 4Z_\mu\bar{Z}^\mu + \frac{1}{2}L^{-2}\text{E}^\mu\text{E}_\mu = 0\,, \tag{8.8}$$

where we underline that the GB invariant is independent of the dilaton field L. Using the equation above, the Einstein equation with Gauss-Bonnet correction can be written as

$$LR_{\mu\nu} = \nabla_\mu\nabla_\nu L - L^{-1}\partial_\mu L\partial_\nu L + \frac{1}{4}LH_{\mu\rho\lambda}H_\nu{}^{\rho\lambda} + 4LZ_{(\mu}\bar{Z}_{\nu)} + \frac{1}{2}L^{-1}\text{E}_\mu\text{E}_\nu$$
$$- \sqrt{2}\text{E}_{(\mu}V_{\nu)} + \frac{1}{2}g_{\mu\nu}(\sqrt{2}\text{E}_\rho V^\rho - L^{-1}\text{E}_\rho\text{E}^\rho) - \frac{\alpha'}{16}\mathcal{E}_{\mu\nu}\,, \tag{8.9}$$

where $\mathcal{E}_{\mu\nu}$ is the contribution from supersymmetric Gauss-Bonnet action and takes the form

$$\mathcal{E}_{\mu\nu} = 2RR_{\mu\nu} - 4R_{\mu\lambda}R^\lambda{}_\nu - 4R_{\mu\rho\nu\lambda}R^{\rho\lambda} + 2R_\mu{}^{\rho\lambda\sigma}R_{\nu\rho\lambda\sigma}$$
$$- \nabla_\sigma\nabla_\rho\big(H_{(\mu}{}^{\sigma\lambda}H^\rho{}_{\nu)\lambda}\big) + \frac{1}{2}R^\alpha{}_{\beta\rho\sigma}H_\alpha{}^\beta{}_{(\mu}H^{\rho\sigma}{}_{\nu)} + \frac{3}{2}R^\alpha{}_{\beta\sigma(\mu}H_{\nu)\lambda}{}^\sigma H_\alpha{}^{\beta\lambda}$$
$$- 2R_{\rho(\mu}H^2_{\nu)}{}^\rho - 2R^{\rho\sigma}H_{\mu\rho}{}^\lambda H_{\nu\sigma\lambda} - \frac{1}{2}\Box H^2_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\nabla_\rho\nabla_\lambda H^{2\rho\lambda} + \nabla_\rho\nabla_{(\mu}H^2_{\nu)}{}^\rho$$
$$+ \frac{1}{6}R_{\mu\nu}H^2 + \frac{1}{2}RH^2_{\mu\nu} + \frac{1}{6}g_{\mu\nu}\Box H^2 - \frac{1}{6}\nabla_\mu\nabla_\nu H^2 + \frac{1}{24}H^2H^2_{\mu\nu}$$
$$- \frac{1}{4}H^2_{\mu\lambda}H^{2\lambda}{}_\nu - \frac{1}{2}H_{\mu\lambda}{}^\alpha H_{\nu\sigma\alpha}H^{2\lambda\sigma} + \frac{5}{12}H_\mu{}^{\alpha\lambda}H_\nu{}^{\beta\sigma}H^2_{\alpha\beta,\lambda\sigma} + \frac{5}{6}H^2_{\mu\lambda,\beta\sigma}H_\nu^{2\beta,\lambda\sigma}$$
$$- 2\nabla_\beta\big(R^{+\lambda\tau\beta}{}_{(\mu}*H_{\nu)\lambda\tau}\big) - R^+{}_{\lambda\tau}{}^\beta{}_{(\mu}H_{\nu)\beta\sigma}*H^{\sigma\lambda\tau} - \frac{1}{2}g_{\mu\nu}\mathcal{L}_{\text{GB P-even term}}\,. \tag{8.10}$$

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[17]The fact that in (8.5) $R_{ab}{}^{cd}(\omega_+)$ appears instead of $R_{ab}{}^{cd}(\omega_-)$, as in (6.4), is simply due to the sign difference in the definition of the Lorentz connection and curvature in this section.

In the expression above, $\mathcal{L}_{\text{GB P−even term}}$ indicates the parity-even part of the GB Lagrangian (8.5) (meaning every term in (8.5) except the last three) and we have defined

$$H^{2\,\mu\lambda,\nu\sigma} := H^{\mu\lambda\rho}H_\rho{}^{\nu\sigma}, \quad H^{2\mu\nu} = g_{\lambda\sigma}H^{2\,\mu\lambda,\nu\sigma}\,. \tag{8.11}$$

The two-form equation of motion takes the form

$$0 = \nabla_\rho\big(LH^{\rho\mu\nu} + \frac{\alpha'}{8}S^{\rho\mu\nu}\big)$$
$$+ \frac{\alpha'}{16}\epsilon^{\mu\nu\rho\sigma\lambda\tau}\left(F_{\rho\sigma}(V)F_{\lambda\tau}(V) - 4F_{\rho\sigma}(Z)F_{\lambda\tau}(Z) + \frac{1}{4}R_{\rho\sigma}{}^\alpha{}_\beta(\omega_+)R_{\lambda\tau}{}^\beta{}_\alpha(\omega_+)\right)\,, \tag{8.12}$$

where $S^{\rho\mu\nu}$ is given by

$$S^{\rho\mu\nu} = -3R^{\lambda\sigma[\rho\mu}H^{\nu]}{}_{\lambda\sigma} + 6R^{\lambda[\rho}H_\lambda{}^{\mu\nu]} - RH^{\rho\mu\nu} - \frac{1}{12}H^2H^{\rho\mu\nu} + \frac{3}{2}H^{2\,\lambda[\rho}H_\lambda{}^{\mu\nu]}$$
$$- \frac{5}{2}H^{[\rho}{}_{\lambda\sigma}H^{2\,\mu|\lambda|,\nu]\sigma} + 3R^+{}_{\lambda\sigma}{}^{[\mu\nu}*H^{\rho]\lambda\sigma}\,. \tag{8.13}$$

The equations of motion for the auxiliary fields V_μ, Z_μ and its complex conjugate, and the gauge 4-form $b_{\mu\nu\rho\lambda}$, whose field strength is associated to E_μ, are given as follows:

$$LZ^\mu - \frac{\alpha'}{48}\epsilon^{\tau\rho\sigma\mu\nu\lambda}H_{\tau\rho\sigma}F_{\nu\lambda}(Z) = 0\,, \tag{8.14a}$$

$$\sqrt{2}V_\mu - L^{-1}\mathrm{E}_\mu = 0\,, \tag{8.14b}$$

$$\sqrt{2}\mathrm{E}^\mu - \frac{\alpha'}{24}\epsilon^{\tau\rho\sigma\mu\nu\lambda}H_{\tau\rho\sigma}F_{\nu\lambda}(V) = 0\,. \tag{8.14c}$$

It is well known that the off-shell 6D $\mathcal{N} = (1,0)$ Einstein-Hilbert action admits a maximally supersymmetric $\mathrm{AdS}_3 \times \mathrm{S}^3$ vacuum with a metric, 3-form flux, and dilaton field of the form

$$ds_6^2 = \varrho^2\big(ds_{\mathrm{AdS}_3}^2 + ds_{\mathrm{S}^3}^2\big)\,, \quad H_{(3)} = 2\varrho\big(\Omega_{\mathrm{AdS}_3} - \Omega_{\mathrm{S}^3}\big)\,, \quad L = 1\,, \tag{8.15}$$

where $ds_{\mathrm{AdS}_3}^2$ and $ds_{\mathrm{S}^3}^2$ are the metrics on the unit radius AdS_3 and S^3, and Ω_{AdS_3} and Ω_{S^3} denote their volume forms. This metric has vanishing Weyl tensor and Ricci scalar

$$C_{\mu\nu\rho\lambda} = 0\,, \quad R = 0\,, \tag{8.16}$$

similarly to the maximally supersymmetric $\mathrm{AdS}_5 \times \mathrm{S}^5$ solution in 10D IIB superstring theory. By substituting the ansatz (8.15) into the α'-corrected equations of motion given above, we have explicitly checked that (8.15) remains a solution. Moreover, the supersymmetry of the solution is also unaffected by the inclusion of the Gauss-Bonnet supergravity action, because the off-shell supersymmetry transformations are independent of the equations of motion.

In the following, we shall study the spectrum of fluctuations around the vacuum solution (8.15). We split the six-dimensional indices into to 3 external indices on AdS_3 labeled by $\hat\mu, \hat\nu, \hat\rho, \cdots$ and 3 internal indices on S_3 labeled by $\hat m, \hat n, \hat p, \cdots$. The metric fluctuations

around the supersymmetric AdS$_3$ × S^3 vacuum are parametrized as (here we adopt the strategy used in [43])

$$h_{\hat{\mu}\hat{\nu}} = H_{\hat{\mu}\hat{\nu}} + \bar{g}_{\hat{\mu}\hat{\nu}}M \,, \qquad \bar{g}^{\hat{\mu}\hat{\nu}}H_{\hat{\mu}\hat{\nu}} = 0 \,, \tag{8.17a}$$

$$h_{\hat{\mu}\hat{m}} = K_{\hat{\mu}\hat{m}} \,, \tag{8.17b}$$

$$h_{\hat{m}\hat{n}} = L_{\hat{m}\hat{n}} + \bar{g}_{\hat{m}\hat{n}}N \,, \quad \bar{g}^{\hat{m}\hat{n}}L_{\hat{m}\hat{n}} = 0 \,, \tag{8.17c}$$

where $\bar{g}_{\hat{\mu}\hat{\nu}}$ and $\bar{g}_{\hat{m}\hat{n}}$ are the metric of unit radius on AdS$_3$ and S^3, respectively. The zero- and two-form fields fluctuations are parametrized as

$$L = 1 + \phi \,, \quad b_{\hat{\mu}\hat{\nu}} = \epsilon_{\hat{\mu}\hat{\nu}\hat{\rho}}X^{\hat{\rho}} \,, \quad b_{\hat{m}\hat{n}} = \epsilon_{\hat{m}\hat{n}\hat{\rho}}U^{\hat{\rho}} \,, \quad b_{\hat{\mu}\hat{m}} = W_{\hat{\mu}\hat{m}} \,, \tag{8.18}$$

and for fluctuations associated with the one-forms V_μ and Z_μ, we retain the same symbols. Since E$_\mu$ is fully determined by V_μ via (8.14b), which should be used in (8.14c) to obtain a closed equation similar to (8.14a), we do not give a separate discussion on E$_\mu$. We impose the following gauge fixing conditions

$$\nabla^{\hat{m}}h_{\{\hat{m}\hat{n}\}} = 0 \,, \quad \nabla^{\hat{m}}h_{\hat{m}\hat{\mu}} = 0 \,, \quad \nabla^{\hat{m}}b_{\hat{m}\mu} = 0 \,, \quad \nabla^{\hat{m}}V_{\hat{m}} = 0 \,, \tag{8.19}$$

where $\{\cdots\}$ denotes the complete traceless symmetrization of a set of indices. As a consequence of the previous gauge conditions, the fluctuations can be expanded in terms of harmonic functions on S^3 as

$$H_{\hat{\mu}\hat{\nu}}(x,y) = \sum H_{\hat{\mu}\hat{\nu}}^{(\ell,0)}(x)Y^{(\ell,0)}(y) \,, \tag{8.20a}$$

$$M(x,y) = \sum M^{(\ell,0)}(x)Y^{(\ell,0)}(y) \,, \tag{8.20b}$$

$$K_{\hat{\mu}\hat{m}}(x,y) = \sum K_{\hat{\mu}}^{(\ell,\pm1)}(x)Y_\mu^{(\ell,\pm1)}(y) \,, \tag{8.20c}$$

$$L_{\hat{m}\hat{n}}(x,y) = \sum L^{(\ell,\pm2)}(x)Y_{\hat{m}\hat{n}}^{(\ell,\pm2)}(y) \,, \tag{8.20d}$$

$$N(x,y) = \sum N^{(\ell,0)}(x)Y^{(\ell,0)} \,, \tag{8.20e}$$

$$\phi(x,y) = \sum \phi^{(\ell,0)}(x)Y^{(\ell,0)} \,, \tag{8.20f}$$

$$X_{\hat{\mu}}(x,y) = \sum X_{\hat{\mu}}^{(\ell,0)}(x)Y^{(\ell,0)}(y) \,, \tag{8.20g}$$

$$U_{\hat{m}}(x,y) = \sum U^{(\ell,0)}(x)\partial_{\hat{m}}Y^{(\ell,0)}(y) \,, \tag{8.20h}$$

$$W_{\hat{\mu}\hat{m}} = \sum W_{\hat{\mu}}^{(\ell,\pm1)}(x)Y_{\hat{m}}^{(\ell,\pm1)}(y) \,, \tag{8.20i}$$

$$Z_{\hat{\mu}}(x,y) = \sum Z_{\hat{\mu}}^{(\ell,0)}(x)Y^{(\ell,0)}(y) \,, \tag{8.20j}$$

$$Z_{\hat{m}}(x,y) = \sum \left[Z^{(\ell,\pm1)}(x)Y_{\hat{m}}^{(\ell,\pm1)}(y) + Z^{(\ell,0)}(x)\partial_{\hat{m}}Y^{(\ell,0)}(y) \right] \,, \tag{8.20k}$$

$$V_{\hat{\mu}}(x,y) = \sum V_{\hat{\mu}}^{(\ell,0)}(x)Y^{(\ell,0)}(y) \,, \tag{8.20l}$$

$$V_{\hat{m}}(x,y) = \sum V^{(\ell,\pm1)}(x)Y_{\hat{m}}^{(\ell,\pm1)}(y) \,, \tag{8.20m}$$

where, in the parametrization we are using, we have denoted with x the coordinates of AdS$_3$ and with y the coordinates of S^3, respectively. Here we omitted several mathematical details with regard to harmonic expansion of the fluctuations which can be found in [43] (for

instance, all longitudinal gauge modes can be removed by a field dependent gauge transformation generated by the harmonic non-zero modes). The harmonic functions $Y^{(\ell_1,\ell_2)}$ with various spins satisfy

$$\nabla_y^2 Y^{(\ell,0)} = -\ell(2+\ell)Y^{(\ell,0)}\,, \tag{8.21a}$$

$$\epsilon_{\hat{m}}{}^{\hat{n}\hat{p}}\partial_{\hat{n}}Y^{(\ell,\pm1)}_{\hat{p}} = \pm(\ell+1)Y^{(\ell,\pm1)}_{\hat{m}}\,, \tag{8.21b}$$

$$\epsilon_{\hat{m}}{}^{\hat{p}\hat{q}}\nabla_{\hat{p}}Y^{(\ell,\pm2)}_{\hat{q}\hat{n}} = \pm(\ell+1)Y^{(\ell,\pm2)}_{\hat{m}\hat{n}}\,. \tag{8.21c}$$

Since the isometry group of S^3 is $SU(2)\times SU(2)$, the harmonic functions can also be labelled using the two $SU(2)$ quantum numbers denoted by $(j,\bar{j})$. The relation between $(j,\bar{j})$ and (ℓ_1,ℓ_2) is given by

$$j = \frac{1}{2}(\ell_1+\ell_2)\,, \quad \bar{j} = \frac{1}{2}(\ell_1-\ell_2)\,. \tag{8.22}$$

The AdS$_3$ part of the fluctuations can also be expanded in terms of harmonics on AdS$_3$. We denote the AdS$_3$ harmonics by $\Xi^{(E,s)}(x)$ where (E,s) are related to the $SL(2,\mathbb{R})\times SL(2,\mathbb{R})$ quantum numbers $(h,\bar{h})$ by

$$h = \frac{1}{2}(E+s)\,, \quad \bar{h} = \frac{1}{2}(E-s)\,. \tag{8.23}$$

Then we have

$$H^{(\ell,0)}_{\hat{\mu}\hat{\nu}}(x) = \sum \left[H^{(E,\pm2)\otimes(\ell,0)}\Xi^{(E,\pm2)}_{\hat{\mu}\hat{\nu}}(x) + H^{(E,\pm1)\otimes(\ell,0)}\nabla_{\{\hat{\mu}}\Xi^{(E,\pm1)}_{\hat{\nu}\}}(x) \right.$$
$$\left. + H^{(E,0)\otimes(\ell,0)}\nabla_{\{\hat{\mu}}\nabla_{\hat{\nu}\}}\Xi^{(E,0)}(x) \right]\,, \tag{8.24a}$$

$$M^{(\ell,0)}(x) = \sum M^{(E,0)\otimes(\ell,0)}\Xi^{(E,0)}(x)\,, \tag{8.24b}$$

$$K^{(\ell,\pm1)}_{\hat{\mu}} = \sum \left[K^{(E,\pm1)\otimes(\ell,\pm1)}\Xi^{(E,\pm1)}_{\hat{\mu}} + K^{(E,0)\otimes(\ell,\pm1)}\partial_{\hat{\mu}}\Xi^{(E,0)} \right]\,, \tag{8.24c}$$

$$L^{(\ell,\pm2)}(x) = \sum L^{(E,0)\otimes(\ell,\pm2)}\Xi^{(E,0)}(x)\,, \tag{8.24d}$$

$$N^{(\ell,0)}(x) = \sum N^{(E,0)\otimes(\ell,0)}\Xi^{(E,0)}(x)\,, \tag{8.24e}$$

$$\phi^{(\ell,0)}(x) = \sum \phi^{(E,0)\otimes(\ell,0)}\Xi^{(E,0)}(x)\,, \tag{8.24f}$$

$$X^{(\ell,0)}_{\hat{\mu}} = \sum \left[X^{(E,\pm1)\otimes(\ell,0)}\Xi^{(E,\pm1)}_{\hat{\mu}} + X^{(E,0)\otimes(\ell,0)}\partial_{\hat{\mu}}\Xi^{(E,0)} \right]\,, \tag{8.24g}$$

$$U^{(\ell,0)}(x) = \sum U^{(E,0)\otimes(\ell,0)}\Xi^{(E,0)}(x)\,, \tag{8.24h}$$

$$W^{(\ell,\pm1)}_{\hat{\mu}} = \sum \left[W^{(E,\pm1)\otimes(\ell,\pm1)}\Xi^{(E,\pm1)}_{\hat{\mu}} + W^{(E,0)\otimes(\ell,\pm1)}\partial_{\hat{\mu}}\Xi^{(E,0)} \right]\,, \tag{8.24i}$$

$$Z^{(\ell,0)}_{\hat{\mu}} = \sum \left[Z^{(E,\pm1)\otimes(\ell,0)}\Xi^{(E,\pm1)}_{\hat{\mu}} + Z^{(E,0)\otimes(\ell,0)}\partial_{\hat{\mu}}\Xi^{(E,0)} \right]\,, \tag{8.24j}$$

$$Z^{(\ell,\pm1)} = \sum Z^{(E,0)\otimes(\ell,\pm1)}\Xi^{(E,0)}\,, \tag{8.24k}$$

$$Z^{(\ell,0)} = \sum Z^{(E,0)\otimes(\ell,0)}\Xi^{(E,0)}\,, \tag{8.24l}$$

$$V^{(\ell,0)}_{\hat{\mu}} = \sum \left[V^{(E,\pm1)\otimes(\ell,0)}\Xi^{(E,\pm1)}_{\hat{\mu}} + V^{(E,0)\otimes(\ell,0)}\partial_{\hat{\mu}}\Xi^{(E,0)} \right]\,, \tag{8.24m}$$

$$V^{(\ell,\pm1)} = \sum V^{(E,0)\otimes(\ell,\pm1)}\Xi^{(E,0)}\,, \tag{8.24n}$$

where the harmonic functions $\Xi^{(E,s)}$ on AdS$_3$ satisfy

$$\nabla_x^2 \Xi^{(E,0)} = -E(E-2)\Xi^{(E,0)}\,, \tag{8.25a}$$

$$\epsilon_{\hat{\mu}}{}^{\hat{\nu}\hat{\rho}}\partial_{\hat{\nu}}\Xi_{\hat{\rho}}^{(E,\pm1)} = \pm(E-1)\Xi_{\hat{\mu}}^{(E,\pm1)}\,, \tag{8.25b}$$

$$\epsilon_{\hat{\mu}}{}^{\hat{\rho}\hat{\sigma}}\nabla_{\hat{\rho}}\Xi_{\hat{\sigma}\hat{\nu}}^{(E,\pm2)} = \pm(E-1)\Xi_{\hat{\mu}\hat{\nu}}^{(E,\pm2)}\,. \tag{8.25c}$$

The harmonic functions on S^3 and AdS$_3$ are known explicitly, and we have implemented them in a *Mathematica* code. After substituting the harmonic expansion of the fluctuations to the equations of motion and noticing that modes with different quantum numbers decouple from each other, we can derive a set of algebraic equations relating E to ℓ, from which we can solve E in terms of ℓ. The results are schematically listed below where we have also introduced the dimensionless parameter

$$\tilde{\alpha} := \alpha'/\varrho^2\,. \tag{8.26}$$

Without any ambiguity and for succinctness, in the following we suppress the (E,ℓ) in the labelling of various expansion coefficients.

- $(|s|,\ell_2) = (2,0)$ **sector**

  We obtain

$$(E-2-\ell)\left(1+\tilde{\alpha}-\frac{1}{2}E\tilde{\alpha}\right)H^{(2,0)} = 0\,, \quad (E-2-\ell)\left(\frac{1}{2}E\tilde{\alpha}+1\right)H^{(-2,0)} = 0\,, \tag{8.27}$$

  from which we find 4 towers of propagating degrees of freedom labelled by their quantum numbers

$$\left((E=2+\ell,2)\oplus\left(E=2+\frac{2}{\tilde{\alpha}},2\right)\oplus(E=2+\ell,-2)\oplus\left(E=-\frac{2}{\tilde{\alpha}},-2\right)\right)\otimes(\ell,0)\,. \tag{8.28}$$

- $(|s|,|\ell_2|) = (1,1)$ **sector**

  In this sector, from the $g_{\hat{\mu}\hat{\nu}}$ and $B_{\hat{\mu}\hat{\nu}}$ equations, we obtain

$$\begin{pmatrix} A_{\pm\pm} & B_{\pm\pm} \\ B_{\pm\pm} & A_{\pm\pm} \end{pmatrix}\begin{pmatrix} K^{(\pm1,\pm1)} \\ W^{(\pm1,\pm1)} \end{pmatrix} = 0\,, \tag{8.29}$$

where the elements of the 2×2 mixing matrices are given by

$$A_{++} = 2 - E + \ell + \left(1 - \frac{1}{2}E + \frac{3}{2}\ell + \frac{1}{4}(E - \ell)^2\right)\tilde{\alpha}, \tag{8.30a}$$

$$A_{+-} = (E - 2 - \ell)\left(E + \ell + \left(1 - \frac{1}{2}(E - \ell) - \frac{1}{4}(E + \ell)^2\right)\tilde{\alpha}\right), \tag{8.30b}$$

$$A_{-+} = (E - 2 - \ell)\left(E + \ell + \left(1 - \frac{1}{2}(E - \ell) + \frac{1}{4}(E + \ell)^2\right)\tilde{\alpha}\right), \tag{8.30c}$$

$$A_{--} = 2 - E + \ell - \left(1 - \frac{3}{2}E + \frac{1}{2}\ell + \frac{1}{4}(E - \ell)^2\right)\tilde{\alpha}, \tag{8.30d}$$

$$B_{++} = \frac{1}{4}\left(E^2 - (2 + \ell)^2\right)\tilde{\alpha} - 2, \tag{8.30e}$$

$$B_{+-} = (E - 2 - \ell)\left(2 + \frac{1}{4}(\ell^2 - E^2)\tilde{\alpha}\right), \tag{8.30f}$$

$$B_{-+} = -(E - 2 - \ell)\left(2 + (E + \ell)\left(1 - \frac{1}{4}(E - \ell)\right)\tilde{\alpha}\right), \tag{8.30g}$$

$$B_{--} = 2 + \frac{1}{4}\left(\ell^2 - (E - 2)^2\right)\tilde{\alpha}. \tag{8.30h}$$

The existence of nontrivial solutions for $K^{(1,1)}$ and $W^{(1,1)}$ requires

$$(E - 4 - \ell)(E - \ell)\left(-1 - \frac{1}{2}\tilde{\alpha} + \frac{1}{2}E\tilde{\alpha}\right) = 0, \tag{8.31}$$

which leads to 3 infinite towers of propagating degrees of freedom

$$\left((E = 4 + \ell, 1) \oplus (E = \ell, 1) \oplus \left(E = 1 + \frac{2}{\tilde{\alpha}}, 1\right)\right) \otimes (\ell, 1). \tag{8.32}$$

The existence of nontrivial solutions for $K^{(1,-1)}$ and $W^{(1,-1)}$ requires

$$(E - 2 - \ell)^2\left(-1 - \frac{1}{2}\tilde{\alpha} + \frac{1}{2}E\tilde{\alpha}\right) = 0, \tag{8.33}$$

which leads to 3 infinite towers of propagating degrees of freedom

$$\left(2 \times (E = 2 + \ell, 1) \oplus \left(E = 1 + \frac{2}{\tilde{\alpha}}, 1\right)\right) \otimes (\ell, -1). \tag{8.34}$$

The existence of nontrivial solutions for $K^{(-1,1)}$ and $W^{(-1,1)}$ requires

$$(E - 2 - \ell)^2\left(1 - \frac{1}{2}\tilde{\alpha} + \frac{1}{2}E\tilde{\alpha}\right) = 0, \tag{8.35}$$

which leads to 3 infinite towers of propagating degrees of freedom

$$\left(2 \times (E = 2 + \ell, -1) \oplus \left(E = 1 - \frac{2}{\tilde{\alpha}}, -1\right)\right) \otimes (\ell, 1). \tag{8.36}$$

The existence of nontrivial solutions for $K^{(-1,-1)}$ and $W^{(-1,-1)}$ requires

$$(E - 4 - \ell)(E - \ell)\left(1 - \frac{1}{2}\tilde{\alpha} + \frac{1}{2}E\tilde{\alpha}\right) = 0, \tag{8.37}$$

which leads to 3 infinite towers of propagating degrees of freedom

$$\left((E = 4 + \ell\,, -1) \oplus (E = \ell\,, -1) \oplus \left(E = 1 - \frac{2}{\tilde{\alpha}}\,, -1 \right) \right) \otimes (\ell\,, -1)\,. \qquad (8.38)$$

- $(|s|\,, |\ell_2|) = (1\,, 0)$ **sector**

  In this sector we obtain the following conditions

  $$\tilde{\alpha} X^{(1,0)} - \left(4 - \tilde{\alpha}(E-3) \right) H^{(1,0)} = 0\,, \qquad (8.39\text{a})$$

  $$\tilde{\alpha}(E-3)(E+1) H^{(1,0)} - \left(4 - \tilde{\alpha}(E+1) \right) X^{(1,0)} = 0\,, \qquad (8.39\text{b})$$

  $$\left(4 + \tilde{\alpha}(E+1) \right) H^{(-1,0)} - \tilde{\alpha} X^{(-1,0)} = 0\,, \qquad (8.39\text{c})$$

  $$\tilde{\alpha}(E-3)(E+1) H^{(-1,0)} - \left(4 + \tilde{\alpha}(E-3) \right) X^{(-1,0)} = 0\,. \qquad (8.39\text{d})$$

  Nontrivial solutions for $H^{(\pm 1,0)}$ and $X^{(\pm 1,0)}$ exist if

  $$E = 1 \pm \frac{2}{\tilde{\alpha}}\,. \qquad (8.40)$$

  From the equations of motion of Z_μ and V_μ, it can readily be deduced that nonvanishing $Z^{(\pm 1,0)}$ and $V^{(\pm 1,0)}$ requires

  $$E = 1 \pm \frac{2}{\tilde{\alpha}}\,. \qquad (8.41)$$

  In total, this sector contains 8 infinite towers of propagating degrees of freedom

  $$\left(4 \times \left(E = 1 \pm \frac{2}{\tilde{\alpha}}\,, \pm 1 \right) \right) \otimes (\ell\,, 0)\,. \qquad (8.42)$$

- $(s\,, |\ell_2|) = (0\,, 2)$ **sector**

  We obtain the conditions

  $$(E - 2 - \ell) \left(\frac{1}{2} \ell \tilde{\alpha} + 1 \right) L^{(0,2)} = 0\,, \quad (E - 2 - \ell) \left(1 - \tilde{\alpha} - \frac{1}{2} \ell \tilde{\alpha} \right) L^{(0,-2)} = 0\,. \qquad (8.43)$$

  For $L^{(0,\pm 2)}$ to be nonvanishing, it is necessary to impose

  $$E = 2 + \ell\,. \qquad (8.44)$$

  Thus this sector contains 2 infinite towers of propagating degrees of freedom

  $$(E = 2 + \ell\,, 0) \otimes (\ell\,, \pm 2)\,. \qquad (8.45)$$

- $(s\,, |\ell_2|) = (0\,, 1)$ **sector**

  Explicit computation shows that this sector does not contain any dynamical degrees of freedom.

- **$(s\,,\ell_2) = (0\,,0)$ sector**

  In this sector we obtain the following conditions

$$0 = \big(1 + \tfrac{\tilde{\alpha}}{2}\big)\Big(E(E-2) + 3\ell(\ell+2)\Big)H^{(0,0)} - 3(1+2\tilde{\alpha})M^{(0,0)} - 9N^{(0,0)}$$
$$- 6\phi^{(0,0)} + \tfrac{3}{2}(E+\ell)(E-\ell-2)\tilde{\alpha}X^{(0,0)}, \tag{8.46a}$$

$$0 = \Big(24 + 6E - 3E^2 + 8\ell + 4\ell^2 + \big(24 + 6E - 3E^2 + 14\ell + 7\ell^2\big)\tilde{\alpha}\Big)N^{(0,0)}$$
$$- \Big(12\ell(\ell+2) + \tilde{\alpha}\ell(\ell+2)\big(12 + 2E - E^2 + 2\ell + \ell^2\big)\Big)U^{(0,0)}$$
$$+ 3(\ell+2)\ell M^{(0,0)} + 2\ell(\ell+2)\phi^{(0,0)}, \tag{8.46b}$$

$$0 = 3M^{(0,0)} + (1-2\tilde{\alpha})N^{(0,0)} + 2\phi^{(0,0)} - \tfrac{\tilde{\alpha}}{2}(E+\ell)(E-\ell-2)U^{(0,0)}, \tag{8.46c}$$

$$0 = (E-3)(E+1)H^{(0,0)} - 3\big(M^{(0,0)} + N^{(0,0)} + \phi^{(0,0)} - V^{(0,0)} - U^{(0,0)}\big), \tag{8.46d}$$

$$0 = \tfrac{\tilde{\alpha}}{2}(E+\ell)(E-\ell-2)(E+1)(E-3)H^{(0,0)} - 9N^{(0,0)} - 6\phi^{(0,0)}$$
$$- 3\left((E+\ell)(E-\ell-2) - \tfrac{\tilde{\alpha}}{2}(5E^2 - 10E - 2\ell - \ell^2)\right)V^{(0,0)}$$
$$+ \left(9 - \tfrac{3\tilde{\alpha}}{2}(12 - 2E + E^2 - 2\ell - \ell^2)\right)M^{(0,0)}, \tag{8.46e}$$

$$0 = -3M^{(0,0)} - 2\phi^{(0,0)} + \left(3 + \tfrac{\tilde{\alpha}}{2}(12 + 2E - E^2 + 2\ell + \ell^2)\right)N^{(0,0)}$$
$$+ \left((E+\ell)(E-\ell-2) + \tfrac{\tilde{\alpha}}{2}(E^2 - 2E - 5\ell^2 - 10\ell)\right)U^{(0,0)}. \tag{8.46f}$$

After diagonalizing the equations above, we obtain the following condition

$$(E-\ell-4)^2(E-\ell)^2(E+\ell-2)^2(E+\ell+2)^2\left(\tfrac{\tilde{\alpha}}{2}E - 1\right)\left(1 - \tilde{\alpha} + \tfrac{\tilde{\alpha}}{2}E\right)\left(\tfrac{\tilde{\alpha}}{2}\ell - 1\right)\left(1 + \tilde{\alpha} + \tfrac{\tilde{\alpha}}{2}\ell\right) = 0. \tag{8.47}$$

Therefore, this sector contains 6 infinite towers of propagating degrees of freedom

$$\left(2 \times \big(E = 4 + \ell\,,0\big) \oplus 2 \times \big(E = \ell\,,0\big) \oplus \left(E = \tfrac{2}{\tilde{\alpha}}\,,0\right) \oplus \left(E = 2 - \tfrac{2}{\tilde{\alpha}}\,,0\right)\right) \otimes (\ell\,,0). \tag{8.48}$$

We now proceed to arrange the states above into multiplets of $\mathrm{SU}(1,1|2)$. The spectrum of the 2-derivative theory (8.3) has been studied before in various works [42, 43]. It contains only the short multiplets of $\mathrm{SU}(1,1|2)$ dressed by irreducible representations of the extra $\mathrm{SL}(2,\mathbb{R}) \times \mathrm{SU}(2)$. A short multiplet of $\mathrm{SU}(1,1|2)$ has the structure

$$(h,j)\,, \quad 2 \times \left(h + \tfrac{1}{2}, j - \tfrac{1}{2}\right), \quad (h+1, j-1)\,, \quad h = j, \tag{8.49}$$

where h and j label the representations of the $\mathrm{SL}(2,\mathbb{R}) \times \mathrm{SU}(2)$ bosonic subgroup inside $\mathrm{SU}(1,1|2)$. Since the total isometry group associated with the maximally supersymmetric $\mathrm{AdS}_3 \times \mathrm{S}^3$ vacuum is $\mathrm{SU}(1,1|2) \times \mathrm{SL}(2,\mathbb{R}) \times \mathrm{SU}(2)$, we also introduce $(\bar{h}, \bar{j})$ to label the

irreducible representations of the extra $\mathrm{SL}(2,\mathbb{R}) \times \mathrm{SU}(2)$ group. Eventually, we denote the short multiplet (8.49) by $\mathrm{DS}^{(\bar{h},\bar{j})}(h,j)_\mathrm{S}$. The spectrum of the 2-derivative theory (8.3) consists of the following multiplets labeled by an integer $n \geq 0$

$$
\mathrm{DS}^{(\frac{n+4}{2},\frac{n}{2})}\left(\frac{n}{2},\frac{n}{2}\right)_\mathrm{S}, \qquad \mathrm{DS}^{(\frac{n+1}{2},\frac{n+1}{2})}\left(\frac{n+3}{2},\frac{n+3}{2}\right)_\mathrm{S}, \quad \mathrm{DS}^{(\frac{n+3}{2},\frac{n+3}{2})}\left(\frac{n+1}{2},\frac{n+1}{2}\right)_\mathrm{S},
$$
$$
\mathrm{DS}^{(\frac{n+4}{2},\frac{n}{2})}\left(\frac{n+2}{2},\frac{n+2}{2}\right)_\mathrm{S}, \quad \mathrm{DS}^{(\frac{n+2}{2},\frac{n+2}{2})}\left(\frac{n+2}{2},\frac{n+2}{2}\right)_\mathrm{S}, \quad \mathrm{DS}^{(\frac{n+4}{2},\frac{n}{2})}\left(\frac{n+4}{2},\frac{n+4}{2}\right)_\mathrm{S}.
\tag{8.50}
$$

We see from the spectrum of the Einstein-Gauss-Bonnet supergravity that the $\tilde{\alpha}$-independent spectrum fits nicely into the multiplets structure above. In fact the short multiplets are still present in the EGB theory and are unaffected by α'-corrections, which has to be the case since BPS conditions render these multiplets protected. Note that we have not studied the fermionic spectrum here. However, there seems to be a unique way to arrange the bosonic spectrum into the supermultiplets which strongly indicates that the fermions should just naturally arrange to complete the supermultiplets we have obtained. We leave the explicit calculation of the fermionic spectrum to future work.

On the top of the infinite tower of short multiplets described above, the inclusion of the supersymmetric Gauss-Bonnet invariant gives rise to 4 new long multiplets possessing the following structure

$$
(h,j), \qquad 2 \times \left(h + \frac{1}{2}, j - \frac{1}{2}\right), \quad (h+1, j-1),
$$
$$
\left(h + \frac{1}{2}, j + \frac{1}{2}\right), \quad 2 \times (h+1, j), \qquad \left(h + \frac{3}{2}, j - \frac{1}{2}\right).
\tag{8.51}
$$

The 4 long multiplets whose AdS_3 energies are independent of the KK level are denoted by

$$
\mathrm{DS}^{(\frac{1}{\tilde{\alpha}},\frac{n}{2})}\left(\frac{1}{\tilde{\alpha}} + \frac{1}{2}, \frac{n+1}{2}\right)_\mathrm{L}, \qquad \mathrm{DS}^{(1-\frac{1}{\tilde{\alpha}},\frac{n}{2})}\left(-\frac{1}{\tilde{\alpha}} - 1, \frac{n}{2}\right)_\mathrm{L}
$$
$$
\mathrm{DS}^{(1-\frac{1}{\tilde{\alpha}},\frac{n}{2})}\left(-\frac{1}{\tilde{\alpha}} - \frac{1}{2}, \frac{n+1}{2}\right)_\mathrm{L}, \quad \mathrm{DS}^{(\frac{1}{\tilde{\alpha}},\frac{n}{2})}\left(\frac{1}{\tilde{\alpha}}, \frac{n}{2}\right)_\mathrm{L}, \qquad n \geq 0.
\tag{8.52}
$$

For a $\mathrm{SU}(1,1|2)$ long multiplet, h is not equal to j and unitarity requires $h > j$.

In summary, (8.50) and (8.52) comprise the full spectrum of the Einstein-Gauss-Bonnet supergravity about the supersymmetric $\mathrm{AdS}_3 \times \mathrm{S}^3$ vacuum, where it is understood that states with negative $\mathrm{SU}(2)$ label should be removed. The massless states are the usual graviton and a massless scalar residing in the two multiplets

$$
\mathrm{DS}^{2,0}(0,0)_\mathrm{S}, \qquad \mathrm{DS}^{1,1}(1,1)_\mathrm{S},
\tag{8.53}
$$

consistent with the results of [43]. Although gauge fields in AdS_3 do not have local degrees of freedom, we prefer to keep the massless graviton in the spectrum as it carries information about the non-trivial boundary dynamics. The fact that the α' dependent states can be nicely arranged into $\mathrm{SU}(1,1|2)$ long multiplets provides a further check of the supersymmetry of the GB invariant.

9 Conclusion

In this paper we have described the supersymmetric completion of several curvature-squared invariants for $\mathcal{N} = (1,0)$ supergravity in six dimensions. By employing the dilaton-Weyl multiplet of conformal supergravity we have described supersymmetric completions for the three possible purely gravitational curvature-squared terms, Riemann, Ricci, and scalar curvature-squared, where in the last case a coupling to a conformal compensator, which we have chosen to be a linear multiplet, was necessary.

We also constructed a novel locally superconformal invariant based on a higher-derivative action for the linear multiplet which can be defined both in the standard-Weyl or dilaton-Weyl multiplet for conformal supergravity leading to new classes of curvature-squared terms in both cases. In the case of the dilaton-Weyl multiplet, the new invariant leads to the Lagrangian (7.27), which includes Ricci and scalar curvature-squared terms together with a nontrivial dependence on the dilaton field, as for instance an overall multiplicative factor of e^{-v}, which clearly distinguishes this invariant from the other three invariants described in section 5.

To our knowledge, our analysis of curvature-squared invariants for $\mathcal{N} = (1,0)$ supergravity in six dimensions is the most complete to date and it has already allowed us to study some interesting applications of these results. For instance, by extending the results presented in [40], we described the Gauss-Bonnet invariant in detail and added to it the off-shell supersymmetric extension of the Einstein-Hilbert term to obtain the Einstein-Gauss-Bonnet supergravity, which plays a central role in the effective low-energy description of α'-corrected string theory compactified to six dimensions. We gave the supersymmetry transformations for the on-shell Einstein-Gauss-Bonnet supergravity for the first time, showing that up to cubic fermion terms there is no α' correction. Moreover, we provided a detailed analysis of the spectrum about the $\mathrm{AdS}_3 \times \mathrm{S}^3$ solution which is relevant to holographic studies.

As an application of the new invariant described in section 7, we have shown how a linear combination of such an invariant and the supersymmetric Einstein-Hilbert term leads to a dynamically generated cosmological constant and non-supersymmetric $(\mathrm{A})\mathrm{dS}_6$ solutions. This result was based on the fact that in a standard-Weyl multiplet there exist D^2 terms in the action for the auxiliary field D. Such a term is remarkable since, unlike the pure Einstein-Hilbert supergravity, the equation of motion for D is consistent even in a standard-Weyl multiplet background and remains the case when coupled to the supersymmetric Einstein-Hilbert term. Moreover, D remains an auxiliary field even in the higher-derivative theory and can be algebraically integrated out leading to a cosmological constant term on-shell. To underline the importance of this mechanism in regards to the cosmological constant, we should stress that, to our knowledge, no supersymmetric pure cosmological constant term has ever been constructed in the literature for 6D $\mathcal{N} = (1,0)$ minimal supergravity.

We believe that our studies open the avenue for various generalizations and, as already pointed out in the introduction, we expect the results in our paper might find several applications. Let us now briefly comment on some of these possibilities.

The fact that all the invariants presented in this paper possess manifest off-shell supersymmetry makes it trivial to add them to other known off-shell models and matter couplings. For instance, it is straightforward to consider vector multiplets coupled to the higher-derivative supergravity invariants in our paper. Since in our paper we have focused on Poincaré supergravity models based on the use of a compensating linear multiplet, which preserves a $\mathrm{U}(1)_R$ symmetry off-shell, it is clear that in general the addition of a coupling between the vector multiplets and the linear multiplet [48] on-shell will generate a potential for the dilaton field together with a gauged R-symmetry exactly as in [66]. It was noted in [66] that the simple case of the gauged minimal 6D supergravity, where only one vector multiplet is coupled to the minimal ungauged supergravity (reviewed in section 4), leads to a version of the Salam-Sezgin model. It is straightforward to show that the supersymmetric Minkowski$_4 \times$ S^2 solution is preserved also when any of the three curvature-squared invariants of section 5, including the Gauss-Bonnet one, is added to the two-derivative gauged model but it would be interesting to study the stability properties of this background in the presence of the higher-derivative terms. For instance, the perturbative stability of the Salam-Sezgin model extended by the Riemann squared invariant was studied in [67], where it was found that the inclusion of the supersymmetric Riemann squared term introduces tachyonic modes around the supersymmetric Minkowski$_4 \times$ S^2 solution. We expect that this will not be the case if the Gauss-Bonnet invariant instead of the Riemann squared invariant is added to the original Salam-Sezgin model.

More generally, it would be of interest to extend the analysis of our paper by considering the higher-derivative dynamics of general systems of vector multiplets, hypermultiplets, and tensor multiplets coupled to conformal supergravity. For instance, a fully α'-corrected description of the gauged supergravity of the Salam-Sezgin model might include many others curvature-squared terms including F^4 interactions. It is then natural to seek off-shell $(1,0)$ supersymmetric extensions of four-derivative terms for vector multiplets by using our techniques and extending the 5D analysis of [131]. To study general couplings, including higher-derivative ones, of off-shell hypermultiplets to conformal supergravity one could use the projective superspace approach of [83], based on the formalism developed in the last decade for supergravity theories with eight supercharges in $2 \leq \mathrm{D} \leq 5$, see, e.g., [61, 87, 132–141] for relevant papers on the subject. On the other hand, the off-shell description of a general system of $(1,0)$ tensor multiplets is less developed and would be a very interesting avenue of research. There have been proposals for an off-shell extension of the tensor multiplets which, similarly to the off-shell 6D charged hypermultiplets of [83], includes an infinite number of auxiliary fields, see [116] and more recently [83], which could be used to construct general higher-derivative interactions for the tensor multiplets in a standard-Weyl multiplet background for conformal supergravity. It is natural to then wonder if and how one could use these multiplets to describe general higher-derivative interactions, including the curvature-squared ones, based solely on a standard-Weyl multiplet and not the dilaton-Weyl used for the invariants in section 5. In this framework it would be intriguing to verify whether the mechanisms of consistency of the dynamics of D and for dynamical generation of a cosmological constant described in section 7 remain general features of curvature-squared models in the standard-Weyl multiplet.

Compactifications of the 6D GB invariant to 5 and 4 dimensions are also potentially interesting for various purposes. First of the all, the dimensional reduction leads to lower dimensional GB invariants coupled to matter multiplets, which can be viewed as super-symmetrizaton of a particular class of the Horndeski scalar-tensor model widely studied in cosmology. The matter multiplets can be consistently truncated out preserving off-shell supersymmetry and the resulting invariants based solely on the off-shell supergravity multiplet can be compared to the existing supersymmetrization of GB invariants in 5D [60] and 4D [58]. In the 4D STU model, [142] proposed a string-string-string duality based on the properties of the 2-derivative Lagrangian. In the context of string theory, this duality should persist to all order in the α' expansion. A first nontrivial check can be performed using the 2-torus reduction of the 6D Einstein-Gauss-Bonnet invariant.

Finally, we recall that the 6D two-derivative supergravity admits supersymmetric black strings and black rings with AdS_3 near horizon geometry. Using the results presented in this paper, one can extend those results to include the leading higher-derivative corrections. Upon circle reductions, the 6D solutions give rise to black holes, black strings and black rings in 5D supergravities. The recent work [143] has made an attempt to systematically analyze the geometry of black hole solutions in the presence of supersymmetric curvature squared terms. Their work utilized off-shell curvature squared invariants based on the standard-Weyl multiplet which are different from the curvature squared invariants coming from the reduction of the 6D invariants. However, on physical grounds, the different formulations of the curvature squared invariants should yield the same physical quantities for the same solution. Therefore, it might be worthwhile to carry out this comparison in more detail. Due to the close relation between 6D and 5D, one can also look for the 6D analog of the non-renormalisation theorem for 6D black string entropy. For 5D black string with AdS_3 near horizon geometry, the non-renormalisation theorem ensures that the entropy of such objects does not receive corrections from terms with more than 4-derivatives (see [144] for a review and references therein). Eventually, for a better understanding of the microstates underlying the black strings, it is indispensable to embed the 6D solution into 10D string theory. In fact, such an embedding is not unique and may lead to interesting physics regarding the apparent different 10D descriptions of the same 6D solutions. We leave these interesting problems for future investigation.

Acknowledgments

A Conformal superspace identities

In this appendix we collect results about conformal superspace in the traceless frame of [70] that are of importance for this paper.

The Lorentz generators act on the superspace covariant derivatives $\nabla_A = (\nabla_a, \nabla_\alpha^i)$ as

$$[M_{ab}, M_{cd}] = 2\eta_{c[a}M_{b]d} - 2\eta_{d[a}M_{b]c}\,, \tag{A.1a}$$

$$[M_{ab}, \nabla_c] = 2\eta_{c[a}\nabla_{b]}\,, \tag{A.1b}$$

$$[M_\alpha{}^\beta, \nabla_\gamma^k] = -\delta_\gamma^\beta \nabla_\alpha^k + \frac{1}{4}\delta_\alpha^\beta \nabla_\gamma^k\,, \tag{A.1c}$$

where $M_\alpha{}^\beta = -\frac{1}{4}(\gamma^{ab})_\alpha{}^\beta M_{ab}$. The $\mathrm{SU}(2)_R$ and dilatation generators obey

$$[J^{ij}, J^{kl}] = \varepsilon^{k(i}J^{j)l} + \varepsilon^{l(i}J^{j)k}\,, \quad [J^{ij}, \nabla_\alpha^k] = \varepsilon^{k(i}\nabla_\alpha^{j)}\,, \tag{A.1d}$$

$$[\mathbb{D}, \nabla_a] = \nabla_a\,, \qquad\qquad [\mathbb{D}, \nabla_\alpha^i] = \frac{1}{2}\nabla_\alpha^i\,. \tag{A.1e}$$

The Lorentz and $\mathrm{SU}(2)_R$ generators act on the special conformal generators $K^A = (K^a, S_i^\alpha)$ as

$$[M_{ab}, K^c] = 2\delta_{[a}^c K_{b]}\,, \quad [M_\alpha{}^\beta, S_k^\gamma] = \delta_\alpha^\gamma S_k^\beta - \frac{1}{4}\delta_\alpha^\beta S_k^\gamma\,, \quad [J^{ij}, S_k^\gamma] = \delta_k^{(i}S^{\gamma j)}\,, \tag{A.1f}$$

while the dilatation generator acts on K^A as

$$[\mathbb{D}, K^a] = -K^a\,, \quad [\mathbb{D}, S_i^\alpha] = -\frac{1}{2}S_i^\alpha\,. \tag{A.1g}$$

Among themselves, the generators K_A obey the only nontrivial anti-commutation relation

$$\{S_i^\alpha, S_j^\beta\} = -2\mathrm{i}\varepsilon_{ij}(\tilde\gamma_c)^{\alpha\beta}K^c\,. \tag{A.1h}$$

The algebra of K^A with ∇_A is given by

$$[K_a, \nabla_b] = 2\eta_{ab}\mathbb{D} + 2M_{ab}\,, \tag{A.1i}$$

$$[K^a, \nabla_\alpha^i] = -\mathrm{i}(\gamma^a)_{\alpha\beta}S^{\beta i}\,, \tag{A.1j}$$

$$\{S_i^\alpha, \nabla_\beta^j\} = 2\delta_\beta^\alpha \delta_i^j \mathbb{D} - 4\delta_i^j M_\beta{}^\alpha + 8\delta_\beta^\alpha J_i{}^j\,, \tag{A.1k}$$

$$[S_i^\alpha, \nabla_b] = -\mathrm{i}(\tilde\gamma_b)^{\alpha\beta}\nabla_{\beta i} + \frac{1}{10}W_{bcd}(\tilde\gamma^{cd})^\alpha{}_\gamma S_i^\gamma - \frac{1}{4}X_i^\alpha K_b$$
$$+ \left[\frac{1}{4}(\tilde\gamma_{bc})^\alpha{}_\beta X_i^\beta + \frac{1}{2}(\gamma_{bc})_\beta{}^\gamma X_{\gamma i}{}^{\beta\alpha}\right]K^c\,. \tag{A.1l}$$

The anticommutator of two spinor derivatives, $\{\nabla_\alpha^i, \nabla_\beta^j\}$, has the following non-zero torsion and curvatures

$$T_{\alpha\beta}^{ij\,c} = 2i\varepsilon^{ij}(\gamma^c)_{\alpha\beta}\,, \tag{A.2a}$$

$$R(M)_{\alpha\beta}^{ij\,cd} = 4i\varepsilon^{ij}(\gamma_a)_{\alpha\beta}W^{acd}\,, \tag{A.2b}$$

$$R(S)_{\alpha\beta\gamma}^{ij\,k} = -\frac{3}{2}\varepsilon^{ij}\varepsilon_{\alpha\beta\gamma\delta}X^{\delta k}\,, \tag{A.2c}$$

$$R(K)_{\alpha\beta c}^{ij} = i\varepsilon^{ij}(\gamma^a)_{\alpha\beta}\left(\frac{1}{4}\eta_{ac}Y - \nabla^b W_{abc} + W_a{}^{ef}W_{cef}\right)\,. \tag{A.2d}$$

The non-zero torsion and curvatures in the commutator $[\nabla_a, \nabla_\beta^j]$ are:

$$T_{a\beta k}^{j\,\gamma} = -\frac{1}{2}(\gamma_a)_{\beta\delta}W^{\delta\gamma}\delta_k^j\,, \tag{A.3a}$$

$$R(\mathbb{D})_{a\beta}^{j} = -\frac{i}{2}(\gamma_a)_{\beta\gamma}X^{\gamma j}\,, \tag{A.3b}$$

$$R(M)_{a\beta}^{j\,cd} = i\delta_a^{[c}(\gamma^{d]})_{\beta\gamma}X^{\gamma j} - i(\gamma_a{}^{cd})_{\gamma\delta}X_\beta^{j\gamma\delta} + 2i(\gamma_a)_{\beta\gamma}(\gamma^{cd})_\delta{}^\rho X_\rho^{j\gamma\delta}\,, \tag{A.3c}$$

$$R(J)_{a\beta}^{j\,kl} = 2i(\gamma_a)_{\beta\gamma}X^{\gamma(k}\varepsilon^{l)j}\,, \tag{A.3d}$$

$$R(S)_{a\beta\gamma}^{j\,k} = -\frac{i}{4}(\gamma_a)_{\beta\delta}Y_\gamma^{\delta jk} + \frac{3i}{20}(\gamma_a)_{\gamma\delta}Y_\beta^{\delta jk} - \frac{i}{8}(\gamma_a)_{\beta\delta}\nabla_{\gamma\rho}W^{\delta\rho}\varepsilon^{jk}$$
$$+ \frac{i}{40}(\gamma_a)_{\gamma\delta}\nabla_{\beta\rho}W^{\delta\rho}\varepsilon^{jk} - \frac{i}{8}(\gamma_a)_{\delta\epsilon}\,\varepsilon_{\beta\rho\tau\gamma}\,W^{\delta\rho}W^{\epsilon\tau}\varepsilon^{jk}\,, \tag{A.3e}$$

$$R(K)_{a\beta c}^{j} = \frac{i}{4}(\gamma_c)_{\beta\gamma}\nabla_a X^{\gamma j} - \frac{i}{4}(\gamma_{acd})_{\gamma\delta}\nabla^d X_\beta^{j\gamma\delta} + \frac{i}{3}(\gamma_a)_{\beta\delta}(\gamma_{cd})_\rho{}^\gamma\nabla^d X_\gamma^{j\delta\rho}$$
$$- \frac{i}{8}(\gamma_a)_{\beta\gamma}(\gamma_c)_{\delta\rho}W^{\gamma\delta}X^{\rho j} + \frac{5i}{12}(\gamma_a)_{\beta\rho}(\gamma_c)_{\gamma\epsilon}W^{\gamma\delta}X_\delta^{j\rho\epsilon}$$
$$+ \frac{i}{4}(\gamma_a)_{\gamma\rho}(\gamma_c)_{\beta\epsilon}W^{\gamma\delta}X_\delta^{j\rho\epsilon} - \frac{i}{2}(\gamma_a)_{\gamma\rho}(\gamma_c)_{\delta\epsilon}W^{\gamma\delta}X_\beta^{j\rho\epsilon}\,. \tag{A.3f}$$

The commutator of two vector derivatives, $[\nabla_a, \nabla_b]$, has the following non-vanishing torsion and curvatures:

$$T_{abk}^{\gamma} = (\gamma_{ab})_\beta{}^\alpha X_{\alpha k}{}^{\beta\gamma}\,, \tag{A.4a}$$

$$R(M)_{ab}{}^{cd} = Y_{ab}{}^{cd} = \frac{1}{4}(\gamma_{ab})_\gamma{}^\alpha(\gamma^{cd})_\delta{}^\beta Y_{\alpha\beta}{}^{\gamma\delta}\,, \tag{A.4b}$$

$$R(J)_{ab}{}^{kl} = \frac{1}{2}(\gamma_{ab})_\delta{}^\gamma Y_\gamma{}^{\delta kl} = Y_{ab}{}^{kl}\,, \tag{A.4c}$$

$$R(S)_{ab\gamma}^{k} = -\frac{i}{3}(\gamma_{ab})_\delta{}^\alpha\nabla_{\gamma\beta}X_\alpha^{k\beta\delta} - \frac{i}{6}(\gamma_{abc})_{\alpha\beta}\nabla^c X_\gamma^{k\alpha\beta} - \frac{i}{6}\varepsilon_{\gamma\beta\epsilon\rho}(\gamma_{ab})_\delta{}^\rho W^{\alpha\beta}X_\alpha^{k\delta\epsilon}\,, \tag{A.4d}$$

$$R(K)_{abc} = \frac{1}{4}\nabla^d Y_{abcd} + \frac{i}{3}X_\alpha^{k\beta\gamma}X_{\beta k}{}^{\alpha\delta}(\gamma_{abc})_{\gamma\delta} + i(\gamma_{ab})_\epsilon{}^\alpha(\gamma_c)_{\gamma\delta}X_\alpha^{k\beta\gamma}X_{\beta k}{}^{\delta\epsilon}$$
$$+ \frac{i}{4}X^{\alpha k}X_{\beta k}{}^{\gamma\delta}(\gamma_{ab})_\gamma{}^\beta(\gamma_c)_{\alpha\delta}\,. \tag{A.4e}$$

Remember that the descendant superfields $X^{\alpha i}$, $X_{\alpha i}{}^{\beta\gamma}$, Y, $Y_\alpha{}^{\beta kl}$ (and equivalently $Y_{ab}{}^{kl}$), $Y_{\alpha\beta}{}^{\gamma\delta}$ (and equivalently $Y_{ab}{}^{cd}$), were defined in (2.6) and (2.7).

By using (2.5) and the previous definitions, one can derive the following relations for the descendant superfields of the super-Weyl tensor:

$$\nabla^i_\alpha X^{\beta j} = -\frac{2}{5} Y_\alpha{}^{\beta ij} - \frac{2}{5} \varepsilon^{ij} \nabla_{\alpha\gamma} W^{\gamma\beta} - \frac{1}{2} \varepsilon^{ij} \delta^\beta_\alpha Y \,, \tag{A.5a}$$

$$\nabla^i_\alpha X^{j\gamma\delta}_\beta = \frac{1}{2} \delta^{(\gamma}_\alpha Y_\beta{}^{\delta)ij} - \frac{1}{10} \delta^{(\gamma}_\beta Y_\alpha{}^{\delta)ij} - \frac{1}{2} \varepsilon^{ij} Y_{\alpha\beta}{}^{\gamma\delta} - \frac{1}{4} \varepsilon^{ij} \nabla_{\alpha\beta} W^{\gamma\delta}$$
$$+ \frac{3}{20} \varepsilon^{ij} \delta^{(\gamma}_\beta \nabla_{\alpha\rho} W^{\delta)\rho} - \frac{1}{4} \varepsilon^{ij} \delta^{(\gamma}_\alpha \nabla_{\beta\rho} W^{\delta)\rho} \,, \tag{A.5b}$$

$$\nabla^i_\alpha Y = -2\mathrm{i} \nabla_{\alpha\beta} X^{\beta i} \,, \tag{A.5c}$$

$$\nabla^k_\gamma Y_\alpha{}^{\beta ij} = \frac{2}{3} \varepsilon^{k(i} \Big(-8\mathrm{i}\, \nabla_{\gamma\delta} X^{j)\beta\delta}_\alpha - 4\mathrm{i}\, \nabla_{\alpha\delta} X^{j)\beta\delta}_\gamma + 3\mathrm{i}\, \nabla_{\gamma\alpha} X^{\beta j)} + 3\mathrm{i}\, \delta^\beta_\gamma \nabla_{\alpha\delta} X^{\delta j)}$$
$$- \frac{3\mathrm{i}}{2} \delta^\beta_\alpha \nabla_{\gamma\delta} X^{\delta j)} - 3\mathrm{i}\, \varepsilon_{\alpha\gamma\delta\epsilon} W^{\beta\delta} X^{\epsilon j)} + 4\mathrm{i}\, \varepsilon_{\alpha\gamma\epsilon\rho} W^{\delta\epsilon} X^{j)\beta\rho}_\delta \Big) \,, \tag{A.5d}$$

$$\nabla^l_\epsilon Y_{\alpha\beta}{}^{\gamma\delta} = -4\mathrm{i}\, \nabla_{\epsilon(\alpha} X^{l\,\gamma\delta}_{\beta)} + \frac{4\mathrm{i}}{3} \delta^{(\gamma}_{(\alpha} \nabla_{\beta)\rho} X^{l\delta)\rho}_\epsilon + \frac{8\mathrm{i}}{3} \delta^{(\gamma}_{(\alpha} \nabla_{|\epsilon\rho|} X^{l\ \delta)\rho}_{\beta)} + 8\mathrm{i}\, \delta^{(\gamma}_\epsilon \nabla_{\rho(\alpha} X^{l\ \delta)\rho}_{\beta)}$$
$$- \frac{4\mathrm{i}}{3} W^{\rho\sigma} \delta^{(\gamma}_{(\alpha} \varepsilon_{\beta)\epsilon\sigma\tau} X^{l\delta)\tau}_\rho - 8\mathrm{i}\, \varepsilon_{\epsilon\rho\sigma(\alpha} W^{\rho(\gamma} X^{l\ \delta)\sigma}_{\beta)} \,. \tag{A.5e}$$

These relations define the Q-supersymmetry transformations of the descendant superfields of the super-Weyl tensor. Their S-supersymmetry transformations are instead given by the following relations [69]:

$$S^\alpha_i X^{\beta j} = \frac{8\mathrm{i}}{5} \delta^j_i W^{\alpha\beta} \,, \qquad S^\alpha_i X^{j\gamma\delta}_\beta = -\mathrm{i}\delta^j_i \delta^\alpha_\beta W^{\gamma\delta} + \frac{2\mathrm{i}}{5} \delta^j_i \delta^{(\gamma}_\beta W^{\delta)\alpha} \,, \tag{A.6a}$$

$$S^\gamma_k Y_\alpha{}^{\beta ij} = -\delta^{(i}_k \left(16 X^{j)\gamma\beta}_\alpha - 2\delta^\beta_\alpha X^{\gamma j)} + 8\delta^\gamma_\alpha X^{\beta j)} \right) \,, \tag{A.6b}$$

$$S^\rho_j Y_{\alpha\beta}{}^{\gamma\delta} = 24 \left(\delta^\rho_{(\alpha} X^{\ \gamma\delta}_{\beta)j} - \frac{1}{3} \delta^{(\gamma}_{(\alpha} X^{\ \delta)\rho}_{\beta)j} \right) \,, \qquad S^\alpha_i Y = -4 X^\alpha_i \,. \tag{A.6c}$$

The descendant superfields also satisfy the following indentities

$$\nabla^{\delta(\alpha} X^{i\beta\gamma)}_\delta = W^{\delta(\alpha} X^{i\beta\gamma)}_\delta \,, \tag{A.7a}$$

$$\nabla_{\gamma(\alpha} Y_{\beta)}{}^{\gamma ij} = 0 \,, \qquad\qquad \nabla^{\gamma(\alpha} Y_\gamma{}^{\beta)ij} = 8\mathrm{i} X^{\gamma(i} X^{j)\alpha\beta}_\gamma \,, \tag{A.7b}$$

$$\nabla^{\delta(\alpha} Y_{\beta\gamma)}{}^{\delta\epsilon} = 0 \,, \qquad\qquad \nabla^{\delta(\alpha} Y_{\delta\epsilon}{}^{\beta\gamma)} = 24\mathrm{i} X^{k\delta(\alpha}_\epsilon X_{\delta k}{}^{\beta\gamma)} - 8\mathrm{i} X^{k\delta(\alpha}_\rho \delta^\beta_\epsilon X_{\delta k}{}^{\gamma)\rho} \,. \tag{A.7c}$$

We conclude by underlining that, compared to the frame chosen in [69], the superspace geometry in the traceless frame described in this appendix is simply given by the following redefinition of the vector derivative

$$\nabla_a \to \nabla_a - W_a{}^{bc} M_{bc} + \frac{3\mathrm{i}}{8} (\gamma_a)_{\alpha\beta} X^{\alpha j} S^\beta_j - \frac{1}{8} Y K_a + \frac{1}{2} \nabla^b W_{abc} K^c - \frac{1}{2} W_a{}^{ef} W_{efc} K^c \,. \tag{A.8}$$

Here on the left hand side ∇_a is the vector derivative of [69] while the operator on the right hand side is the vector covariant derivatives in the traceless frame of [70], which we used everywhere in the paper, defined in terms of the vector derivative of [69].

B Useful descendant components of the composite gauge 3-form (5.17)

For the composite gauge 3-form (5.17), in this appendix we collect the descendant components relevant for the derivation of the invariant (5.19). They are

$$
\begin{aligned}
\Lambda_{\alpha a}{}^i &= \frac{4}{3}(\gamma_a)_{\alpha\beta}\left(\frac{2}{15}Y_\gamma{}^{\beta ij}\chi_j^\gamma - X_{\gamma j}{}^{\delta\beta}Y_\delta{}^{\gamma ij}\right) + 4(\gamma_a)_{\beta\gamma}\left(X_{\alpha j}{}^{\delta\beta}Y_\delta{}^{\gamma ij} - \frac{1}{15}Y_\alpha{}^{\beta ij}\chi_j^\gamma\right) \\
&\quad + \frac{2}{45}(\gamma_a)_{\alpha\beta}D\chi^{\beta i} + 8(\gamma_a)_{\beta\gamma}X_\delta^{i\epsilon\beta}Y_{\alpha\epsilon}{}^{\delta\gamma} + 2(\gamma_a)_{\beta\gamma}\left(\nabla_{\delta\epsilon}W^{\beta\delta}X_\alpha^{i\,\gamma\epsilon} - \frac{4}{3}W^{\beta\delta}\nabla_{\delta\epsilon}X_\alpha^{i\,\gamma\epsilon}\right) \\
&\quad + 2(\gamma_a)_{\alpha\beta}\left(\nabla_{\gamma\delta}W^{\epsilon\gamma}X_\epsilon^{i\beta\delta} - \frac{2}{15}W^{\beta\gamma}\nabla_{\gamma\delta}\chi^{\delta i}\right) \\
&\quad + 4(\gamma_a)_{\beta\gamma}\left(\frac{1}{15}\nabla_{\alpha\delta}(W^{\beta\delta}\chi^{\gamma i}) - \frac{4}{3}W^{\epsilon\beta}\nabla_{\alpha\delta}X_\epsilon^{i\gamma\delta} - \nabla_{\alpha\delta}W^{\epsilon\beta}X_\epsilon^{i\gamma\delta}\right) \\
&\quad + \frac{4}{3}(\gamma_a)_{\beta\gamma}\varepsilon_{\alpha\delta\epsilon\rho}\left(2W^{\sigma\delta}W^{\beta\epsilon}X_\sigma^{i\,\gamma\rho} + \frac{1}{5}W^{\beta\delta}W^{\gamma\epsilon}\chi^{\rho i}\right),
\end{aligned}
\tag{B.1a}
$$

and

$$
\begin{aligned}
C_{ab} &= (\gamma_b)_{\alpha\beta}\left(\frac{1}{8}(\gamma_{acd})_{\gamma\delta}\nabla^c W^{\alpha\gamma}\nabla^d W^{\beta\delta} - \frac{2}{3}(\gamma_{ac})_\epsilon{}^\delta W^{\gamma\alpha}\nabla^c Y_{\gamma\delta}{}^{\beta\epsilon} - (\gamma_{ac})_\epsilon{}^\delta \nabla^c W^{\gamma\alpha}Y_{\gamma\delta}{}^{\beta\epsilon}\right) \\
&\quad + \frac{1}{2}(\gamma_c)_{\alpha\beta}\left((\gamma_{ab})_\epsilon{}^\delta\nabla^c W^{\gamma\alpha}Y_{\gamma\delta}{}^{\beta\epsilon} - (\gamma_{abd})_{\gamma\delta}W^{\alpha\gamma}\nabla^c\nabla^d W^{\beta\delta} + \frac{3}{4}(\gamma_{abd})_{\gamma\delta}\nabla^c W^{\alpha\gamma}\nabla^d W^{\beta\delta}\right) \\
&\quad + 4i(\gamma_{abc})_{\gamma\delta}\nabla^c X_{\alpha i}{}^{\beta\gamma}X_\beta^{i\,\alpha\delta} - \frac{16i}{45}(\gamma_{abc})_{\beta\gamma}\nabla^c X_{\alpha i}{}^{\beta\gamma}\chi^{\alpha i} - \frac{4i}{15}(\gamma_{abc})_{\beta\gamma}\nabla^c\chi_i^\alpha X_\alpha^{i\,\beta\gamma} \\
&\quad + \frac{14i}{225}(\gamma_{abc})_{\alpha\beta}\nabla^c\chi_i^\alpha\chi^{\beta i} + \frac{1}{45}\eta_{ab}D^2 + \frac{1}{15}(\gamma_{abc})_{\alpha\beta}\nabla^c(DW^{\alpha\beta}) \\
&\quad + \frac{1}{2}\eta_{ab}\left(\frac{1}{4}\nabla_{\alpha\beta}W^{\alpha\gamma}\nabla_{\gamma\delta}W^{\beta\delta} - W^{\alpha\gamma}\nabla_{\alpha\beta}\nabla_{\gamma\delta}W^{\beta\delta}\right) \\
&\quad + (\gamma_{(a})_{\alpha\beta}\left(W^{\alpha\gamma}\nabla_{b)}\nabla_{\gamma\delta}W^{\beta\delta} + \frac{1}{4}\nabla_{b)}W^{\alpha\gamma}\nabla_{\gamma\delta}W^{\beta\delta}\right) \\
&\quad + (\gamma_a)_{\alpha\beta}(\gamma_b)_{\gamma\delta}\left(-\frac{1}{2}W^{\alpha\gamma}\Box W^{\beta\delta} - \frac{5}{8}\nabla^c W^{\alpha\gamma}\nabla_c W^{\beta\delta} + \frac{2}{15}DW^{\alpha\gamma}W^{\beta\delta} + W^{\epsilon\rho}W^{\alpha\gamma}Y_{\epsilon\rho}{}^{\beta\delta}\right) \\
&\quad - (\tilde\gamma_a)^{\gamma\delta}(\gamma_b)_{\alpha\beta}Y_{\epsilon\gamma}{}^{\rho\alpha}Y_{\rho\delta}{}^{\epsilon\beta} + (\gamma_a)_{\alpha\beta}\left(4i\nabla_b X_{\gamma i}{}^{\delta\alpha}X_\delta^{i\gamma\beta} + \frac{2i}{75}\nabla_b\chi_i^\alpha\chi^{\beta i}\right) \\
&\quad + (\gamma^c)_{\alpha\beta}\left(-8i\eta_{ab}\nabla_c X_{\gamma i}{}^{\delta\alpha}X_\delta^{i\gamma\beta} - 4i(\gamma_{ab})_\epsilon{}^\gamma\nabla_c X_{\gamma i}{}^{\delta\alpha}X_\delta^{i\beta\epsilon} + \frac{8i}{45}(\gamma_{ab})_\delta{}^\gamma\nabla_c X_{\gamma i}{}^{\alpha\delta}\chi^{\beta i}\right. \\
&\qquad\left. - \frac{4i}{15}(\gamma_{ab})_\delta{}^\gamma\nabla_c\chi_i^\alpha X_\gamma^{i\,\beta\delta} - \frac{22i}{225}\eta_{ab}\nabla_c\chi_i^\alpha\chi^{\beta i}\right) \\
&\quad + (\gamma_b)_{\alpha\beta}\left(-\frac{4i}{3}\nabla_a X_{\gamma i}{}^{\delta\alpha}X_\delta^{i\gamma\beta} + \frac{2i}{75}\nabla_a\chi_i^\alpha\chi^{\beta i} - \frac{28i}{3}(\gamma_{ac})_\delta{}^\epsilon\nabla^c X_{\gamma i}{}^{\alpha\delta}X_\epsilon^{i\gamma\beta}\right. \\
&\qquad\left. + 4i(\gamma_{ac})_\epsilon{}^\gamma\nabla^c X_{\gamma i}{}^{\delta\alpha}X_\delta^{i\beta\epsilon} - \frac{16i}{45}(\gamma_{ac})_\delta{}^\gamma\nabla^c X_{\gamma i}{}^{\alpha\delta}\chi^{\beta i}\right. \\
&\qquad\left. + \frac{8i}{15}(\gamma_{ac})_\delta{}^\gamma\nabla^c\chi_i^\alpha X_\gamma^{i\,\beta\delta}\right) \\
&\quad - \frac{1}{6}(\gamma_{ab})_\sigma{}^\rho\varepsilon_{\gamma\delta\epsilon\rho}W^{\alpha\gamma}W^{\sigma\delta}\nabla_{\alpha\beta}W^{\beta\epsilon} + \frac{5}{12}\eta_{ab}Y_\alpha{}^{\beta ij}Y_\beta{}^\alpha{}_{ij} + \frac{1}{4}(\gamma_{ab})_\gamma{}^\alpha Y_\alpha{}^{\beta ij}Y_\beta{}^\gamma{}_{ij}
\end{aligned}
$$

$$+(\gamma_{ac})_\sigma{}^\rho(\gamma_b)_{\alpha\beta}\varepsilon_{\gamma\delta\epsilon\rho}\left(\frac{1}{2}W^{\alpha\gamma}W^{\beta\delta}\nabla^c W^{\sigma\epsilon}-\frac{1}{3}W^{\alpha\gamma}W^{\sigma\delta}\nabla^c W^{\beta\epsilon}\right)$$

$$+\varepsilon_{\alpha\beta\gamma\delta}\left(\frac{4\mathrm{i}}{3}(\gamma_{ab})_\sigma{}^\delta W^{\epsilon\alpha}X_{\epsilon i}{}^{\rho\beta}X_\rho^{i\sigma\gamma}+\frac{2\mathrm{i}}{5}(\gamma_{ab})_\rho{}^\delta W^{\epsilon\alpha}X_{\epsilon i}{}^{\rho\beta}\chi^{\gamma i}\right)$$

$$+\frac{1}{6}\varepsilon_{\alpha\beta\gamma\delta}\varepsilon_{\epsilon\rho\sigma\tau}\left(\eta_{ab}W^{\alpha\epsilon}W^{\beta\rho}W^{\gamma\sigma}W^{\delta\tau}-(\gamma_{ab})_\lambda{}^\tau W^{\lambda\alpha}W^{\beta\epsilon}W^{\gamma\rho}W^{\delta\sigma}\right)$$

$$+\frac{1}{3}(\gamma_a)_{\alpha\beta}(\gamma_b)_{\gamma\delta}\left(2W^{\alpha\epsilon}W^{\gamma\rho}\nabla_{\epsilon\rho}W^{\beta\delta}+W^{\alpha\gamma}W^{\beta\epsilon}\nabla_{\epsilon\rho}W^{\delta\rho}-2W^{\alpha\gamma}W^{\delta\epsilon}\nabla_{\epsilon\rho}W^{\beta\rho}\right)$$

$$+(\gamma_a)_{\alpha\beta}(\gamma_b)_{\gamma\delta}\left(\frac{16\mathrm{i}}{3}W^{\alpha\gamma}X_{\epsilon i}{}^{\rho\beta}X_\rho^{i\epsilon\delta}-12\mathrm{i}W^{\epsilon\alpha}X_{\epsilon i}{}^{\rho\gamma}X_\rho^{i\beta\delta}+\frac{44\mathrm{i}}{3}W^{\epsilon\gamma}X_{\epsilon i}{}^{\rho\alpha}X_\rho^{i\beta\delta}\right.$$

$$-\frac{20\mathrm{i}}{3}W^{\epsilon\rho}X_{\epsilon i}{}^{\alpha\gamma}X_\rho^{i\beta\delta}-\frac{8\mathrm{i}}{5}W^{\alpha\gamma}X_{\epsilon i}{}^{\beta\delta}\chi^{\epsilon i}-\frac{22\mathrm{i}}{45}W^{\epsilon\alpha}X_{\epsilon i}{}^{\beta\gamma}\chi^{\delta i}$$

$$\left.+\frac{14\mathrm{i}}{45}W^{\epsilon\gamma}X_{\epsilon i}{}^{\alpha\delta}\chi^{\beta i}+\frac{2\mathrm{i}}{75}W^{\alpha\gamma}\chi_i^\beta\chi^{\delta i}\right)$$

$$+\frac{1}{2}(\gamma_a)_{\alpha\beta}(\gamma_b)_{\gamma\delta}\varepsilon_{\epsilon\rho\sigma\tau}W^{\alpha\epsilon}W^{\beta\rho}W^{\gamma\sigma}W^{\delta\tau}+\frac{20\mathrm{i}}{3}(\tilde{\gamma}_a)^{\epsilon\rho}(\gamma_b)_{\alpha\beta}\nabla_{\gamma\delta}X_{\epsilon i}{}^{\alpha\gamma}X_\rho^{i\beta\delta}$$

$$-\frac{1}{4}(\tilde{\gamma}_a)^{\gamma\delta}(\gamma_b)_{\alpha\beta}Y_\gamma^{\alpha ij}Y_\delta{}^\beta{}_{ij}. \tag{B.1b}$$

Inserting the expression for $B_a{}^{ij}$, eq. (5.17), and its corresponding descendants $\Lambda_{\alpha a}{}^i$ and C_{ab} presented in this appendix into the action principle (3.22), one can obtain the action (5.19) together with all the fermionic terms that complete the new supersymmetric invariant described in subsection 5.3.

C   Full bosonic terms of the invariant (7.8)

In this appendix we present the full bosonic contribution to the new higher-derivative locally superconformal invariant constructed in section 7. In a general gauge and conformal supergravity background, up to fermionic terms, the completion of the Lagrangian (7.8) reads

$$e^{-1}\mathcal{L}_{\text{new-linear}}=-\frac{1}{12}L^{\frac{1}{2}}\Box D-\frac{1}{20}L^{\frac{1}{2}}D^2+\frac{1}{64}L^{-\frac{3}{2}}\mathrm{E}^a\mathrm{E}_a D-\frac{1}{24}L^{-\frac{3}{2}}DL^{ij}\Box L_{ij}$$

$$+\frac{1}{192}L^{-\frac{3}{2}}D(\nabla^a L^{ij})\nabla_a L_{ij}+\frac{1}{128}L^{-\frac{7}{2}}DL^{ij}L^{kl}(\nabla^a L_{ij})\nabla_a L_{kl}$$

$$+\frac{15}{32}L^{-\frac{3}{2}}L^{ij}L^{kl}Y^{ab}{}_{ij}Y_{abkl}-\frac{5}{16}L^{-\frac{3}{2}}L^{ij}\Box^2 L_{ij}+\frac{5}{32}L^{-\frac{7}{2}}L^{ij}L^{kl}(\Box L_{ij})\Box L_{kl}$$

$$+\frac{5}{32}L^{-\frac{3}{2}}(\Box L^{ij})\Box L_{ij}+\frac{5}{32}L^{-\frac{7}{2}}L^{ij}L^{kl}(\nabla^a\nabla^b L_{ij})\nabla_a\nabla_b L_{kl}-\frac{15}{64}L^{-\frac{3}{2}}(\nabla^a\nabla^b L^{ij})\nabla_a\nabla_b L_{ij}$$

$$+\frac{15}{256}L^{-\frac{7}{2}}\mathrm{E}^a L^{ij}(\nabla_a L_i{}^k)\Box L_{jk}-\frac{15}{64}L^{-\frac{7}{2}}\mathrm{E}^a L^{ij}(\nabla^b L_i{}^k)\nabla_a\nabla_b L_{jk}$$

$$-\frac{105}{256}L^{-\frac{7}{2}}L^{ij}(\nabla^a\mathrm{E}^b)(\nabla_a L_i{}^k)\nabla_b L_{jk}-\frac{105}{256}L^{-\frac{11}{2}}\mathrm{E}^a L^{ij}L^{kl}(\nabla_a L_i{}^p)(\nabla^b L_{jp})\nabla_b L_{kl}$$

$$+\frac{5}{64}L^{-\frac{3}{2}}\mathrm{E}^a\Box\mathrm{E}_a+\frac{25}{128}L^{-\frac{3}{2}}(\nabla^a\mathrm{E}^b)\nabla_a\mathrm{E}_b-\frac{35}{1024}L^{-\frac{7}{2}}\mathrm{E}^a\mathrm{E}_a L^{ij}\Box L_{ij}$$

$$-\frac{15}{512}L^{-\frac{7}{2}}\mathrm{E}^a\mathrm{E}^b L^{ij}\nabla_a\nabla_b L_{ij}-\frac{15}{512}L^{-\frac{7}{2}}\mathrm{E}^a(\nabla_a\mathrm{E}^b)L^{ij}\nabla_b L_{ij}$$

$$-\frac{155}{512}L^{-\frac{7}{2}}\mathrm{E}^a(\nabla^b\mathrm{E}_a)L^{ij}\nabla_b L_{ij}+\frac{35}{256}L^{-\frac{11}{2}}\mathrm{E}^a\mathrm{E}_a L^{ij}L^{kl}(\nabla^b L_{ij})\nabla_b L_{kl}$$

$$
- \frac{275}{4096} L^{-\frac{7}{2}} \mathrm{E}^a \mathrm{E}_a (\nabla^b L^{ij}) \nabla_b L_{ij} + \frac{105}{2048} L^{-\frac{11}{2}} \mathrm{E}^a \mathrm{E}^b L^{ij} L^{kl} (\nabla_a L_{ij}) \nabla_b L_{kl}
$$

$$
- \frac{105}{2048} L^{-\frac{7}{2}} \mathrm{E}^a \mathrm{E}^b (\nabla_a L^{ij}) \nabla_b L_{ij} - \frac{45}{16384} L^{-\frac{7}{2}} (\mathrm{E}^a \mathrm{E}_a)^2
$$

$$
+ \frac{35}{64} L^{-\frac{7}{2}} L^{ij} L^{kl} (\nabla^a L_{ij}) \nabla_a \Box L_{kl} - \frac{5}{32} L^{-\frac{3}{2}} (\nabla^a L^{ij}) \nabla_a \Box L_{ij}
$$

$$
- \frac{25}{64} L^{-\frac{11}{2}} L^{ij} L^{kl} L^{pq} (\nabla^a L_{ij})(\nabla_a L_{kl}) \Box L_{pq} - \frac{25}{256} L^{-\frac{7}{2}} L^{ij} (\nabla^a L_{ij})(\nabla_a L^{kl}) \Box L_{kl}
$$

$$
+ \frac{155}{512} L^{-\frac{7}{2}} L^{ij} (\nabla^a L^{kl})(\nabla_a L_{kl}) \Box L_{ij} - \frac{65}{128} L^{-\frac{11}{2}} L^{ij} L^{kl} L^{pq} (\nabla^a L_{ij})(\nabla^b L_{kl}) \nabla_a \nabla_b L_{pq}
$$

$$
+ \frac{85}{128} L^{-\frac{7}{2}} L^{ij} (\nabla^a L_{ij})(\nabla^b L^{kl}) \nabla_a \nabla_b L_{kl} + \frac{5}{256} L^{-\frac{7}{2}} L^{ij} (\nabla^a L^{kl})(\nabla^b L_{kl}) \nabla_a \nabla_b L_{ij}
$$

$$
+ \frac{15}{64} L^{-\frac{3}{2}} T^{-abc} \mathrm{E}_a \nabla_b \mathrm{E}_c + \frac{15}{32} L^{-\frac{3}{2}} L^{ij} Y^{ab}{}_{ij} \nabla_a \mathrm{E}_b + \frac{15}{16} L^{-\frac{3}{2}} L^{ij} (\nabla_a L_i{}^k) \nabla_b Y^{ab}{}_{jk}
$$

$$
+ \frac{105}{256} L^{-\frac{7}{2}} \mathrm{E}^a Y_{abij} L^{ij} L^{kl} \nabla^b L_{kl} - \frac{45}{128} L^{-\frac{3}{2}} Y^{abij} \mathrm{E}_a \nabla_b L_{ij}
$$

$$
- \frac{45}{256} L^{-\frac{7}{2}} T^{-abc} \mathrm{E}_a L^{ij} (\nabla_b L_i{}^k) \nabla_c L_{jk} - \frac{165}{256} L^{-\frac{7}{2}} L^{ij} L^{kl} Y^{ab}{}_{ij} (\nabla_a L_k{}^p) \nabla_b L_{lp}
$$

$$
- \frac{45}{128} L^{-\frac{3}{2}} Y^{abij} (\nabla_a L_i{}^k) \nabla_b L_{jk} - \frac{15}{8} L^{\frac{1}{2}} (\nabla_a T^{-acd}) \nabla^b T^-_{bcd} + \frac{15}{32} L^{-\frac{3}{2}} \mathrm{E}^a L^{ij} T^-_{abc} Y^{bc}{}_{ij}
$$

$$
+ \frac{15}{256} L^{-\frac{3}{2}} \mathrm{E}_a \mathrm{E}^b T^{-acd} T^-_{bcd} - \frac{15}{32} L^{-\frac{3}{2}} T^{-acd} T^{-b}{}_{cd} L^{ij} \nabla_a \nabla_b L_{ij}
$$

$$
+ \frac{45}{128} L^{-\frac{7}{2}} T^{-acd} T^-_{bcd} L^{ij} L^{kl} (\nabla_a L_{ij}) \nabla^b L_{kl} - \frac{45}{128} L^{-\frac{3}{2}} T^{-acd} T^-_{bcd} (\nabla_a L^{ij}) \nabla^b L_{ij}
$$

$$
+ \frac{15}{32} L^{-\frac{3}{2}} L^{ij} T^{-abc} Y_{abi}{}^k \nabla_c L_{jk} - \frac{15}{16} L^{-\frac{3}{2}} L^{ij} T^{-acd} (\nabla_a L_{ij}) \nabla^b T^-_{bcd}
$$

$$
- \frac{15}{32} L^{\frac{1}{2}} T^{-acd} T^-_{bcd} T^-_{aef} T^{-bef} + \frac{165}{512} L^{-\frac{15}{2}} L^{ij} L^{kl} L^{pq} L^{rs} (\nabla^a L_{ij})(\nabla_a L_{kl})(\nabla^b L_{pq}) \nabla_b L_{rs}
$$

$$
- \frac{135}{256} L^{-\frac{11}{2}} L^{ij} L^{kl} (\nabla^a L_{ij})(\nabla_a L_{kl})(\nabla^b L^{pq}) \nabla_b L_{pq}
$$

$$
+ \frac{5}{1024} L^{-\frac{11}{2}} L^{ij} L^{kl} (\nabla^a L_{ij})(\nabla_a L^{pq})(\nabla^b L_{pq}) \nabla_b L_{kl}
$$

$$
+ \frac{1005}{4096} L^{-\frac{7}{2}} (\nabla^a L^{ij})(\nabla_a L_{ij})(\nabla^b L^{kl}) \nabla_b L_{kl}
$$

$$
- \frac{325}{2048} L^{-\frac{7}{2}} (\nabla^a L^{ij})(\nabla_a L^{kl})(\nabla^b L_{kl}) \nabla_b L_{ij}
$$

$$
+ \text{fermions} \,. \tag{C.1}
$$

D   Different 6D $\mathcal{N} = (1,0)$ notation and conventions

In this appendix we describe the differences between our notation and conventions, based on [69, 70], and the ones used in [48] and [76]. By using the map between notations described here the reader can rewrite the results of our paper in the notation of [48] and [76].

Throughout our paper we have used chiral four-component spinor notation while in [48, 76] eight-component spinor notation is used. To translate our results, the reader should first reinterpret our formulae using eight component spinors. According to appendix

Our notation	Bergshoeff et al. [48]	Coomans & Van Proeyen [76]
$\eta^{ab}, \varepsilon^{abcdef}, \Gamma^a, \Gamma^{a_1\cdots a_n}, \cdots$	$\delta^{ab}, \mathrm{i}\varepsilon^{abcdef}, -\mathrm{i}\gamma^a, (-\mathrm{i})^n\gamma^{a_1\cdots a_n}, \cdots$	$\eta^{ab}, -\varepsilon^{abcdef}, -\mathrm{i}\gamma^a, (-\mathrm{i})^n\gamma^{a_1\cdots a_n}, \cdots$
P_a, K^a, Q^i, S_i	$P_a, K^a, -2Q^i, 2S_i$	$P_a, K^a, -2Q^i, 2S_i$
$M_{ab}, J_{ij}, \mathbb{D}$	$-2M_{ab}, 2U_{ij}, D$	$-M_{ab}, -U_{ij}, D$
$e_m{}^a, \psi_m{}^i, \mathcal{V}_m{}^{ij}, b_m$	$e_\mu{}^a, \psi_\mu{}^i, \frac{1}{2}V_\mu{}^{ij}, b_\mu,$	$e_\mu{}^a, \psi_\mu{}^i, -\mathcal{V}_\mu{}^{ij}, b_\mu,$
$\omega_m{}^{ab}, \phi_m{}^i$	$-\omega_\mu{}^{ab}, \phi_\mu{}^i + \frac{1}{60}\gamma_\mu\chi^i$	$-\omega_\mu{}^{ab}, \phi_\mu{}^i + \frac{1}{60}\gamma_\mu\chi^i$
$\mathfrak{f}_{ma}$	$f_{\mu a} - e_\mu{}^a\left(\frac{1}{240}\delta_{ab}D - \frac{1}{8}T_a^{-cd}T_{cdb}^-\right)$	$f_{\mu a} - e_\mu{}^a\left(\frac{1}{240}\eta_{ab}D - \frac{1}{8}T_a^{-cd}T_{cdb}^-\right)$
∇_a	$\hat{\mathscr{D}}_a + \frac{1}{60}\chi^i\gamma_a S_i + \left(\frac{1}{240}\delta_{ab}D - \frac{1}{8}T_a^{-cd}T_{cdb}^-\right)K^b$	$\mathcal{D}_a + \frac{1}{60}\chi^i\gamma_a S_i + \left(\frac{1}{240}\eta_{ab}D - \frac{1}{8}T_a^{-cd}T_{cdb}^-\right)K^b$
T_{abc}^-, χ^i, D	T_{abc}^-, χ^i, D	T_{abc}^-, χ^i, D
$R(P)_{ab}{}^c = R(\mathbb{D})_{ab} = 0$	$\hat{R}(P)_{ab}{}^c = \hat{R}(D)_{ab} = 0$	$\hat{R}(P)_{ab}{}^c = \hat{R}(D)_{ab} = 0$
$R(J)_{ab}{}^{ij}, R(Q)_{ab}{}^i$	$\frac{1}{2}\hat{R}(U)_{ab}{}^{ij}, \frac{1}{2}\hat{R}(Q)_{abk} + \frac{1}{60}\gamma_{ab}\chi_k$	$-\hat{R}(U)_{ab}{}^{ij}, \frac{1}{2}\hat{R}(Q)_{abk} + \frac{1}{60}\gamma_{ab}\chi_k$
$R(M)_{ab}{}^{cd}$	$-\hat{R}(M)_{ab}{}^{cd} - \frac{1}{30}D\delta_a^{[c}\delta_b^{d]} - T_{abf}^-T^{-fcd}$	$-\hat{R}(M)_{ab}{}^{cd} - \frac{1}{30}D\delta_a^c\delta_b^d - T_{abf}^-T^{-fcd}$
$R(S)_{abi}$	$\frac{1}{2}\hat{R}(S)_{ab}{}^k - \frac{1}{60}\gamma_{[a}\hat{\mathscr{D}}_{b]}\chi^k + \frac{19}{600}T_{abc}^-\gamma^c\chi^k$	$\frac{1}{2}\hat{R}(S)_{ab}{}^k - \frac{1}{60}\gamma_{[a}\mathcal{D}_{b]}\chi^k + \frac{19}{600}T_{abc}^-\gamma^c\chi^k$
$R(K)_{abc}$	$\left[\hat{R}(K)_{abc} + \frac{1}{120}\eta_{c[a}\hat{\mathscr{D}}_{b]}D + 2T_c^{-de}\hat{\mathscr{D}}_{[a}T_{b]de}^-\right.$	$\left[\hat{R}(K)_{abc} + \frac{1}{120}\eta_{c[a}\mathcal{D}_{b]}D + 2T_c^{-de}\mathcal{D}_{[a}T_{b]de}^-\right.$
	$+\frac{1}{1440}\chi^i\big(\gamma_{[a}\hat{R}(Q)_{b]ci} - 7\gamma_c\hat{R}(Q)_{abi}\big)$	$+\frac{1}{1440}\chi^i\big(\gamma_{[a}\hat{R}(Q)_{b]ci} - 7\gamma_c\hat{R}(Q)_{abi}\big)$
	$+\frac{1}{900}\chi^i\gamma_{abc}\chi_i\big]$	$+\frac{1}{900}\chi^i\gamma_{abc}\chi_i\big]$
$b_{mn}, \sigma, \psi_i, H_{abc}$	$B_{\mu\nu}, \sigma, -2\psi_i, (F_{abc}^+ + 2\sigma T_{abc}^-)$	$B_{\mu\nu}, \sigma, -2\psi_i, \hat{F}(B)_{abc}$
$v_m, \Lambda^i, X^{ij}, F_{ab}$	$-\frac{1}{2}W_\mu, -\mathrm{i}\Omega^i, -\frac{1}{2}Y^{ij}, -\frac{1}{2}\hat{F}(W)_{ab}$	$-\frac{1}{2}W_\mu, -\mathrm{i}\Omega^i, -\frac{1}{2}Y^{ij}, -\frac{1}{2}\hat{F}(W)_{ab}$
$b_{mnpq} L^{ij}, \varphi^i, H_a, L$	$E_{\mu\nu\rho\tau} L^{ij}, \varphi^i, E_a, \frac{1}{\sqrt{2}}L$	$E_{\mu\nu\rho\tau} L^{ij}, \varphi^i, E_a, \frac{1}{\sqrt{2}}L$

Table 1. Different notation and conventions.

A of [69], our 8×8 Dirac spinors Ψ and matrices Γ^a are

$$\Psi = \begin{pmatrix}\psi^\alpha \\ \chi_\alpha\end{pmatrix}, \qquad \Gamma^a = \begin{pmatrix} 0 & (\tilde{\gamma}^a)^{\alpha\beta} \\ (\gamma^a)_{\alpha\beta} & 0\end{pmatrix}, \qquad \Gamma_* = \begin{pmatrix}\delta^\alpha_\beta & 0 \\ 0 & -\delta^\beta_\alpha\end{pmatrix}, \qquad (\text{D.1})$$

where Γ_* obeys $\Gamma_{[a}\Gamma_b\Gamma_c\Gamma_d\Gamma_e\Gamma_{f]} = \varepsilon_{abcdef}\Gamma_*$. Similarly, there is a direct relation between $\gamma^{a_1\cdots a_n}$, $\tilde{\gamma}^{a_1\cdots a_n}$ and $\Gamma^{a_1\cdots a_n} := \Gamma^{[a_1}\Gamma^{a_2}\cdots\Gamma^{a_n]}$ since a product of chiral γs are straightforwardly lifted to a product of Dirac Γs. For example, the eight component spinor generators of the 6D $\mathcal{N} = (1,0)$ superconformal algebra are $Q^i = \begin{pmatrix} 0 \\ Q^i_\alpha\end{pmatrix}$ and $S_i = \begin{pmatrix} S^\alpha_i \\ 0 \end{pmatrix}$. Similarly, all the (anti)chiral spinor fields are straightforwardly lifted to eight components, such that, e.g. $\psi_{m\,i}^{\alpha}Q^i_\alpha \to \bar{\psi}_{mi}Q^i$, $\phi_{m\alpha}^{i}S^\alpha_i \to \bar{\phi}_m{}^i S_i$. To map our results to the notation of [48] and [76], the reader should then use the table 1. This table shows how our gamma matrices, generators of the superconformal group, connections, curvatures, fields of matter multiplets, etc, described in the first column, should be replaced with the terms in the second and third columns to match the notation and conventions of [48] and [76], respectively. This table can be obtained by using results of appendix A of [70] which describes in general how to change frames for the local gauging of the superconformal algebra.

Note also that our definitions of local superconformal transformations are different from the ones of [48] and [76]. If we restrict only to $\delta_Q + \delta_S + \delta_K$ transformations, it can be then shown that the differences between our notation and the ones of [48] and [76] only amount to a simple rescaling of the local parameters ξ_i, η^i and λ^a as described in table 2.

Traceless frame	Bergshoeff et al. [48]	Coomans & Van Proeyen [76]
$\xi_i,\ \eta^i,\ \lambda^a$	$\frac{1}{2}\varepsilon_i,\ \frac{1}{2}\eta^i,\ \Lambda^a_K$	$\frac{1}{2}\varepsilon_i,\ \frac{1}{2}\eta^i,\ \lambda^a_K$

Table 2. Transformation parameters for $\delta_Q + \delta_S + \delta_K$.

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