# Science in Court Society

Giovan Battista Benedetti's

*Diversarum speculationum mathematicarum et physicarum liber*
(Turin, 1585)

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*Diversarum speculationum mathematicarum et physicarum liber*
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Pietro Daniel Omodeo and Jürgen Renn

**Sources 11**

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**Sources 11**

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Cover: The frontispiece of Benedetti's *Diversarum speculationum mathematicarum, et physicarum liber* (1585)

Edition: Giovanni Battista Benedetti. *Diversarum speculationum mathematicarum, et physicarum liber*. Taurini: Bevilaqua, 1585. Courtesy of the Library of the Max Planck Institute for the History of Science:
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# Contents

# List of Figures

**Benedetti: Life, Work, Contexts**

# Introduction

Giovanni Battista Benedetti is today a lesser known figure in the history of early modern science. This relative oblivion is in striking contrast to the fame he enjoyed during his lifetime as a prominent mathematician and mathematical philosopher of Venetian origin and reputable courtier to the Savoy dukes in Turin. Among his admirers, the astronomer Johannes Kepler regarded him as one of the few Italians to significantly contribute to the advancement of mathematics in his time: "The Italians are asleep with the sole exceptions of Commandino and Giovanni Battista Benedetti. And in fact, Clavius is German."[1] For his part, the mathematician of the *Collegio Romano*, Christopher Clavius, extolled Benedetti's scientific merits in the 1589 edition of his reputed commentary of Euclid, *Euclidis Elementorum Lib[ri] XV*. In the dedicatory letter to Carlo Emanuele I of Savoy (*Serenissimo Principi ac Domino D. Carolo Emmanueli Sabaudiae Duci*), he praised "his court mathematician" Benedetti as "very expert in mathematics" (*scientissimus rerum Mathematicarum*).[2] In contrast to the eulogies of the Imperial mathematician Kepler and the most prominent astronomers of the Jesuit order, the Urbino school gathered around the mathematical purist Federico Commandino was rather reluctant to acknowledge his achievements, probably due to rivalry, reciprocal misunderstandings, and different philosophical and cultural choices. Among Commandino's pupils, Guidobaldo Del Monte severely criticized Benedetti's approach to mechanics and his claim to originality, as documented by his manuscript annotations on mathematical issues.[3] Another member of the Urbino school, Bernardino Baldi, gave an extremely negative and reductive portrait of Benedetti in his collection of short biographies of mathematicians from all epochs, *Cronica de matematici*.

> The Venetian Giovanni Battista Benedetti occupied himself with mathematics, a field in which he served the dukes of Savoy. He wrote a book on gnomonics, which dealt with many proofs belonging to this discipline. However, he has been criticized by the most exquisite scholars for not having respected the method and the purity of explanation which mathematics requires and which was respected by the Greek masters and by their followers. He also wrote some light things of no great import.[4]

Such a harsh judgment can only be explained on the basis of a profound enmity held by Commandino's followers against Benedetti. This deserves attention since it also influenced the reception of his work. Therefore, we will offer a reconstruction of Benedetti's

[1] Kepler 1937–2001, 390: "Itali somniant (preter unum Commandinum et Joh[annem] Baptistam Benedictum, Clavius enim Germanus est)."

[2] Clavius 1589, ff. *4*r*–*5*r*. The list of Benedetti's admirers also includes the Pisa philosopher Jacopo Mazzoni, the Venetian intellectual leader Paolo Sarpi, and the French scholar Marin Mersenne. See Cappelletti 1966, 262.

[3] See Renn and Damerow 2012 and Renn and Omodeo 2013.

[4] Baldi 1707, 140: "GIO[VANNI] BATTISTA Benedetti veneziano attese alle matematiche, nelle quali servì i Duchi di Savoia. Scrisse un libro di gnomonica, il quale toccò molte cose appartenenti alle dimostrazioni della detta disciplina, se non che viene ripreso da più esquisiti di non haver'osservato quel metodo, e quella purità dell'insegnare, che ricercano le matematiche, et è stato osservato dagl'ottimi Greci, e dagl'imitatori loro. Scrisse anco alcune altre cose leggiere, e di nessun momento." Here and in the subsequent pages, Italian and Latin grammar (e.g., capitalization and punctuation) has been modernized.

cultural environment and the scientific culture of Renaissance Italy in order to understand his work, its roots, and legacy.

Although Benedetti was recognized by his contemporaries, in many respects the theoretical and historical relevance of his work remains obscure. The obscuration of Benedetti is also the result of the prominence of the Urbino school and their bias towards mathematical purism. Moreover, the hagiographic historiography of science sought out the precursors to Galileo and therefore almost naturally adopted the criticism of Benedetti by Galileo's patron Guidobaldo del Monte. Intrinsic factors also played a role in the eclipse of Benedetti's fame, principal among which is the relative rarity of his major work, *Diversarum speculationum mathematicarum et physicarum liber* (Book Including Various Mathematical and Physical Speculations, 1585), not to mention that of most of his earlier publications. Benedetti's fragmentary style is a special difficulty faced by the reader, a defect that was overemphasized by Baldi and denounced by early modern scholars such as Claude-François Milliet Dechales. Dechales dealt with Benedetti's geometry in his *Cursus seu mundus mathematicus* (1690), observing that "the end of his work [the *Diversae speculationes*] comprises many miscellaneous geometrical remarks, some of which are good, in particular with regard to their special concern, but [they are] disordered."[5]

Following historiographic commonplaces and their nineteenth-century crystallization, recent historians of science have tended to neglect Benedetti's work due to the lack of extensive translations of his writings into modern languages, with the exception of the excerpts included in Drake and Drabkin, *Mechanics in Sixteenth-Century Italy* (1969). This anthology isolated passages that were exclusively devoted to what the editors saw, in hindsight, as the most relevant contributions for the progress of mechanics. Due to its selective nature, this publication did little justice to the complexity and richness of Benedetti's stature in the history of science. In fact, isolating certain results elides recognition of the multilayered architecture of a book such as the *Diversae speculationes*, which is characteristic of Renaissance science. Benedetti's showcase of mathematical erudition and scholarship is thus omitted and obscured. Drake and Drabkin's selection could only yield a reductive and rather misleading image of Benedetti as a scientist and thinker. As we will argue, entire paragraphs or chapters expunged from Drake and Drabkin's translation, for instance those concerning philosophy and cosmology, are relevant for an understanding of the author's general conception of mechanics and physics. From a historical viewpoint, it is hardly possible to trace disciplinary boundaries in the Renaissance that fit those established today. Mechanics was an emerging discipline at the crossroads of mathematics, engineering, and natural philosophy. Hence, a reassessment of Benedetti's work in its entirety is necessary not only to understand his personality but also to grasp the scientific culture of his age as the result of interdisciplinary controversies.

This open access edition makes the *Diversarum speculationum mathematicarum et physicarum liber* accessible to a large scholarly readership. Benedetti's volume is a major contribution to Renaissance science, especially due to its insights into mechanics, the mathematization of (or geometrical approach to) natural investigation, and the connection of celestial and terrestrial dynamics in a post-Copernican perspective. The first edition of this work was an elegant folio, which included heterogeneous writings on technical and philosophical issues as well as on mathematics and physics. Benedetti presented them as short treatises (*tractatus*) or letters (*epistolae*) addressed to gentlemen, courtiers, scholars, engineers, and practitioners of different arts. The volume was printed by Niccolò Bevilac-

[5] Ventrice 1985, 188: "in fine sui operis multa habet miscellanea geometrica, quorum nonnulla ad sectiones praesertim pertinentia bona sunt, sed inordinata."

qua's heir (*apud haeredem Nicolai Bevilacquae*), who was the owner of the main printing house in Turin, which was directly supported by the dukes of Savoy. The *Diversae speculationes* appeared in a series of prestigious volumes aimed at celebrating the magnificence of the court and the capital, including the works of the court historian Emanuele Filiberto Pingone: *Augusta Taurinorum* (1577), on Turin, *Inclytorum Saxoniae Sabaudiaeque principum arbor gentilitia* (1581), on the genealogy of the ruling family, and *Sindon evangelica* (1581), on Christ's shroud, which had been recently transferred from Chambéry to Turin. The *Diversae speculationes* exhibits the same celebratory intention. The volume aimed to make the quality of the court mathematician's research and skills publicly appreciable. It also bore witness to the intensity of the cultural debates going on in Turin, and connected this city with other centers of learning, especially Venice.[6] Its miscellaneous and epistolary form was suitable for displaying the variety of the author's interests and for praising his patrons, friends, and colleagues by naming them as dedicatees or addressees of the different sections and letters.

The structure of the *Diversae speculationes*—its occasional and fragmentary character, its celebratory purpose, and the epistolary display of a network of personal connections—mirrors the socio-cultural embedment of Benedetti's work. We regard it as exemplary of "science in court society." As Norbert Elias has pointed out, the *höfische Gesellschaft*, or court society, is a particular social configuration (*gesellschaftliche Figuration*) characteristic of the transitional phase to an industrial and capitalist Europe, which we conventionally refer to as the early modern period.[7] During the Renaissance and the Ancien Régime, the court was (or became) a hegemonic center from which powerful elites mediated between the ruler and the socio-political environment. Benedetti's activities as a court mathematician exemplify such a function. In his role he was expected to interact with the upper classes and respect aristocratic etiquette, and to act as the Savoy "expert" on a wide range of pedagogical and technical issues linked to his profile as a mathematician and mathematical philosopher. He was required to teach geometry to the offspring of the ducal family, to supervise engineering and architecture projects, to produce instruments or machines for practical purposes, warfare, and recreation (such as fountains, sundials, or nautical instruments). He had to adhere to shared court values, norms, and behaviors, primarily those linked to honor and prestige. These courtly principles are reflected in the epistemic values permeating his scientific production, for instance in the value of scientific disinterestedness that marks his theoretical approach to practical as well as to speculative problems. In a hierarchical and aristocratic society, his theoretical attitude marked at once the continuity and the distance between his role as a court mathematician and those involved in practical activities. Moreover, the primacy of courtly interests over those of science as a purely scholarly endeavor (as it was pursued at universities and academies) is evident from Benedetti's networking strategies, which were aimed at not so much exchange with other scholars as at giving advice to a wide range of people, beginning with the ruling elites of the country. In other words, he was not primarily concerned with establishing a réseau, as was typical for the Republic of Letters. As we will show, he did not regard himself and his activity as part of a learned network but rather as the center of courtly interaction. This center-periphery structuring of his network mirrors—in two senses—the "knowledge economy" his work is embedded in. Sociologically, the central-

[6] Cecchini and Roero 2004.

[7] As Norbert Elias put it (Elias [1969] 2002, 73): "Durch das Bemühen um die Struktur der höfischen Gesellschaft und damit um das Verständnis einer der letzten großen nicht-bürgerlichen Figurationen des Abendlandes eröffnet man sich also mittelbar zugleich einen Zugang zum erweiterten Verständnis der eigenen berufsbürgerlich-städtischen-industriellen Gesellschaft."

izing character of court society is reproduced in scientific policies through Benedetti's function as a technocrat; epistemologically, the fluid style and fragmentary form of the *Diversae speculationes* is an expression of the expert-advice character of his scientific work. Thus, in order to properly understand his work, we deem it necessary not only to investigate the technical and theoretical dimensions of Benedetti's science, but also to analyze evidence concerning the ties between these dimensions and the social and cultural environment.

Among the studies on Benedetti, Giovanni Bordiga's monograph *Giovanni Battista Benedetti: filosofo e matematico veneziano del secolo XVI* (1926) still stands out as one of the most important references as far as prosopographical information is concerned.[8] The excellent research accomplished by Carlo Maccagni and the proceedings of the conference on Benedetti held in Venice in 1985 at the *Istituto Veneto di Scienze, Lettere ed Arti* investigated many aspects of Benedetti's contribution.[9] In spite of the accuracy of these relatively recent Italian studies, Alexandre Koyré's evaluation of Benedetti's role in the first stages of modern science, in the *Études galiléennes* (1939), has had a more direct influence on his international reception. Koyré emphasized the link between the incipient mathematical science of motion and heliocentrism in Benedetti's speculations. On account of this, Paul Lawrence Rose regarded him as a herald of the "Italian Renaissance of mathematics".[10] Koyré's grand narrative of the Scientific Revolution, which he conceived as a development with a "prologue" (Copernicus) and "epilogue" (Newton) in the heavens, included Benedetti as a precursor to Galileo because of the interconnection of mathematical and physical themes in the former's work. Koyré's main thesis was that classical physics (the mathematical science of nature of Galileo, Descartes, and Newton) emerged as a direct consequence of Copernicus's geokinetic system, which undermined the traditional (Aristotelian and Ptolemaic) worldview. Although Benedetti's relevance as a source for Galileo cannot be denied, scholars now view Koyré's narrative as dubious due to its abstract treatment of the history of science, conceived of as an internal development of ideas. In the years of the Cold War (or shortly before it), this viewpoint embodied the ideological reaction to Marxist or materialist-oriented accounts, which stressed the technological, empirical, and social roots of modern mechanics, as was the case with Leonardo Olschki and Edgar Zilsel.[11] Bendetti's approach to mechanics and post-Copernican astronomy therefore appears to be an appropriate case study for reconsidering this general historical *problematique*, beginning with a reassessment of the relation between mechanics and astronomy in early modernity. This implies a reconsideration of the basic questions of the historiography of science and of historical epistemology, such as the role of material and intellectual factors in the so-called Scientific Revolution.

In this edition of the *Diversae speculationes*, we aim to present Benedetti's achievement in its rich complexity. Benedetti is emblematic both of his time and of the nonlinearity of the historical process of Renaissance science with its multicentric institutions and scientific networks. We will show that the apparently fragmentary nature of his work is expressive of the peculiar character of science in court society and, in spite of this form, it conceals a fundamental unity of his conception of nature and method, both of which rest on geometry. To be sure, Benedetti regarded mechanics as a model, but he enlarged his

[8]Bordiga 1985.

[9]Bordiga 1926, repr. Bordiga 1985, Maccagni 1967b, Maccagni 1967a, Maccagni 1983, and Istituto Veneto di Scienze 1987.

[10]Rose 1975, 154–156.

[11]The cultural-political intentions of Koyré's approach emerge most vividly from his 1943 article on Galileo's Platonism, see Koyré 1943. Lefèvre stresses it in Lefèvre 2001.

perspective to include the most varied fields of investigation in order to concretely demonstrate the fruitfulness of his approach to universal knowledge about astronomy, physics, meteorology, and even literature and ethics.

## Overview and Acknowledgments

Our first chapter is devoted to Benedetti's biography and comprises an overview of his publications. We consider the various dimensions of his identity, such as his nobility as both a Venetian patrician and a Savoy aristocrat in the retinue of Emanuele Filiberto and Carlo Emanuele at the Turin court. In particular, we discuss his role as a courtier, the tasks he was entrusted with, and the relations he established in this context. We also deal with his extra-academic education and his attitude towards mathematics, which he initially saw as an intellectual instrument to be used against the "bookish" culture of the universities and the "idle" rhetoric of the humanists. Later he softened the polemical tone that characterizes his early writings. Moreover, since he saw himself as a philosopher, more precisely as a court philosopher to the Dukes of Savoy, a discussion of Benedetti's mathematics cannot be separated from his philosophical project. Benedetti claimed for himself the right to discuss in mathematical terms issues of natural philosophy that traditionally belonged to the rather qualitative and conceptual approach of the peripatetics. The *Diversae speculationes* is an altogether magisterial example of this merging of philosophical and mathematical perspectives.

Chapter 2 is a reconstruction of the cultural life of Renaissance Turin, the town in which Benedetti spent his mature years and where he composed his major work. His achievement was embedded in the cultural ferment of the new capital of Savoy, a place of ambitious town planning and civil reforms. It was a time in which the arts, literature, and philosophy received a new impetus. Editorial projects were launched; the university was reopened and illustrious scholars were attracted there. The dukes' religious politics was informed by a sense of pragmatism, which is mirrored in the fluctuating relations between the ruling family, the Jesuits, and Rome. Benedetti's secular attitude towards science and philosophy mirrors the cultural politics of his patrons. In addition we discuss his involvement in various scientific debates divided into courtly conversations, academic controversies, and controversies going beyond the settings of the court and the university. Among such extra-academic public controversies, the most important was Benedetti's public defense of the reliability of astronomical calculation against a polemist, Benedetto Altavilla, who indirectly attacked his and others' astrological practice. Newly discovered documents show that Benedetti's successor as court mathematician, Bartolomeo Cristini, continued that polemic after Benedetti's death. Cristini discredited Benedetti's use of astronomical tables to cast horoscopes, in order to ingratiate himself with the dukes and successfully start a career at court. We trust that this chapter offers new insights into the scientific culture of the Renaissance by bringing Turin into focus, a cultural centre that has so far escaped in-depth consideration by historians of early modern science.

In chapter 3 we offer an overview of the structure of the *Diversae speculationes*. We introduce Benedetti's mathematical sections in general terms, focusing on his geometrical demonstrations for the solutions of problems of arithmetic—which were the result of his private teaching of mathematics to the Savoy prince—his sketchy annotations on the theory of proportions based Book 5 of Euclid's *Elements*, and his considerations on linear perspective aimed at supporting the work of painters and architects. The sections on physics, mechanics, and natural philosophy are not discussed in this chapter as they receive special treatment in other chapters. The last part of the *Diversae speculationes* was a miscellanea of scientific letters. We discuss them vis-à-vis their significance as a mirror of Benedetti's social capital. His epistles were mostly directed to aristocrats, beginning with his patrons, other courtiers, and diplomats, especially those from Venice. He also corresponded with professors, artists, engineers, and practitioners, some of whom lived north of the Alps. His network was markedly a center-periphery one, in which the court expert shared his views on the most varied topics with others seeking his advice or opinion. Thus, it was not a scholarly network implemented for the sake of exchange and the advancement of knowledge. Rather than a networking activity establishing a Republic of Letters, Benedetti's correspondence reflected court-society centralism.

Chapter 4 addresses Benedetti's epistemology on the basis of passages regarding the certitude of mathematics and his effective use of mathematics in physics. His role as an early champion of what would later become known as "physico-mathematics" is understood here against the background of the philosophy of mathematics in the Renaissance. Moreover, we deem the modal epistemology underlying his science of particular interest: his treatment of nature in mathematical terms did not imply the necessary or deterministic nature of physical processes. Rather, he embraced an ontology and an epistemology of contingency that constituted a bridge between medieval scholastic views on nature and the mathematical physics of the time of Galileo and Descartes. We dedicate an excursus to the vision of nature as the realm of contingency in the period extending from the medieval science of weights to seventeenth century mechanics, and ascribe to Benedetti a central position in this intellectual process.

Chapter 5 deals with the field in which Benedetti has received the most credit from historians: mechanics. Actually, Benedetti himself emphasized the importance of his contribution to mechanics as what would secure his fame in posterity. We summarize his theories on equilibrium and his critical reworking of earlier theories such as those developed by Jordanus Nemorarius and Niccolò Tartaglia. We consider Guidobaldo Del Monte's negative reaction to Benedetti's mechanics in detail, as well as the weaknesses and strengths of both authors. We regard this pluralism of clashing and integrating views as revealing the complex paths of discovery undertaken by students of mechanics in a period of the utmost relevance to its modern systematization. Moreover, the subterranean conflict of views and approaches between Benedetti and Del Monte affected Galileo's work. His mechanics drew from both authors, although he did not acknowledge Benedetti explicitly due to circumstances and opportunity.

Chapter 6 summarizes Benedetti's astronomical work. Although he did not see himself as an astronomer, his contribution is quite interesting. He should be acknowledged for his effort to develop a new mathematical physics in accordance with post-Copernican astronomy. His discussion of astronomical theory against the background of a general philosophical reform was strikingly innovative. His specific polemics on the reliability of astronomical calculation also receive close treatment here. Furthermore, in an appendix Günther Oestmann offers an assessment of Benedetti's astrological calculations on the basis of so-far neglected manuscript sources containing two of his horoscopes.

In chapter 7 we deal with Benedetti's natural philosophy as he presented it in Book 4 of the *Diversae speculationes*. Although he entitled it "Disputations on Some Opinions Held by Aristotle" (*Disputationes de quibusdam placitis Aristotelis*), it was a polemic directed "against" fundamental Aristotelian theses on motion, time, space, matter, and cosmology. This is the section in which Benedetti's commitment to "the system of Aristarchus and Copernicus" most clearly emerges. It is also a fundamental section on the existence of the physical void as the necessary presupposition of any local displacement and on free fall through different media. We see this book of the *Diversae speculationes* as a major contribution to the Renaissance debate on the foundations of physics, going far beyond the treatment of mechanics and cosmology *strictu sensu*. Hence, we take into consideration Benedetti's definition of space as an "inter-bodily gap" (*intervallum corporeum*), his defense of the possibility of actual infinity in nature against Aristotle's veto, his understanding of time as an absolute frame complementary to space and its place in the philosophical debates of the Renaissance, the revision of the concepts of natural and violent motion, and finally, the surprising conclusion of the "Disputations on Some Opinions Held by Aristotle" with a Copernican note.

This volume is a continuation of an Edition Open Access project aimed at the publication and scholarly reassessment of the fundamental sources of Renaissance mechanics. This project began with Jürgen Renn and Peter Damerow's *Guidobaldo del Monte's Mechanicorum Liber* in 2010. Elio Nenci's open-access publication of Bernardino Baldi's *In mechanica Aristotelis problemata exercitationes* appeared in 2011 and, in 2013, Matteo Valleriani's *Metallurgy, Ballistics and Epistemic Instruments*, including a transcription and an English translation of Nicolò Tartaglia's *Nova scientia*. Ideas that were crucial for the writing of this introduction to Benedetti's *Diversae speculationes* are derived from another volume by Renn and Damerow, *The Equilibrium Controversy: Guidobaldo del Monte's Critical Notes on the Mechanics of Jordanus and Benedetti and their Historical and Conceptual Backgrounds* (2012).

We are very thankful to our colleagues at the Max Planck Institute for the History of Science in Berlin for their insightful remarks and the precious exchanges about early modern science that took place in the many workshops and regular meetings over the past years: Angela Axworthy, Sonja Brentjes, Jochen Büttner, Giulia Giannini, Peter McLaughlin, Matthias Schemmel, Joyce Van Leeuwen, Matteo Valleriani, and many others. We are especially thankful to Antonio Becchi and Elio Nenci for their accurate reading of the manuscript of this book and their expert advice. Rodolfo Garau kindly took pictures of manuscripts and rare books that are preserved in the Biblioteca Nazionale Universitaria di Torino and are published here. Stefan Paul Trzeciok helped us with the translation of several Latin passages. We would like to acknowledge the Collaborative Research Centre "Episteme in Motion" (Freie Universität Berlin), which has constituted an important intellectual framework for this collaboration, and the Deutsche Forschungsgemeinschaft for financing Pietro Daniel Omodeo's research in Berlin for several years as well as a research trip to Turin in 2012. We are grateful to the librarians of the Max Planck Institute for the History of Science for their valuable support with bibliographical research and the digitization of sources, and to Louis Berger, Lindsay Parkhowell, and Charlie Zaharoff for their copyediting work and linguistic support. The last phase of the authors' collaboration on the manuscript of this volume is part of a project based at the Ca' Foscari University of Venice that has received funding from the European's Horizon 2020 Research and Innovation Programme (GA n. 725883 *EarlyModernCosmology*).

# Chapter 1
# Prosopography

## 1.1 The Life and Career of a Renaissance Man

Giovanni Battista de Benedetti came from a patrician family of Venice. Although the title of nobility may appear superfluous to the historian of science, it was not so for him and his contemporaries. Benedetti often noted this in his publications, adding to his name the honorific "Patritius Venetus." Evidence for Benedetti's noble origins can be found in a document dated January 14, 1570. This is a patent through which Duke Emanuele Filiberto of Savoy conferred upon Giovanni Battista the privileges of Imperial nobility in addition to his previous titles:

> We make, create, and constitute the aforementioned Giovanni Battista Benedetti as a true noble of the Holy Roman Empire and of our Empire forever, alongside all his legitimate and natural sons and daughters (those who are already born and those that will be born). We will call and fully declare them such [nobles of the Holy Roman Empire]—although he and his predecessors are noble and were born from an ancient and noble progeny, as we are very well informed.[1]

In those years, the establishment of the Savoy court in Turin brought about a general transformation of the urban *patriziato* into an aristocratic class gravitating around the dukes.[2] This trend was parallel to the more general political-social shift from the civil humanism of the medieval municipalities toward the courtly culture of centralized territorial States.

On the occasion of the conferral of the patent on Benedetti, the cross of Savoy was added to his heraldic design along with the motto "sic vita veritas."[3] This motto, which indicated a conduct of a life dedicated to the search for truth, was the acknowledgment of his mathematical and philosophical excellence. In the preamble to the duke's patent of nobility, it was precisely Benedetti's devotion to the mathematical disciplines, the *humanae litterae*, and the philosophy that was extolled as an example to be imitated and a reason for the conferral of aristocratic privileges on him and his heirs.[4] In this case, scientific distinction led to higher social recognition and even served as a legitimation for it.

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[1] Bordiga 1985, 752: "Habbiamo creato, fatto et costituito, facciamo creamo et costituiamo il detto Giovan Battista de Benedetti con tutti i suoi figliuoli maschi e femine legittimi, et naturali, nati et che nasceranno, et saranno procreati di legittimo matrimonio, con tutti loro posteri et heredi et successori in perpetuo veri nobili del Sacro Romano Imperio et nostri, et per tali li chiamiamo et dicchiariamo per dabondante (ancora ch'egli insieme coi suoi predecessori siano nobili e nati di antica prole nobili come siamo benissimo informati)."

[2] Stumpo 1998, 138.

[3] Bordiga 1985, 601.

[4] Bordiga 1985, 752: "Emanuele Filiberto per gratia di Dio Duca di Savoia Principe di Piemonte etc. Essendoche le attioni che tendono alla Virtù, come che da quella prendano accrescimento et perfettione, sono ammirate et havute in pregio: così gl'huomini che in quelle di continuo si essercitano vengono da ogniuno istimati et tenuti in particolare consideratione, la onde havendomi sempre fatto conto delle persone che dirizzassero ogni loro pensiero al bene operare, et quanto più si potrà, cercassero col mezo delle scienze, et arti liberali sicure et vere guide alla virtù di venire alla cognizione di esso doppo l'haver noi ricercato che

During the Renaissance, nobility was more important than professional appurtenances or academic titles. For instance, the celebrated Danish astronomer Tycho Brahe, himself an appreciative reader of Benedetti, held aristocratic lineage in higher esteem than any status linked to university professorship, including the position of imperial mathematician—an appointment which, by contrast, raised the status of his fellow countryman and opponent Nicolaus Reimarus Ursus, who was of low extraction.[5] Accordingly, Brahe always emphasized Benedetti's lineage when citing his work, for instance his letter on the superlunary location of the supernova of 1577. The capitalization as well as the reverence in this passage from the *Astronomiae instauratae progymnasmata* (posthumous, 1602) is telling:

> The small star of Cassiopeia would not shine as brightly as this nova over the whole surface of the Earth because of the dry fumes placed in-between, if they had been only under that one, and did not affect in the same manner the other stars next to it and augmented that unusual light. But the most excellent philosopher GIOVANNI BATTISTA BENEDETTI, THE VENETIAN PATRICIAN, eminently and skillfully demonstrated this with geometric arguments, in [his] outstanding work concerning mathematical and physical speculations (around the end of his letters). Writing to Annibale Raimondo […] he clearly showed the absurdity which necessarily follows from his false assumption [i.e., the sublunary position of the nova].[6]


Figure 1.1: An example of the titles Benedetti added to his name in his publications. In the title page of *De gnomonum umbrarumque solarium usu* (1574), he called himself "Venetian Patrician, Philosopher." (Max Planck Institute for the History of Science, Library)

The prominence accorded to lineage is evident from Brahe's self-representation in the portrait at the beginning of his *Epistolarum astronomicarum libri* (1596), a collection of

in questo ne sotisfacesse, massime nelle discipline matematiche. Al fine ci è pervenuto nelle mani il nobile messer Giovanni Battista de Benedetti venetiano, nostro mattematico il quale havendo consumato la maggior parte dell'età sua nelle bone lettere et studij di filosofia, et fatto professione delle dette mattematiche, et così divinamente et per eccellenza riuscito che si può dire in quelle (tra gl'altri) essere singolare cosa che si porge tal contento, et la sua servitù a noi molto grata tale soddisfattione che lo giudichiamo degno che partecipi de gl'honori dovuti alle sue virtù acciò che gl'accresca l'animo di perseverare et altri siano invitati a seguitare li suoi vestigij."

[5] This is why Brahe was not and could not desire to be imperial mathematician to Rudolph II, as has often been wrongly thought. See Voelkel 1999.

[6] Brahe 1916, 250: "Accedit et hoc, quod Stellula illa Cassiopeae in toto Orbe Terrarum ob siccas illas fumositates interpositas non tam splendide apparuisset atque haec Nova, si sub hac sola constitissent, et non reliquas illi vicinas pari modo attingissent, lumineque insueto auxissent. Hoc vero ultimum egregie et solerter ex excellentissimo Philosopho IOHANNE BAPTISTA BENEDICTO PATRICIO VENETO in praeclaro illo Opere quod de speculationibus Mathematicis et Physicis inscripsit, circa finem inter Epistolas eius evidenter et dilucide, Geometricis rationibus demonstratur. Ubi ad hunc ipsum Annibalem Raimundum scribens, absurdum, quod ex eius falsa assumptione necessario sequitur, dilucide ostendit."

epistles that arguably took Benedetti's collection in the *Diversae speculationes* as a model. Brahe's image is encircled by the heraldic designs of the family and makes the signs of his nobility very visible. In the same epistolary, Brahe's letters directed to aristocrats appear more prominently than those addressing "simple" professors or practitioners. He attached greater importance to his correspondence with the patron of sciences, Landgrave William IV of Hesse-Kassel, than to exchanges with the latter's court mathematician Christoph Rothmann.[7] Similarly, in the *Diversae speculationes*, Benedetti published with pride his letters to dukes or to illustrious aristocrats.

Apart from his nobility, we do not know much about Benedetti's origins. According to a horoscope that he cast for himself (Figure 1.2), and was printed by the Neapolitan astrologer Luca Gaurico in *Tractatus astrologicus* (*Astrological Treatise*, 1552),[8] Benedetti's father was a learned *Hispanus*, or Spaniard. Based on this thin evidence, his biographer, Giovanni Bordiga, speculated that his family could have been merchants trading with Spain.[9] Other archival documents caused him to speculate about Benedetti's marriage, around 1585, and about the existence of a daughter called Lodovica from an earlier relationship or marriage. She married a certain Domenico Pipino of Racconigi. Benedetti built a sundial for this son-in-law (*magnificus Dominus Dominicus Pipinus generus meus*), as indicated in *De gnomonum… usu* (1574). Lodovica died young, long before her father, in 1580.[10]

For the greater part of his life Benedetti was a courtier. For several years he served duke Ottavio Farnese of Parma, whom he joined in 1558 as "*lettore di filosofia e mathematica*."[11] Later, from 1567 up to his death on January 20, 1590, Benedetti served the Dukes of Savoy Emanuele Filiberto and Carlo Emanuele I. His duties were typical for a Renaissance court mathematician and are akin to those of Leonardo da Vinci in Milan, Guidobaldo del Monte in Urbino, Galileo in Florence, and Kepler in Prague, to mention only a few well-known names.[12] Benedetti was required to advise his patrons on issues of mathematical expertise. His fields of competence included engineering and architecture.[13] In Parma and Turin he built sundials (such as the modern one in Figure 1.3). He was also responsible for the construction of a fountain in the ducal park (*Parco di Viboccone*, later *Parco Regio*), which was destroyed by the French army during the siege of 1706.[14] Moreover, he was consulted on astronomy and music, both traditionally considered mathematical disciplines. In Parma he carried out astronomical observations, which he also reported on in the *Diversae speculationes*. In two letters to the Parma choirmaster de Rore, Benedetti explained musical consonance and dissonance of two tones by the ratio of oscillations of waves of air generated by the strings of musical instruments.[15] He claimed that the frequency of two strings of equal tension must have an inverse ratio to the lengths of the strings, and thus proposed to describe the degree of consonance or dissonance of two tones mathematically. In Turin he wrote a proposal for the calendar reform in 1578, *De temporum emendatione*, later reprinted in the *Diversae speculationes* as the

[7] See Mosley 2007.
[8] Gaurico 1552, f. 76*r*.
[9] Bordiga 1985, 588.
[10] Bordiga 1985, 604–605.
[11] Bordiga 1985, 593–595.
[12] For the broad European context of patronage and the arts in the Early Modern Period, see Bedini 1999, Moran 1981, and Moran 1991.
[13] See Roero 1997 and Mamino 1989.
[14] Maccagni 1967a, 353–354.
[15] Benedetti 1585, 277–278.

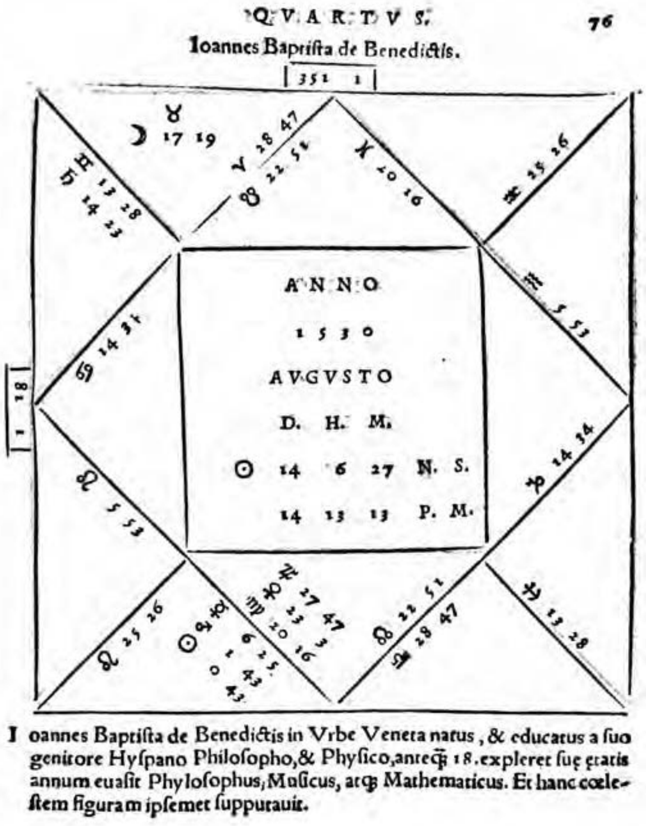


Figure 1.2: Benedetti's own horoscope, in Luca Gaurico, *Tractatus astrologicus* (1552), f. 76*r*. (Bayerische Staatsbibliothek)

first of his epistles (to Duke Emanuele Filiberto).[16] This proposal was also sent to Rome and was meant as an aid to Clavius's efforts to correct the calendar.[17] At the same time, he taught mathematics to Emanuele Filiberto and his son Carlo Emanuele I.

Courtly life included participation in literary culture. Baldassar Castiglione, in his idealization of the court of Urbino in *Il libro del Cortegiano* [*The Book of the Courtier*] (1528), launched the model of a courtier with a refined literary education.[18] Following such cultural dispositions, a courtier versed in mathematics could advocate the usefulness of his expertise for the interpretation and assessment of "scientific" questions raised by classical sources, even poems. This attitude explains the inclusion of a letter on Ovid in the collection of epistles in the *Diversae speculationes*.[19] It was addressed to a certain Pancrazio Mellano, perhaps a courtier, asking Benedetti's opinion about the astronomical references in Book 2 of the *Metamorphoses*, in which Ovid tells the myth of Phaeton. According to the myth, Phaeton rode his father Apollo's chariot one day but he was unable to control the horses and keep the sun on its regular path. Finally, he was thrown out of the chariot, took a bad fall, and died. In the poem Ovid described the solar path in some detail but, according to Benedetti, he mixed up daily rotation and annual motion along the ecliptic: "Ovid unduly passes from the daily motion to the annual" (*Quod Ovidius transcurrit*

[16] Benedetti 1585, 205–210.

[17] Benedetti's advice on the calendar reform is preserved in the Biblioteca Apostolica Vaticana under the signature cod. Vat. lat. 5645, 148*r*–150*r*. See Ziggelaar 1983, 211–214.

[18] Baldassar Castiglione, *Il libro del Cortegiano*, ed. Walter Barberis (Torino: Einaudi, 2017).

[19] Benedetti 1585, 417–418.


Figure 1.3: A modern sundial on the Church of San Lorenzo in Turin reminiscent of those designed by Benedetti. (Own photography)

*a motu diurno, ad motum annuum praeter rem*). To make his point clear, Benedetti listed the passages dealing with one or other of the two motions ascribed to the sun in ancient astronomy.[20]

As an exponent of the Turin elite, he was himself devoted to poems. For instance, the Milanese painter and writer Giovanni Paolo Lomazzo, who was linked to Savoy's court, celebrated Benedetti in verse as a philosopher, mathematician, and astrologer. In the first lines of a poem dedicated to him, Lomazzo declared himself delighted that Benedetti appreciated his paintings and cast his birth horoscope. Lomazzo's poem paints a vivid picture:

> Prudence and knowledge descend
> From Philosophy into [human] intellects;
> Which are perfect as far as their disposition is concerned,
> As each one receives its part of justice and reason.
> To Benedetti, he so wise
> And precious in the world,
> Belongs so much of this [philosophy]
> That it would be vain to try to equal him:
> So sublime does his value shine.
> All the more am I delighted that he appreciated

[20] Omodeo 2012b.

> My painting so much so that he considered
> The time and the point in which I was born in the world.
> Oh splendor of our time, the sound [of your voice] silenced
> Every scholar of your art, who had to direct his judgment elsewhere,
> As it was overshadowed by yours, which is so deep.[21]

Benedetti received no formal or academic education. Like other Renaissance self-taught men (e.g., Niccolò Tartaglia and Tommaso Campanella), Benedetti was even proud of being removed from the academic habitus and training centers. This is particularly evident from the anti-academic tone of some of his polemics. In the preface to his first scientific treatise, *Resolutio omnium Euclids problematum* (1553) (On the solution to geometrical problems using a compass with a fixed opening), the twenty-three year old Benedetti emphasized the fact that he had not had a "common" (*quod vulgus solet*) education at some gymnasium or school. He boldly wrote to his patron, the Dominican abbot and diplomat Gabriel Guzman, that:

> Until now I have advanced without any mentor or teacher (under the guidance of God). I have never frequented any gymnasium or school. I have not learned what the vulgar (I mean this word without arrogance) use to estimate erudition, [such as limiting it] to the time spent at school, thus setting an end to learning when the seven years [of regular studies] are ended. As long as I live, I will continue [learning].[22]

It is possible that Benedetti was educated privately by his father, depicted in Gaurico's *Tractatus astrologicus* as "*Hyspanus, Philosophus, et Physicus*" (see Figure 1.2). For his part, Benedetti acknowledged only one teacher, namely the reputed mathematician and scientist-engineer Niccolò Tartaglia (ca. 1500–1557), for introducing him to the first four books of Euclid's *Elements*, probably between 1546 and 1548. In the *Diversae speculationes mathematicae et physicae*, Benedetti mentioned Tartaglia again as one of the very few authors of mathematical works whom he deemed worth reading.[23] However, in the

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[21] Lomazzo 2006, 177–178, III, 19, "Del Sig. Gio. Battista Benedetti Matematico":

> "De la Filosofia nasce e discende
> La prudenza e 'l saper de gli intelletti;
> Co' quali essendo nel dispor perfetti,
> A ognuno suo diritto e sua ragion si rende
> Di questa sì gran parte se ne prende
> Il saggio e raro al mondo Benedetti,
> Che d'agguagliarlo in vano è chi s'affetti:
> Tanto sublime suo valor s'estende.
> Però tanto godo io che sì gli piacque
> La mia pittura, e perciò egli volse
> L'ora et il punto nel qual nacqui al mondo.
> Splendor di questa etade al tuo suon tacque
> Ogn'un de l'arte tua, e altrove volse
> Il suo dir vinto dal tuo sì profondo."

[22] Benedetti 1553, f. 5*r*: "[…] huc usque progressus sum (Deo duce) sine monitore praeceptoreque ullo, nullum gymnasium unquam, nullamque scholam frequentavi, neque hoc studui, quod vulgus solet (sed absit verbo arrogantia) pro tempore in scholis transacto, eruditionem estimare, ac septennario finito finem studiis imponere, sed dum vivo, illa prosequi."

[23] One reads in the preface *ad lectorem* of the *Diversae speculationes* the following declaration: "In his autem meditandis, ex arithmeticis authoribus quos inspexi praecipuus fuit Nicolaus Tartalea, quippe quem

*Resolutio omnium Euclidis problematum*, he was quick to add that he had learned the rest of the *Elements* by himself:

> As it is honest and right to attribute to everybody his own merit, [I should acknowledge that] Niccolò Tartaglia taught only the first four of Euclid's books to me. I studied the rest alone with effort and diligence. In fact, for the one who wants to know, nothing is [too] difficult.[24]

Bordiga described such self-celebration as a sign of Benedetti's "pride in the assumed independence of his own thinking" (*orgoglio di creduta indipendenza del proprio pensiero*).[25] This is the same pride that would later lead to animosity with other prominent mathematicians such as Del Monte.

Moreover, in the preface to the *Resolutio*, Benedetti contrasted the simplicity of mathematics with the vanity of rhetoric. He went so far as to accuse learned and eloquent doctors of corrupting the sciences.

> Furthermore, mathematics does not require much [stylistic] splendor. If some language expert tried to improve its elegance, this would have no value, because a change of the mathematical language and of the scientific terminology could easily confuse the sense [of the reasoning] and render everything obscure. Therefore, I will follow the scholarly tradition and use plain words in my demonstrations, as I disapprove of deceptive elegance. In this respect, I follow the steps of the ancients who taught the sciences and the subjects themselves using plain words. Petty teachers (indeed, charlatans and babblers) corrupted this manner of teaching. Although they do not understand the subject, their babbling obtains the highest praise by the vulgar who regard them as learned scholars. This should not be surprising, considering that the most perfect and distinguished expertise in the sciences is attained by very few—despite the fact that many people write a great deal in all kind of sciences and arts, babbling a lot and capturing the attention of the uneducated with illusions and bombastic words.[26]

The same tone characterized Benedetti's next publication. Its title was intentionally polemical: *Demonstratio proportionum motuum localium contra Aristotilem et omnes philosophos* (1554). In fact, this booklet put forward a novel theory of motion. He

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fere omnia ab aliis scripta collegisse constat, nec alios ex praecipuis quos legere potui omittendos duxi, inter quos sunt Hieronymus Cardanus, Michael Stifelius, Gemma Frisus, Ioanna Novimagus, Cuthbertus Tonstallus, caeterique huiusmodi."

[24]Benedetti 1553, f. 5*v*: "Caeterum quia cuiusque quod suum est reddi debet, nam et pium et iustum est, Nicolaus Tartalea, mihi quatuor primos libros solos Euclidis legit, reliqua omnia, privato et labore et studio investigavi, volenti namque scire, nihil est difficile."

[25]Bordiga 1985, 588 (4).

[26]Benedetti 1553, f. 5*v*: "Adde quod Mathematicae disciplinae, neque tantum requirunt splendorem, neque si quis peritus linguarum contendat ad elegantiam rem reducere, egregium quid effecerit, quia mutato usu Mathematicae loquendi, ipsiusque scientiae terminis, sensum facile perturbaverit, et ex nihilo nihil apprehensum obtinuerit. Quare morem scholarum sequutus, obstentatione elegantiae explosa, verbis nudis in demonstrationibus usus sum, hac in parte veterum vestigia sequutus, qui nudis verbis scientias resque ipsas docebant, quem modum docendi, nobis devastarunt scioli vel potius circulatores, garruli, rebus ipsoque iudicio destituti, garrulitate siquidem apud vulgus, laudem summam consequuntur, et pro doctis circunferuntur, nec mirum, cum scientiarum perfecta exquisitaque perita, paucissimis detur, non obstante quod multi permulta de omnis generis et scientiis et artibus scribant, permultaque garriant, fucis suis, et ampullis imperitorum oculos perstringentes [...]."

argued that bodies of the same material fall through a given medium with the same speed, and not with speeds proportional to their weights, as Aristotle held. This is the reason for Benedetti's declaration of war "against Aristotle and all philosophers" in the title. Benedetti employed the Archimedean concept of buoyancy to account for the dependence of the motion of fall on their specific rather than absolute weight. As we shall see, these ideas played an important role in the *Diversae speculationes*. The use of Archimedean notions to improve on Aristotle's physics was probably stimulated by Tartaglia's Italian translation (1543) of Book 1 of Archimedes's treatise on bodies in water.[27] Benedetti's challenge to Aristotle must have raised considerable discussion, as is shown by the fact that, in his *Demonstratio*, he discussed Aristotle's views and responded to his critics at length. In the second edition of the *Demonstratio* (13 February 1554 *more veneto*, in fact, 1555), he showed that the resistance encountered by a falling body in a medium depends not on its volume, but on its surface area. Benedetti moreover explained the acceleration of the motion of fall in terms of an increasing impetus of the falling body. He had already outlined his theory of fall in the dedicatory letter of the *Resolutio*, explaining this anticipation as a means of avoiding plagiarism.[28] Still, in spite of his efforts to secure priority for his ideas by repeated publication, they were plagiarized by the Flemish polymath Jean Taisner in 1562 and spread through Europe with no clear acknowledgement of their origin.[29] This prompted Benedetti to express his indignation and rage at Taisner in the dedicatory letter of his *De gnomonum… usu* (1574).[30]

As was to be expected by his irreverent tone, some of the first reactions to Benedetti's early writings were rather critical. As he reports in the preface to the second edition of the *Demonstratio* (1555), some Roman scholars objected that his treatment of motion was in disagreement with Aristotle (*illam [meam propositionem] neutiquam esse iuxta mentem Aristotelis*). Benedetti was informed about their disapproval by a Dominican friend, Petrus Arches, an expert of Hebrew and Greek letters cultivated in philosophy and mathematics.[31] Benedetti replied that those scholars worshipped Aristotle like a pagan god (*veluti coeleste quoddam numen*) and did not admit that their *auctor* could make mistakes. He claimed that he had not misunderstood Aristotle; rather, that he simply disagreed with him.

> I remember that he [the very educated Doctor Peter Arches]—after many different conversations on various subjects—told me that many in Rome considered that proposition of mine (which I sent to you, Reverend Mr. Guzman, among other ones) and they mostly reacted with surprise for I did not specify that it was by no means in accordance with Aristotle's mind. Such was the reaction of those who considered my demonstration very attentively.
>
> They could not concede that Aristotle was mistaken in any way, because they do not regard him as a human being. Rather, they confer upon him the celestial condition of a pagan divinity. And they see even slight disagreement as a sin. Therefore [they believe that] I committed (and still commit) heresy if, according to their judgment, I do not follow the pure and authentic mind of Aristotle's doctrine in any manner.
>
> Thus, in order to escape the allegation of such an error or [the rumor] that I am dissimulating and hiding something, especially as far as this issue is con-

[27] Archimedes 1543.
[28] Benedetti 1553, f. 10*v*. See Maccagni 1967a, 338–340 and Maccagni 1967b, 14–15.
[29] Taisner 1562, see the discussion in Maccagni 1967a, 344–455, n. 13.
[30] Benedetti 1574, f. 4*v*.
[31] Maccagni 1967b, 20–21, and 20, n. 14.

> cerned, I decided to publish this new booklet in which I present my opinion more clearly. In this manner, everybody should become aware that I correctly understood Aristotle and that I disagree with him on a particular issue with considered reason. This is an unpleasant task for me. In fact, it is only unwillingly that I dissent with such a great man. I know nobody who could rival his excellence in all kind of doctrines. Nevertheless, his teaching is to take as true that which is supported by stronger reasons. He himself followed this precept, as he stated in the *Ethics*: "Plato is my friend, Socrates is my friend, but truth is even more friend to me."[32]

It is evident from these passages that Benedetti regarded mathematics as a support for conclusive rational argumentation in the treatment of natural issues. Therefore, as a *mathematicus* he claimed for himself the right to be called a *philosophus*. Already in the short biographical indication accompanying his birth horoscope, he was said to be a "*Phylosophus, Musicus, atque Mathematicus*" (see Figure 1.2). In his publications, Benedetti often stressed his quality as "*philosophus*" or "*filosofo*." Galileo would later add the title of "philosopher" to that of "court mathematician" in Medici's Florence.[33] However, in Benedetti's case, it is evident that adding the title of "*philosophus*" was not part of a strategy aimed at social advance but rather mirrored his cultural and philosophical commitment to a mathematical philosophy of nature with all its consequences, among them that Aristotelian physics was open to critique by means of mathematical reasoning.

Thus, Benedetti not only dealt with fields of mathematical inquiry that traditionally belonged to the domain of mathematics (such as mechanics, optics, mathematical astronomy, and musical theory), but also addressed issues considered beyond the limitations of mathematics, especially terrestrial and celestial physics. The title of the *Diversae speculationes mathematicae et physicae* is itself provocative, as it brings together mathematics and natural philosophy (or *physica*), considered to be separate fields, one dealing with the *quia* (the "phainomena") and the other with the *propter quid* (the "causes"). In this respect, Benedetti's methodology is very close to that of Nicolaus Copernicus, whose heliocentric system he admired. In Book 1 of *De revolutionibus orbium coelestium* (1543) and in the *Narratio prima* (1540), Copernicus and his pupil Georg Joachim Rheticus (1514–1574) reversed the Peripatetic hierarchization of physics over mathematics, urging a reform of natural philosophy and celestial and terrestrial physics in order to bring them into accord with the geokinetic and heliostatic innovations in mathematical astronomy. Beyond astronomy, the issue of the status of mathematics and its role in natural investigations was

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[32] Maccagni 1967b, 20–21: "Memini eum [eruditissimum Doctorem Petrum Arches], post varia et diversa colloquia utro citroque inter nos habita, mihi retulisse quamplurimos Romae, conspecta mea illa propositione quae ultra reliquas tuae R[everende] D[omine] [Guzman]a me mittebatur, valde mirari solitos me addidisse illam neutiquam esse iuxta mentem Aristotelis, idque ab eis dictum ubi meam demonstrationem attentius considerarunt.

Ne vero Aristotelem ullo modo errasse concederent, cum illum non infra humanae conditionis terminum habeant, sed potius veluti coeleste quoddam numen sibi proponant, censeantque nefas esse si vel latum quidem unguem ab eo quis dissentiat, in hac potius haeresi fuisse, ac etiamnum esse, ut me germanum et genuinum sensum Aristotelicae opinionis nequaquam ex authoris mente assecutum existiment.

Ego vero ne mihi diutius talis impingatur error, neve quid maxime super hac re sentiam, aut dissimulem, aut reticeam, statui, hoc novo libello edito, meam sententiam clarius aperire, ut omnes intelligant me et Aristotelem ipsum antea recte intellexisse, et non temere hoc in loco ab eo discrepare, quod sane quanquam invitus facio (nec tamen libenter a tanto viro diversum sentio, quippe qui norim quam ille praeclarus extiterit in omni doctrinarum genere), docet tamen maiorem ratione veritatis habere, quo ipsemet facendum censuit, quam inquit in *Ethicis*: 'Amicus Plato, amicus Socrates, at magis amica veritas.'"

[33] Biagioli 1989, 49–50.

heatedly debated by philosophers and mathematicians during the Renaissance.[34] One ancient predecessor to praise mathematical physics was the Hellenistic "prince of astronomy and geography," Claudius Ptolemy. In the beginning of the *Almagest*, he pointed out the superiority of mathematics over theology and physics, and even argued for a possible extension of the method of mathematical astronomy to include the treatment of local motion in general, as well as theology and ethics.

> Only mathematics can provide sure and unshakeable knowledge to its devotees, provided one approaches it rigorously. For its kind of proof proceeds by indisputable methods, namely arithmetic and geometry. Hence we were drawn to the investigation of that part of theoretical philosophy, as far as we are able to the whole of it, but especially to the theory concerning divine and heavenly things. For this alone is devoted to the investigation of the eternally unchanging. For that reason it too can be eternal and unchanging (which is a proper attribute of knowledge) in its own domain, which is neither unclear nor disorderly. Furthermore it can work in the domains of the other [two divisions of theoretical philosophy, physics and theology] no less than they do. For this is the best science to help theology along its way, since it is the only one which can make a good guess at [the nature of] that activity which is unmoved and separated; [it can do this because] it is familiar with the attributes of those beings which are on the one hand perceptible, moving and being moved, but on the other hand eternal and unchanging, [I mean the attributes] having to do with motions and the arrangements of motions. For almost every peculiar attribute of material nature becomes apparent from the peculiarities of its motion from place to place. [Thus one can distinguish] the corruptible from the incorruptible by [whether it undergoes] motion in a straight line or in a circle, and heavy from light, and passive from active, by [whether it moves] towards the centre or away from the centre.[35]

Even after Copernicus, Ptolemy's methodological insights maintained their full importance and could guide scholars who intended to expand the realm of the application of mathematics far beyond the limits established by traditional philosophy. In the *Diversae speculationes*, Benedetti deepened the discussion of issues of natural philosophy such as the concepts of space, time, and motion, claiming for a mathematician a better and clearer insight into foundational problems of physics.

Astrology was another area of expertise for Benedetti. During the Renaissance, astronomy and astrology were never separated. Benedetti was expected to cast horoscopes and give astrological advice to his patrons, just as Brahe astrologically advised the King of Denmark, Kepler the Emperor, and Galileo the grand dukes of Tuscany.[36]

In Venice Benedetti frequented celebrated exponents of the astrological culture of the time, among them Annibale Raimondo of Verona and Francesco Giuntini. Raimondo reported about a meeting they had in the residence of the senator and poet Domenico Venier. On that occasion he and Benedetti tested Giuntini's astrological preparation:

> We gathered at Mr. Domenico Venier's place; his magnificence [came] first, followed by the most excellent Mr. Giovanni Battista Benedetti, many other

[34] De Pace 1993.
[35] Ptolemy 1984, 35–37.
[36] A very informed case study on astrology at Italian Renaissance courts is Azzolini 2013.

> gentlemen, myself (Annibale Raimondo), and finally the ex-reverend father Pacifico of Florence (now, as an ex-friar, known as 'excellent Mr. Francesco Giuntini'). As soon as the latter arrived, he was given the simple astrological chart of the revolution of the magnificent Venier, without any written indication around or below. The good father took countless and endless texts and aphorisms out of his scapular. He related them to the revolution as good as a physician might give prescriptions to sick people by saying 'God might help you.' Since the most excellent Mr. Benedetti and myself laughed uncontainably—thereby making the father believe that he could not have better done—the good father, who was already trotting, was spurred by our laugher to gallop so quickly that it became extremely difficult to bring him back to silence and prevent him from telling more stupidities.[37]

An astrological report by Benedetti, cast for Carlo Emanuele I (Turin, 19 October 1589), is still extant and preserved in the Civic Library of Turin (Coss. 4, ff. 1*r*-2*v*). It contains a day-by-day personalized astrological forecast for the month of November 1589. The days are qualified with adjectives such as "*buono*" (good), "*mediocre*," or "*cattivo*" (bad), but some are treated more specifically (the 9th of November is indicated as apt to "*negotii ingeniosi*," ingenious endeavors, whereas the 10th of November as "*buono in cose femminili ma nel resto cattivo*," that is, bad except for women's affairs). Benedetti signed this astrological letter as "*Matematico e Astrologiaro*."[38] This signature shows that his "professional" profile could vary depending on circumstances, since it depended in part on the kind of advice requested from him.

In the concluding letter of the *Diversae speculationes*, Benedetti envisaged a reform of astrology. He directed this letter to a German correspondent whose name he awkwardly Latinized as *Volfardus Aisestain*.

> As for the question whether or not I regard as true all that is written in the books of judicial astrology, I respond that I do not. I even believe that much is wrong […]. But you will be informed about all this in a special tract of mine, about which I told you on another occasion. In it, you will find many things I have proven through the evidence of many observations. I intend to publish that tract along with some other speculations of mine, if only I will have enough time to do that, before I meet the body of the adverse Mars as indicated by my horoscope. This is going to happen in 1592.[39]

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[37]Raimondo 1574: "Ritrovandosi nella camera del Clariss. M. Dominico Veniero prima la sua Mag. [,] lo eccellentissimo M. Gio. Battista Benedetti, molt'altri gentilhuomini, et Annibale Raimondo, che son quel io, vi sopraggiunse al'hora il Reverendo Padre Frate Pacifico Fiorentino de gli bene inculati, adesso per essersi sfratato lo Eccellente M. Francesco Giuntini, alquale, subito giunto, fu dato in mano la figura simplice del cielo della Revolutione del detto Mag. Veniero, senz'altra scrittura intorno, né appresso, il buono padre alhora mise mano al suo scapolario et cavò fuori testi, et afforismi senza fine, et senza fondo, allegandoli tanto a proposito della Revolutione, quanto facea quel buon medico le ricette che 'l dava ai suoi infermi, quando le dicea Dio te la mandi buona, et perché lo Eccell. M. Gio. Battista Benedetti et io se smassellavamo dalla risa, ben però in modo di maravigliarsi, come non fusse possibile a dir meglio di quello che dicea sua paternità, il buon padre per il nostro ridere sì come prima andava trottando, si misse a correr' de modo che fu gran fatica a poterlo tenere et farlo tacere che'l non dicesse più minchionerie." Cf. Corradeschi 2009, 111, n. 46. On Raimondo and Giuntini, see Ventrice 1989, 140–145.

[38]Roero 1997, 57–58.

[39]Benedetti 1585, 425–426: "Circa vero id de quo me interrogas, scilicet, utrum putem omnia vera esse, ea quae scripta reperiuntur in libris Astrologiae iudiciariae, respondeo quod non, imo puto plurima falsa esse […]. Sed diffusius haec omnia videbis in meo illo particulari tractatu, de quo tibi alias dixi, in quo multa

This passage concludes his major work. In it, Benedetti predicted, using astrological means, his own death for the year 1592, but he actually died in January 1590.[40] This fact aroused some doubts about his proficiency as an astrologer, especially from his successor as court mathematician, Bartolomeo Cristini.[41]

To sum up, Benedetti's persona and work had various facets, his interests ranging from mathematics to cosmology and from natural philosophy to literature. In a certain sense, he can be seen as a Renaissance polymath. However, his profile can be better encompassed by the title of "*mathematicus*," as long as we do not take it too restrictively. A Renaissance mathematician like Benedetti was an engineer and a technical inventor, as well as a theoretician and a natural philosopher; someone with teaching and civil duties who served as a counsellor, also for astrological matters. Being a court mathematician implied benefiting from high recognition and visibility in society. Thus, this professional and intellectual appurtenance had nothing to do with the rather low acknowledgment that mathematicians received at universities, where physicians, lawyers, and theologians were higher placed and received better salaries.[42] The cultural environment of Turin, with which Benedetti interacted in the most important years of his career, shall be addressed in the next section.

## 1.2 Benedetti's Works and Publications

Benedetti published his first work at the age of 23, the *Resolutio omnium Euclidis problematum* (Resolution to All of Euclid's Problems, Venice 1553), which offered the solution to "all" geometrical problems using a compass with a fixed opening. The work reacted to a challenge that emerged from a controversy between Niccolò Tartaglia and Lodovico Ferrari in the years 1546–1548 and inserted Benedetti into the scientific debates of his time. One year earlier the astrologer Luca Gaurico had already paid homage to him, including in his *Tractatus astrologicus* a horoscope of the promising mathematician cast by Gaurico himself.

In 1554 Benedetti published a *Demonstratio proportionum motuum localium contra Aristotilem et omnes philosophos* (Demonstration Concerning the Proportions of Local Motions against Aristotle and All Philosopers), which is not as famous for its polemical verve as for the presentation of an innovative theory of fall. As we have discussed in the preceding section, in this treatise Benedetti developed a theory of the motion of fall, first proposed in the dedicatory letter of the *Resolutio* of 1553. Benedetti maintained that bodies of the same material fall through a given medium with the same speed and not with speeds in proportion to their weights, as Aristotle and his followers claimed. Benedetti tried to overcome the fallacies of the Aristotelian theory of fall by employing the Archimedean concept of buoyancy, assuming that the motion of fall depends on their specific rather than absolute weight. As we have also discussed above, in the second edition of the *Demonstratio*, published in Venice in 1555,[43] Benedetti argued that the resistance incurred by a

videbis, quae omnia ab experientia, ex multis a me observatis, comprobata sunt, quem quidem tractatum cum quibusdam aliis meis speculationes in lucem producere cupio, si fieri poterit, antequam ad directionem mei Horoscopi cum corpore Martis Anaeretae perveniam, quae quidem directo circa annum millesimum quingentesimum nonagesimum secundum eveniet."

[40]Benedetti was not the first mathematician who tried to forecast his own death. Among his predecessors are famous the cases of Johannes Stöffler and Girolamo Cardano. Cf. Omodeo 2014b, 3–4.

[41]Vernazza 1783, 16–18.

[42]On the lower status of mathematicians, see Henry 2011.

[43]Benedetti [1554] 1555, see Benedetti 1985.

falling body in a medium depends not on its volume, but on its surface area. This is also the view that he presented in the *Diversae speculationum mathematicarum et physicarum liber*, published in Turin in 1585. He explained the acceleration of the motion of fall in terms of an increasing impetus of the falling body. Such examples show how he dealt with new challenging problems, which were difficult and sometimes impossible to solve using the mainstream theories of his time, by bringing forth and promoting new ideas.

After the *Resolutio omnium Euclidis problematum* and the *Demonstratio proportionum motuum localium*, composed when Benedetti was still in Venice, the next extant works stem from the time when he had already settled in Turin. First, he composed two works on gnomonics, one in Italian and one in Latin. The former is a manuscript preserved in the Civic Library of Carignano (Turin, Italy), entitled *La generale et necessaria instruttione per l'intelligentia et compositione d'ogni sorte [di] Horologij Solari*, which was presumably written between 1567 and 1573. The latter was printed under the title *De gnomonum umbrarumque solarium usu liber* (1574). Here Benedetti dealt at length with the construction of sundials with faces of varying inclinations and also with cylindrical and conical surfaces. At ff. 107*r-v* one finds a discussion of a sundial that perhaps can still be seen today on a wall of the Royal Palace in Turin.[44]

In 1574 Benedetti also wrote about a trigonometrical measuring instrument of his own invention, *Descrittione, uso, et ragioni del Trigonolometro*. It was never printed and is preserved in manuscript form in the Civic Library of Carignano along with the Italian work on sundials, *Intelligentia et compositione d'ogni sorte [di] Horologij Solari*.[45] His next scientific treatise, *De temporum emendatione opinio* (1578), proposed correcting and reforming the calendar. In 1578 the duke initiated a public disputation at the University of Turin where Benedetti argued with Antonio Berga about whether there was more water or more land on the earth, following an argument by Alessandro Piccolomini. The views which Benedetti brought forth against his opponent were published in Turin in 1579 under the title *Consideratione… d'intorno al discorso della grandezza terra et dell'acqua del eccellent[e] sig[nor] Antonio Berga*. This polemic was renowned, as can be seen in the Italian translation and commentary of Sacrobosco's *Sphere* by the theologian, astronomer, and astrologer Francesco Giuntini in Lyon: "The excellent philosopher, Mr. Giovanni Battista Benedetti, mathematician to the serene duke of Savoy, resolved this question very aptly, arguing against the philosopher Berga, a famous reader at the University of Turin. The latter argues against Mr. Piccolomini that there is more water than earth. Benedetti defends the opposite view, which corresponds to truth: that there is less water than earth."[46]

Next came Benedetti's defense of the reliability of the mathematical computations underlying astrological predictions in the context of a heated polemic on this issue that burst out in Turin 1580–1581. Benedetti first communicated his views in Italian, in epistolary form: *Lettera per modo di discorso… all'illustre sig. Bernardo Trotto. Intorno ad alcune nuove riprensioni, et emendationi contra alli calculatori delle effemeridi* (Letter

[44]Roero 1997, 47.

[45]Clara Silvia Roero published Benedetti's letter to Carlo Emanuele I (Turin, 19 October 1589), the index of the manuscript on gnomonics, as well as an excerpt from the manuscript on the mathematical instrument *trigoniometro* as appendices II and III of Roero 1997.

[46]Giuntini 1582, 95–96: "La qual questione ha resoluta molto dottamente lo eccellente filosofo, il signor Giovambattista Benedetti mathematico del serenissimo signor Duca di Savoia, contra il filosofo Berga, famoso lettore nella università di Turino: il quale contra l'opinione del signor Piccolomini defende che l'acqua è maggiore della terra: e il Benedetti defende il contrario in favore della verità: cioè che l'acqua è minore della terra."

in the Form of a Discourse… Addressed to the Illustrious Mr Bernardo Trotto Concerning Some New Criticism and Corrections against the Ephemerides Calculators) (1581). Benedetti later included a Latin translation of this letter in the *Diversae speculationes* (1585).[47] His commitment to astrological practice is testified to by an astrological report he wrote for Carlo Emanuele I, a handwritten letter (Turin, 19 October 1589) preserved in the Civic Library of Turin (Coss. 4, ff. 1*r*-2*v*).[48]

Finally, Benedetti had his major work, *Diversarum speculationum mathematicarum, et physicarum liber*, printed in 1585. It was issued again under slightly different titles in Venice in 1586 (*Speculationum mathematicarum et physicarum tractatus*) and, posthumously, in 1599 (*Speculationum liber*).

Two of Benedetti's manuscripts, preserved in the Biblioteca Nazionale Universitaria of Turin until 1904, are irreparably lost due to a fire that burst out in that year, destroying many valuable manuscripts. The first one was a collection of his letters, *Lettere di Giovanni Battista Benedetti, Veneziano, matematico del Duca Emanuele Filiberto e Carlo Emanuele I, in risposta ai quesiti fattigli dal Duca e da altri personaggi intorno alla matematica, fisica, musica e filosofia*.[49] The second one held similar content and was entitled *Lettere di Giovanni Battista Benedetti in risposta a quesiti di fisica e matematica* (Letters by Giovanni Battista Benedetti answering questions on physics and mathematics).[50]

Reprints of Benedetti's works are rather scarce. Excerpts on mechanics from Benedetti's work were included by Stillman Drake and Israel Edward Drabkin in their *Mechanics in Sixteenth-Century Italy: Selections from Tartaglia, Benedetti, Guido Ubaldo and Galileo* (Madison, Wisc.-Milwaukee-London, 1969). Carlo Maccagni's *Le speculazioni giovanili "de motu" di Giovanni Battista Benedetti* (Pisa, 1967) includes excerpts from the dedicatory letter of the *Resolutio omnium Euclidis problematum* and the text of the two editions of the *Demonstratio proportionum motum localium contra Aristotilem et omnes philosophos*.

[47] Benedetti 1585, 228–248, "Defensio ephemeridum."
[48] See Roero 1997, Appendix I.
[49] Peyron 1904, 73–74, Codex 83, N. II. 50.
[50] Peyron 1904, 95, Codex 94, N. III. 27.

# Chapter 2
# The Subalpine Environment

Benedetti's life, work, and reception are indissolubly linked to Turin and the Duchy of Savoy. As one reads in the *Diversae speculationes*, he chose to live in this town until the end of his life. There he benefited from the patronage of Duke Emanuele Filiberto (Figure 2.1) and, as a consequence, from a prominent social position and recognition. In the dedicatory epistle of his major work, addressed to Filiberto's successor, Carlo Emanuele I, Benedetti extolled the merits of the deceased duke, who had invited him to Turin almost two decades earlier:

> Nineteen years have passed since I was sent for by a letter of the most serene [Emanuele Filiberto] father of Your Highness [Carlo Emanuele I] and I moved from the town Parma to this municipality. Upon my arrival, he received me so humanely, and later I met with so much generosity as a reward for my services, that I began to desire vehemently that I could spend the rest of my life under his authority.[1]

As one reads, Benedetti and Emanuele Filiberto were so close that the patron even wanted his court mathematician to accompany him during his periods of residence in the countryside. On such occasions they often discussed scientific matters:

> His benevolence toward me, as well as my respect toward him, consolidated through the time we spent together, and our familiarity [grew] to the point that the duke wanted me to accompany him when he resided in the countryside. [He] often [even invited me] to stay with him overnight. In that time he discussed mathematics with me. He used my work in order to learn those sciences, asking questions on arithmetic, geometry, optics, music and astronomy [*astrologia*].[2]

Emanuele Filiberto's passion for mathematics was well known in his day. The Venetian ambassador to Turin, Giovanni Correr, reported on this singular aspect of his personality in 1566:

> That Duke is no man of letters but he loves the virtuosi. Hence, he has many of them by him; he likes to listen to their reasoning and he asks them questions. However, there is no subject that delights him more than mathematics, as

[1] Benedetti 1585, f. A2*r*: "Agitur nonus decimus annus ex quo litteris Serenissimi patris tuae Celsitudinis, accersitus ex urbe Parmensi in hanc me civitatem contuli. Is advenientem tam humane excepit, tanta deinde liberalitate fuit complexus ego vicissim ei deserviendi, tam vehementi cupiditate fui accensus, ut sub eius ditione quod superesset vitae agere constituerem."

[2] Benedetti 1585, f. A2*r*: "Cuius in me banignitas, mea in illum observantia mirum in modum mutuo usu, et consuetudine est adaucta, ut idem Dux me secum dum rusticaretur esse vellet, saepe etiam secum pernoctare; quo quidem tempore de Mathematicis scientiis mecum agebat, in quibus perdiscendis mea opera utebatur, quaestiones, Arithmeticam, Geometriam, Opticen, Musicam, aut Astrologiam spectantes proponens."

> this science is not only apt but also necessary to the profession of military commander.[3]

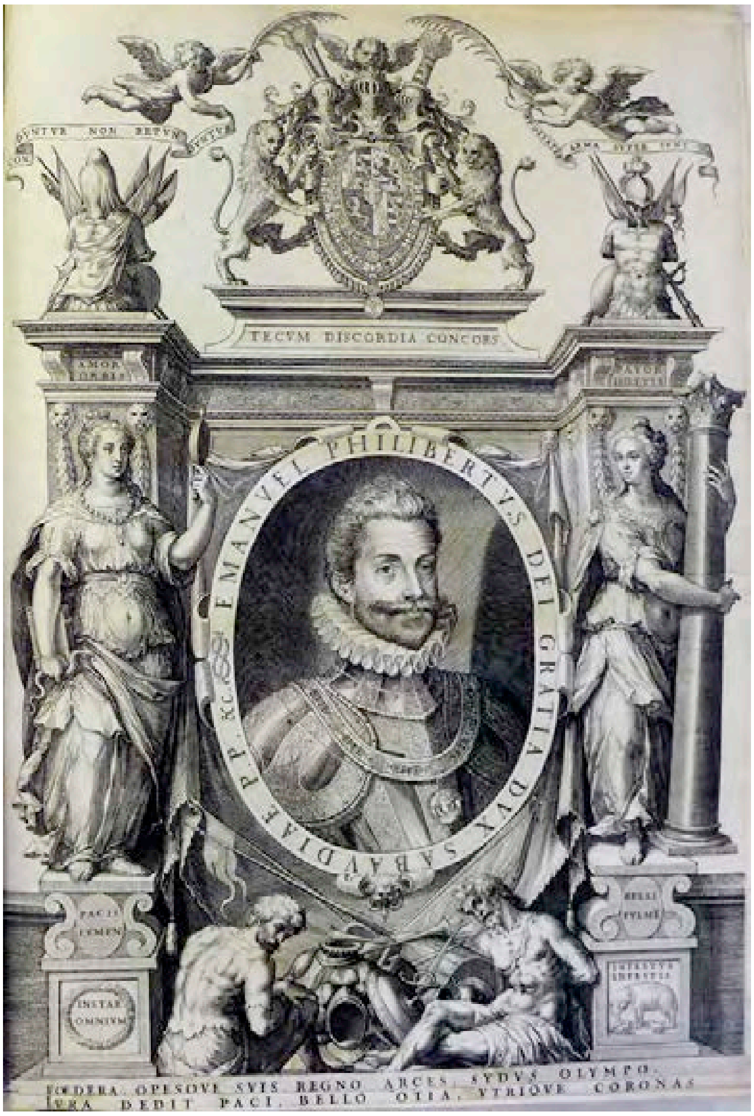


Figure 2.1: Portrait of Emanuele Filiberto from Tonso, *De vita Emmanuelis Philiberti* (1596). (Biblioteca Nazionale Universitaria di Torino)

The duke's passion for science and his special relation with his court mathematician is further confirmed by the Venetian ambassador Giovanni Francesco Morosini, who mentioned Benedetti in a speech delivered to the Senators of Venice in 1570:

> The duke of Savoy has a wonderful mind apt to every kind of science. However, he did not learn the sciences [*le lettere*] with the diligence that is necessary to become an expert, as his passion has always been the profession of

[3]Firpo 1983, 123: "Non è quel duca litterato, ma ama li virtuosi, et però ne tiene alquanti appresso di sé, sente piacere a udirli ragionare, egli stesso li fa de quesiti, ma nessun ragionamento più li diletta, che quello delle matematiche, come scientia, che non solo è conveniente ma ancora è necessaria alla professione del capitano."

> war […]. But since mathematics is very useful and [even] necessary to professional warfare, His Excellency [Emanuele Filiberto] learns [mathematics] with much pleasure and knows more of it than the average man. He is aware that to receive substantial knowledge in any science one has to be in contact with it and learn it continuously; therefore a certain Mr. Giovanni Battista Benedetti of Venice imparts to him a lesson either on Euclid or on another writer of those sciences every day. In my opinion, as well as according to many other gentlemen, he is the most excellent scholar in this discipline in our times. The duke likes him very much. In fact, not only has [Benedetti] mastered this science, but he is also able to transmit it very well to others in his lessons.[4]

However, Benedetti's activities in Turin cannot be fully grasped if we limit our consideration to his relationship with the dukes. Rather, we should consider the wider political and cultural environment in which this relationship was established.

## 2.1 Turin's Economy and Politics between Italy and Europe

From the point of view of economic exchanges as well as of the European balance of power, Turin was located in a delicate and strategic position. It was in fact an obligatory station on the commercial road connecting Italy and France through the *Val di Susa*. For many centuries it had served as a transit point for merchants from Liguria, Lombardy, and Piedmont on their way to Lyon and the French and Flemish markets, and vice versa. Merchants were not the only visitors, as scholars from France, Flanders, and the British Islands began their *iter Italicum* from Turin. Turin was also the first station in Italy of Erasmus of Rotterdam, a key figure of the European Renaissance. On that occasion, on September 4, 1506, he received an "Italian" degree in theology from the University of Turin.

Its intermediate position between Italy and France made the town relevant not only from the point of view of economics and culture but also for military reasons. When Francis I of France and Charles V of Spain fought over Italian and European supremacy, Turin acquired fundamental strategic importance. The French army conquered it in 1536, together with most of Savoy and Piedmont, at the expense of Charles II of Savoy, brother in law to Charles V of Spain. The King of France made Turin the most important center in the region and a bulwark that was fundamental for consolidating his position on the Italian peninsula. Some of the political and administrative reforms promulgated by the new ruler were long-lasting. The most important of them were the creation of a parliament and of a *Camera dei Conti* for the supervision of public finances.[5]

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[4]Firpo 1983, 211: "Ha il signor duca di Savoja un bellissimo ingegno capace d'ogni scienza: ma non ha atteso alle lettere con quella diligenza, che si converria a chi ne volesse sapere, essendo la sua principal professione il mestiero della guerra […]. E perché la scienza delle matematiche è molto utile e necessaria a chi vuole fare questa professione de l'arme, però se ne diletta assai Sua Eccellenza [Emanuele Filiberto] e di quella sa assai più che mediocremente. Con tutto questo sapendo che l'uomo tanto sa di ogni scienza quanto continua in vederla e studiarla, però usa di udire ogni giorno una lezione o d'Euclide o d'altro scrittore di quelle scienze da un messer Giovan Battista Benedetti veneto; uomo, per opinione non solamente mia, ma di molti valentuomini ancora, il maggiore che oggi faccia professione, e di grandissimo gusto del Signor Duca; perché oltre a possedere lui quella scienza eccellentissima sa anco così bene insegnarla ad altri che con molta facilità ne fa restar capacissimo chi lo ascolta."

[5]Merlin 1998, 16.

Emanuele Filiberto, known as “testa di ferro” for his energy and capacity in military affairs, retook Turin on the battlefields. He conducted the campaign against the French as a captain in the service of the Habsburgs. In 1553 he was the supreme commander of Charles V’s imperial army in Flanders and was nominated governor of the Netherlands by Philip II in 1556. His victory in the battle of Saint-Quentin led to the Peace of Cateau-Cambrésis (1559), according to which the Savoy and Piedmontese territories had to be restored. The French agreed to give them back to the dukes of Savoy with the significant exception of five fortified towns, occupied by their troops.

Turin was one of them. Therefore, it took some years before it was eventually returned to Emanuele Filiberto in 1562. In 1563 the duke entered the town and choose it as the new capital of his duchy instead of Chambéry. In this manner, he conferred an Italian identity to his duchy. This transfer set in motion political, social, and economic transformations, which were still in progress when Benedetti arrived in Turin in 1567. Moreover, the Piedmontese territories were politically fragmented. Apart from the centers under French control (Chieri, Pinerolo, Chivasso, and Villanova d’Asti), the region included the *Marchesati* of Monferrato and of Saluzzo. Moreover, the county of Tenda, connecting Piedmont with the Savoy possession of Nice, was an imperial fief. As for Geneva, a former possession of Savoy, it had become the “Jerusalem” of the Calvinists and would never be regained.

Within this difficult territorial and political constellation it was imperative that Emanuele Filiberto reestablish his authority after years of wars and foreign domination. In the European context, this meant striking a balance between the interests of Spain and France, who both wanted to annex the territories of the duchy either as a part of France or as a continuation of the Milanese territories. Piedmont was already split into a faction favorable to the French and one favorable to the Spaniards during the years of the war, and this division would also continue during the reigns of Emanuele Filiberto and Carlo Emanuele I.[6]

International diplomacy was comprised of marriage politics. Emanuele Filiberto received a French spouse, Margret of Valoys, daughter of Francis I of France and sister of King Henry II. This meant a strong political and cultural link to Paris. Margret was well known for her patronage of literati and artists, among them the poets of the Pléiade, Pierre de Ronsard, and Joachim Du Bellay. However, her son Carlo Emanuele I married a Habsburg, the daughter of Philip II of Spain, *infanta* Catherine Michelle, who arrived in Turin in 1585. This liaison was strongly encouraged by the pro-Spain party. Its leader was Andrea Provana of Leyní (1511–1592), with whom Benedetti was well acquainted. Four of the letters included in the epistolary of the *Diversae speculationes* are addressed to him. Benedetti judged the importance of his correspondence with this exponent of the Savoy aristocracy to be second only to those with Emanuele Filiberto (first epistle of his collection) and Carlo Emanuele I (second epistle). We can assume, taking his origins as a guide, that Benedetti supported Provana’s pro-Spain party.

In his relations to other Italian States the duke also followed a politics of balance. He was particularly keen on having good relations with Venice, which he visited in 1566 and 1574. On the latter occasion he was even endowed with the title of *patrizio* of the town. In turn, a Venetian embassy was established in Turin. The Savoy relation with Rome was also cordial. The papacy regarded Turin as a bulwark to stop the dissemination of reformed ideas in Italy, especially from the Swiss cantons. For his part, Emanuele Filiberto saw “heresy” as a danger to the unity of his state and his authority. Therefore, on

[6]Merlin 1998, 33 and Merlin and Stango 1998, 266–267.


Figure 2.2: Portrait of Carlo Emanuele I by Francesco Maria Ferrero di Labriano, *Augustae Regiaeque Sabaudae Domus Arbor Gentilitia* (Turin, 1702), p. 174. (Biblioteca Nazionale Universitaria di Torino)

matters of faith, the Roman interests and his own converged. Against the background of the confessional tensions of those years, his support for the Jesuits is comprehensible. Yet he was no fanatic of orthodoxy. He was influenced by the Imperial policy of mediation, as is shown by his ratification of a compromise with the Valdesans in 1561, in which he accorded to them religious freedom in their valleys.

The ties with Rome and Venice were reinforced through Savoy support for expeditions against the Turks. In 1565 Andrea Provana was sent with three galleys to Malta, as the court historian Pingone recounted in his history of Turin, *Augusta Taurinorum*:

> When Malta was besieged by the Turks, in June 1565, duke Emanuele [Filiberto] sent Andrea Provana of Leinì with four well-equipped triremes to bring supplies to the isle together with triremes from the Pope, Spain, and other [states]. First, Provana [*Leniacus*] arrived and assessed the difficulties. Then, he conveyed others [to the battle] and broke the siege with divine favor. The holy and vigorous order of the knights of Jerusalem was liberated under the superior command of the French Jean of Valetta. Public demonstrations of

> immense joy and pious celebrations of thanks to God for the victory were displayed in Turin.[7]

In 1571 Provana was enlisted to defend Cyprus and contributed to the “holy” victory in the battle of Lepanto.

> In 1571, when duke Emanuele [Filiberto] ruled over Turin and a confederation was established between Pope Pius V, the king of Spain and the Venetian Republic, he was asked to command the fleet with everybody’s agreement. But he had to renounce the offer owing to the present danger to his country engendered by local conflicts. [In his place] John of Austria, offspring of emperor Charles V, of great spirit and promising youth, was made commander. Chief Andrea Provana of Leyní joined this expedition with three triremes. It was fought near Nauplia with the support of the Greeks. The Christians had hardly two hundred triremes and the Ottomans more than three hundred. The battle [*Mars*] was undecided for a long time but finally victory was given to the Christians, with the favor of God or even as a miracle. Provana, who fought bravely in the commanding trireme, was hit by a gun bullet and could hardly escape under the protection of a galley. One of the [Savoy] triremes, named Margara, was scattered and sunk into the depth; [another one], Pedemontana, was saved many times from the enemy. That victory was celebrated in Turin with thanks given to God and holy days set aside for the people.[8]

On these occasions Benedetti served as an advisor to Provana. Three of the four epistles of the *Diversae speculationes* addressed to him deal with mathematical issues related to navigation. As one reads, Benedetti undertook to give Provana suggestions concerning navigation and the employment of navigational instruments.[9] The first epistle is entitled *Per eundem parallelum absque correctione semper navigari non posse ubi notantur Petri Nonii lapsus in correctione erroris navis et alii Petri Medinae errores* (That one cannot always navigate along the same parallel without correction, where an error by Petrus Nonius concerning the correction of the deviation of a ship and other [errors] by Petrus Medina are considered). The second and the third letter deal with a navigation instrument invented by Benedetti based upon the design of Gerardus Mercator (Figure 2.3). They are a description of the instrument accompanied by technical drawings and an explanation of its use. They

[7]Pingone 1577, 85: “Anno Christi 1565 mense Iunio, Dux Emanuel, obsessa a Turcis Melita, Andream Provanam Leniacum cum triremibus quatuor instructissimis mittit, qui una cum Pontificiis, Hispanis et aliis triremibus suppetias insulae afferret. Prior Leniacus applicuit, difficultates exploravit, alios postea advexit, et soluta tandem faventibus superis obsidione, Hierosolymitanorum militum sacer, et strenuus ordo liberatus, Ioanne Valleta Gallo summum magisterium gubernante. Quam ob victoriam Taurini immensae laetitiae publica significatio reddita, et devotae superis gratiarum actiones.”

[8]Pingone 1577, 88: “Anno Christi 1571 Emanuel Dux Taurini agens, confoederatione inita in Turcam Cypri vastatorem, inter Pium quintum Pontificem, Hispaniarum Regem, et Venetam Rempublicam, qui classi praeesset ab omnibus exposcitur: sed ob imminentiam a vicinis discordiis patriae discriminis, excusatus habetur. Ioannes vero Austriacus Caroli quinti Caesaris soboles, magni animi, et expectationis iuvenis praeficitur. At Dux Andream Provanam Leniacum tribus cum triremibus in eam expeditionem adiungit. Apud Naupactum Achaicum concursum, et decertatum. Christianorum vix ducentum triremes: Turcarum vero plusquam trecentum: Mars diu anceps, tandem Deo maximo favente, et quodam potius miraculo ad Christianos inclinavit victoria. Leniacus ex triremi Praetoriam fortiter dimicans sclopeto ictus in capite vix galeae praesidio evasit: triremium una Margaris nomine dissipata, mersaque penitus, Pedemontana semel atque iterum ab hostibus recepta. Ob eam victoriam, Taurini supplicationes superis, feriae mortalibus indictae..” See also Tonso 1596, 142, 161 and 177–179.

[9]Benedetti 1585, 214–216.

are entitled *De armilla nautica* (On the armillary nautical sphere) and *De usu armillae nauticae* (The utilization of the armillary nautical sphere), respectively.[10] As one reads, the letters follow private discussions with Provana on the difficulties linked to navigation using nautical maps.

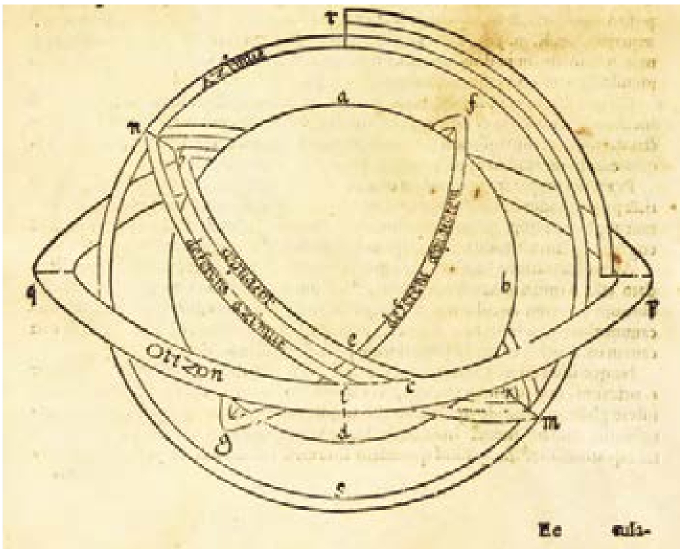

Figure 2.3: An armillary nautical sphere invented by Benedetti for Andrea Provana for navigation purposes, presumably in the Savoy military expeditions against the Turks. (Max Planck Institute for the History of Science, Library)

## 2.2 Civil Reforms and Military Policy

Emanuele Filiberto and his son were very different rulers. While the court of the former has been depicted as "*funzionale, ristretta e popolata di homines novi*" (functional, small and composed of *homines novi*) the latter's court was "*fastosa, aristocratica, centro propulsore di una politica culturale oramai intensamente barocca*" (pompous, aristocratic, irradiating center of a deeply baroque cultural politics).[11] Their common efforts were directed towards the consolidation of their state. Emanuele Filiberto implemented profound administrative, financial, and military reforms. He issued tax reforms and imposed the use of the vulgar tongue in official documents. As to his military policy, Emanuele Filiberto introduced the obligatory conscription of all men aged between 18 and 50 years. Thanks to this reform, which followed the Swiss example and Machiavelli's theory, Emanuele Filiberto trained his subjects to defend their territories in case of invasion, disposed of many thousands of soldiers, and limited the use of mercenary troops.[12] Part of his defensive strategy was the erection of new fortifications in Nice, Bourg-en-Bresse, Saint-Julien, and Montmélian (see Figure 2.4). The construction of the *cittadella* of Turin was particularly important and rapid. It was achieved in only two years, between 1564 and 1566, and was celebrated by the official town historian Emanuele Filiberto Pingone in the following terms:

[10] Benedetti 1585, 217–219 and 219–220.
[11] Ricuperati 1998, XXII.
[12] See Stumpo 1993, 561.

> In that year [1564], the duke began building a fortification, which is commonly called the citadel, in the most sacred part of the town on the ruins of the temple of the divine Savior.[13]

> On March 15, 1566, the citadel of Turin was finished after a few months of work. It was admirable with its five bulwarks, serving all military purposes and built according to the art of architecture. He [the duke] let it be blessed with religious and pious blessings (Archbishop Della Rovere was in charge of the rite). Soon he organized the defenses, entrusting them to Giuseppe Caresana of Vercelli, a subject of his [*benemeritus*] and a man very expert in the military art.[14]

Francesco Horologi was responsible for the construction of the citadel, designed by engineer Francesco Paciotto, whom Emanuele Filiberto recruited in Flanders. Its pentagonal structure, responding to recent developments in warfare, was the model for later fortifications, such as the citadels created by the same Paciotto in Antwerp (1567) and Parma (1591).

As often occurred during the Renaissance, the military-political function of the citadel had two sides. On the one hand, it served to defend the town from possible assaults from outside. On the other, it affirmed the supremacy of the dukes over the new capital and had the function of dissuading the subjects from claiming too much autonomy.[15] As Martha Pollak remarked, "Paciotto proposed a five-sided fortress, with three bastions oriented towards the countryside, defending the approach to the city from the west, and two bastions facing the city, ready to bring it under control in case of riotous uprisings against the duke."[16] The new urban arrangement transformed Turin along with its political balance of power. During these changes the relations between the dukes and the local *patriziato* were often strained. In fact, all decisions had to be negotiated between civic administrators, state functionaries, and the court. Whereas the town council was eager to keep its medieval privileges, the dukes made the opposite effort of centralizing power in order to grasp control firmly in their hands.

A thorough knowledge of the surrounding territory through cartography, alongside fortification and military reforms, was also seen as an important element of defense. The intensity of mapping efforts in the Savoy in the second half of the sixteenth century is a noteworthy example. A large number of maps of great quality were made, for instance Forlani's *Savoia* (1552), Boileau de Boullion's work on the road from Lyon to Turin (1556), Nicolais's maps *Boulonnais* (1558), *Berry* (1566) and *Bourbonnais* (1569)[17] and, above all, Giacomo Gastaldi's *Pedemontanae vicinorumque regionum… descriptio* (1574). Many of the maps made in those years are still preserved in Turin, in the *Biblioteca Reale*, the *Biblioteca Nazionale Universitaria*, the *Archivio di Stato*, and in the wide collection of the *Archivio Storico della Città*.[18] Benedetti shared this interest in

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[13] Pingone 1577, 85: "Eo anno [1564] Dux in aeditiore parte civitatis, in ipsis templi Divi Solutoris ruinis Acropolis aedificare coepit, Cittadellam vulgo dicunt."

[14] Pingone 1577, 86: "Anno Christi 1566 idibus Martiis, absoluta paucis mensibus Taurinensi acropoli, quinis propugnaculis admiranda, servata omni rei militaris, et architectonicae artis ratione eam religiosa ac pia benedictione communiri curat, Archiepiscopo Rovereo sacris praeeunte: mox praesidiis firmat, eique praeficit Iosephum Caresanam Vercellensem de se benemeritum, ac rei militaris peritissimum."

[15] Merlin and Stango 1998, 118–119.

[16] Pollak 1991b, 16.

[17] See Broc and Greppi 1989, 113.

[18] The Archivio di Stato preserves Carracha's maps of Turin: *Augusta Taurinorum* (1577) and *Turino* (ca. 1580)—see Archivio Storico della Città di Torino 1982.

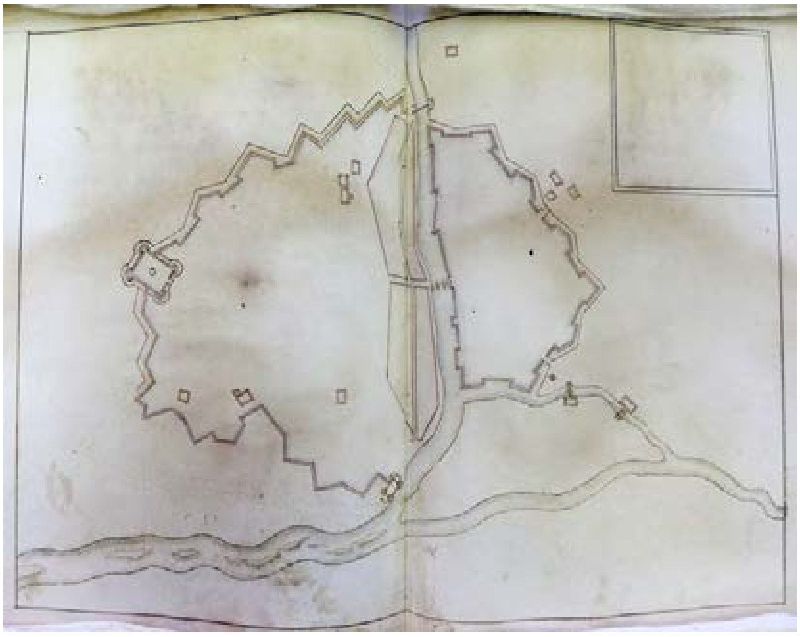

Figure 2.4: Fortification projects in a drawing by Benedetti's follower as court mathematician, Bartolomeo Cristini. (Biblioteca Nazionale Universitaria di Torino)

geography and topography. This especially emerges from some of his epistles, for instance those to the architect Gabriele Busca on topography and measuring instruments, to the imperial land surveyor (*agrimensor*) Anselm Rosenburg (presumably of the Bohemian aristocrat family Rožmberk) on measuring techniques, and to the Turin physician and natural philosopher Giovanni Battista Femello concerning cartographic errors about the position of islands, in particular Iceland.[19]

## 2.3 Engineering and Architecture

Countless engineers worked in Turin under Emanuele Filiberto and Carlo Emanuele I to implement the defenses and the hydraulic system, among them Francesco Paciotto, Ferrante Vitelli, Ascanio Vitozzi, and Vitozzo Vitozzi.

The leading Italian architect of that time, Andrea Palladio, visited Turin between May and June 1568. He might have been the architect behind the park of Viboccone (later known as *Regio Parco*) between the rivers Dora, Po, and Stura. Benedetti is said to have constructed his fountain there. It is also likely that Palladio gave Emanuele Filiberto advice on the organization of his popular militia according to the classical Roman model.[20] Later, he dedicated to the duke of Savoy the third of his four books on architecture, *Quattro libri dell'architettura* (Venice, 1570). This section deals with public constructions, streets, bridges, squares, basilicas, and gymnasia. In the letter to the reader, the author stressed the prestige deriving from public buildings, and the fruitful collaboration between Renaissance princes and architects: "[Public buildings] are bigger and more embellished with rare ornaments than the private ones; and they aim to be used by everybody. Therefore, through them, princes can display the greatness of their generosity to the world and

[19] Benedetti 1585, 271–274, 405–408 and 267.
[20] Tessari 1993.

architects have the occasion to show how capable they are through beautiful and wonderful inventions."[21] The dedication to Emanuele Filiberto by Palladio was motivated by his "heroic spirit"[22] as well as by his interest in and deep understanding of architecture:

> As your Highness is familiar with the most noble arts and sciences related to these issues [concerning architecture], you will have much pleasure and relief by considering the subtle and beautiful inventions of humankind as well as the true science of this art, which you understand very well and which has been brought to the most rare and almost absolute perfection. This is witnessed by the illustrious and royal buildings that have been constructed in many parts of your large and most happy state.[23]

Urban and military developments were accompanied by a flourishing literature on war and defense theory. Emanuele Filiberto was a great supporter and collector of such writings:[24] among other examples, Benedetti's correspondent Busca authored the treatise *Della espugnazione et difesa delle fortezze* (On the conquest and defense of fortresses, Turin, 1585), which followed the *Istruttione de' bombardieri* (Education of the bombardiers, Carmagnola, 1584). He would later publish the tract *Architettura militare* (1601) in Milan. Another acquaintance of Benedetti's, Giacomo Soldati composed *Discorso intorno al fortificare la città di Torino* (Discourse on fortifying the town of Turin).[25]

In this context of military reforms and architectural changes aimed at transforming Turin into the capital of an absolutist state, the skilled engineers implementing the dukes' vision gained high social recognition. An example is the career of the mathematician Cristini. In 1569 he was courtly librarian and later "calculatore" or "controlore delle fabriche," that is, supervisor of architectural projects. In this capacity, he become closely connected to the celebrated military and civil architect Ascanio Vitozzi. On December 13, 1582, he became "re d'arme dell'ordine dell'Annunziata," a honorific and administrative title that implied responsibility for the organization of courtly tournaments, feasts, and balls.[26] He entered the court as Benedetti's successor in the position of ducal mathematician. Cristini's career shows the enhanced status of Renaissance scientist-engineers bringing together mathematical and natural expertise, technical skills, administrative duties, and courtly honors. Benedetti addressed one of the letters of the *Diversae speculationes* to him (the one dealing with geometrical problems encountered in Ptolemy's *Geography*) calling him "Bartholomeo Christino Serenissimi Sabaudiae Ducis apparitor." The term *apparitor* can mean either "servant" or "functionary." An unknown hand corrected this title in a copy preserved in the Royal Library of Turin, substituting it for the more

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[21] Palladio 1570, III, 5: "Ne' quali [edificii publichi], perché di maggior grandezza si fanno, e con più rari ornamenti, che i privati, e servono a uso, e commodo di ciascun; hanno i Principi molto ampio campo di far conoscere al mondo la grandezza dell'animo loro; e gli Architetti bellissima occasione di dimostrar quanto essi vagliano nelle belle, et meravigliose invenzioni."

[22] Palladio 1570, III, 3: "Principe, il qual solo a tempi nostri con la Prudenza, e co'l valore s'assimiglia a quelli antichi Romani Heroi, le virtuosissime operationi de' quali si leggono con maraviglia nell'historie, et parte si veggono nell'antiche ruine."

[23] Palladio 1570, III, 3: "Delle qual cose [concernenti l'architettura] essendo l'A[ltezza] V[ostra] dotata delle più nobili arti, e scientie; piglierà non poca contentezza, e consolazione considerando le sottili, e belle invenzioni degli huomini, e la vera scienza di quest'arte, da lei molto bene intesa, e ridotta a rara, e perfetta perfezione; come dimostrano gli illustri, e reali edifici fatti fare, e che tuttavia si fanno in diversi luoghi dell'amplissimo, e felicissimo suo stato."

[24] Pollak 1991a, 18–26.

[25] See Viglino Davico 2005, Pollak 1991a and Signorelli 1969–1970.

[26] Vernazza 1783, 8 and 11.

emphatic "P[rim]o Feciali," that is to say, "First Herald."[27] This was in fact the most appropriate title for the "Roy d'armes."[28] It is possible that this correction was inserted by Benedetti himself.

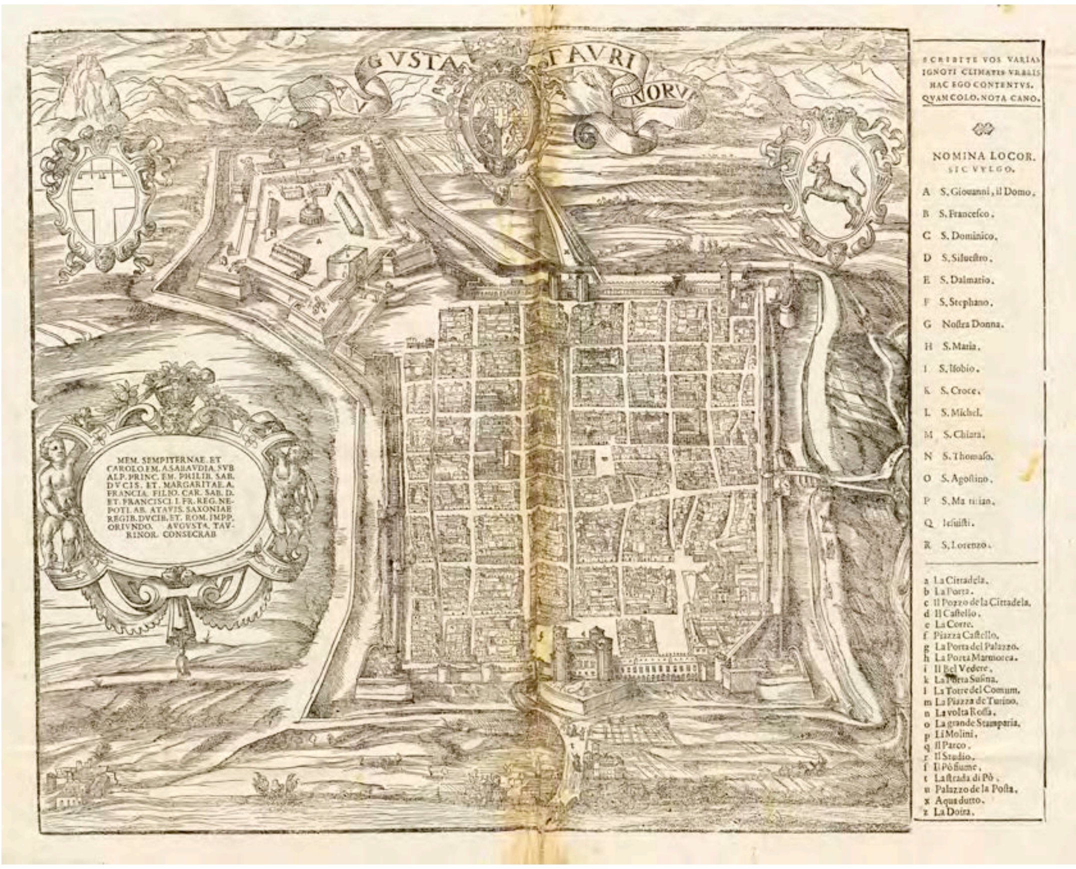


Figure 2.5: Map of Turin in Benedetti's times, from Pingone's *Augusta Taurinorum* (1577). (Staatsbibliothek zu Berlin)

Benedetti interacted with architects and engineers, as can be seen in his correspondence. Four of the scientific letters included in the *Diversae speculationes* are addressed to the architect Busca. Their topics, however, are not strictly related to building or engineering. In fact, their topics range from meteorology to instruments, topography, and natural philosophy.[29] Benedetti was closely related to the ducal "architect and cosmographer" (*Architetto e Cosmografo*) Soldati, who had worked as a hydraulic engineer and architect in Milan and in Lombardy, and joined the Turin court in 1576. Benedetti held him in great esteem, judging by the dedication to him in one of the most technical parts of the *Diversae speculationes*, that is, the entire second book, which is entitled "Explanation on Operations of Perspective" (*De rationibus operationum perspectivae*).[30] Besides, Benedetti's interest in engineering and measuring instruments emerges from many writings, especially from his work on gnomonics (1574) and from a manuscript analysing a measuring instrument that

[27]The letter is to be found in Benedetti 1585, 330–331. The collocation of the volume in the *Biblioteca Reale di Torino* is G 43 8.
[28]Vernazza 1783, 37, n. 31.
[29]Benedetti 1585, 271–277.
[30]Benedetti 1585, 119–140. See Mamino 1989.

is preserved in the *Biblioteca Civica di Carignano*, entitled *Descrittione, uso, et ragioni del Trigonolometro* (1578).[31]

## 2.4 Intellectual Ferment: Arts, Literature, and Philosophy

Renaissance Turin was as appealing to mathematicians as to all other types of intellectuals, including artists, literati, and scholars in general. One could mention the names of two reputed theorists of art who were closely linked with the court: Federico Zuccari and Lomazzo. The former wrote *L'idea de' pittori, scultori, et architetti* (The idea of the painters, sculptors and architects, 1607) and the latter *Trattato dell'arte della pittura* (Treatise on the art of painting, 1584), dedicated to Carlo Emanuele I. Lomazzo also dedicated to the duke of Savoy his collection of poems, *Rime* (1587), including one for Benedetti. Among the artists appointed by the duke, the Flemish Jan Kraeck or "Carracha," who resided in Chambéry, not only painted; he also worked as a cartographer and made a large map of Turin in 1572 (385x397 cm) as part of a wider cartographic program.

Illustrious scholars came to Emanuele Filiberto's court or to the reopened university, first in Mondovì and later in Turin. Among them, the man of letters from Ferrara, Giovanni Battista Giraldi Cinzio, was appointed to teach humanities. His collection of *novelle*, the *Ecatommiti* (Mondovì, 1565), ended with a long celebratory poem mentioning the most visible intellectuals of the Peninsula. Many of them were linked to the duke of Savoy, either as professors or as courtiers.[32] For instance, Francesco Ottonaio of Florence, who taught mathematics at the university in Benedetti's years and exchanged views with him, is extolled for his expertise in astronomy, meteorology, and astrology:

> My Ottonaio moves his feet towards you along with the others.
> He received the gift of scrutinizing the heavens,
> of knowing the reasons for warmth and coldness,
> why the days are short or long,
> and what layer veils the Sun making it dark,
> the manner in which the year becomes adorned of beautiful flowers again,
> what nativity is a sign of honor and merit
> or of shame and disgrace,
> and what is the star presiding over
> a man's state from his birth
> until his vital light is extinguished
> one circle after the other.[33]

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[31] Mamino 1989, 432–433 and Roero 1997.
[32] Villari 1988, 93–95 and 107–110. See Doglio 1998, 599ff.
[33] Villari 1988, 93–94:

> "Move insieme con lor verso te il piede
> il mio Ottonaio, a cui scorrere il cielo, per grazia, diede.
> Del caldo la cagion saper, del gelo,
> e perché breve sia, sia lungo il giorno,
> e quale offoschi il sole oscuro il velo;
> come ritorni di bei fiori adorno
> l'anno e chi debba aver dal nascimento
> onore e pregio, e qual ingiuria e scorno;
> e da che stella prender de' argomento
> de lo stato suo l'uom, poi ch'egli è nato,
> insin che il suo vital lume sia spento

Giraldi Cinzio did not mention Benedetti as he had not yet come to Savoy, but he included the Turin physician and professor of medicine Antonio Berga, with whom Benedetti would later enter a controversy over the proportion of water and earth in the terraqueous globe:

> With his gentle and beautiful works
> he tries to subtract his name from the oblivion,
> defeating the stealing forces of greedy time.
> I refer to my gentle Antonio Berga,
> who shows the way to those who wish to learn
> by writing his papers for the common good.[34]

Two famous authors who visited Turin between 1576 and 1579 are the poet Torquato Tasso and the philosopher Giordano Bruno. Both arrived in the town as fugitives and both enjoyed their stay. Tasso interacted with the cultivated elites. The Turin philosopher and physician Agostino Bucci appears as *persona dialogans* in three dialogues of his (*Il Forno ovvero della nobiltà*, *Della dignità*, and *Della precedenza*). His connection with the court is further confirmed by the dedication of the 1581 edition of the *Gerusalemme liberata* to the prince of Savoy.[35]

As for Bruno, he did not establish lasting contacts in town. He visited Turin in 1576 (or at the beginning of 1577) for the first time after abandoning the Dominican cloister of Naples, where he was accused of heresy. However, as he stated later in his Inquisition trial, "non trovando trattenimento a mia satisfattione, venni a Venezia per il Po [as I did not find sufficient means, I came to Venice along the Po]."[36] He visited Turin again in 1578 and went to Chambéry, where he spent the winter of that year as a guest of the Dominicans. On that occasion he possibly carried a booklet, now lost, entitled *De' segni de' tempi*, that he had printed in Venice and that probably dealt with the comet of that year.[37] We will refer to his possible involvement in some polemics on the comet of 1577–1578 later. It is hard to say whether Bruno and Benedetti ever met or were informed of each other's views. In spite of the fact that they belonged to very distant milieus, there is some affinity between their outlooks. Both shared an aversion to Aristotle, the project of reforming natural philosophy, the support for the Copernican system, and other cosmological views.

The case of the philosophical poet Pandolfo Sfondrati also deserves our attention. He was active in Turin as a poet at the same time as Benedetti and authored poems that were often inserted in the first pages of books printed by the Bevilacqua printing house. Pandolfo made himself visible in Turin with celebratory poems that were included in important local publications, in particular in the works of the court historian Emanuele Filiberto Pingone: *Augusta Taurinorum* (1577), *Inclytae Saxoniae Sabaudiaeque principum arbor gentilitia*

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di cerchi in cerchio."

[34] Villari 1988, 95:

> "E quel che, con gentil opre, e leggiadre,
> tenta che il nome suo da l'oblio s'erga,
> vinte del tempo avar le forze ladre,
> i' dico il mio gentile Antonio Berga,
> che addita, a chi imparar cerca, la strada,
> mentre, ad util comun, le carte verga."

[35] Doglio 1998, 621 and 625.
[36] Firpo 1993, 159. See Ricci 2000.
[37] Ernst 1992.

(1581) and *Sindon evangelica* (1581).[38] Hence, Sfondrati frequented the same courtly and cultural environment as Benedetti. It is likely that they discussed natural issues together, especially when considering that Sfondrati composed philosophically minded poems such as the Copernican *Inferiora regi dum syderis omnia motu*, which opens the *Animadversiones in Ephemeridas* by Benedetto Altavilla (Turin, 1580). There is also evidence that Benedetti was familiar with the Sfondratis, in particular with Paolo Sfondrati, who was a senator of Milan and an ambassador of Filippo II in Turin.[39] Moreover, Pandolfo Sfondrati authored the atomistic poem *Democriti prohibent nosci corpuscula formas* and a treatise on the tides, which he explained in mechanical terms as the result of the interaction of water particles warmed up by solar rays.[40]

## 2.5 Religious Policy

### 2.5.1 Pragmatic Counter-Reformation

The relics were moved from the old to the new capital: Christ's shroud traversed the Alps together with the court. Religion was an essential stabilizing factor. According to the report of the political thinker Giovanni Botero, Emanuele Filiberto declared that piety was essential to guarantee his authority in the state: "Those people who are zealous in their devotion [to religion] are more moderate: in consequence, they obey their Prince better than licentious people."[41] His religious politics were characterized by pragmatism. He undertook measures against the dissemination of the Reformation in his country and repressed the reformed communities only insofar as they jeopardized the integrity of the state or its relations with Rome or with other Catholic countries. The persecution of the Valdesans in the Alpine valleys, between 1559 and 1560, was part of a wider endeavor aimed at establishing a Catholic league that could favor his conquest of Geneva. However, when the prospect of an anti-Protestant confederation vanished, Emanuele Filiberto interrupted the aggression. The resistance of the Valdesans had been strong and persistent. It was a destabilizing factor for the duchy. A compromise was reached on June 5, 1561, when Emanuele Filiberto promulgated an edict, known as the Edict of Cavour, conceding to the Valdesans freedom of worship in their mountains. In exchange, he imposed the construction of new fortifications and strict military control of the Alpine passes.[42]

The other pole of Savoy religious politics was external. It concerned the regulation of jurisdictional issues with the Roman Church. A reason for friction was the taxation of the ecclesiastics, which Rome was reluctant to grant. The economic stake was high, since the estates belonging to the Church amounted to a third of the land; in some areas, it even reached values comprised between 40 and 70 percent. The ancient privileges of the dukes to select the candidates for the principal ecclesiastical positions had to be negotiated anew. It was only in 1573, under Gregory XIII, that these were confirmed. Finally, the introduction and the reinforcement of the Inquisition in the duchy fostered confessional uniformity but interfered with secular jurisdiction.[43]

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[38] See Griseri 1998.

[39] In a letter to Benedetti, Francesco Patrizi asked him to give his regards to Baron Sfondrati. See Patrizi 1975, 42–43.

[40] See Omodeo 2008b and Omodeo 2012a.

[41] "La gente infervorata di devotione è molto più regolata: e per consequenza più ubidiente al Suo Prencipe, che la dissoluta." Botero 1608, 241.

[42] De Simone 1958.

[43] See Merlin 1995, 238–267, chap. IX, "Tra Controriforma e Ragion di Stato."

The new Inquisition, established in the wake of the Council of Trent in order to counter the Reformation and reaffirm Catholic hegemony in Italy, had a centralized structure with ramifications for the entire Peninsula. The Holy Office lay at its heart. The various Italian states accepted it as a preventative measure against public disorder, especially against confessional conflicts such as the civil wars affecting France and other European areas. The Inquisition was a repressive control system binding Rome and the local powers. It was a compromise in which, in Adriano Prosperi's words, "l'aiuto era offerto e richiesto in nome della conservazione del potere, quello politico dei principi e quello della corporazione ecclesiastica" (The aid was offered and requested in order to preserve the power—the political one of the princes as well as that of the ecclesiastical corporation.)[44] Nonetheless, the jurisdiction problem remained acute: what was the legitimacy of a foreign tribunal—the Roman one—trying and condemning the subjects of other countries? In Piedmont, the interests of Turin and Rome were guaranteed through the mediation of the Papal nuncio, who was obliged to inform both the duke and the Holy Office in Rome about Inquisition trials.[45] For their part, these authorities could intervene in the trials and had the right to give their consent concerning the opportunity to carry them out. However, the opposition to the establishment of the Inquisition was strong, particularly in the French-speaking areas. Relying on its Gallican tradition, the local church in Savoy did not accept a form of direct control from Rome.[46]

The relations between Emanuele Filiberto and the Papacy were not always calm, and became strained after the Cavour edict of tolerance in 1561. Its promulgation provoked the protests of Rome and the commission of the intransigent Cardinal Inquisitor, Michele Ghisleri, to the diocese of Alexandria as Bishop of Mondovì, with the aim of reinforcing religious control. The relations between this champion of orthodoxy and the pragmatic politics of the Savoy dukes were tense, including after Ghisleri was elected pontiff as Pius V in 1566.

### 2.5.2 Jesuit Colleges in Piedmont

In 1561 Emanuele Filiberto supported the foundation of the first Jesuit college in his territories, in Mondovì, where the university was located at that time. A second college was established in Chambéry (1564), which was the former capital of the duchy. Among the Jesuit teachers, Antonio Possevino is one of the most renowned. The future compiler of the *Bibliotheca selecta* (1593) sojourned in Piedmont between 1560 and 1562.[47] From 1564 to 1567 the future cardinal and inquisitor Robert Bellarmine was also there but could not be appointed professor of philosophy in the Jesuit college of Turin, opened in 1567, due to the intransigent opposition of the university against the attempts to transfer this chair from the university to the Jesuit institution.[48] Both in Mondovì and in Turin the Jesuits took over the so-called "public schools," which essentially taught the rudiments of grammar.[49]

In those years, the chair of letters belonged to the Ferrara humanist Giovanni Battista Giraldi Cinzio, whose religious tendencies could be called Erasmian.[50] At the moment of the establishment of the Jesuit college in Turin, he sided with the humanistic legacy

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[44] Prosperi 1996, 57–58.
[45] See Prosperi 1996, III, "Inquisizione romana e stati italiani" and Black 2013, 30.
[46] Prosperi 1996, 103–105.
[47] Longo 1998, 475 and Scaduto 1959, 52.
[48] Grendler 2002, 42.
[49] Vallauri 1846, 19.
[50] For instance, Giraldi Cinzio defended the famous commentator on Aristotle's *Poetics*, Ludovico Castelvetro, who was excommunicated in 1560 as "eretico fuggitivo e impenitente" for his alleged bias towards Melanchthon. On this occasion Giraldi Cinzio argued that violence and coercion could only produce the

against their pedagogy. He paid the price of their expansion, as his chair was suppressed and transferred to the Jesuit college.[51] From 1567 to 1574, the Jesuits received 200 *scudi* per year to teach Greek and Latin grammar, humanities, and rhetoric to the youth (half of Giraldi's salary). Thus, the humanist had to abandon Turin for Pavia in 1569. In a letter to the Florentine philologist Pietro Vettori (Pavia, March 20, 1569), he lamented his expulsion, although he expressed his gratitude to the dukes for the donation of 500 *scudi* for his leave.[52] He particularly protested that his teaching as a learned humanist was being substituted by the teaching of elementary grammar.

The opening of the Turin college set off enduring hostilities between the Jesuits, on the one side, and the university and the municipality on the other. According to Grendler's reconstruction, between 1570 and 1572 the duke and the rector, Achille Gagliardi, made an agreement according to which nine chairs would be given to Jesuit professors. However, the project did not succeed owing to obstruction on the part of the town and the university.[53] In these struggles, the Archbishop Gerolamo della Rovere was among the most strenuous opponents of the Jesuits. The position of Emanuele Filiberto fluctuated. Initially, he supported the "reverend fathers" but later distanced himself from their educational projects. In 1575 he even reintroduced the teaching of humanities in the university. There is indirect evidence that Benedetti sided with the humanists in this anti-Jesuit controversy. In 1583 Francesco Patrizi, who belonged to the same Ferrara cultural environment as Giraldi Cinzio, asked him to support the candidacy of his friend Giovanni Giacomo Orgiazzo for the position of professor of humanities in 1583.[54]

Apart from the political interests at stake (the privileges of the town and of the university), the professors' resistance concerned the contents of the teaching, as one reads in a document from 1593, "Raggioni perché non sia bene che gli Rev[erendi] Padri Gesuiti leggano la filosofia tutta, et la logica nel loro Comento, et si lasci a leggerli nello Studio et pubbliche scuole, come sempre insino a qui si è fatto" (Reasons why it is not good that the Jesuit Reverend Fathers teach all philosophy and logic in their commentary and are allowed to teach at the university and in public schools, as has been the case until now).[55] According to the academics, philosophy should be imparted to students as the fundamental tenet of the study of medicine. Therefore, the focus should be set on Aristotle's natural philosophy and not on metaphysics and logic, as was the case with the Jesuits. Metaphysics, as one reads in the document, is the "last" and not the "first" part of philosophy. By contrast, the Jesuits began their teaching with the most abstract issues, e.g., the statute of ideas and universals, and divine ineffability ("*utrum Deus sit in praedicamento*" or "*utrum Deus sit infinitus*").

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opposite effects than those wished for by the defenders of orthodoxy. See Cinzio 1996, Letter n. 101, 371, n. 3.

[51] Vallauri 1846, 19 and Grendler 2002, 42–43.

[52] Cinzio 1996, Letter n. 127, 425: "Sed Taurino iam menses quatuor absum, Ticinique publice profiteor. Nam, praeter iacturam valetudinis, quam ibi quotidie faciebam, me ad abeundum urgentem, natio illa haec nostra studia nihil quidem facit. Hinc Princeps ille, qui oratoriam ac poeticam facultatem profiteretur, in Academia sua habere constituit neminem, quod satis esse censuerit Iesuitas nescio quos, suo in collegio, hoc muneris cum puerilis ac infantibus obire; qui, cum Deuspaterio quodam, barbaro plane auctore, mollia ingenia, obscurissima, ne dicam foedissima, imbuunt barbarie. Me tamen abeuntem, praeter annuam quadrigentorum aureorum nummum stipe, quam liberaliter exsolvit, centum etiam scutatis aureis donavit."

[53] Grendler 2002, 42–44.

[54] Patrizi to Benedetti (Ferrara, 21 March 1583), Patrizi 1975, 39.

[55] Archivio di Stato di Torino, Istruzione Pubblica/ Regia Università di Torino/ Mazzo 1 (1267–1701), Fascicolo 7/2. The document is included as an appendix to Omodeo 2014d.

### 2.5.3 Benedetti and the Counter-Reformation

What can be said about Benedetti's attitude toward the culture of the Counter-Reformation emerging after the Council of Trent? We can assume that his scorn for Aristotelian philosophy was not only a dispute with the university professors of his day, but also with the theology-oriented Aristotelianism propagated through the cultural apparatus of the post-Tridentine Church. We have no evidence that Benedetti frequented exponents of the Jesuit order, in spite of their presence in Turin. Rather, we have evidence of his connections with scholars who were not in the mainstream of the official Catholic culture of those years.

Among others, he corresponded with Francesco Patrizi of Cherso, whose Platonism was regarded with suspicion and even censured in Rome. The closeness between them is witnessed by the fact that Benedetti acted as a mediator between the philosopher and the duke of Savoy on at least one occasion. When Patrizi issued his *Della nuova geometria libri XV* (Fifteen books on the new geometry, Ferrara, 1587) with a dedication to Carlo Emanuele I, Benedetti passed on a copy to his patron. Patrizi expressed his gratitude with the following words (Ferrara, April 6, 1587):

> Very Magnificent and Excellent Signore,
>
> I rejoice with your Lordship that you recovered from sickness quicker than believed. And I am very thankful to you for presenting my book to the very serene Prince and promising to inform me about his remarks after he has read it. If by chance the book will be forgotten, due to his many duties [negozii], I hope at least that you will remember me. If his High Serenity will give some sign that he appreciated it [my book], I will be very glad and I will be particularly grateful to your Lordship for your benevolence.[56]

In exchange, Benedetti sent him a copy of his discussion on the relative sizes of the elements of earth and water, as witnessed by a letter from Patrizi (Ferrara, 18 January 1588).[57]

The two scholars shared views on cosmology that were to be censured by the Inquisition in the 1590s. It is thus expedient to briefly recall Patrizi's natural and cosmological views, as they are close to those Benedetti expressed in his writings, especially in the *Diversae speculationes*. Already in his *De rerum natura libri I priores. Alter de spacio physico, alter de spacio mathematico* (First Two Books on Nature, One on Physical Space and One on Mathematical Space, 1587), Patrizi embraced the neo-Stoic doctrine of the fluidity of the heavens, the infinity of space beyond the sphere of the stars, and the free motion of planets through cosmic space following an inner drive. He later expanded on that in *Pancosmia*, which is the cosmological section of his philosophical masterwork, *Nova de universis philosophia* (New Universal Philosophy, 1591). In it, he did not limit himself to asserting the infinitude of luminous space beyond the visible stars, to repeating the thesis of planetary self-motion, and to rejecting the existence of celestial spheres responsible for the transportation of the heavenly bodies. He also ascribed to Earth the daily motion around its axis without renouncing its central position in the cosmos and in

[56] Patrizi 1975, XXVII, 53: "Molto Magnifico et Eccellentissimo Signore, mi rallegro con Vostra Signoria, che più tosto che non credea si è rilevata dal male, e li rendo moltissime gratie dell'haver presentato il mio libro a quel Serenissimo Prencipe, e ricevuto il favore, che Ella mi avvisi ciò che haverà detto, dopo che l'havrà letto. Et se per sorte per li molti negozii il libro andasse in oblio, spero da Lei il rimedio di un poco di ricordanza, la quale, se partorirà alcun segno che Sua Altezza Serenissima l'habbia havuto caro, mi sarà carissimo e tutto l'obbligo l'haverò a Vostra Signoria e all'amor suo verso me."

[57] Patrizi 1975, 57–58. In the letter Ottonaio is also mentioned as a common acquaintance and an intellectual partner.

the planetary system.[58] All of these theses—which, as we shall see, are also present in Benedetti's *Diversae speculationes*—were censured by the Inquisition in the 1590s, in the course of an attack directed against Patrizi's *Nova de universis philosophia*.

The censure of Patrizi's philosophy occurred after Benedetti's death—he was lucky enough to avoid witnessing the prohibition of theses to which he subscribed. On October 8, 1592, the Master of the Sacred Palace, Bartolomé de Miranda, and his fellow, Pedro Juan Saragoza, wrote a document of censure, attacking many passages and doctrines of the *Nova philosophia*. The same Saragoza would later be one of the two censors of Bruno's work during his Inquisition trial in Rome.[59] The two censors of Patrizi did not limit their criticism to theology (especially Patrizi's theses on the Trinity) but also scrutinized natural philosophy and cosmology. They rejected the idea that there is only one unique heaven and judged this to be an opinion at odds with accepted philosophical doctrines and against theology (*In lib. 13 Pancosmias tenet unum tantum esse caelum*). Furthermore, Patrizi was accused of following Copernicus, whose doctrine of terrestrial motion was seen as incompatible with the Bible:

> In *Pancosmia*, Book 17, f. 103, p. 1, column 2a, he [Patrizi] states 'that the motion of the Earth is by far in better agreement with reason than the motion of the heavens or the uppermost celestial bodies.' And he refers to Nicolaus Copernicus's sentence according to which the sidereal heaven is immobile, along with the stars, while the Earth moves.[60]

Further theses to be censured were his vitalistic concept of celestial bodies and celestial infinity. The criticism of the latter point goes as follows:

> This [to sustain this view] is to dream in very deep obscurity and fall down a precipice after abandoning the common way. In fact, the best and greatest God created everything according to weight, number, and measure. Therefore, everybody agrees that no infinite body is possible in act and no existing multiplicity can be infinite in act. On the empyrean heaven see the Fathers and Thomas Aquinas.[61]

Patrizi replied with an *Emendatio in libros suae novae philosophiae* (Correction of the Books of His New Philosophy), written before Christmas 1592. As far as Earth's motion is concerned, he clarified that he advocated its motion but not its eccentricity (*Non tamen dixi, eam de medio suo et naturali loco exire*). Furthermore, he stressed that terrestrial motion was supported by many arguments and several philosophers, and claimed that this view does not contrast with theology. However, he declared himself ready to erase passages referring to terrestrial motion, if necessary (*Delebo tamen si iubetis*). He also defended his views about the infinity of space but was ready to renounce this thesis as well, if he was ordered to do so: "Delebo hanc etiam si iubetis."[62]

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[58] Seidengart 2006, 116–124 and Omodeo 2014a, 174–175.

[59] See Bruno 2000b, doc. 45, 225.

[60] Baldini and Spruit 2009, Vol. I, 3, 51, doc. 1, 2216: "Lib. 17 Pancosmias fol. 103, pag. 1, col. 2.a ait *quod Terram revolvi longe videtur esse rationi consonantius, quam Coelum, vel suprema astra moveri. Et refert sententiam Nicolai Copernici dicentis Coelum sydereum stare simul cum* stellis, Terram vero moveri."

[61] Baldini and Spruit 2009, Vol. I, 3, 51, doc. 1, 2219: "Hoc est somniare per altissimas tenebras, et a via communi declinando in praecipitia ruere, nam cum Deus opt. Max. omnia in pondere, numero, et mensura produxerit, nullum infinitum corpus actu dari nullamque rerum subsistentium multitudinem actu infinitam omnes viri fatentur. De Coelo empyreo consultat Patres, et sanctum Thomam."

[62] Baldini and Spruit 2009, Vol. I, 3, 51, doc. 7, 2231.

The Jesuit Benedetto Giustiniani proved a more open-minded censor of Patrizi's natural views in 1593.[63] The same person, however, would be one of the eleven theologians who decided that the Copernican theory was not reconcilable with the Catholic religion, in 1616. As to Patrizi's work, it was downright (*omnino*) prohibited in 1594, and placed in the Clementine Index of 1596 as well as in later Indexes.[64]

Not only did Benedetti correspond with intellectuals engaged against the mainstream in Rome, but he himself had differences with Roman Aristotelians, as he mentioned in the preface to the second edition of the *Demonstratio motuum localium contra Aristotelem* (1555). On some occasions, Benedetti even allowed himself to be ironical about theological dogmas. For instance, in one of the letters of the *Diversae speculationes*, he accompanied his adherence to methodological Pythagoreanism (a mathematical approach to the investigation of nature) with a joke about reincarnation and his earlier life:

> If the souls' transmigration imagined by the father of Italian wisdom, Pythagoras, were true, I believe that your soul and mine were once the souls of hunting dogs.[65]

Another indicator of Benedetti's attitude towards the Counter-Reformation and the confessional quarrels of his time emerges from his approach to the calendar reform. This was a very divisive issue. The pope imposed upon all Christianity an emendation of the calendar in an age when it was affected by profound divisions. In this climate, the pope's political and religious legitimacy and his authority in such matters was cast into doubt by many, especially in the reformed countries. Reputed Lutheran astronomers such as Michael Maestlin opposed the calendar reform implemented by Christopher Clavius and promulgated by Gregory XIII in 1582. The elimination of ten days to make the spring equinox date correspond to its date at the time of the Council of Nicaea was particularly controversial, since it symbolically legitimized the universality of the Roman Church through Constantine I. Benedetti wrote a reform proposal on April 1, 1578, and Emanuele Filiberto sent it to Rome on May 31. The text was printed two times, first in 1578, and then reprinted in 1585 as the first of the letters in the epistolary section of the *Diversae speculationes*. The most striking feature of Benedetti's proposal is its technical radicalism and extreme rationalism. August Ziggelaar's perspicuous description of Benedetti's proposal here follows: "Benedetti prefers the Eastern date to be fixed according to the true motions of Sun and Moon rather than by cycles. He finds that the Prutenic tables are sufficiently exact for this purpose. Furthermore not ten days, not even 14, but 21 days should be left out in order to make the first day of January the winter solstice. The lengths of the months are to be adjusted so that they coincide with the presence of the Sun in each of the twelve zodiacal signs. Surely, these time intervals change their lengths in the course of time because of the motion of the perihelion of Earth, but Benedetti assures us that only after 24,000 years will an adjustment be necessary. The proposal is not only interesting and original but also very rational because, by eliminating all reference to the Moon, it makes the year entirely solar."[66] The extreme technicality of such a proposal, one can argue, was

[63] Baldini and Spruit 2009, Vol. I, 3, 51, doc. 10.

[64] For a reconstruction of the anti-Platonic reaction also affecting the reception of Patrizi, see Rotondò 1982. On the censure of 1616, see Bucciantini 1995, Bucciantini, Camerota, and Giudice 2011 and Omodeo 2014a, chap. 7.

[65] Benedetti 1585, 285: "Si vera esset animorum illa transmigratio quam sibi Italicae sapientiae Pater Pythagoras effinxerat; tuam, meamque existimarem animam canis, quandoque venatici fuisse."

[66] Ziggelaar 1983, 211 and 214.

also a means to de-ideologize the issue. Therefore, it was not only rejected for its "scientific radicalism" but also for its rationalistic distance from confessional struggles. This positioning is in line with late-humanistic universalism and signals Benedetti's distance from the Counter-Reformation and the militant cultural production of those years.[67]

## 2.6 Cultural Institutions: University, Academies, Collections, and the Press

The reformation of the *Studio* was a cornerstone in Emanuele Filiberto's and his successor's cultural policy. It was reopened in Mondovì in 1560 after the French closed it as a potential center of dissent and revolt. It was transferred to the new capital on October 12, 1566, after long discussions and a dispute between Turin and Mondovì. According to the new statutes, issued in 1571, the direction of the university was conferred to nine *reformatores*, among whom were the Archbishop, the ducal chancellor, the first and second presidents of the Senate of Turin, and the court physician. These men were responsible for the scientific and administrative direction, and for academic discipline.[68]

Most of the professors (about thirty people) were jurists. Among them, the most reputed was the professor of civil law Guido Panciròli. The physicians Francesco Valleriola and Giovanni Argenterio were also illustrious professors, known and appreciated by Conrad Gesner and Michel de Montaigne. The reopening of the university offered Argenterio the opportunity to come back to Piedmont after many years of practice as a physician in Lyon, and of teaching in Antwerp, Bologna, Pisa, Rome, and Naples.[69] Teodoro Rendio of Chio was appointed to teach Greek grammar and, as mentioned before, the poet Giraldi Cinzio became professor of humanities. At the University of Turin, the teaching of philosophy on the basis of the *corpus Aristotelicum* received high recognition, as the professors in this discipline had a better salary than their colleagues of medicine. Giacomo Castagneri taught on Aristotle's *Physica*, *De generatione et corruptione*, and *De Anima*. Other celebrated scholars in philosophy were Antonio Berga and Agostino Bucci, both Piedmontese educated at Padua.[70]

One of the most reputed professors appointed in Mondovì was Francesco Vimercato of Milan, with whom Benedetti later corresponded. Vimercato was a thoughtful Renaissance commentator on Aristotle, whose work he read in the original language. He published commentaries on *De anima* (1543), on *Metaphysics* (1551), in particular on book lambda and on *Meteorologica* (1556). However, his commentary on *Physics* has to be seen as his magnum opus. After his studies in renowned Italian universities, he was appointed in Paris as the first royal lector in philosophy. There, he was first involved as a judge in the Ramist disputes and later was a colleague of Petrus Ramus. In 1561 he came to Piedmont to serve the Savoy family as a university professor, a councillor, a tutor to Carlo Emanuele I, and, from 1567 to 1570, as diplomat in Milan.[71]

Bordiga argued that Benedetti might have taught at the reopened university, first in Mondovì and then in Turin, by relying on some indirect sources. In fact, the information about Benedetti's teaching activity at Mondovì was derived from the sixteenth-century biographer of Emanuele Filiberto, Giovanni Tonso, who included his name on the list of

[67] Steinmetz 2011.
[68] See Bonino 1824–1825, Naso 1993, and Catarinella and Salsotto 1998.
[69] On Argenterio, see Temkin 1974, 141–144 and 149–152 and Mammola 2012, 185–193.
[70] On the philosophical culture of Turin of those years, in particular on Bucci, see Mammola 2013.
[71] See N. W. Gilbert 1965.

those who made that university eminent (*Qui viri insignes publice docuerint*).[72] Another contemporary of Benedetti, the mathematician Cristini, mentioned him as one of those whom Emanuele Filiberto called to Turin in order to increase the prestige of his university.[73] Still, the role that he could have played in the academic life of Turin remains obscure.[74]

We could find no direct evidence that Benedetti served as a professor in the documents preserved at the *Archivio di Stato di Torino*. We considered the acts gathered under the signature "Istruzione Pubblica/ Regia Università di Torino/ Mazzo 1 (1267–1701)," which include the statutes of 1571 and other precious sources concerning the first years of the univeristy. A dossier entitled "1571, Costituzione de' Riformatori dell'Università dello Studio di Torino, coll'Istruzione da osservarsi da medesimi, colle distribuzioni delle ore per la Lettura, e Rolli de' Stipendi de' Lettori" (*fascicolo 7 primo*) includes decrees concerning the reform of the university, the names of those responsible for accomplishing it, and a list of the chairs with the corresponding salaries and the names of the professors. These documents indicate that the professor of mathematics was Francesco Ottonaio of Florence, who had occupied the same chair at Mondovì. Other documents testify that the same person held the chair of mathematics in 1573 and in 1585/6. It is therefore hard to believe that Benedetti was professor in this subject before 1586, as several secondary sources contend.[75] As to his alleged position at Mondovì, that university was opened by Emanuele Filiberto in 1560 and then transferred to Turin in 1566, that is, before Benedetti's arrival at the Savoy court. Thus, it must be excluded as a possibility that he ever taught there, contrary to the claim made by Tonso. It is possible though, even likely, that he acted as an external advisor on matters concerning the university.[76]

Turin had fewer academies than other centers such as Rome, Naples, and Florence, although it was a typical Renaissance phenomenon to establish academies, circles of learned men who met to discuss issues pertaining to literature, the arts, or the sciences, and who published works representative of their common intellectual efforts. Apart from two minor academies ("*de' Solinghi*" and "*degl'Impietriti*"), Carlo Emanuele I conceived the project of forming an academy guided by the Jesuits around 1585. Bonifacio Vannozzi, a man of letters from Pistoia, described it as follows:

> His Highness, the very serene [duke] of Savoy, had the wish to found an academy in this august town of Turin. He charged three Jesuit Fathers of the renowned College with the task. Although they are generally sober of mind, in this case they were so intemperate as to entrust myself [with this endeavor] although the overwhelming responsibility [*machina da incurvar le spalle*] would be excessive for even the most competent person. His Highness has made himself Prince, Protector, and Head [of the academy], in order to attract a good deal of his courtiers [into the academy] who are so cultivated

[72] Tonso 1596, 141: "Neque vero liberalium disciplinarum omniumque artium colendarum quam susceperat cogitationem unquam deposuit: nam et publicum earum Gymnasium pro tempore in oppido Monteregali instituit: et qui viri in quacunque scientia excellerent undique conquisuit. […] Mathematicos illustres Franciscum Othonarium, et Io. Baptistam Benedictum Venetum."

[73] Bordiga derived this information from a manuscript of Cristini's preserved in the Biblioteca Marciana in Venice. See Bordiga 1926, 596–597.

[74] The historian of Piedmontese Universities Silvio Pivano complained already in the 1920s about the lack of relevant documents. Pivano 1928, 19–22.

[75] See, e.g., Bauer 1991, 156–157.

[76] Roero 1997, 65, n. 5. Evidence for Benedetti's role as an advisor in university matters can be found in Patrizi's correspondence, as already mentioned.

> and refined that, if one adds to it the splendor of the arts [lettere], there will be no court in Europe more illustrious than this one. Our name is 'Incogniti.'[77]

In spite of the initial impetus, this academy was not particularly successful and did not leave significant traces of its activities. Perhaps it was negatively affected by the fluctuating relations between the Crown and the Jesuit order.

Emanuele Filiberto also created a *Theatrum omnium disciplinarum*, which was realized for him by Ludovic Demoulin de Rochefort. It is not completely clear what this theater contained. It was probably a *Wunderkammer*, as Mamino argued.[78]

> In March 1572, duke Emanuele [Filiberto] established in Turin a museum [*theatrum*] of all disciplines [organized] in marvelous order and at a very high cost. Archbishop Gerolamo della Rovere and the philosopher Ludovic Demoulin de Rochefort, the most educated men in all fields, cared for it.[79]

Moreover, the dukes supported editorial activities. Emanuele Filiberto first called the Flemish printer Laurens Leenaertsz van der Beke, better known as Torrentinus, to Piedmont. Torrentinus had already initiated a printing house in Florence under Cosimo I., but he died shortly after his arrival in Mondovì (1561). Hence, in 1570, Emanuele Filiberto employed another printer, Niccolò Bevilacqua. This pupil of Manuzio founded the *Compagnia della Stampa* (Printing Company), which benefited from ducal privileges (a sort of editorial monopoly). The activity of Bevilacqua and his heirs culminated in 71 editions produced between 1578 and 1580, most of them on juridical subjects. Scientific issues were not neglected in Turin. Among the publications issued during the years of Emanuele Filiberto, between 1563 and 1580, books on scientific and natural subjects constitute about one fifth of the entire production, that is, almost as much as literary publications.[80] They also printed several books by Benedetti, including the *Diversae speculationes*.

## 2.7 Scientific Debates

### 2.7.1 Courtly Conversations

Renaissance Turin was a center of lively cultural and scientific debates taking place in different institutional settings, at court, at the university, and in town. The printing press was a powerful means for public discussion, dissemination of ideas, and criticism. Printed sources are evidently our main source of information about the intellectual debates or polemics that animated Turin in Benedetti's time.

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[77] Tiraboschi 1824, 289–290: "L'Altezza di questo Serenissimo di Savoia ha desiderato, che si dia principio a fondar un'Accademia in questa sua Augusta cittá di Turino, et n'ha data la cura a tre Padri del Gesù di questo insigne Collegio, i quali, non so da che allucinati, soliti però a non s'abbagliare, hanno fatto gran fondamento nella persona mia, caricandomi d'una macchina da incurvar le spalle, quantunque gigantesche. S.A. se n'è fatto Principe, e Protettore, e Capo, per tirarvi buon numero de' suoi Cortigiani, tanto culti e fioriti nel resto, che, se vi si aggiugne l'ornamento delle belle e delle pulite lettere, non sarà Corte in Europa più rilucente di questa. Il nostro nome è degli Incogniti […]."

[78] Mamino 1992 and Mamino 1995. By contrast, Cibrario thought that it was an encyclopedic project. See (Cibrario 1839).

[79] Pingone 1577, 88: "Anno Christi 1572 mense Martio, Emanuel Dux Taurini theatrum omnium disciplinarum miro ordine, nec minimis sumptibus instituit, curantibus Hieronymo Ruvereo Archibiscopo, et Ludovico Molineo Rochefortio Philosopho, viris in omni doctrinae genere absolutissimis."

[80] On Renaissance publications in Piedmont, see Bersano Begey 1961, especially vol. 1. See also Merlotti 1998.

First of all, we should consider courtly debates. A circle of intellectuals gathered around the Savoy family. The professor of jurisprudence, Bernardo Trotto, depicted the relationship between the rulers and their learned courtiers as follows:

> These learned men, played by the Prince like well-tuned musical instruments, immediately give out their specific sounds with words. And they give it their best to be clearly understood in conversations, to please the others with good arguments and to convince them of their opinions. It is like the consonance of truth. In fact, everyone says what one knows or, at least, considers to be true. Hence they discuss natural issues and at times moral ones and mathematical ones. In conclusion, one can regard him [the prince] as Apollo surrounded by the Muses near the water spring that was born from the hoof of Pegasus.[81]

A reflection of the intellectual climate and the topics addressed in such informal meetings is a poem by the court physician Arma, *Proposte tenute co 'l Serenissimo Prencipe* (Issues Discussed with the Very Serene Prince), printed in Turin in 1580. In this tiny book, addressed to Emanuele Filiberto, Arma reported a discussion on meteorological problems that took place between Carlo Emanuele I, Benedetti, Ottonaio, Berga, and himself during the prince's meal for three successive days. On the first day, Carlo Emanuele I asked the reason why the sun heats. Arma used Plato to argue that its body is not warm but that it heats through the percussion of its rays on terrestrial elements.[82] The next day, Arma had a quarrel with Benedetti, Berga, and Ottonaio concerning "solar attraction."

> The day after, Mister Benedetti
> And Mister Berga, along with Ottonaio
> Expressed opinions that are far from mine:
> That the Sun attracts everything to itself with its great brightness
> As if it had hands.[83]

Arma objected that, if the sun attracts other bodies, this would be very inconvenient for its inhabitants, the solarians (*solari*). The argument is not clear and the reader is only informed about the fact that Benedetti agreed with him.

During the conversation, Ottonaio mentioned the vapors brought upward by the sun's action. Accordingly, the next topic of discussion concerned natural places and elemental displacements with reference to the behavior of vapors.

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[81] Trotto 1625, 2–3: "[…] questi huomini saputi, tocchi dal Prencipe, come instrumenti musici bene accordati, subito rendono ciascuno il suo suono con le parole et quanto meglio possono procurare d'essere intesi discorrendo, e di dar diletto con le buone ragioni, et anco di tirare gli altri al suo parere, come ad una consonanza della verità: perché ognuno dice quello ch'egli sa o crede almeno sia vero. E quindi si veggono trattare hor cose naturali, hor morali, hor mathematiche. Sì che egli quasi come uno Apolline si può dire, che sta fra le Muse, intorno al fonte, che uscì dal colpo del piede del cavallo alato." On Trotto's teaching, see Vallauri 1846, 28 and 48–49.

[82] Arma 1580a, f. A2*r*: "Scalda co raggi […]/ Sbattendo la Terra di caldo priva. Sì com'il martel che bate l'incudine,/ Riscalda l'un e l'altr'in certitudine."

[83] Arma 1580a, f. A2*v*:

> "Il Signor Benedetti l'indomani
> Col signor Berga, insiem' a l'Ottonaglio
> Forn'in pensier' a me d'assai lontani,
> Che'l Sol tirass'a sé com grand'abbagio
> Ogni cosa si com'havesse mani."

On the third day, the prince asked about the origin of lightning, and why we perceive their light before we hear the thunder. Arma answered that our sight is much quicker than our hearing, but this remark was received with skepticism by his colleagues. No details are reported about the objections that should have concerned the theory of perception, light, and optics.

The next issue was colors and the rainbow; Benedetti asked about the center of the rainbow's arc and Arma offered the following answer:

> Benedetti, as an expert master of his art,
> Asked me about the center of the arc [of the rainbow].
> I answered that it was on the vertical line
> Descending downward from the center of the heavenly body,
> As was the opinion of Zoroaster.
> And with this answer I got rid of him.[84]

At the end of this three-day conference, all opinions were written down for the prince and signed by the ducal advisors:

> All of this was presented in written form
> To His Highness, reporting all speeches.
> Dr Berga confirmed it [the rightness of the report].
> Benedetti did the same.
> After that we discussed other issues,
> Occult things and their effects.[85]

Other publications also mention such table talks at court. For instance, the physicist and philosopher Bucci wrote in the preface to his book on physiology, *Disputatio de principatu partium corporis* (Disputation on the Superiority of Parts of the Body, 1583), that his discussion about whether the heart or the brain held superiority in the body, and about the localization of the soul, originated from a table talk. On a certain occasion, in fact, Carlo Emanuele I had gathered his learned courtiers and addressed these issues. Among the participants, including several physicians, Bucci also mentions the "mathematicians," Ottonaio and Benedetti.[86]

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[84] Arma 1580a, f. A4*r*:

> "Il Benedetti, come degno maestro,
> Mi dimandò d'il centro di tal arco.
> Dissi, che gliera col centro de l'Astro,
> Ne la medema linea giù scarco.
> Si com'anchora volse Zoroastro.
> E con tal dire di lui mi discarco."

[85] Arma 1580a, f. A4*r*:

> "E tutto quest'in scritti fu donato
> A Sua Altezza, con tutti soi detti.
> E fu dal Dottor Berga conformato.
> Il che fece'l signore Benedetti.
> Fu poi d'altre proposte ragionato
> E de gl'occolte cose, e soi effetti."

[86] Bucci 1583, 7–8. See Mammola 2013, 6–8.

### 2.7.2 Academic and Scholarly Controversies

Scholarly controversies and polemics on various issues and with very different tones were printed in Benedetti's years. While courtly debates had a polite and entertaining character, academic disputes could be more vehement. However, the two contexts were not always neatly divided. In 1572 two professors of philosophy, Berga and Bucci, held divergent opinions concerning the interpretation of Aristotle's *De anima*.[87] As Simone Mammola has argued, their disputes on that issue should be understood against the background of the Averroistic-Simplician debates on Aristotle's psychology started at Padua and continued at Turin by scholars such as Filateo and Vimercato. The Turin controversy over Aristotle's soul doctrine, which developed through academic disputations and publications, ranged from cognitive problems linked to the functions of the soul (such as the problem of the relationship between imagination and cogitation) to metaphysical and theological issues, e.g., the legitimacy of a reading of Parmenides's theory of being as a form of *prisca theologia*. The real issue at stake was the correct approach to the Aristotelian corpus based on different commentators.[88]

The court physician Arma was at the center of several public controversies as well. In 1575, he defended the scientific status of medicine, traditionally considered "only" an art, in the programmatic book *Quod madicina sit scientia et non ars* (That Medicine is a Science and Not an Art). This writing was considered worthy of a second edition in 1585. In 1579, Arma entered a dispute against the professor of medicine Giovanni Costeo over the healthiness of bread made out of rice ("*pane fatto col decotto di riso*"). The two parties then issued a series of publications on this controversial topic.[89]

Another polemic opposed Berga and Benedetti regarding the proportion between the earthly and the watery element of our globe. The debate was initiated by some questions Carlo Emanuele I asked his courtiers. While Berga stuck to the Scholastic view that the elements have an increasing quantity proportional to their distance to the center, Benedetti favored the merging of empirical and mathematical arguments as proposed by Alessandro Piccolomini. Piccolomini had come to the conclusion that the quantity of the earthly element is superior to that of the water element, as one can read in his *Della grandezza della terra e dell'acqua* (Venice, 1558).[90] Although Benedetti regarded Piccolomini's arguments as conclusive, Berga undertook to compose a Scholastic refutation of them, *Discorso… della grandezza dell'acqua e della terra contra l'opinione dil S. Alessandro Piccolomini* (Turin, 1579). Part of his strategy was to eliminate arguments derived from the "misure dei cieli e della terra, dalla Scuola dei matematici immaginate," that is, from mathematical and empirical methods applied to this issue ("measurements of the heavens and the earth imagined by the school of the mathematicians"). Benedetti reacted with his *Considerazione… d'intorno al discorso della grandezza della Terra, et dell'Acqua. Del Eccellent. Sig. Antonio Berga Filosofo nella Università di Torino* (Consideration… on the discourse concerning the dimensions of earth and water by the excellent Mr. Antonio Berga, philosopher of the University of Turin, Turin, 1579). In it, he applied mathematics to show that, even if the entire earth were covered by water, the volume of the terrestrial

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[87] Bucci 1572 and Berga 1573.

[88] See Mammola 2013.

[89] Merlotti 1998, 585: "Come s'è visto per la polemica fra Costeo e Arma […] non si trattava di isolati testi a stampa che generavano dibattiti destinati a rimanere manoscritti e chiusi nell'ambito degli eruditi, ma semmai del contrario: di discussioni, cioè, sorte in circoli ristretti di medici e scienziati, prima affidate a manoscritti e poi trasportate a stampa a vantaggio d'un più vasto pubblico."

[90] For an accurate reconstruction of the polemic and its cultural and scientific context, see Ventrice 1989, 103–145 and Mammola 2014.

element would not be inferior to that of water since the depth of seas and oceans is small in comparison to the terrestrial radius. The oceanic navigations, the geographical explorations, and the European colonization of the globe during the fourteenth and fifteenth centuries had indisputably demonstrated that water and earth constitute one single globe, a unique *globus terraquaeus*.[91] This cosmographical advance had been the basis for Piccolomini's considerations, which met with considerable success and agreement among scholars. On such issues, as Benedetti stressed, experience and mathematical reasoning should be joined:

> Very serene Prince, the discovery, after two thousand years, that the [element] earth is much more than the [element] water (for which we are greatly indebted to the very learned Mr. Alessandro Piccolomini) very much pleased the spirits of the most renowned philosophers of our time. In the past, they did not dare to depart from the false doctrine they had imbibed for many centuries, although it was sustained by implausible reasons. Today they are glad to embrace the opposite opinion [concerning water and earth], because both the senses and reason are in accordance with the [new] demonstration of the truth. The ancient mistake has been unveiled by the mathematical school with very certain proofs that offer a firm foundation of the measurement of the heavens and the Earth.[92]

The dispute continued with the Latin translation of Berga's writing by Francesco Maria Vialardi (1580) and a skeptical intervention by Arma. The latter was a poem dedicated to Carlo Emanuele I, entitled *Stanze del dottore Arma al serenissimo Carolo Emanuele di Savoia et Piemonte Prencipe, etc. suo signore sempre osservandissimo. Che l'acqua e la terra non si possono a modo alcuno misurar* (Doctor Arma's Stanzas to the Most Serene Prince Carlo Emanuele of Savoy and Piedmont, His Perpetually Honored Lord, 1580). The composition is poor both from a stylistic viewpoint and a scientific one. The courtly physician could only point out the uncertainty of human knowledge and the wisdom in measure gifted by God to creation, although he also stated we cannot grasp the latter: "The Creator gave it a measure,/ Which cannot be grasped by any creature."[93] Benedetti addressed the issue again in one of the letters of the *Diversae speculationes*.[94]

### 2.7.3 Astronomical-Astrological Polemics

In Renaissance Turin, astronomical and astrological issues were at the center of intense debates and even polemics. In 1578 the protophysician Arma was the target of a denigrating pamphlet in twenty points. An anonymous author attacked a booklet of his on the comet that had just appeared. As we know from indirect evidence, Arma had written one or two treatises on this issue. One was entitled *De significatione stellae crinitae*.[95] Perhaps it has

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[91] Vogel 1993.

[92] Benedetti 1579, 3: "[…] l'essersi doppò due mila e più anni scoperto con trionfo della verità, che la terra è molto maggiore dell'acqua, (del che si ha da haver grande obligo tra gl'altri al dottissimo Signor Alessandro Piccolomini) ha non poco rasserenato, Serenissimo Principe, l'animo de' più famosi Filosofi di nostra età; i quali, sì come prima non intendeano dipartirsi dalla già imbevuta falsità, e per molti secoli adietro, benché con inefficaci ragioni difesa, così hora si lasciano volentieri persuadere il contrario; poiché il senso, e la ragione s'accorda alla dimostratione del vero. E nella scuola de Mathematici per certissime prove si scuopre l'antico errore, puotendosi far fondamento stabile delle misure de cieli, e della terra."

[93] Arma 1580b: "Il Creator gli diede tal misura./ Che saper non si può da creatura."

[94] Benedetti 1579, 397–405, "Defensio nostra contra Antonium Bergam, et Alexandrum Piccolomineum." See Ventrice 1989, 131–134.

[95] This information stems from Bonino 1824–1825.

to be identified with a composition in verses, referred to as *πωγωνία* (bearded comet) in the anonymous pamphlet. The only remaining evidence of the cometary polemics is a defence of Arma, *La Stravagantographia del Sig. Filosofo stravagante, in difesa de la πωγωνία d'il Dottore Arma* (The Stravagantography of Mr. Stravagant Philosopher in Defence of Dr Arma's Bearded Comet).[96] The apologist was one Monsignor Della Torre linked to the court. His apology offers an insight into the controversy: It concerned astronomical and meteorological issues relating to the nature of the comet, its location below or above the sublunary sphere, the criticism of astrological interpretations of the celestial phenomenon, as well as personal insults. The provocateur who had published against Arma is depicted as follows:

> I cannot stop wondering who this person is. I cannot understand why he sometimes presents himself as a scholar, sometimes as a cook, as a Roman courtier, or as a practicing friar [*frate osservantino*] (as he speaks about the *osservantini*). I cannot believe that he is a practicing [man of religion], as the ecclesiastics speak in a correct manner and not heedlessly like him (who behaved heedlessly). Moderation has always been praised. Therefore, moderate people will always damn this person. I will never believe that he is a scholar. In fact, today's scholars are well-educated and would never indulge in such excesses, especially against such a man [Dr. Arma] from whom they did not receive anything but pleasure, honor, and courtesy. Although he seems to come from the area of Rome, in the end he shows himself to be a dishwasher because even a cook would behave better than him. Whoever the hell he is, if he will not control himself better in the future, I will repay him as he deserves.[97]

Possibly the identity of this mysterious denigrator was the philosopher Giordano Bruno, who was in Turin in 1578 on his way to Chambéry, in Savoy. At that time he wore the Dominican habit and had just published, as mentioned above, a booklet on meteorology in Venice entitled *De' segni de' tempi* (On the Signs of the Times) that presumably dealt with the comet.[98] The reference to the provocateur as a Roman courtier could correspond to an episode of Bruno's life. In Paris, in 1585, he told the librarian Guillaume Cotin that he had been once received at the Roman court by Pius V and the Cardinal Rebiba to whom he demonstrated his technique of the art of memory.[99] Moreover, the Turin episode resembles a *querelle* that burst out in Geneva in 1579, after the publication of a pamphlet in twenty points ridiculing a professor of philosophy. The detractor, Giordano Bruno, was discovered and condemned to exclusion from communion. As one reads in a document (6 August 1579), " Philippe Jordan, dit Brunus, Italien [était] détenu pour avoir faict imprimer

[96] It is preserved in the Biblioteca Reale of Turin, coll. G 25–67.

[97] Della Torre 1578: "[…] non mi posso quietare pensando chi possi esser costui. Non posso capire, perché quando fa d'il scuolaro, quando del cuogo, quando del corteggiano di Roma, quando del frate osservantino, poi che di osservantini parla. Di esser osservante, nol posso pensare, perché li religiosi parlano correttamente, e non si sgovernano nel parlare, come ha fatto costui, il quale mattamente si è sgovernato. Fu sempre lodata la modestia. Sarà donque dalli modesti dannato costui. Che sij scuolaro, non lo crederò mai, perché hoggi dì li scuolari sono ben creati e non farebbono tale scappate specialmente contra di un'huomo tale da cui mai hebbero altro che apiacere, honor e cortesia. Par bene che habbi del Romanesco nel principio, ma il fine dimostra più presto haver del sguattero, perché il cuogo si sarebbe meglio deportato che non fa costui. Sij chi diavol esser si voglia. Se esso per avanti meglio non si governarà, tale e tanto mi ritrovarà, quale e quanto mi ricercarà."

[98] See Omodeo 2008a. On Bruno's lost meteorological-cometary work, see Ernst 1992.

[99] Spampanato 1921, 654–655 and Ricci 2000.

certaines responses et invectives contre Mr. de la Faye, cottans 20 erreurs d' iceluy en une de ses leçons." [100]

Only one year later, between 1580 and 1581, Benedetti was involved in an astronomical-astrological quarrel with a certain Benedetto Altavilla of Vicenza concerning the reliability of ephemerides and astrological prognostication. The controversy was sparked by the publication of Altavilla's *Animadversiones in ephemeridas* (Remarks against Ephemerides, 1580) and was continued with two further publications by the same author.[101] They cast the reliability of ephemerides' calculations and astrological forecasting into doubt, based on the alleged theoretical flaws of mathematical astronomy. Since they appear to have been directed against scientists linked to the court, Benedetti eventually intervened with a printed letter, *Lettera per modo di discorso... intorno ad alcune nuove riprensioni, et emendationi, contra alli calculatori delle effemeridi* (Turin, 1581). He later translated it into Latin as *Defensio ephemerides* and included it in the miscellanea of epistles of the *Diversae speculationes*. We will expand on this polemic later, in the section on Benedetti's astronomy.

### 2.7.4 Posthumous Criticism: Cristini on Benedetti

Benedetti died on January 20, 1590, two years before his own astrological prediction. This untimely death did not leave him the time to complete the astrological work that he announced at the end of the *Diversae speculationes*. What is worse, the fact that his own prediction was wrong awakened doubts and rumours about his scientific talent. The mathematician Cristini was quick to compose a critical essay, entitled "Examination of the mistake and emendation of the nativity of the very excellent mathematician, Mr. Giovanni Battista Benedetti, now deceased, to account for his [wrong] prognostication of his own death two years later than it in fact occurred" (*Essaminatione dell'errore, della rettificatione de tempo della natività del fu S[ignor] Gio[vanni] Battista Benedetti mathematico eccellentissimo, per cagion del quale esso s'era pronosticato morte due anni appresso in circa al tempo che gl'è avvenuta*). Although the manuscript was lost in the fire of the Biblioteca Nazionale of Turin in 1904,[102] a transcription of significant parts by the eighteenth-century biographer of Cristini, Giuseppe Vernazza, is still extant in the Biblioteca Reale of Turin among the documents that Vernazza gathered for the composition of his *Notizie di Bartolommeo Cristini* (Notes on Bartolomeo Cristini, 1783).[103]

Cristini started his examination of Benedetti's errors with specific reference to his mistaken self-prognostication in the *Diversae speculationes*. He also reported that Benedetti himself acknowledged an error in his nativity, amounting to four minutes. After that, Cristini took upon himself the task of recalculating that horoscope on the basis

[100] Spampanato 1921, 132.

[101] On astronomical-astrological quarrels in Renaissance Italy and Turin, see Omodeo 2008a and Tessicini 2013.

[102] Bordiga 1985, 609, n. 6. See Omodeo 2014c.

[103] Vernazza 1783. Two manuscript copies of Vernazza's biography of Cristini are still extant. One is preserved in the Turin State Archive (Archivio di Stato di Torino, coll. Miscellanea J.b.VIII. 9), the other is kept in the Biblioteca Reale of Turin (Vernazza manuscript, misc. 67.5). The latter is a good copy, ready for the printer. It contains an appendix of "documents" for the personal use of the author. These are transcriptions or translations of significant passages of documents by Cristini that were lost or seriously damaged after the fire at the Turin National Library in 1904. They comprise the dedication and table of contents of the *Revolutione trentesimaterza del Ser[enissi]mo Sig[nor] il Signor Carlo Emanuele duca di Savoia* (1596), notes from various astrological diaries, an Italian version of the beginning of *La rithmomachia o sia gioco di Pithagora* and, most importantly, a long extract from the *Essaminatione dell'errore... della natività del fu S[ignor] Gio[vanni] Battista Benedetti mathematico*.

of the figure published by Gaurico and came to the conclusion that the mistake was even bigger. It amounted to eight minutes.

> Benedetti published his prognostication of the moment of his death in the work entitled *Diversarum speculationum mathematicarum et physicarum liber* (published in 1585), in a letter to the most illustrious Wolfhard Eisenstein [Volfardus Aisenstain], which is to be found at the end of this work. After a brief assessment of those things of the judicial art that he regarded as vain or false, and after announcing to Wolfhard that he would expand [on astrology] in that tract with his astrological observations, which he wished he could publish before his death, he added the indication of the time in which, according to him, [his death] was to happen (that is, [the date] before which he wished he could publish the aforementioned tract). These are his words: "antequam ad directionem mei horoscopi cum corpore Martis anaeretae perveniam, quae quidem directio circa annum millesimum quingentesimum nonagesimum secundum evenienti" [as indicated by my horoscope, before I meet the body of the adverse Mars. This is going to happen in 1592].
>
> As we can see, he was certain that he would die when the direction of his ascendant and Mars would meet. He calls [Mars] "anaereta," that is, giver or announcer of his death. He confirmed this when […], just before his death, he felt that the disease was attacking him and declared that he made a mistake of four minutes in the rectification of the time of his birth horoscope [*natività*]. This is as if he would say that, by augmenting by four minutes the time of his birth horoscope, he would have predicted the direction [of his ascendant sign meeting Mars] at about the time when he became sick. Hence, he believed he was dying, and this [his death] in fact occurred at the end of the ongoing year 1590, at 17:00 of 20th January according to [the calendar of] Gregory, which corresponds to the 10th of the old [calendar]. I had to know the time in which he believed he was born in order to assess by how much time he was mistaken in the rectification of his birth horoscope, so that the direction of his horoscope relative to Mars corresponded to the days when he left this world. Therefore, at Benedetti's death, I immediately began to compute the error of the aforementioned time, though only approximately, because I did not know Mars's latitude. And I found that it [the mistake] amounted to eight minutes […]. Later, when the same person who told me that Benedetti had acknowledged a mistake of only four minutes according to his calculations, openly accused me of not being able to do this calculation, as my mistake was two times [that of Benedetti], I began the calculation in the following manner. First, I determined the time attributed to his birth […] Etc.[104]

[104] From Vernazza's papers accompanying his manuscript of his *Notizie di Bartolommeo Cristini*. Biblioteca reale di Torino, Misc. 67.5, *Vita di Bartolomeo Cristini con documenti*, "M.S. L.1.10, 11.493, di pag. 42." See Omodeo 2014b: "Ha pubblicato il Benedetti, il pronostico fattosi del tempo di sua morte nell'opera sua titulata *Diversarum speculationum mathematicarum et physicarum liber* stampata dell'anno '85 in una lettera scritta all'ill.mo Volfardo Aisestain, posta nel fine d'ess'opera, percioché appresso haver brevemente dichiarato quali cose egli stimava vere nella giudiciaria e quali vane o false, et detto com'esso Volfardo potrà veder poi meglio in quel trattato dell'osservationi sue astrologiche, quale sperava dar in luce avanti la sua morte, soggiunge il tempo il quale giudicava essa doverli avvenire, o sia avanti al quale desiava pubblicar detto trattato, con queste istesse parole: "antequam ad directionem mei horoscopi cum corpore Martis anaeretae perveniam, quae quidem directio circa annum millesimum quingentesimum nonagesimum secundum evenienti." Donde appare ch'esso teniva per certo d'haver a morire, quando giongerebbe alla

In his transcription of Cristini's *Essaminatione dell'errore*, however, Vernazza omitted numbers and calculations. These can be found in another astronomical-astrological assessment and criticism of Benedetti entitled "Thirty-first revolution of the very serene Sir, Duke Carlo Emanuele of Savoy, for the year 1592, very diligently and reliably calculated and explained by Bartolomeo Cristini, scholar of mathematical disciplines in the service of His Highness, according to the best opinion of the main judiciary astrologers" (*Revolutione trentesima prima del Serenissimo Signore il Signor Carlo Emanuel Duca di Savoia corrente dell'anno 1592 con ogni diligenze et fedeltà calculata et decchiarata secondo le migliori intelligenze de più principali autori dell'astrologia giundiciaria per Bartolomeo Crestino studioso delle mathematiche discipline in servitio di Sua Altezza*). Although the manuscript was damaged by the fire of the Turin library in 1904, it is still readable. The dedicatory letter shows that, at the time of its composition (June 8, 1592), Cristini was striving to obtain a stable appointment at court:

> But your very generous Highness awoke in my spirit the desire of mathematical virtues and of undertaking the present endeavor. Your request woke up and unveiled in me the desire (which is always alive) to serve [Your Highness]. However, my desire has been impeded by the difficulties of my continuous poverty and adverse times owing to the fact that no treasurer (or any monetary and financial administrator) regards me as an ordinary servant of Your Highness. [I have been acknowledged as a servant] only in exceptional cases, when my capacity, readiness and knowledge in making calculations has proved useful—as has happened several times, when I was required to serve Your Highness. […]
>
> Therefore, I place growing hope only in Your Highness the more [you] require my services, the more efforts I make for You and the fewer are the number of [benefactors] by whom I can hopefully be supported[105]

direttione del suo ascendente al corpo di Marte, quale chiama anaereta cioè datore, o promissore de la morte sua. Il che pare habbi volsuto confirmare quando che, come dice, poco avanti la sua morte ei si sentì carrigar dal male, disse d'essersi fallato di quattro minute nel rettificare il tempo di sua natività, perché questo è come s'havesse detto che quando egli havesse accresciuto tempo di sua natività per quattro minute havrebbe conosciuto la direttione predetta essere minore di quello [che] l'haveva fatta, et periciò il tempo della sua morte caggionata da essa direttione dover essere circa questo tempo, ch'egli s'era infermato, et credeva di morire come è pur avvenuto, essendosi occorso ciò fare dell'anno presente 1590 circa le 17 hore del 20 giorno di genaro secondo Gregorio, che viene ad essere il dieci dell'anno antico. Perciò volendo io essaminare di quanto tempo egli habbi fallato nella rettificatione di essa sua natività, accioché giustamente la direttione predetta dell'horoscopo suo al corpo di Marte venisse a cadere nel giorno istessi ch'egli partì da questo secolo, m'è stato necessario sapere il tempo ch'egli havea presupposto fosse quando nacque […]. Perciò mi posi subito seguita la morte del Benedetti a far conto dell'errore del tempo predetto, così alquanto alla grossa, per non haver nota la sopradetta latitudine di Marte, et ritrovai detto errore essere di minute otto in circa di hora […] Ma perché ho dipoi inteso che chi mi ha riferto il Benedetti haver confessato il detto fallo di min. 4 et haver solamente ritrovato tanto per calculo ha espressamente detto che io errava del doppio et non sapea far questo conto […] mi posi a calculare di questa maniera. Prima ho ritrovato il tempo presupposto della natività […] Etc."

105 Cristini, *Revolutione*, Biblioteca Nazionale Universitaria di Torino, N. VII. 10, f. 4*r*–*v*: "Ma V[ostra] Alt[ezza] benignissima sì come è stata cagione d'eccitar nell'animo mio il desio delle vertù matematiche, et di farmi fare la presente fatica; così ancora co'l chiamarmela adesso ha risvegliato, o riscoperto le sempre vive brame mie di servirla, le quali erano tenute sepolte dai disaggi che queste carestie et mali tempi mi causano maggiori giornalmente, percioché non sono conosciuto per servitore ordinario di V[ostra] Alt[ezza] da Tesoriere alcuno, né da ministro di suoi dinari o finanze; se non ne' casi che la vertù et prontezza, o cognizione mia ne' conti, può reccarli qualche giovamento come ha fatto più volte quando per servitio di

In this case, the allegation against Benedetti is for using the Alfonsine tables to determine the nativity of Carlo Emanuele I, instead of more reliable Copernican tables. Such inaccuracy invalidated his astrological judgments.

> I took into account the places where they [the planets] are to be found in the horoscope made according to the true time calculated on the basis of Copernicus, following the teaching of the major authors on astrology. In general, since scholars are in disagreement concerning the employment of different tables to compute their horoscopes [*revolutioni*] and although I have demonstrated (in the calculations at the beginning of my tract) that only one set [of tables] is true, I calculated the astrological figures of the heavens according to both tables—in fact, false ones were also in use by many and in particular by Benedetti—and I offered double astrological judgments depending on the places assigned according to the different figures. In this manner, your Highness will possibly compare them and see which ones are in better agreement with the truth.[106]

The terms of Cristini's polemics echoed those of the controversy of the years 1580–1581 between Altavilla and Benedetti. In fact, Cristini examined two astrological figures calculated by Benedetti: a nativity based on the "Copernican" tables of the German astronomer Erasmus Reinhold, *Prutenicae tabulae* (1551), and a prognostication based on the Alfonsine tables.[107] He pitted these figures against a "figure that is computed from Giovanni Antonio Magini's tables of the second celestial mobiles" (*figura della natività di novo da me calculata con le tavole de secondi mobili celesti di Antonio Magini*) and a "figure of the revolution that I calculated according to the time of the real motion indicated in Magini's's ephemerides" (*figura della revolutione da me calculata sotto il tempo che si trova per il moto vero insegnato nell'effemeridi del Magini*).[108] Finally, he discussed the differences between his and Benedetti's calculations and concluded with an accusation directed against Benedetti for being careless and opportunist:

> But I believe that he [Benedetti] followed the calculation of Alfonso X rather than the true one only owing to its simplicity. In fact, before [the publication of] the ephemerides of Magini it was very difficult to establish the true time of the revolution. Before him, nobody calculated the Sun up to the seconds in any ephemerides, which is the presupposition for more exact and true computations […]. It is only in consideration of Benedetti's authority that I did not omit to compare his horoscope with the other one.[109]

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V[ostra] Alt[ezza] sono stato da loro richiesto […].

Et per questo sempre cresce maggiore la speranza mia, in solo vostra Altezza quanto ch'essa più m'incita a servirla, et che maggior è fatica che faccio per lei, et minor il numero di quelli in quali posso haver spernaza di soccorso."

[106] Cristini, *Revolutione*, Biblioteca Nazionale Universitaria di Torino, N. VII. 10, f. 8*r*: "[Ho] havuto riguardo ancora ai luoghi ne' quali cadono essi [pianeti] nella figura della revolutione fatta secondo il vero tempo dato dal Copernico, come è insegnato da principalissimi scrittori dell'astrologia. Et nell'universal giudicio perché ho conosciuto tra scrittori essere certa diversità seguendo alcuni un tempo et altri un altro nel fare delle revolutioni delli quali ancor ch'io provi (come per i calculi di ciascuno posti al principio di questa opera) l'uno solo essere il vero, ho fatto le figure del cielo che si mostrano sotto ambi essi tempi (atteso che ancor la falsa era seguita da diversi et particolarmente dal Benedetti), ho radopiato essi giudici per i luoghi che diversi significati fanno havere esse figure. Accioché V[ostra] Alt[ezza] provandole ambidue conosca ancor lei quale meglio secondi la verità."

[107] Cristini, *Revolutione*, Biblioteca Nazionale Universitaria di Torino, N. VII. 10, ff. 11*v*–12*r*.

[108] Cristini, *Revolutione*, Biblioteca Nazionale Universitaria di Torino, N. VII. 10, ff. 12*v*–13*r*.

[109] Cristini, *Revolutione*, Biblioteca Nazionale Universitaria di Torino, N. VII. 10, f. 16*v*–17*r*: "Ma io tengo ch'egli seguisse più tosto il calculo d'Alfonso che il vero; solo per causa della facilità d'esso percioché avanti

In this second criticism, Cristini continued to discredit Benedetti. This time he cast his capacity as both an astrologer and a mathematical astronomer into doubt. Cristini suggested, in fact, that Benedetti misused his prestige to disguise the lack of accuracy in his astrological computations. Mistakes affected not only the horoscope he carried out for himself but also those cast for his patrons.

Once he had established himself as an expert in the field, Cristini continued to prepare prognostications for the ruling family from 1592 to 1595, as testified to by the titles of several manuscripts, such as "*diari*" and "*revolutioni*," which are for the most part lost.[110] He obtained the position at court that he desired in 1594. Carlo Emanuele I designated him "as our and our princely children's mathematician, follower of Giovanni Battista Benedetti, who has recently passed away" (*per mathematico nostro et dei principi nostri figliuoli in luogo del fu Gio[vanni] Battista Benedetti ultimamente defonto*). He moreover accorded to the new court mathematician a "reasonable stipend" (*un ragionevole stipendio*) of three hundred scudi per year, "so that, according to our wish, he will cover the efforts of his studies, and will be in condition to serve us with more ease and comfort in all the duties we will entrust him" (*acciò che possi comportar alle fatiche delli studi, et trattenersi al servitio nostro più agevolmente et commodamente come desideriamo in tutti li carighi che ha da noi*).[111]

## 2.8 Strengths and Limitations of the Institutional Framework of Benedetti's Science

Benedetti's life, career, and work, as well as his legacy, fortunes, and misfortunes should be understood against the background of the Renaissance world he was part of, in particular the Italian and Turin environments. His case is paradigmatic of both the strength and the limitations of Renaissance science. On the one hand, the cultural and economic flourishing of centers such as Turin, new and challenging engineering and architectural projects, and the establishment of a court and of a modern state apparatus with its need for technical advice and cultural grandeur created an exceptional environment, favorable also to the pursuit of science and philosophical speculations. The constraints of Counter-Reformation culture did not affect the speculative freedom of Benedetti. This is especially due to the pragmatic cultural and religious politics of the Savoy dukes, who were trying to establish a balance between their state and international diplomacy and confessional tensions. On the other hand, however, the fragility of Renaissance knowledge institutions also comes into view. Universities were teaching institutions instead of research centers. Professors were concerned with the transmission of knowledge rather than with the implementation of new knowledge and theories. The intended mission of early-modern universities was preservation, namely the transmission of traditional knowledge to future generations, not producing change. Thus, epistemic processes in institutional settings were often imperceptible and transformations of knowledge often occurred against the explicit intentions of the historical actors. Lectures and commentaries on authoritative sources—the teaching of which was sanctioned by academic statutes and curricula—were not expected to alter the knowledge preserved in the classics and in the textbooks. The Savoy dukes tried to

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l'effemeridi del Magini molto difficil cosa era trovar il tempo vero della revolutione percioché nissuno avanti lui havea nell'effemeridi calculato il Sole sino alle seconde onde ne seguono i calculi più sottili e veri […]. Con tutto ciò solo per l'autorità d'esso Benedetti non ho volsuto lasciar del tutto la consideration delle figura sua con l'altra come vedevasi."

[110] See Peyron 1904, 617–618.

[111] Vernazza 1783, 20–21.

attract prestigious professors to Turin and also supported, at least initially, the teaching of humanities according to the new standards set by humanistic philology. However, the place for free inquiry and innovation was outside universities. Benedetti's works, marked by original and unorthodox conceptions in physics, mathematics, and other disciplines, emerged from a courtly environment. Yet, this institutional frame proved ephemeral as it was dependent on patronage. It also had a strongly personal character, as it depended on informal exchanges within a system of unsystematic patronage. In Turin, modern scientific academies, with a stable body of investigators and statutes, had not yet made their appearance. Not even literary and artistic academies met with a particularly favorable environment.

Given this context, Benedetti's scientific activity, accomplished outside university and institutionalized settings, cannot but appear as occasional. In fact, it was linked to the contingency of courtly life, for instance to the requests for advice by the Savoy rulers or other patrons. This is the case with all of Benedetti's letters and with other publications, such as his writing on the calendar reform. He appears to have given expert advice on issues of cultural policy, such as university appointments, as well as on technical issues, and not least on matters of astrology. His construction of a fountain and of sundials, as well as his writings on technologies and gnomonics are directly connected to his role as a mathematical expert at the court. In the same function, he also entered debates and polemics animating Turin. Some of his interventions were friendly, for instance his exchanges on meteorology with other courtiers in the presence of members of the ruling family. His controversy with Professor Berga over the quantity of water and earth in our globe was more vehement but never harsh. Astrological polemics were the most virulent ones, as evidenced by Benedetti's publications against the critic of astrology, Altavilla. A constant feature of Benedetti's scientific work remains its occasional character. This is also reflected in the lack of systematic order in his *magnum opus*, the *Diversae speculationes*. Benedetti probably saw himself primarily as a courtier, participating in the cultural life of Turin as an exponent of the Savoy elite, and not as a scientist pursuing the immaterial glories of scholarly achievements. As a matter of fact, he did not primarily take upon himself the burden of a scientific effort going beyond the deliverance of brilliant booklets, short judgments, and advice on specialistic issues.

One astonishing aspect of Benedetti's intellectual activities is the lack of an enduring and explicit legacy. On the one hand, his conceptions clearly influenced contemporaries and followers in Italy and abroad. Among others, his impact is reflected in the positive opinions of Brahe and Kepler, in Galileo's reception of several insights of his mechanics and physics, and in the European circulation of his ideas on physics through Taisner's plagiarism. On the other hand, the lack of an enduring acknowledgment of his work is equally evident and seems to be linked to the fact that he was not able, and perhaps not even willing, to establish a school like the one set up by Commandino in Urbino, or by those later set up by Galileo in Padua and Tuscany. Not even in Turin did he benefit from lasting recognition. As we have seen above, his immediate successor as court mathematician, Cristini, even saw the denigration of his astrological and astronomical skills as an opportunity to obtain a visible position in town and start a courtly career.

In many ways, Benedetti is the mirror of his world, in particular of the courtly society he belonged to. His work can be seen as the embodiment of this context. His case is different from that of many other Renaissance scholars, who strongly identified themselves with their scientific work and output. For scholars like Galileo, for instance, the publication of their works had a functional aim in accessing the courtly milieus. In the case of Benedetti, he was already part of the patrician and aristocratic milieu for many reasons. His work is the product of courtly life rather than his entry ticket to it. The author disappears (or almost vanishes) and leaves in his place a sort of collective author, which is not the scientific Republic of Letters, but rather the court itself, its institutions, its elites, its participants, and its networks. In this respect, Benedetti differs from the great protagonists of Italian Renaissance science. He is very far from the self-celebration of intellectuals like Cardano and Galileo. His work is no monument to himself but rather to his environment, ranging beyond the local boundaries of Piedmont and the Savoy. The Urbino school was also populated by scholars less concerned with their own ego than with science. However, in contrast with this school Benedetti conceived of himself as an innovator, rather than as a restorer of antiquity and classicism.

# Chapter 3
# Structure of the Book and Main Issues

## 3.1 Benedetti Introduces His Physico-Mathematical Speculations

The *Diversarum speculationum... liber* is composed of six books, indicated as follows in the table of contents:

1. "Arithmetic Theorems" (*Theoremata arithmetica*);
2. "Explanation of the Operations of Perspective" (*De rationibus operationum perspectivae*);
3. "Mechanics" (*De mechanicis*);
4. "Disputations on Some Opinions Held by Aristotle" (*Disputationes de quibusdam placitis Arist[otelis]*;
5. "On Euclid's Fifth Book [of the *Elements*]" (*In quintum Euclidis librum*);
6. "Epistolary Answers on Physics and Mathematics" (*Physica et mathematica responsa per epistolas*).

It should be noted that this partition of themes does not mirror the relative relevance of the issues according to the author. In fact, the length and the importance of the books does not exactly correspond to the subdivision indicated in the table of contents. Two sections are quite short: Book 2 on perspective and Book 5 on Euclid. By contrast, the first one, on arithmetic, and the last one are much longer. Chapter 6 is an extremely diverse collection of letters on the most different subjects.

The dedicatory epistle of the *Diversae speculationes* begins with an acknowledgment of the generosity of Emanule Filiberto of Savoy, with whom Benedetti had particularly good relations. As one reads, they often talked about mathematical issues pertaining to arithmetic, geometry, optics, music, and astrology.[1] Since the *Diversae speculationes* appeared after Emanuele Filiberto's death, it was dedicated to his successor, Carlo Emanuele I. Benedetti reports that both dukes encouraged his inquiries and their questions motivated his investigation of specific questions. This is the reason why Benedetti's *Speculationes* have an occasional character and are not ordered in a systematic manner. The *Theoremata arithmetica* (book one) clearly originated from conversations with Carlo Emanuele I as the theorems are presented as answers to the patron's questions. For instance, the first one is introduced as follows: "The very serene duke of Savoy asked me to prove by means of science and speculation (as one says) that the product of two fractions is inferior to either factors."[2] The curiosity the dukes held for mathematical matters was not idle but rather rooted in a deep comprehension of the importance of practical mathematics in military and civil affairs. However, courtly etiquette required that serious matters be

[1] Benedetti 1585, f. A2*r*. Note that the pagination A1–A4 is doubled: the first installment A1–A4 contains frontispiece, table of contents, dedicatory epistle, and preface to the reader, whilst the second installment includes the first eight pages of the *Theoremata arithmetica*. Since the latter has also a pagination number (1–8) we will quote from the first fascicle indicating the folio and from the second giving the page.

[2] Benedetti 1585, 1: "Interrogavit me Serenissimus Dux Sabaudiae, qua ratione cognosci posset scientifice et speculative (ut dicitur) productum ex duobus fractis numeris, quolibet producentium minus esse."

concealed under the mask of aristocratic detachment and disinterestedness. The court was a refined cultural center that also functioned as a political headquarters, where the most important decisions had to be taken. "Reality and imagination—as has been remarked—prescribed that at court the weight of the duty of government be mirrored by the lightness of amusement."[3]

> Following his example [that of the Duke] [...] many asked my advice either in person or by mail on those mathematical problems. As I never avoid work in support of friends, it happened that, after so many years, looking at my paper boxes [*scrinia*] I found so many solved problems that they could be gathered in a fairly big volume.[4]

Benedetti expressed his admiration for the stimulating intellectual environment made possible by the magnificence of Carlo Emanuele I and the legacy of his father. The wide range of scientific interests shared by both dukes is emphasized in the concluding remark of the dedicatory letter:

> Therefore, the glory [of your Highness] will equal that of the ancient Persian kings, and we can expect great happiness in this century if Plato's prophecy is correct: the future State in which princes philosophize will be blissfully happy.[5]

Benedetti's preface to the reader,[6] following the dedicatory epistle, provides some more information related to his mathematical-physical work. The author repeats that the *Diversae speculationes* are a miscellanea of thoughts on various subjects brought about by his own curiosity or by that of patrons and friends. He declares himself confident that, in spite of their disordered format, his speculations will be considered useful, and makes a further claim for the absolute originality of his ideas: "*non dubitans quin illis* [*meis scriptis*] *in illis scientiis aliquid commodi atque utilitatis allatura sint, praesertim cum in eiusmodi quaestionibus investigandis atque perpendendis, nemo* (*quod sciam*) *hactenus elaboraverit*" (as I have no doubt that these [writings of mine] will bring something pleasant and useful, especially because (to my knowledge) nobody has so far sufficiently investigated and pondered such issues).[7] The enthusiasm for novelty that was to motivate the supporters of modernity in the *querelle des anciens et des moderns* can be sensed in these words. Moreover, Benedetti does not exclude the possibility that different people, at different times and in different places, could have made the same discoveries by treating similar problems:

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[3] Barberis 2017, xvii.

[4] Benedetti 1585, f. A2*r*: "Illiusque imitatione [...] non pauci aut praesentes, aut per litteras me de his, atque illis mathematicis quaestionibus consuluerunt. Cumque ego nunquam laborem amicorum causa defugerim, evenit ut post tot annorum curricula, mea scrinia scrutatus, invenerim tot absolutas quaestiones, ut ex eis corpus mediocre effici posse videretur."

[5] Benedetti 1585, f. A2*v*: "Quare, et veterum Persarum Regum gloriam [tua celsitudo] aequavit, et nos veluti in spem certam faelicitatis huius saeculi induxit, si verum est Platonis vaticinium, beatam eam futuram Rempublicam in qua Principes philosophentur."

The comparison between the Duke of Savoy and the ancient Persian kings is in line with an established Renaissance topos derived from classical sources, in particular Xenophon's *Life of Cyrus*, as discussed by Vester 2007, 228-229.

[6] Benedetti 1585, "Ad lectorem," ff. A3*r*–A4*r*.

[7] Benedetti 1585, f. A3*r*.

> In fact, I included [*traditum est*] nothing in these books that, as far as I remember, I read or heard from others. If I picked up on the suggestions of others, either I offered a somehow different demonstration or I wrote on the same subject more clearly. In the case that somebody else wrote the same ideas [*eadem tradidit*], either I was not informed about this person's speculations or the memory of these readings has vanished. Aristotle himself remarked that it can easily happen that many come to the same ideas. It can even happen that, writing so much, I repeat a certain issue having forgotten that I already wrote on it. Indeed, this has happened to me a few times.[8]

Benedetti adds that very few people ever wrote books which are entirely and solely the fruit of their own mind, except perhaps for Archimedes. Following in the footsteps of this illustrious predecessor, he presents the results of his personal investigations as independent from any authority. He only acknowledges some influence from Tartaglia and a few other authors on whose works he relies: "Hieronymus Cardanus, Michael Stifelius, Gemma Frisus, Ioannes Novimagius, Cuthebertus Tonstallus, caeterique huiusmodi." The omission of Del Monte as a source on mechanics is striking and telling about the enmity between the two men, which is well documented and will be discussed later.

Benedetti envisages possible criticism of his work and seeks to anticipate objections with a remark inspired by a humanistic sense of relativism: "Quot capita, tot sententiae (As many heads, so many opinions)."[9] Since a book will never receive universal approbation, he writes, it must suffice to provide fruitful insights which will encourage others to undertake further investigations. He mentions only three ancient models: Ptolemy, Euclid, and Pythagoras. Whereas the reference to the first two men can be seen as an appreciation of the most valuable ancient sources on geometry, mathematical astronomy (and astrology), and geography, the mention of Pythagoras explicitly refers to astronomy and should be understood as a reference to Copernicus's "restoration" of heliocentrism.[10]

The letter to the reader ends up with a reflection on and a rebuttal of the principle of authority. Benedetti claims that mathematical and natural investigation should only be inspired by the love for truth. All considerations which do not derive from an open-minded philosophical attitude ought to be dismissed as unfounded:

> In order to establish the truth, I occasionally had to oppose the opinions of others in many places, but I do not want you to ascribe it to some vice of mine nor to call me a malevolent and a sycophant as I display the errors of others. They should rather be thankful to me, since I aim to erase wrong opinions while I am dealing with the same issues—according to Antisthenes, it is indeed necessary "to begin by unlearning errors." I show the truth, which all philosophers, beginning with Aristotle, should hold in higher esteem than any human authority or favor. As you will encounter something of this sort in my volume, I beg you to abandon all passions in your judgement, keeping in mind Sallust's admonishment: "Those who assess controversial issues should avoid all hate, friendship, rage, and compassion." Thus, always favor truth,

[8] Benedetti 1585, f. A3*r*: "Nihil enim his libris a me traditum est, quod aut legisse, aut ab aliis audivisse meminerim, nam si aliena attigi, ea, aut cum aliqua differentia demonstrationis, aut dilucidius scripsi, quod si forte alius eadem tradidit, aut eius lucubrationes ad me non pervenerunt, aut earum perlectionis memoria excidit. Ut etiam Aristoteles ipse sensit facile fieri potest, ut pluribus eaedem opiniones in mentem veniant. Immo multa scribendi evenire potest, ut cum iamdiu aliquid scripserit, iam oblitus, idem repetat, quod mihi etiam nonnunquam accidit."

[9] Benedetti 1585, f. A3*v*.

[10] Omodeo 2014a, "The Invention of the Pythagorean Cosmology," 167ff.

> which is worthy of the greatest efforts, instead of some person, as too many do. Hopefully you will benefit from my work and, in case you will pick some fruit, in the first place you should be thankful to Him from Whom all sciences descend.[11]

## 3.2 Mathematical Sections

Benedetti proved his capacity as a mathematician in his early writings, especially in the *Resolutio omnium Euclidis problematum* (1553). The relevance of this publication for the ongoing debates of the time, involving Ferrari, Cardano, and Tartaglia has often been stressed. For instance, Moritz Cantor, in his classic history of mathematics, *Geschichte der Mathematik*, regarded Benedetti as "*ein wirklicher Geometer*" on account of his treatment of geometrical problems using a compass with fixed opening.[12] Special mathematical problems scattered in the *Diversae speculationes* have been summarized by Bordiga. We will deal with some of these problems in other sections, in connection with other aspects of Benedetti's work such as mechanics, natural philosophy, and astronomy. For the time being, we will limit our treatment to the two books of the *Diversae speculationes* specifically dealing with mathematics.

### 3.2.1 Geometrical Demonstrations for the Solutions of Arithmetic Problems

Two books of the *Diversae speculationes* are devoted to mathematics in the strict sense, Book 1 to arithmetics and Book 5 to proportions. Additionally, several letters in Book 6 are dedicated to mathematical problems and some metaphysical and physical issues are also treated as geometrical problems. In Book 4 Benedetti discusses examples such as the perfection of the circle, i. e., whether the circle, owing to its properties, has to be seen as the "first" or the "last" of the geometrical figures, and the possibility of a never-ending motion on a finite line or the possibility of an infinite motion on a finite line.

Book 1 of the *Diversae speculationes* has the form of a collection of mathematical exercises. Although Benedetti did not give them a systematic order, he claimed that his readers might find many useful explanations and remarks.[13] He chose to call his propositions "theorems" instead of "problems" to stress their originality, as Benedetti believed they deserved higher recognition than mere problem-solving. He also apologized for the brevity of his treatment by saying that he had confidence in the intelligence and expertise of his readers (the same remark can be generalized for many other sections of the work). In Book 1 he approached arithmetic problems geometrically following Euclid's example in Book 2 of the *Elements*. This geometrical approach finds a legitimization in epistemological considerations about the role of geometrical visualization for the comprehension

[11] Benedetti 1585, ff. A3*v*–A4*r*: "Quoniam vero multis in locis accidit, ut veritatis iudicandae causa necesse mihi fuerit quorundam sententiis adversari nolim te hoc mihi vitio tribuere, meque hoc nomine carptorem maledicumque habere quod alienos errores aperiam, cum potius habenda sit mihi gratia, quod in iis interdum laborans (quae Antisthenes in disciplinis magis necessaria esse dixit, *ut mala scilicet prius dediscantur*) falsas opiniones evellere studeam, veritatemque ostendere, quam omnis philosophus, Aristotelis exemplo, pluris quam cuiusvis hominis authoritatem, aut gratiam facere debet. Cumque in hoc volumine aliquid eiusmodi legeris te oratum volo, ut in iudicando, affectum omnes exuas, Sallustianum illud prae oculis habens. *Omnes qui de rebus dubiis consultant, ab odio, amicitia, ira, atque misericordia vacuos esse decet.* Hinc fiet, ut non personae (ut multi solent) sed veritati, quae summo studio dignissima est, semper potius faveas. Vale nostrisque laboribus utere, si quem inde fructum, sicuti spero tuleris, illi precipue habes gratiam a quo omnes fluunt scientiae."

[12] Cantor 1892, 521–525.

[13] Benedetti 1585, f. A3*v*.

of truth: "images are necessary to the intelligent one in order to speculate" (*quoniam oportet intelligentem phantasmata speculari*). In Benedetti's eyes, the process of learning and thinking requires images (*phantasmata*).

Moreover, Benedetti informs his reader about the process through which knowledge is acquired. In other words, his treatment is not restricted to theory but intentionally expands on heurstics. He complains that ancient mathematical knowledge was often transmitted in a very concise manner. It often lacked demonstration and definitions, or a clarification of the fundamental concepts. For this reason modern readers are often forced to investigate the hidden reasons of "the numbers" and "their effects," with huge effort:

> As we know, ancient mathematician-philosophers discovered many properties of the numbers but transmitted them to posterity either without reasons or with too few ones. Hence, several mathematical problems emerged, which have been addressed by the duke of Savoy. I consider some of the ensuing reflections on the ancients' propositions to be worth transmitting to posterity. In this manner, my speculations will not fall in oblivion. Rather, I will offer to many an occasion to investigate abstruse themes which are entailed in problems and theorems and could hardly find an explanation so far.[14]

In order to examine Benedetti's geometrical approach to arithmetic problems, let us consider theorem 120. Here is the problem:

> The ancients already addressed this problem: three associates have an amount of money. The sum of the [money of] the first and the second is known, as well as the sum of the first and the third and the sum of the second and the third. From such three aggregates [the ancients] derived the particular [amount of money] of each one of them.[15]

Following a method of resolution, which Benedetti ascribes to Gemma Frisus, he offers the solution to a case chosen arbitrarily:

> Gemma Frisus solves this problem applying the *regula falsi* [rule of the false]. I will follow the same path. Suppose that the addition of the first with the second is 50, that of the second with the third 70 and that of the first with the third 60. From those sums take any two, for instance 50 and 70, whose addition is 120. Subtract from this sum the other one, that is, 60. The result is 60. Its half is 30. This is the amount of money of the second associate. It you subtract this number from 70 (which is the addition of the second with the third) you will get 40. This is the amount of the third associate. Finally, from this number taken away from 60 you will be able to derive the amount of the first associate.[16]

[14]Benedetti 1585, 1: "Praeclare multa veteres mathematici philosophi de numeris eorum effectibus excogitata posteris tradiderunt, quorum cum vix ullam rationem reddiderint, aut certe per exiguam, occasione diversorum problematum mihi Serenissimo Sabaudiae Duce propositorum praebita, de iis quae ab antiquis proposita fuerunt contemplanda nonnulla occurrerunt, quae posteritate commendare non inutile arbitratus sum, ne heae meae cogitationes interciderent, et occasionem praeberem quamplurimis abstrusa haec indagandi, quae problematibus et theorematibus involuta, vix aliquem qui evolveret nacta sunt."

[15]Benedetti 1585, 81: "Supponunt etiam antiqui tres socios nummos habere, quorum summa primi et secundi cognita sit, item summa primi et tertii cognita et summa secundi et tertii item cognita, atque ex huiusmodi tribus aggregatis veniunt in cognitionem particularem uniuscuiusque illorum."

[16]Benedetti 1585, 81: "Gema Frisus solvit hoc problema ex regula falsi. At ego tali ordine progredior. Sit verbi gratia, summa primi cum secundo 50 et secundi cum tertio 70 et primi cum tertio 60; harum trium

If we express the problem in modern form, this is the set of equations Benedetti is dealing with:

$$x + y = 50$$
$$y + z = 70$$
$$x + z = 60$$

The algebraic solution devised by Benedetti is the following:

$$2y = 50 + 70 - (x + z)$$
$$y = \frac{120 - 60}{2} = 30$$

Thus,

$$z = 70 - 30 = 40$$

and

$$x = 60 - 40 = 20$$

After this solution, Benedetti offers a geometrical demonstration of the validity of this procedure, referred to as "Gemma Frisus's *regula falsi*." For this purpose he draws a triangle with an inscribed circle (Figure 3.1) and supposes that the three sides correspond to the sums that we have expressed as a set of equations.

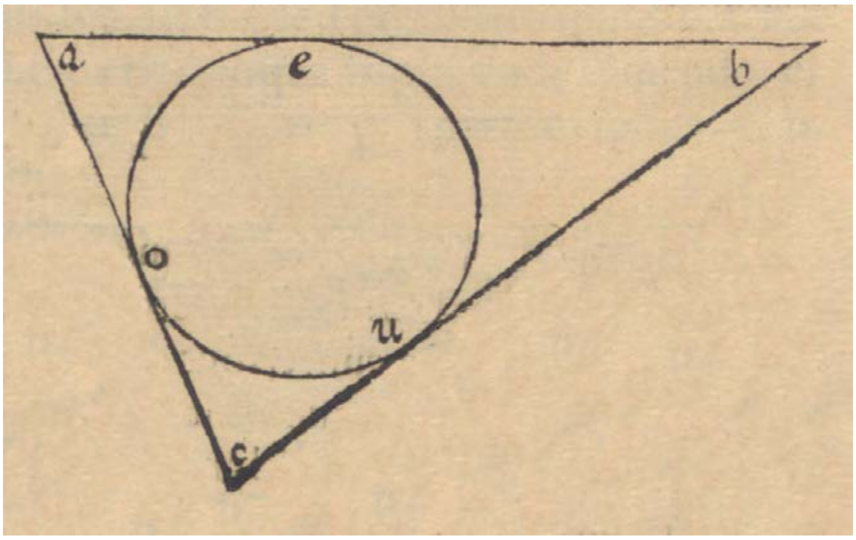


Figure 3.1: Geometrical demonstration for the solution of a particular set of equations. Benedetti, *Diversae speculationes*, Book 1, Theorem 120, p. 82. (Max Planck Institute for the History of Science, Library)

summarum accipiantur duae quaevis, ut puta 50 et 70, quae coniunctae simul dabunt 120 a qua summa detrahatur reliqua, idest 60. Et restabit nobis 60, cuius medietas ergo 30, hoc est numerus nummorum secundi socii; quo numero detracto a 70 (hoc est a summa secundi cum tertio) remanebit 40, hoc est numerus tertii socii; et adhic numerus desumptus a 60 residuus ergo numerus primi socii."

Benedetti's demonstration here follows:

> In order to grasp this problem, let us consider the triangle here indicated as *ABC* each side of which corresponds to the addition of [the money of] two associates—for instance, assume that the side *AB* corresponds to the addition of the first with the second whereas the side *BC* to the addition of the second with the third and the side *AC* to the addition of the first with the third. Thereby *AE*, or *AO*, should be the number [i.e., the amount of money] of the first associate; *EB*, or *BU*, should be the second associate; and *CU*, or *CO*, should be the third. Furthermore, since *AE* is equal to *AO*, *BE* to *BU* and *CU* to *CO*, according to premises, if we derive the sum or side *AC* resulting from the addition of the side *AB* with *BC* corresponding to the other sums, we will know the addition of *BE* with *BU*. Hence, we will also know the half, which is *BE* or *BU*. Once we subtract it from the sum *BA* the resultant will give us the number [corresponding to] *AE*. By subtracting the number of *AE*, that is *AO*, from the sum (or side) *AC*, and [by subtracting] *BU* from *BC* we will know as a result *OV* and *CU*.[17]

We can (anachronistically) formalize Benedetti's reasoning in the following manner.

$$\begin{aligned} ab &= x + y \\ bc &= y + z \\ ac &= x + z \\ ae = ao &= x \\ eb = bu &= y \\ cu = co &= z \end{aligned}$$

The geometrical visualization of Frisus's method, the so-called *regula falsi*, corresponds to the addition of two sides of the triangle and the subtraction of the third one:

$$ab + bc - ac$$

This corresponds to the following addition:

$$\begin{aligned} ab + bc - ac &= \\ = (ae + eb) + (bu + cu) - (co + ao) &= \\ = ae + eb + eb + cu - cu - ae = 2eb &= 2y \\ c.v.d.& \end{aligned}$$

[17]Benedetti 1585, 82: "Pro cuius ratione consideremus triangulum adhic subnotatum *abc* cuius unumquodque latus significet summam duorum sociorum, ut puta latus *ab* significet summam primi cum secundo, latus vero *bc* summam secundi cum tertio, latus autem *ac* summam primi cum tertio, et *ae* seu *ao* sit numerus primi socii, et *eb* vel *bu* sit secundi socii, et *cu* seu *co* sit tertii, cum autem *ae* aequalis sit *ao* et *be* aequalis *bu* et *cu* aequalis *co* ex supposito si de<su>mpta fuerit summa seu latus *ac* datum ex aggregato laterum *ab* cum *bc* reliquarum summarum, relinquet nobis cognitum aggregatum ex *be* cum *bu*. Quare et eius medietas *be* sive *bu* nobis cognita erit, qua detracta ex summa *ba* relinquetur nobis cognitus numerus *ae*, detracto vero numero *ae* hoc est *ao* ex *ac* summa, seu latus, aut *bu* ex *bc* remanebit *oc* seu *cu* cognitus."

### 3.2.2 Theory of Proportions

Book 5 of the *Diversae speculationes* deals with the fifth book of Euclid's *Elements*, on proportions. This book of the *Elements* was a notorious case in medieval translations, as the definitions were often misunderstood, inconsistent interpolations had been added, and flaws introduced in the demonstrations. In his Italian version, Tartaglia tried to emend these flaws. Commandino's and Clavius's attempts followed suit. Moreover, a certain number of Euclidian definitions (especially the fifth and the seventh) could not be simply taken for granted, but required analysis and justification.[18]

Hence, Benedetti proposed to introduce new postulates which he considered to be clearer (*intellectui commodiora*) in order to render the theorems that rely upon them more easily understandable (*quandoquidem iis nostri postulatis admissis, sequentia Theoremata perfacillima reddentur*). Actually, what he undertook was a revision of Euclid's theory of proportions, based on a reorganization of postulates and theorems. As Enrico Giusti stressed, despite its brevity, this booklet by Benedetti stands out as "the first modern attempt to accomplish an organic reform of the fifth book of Euclid's elements."[19] The difficulty that he and his contemporary readers of Euclid had to face was not technical but rather foundational, as it resided in the clarity of the definitions and the internal coherence of the theory. For the sake of intelligibility, Benedetti thus inserted two preliminary axioms and reversed the relation between the most simple of the Euclidean theorems and the less intuitive of the axioms in order to establish a gradual progression from the most simple to the more complex propositions. The first two postulates are derived from Euclid's common notions II and III concerning the addition and subtraction of equal parts to equal quantities.[20]

Benedetti rephrases Euclid's common notions translating the addition (or subtraction) of parts as the composition of ratios:

> [First:] All additions [*composita*] of an equal number of equal parts are equal. Similarly, all proportions are equal that result from the addition of an equal number of other proportions, which are equal among themselves. Euclid tries to demonstrate this in [*Elements*] V 22 and 23.
>
> Second: If one subtracts equal parts from equal wholes, the remaining parts are equal among them. Conversely, if one adds equal parts to equal wholes, the additions will be equal among them. One should consider these considerations as referred to proportions.[21]

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[18] In Commandino's Italian translation (Euclid 1575, ff. 63*r*–*v*), these definitions go as follows:

"V. Le grandezze si dicono essere nella medesima proporzione, la prima alla seconda, et la terza alla quarta, quando le ugualmente molteplici della prima, et della terza, o vero insieme avanzano le ugualmente molteplici della seconda, et della quarta secondo qual si voglia moltiplicatione, o vero insieme le pareggiano e vero insieme sono avanzate da loro."

"VII. Quando delle ugualmente moltiplici, la moltiplice della prima vanterà la moltiplice della seconda, et la moltiplice della terza non avanzerà la moltiplice della quarta, allhora la prima alla seconda si dirà haver maggior proportione che la terza alla quarta."

[19] Giusti 1993, 22.

[20] In Commandino's Italian translation, these definitions go as follows. See Euclid 1575, f. 7*r*:

"II. Se alle cose uguali, si aggiungono cose uguali, tutte sono uguali fra loro.

III. Se dalle cose uguali si traggono cose uguali, etiamdio le rimanenti sono uguali tra loro."

[21] Benedetti 1585, 198: "[Primum.] Quod tota composita ex aequali numero partium aequalium, sunt invicem aequalia. Ut si quis diceret omnes proportiones quae compositae sunt ex aequali numero aliarum proportionum invicem aequalium, sunt etiam invicem aequales, quod Euclides conatur demonstrare in 22. et 23. quinti libri.

The postulates that follow (3–9) are Euclid's propositions 7–13 with minimal changes in the order (the inversion of Euclid's propositions 8 and 9). Benedetti adds three additional postulates (10–12) before he tackles an explanation of Euclidian theorems on proportions. What is lacking is an explicit treatment of the definitions underlying Benedetti's "reform" of the theory of proportions.[22]

Bordiga underscored the didactic character of Benedetti's treatment of Book 5 of the *Elements* by remarking that his concern about clarity and simplicity might have emerged from the teaching of mathematics to his princely pupils in Turin.[23] This might be true; however, the theoretical relevance of this section and of Benedetti's effort to reform the theory of proportions should not be neglected. During the sixteenth century, geometry, in particular the theory of proportions, was gradually becoming the fundamental tool for the treatment of physics in the process toward a mathematical inquiry and explanation of natural phenomena. In this context, Book 5 of the *Elements* acquired a higher theoretical status by offering a sort of "meta-geometry" or *mathesis universalis*.[24]

The universal meaning of geometry as the foundation of rationality itself had been emphasized in the generation before Benedetti in an emphatic *Encomium geometriae* (*Eulogy of Geometry*) delivered by Girolamo Cardano at the *Academia Platina* of Milan in 1535. In this talk he presented geometry as the highest science, or as a sort of *prisca scientia*, by contending that geometrical rationality, based on quantity and proportion, is the source for all arts and disciplines. Indeed, the *modus geometricus* is the essence of rationality and even the *a priori* of God's Creation. In Cardano's eyes, geometry was also a practical discipline insofar as it included statics, mechanics, and architecture as subordinate disciplines. Actually, Cardano's list of arts and sciences that depend on geometrical rationality (committed to the study of quantities and proportions) is long. Geometry, as one reads in his *Encomium*, is fundamental for arithmetic, music, astronomy, and optics. It is the *conditio sine qua non* of architecture (*non aedificare sine illa licet*) and of the plastic arts, painting and sculpture. Geometry is necessary for the construction and understanding of clocks and machines (*horologiorum, machinarumque structura*). It is further presupposed by natural magic, by the science of weights, by aesthetics (*pulchritudo... tota geometrica ratione constet*), and by countless other fields of human activity and knowledge.[25]

Benedetti agreed on the fundamental relevance of geometry as the cornerstone of natural inquiry. The fact that his treatment of proportions, although it was very short and condensed, was printed as a book in its own right in the *Diversae speculationes* bears witness to the relevance he attached to this part of mathematics. In fact, it was crucial for his treatment of weights and thus a close examination of Book 5 of the *Elements* was an indispensable premise of his mechanics. As Giusti emphasized, Benedetti's treatment of composite propositions (relating to Euclid's proposition 17) was the most significant

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Secundum. Quod si a totis aequalibus detractae fuerint aequales partes, quae remanent eruntque partes invicem aequales. Et e converso si aequalibus aequalia addas composita erunt invicem aequalia. Quod in ipsis proportionibus hoc loco semper intelligendum est."

[22] Cf. Giusti 1993, 27 quoting from Antonio Nardi: "Il Benedetti, Geometra insigne non si accorse, che volendo riformare il 5° libro di Euclide, trascurò la definitione della uguale, e disuguale ragion, quale principio e fondamento dell'opera. Stupiscomi certo di tale inavertenza."

[23] Bordiga 1985, 629.

[24] Giusti 1993, 22.

[25] Cardano 1966, vol. 4, 440–445.

aspect of his theory insofar as it offered an elegant computational tool.[26] As a matter of fact, Galileo and his school would follow the same track by applying composite propositions to the analysis of functional relations in physics, in particular to motion. Hence, from the viewpoint of natural inquiry, Benedetti's Book 5 is not exclusively motivated by an abstract interest in pure mathematics but by the challenging problems of contemporary physics.[27]

## 3.3 The Geometrical Theory Underlying Linear Perspective

Book 2 of the *Diversae speculationes*, entitled *De rationibus operationum perspectivae* (Reasons of Perspective Operations), is a short treatise on linear perspective, that is, the optical discipline dealing with the construction of perspective to give the illusion of depth. Its main task was to recreate the "cone of vision." Historically, it originated from the practical problems of three-dimensional representation in the fine arts but also had relevance for architecture (e.g., in surveying or in theoretical treatises), the military art (e.g., the derivation of the structure of the enemy's fortification from a scout's sketches), and found special application in theatrical stage scenery.

Benedetti's book on perspective begins with a claim of originality: "To my knowledge nobody has so far taught the true and inner causes of the operations in perspective in an accomplished manner. Thus, I deemed it to be worth undertaking some reflection [*disputationem*] in this field."[28] The fact that "nobody" (*nullus*) has adequately treated perspective before is attested to by the great number of misunderstandings and widespread errors in this discipline: "In fact, many of those who prescribe the rules of such operations ignore the implications of the true causes, therefore they make various mistakes, as for instance in the following plane figure A [etc.]."[29] Benedetti's treatment was indeed different from the tradition of practical treatises (from Piero della Francesa's *De prospectiva pingendi* onwards) explaining *how* to construct a perspective picture owing to its higher mathematical sophistication and theoretical depth. Moreover, it took a different angle than Federico Commandino's discussion of perspective in connection with mathematical astronomy and geography in his edition of Ptolemy's *Planispherium* (1558), which treats it as mathematically equivalent to stereographic projection. Benedetti's focus was the explanation of the reasons underlying perspective, as well as errors that might occur in theory and practice. In a manner that is similar to his treatment of problems of arithmetic, geometry offers the conceptual tools to formalize knowledge embedded in practice.

Like most parts of the *Diversae speculationes*, this book on perspective originated from extemporary sources of inspiration. In fact, it begins *ex abrupto* with the examination of an error in linear perspective which leads Benedetti to more general considerations. Moreover, the dedication of its seventh chapter to the architect Giacomo Soldati strengthens the impression of the occasional character of the writing.[30]

[26] Giusti 1993, 33: "D'altronde, l'interesse della teoria di Benedetti non sta nel risultato globale, ma soprattutto nel ruolo chiave della proporzione composta, e in particolare nella creazione di un algoritmo di calcolo agile ed elegante." See Benedetti 1585, 202.

[27] See Giusti 1993, chap. 2, section 2.

[28] Benedetti 1585, 119: "Cum nullus adhuc (quod sciam) veras internasque causas operationis perspectivae perfecte docuerit, operaeprecium existimavi aliqua de iis disputationem suscipere."

[29] Benedetti 1585, 119: "Multi enim eorum, qui huiusmodi operationis regulas praescribunt, cum eius effectuum veras causas ignorent, varios diversosque errores committunt, ut exempli gratia in subscripta figura superficiali A [etc.]." See Figure 3.2.

[30] Benedetti 1585, 133: "Superioribus diebus non diu postquam de perspectivis inter nos sermonem habuimus, dum animus totus adhuc in his esset. Illud in mentem venit quod eximius ille vir, et profundissimae

The error discussed in the *incipit* of the *De rationibus operationum perspectivae* is represented in a two-dimensional figure (Figure 3.2).

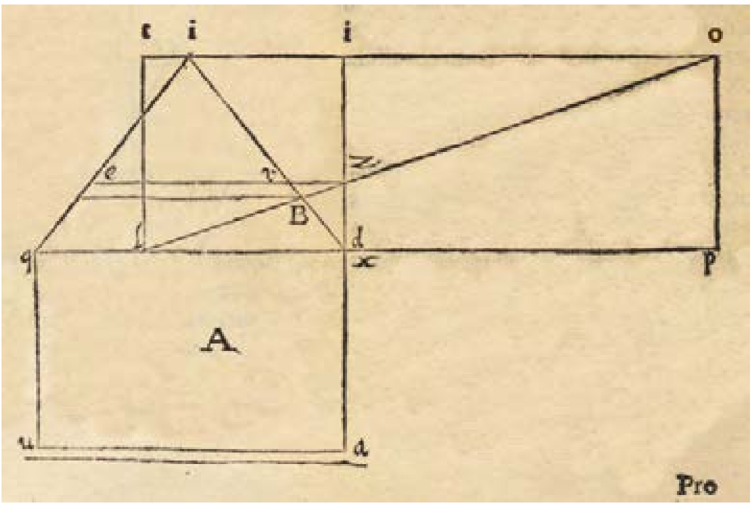


Figure 3.2: *Figura superficialis*, or plane diagram, pointing out an error of linear perspective. From Benedetti, *Diversae speculationes* II, p. 119. (Max Planck Institute for the History of Science, Library)

Benedetti explains how to correctly construct the linear perspective "inscribed" in the given triangle *IDQ*:

> [...] in the plane figure *A* (here included), in order to 'degradate' (as one says) [i.e., to put it in perspective] the rectangle *QA* [*QXAU*] in the triangle *IDQ*, they [the practitioners] draw a line parallel to *QD* from point *B* (at the intersection of line *OL* with the side *ID* of the triangle) or, indifferently, they draw it from point *Z* (the intersection of the same *OL* with the perpendicular *XI*). They are not aware that the latter method is the only correct one, not the former. Conversely, if the former were correct, the latter could not be true. They might excuse themselves by arguing that one draws the aforesaid parallel line from *B* supposing that the plane *IDQ* is inclined horizontally relative to the rectangle *QA* according to the angle *IDQ*. But this excuse cannot be accepted because, according to their supposition, if one assumes that *IDQ* is inclined, the inferior angles of the rectangle put in perspective should not be as acute as *IDQ* and *IQD*. This can be easily understood considering their construction [*ratio*], which I show in the plane figure *A* here included. However, if one wants to see the rectangle in perspective, one should locate this plane relative to the eye in the same manner as the line *ID* relative to *O*. But this is too difficult [to represent].

doctrinae, nec unquam satis laudatis Daniel Barbarus se accepisse profitetur a Ioanne Zamberto patritio Veneto, qui ad verbum omnia desumpserat a Ioanne Cusino Parisiense. Nec parum mirabar peritissimus illum Cusinum, quod in capite quarto secundae partis perspectivae, ut quod piam planum quadrilatam composuisse. [...]." See Mamino 1989.

> To summarize, the correct manner [to put a rectangle in perspective] is to draw a line *ER* parallel to *QD* from point *Z*, which is common to *OI* and *XI* (perpendicular to *LP*).[31]

In order to visualize the construction, Benedetti produces an additional diagram (Figure 3.3) offering the tridimensional correspondent of the plane diagram he has just examined. Note that point *O* is the point of departure of the cone of sight. The observer is thus lying with his feet in point *P*.

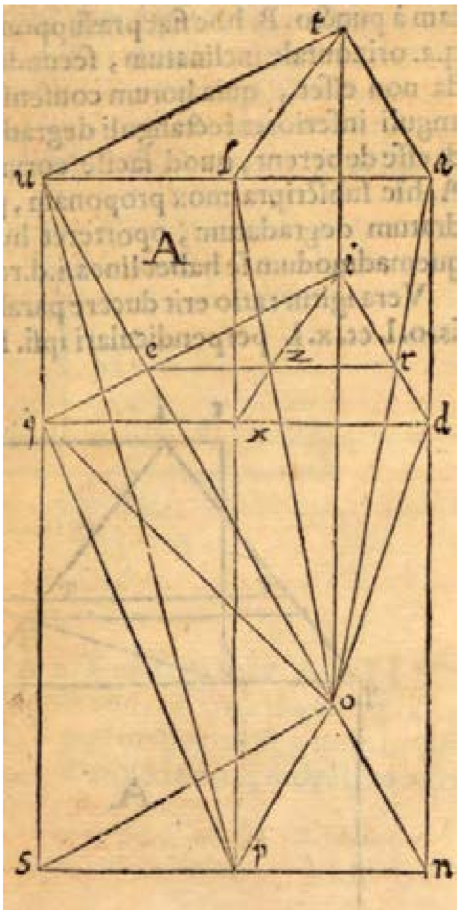

Figure 3.3: *Figura corporea*, or tridimensional diagram, visualizing the same problem of linear perspective as in Figure 3.2. From Benedetti, *Diversae speculationes* II, p. 120. (Max Planck Institute for the History of Science, Library)

Departing from these considerations, Benedetti treats various cases (including the one in which the projection plane is not perpendicular to the observer's line of sight) using the same approach. That is to say, he first draws bi-dimensional diagrams showing the construction and then offers tridimensional geometrical explanations providing an insight into the physical reality underlying the construction. He even offers a sort of virtual instrument

[31] Benedetti 1585, 119: "[…] in subscripta figura superficiali *A* volentes degradare (ut dicunt) rectangulum *qa* in triangulo *idq* ducunt parallelam ipsi *qd* a puncto *B* intersecationis lineae *ol* cum latere *id* trianguli, et (idem) indifferenter, eandem quoque a punctoque *Z* intersecationis ipsius *ol* cum perpendiculari *xi* ducunt, nescientes hunc solum esse verum modum, non item alium, quia si alius, talis esset, hic, verus non existeret, nam si vellent sese excusare, quod ducendo dictam parallelam a punctoque *B* hoc fiat praesupponendo planum ipsius *idq* versus rectangulum *qa* orizontale inclinatum, secundum angulus *idq*, haec excusatio accipienda non esset, quia horum consensu, praesupponendo planum *idq* inclinatum, anguli inferiores rectanguli degradati, non tam acuti, quae sunt duo *idq* et *iqd* esse deberent, quem facile eorum ratione innotescet, quae de figura corporea *A* hic subscripta mox proponam, praeter id, quod volentes deinde aspicere quadratum degradatum, oporteret huiusmodi planum respectu oculi ita collocare, quemadmodum se habet linea *id* respectu *o* quem factu nimis arduum esset.

Vera igitur ratio ergo ducere parallelam *er* ad *qd* a puncto *Z* communi ipsis *ol* et *xi* perpendiculari ipsi *lp*."

to guide constructions in linear perspective.[32] As Judith V. Field has argued, Benedetti "shows the applied mathematician's concern with particular problems, rather than the pure mathematician's interest in theoretical developments. The significance of his work lies not only in its mathematical insight but also in the fact that it shows us innovative work in the purest of sciences, namely pure mathematics, being carried out within an applied tradition."[33] As has been argued, it is plausible that Benedetti's constructions were included in later works on perspective, in particular by Simon Stevin and Guidobaldo del Monte, although none of them explicitly referred to Benedetti. Apart from Del Monte's enmity towards him, it was typical of Renaissance mathematicians who wrote on optics to leave their sources unmentioned.[34]

### 3.4 Sections on Physics: Mechanics and Natural Philosophy

Book 3 of the *Diversae speculationes* deals with mechanics. Benedetti remarks that many learned people have already written extensively on this issue, but that their achievements are not sufficient because nature and practice (*usus*) awaken many doubts concerning the validity and the completeness of previously acquired knowledge. New problems have emerged. His intention is to present many novelties and to propose theses that either have never been treated or have never been adequately demonstrated (*nusquam ante hac tentata, aut satis exacte explicata*). In the final words of his introduction to Book 3, we can see that he attaches great importance to his speculations on mechanics; he even states that he will be especially remembered after his death for his contribution to the advancement of this discipline:

> Many man have written a great deal about mechanics, and they have written most ably. But nature and experience are always wont to bring to light something new or previously unknown. And it is therefore incumbent upon a high-minded and grateful individual, if he happens to discover something previously shrouded in darkness, not to begrudge it to posterity. For he himself gained a great deal from the work of others. Now it is my desire to publish a few items that will, I believe, prove not unwelcome to those who concern themselves with mechanics, items which have never before been dealt with or have not been sufficiently well explained. I may thus either show my desire to be helpful or at least give some evidence of possessing a bit of talent and industry. And perhaps in this way alone may I leave behind me proof that I ever lived at all.[35]

Book 4 essentially deals with Aristotle's opinions on local motion in *Physica* and *De caelo*, although critical references to *Metaphysica* and *Meteorologica* are also present. In this section, Benedetti seeks to provide new foundations to the theory of motion from a

[32] Benedetti 1585, 131.
[33] Field 1987, 247.
[34] See Andersen 2007, 152
[35] Translation from Drake and Drabkin 1969, 166. Cf. Benedetti 1585, 141: "Scripserunt multi multa, et quidem scitissime, de mechanicis, at cum natura ususque, aliquid semper vel novum, vel latens in apertum emittere soleant, nec ingenui aut grati sit animi, posteri invidere, si quid ei contigerit comperuisse priis tenebris involutum: cum tam multa ipse ex aliorum diligentia sit consequutus. Paucula quaedam futura, ut reor, non ingrata his qui in hisce mechanicis versantur, nusquam ante hac tentata, aut satis exacte explicata in medium proferre voluit: quo vel iuvandi desiderium, vel saltem non ociosi ingenioli argumentum aliquod exhiberem: atque vel hoc uno modo me inter humanos vixisse testatum reinquerem."

philosophical perspective based on a mathematical approach to nature (*inconcussa mathematicae philosophiae basis*). He revises basic concepts of physics, such as place and time, as well as natural issues, for example the existence of a physical void. Eventually, he deals with cosmological aspects linked with the theory of motion, including a defense of the Copernican system.

We will treat books 3 and 4 of the *Diversae speculationes* separately and with particular attention (in chapters 5 and 6), owing to their historical and theoretical relevance.

## 3.5 The Epistles: Miscellanea Mirroring a Scientific Network

The last book of the *Diversae speculationes* is a large collection of letters, "Physica et mathematica responsa" (Epistolary Answers on Physics and Mathematics). The autonomy of this section is underscored by the insertion, at its beginning, of a second preface.

> To the reader.
>
> Nothing is more suited to virtue than being active and present through constant motion. Just like a shining star shimmering for the eyes of the spectators. It occurred to me that this or that person invited me with words or stimulated me with letters to dispute on those issues in which I am versed. In fact, I was devoted to mathematics and to highly philosophical speculations while I sojourned in the most splendid princely courts and illustrious cities, where many intelligent people displayed admirable curiosity, desire for knowledge and conversation. I collected part of those disputes and answers, because I judged them to be worth communicating. I planned to reread and revise them, if I had some spare time, [having in mind two goals]: on the one hand, that science itself circulates as much as possible and grows; on the other hand, that the names of those excellent men, who stimulated me with their questions, are made immortal, as far as it is in my power to do so, and that I can lead others to follow their example, abandon the idle sloth (which is able to corrupt even the most talented) and start investigating, exchanging, and discussing serious subjects that could prove useful at some point and worth disseminating. For the time being, please, approach our efforts with a benevolent and judicious attitude. Regards.[36]

Significant elements emerge from this passage: the courtly environment influencing Benedetti's activity, the interest in applied knowledge, the recognition of the importance

[36] Benedetti 1585, 204: "Ad lectorem. Ut nil magis virtutis est proprium, quam agitari, et incessabili motu prodesse. Ac veluti fulgidum sydus ante oculus spectantium commicare. Ita mihi mathematicis iisque maxime philosophicis speculationibus dedito, saepissime, ut in principium summorum aulis, et amplissimis civitatibus degenti, ubi multa semper Nobilium mira curiositate, sciendi desiderio, et conferendi cupiditate referta, versantur, ingenia, contigit, modo ab his, modo ab illis, aut verbis tentari, aut literis provocari ad disserendum, de his, in quorum studiis versamur. Quarum concertationum et responsionum, quoniam non omnino indigna existimavi, quae memoria comendarentur, partem aliquam apud me conseruivi. Ubi vero per ocium licuit, relegi, ac tandem de manu mittere decrevi. Tum ut scientia ipsa quo magis diffundetur, crescat; et quicquid valeo, sine invidia in communem utilitatem conferam. Tum ut virorum praestantissimorum, qui me suis interrogationibus excitaverunt, quantum in me erit, gratitudine ergo, nomina reddam immortalia, et eorum exemplo alios, ocio sordidiore abiecto, quod solet iurialium praecipue excelsa ingenia corrumpere, ad sciscitandum conferendum, et disserendum, de rebus seriis, et quae usui aliquando esse possint, et quandoque evulgari mereantur, alliciam. Tu interim nostris laboribus fruere, et nostram diligentiam boni, et aequi consule, et Vale."

of dissemination, as well as the celebratory intention of the work (to render the dedicatees immortal). For us, this epistolary is the most tangible evidence of a network of social relations and its scope.

The epistles are not organized chronologically (actually the dates are almost always omitted) but rather according to the importance of the addressees, some of whom were already dead at the time of publication. The first letter was directed to the duke Emanuele Filiberto, the second to his son Carlo Emanuele I, and the following four letters to the powerful nobleman Andrea Provana de Leyní. The topics are linked to Benedetti's role as court mathematician and mathematical advisor.[37] The first letter is an expert report on the reform of the calendar, the second deals with a geometrical problem (the determination of the circle circumscribing a given square) that probably emerged from the lessons imparted to the prince, and the letters from three to six address practical problems of navigation (such as the use of astronomical instruments at sea).

Although it is not possible to identify all of Benedetti's correspondents, one can say with certainty that most of the epistles are tied to the northern Italian environment, specifically to Turin and to Venice. Many correspondents were in fact Subalpine or Venetian patricians or courtiers and artists linked to one of these two centers. Sometimes they were linked to both, as was the case with ambassadors such as Domenico Morosini (*Dominicus Moresinus*), Pietro Pizzamano (*Petrus Pizzamanus*), and Francisco Mendoza. There is great variety in the status and professions of the correspondents. Michela Cecchini and Clara Silvia Roero, in their accurate reconstruction, came to the following assessment: "The variety of themes that were discussed and of the professions of the participants in the debates shows that Giovanni Battista Benedetti was a man of culture and practice. He was ready to engage in a fruitful debate with exponents of the scientific world in the broadest sense (such as mathematicians, physicians, jurists, and philosophers) and with politicians, diplomats, and ambassadors, as well as with experts of military art and religion. Moreover, he did not dislike architects, artisans, constructors of instruments and fortifications, surveyors, and astrologers."[38]

Among his correspondents, the list of Savoy patricians and courtiers is long. Those who emerge most prominently are the orator Francesco Barbaro, who sojourned in Turin between 1578 and 1581, the Turin professor of jurisprudence Bernardo Trotto, the court historian Emanuele Filiberto Pingone, and the functionary and artillery general Giuseppe Cambiani. Benedetti advised Cambiani on ballistics, commenting upon Tartaglia to determine the most effective inclination for a cannon.[39] Another member of the Savoy milieu was Giovanni Battista Femello, philosopher, physician, and ordinary professor of practical medicine at Turin. Benedetti wrote to him on a very special geographical issue: the determination of the position of Iceland. By contrast, the mathematical advice directed to his friend, the jurist Francesco Ferrari, concerned ethics and justice. Benedetti explained

[37]"Mathematics" has to be here understood in the wide and interdisciplinary Renaissance meaning. It comprised arithmetic and geometry, as well as practical mechanics, architecture and engineering, astronomy and meteorology, optics, physics, and even metaphysical and epistemological issues.

[38]Cecchini and Roero 2004, 32: "Dal quadro variegato dei temi discussi e dalle professioni dei protagonisti dei dibattiti emerge la figura di Giambattista Benedetti come uomo di cultura e di pratica, disposto ad instaurare un dialogo proficuo sia con esponenti del mondo scientifico in senso lato, come matematici, medici, giuristi e filosofi, sia con politici, diplomatici e ambasciatori, come pure con uomini d'arme e di religione, non disdegnando neppure architetti e artigiani, costruttori di strumenti e di fortificazioni, agrimensori e astrologi." In the following we will often rely on Cecchini and Roero for an overview of Benedetti's correspondents.

[39]Benedetti 1585, 258–259.

to him how to make use of proportions in order to explain "commutative justice."[40] An epistle on the quantification of the time necessary to spill the water out of a vase through three tubes, alongside another two epistles on geometry and the application of geometry to the study of solar radiation, are directed to the Savoy secretary Ludovico Niccolò Calusio. The letters on mechanics addressed to Giovanni Paolo Capra of Novara are better known to historians of science. They intermingle considerations on turning wheels with others on astronomy and the boundaries of the cosmos. Other noblemen belonging to the Turin milieu who corresponded with Benedetti were Franchino Trivulzio and Demoulin de Rochefort, responsible for the ducal *Wunderkammer*.

Benedetti had intense exchanges not only with patricians but also with artists, engineers, and practitioners. Four letters are addressed to the architect Gabriele Busca and their issues range from natural philosophy and meteorology to the use of instruments and topography. The Savoy clock-maker Jacopo Mayeto (perhaps Maletto) shared Benedetti's interest in gnomonics, sundials, and mathematical instruments, as witnessed by one of the letters in the collection.[41] Furthermore, a letter to the Savoy surveyor Angelo Ferrario is at once technical and playful: it is a discussion of the curve described by a hanging rope that Ferrario and Benedetti observed together in the park of the ducal castle of Lucento by Turin.[42] Additionally, Benedetti wrote on music to the Flemish madrigalist Cipriano de Rore, who had been appointed in Venice, Ferrara, and Parma and whom Benedetti had probably already met in his hometown Venice. Among the artists, the Venetian poet Girolamo Fenarolo is the addressee of two letters on geometry.

Some epistles tackle the philosophical and epistemological issues underlying mathematics and their cognition. For instance, those to his Venetian friend Domenico Pisani deal with the issues "de philosophia mathematica" (on mathematical philosophy) and "de imaginatione specierum" (on the imagination of the species).[43] Another philosopher corresponding with him was the Turin professor Francesco Vimercato to whom several letters are addressed. While these mainly deal with optics, theses on natural philosophy and cosmology are discussed in a brief letter to the theologian Gerolamo Cordero (*Hyeronymus Condrumerius*). It is entitled "Quod recte Arist[otelis] senserit coelum casu non esse productum" (Aristotle correctly saw that the heavens are not produced in a casual manner).[44] Benedetti moreover corresponded with the Paduan professor Pietro Catena, known as a keen supporter of the *certitudo mathematicarum* (see the next chapter) and the mathematical method applied to natural inquiry.

The identity, biographies, and work of many of Benedetti's correspondents are still obscure. This is especially true for foreign scholars, presumably Germans, whose names were latinized as follows: Theodosius Raisestaim, Paulus Aemilius Raisestaim, Volfardus Aisestain, and Conradus Neubart. Benedetti does not give these names any titles. An exception to this is the correspondent Anselm Rosenburg, who is designated as Imperial surveyor (*agrimensor Cesareus*).

[40]Benedetti 1585, 284.
[41]Benedetti 1585, 423–425.
[42]Benedetti 1585, 361–363.
[43]Benedetti 1585, 298 and 298–299.
[44]Benedetti 1585, 298 and 298–299. For the identification of this person, see Cecchini and Roero 2004, 58–59.

# Chapter 4
# Epistemology

One of the most challenging aspects of Benedetti's endeavor was his attempt to merge mathematical and physical speculations, as is clearly stated in the title of the *Diversae speculationes mathematicae et physicae*. In order to understand his way to "physico-mathematics," we will discuss his mathematical epistemology starting from some statements scattered in his major work and then look at the premises implicit in his treatment of nature. We will briefly review the Renaissance reflections on mathematics linked to practical developments in technological fields as well as to eclectic reassessments of Pythagorean and Aristotelian debates on the certainty of mathematics and their applicability to natural philosophy. Focusing on the epistemological premises underlying Benedetti's mechanics, we will discuss medieval and early modern approaches to natural knowledge, which, in spite of their mathematical rigor, rested on a physics and metaphysics of contingency. For many centuries, it was assumed that the mathematical regularity of the phenomena does not imply their causal necessity.

## 4.1 The Certainty of Mathematics

In the letter to the Venetian patrician Domenico Pisani included in the collection of the *Diversae speculationes* and entitled *De philosophia mathematica* (On Mathematical Philosophy), Benedetti emphasized the philosophical dignity of his discipline, placing it at the same rank as physics, metaphysics, and ethics—if not higher than them, considering the certainty of its demonstrations (*certitudo suarum conclusionum*):

> I am surprised that, although you are well-versed in Aristotelian philosophy, nonetheless you make a distinction between the philosopher and the mathematician in your writings, as if the mathematician were not as much a philosopher as the naturalist and the metaphysician. In fact, as far as the certainty of his conclusions is concerned, he deserves the title of philosopher much more than them.[1]

This reference to mathematical *conclusiones* reveals Benedetti's methodological focus on the dignity and validity of his discipline. In his connection of mathematical and physical speculations, he seems to put the emphasis on the method rather than on ontology and to seek for the certainty of mathematics and its applications by way of its specific logic. This was the position of his correspondent, the Paduan professor Pietro Catena.[2] Along with him, Benedetti maintained that the certainty of mathematics has an extra-sensible and intelligible character.[3] As Benedetti added in his letter to Pisani:

[1] Benedetti 1585, 298: "Miror quod cum in Aristotele sis versatus, in tuis tamen scriptis philosophum a Mathematico separes, quasi mathematicus non sit adeo philosophus, ut est naturalis, et metaphysicus, cum multo magis quam ii philosophus sit appellandus, si ad veritatem suarum conclusionum respiciamus."

[2] Benedetti includes a letter to Catena in Benedetti 1585, 371.

[3] See on this De Pace 1993, 228–229.

> Actually, you are not the only one who makes this mistake, but this is more grave in consideration of the fact that, although you [Aristotelians] even label ethics as a philosophical discipline, you do not acknowledge that the divine mathematical sciences also should be adorned with the name of philosophy. In fact, if we consider this name more attentively we will clearly see that it is in itself more suited to the mathematician than to anyone else, since none of the others is more certain in his affirmations than the mathematician. And no one is more driven by the love of science in his cognition. This is evident. In fact, [the mathematician] does not rely on the senses nor accepts any presupposition that is not so true and evident to the intellect that no power whatsoever could show that it is false.[4]

Benedetti was acquainted with scholars quarreling over the status of mathematics, its demonstrative methods, and its legitimacy in the treatment of natural issues.

In his time such debates on the foundations and status of mathematics were intense. As an instance of epistemological reflections on the philosophy of mathematics, historians often mention the controversial theses by the Paduan professor of philosophy Alessandro Piccolomini, with whose work Benedetti was familiar. Piccolomini authored, among other writings, a treatise *De certitudine mathematicarum* (On the Certainty of Mathematics, 1547) affixed to his paraphrases of pseudo-Aristotelian mechanics, *In mechanicas questions Aristotelis paraphrasis*. As one reads in this sort of appendix, one ought not to cast into doubt the certainty of mathematics. However, this does not depend on demonstrative methods but rather on the subject of inquiry: "Mathematical disciplines are certain not due to the force of their demonstrations but rather to their subject matter itself."[5] Their special subject is quantity, connected to matter. Hence, the certainty of mathematics, for an Aristotelian such as Piccolomini, rests on the fact that it deals with universal properties of nature that can be extracted from concrete reality by means of abstraction (*res mathematicae sunt ex abstractione*).

> The cause of the certainty of mathematics is evident from Aristotle's statements. Simplicius is of the same opinion when he states (in *De anima* I 11) that the cause of the certainty of mathematics is due to the fact that they refer to quantity. In fact, as he argues, quantities are sensible things, they have sensible causes and they are known to us as such.[6]

This consideration led Piccolomini to argue that motion can become a mathematical object, if one abstracts from materiality:

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[4]Benedetti 1585, 298: "Verum quidem est, te in huiusmodi errore solum non versari; sed gravius est, quod cum vos videatis etiam res morales sub philosophiae appellationem cadere, non animadvertatis divinas scientias mathematicas etiam philosophiae nomine ornandas esse. Quod si eiusdem nomen penitius considerare velimus, inveniemus aperte, mathematico magis illud ipsum quam cuilibet alio convenire, cum nullus ex aliis tam certo sciat id quem affirmat quam mathematicus, neque aliquis sit, qui in cognitionis, et scientiae cupiditatem magis ducantur, ut aperte patet, cum nec etiam ipsi sensui det locum, neque aliquid praesupponat, quem non sit ita verum et intellectui notum, ut nulla quaevis potentia, illud esse falsum ostendere queat."

[5]Piccolomini 1565, f. 107*v*: "Mathematics disciplines esse certas non vi demonstrationis, sed ex subjecti ipsius ratione."

[6]Piccolomini 1565, 106*v*: "Patet igitur ex dictis Aristotelis causa certitudinis mathematicae. Hoc idem sensit Simplicius, qui primo de Anima 11. dicit causam certitudinis mathematicarum esse, quia versantur circa quantum. Quantitates enim ut dicit ipse, sunt res sensatae, et causas sensatas habent, et ideo nobis notas."

> One could argue that, just like magnitude, motion is a common sensible, too. Moreover, it has its effects and causes (see *Physics* V and VI). Thus, there can be a science of motion (a natural one), which is certain, similar to the science of quantity, that is, mathematics.
>
> We can answer to this [apparent objection], that if we consider motion in general, as separated from matter and insofar as it is a continuum […], our consideration will be mathematical. This is not in contrast with our principles.[7]

The "ontological" and not only "epistemological" dimension of mathematical physics would concern later scholars such as Kepler and Galileo, going beyond the shared Aristotelian discourse in their investigations of the mathematical properties of material processes.[8] Benedetti was rather concerned with mathematics as an intellectual tool, a sort of "logic of scientific inquiry." In the above-mentioned letter to Pisani on his mathematical philosophy, he stressed the certainty of mathematical reasoning rather than that of its "objects." Nonetheless, he was interested in the question raised by Piccolomini as to the usefulness of mathematics in the study of motion. As we will discuss, Benedetti's insight concerning the generalization of the methods already in use in mechanics, in the science of weights, established the premises for the conceptualization of problems in dynamics.

Benedetti's interest in mathematics as a conceptual instrument accords with the interest in the demonstrative power of mathematics shown by many scholars entering the debates about mathematical certainty. The publication of Piccolomini's *De certitudine mathematicarum* led to a series of negative or sympathetic reactions, among them the criticism made by the translator of Proclus's *Commentary* on Euclid, Francesco Barozzi, as well as those by the Paduan professors Pietro Catena and Giuseppe Moletti. Barozzi, in his 1560 *Quaestio de certitude mathematicarum*, and Catena, in his 1563 *Oratio pro idea methodi*, argued in favor of the demonstrative certainty of mathematics, contra Piccolomini's exclusive focus on mathematical objects. The theoretical discussion regarding the status of mathematics, the certainty of its demonstrations, their applicability to the investigation of nature, and the hierarchy between natural philosophy and mathematics continued for a while. It also produced frictions among Jesuit scholars such as the philosopher Benito Pereira and the mathematician Clavius, who were inclined to assign different levels of importance to the study and teaching of mathematics in the colleges of their order.[9]

As far as the institutional side of the defence of mathematics is concerned, it opposed scholars and intellectuals benefiting from varying social status, such as mathematicians, philosophers, and theologians. Benedetti's self-perception and, later, Galileo's self-presentation as "philosophers" involved polemical stances. They claimed for their math-

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[7]Piccolomini 1565, 107*r*: "Si vero adhuc replicaretur, quod motus etiam est sensibile quoddam commune, sicut magnitudo; habet autem motus suas passionet, et suas causas, ut patet 5. et 6. Phys. ergo ita erit certa de motu scientia, naturalis scilicet, sicut scientia de quantitate, quae Mathematica est. Ad hoc respondere possumus, quod si motum consyderabimus, in communi, abstractu a materia quatenus continuum quoddam est, […] tunc consyderatio erit mathematica, et nihil contra nos."

[8]As Ofer Gal and Raz Chen-Morris recently stressed: "It is not epistemology that worries the two court mathematicians here, but ontology. Neither of them questions the power of mathematics to provide the knowledge they seek; it is the objects that mathematics can be true about that they both feel forced to establish." See Gal and Chen-Morris 2013, 118–119.

[9]The literature on the Renaissance debates on the philosophical status of mathematics is wide. Among other sources, see Giacobbe 1972, Giacobbe 1973, Carugo 1983, Jardine 1990, 693–697, De Pace 1993, Cozzoli 2007, and Axworthy 2016, chap. 2. For the Jesuit debates on mathematics, see Romano 1999. For the seventeenth century, cf. Mancosu 1996, 8–33.

ematical and physical investigations a wide cultural meaning against critics who downplayed such investigations as merely technical and specialistic.

Early polemics over the viability of the *mos geometricus* were not purely intellectual and academic but were also rooted in the rising recognition of the practical import of mathematics in engineering, architecture, mechanics, and warfare. A new class of intellectuals was emerging composed of "scientist-engineers," so to speak, both expert in practical disciplines and trained in letters.[10] Edgar Zilsel already remarked that the Renaissance exaltation of mathematics went far beyond purely Platonic and Pythagorean influences. At that time new mathematical writings were composed and published dealing with the practical problems of commerce, topography, architecture, and the arts.[11] Moreover, the emergence of mathematical and natural conceptions dependent on the advance of technology was reinforced by the growing self-consciousness of new social groups.[12] As an example of the awareness of the status of the practical arts one could mention Filippo Pigafetta's introduction to the Italian edition of Del Monte's work on mechanics. Here he reversed the assessment of craftsmen and practical knowledge, which had been marked by the contempt of aristocrats and traditional intellectuals, as follows:

> 'Mechanic' is a very honored title. According to Plutarch it refers to a profession linked with warfare. It is suited to a man of high rank who is also capable of using his hands and his intelligence to realize wonderful works of rare usefulness and pleasure for human life.[13]

This judgment well expresses the shifting opinion on practical knowledge which also marked Benedetti's environment. We have already stressed the centrality of practical mathematics for the Savoy dukes, in particular Emanuele Filiberto, in their construction of the new capital, Turin.

## 4.2 Physico-Mathematics

As a direct consequence of this mathematizing epistemology Benedetti dismissed the well-established separation between physics and mathematics in cosmology, that is, he refused to separate the investigation of "causes" and calculation.[14] This anti-fictionalist perspective implied a realist commitment related to the Copernican system and its embedding within a renewed cosmology. As we will discuss in the section on Benedetti's views on the universe, he praised the system "of Aristarchus and Copernicus" as it avoided the absurdities of an anthropocentric conception according to which the immensity of the firmament was created only for us. Rather, all planets are like Earth or, better, like moons reflecting the solar light. Among the direct consequences of the Copernican view was accepting that the fixed stars do not rotate around the center of the world within one day; rather, they are immobile.[15]

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[10] See Valleriani 2010 and Valleriani 2013.

[11] Zilsel 1942.

[12] See Lefèvre 1978.

[13] Pigafetta in Del Monte 1581, *Ai lettori*: "Mechanico è vocabolo honoratissimo, dimostrante, secondo Plutarco, mestiero alla Militia pertinente, et convenevole ad huomo di alto affare, et che sappia con le sue mani et co'l senno mandare ad esecutione opre maravigliose a singulare utilità et diletto del vivere humano."

[14] Hypotheses on conventionalism already emerged from the debate on the conflict between Ptolemy's geometrical models and Aristotle's homocentric cosmology. See Di Bono 1990 and Granada and Tessicini 2005.

[15] Most of these cosmological views are discussed in Benedetti 1585, Book 4. We deal with the details in chapter 6 as well as, partly, in chapter 7.

From this viewpoint, Benedetti's understanding of mathematics is not too removed from that of a mathematician such as Copernicus, who, in Book 1 of *De revolutionibus*, indicated that the mathematical superiority (simplicity and intelligibility) of his own planetary system was such that natural philosophy had to be subordinated to mathematical astronomy and not vice versa. The theologian who wrote the anonymous introduction to Copernicus's work, Andreas Osiander, tried to reaffirm the hypothetical character of mathematical astronomy, and its subordinate position as a discipline relative to physics and theology. By contrast, Renaissance scholars who appreciated the physical meaning of the Copernican system called it "Pythagorean" to underscore at once its natural philosophical and mathematical character.[16] As an extreme case one could mention Bruno's declarations during his Inquisition trial. In order to defend his cosmological views, and in particular the motion of Earth, he did not mention Copernicus but the ancient philosophical school of Pythagoras: "I affirmed [the existence of] infinite individual worlds [i.e., planetary systems] similar to that of the Earth. Following Pythagoras, I regard the latter as a celestial body. The Moon is similar to it, as well as other planets and stars, which are infinite [in number]."[17] Pythagorean cosmology was regarded with suspicion by the Inquisitors and the doctrine of the plurality of worlds became one of the allegations against Bruno, who would be eventually executed as a heretic in Rome. In the same years in which Bruno was a prisoner of the Holy Office in Rome and his works were examined for censure, the censors also attacked Patrizi for his natural views, including the doctrine of terrestrial motion. Although Benedetti shared similar views about the plurality of worlds and the possibility of terrestrial motion, he did not incur any censure. We dare say that he was one of the last Renaissance authors who could freely speculate on nature in Italy before natural philosophy became a highly ideological issue in the religious repression escalating in the 1590s.

Benefiting from his subalpine freedom, Benedetti reflected on Pythagorean cosmology in a section entitled *Pythagoreorum opinionem de sonitu corporum coelestium non fuisse ab Aristotele sublatam*, where he excluded the possibility that the "sound of celestial bodies" is the production of any physical sounds. Rather, he identified the Pythagorean doctrine of the world harmony with divine providence:

> As to motions, dimensions, distances, and influences there is nothing that corresponds to such proportions, but, since all of them depend upon the infinite Divine Providence of God, these velocities, those dimensions, distances, and influences must have the most perfect order and relations among them and relative to the universe.[18]

According to Benedetti's outlook, the harmony of the heavens does not correspond one to one to musical harmony in the strict sense. From this viewpoint, Kepler's later effort to translate heavenly geometries into musical melodies in the *Harmonices mundi libri V* (1619) can be seen as a radicalization of similar "Pythagorean premises."

Most significantly, Benedetti and Kepler shared a commitment in favor of the fusion of mathematical and physical accounts of nature in the frame of an early modern transfor-

[16] Omodeo 2014a, 167–170.

[17] Bruno 2000b, doc. 13, 67: "Ho dechiarato infiniti mondi particulari simili a questo della Terra; la quale con Pittagora intendo uno astro, simile alla quale è la Luna, altri pianeti et altre stelle, le qual sono infinite."

[18] Benedetti 1585, 191: "Quod autem attinet ad motus, ad magnitudines, ad distantias et ad influxus, nihil est, quod hisce proportionibus conveniat, sed quia haec omnia dependent ab infinita et divina providentia Dei, necessario sit ut istae velocitates, eae magnitudines, distantiae et influxus, talem ordinem et respectum inter se ipsa et universo habeant, qualis perfectissimus sit."

mation of natural science in which the methods of the physico-mathematical disciplines gained a paradigmatic status. The epistemological shift also involved well-established disciplines such as astronomy. Kepler's astrophysics, first illustrated in the *Astronomia nova* (1609), was a significant step toward the derivation of celestial geometries from physical forces. Kepler translated a geometrical discipline (Ptolemaic and Copernican mathematical astronomy) into a physico-mathematical one. In fact, he explained the elliptical path of planetary orbits as the effect of interactions of forces. He emphasized the double bound of his astronomy, inseparably intertwining physics and mathematics, in the title of the work: *Astronomia nova αἰτιολογιτός seu physica coelestis de motibus stellae Martis* (New Astronomy Investigating the Causes, or Celestial Physics Concerning the Motions of Mars). As Kepler announced in the introduction: "In this work I mixed celestial physics with astronomy."[19] He meant to launch a new discipline, "celestial physics," that merged mathematical modeling with causal physics.[20] Kepler remarked that the ignorance of physical causes compels scholars to settle for conjectures since no choice can be made between mathematically equivalent hypotheses. By contrast, physical arguments are decisive in deciding between mathematically equivalent models. Therefore, celestial physics and astronomy should be unified. The result was a mixed science (*scientia mixta*) whose data came from the senses and whose demonstrations are expressed in mathematical terms. This physicalization is well shown in Kepler's physico-mathematical concept of "orbit" (*orbitae*) substituting that of orbs (*orbes*) (that is, the material spheres transporting celestial bodies). According to him orbit is "the path together with its physical causes—expressed as physical laws."[21] Shape and velocity of astronomical orbits depend on the force (*vis*) emanating from the sun, that is, on a physical cause of geometrical effects.[22]

Descartes's *Traité du monde et de la lumière* (completed in 1632–1633, but printed posthumously, in 1664) and the *Principia philosophiae* (1644) marked a culminating point in the move toward the reduction of natural disciplines (such as optics and astronomy but also physiology) to material interactions of corpuscles in motion. Descartes's philosophy was particularly influential as it legitimized a mathematical treatment of nature with the advances of physics in his time. At the same time, he connected his explanations to views on matter and causality irreconcilable with the qualitative, essentialist, and teleological accounts of the Scholastic tradition. In particular, his mechanization elevated the results of Renaissance mechanics to a higher and more generalized level.

Benedetti's place is rather at the beginning than at the end of this process. As the title of his major work hints, he was committed to a mathematical-physical investigation of nature. He did not limit his application of a mathematical method to those fields where this approach was already established, but extended it to the treatment of all realms of natural inquiry.

## 4.3 The Contingency of Nature and Mechanics

Benedetti's mathematical approach to nature did not lead him to the belief that physical phenomena are ruled by necessity. Rather, he shared a medieval and early-modern ontology and epistemology of contingency enabling a particular cohabitation of mathematized physics and indeterminism (in other words, formal determination without causal neces-

[19]Kepler 1937–2001, vol. 3, 19.
[20]Gingerich 1975, 261–278.
[21]Goldstein and Hon 2005, 76.
[22]On Kepler's discovery, see Donahue 1988, Donahue 1993 and Wilson 1968.

sity). In order to better understand it one has to look at Scholastic motives informing his physics, in particular his mechanics, and the scientific and philosophical work of his successors. This will require a short excursus.[23]

### 4.3.1 Scholastic Treatments of Nature as the Realm of Contingency

It would be misguided to think that a mathematical approach to nature in Renaissance science implies the assumption that natural causation is ruled by necessity. This was indeed not the case for well-established medieval and Renaissance views. Only in the course of the seventeenth century would contingency be banned from the realm of natural causation in the developments of post-Cartesian mechanism. For philosophers such as Baruch Spinoza and Gottfried Wilhelm Leibniz contingency marked the limitations of our knowledge and not an ontological limitation of nature. As one reads for instance in Spinoza's *Ethica ordine geometrico demonstrata* (Ethics, demonstrated in geometrical order) I 29: "There is no contingency in nature. All natural beings are determined by divine necessity to exist and operate in a special manner." (*In rerum natura nullum datur contingens, sed omnia ex necessitate divinae naturae determinata sunt ad certo modo existendum et operandum*). By contrast, in the Renaissance a mathematical treatment of natural phenomena underlaid no principle of sufficient reason, hence it did not imply the necessity of natural causation. In particular, mixed mathematical disciplines that had received a Scholastic embedment or systematization rested on a well-established Aristotelian conception, according to which sublunary phenomena are determined without necessity.

Historically, *contingentia* is the Latin variant translation of the Aristotelian concept of "possibility," both as modal logical *endechomenon* as well as physical-metaphysical *dynamis* within a hylemorphic framework. In the context of the Christian reception, this terminus received an onto-theological connotation in a frame of creationist theology. In late Scholasticism, *contingentia* came to signify the worldly reality, or nature as Creation. Nature was deemed to be contingent. It exists *de facto* but could also not exist because it depends on God's will. As John Duns Scotus put it,

> So then, the first issue has become clear: how there is contingency in things—because it comes from God—and what is in God which is the cause of this contingency—because it is his will.[24]

In Aristotle, there was a tension between two meanings of "possibility." According to *Analytica Priora* (13: 32 a 18–20) the possible is that which is "neither necessary nor impossible," whereas according to *De interpretatione* (13: 22 a 14–13 a 26) possibility is exclusively that which is opposed to "impossibility" and therefore includes also that which is necessary. As a reminiscence of this original tension, one can find in Scholastic philosophy two different definitions of contingency either as "quod est nec impossibile nec necessarium" (that which is neither impossible nor necessary) or "quod non est impossibile" (that which is not impossible).[25] Both meanings were kept in the Latin rendering of the Aristotelian possibility as *contingentia* by Gaius Marius Victorinus (III–IV cent.

[23] We have first discussed contingency and mechanics in the Renaissance in Omodeo and Renn 2015. A volume entirely devoted to ontological and epistemological contingency in the natural debates of early modernity is Omodeo and Garau 2019.

[24] Duns Scotus 1994, 140: "Sic igitur apparet primum, quomodo est contingentia in rebus, quia a Deo, – et quid est in Deo quod est causa huius contingentiae, quia voluntas eius."

[25] Cf. Vogt 2011, 52. The entire first chapter is relevant for a historical overview of the reception and transformation of the Aristotelian concept of "possibility" as "contingency" in the Latin tradition.

CE) and Boethius (IV–V cent. CE), but the Latin expression also suggested affinity between that which is contingent (*contingit*) and that which occurs (*evenit* or *accidit*).[26] This third connotation would eventually prevail through the late-Scholastic differentiation between *contingentia* and *possibilitas* and its reception in the philosophical systems of the seventeenth century (and most notably by Leibniz).[27] Unlike abstract (purely logical) possibility, contingency referred only to that which is real but not so by necessity: "id, quod [est sed] potest non esse" (that which [is but] could not be). In the Christian perspective of the Almighty's Creation, contingency happened to include all that is not God himself, that is to say, nature, or the universe.

This background is fundamental to understand not only theological disputes but also natural philosophical and scientific developments during the Middle Ages and the Early Modern Period. The connotation of nature as contingent—as that "which could not be"—is theological and metaphysical in its essence, since it points to the dependency of the world on God. However, from the point of view of natural conceptualizations, not only the "vertical" dimension of metaphysics is relevant but also the "horizontal" dimension of causality within nature. On the horizontal plane of the interrelation among finite beings, contingency refers to a degree of indetermination, and a certain unpredictability in the connection between causes and effects. Moreover, whereas a theological perspective focuses on the radical contingency of that which exists as created being, natural philosophy addresses the *relationship* between contingency and necessity within nature, that is, between divine order and phenomenal imperfection. This relationship between that which is not necessary and that which is necessary *had to* be conceptualized and indeed was conceptualized as the relationship between the *absolutum* and the *conditionale* or *secundum quid*.

In Book 1 of the *Summa contra gentiles*, Thomas Aquinas defined contingency through its distinction from necessity. In the case of the contingent beings, as one reads in *Summa contra gentiles* I 67, a cause can produce its effect or not, whereas in the case of necessary beings, their cause cannot not produce them:

> The contingent differs from the necessary according to the way each of them is found in its cause. The contingent is in its cause in such a way that it can both not-be and be from it; but the necessary can only be from its cause. […] Just as from a necessary cause an effect follows with certitude, so it follows from a complete contingent cause if it be not impeded.[28]

A contingent cause, as one reads, will fulfill its tendency to produce a certain effect "si non impediatur," that is, if no impediment hinders its realization.

In Book 2 of the *Summa contra gentiles*, Thomas dealt extensively with the contingent being ("*omne quod est possibile esse et non esse*" and "[*id quod*] *ad utrumlibet se habet*").[29] According to him, the world is contingent insofar as it is created. In this general sense, "God is to all things the cause of being" (*Summa contra gentiles* II 15).[30]

[26] Vogt 2011, 50.
[27] Schepers 1965.
[28] Aquinas 1975, 221f: "Contingens a necessario differt secundum quod unumquodque in sua causa est: contingens enim sic in sua causa est ut non esse ex ea possit et esse; necessarium vero non potest ex sua causa nisi esse. […] Ex causa necessaria certitudinaliter sequitur effectus, ita ex causa contingenti completa si non impediatur."
[29] Thomas, *Summa contra gentiles* II,15. Cf. Aquinas 1975, 48: "everything that can be and not-be" and "it is indifferent to either."
[30] Aquinas 1975, 46: "Deus est omnibus causa essendi."

In particular, God's free will is the origin of this world. Nonetheless, Thomas does not exclude that natural reality is populated by both necessary and contingent beings. Absolute necessity (*necessitas absoluta*), he writes in *Summa contra gentiles* II 29, does not pertain to God, since His decision and action is independent from any constriction (*debitum*). Rather, absolute necessity pertains to the immaterial, or "separated" beings as well as to those bodies in which the form fulfills all potentialities of their matter, as is the case with the heavenly bodies transported in circles. As for terrestrial (sublunary) bodies, their forms are imperfectly realized. Matter, as the potentiality to take different forms, is at the origin of their contingency, that is, it is the source of the possibility to realize or not to realize a certain effect: "But in things whose form does not fulfill the total potentiality of the matter, there still remains in the matter potentiality to another form."[31] For the low realm of birth, corruption, and change, Thomas speaks of conditional necessity (*necessitas conditionalis*). In the sublunary sphere, contingency cohabits with absolute necessity (e.g., the inevitability of death for all animals and the hylemorphic composition of all bodies). Whereas necessity pertains to the formal determinations of natural phenomena, contingency is the partial fulfillment of necessary tendencies.[32]

According to Scholastic terminology, there is always a "quid" producing the deviation of material phenomena from their formal rule. We will call this outlook an "ontology and epistemology of contingency."[33] The Pythagoreanism of many Renaissance scholars such as Benedetti did not depart from a view stressing the contingent character of natural phenomena in general. As we will argue, one encounters in Benedetti's physics and mechanics a systematic use of theoretical tools implying natural contingency in the form of a distinction and interrelation between formal mathematical necessity and its material realization. In order to understand Benedetti's mathematical treatment of contingency it is useful to consider the medieval approaches to contingency, especially the science of weights (*scientia de ponderibus*) he relied upon.

The idea of contingency informing physics and mechanics was related to its use in other disciplines, even ethics. Whereas there can be no obstacle impeding the realization of God's will, which is therefore "absolute" (*voluntas absoluta*), human will, or *voluntas secundum quid*, is conditioned by circumstances. In other words, the realization of the highest aims of humankind is intrinsically contingent, as Dante expressed in the *Divine Comedy*:

> But utterance and feeling among mortals,
> For reasons which are evident to you,
> Have different feathers making up their wings.
> I, too, as man feel this disparity […].[34]

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[31] *Summa contra gentiles* II 30: "In quibus [rebus] vero forma non complet totam potentiam materiae, remanet adhuc in materia potentia ad aliam formam." Cf. Aquinas 1975, 87.

[32] *Summa contra gentiles* II 23: "Omnis enim agentis per necessitatem naturae virtus determinatur ad unum effectum. Et inde est quod omnia naturalia semper eveniunt in eodem modo, nisi per impedimentum: non autem voluntaria. Divina autem virtus non ordinatur ad unum effectum tantum […]. Deus non agit per necessitate naturae, sed per voluntatem." Cf. Aquinas 1975, 68: "For the power of every agent which acts by natural necessity is determined to one effect; that is why all natural things invariably happen in the same way, unless there be an obstacle; while voluntary things do not. God's power, however, is not ordered to one effect only […]. Therefore, God acts, not out of natural necessity, but by His will."

[33] Omodeo and Renn 2015.

[34] Alighieri 1984, 94; Dante Alighieri, *Paradiso* XV 79–83:

> "Ma voglia e argomento ne' mortali,
> per la cagion ch'a voi è manifesta,

Apart from ethical contingency, Scholastic authors also used *secundum quid* in logic. For instance, Petrus Hispanus explained the meaning of the so-called *secundum quid* fallacy in his *Tractatus sive summule logicales*, commenting on Aristotle's *On Sophistical Refutations* V (166b36–167a14).[35]

In logic, *secundum quid* meant either a "diminution" of a concept through restriction of its definition (*secundum quid et simpliciter*), or the designation of a subject through one of its parts or characteristics (*denominatio totius per partem*). A *secundum quid* fallacy occurs if an identity is established between something considered in a particular respect and the same thing considered absolutely (or *simpliciter*). For instance, the existence of a depicted animal does not imply the existence of the animal *simpliciter*. Thus, the argument "est animal pictum, ergo est animal" is not correct. In this case, there is a quid signalizing the gap between universal necessity and particular or concrete contingency.

### 4.3.2 Contingent Causation in the *scientia de ponderibus*

The *scientia de ponderibus* heavily drew on the idea of the conditional limitation of natural necessity depending on circumstances (*secundum situationem*, also literally meant as "depending on the position"). In particular, the concept of *gravitas secundum quid*, or positional heaviness, had a powerful explanatory function, most notably in the Aristotelian treatment of weights by Jordanus Nemorarius, and continued to be essential during the Renaissance in the reflections on mechanics by scholars such as Tartaglia, Cardano, and Benedetti himself.[36]

In mechanics the "limitation" or "determination" *secundum quid* implied that the dynamic tendency of a body was reduced or enhanced depending on intervening constraints or circumstances, in particular mechanical ones. The rotations of a lever around a pivot or of a balance around its fulcrum were conceptualized as constrained motions. In such displacements, the inherent ("necessary") vertical tendency of a weight resulted in a circular motion due to external constraints. Similarly, the heaviness (*gravitas*) of the bodies suspended at the extremities of a simple machine varied in relation to their changing positions within the system. In such cases, a "necessary" straightforward motion in accordance with natural order resulted contingently in a circular one. The implicit mental model for this kind of displacement was that "circular motion is constrained rectilinear motion." This means that, in the sublunary sphere of contingency, straightforwardness and rectilinear tendency had a higher onto-epistemological status than circularity since straightforwardness was necessarily rooted in natural order. By contrast, circularity, as the deviation from such order, had to be explained. As a consequence, circularity (in the elementary sphere) was allotted a derived and subordinated onto-epistemological status. In other words, circularity was an instance of nature departing from necessity owing to some rather elusive factor or *secundum quid*. From this viewpoint, it was seen as a deviant realization of given potentialities similar to moral deviation from the necessary laws of uprightness. In order to stress that the mechanical treatment of the *scientia de ponderibus* was embedded in the framework of contingency, we could also formulate the principle in this way: "circular motion is rectilinear motion modified by a contingency."

---

diversamente son pennuti in ali;
ond' io, che son mortal, mi sento in questa
disagguaglianza […]."

Also, see *Paradiso* IV, 87, IV, 109, IV, 113, and *Purgatorio* VII, 57.

[35] Hispanus 1972, 157–158.

[36] See Renn and Damerow 2012, especially the sections from 3.6 to 3.8.

Almost at the beginning of his small treatise "on the weights," Nemorarius stressed his Aristotelian commitment. In fact, his approach was based on the opposition between the *natural* vertical motion of the elements and the *violent* hindrances producing circular deviation. At the same time, he introduced the key concept of *gravitas secundum quid* (in some cases, also *levitas secundum quid*), which we will refer to as "positional heaviness."

> [...] if equal arcs are taken on a greater circle, and on a smaller one, the chord of the arc of the greater circle is longer. From this I can then show that a weight on the arm of a balance becomes lighter, to the extent that it descends along the semicircle. For let it descend from the upper end of the semicircle, descending continuously. I then say that since the longer arc of the circle is more contrary to a straight line than is the shorter arc, the fall of the heavy body along the greater arc is more contrary to the fall which the heavy body would have along the straight line than is a fall through a shorter arc. It is therefore clear that there is more violence in the movement over the longer arc than over the shorter one; otherwise the motion would become heavier. Since something moves with more violence in the ascent [along the arc], it is apparent that there is more positional heaviness [*gravitas secundum situm*] and, as it is like that depending on position [*secundum situationem*], one can aptly call it 'positional heaviness' [*gravitas secundum situm*].[37]

In its circular descent along a circular path, a weight deviates from its natural tendency, or *intentio*, the more the arm of the balance departs from the horizontal position. Therefore, the "violence" is greater when the arc of displacement is longer, while the weight progressively loses its weight insofar as the vertical component in its motion is reduced.

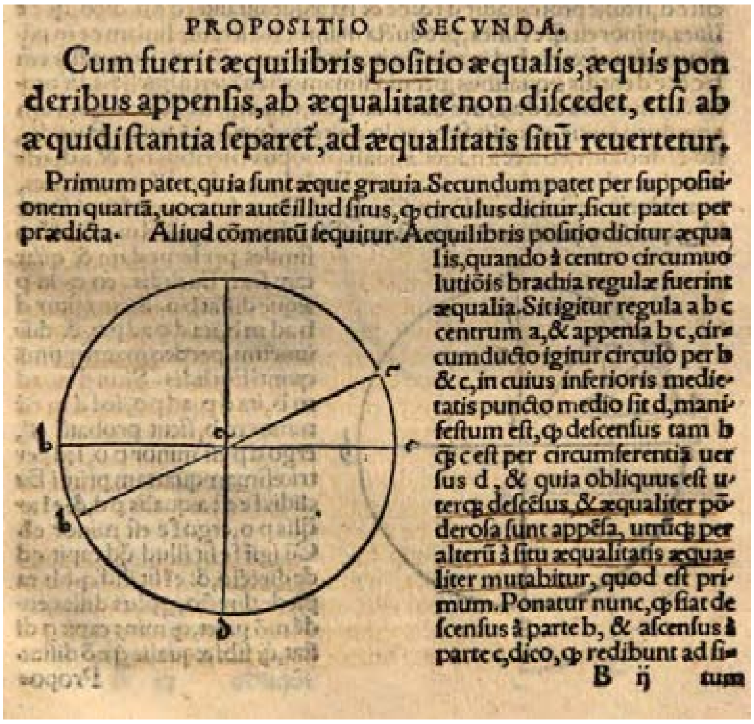
According to Nemorarius, a weight that reaches the bottom of the circular arc described by the arm in its displacement is not "at rest" but only "lighter." In fact, a natural being is at rest only if it is fully accomplished, that is, once it has realized the aim, or act, toward which its power is directed teleologically. By contrast, a body is always in motion, or striving to move, until it has reached its end: "All motion strives toward its aim—indeed the whole nature strives towards actuality and is realized [in it]—hence the opposition occurs against [a displacement] contrary [to the natural tendency]."[38]

A body on one arm of the balance becomes lighter during its downward motion than an equal one located on the other extremity. Thus, as Nemorarius assumes, or tries to demonstrate, a balance removed from its state of equilibrium will tend to restore the original state. As one reads in the *propositio secunda* (with reference to the diagram in Figure 4.1), which is the second of a series of propositions developing the details of Nemorarius's doctrine of weights,

> Suppose now that the descent occurs on the side *B* and the ascent on the side *C*. I say that both will go back to the [horizontal] position of equality. In fact,

[37]Nemore 1533, f. A3*v* (emphasis added): "[...] si sumantur de circulo maiori et minori arcus aequales, corda arcum maioris circuli longior est. Propeterea posset ex hoc ostendi, quod pondus in libra tanto sit levius, quanto plus descendit in semicirculo. Incipiat igitur mobile descendere a summo semicirculi, et descendat continue. Dico tunc quod maior arcus circuli plus contrariatur rectae lineae quam minor, et casus gravis per arcum maiorem, plus contrariatur casui gravis, qui per rectam fieri debet, quam casus per arcum minorem. Patet ergo *maior est violentiam in motus secundum arcum maiorem*, quam secundum minorem. Aliter enim fieret motus magis gravis. Cum ergo plus in ascensu aliquod movetur violentiae, patet, quam maiore est *gravitas secundum situm, et quia secundum situationem* talium sic sit, dicatur gravitas secundum situm."
[38]Nemore 1533, ff. A3*v*–A3*r*: "In termino enim cuiscunque motus intenditur, intenditur et viget tota natura in actu, qui in motu sit quasi in potentia, secundum quem fiebat contrarietatis suae oppositio."

*B* will not further descend, because its descent towards *D* is more oblique than the ascent of *C* towards the [horizontal position of] equality; in fact, *B* and *C* are equidistant from the place of equality.[39]


PROPOSITIO SECVNDA.

Cum fuerit æquilibris positio æqualis, æquis ponderibus appensis, ab æqualitate non discedet, etsi ab æquidistantia separetur, ad æqualitatis sitũ reuertetur.

Primum patet, quia sunt æque grauia. Secundum patet per suppositionem quartã, uocatur autẽ illud situs, ꝙ circulus dicitur, sicut patet per prædicta. Aliud comentũ sequitur. Aequilibris positio dicitur æqualis, quando à centro circumuolutiõis brachia regulæ fuerint æqualia. Sit igitur regula a b c centrum a, & appensa b c, circumducto igitur circulo per b & c, in cuius inferioris medietatis puncto medio sit d, manifestum est, ꝙ descensus tam b ꝙ c est per circumferentiã uersus d, & quia obliquus est uterꝗ descensus, & æqualiter põderosa sunt appẽsa, utrũꝗ per alterũ à situ æqualitatis æqualiter mutabitur, quod est primum. Ponatur nunc, ꝙ fiat descensus à parte b, & ascensus à parte c, dico, ꝙ redibunt ad situm

B ij tum

Figure 4.1: Diagram accompanying preposition two in Apianus's 1533 edition of Nemorarius's *Liber de ponderibus* (1533, f. B2*r*). (Bayerische Staatsbibliothek)

Nemorarius's reasoning becomes clearer in light of propositions four and five:

> Fourth [proposition]: It is positionally heavier, insofar as its descent, in the same position, is less oblique.
>
> Fifth [proposition]: But a more oblique descent partakes less of the straight [descent], for the same quantity [of the path].[40]

In proposition five, it is suggested that the vertical components of the potential descents of the two beams could be identified and compared. This was the source of the idea that the variation of heaviness could also be determined by comparing the straightness of the descents. A similar procedure was later taken up and explained in detail in Niccolò Tartaglia's considerations in the *Questiti et inventioni diverse* (1546) about the manner of ascertaining the positional heaviness of two weights on the basis of the so-called angles of contact. These are the "curvilinear" or "mixed" angles between the circular path of the

[39]Nemore 1533, ff. B2*r*–*v*: "Ponatur nunc, quod fiat descensus a parte B, et ascensus a parte C, dico quod redibunt ad situm aequalitatis. Non enim ulterius descendet B, eo quod descensus eius versus D magis obliquus est, quam ascensus C ad aequalitatem; B enim et C iam aequaliter distant a situ aequalitatis."

[40]Nemore 1533, f. A3*r*: "Quarta [propositio]: Secundum situm gravius esse, quanto in eodem situ minus obliquus est descensus. Quinta [propositio]: Obliquiorem autem descensum minus capere de directo, in eadem quantitate." Translation from Renn and Damerow 2012, 63. For proposition four, see Nemore 1533, f. B3*v*–B4*r* and, for proposition five, Nemore 1533, f. B4*r*–C2*v*.

arms of a balance and the vertical lines connecting the weights to the cosmological center of gravity (see Figure 4.2). Tartaglia compared the angles of contact of two equal weights located on the extremes of a balance, and argued that the lifted one is always smaller than the lowered one. Thus, the lifted weight would face a descent that is more oblique. It would acquire a greater positional heaviness than its lowered counterweight and, as a further consequence, the inclined system would reestablish its horizontal balance, if not hindered to do so.

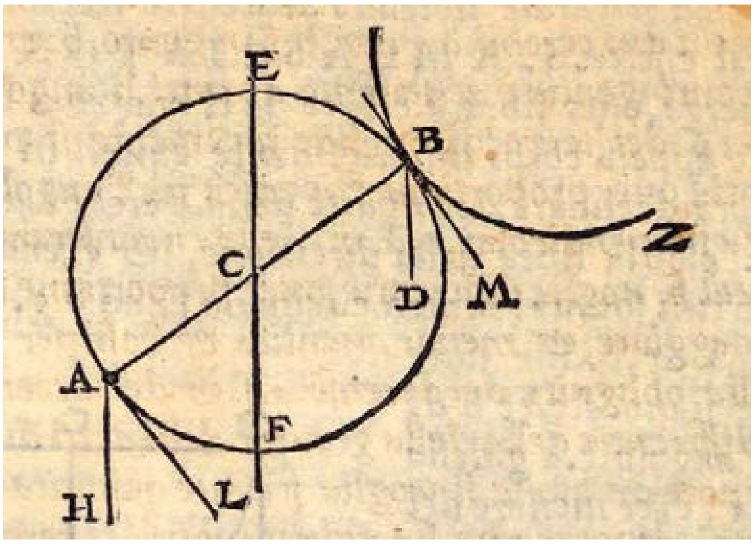



Figure 4.2: In the *Quesiti et invenzioni diverse*, Tartaglia argued that the relative positional heaviness of the weights A and B on a balance could be determined on the basis of the "mixed" angles of contact HAF and DBF. Since it is argued that DBF < HAF, the weight B will be heavier than A. Thus, the inclined system will strive toward the restoration of a horizontal equilibrium. (Max Planck Institute for the History of Science, Library)

In spite of his attempt to quantify the *quid* accounting for the alleged restorative motion of the inclined balance, Tartaglia's geometrical quantification maintained a margin of indeterminancy. As he stated, the ratio between the two mixed angles is less than that between any determined quantities. Therefore, it is impossible to stabilize the system in its inclined position by adding a small (no matter how small) weight on the lowered side of the balance. According to Tartaglia, it is impossible to counterbalance the positional heaviness of the lifted weight. Quite on the contrary, any additional weight added to the lowered side would make the balance rotate and reach the vertical position.[41]

## 4.4 The Epistemological Import of Benedetti's Generalization from Weights to Forces

As we have argued so far, in the medieval *scientia de ponderibus* circular motion is conceived of as constrained linear motion. Yet, within an Aristotelian cosmology, this mental model is restricted to the sublunary sphere, where motions cannot fulfill their nature. This is indeed the sphere of contingency, where a gap is to be witnessed between the necessary

[41] Tartaglia's approach was controversial, already in his time. See Renn and Omodeo 2013, sec. 3.6.

order of things (or "nature" as actuality) and the effective phenomena (subjected to "violence" or to external constraints). The four elements naturally tend toward their places through a straightforward descent or ascent. Heavy bodies, for instance, strive toward the center of gravity, which is, at the same time, the center of the cosmos. If their motion is hindered, as is the case with mechanical constraints, a certain factor or *quid* has to be taken into account, which explains the deviation from the rule. In this theoretical context, contingency is the concept expressing the relationship between the natural law and phenomenal reality, which follows a norm while deviating from it. The *secundum quid* is that which explains this deviation. Possibly, it has to be expressed through geometrical means, although it might prove unintelligible or infinitesimal, as was the case with Tartaglia's ratio between mixed angles accounting for the *gravitas secundum quid* of the weights of a balance. In the treatment of weights, in particular of those on a balance, Nemorarius and his followers made a limited use of the mental model of curvilinear motion as constrained linear motion. In fact, they employed it to account for phenomena linked to gravity (i.e., the vertical fall of bodies explained in Aristotelian terms). It was Benedetti who made the decisive step toward the generalization of this model in the direction of inertial dynamics. Let us consider his application of it first to balances and then to centrifugal forces.

In the section on mechanics of the *Diversae speculationes*, Benedetti picked up and revised the Scholastic concept of *gravitas secundum quid*. Guidobaldo del Monte had already criticized Nemorarius's and his followers' conclusion that an inclined balance hinged on its fulcrum as its center of gravity would return to the horizontal position, but his criticism went so far as to renounce the concept of positional heaviness altogether.[42]

Relying on the Archimedean concept of the center of gravity of a body, Del Monte concluded that an equal-arms balance hinged on its fulcrum would remain stable in any position (a correct conclusion only if it is assumed, in modern terms, that the gravitational field is homogeneous): "*Propositio IV: Libra horizonti aequidistans aequalia in extremitatibus, aequaliterque a centro in ipsa libra collocato, distantia habens pondera; sive inde moveatur, sive minus, ubicunque relicta manebit*." (Fourth Proposition: Take a balance that is equidistant from the horizon and that has weights in its extremities which have the same weight and equally distant from the center (the latter being located in the balance itself). Whether it is displaced or not, it will remain in the same position in any position.)[43]

Benedetti shared the criticism of Nemorarius and Tartaglia with regard to their specific argumentation about the tendency of such an inclined balance to reach the horizontal position but based his judgement on a novel treatment of positional heaviness. The first chapter of Benedetti's *De mechanicis* begins with the statement: "Every weight placed at the end of an arm of a balance has a greater or a lesser heaviness depending on differences in the position of the arm itself."[44]

Hence, he clearly committed himself to a mechanical theory of equilibrium based on positional heaviness. Benedetti's technical terms are not always employed in a rigorous and consistent manner. He treats the *pondus* at times as the varying quantity to be taken into consideration, as is shown by expressions like "*proportio ponderis in C ad idem pondus in F*" and "*unde fit… pondus magis aut minus grave*," in *De mechanicis* II (Benedetti 1585, 142). Given these semantic fluctuations, we will translate *pondus* as "body" or as

[42] Renn and Damerow 2012, 86–92. We will discuss the divergent interpretations of Benedetti and Del Monte later, in chapter 5.
[43] Damerow and Renn 2010, 65.
[44] Drake and Drabkin 1969, 166. Benedetti 1585, 141: "Omne pondus positum in extremitate alicuius brachii librae maiorem, aut minorem gravitatem habet."

"weight" and *gravitas* as "heaviness" or as "weight," depending on the context. At the beginning of chapter 1 of his book on mechanics, Benedetti talks of a varying quantity of heaviness, or gravity (*gravitas*), belonging to a weight (*pondus*) or a body placed on a balance beam. Hence, he makes a terminological distinction between *pondus*, as a kind of absolute weight or heavy thing, and *gravitas*, as a downward tendency that can act with more or less force on the body (depending on the inclination of the beam). In this case (as in most cases in the text), *pondus* has the essentialist meaning of a substance (a substratum or ὑποκείμενον). It is the body or weight on the balance, whose special property of being heavy, namely the *gravitas*, varies depending on a *quid*. This *quid* is the position, or *situm*.

Benedetti seeks to quantify it by means of a method he invented. He considers the line, which he calls *linea inclinationis* or *linea itineris*, connecting a weight on an inclined balance beam to the cosmological center of gravity. Note that Benedetti calls the elementary downward tendency an *iter* from a merely kinematic viewpoint, but also an *inclinatio* from a physical and more proper one. According to him, the major or minor heaviness of the weight can be assessed through the projection of the *linea inclinationis* on the horizontal line passing through the fulcrum (Figure 5.1). The more distant it is from the fulcrum, the heavier the positional heaviness becomes. Thus, the weight reaches a maximum of heaviness when the balance is horizontal, and its minimum when it is vertically resting (*nititur*) on the fulcrum or hanging (*pendet*) from it. Notably, this approach anticipates the one based on the determination of the torque in classical physics, and comes to the same conclusions.[45]

Additionally, Benedetti equates the heaviness to a *virtus*, *vis*, or *vigor*, i.e., a force, which might also act in different directions (in *De mechanicis*, Ch. 3) and is applied to the extremity of a constrained mechanical system, like a lever or a balance. This is a significant generalization from weights to forces, but for our present discussion the most important generalization relates to rectilinear tangential tendencies in systems set in circular motion.[46]

The relevant treatment is the epistle to Capra and is included in the *Diversae speculationes*. It deals with the rotation of a millstone and the question of whether its motion could be perpetual. Benedetti denies this by arguing that the rotation is impeded first by the friction of the air and, second and more importantly, by the resistance of the millstone's parts. The latter have a straightforward tendency, an *inclinatio recte eundi*, along the tangential lines of their rotation (Figure 4.3). As one reads, this rectilinear inclination or impulse (*impetus*) can be bent only by violence. Moreover, the centrifugal tendency grows in proportion to the augmentation of the velocity, as witnessed by other cases, among them the rotation of a catapult or a sling (*machina missilis*). A centrifugal tendency is seen as a rectilinear natural inclination (*naturalis inclinatio recte eundi*).

> You ask me this question in your letter. Suppose a millstone rested on a virtually mathematical point and was set in circular motion, could that circular motion continue without end, assuming that the millstone is perfectly round and smooth?
>
> I answer that this kind of motion will certainly not be perpetual and will not even last long. For apart from the fact that the wheel is constrained by the air which surrounds it and offers resistance to it, there is also resistance from

[45] Renn and Damerow 2012, 138. We will deal with the details of Benedetti's mechanics in the next section.
[46] Cf. Büttner 2008.

> the parts of the moving body itself. When these parts are in motion, they have by nature a tendency [*impetus*] to move along a straight path. Hence, since all the parts are joined, and any one of them is continuous with another, they suffer constraint in moving circularly and they remain joined together in such motion only under compulsion. For the more they move, the more there grows in them the natural tendency to move in a straight line, and therefore the more contrary to their nature is their circular motion. And so they come to rest naturally: for, since it is natural to them, when they are in motion, to move in straight line, it follows that, the more they rotate under compulsion, the more does one part resist the next one and, so to speak, hold back the one in front of it.[47]

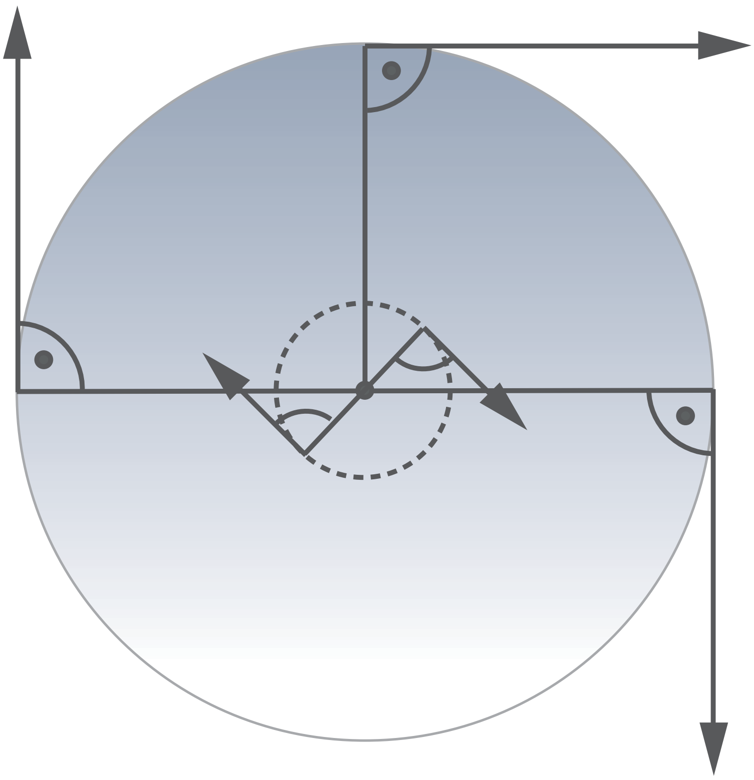


Figure 4.3: A diagram showing Benedetti's considerations on the rotating millstone stressing the centrifugal tendencies of its parts. (Drawing by Irina Tupikova)

The mental model of circular motion as constrained straight motion receives in Benedetti's treatment a higher degree of generalization. In this case, he argues that, since it contrasts

[47] Drake and Drabkin 1969, 229. Benedetti 1585, 285 (emphasis added): "Quaeris a me literis tuis, an motus circularis alicuius molae molendinariae, si super aliquod punctum, quasi mathematicum, quiesceret, posset esse perpetuus, cum aliquando esset mota, supponendo etiam eandem esse perfecte rotundam, et laevigatam. Respondeo huiusmodi motum nullo modo futurum perpetuum, nec etiam multum duraturum, quia praeterquam quem ab aere qui ei circumcirca aliquam resistentiam facit stringitur, est etiam resistentia partium illius corporis moti, quae cum motae sunt, natura, impetum habent efficiendi iter directum, unde cum simul iunctae sint, et earum una continuata cum alia. *Dum circulariter moventur patiuntur violentiam, et in huiusmodi motu per vim unitae manent, quia quanto magis moventur, tanto magis in iis crescit naturalis inclinatio recta eundi, unde tanto magis contra suammet naturam volvuntur*, ita ut secundum naturam quiescant, quia cum eis proprium sit, quando sunt motae, eundi recta, quanto violentius volvuntur, tanto magis una resistit alteri, et quasi retro revocat eam, quae antea reperitur habere."

with a natural inclination, it cannot be eternal. Note that this assumption (violent motion cannot be eternal) is Aristotelian but emerges in a context in which this legacy is meant to be rejected.[48]

Another Aristotelian echo looms over Benedetti's statement that the linear tendency makes a body "lighter," since if it were freed from the constraint hindering its projection, it would not fall vertically but rather travel through a more or less rectilinear trajectory tangent to the circular motion of the constrained rotation. In the conclusion of his reflection on the natural rectilinear striving of the parts of a body set in circular motion, Benedetti stressed the originality of his treatment "without precedents" and its opposition to Aristotelian dynamics (according to which the projection of a body through a medium presupposes the support of the medium itself).

> But if you wish to see this truth more clearly, imagine that while the body, i.e., the top, is spinning around very rapidly, it is cut up or divided into many parts. You will observe not that those parts immediately fall toward the center of the universe, but that they move in a straight line, and, so to speak, horizontally. No one, so far as I know, has previously made this observation on the subject of the top.
>
> From such motion of the top or of a body of this kind it may be clearly seen how mistaken are the Peripatetics on the subject of the forced motion of a body. They hold that the body is driven forward by the air which enters [behind it] to occupy the space left by the body. But actually the opposite effect [that is to say, resistance] is produced by the air.[49]

We have so far observed two instances in Benedetti's work on mechanics in which a tension between mathematical laws of nature and their empirical realization emerges: his treatment of the rotation of a beam about its pole and that of a turning wheel. In both cases, natural straightforward tendencies are constrained and deviated into violent circular ones. The epistemological meaning of these concepts lies in the possibility of a geometrical treatment of natural contingency seen as the connection between the *necessity* of the rules and of the principles and their *necessitation*, that is, their deviation, as witnessed by the empirical reality of curvilinear motions.

## 4.5 From *inclinatio* to *inertia* and Beyond: Mechanistic Perspectives

René Descartes generalized the insights implicit in the idea that curvilinear motion is contingent rectilinearity at an epistemic level (through the expansion of their realm of application) as well as at an epistemological and ontological level (giving them a foundational meaning). In *Le Monde*, circular motion is treated as a deviation from rectilinear motion.

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[48] On Benedetti's anti-Aristotelianism, see Maccagni 1983.

[49] Drake and Drabkin 1969, 229–230. Benedetti 1585, 285: "Sed si clarius, hanc veritatem videre cupis, cogita illud corpus, trochum scilicet, dum velocissime circunducitur secari, seu dividi in multas partes, unde videbis illas omnesque, non illico versus mundi centrum descendere, sed recta orizontaliter, ut ita dicam, moveri. Id quem a nemine adhuc (quem sciam) in trocho est obseruatum. Ab huiusmodi motu trochi, aut huius generis corporis, clare perspicitur, quam errent peripatetici circa motum violentum alicuius corporis, qui existimant aerem qui subintrat ab occupandum locum a corpore relictum, ipsum corpus impellere, cum ab hoc, magis effectus contrarius nascatur."

Descartes develops a general theory of the world in which circularity is the main characteristic of the motions of both the particles of matter as well as of planets revolving about the centers of their orbits.[50]

> […] when a body is moving, even if its motion most often takes place along a curved line and, as we said above, it can never make any movement that is not in some way circular, nevertheless each of its parts individually tends always to continue moving along a straight line. And so the action of these parts, that is, the inclination they have to move, is different from their motion.[51]

This is the third of Descartes's three laws of nature (*loix or règles de la Nature*) as exposed in chapter 7 (" *Des loix de la nature de ce nouveau Monde*" ). It follows the inertial law of conservation of the state of the bodies and that of the conservation of the quantity of motion. The third law is particularly relevant from the viewpoint of our epistemological inquiry into mathematics without necessity, since it clearly expresses the gap between law and effective reality, between the straightforward tendency of all bodies and their real circular motions, in a manner that is akin to medieval and Renaissance predecessors such as Benedetti. Note that Descartes calls the rectilinear tendency " *inclination*" just as Benedetti called it "*inclinatio recte eundi*." This terminological choice is apt to express its character as a natural inner tendency. The examples that Descartes choses to illustrate his claim are familiar to readers of Renaissance sources on mechanics: the wheel (*une roue*) and the sling (*fronde*) (Figure 4.4).

In the *Études galiléennes*, Koyré affirmed the complete independence of the law of inertia, which is only *in nuce* in Galileo's physics, from experience, since rectilinear motion is never observed in nature. "Contrairement à ce qu'on affirme bien souvent, la loi d'inertie n'a pas son origine dans l'expérience du sens commun et n'est ni une généralisation de cette expérience, ni même son idéalisation. Ce que l'on trouve dans l'expérience, c'est le mouvement circulaire ou, plus généralement, le mouvement curviligne. On n'est jamais—sauf le cas exceptionnel de la chute, qui n'est justement pas un mouvement inertial—en présence d'un mouvement rectiligne."[52]

In light of our reconstruction, this statement proves quite inaccurate. As we have seen, the vertical fall of a heavy body is not the only observable straight motion: the beginning of the trajectory of a projectile thrown with great speed also looks rectilinear. Slings and catapults are in fact the instruments with which turning wheels and rotating millstones were compared, and it was from these instruments that Benedetti, Descartes, and also Galileo in the Second Day of the *Dialogo sopra i massimi system del mondo*, derived the centrifugal tendencies of the parts of rotating objects. Is this not a generalization from experience? Such generalization went so far as to include the explanation of the behavior of bodies on a rotating Earth, in the case of Galileo, and the conceptualization of corpuscular and planetary motions, as was the case for Descartes. Moreover, before the classical law of inertia was defined, what took center stage was the observation of rectilinear motions—either the vertical fall or centrifugal tendencies—and of their circular deviations. A major physical problem faced by Scholastic and post-Scholastic mechanics

[50] On the Cartesian cosmos, see Aiton 1972, 30–64 and Gaukroger 2006, 304–317.

[51] Descartes 1998, 29. Descartes 1986, 43–44: " Lors qu' un corps se meut, encore que son mouvement se fasse plus souvent en ligne courbe, et qu' il ne s' en puisse jamais faire aucun, qui ne soit en quelque façon circulaire […], toutesfois chacune de ses parties en particulier tend toujours à continuer la sien en ligne droite. Et ainsi leur action, c' est à dire l' inclination qu' elles ont à se mouvoir, est differente de leur mouvement."

[52] Koyré 1986, 206.

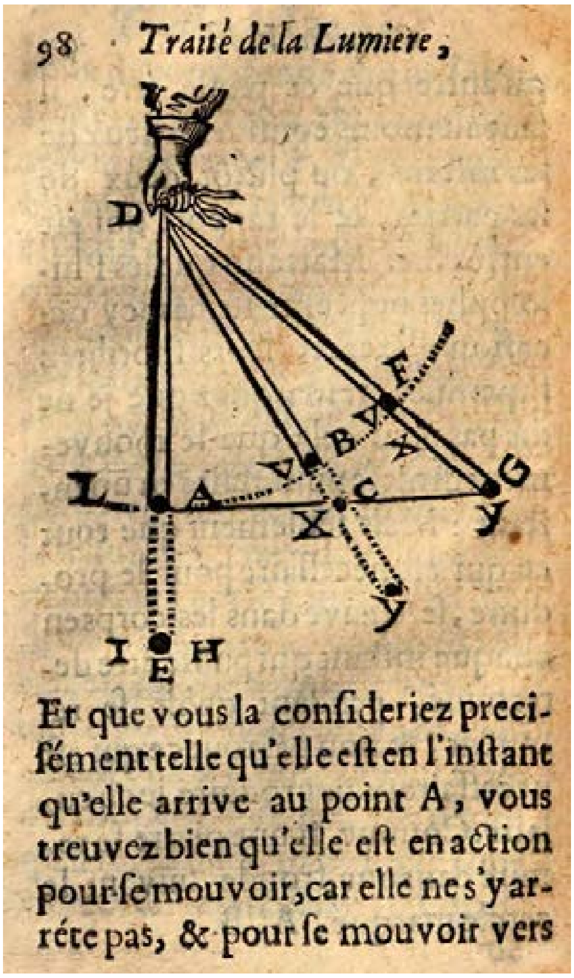


Figure 4.4: Descartes's visualization of the centrifugal tendency of bodies thrown by a sling, in *Le Monde*, Ch.7. (Bayerische Staatsbibliothek)

was precisely that of conceptualizing the relationship between curves and straight lines. In particular, against the backdrop of Aristotelian philosophy, curvilinear motion appeared as constrained. It was a derived displacement resulting from a *violent* external intervention bending the straightforward *natural* tendency of a moving body. In such an Aristotelian and post-Aristotelian context, circular motion was seen as contingent. That is to say, it was the deviation from natural order depending on an obstacle which was called the "secundum quid." As we have argued, the concept of "secundum quid" is embedded in the Scholastic reflections upon natural necessity, order, and contingency. It was referred to as a model of causality in which the observed phenomena represent a partial fulfillment of an underlying order, or of natural laws. Accordingly, elementary bodies express their necessary laws in a limited manner, that is, they have to be explained through the so-called *necessitas conditionata* or *necessitas secundum quid*. Contingency is the relation between necessary order and phenomenal reality. The gap has to be explained, and was explained with a quid, a factor, or a determination. Accordingly, a quid was introduced into mechanics to account for circular motions in terms of mechanical constraints.

In the medieval *scientia de ponderibus*, two determinations were considered for the equilibrium of a balance: first, the circle resulting from the inclusion of the vertical motions of the weights in a mechanical system, and second, the *situm* (location) of the weights in a mechanical system determining a variation in heaviness. The reflection on *gravitas secundum situm* (positional heaviness) from Nemorarius to Benedetti presupposes this twofold *quidditas* and focuses on the latter aspect (the variation of the heaviness).

The conviction that circular motion, as a violent motion, requires an explanation is based on the mental model that "circular motion is constrained (or *contingented*) straight motion." Although they were embedded in the medieval discourse on contingency, the several attempts to quantify the *quid* accounting for the deviation testify to the common effort to overcome the qualitative and indeterminable characterization of contingency as a form of causality. What was maintained, for instance in Descartes, was the idea of a gap between law and phenomenon. Yet, if the deviation from the law can be perfectly quantified, then the separation between the order of nature and its realization is virtually eliminated, that is, the fracture between absolute necessity and conditional necessity is recomposed. To be sure, this step toward the necessitation of nature, resulting from the abandonment of contingency in both senses (causal and epistemological), was accomplished only later, in the course of the seventeenth century.

The work of Benedetti and his onto-epistemology of contingency are representative of an age of transition from Scholastic and Renaissance natural philosophies to the various instantiations of the classical science of the next century. Benedetti's Pythagorean commitment to mathematics, seen as the most powerful logical means applied to all fields of knowledge and to nature in particular, is an illustrative case of the complex and non-linear history of scientific thought. His efforts to overcome Aristotelian conceptions could not really renounce the crucial assumption of the Aristotelian outlook under attack. This particularly concerns the ontology and epistemology underlying his scientific theories and practices. Mathematical determination, both in science and nature, did not imply necessity, neither at the level of material causation nor of explanation. The gap between the laws of nature and the effective processes reflected a Scholastic distinction between formal necessity and material imperfection. Such philosophical assumptions underpinned medieval treatments of phenomena, including statics, and Renaissance developments, especially in the line connecting Tartaglia and Cardano to Benedetti and Descartes. The distinction between formal necessity and phenomenal contingency offered them a horizon within which they could conceptualize general laws as well as their empirical instantiation. In particular, Benedetti could extend the area of application for the mental model that circular motion is a constrained (violent) deviation from the law of rectilinear motion. He did this by applying a model originating from statics to the area of dynamics, thus paving the way for the classical concept of inertia. However, we should not neglect the practical roots of his work in a Scholastic-embedded science of weights, which generalized observations of mechanical systems in order to make universal statements about nature.

# Chapter 5
# Mechanics

The book on mechanics, *De mechanicis*, the third of the *Diversae speculationes*, is divided into twenty-five chapters. Mechanical issues and references to mechanics can also be found in the epistles. As to the discussion of the motion of fall through media and of hydraulic problems, these are not part of this book. *De mechanicis* begins with a brief preamble in which Benedetti claims that he treats topics that have never been dealt with before or have not been sufficiently explained. In this section we will discuss the positioning and controversies implicit in this strong statement in an age when mechanical studies were very lively in the Italian peninsula and abroad. We will first offer an overview of Book 3 of the *Diversae speculationes*. Second, we are going to look more closely at the first foundational chapters of the treatise. Third, we will consider the rivalry with Del Monte, emerging from the latter's harsh criticism of Benedetti and, in part, his misunderstanding of some crucial elements of Benedetti's theory. The context of these lively disputes is the reaction to the publication of Tartaglia's eclectic work on this subject, the *Quesiti, et inventioni diverse* (1546), and his re-issue of the medieval classic on the science of weights. Benedetti, as a critical pupil of Tartaglia, could not sympathize with the absolute rejection of Tartaglia and the medieval tradition his approach rested upon. At the same time, he felt the need to distance himself from several aspects of Tartaglia's treatment, as we will reconstruct in detail in this section. The debates between Benedetti and Del Monte arguably culminated with Galileo's work, which stands out as a sort of synthesis of earlier positions. Understanding these historical developments, as well as the intellectual triangle Benedetti-Del Monte-Galileo, is fundamental in order to trace Benedetti's influence on his contemporaries and on the young Galileo.[1]

## 5.1 An Overview of *De mechanicis*

### 5.1.1 The Foundations of the Theory of the Balance

Chapters 1 to 6 of *De mechanicis* contain a systematic account of the foundation on which Benedetti built his mechanics. Chapter 1 clarifies qualitatively how the variable weight changes depending on the obliqueness of the balance beam. While a body attached to the end of the beam has a maximum weight if the beam is in a horizontal position, it vanishes when the beam is in a vertical position. Benedetti explained this behavior as a consequence of the different extent to which the attached weight rests on the center of the balance. If the position of the beam is close to the vertical, the weight of a body attached to the end of the beam is close to zero since it rests nearly completely on the center of the balance.

Chapter 2 clarifies the positional changing of the weight quantitatively. Benedetti related the balance with an oblique position of the beam to a bent lever with one horizontal and one oblique arm, thus providing the precondition for a generalization of his result. A

[1] Section 5.1 is derived from Renn and Damerow 2012, chap. 6.1–6.3 and section 5.2 from Renn and Omodeo 2013.

generalization of this kind is indeed required if the lines of inclination of the bodies at the end of a balance are conceived as being directed to the center of the earth and hence no longer as being parallel to each other. Benedetti mentioned this possibility at the end of this chapter, but considered the angle between the two directions as being too small to be measured and thus not necessary to be taken into account.

In chapter 3 Benedetti generalized from the downward inclination of a body attached to the balance beam to forces acting upon the body not vertically but making an acute or obtuse angle with the horizontal beam. Accordingly, he replaced the bodies at the end of the balance beam with two weights or two moving forces (*duo pondera, aut duae virtutes moventes*), as he formulated somewhat ambiguously. His derivation of their quantities was based on a reinterpretation of the horizontal distances between the center of the balance and the vertical projections of the bodies at the end of a beam in an oblique position (Figure 5.1). He interpreted these distances as perpendicular distances from the center of the balance to the lines of inclination, and was thus also able to apply the result he achieved for vertically descending weights to lines of inclination caused by forces that are not vertical.

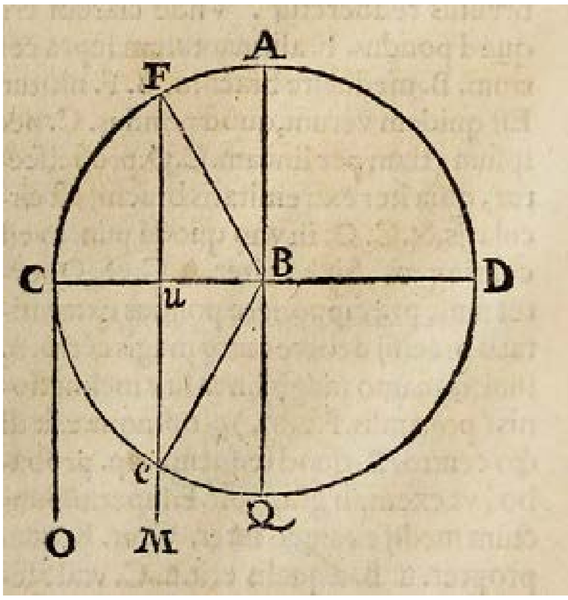


Figure 5.1: Benedetti's diagram showing a balance CBD or FBD. The lines CO and FUEM are the so-called lines of inclination connecting the weights C and F with the center of the elements. The length of the projection on the horizontal is proportional to the positional heaviness. (Max Planck Institute for the History of Science, Library)

Benedetti maintained that his arguments in chapters 1 to 3 clarify all the causes operating on balances and levers. To demonstrate this, he discussed in chapters 4 and 5 the validity of his results if applied to material balances and levers, taking into account that they have a beam with finite extension. This, however, does not imply that he calculated the influence of the weight of the beam itself. His discussion was rather restricted to a justification of his claim that the geometry of a rectangular beam does not require a modification of his propositions. In chapter 5 he treated the case of a lever whose fulcrum is at one of its ends.

Finally, in chapter 6, Benedetti added the description of an instrument used in bakeries for treating the dough. He explained the function of the instrument by applying his proposition from chapter 3.

The systematic approach used by Benedetti in this first part of his treatise is complemented by chapter 9, in which he explained the division of the scale of a steelyard into equal intervals.

### 5.1.2 Criticism of Tartaglia and Nemorarius

In chapters 7 and 8 Benedetti criticized the theorems of his former teacher Tartaglia, in particular those that Tartaglia adapted from Jordanus Nemorarius. Both chapters deal exclusively with some propositions of Book 8 of Tartaglia's *Quesiti, et inventioni diverse*,[2] which is concerned with the science of weights and is entitled, accordingly, *Sopra la scientia di pesi*. In those cases in which Tartaglia's propositions are adapted from Nemorarius, Benedetti mentioned explicitly the corresponding proposition in the edition of Nemorarius' *De ratione ponderis*, corrected and illustrated by Tartaglia, and published under the title *Iordani opusculum de ponderositate Nicolai Tartaleae studio correctum novisque figuris auctum*.[3]

Chapter 7 starts with some brief critical remarks on Tartaglia's propositions 2 to 5. Tartaglia's proposition 2 essentially paraphrases and modifies the Aristotelian claim that the speed of moving bodies is proportional to the driving force. Following Nemorarius, Tartaglia maintained that the velocities of descending heavy bodies of the same kind are proportional to their power (*potentia*), while in the case of ascending bodies their velocities are inversely proportional to their power. For bodies of the same kind their power is conceived here as proportional to their sizes, that is, to their weights. Descending bodies are thus simply falling bodies with velocities proportional to their weights, while in the case of ascending bodies their weight acts as a resistance. Tartaglia's proposition 3 generalizes proposition 2 for bodies with equal weights but unequal positional heaviness. His proposition 4 maintains that in the latter case the power of bodies attached to a balance is proportional to the distances from the center.

Benedetti's critical remarks are somewhat eclectic. He argues that Tartaglia, in his second proposition, does not take into account the quantity of external resistance (*quanti momenti sint extrinsecae resistentiae*). With regard to Tartaglia's third proposition, Benedetti points to its assumptions, namely that the bodies have to be homogenous and must have the same shape. He criticizes Tartaglia's proof as it does not actually require these assumptions, but would also be true for heterogeneous bodies or for bodies with differing shapes. Concerning the fourth proposition, he criticizes Tartaglia for not proving what he claimed to prove. Instead, he should have followed Archimedes's proof of the law of the lever.

Benedetti's chapter 7 continues with a detailed discussion of the second part of Tartaglia's proposition 5 and the following two corollaries and is thus directly concerned with the equilibrium controversy, that is, the controversy about whether or not a balance in equilibrium removed from its horizontal position will automatically return to this position. Tartaglia maintained in this proposition that a balance that is in equilibrium in a horizontal position will necessarily return to this horizontal position when moved into an oblique position. In a first corollary, he claimed that the more the balance beam is brought into an oblique position, the more the bodies attached to it become positionally lighter. In a second corollary, he claimed that while both bodies in this case become positionally lighter, the lifted body loses less of its positional heaviness than the body moving down. He concluded that the beam will return to a horizontal position. Benedetti

[2] Tartaglia 1546.
[3] Nemore 1565.

questioned Tartaglia's approach by referring to the first three chapters of his own treatise, arguing in particular that Tartaglia's second corollary must be wrong. He discussed once more the balance beam in an oblique position, but now without the assumption that the lines of inclination of bodies attached to the balance beam are parallel. Rather, he considered the case that these lines are directed to the center of the world, showing, as we have discussed, that it is not the lifted body, but rather the body that is moved down, which loses less of its positional heaviness.

Benedetti continued in chapter 8 with critical comments on Tartaglia's propositions 6, 7, 8, and 14. Tartaglia's proposition 6 contains the proof of his fallacious claim that the lifted body of an oblique balance beam loses less of its positional heaviness than the body moving down, now modified by the further claim that the difference is smaller than any finite quantity. Tartaglia claimed:

> [...] that the differences between the heaviness of these two bodies is impossible to give or find between two unequal quantities.[4]

Like Del Monte had done before him, but with different results, Benedetti criticized Tartaglia for not taking into account that the lines of inclination are not parallel.

Tartaglia's proposition 7 contains the simple statement that if the arms of a balance are unequal and bodies with equal weights are attached to the ends of the beam the balance will tilt on the side with the longer arm. Benedetti criticized Tartaglia again for not taking into account that the lines of inclination are not parallel, and claimed that in any case Tartaglia did not give the correct cause of the effect.[5]

Tartaglia's proposition 8 formulates, following Nemorarius, the law of the lever in terms of positional heaviness, stating that if the lengths of the parts of the balance beam with unequal arms are inversely proportional to the weights of the bodies attached to them, their positional heaviness will be equal. Benedetti criticized that this proposition is much better demonstrated by Archimedes.

Finally, Tartaglia's propositions 14 and 15 concern Nemorarius's proof of the law of the inclined plane, which from a modern perspective is essentially correct. Benedetti criticized Tartaglia's argument by attributing to it an interpretation of the inclined plane as a balance, with the top of the plane being its center. His criticism, based on the propositions of his chapters 1 to 3, thus completely missed the point of Tartaglia's argument.

### 5.1.3 Criticism of Aristotle's Mechanics

Benedetti's treatise on mechanics continues mainly with critical notes on the Aristotelian *Mechanical Problems*.[6] His notes are as diverse as the Aristotelian *Mechanical Problems* themselves.

Before he embarked on this criticism, Benedetti dealt with the problem of why a steelyard carries a linear gradation in chapter 9.[7] He took into account the weight of the beam and that of the scale by postulating the equilibrium of the balance when no extra weight is added. Then he added weights of one pound on both sides, arguing that, by

[4] Tartaglia 1546, 91*r*: "[...] che la differenzia ch'è fra le gravità de questi dui corpi egli è impossibile a poterla dar, over trovar' fra due quantità inequali." Translation in Drake and Drabkin 1969, 130.
[5] We will discuss Benedetti's criticism in more detail later.
[6] Aristotle 1980. See Rose and Drake 1971 and also the introduction to Nenci 2011.
[7] Benedetti 1585, 152. See Drake and Drabkin 1969, 178.

common science (*scientia communis*),[8] the balance stays in equilibrium if they are placed at equal distances from the fulcrum. He had thus found the mark on the beam that indicates a magnitude of one pound. He then successively placed further weights onto the scale, now arguing from the law of the lever that they must be compensated by distances proportional to their number. He thus avoided the problem of applying the law of the lever directly to a material steelyard, just as one does in practice when gauging such a balance.[9]

In chapters 10 and 11 Benedetti started with critical remarks on Aristotle's first problem. Aristotle asked why larger balances are more accurate than smaller ones.[10] Actually, this concrete physical question is not the focus of the extensive answer the author gave to this problem. Instead, he provided a long proof of the basic explanatory principle which plays a major role in the whole treatise. At the end of the proof Aristotle argued that the same load will move faster on a larger balance, thus making such balances more accurate.[11]

The criticism Benedetti applied to Aristotle's argument has two parts. In chapter 10 Benedetti began by rejecting Aristotle's claim that the circumference of a circle combines concavity with convexity. He then argued against a specific part of Aristotle's proof of his principle which involves the superposition of motions. In this part Aristotle showed that:

> [...] whenever a body is moved in two directions in a fixed ratio it necessarily travels in a straight line.[12]

He concluded:

> [...] if a body travels with two movements with no fixed ratio and in no fixed time, it would be impossible for it to travel in a straight line.[13]

For the Aristotelian author this proposition served as a means to describe circular motion as a result of two movements with no fixed ratio. Benedetti, however, did not relate his criticism to this context. He argued only that Aristotle's inference concerning movements in two directions is not sufficient since a straight movement can result from two quite different motions. This criticism does not really relate to the Aristotelian argument, other than by showing that his entire attempt to derive the behavior of a balance from a principle of circular motion is misguided.

In the same vein, Benedetti's criticism in chapter 11 then deals directly with Aristotle's answer to the question of why larger balances are more accurate than smaller ones. He argued that Aristotle's argument is not well founded since the greater accuracy has nothing to do with the motion of the balance beam but only with the geometrical constellation.[14] To conclude he added a consideration of material balances, arguing according to his own principles that a weight on the larger balance will be positionally more effective.

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[8] In the sixteenth century the term *scientia communis* was used to designate knowledge common to all mathematical sciences, its core being the Euclidean theory of proportions. See Sepper 1996, 153–154.

[9] See the discussion in Damerow, Renn, et al. 2002.

[10] Aristotle 1980, 1, 848 b 1–850 a 2 (337–347).

[11] Aristotle 1980, 1 (347).

[12] Aristotle 1585, 507: "Quandoquidem igitur in proportione fertur aliqua id, quod fertur, super rectam ferri necesse." Translation in Aristotle 1980, 1, 848 b 11–848 b 13 (337).

[13] Aristotle 1585, 508: "Si autem in nulla fertur proportione secundum duas lationes nullo in tempore, rectam esse lationem est impossibile." Translation in Aristotle 1980, 339.

[14] Benedetti 1585, 153; Drake and Drabkin 1969, 180–182.

Benedetti's chapter 12 concerns problems 2 and 3 of the Aristotelian *Mechanical Problems*.[15] Problem 2 raises the question that forms the starting point of the equilibrium controversy:

> If the cord supporting a balance is fixed from above, when after the beam has inclined the weight is removed, the balance returns to its original position. If, however, it is supported from below, then it does not return to its original position. Why is this?[16]

Aristotle implicitly assumed that the balance beam has a certain thickness and weight. It follows as a result of the geometry of the balance in an oblique position that if the beam is fixed from above, a greater part of the beam is on the lifted side of the perpendicular line across the suspension point. Consequently the beam will move back by itself into the horizontal position. The opposite is true for a beam fixed from below. In this case, the greater part of the beam is on the lower side so that it cannot move back into a horizontal position by itself.

Benedetti criticized the first case by arguing that it is not only the weight of the beam that causes it to return to the horizontal position, but also the different distances of the weights in an oblique position from the vertical through the point where the beam is fixed. According to his theory of the dependency of the weight on the obliqueness of the beam, the weights must be different on both sides. Benedetti thus generalized Aristotle's argument to the case of a balance without a material beam carrying weight itself.

In the second case of a beam supported from below, he argued that Aristotle is completely mistaken. Benedetti maintained that the beam will not remain in its oblique position, but that the lower part will move down until the beam is in the vertical position.

Problem 3 of the Aristotelian *Mechanical Problems*[17] concerning an explanation of the effect of a lever is, for Benedetti, not worth the effort of a detailed criticism. He only briefly notes that Aristotle did not give the true cause, which one will find in his own theory presented in chapters 4 and 5.[18]

In the very short chapter 13, Benedetti criticized problem 6 of the Aristotelian *Mechanical Problems*:

> Why is it that the higher the yard-arm, the faster the ship travels with the same sail and the same wind?[19]

The Aristotelian answer provided in the *Mechanical Problems* is based on an interpretation of the yard-arm as a lever that has its base at the point where the yard-arm is fixed as the fulcrum. Benedetti maintained that this interpretation of the yard-arm as a lever:

> […] does not give the true explanation. For on this kind of explanation the ship would have to move more slowly rather than more swiftly. For the higher

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[15] Aristotle 1980, 347–355; Drake and Drabkin 1969, 182–183.

[16] Aristotle 1585, 511: "Cur siquidem sursum fuerit spartum, quando deorsum lato pondere, quispiam id admovet, rursum ascendit libra: si autem deorsum constitutum fuerit, non ascendit, sed manet?" Translation in Aristotle 1980, 347–349.

[17] Aristotle 1980, 353–355.

[18] Benedetti 1585, 154; Drake and Drabkin 1969, 183.

[19] Aristotle 1585, 515: "Cur quando antenna sublimior fuerit, iisdem velis, et vento eodem celerius feruntur navigia?" Translation in Aristotle 1980, 361.

> the sail that is struck by the force of the wind, the more the ship's prow will be submerged in the water.[20]

Benedetti added one sentence with his own explanation, according to which the ship with a higher sail moves more swiftly because the wind blows more strongly in the higher region.

Chapter 14 provides a long discussion of problem 8 of the Aristotelian *Mechanical Problems*. The question posed in this problem is why round and circular bodies are easiest to move. Three examples are mentioned and later discussed: the wheels of a carriage, the wheels of a pulley, and the potter's wheel. Benedetti claimed that Aristotle's answer to the question he posed is not sufficient. Nevertheless, Benedetti himself argued essentially in a similar manner, only somewhat more extensively. Both of them argued that the circle, contrary to differently shaped bodies, touches a plane only at one point which can be considered as the fulcrum of a lever. But Benedetti added a further argument which is not given by Aristotle. He argued that a circle can be pulled along a plane without difficulty and resistance:

> [...] because in such a case the center will never change its position by moving upward from below, i.e., will never change its position with respect to the distance or interval which lies beween it and line *AD*.[21]

At the end of the chapter, Benedetti discussed the question of why a potter's wheel set into motion by an external force will continue to rotate for a time but not forever. In his response he took into account the friction with the support of the wheel and with the surrounding air. But he also discussed reasons that are more deeply concerned with the nature of such motion, as we have discussed above. He claimed, in particular, that the rotational motion is not a *natural motion* of the wheel, evidently making reference to the Aristotelian distinction between natural and violent motions. He also claimed that a body moving by itself because an *impetus* has been impressed upon it by an external force has a natural tendency to move along a rectilinear path. This statement seems to comes close to the principle of inertia of classical physics, but it actually deals with rectilinear motion as a forced motion and does not involve any assertion about its uniformity. Benedetti seems to suggest, in any case, that this natural tendency is in conflict with the forced rotational motion of the wheel, which in turn slows it down. The smaller the wheel and the more its parts are constrained to deviate from the rectilinear path, the greater the decrease in speed will be.[22]

In chapters 15 and 16 Benedetti dealt with issues of scale as they are brought up by the Aristotelian *Mechanical Problems*. In chapter 15, consisting merely of one short sentence, Benedetti referred to his own earlier treatment of Aristotle's question of why larger balances are more exact (erroneously citing chapter 10 instead of chapter 11 of his treatise) in order to deal with the ninth problem of the Aristotelian *Mechanical Problems*, which reads:

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[20]Benedetti 1585, 155: "[...] verum non est. Huiusmodi enim ratione navis tardius potius, quam velocius ferri deberet, quia quanto altius est velum, vi venti impulsum, tanto magis proram ipsius navis in aquam demergit." Translation in Drake and Drabkin 1969, 183.

[21]Benedetti 1585, 155: "[...] quia huiusmodi centrum ab inferiori parte ad superiorem, nunquam mutabit situm respectu distantiae seu intervalli, quae inter ipsum lineamque *AD* intercedit." Translation in Drake and Drabkin 1969, 184.

[22]For the historical context, see Büttner 2008.

> Why is it that we can move things raised and drawn more easily and more quickly by means of greater circles?[23]

In chapter 16 he discussed the tenth problem of the Aristotelian *Mechanical Problems*, which reads:

> Why is a balance moved more easily when it is without a weight than when it has one?[24]

In his detailed response to this problem—indeed much more detailed than the one found in the Aristotelian text—Benedetti compared balances that are alike with different sets of weights on their scales, one with two weights of one ounce, the other with two weights of one pound. He then added a half-ounce weight on one side of each balance and observed that the balance with the smaller weights moves more rapidly. He explained this effect by referring to the dynamical assumption that one always has to consider *the ratio of the moving force to the body moved*.

In chapter 17 Benedetti addressed the twelfth problem of the Aristotelian *Mechanical Problems*, which reads:

> Why does a missile travel further from the sling than from the hand?[25]

Benedetti's response is based on the concept of *impetus*, conceived as an intrinsic cause of motion originally acquired by the action of an external force that then gradually decreases after separation from the original mover. He argued that a greater impetus can be impressed by the sling due to the repeated revolutions which evidently lead to an accumulation of this intrinsic force. He observed that the impetus would lead, if not impeded by the sling or the hand, to a straight motion of the projectile along the tangent to the circle of its forced motion. He also noted—distancing himself from a claim made by Tartaglia—that the motion due to the impressed force can mingle with the projectile's natural motion downward, thus leading to a curved trajectory. It may well be the case that it was this claim that later convinced Galileo and Del Monte to perform their experiment on projectile motion from which they drew the conclusion that such a mixture of motions indeed takes place.[26]

In chapter 18 Benedetti considered problem 13 of the Aristotelian *Mechanical Problems* dealing with the question of why larger handles can be moved more easily around a spindle than smaller ones.[27] In his short response Benedetti simply referred to the fourth and fifth chapters of his own treatise, stressing that everything depends on the lever. He was evidently convinced that the Aristotelian reduction of such problems to properties of the circle is superfluous, if not misguided.

In chapter 19 he handled problem 14 of the Aristotelian *Mechanical Problems* in the same way. It reads:

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[23] Aristotle 1585, 517: "Cur ea, quae per maiores circulos tolluntur et trahuntur, facilius et citius moveri contingit [...]?" Translation in Aristotle 1980, 365.

[24] Aristotle 1585, 517: "Cur facilius quando sine pondere est, movetur libra, quam cum pondus habet?" Translation in Aristotle 1980, 365.

[25] Aristotle 1585, 518: "Cur longius feruntur missilia funda, quam manu missa [...]?" Translation in Aristotle 1980, 367.

[26] See the discussion in Renn, Damerow, and Rieger 2001.

[27] Aristotle 1980, 367.

> Why is a piece of wood of equal size more easily broken over the knee, if one holds it at equal distance far away from the knee to break it, than if one holds it by the knee and quite close to it?[28]

Again, Benedetti just referred to the earlier chapters of his treatise.

In chapter 20 Benedetti reconsidered problem 17 of the Aristotelian *Mechanical Problems*, which reads:

> Why are great weights and bodies of considerable size split by a small wedge, and why does it exert great pressure?[29]

In the Aristotelian text, the answer is based on interpreting the wedge as two levers opposite to each other, their fulcra being placed at the entry points of the wedge into the wood. Benedetti, however, disagreed with the identification of the two levers allowing the action of the wedge to be interpreted in terms of force, fulcrum, and resistance. He claimed that the fulcrum is actually placed just underneath the deepest point of the opening produced by the wedge entering a block of wood.

In chapter 21 Benedetti claimed to provide the true explanation of compound pulleys. He reduced a compound pulley to a chain of balances by appropriately identifying forces and fulcra, each wheel of the pulley corresponding to one balance.

In chapter 22 Benedetti discussed Aristotle's wheel, i.e., problem 24 of the Aristotelian *Mechanical Problems*, which reads:

> A difficulty arises as to how it is that a greater circle, when it revolves, traces out a path of the same length as a smaller circle, if the two are concentric.[30]

While the author of the *Mechanical Problems* referred to dynamical reasons in explaining this apparent paradox, Benedetti resorted to a kinematic argument, a pointwise reconstruction of the trajectory of the motion of a point on the circumference, arguing that it results from a superposition of two motions. In the case in which the motion is controlled by the larger circle, a point on the circumference of the smaller circle traverses a path resulting from an *addition* of two motions. In the case in which the motion is controlled by the smaller circle, a point on the circumference of the larger circle traverses a path resulting from a *subtraction* of two motions.

Chapter 23 of Benedetti's treatise does not exist.[31] In chapter 24 Benedetti discussed problem 30 of the Aristotelian *Mechanical Problems*, which reads:

> Why is it that when men stand up, they rise by making an acute angle between the lower leg and the thigh, and between the trunk and the thigh?[32]

In his response Benedetti suggested that the reason for this behavior is to create an equilibrium of the body with regard to the line that serves as support underfoot.

In chapter 25 Benedetti addressed the last problem, problem 35 of the Aristotelian *Mechanical Problems*, which reads:

[28] Aristotle 1585, 518: "Cur eiusdem magnitudinis lignum facilius genus frangitur, si quispiam aequi diductis manibus extrema comprehendens fregerit, quam si iuxta genu?" Translation in Aristotle 1980, 369.

[29] Aristotle 1585, 520: "Cur a parvo existente cuneo magna scinduntur pondera, et corporum moles, validaque sit impressio?" Translation in Aristotle 1980, 371.

[30] Aristotle 1585, 525: "Dubitatur quam ob causam maior circulus aequalem minori circulo convolvitur lineam, quando circa idem centrum fuerint positi." Translation in Aristotle 1980, 387.

[31] In Drake and Drabkin 1969, 193; chapter 22 is erroneously numbered as chapter 23.

[32] Aristotle 1585, 532: "Cur surgentes omnes, femori crus ad acutum constituentes angulum, et thoraci similiter femur, surgunt?" Translation in Aristotle 1980, 403–405.

> Why do objects which are travelling in eddying water all finish their movement in the middle?[33]

Benedetti's answer simply referred to the fact that whirlpools are depressed in their middle without giving an explanation of this phenomenon. He could thus restrict himself to arguing that the motion of an object to the center of such a whirlpool is simply its natural downward motion. The final comment by Benedetti is a remarkable conclusion to his criticism of Aristotle as well as his treatise on mechanics:

> But in the case of all those other problems that I have omitted, Aristotle's explanations are correct.[34]

## 5.2 The Beginning of Benedetti's Mechanics

After our overview of Benedetti's book on mechanics, we concentrate on the theses he expounded in the first chapters because they have a foundational character and proved particularly controversial, at least in light of Del Monte's criticism, which we are aware of from the comments he made in one of his notebooks and from marginal notes in his own copy of Benedetti's book.

### 5.2.1 *De mechanicis* I: "On the different positions of balance beams"

In chapter 1, Benedetti notes that "a body (*pondus*) […] acquires a larger or smaller weight (*gravitas*) depending on the different ratio of the beam's position" (*pondus… maiorem, aut minorem gravitatem habet, pro diversa ratione situs ipsius brachii*). According to him, a body has the greatest heaviness when the beam at whose extremity it is loaded is in the horizontal position. His idea is based on a simple common-sense intuition: if one considers an equal-arms balance suspended at its center, the weight of a loaded body is:

- borne entirely by the fulcrum when resting vertically upon it,
- entirely hanging on the fulcrum when suspended vertically below it,
- not supported in any way by the fulcrum when the beam is in the horizontal position.

In the first case, the body completely rests or leans on the center (*nititur*), and the center in turn hinders (*impellet*) the downward tendency of the weight. In the second case, the body is suspended vertically (*pendet*) and the center "attracts" it (*attrahet*), in the sense that it hinders its natural tendency to fall down (*inclinatio*). Hence, the body attains its maximum weight in the third case. If the balance beam moves upward, departing from the horizontal position, the weight slowly decreases and reaches its minimum at the top when the beam is in the vertical position. If the rotatory motion around the fulcrum continues, now downward, the weight increases again until it reaches its maximum in the horizontal position. It then diminishes until it is suspended entirely below the fulcrum. Benedetti visualizes these variations of weight depending on the position (*situs*) in a diagram comparing the lines connecting the weight to the center of the world in different cases, more specifically if the beam is:

- horizontal,

[33] Aristotle 1585, 533: "Cur ea quae in vorticosis feruntur aquis, ad medium tandem aguntur omnia?" Translation in Aristotle 1980, 409.

[34] Benedetti 1585, 167: "[…] a quo aliarum omnium quaestionum, quas ego omisi rationes sunt bene propositae." Translation in Drake and Drabkin 1969, 196.

- raised upward, or
- moved downward with the same angle as in the second case (which is equivalent to 2).

The parallel lines, called *lineae inclinationis* or *lineae itineris*, indicate the direction in which a body would fall if it were free. The closer these lines are to the center of the beam, Benedetti says, the "less heavy" the body becomes.

In his own copy of Benedetti's book, Del Monte wrote a brief annotation in the margin of chapter 1: "this first chapter is derived entirely from our treatise on the balance in the *Mechanicorum liber*."[35] Clearly, he sought to assert the relevance of his treatise for Benedetti's speculations, in spite of the latter's claims of originality. It should be remarked, however, that Del Monte's treatment of the balance, based on the concept of center of gravity, was significantly different from Benedetti's, which was based on an original reworking of *positional heaviness*. Del Monte merely reassessed a concept received from authors such as Jordanus Nemorarius, Tartaglia, and Cardano, all of whom he personally opposed. In his book on mechanics, Del Monte had in fact criticized the concept of *positional heaviness*. Downplaying Benedetti's theory as a repetition of his predecessor's theories, he could therefore claim that his own treatment already included a summary (as well as a criticism) of Benedetti's approach.

### 5.2.2 *De mechanicis* II: On the proportion of weights at the extremities of a balance beam in a position other than the horizontal

In chapter 2, Benedetti deals with the proportions of a weight placed at the extremity of a balance beam if its position is not horizontal (*De proportione ponderis extremitatis brachii librae in diverso situ ab orizontalis*). The thesis to be demonstrated is the following: "The proportion between [the weight of] a body (*pondus*) at *C* and [the weight of] the same body (*pondus*) at *F* corresponds to that between the whole beam *BC* and its part *BU*, which is [set on the beam *BC* and is] delimitated by the fulcrum and the [intersection between the beam and the] inclination line *FUM* that connects the weight at *F* to the center of the world" (Benedetti 1585, 142). For the sake of simplicity, we will represent these relations symbolically in modern terms:

$$C : F = BC : BU$$

where *C* is the weight in the horizontal position and *F* in the inclined position; *BC* is the beam and *BU* the part of the beam *BC* between the center *B* and the perpendicular line drawn from *F*.

Benedetti's demonstration is as follows. He imagines placing a weight *D* on the other extremity of the balance that has the same proportion to *C* as *F*, that is, the following proportion expressed in modern terms:

$$D : C = BU : BC.$$

In accordance with Archimedes's *De ponderibus* I. 6, the balance will be stable if the weight *C* is loaded at *U*, since weights and distances from the fulcrum are proportional by supposition.

[35]"Hoc primum caput to[tum] desumptum est a n[ostro] *Mechanicorum libri* tractatu de lib[ra]."

The next step is to show that *F* : *C* = *BF* : *BU* (where *BF* is the beam, hence *BF* = *BC*). In order to demonstrate this, Benedetti resorts to the mental model (*imaginemur*) of a string hanging vertically from *F*, to which a weight equal to *C* is suspended. He claims that it is visually evident that the weight has the same effect at *F* as at *U*. The same is valid for the case in which the weight is suspended from *U* and intersects the circumference described by the rotation of the beam at a point *E*. In both cases, the balance would remain horizontal since the weight *C* at *F*, *U*, or *E* would balance the weight at *D*. Benedetti further argues that the balance under consideration can be treated like a bent lever with a horizontal and an inclined arm (*FBD* or *EBD*): "si brachium BE consolidatum fuisset [...]" (If the beam *BE* was made solid [...]).

The author concluded that his reasoning has satisfactorily demonstrated his thesis: "A body (*pondus*) is more or less heavy (*grave*) the more or less it hangs from (*pendet*) or rests on (*nititur*) the fulcrum" (Benedetti 1585, 142). And he deems this resting on or hanging from the fulcrum to be the most direct cause (*haec est causa proxima, et per se*) of the positional changing of a weight.

As an additional commentary, Benedetti remarks that in his diagram he supposes the inclination line *CO* to be perpendicular to *CB* and parallel to *BQ*, whereas *CO* and *BQ* in fact converge at the center of the sphere of the elements (*centrum regionis elementaris*), that is, the earth. But for the sake of his present argumentation, this angle is negligible and one may simply assume perpendicularity and parallelism. Benedetti thus developed a method to quantify positional heaviness that corresponds to the modern concept of "torque."

## 5.3 Del Monte's Criticism Concerning the Non-Negligibility of the World's Center

As will be shown in the following section, it was only in his initial treatment of the inclined balance, in chapter 1 of *De mechanicis*, that Benedetti neglected to consider the convergence of the inclination lines to the center of the elements. This omission gave rise to criticism. Del Monte severely criticized both this assumption and Benedetti's reasoning in general in *De mechanicis*, in his handwritten notes on scientific and technical matters known as *Meditatiunculae de rebus mathematicis*. In his notes he assessed Benedetti's arguments from his perspective, relying on the concept of the center of gravity as it was developed in his own book on mechanics.

In a marginal note to the *Diversae speculationes* (Figure 5.2), Del Monte expressed his disagreement with Benedetti's conclusion: "Thus, in this manner, a weight (*pondus*) more or less hangs from or rests on the center; this is the next cause and the [cause] in itself [of the variation in heaviness]."[36] His disagreement reads as follows:

> because that [that is, the greater or smaller extent to which a weight rests at the center] is neither the next [cause] nor the [cause] in itself. For the weight at *F* of the arm *BF* is not equally heavy as the weight *U* of the arm *BU*; nor is the weight at *E* of the arm *BE* equally heavy as the weight at *U* of the arm *BU*. Thus, this entire demonstration is false.[37]

[36]"[...] unde fit ut hoc modo pondus magis aut minus a centro pendet aut eidem nititur: atque haec est cause proxima, et per se [...]."

[37]See Renn and Damerow 2012, 207: "non est neque proxima neque per se; nam [pond]us in *F* brachii [*BF*] non est equegrave ut pondus in *U* brachii *BU*; [nec] pondus in *E* brachii *BE* est equegrave ut pondus [in] *U* brachii *BU*. Unde tota haec demonstratio falsa est."

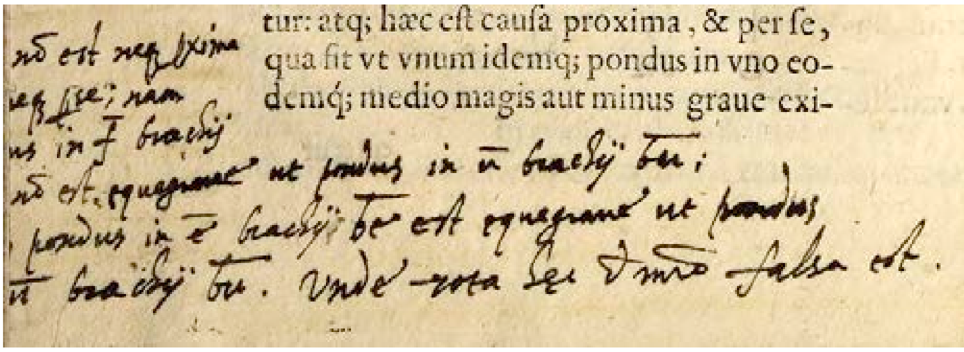
tur: atq; hæc est causa proxima, & per se,
qua fit vt vnum idemq; pondus in vno eo-
demq; medio magis aut minus graue exi-

Figure 5.2: Del Monte's marginal note to *De mechanicis*, II. (Max Planck Institute for the History of Science, Library)

This means that Del Monte did not accept the claim that a weight is equally heavy in different positions on the balance beam, provided the projections of the beam along the horizontal are the same length or rather, as Benedetti writes, the distances between the projections of the beam on the horizontal and the center have the same lengths.

To find Del Monte's counter-arguments, one must look to the *Meditatiunculae*, f. 145, *Contra Cap. 2 Jo. de Benedicti de Mechanicis*. As mentioned, he basically rejected Benedetti's perspective by objecting that he did not take into due account the finite distance of the weights from the center of the world and hence the fact that the plumb lines are not parallel to each other, as Benedetti assumed in this part of his treatise.

In his diagram (Figure 5.3) Del Monte compared the line *LUS* (parallel to the line *AQ*, connecting the fulcrum *B* of the balance with the center of the world *M*) with the line *FM* (connecting the upper weight *F* and the lower weight *E* with the center of the world *M*). *S* is the point at which the line *LUS* meets the circle that the beam makes around the fulcrum, which is above the position of the lower weight *E*.

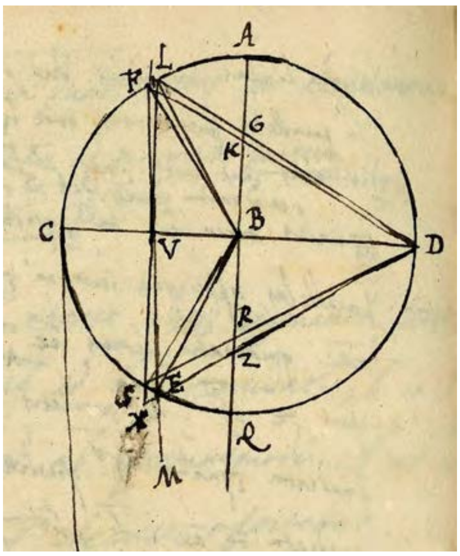


Figure 5.3: Del Monte's critical reworking of Benedetti's diagram in *Meditatiunculae*, f. 145. (Bibliothèque Nationale de France)

Next, he considered a bent lever made of the oblique arm *BS*, rigidly connected to the straight arm *BD*, assuming that *BU* is half *BD*. If a weight is now placed at *S* that is double the weight at *D*, the bent lever will be in equilibrium, as Del Monte showed with reference to his book, because the center of gravity of the weights at *S* and at *D* will be at the point *R*, which will be in its lowest place on the vertical line *BQ*. He therefore concluded that it is the weight at *S*, but not the lower weight *E*, that will be equally heavy as the weight at *U*.

He proceeded to demonstrate this in greater detail by considering the proportions into which the line connecting the two weights is cut by the perpendicular *BQ* for the two cases, that is, the weight placed at *S* and the weight placed at *E*. Del Monte concluded that the same weight is heavier at *S* than at *E*. He then turned to a closer consideration of the upper weight *F*. Again he constructed a bent lever *LBD* in equilibrium in order to compare it with the bent lever formed with the upper weight *F*. Again he showed that the weight is heavier at *L* than at *F*.

Del Monte concluded by summarizing that the entire fallacy is due to Benedetti assumption that the weight at *F* would gravitate in the same way as at *U*, which would only be the case, according to Del Monte, if it were to hang freely.

## 5.4 Benedetti on Weights and Forces Acting on a Balance

Chapter 3 of Benedetti's *De Mechanicis* contains a generalization of the results of chapter 2 or, rather, presents a general rule concerning the action of forces (*virtutes*) on balance beams, including in the case that they do not act vertically downward but also with an acute or obtuse angle. Benedetti moves forward from the result of the previous chapter as follows: the length of the line perpendicularly connecting the center to the line of inclination (the line *BU* in the diagram) allows the quantity of the positional force (*quantitas virtutis... in... situ*) of a weight (*F* in the diagram) to be established. Thus, Benedetti calls the positional weight a force, and this is the presupposition that allows him to generalize from *gravitas* the action which he calls *virtutes moventes*, or "moving forces." The thesis of this chapter is summarized in its title: "That the quantity of any given weight (*pondus*) or moving force in relation to another quantity can be determined thanks to the perpendicular projections connecting the center of the balance to the line of inclination."

Benedetti draws two diagrams showing a balance at whose extremities two weights or forces act in different directions (Figure 5.4). At the left extremity *B*, a weight *E* has a downward tendency, while at the right extremity, a weight *C* acts making an acute or an obtuse angle. According to Benedetti, the length of the perpendicular projection drawn from the center to the inclination line, *OT*, permits the determination of the distance *OI* on the beam at which the same force acting vertically downward produces the same effect. Given this equation, Benedetti can determine how much the force acting in a non-perpendicular direction has to be augmented in order to balance an equal weight acting perpendicularly on the opposite beam. This measure is given according to the following proportion (expressed in modern terms):

$$E : C = BO : OI$$

where *E* is the weight acting vertically on the extremity *B*; *C* is the *virtus movens* acting on the opposite extremity *A* at an angle; *BO* is the left beam and *OI* the part of the right beam *OA* determined as explained above.

In his argumentation, Benedetti thus equates a balance (*BOI*) with a bent lever (*BOT*). Accepting this equation, he concluded that, according to commonly shared knowledge (*communi quadam scientia*), the weights or forces that are required to obtain a perfect balance can easily be calculated.

The chapter ends with a cosmological corollary: "The closer the center *O* of the balance is to the center of the elementary sphere, the less heavy (*minus grave*) it becomes." In fact, the angles between the beam and the inclination lines become progressively smaller.

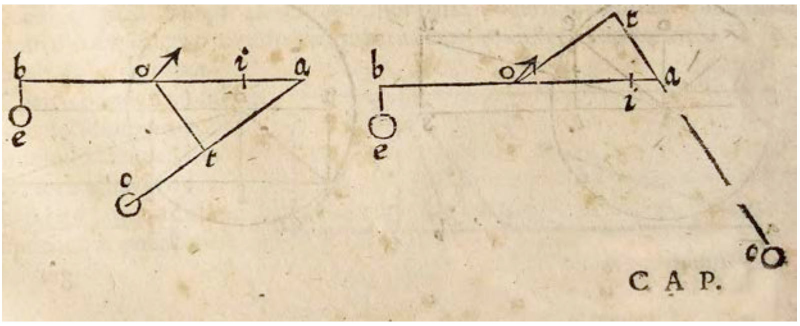


Figure 5.4: Benedetti's representation of forces acting on a balance in arbitrary directions. (Max Planck Institute for the History of Science, Library)

## 5.5 Del Monte's Misunderstanding

In his notes on folio 146 of the *Meditatiunculae*, Del Monte grappled with Benedetti's instructions on how to determine positional heaviness in the case of forces acting in an arbitrary direction. These he refuted at length under the erroneous assumption that Benedetti had claimed forces can be indiscriminately replaced by weights. Like Benedetti, Del Monte considered a bent lever *BOAC* with fulcrum *O*, weights *E* and *C*, a straight arm *BO*, and a bent arm *OAC* to discuss the two cases of an acute and an obtuse angle *BAC* (Figure 5.5).

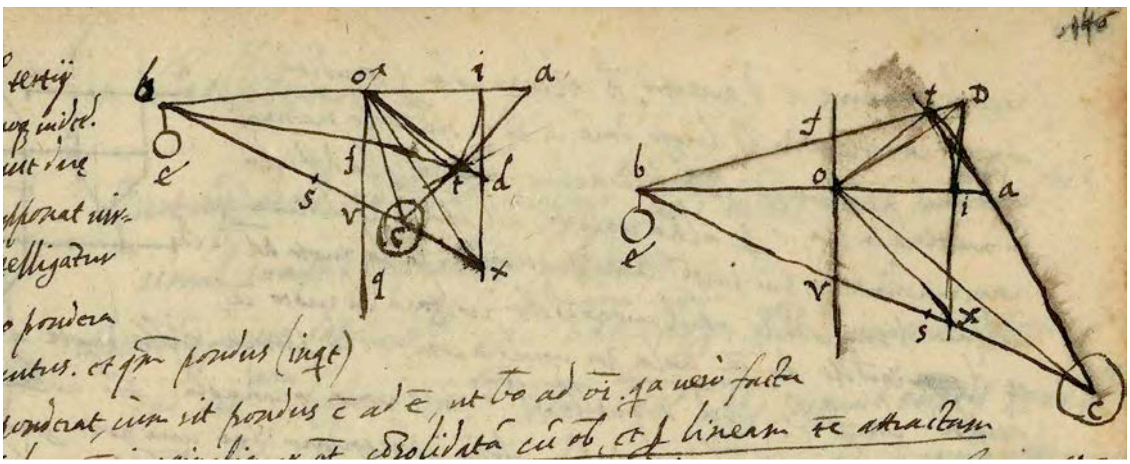

Figure 5.5: Del Monte's critical reworking of Benedetti's representation of forces acting on a balance in arbitrary directions. (Bibliothèque Nationale de France)

He first recapitulated Benedetti's procedure, assuming that a vertical line *OT* drawn from the fulcrum to the line *AC* represented the oblique arm of the bent lever. He stated that when the weight *C* is placed at the end of the horizontal line *OI*, whose length is the same as that of the perpendicular *OT*, according to Benedetti it will be in equilibrium with the weight *E* if the weight *C* is to the weight *E* as is *BO* to *OT* or *OI*. Del Monte then summarized Benedetti's claim that when a force represented by the weight *C* acts along the line *TC*, the bent lever formed by the straight arm *BO* and the oblique arm *OTC* will also be in equilibrium, which he doubted.

Del Monte reformulated this claim by stating that the same weight *C* will be in equilibrium with the weight *E* whether it is placed on the straight balance *BOI* or on the broken bent lever *BOTC*. He thus replaced Benedetti's conception of a force acting along an oblique line with that of a weight always tending downward and as a result arrived at absurd conclusions.

Del Monte then showed that the same weight will be heavier on the horizontal at point *I* than along the bent lever at *T*, demonstrating that the bent lever *TOB* will not be in equilibrium if the straight lever *BOI* is in equilibrium. To show this, Del Monte again proceeded by finding the center of gravity of the weights *E* and *C* placed at *T*. More precisely, Del Monte determined a position for the weight *C* where the bent lever is in equilibrium, a position, however, that is distinct from *T*. Thus it follows that *T* cannot be the position of equilibrium. For this purpose, he extended the line *BT* to *D*, just beneath *I*, so that it is immediately evident that if the weight *C* is placed at *D*, the center of gravity of the two weights will be just beneath the fulcrum.

Using the same pattern, he continued by showing that the bent lever *BOC* cannot be in equilibrium because its center of gravity *S* can never fall on the perpendicular line *OU* through the fulcrum. Finally, he applied this argument to the broken bent lever *BOTC*. Del Monte next addressed the case in which the bent lever is characterized by an obtuse angle *BAC*, showing that the weight at *T* is lighter than the weight at *I*. In his concluding remarks, however, he began to waver. Once again, he stated that Benedetti is completely mistaken when applying his procedure to weights. But he did admit that this may be true when dealing with a force.

As an afterthought, Del Monte once again criticized Benedetti's appeal to common sense: he did not feel this to be worthy of an expert mathematician. And as a second afterthought, he constructed an extreme case in which it is immediately clear that the broken bent lever cannot be in equilibrium if weights are attached to it rather than forces.

The following considerations enable Del Monte's marginal annotations to Benedetti's *De Mechanicis* III to be understood. These are not perfectly legible, but nonetheless their meaning becomes clear in light of the *Meditatiunculae*:

> If we understand that a weight is at *C*, as we can assume from his own words, then *CT* must also be understood as being solid [and connected with] the solid lines *TO* […] If we hence understand that *C* is a weight and not moving, [the proposition] is false. If it is understood that *C* moves as […] of a man, it can be true, since what moves is not a weight. [But] if he himself assumes in the following that [this] can be demonstrated [also for a weight], nothing […] therefore as is evident in chapter 7. All demonstrations of the author are founded on these two chapters inasmuch as they are the first fundaments of mechanics; once their falsity is recognized, everything is rejected.[38]

[38]See Renn and Damerow 2012, 213: "si intelligamus p[ondus] in *C*, ut supponi p[otest] ex verbis ipsius, intelligendum est *C*[*T*] quoque consolidatam consolidatis *TO* […]. Unde si intelligamus *C* pondus et non

## 5.6 Diverging Approaches to Tartaglia

Del Monte's and Benedetti's criticisms of Tartaglia's conception of positional heaviness help us to understand where these two scholars converge and diverge on the issue of the equilibrium (or lack of equilibrium) of a balance deflected from its horizontal position, and also the reasons for the presumed equilibrium or tendency to restore it. Moreover, their arguments reveal a different attitude toward the medieval tradition of the *scientia de ponderibus* and the *gravitas secundum situm*.

### 5.6.1 The Tradition of Nemorarius, Tartaglia, and Cardano

The concept of *gravitas secundum situm*, or positional heaviness, was extensively employed in Jordanus Nemorarius's *Liber de ponderibus*. Del Monte owned and annotated a sixteenth-century Nuremberg edition of the book, commented, and illustrated by Petrus Apianus. Del Monte's handwritten annotations document his general disagreement with the approach of this medieval scholar, who did not know the Archimedean concept of the center of gravity and therefore tried to develop a deductive science of weights relying solely on the Aristotelian theory of motion and its development in the Arabic tradition of the science of weights. We have already hinted at the Aristotelian framework underlying the concept of *gravitas secundum situm*. In his book, Jordanus stated that a deflected balance would return to the horizontal position (his second proposition) (Nemore 1565, B2 *r*). According to Jordanus, the upper weight acquires more positional heaviness than the lower one due to the fact that its descent is less oblique. In fact, he postulated that positional heaviness depends on the obliqueness of descent of a weight (his fourth postulate) and that "a more oblique descent partakes less of the straight [descent] for the same quantity [of path]" (fifth postulate) (Nemore 1533, A4 *r*). The determination and possibly the quantification of obliqueness was therefore essential to establish the behavior of a deflected balance.

In the sixteenth century, Tartaglia in *Quesiti, et inventioni diverse* (1546), and Cardano in Book 1 of *De subtilitate* (first edition, 1550) and in *Opus novum de proportionibus* (1570), expounded their own versions for determining descent and reinforced Jordanus's second proposition (that the deflected balance returns to the horizontal position). A brief account of three ways to determine positional heaviness is given in the following pages. The first two are derived from Tartaglia and the last from Cardano.

*Descent*: A first method of dealing with positional heaviness consisted in comparing the lengths of the projections of the equal arcs described by the motion of opposite balance beams—one ascending and one descending—on the vertical line of descent to the center of the world.

As Tartaglia's diagram in Figure 5.6 shows, the vertical component of descent of the upper weight is always larger than that of the lower. Thus, the former acquires more heaviness (*secundum situm*) than the latter and the balance returns to the horizontal position.

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movens, falsa est i[ta]que si intelligatur *C* movens ut homi[…] vera esse pote[st] quod [deleted: non] moveat non esse pondus s[i…] ipse [vero] in sequenti accipiat [hoc atque ponderi?] posse demonstratum quare nihil […] ut patet in 7 cap. In his duobus cap. fundantur omnes authoris demonstrationes ita ut sunt praecipua mechanicorum fundamenta quorum cognita falsitate omnia rem[oventur]."

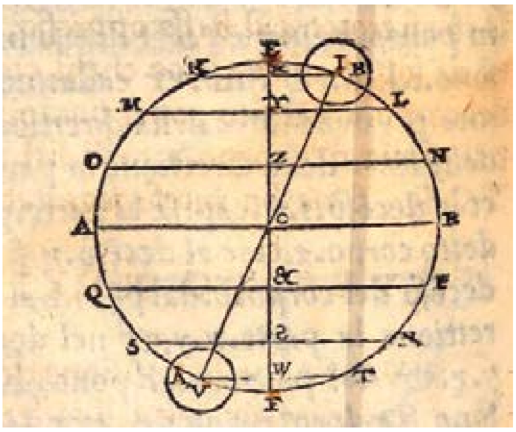

Figure 5.6: According to Tartaglia, the body at *I* is positionally heavier than the body at *V*, since the projection of the arc *IL* on the vertical *XY* is greater than the projection of *VF*, *WF*. (Max Planck Institute for the History of Science, Library)

*Angle of contact*: Tartaglia's second method of determining positional heaviness consists in comparing the angles between the circular path of the beams and the perpendicular lines connecting the weights to the center of the elements (as already mentioned in chapter 4). These angles "of contact" are also called "curvilinear angles" or "mixed angles" since they result from the intersection of a straight line downward and a curved line, that of the circle circumscribing the balance (Figure 5.7).

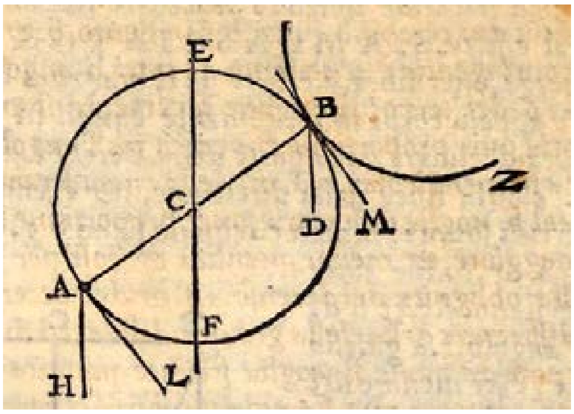


Figure 5.7: Tartaglia's representation of the angle of contact for the determination of positional heaviness. (Max Planck Institute for the History of Science, Library)

By comparing the angles of contact of the two weights, Tartaglia could establish that the higher angle is always smaller than the lower; therefore, the higher weight has a straighter descent and is positionally heavier. The inclined balance would therefore return to the horizontal position. It should be noted that Tartaglia perceived the comparison of curvilinear angles as problematic. He considered the ratio of two such angles to be less than any ratio between determined quantities. As a consequence, no weight placed on the positionally lighter side of the deflected balance could compensate for the other weight and keep the balance inclined. On the contrary, any additional weight—no matter how small—would have produced an opposite displacement of the balance beam toward the vertical.

*The angle between the support and the beams*: We have so far considered two ways of determining positional heaviness on the basis of Tartaglia's *Quesiti*. Assuming that positional heaviness depends on the obliquity and straightness of descent, positional heaviness can be determined either from the projections of the descents on the vertical, or the curvilinear angles that are produced by the intersection of the descent arcs and the lines connecting the weights to the center of gravity. Cardano considered three criteria for establishing positional heaviness which he mistakenly regarded as equivalent: first, the distance of the beam from the vertical; second, its distance from the horizontal; and third, an angle that he called *meta*. This was the angle between the support of the balance and the beam. Commenting on the diagram that is reproduced here as Figure 5.8, he explained:

> Aristotle says that this happens when the support is above the balance, because the angle *QBF* of the *meta* is larger than the angle *QBR*. And similarly, when the support is *QB*, the *meta* will be *AB*, and thus the *RBA* will be larger than the angle *FBA*, but the larger angle will render the weight heavier. […] The general reason is hence this: the more the weights are removed from the *meta* or from the line of descent along a straight or an oblique line, that is, [as measured] by an angle, the heavier they are.[39]

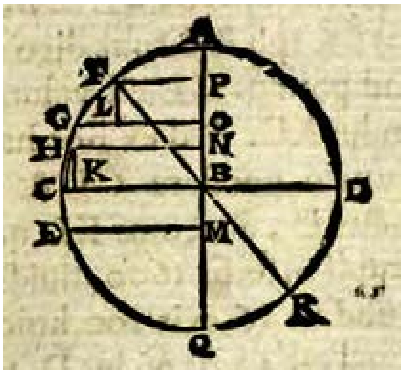


Figure 5.8: According to Cardano, there are three ways to determine positional heaviness. The positional heaviness at point *F*, for instance, may be determined by the horizontal *FP*, by the vertical *FL*, or by the angle *QBF*. (Max Planck Institute for the History of Science, Library)

Given these premises, Cardano contended that a weight will reach its maximum positional heaviness in the horizontal position. He therefore shared Nemorarius's and Tartaglia's opinion about the return of an inclined balance to the horizontal position.

### 5.6.2 Del Monte's Critical Remarks on Positional Heaviness

Del Monte's criticism of Benedetti, in the *Meditatiunculae* as well as in the marginal remarks of his copy of *Diversae speculationes*, are closely related to his criticism of Nemorarius, Cardano, and Tartaglia in his *Mechanicorum liber* (1577). Here he dealt

[39] Cardano 1550, 17–18: "Aristoteles dicit hoc contingere, quum trutina est supra libram, quia angulus *QBF* metae, maior est angulo *QBR*. Et similiter quum trutina fuerit *QB*, erit meta *AB*, et tunc angulus *RBA*, maior erit angulo *FBA*, sed maior angulus reddit gravius pondus. […] Generalis igitur ratio haec sit: pondera quo plus distant a meta seu linea descensus per rectam aut obliquum, id est, per angulum, eo sunt graviora."

extensively with the balance and provided a detailed discussion of the theories of these scholars which he judged to be irremediable. These theories supported the idea that an inclined balance returns to the horizontal and were thus at odds with his own treatment of the matter, which he based on the Archimedean concept of center of gravity. Del Monte believed that an ideal balance would remain in any position as long as it had equal arms, was hinged on its fulcrum and was loaded with equal weights. The only difficulty in testing this theory, he asserted, was the technical difficulty in constructing a perfect balance. It should be noted, moreover, that he assumed that a center of gravity meeting the requirement of his (and Pappus's) definition of the center of gravity always exists:

> The center of gravity is a certain point within it, from which, if it is imagined to be suspended and carried, it remains stable and maintains the position which it had at the beginning, and is not set to rotation by that motion.[40]

Apart from the conceptual irreconcilability between his own approach and that of the Nemorarius school, Del Monte tried to demonstrate the inconsistencies of positional heaviness also within the conceptual framework of his adversaries. One of his main objections was based on a consideration of the cosmological context, which he considered relevant to correctly treat the inclined balance, at least with regard to positional heaviness. Of course, this aspect indeed matters when considering Tartaglia's remark that the difference in positional heaviness is infinitesimally small and cannot be compensated by any finite weight resulting from the infinitesimal difference between curvilinear angles.

Contrary to the assumptions of Nemorarius and his successors, Del Monte noted that the downward tendencies of the weights are not parallel but converge at the center of the world. Since the directions toward the center of the world from different points on the circular path of the end of the beam cannot be parallel, they are inappropriate for representing positional heaviness. From the fact that those lines converge, he argued further that the lower weight should actually become positionally heavier than the higher one. His idea is clearly illustrated by the diagram in Figure 5.9.

Del Monte objected that, from the point of view of positional heaviness, it is not in the horizontal position that a body weighs the most but at that point where a straight line drawn from the center of the world touches the circle described by the balance arm. Certainly, if the center of the world were infinitely distant and all lines of direction converging at it were perpendicular and parallel to each other, then the extreme point would mark the horizontal position of the balance arm. Still, for a finite distance from the center of the world, the point where the weight is heaviest lies instead slightly below the horizontal through the fulcrum. Del Monte even demonstrated that the closer the balance is to the center of the world, the further this "extreme point" (where the weight is heaviest) will lie from the horizontal position of the balance arm (as seen from the fulcrum).

Del Monte's crucial objection to the Nemorarius school was that one should not consider both weights separately, but rather in terms of their connection by the balance beam. He drew attention to the fact that one must not compare two descents, but rather a descent on one side with a rise on the other. With regard to their positional heaviness the two weights are then equal. Thus Del Monte could claim, using the premises of his adversaries, that the deflected balance does not return to the horizontal.

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[40] Del Monte 1577, 1*r*: "Centrum gravitatis uniuscuiusque corporis est punctum quoddam intra positum, a quo si grave appensum mente concipiatur, dum fertur, quiescit; et servat eam, quam in principio habebat positionem: neque in ipsa latione circumvertitur." Translation in Drake and Drabkin 1969, 259, revised in Damerow and Renn 2010, 57.

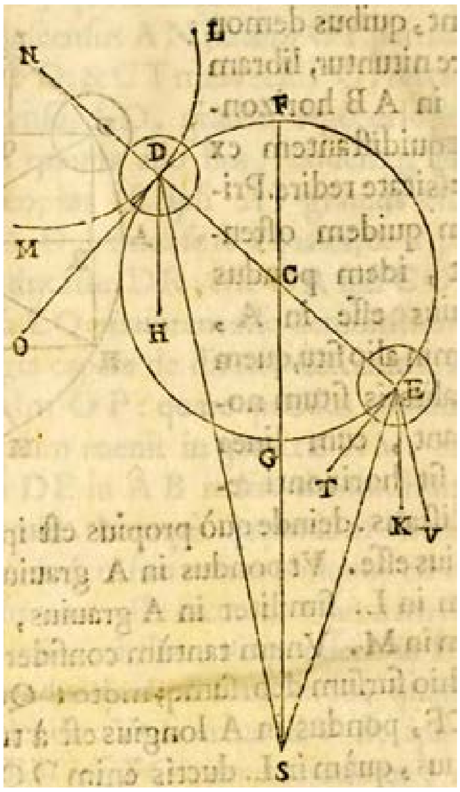

Figure 5.9: According to Del Monte, if *S* represents the center of the world, then the mixed angle *SEG* between the circular path of the weight at *E* and the direction to the center of the world is less than the mixed angle *SDG*. Thus, contrary to what his adversaries claim, by their own suppositions the weight placed at *E* must be heavier than that at *D*. (Max Planck Institute for the History of Science, Library)

### 5.6.3 Benedetti on Tartaglia's and Nemorarius's Shortcomings

Benedetti addressed the ideas of Tartaglia and Nemorarius on positional heaviness in section seven of his *De mechanicis*. There, Benedetti stressed that his approach to positional heaviness, focusing on the distance from the fulcrum to the line of inclination, was distinct from and superior to Tartaglia's approach in the Jordanus tradition of straightness of descent.

More specifically, Benedetti refuted several of Tartaglia's claims. In particular, he disputed the central thesis that when a balance is moved from its horizontal position, it will return to this position because the body that has moved upward will attain greater positional heaviness than the body which has moved downward. As we have seen above, Jordanus's and Tartaglia's arguments were based on a comparison of the descents of the two weights. In other words, the balance would have to break in the middle to visualize these descents. Benedetti now pointed to the simple fact, already emphasized by Del Monte, that when one weight descends, the other must ascend, and that the corresponding arcs will always be similar to each other and positioned in the same way. He concluded that no positional difference in heaviness can be produced in the way that Tartaglia argued.

Nevertheless, Benedetti did not believe in an indifferent equilibrium of such a balance when considered in a cosmological context. In the continuation of his argument, he came to the conclusion that when such a balance in equilibrium is displaced from its original horizontal position, the weight that has been lowered will actually assume a greater positional heaviness than the one that has been lifted up:

> Therefore the weight of *A* in this [lower] position will be heavier than the weight of *B*.[41]

[41] Benedetti 1585, 148: "Pondus igitur ipsius *A* in huiusmodi situ, pondere ipsius *B* gravius erit." Translation in Drake and Drabkin 1969, 176.

He reached this conclusion by taking into account that the lines of inclination of the two weights are not parallel to each other but must converge at the center of the elements. The effective lever arms of the two weights must hence be determined by perpendicular lines drawn from the center of the balance to these lines of inclination. It now turned out that the perpendicular line corresponding to the weight that had been lowered is longer than the line corresponding to the weight that had been lifted. Consequently, the lower weight had become heavier positionally, so that one would expect the balance to tilt into a vertical position.

Benedetti added some more critical remarks on Tartaglia's consideration of positional heaviness. As we have seen, Tartaglia had argued in *Quesiti* that the upper weight attains a greater positional heaviness than the lower one, but that this difference is arbitrarily small and can therefore not be compensated by any finite weight. This conclusion was reached by comparing curvilinear *angles of contact* on each side of the balance. In his analysis of this argument, Benedetti again emphasized that the lines of inclination are not parallel to each other but must converge toward the center of the elements, just as Del Monte had done before him. Clearly, since Tartaglia's argument hinges on angles of contact, which are infinitesimally small compared to ordinary angles, even such a small deviation from the parallel must be relevant. Taking this into account, Benedetti was able to construct a contradiction, thus refuting Tartaglia's argument. He concluded:

> Now the whole error into which Tartaglia and Jordanus fell arose from the fact that they took the lines of inclination as being parallel to each other.[42]

In summary, Benedetti introduced a way of determining the positional effect of a weight or a force that, in the cases he considered, essentially produces the same results as the application of the modern concept of torque. In particular, Benedetti had managed to go beyond the consideration of weights tending downward to include forces acting in an arbitrary direction. In this way, he was also able to take into account the fact that, on a spherical earth, the lines of inclination of weights on a balance are not parallel. He did not manage, however, to successfully apply his measure of positional heaviness to challenging objects such as the inclined plane.

## 5.7 The Triangulation Benedetti-Del Monte-Galileo

In this chapter, we have dealt with Del Monte's and Benedetti's different approaches to mechanics emerging from their reflection on the balance and their treatment of earlier authors. Relative to the issue of positional heaviness, Del Monte's self-positioning was essentially external whereas Benedetti positioned himself (albeit critically) within the tradition of the Nemorarius school. He explicitly mentioned Tartaglia and Cardano as relevant sources for his treatment, whereas he omitted any mention of Del Monte.[43] In spite of their opposite intentions and mutual suspicion, Benedetti and Del Monte shared several opinions and sometimes reached the same conclusions, albeit following different paths: both considered the cosmological center of gravity relevant for an evaluation (and criticism) of Tartaglia's concept of positional heaviness, and both remarked that one cannot treat the two balance beams separately, but rather emphasized that they must be considered simultaneously. Moreover, both stressed the ambiguity of the concept of a mixed angle

[42] Benedetti 1585, 150: "Omnis autem error in quem Tartalea, Iordanusque lapsi fuerunt ab eo, quod lineas inclinationum pro parallelis vicissim sumpserunt, emanuit." Translation in Drake and Drabkin 1969, 177.
[43] Benedetti 1585, f. A3*r*.

and the difficulty of its determination. Nevertheless, their approaches were quite different. As mentioned, Benedetti still worked within the framework of the *gravitas secundum situm*, while Del Monte renounced it in favor of the concept of *centrum gravitatis*. For Del Monte, the displacement of the balance toward the vertical position was an absurdity that revealed the untenability of Tartaglia's premises. Benedetti deemed this vertical tilt to be the consequence of a correct analysis of the balance based on a concept close to the modern idea of torque, in consideration of the cosmological context. Furthermore, one should stress the importance of Benedetti's attempt to determine the quantity of positional heaviness, a fact that distinguishes him from his predecessors. Additionally, unlike Del Monte, he treated the balance by also taking into consideration the general case of forces acting arbitrarily on the beams.

In conclusion, it may be useful to recall the problems linked to the triangulation Benedetti-Del Monte-Galileo, on which the equilibrium controversy sheds new light. The remarkable proximity of these authors on several issues is well known in the history of mechanics. Nevertheless, recent accounts tend to neglect or even deny a possible influence of Benedetti on Galileo.[44] By contrast, the influence of Benedetti on Galileo was assumed and underscored by earlier scholars like Raffaello Caverni, Pierre Duhem, Emil Wohlwill, and Ernst Mach.[45] It is helpful to mention the most important issues common to these authors: the attempt at a theory of motion based on Archimedean hydrostatics, the treatment of the acceleration of fall and its causes, the formulation of what in hindsight appear as proto-inertial principles, a similar treatment of the bent lever, the analysis of the relation between vibrating strings and musical tones, their views on the irradiation of surfaces and on thermal and hydrostatic phenomena, and, last but not least, their support of the Copernican world system.[46] Although many of these themes and ideas belonged to the shared knowledge of preclassical mechanics, in some respects the agreement of their approaches is so striking that one may suspect that this is not mere coincidence.[47] Another potential intermediary was Galileo's friend Paolo Sarpi who discussed Benedetti's theory of fall in *Pensieri naturali e metafisici*. In any case, the strongest evidence of Galileo's acquaintance with Benedetti's insights is provided by Del Monte's *Meditatiunculae*.

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[44]See the discussion by Ventrice in Bordiga 1985, 732–736. He mentions Drake, Drabkin, Fredette, and Galluzzi among those who are skeptical about a concrete influence of Benedetti on Galileo. Notable exceptions are the commentaries by Carugo and Geymonat in their edition of Galileo's *Discorsi*, see Carugo and Geymonat 1958. Bertoloni Meli even considers the possibility of Del Monte and Galileo discussing Benedetti, but nevertheless rejects any substantial influence by the latter on Galileo's thinking because that influence supposedly would have arrived too late, see Bertoloni Meli 2006, 61–65.

[45]Cozzi and Sosio 1996. For an overview of such potential connections, see the discussion in Bordiga 1985, 732–736 who also mentions Mersenne, Clavius, and Cardinal Michelangelo Ricci as possible intermediaries.

[46]For an overview, see Bordiga 1985.

[47]See, for instance, Drake and Drabkin 1969, 36. Yet, the question of Benedetti's direct impact on Galileo remains unclear, in particular as Benedetti's work was never mentioned by Galileo.

There are several possible connections between Benedetti and Galileo that have been considered in the past. For instance, Benedetti is referred to by Galileo's Pisan colleague Jacopo Mazzoni in *In universam Platonis et Aristotelis philosophiam praeludia* from 1597. See Mazzoni 1597. He is often mentioned in the Galileo Studies as the addressee of a famous letter by Galileo arguing for the Copernican system (May 30, 1597). See Galilei 1968, vol. 2, 194–202. In his book Mazzoni referred to Benedetti's discussion of the possibility that motion along a straight line can be continuous. See Benedetti 1585, 183–184. For a historical discussion of the context of this argument in contemporary technology, see Freudenthal 2005, a theme that was later taken up by Galileo in chapter 20 of *De Motu*, which also refers explicitly to Copernicus. See Mazzoni 1597, 193 and Galilei 1960, 326. It is conceivable that such issues had been discussed, inspired by Benedetti's work, between Galileo, Mazzoni, and Del Monte during Del Monte's stay in Tuscany in 1589. We would like to thank Pier Daniele Napolitani for drawing our attention to this possibility and to the above-mentioned passages.

An important clue is page 145*bis* of the *Meditatiunculae* (Figure 5.10), which is the page opposite the one containing the detailed criticism of Benedetti dealt with in this chapter. This page shows Galileo's construction of the inclined plane, reducing it to a bent lever.

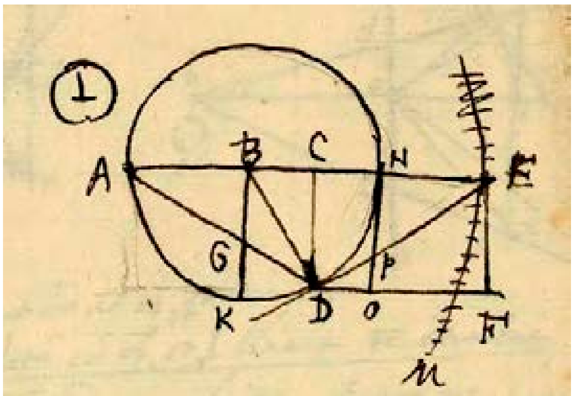


Figure 5.10: Del Monte, *Meditatiunculae*, p. 145*bis* showing Galileo's construction relating the bent lever to the inclined plane. (Bibliothèque Nationale de France)

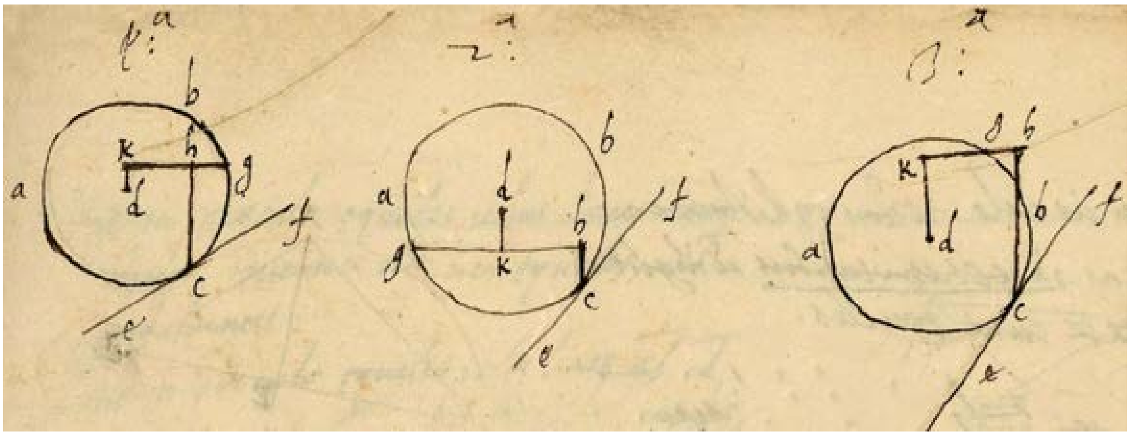

Figure 5.11: Del Monte's construction related to the inclined plane on p. 64 of his notebook. The construction was adapted from Pappus's erroneous solution. (Bibliothèque Nationale de France)

This fact is all the more noteworthy since Del Monte's notebook, on an earlier page, also contains his own problematic adoption of Pappus's analysis of the inclined plane (Figure 5.11).[48] In his writings, Galileo had criticized this analysis, substituting it with his own solution of the problem, which makes use of the bent lever conceptualized in the same way as Benedetti had done.[49] Del Monte therefore must have learned about this proof from Galileo, and he must also have seen the connection to Benedetti's methods. In any case, it is likely that the two scientists discussed this connection and quite plausible that Galileo became familiar with Benedetti's work through Del Monte. Galileo began to correspond

[48] Del Monte 1587, 64.
[49] Galilei 1960, 172.

with Del Monte in 1588, three years after the publication of Benedetti's *Diversae speculationes* and shortly before he embarked on the writings that later became known as *De Motu*.[50] Galileo first wrote a dialogue version of *De Motu* and then an essay in twenty-three chapters. Only the second essay version of these writings contains his proof of the law of the inclined plane, the argument about continuity of motion along a straight line, and a mention of Copernicus. This version was most likely written after Galileo became familiar with Benedetti's work. His treatise on mechanics, which for the first time discussed explicitly the problem of the effective lever arm, was written much later, certainly after he had visited Del Monte in 1592 during his journey to Padua. Hence, it seems most likely that Galileo was already familiar with Benedetti's key ideas at the time of writing these works.

Recent research into Del Monte's biography has shown that Del Monte and Galileo must have met as early as 1589 in Tuscany.[51] They might even have met jointly with Galileo's teacher, Mazzoni, who, as mentioned earlier, cited Benedetti in his work. Thus, Del Monte, Mazzoni, and Galileo may have discussed Benedetti's *Diversae speculationes*, leading Galileo to reconsider his work in progress on motion and, in particular, his treatment of motion along inclined planes, making use of Benedetti's theory of the bent lever that was mentioned in Del Monte's notebook. But Benedetti's impact on Galileo probably went even further than that. Galileo may have started taking the Copernican hypothesis much more seriously after his encounter with Benedetti's work, discussing this as well as other subjects with Mazzoni. In the above-mentioned letter of 1597, Galileo praised Mazzoni for his *Praeludia* and reminded him of the controversial issues on which they had meanwhile reached an agreement, and also tried to press him on the Copernican hypothesis.

In particular, Galileo's concept of *momento*[52] and his analysis of the bent lever—crucial to both his mechanics and his theory of motion—evidently emerged from the midst of the controversy about positional heaviness. In that debate, Galileo took a position much closer to Benedetti than to Del Monte. Rather than *gravitas secundum situm*, Galileo used the concept of *momento* or *momentum* that Del Monte had introduced in his book by quoting Commandino's definition of the center of gravity. But while Del Monte made no further use of this in his mechanics, Galileo took this concept from the respected Urbino school, gave it a new meaning that was taken from Benedetti, and made it a pillar of his own conception, which included Commandino's definition of the center of gravity:

> Center of gravity is defined as that point in every heavy body around which parts of equal moments are arranged.[53]

The evidence for this claim concerning Benedetti's legacy in Galileo's work derives from the marginal notes Del Monte made in his copy of Benedetti's book, as well as from his entries in the *Meditatiunculae* which contain traces of Galileo's intervention in this controversy.[54]

According to Benedetti and Galileo (and contrary to Tartaglia and Del Monte), the effective length of the lever arm, obtained by drawing a perpendicular from the fulcrum

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[50] Galilei 1960. For a thorough discussion of the chronology of these writings, see Giusti 1998.
[51] Menchetti 2012.
[52] See the extensive discussion in Galluzzi 1979.
[53] Galilei 1968, vol. 2, 159: "Centro della gravità si diffinisce essere in ogni corpo grave quel punto, intorno al quale consistono parti di eguali momenti." Translation in Galilei 1960, 151. See also Galilei 2002.
[54] Del Monte 1587.

of the balance to the line of inclination, determines the effectiveness of a weight or a mechanical constellation. In his *Mechanics*, Galileo later stressed how important it is to carefully define the effective distances of weights from their support:

> There is one thing that must be considered before proceeding further, and this concerns the distances at which heavy bodies come to be weighed; for it is very important to know the sense in which equal and unequal distances are to be understood, and in what manner they must be measured.[55]

In his analysis of the inclined plane using the bent lever, Galileo also made clear that this procedure is critical for determining the *momento* of a given weight.[56] As discussed earlier, in his *Diversarum speculationum [...] liber*, Benedetti convincingly demonstrated the efficacy of this method for determining the magnitude of a force or weight according to its position.

In conclusion, the very existence of Del Monte's annotations on his copy of Benedetti's *Diversae speculationes* provides a definitive answer to the question of whether Del Monte had read this book or not.[57] It is also difficult to imagine that he did not discuss his views on Benedetti's mechanics with Galileo, views that he considered both misguided and profoundly challenging, as is made evident in his handwritten notes. It was most probably Del Monte, Benedetti's fervent opponent in matters of mechanics, who served as a conduit to Galileo. At the same time, he also made it virtually impossible for Galileo to openly admit to Benedetti's influence if he did not also want to jeopardize the protection of the most important patron of his early career.

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[55] Galilei 1968, vol. 2, 164: "Un'altra cosa, prima che più oltre si proceda, bisogna che sia considerata; e questa è intorno alle distanze, nelle quali i gravi vengono appesi: per ciò che molto importa il sapere come s'intendano distanze eguali e diseguali, ed in somma in qual maniera devono misurarsi." Translation in Galilei 1960, 156–157.

[56] See Galilei 1968, vol. 2, 181. Translation in Galilei 1960, 173.

[57] The knowledge that he had read it, however, is not entirely new. See Renn, Damerow, and Rieger 2001, 74.

# Chapter 6
# Astronomy

## 6.1 Benedetti as an Astronomer

Benedetti's astronomical considerations are not systematic. They are scattered throughout the volume in different sections. In spite of the difficulty of ordering them and obtaining an overview, they were very much appreciated among his contemporaries. Apart from Kepler's eulogy of Benedetti's ingenuity, the broad European success of the astronomical parts of this work is documented in other references. A few years after the publication of the *Diversae spaeculationes*, Brahe must have had a copy of it in Denmark, as he quoted it extensively and accurately on two occasions. In his correspondence with Landgrave William IV and the Hesse-Kassel court mathematician Christopher Rothmann, he referred to Benedetti's observation of the light of Venus reflected on the part of the lunar disc not presently enlightened by the sun:

> In fact, I sometimes saw that Venus illuminated in a rather sensible manner that part of the Moon that was most distant and opposed to the Sun, although the Moon is by far more distant from Venus's circuit than the comet. I remarked that the Venice patrician Giovanni Battista Benedetti, the most excellent philosopher and mathematician, noted something similar in that erudite work which he wrote on mathematical and physical speculations. At the end of an epistle to a certain Savoy baron, Filiberto, he says: "[…] that the part of the Moon which is deprived of the Sun's light is sometimes partially illuminated by Venus's light. I observed this often and showed it to many people."[1]

Brahe quotes this passage correctly from Benedetti's letter to Baron Emanuele Filiberto Pingone "*De Luce, Lumine, et Colore, De obiectu oculi, De lumine Lunae, et Rubedine nubium*" (On light, lumen, and color; on the eye's object, on the lunar lumen, and the redness of the clouds).[2]

A second long direct quotation of Benedetti can be found in Brahe's book on the nova of 1572, which was part of the *Astronomiae Instauratae Progymnasmata*, posthumously published in Prague in 1602.[3] The Danish astronomer here praised Benedetti as a "philosophus et mathematicus inprimis excellentem," and his work as "praeclarum Opus." He entirely reproduced Benedetti's letter and diagrams on the star in Cassiopeia.[4] This letter

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[1]Brahe 1919, 172: "Veneris enim Stella, visa est mihi aliquando eam partem Lunae, quae a Sole aversa erat, et ipsi obiecta, satis sensibiliter illuminare, utut Luna longe remotius a Veneris circuitus distiterit, quam Cometa. Simile quid Ioannem Baptistam Benedictum, Patricium Venetum Philosophum et Mathematicum inprimis excellentem, animadvertisse reperio, in erudito illo Opere, quod de Mathematicis et Physicis speculationibus inscripsit. Sic enim in fine Epistolae, ad Baronem quendam Sabaudarum Philibertum scribens, ait: '[…] quod pars Lunae lumine Solis destituta, a lumine Veneris aliquantulum illustratur, quod ego saepe vidi, et multis ostendi.'"

[2]Benedetti 1585, 256–257.

[3]Brahe 1916, 251–253.

[4]Benedetti 1585, 371–374.

was directed against Annibale Raimondo—an author whom Brahe also criticized—and demonstrated that the nova appeared above the sublunary sphere. Brahe commented:

> Here follows the epistle which I referred to. It is taken from the aforementioned book by [Giovanni] Battista Benedetti alongside the demonstrative diagrams offered by the same author. Afterwards I will consider others, who discussed that star [i.e., the nova of 1572] in an extraordinarily incompetent manner. This [quotation from Benedetti] (as mentioned) will cast light on these issues through a synthetic and wise geometrical truth, so that no significant doubt will survive.[5]

Another reader of the *Diversae speculationes* was the English scholar of magnetism William Gilbert. In *De mundo nostro sublunari philosophia nova* (New Philosophy on Our Sublunary World, written about 1600 but published long after the author's death, in Amsterdam in 1651), he in fact discussed Benedetti's views on the spots on the surface of the moon, in a chapter trying to determine which parts of it were seas and continents.[6] It is evident that the *Diversae speculationes* had a wide European circulation, and that the astronomical part attracted the attention of many scholars dealing with mathematical and physical issues.

Benedetti's treatment of astronomical matters ranges from the calendar reform to the nova of 1572, sundials, and astrology. We would like to focus on a special issue: Benedetti's defense of ephemerides, *Defensio ephemerides*, and the quarrel that motivated its writing. This defense of ephemerides figures as one of the epistles of the *Diversae speculationes*. It is the Latin translation of an Italian letter, *Intorno ad alcune nuove riprensioni… contra alli calculatori delle effemeridi* (Letter in the Form of a Discourse… Addressed to the Illustrious Mr Bernardo Trotto Concerning Some New Criticism and Corrections against the Ephemerides Calculators, Turin, 1581), addressed to Trotto, which Benedetti had already published when a heated quarrel on the reliability of ephemerides burst out in Turin between 1580 and 1581. In the following pages we will give an account of these facts.[7]

## 6.2 The Controversy over the Reliability of Ephemerides

The ephemerides controversy began with the publication of Altavilla's *Animadversiones in ephemeridas* (Remarks against Ephemerides, Turin, 1580). This lesser-known author from Vicenza intended to denounce the inexactitude of all existing astronomical computations.[8] For this purpose he compared predictions and horoscopes cast using different sets

[5]Brahe 1916, 251: "Nunc igitur epistolam, quam pollicitus sum, subiungam, verbotenens e praedicto Baptistae Benedicti libro desumptam, una cum demonstrationum delineationibus, quas ipse author assignavit. Deinde ad caeteros qui de hac stella nimis incompetenter, sententiam tulerunt, calamum dirigam. Ex quo (uti dixi) haec adeo succinte et scite geometricam veritatem redoleant, ut nullum, quod alicuius sit momenti, super esse queat, dubium."

[6]W. Gilbert 1651, 173: "Luna maculas quasi ostendit substantiae et peripheriae differentia: ita Tellus erga Lunam maculas repraesentat, terrarum continentium minus relucentium; aquarum vero et Oceani, propter laeviorem et luminis apprehensivam naturam magis splendentem. […] Non enim maculae Lunae existunt a partibus Lunae magis perspicuis, ut Iohannem Benedictus contendit, in quibus lumen non reflexum sed penetrans nobis occultatur." See Pumfrey 2011, 193–203.

[7]Section 6.2 is a revision of Omodeo 2014a, chap. 3.8–9 and chap. 6.3 of Omodeo 2014a, chap. 4.7.

[8]This Benedetto Altavilla could be the same person involved many years later, in 1606, in a gunpowder plot in Venice; he pretended to have discovered it by astrological means and was tortured by the Venice authorities in order to obtain information about the perpetrators. Cf. L. P. Smith 1907, vol. 1, 364–365.

of tables and ephemerides. In particular, he pointed out that ephemerides diverged from each other even more than the astronomical tables, Alfonsine or Copernican, from which they were derived. In his opinion, this fact undermined the reputation of astronomy in general, regardless of whether its cause was the inaccuracy of the compilers (*calculatores*) or the inexactitude of the tables themselves: "We consider nothing to be more odious than an unreliable person who is regarded by many as trustworthy."[9] Altavilla declared himself unwilling to decide between Alfonsine or Copernican computations. However, he himself was probably interested in the cosmological issue, judging by the fact that the *Animadversiones* were introduced with a poem by Pandolfo Sfondrati in favor of a new world system with the earth in motion.[10]

Altavilla had established by observation that both Alfonsine ephemerides and Johannes Stadius's Copernican computations were in disagreement with the heavens. Still, Stadius's computations proved to be in better agreement with the heavens. The reference to Stadius is not casual, since the Flemish astronomer had been a protégé of duke Emanuele Filiberto of Savoy, as one can read in the *Ephemerides novae* of 1556, where the author gave himself the title "mathematician to the King [of Spain] and the Duke of Savoy" (*Regius et Ducis Sabaudiae mathematicus*). Altavilla listed predictive errors of Ptolemaic astronomers (Regiomontanus, Stöffler, Leowitz) as well as those of post-Copernican ephemerists (Stadius and Giuntini). This led him to skepticism toward predictions in general: "You see, dear reader, how reliable ephemerides are."[11] Altavilla invited scholars (*magistri*) to trust only their eyes and to correct astronomy through observational campaigns with no regard for any authority: "Posterity should learn how dangerous it is to blindly adhere to the opinions of the ancients without [perfecting the art through] daily observations of the heavens, and to prefer their opinions to truth."[12]

The *Animadversiones* were soon followed by a second publication in Italian: *Breve discorso intorno gli errori dei calculi astronomici* (Brief discourse on the mistakes of astronomical calculations, 1580). A poem by a certain Francesco Onto of Pinerolo, inserted as a preface to the *Breve discorso*, made its polemical target explicit: "Altavilla has unveiled the astrologers' fallacy, as they think to cast certain [astrological] judgments about our lives relying on flawed ephemerides."[13] Altavilla's criticism was directed mainly against astrology, whose validity he considered to be doubtful due to the inaccuracy of predictions. His argumentative strategy was no different than that of Pico della Mirandola in books 8 and 9 of the *Disputationes in astrologiam divinatricem* (Disputations against divinatory astrology, 1496): an attack on mathematical astronomy aimed to discredit astrological forecasting. Altavilla even claimed that astrologers and ephemerists should renounce their activity, as they were not capable of superseding the flaws of their discipline: "Since it is impossible for the scholars in those sciences (especially those who are not capable of using the tables) to renounce ephemerides, and they know that they will encounter irremediable errors, they should be forced to abandon their studies."[14]

In his second publication, the *Discorso*, Altavilla complained that many scholars (who were not named) pretended to ignore his criticism. He explained that the decision to write another booklet, this time in Italian instead of Latin, originated from the desire to reach readers outside academic and scholarly circles, probably also at the Savoy

[9] Altavilla 1580a, f. A2*r*.
[10] See Omodeo 2008b and Omodeo 2012a.
[11] Altavilla 1580a, *Conclusio*.
[12] Altavilla 1580a.
[13] Altavilla 1580b, 2.
[14] Altavilla 1580b, 4–5.

court: "In these few pages, I aimed at demonstrating not only to the learned man, but also to everybody else, that the errors [of the ephemerides] are worthy of consideration."[15] He first reassessed the inadequacy of Alfonsine tables and Alfonsine ephemerides (those of Peuerbach, Prugnerus, Bianchini, Regiomontanus, Stöffler, Schöner, Gaurico, Pitati, Simi, Carelli, Moletti, Leowitz, and others). He moreover stressed the superiority of the Copernican tables in order to show the inconsistency of some unnamed Turin ephemerists who used Alfonsine ephemerides for their predictions although they claimed to prefer Copernicus. To illustrate this inconsistency, he analyzed some astrological figures on the basis of Stadius's and Giuntini's tables. In the last section Altavilla turned on the Copernican ephemerists, denouncing the excessive difference between computations based on Stadius and Giuntini: "And the difference between one computation and the other is really great and monstrous."[16]

This attack on the reliability of astronomical computations and astrology provoked negative reactions both at the university and at the court. Altavilla thus felt compelled to challenge his critics to an academic debate on August 14 and 15, 1581, announcing it through a broadside that is still preserved in the libraries of Turin, along with copies of his *Animadversiones*.[17] The public dispute concerned the theory of Mars for which, as one reads, some scholars blamed him. He maintained, in fact, that Mars cannot stay in a zodiacal sign for more than two months, considering that its entire revolution lasts twenty-four months. He argued that ephemerides are wrong if they forecast that it would spend six or even seven months in the same zodiacal constellation. This incorrect opinion presented the court mathematician and philosopher Benedetti with an occasion to intervene and criticize Altavilla on this and other issues related to astronomical theory, computation, and astrological prediction.

Soon after Altavilla's public dispute, Benedetti published an epistle "on some recent remarks and emendations directed against ephemerists" (Turin, 1581). At the beginning, Benedetti indicated Altavilla's intentions: "I assume […] that his intention was only to demonstrate that [different] ephemerides assigned a different place to the planet at the same point of time […] and that, as a consequence, they offer no certain ground on the basis of which the future can be judged or predicted."[18] In his account, Benedetti rejects Altavilla's complaint that Copernican and Alfonsine ephemerides diverge from each other more than the tables from which they are derived. He assures the reader that "the people who calculated have been very accurate and trustworthy" (*i calcolatori sono stati diligentissimi e fedeli*) and they are exact in their calculations, although some minor and accidental mistakes can occur.[19]

Moreover, he accuses Altavilla of misunderstanding Ptolemy's astrology, interpreting it in light of Abu Ma'shar and Al-Qabisi (*Alcabitius*). In particular, Altavilla draws from these sources the rule of the "triplicity" of the conjunctions of Jupiter and Saturn, according to which these planets meet four times in the same three astrological signs, or trine, before they can meet in the next trine. However, although the mean motions of two planets should meet in the triplicity sign, nonetheless their "real" motions (those observed and calculated by the ephemerides upon which astrological predictions rely) may meet elsewhere. This is an obvious consequence of planetary theory. In fact, it distinguishes between "mean" motions, which correspond to the revolutions of the deferents, and "real" motions, which

[15] Altavilla 1580b, 3.
[16] Altavilla 1580b, 6.
[17] In Turin: Biblioteca Nazionale di Torino, coll. Q.V.191, and Biblioteca Reale di Torino, coll. G.25.12.
[18] Benedetti 1581, 5.
[19] Benedetti 1581, 6.

correspond to observable phenomena and are the product of moving epicycles. Benedetti calculates the period of triplicity to be 794 years and 138 days, whereas the Arabs on whom Altavilla relies overestimated it at 960 years.[20] He furthermore remarks that Altavilla neglected planetary theory by criticizing those who let Mars run too fast or too slowly along the signs of the zodiac. Simple observations would show the correctness of the theory according to which the planet can remain in the same sign for six or even seven months. Benedetti explains that the amplitude of Mars's epicycle accounts for its complex phenomenology, in particular the long period of retrograde motion. On this account, he reports an observational campaign accomplished between 1565 and 1566 in order to check Stadius's ephemerides:

> Yet, he [Altavilla] dared too much, seeking to reprimand so many talented ancient and modern men who, as is required by diligent observers of the heavens, checked with their own eyes these appearances of Mars as well as of the other [planets]. From those [observations], they were forced to "imagine" such a large [Martian] epicycle. By contrast, he has never observed the motions of either this or any other planet, but rather limited himself to look at what is written in the ephemerides. In fact, if he had at least said that he observed Mars's journey for a certain period, and that he found that the others' opinion was false, he would have at least given some "color" to his opinion. In my assessment, however, if he had made an observation of the path of Mars, he would not have held the contrary view. In fact, the truth is the following: in every revolution of its epicycle, Mars in the lower part of its epicycle always stays many months (six or seven, or more) in a twelfth [*duodecatemerio*] of the zodiac. I observed this many times, for instance, in the years 1565 and 1566. First, consulting Stadius's ephemerides, I found that Mars would finish its retrograde motion on about 12 January 1566, in 16° of Gemini, and that, equally, Mars would be in the same place on the last day of August 1565, before it began its retrograde motion. Second, I found that, after that retrograde motion, on 11 April 1566, Mars would be in 16° of Cancer, so that it would take [Mars] seven months and eleven days [to move] those thirty degrees, from 16° of Gemini to 16° of Cancer. After these computations, I took the instruments and got ready to make a test. And I found that the last night of August of the year 1565 Mars was in the aforesaid 16° of Gemini, as Stadius had noted. I then made observations every week, in order to see the retrograde motion, and I saw that, at about the end of October, the [planet] began its retrograde motion and that retrograde motion lasted until January (or about January) 1566. I later observed the position of that planet on 11 April, and I found it in 16° of Cancer, that is, the place where Stadius had located it. Thus, my experience confirmed Stadius's computations and I found that he was not mistaken. In the same manner, everybody can ascertain the truth every two years by carrying out observations.[21]

Benedetti thus demonstrated not only the theoretical incompetence of his opponent, but also his lack of empirical verification. Altavilla's appeal to base astronomy using observation backfired. Benedetti challenged his opponent to observe Mars's backward motion in Cancer which, according to Stadius's tables, would begin on November 20, 1582 and

[20] See Bonoli 2012, 49–55.
[21] Benedetti 1581, 17–19.

last until the end of February 1583. He furthermore observed that everyone familiar with planetary theory would understand the reasons for the orbit of Mars and other planets. For the theory, he added, it did not matter whether one relied on Ptolemy's *Almagest* or on the "*Rivolutioni de gl'orbi celesti* dell'eccellentissimo Copernico."[22] Of course they were only equivalent as far as the understanding of a system of deferents and epicycles was concerned, but not in their general hypotheses, since Benedetti himself tended toward heliocentrism.

As to the difference between Leowitz's and Stadius's computations, Benedetti traced this back to the contrast between the theories underlying the Alfonsine and the Copernican tables. Nonetheless, he ensured that ephemerides never diverged by more than three degrees. Thus, if Altavilla detected greater discrepancies, this was due only to false computations. Benedetti added that Stadius's superiority over Leowitz was a consequence of him employing better parameters. He advised Altavilla to always rely on the most recent observations and tables.[23] In fact, he judged the progress of astronomy to be such that more recent tables would inevitably be superseded by new ones, augmented and perfected through new observations, just as Copernicus had superseded Alfonso's astronomers. Divergence between ephemerides was not a shortcoming, but a necessary and desirable sign of the advancement of knowledge and predictive accuracy.

As a courtier expert of mathematics, Benedetti defended the validity of some astrological figures that Altavilla criticized in his second published work, *Breve discorso*. These horoscopes had probably been cast by somebody that he knew well. Altavilla complained that some astrological figures had not been calculated on the basis of Copernican tables. Benedetti replied that it was not always necessary to use the best tables for predictions, especially if a generic horoscope was expected and if the astrologer had no Copernican tables to consult. He showed, moreover, that Altavilla himself was not able to employ Giuntini's tables properly and made mistakes of computation. He concluded: "And such monsters [those denounced by Altavilla] are not generated by different tables or ephemerides but, instead, they are the offspring of this author."[24] He added as a remark: "As to the difference of the Sun according to Copernicus and Alfonso, no learned man, [expert] in these sciences, ignores it, and, as a consequence [everybody knows] the different place [assigned to it] in the heavens during the annual revolutions."[25] In 1581, the general views of *De revolutionibus* were so well known in Benedetti's environment that he deemed it unnecessary to expand on them in the context of a polemic on the accuracy of heavenly computations. The cosmological implications of these different hypotheses were not addressed explicitly in this dispute. However, the defense of mathematical astronomy could not avoid a reference to Copernicus as a source for tables (Reinhold, Stadius, Giuntini) and theory. In this context, "Copernican" and "not Copernican" are expressions that merely mean "based on Copernican tables" or not. Altavilla's criticism would have been more effective if it had been directed against astrological beliefs as such, rather than attempting to show the inconsistency of the mathematical basis of astrology without sufficient preparation. On the other hand, Benedetti, in his *Lettera*, focused on the mathematical aspects and cautiously avoided expanding on ethical issues related to astrology.

Altavilla never responded to the court mathematician who had rebutted his arguments so forcefully. The epilogue to their quarrel was the inclusion of a Latin translation of the

[22] Benedetti 1581, 20.
[23] Benedetti 1581, 32–33.
[24] Benedetti 1581, 37.
[25] Benedetti 1581, 37–38.

*Lettera*, as *Defensio ephemeridum* (A defense of Ephemerides), in Benedetti's *Diversae speculationes*.[26]

## 6.3 The System of the World

Benedetti did not limit himself to considering astronomy from a computational point of view, but also expanded on cosmological aspects. The epistle "De fine corporum coelestium, et eorum motu" (On the Aim of Celestial Bodies, and their Motions),[27] addressed to Pingone, bears witness to his interest in cosmology and his realist interpretation of Copernicus's hypotheses. Benedetti remarks that it is not reasonable (*si [...] humanam rationem sequi volueris*) to believe that the heavens were created only for the sake of terrestrial life, "as these [celestial] bodies are divine, uncountable, and endowed with the greatest dimensions" (*cum ea corpora sunt divina, in numero incompraehensibilia, maximis magnitudinibus, et motibus velocissimis praedita*).[28] This absurdity can be avoided, as Benedetti claims, if one accepts the planetary doctrine of Aristarchus and Copernicus:

> […] this will hardly be believed by those who embrace the doctrine of Aristarchus of Samos and Nicolaus Copernicus. Following their approach it is impossible to make them believe that the rest of the universe has no other aim than to rule over this center of the lunar epicycle [the earth] (to use their way of speaking).[29]

Although he speaks in the third person, as if he were reporting the views of someone else, these are his own views. He is inclined to accept the Copernican system or some variation of it, as the following pages of the letter and the force of the arguments show. Firstly, he assumes a principle of cosmological homogeneity according to which there is no reason why other planets should not be subjected to alterations (*ab ortu, et interitu*), as the Aristotelians suppose. The peripatetic argument that no change in the heavens was ever observed is not valid, because the distance does not permit verification of whether there is any life or alterations on distant bodies (*unde etiam fieri potest, ut in coelo sint particulares alterationes, quae a nobis tamen, qui ab illis longe distamus, non compraehendantur*).[30] Benedetti even surmises that other planets are moons reflecting the solar light to dark planets invisible to us.[31] He ascribes this opinion to the followers of Copernicus. This is a free interpretation on his part. Perhaps he aimed to explain the epicyclic motions of other planets through an analogy with the lunar epicycle around the earth. Benedetti also rejects Ptolemaic and Aristotelian arguments against terrestrial motion. Following Copernicus (*De revolutionibus* I 8), he stresses that the axial rotation avoids the otherwise enormous motion of the fixed stars: "which is eliminated by the rotation of the Earth about its axis (as they say) as it is sufficient to receive the light and the influences of the [celestial] bodies."[32] Moreover, the annual revolution respects the dignity of the "divine body of the

[26]Benedetti 1585, 228–248, "Defensio ephemeridum."
[27]Benedetti 1585, 255–256.
[28]Benedetti 1585, 255.
[29]Benedetti 1585, 255: "[…] id etiam minus putabunt hii, qui opinionem Aristarchi Samii, et Nicolai Copernici sequuntur, quorum ratione fieri non potest, ut credant eius, quod ex universo reliquum est, alium finem non habere, quam regimen huius centri [Tellus] epicycli Lunaris, ut illorum more loquar."
[30]Benedetti 1585
[31]The same thesis is presented in Benedetti 1585, 195–196.
[32]Benedetti 1585, 255–256: "quae quidem omnia [phaenomena], cum simplici gyro terrae circa suum axem (ut dicunt) tolluntur, quod sufficit ad recipiendum lumen, et influentias illorum corporum."

Sun" (*divinum corpus solare*), which stands still at the center of the planetary circles.[33] Note Benedetti's astrological concern. In the final passage of his letter, he reassesses Copernicus's objection to Ptolemy's view of how bodies suspended in the air are affected by terrestrial motion:

> Ptolemy's objections are not valid for them [astronomers who assume that the earth moves]. As they say, every part maintains the nature of the whole, apart from the fact that the air and water circumscribing the earth receive the same natural impulse of motion [*impetum motus*]. This is slower the further the air is distant from the earth. According to the same doctrine, there is no necessity that the place of the fixed stars has (either convex or concave) superficial boundaries.[34]

According to this passage, the air close to the earth is transported by the motion of the planet and slows down the more it is distant from it. The fixed stars are placed in a motionless air whose place (*locus*) has no boundaries, either convex or concave.

In a letter to the courtier Capra, Benedetti confronts the issue of the form of the heavens.[35] This is said to be a sphere encompassed by infinite space. Accordingly, Benedetti distinguishes between *spacium* (space) and *coelum* (heavens), a distinction that can be traced back to Stoic cosmology or to the more recent views of Marcellus Palingenus Stellatus. The idea of the infinity of space beyond the starry vault can be found also in Patrizi's *Nova de Universis Philosophia* (1591).[36]

Furthermore, Benedetti rejects the existence of material spheres with the role of transporting the planets:

> That you do not accept that distinction of spheres, which was well-established in the past, but rather that you believe that the whole is a continuum accommodating the stellar bodies, this is not new. In fact, some philosophers of solid doctrine were of the same opinion.[37]

The motion of celestial bodies is accompanied by that of transparent bodies similar to vapors (*fumi*). Their motion is the cause of the apparent sparkling of the most distant stars.[38] The sparkling of the new star in Cassiopeia in 1572 bears witness to its great distance above the moon, which Benedetti also demonstrates through geometry.[39]

[33] Benedetti 1585, 256.

[34] Benedetti 1585: "Rationes autem a Ptolomeo in contrarium adductae apud ipsos, nullae sunt, quia quaelibet pars (ut inquiunt) retinet naturam totius, praeterquam quod aer, et aqua, quae ipsam terram circundant, plane eundem naturalem impetum motus obtineant, qui tanto lentior est, quanto longius distat aer, ab ipsa terra, secundum etiam talem opinionem, nulla necessitas, ut locus fixarum terminaretur aliquibus superficiebus, convexa scilicet, et devexa."

[35] Benedetti 1585, 285–286, "De motu molae, et trochi, de ampullis, de claritate aeris, et Lunae noctu fulgentis, de aeternitate temporis, et infinito spacio extra Coelum, Coelique figura."

[36] For Benedetti's correspondence with Patrizi, see Claretta 1862.

[37] Benedetti 1585, 411: "Quod eam distinctionem orbium, quae iam invaluit, non teneas, sed putes totum esse quoddam continuum excipiens corpora stellarum, novum non est, nam nonnulli solidae doctrinae philosophi idem confuerunt."

[38] Benedetti 1585, in the section entitled "Disputationes de quibusdam placitis Arist[otelis]," n. 38: "Occultam fuisse gravissimo Stagiritae causam scintillationis stellarum," 186: "Scintillatio ergo stellarum, neque aspectus nostri ratione, neque alicuius mutationis earundem stellarum, sed ab inaequalitate motus corporum diaphanorum mediorum nascitur, quemadmodum clare cernitur, quod si inter aliquod obiectum, et nos, aliquis fumus, qui ascendat, intercesserit, videbimus obiectum illud quasi tremere. Hoc autem tanto magis fiet, quanto magis distabit obiectum ab ipso fumo; unde admirationi locus non erit, si stellas fixas magis scintillare, quam errantes cernamus. Lumen stellae ad oculum nostrum accedens, perpetuo per diversas diaphaneitates penetrat, medio continuorum motuum corporum mediorum, unde continuo eorum lumen variatur, et hoc in longitudinis magis, quam in propinquis stellis apparet."

[39] Benedetti 1585, 371–374.

One of the books of the *Diversae speculationes* entails a discussion and a refutation of Aristotelian physical and celestial theses *de motu*. It has the rather neutral title *Disputationes de quibusdam placitis Arist[otelis]* (Disputations on Some Opinions Held by Aristotle) but it is indeed an attempt to revise basic concepts of natural philosophy such as *locus* (place) and *tempus* (time). We shall deal with this issue in detail in the next section. For now, it is important to anticipate that this anti-Aristotelian section entails Benedetti's most explicit defense of Copernican planetary hypotheses. Another remarkable thesis of these *Disputationes* on Aristotle is the statement of a principle of relativity according to which planets appear to us as we appear to them:

> Aristotle did not consider that one could affirm the same about the Earth as seen from great distance. There is no doubt that, even if the Earth had the light of the Sun and somebody tried to observe it from the eighth sphere, he would not be able to perceive it. In fact, those celestial bodies that are said to be of the first magnitude and that are believed to be more than a hundred times bigger than the Earth look just like points.[40]

Benedetti supports the plurality of worlds as well (*Minus sufficienter explosam fuisse ab Aristotele opinionem credentium plures mundos existere*). Every planet should be regarded as another Earth with its elements and natural places: "If those worlds existed, each of them would have its own center and its own circumference and the earths and fires would have an inclination towards the centers and the circumferences of their worlds, respectively."[41]

[40]Benedetti 1585, 197, "Disputatio XXXIX, Examinatur quam valida sit ratio Aristotelis de inalterabilitate Coeli: Aristo[teles] non consideravit, quod similiter de terra dici posset, quando ipsa ita eminus prospiceretur, imo absque dubio putandum est, quod si terra luce Solis praedita esset, et aliquis ipsam ab octavo orbe vellet videre, nullo pacto cerneret, cum sidera illa quae primae magnitudinis vocantur, et quae plusquam centies maiora ipsa terra putantur non nisi ut puncta videantur."

[41]Benedetti 1585, 195: "Si essent dicti mundi, eorum quilibet suum proprium centrum, suamque propriam circunferentiam haberet, terraeque et ignes haberent inclinationem ad centra circunferentiasque suorum mundorum."

## 6.4 Appendix: An Assessment of Benedetti's Horoscopes (by Günther Oestmann)

For the recalculation of a historical horoscope, the same methods and means the author had at his disposal must be employed, that is, the use of modern parameters or tables is not allowed.[42] In the following disposition, planetary positions are rendered in ecliptic longitude (degrees ; minutes) for each zodiacal sign (0–30°), geographical coordinates likewise in degrees ; minutes, and time in hours ; minutes. Latitude is denoted as φ.

### 6.4.1 Nativity Cast by Benedetti for Duke Carlo Emanuele I of Savoy

January 11, 1562 (Julian date), 16;23 p.m., φ = 45°; Planetary positions according to the *Prutenicae Tabulae* by Erasmus Reinhold (1551).

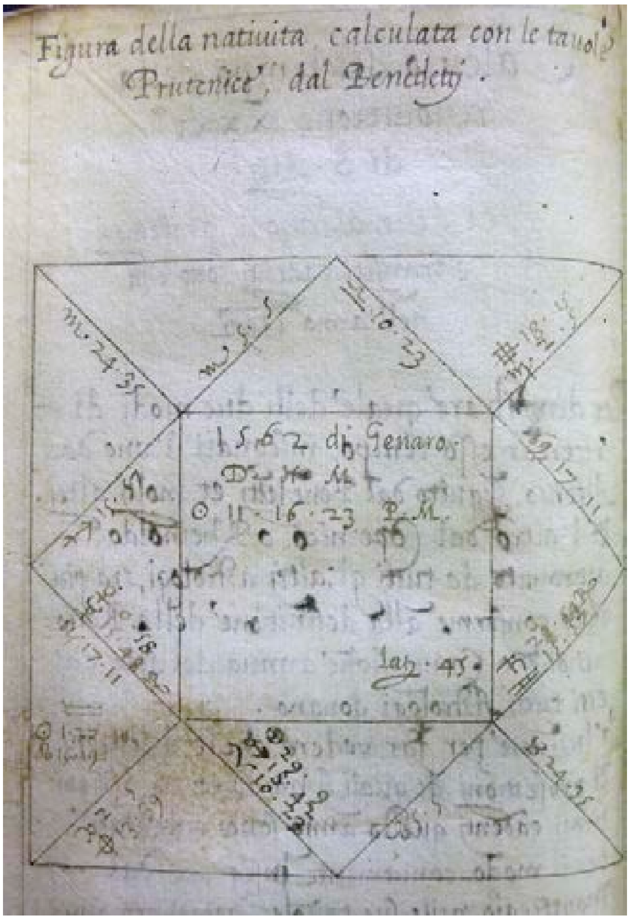


Figure 6.1: The horoscope cast by Benedetti for Duke Carlo Emanuele I of Savoy. This was calculated with the Prutenic tables, as transcribed by Bartolomeo Cristini in *Revolutione trentesima prima del Serenissimo Signore il Signor Carlo Emanuel Duca di Savoia corrente dell'anno 1592 con ogni diligenze et fedeltà calculata et decchiarata secondo le migliori intelligenze de più principali autori dell'astrologia giundiciaria*, Turin. (Biblioteca Nazionale Universitaria: Coll. N VII 10, f. 11*v*)

[42] Here a convenient, unfortunately little-known computer program created by Peter Schiller especially for the needs of historians has been used. See Schiller 2001. There is not sufficient room here for a detailed analysis of the choice of appropriate historical parameters; for a concise description, see Oestmann 2002 and Eade 1984, 1–103.

Carlo Emanuele I of Savoy was born in the Castle of Rivoli (c. 15 km west of Turin) on January 12, but here the "noon epoch" is indicated in the manner commonly used by astronomers/astrologers: the date changes at 12:00 local time, and the hours are counted from there to 24—contrary to civil use, where sunrise or sunset often marked the change of day. With the proliferation of mechanical clocks in the late Middle Ages, the date change at midnight gradually became predominant and hours were counted from 1–12.[43]

The geographical coordinates of the nearest town to Turin listed in the *Catalogus locorum* in Reinhold's *Prutenicae Tabulae* is Venice, which is 0 h 50 m (12;30) west of Königsberg, the reference meridian of the Prutenic tables. In Petrus Apianus's *Cosmographicus liber* (1533), the following specifications are given: Turin 30;30 and Königsberg 42;16 east of the island Porto Santo near Madeira (f. XLII*r*, XXXIX*r*) → the difference of longitude is 11;46 (modern value: 12;46). In the following recalculation, a longitude of 12;00 west of Königsberg has been assumed:

Table 6.1: Planets

| Planets | Original Source | Recalculation |
|---|---|---|
| Sun | 1;27 Aq | 1;27 Aq |
| Moon | 29;09 Ar | 29;16 Ar |
| Saturn | 28;54 Ge retrograde | 28;55 Ge |
| Jupiter | [missing] | 21;02 Ta |
| Mars | 18;42 Ar | 18;41 Ar |
| Venus | 0;58 Cp | 0;58 Cp |
| Mercury | 15;48 Cp retrograde | 15;48 Cp |
| Lunar node (asc.) | 15;19 Aq | 15;16 Aq |

Table 6.2: Houses

| Houses (Regiomontanus) | Original Source | Recalculation |
|---|---|---|
| X | 10;23 Li | 10;16 Li |
| XI | 5;05 Sc | 4;59 Sc |
| XII | 24;35 Sc | 24;32 Sc |
| I | 15;57 Sa | 15;54 Sa |
| II | 17;11 Cp | 17;07 Cp |
| III | 2;05 Pi | 1;58 Pi |
| | | |
| Lot of Fortune (Night) | 18;15 Vi | 18;05 Vi |
| Lot of Fortune (Day) | 13;39 Pi | 13;43 Pi |

The Lot of Fortune (*Pars Fortunae*; named for the Roman goddess of luck and wellbeing) is calculated in diurnal charts by subtracting the ecliptic longitude of the sun from the

[43] For details, see Bilfinger 1888, 262–286 and Ginzel 1914, 94–96.

longitude of the moon. Then the difference is added to the longitude of the Ascendant: Lot of Fortune = Ascendant + Moon – Sun. For nocturnal charts, the calculation is Ascendant + Sun – Moon.

Although this is a night-time birth chart, Benedetti has marked the Lot of Fortune for night and day.

The sign and degree occupied by the moon when crossing the ecliptic from southern to northern latitude is the ascending node (*Caput Draconis*). When the moon is moving in the opposite direction (crossing the ecliptic from north to south), the point of intersection is called the South Node (*Cauda Draconis*). The nodes are not fixed, but have a retrograde movement (a complete revolution of the nodes in the ecliptic takes 6798 days/18.61 years). To both points (which are important in the interpretation of a chart), the strength of a planet has been assigned. The Dragon's Head is considered beneficial, the Dragon's Tail malefic. (In Hindu astrology, the ascending node is called *Rāhu* and the descending node *Ketu*; both are considered malefic planets.[44].)

Benedetti forgot to inscribe Jupiter. Apart from this flaw everything has been calculated accurately.

### 6.4.2 Revolution or Solar-Return Horoscope

> January 21, 1592 (Gregorian Date), 23 h 15 m 30 s p.m., $\varphi = 45°$.
> Geographical coordinates of Turin according to Petrus Apianus in *Cosmographicus liber* (1533): f. XXXV*r* – Toledo 9;04 East of Porto Santo; f. XLII*r* –Turin 30;30 → 21;26 East of Toledo (the reference meridian of the Alfonsine tables).

Table 6.3: Planets

| Planets | Original Source | Recalculation (Alfonsine tables) | Recalculation (Prutenic tables) |
|---|---|---|---|
| Sun | 1;27 Aq | 2;08 Aq | 1;12 Aq |
| Moon | 9;27 Ta | 11;20 Ta | 8;12 Ta |
| Saturn | 7;29 Ca retrograde | 10;30 Ca | 7;30 Ca |
| Jupiter | 11;44 Sa | 10;31 Sa | 11;42 Sa |
| Mars | 3;05 Ar | 4;23 Ar | 3;00 Sa |
| Venus | 25;09 Sa | 23;00 Sa | 25;08 Sa |
| Mercury | 9;36 Aq | 5;43 Aq | 9;25 Aq |
| Lunar node (asc.) | 4;54 Ca | 5;05 Ca | 4;53 Ca |

[44] See Hartner 1938, 131–134

Table 6.4: Houses

| Houses (Regiomontanus) | Original Source | Recalculation I (φ = 45;00; 23;15 p.m.) | Recalculation II (φ = 45;00, 23;17 p.m.) |
|---|---|---|---|
| X | 21;00 Ca | 20;30 Cp | 20;59 Cp |
| XI | 1;00 Aq | 10;36 Aq | 11;11 Aq |
| XII | 17;00 Pi | 16;26 Pi | 17;18 Pi |
| I | 10;49 Ta | 9;56 Ta | 10;43 Ta |
| II | 15;00 Ge | 14;36 Ge | 15;08 Ge |
| III | 5;00 Ca | 4;18 Ca | 4;45 Ca |
| | | | |
| Lot of Fortune (Day) | 18;49 Le | 16;55 Le | 17;44 Le |

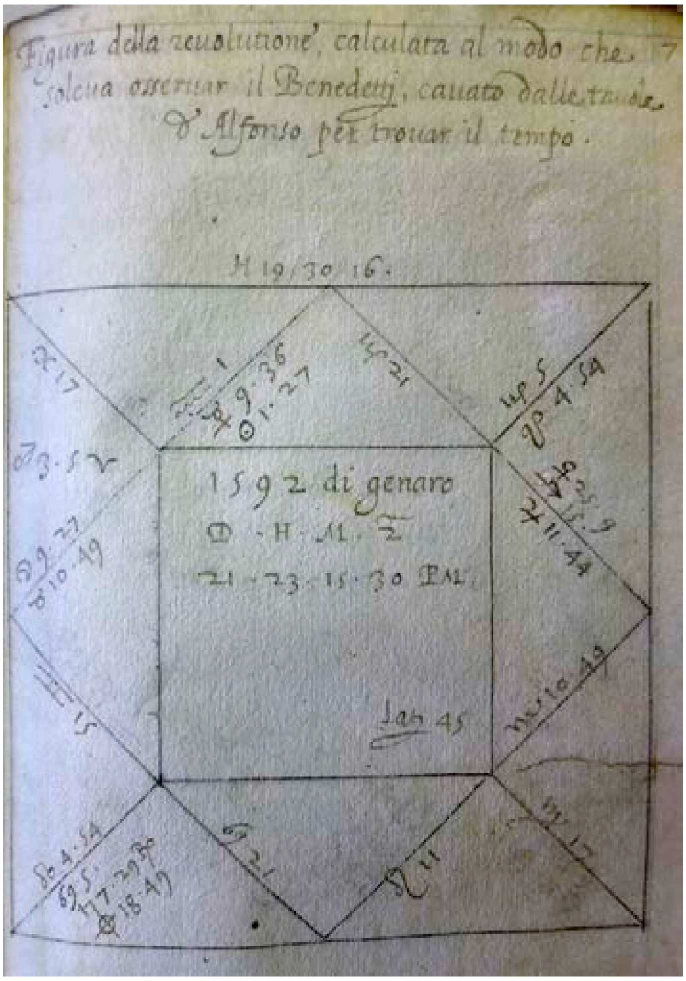


Figure 6.2: Benedetti's horoscope for Carlo Emanuele I, calculated with the Alfonsine tables, as transcribed by Cristini in *Revolutione trentesima prima* (1592), f. 12*r*. (Biblioteca Nazionale Universitaria di Torino, coll. N VII 10)

The second of Benedetti's horoscopes to be considered is also taken from Bartolomeo Cristini, *Revolutione trentesima prima del Serenissimo Signore il Signor Carlo Emanuel Duca di Savoia corrente dell'anno 1592 con ogni diligenze et fedeltà calculata et decchiarata secondo le migliori intelligenze de più principali autori dell'astrologia giundiciaria*, Turin, Biblioteca Nazionale Universitaria: Coll. N VII 10, f. 12*r* (Figure 6.2).

This is a chart constructed for the moment in which the sun returns to the degree and minute of its longitude at nativity (i.e., transiting the position of the "natal" sun) for the respective location. A revolution horoscope indicates the course of events during the ensuing year.

Contrary to Benedetti's caption (*Figura della revolutione [...] cavato dalle tavole d'Alfonso per trovar il tempo*), he has obviously used the Prutenic tables for calculating the planetary positions of this chart. But the moon's position is off by c. 1°, and the cusps of the houses deviate somewhat. Calculating with a time of 23;17 p.m. gives a reasonably good compliance, however. The cusp of house XI (1;00 Aq instead of 11 Aq) is most likely a scribal error.

It is noteworthy that minutes for an arc are only provided for the first house (i.e., the ascendant). For the other cusps, only whole degrees are noted. Benedetti simply cut off the minutes, which was a common rounding procedure at his time.

### 6.4.3 Natal Horoscope of Giovanni Battista Benedetti

August 14, 1530 (Julian Date), 13 h 13 m p.m., Venice; planetary positions according to the Alfonsine tables. Geographical coordinates of Venice according to Petrus Apianus (1533): Toledo 9;04 East of Porto Santo (f. XXXV*r*); f. XLII*r*: Venice 32;30, Latitude φ = 44;50 → 23;26 East of Toledo (the reference meridian of the Alfonsine tables).

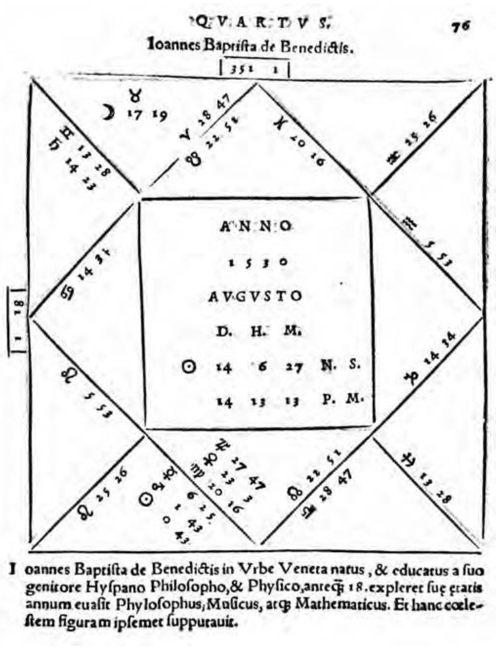


Figure 6.3: Benedetti's own horoscope, detailed in Luca Gaurico's *Tractatus astrologicus* (1552, f. 76*r*). (Bayerische Staatsbibliothek)

Table 6.5: Planets

| Planets | Original Source | Recalculation |
|---|---|---|
| Sun | 0;43 Vi | 0;42 Vi |
| Moon | 17;19 Ta | 15;02 Ta |
| Saturn | 14;23 Ge | 14;23 Ge |
| Jupiter | 27;47 Vi | 27;46 Vi |
| Mars | 1;43 Vi | 1;45 Vi |
| Venus | 23;03 Vi | 23;02 Vi |
| Mercury | 6;25 Vi | 6;25 Vi |
| Lunar node (desc.) | 22;51 Ar | 22;52 Ar |

Table 6.6: Houses

| Houses (Regiomontanus) | Original Source | Recalculation |
|---|---|---|
| X | 20;16 Pi | 20;14 Pi |
| XI | 28;47 Ar | 28;36 Ar |
| XII | 13;28 Ge | 13;17 Ge |
| I | 14;34 Ca | 14;23 Ca |
| II | 5;53 Le | 5;46 Le |
| III | 25;26 Le | 25;23 Le |

Apart from the moon's position (which is about 2° off) the horoscope is correct. In all three horoscopes the houses have been constructed according to the so-called "rational method," commonly—but erroneously—attributed to Regiomontanus:[45] Circles of position joining in the north and south point of the observer's horizon are laid at distances of 30° through the celestial equator, thus giving unequal sections of the ecliptic. This method of house division was widely used by astrologers during the sixteenth and seventeenth centuries.

[45]It was already known in the Maghreb in the eleventh century, see Kennedy 1996, 543. For a profound treatment of the history of house division, see North 1986, although the way this text coins new designations is awkward and may lead to confusion.

# Chapter 7
# Foundations of Physics

In this chapter we analyze Book 4 of the *Diversae speculationes*, entitled *Disputationes de quibusdam placitis Aristotelis* (Disputations on Some Opinions Held by Aristotle). We will refer to this section as *Physical Disputations*. Benedetti here developed his theory of motion and clarified his physical conceptions by means of a discussion and criticism of Aristotle's physics. He dealt with fundamental concepts such as place and time. Moreover, it is here that the interdependency of physics and cosmology in his conception most clearly emerges.

## 7.1 Sections of the *Physical Disputations*

The *Physical Disputations* are a discussion of and an objection to Aristotle's theses on local motion and cosmology as presented in *Physics* and *De caelo*, and partly also in *Meteorologica* and *Metaphysica*. Benedetti does not limit himself to criticism but rather seeks to provide a new approach to and foundation of physics and cosmology, beginning with the theory of motion. He calls his approach mathematical (*inconcussa mathematicae philosophiae basis*). In actual fact, his conceptual tools for the treatment of motion are principally derived from Archimedes's *Floating Bodies* and Euclid's Book 5 on proportions. As we have seen, the reader of Benedetti's *Diversae speculationes* is provided with a brief book dedicated to Book 5 of Euclid's *Elements*, directly following the *Physical Disputations*.[1] Benedetti moreover reworks and transforms basic concepts of physics, such as place and time, and propounds anti-Aristotelian natural views such as spatial infinity and the existence of the void. His treatment culminates with cosmological speculations, including a defense of the Copernican system.

One can conveniently subdivide the *Physical Disputations* into five sections (see table 7.1 below). The first twelve chapters are a lengthy discussion of Aristotle's *Physics* IV 8. This section deals with the ratio of velocities of bodies moving through different media or the void. Secondly, from chapter 13 to chapter 18, Benedetti challenges *Physics* VII 5 on further problems linked with the theory of motion. The third subdivision (chapters 19 to 22) deals with basic philosophical matters (the void, infinity, place, and time). This section is extremely important for an overarching understanding of Benedetti as it connects the investigation of motion with a general reform of natural philosophy. Whereas Drake and Drabkin, in *Mechanics in Sixteenth-Century Italy*, translated the first two subdivisions (chapters 1–18), they neglected the foundational one, except for chapter 19 on the void. Their choice to expunge this part of the *Physical Disputations* deprived the English-speaking readership of some of the most daring pages in Benedetti's work.

Another subdivision (chapters 23–26, entirely translated in Drake and Drabkin 1969) deals with local motion and the shortcomings of the Aristotelian theory of natural places. The fifth and last subdivision, only partly included in the English edition, presents cosmological ideas. It deals with the "sphere" as a geometrical-cosmological figure, as well

[1] Benedetti 1585, 198ff.

as with the (apparent) motion of the sun, with stars, meteorological aspects linked to the sun, the propagation of light in the cosmos, and other issues connected with astronomy in a broad sense. The Copernican system is discussed in the second part of this last section (chapters 35 to 39), along with other innovative theses such as the plurality of inhabited worlds akin to the earth and the reciprocity of the observational points in the universe.

We will concentrate on sections 1 (on motion), 3 (on the foundations of physics), and 5, part 2 (on cosmology). As one sees from our overview (table 7.1), the *Physical Disputations* are no less complex and heterogeneous than the volume of which they are part, the *Diverse speculationes*. Therefore, we will review Benedetti's arguments in enough depth to enlighten the thematic interdependency and his approach in general, deliberately leaving aside an excessive analysis of details that would not help to understand his intellectual endeavor as a whole.

Table 7.1: An overview of the *Physical Disputations* and of their English translation in Drake and Drabkin 1969.

| Sections and their subjects | Chapters | Details | Presence in Drake and Drabkin 1969 |
|---|---|---|---|
| 1. Phys. IV 8 | §1–12 | Discussion on the void and down- and upward motion in different media | ✓ |
| 2. Phys. VII 5 | §13–18 | On the proportions of velocities, volumes, and surfaces | ✓ |
| 3. Foundations of physics revised | §19–22 | On the void (XIX ≈ Phys. IV 8), place (XX ≈ Phys. IV 4), the infinite (XXI ≈ Phys. III 5, De Caelo I,9 etc.,), and time (XXII) | Only §19 |
| 4. Violent and natural motion | §23–26 | Rejection of the Aristotelian theory of natural places | ✓ |
| 5. The sphere (geometrical and cosmological) | §27–34 | On the sphere (§29), on starry light (§30), on infinite motion (§31), on the Sun, its warmth and seasonal changes (§30–31 and §34), celestial music, and harmony (§32–33) | Only §28 and §29 |
| | §35–39 | On the composition of circular and rectilinear motion and Copernicus's hypotheses (§35), plurality of worlds (§36), cosmic propagation of light (§37), geometrical aspects relative to elements (§38), and relativity of the point of observation (§39) | ✓ (only §38 missing) |

## 7.2 An Archimedean Theory of Motion

The Renaissance rediscovery of Archimedes's work can be seen as a crucial contribution to the theoretical advance of modern physics.[2] The wide dissemination of this ancient work produced a renewed interest in mathematical methods for the investigation of nature. In the *Questiti et inventioni diverse* of 1554, Tartaglia's Archimedean perspective implied a critical approach to Aristotle's mechanics. By contrast, Del Monte had a rather syncretistic approach merging Archimedean and Aristotelian elements. He even argued for the possibility of deriving terrestrial motion from Aristotelian premises (probably referring to geological motions rather than to an astronomical phenomenon).[3] Benedetti's feelings toward Aristotelian philosophy are rather hostile; nonetheless, it is clear that his anti-Aristotelian theory of motion is embedded in the Renaissance discourse on natural philosophy, mechanics, and motion among mathematically-trained scholars.[4]

### 7.2.1 Aristotle's *Physics* IV 8

Benedetti's considerations on motion are presented as a criticism of Aristotle's *Physics* IV 8. In that section Aristotle argued against the existence of the void and infinity of space and presented some reflections on the motion of bodies through different media. According to Aristotle, the void and infinity would undermine any reasonable theory of natural motion (indeed, his own theory of the natural and violent motions). His polemic is directed against "those," probably Democritus and his followers, who held the void to be the condition of motion. Aristotle claims that in an infinite and void space there would be no absolute directions; an up- or downward motion would be conceptually impossible. By contrast, he keeps to the "observation" that the elements display natural tendencies upward or downward (water and earth downward, air and fire upward), which, according to him, falsifies the thesis of an infinite and void space. Additionally, this assumption would lead to "absurd" consequences:

> But in vacancy […] nothing could go on moving unless it were carried. Nor (if it did move) could a reason be assigned why the projectile should ever stop—for why here more than there? It must therefore either not move at all, or continue its movement without limit, unless some stronger force impedes it.[5]

As far as motion is concerned, Aristotle remarks that the difference of speeds between two bodies depends either on their different weight or lightness or on the different density of the media through which they travel. He adds that the ratio of the speeds is reciprocally proportional to that of the densities, whereas it is directly proportional to that of the times.

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[2] Paolo D'Alessantro and Pier Daniele Napolitani have emphasized the impact of its sixteenth-century reedition in their studies on the Latin reception of Archimedes: "Nel 1544 usciva a Basilea l'*editio princeps* greco-latina di gran parte dei testi di Archimede oggi noti. Senza tema di esagerare, si può sostenere che questo avvenimento – al pari della pubblicazione del *De revolutionibus orbium coelestium* di Niccolò Copernico, apparso l'anno precedente a Norimberga – abbia fornito impulso alla nascita della scienza moderna." D'Alessandro and Napolitani 2012, 9.

[3] See Omodeo 2015.

[4] As has been argued in *Exploring the Limits of Preclassical Mechanics*, the wider conceptual framework of Archimedean theories of motion was in fact deeply rooted in Aristotelianism. See Damerow, Freudenthal, et al. 2004.

[5] Aristotle 1995, IV 8, 215a 17–22 (351).

For the sake of simplicity, we could represent these relations symbolically, in a modern way. Heath, in his study on *Mathematics in Aristotle*, put it as follows:[6]

$$\frac{speed\ in\ B}{speed\ in\ D} = \frac{density\ of\ D}{density\ of\ B}$$

Furthermore:

$$\frac{time\ taken\ in\ B}{time\ taken\ in\ D} = \frac{density\ of\ B}{density\ of\ D}.$$

On this basis, Aristotle concludes that motion in the void would be impossible, since “a body would move through the void with a speed beyond any ratio.”

According to Aristotle, differences depending on the weights and on the dimensions of the bodies would disappear *in vacuo* with “very unpleasant” consequences:

> What reason can be assigned for this greater velocity [of a heavier falling body]? If the passage is through a medium, there must be such a difference [in the velocity of fall between heavier and lighter bodies]; for when there is anything there to cleave, the body superior in force of its thrust will necessarily cleave the medium faster, since either its more suitable shape or the natural thrust it exercises, whether following its natural movement or being thrown, makes it cleave the better. Where there is nothing to cleave, therefore, all bodies will move at the same velocity; which is impossible.[7]

Aristotle assumes that the speed of falling bodies is proportional to their weight (or dimension). For instance, one reads in *De caelo*:

> His must surely be a careless mind who does not wonder how it is that a small particle of the earth, if raised to a height and then set free, should refuse to remain where it was but begin to travel, and travel the quicker the bigger it is, whereas if one held the whole Earth in the air and let it go, it would not move. But in fact, for all its weight, it is at rest.[8]

This argument was repeated by Ptolemy in *Almagest* I 7: “If the Earth had a single motion in common with other heavy objects, it is obvious that it would be carried down faster than all of them because of its much greater size.”[9]

In summary, *Physics* IV 8 provides Benedetti with a series of entangled problems involving the void, infinity, and motion. He begins his reform of physics by dealing with the last issue and then moving to a treatment of the other ones. In relation to motion through a medium, he enlists some commonly accepted assumptions (*primo verissima et obiecta intellectus per se cognita*) in chapter 2:

> Therefore, whenever two bodies are subjected to or receive one and the same resistance to [the motion of] their surfaces, [the speed of] their motions will turn out to be to each other in precisely the same proportion as their motive forces. And, conversely, whenever two bodies have one and the same heaviness or lightness, but are subject to different resistances, [the speed of] their

[6] Heath 1949, 116.
[7] Aristotle 1995, IV 8, 216a 17–21 (357).
[8] Aristotle 1986, II 13, 294 a (223–225).
[9] Ptolemy 1984, 44.

> motions will have the same ratio to each other as the inverse ratio of the resistances.[10]

The cases that have to be considered are basically two: first, different bodies in the same medium, and second, equal bodies in different media. A decisive difference between two bodies is their *gravitas* (gravity, weight) or *levitas* (lightness). For Benedetti, *gravitas* and *levitas* act as moving forces. He calls them *virtutes moventes* or *facultates moventes*. For the sake of brevity, we will refer to them as *virtus/virtutes*.

If we use V like *velocitas* for speed, P like *pondus* for *virtus*, and R like *resistentia* for resistance, we can formalize the previously mentioned general assumptions in the following way:[11]

I. Case in which R is constant:

$$\frac{V_1}{V_2} \sim \frac{P_1}{P_2}$$

II. Case in which P is constant

$$\frac{V_1}{V_2} \sim \frac{R_2}{R_1}$$

### 7.2.2 Speed in Different Media

Benedetti regards the Aristotelian theory of motion as inadequate and prefers to rely on Archimedes. He conceives of motion as analogous to the behavior of bodies in water. Following the *Floating bodies*, he holds that weight and lightness are not absolute but relative properties because they depend on the medium: "*quia in medio se densiore si poneretur, non grave esset, sed leve, quemadmodum Archimedes ostendit*" (in fact, if it is put in a denser medium it is not heavy but light, as Archimedes demonstrates).[12] Thus, the direction of a vertical motion and, as we will see, the speed depend on the matter of a body as well as on the fluid (air, water, or whatever) in which it is merged.

In the second *disputatio*, Benedetti declares that the *virtus* (weight or lightness)[13] of a body varies depending on the *densitas* (density) of the medium. Here he has in mind the three different cases considered by Archimedes in Book 1 of the *Floating bodies*: a body the weight of which is equal to that of the fluid, lighter, or heavier. Archimedes's seventh proposition, concerning the last case, might illustrate Benedetti's mental model for motion in a medium:

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[10] Drake and Drabkin 1969, 198. Cf. Benedetti 1585, 169: "Quotiescunque igitur duo corpora unam eandemque resistentiam ipsorum superficiebus, aut habebunt aut recipient, eorum motus inter seipsos eodem plane modo proportionati consurgent, quo erunt ipsorum virtutes moventes: et e converso, quotiescunque duo corpora unam eandemque gravitatem, aut levitatem, et diversas resistentias habebunt, eorum motus inter seipsos eandem proportionem sortientur, quam habebunt eorum resistentiae converso modo […]."

[11] It should be remarked that this algebraic rendering of Benedetti's thoughts does little justice to contemporary efforts to represent functional dependencies among different magnitudes with the help of the available mathematical techniques, such as compound proportions, as will become clear from the following discussion; see also Damerow, Freudenthal, et al. 2004.

[12] Benedetti 1585, 170.

[13] In this passage, Benedetti employs the word *pondus*, but in the discussion he also talks of "lightness"; therefore, we prefer to use the term *virtus movens* or, more briefly, *virtus*, which is the term used by Benedetti.

> Solids heavier than the fluid, when thrown into the fluid, will be driven down as far as they can sink, and they will be lighter in the fluid by the weight of a portion of the fluid having the same volume as the solid.[14]

The actual *virtus* (either weight or lightness) of a body results by subtracting from the total virtue (*virtus totalis*) a quantity which Benedetti calls *resistentia extrinseca* (resistance, for instance in disp. 9) and depends on the *densitas* (density, for instance in disp. 3), which we could regard as an expression indicating the specific weight:[15]

*virtus in medio = virtus totalis – resistentia*

Accordingly, he holds the position against Aristotle that the ratio of the speeds in different media is not inversely proportional to the densities of the media but directly proportional to the *virtutes* in different media.

$$\frac{\textit{speed in A}}{\textit{speed in B}} = \frac{\textit{virtus totalis} - \textit{resistance of A}}{\textit{virtus totalis} - \textit{resistance of B}}$$

Given this equation, Benedetti shows (chapters 3–4) that the thesis of *Physics* IV 8, namely that the ratio of the speeds is reciprocally proportional to that of the densities of the media, is only true in a special case, which can be constructed *ad hoc*: if the ratio of an object's weight (or the corresponding speed) in one medium (for instance air) to its weight in another medium (for instance water) is equal to the ratio of the first medium (air) to the second (water). However, it is incorrect to claim that the ratio of the speeds of all bodies moving through different media remains the same. In chapter 6, Benedetti demonstrates moreover that the ratios of the weights (or speeds) of a body through different media are not maintained as constant in different media (*Quod proportione ponderum eiusdem corporis in diversis mediis proportiones eorum mediorum densitatum non servant. Unde necessario inaequales proportiones velocitatum producuntur*).[16]

We should now add some considerations on Benedetti's argumentative strategy. He begins chapter 2 with a reference to principles generally taken for granted: *primo verissima et obiecta intellectus per se cognita*. The adverb "primo" can have two meanings in this context: it could indicate either that the author is proposing the "first" principles of the motion theory or, quite on the contrary, that he is presenting theses that are universally true "only at a first glance." These "very true assumptions," these *primo verissima*, are in fact the Aristotelian statements concerning the proportion of the ratios of velocities and weights as well as the proportion of the ratios of velocities and resistances. In the second part of chapter 2, Benedetti brings forward his own conception of the proportion between the ratio of velocities and that of weights in a medium. He begins the paragraph on his theory with these words: "Aliud quoque supponendum est." This *incipit* can be interpreted in two different and rather opposite ways: as "Another proposition must also be presupposed" (which is Drake and Drabkin's translation) or as "We must presuppose a different proposition." As a matter of fact, the thesis that follows these words, that on the subtraction of the resistances from the total *virtutes*, is meant as a general truth. Benedetti himself intends to show that the Aristotelian laws are not true universally but only for

[14]Dijksterhuis 1956, 376.

[15]See Benedetti 1585, 174: "dictis corporibus subtrahitur proportio resistentiarum extrinsecarum." For a discussion of Benedetti's employment of the termini 'pondus,' 'densitas,' and 'resistentia,' see Helbing 1987, 155–168.

[16]The details, the text, and the notes in the English edition by Drake and Drabkin are clear enough, so we will not expand on these aspects here.

special cases. Benedetti presents his theory of motion in opposition to that of Aristotle, so we tend to interpret chapter 2 as an opposition between two contrasting perspectives. Yet a certain ambiguity in his words cannot be denied. Perhaps it is a rhetorical strategy consciously employed by Benedetti: he first presents to the reader some propositions with which natural philosophers should be familiar, and then leads his reader to reject these common statements as not universally true rather than absolutely false.

### 7.2.3 Motion *in pleno* and *in vacuo*

After these considerations on motion in different media, Benedetti deals with the motion of bodies with different dimensions (but of the same material) through a medium as well as through the void. Chapter 7 considers the case in which two bodies A and O, made of the same matter and having the same figure, traverse the same medium. According to Aristotle the ratio of their speeds should be directly proportional to that of their weights. Benedetti demonstrates that this is false, since one could imagine a third body U, made of a different material, with the same *virtus* (here: *gravitas*) as O and the same dimensions (*magnitudo et figura*) as A. If V is the speed, M the *magnitudo* corresponding to a certain weight (or more in general, a *virtus*), then:

$$\frac{VA}{MA} = \frac{VO}{MO}$$

(according to Aristotle)[17]

$$\frac{VA}{MA} = \frac{Vu}{Mu}$$

(case of two bodies with equal dimensions and different weights).

Since by assumption MO = Mu, it follows that Vu = VO, but this is not consistent with the assertion that two bodies of equal weights and different dimensions will move with different speeds in a medium because a bigger body needs more "force" to penetrate the medium than a smaller one:

> The body which, when compared to the other, is of equal weight or lightness, but is subject to smaller resistance, moves [in natural motion] more swiftly than the other in the same proportion as its surface is subject to a smaller resistance than that of the other body, by reason of its being able more readily to divide the continuity of the air or water.[18]

Therefore, the ratio of the velocities of two bodies having equal material composition and figure but different dimensions is untenable, *pace* Aristotle.

Chapter 9 introduces the issue of motion through the void, asserting, against Aristotle, that the ratio of two bodies *in pleno* and *in vacuo* cannot be the same:

> In a plenum the ratio of the external resistances in the case of these bodies is subtracted from the ratio of the weights, so that the ratio of the speeds

[17] It corresponds to the above-mentioned equation: $\frac{V1}{P1} = \frac{V2}{P2}$.

[18] Drake and Drabkin 1969, 198. See Benedetti 1585, 169: "Corpus illud quod alteri comparatum, aequalis erit ponderis, aut levitatis, sed minoris resistentiae, existet velocius altero, in eadem proportione, cuius superficies resistentiam suscipit minorem ea quae alterius est corporis, ratione facilioris divisionis continuitatis aeris, aut aquae […]."

remains. And this last ratio would be annuled if the ratio of these resistances were equal to the ratio of the weights.[19]

The argumentation is not immediately intelligible to the modern reader. Contrary to appearance, in fact, this passage does not describe the following subtraction: $\frac{P_1}{P_2} - \frac{R_1}{R_2}$ (V is the speed, P the weight, and R the resistance).

$$\frac{V_1}{V_2} \nsim \frac{P_1}{P_2} - \frac{R_1}{R_2}$$

According to the theory of proportions, the addition and subtraction of a ratio to or from another ratio can be conveniently represented in a modern fashion as follows:[20]

I. Meaning of ratios addition:

$$\frac{a_1}{a_2} + \frac{b_1}{b_2} \sim \frac{a_1 b_1}{a_2 b_2}$$

II. Ratios subtraction:

$$\frac{a_1}{a_2} - \frac{b_1}{b_2} \sim \frac{a_1 b_2}{a_2 b_1} \sim \frac{a_1}{a_2}\frac{b_2}{b_1}$$

Given this, the relation indicated by Benedetti in the above-mentioned passage can be rendered through the following symbolic expression:

$$\frac{V_1}{V_2} \sim \frac{P_1}{P_2}\frac{R_2}{R_1}$$

This relation expresses in a concise form the Aristotelian perspective concerning the relation of velocities, weights, and resistances, as already taken up by Benedetti at the beginning of the *disputatio* number two. Benedetti notices that the ratio of the velocities is annuled (*nulla esset*) if the ratio of the *virtutes* is equal to that of the resistances ($\frac{P1}{P2} \sim \frac{R1}{R2}$). In fact, in this case (and only in this case) the speeds of bodies with different weights would be the same. As a consequence, there would be "no proportion" between different quantities but rather an equality of speeds. Benedetti indicates that this special case of motion *in pleno*, in which bodies with different weights have equal speeds, is the rule for motion *in vacuo*.

Chapter 10 expands on motion *in vacuo*. It presents the famous thesis that bodies of the same material move with the same speed *in vacuo*, although they might have different dimensions. Benedetti remarks that a body O can be divided into halves A and E, each of the same weight (or *virtus*). If an ideally weightless bar connects them, the weight at the center of the connection should be the sum of the partial weights and thus equal O. Therefore, bodies composed of the same material will fall with the same speed, independently of their weight. In chapter 11, one reads that motion *in pleno* is different as a consequence of the friction of the medium, except for the special case in which the parts travel through

[19]Drake and Drabkin 1969, 205. See Benedetti 1585, 174 (emphasis is ours): "In pleno dictis corporibus *subtrahitur* proportio resistentiarum extrinsecarum a proportione ponderum, ut velocitatum proportio remaneat, quae nulla esset, si dictarum resistentiarum proportio, ponderum proportioni aequalis esset, et hanc ob causam diversarum velocitatum proportionem in vacuo haberent ab ea, quae est in pleno."

[20]The same concept of addition and subtraction of ratios can be found in the manuscripts of Thomas Harriot, who called them "compositio" and "subductio" (or "compositio contraria"). See for instance Schemmel 2008, 635, reference letters *g* and *o*.

media whose resistances have “the inverse proportion of the weighs” (*si duo corpora… suas resistentias ad invicem proportionatas haberent, ut sunt eorum pondera*).[21]

### 7.2.4 A Note on the Historical Relevance of Benedetti’s Reflections on Motion

Our analysis of the first part of the *Physical Disputations* should be sufficient to understand Benedetti’s approach to motion and the scope of his investigation. Subdivision two tackles Aristotle’s *Physics* VII 5 and deals with the proportions of velocities, volumes, and surfaces.[22] We are not going to analyze in further detail Benedetti’s views on motion. Instead, we deem it important to consider the “philosophical” part of the *Physical Disputations* (which we have indicated as section 3), as it introduces novel prospects on the foundations of physics. It extends from chapter 19 to chapter 22 and is an attempt to revise basic concepts of physics from an anti-Aristotelian perspective.

Before we tackle these foundational aspects, we would like to recount the influence that Benedetti’s theory of motion exerted on the young Galileo. We have already hinted at Benedetti’s triangulation with Galileo via Del Monte. At a theoretical level, one of the strongest pieces of evidence of this influence is the affinity between Galileo’s early speculations on motion and the theses that Benedetti propounded in the *Diversae speculationes*. This link is evident and well known, beginning with the hydrostatic analogy to explain the motion through a medium, the relativity of heavy and light, and the subtraction of resistance from weight, which allows motion through a vacuum to be accepted and makes it physically plausible.[23] For instance, several chapters of Galileo’s first manuscript among those gathered by Favaro under the title of *De motu* are very close to Benedetti’s treatment; above all Galileo’s chapter 8, “in which it is shown that different bodies moving in the same medium maintain a ratio [of their speeds] different from that attributed to them by Aristotle”; chapter 10: “in which, in opposition to Aristotle, it is proved that, if there were a void, motion in it would not take place instantaneously, but in time”; chapter 12: “in which, in opposition to Aristotle, it is concluded that the absolutely light and the absolutely heavy should not be posited; and that even if they existed, they would not be earth and fire, as he believed”; and chapter 15: “in which, in opposition to Aristotle, the conclusion is reached that rectilinear and circular motions have a ratio to each other.” This connection between the work of the young Galileo and Benedetti’s insights on motion is significant, the more so since Galileo’s patron Del Monte was skeptical about the possibility of quantifying motion.[24] Actually, in his manuscripts, Del Monte took some annotations on falling bodies in different media. This might have been an issue of discussion between him and Galileo.[25]

Moreoever, it should be remarked that the Copernican cosmological element is absent from Galileo’s early manuscript *De motu*, although this would become a crucial aspect of

[21] See Drake and Drabkin 1969, 206, n. 119: “Benedetti asserts that the speeds are equal *only in the void*, on the ground that in a plenum there would be an additional frictional resistance that would disturb this equality (unless this resistance itself were proportional to the weights of the bodies: Ch. 11).”
[22] See Helbing 1987, 162.
[23] Apart from Koyré 1986, see also Drabkin, “Introduction” to Galilei 1960, 9–10.
[24] The ongoing debate between Galileo and Del Monte on the possibility of developing a mathematical dynamics is seen in Galileo’s letter of November 29, 1602: “V.S. Ill.ma scusi la mia importunità, se persisto in voler persuaderle vera la proposizione dei moti fatti in tempi uguali nella medesima quarta del cerchio; perché essendomi parsa sempre mirabile, hora viepiù mi pare, che da V.S. Ill.ma vien reputata come impossibile: onde io stimerei grand’errore e mancamento il mio, s’io permettessi che essa venisse repudiate dalla di lei speculazione, come quella che fusse falsa.” See Galilei 1968, 97.
[25] See Tassora 2001, 281–283.

his later investigations. Also, the alliance of mechanics and Copernican astronomy, which emerged only later in Galileo, bears witness to Benedetti's influence on his work.[26]

## 7.3 On the Void: Atomistic Prospects

Benedetti's considerations on motion are followed by a treatment of the void which, from the perspective of Aristotle's *physics*, is directly connected with the former issue. His theory of motion through media and through the void is the basis upon which he casts Aristotle's rejection of the void into doubt. Chapter 19 of the *Physical Disputations*, *Quam sit inanis ab Aristotle suscepta demonstratio quod vacuum non detur*, is a transition from the Archimedean theory of motion (chapters 1–18) to the reconceptualization of natural philosophy in general. The discussion on the void is directed against Aristotle's *Physics* IV 8, in which the theory of motion serves to reject the physical void and infinity. Benedetti claims that the void is conceptually possible as a consequence of his previous demonstration that the Aristotelian assumptions on the relation between speed and density of the medium are wrong or, at least, not universally valid. "*Ex iis, quae superius demonstravimus facile cognosci potest irritam esse eam rationem, quam Aristoteles 8 cap. lib. 4* Physicorum *ad destruendum vacuum, confinxit*" (From the demonstrations above it is easy to see that the argument that Aristotle devised in Physics VIII 4 is vain).[27] According to Benedetti's Archimedean dynamics, motion through a void is not absurd at all. He explains that such a motion is simply quicker than one taking place through any medium, because no resistance has to be subtracted from the body's *virtus*.

Hence, Benedetti reintroduces the concept of the void into physics, which was excluded by Aristotle as a criticism of Democritean atomism.[28] Benedetti's move is in line with the rehabilitation of atomistic philosophy during the Renaissance and the early modern period. The void and atoms are the two ingredients of the same matter theory inspired by the rediscovery of Lucretius and a reassessment of Epicureanism.

Among Benedetti's contemporaries the most committed natural philosopher supporting atomism was Bruno. Although they probably developed their theories independently of each other, they have many points of contact. In his Italian philosophical dialogues (published in London in the years 1584–1585), Bruno widely drew on ancient atomistic doctrines. He called Democritus and Epicurus those "who contemplated nature with open eyes" (*con occhi più aperti han contemplata la natura*).[29] He derived from them (often relying on Lucretius) a vision of the universe as infinite, the idea of a countless number of worlds (similar to the solar system), as well as the principle of cosmological homogeneity and the atomic theory of matter and of minima moving through a void. In *De l'infinito universo e mondi*, he celebrated Epicurus's theory of matter, summarizing it as follows:

> Epicure similarly nameth the whole and the universe a mixture of bodies and of the void; and in this universe and in the capacity thereof to contain the void and the empty, and furthermore in the multitude of the bodies contained therein he maintaineth that the nature of the world, which is infinite, doth exist.[30]

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[26] Drabkin 1964, Drake 1976, and Renn and Damerow 2012, 142–155.

[27] See Benedetti 1585, 179.

[28] On the medieval debates on the void and on the historical place of Galileo, and Benedetti before him, see Grant 1981, 60–66.

[29] Bruno 2000a, 374.

[30] Bruno 1968, 272–273.

However, the concept of the void assumes in Bruno a very special meaning since, in his philosophical terminology, the terms 'space,' 'void,' and 'aether' are used as synonyms. The ethereal void is the medium that makes the motion of bodies possible.[31] In the second dialogue of *De l'infinito*, he recapitulates the following three meanings of *vacuo*:

- First, the void refers to all which is not bodily and thus does not have the property of resisting penetration. In this sense, there is a "boundless ethereal region" (*eterea regione inmensa*), in which the worlds are plunged. The countless worlds populating the universe are themselves composed of matter and the void.[32]
- Second, the void has the meaning of infinite space (*spacio infinito*). In it everything is included. It is the container that cannot be included in anything else.
- Third, space can be assumed to be nothingness, in accordance with Aristotle. Bruno calls this a metaphysical meaning, or rather lack of meaning. According to him, this extra-physical meaning was a sophism that served Aristotle to refute the possibility of the void in nature. For Bruno, only the first and the second meaning of the void make sense. They could and should be embraced in natural philosophy. By contrast, the metaphysical void has to be rejected as meaningless.

Bruno's reflections on the physical void are part of his program to revive an atomistic conception of nature. He combined it with cosmological infinity and the post-Copernican idea that a plurality of worlds exists, each one of them a solar system centered on a star similar to ours. This vision, which shows similarities to Benedetti's, paved the way for seventeenth-century cosmologies, in particular for the Cartesian multi-centric universe.

Other Renaissance scholars supported combinations of atomism and heliocentric astronomy as well. A case in point is the circle of scientists gathered around Henry Percy of Northumberland in London. Just like Benedetti and Bruno, they brought together heliocentrism, atomism, an empirical and mathematical approach to nature, Renaissance naturalism, and anti-Aristotelianism.[33] Thomas Harriot, for example, was an atomist and a supporter of the infinity of the universe.[34] Nicolas Hill, another member of the "Northumberland circle," authored an apology of Epicureanism entitled *Philosophia Epicurea* (1601), which was directly inspired by Bruno. It included Copernican arguments in favor of terrestrial motion and many others in favor of the earth's magnetism, in accordance with Gilbert.[35] Following ancient and modern atomists, Hill affirmed the boundlessness of the universe and the plurality of worlds.[36]

Atomism was to be embraced by celebrated exponents of sixteenth-century science and philosophy such as Galileo and Pierre Gassendi. At the same time, corpuscular alternatives were spread by Descartes and his followers.[37] Atomism and, more generally, corpuscular theories of matter encountered much censorship, especially owing to theological difficulties, since they appeared to be irreconcilable with Scholastic accounts of the

[31] See Del Prete 1999, 61 and Michel 1962.

[32] Bruno 2000a, 348: "[…] perché questo spirito, questo aria, questo etere non solamente è circa questi corpi, ma ancora penetra dentro tutti, e viene insito in ogni cosa."

[33] Kargon 1966.

[34] On his intellectual stature and achievement, see Schemmel 2008, who stressed that Harriot's unpublished manuscripts reveal that his research activity was similar to that of Galileo in the same years. Moreover, Harriot's reflections on infinity and the minimum bear witness to his familiarity with the work and ideas of Bruno. See Fox 2000 and Henry 1982.

[35] N. Hill 2007, 155–157. See Plastina 1998, Plastina 2004 and Omodeo 2014a, 372–377.

[36] For an overview of English science in that period, see C. Hill 1997, 15–76 and 118–200.

[37] On medieval and early-modern corpuscular matter theories, see Lüthy, Murdoch, and Newman 2001. On Galileo, see Galluzzi 2011.

Eucharist.[38] Nonetheless, many modern scholars were attracted, just as Benedetti was, to the prospect of connecting the advances of mechanics with a physics and a theory of motion rooted in corpuscularism alongside a post-Copernican cosmological outlook. As Galluzzi has recently pointed out, this was a guiding idea for Galileo already in his *Iuvenilia*.[39] Theories of motion, atomism, and Copernican astronomy are three interconnected issues in his as well as Benedetti's work. In *De motu*, Galileo was committed to the homogeneity of matter and reflected on it in connection with motion, in particular with the Benedettian thesis that "*si vacuum esset, motuum in instant non contingere, sed in tempore*" (If the void exists, motion does not occur instantaneously but rather in time).[40] In the tenth section of this manuscript, Galileo followed in Benedetti's footsteps and came to the same conclusion: "Hence it follows, not that motion in a void is instantaneous, but that it takes place in less time than the time of the motion in any plenum."[41]

Benedetti shared many ideas with contemporary and later scholars in his criticism of Aristotle's natural philosophy as well as in his effort to construct a new physics. Whereas he developed his conceptions on matter and the cosmos independently of Bruno, and probably did not directly influence the English circle of Harriot and Hill, nonetheless he must have influenced the views of Italian scholars such as Galileo who were familiar with the *Diversae speculationes*. Benedetti showed them that a revision of dynamics could not be fulfilled independently of a broader program of philosophical reform.[42]

## 7.4 On Place: Space as *intervallum corporeum*

Chapter 20 of Benedetti's *Physical Disputations* deals with the concept of place. Aristotle, in *Physics* IV 4, defines it as the surface in which a body is included and denies that it could be conceived as an *intervallum*. By contrast, Benedetti defines it precisely as *intervallum corporeum*, an expression which could be roughly translated as an inter-bodily gap. Aristotle says that *locum* (the place) and *locatum* (the placed) must be coextensive but, as Benedetti notices, a spherical surface contains more "extension" (*intervallum*) than any other figures with an equal surface. As a consequence, one can imagine two places (in the sense given to the term by Aristotle) occupied by bodies of different dimensions or, the other way round, two bodies of different dimensions which suit the same surface. By contrast, an *intervallum* of space contains only bodies of equal volume, thus respecting the bijective relation between the volume of the place and that of the placed body: "But equal bodily extensions [*intervalla*] delimitated by any figure will always contain equal bod-

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[38] See Redondi 1983, chap. 7, 203–226. Also see Ariew 2013.

[39] Galluzzi 2011, 9.

[40] Galilei 1968, vol. I, 276.

[41] Galilei 1960, 47. See Galilei 1968, vol. I, 282: "[…] et ita non est necessarium, motum in vacuo fieri in instanti, sed in tempore minori quam sit motus in quovis pleno."

[42] Thus, Galluzzi's remarks on the scientific production of the young Galileo could be conveniently extended to Benedetti (Galluzzi 2011, 19): "Il laboratorio del *De motu antiquiora* servì a Galieo per prendere definitivamente coscienza che la cosmologia e la fisica aristoteliche non potevano essere riformate con interventi limitati ad ambiti di indagine circoscritti. In particolare, lo scritto pisano evidenziava la raggiunta consapevolezza dell'importanza strategica di una radicale riforma della concezione aristotelica del movimento. Era questo, tra l'altro, l'ostacolo più serio da superare perché potesse essere accolta la visione copernicana, intesa non come semplice ipotesi matematica, ma come dottrina fisica. […] Né si può escludere che abbia tratto anche da esse [da tali problematiche] l'impulso per dedicarsi, con l'impegno proprietario esibito dai documenti dell'attività giovanile, all'impresa ambiziosa di una radicale riforma della concezione tradizionale del movimento e della struttura della materia."

ies."[43] The definition of place as *intervallum* has the advantage that it allows us to assign a place to every body, "in the world or outside it, *pace* Aristotle."[44] This remark anticipates the cosmological treatment of infinite space, or of the possibility of space beyond our worldly system (*mundus*). In chapter 20, on place, Benedetti remarks that Aristotle is wrong when he rejects the concept of place as *intervallum* because it could lead to infinity. Benedetti objects that this is not inconvenient at all, since one could conceive of *infinita loca*. To grasp this infinity is no more difficult than to understand the never-ending process which Aristotle calls "potential infinity" (*infinitum in potentia*) and concerns the division of a body *ad infinitum*:

> That infinite places must exist, if place is [conceived of as] *intervallum*, is not inconvenient. In fact, just as any body can be [divided into] infinite bodies (which he [Aristotle] would call "*in potentia*"), so can any *intervallum* be [divided into] infinite *intervalla* as well.[45]

Benedetti is always brief in his treatment of philosophical issues and leaves much implicit. Therefore, the reader is challenged to reconstruct the complete vision implied in his short statements and critical points. Reference to Renaissance philosophical sources discussing the same theses from an Aristotelian viewpoint can help us to better understand Benedetti's conception. In this case, Alessandro Piccolomini's refutation of the conception of space as a receptacle of bodies can shed light on the opposite viewpoint defended by Benedetti. The following quotation is taken from the first part of Piccolomini's *Della filosofia naturale* (On natural philosophy), a very successful introduction to natural philosophy in Italian, which Benedetti might have known:

> Other philosophers (and not unimportant but very reputed ones) asserted that there is void space among natural beings and that it is not only distinguished in essence [*per sua natura*] from the bodies it accommodates, but also it is so separated from them that it can remain completely naked and really deprived of them. Furthermore, many supposed that such a space (which is completely void, without any bodies) is mixed and situated between the beings of this world and their parts. Additionally, they believed that it infinitely extends outside the heavens. Thus, these philosophers considered the void to be that being which not only can be deprived of all bodies and substances […] but also is apt and capable of receiving the bodies, giving them a place (something which does not occur to lines, points, and other accidents). One of the main reasons for holding such doctrines was, as they said, that, if there is no space in nature (or a void place without bodies), the motion from place to place would be impossible, although the motion of alteration would be possible. In fact, all beings, in order to move to some other place, need some in-between space through which they can move. If this space were not void, it would be necessary that, as it is filled with other bodies, different bodies

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[43] Benedetti 1585, 180: "Sed intervalla corporea aequalis a quavis figura terminata, continebunt semper corpora aequalia."

[44] Benedetti 1585, 181: "Et hoc modo nullum est corpus, quod in mundo aut extra mundum (dicat autem Aristoteles quicquid voluerit) locum suum non habeat."

[45] Drake and Drabkin 1969, 198. Cf. Benedetti 1585, 180: "Quod si locus intervallum esset, infinita loca existerent […] inconveniens [non] existit, quia eodem plane modo quo aliquod corpus potest esse infinita corpora (quod ipse diceret in potentia), sic etiam intervallum aliquod posset esse infinita intervalla."

> would penetrate each other while crossing that full space. And this is impossible.[46]

Thus Piccolomini presented the views of the supporters of a natural void in order to reject them. Yet his discussion is helpful as it indirectly presents us with the views of such thinkers as Benedetti who defended void and infinity. Benedetti was in fact favorable to the notions of the void and infinity. For him, space is prior to and independent of bodies. It should be remarked that, in Benedetti's perspective, infinite space does not imply cosmological infinity or the boundlessness of the material universe, precisely because space is independent of matter. In this respect, his conception is different from the one that Bruno defended in those years. Rather, it is closer to that of his correspondent Patrizi. As De Risi has pointed out, Patrizi's conception of space is marked by its ontological autonomy from bodies. It is in fact an "incorporeal and immaterial extension, tridimensional and infinite, which receives and precedes all created bodies."[47] Patrizi saw space as a sort of Plotinian *hypostasis* (*extensio hypostatica*), that is, a pre-corporeal quantity quantifying reality. He saw this idea as the lever to be employed in order to radically reform Scholastic metaphysics, a project that he developed in a series of publications, *De spacio physico et mathematico* (On physical and mathematical space, 1587), *Della nuova geometria* (On the new geometry, 1587), and eventually in the *Nova de universis philosophia* (1591).[48] It also served him to set the foundations of an epistemology of mathematics according to which the ancient science dealing with figures had to be substituted for a new science of space itself. This is the concept of his *New Geometry*. Patrizi sent a copy of this book to Benedetti, probably before the publication, to gain the favor of Carlo Emanuele I, to whom the book was dedicated.

To sum up, Benedetti defines the *locus*, against Aristotle, as *intervallum corporeum*, or bodily extension. He regards it as an invariable empty extension capable of being occupied by material bodies, an idea that becomes clearer from the related discussion on time. He basically rejects the Aristotelian definition of place as the *superficies corporis ambientis* (surface of the containing body), remarking that the place is a measure, actually an *intervallum*, and not a surface. Moreover, he explains that only his understanding of locus as a kind of objective space guarantees that two bodies of equal material constitution necessarily occupy the same quantity of "place" as *intervallum* or empty extension, independently of their form. By contrast, the definition given by Aristotle is inconsistent with the assumption of a bijective relation between *locus* and *locatus*. In fact, as geometry shows, if one accepts Aristotle's definition "a great absurdity [*maximum inconveniens*]

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[46]Piccolomini 1585, I 3, chap. 5, f. 44*r*: "Altri filosofi sono stati poi, et non già minimi, ma di gran fama, che han voluto che tra le cose della natura, si trovi spatio voto, non solamente distinto per sua natura dai corpi ch'egli riceve; ma intanto separato da quelli, che ignudo in tutto, et attualmente spogliato ne può restare. Né son mancati molti, che cotale spatio, voto in tutto d'ogni corpo, han posto, non solo meschiato, et interposto tra le cose che sono al mondo, e tra le parti loro; ma ancora fuor dei Cieli, infinitamente han creduto che si distenda. Questi filosofi adunque per il voto intendevano quella cosa, che non solo fusse privata d'ogni corpo et d'ogni sostanza […] ma fosse ancora atta, et capace, a ricevere i corpi, et dar lor luogo, il che delle linee, et de punti, et altri accidenti, non avviene. Tra le ragioni principali, che gli spingeva a credere una tal cosa, l'una era questa nella qual dicevano, che se non si potesse trovare tra le cose della natura spatio alcuno, o luogo voto d'ogni corpo; allora, se bene il movimento dell'alteratione potrebbe forse restare al mondo, tuttavia il movimento di luogo a luogo, non potrebbe senza 'l voto restar giammai. Conciosia c'havendo bisogno quella cosa, che ha da muoversi ad alcuno altro luogo, di alcuno spazio di mezo, per il quale si muova; se cotale spatio non fosse voto, saria forza che essendo pieno di altro corpo, nel passar per quello spatio pieno, venissero a penetrarsi più corpi insieme; adunque è cosa al tutto impossibile."

[47]De Risi 2014, 282.

[48]De Risi 2014, 276–277.

would follow, namely that equal places can contain unequal bodies or equal bodies can occupy unequal places."[49] Additionally, Benedetti remarks that his concept of place admits an infinite universe, since it is capable of containing *infinita corpora* (infinite bodies).

## 7.5 In Defense of Infinity

Chapter 21 is devoted to infinity: *Utrum bene Aristoteles senserit de infinito* (Whether Aristotle judged correctly about infinity). Needless to say, Benedetti holds that Aristotle's arguments on this issue are idle. The Greek philosopher rejects the possibility of an infinite body (*infinitum corpus*), that is to say, the infinity of the universe because—as the argument goes—there is no *locus infinitus* which could include it. Benedetti protests that this is a *petitio principi* since this rejection is based on a questionable definition of *locus*: "*cum Aristoteles debuerit beneficio loci destruere infinitum, ordine perverso de infinito prius*" (Since Aristotle had to destroy infinity with the benefit of [a correct understanding of] place, he wrongly started with [a discussion of] infinity).[50] By contrast, Benedetti's redefinition of place as *intervallum* entails no conceptual hindrances to the acceptance of the infinity of the universe. As one reads: "*hoc modo nullum inconveniens sequeretur, quod extra caelum reperiri possit corpus aliquod infinitum*" (In this manner it is not inconvenient to assume that one can find an infinite body outside the heavens).[51]

Additionally, Aristotle writes (*Physics* VIII 8 and *De caelo* I 9) that a continuum can include infinite parts only *in potentia* (potentially) and not *in acto* (in act). Benedetti does not agree with him. He argues that, if the continuum is *in acto*, its infinite parts should be *in acto* as well, because it is foolish (*stultum*) to believe that something actually existing could be composed of parts which exist only potentially (*quae potentia existunt*).

The weakest argument brought forward by Aristotle is, according to his critic, that the infinite cannot be considered to be a quantity (*Physics* III 5), because only a finite quantity is thinkable, for instance the dimensions of a square or a cube. Benedetti objects that the definition of number (which he does not provide, however) does not include finitude (*necessitas terminorum*). Numbers are not necessarily terminated (*determinati*) and one can conceive an infinite multitude exactly like a finite one:

> Aristotle's arguments in the same part of *Physics* III 5 are even worse. He denies that infinity can be regarded as a quantity by saying that only one defined quantity is intelligible, such as a cubit, a three-cubit, etc. Thereby, he does not consider that in the same manner the quantity of infinite cubits is intelligible as well. Moreover, the definition of quantity does not imply limitation by necessity. For instance, the definition of number does not imply the necessity of any determined number, since an infinite multitude is not less intelligible than a finite.[52]

[49]Benedetti 1585, 180: "maximum inconveniens sequeretur, scilicet aequales locos capere inaequalia corpora, aut corpora aequalia, locos inaequales occupare."

[50]Benedetti 1585, 181.

[51]Benedetti 1585, 181.

[52]Benedetti 1585, 181: "Sed peius etiam sensit Aristotles eodem loco capitis quinti lib. 3 Physicorum, negando infinitum posse connumerari inter quantitates, dicens unam aliquam quantitatem intelligi ut cubitum, tricubitum, et cetera; ubi non considerat eadem etiam ratione intelligi posse aliquam quantitatem infinitorum cubitorum, et in quantitatis definitione nullam esse necessitatem terminorum, ut exempli gratia in definitione numeri, non est necessitas alicuius determinati numeri, quia multitudo, non minus infinita, quam finita, intelligi possit."

The last false Aristotelian assumption on infinity is the equation of *infinitum* and *vacuum* in *Physics* IV 8. Benedetti's commentary is harsh: "Later, in *Physics* IV 8, he says that there is no difference between infinity and the void. Indeed, he could not assert and imagine anything more absurd than this."[53]

Like Bruno, the infinitist *par excellence*, in *De l'infinito universo e mondi* (1584), Benedetti remarks that Aristotle's arguments are not compelling. Aristotle denied the possibility of an unbounded space on the basis of a finite cosmology (i.e., the theory of the spherical and geocentric heavens along with the theory of the natural places), which excludes cosmological infinity by definition. Yet his assumptions are not self-evident.

In summary, in chapter 21 of the *Physical Disputations*, Benedetti defends the mathematical and physical possibility of the infinite. The question of the title "Whether Aristotle Judged Correctly about Infinity" is rhetorical. It calls for a negative answer.

### 7.6 On Time: Toward an Absolute Frame for Physics

Benedetti's definition of *tempus* (time) is closely connected with that of *locus* (place). He deals with it in *Physical Disputations*, chapter 22, *Exagitatur ab Aristotele adducta temporis definitio* (Rejection of Aristotle's Definition of Time). Benedetti questions the definition of time as *motus mensura numerusque* (the measure and number of motion) and offers an alternative conceptualization. But before we discuss his opinion, we will recount standard conceptions of time from antiquity.

It should be noted as a preliminary remark that the understanding and definition of time was regarded as a natural as well as a psychological issue. This should not be surprising, since the doctrine of the soul, or *psychologia*, was an integral part of natural philosophy, or *physica*, in standard university curricula. A standard reference on time was the *Timaeus*, where Plato called time, rather poetically, the "moving image of eternity" (εἰκὼ κινητός αἰῶνος).[54] According to his myth, the divine Demiurge created time together with the heavens (οὐρανὸς), making them inseparable. Plato's pupil Aristotle, in Book 4 of the *Physics*, then proposed the definition of time as the "dimension of movement in its before-and-afterness."[55] He also noticed that χρόνος (time) had generally been connected with the motion of the celestial sphere and was referred first and foremost to the heavens because all measurements of motion and time depend on celestial regularities:

> Neither qualitative modification nor growth nor genesis has the kind of uniformity that rotation has; and so time is regarded as the rotation of the sphere, inasmuch as all other orders of motion are measured by it, and time itself is standardized by reference to it.[56]

For both philosophers, Plato and Aristotle, the tie between time and cosmic order was unquestionable. This interconnection was later refuted by a third authoritative source. At the downfall of the ancient world, Augustine, in Book 11 of the *Confessiones*, denied the commensurability of time and local motion: "The motion of a body is one thing, and that by which we measure how long it is, another thing."[57] He pointed out the transient

[53] Benedetti 1585, 181: "Ubi postea cap. 8 lib. 4 Physicorum ait nullam esse differentiam inter infinitum, et vacuum, revera nihil absurdius hoc dicere fingereve poterat."
[54] Plato Timaeus 37C–39E
[55] Aristotle 1995, IV 11, 220 a 25–26 (395).
[56] Aristotle 1995, IV 14, 223 b 21–22 (425).
[57] Augustine 1988, XI 14, 265: "aliud sit motus corporis, aliud quo metimur quamdiu sit."

character of the temporal flux and noticed that the three dimensions of time are a product of our mind (*animus*). In the mind, time is constituted as the memory (*memoria*) of the past, the intuition (*attentio*) of the present, and the expectation (*expectatio*) of the future.[58] In other words, Augustine underscored the subjectivity of time, conceived of as a *distensio animi*, a "stretching" of the mind independent of heavenly cycles.

According to Benedetti's criticism of Aristotle, the definition of time as "the measure and number of motion" is intrinsically wrong because measuring presupposes commensurability. But because time and motion are heterogeneous, they cannot be compared. A line is measured by a line, a surface by a surface, and a *corpus*—that is, a three-dimensional body—by a *corpus*. Similarly, motion is measured by motion and not by time: "Time cannot be the measure of motion, but only motion can measure motion, precisely a faster one [measures] a less fast one, and a shorter one [measures] a longer one, whereas a number is measured by a number, and time by time inasmuch as it is long or short, and not inasmuch as it is fast or slow."[59] Hence, time can only be measured extrinsically (*per accidens*) through motion, as is the case with common expressions like "two hours, or two days, or two years," referring to astronomical displacements. These are only metaphors that refer to heavenly motions as "placed" in the interval of time that corresponds to their motion. In the following passage, presenting these reflections, time is called the "place of motion" (*locus motus*):

> It could seem to somebody that, to indicate [*significare*] a certain quantity of motion, one has to assume as much time as if one says, for instance, that a certain operation has been carried out in the space of two hours, or two days, or two years. Still, it should be remarked that this is not literally [*simpliciter*] true, since the imagination conceives the interval of two hours, days, or years as the motion of celestial bodies without which neither years, not days, not hours would exist, even though time is placed, so to speak, in time, just as a body in a place. Thus, motion is measured by motion and time by time, and not the one by the other.[60]

Benedetti claims that time, unlike motion, is eternal: "Necessarily, from a philosophical point of view, time is eternal and motion not."[61] Whereas a motion can be extinguished and a body can be at rest after a displacement, time goes on inexorably. It is always present to our senses and always escapes them because it is the never-ending passing of a single instant. Benedetti makes clear that he intends the *instans* to mean "one in species" (*unum in specie*), i.e., the essence underlying the vanishing flux of time that cannot be experienced in itself as a given and measurable succession (*non in numero*).[62] This Augustinian

[58] Augustine 1988, XI 28.

[59] Benedetti 1585, 183: "Tempus non erit mensura motus, sed motus quidem potest mensurare motum, videlicet velocior minus velocem, et brevior longiorem; et numerus mensuratur numero, et tempus tempore in quantum longum est, aut breve, non in quantum velox, aut tardum."

[60] Benedetti 1585, 182: "Si alicui videtur, quod ad significandam aliquam quantitatem motus, dicere huiusmodi operationem duarum horarum, aut duorum dierum, aut duorum annorum spatio completam esse, sit ponere tantum tempus, animadvertere debet hoc simpliciter non esse verum, quia horarum, dierum, et annorum intervalla, imaginatione concipiuntur ut motus corporum caelestium, sine quibus neque anni, neque dies, neque horae existerent, etiam si omnis motus sit (ut ita dicam) locatus in tempore, ut corpore in loco, unde motus motu, et tempus tempore, non autem aliud ab alio mensuratur."

[61] Benedetti 1585, 182: "Tempus ex necessitate, philosophice tamen loquendo, res est aeterna, motus non item."

[62] Benedetti 1585, 182: "Tempus igitur potius locus motus erit dicendum, quam numerus aut mensura eius, et tale est, ut consumatum videatur a continuo quodam fluxu unius instantis [...] et cum dico ab uno instanti,

paradox helps Benedetti to stress the heterogeneity of time and motion. Still, he does not renounce an objective meaning, which is essential to his dynamics. Motion and time, he writes, are *continua successiva*, continuous and successive quantities. Their relation can be conveniently described as that between a place and the bodies that it contains. His explanation sheds light on the concept of *locus* as well as on that of *tempus*:

> Just as a dense body occupies a lesser interval of place than a less dense [*rarus*] body, similarly a quick motion is accomplished [*peragetur*] in a shorter space of time than a slower motion.[63]

From this passage it is clear that Benedetti's idea of place as *intervallum corporeum* goes in the direction of an empty homogeneous space which can be occupied by bodies of greater or lesser density. Time has the same absolute character as place. It can contain more or less rapid motions without being affected by them. Space and time or, more precisely, *intervallum corporeum* and *tempus* are objective and independent coordinates of natural phenomena.

In order to understand Benedetti's considerations on time, it is useful to delve into the views of his immediate forerunners, in particular Cardano and Scaliger.

### 7.6.1 Cardano's Subtleties on Time

In the letter to the reader of the *Diversae speculationes*, Benedetti indicated Cardano as one of his principal sources.[64] However, concerning the concept of time their opinions are not in agreement. Cardano tackles the issue *tempus quid sit* (What is time?) in Book 18 of *De subtilitate*. Unlike Benedetti, he does not explicitly question Aristotle, but rather quotes his definition as a common truth: "in fact, motion is the measure of time" (*motus enim tempus est mensura*).[65] Still, he reworks the Aristotelian concept within a rather Augustinian perspective, which leads to original results.

He introduces the problem of time in connection with his treatment of the dream (*somnium*), beginning with the question: "Why does time appear much longer while dreaming than in reality?"[66] He reports that once he dreamed that he visited an unknown city far away from his home in Milan. He traveled through mountains, valleys, and fields. In order to cover that distance six days of travel would be barely sufficient. Thus, when he woke up, he thought that he had slept for a long time but in actual fact his nap had taken less than one hour. The reason for this expansion of time, Cardano explains, is due to the fact that dreamed activities (*operationes*) are accomplished independently of any bodily effort (*absque corporis labore*) and therefore very rapidly. A correct judgment about time depends on bodily movement. This is why the perception of time is not altered in our mind when we imagine long-lasting processes while awake. "During sleep, time is contracted in the opposite manner than if we are awake: in fact, motion is the measure of time."[67] Surprisingly, Cardano's Aristotelian conclusion, "motion is the measure of time," does

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unum in specie, et non in numero intelligo, quod a sensibus nostris percipi non potest, neque etiam notari, quia novum semper instans nobis occurrit."

[63] Benedetti 1585, 183: "Quemadmodum corpus densum occupat minus intervallum loci, contra quam fiat in corpore raro; sic etiam motus velox breviori temporis spatio peragetur, quam tardus."

[64] Benedetti 1585, *Ad lectorem*, f. A3*r*.

[65] Cardano 1966, vol. 3, 651.

[66] Cardano 1966, vol. 3, 651: "Cur somnium tempus longius multo ostendat quam sit."

[67] Cardano 1966, vol. 3, 651: "Contraria ratione tempus in somnio contrahitur, cum vel non somniamus […]: motus enim tempus est mensura."

not refer to heavenly motions, but to corporeal activity. In other words, he extrapolates and isolates the peripatetic definition from its original context. In fact, from an objective, physical, and cosmological meaning, time acquires a rather subjective meaning, connected with physiology and perception. Time, as Augustine would say, is an "expansion of the mind."

It should be added that there is a passage of the *Physics* where Aristotle also hinted at the subjective dimension of time, even at how the soul grasps it. This passage might have been a source of inspiration for Cardano:

> Time cannot be disconnected from change; for when we experience no changes of consciousness, or, if we do, and are not aware of them, no time seems to have passed, any more than it did to the men in the fable who 'slept with the heroes' in Sardinia, when they awoke; for under such circumstances we fit the former 'now' to the later, making them one and the same and eliminating the interval between them, because we did not perceive it. So, just as there would be no time if there were no distinction between this 'now' and that 'now,' there appears to be no time between two 'nows' when we fail to distinguish between them. Since, then, we are not aware of time when we do not distinguish any change (the mind appearing to abide in a single indivisible and undifferentiated state), whereas if we perceive and distinguish changes, then we say that time has elapsed, it is clear that time cannot be disconnected from motion and change.[68]

These considerations are not aimed at isolating a subjective meaning of time by eliminating any objective references. Still, it is plausible that Cardano took into account such passages in order to freely speculate on time, in Book 18 of *De subtilitate*, from a perspective that owes more to Augustine than to Aristotle.

This Augustinian influence is particularly evident from the following passage, in which Cardano tries to better define time:

> But what is time? Although nothing of it is ever [given], nonetheless everything is in it and it accompanies [*assistit*] everything always. It itself generates and destroys everything; it is the source [*auctor*] of life and death. Its expectation is always very long, while its memory is always very short. Although it is always with us, we never grasp it. Even though there is such an abundance of it, nonetheless no restoration [*reparatio*] of it is ever conceded, thence the waste of no other thing is greater or worse.[69]

In this passage, Cardano brings together ideas derived from erudite lectures, as well as from commonsense, experience, and even trivial commonplaces. Additionally, he recounts the Augustinian paradoxes: time is everywhere and nowhere, it is made out of expectation and memory, and it is for us the most familiar and unknown mental presence. To quote from the *Confessiones*: "What is time then? If nobody asks me, I know: but if I

[68] Aristotle 1995, IV 11, 218 b 20–219 a 1 (383).

[69] Cardano 1966, vol. 3, 651: "Sed quid tempus est? Cuius cum nihil unquam sit, omnia tamen in illo sint et semper omnibus assistit. Illud idem omnia generat et occidit, auctor vitae ac mortis. Utque illius expectatio longissima, ita semper memoria brevissima. Cumque nos semper comitetur, nunquam ipsum tamen agnoscimus. Nec cum eius tanta sit copia, reparatio tamen ulla conceditur: unde fit, ut nullius alterius rei iactura sit maior et vilior."

were desirous to explain it to one that should ask me, plainly I know not."[70] Cardano adds to this paradox a popular sense of the caducity of life, according to which a discourse on time is a kind of *memento mori*. Time itself is said to be the cause of life and death.

Hence, for Cardano, the assumption that "time is the measure of motion" does not mean maintaining the cosmological dependency of time on astronomical cycles. Quite to the contrary, time transcends motions and changes. In fact, we do not perceive it in itself but rather that which happens in it. What we know about time is a product of the mind, precisely of its imaginative faculty:

> Thus, we do not comprehend time but rather that which occurs, or occurred, and endures in it. But time itself per se is out of reach [*ignotum*] to the senses. In fact all that we know [about it] is constituted through imagination.[71]

Like Benedetti, Cardano denied the interdependency of cosmological space and time, although from a different standpoint. He was not concerned with reformulating the physical space and time framework of motion in mathematical terms. Instead, he concentrated on the psychological and physiological dimension of time as a construction of the *imaginatio*. Therefore, this treatment was connected with that of the mind and was introduced by considerations on sleeping and dreaming. This even led to considerations on altered states of mind such as ecstasies and hallucinations, as well as the divine and demoniac visions of the hermits—Cardano went so far as to report some hallucinations that he had when he was a child. His mental treatment of time, as well as this excursus on altered states of mind, were both harshly criticized by the Flemish humanist Julius Caesar Scaliger, as we shall now discuss.

### 7.6.2 Scaliger's Aristotelian Restoration

Scaliger conceived of his *Exotericae exercitationes* as a critical confrontation with Cardano's *De subtilitate*, although he formally presented them as a reverent discussion of some points that arose from the lecture on the "subtleties" of that *doctissimus vir*, as one reads in the dedicatory epistle, "who will never be praised enough" (*nunquam satis laudatus*). Among the most notable theses indicated in the *Index acutiorum sententiarum* at the end of the book, one is directly relevant to our discussion: the *exercitatio* number 352.2, which explains why "time is not the measure of motion." That *exercitatio* deals with the passage of *De subtilitate* on time that we have discussed, but reverses Cardano's viewpoint.

Scaliger tries to answer the question of "whether time is the measure of time," remarking that most people just repeat this definition without properly understanding its meaning.[72] The allusion is clear: Cardano repeats a commonplace without any thorough reflection on its meaning. In fact, even though time might be regarded as the measure of "our motion," it is definitely not the measure of worldly motion, in particular not of the first motion of the heavens, i.e., the daily one.[73] Scaliger therefore denounces Cardano's

[70] Augustine 1988, XI 14, 238: "Quid est ergo tempus? Si nemo ex me querat, scio; si querenti explicare velim, nescio."

[71] Cardano 1966, vol. 3, 651: "Ergo nos non tempus, sed quod in eo fit, factumque est, atque manet, comprehendimus. Tempus vero ipsum per se est sensui ignotum: nam imaginatione constat quod a nobis cognoscitur."

[72] Scaliger 1557, f. 458*v*: "Hoc quidem ab omnibus iactatur: pauci introspectant."

[73] Scaliger 1557, f. 458*v*: "Nostri sane motus mensura sit: primi motus mensura non erit."

subjective reading of the Aristotelian definition. According to him, the objective dimension of time cannot be dismissed. From a cosmological-ontological perspective it is in fact a dependent (*affectus*) of heavenly motion;[74] to be precise, it descends from the "first motion," or the daily rotation of the starry sphere. The first body (*primum corpus*), that is, the heaven of the fixed stars, is the measure of all bodies. Similarly, its motion, the *primus motus*, is the yardstick of all motions. In accordance Book 12 of the *Metaphysica*, where Aristotle advocates a spherical and geocentric cosmology of concentric spheres, Scaliger states that "time is either the same thing as motion or its affection."[75] In other words, he intends to restore an Aristotelian objective conception of time in accordance with a metaphysical perspective that attaches an ontological priority to motion over time.

Scaliger acknowledges that Aristotle ambiguously defined the interrelation between time and motion by accepting both possibilities: "time is the measure of motion and, in turn, motion is the measure of time."[76] In his attempt to dispel this paradoxical interdependency, Scaliger distinguishes between two fundamental dimensions of time: the objective and the subjective dimension or, as one reads, the understanding "according to nature" (*a Natura*) and that "according to us" (*mensura nostra*). In nature, motion is the measure of time as well as its source. By contrast, time is the measure of motion only for us, which is an aspect that Cardano allegedly overemphasizes. This is clearly documented by expressions like "the equinoxial circle accomplishes its motion in this much time."[77] In actual fact, things are the opposite of what our way of speaking suggests: time is derived from the translation from "here" (*ubi*) to "there" (*ubi*) in space (*in loco*). The *ubi* (where), to which people improperly refer in order to express a quantity of time, is a naive way of thinking that reduces time to certain spatial determinations. As to the definitions: time is a transient "now" (*nunc*), whereas place (*locus*) is a continuous "where" (*ubi*):

> Since the quantity of time corresponds to that of a motion between a 'here' [*ubi*] and a 'there' [*ubi*], the 'where' [*ubi*] that we use for time is deduced [*transumptum est*], without inconvenience, from that 'where' [*ubi*] which, in fact, pertains to [a determination of] place [*locus*]. Actually, time is a transient *now*, whereas place is an enduring *where* [*ubi*].[78]

Scaliger's criticism of Cardano is not limited to a vindication of the objective meaning of time, its interconnection with space (or more accurately, place), and the priority of cosmological motions over time, i.e., time as something derived from celestial motions. He additionally criticises Cardano's hint that time is the cause of generation and corruption, dealing with the question of "Whether time generates and corrupts [things]" (*An tempus generet et corrumpat*). In Scaliger's assessment—and according to the Aristotelian authority on which he relies—this is impossible. Time cannot generate or corrupt anything, since it is not a substance but a quantity, that is, a property of a substance. Therefore, it cannot produce any effects.[79] Scaliger opposes his "very subtle" (*subtilissime*) opinion to Cardano's superficiality: "Our life is the act of the soul. In it, time has neither jurisdiction

[74] Scaliger 1557, f. 458*v*: "At tempus est posterius motu primo. Est nimirum affectus eius, ab illo pendens."
[75] Scaliger 1557, f. 458*v*: "Tempus, inquit, aut est idem, quod motus, aut affectus eius."
[76] Scaliger 1557, f. 458*v*: "Tempus esse mensuram motus, et vicissim motum mensuram temporis."
[77] Scaliger 1557, f. 458*v*: "[…] dicimus tot partes aequinoctialis tanto moveri tempore."
[78] Scaliger 1557, f. 458*v* (punctuation and emphases have been standardized and modernized): "Quia tempus tantum est, quantus est motus inter 'ubi' et 'ubi' in loco. Iccirco 'ubi' quod pro tempore usurpamus, transumptum est, haud iniuria, ab eo 'ubi' quod est vere loci. Erit ergo tempus nunc fluens; et locus erit ubi continuatum."
[79] Scaliger 1557, f. 458*v*: "Cum enim tempus sit quantitas, nihil agit."

nor power."[80] Scaliger adds that Cardano's words "are suited to the vulgar" (*vulgo proprior*), not to philosophers. What's more, his references to visions and alternative states of mind should be regarded as only insignificant phenomena which concern children and *melancholici* like Cardano himself.[81]

### 7.6.3 Benedetti and the Renaissance Concepts of Time

Our excursus on Renaissance concepts of time is far from exhaustive, but it helps us to grasp the scientific context out of which Benedetti's own position emerged. It also permits us to point out some major problems in the conceptualization of time, in particular its subjective and objective dimensions. The interrelation between *tempus* and *locus* was at the center of the reflections, the debates, and even the polemics of scholars investigating nature from various angles. The debate on time and on its relation to motion has meanings that are, at the same time, physical (Benedetti), psychological-physiological (Cardano), and cosmological-metaphysical (Scaliger). Finally, for a more traditional cosmological perspective on time as the measure of celestial motions and of the motion of the first mobile as the standard measure for all other measurements of time, one can refer to Alessandro Piccolomini's *Della filosofia naturale*, where one finds the following definition:

> Hence, time, which is the measure of all movements, mainly has to measure a motion that is the most regular and enables the measurement of all other [motions] that do not have the same regularity in their components. As this motion is that of the first heaven, one has to conclude that time (which is like [a property] of a substance [*in proprio soggetto*],) is first of all measured by it through that motion of the first mobile. Through it all other motions are ruled. Hence, although time can be called the rule and measure of any motion, it will be reasonable not to pluralize it alongside the plurality of motions. Rather, it will remain one and the same for the whole world, just as the first motion, which is its proper and true subject [*soggetto*], is singular.[82]

In the generation before Benedetti, Cardano affirmed that time is independent of cosmological space on the basis of his assumption that time is a "stretching of the mind" connected with the perception of corporeal activity. Cardano formally accepted the traditional Aristotelian definition of time as the measure of motion, but only as the consciousness of physiological motion. Scaliger criticized this perspective and accused Cardano of misunderstanding Aristotle. In the *Exercitationes* he tried to restore a peripatetic metaphysical conception of time as a product of celestial motions. In a sense, his conception of time has a "conservative" character for his commitment to the Aristotelian tradition. However, the distinction of the subjective and objective dimensions of the issue permits us to highlight a problematic aspect of the Renaissance definition of time. According to Scaliger, time can

[80] Scaliger 1557, f. 458*v*: "[…] vita nostra est actus animae: in quem nihil habet tempus aut iuris, aut imperii."
[81] Scaliger 1557, f. 459*r*, *Exercitatio* 353, "De tua visione."
[82] Piccolomini 1585, I 3, chap. 7, ff. 52*v*–53*r*: "Di qui è, che'l tempo, c'ha da esser la misura di tutti i movimenti, bisogna che principalmente si applichi a misurare alcun movimento regolato sopra tutti gli altri, mediante il quale possa poi far da misura de gli altri tutti, che tal regola […] non hanno nelle parti loro. E tal movimento […] essendo […] quello del primo Cielo: si deve concludere, che'l tempo sia come in proprio soggetto da lui primamente misurato, nel detto movimento del mobil primo: mediante il quale, regolandosi tutti gli altri movimenti: ragionevol cosa sarà, che'l tempo, se ben si può chiamar regola, et misura d'ogni movimento, come di sopra habbiam lungamente detto, tuttavia egli non si multiplichi con la multiplicatione delli movimenti, ma resi uno stesso per tutto il mondo, sì come uno solo è quel movimento primo, che è il proprio, et vero soggetto suo."

be regarded as the measure of motion only from a subjective viewpoint, not from a natural one. According to nature, the relation between time and motion is the reverse: motion is the measure and time the measured thing.

For Benedetti, as a mathematician advancing a new Archimedean theory of motion and a post-Aristotelian physics, time is the *locus motus*, the place of motion, that is to say, an objective—we dare say, absolute—measure independent of its content and of spatial determinations. He shared with Cardano the independence of time from matter but not his subjectivism, while he maintained with Scaliger the objectivity of time without assuming the Aristotelian dependency of time on motion.

### 7.7 Natural and Violent Motions Revisited

After his revision of the physical concepts of the void, infinity, space, and time, Benedetti moves on to discard the Aristotelian theory of natural and violent motions in the section from chapter 23 to chapter 26. Chapter 23, "*Motuum rectum esse continuum, vel dissentiente Aristotele*" (Rectilinear motion is continuous although Aristotle is in disagreement), shows that continuity is not exclusive to circular motion, therefore Aristotle's distinction between circular and rectilinear motion must be corrected. In *Physics* VIII 8 one reads that "the motion of a body moving on a finite straight line cannot be continuous."[83] By contrast, Benedetti demonstrates that the projection of a circular motion has the same continuity as the circular motion itself, although it is a forward and backward motion on a line. This is the case with planetary appearances produced by the displacement onto an epicycle as seen from the earth.[84] In a way, this section is a rehabilitation of the epicyclic model against Averroist-Aristotelian criticisms concerning the physical tenability of a non-concentric model of heavenly circles. This contrast between mathematical and physical astronomy received much attention from sixteenth-century Aristotelian scholars who received their education at Padua. In the 1530s Giovan Battista Amico and Girolamo Fracastoro promoted a fleeting rebirth of homocentric astronomy through the publication of *De Motibus corporum coelestium iuxta principia peripatetica, sine eccentricis et epicyclis* (Venice, 1537 and Paris, 1540) and *Homocentrica* (Venice, 1538), respectively.[85] More directly, Benedetti's assertion that a continuous rectilinear motion is possible seems to be directed at the opposite statement in Piccolomini's Aristotelian *Della filosofia naturale* (Book 2, Part 2, chapter 3, "*Come esser non possa infinito corpo alcuno di quelli che per natura loro si muovano per retta linea*" (There can be no infinite body among those that move straightforeward following their own nature)). Just as in the *Diversae speculationes*, this section is complemented with diagrams.[86]

[83] Aristotle 1995, VIII 8, 261b 32–33 (369).

[84] Benedetti 1585, 183: "[…] ut accidit lineae directionis, & retrogradationis planetarum, unde circulus uan erit ut epiciclus et b ut terrae centrum."

[85] Pierre Duhem pointed out the tension between physical and mathematical astronomy, albeit in a very inadequate way. See Duhem 1908. For a better understanding of the matter, see Di Bono 1990.

[86] Piccolomini argues as follows: "Non è a punto manco sconvenevole il concedere cotale infinità in qual si voglia corpo semplice, che o salendo, o scendendo si muova per retta linea; che si sia veduto disopra esser di quello che in circolo si rivolge. Percioche essendo il partirsi salendo dal mezo dell'universo, e' l venir discendendo a quello, che movimenti di luogo a luogo tra di lor contrarii: e ricercando li contrarii movimenti, luoghi contrarii parimenti; confessaremo esser opposti tra di loro il luogo disopra e questo qua giù da basso. E perché sempre tra due contrarii, se l'uno e per natura sua finito, e determinato, non sopporta il giustissimo governo della natura, che l'altro sia infinito e senza termine, secondo che discorrendo per tutte le contrarietà si può vedere." See Piccolomini 1585, II 2, chap. 3, f. 24*v*.

Chapter 24 is a refutation of a series of Aristotelian assumptions on natural and violent motion. In the title of this chapter, Aristotle is called *vir gravissimus*, but this attribution sounds quite ironic, since the theses of this "very grave man" are here refuted. The first criticism concerns the idea that a projectile is transported by air once separated from its thrower. According to Benedetti, it is rather the contrary: air is a hindrance to motion because it resists the penetrations. Secondly, Aristotle writes in *De caelo* I 8 that a body accelerates the closer it is to its aim. Instead, one should say that a body moves (e.g., falls) quicker the further it is from its aim (e.g., the ground). In fact, the longer the distance it covers, the more it is pushed (*maior fit semper impraessio*) by its inner *impetus*, which is due to the spontaneous inclination toward its proper place (*inclinatio ad locum suum eundi*). Chapter 26 is a rejection of Aristotle's statement that a body is not "heavy" in its proper place. In fact, air in air, or water in water, has no weight at all, except for when one artificially compresses an element (for instance inflating air in a bladder). The difference in density of the compressed element produces a difference in weight. Chapter 25 denies that vertical motion could legitimately be called natural. In fact, only perpetual circular motion is natural. An entire (i.e., spherical) body and its parts spontaneously move in circles. By contrast, rectilinear motion is that of a part separated from its whole. The cosmological significance of these remarks should be stressed. It is implicit but can be demonstrated by comparison with Copernicus's *De revolutionibus* I 8, where the same distinction between the motion of the whole and that of its parts accounts for the difference between the natural circular motion of a planet, basically the earth, and the vertical fall of heavy bodies.[87]

Copernicus presented his considerations on natural and violent motions in *De revolutionibus* I 8, which is the chapter aimed at rejecting Ptolemaic and Aristotelian arguments against terrestrial motion. There Copernicus attacked the Aristotelian theory of natural and violent motion and sought to abandon the doctrine of natural places. Benedetti's undertaking is very close, even though the cosmological theme has not emerged yet.

## 7.8 The Cosmological Perspective of the *Physical Disputations*

The cosmological dimension of Benedetti's anti-Aristotelian discussion is documented in the last part of the *Physical Disputations*.

### 7.8.1 Physico-Mathematical Astronomical Issues

The astronomical-cosmological section begins (chap. 28) with a reflection on the sphere that goes against the opinion of Aristotle, ironically called *Princeps Peripateticorum*. Whereas the ancient philosopher regarded the circle as the "first plane figure" (*prima figurarum superficialium*) and the sphere, the form of the heaven, as the "first body" (*prima corporearum figurarum, that is, the first three-dimensional figure*), Benedetti claims that they are the "last" figures. In fact, they can be regarded as polygons with infinite sides:

[87] One can compare the text of Benedetti, *Diversae speculationes*, disp. XXV, with that of Copernicus, *De revolutionibus*, I 8. The latter wrote: "Igitur quod aiunt, simplicis corporis esse motum simplicem (de circulari in primis verificatur) quamdiu corpus simplex in loco suo naturali, ac unitate sua permanserit. In loco, siquidem non alius, quam circularis est motus, quo manet in se totus quiescenti similis. Rectus autem supervenit iis, quae a loco suo naturali peregrinantur, vel extruduntur, vel quomodolibet extra ipsum sunt. Nihil autem ordinationi totius et formae mundi, tantum repugnat, quantum extra locum suum esse. Rectus ergo motus non accidit, nisi rebus non recte se habentibus, neque perfectis secundum naturam, dum separantur a suo toto, et eius deserunt unitatem."

"the triangle is the first plane figure and the circle the last one."[88] He adds that the *principium*, the beginning and the origin of everything, is its center and those figures which equally encircle it in all directions can be said to be perfect. The author concludes as follows: "That which is perfect, although it is [qualitatively] first as to its essence [*natura*], is last as to its generation."[89] The circle, according to him, is perfect because it is, in a sense, an "infinite figure." If one considers it as a polygon of infinite sides, one can say that the sum of its angles is equal to an infinite number of right angles. One can interpret this statement as follows: every polygon inscribed in a circumference can be divided into several triangles whose vertices touch the center of the circumference and whose bases coincide with the sides of the polygon. In the case of the circle, taken as the "last" polygon, the triangles decomposing it are infinite in number. Since the angles at the vertices are zero and the sum of all internal angles must be 180°, it follows that the angles at the base must be two square angles. Thus, Benedetti feels vindicated: "The circle and the sphere are not constituted of one single angle, as Aristotle believes [...]. Rather, these are figures of infinite right angles. For that reason I call them last and perfect, because one cannot add anything to infinity."[90] To sum up this reasoning, Benedetti shares Aristotle's opinion that the sphere is the perfect figure, but adduces geometrical-metaphysical reasons. For both authors the sphere is the form of the world (Benedetti would say, "of the *caelum*" surrounded by an infinite empty space) for aesthetic and metaphysical reasons.

In the following chapters, Benedetti reviews a series of astronomical and meteorological issues on which he accused Aristotle of being wrong. Chapter 28 deals with the sparkling of the stars, which is, according to Benedetti, the effect of the motion of heavenly transparent media (*ab inaequalitate motus corporum diaphanorum mediorum nascitur*).[91] Among other things, Benedetti denies (chap. 30) that the warmth of the sun can be produced by its motion rather than by its light and subsequently (chap. 31) explains the seasonal variations. Chapter 33 reassesses, against Aristotle, the plausibility of the Pythagorean doctrine of celestial harmony. This has nothing to do with the production of sounds, nor with any harmonic proportions between the "aspects" of the heavenly bodies. Rather, it is the secret order imparted by to the world by divine providence.[92] Chapter 33 comprises a lengthy discussion on meteorology, in which atmospheric phenomena are essentially explained through the variations of air density.

### 7.8.2 The Copernican Conclusion of the *Physical Disputations*

Chapter 35, *Motum rectum curvo posse comparari* (Straight and curvilinear motions are comparable), is a crucial chapter for our analysis, since it is here that Benedetti, almost

[88] Benedetti 1585, 186.
[89] Benedetti 1585, 185: "Quod autem perfectum est, licet natura sit primum, est tamen ultimum generatione."
[90] Benedetti 1585, 185–186: "Circulus sphaeraque non ex uno solo angulo recto constat, ut idem Aristoteles putat [...] sed sunt figurae infinitorum angulorum rectorum, et hanc ob causam a me dicuntur ultimae et perfectae, quia infinito nihil addi potest." The authors would like to thank here Irina Tupikova for suggesting this interpretation of Benedetti's argument.
[91] Benedetti 1585, 189.
[92] The Pythagorean harmony was ridiculed by Alessandro Piccolomini among others. See Piccolomini 1585, II 4, chap. 12, "Del suono, et armonia, che i pitagorici pensavano, che nascesse per li movimenti de' corpi celesti," f. 105*v*: "[...] Quando i corpi celesti movendosi facesser suono avvenir dovrebbe a noi poscia che da si alto, et soverchio strepito, et sproportionato all'odito nostro quasi assordati, né quello né altro suono che qua giù si faccia, odire in modo alcuno dovremmo giamai. Per la qual cosa essendo manifesto che tante diversità di strepiti, che tra questi corpi inferiori si fanno, ancora che piccolissimi sieno, son da noi odite distintamente, è forza dire, per la ragione ultimamente fatta che né suono, né armonia, non può causarsi per li movimenti de gli Orbi, o delle stelle che volgendosi faccin la su in cielo."

at the end of his *Physical Disputations*, introduces the Copernican theory. Although we have already dealt with Benedetti's astronomical views in the previous chapter, it is useful to recount here the most important features of his cosmology in the context of the philosophical section of the *Diversae speculationes* (IV) and add some more details.

The subject matter is the comparability of rectilinear and circular motion: "[Aristotle] is wrong when he says that straight motion cannot be compared to the curvilinear (*Physics* VII 4), where he mistakenly also says that one cannot find any lines equal to the circumference of a circle."[93] It is directed against Aristotle's denial that a straight and a circular motion could be compared, thus hinting at the qualitative difference between celestial circular motions and the vertical tendency of the elements in the sublunary sphere. From a Copernican perspective, Aristotle's words could be considered to be an implicit rejection of terrestrial motion. In fact, if the earth rotates, one should assume that the trajectory of a falling body is rectilinear for an observer on the earth but has a circular component as well, if considered in relation to the outside world.

Benedetti first appeals to Archimedes's *De quadratura circuli* (On the quadrature of the circle) to argue that the circle and the straight line are comparable: "If, then, this quadrature can exist, there can also exist, for the reason already given, a straight line equal to the circumference of that circle."[94] Thus, a geometrical problem, the squaring of the circle, attains a direct cosmological meaning. If the issue at stake is the distinction between celestial and elementary motions, they are of course different, but this difference does not lie in the circularity of the former and the straightness of the latter, but rather in the uniformity of speed opposed to acceleration.

These considerations offer Benedetti the occasion to expand on the velocity of celestial motions. According to the commonly held opinion (*secundum opinionem cummunem*), the heavens would have to cover an immense distance within the 24 hours of the daily rotation. Close to the equator, the sun would cover 1,000 Italian miles per minute and Saturn 260,000 miles per minute, not to speak of the rapidity of the fixed stars. The assumption of this inconvenient velocity would of course be avoided if one assumed "the most beautiful theory" (*pulcherrima opinio*) of Aristarchus, "divinely" restored by Nicolaus Copernicus:

> And as for the speed of the fixed stars situated near the equator, one may make one's estimate, and, in fact, this will seem very difficult to some. But this difficulty does not occur in the most beautiful system of Aristarchus of Samos that has been so divinely expounded by Nicolaus Copernicus.[95]

From a Copernican perspective, the sun would cover "only" 48 miles per minute and Saturn 24, whereas the heavens would be stationary.

In the subsequent chapter (chap. 36), Benedetti reworks the doctrine of the *doctissimus Aristarchus*. It is entitled "*Minus esse explosam ab Aristotele opinionem credentium plures mundos existere*" (The view of those who hold that many worlds exist was not adequately refuted by Aristotle) and deals with the plurality of worlds. According to Aristotle,

[93]Benedetti 1585, 194: "[Aristoteles] recte dicere non potest motum rectum ad curvum comparabilem non esse 4. cap. lib. 7 *Physicorum* ubi errat quoque dicens reperiri non posse lineam aliquam rectam alicuius circuli circunferentiae aequalem."

[94]Drake and Drabkin 1969, 220. Cf. Benedetti 1585, 194: "Si igitur dicta quadratura dari potest, potest etiam dari una recta linea aequalis circunferentiae eiusdem circuli."

[95]Drake and Drabkin 1969, 221. Cf. Benedetti 1585, 195: "Et amplius de stellis autem fixis circa aequatorem posits quivis cogitet; quod revera difficillimum quibusdam videbitur, quod quidem non occurrit secundum pulcherrimam Aristarchi Samii opinionem, divinitus a Nicolao Copernico expressam, contra quam nil plane valent rationes ab Aristotele, neque etiam a Ptolomeo propositae."

a universe with a plurality of worlds similar to the earth would be unstable and eventually collapse, since the earthly parts of the other worlds would fall toward the cosmological center and the fiery parts would eventually become part of the fiery sphere of our sublunary world. This Aristotelian objection is based on an *a priori* assumption of the theory of the natural places. It is therefore easy for Benedetti to contradict him by arguing that all worlds (that is, planets) would have their elements and their places.[96]

Apart from that, as we have already seen, Benedetti proposes a bizarre transformation of the Copernican system based on an analogy between the moon and the other planets. Like our satellite, all these light-mirroring and wandering bodies are supposed to turn around dark earths which, in turn, spin about their axis:

> If the system of the learned Aristarchus is correct, it will be perfectly logical for that which takes place in the case of the Moon to take place also in the case of any of the five other planets. Thus, just as the Moon with the help of its epicycles revolves around the Earth as if on the circumference of a certain other epicycle of which the Earth is like a natural center (i.e., in the middle), carried around the Sun by the sphere of annual motion, so too may Saturn, Jupiter, Mars, Venus and Mercury revolve about some body situated in the center of their major epicycle. And this body, also having some motion about its axis, may be opaque, possessing conditions like those of the Earth, with conditions on the epicycle in question similar to those on the lunar epicycles described.[97]

This conception could provide an explanation for the existence of planetary epicycles, whose physical tenability has been already stressed in the *disputatio* 23. We could also regard these views of Benedetti as a cosmological reading of Copernicus focused on possible cosmological and physical consequences of the planetary theory. The plurality of worlds and the analogy between the moon and the planets are not the only innovative elements in comparison with the theses of *De revolutionibus*. After a section on the motion of light through the cosmic void (chap. 37) and one on the geometry of the elements (chap. 38), the conclusive section of the *Physical Disputations* (chap. 39) attacks a Peripatetic dogma: the unalterability of the heavens. In *De caelo* I 22 Aristotle remarked that no change in the heavens was ever observed. This is, according to Benedetti, not a valid argument. One should rather assume a principle of relativity of the point of observation. In fact, the earth would be invisible from the eighth heaven (that of the fixed stars), even though, by supposition, it was endowed with a light equal to that of the sun. The distance hinders us from perceiving changes that occur on other worlds.[98]

With this rejection of the distinction betwee a sublunary and a heavenly realm, Benedetti's criticism of Aristotelian physics is complete. It should be noticed that this final objection hits at the core of the Peripatetic natural philosophy, since it is a denial of the fundamental distinction between a terrestrial and a celestial physics, on which the entire physics and cosmology of the Aristotelians relies.

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[96] Benedetti 1585, 195.

[97] Drake and Drabkin 1969, 222. Cf. Benedetti 1585, 195–196: "Si doctissimi Aristarchi opinio est vera, rationi quoque consentaneum erit maxime, ut quod Lunae contingit, cuilibet etiam ex aliis quinque planetis eveniat, idest, ut quemadmodum Luna suorum epicyclorum ope circum terram volvitur, quasi per circunferentiam alterius cuiusdam epicycli, in quo terra sit instar centri naturalis (idest sit in medio) delati ab orbe annuo circa Solem; sic etiam Saturnus, Iupiter, Mars, Venus, atque Mercurius, circum aliquod corpus in medio sui epicicli maioris, situm habens, volvantur; quod quidem corpus, et aliquem quoque habeat motum circa suum axem, sit opacum, iis conditionibus, quae terrae sunt similes, praeditum existat, et in dicto epyciclo sint res similes istis lunaribus."

[98] Benedetti 1585, 197.

## 7.9 An Evaluation: Benedetti's Path to Natural Philosophy

The *Disputationes de quibusdam placitis Aristotelis* is a complex book within the larger book. It concerns at least three main fields of investigation: motion, the foundations of physics, and astronomy, in particular cosmology. It begins with a rejection of the theory of the natural places (violent and natural motion) based on an Archimedean relativization of heaviness and lightness as well as on a mathematical approach derived from the Euclidean theory of proportions. It deals subsequently with basic concepts of physics. It defines space and time anew as an absolute framework for the investigation of natural phenomena, in particular motion. This part of the *Physical Disputations* also aims at demonstrating actual infinity and the void, which are Democritean theses rejected by Aristotle in *Physics* and *De caelo*. The astronomical part then follows, which confronts many special issues and illustrates what we shall call a "post-Copernican cosmology." Benedetti advocates the heliocentric system (albeit modified relative to the model of Copernicus's *De revolutionibus*), the plurality of worlds, the inter-changeability of the observational viewpoint in the universe, and, last but not least, the homogeneity and continuity of the sublunary and the heavenly realm, contrary to one of the most fundamental assumptions of Aristotelian philosophy.

Let us summarize the Copernican considerations that could have influenced Benedetti and consider the extent to which he went beyond them. First of all, Copernicus abandons the theory of natural and violent motions because, "if anyone believes that the Earth rotates, surely he will hold that its motion is natural not violent."[99] Additionally, the daily rotation of the heavens is more absurd than that of the earth because it would be excessive compared to that required of the relatively small earth. A third Copernican remark concerns the infinity of space. It is directed against the Aristotelian assumption that there is nothing, "no space, no body, no void," outside the heavens (*dicunt quod extra caelum non esse corpus, non locum, non vacuum*). Copernicus remarks that the axial rotation of the earth undermines the strongest argument in favor of cosmological finiteness: "For the chief contention by which it is sought to prove that the universe is finite is its motion."[100] As to the objections against the earth's motion based on considerations of the effects to be expected for flying and thrown objects, Copernicus assumes, against Aristotle's claim for the simplicity of motion, that things on Earth participate in the planetary motion and, therefore, the vertical displacement of light and heavy bodies (*cadentium vero et ascendentium*) is a composite motion (*duplicem*) relative to the whole (*mundi comparatione*), with a rectilinear and a circular component. Copernicus holds that only circular motion is natural and it does not only pertain to celestial bodies but also to the elements in their natural place. As we have seem, he defines rectilinear motion as the tendency of bodies to reach their whole if they have been separated from it. This vertical appetency is not uniform but accelerated. Copernicus also criticizes Aristotle's opinion that bodies are heavy (or light) in their proper place, since weight depends exclusively upon the tendency of the part towards their whole.

Many of these Copernican ideas and suggestions appear also in Benedetti's *Physical Disputations*: the rejection of the theory of natural places and of violent and natural motions, the excessive rapidity of the rotation of the heavens, the void, infinity, the naturalness of circular motion against the unnaturalness of the vertical motion of the parts separated from their whole, and the criticism of Aristotle's assertion about the weight of

99 Copernicus 1978, 15
100 Copernicus 1978, 15–16.

the bodies in their natural place. However, it should be remarked that Copernicus does not expand on these ideas for the most part and cursorily presents them only for the sake of his apology for terrestrial motion. Benedetti's treatment is much more explicit and, what is more, his motivations and presuppositions appear to be quite different. His Archimedean and Euclidean treatment of motion is the basis of his rejection of the distinction between natural and violent motions. No consideration of this kind is present in Copernicus's work. Moreover, the reference to spatial infinity in *De revolutionibus* is limited to a remark. Copernicus himself does not explicitly support this thesis and leaves the discussion to the natural philosophers or, as he calls them, the *physiologi*. Actual infinity receives a substantially different treatment in Benedetti since it is closely related to the attempt to define space anew as *intervallum corporeum*. It is precisely this broad, natural philosophical dimension which is absent in Copernicus's work and which, in our opinion, Benedetti did not derive from his reading of *De revolutionibus* or from general astronomical concerns. It seems, by contrast, that he was primarily interested in the physical issue of a mathematical treatment of motion and that the criticism of the Aristotelian philosophy led him in a quite natural way to also confront cosmology. Nor could issues like the void and atomism be reasonably derived from Copernicus. Even the planetary theory of Benedetti departs from *De revolutionibus* as it includes theses like the plurality of worlds and the corruptibility of the heavens. However, it is clear that the heliocentric and geokinetic theories fit perfectly into Benedetti's worldview. In light of his general theory, as he writes, Aristotelian and Ptolemaic arguments against Copernicus's theory appear extremely weak: "contra quam [doctrina] nil plane valent rationes ab Aristotele, neque etiam a Ptolomeo propositae."[101] Koyré wrote that Bruno's *La cena de le Ceneri* (London, 1584) was the best defense of the Copernican system from a natural and physical point of view before Galileo's *Dialogo sopra i due massimi sistemi del mondo* (Florence, 1632). However, this statement seems to underestimate the force of Benedetti's *Physical Disputations*, which are perhaps less speculative than Bruno's dialogues but should nonetheless be regarded as an extremely strong apology for the physical tenability of the Copernican system. A reciprocal influence between Bruno and Benedetti cannot be excluded, since the wandering philosopher from Nola stayed for a period in Turin and the Savoy around 1578 and probably participated in a debate concerning the comet of 1577.[102] At any rate, the *Diversae speculationes* encountered much more acknowledgment among astronomers of the time than Bruno's work. As we have seen, Brahe extensively quoted Benedetti both in his *Epistolae astronomicae* of 1596 and in his book on the nova of 1572. Kepler's admiration for Benedetti was no less and was only equaled by his admiration for Commandino and Clavius.[103] The proximity of many themes of the *Diversae speculationes* and those of the young Galileo are a well-known issue in the history of mechanics; in light of our discussion, it is plausible to assume that Benedetti's influence on Galileo also concerned the insight into the close relation between the heliocentric theory and a new mechanics.[104]

Our analysis has shown that the heliocentric system is not the main issue at stake in the *Physical Disputations*, although that theory becomes part of a general program of reform for natural philosophy. Far from being a mere "Copernican enterprise," Benedetti's visionary project is much more complex. It is an ambitious attempt to build a new physics, in the wide Renaissance meaning of the term, out of a criticism of Aristotelian physics. Concerning Aristotle, it is clear that the *princeps peripateticorum* provides him with a

[101] Benedetti 1585, 195.
[102] Omodeo 2008a.
[103] Kepler 1937–2001, 390.
[104] Damerow and Renn 2010.

model, albeit a negative one, in which the theory of motion, cosmology, astronomy, meteorology, natural philosophy, and metaphysics are closely interrelated. Benedetti's undertaking is precisely a revision and a restructuring of these interdisciplinary ties on the basis of new insights and a mathematical approach. Although his investigation intentionally and explicitly departs from Peripatetic physics, it is historically possible only in the form of a thorough confrontation with Aristotelian themes. Indeed, the *Physical Disputations* have the form of a dispute on Aristotelian places. Benedetti's familiarity with Aristotle's *Physics*, *De caelo*, and *Meteorologica* should also be underscored. This apparently contradictory aspect of early modern physics in its ambiguous relation to Aristotelianism has already been stressed by Anneliese Maier in her studies on the medieval contributions to classical science.[105] In a sense, the development of a new physics required a thorough confrontation with Aristotle and his concepts, as also the examples of Bruno and Galileo bear witness to in different ways.

[105]Maier 1951, 304–305.

# Concluding Remarks

Giovanni Battista Benedetti, the Renaissance scientist, has received ambivalent historical judgements by scholars in the past. The historian of medieval science and philosophy Anneliese Maier, for one, viewed him with mixed feelings. To her, Benedetti appeared to be a sort of intellectual companion of Galileo Galileo, at the same time his "forerunner" in mathematical physics and an epigone who was disrespectful to his own medieval *Vorläufer* or predecessors. Maier wrote that Parisian scholastics such as Nicole Oresme and Jean Buridan had provided Benedetti and Galileo with the concepts they needed to inquire into physics—she particularly had the concept of *impetus* in mind—which they did not acknowledge in their fierce attacks on Aristotelian philosophy.[1] Maier shared Koyré's view that modern mechanics was constructed around a few central concepts and authors relevant for Newton's *Principia mathematica*. They were perplexed by the concomitant reception and rejection of medieval physics by Renaissance scientists. In our view, however, it is too narrow a point of view to just consider individuals and sets of ideas and their genealogies. Instead, one should consider the wider intellectual currents and the shared knowledge they generated. The incipient *querelle des anciens et des moderns*[2] is an example of a debate transcending specific questions and problems, even approaches and methodologies, towards a larger reflection on the relation between past and present. The problems inherent in this gap between the individual perception of change and the intellectual transitions of the time are exhibited by the astronomy of Nicolaus Copernicus, a sort of "unaware revolutionary,"[3] who saw himself (or at least presented himself) as a Renaissance restorer of planetary theories defended in antiquity by the legendary Pythagoreans. By contrast, his scholastic counterpart, the Padua-trained physician and natural philosopher, Girolamo Fracastoro, presented his homocentric reform of mathematical astronomy as a radical innovation, comparable with Amerigo Vespucci's discovery of the New World.[4] Fracastoro's work was based on the modeling of all celestial motions through concentric spheres (in line with a well-established Aristotelian tradition). In Benedetti's case, the rejection of the *philosophia naturalis* taught in the universities was achieved with intellectual means descending from that very philosophical tradition. Rather than viewing this fact as a paradox, it should be regarded as a sign of a profound tension in Renaissance science between past and present and a hallmark of what we have called preclassical mechanics.[5] The *intention* to outdo traditional authorities in order to move beyond their legacy had to rely on the shared knowledge of the time, which was marked by Aristotelian thought. In our introduction we delved into Benedetti's conceptions and reconstructed their socio-cultural coordinates, characterized by the Renaissance tension between conceptual heritage and novelty. Maier's perplexity thus rests upon a lack of reflection on the embedment of in-

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[1] Anneliese Maier established a connection between Benedetti's treatment of motion and that of Galileo in Maier 1951, 304–305.

[2] Lehner and Wendt 2017.

[3] Copernicus's revolutionary role *malgré soi* already puzzled Thomas S. Kuhn, who called him at once "radical" and "conservative" and regarded *De revolutionibus orbium coelestium*, the book propounding the first modern heliocentric theory in mathematical astronomy, "revolution-making" rather than "revolutionary." Cf. Kuhn 1959, 135 and 148.

[4] Goddu 2010 and Granada and Tessicini 2005. Also see Omodeo 2017.

[5] See Renn, Feldhay, et al. 2018.

tellectuals and their theories in socio-cultural processes. Benedetti in particular ought not to be seen as a link in a chain, but rather as one representative of a complex and comprehensive knowledge economy.[6]

In order to correctly locate Benedetti in the knowledge economy of the Renaissance, it is expedient to consider him against the background of the material and intellectual conditions of early-modern science, and as a figure between the intentions and identities of a new genre of intellectuals who formed the archetype for modern scientists. Benedetti's case helps us to reflect upon the social position and intellectual identity of these new types of scholars as well as on the way socio-cultural coordinates penetrated science, as far as its demarcation, content, form, and justification are concerned. With social coordinates we refer to the institutional setting involving Benedetti's role as a courtier and thus to his function as a court mathematician, which, in turn, was linked to the wider socio-economic interests of a Renaissance territorial state.[7]

In his seminal work on the sociological roots of modern science, Zilsel discussed the scientific relevance of the social transformations taking place in the late Middle Ages and the Renaissance. In particular, he argued that the emergence of modern science depended on the rise of capitalism. We could aptly refer to this phase as a pre-capitalistic or early-capitalistic "knowledge society." Technical knowledge proved to be a key element in the organization of life and production while the status of the artisans, those whom Zilsel called the "artist-engineers," increased and received high recognition among civil and political authorities. The town of Florence is prototypical for these changes, as Leonardo Olschki has forcefully demonstrated in his studies on science and vulgar literature.[8] A wide range of artist-engineers transformed Florentine society and its mentalities. Filippo Brunelleschi, most representatively and symbolically, forever changed the skyline of the same town in which, at the end of the Italian Renaissance, Galileo composed works that irreversibly modified the landscapes of science and scientific culture.[9] In Florence and Europe more generally, in the passage from the Middle Ages to early modernity the "artisan-practitioners" were confirmed as a new class. The codification of their experience and knowledge profoundly changed epistemology and science, most evidently in mechanics. This practical art was first codified as a physico-mathematical discipline, and then as a science in its own right, and was later adopted as a methodological and ontological point of reference in the shift toward the mechanistic world views of the seventeenth century.[10]

According to a corollary of the Zilsel thesis about the social origins of modern science, scientific culture was reshaped by the merging of three intellectual strands: the artisanal/technical, the scholastic/logical, and the humanistic/rhetorical. This fusion was accomplished by mediators, who were social actors with an in-between status bridging different intellectual and social realms. "Hybrid experts" became increasingly necessary because of their capacity to bring together the technical and the theoretical dimensions of knowledge. Their socio-cultural relevance would never diminish from the late Middle Ages to the Industrial Revolution and beyond.[11]

[6] For a recent study accomplished in this vein, see Trzeciok 2016.
[7] For further considerations on Benedetti in light of a discussion on methodological and historiographical approaches, see Renn, Feldhay, et al. 2018.
[8] Olschki [1919–1927] 1965.
[9] On the Florentine prototype, see Renn 2014. Cf. Zilsel 2000, 941.
[10] On artisanal knowledge and its codification, see P. Smith 2004 and Long 2001. On practical knowledge, see Valleriani 2017. On the elevation of mechanics to a worldview, see Renn and Damerow 2010.
[11] Ursula Klein has made this point most forcefully in Klein 2015.

During the Renaissance, this mediation was secured by a new group of "scientist-engineers," a series of court mathematicians of which Galileo is the best-known figure and which also included his protector, Guidobaldo Del Monte. Actually, the description of the Renaissance figure of the "scientist-engineer" suits the intellectual and social profile of Benedetti very well.[12] Galileo and his like were well versed both in the technical as well as in the intellectual dimensions of knowledge production. Renaissance "scientist-engineers" underwent a period in apprenticeship of practical mathematics, in some field of application like architecture or the art of war, but later distanced themselves from artist-practitioners as they aspired to gain higher social recognition, especially as courtiers. They had a high degree of education, as they mastered theoretical mathematics, the language of the learned, Latin, as well as the courtly language, for instance by acquainting themselves with the elegant Italian of the literature of the time. Scientist-engineers thus acted as mediators connecting the centers of power and decision on the one hand and the workshops and building sites on the other. As was the case with Benedetti, these experts could supervise artisanal work or give advice on technical issues.[13] As courtiers they were additionally required to participate in the refined dialogical and literary culture of the elite, to serve as educators as well as to use their astronomical expertise to cast horoscopes for the rulers.

The most specific socio-political aspect of Benedetti's time is the affirmation of court society as a particular social formation whose features show continuities and fractures both with the earlier aristocratic setting of the feudal society and the later capitalist one. A distinguishing feature is the centralization of power and administration around the court. As Norbert Elias argued, this formation culminated in the absolutism of the Ancien Régime but was preceded, on a smaller scale, by early attempts at territorial centralization.[14] Although such social formations apparently gravitated around an individual sovereign who made all decisions (as much of the literature of the time on the *Principe* and its privileges boasted), it was in fact a hierarchical system in which the group of experts surrounding the princely ruler constituted an oligarchy who operated the complex organization of modern states. The Duchy of Savoy is one such case. The dukes strove to create a "modern" capital city partly by following the model of Florence, insofar as culture and prestige are concerned, but also the Spanish and French models, insofar as the suzerainty of the ruling family is concerned. Other models played a role, too, for instance the Netherlands for military technology and Switzerland for military conscription and discipline.

Benedetti shared the enthusiasm of his patrons (especially Emanuele Filiberto) for mathematics and its perceived powerfulness as an instrument for successful navigation in war and peace. He also shared the aristocratic values of the court such as disinterest and prestige. Adherence to these values largely explains his bias toward theory despite the practical origins of his knowledge and the fields of application of his mathematics (ranging from mechanics to navigation, architecture, and perspective). He also ventured into the most general fields, such as cosmology and philosophy (as seen through his criticism of Aristotelian natural conceptions, his favorable opinion on Copernican astronomy and post-Copernican cosmology, and his remarks on "Pythagorean" philosophy of mathematics).

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[12]The figure of the "scientist-engineer" has been introduced into the history of science by Renn 2001, particularly in the contributions by Lefèvre (Lefèvre 2001) and Renn, Damerow, and Rieger (Renn, Damerow, and Rieger 2001). Valleriani discusses it in detail in Valleriani 2010, chap. 6.

[13]Valleriani 2010, 208: "Except for the period of the apprenticeship, an engineer-scientist was almost never personally employed in workshops or building sites, but he was aware of the work procedures followed in these locations and was able therefore either to commission craftsmen or other persons involved with practical activities, to supervise or teach them, or simply be consulted to evaluate their works."

[14]As already discussed in the introduction. The reference work is Elias [1969] 2002.

Actually, he did not hesitate to call his wide and unsystematic work "speculations," an expression that stresses the theoretical character of the endeavor.

From the perspective of a court scientist such as Benedetti, mathematics was the key to practice and theory. It was his specific field of expertise among the Turin courtly elites; through it he acquired a central epistemological status in line with the exaltation of the *certitudo mathematicarum* by many of his contemporaries, among them his correspondent Pietro Catena. At the same time, the practical context surrounding the mathematical approach in many fields such as mechanics led him to emphasize the contingent element of natural phenomena. Thus, the centrality of mathematics in Benedetti's work has a multilayered meaning, including the theoretical, practical, epistemological, and social. The limits of validity and applicability of Benedetti's mathematical science mirror the boundaries of his field of competence in the division of intellectual labor within his courtly environment. Although he used geometry as a sort of universal key, he could not impose his views on other courtiers who were experts in fields such as philosophy and medicine. In this context of enforced openness, Benedetti's criticisms of Aristotelianism appear as a sort of defense of his professional position in the framework of a courtly dialogical pluralism. Such an environment explains the occasional (and fragmentary) character of the *Diversae speculationes*, which brings together occasional materials such as texts for private teaching, letters, short treatises, expert advice, and polemical essays (among others), in which Benedetti made his mathematical expertise manifest and showed its usefulness.

The intellectual distribution of labor in the Renaissance ensured that Benedetti was at the heart of the courtly milieu by virtue of his family's social status and not through his ambition alone. His work exhibits many similarities with the work of other Italian court mathematicians, most eminently that of the aforementioned Del Monte and Galileo, as far as the range of their interests and the overall approach are concerned. Benedetti's most daring passages, which open up unconventional solutions to technical and theoretical problems, and his remarkable disregard for authority qualify him as one of the Italian *novatores*, although he did not make explicit his natural conception as an all-encompassing alternative to the well-established Aristotelian philosophy. His fierce attacks on crucial aspects of the Aristotelian conception—relating to motion, the void, infinite space, time, infinity, and planetary theory—did not result in a systematic new natural philosophy. Rather, he limited himself to collecting results in different areas and to working on the most varied aspects without finding their common denominator. He also made elliptical references to Pythagoreanism and implicitly rehabilitated some aspects of atomist and stoic conceptions, for instance the plurality of worlds and the fluidity of the heavens. Cardano, whom he appreciated, went much further in the inquiry of the common foundations of the sciences (specifically mathematics, practical arts, and medicine) while Benedetti's correspondent Patrizi advanced a systematic natural philosophy inspired by neo-Platonism. In the same years in which Benedetti finished and published his physico-mathematical speculations in Turin, Bruno published philosophical dialogues in London expounding a natural philosophy and an anthropology that led to far more radical consequences for the premises of cosmology, similar to those reached by Benedetti. Another contemporary of Benedetti, Telesio, had offered the first modern attempt to build up a conception of nature on new principles. His *Natura iuxta propria principia* paved the way for the next generations of scholars searching for new foundations in natural science. Among them was his direct follower Campanella, who brought his philosophy to France in the seminal years of the mechanical philosophies of Pierre Gassendi and René Descartes. Benedetti participated in this wide cultural transformation; he contributed to advancing the mathematical and physical disciplines and discarding consolidated theories—but without offering a systematic alternative.

To summarize the most evident features of Benedetti's endeavor: it was courtly, secular, anti-Academic, unsystematic, occasional, elitist, learned, abstract, pleasant, and useful. It was *secular*, that is, non-theological, as it was linked to the interests of the ruling class and the state. It was a *useful and pleasant* science: on the one hand, it was practice-oriented but not purely empirical; on the other hand, it proved witty and fit for courtly sociability. It was *abstract and disinterested*: superior to the vulgar and tuned to aristocratic values. *Learned*: fit to be exhibited at court alongside the other arts. *Elitist*: Benedetti elevated mathematics from a practical discipline of scientist-engineers to a refined cultural activity. *Occasional*: linked to the variegated political and cultural interests of the court. *Unsystematic*: fragmented, lacking the inner coherence of scholasticism. *Anti-Academic*: free from concerns about respect for university scholarly traditions. All of these characteristics of Benedetti's science were the hallmark of court science: it was technical and abstract without losing contact with practice and experience—a mathematical-empirical science *in nuce*; it was (relatively) free from bookish tradition and theology but not from the contingencies of courtly life.

What is the common denominator of the great variety of subjects dealt with by Benedetti? What is the center around which they all gravitate? Is there one unifying principle behind the apparent disorder and heterogeneity? It should be emphasized that Benedetti first established his fame *as a mathematician*. His early treatment of motion by mathematical means was explicitly directed "against Aristotle and all philosophers" (*contra Aristotilem et omnes philosophos*). In his time "mathematics" had a wide scope. It comprised arithmetic and geometry, astronomy and astrology, as well as music, but also reached far beyond the boundaries of the *quadrivium* by encompassing optics, practical mechanics, architecture, and engineering. The expansion of mathematics into the fields of physics, natural philosophy, meteorology, and even metaphysics and epistemology was a crossing of the disciplinary boundaries. Benedetti's time bears witness to several attempts to expand the boundaries of mathematics. Cardano, for one, claimed that geometry had the function of a universal logic fundamental to rational thought, and that the practical disciplines including statics, mechanics, and architecture were its subordinate fields of inquiry.[15]

Benedetti's intellectual identity, however, proves much more complex than his corporate identity as a mathematician.[16] His pronounced titles vary. In a short biographical note accompanying the birth horoscope published by Gaurico, he was referred to as "Phylosophus, Musicus, atque Mathematicus"; on October 19, 1589, he signed an astrological report cast for Carlo Emanuele I as "Matematico e Astrologiaro";[17] contemporary admirers of his such as the Milan painter and poet Lomazzo and the Danish astronomer Brahe called him "matematico" and "philosophus et mathematicus inprimis excellentem," respectively.[18] Probably, Brahe's designation of Benedetti as both philosopher and mathematician best captures the poles of his intellectual activity. Intriguingly enough, Benedetti generally dropped the title of "mathematician," keeping only that of "philosopher" in his

[15]Girolamo Cardano, *Encomium geometriae recitatum anno 1535 in Academia Platina Mediolana* in Cardano 1966, vol. 4, 440–445.

[16]By "corporate" we refer here to the *esprit de corps* of a group that considers itself a bounded entity whose interests are marked as separate from other groups. The guild culture of the Middle Ages originated this particular meaning of corporation, which precedes the modern sense of a professional group or legal body.

[17]Roero 1997, 57–58.

[18]Lomazzo 2006, 177: "Del Sig. Gio. Battista Benedetti Matematico" Brahe 1916, 251–253.

publications. On the title page of his *magnum opus* of 1585, the *Diversae speculationes*, he appears as "patritius Venetus philosophus," exactly the same epithet that appears in *De gnomonum umbrarumque solarium usu liber* (1574). In the publications in the vernacular, he correspondingly appears as "filosofo del sereniss. duca di Savoia," e.g., in the *Consideratione… d'intorno al discorso della grandezza terra et dell'acqua* (1579). In the last publication, his self-presentation as court philosopher is interestingly opposed to the designation of his intellectual opponent, Antonio Berga, as "*filosofo nella Università di Torino*," that is, "university philosopher"—which is equivalent to *scholastic* philosopher. These references are telling for Benedetti's self-perception or, to use an in-vogue expression, his *self-fashioning*.[19] In both cases, the image of court philosopher was his intended identity, whether reflected or purposely constructed (or a mixture of both). As was the case with Galileo, the Florentine courtier, the philosopher's social status and reputation was higher than that of the mathematician. This is why, among the conditions for Galileo's appointment as a courtier to the Medicis, he regarded the designation "philosopher" as relevant.[20]

As for the epistemological debates mirroring the disciplinary and social divides and hierarchies of the time, heated controversies began over the "certainty of mathematics." The determination of the degree of certainty of mathematics also concerned the legitimacy of using mathematics in physics. In the case of Benedetti, the tension between his function as court mathematician and his identity as philosopher—and *patrizio*—lies beneath his science. While philosophical legitimacy was essential for the acknowledgment of the intellectual dignity of his endeavor, the practical dimension of mathematics remained fundamental for the social justification of his function as a court expert.

One could single out the social and the political-cultural coordinates of Benedetti's science as two complementary drives. On the one hand, his position as a court mathematician directly determined much of the content of his writings, occasioned by the requests addressed to him as a court *expert* in technical issues pertaining to mathematics. His position also determined formal aspects of his work, in particular its occasional character and fragmentation. On the other hand, Benedetti's identity as a philosopher was directly related to his cultural ambitions and his engagement aimed to affirm *mathematical philosophy* in the intellectual arena against scholastic thinkers and humanistic literati. His political identity as a lay aristocrat made him an organic part of the centralizing project of the court and marked his distance from Counter-Reformist drives which sought to impose Roman universal interests over territorial states' autonomy. His support for a sort of party of the *politiques* resulted in treatises advising on politically relevant technical and cultural issues (e.g., navigation on the occasion of the battle of Lepanto or the calendar reform). His activity as a lay educator, e.g., his arithmetic teaching to the prince, Carlo Emanuele I, is found in his pedagogical writings, some of which were published in his scientific miscellanea. In summary, both content and form, as well as the demarcation of the fields of his scientific competence as a mathematician and philosopher, depended on social settings and cultural engagement.

The fact that Benedetti never established a scientific school around himself can be seen as an indication of the precarity of patronized science, linked to the person of a particular ruler and not institutionalized at the level of an academic body. In the course of the seventeenth century, these limitations of early court society would be solved by securing scientific continuity for patronized science through the foundation of scientific societies.

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[19] Greenblatt 1980.
[20] Biagioli 1993. Also see Biagioli 1989.

These societies constituted an improvement over the volatility of Renaissance patronage, which depended on the humors and interests of a prince, by replacing him with a corporative *persona ficta* deputed to protect, credit, and promote science. This did not imply a diminution of the political relevance of science. As has been argued, the institution of the Académie Royale des Sciences as a means to patronize all of the sciences also meant the conquest of a new kingdom, *la république des lettres tout entière*, for Louis XIV.[21]

Montesquieu was a perspicacious observer of the courtly society in which Benedetti lived and worked. In his opinion, the "courtly air," or the ethos of the ruling elites of a monarchic state, "consists in putting away one's own greatness for a borrowed greatness. This greatness is more flattering to a courtier than is his own."[22] Such *grandeur empruntée*, or borrowed greatness, was a function of a person's distance from the ruler. Benedetti's greatness could have solely consisted in his mathematical acumen, in his mechanical insights and demonstrations, or in his philosophical discernment; these are the virtues that the historian of science is inclined to observe as principal. However, Benedetti saw himself as a court gentlemen, and only valued his capacity as a mathematician as subordinate. He presented himself as a *court* intellectual, more precisely, as a "philosopher *to the Dukes of Savoy*." He "borrowed his greatness" (in Montesquieu's words) from his proximity to the rulers. In the courtly milieu, it was honor and rank, together with their corollary, ambition, rather than skill, diligence, and measure that marked the character of a nobleman who belonged to the hegemonic class of the new state. Greatness is a major motivation for Benedetti's science, which cannot be confined to technical demonstrations or the solution of specific problems. Rather, his treatment of details never departed from concerns about the big picture; in his work, special issues were constantly elevated and received their meaning on the level of a grand overview, natural and epistemological.

Greatness is not the only courtly quality to enter Benedetti's science. As Montesquieu further observed: "At court one finds a delicacy of taste in all things, which comes from continual use of the excesses [*superfluités*] of a great fortune, from the variety, and especially the weariness, of pleasures, from the multiplicity, even the confusion, of fancies, which, when they are pleasing, are always accepted."[23] To be sure, one cannot say that Benedetti's knowledge was superfluous in the sense that it had no concrete application. In the Renaissance, it was evident to anybody how closely mathematics was connected to practical realms ranging from war technology to fortification, navigation, and administration. Benedetti's work and activities related to these realms; even his astrological consultancies can be appreciated for their practical orientation—as astrology notably coincided with the so-called *astronomia practica*, as opposed to mathematical astronomy, or *astronomia theorica*. Still, Benedetti insisted on his lineage as a "philosopher" (connected with his claims about the Pythagorean universality of his method and the fragmentation of its applications) despite the attention given to practice and concreteness in Renaissance mathematics. Such a contention was aimed at confirming his superiority over the immediate application of knowledge or the material origin of arts such as mechanics.

His stress on theory—on "speculation"—is well attuned to the spirit of court society, which was centered on nobility, that is, on disinterest and rank, rather than efficacy. The "superfluity of Benedetti's science" corresponds to the leisure character of knowledge in general, due to fact that its bond with materiality and practice was sublimated. Whereas corporative and merchant societies like those of the Italian Quattrocento (or, more gen-

[21] Biagioli 1995, 1418 and 1438.
[22] Montesquieu 1989, 33.
[23] Montesquieu 1989, 33.

erally, bourgeois and democratic ones like those emerging in the seventeenth century) would emphasize the practical origin and meaning of science, a court society stresses its symbolic value rather than direct usefulness and economical importance.

Besides the *superfluité* (which applies to Benedetti only if it is not taken too literally), all of the other qualifications Montesquieu attached to the court atmosphere suit his endeavor: good taste (we can add, “wit”), variety, pleasure, multiplicity, even confusion. The main virtue of a court society rested on the sense of honor and ambition: “Honor, meddling in everything, enters into all the modes of thought and all the ways of feeling and even directs the principles.”[24] Norbert Elias, who agreed with this assessment, also pointed out the fatal consequences for budgetary issues of a mentality that is so distanced from a modern bourgeois economy. From an economic viewpoint, court society was intrinsically flawed. It was destined to bankruptcy because form, ritual, and etiquette counted more than parsimony. Similarly, courtly science displayed detachment from monetary return. Elias has also emphasized the centrality of etiquette for this detachment. At court, formal etiquette was decisive, as it served to maintain and reinforce distances and hierarchies.[25]

The sense of honor and superiority typical of such social formations appears in Benedetti’s intended distance (social, intellectual, moral, and epistemological) from artisanal practice and the erudition of university professors. He appropriated the results and methods of both fields, in particular those of the practical arts, but at a higher level of generalization. He particularly envisaged a reformed natural philosophy as the most cherished fruit of his “mathematical-physical speculation.” Such theoretical distance from immediacy is the epistemological parallel of the sense of honor and social distance and, as such, it became an essential ingredient of Benedetti’s science and added symbolic value. As a court intellectual, he did not identify himself with traditional forms of higher culture such as Scholastic Aristotelianism or humanistic rhetoric. He proudly affirmed himself as a courtier, free to think and philosophize in the protected space of the court, independent of the most immediate material needs, of academic constraints dictated by tradition, and concerns about systematicity and completeness. Ambition, the companion of aristocratic honor, “meddled in everything” and directed Benedetti’s search for the most general principles of a new vision of nature, both mathematical and physical. The court protected and promoted a science and philosophy in which disinterestedness was foremost. In its favorable womb, a daring mind could venture out to explore new realms beyond established disciplinary boundaries. The speculative freedom of the court also determined the specific form of Benedetti’s work, its occasional character, and the amazing variety exhibited by his *diversae speculationes mathematicae et physicae*.

The economy of honor in the court society left an enduring epistemological imprint on the social fabric of science. Symbolic capital governed modern science long after it became coupled with economic capital and, in many ways, it still significantly influences science and research. The legacy of courtly ingenuity and leisure has to be acknowledged as a lasting influence upon scientific practice as well. Moreover, the topos of a protected space, so attractive to the emergent category of philosopher-scientists in the sixteenth and seventeenth centuries, contributed to creating the myth of the independence of pure science. Constant claims and controversies about scientists’ autonomy have accompanied the modern path to science in its migration from the court to the scientific academy and from the scientific academy to the laboratory. The connections of modern science to the

[24]Montesquieu 1989, 33.
[25]Elias [1969] 2002, 173.

economy and society at large, politics, and cultural structures can be appreciated by considering the complex historical ties that link knowledge with its material and cultural conditions reaching far beyond the perception of the individual historical actors. The spirit of Benedetti's science can be seen as typical of an age of profound social transformation and political reconstitution, which is reflected in the exceptional re-structuring of knowledge and the transition to novel forms of scientific acquisition, legitimation, and transmission.

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## Index of Names

# Index of Locations

## S



## T



## U



## V

## A Facsimile of Giovanni Battista de Benedetti's
## *Diversarum speculationum mathematicarum, et physicarum liber*
## (Turin 1585)

# IO. BAPTISTAE BENEDICTI

Patritij Veneti Philoſophi.

*DIVERSARVM SPECVLATIONVM Mathematicarum, & Phyſicarum*

Liber.

Quarum ſeriem ſequens pagina indicabit.

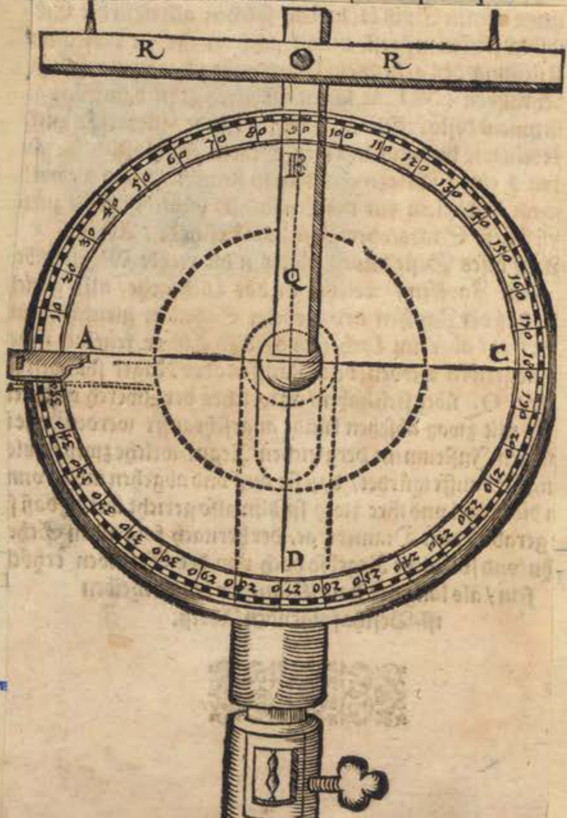
AD SERENISSIMVM CAROLVM EMANVELEM ALLOBROGVM, ET SVBALPINORVM DVCEM INVICTISSIMVM.


TAVRINI, Apud Hæredem Nicolai Beuilaquæ, MDLXXXV.

*Superioribus permiſſum.*

*TRACTATVS QVI IN HOC volumine continentur.*

Theoremata Arithmetica.
De rationibus operationum perſpectiuæ.
De Mechanicis.
Diſputationes de quibuſdam placitis Ariſt.
In quintum Euclidis librum.
Phyſica & Mathematica reſponſa per Epiſtolas.



# SERENISSIMO
## CAROLO EMANVELI
## Sabaudiæ Duci, &c.

AGITVR *nonusdecimus annus ex quo litteris Sereniſsimi patris tuæ Celſitudinis, accerſitus ex vrbe Parmenſi in hanc me ciuitatem contuli. Is aduenientem tam humanè excepit, tanta deinde liberalitate fuit complexus ego viciſsim ei deſeruiendi, tam vehementi cupiditate fui accenſus, vt ſub eius ditione quod ſupereſset vitæ agere conſtituerem. Cuius in me benignitas, mea in illum obſeruantia mirum in modum mutuo vſu, & conſuetudine eſt adaucta, vt idem Dux me ſecum dum ruſticaretur eſſe vellet, ſæpè etiam ſecum pernoctare; quo quidem tempore de Mathematicis ſcientijs mecum agebat, in quibus perdiſcendis mea opera vtebatur, quæſtiones, Arithmeticam, Geometriam, Opticen, Muſicam, aut Aſtrologiam ſpectantes proponens. Cui vt quod in me eſset ſatisfacerem, acrius quàm antea in ea ſtudia (ad quæ tamen ſemper fui propenſiſsimus) incubui. Illiusq; imitatione (vt ferè cæteri Principum ſtudia imitantur) non pauci aut præſentes, aut per litteras me de his, atque illis Mathematicis quæſtionibus conſuluerunt. Cùmque ego nunquam laborem amicorum cauſa defugerim, euenit vt poſt tot annorum curricula, mea ſcrinia ſcrutatus, inuenerim tot abſolutas quæſtiones, vt ex eis corpus mediocre effici poſſe videretur. Quas, cùm rationibus in epiſtola ſubſequenti allatis edere conſtituiſſem, non ſub cuiuſque alterius nomine, & auſpicijs quam tuæ Celſitudinis volui apparere; tum quòd patri debitum libellum filio reddere par erat, tum* quòd in tua Celſitudine paternam in me fouendo, & augendo benignitatem ineſſe ſemper ſum expertus, tum quòd tuæ Celſitudinis interrogationibus excitatus non pauca quæ hoc volumine continentur, elucubraui. Acceſſit, quod ego ſemper in his dedicationibus ſpectandum putaui, tuam Celſitudinem tantos progreſſus in Mathematicis feciſſe, vt vel idonea æſtimatrix mearum vigiliarum eſſe poſſit. Quare, & veterum Perſarum Regum gloriam æquauit, & nos veluti in ſpem certam fælicitatis huius ſæculi induxit, ſi verum eſt Platonis vaticinium, beatam eam futuram Rempublicam in qua Principes Philoſophentur. Tua igitur celſitudo libellum tot ei nominibus debitum, ea qua ſolet humanitate accipere nè grauetur. Deus tuas omnes cogitationes, & conatus ad fœliciſſimos ſemper exitus perducat, teq; diutiſſimè ſeruet incolumem.

# AD LECTOREM.

CVM Varijs temporibus permulta in diuerſis diſciplinis contemplatus ſim, partim à præſtantibus viris patronis ac amicis meis excitatus, qui ſuper eis ſententiam meam exquirebant, partim, ab ingenito mihi deſiderio, rationem, & cauſam eorum percipiendi, committendum non putaui, quin qualiacunque mea ſcripta in illis ſcientijs, ſtudioſis impartirer, non dubitans quin illis aliquid commodi atque vtilitatis allatura ſint, pręſertim cum in eiuſmodi quæſtionibus inueſtigandis atque perpendendis, nemo (quod ſciam) hactenus elaborauerit. Nihil enim his libris à me traditum eſt, quod aut legiſſe, aut ab alijs audiuiſſe meminerim, nam ſi aliena attigi, ea, aut cum aliqua differentia demonſtrationis, aut dilucidius ſcripſi, quod ſi forte alius eadem tradidit, aut eius lucubrationes ad me non peruenerunt, aut earum perlectionis memoria excidit. Vt enim etiam Ariſtoteles ipſe ſenſit facilè fieri poteſt, vt pluribus, eædem opiniones in mentem veniant. Immo multa ſcribenti euenire poteſt, vt cum iamdiu aliquid ſcripſerit, iam oblitus, idem repetat, quod mihi etiam nonnunquam accidit. In his autem libris non ſuſcepi munus integræ alicuius ſcientiæ tradendæ, ne, quæ ab alijs iam tradita ſunt, ipſe inutiliter repeterem, mihique viderer ex alienis laboribus laudem voluiſſe comparare. Singularum enim ſcientiarum volumina, iam ab alijs collecta, atque in ordinem ſunt digeſta, & ſi pauciſſimi ſint libri quorum omnes ſententiæ, omniaque inuenta vnius ſint authoris, excipio Archimedis volumina. Cumque multi ſint, qui vel vnam rem à ſe inuentam in publicum proferre non dubitent, multo magis mihi qui multa excogitaui, & ſi inter ſe hætereogenea, atque vtcunque expreſſa, idem licere ſum arbitratus. In his autem meditandis, ex Arithmeticis authoribus quos inſpexi, præcipuus fuit Nicolaus Tartalea, quippe quem ferè omnia ab alijs ſcripta collegiſſe conſtat, nec alios ex præcipuis, quos legere potui omittendos duxi, inter quos ſunt Hieronymus Cardanus, Michael Stifelius, Gemma Friſius, Ioannes Nouiomagus, Cuthebertus Tonſtallus, cæteriq; huiuſmodi. Quorundam tamen volumina illorum qui à Tartalea citantur, vt Leonardi Piſani, Proſdocimi, Ioannis Infortunati, Fratris Lucæ, Petri Borgi, aliorumque aliquot inſpiciendorum,

facultas

facultas mihi non fuit. Præterea, licet in his libris nonnullę inueniantur propoſitiones, quæ diſiunctam ab alijs habeant rationem, eæ non ſpernendæ tamen ſunt, viam fortaſſe alicui aperient vlterius progrediendi. Quemadmodum enim, exempli gratia, ex ſub contraria coniſectione ſumpta poſtea fuit diuina illa Planisferij delineatio, quæ ſub Ptolomæi nomine legitur, & ſicuti ex penultima primi Euclidis, quam Pythagoras excogitauit propè innumeræ pulchræ conſequentiæ in Aſtronomia, in Architectura, in multisq; alijs ſcientijs deſumptæ ſunt, immo quemadmodum ex ſingulis propoſitionibus à noſtris maioribus excogitatis multa egregia ſunt deducta, ita fortaſſe continget, vt ex mearum inuentionum aliqua, nõnihil in poſterum vtilitatis deſumatur. Si quid verò, hic inueneris, quod tuo genio non arrideat, illa prudentiſſimi hominis ſententia in mentem veniat. *Quot capita, tot ſententiæ*, ac per raro contingere, vt idem omnibus probari, atque placere queat, & perdifficulter inueniri hominem cui placeant omnia quæ alteri ſatisfaciunt. Nec te moueat, quod hęc Theoremata ſiue excogitationes non videas ordine illo diſpoſitas, quo collocari debere exiſtimaueris, tum in Arithmeticis, tum in cæteris. Cum enim in huiuſmodi rebus ordo non ſit neceſſarius, viſum eſt mihi poſſe me, ſine repræhenſione, illum negligere, cum ſpeculationi, ſiue inuentioni pręcipuè adeo mihi incumbendum decreuerim vt in collocatione operam ponere, & tempus abſumere operæpretium non duxerim, quod idem in epiſtolarum collocatione feci, in quibus perſonarum ad quas ſcribo nullus ferè graduum ordo ſeruatus eſt, nec temporis, quo ſunt ſcriptæ, quæſitorum tantummodo ratione habita. Nec admirari quenquam velim, quod in ſpeculandis numerorum paſſionibus, figuris vtar geometricis, ita enim in. 2. libr. fecit Euclides, qui modus, eo magis mihi arridet, quo minus eſt abſtractus, *quoniam oportet intelligentem phantaſmata ſpeculari*, cum pręterea perſpicuum ſit, diſcretum omne, ex continui diuiſione aliquo modo oriri, ſiue actu, ſiue potentia. Deinde ſi forte meis in demonſtrationibus tibi videbor aliquando breuior, illud in cauſa fuiſſe ſcias, quod ibi ad viros ſcribebam in his diſciplinis exercitatos, quibus ſatis fuit rem ſignificare. Libuit autem mihi omnes voluminis Arithmetici propoſitiones potius vocabulo theorematum appellare, quam problematum, quia pars earum ſpeculatiua tantum mea eſt, & ſi ex varijs eiuſmodi propoſitionibus etiam operatiuam adinuenerim. Quoniam verò multis in locis accidit, vt veritatis iudicandæ cauſa neceſſe mihi fuerit quorundam ſententijs aduerſari nolim te

hoc

hoc mihi vitio tribuere, meq; hoc nomine carptorem maledicumq; habere quod alienos errores aperiam, cum potius habenda ſit mihi gratia, quod in ijs interdum laborans (quę Antiſthenes in diſciplinis magis neceſſaria eſſe dixit, *vt mala ſcilicet prius dediſcantur*) falſas opiniones euellere ſtudeam, veritatemq; oſtendere, quam omnis philoſophus, Ariſtotelis exemplo, pluris quam cuiuſuis hominis authoritatem, aut gratiam facere debet. Cumque in hoc volumine aliquid eiuſmodi legeris te oratum volo, vt in iudicando, affectum omnem exuas, Salluſtianum illud præ oculis habens. *Omnes qui de rebus dubijs conſultant, ab odio amicitia, ira, atque miſericordia vacuos eſſe decet*. Hinc fiet, vt non perſonæ (vt multi ſolent) ſed veritati, quę ſummo ſtudio digniſſima eſt, ſemper potius faueas. Vale noſtriſque laboribus vtere, ſi quem inde fructum, ſicuti ſpero tuleris, illi præcipuè habeas gratiam à quo omnes fluunt ſcientiæ.

# IO. BAPTISTAE BENEDICTI PATRITII VENETI SERENISS. CAR. EM. ALLOBROGVM DVCIS PHILOSOPHI.

## *Theoremata Arithmetica.*

PRAECLARE' multa veteres mathematici philoſophi de numeris eorumq́ue effectibus excogitata poſteris tradiderunt, quorum cum vix vllam rationem reddiderint, aut certè per exiguam, occaſione diuerſorum problematum mihi à Sereniſſimo Sabaudiæ Duce propoſitorum præbita, de ijs quæ ab antiquis propoſita fuerunt contemplanda nonnulla occurrerunt, quæ poſteritati comendare non inutile arbitratus ſum, ne hæ meæ cogitationes interciderent, & occaſionem præberem quamplurimis abſtruſa hęc indagandi, quæ problematibus & thæorematibus inuoluta, vix aliquem qui euolueret nacta ſunt.

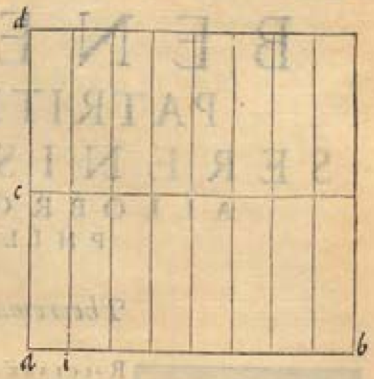
Inter cætera vero à me queſita, hoc fuit theorema.

### THEOREMA PRIMVM.

INTERROGAVIT me Sereniſſimus Dux Sabaudiæ, qua ratione cognoſci poſſet ſcientificè & ſpeculatiue (vt dicitur) productum ex duobus fractis numeris, quolibet producentium minus eſſe. Cui reſpondi, mente & cogitatione concipiendum eſſe fractos producentes cum fractis productis, non vnius eiuſdemq́ue naturæ eſſe, imò longè diuerſæ.

Exempli gratia, fractis numeris propoſitis. a. i. et.a. c. quorum integri ſint. a. b. et. a. d. qui tanquam lineæ cogitentur, apertum ſanè eſſet productum. c. i. ſuperficiale futurum, quod nomen caperet à producto ſuperficiali. d. b. generato ex vno in aliud totorum linearium, nam ſi conſtitueretur.a.i. octauum ipſius.a.b.et.a. c. dimidium. a. d. multiplicato.a. i.cum. a. c. produceretur ſextumdecimum ipſius. d.b. Quare.d.b. eſſet totum relatiuū ipſius. c. i. non aliquod totum producentium. Mirum itaque non eſt ſi productum. c. i. minus videatur ſuis producentibus, cum toto, diuerſæ naturæ à primis conferatur, fractum ſiquidem ab integro eiuſdem naturæ, linearis, ſuperficialis, aut corporeæ denominatur.

Quòd ſi amplioris cognitionis gratia ex ſcientiæ præceptis ſpeculari voluerit ali quis, qua ratione fractus numerus . c. i. minor ſit in ſuo integro . d. b. fracto.a. i. in ſuo integro . a. b. aut fracto . a. c. in ſuo integro . a. d. conſideret is quo pacto proportio.c.i.ad.d.b.minor ſit proportione.a.i.ad.a. b.et.a. c.ad.a. d. hac ratione. Manifeſtum eſt ex prima ſexti de quantitate continua, aut.18.ſeptimi Euclidis de diſcreta, proportionem ipſius.d.i.ad.d. b. eſſe ſicut.a. i. ad.a.b. & cum.c. i. minor ſit.d. i. velut pars ſuo toto, proportio, c. i.ad.d. b. minor erit proportione. d. i. ad.d. b. ex.8. quinti, quare minor erit pariter proportione.a. i. ad.a. b.ex. 12.eiuſdẽ vnà etiam proportio. c. i. ad. d. b. minor erit. a.c.ad.a. d. ex eiſdem cauſis,medio. c.b. Ex quibus patet ratio, cur fracti diuerſarum denominationum ad vnicam reducantur. Cur etiam numeros integros in partes fractis ſimiles frangere liceat, quæ omnia ex ſubſequenti figura facilè cognoſci poſſunt.




## THEOREMA II.

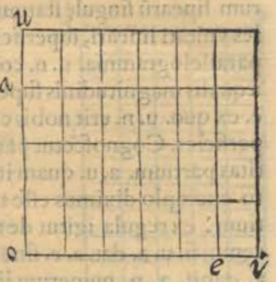
QVAE ſit ratio, cur hi,qui numeros,fractos diuerſarum denominationum colligere volunt,& in ſummam redigere, multiplicent vnum ex numerantibus per denominatorem alterius, & poſtmodum denominatores adinuicem, quorum vltimum productum, commune eſt denominans duorum priorum productorum, quæ collecta in ſummam efficiunt quod quærebatur.

Qua in re ſciendum eſt,denominantes conſiderari tanquam partes vnius eiuſdẽque magnitudinis quantitatis continuæ,linearum (verbigraria) a. b. et.a. d. æqualiũ in longitudine,quarũ. a.b. in quatuor partes diuidatur,et.a.d.in tres. Quare ſi colligere voluerimus duo tertia cum tribus quartis, multiplicabimus. a. c. duo tertia, cum.a. b. diuiſa in 4. partes, producetur que.c. b. octo partium ſuperficialium, dehinc multiplicando. a.i. tres quartas cum. a.d. diuiſa in.3. partes producetur.i.d.primis ſingulis æqualis, nouem partium ſuperficialium, multiplicata deinde a. b. diuiſa in.4.partes per.a.d. in.3. diuiſa, producetur quadratum.d. b. in continuo, in 12. partes diuiſum, quod erit totum commune ſingulis productis, quorum primum erat.c. b. Quare.c. b. ita ſe habet ad totum.d.b.ſicut.a. c.ad.a.d. ex prima ſexti in continuis, aut. 18. ſeptimi in diſcretis quantitatibus, et.d. i. ad.d. b. ſicut. a. i. ad.a. b.ex eiſdem propoſitionibus. Collectis deinde partibus producti. c. b. cum partibus producti. d. i. manifeſtè depræhendetur eiuſmodi ſummam componi ex partibus vnius totius communis ſingulis earum.

## THEOREMA III.

CVR reperturi qualis sit fractus aliquis numerus respectu alterius, multiplicare debeant numeratores adinuicem & ita etiam denominatores, ex quo productum ex numeratoribus nomen capiat à producto denominatorum.

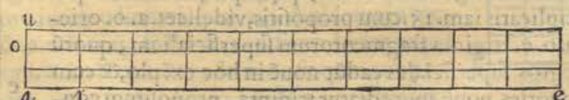
Huius si causam nosce vis, sume.o.i. &. o.u. pro totis denominatoribus, tum.o. e. &.o. a. pro numeratoribus (exempli causa) sit.o.i. senarius. o. u. quaternarius. o.e. quinarius.o. a. ternarius. Si nosce vis quæ sint tres quartę partes quinque sextarum, patet ex regulis practicis oriri quindecim vigesimasquartas. Id quomodo fiat, ex subscripta figura depræhendetur, memores tamen esse oportet, quodlibet productũ considerari tanquã superficiem, producentia autẽ tanquam lineas. In hac igitur figura productum ex totis linearibus est. u.i. aggregatum ex. 24. partibus, &.u.e. productum aggregatum ex. 20. Quod ita se habebit ad productum totale. u. i. sicut. o. e. ad o. i. ex prima sexti aut. 18. septimi, ita. u.e. erunt quinque sextæ partes. u. i. quarum in proposito exemplo, tres quartæ quærũtur. Si itaq; multiplicabitur.o.e. cũ.o.a. orietur productum.a.e. ita proportionatũ ad. u.e. sicut. o.a. ad o.u. reperitur, ex prædictis rationibus. Quòd si statutũ est. o.a. tres quartas partes esse ipsius.u.o. etiã. a.e. tres quartæ partes erũt. u.e. sed. u.e. quinque sextæ sunt ipsius.u. i. ex quo sequitur bonum esse huiusmodi opus.




## THEOREMA IIII.

CVR multiplicaturi fractos cum integris, rectè multiplicent numerantem fracti per numerum integrorum, partianturque productum per denominantẽ fracti, ex quo numerus quæsitus colligitur.

Propter quod mente concipiamus in subsequenti figura, numerum integrorum tanquam lineam.a.e. qui, verbigratia, sit denarius, quorum vnusquisque sit æqualis a. i. cogiteturque productum ipsius.a.e. in. a. i. sitque. u.e. quod quidem erit denarius superficialis, constituta prius. a.u. æqualis.a. i. &.a.o. sint duæ tertiæ.a. u. quarũ duarum tertiarum productum in numerum. a. e. sit.o. e. pariter. u. i. vnitas sit superficialis prout. a. i. vnitas est linearis, quam.u. i. respicere debet productum.o. e. ex quo integer superficialis.u. i. erit tanquam ternarius, & productum.o. i. tanquam binarius, & quia quælibet pars è viginti ipsius.o. e. æqualis est tertiæ parti. u. i. vnitatis superficialis; si cupiamus scire quot integræ vnitates sint in partibus.o. e. consultum est easdem diuidere per denominantem.u. i. compositum ex tribus partibus superficialibus, & cum tam linea u. a. quam superficies.u. i. diuidatur in 3. partes ęquales nosce peroportunum est eiusmodi partitionem numeri. o. e. fieri per numerum ipsius.u.i. non.u. a. ex prædictis causis.



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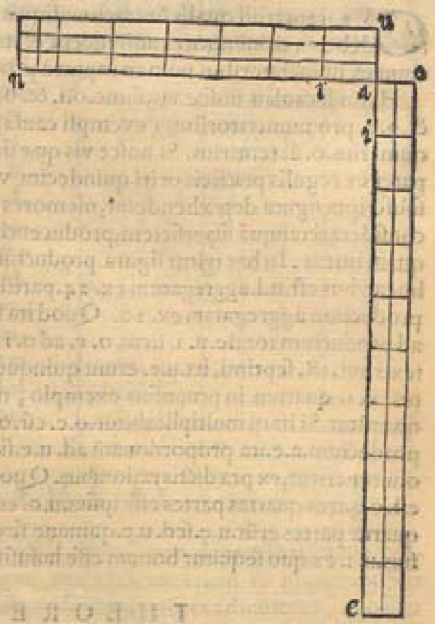
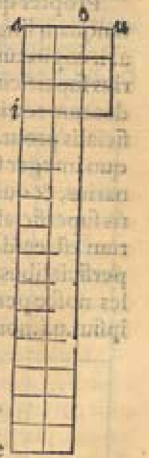



ALIA quoque via prædicti effectus causa, speculando innotescere potest, cuius rei gratia formetur sequens figura. e. o. a. u. n. eiusmodi, vt. a. e. sit numerus linearis integrorum,&c. o. e. productum numerantis ipsorum fractorũ in integris, ex quo. a. o. erunt duæ tertiæ, verbigratia, a. i. aut a. u. quarum linearũ singulę statuuntur æquales vnitati lineari, superficies autem parallelogramma. u. n. constituatur æqualis magnitudinis superficiei. o. e. ex quo. u. n. erit nobis cognita superficies. Cognoscetur pariter quantitas partium. a. u. quam in proposito exemplo diximus esse trium partium. ex regula igitur de tribus, dicemus si. u. a. dat. a. e. sine dubio. o. a. dabit. a. n. numerum linearem. quæ regula ex. 15. sexti in continuis, & ex 20. septimi in discretis, depromitur. rectè igitur multiplicãtur fracti numerantes cum integris, & productum diuiditur per denominantẽ fractorum.

## THEOREMA VI.




ITEM & alia speculatione cognosci potest hoc rectè fieri multiplicantes enim has duas tertias per decem, debemus considerare quantitatem duarum tertiarũ decies produci, ex quo oriuntur 20. tertia, quandoquidem singulæ vnitates, tunc pro duobus tertijs sumuntur, sed cum quilibet integer tria fragmenta contineat, ideo ex ratione partiendi quoties ternarius ingrediatur viginti, statim cognoscemus quod optabamus.

Id ipsum accideret si integri in eiusmodi specie fractorum diuiderentur. quo facto hi multiplicandi essent cum numerante proposito, & partiendũ productum per quadratum denominantis.

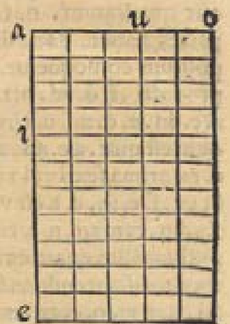
Cuius rei hæc est speculatio. Sit linea. a. e. constans ex quinq; integris numeris, quorum vnusquisq; æqualis sit. a. u. vel. a. i. &. a. o. sint duo tertia vnitatis integræ linearis. cogitemus nunc hos quinq; integros diuidi in sua fragmẽta linearia, quę in proposito exemplo erunt. 15. multiplicatis iam. 15. cum propositis, videlicet. a. o. orietur productum. o. e. triginta fragmentorum superficialium, quorũ in singulos integros superficiales cadũt nouẽ in hoc exẽplo, & cum notauerimus quoties nouẽ ingrediatur triginta, propositum consequemur.

THEO-

## THEOREMA VII.

CVR multiplicaturi integros numeros & fractos, cum integris & fractis, debeant integros reducere ad species fractorum, eos colligendo cum fractis: deinde multiplicare hos vltimos numerantes adinuicem & productum partiri per productum denominantium.

Vt (exempli causa) si volumus multiplicare vnum & duo tertia, per duo & tria quarta, reducentur omnia in fractos, ex quo vna ex parte essent quinque tertia, multiplicanda cum vndecim quartis ex altera, quo facto oriretur productum quinquagintaquinque fractorum, quod diuisum per productum ternarij in quaternarium, videlicet per duodecim, quatuor integri proferentur cum septem duodecimis fractis vniusintegri.

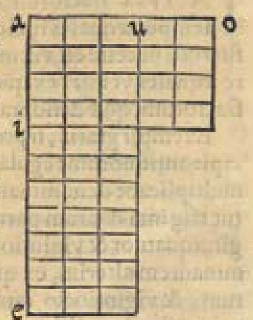



Detur subsequens figura in qua linea a.i. æqualis sit lineæ.u.a. quarum vnaquæq; cōsideretur pro integro numero: cogiteturq;.a.i. valere quatuor in pręsenti exēplo, &.a.u. tria: detur deinde linea.a.o. æquipollens vni integro cū duobus tertijs, &.a.e. æquipollens duobus integris & tribus quartis. Iam si hæ duæ lineæ in suos fractos reducantur, multiplicata (vt in sequenti figura apparet.) a.o. cū a.e. orietur productum. o. e. fractorum superficialium quinquagintaquinq;, quorum integer superficialis valet duodecim, scilicet. u. i. vt cuique manifestum est, ex quo, quærenti media partitione, quoties duodecim ingrediatur quinquagintaquinque, citra errorem, quæsitum occurret.

## THEOREMA VIII.

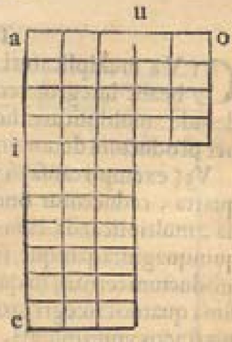
IDipsvm accideret si fracti ad vnam eandemque denominationem reducerentur, qui postmodum simul multiplicarentur, productumque partiremur per quadratum denominantis communis.

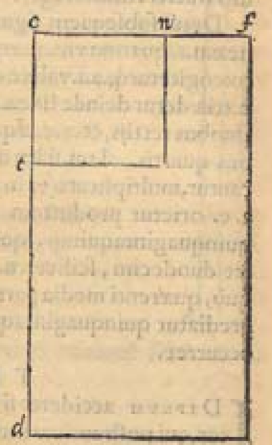
Exempli causa, sint eadem quinque tertia, & vndecim quarta adinuicem multiplicanda, quæ si reducantur ad vnam & eandem denominationem quinarius numerans vnius, multiplicabitur cum quaternario denominante alterius, & vndenarius secundi cum ternario denominante primi. ex quo vna ex parte essent viginti, ex altera 33. numerantia vnius cōmunis denominantis, quod esset productum ternarij in quaternarium, videlicet duodecim, vt ex veteri regula patet. Iam si multiplicentur viginti cum trigintatribus, dabuntur. 660. fracti, quorum integer erit quadratum duodenarij, nempe. 144. quibus quidem. 660. diuisis per. 144. proferentur quatuor integri & septem duodecimi.




Cuius rei gratia sit in subscripta figura linea. a. i. & ei æqualis. a.u. pro integro lineari, quæ. a. i. diuidatur in quatuor partes, &.a.u. in tres, & linea. a. e. sit vndecim partiū talium qualium. a. i. est quatuor, &. a. o. sit quinque prout. a. u. est trium. nunc multiplicato. a. o. &. a. i. orietur productum. o. i. viginti partium superficialium. tum multipli-

cato




cato . a. e. per. a.u. dabitur productum.u. e.trigintatriũ partium . ad hæc quadratum. u. i. constabit ex duodecim partibus eiusdem rationis cum reliquis duobus productis, quod quadratum. u. i. vnitas est superficialis, & communis denominans duorum productorum. quod si in præsentiarum cogitabimus lineam. c. d. trigintatrium partium æqualium, et . c. t. duodecim similium,et . c. f. viginti. c. n. duodecim, multiplicato . c. d. cum . c. f. dabitur superficies. f. d. *660*. fractorum superficialium , quorum vnitas integra superficialis erit quadratum . n. t. 144. partium cuiusmodi . f. d. partes habet . *660*. diuiso itaque . f. d. per . n. t. propositum consequetur. eo quòd eadem proportio erit producti . f. d. ad . n. t. quæ producti eius quòd fit ex. a. e. in . a. o. ad. u. i. nam proportio . c. d. ad . c. t. eadem est quæ . a e. ad . a. i. & c. f.ad . c. n. vt. a. o. ad. a. u. ex prima sexti vel 18. septimi, sed vt. f. d. ad id ꝙ fit ex . f. c. in . c. t. est vt . c. d. ad. c. t. & vt eius ꝙ fit ex f. c. in . c. t. ad . n. t. est vt . f. c. ad . c. n. ex dictis propositionibus quare ex æqua proportionalitate, eodem modo discurrendo in figura . o. a. e. ita se habebit. f. d. ad . n. t. vt. o. e. ad . u. i. Porrò ex ijs, quæ hactenus de fractorum multiplicatione considerata fuerunt, apertè ratio deprehenditur, cur productum, singulis producentibus semper minus sit, cum producta sint superficialia producentia verò semper linearia, omissis productis corporeis, quæ omnia ad superficialia reducuntur.

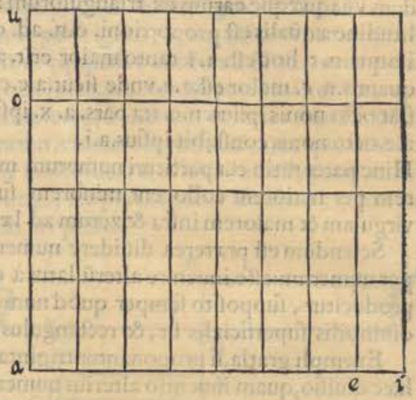
## THEOREMA IX.

IN IPSA fractorum diuisione, animaduertendum est, denominantes numeros semper æquales inuicem esse debere, vnius scilicet speciei, quòd si æquales non fuerint, necesse est via multiplicationis ipsorum denominantium adinuicem effice-re æquales vt sint, ex quo productum oritur eiusmodi, vt aptum sit habere partes fractorum, quæ desiderabantur.

Exempli gratia, si proponerentur diuidenda septem octaua per tria quarta præcipit antiquorum regula, vt ad vnam tantum denominationem reducantur . quare multiplicant denominantes inuicem . ex quo productum in materia proposita oritur triginta duarum partium commune denominans, cuius duo numerantes sunt vigintiquatuor & vigintiocto, producti ex multiplicatione vnius numerantis in denominantem alterius, ex quo dantur vigintiquatuor tamquam tria quarta trigintaduorum, & vigintiocto tanquam septem octaua particularum vniformium, prout ope primæ sexti aut decimæoctauæ septimi in subscripta figura cognosci potest.

Sit

Sit itaque linea.a.i. diuiſa in partes octo,& ei æqualis in longitudine.a.u. in quatuor, productum verò vnius in alteram ſit. u. i. trigintaduarum particularum ſuperficialium ſimilium & æqualiũ adinuicem. ſit deinde. a. e. ſeptem partiũ lineæ. a. i. & . a.o. trium partium. a. u. tunc productum. a. e. in. a. u. erit. u. e. particularum ſuperficialium vigintiocto & productum. a. o. in. a. i. erit. o. i. particularum ſuperficialiũ vigintiquatuor eiuſdem naturæ cum partibus trigintaduabus totius denominantis communis. vnde diuiſo numerante vigintiocto pernumerantem vigintiquatuor, dabitur vnum cum ſexta parte illius vnius.

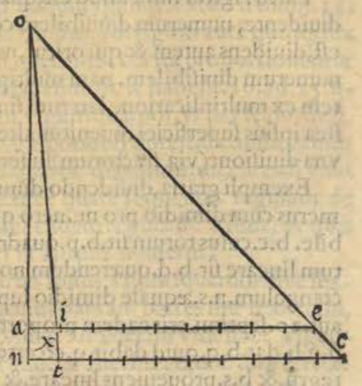



## THEOREMA X.

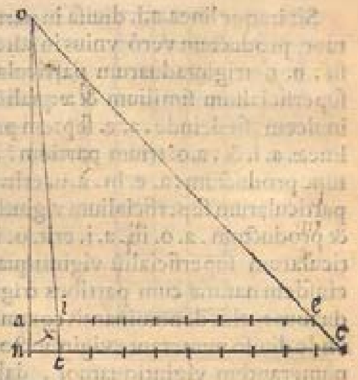
PARTIRI ſeu diuidere vno numero alium numerum, eſt etiam quodammodo eiuſmodi partem numeri diuiſibilis inuenire reſpectu totius numeri diuiſibilis, cuiuſmodi eſt vnitas in diuidente reſpectu totius diuidentis, partem inquam numeri diuiſibilis ſic ſe habentem ad totum numerum diuiſibilem ſicut vnitas ad totum diuidentem, quod ſimiliter ex regula de tribus præſtamus dicentes, ſi tantus numerus diuidens dat vnitatẽ, quid dabit numerus diuiſibilis, quemadmodum ex. 15. ſexti ſeu. 20. ſeptimi licet ſpeculari, Idcircò quotieſcunque minorem numerum per maiorem diuidimus, ſemper qui prouenit fractus eſt.

Exempli gratia, ſi cogitaremus lineam. a. e. diuiſam in octo partes æquales, quarum vna ſcilicet vnitas eſſet. a. i. & cuperemus eam diuidere in nouem partes, ac ſcire quanta ſit nona illius pars; manifeſtum eſſet, nonam partem ipſius. a. e. minorem futuram ipſa. a. i. cum. a. i. diminui debeat à ſua integritate eadem proportione, qua. a. e. minor reperitur vna linea nouem partium æqualium ſingularum. a. i.

Quod vt dilucidè cuiuis innoteſcat, hoc etiam modo licebit videre ſit linea. n. c. nonupla ad.a.i. & parallela ad. a. e. dubium non eſt quin. n. c. maior futura ſit ipſa. a. e. iam ſi earum extrema congiungantur medijs duabus lineis. n. a. et. c. e. quæ ſimul concurrant in puncto. o. (quod eſt probatu facillimum) dabuntur certè duo trianguli ſimiles. a. o. e. et. n. o. c. Sit deinde. n. t. vna è partibus ipſius. n. c. quæ. n. t. æqualis erit. a. i. ex præſuppoſito. ducatur deinde. o. t. quę interſecet. a. e. in puncto. x. dico. a. x. tanto minorem futuram. a. i. quanto. a. e. minor eſt. n. c. neque enim dubium eſſe poteſt quin proportiones. n. t. ad. a. x. et.




n. c.



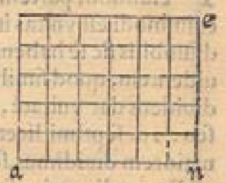

n. c. ad.a.e. ſint æquales inuicem quandoquidem vnaquæque earum ex triangulorum ſimilitudine æqualis eſt proportioni. o.n. ad. o. a. itaque.n. t. hoc eſt. a. i. tanto maior erit. a. x. quanto.n. c. maior eſt.a.e.vnde ſicut.a.e.conſtat octo nonis ipſius. n.c. ita pars. a. x. ipſius. a.e.octo nonis conſtabit ipſius.a.i.

Hinc patet ratio cur partituri numerum minorem per maiorem collocent minorem ſupra virgulam & maiorem infra & zerum ad læuã.

Sciendum eſt præterea diuidere numerum per numerum:eſſe inuenire alterũ latus à quo producitur, ſuppoſito ſemper quòd numerus diuiſibilis ſuperficialis ſit, & rectangulus.

Exempli gratia,ſi proponantur triginta diuidenda per quinarium, nihil aliud erit hæc diuiſio,quam inuentio alterius numeri,qui multiplicatus per quinarium producat triginta ſuperficies rectangulas, huiuſmodi verò eſt ſenarius,cuius ſingulæ vnitates ſuperficiales erunt.

Cuius rei gratia ſit ſubſcriptum rectangulum.a.e. triginta vnitatum ſuperficialiũ, cuius latus. e.n. ſit quinque vnitatum. hinc latus.a.n.erit ſex vnitatum; ita diuidentes rectangulum. e. a. nihil aliud faciemus, quam vt inueniamus quantum valeat latus.a.n. quod erit ſex vnitatum. Sin verò diuiſerimus per latus. a. n. quæremus latus. e. n. quinque vnitatum. ex quo,proportio totius numeri diuiſibilis ad numerum qui oritur, erit ſicut diuidentis ad vnitatem,ex prima ſexti, aut. 18,vel.19. ſeptimi,& permutatim ita ſe habebit diuiſibile ad diuidentem, ſicut numerus qui oritur ad vnitatem.

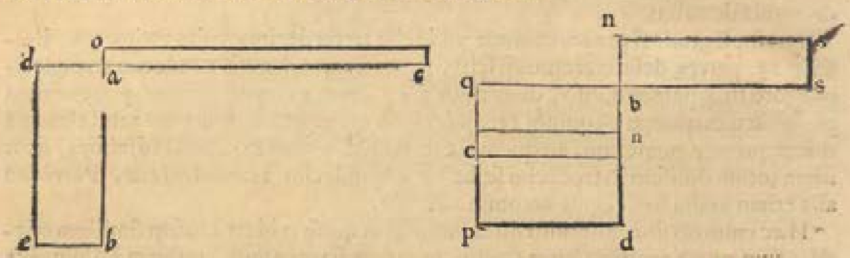



Partiri igitur nihil aliud eſt,quam inuenire latus rectanguli, quod productum in diuidente, numerum diuiſibilem compleat, ex quo numerus diuiſibilis ſuperficialis eſt,diuidens autem,& qui oritur,numeri lineares & latera producentia huiuſcemodi numerum diuiſibilem. nam multiplicare & diuidere opponuntur inuicem, cum autem ex multiplicatione laterum ſiue linearum generatur ſuperficies,ex diuiſione poſtea ipſius ſuperficiei inuenitur alterum latus. quare mirum non eſt, ſi proueniens ex vna diuiſione(via fractorum)ſit ſemper maius numero diuiſibili.

Exempli gratia,diuidendo dimidium per tertiam partem,reſultat vnus integer numerus cum dimidio pro numero qui oritur. Sit itaque dimidium ſuperficiale diuiſibile. b.c.cuius totum ſit.b.p.quadratum:tertium verò lineare diuidens,b.n. cuius totum lineare ſit.b.d.quærendum nobis eſt latus.b.s.quod cum latere.b.n. producat rectangulum.n.s.æquale dimidio ſuperficiali propoſito.b.c. quod ſi fiat, ex. 15. ſexti, aut.20.ſeptimi.erit eadem proportio.b.n.ad.b.q.quæ eſt.q.c.ad. b.s. dicemus itaque ſi.n.b.dat.b,q.quid dabit,q.c?certè.b.s.ſed.n.b.eſt tertium lineare et. b.q. lineare integrũ,& b.s.proueniens lineare.& quia.b.c.dimidium ſuperficiale,producitur à.q.c. dimidio lineari in.q.b.integro lineari.quare cum.n.s. ſit æqualis.b.c.& productum ex b.n.minori.q.c.neceſſe eſt, vt producatur in. b.s.maiore.q.b.quod.q.b. maius eſt.q.c. quod quidem.q.c.ita appellatur ſicut.b.c.quare mirum non eſt ſi proueniens per fractos numeros ex diuiſione, maior ſit numero diuiſibili.

Hinc

Hinc manifeſte patet quamlibet diuiſionē aut partitionem oriri ex regula de tribus, quandoquidem ſinguli diuidentes æquipollent vni integro, & loco illius ſumuntur. Perinde enim eſt diuidere centum per viginti, ac regulā obſeruare de tribus dicētes, ſi viginti æquipollent vni, quibus ęquiualebūt cētum? Hoc autem ex ſubſequenti figura facile deprehendetur, in qua linea.a.b.ſignificat viginti, et.a.o. vnitatē linearē, et.a.c. vnitates lineares centū: o.c.verò centum vnitates ſuperficiales, et.a.d.quinq; vnitates lineares, et.d.b.centum vnitates ſuperficiales, ex quo manifeſtè deprehenditur quòd quemadmodum multiplicare, nihil aliud eſt, quam inuenire productū ex duobus lateribus propoſitis, ita partiri nihil aliud eſt, quam dato vno latere inuenire aliud latus producti propoſiti.




Nam quotieſcunq; ratiocinātes dicimus tantundem numeri, immediate producimus ſuperficiem, mediāte vnitate in huiuſmodi numero, qui numerus antequā producatur in vnitatem, mente concipiendus eſt tanquam linearis, tanquam linea inquam diuiſa in totidem particulas lineares, ſingulas continuas & æquales vnitati propoſitæ. Cū verò productus fuerit numerus in vnitate ſuperficialis, erit ac ſi tot eſſent vnitates quadratæ, quod ſi ita non eſſet, nulla mentio facienda eſſet quorumuis fractorū. Ex eadē regula de tribus reduci poteſt ad praxim tertiū theorema.

Quare cupientes ſcire quæ ſint illæ partes, quæ ſunt tres quartę, ipſarum quinque ſextarum, dicemus ſi quatuor dant tria, quid dabunt quinq; ſextæ? dabunt.15. vigeſimas quartas, quæ quindecim ſunt tres quartæ ipſius.20. viginti autē quinq; ſextæ vigintiquatuor, quandoquidem nos numerum quęrimus, cui ita proportionentur quinq; ſextæ alterius numeri, ſicut quatuor ad tria, vnde ſic ſe habent.20.ad.15.ſicut.4.ad.3. ipſe autem.20.quinq; ſextę partes ſunt vigintiquatuor, vt per ſe notū eſt.

Ex eadem regula de tribus, huiuſmodi quęſito reſponderi poteſt, ſi conſtituamus prædictas quinq; ſextas eſſe numerum, cuius tres quartæ quęrantur, dicentes, ſi vnus integer dat tres quartas, quid dabunt quinq; ſextæ? quare ſequentes regulam de tribus, dabuntur quindecim vigeſimæ quartæ. Valet eadem regula de tribus, vt quis ſcire poſſit, quæ pars aut partes numeri propoſiti ſit aliquis numerus.

Exempli gratia, ſcire cupienti, quæ pars aut partes ipſius vigintiquatuor ſint ſexdecim, conſtituentur.24.tanquam vnum totum, cuius pars aut partes ſint ſexdecim, dicemus igitur ſi.24.dant ſexdecim, quid dabit vnum? ſexdecim videlicet vigeſimasquartas, quæ cum ad primos numeros reductæ fuerint, erunt duæ tertiæ. Eadem ratione qui ſcire uellet, quæ partes aut pars eſſent tres quartæ, octo nonarum, diceret, ſi octo nonæ dant tres quartas, quid dabit vnum? prouenient.27. trigeſimęſecundæ.

Subſeruit pariter ad ſciendū naturā partiū numeri propoſiti. Exempli cauſa, ſi quis quærat, cuius numeri, duodecim ſint duæ tertiæ partes. Dicet ſi duo dant tria, quid dabunt duodecim ? nempe dabunt decemoćto , numerum quæſitum ſcilicet, Tunc autem nil aliud prę́ſtamus quam quòd quærimus numerum ad quem ita ſe habeant duodecim , ſicut duo ad tria. Ita etiam ſi quis quærat, cuius numeri duo tertia ſint tres quintę,dicet, ſi tria dant quinq;, quid dabunt duo tertia? nempe dabunt integrum cum fraćto nono. Hoc erit itaq; quę́rere numerum ad quem ſic ſe habeant duo tertia ſicut tria ad quinq;,quod manifeſtum eſt per ſe.

Eadem ratione qui ſcire vellet,cuius numeri duæ ſeptimæ, eſſent oćto integrarum cum duabus quintis,diceret,ſi duo dant ſeptem quid dabunt oćto integra cum duabus quintis?nempe dabunt.29.integra cum duabus quintis numerum quæſitum. Sic etiam qui transferre uellet fraćtum numerum in fraćtum, id perficeret ex regula de tribus.

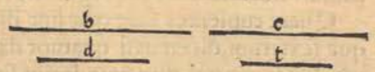
Exempli gratia ſi proponerentur vndecim tertiædecimæ vnius totius, toto diuiſo in.13. partes, deſideraremusq́; ſcire,quot partes totius eſsẽt vndecim tertię́decimæ,toto in.4.partes diuiſo, diceremus ſi. 13 .dant. 11. quid dabunt quatuor?nempe dabũt tres quartas cũ quinq; tertijsdecimis unius quartæ,hoc verò nihil aliud eſt quam querere numerum, ad quem ſic ſe habeat totum in 4. partes diuiſum, ſicut idem totum diuiſum in tredecim ſe habet ad undecim tertiasdecimas, Porrò ad alia etiam multa hæc regula accommodata eſt.

Hæc enim nõ ſine propoſito dićta ſunt,ſed ut quiſq; videat cauſam ſimilium operationum,quæ à praćticis circa fraćtos numeros ſcriptæ ſunt , omnem à diuina illa regula de tribus originem trahere ut etiam in ſequentibus videbimus.

## THEOREMA XI.

CVR produćtum ex eo quod oritur in diuidente, ſemper æquale eſt numero diuiſibili ſi queras ita accipe.

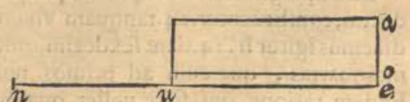
Sit numerus diuiſibilis.b.quod oritur ſit.c.diuidens.d.& vnitas diuidentis.t. cum igitur,vt in præcedenti theoremate dićtum fuit, eadem ſit proportio . b.ad.c.quæ eſt.d. ad.t. manifeſte deprehenditur ex.20. ſeptimi,produćtum ex.b.in.t. æquale eſſe produćto .c.in d.




## THEOREMA XII.

ID ipſum alia ratione contemplari licet.

Numerus diuiſibilis ſignificetur per lineam.n.e.diuidens verò per lineam.a.e. quod oritur linea.u.e vnitas diuidentis.o.e.quã cogitamus eſſe vnitatem linearem; ad hæc produćtum ex.u.e.in.a.e.ſit ſuperficies.u.a.Dico ſuperficiem.u.a. componi ex tot vnitatibus ſuperficialibus quot linearibus conſtat linea.n.e.nam ex ijs quæ diuidendi ratione notauimus,cõſtituitur eandem proportionem eſſe .n.e. ad.u.e. quę eſt.a.e.ad.o.e.At ex prima ſexti aut 18.ſeptimi ſic ſe habet totale produćtũ. u.a. ad partiale. u.o. ſicut.a.e.ad.o. e. quare ſic ſe habebit.u.a.ad.u.o. ſicut.n. e.ad.u.e.ſed.u.e.et.u.o.numero non differunt,cum ſint vnius & eiuſdem ſpeciei,(tamet ſi numerus.u.o.ſit ſuperficialis et.u.e.linearis). Itaq; ex nona quinti numerus. u.a. æqualis erit numero.n.e.

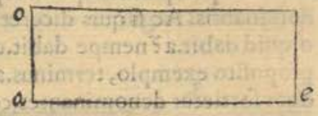



THEO-

## THEOREMA. XIII.

CVR diuidentibus numerum diuiſibilem per prouenìens,oritur numerus diuidens?

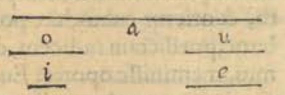
Sit ſubſcriptus rectangulus.o.e.numerus diuiſibilis,qui producitur,tam ex.a.o.in.a.e.quàm ex.a.e.in.a.o.quare ſi.a.o. diuidens fuerit.a.e. proueniens erit, ſi verò.a.e.diuidens extiterit,a.o. proueniens erit futurum.




## THEOREMA. XIIII.

HOc ipſum,alia quoq; uia licebit ſpeculari.

Sit linea.a.denotãs numerum diuiſibilem,et.o.primi prouenientis linea.e.primi diuidentis .u. ſecundi prouenientis ideſt cum.o.pro diuidente ſumetur. Iam ex indicata definitione diuiſionis nono theoremate huius libri,dabitur proportio.a.ad. o. prout datur. e. ad vnitatem ſignificatam linea. i. & permutatim .a. ad. e.ſicut .o. ad.i. ſed.a. ad.u. ſic ſe habet prout. o.ad.i. ex eadem definitione diuiſionis, itaq; ſic ſe habebit.a.ad.u.ſicut.a.ad. e.vnde.u.æqualis erit.e.ex.9. quinti



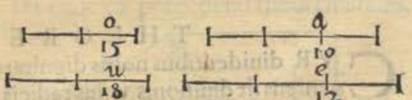

## THEOREMA. XV.

VNde prouenit, vt qui velit cognoſcere cuius numeri quatuor quintæ partes, ſint, duæ tertię,aut quid ſimile, cõſultiſſime faciat, ſi ad unam eandemq; denominationem reduxerit.

Prout in propoſito exemplo,cũ denominãs cómunis ſit quindecim,cuius duæ tertiæ ſunt decẽ,& quatuor quintæ duodecim,cõmunis autẽ denominans. 15. multiplicandus ſit per quatuor quintas, ſcilicet duodecim, & productum diuidendum per duas tertias,hoc eſt decem,ex quo oriantur decemocto quęſitus numerus?

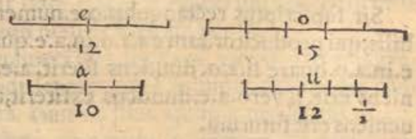
Quod ad reductionẽ numeratorũ ad vnam & eandem denominationem attinet, ea de cauſa fit quo uti poſſimus regula de tribus, quæ tribus tantummodo notis terminis indiget, quo quartus à prędictis dependens, inueniri poſſit, quandoquidem bini illi reſpectus, tribus terminis comprehendi poſsũt. At quod ad multiplicationem ſpectat denominantis cómunis cũ numerante denominantis incogniti & diuiſionem producti per numerantem cognitũ illę nihil aliud ſunt, quam quartũ terminũ inuenire, ita proportionatum tertio, vt ſecundus primo.

Exempli gratia,ſit. a. denotãs numerantem denominantis cogniti, qui ſignificetur linea.o. et.e.ſit denominantis incogniti numerans, denotati linea.u.imò verò & cogniti.o. nempe quatuor quintæ, Iam ſi.o.cum.e.multiplicemus, & productum per .a. diuidemus dabitur.u. ſic ſe habens ad.e. ſicut.o.ad.a.ex.20. ſeptimi.



## THEOREMA XVI.

INuenire autem cupienti cuius numeri, duæ tertiæ, ſint quatuor quintę partes, multiplicandę eſſent duæ tertiæ per denominantem communem, & productum diuidendum per quatuor quintas ipſius denominantis. Ac ſi quis diceret ſi.e.dat.o.quid dabit.a? nempe dabit.u.nam in propoſito exemplo, terminus.a.loco.e. duos ſortietur denominantes; cognitum videlicet .o. et.u. incognitum quod poſtea cognitum oritur ex regula de tribus, vt dictum eſt.

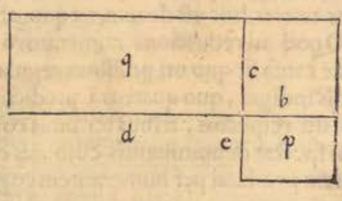



## THEOREMA XVII.

QVA ratione cognoſci poterit proportionem quantitatis cenſicæ cenſicæ ad ſimilem quantitatem quadruplam eſſe ad eam, quæ eſt ſuarum radicum; proportionem autẽ primarum relatarum eſſe quintuplam, atq; ita deinceps?

Cuius rei gratia, ſciẽdus eſt modus ꝓductionis harũ dignitatũ qui oritur ex productione primæ radicis in ſeipſam, prout qui cubũ requirit, ducat radicẽ in ſuo quadrato, & orietur cubus, hæc poſtea ducta in cubum, quantitatẽ cenſicam cenſicã, et in hanc, prædictam radicem, dabit quantitatem primam relatam. Quod vbi ſciuerimus, meminiſſe oportet Euclidem decimaoctaua ſexti aut.11.octaui docere, proportionem quadrati ad quadratũ, duplam eſſe proportioni ſuarum radicum, &.36. vndecimi aut.11.octaui, cubi ad cubũ triplam eſſe, ego verò nunc aſſero, cenſici cenſici ad radicum proportionem quadruplam eſſe, primi verò relati ad primum relatum quintuplam atq; ita gradatim.

Cuius ſpeculationis gratia, detur linea.d.quæ cubum maiorem ſignificet.et.b. minorem.c. verò ſit radix ipſius.d.et.e.ipſius.b. ita ordinate adinuicem, vt in ſubſcripta figura cernitur. Iam.c.cum.d. producatur proueniatq;.q. cenſicum cenſicum, tum producatur.c. cum.b. et dabitur.p. alterum cenſicum cenſicum. Dico igitur proportionem.q. ad.p. quadruplam eſſe proportioni.c. ad.e. hac de cauſa quòd proportio.q. ad.p. componatur ex proportione.d.ad.b.et.c.ad.e. prout facile ex.24.ſexti, aut quinta octaui deprẹhenditur. Quare cũ proportio.d.ad.b. proportioni.c.ad.e. tripla ſit, patet proportionem.q. ad.p. quadruplam eſſe proportioni.c.ad.e. Idem de cæteris dignitatibus dico, ſumptis ſemper.d.et.b. pro duobus cenſibus cenſuum, aut duobus primis relatis, aut alio quouis axiomate.




## THEOREMA XVIII.

CVR diuidentibus nobis dignitatem, per dignitatem, radix prouenientis: proueniens ſit diuiſionis vnius radicis per alteram?

Sint exempli gratia duę lineæ.b.q.et.f.g. quæ ſignificent duas radices cuiuſuis dignitatis; demusq; eſſe radices duorum quadratorum, quadratumq; ipſius b.q. per quadratum ipſius.f.g. diuidatur; quadrataque radix prouenientis ſit.d.q. vnitas verò linearis ſit.i.g. Dico ipſam.d.q. eſſe proueniens ex diuiſione.b.q. per.f.g. Patet enim ex definitione diuiſionis nono theoremate tradita quadratum

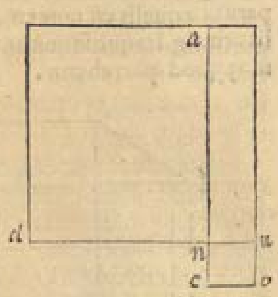
tum

tum ipsius.d.q.talem esse partem quadrati ipsius.b.q.qualis quadratum ipsius.g.i. est quadrati ipsius.f.g. Scimus prẹterea ex.19.sexti,aut vndecima octaui, proportioné quadrati ipsius.b.q.ad quadratũ ipsius.d.q. duplam esse proportioni. b. q. ad. d.q. suarum radicum (cuborum enim tripla esset & censuum censuum, quadrupla, atq; ita deinceps ex praecedenti theoremate) Id ipsum dico de dignitatibus ipsius. f.g.et.i.g.respectu radicum.f.g.et.i.g.Vnde cum proportio dignitatis ipsius.b.q. ad illam.d.q. ẹqualis sit proportioni dignitatis ipsius.f.g.ad illam.g.i.ex communi scientia apertè cognoscemus simplices proportiones esse interse æquales,nempe eam quẹ est.b.q.ad.d. q.æqualem esse ei, quæ est.f. g.ad.i.g.itaq; sequitur ex definitione diuisionis.d.q.esse proueniens ex diuisione. b.q.per.f.g.


## THEOREMA XVIIII.

CVR productum ex duabus radicibus quadratis, est quadrata radix, producti suorum quadratorum simul?

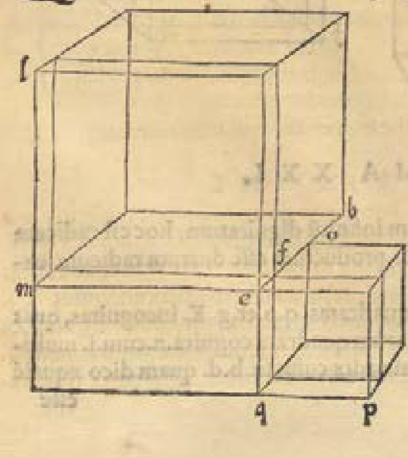
In cuius rei gratiam,sint duo quadrata.d.a.et n.o. coniuncta simul, prout in subscripta figura apparet,ita tamen vt angulus.a.n.u.sit rectus, quare ex quartadecima primi,duo latera. n.c.et. n.a.directe coniũgentur adinuicem, prout etiam reliqua duo latera.n.u. et. n.d.Cogitato deinde.a.u.producto ipsius.a.n.in.n. u. duarum videlicet radicum quadratarum simul, dabitur ex prima sexti, aut decimaottaua septimi,productum.a.u.medium proportionale inter quadratum.a. d. et.u.c. quod si cogitemus has tres superficies, tres numeros esse, patebit ex vigesimaprima septimi productum. a. u.in seipsum,quadratum scilicet. a. u. æquale esse producto. a. d. in. u.c.ex quo propositi euidentia consequetur.

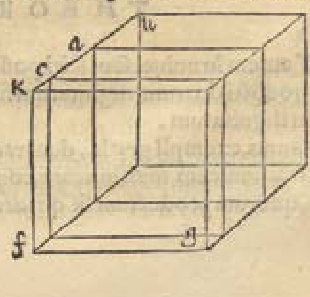



## THEOREMA XX.

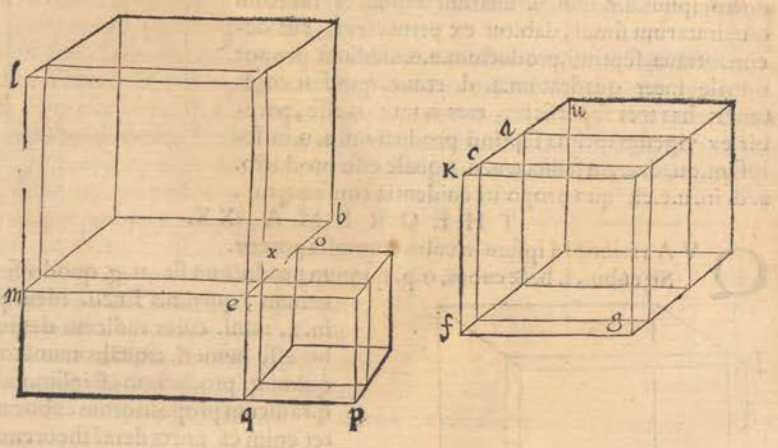
QVA ratione id ipsum in cubis cognosci poterit.

Sit cubus. l. b. & cubus. o.p. quorum productum sit. u. g. quod assero esse cubum, quamuis Eucli. idem probet in.4. noni. cuius radicem demonstrabo esse numeri æqualis numero.m. q. qui.m.q. productum est ipsius.m.e.in.e. q.radicum propositorum cuborum.Patet enim ex præcedenti theoremate.m.



q. radicem esse quadratam producti .l. e. in. e. p. quod productũ sit quadratum corporeum. c. g. cogitemus pariter duo quadrata. l. e. et. e. p. esse pariter corporea, tantę profunditatis, quantam, vnitas linearis radicum. m. e. et. e. q. requirit. Hæc duo corpora producentur à superficie in vnitatem, vocenturq́;.l.x.et. x. p.quo facto,cogitemus corpus.a.g.tamquam productum cubi.l.b. in quadratum.e. p. Vnde ex decimaoctaua,aut decimanona septimi, eadem erit proportio. a. g. ad. c. g. quæ est.l.b.ad.l.x.corporeum,sed ex. 25. vndecimi & prima sexti,ita se habet.a. K. ad.K.c.vnitatem linearẽ sicut.a.g.ad.c.g.& ex eisdẽ ita se habebit.b.e.ad.e.x.vnitatem linearem, sicut.l.b.ad quadratum.l.x.corporeum.Itaque sic se habebit. b.e. ad vnitatem linearem.e.x.videlicet.K.c.sicut.a.K.ad ipsam.K.c.Vnde ex nona quinti. a.K.æqualis erit.e.b.& consequenter æqualis.m.e.Iam verò sit.u.g. productum. l.b. cubi,in cubum.o.p.vt supra dictum est, Hinc patebit ex quauis duarum propositionum,decimaoctaua,aut decimanona septimi,eandem futuram proportionem. u. g. ad.a.g.quæ est.o.p.ad.x.p.quadratum corporeum.Quare ex postremis,dictis rationibus,eadem erit proportio.u.K.ad.a.K.quæ est.o. e. ad vnitatem linearem. e. x. at ex dictis decimaoctaua & decimanona septimi, ita se habet numer⁹.m.q.ad numerũ superficialẽ.m.e.qui ꝓducitur à lineari.m.e.in vnitatẽ linearẽ ipsius. e.q.sicut numerus.q.e.ad suam vnitatẽ,sed cũ numerus.a.K.æqualis sit numero.m.e. vt probatũ est erit ergo ex vndecima & nona quinti,numerus .u.K.æqualis numero. m. q. At. f. g. pariter æqualis est numero.m.q.ex præcedenti theoremate,vnde.K.u.pariter æqualis erit.f.g.Itaque sequitur.u.g.cubum esse,& f.g.radicem ipsius, æqualem numero. m.q.quod quærebatur.



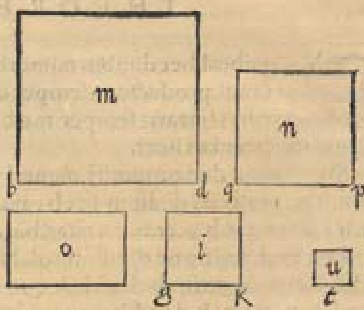

## THEOREMA XXI.

VT autem in uniuersum sciri possit totum infinitũ dignitatum, hoc est radicem producti duarum dignitatum similium, productum esse duarum radicum earundem dignitatum.

Ponamus, exempli gratia, duas radices quadratas. q.p.et.g. K. incognitas, quas qui velit adinuicem multiplicare,cogatur earum quadrata cognita.n.cum.i. multiplicare, quorum productum sit quadratum.m.radix cuius sit.b.d. quam dico æqualẽ

esse

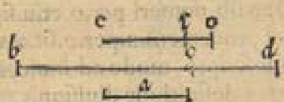
esse ꝑducto.q.p.in.g.κ. qđ autē sit.o. Patet enim proportionē.o.ad.q.p.eandē esse cum proportione.g.κ.ad suam vnitatem linearem, ex decimaoctaua, aut decimanona septimi, hæc vero vnitas linearis sit.t.cuius superficialis sit.u.vnitas scilicet toties in seipsam multiplicata quoties proposita dignitas patitur, tametsi in præsenti exemplo quadrata dignitas sumatur. Itaq; ex eisdem propositionibus decimaoctaua aut decimanona, sic se habet.m.ad.n.sicut.i.ad.u. Scimus pręterea proportionē.m.ad.n.(eo quod in proposito exemplo sint quadrata) duplam esse proportioni.b.d.ad.q.p.et ipsius.i.ad.u. pariter duplam proportioni.g.κ.ad.t. iam autem dictum fuit sic se habere.m.ad.n.sicut.i.ad.u.Itaq;.b.d. sic se habebit ad. q.p. sicut. g.κ.ad.t. quandoquidem sic se habeat totum ad totū, sicut pars ad partē, dū similes sint, probatū autē est superius ita se habere.o.ad.q.p. sicut.g.κ.ad.t.itaq;.o.sic se habebit ad.q.p. sicut. b.d.ad.q.p. vnde.o. æqualis erit.b.d. Hoc ipsum cęteris dignitatibus conueniet, mutatis tantummodo proportionibus. m. n. ad proportionem.b.d:q.p. sic proportionibus duarum dignitatum.i.u. ad proportionem suarum radicum.g.κ.t.




## THEOREMA XXII.

DOCENT veteres, quòd si quilibet numerus in duas partes inæquales diuisus fuerit, totumq; diuisum per vnā partium, & per eandem pars altera diuisa fuerit: differentia prouenientium semper vnitas erit. quodquidem verissimum est.

Detur enim.b.d. propositus numerus in duas partes inæquales diuisus.b.c.et.c.d. & in primis totū.b.d.per.c.d.diuidatur, ex quo oriatur e.o.vnitas autem.p.i.o.significetur, tum pars ipsa.b.c.p.eādem.c. d.diuidatur, sitq; proueniēs.a. Sanè ex definitione diuisionis, eadem erit proportio.b.d.ad.e.o.quæ est.c.d.ad.i.o.et ita.b.c.ad.a. sicut.c.d.ad.i.o.Ex.19.autem quinti, ita se habet.b. c.ad.e.i.sicut.b.d.ad.e.o.at.b.d. ad.e.o.sic se habet sicut.c.d.ad.i.o.hoc est sicut.b.c.ad. a.Quare ex.11.quinti sic se habebit.b.c.ad.e.i.sicut.ad.a.ex quo ex.9.pręd iecti.a. æqualis erit. e.i. sed.e.i. minor est.e.o. per.i.o. Quare sequitur propositum verum esse. Quod ipsum paucissimis verbis sic definiri potest, si dixerimus, eiusmodi diuidens. in parte diuisibili, quā in toto, semel minus ingredi, quandoquidem altera pars est, ex qua totum integrum perficitur.




## THEOREMA XXIII.

HOc ipsum alia ratione contemplari poterimus.


Significetur enim totalis numerus per.a.e. in duas partes diuisus.a.u.et.u.e.totius autem diuidens sit.u. e. & partis alterius.a.u. totius verò proueniēs sit.a.c.partis autē, sit proueniēs.a.n.tum differentia sit.n.c.vni

tas

tas vero cui differentiã.n.c.æquari dico,ſit.a.i.Patet enim in primis,eandem propor tionem eſſe.a.e.ad.a.c. quæ eſt. u.e.ad.a.i.ex definitione diuiſionis, et eandem eſſe.a.u.ad.a.n.quæ eſt.u.e.ad.a.i.vnde ex.

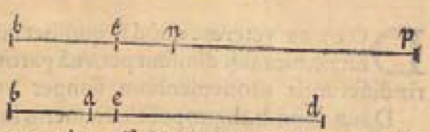

11.quinti ſic ſe habebit.a.e.ad.a.c.ſicut.a. u.ad.a.n.et ex.19.eiuſdem ſic ſe habebit.u.e.ad.n.c.ſicut.a.e.ad.a.c.ſed.ſic ſe habebat.u.e.ad.a.i.Itaq; ex prædicta.11.quinti,ſic ſe habebit.u.e.ad.n.c.ſicut ad.a. i.Quare ex.9.eiuſdem.n.c.æqualis erit.a.i.et idcirco.n.c.pariter vnitas erit.

## THEOREMA XXIIII.

CVR quibuslibet duobus numeris diuiſis adinuicem,multiplicatisq; prouenientibus ſimul,productum,ſemper eſt vnitas ſuperficialis?Nempe ex.20.ſeptimi, quoniam vnitas linearis ſemper media proportionalis eſt inter bina prouenientia. Quod ita ſpeculari licet.

Significetur duo propoſiti numeri per.b.p.et.b.d.mutuo diuiſi, proueniens autem.b.p.per.b.d.diuiſum ſit.b.n.tum proueniens.b.d.diuiſum per.b.p.ſit.b.a. et.b.t.ſit vnitas.b.p.et.b.e.vnitas.b.d.ex quo.b.t.æqualis erit.b.e.

Iam ex definitione diuiſionis,dabitur eadem proportio.b.p.ad.b.n.quæ eſt.b.d. ad.b.e.et proportio.b.d.ad.b.a.quæ eſt.b.p.ad.b.t.Sed cum ſic ſe habeat.b. p.ad.b.n.ſicut.b.d.ad.b.e.permutando ſic ſe habebit.b.p.ad.b.d.ſicut.b.n.ad.b. e.hoc eſt ad.b.t.et cum ſic ſe habeat.b.d.ad.b.a.ſicut.b.p.ad.b.t: permutando ſic ſe habebit.b.d.ad.b.p.ſicut.b.a.ad.b.t.

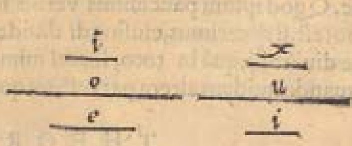



Quare eueſim ſic ſe habebit.b.p.ad. b.d.ſicut.b.t.ad.b.a.ſed.b.n.ad.b.t.ſic ſe habebat vt.b.p.ad.b.d.Itaq; ex.11. quinti ſic ſe habebit.b.n.ad.b.t.ſicut.b. c.ad.b.a.Dictum autem eſt.b.e.et.b.t.idem omnino eſſe. Quare ex.20.ſeptimi propoſiti veritas innoteſcet.

## THEOREMA XXV.

IDipſum & hac altera uia patebit.

Duo illi numeri per.o.et.u.ſignificentur mutuo diuiſi,proueniens autẽ.o.per. u.ſit.e.et proueniens.u.per.o.ſit.x.vnitas uerò per.i.ſignificetur, quas tamen quantitates ſubſcripto modo ad inuicem diſponito.Itaq; ex definitione diuiſionis, eadem erit proportio.o.ad.e.quę eſt.u.ad.i.et.o.ad.i.quę eſt.u.ad.x.Quare ex æqualitate proportionũ. e.ad.i.ſic ſe habebit ſicut.i.ad.x.erit enim.i. media proportionalis inter.e.et.x.ex.20.autẽ ſeptimi propoſitum concludetur. Huiuſmodi rei cauſa etiam eſt, quod proueniens diuiſionis vnius eſt numerator æqualis denominatori diuiſionis alterius.



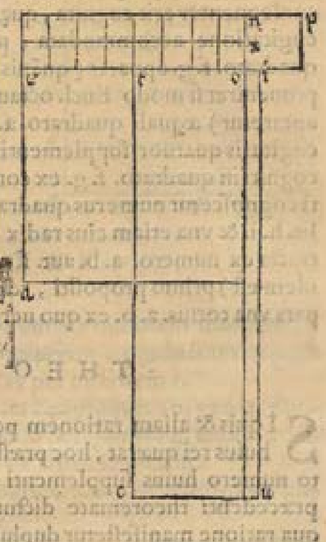

## THEOREMA XXVI.

CVR duobus numeris mutuo diuiſis,sũptis deinde prouenientibus ſimul et adinuicem,& per hanc ſummam,diuiſa ſumma quadratorum dictorum propoſitorũ nume-

numerorum, proueniat numerus æqualis numero producti duorum primorum numerorum ſimul.

Sint exempli gratia propoſiti numeri.2.et.8.qui mutuo diuiſi in primis dent prouenientia quatuor integra, tum quartam partem pro altero proueniente, hæc collecta dabunt ſummam quatuor integrorum et quartæ partis vnius, ſumma autem quadratorum binarij & octonarij erit.68. qui quidem numerus per quatuor & quartam partem vnius diuiſus dabit.16.pro proueniente, quæ.16.æqualia erunt producto binarii in octonarium.

Cuius rei hæc erit ſpeculatio, ſint duæ lineæ.o.e.et.o.n. quæ duos numeros propoſitos ſignificent, inuicem ad angulum rectum.o.coniunctæ, quarum quadrata ſint.o.a.et.o.p.ipſorum productum ſit.n.e.tum.o.t.ſit proueniens ex diuiſione.o.e. per.o.n.Hęc ſingulatim conſideremus (nã ſi in partibus ſimplicibus quod dicimus acciderit, id ipſum in compoſitis conſequenter eueniet)quamobrem ex definitione diuiſionis dabitur eadem proportio.o.e.ad.o.t.quæ eſt.o.n.ad vnitatem, quæ ſit.o.x. Nunc cogitemus ſuperficiẽ rectangulã.o.c.æqualẽ quadrato.o.a.tunc numerus.c.t.proueniens erit, ut patet, ex diuiſione numeri quadrati.o.a.per numerũ.o.t.eritq; eadẽ proportio.c.t.ad.o.e.quæ eſt.o.e.ad.o.t.ex ſecunda parte quintædecimæ ſexti, aut.20.ſeptimi.Iã autẽ dictum eſt.o.e.ad.o.t.ſic ſe habere ſicut.o.n.ad.o.x.Itaq; ex.11.quinti ſic ſe habebit.c.t.ad.o.e.ſicut.o.n.ad.o.x.Sed ex prima ſexti, aut.18.vel.19.ſeptimi, ſic ſe habet ꝓductum.n.e.ad.e.x.ſicut.o.n.ad.o.x.quare denuo ſic ſe habebit numerus.c.t.ad numerum.o.e.ſicut numerus.n.e.ad numerum.x.e.Sed numerus.o.e.cum numero.x.e.ſpecie idem eſt, igitur ex.9.quinti numerus.c.t.numero.n.e.æqualis erit.



Id ipſum de quadrato ipſius.o.n.videlicet.p.o.dico. Nam ſi proueniens.o.n.diuiſo per.o.e.ideſt.o.i.proportionale reſpondens ad.o.t.cum.o.t.coniunctũ fuerit, et per hãc ſummam diuiſa ſumma quadratorum.o.a.et.o.p.patet per ſe proueniens futurum eiuſdem numeri.c.t.ipſumq;.c.t.proueniens ſemper futurum.

Quo autem lucidius res hæc innoteſcat. Cogitemus proueniens quadrati.o.p. diuiſi ab.o.i.reſpondentisq;.o.t.eſſe.i.u. quod via prædicta inuenitur æqualis eſſe numero.n.e.ex quo conſequenter æquale.c.t: cogitato deinde rectangulo.o.u.æquali.o.p.coniuncto.o.c:totum.t.u.æquale erit compoſito duorum quadratorum.o.a.et.o.p.cum in nullo numerus.c.t.mutetur, tam ex compoſito.t.u.quã ex ſimplici.o.c.ex quo propoſiti ſeſe ueritas profert.

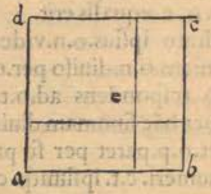
## THEOREMA XXVII.

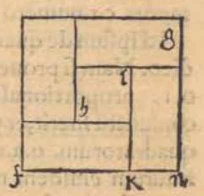
PROPOSVERVNT veteres nobile quidem problema, ſed quod tamen citra algebraticam effectionem, aut neſcierunt, aut noluerunt diſſoluere, quod nihilominus facillimum eſt.

Proponunt hi numerum in binas eiuſmodi partes diuidendum, vt ſumma quadratorum dictarum partium, alteri numero poſsibili propoſito æqualis ſit, poſſibili inquam, etenim ſi eiuſmodi numerus propoſitus, minor eſſet producto totius primi in ſuum dimidium, eſſet huiuſmodi factum impoſſibile. Quod nos exequi cupientes, ſumamus primum numerũ propoſitum, quem in ſe ipſum multiplicemus. ab hoc quadrato deducamus ſecundum numerum propoſitum, tum quod remanſerit duplicemus, quod duplum denuo iubeo ex eodem primo quadrato detrahi, accepta poſtea radice quadrata reſidui & dempta ex priori numero propoſito, tunc dimidium reſidui vna pars erit ex duabus primi numeri quæſita.

Exempli gratia proponantur. 20. diuidenda in duas eiuſmodi partes, vt ſumma quadratorum ipſarum partium æqualis ſit. 272. qui numerus maior eſt. 200. maior inquam dimidio quadrati. 400. ipſorum. 20. hic autem numerus. 272. è quadrato. 400. deducatur, remanebũt enim. 128. quod duplicari iubeo, producẽtur ſiquidẽ. 256. quæ pariter deducta è quadrato totali, remanebunt. 144. cuius radicem ſumi volo, quæ erit. 12. & dempta ex. 20. priori numero dato remanebit. 8. cuius dimidium erit. 4: pars vna ex quæſitis, quæ ex primo numero propoſito. 20. detrahetur, remanebitq́;. 16. pro altera parte.

Cuius demonſtrationis cauſa, in primis cogitemus quadratum. a. c. cognitum numeri. a. b. primò propoſiti, qui cogitetur diuiſus in duo quadrata. d. e. et. e. b. duoq́ue ſupplementa. a. e. et. e. c. numerus autem ſummæ duorum quadratorum. d. e. b. pro ſecundo propoſito datur; ex quo, ſumma duorum ſupplementorum. a. e. c. conſequenter erit cognita, quę cum duplicata fuerit, & quatuor hæc ſupplementa cogitatione accommodata, prout in quadrato. f. g. apparet (quãuis idipſum proueniret ſi modo Eucl. octaua ſecũdi aptaretur) æquali quadrato. a. c. ita vt cogitatis quatuor ſupplementis numeri cogniti in quadrato. f. g. ex conſequenti cognoſcetur numerus quadrati partialis. h. i. & vna etiam eius radix qua detracta ex numero. a. b. aut. f. n. (quod idem eſt) primo propoſiti, relinquetur numerus cognitus duplum. k. n. aut. t. b. pars vna totius. a. b. ex quo uerum erit hoc meum problema.

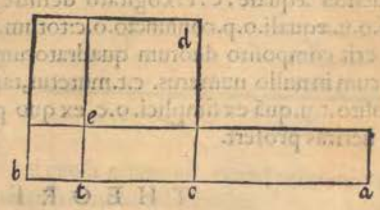



## THEOREMA XXVIII.

SI quis & aliam rationem perficiendæ huius rei quærat, hoc præſter inuento numero huius ſupplementi, cum in præcedenti theoremate dictum fuerit, qua ratione manifeſtetur duplum ſupplementi ipſius.

Cogitemus in ſubſcripta figura lineam. a. b. tanquam primum numerum propoſitum, & productum. a. e. ſupplemento. a. e. primæ præcedentis figuræ æquale ſit, ac deinde ordine ab antiquis tradito procedatur, ad quadratum reducto dimidio. a. b. videlicet. b. c. quod erit. b. d. ex quo detrahatur deinde. a. e. quare remane-

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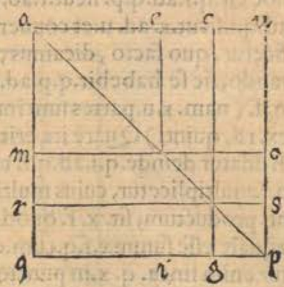
bit quadratum. e. d. cognitum, cuius radix æqualis erit. c. t. qua coniuncta dimidio. c. a. ex quinta secundi Eucli. dabit quod propositum erat.

## THEOREMA XXIX.

QVID causæ est, cur subtracto duplo producti duorum numerorum ad inuicem multiplicatorũ ex summa suorum quadratorum, semper quod super est duorum numerorum quadratum differentiæ sit?

Exempli gratia si proponerentur duo numeri. 16. et. 4. duplum producti eorum esset. 128. quò detracto ex summa suorum quadratorum, nempè ex. 272. remaneret. 144. cuius quadrati radix esset. 12. tanquam differentia inter. 4. et. 16.

Id vt sciamus, duo numeri propositi, duabus lineis significentur, maiore. q. g. et minore. g. p. directè coniunctis, super quas, totale quadratum extruatur. a. p. in quo cogitetur diameter. a. p. et à puncto. g. ducatur parallela. g. n. c. et à puncto. n. parallela. n. s. r. ex quo duo producta dabũtur. q. n. et. n. u. singula æqualia producto. q. g. in g. p. et. a. n. et. n. p. duo quadrata dictorum numerorum propositorum, quod satis superq; probatur quarta secundi Eucli. Cogitemus deinde. n. o. æqualem. n. p. et à puncto. o. ducatur. o. m. t. parallela. r. s. et. o. e. ad. n. c. quare ex allatis ab Eucli. octaua secundi, dabitur quantitas. m. n. æqualis. q. n. producto. q. g. in g. p. et quantitas. o. c. minor ipso producto, ex quantitate quadrati. n. p. ex quo quantitas. m. n. e. vnà cum quadrato. n. p. æqualis erit duplo producti. q. g. in. g. p. sed hæ duæ quantitates, sunt partes duorum quadratorum dictorum, & quæ super est. m. e. quadratum differentiæ vnius numeri propositi ab altero, prout in subscripta figura licebit cui libet considerare. Itaque veritas hæc manifesta erit.

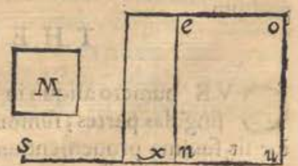



## THEOREMA XXX.

CVR ij qui ex duobus numeris propositis maiorem per minorem diuidunt, si proueniens per maiorem numerum multiplicauerint, productum æquale erit prouenienti ex diuisione quadrati maioris numeri per minorem?

Exempli gratia si proponantur duo numeri. 20. et. 4. ipseque. 20. per. 4. diuidatur, dabit quinque, tum. 400. quadrato. 20. diuiso per priorẽ. 4. dabit. 100. quod proueniens, producto ex. 20. in. 5. primo prouenienti adæquatur.

Cuius speculationis causa, sint duo numeri, qui lineis. x. u. et. x. s. maiore atq; minore significẽtur, tum. u. x. numerus per. s. x. diuidatur, sitque proueniens. x. n. postmodum quadratum. u. x. fit. x. o. et productum ex. n. x. in. u. x. fit. x. e. quod æquale esse dico prouenienti ex diuisione quadrati. o. x. per. s. x. quod fit. m. Patet enim ex definitione diuisionis, talem futuram proportionem. u. x. ad. n. x. qualis est. s. x. ad vnitatem, & quadratum. o. x. ad rectangulum. e. x. ita se ha-

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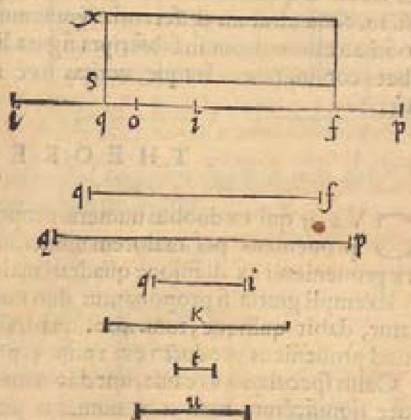
biturum, ſicut. u. x. ad.n.x. ex prima ſexti aut. 18.vel. 19. ſeptimi, quare ex 11. quinti ita ſe habebit. o. x.ad. e. x. ſicut. s. x. ad vnitatem; ſed ſicut ſe habet. s.x. ad vnitatem, ita ſe habet pariter. o.x. ad. m. vnde ex. 11. prædicta ita ſe habebit. o. x. ad. m. ſicut idipſum. o. x.ad.e.x. itaque ex. 9. prædicti quinti.m. æqualis erit.o.x.

## THEOREMA XXXI.

CVR propoſito aliquo numero in duas partes inæquales diuiſo, ſi rurſus per quamlibet ipſarum diuidatur, prouenientia tantumdem coniuncta quantum multiplicata efficiant.

Exempli gratia, ſit denarius propoſitus numerus, per binarium & octonarium diuiſus, prouenientia erunt quinque & vnum cum quarta parte, quæ coniuncta erunt. 6. cum quarta parte lineari, quæ ſimal multiplicata, pariter erunt. 6. cum quarta parte ſuperficiali.

Cuius ſpeculationis cauſa, totalis numerus, linea. q. p. ſignificetur, eius duæ partes, per. κ. maiorem et. u. minorem, ipſa vnitas per. t: proueniens ex diuiſione. q.p.per. κ. ſit.q.i. proueniens autem ipſius. q.p. per.u. ſit.q. f. quare ex definitione diuiſionis ita ſe habebit. q.p. ad.q. i. ſicut. κ. ad.t. et.q.p.ad.q.f.ſicut. u.ad.t. hoc eſt.q.f.ad.q.p. ſicut.t.ad.u. vnde ex æqualitate proportionũ ſic ſe habebit. q. f. ad.q.i.ſicut.κ. ad. u.et conuerſim. Ad hæc in linea.q.p.vnitas, per lineam.q.o.ſignificetur, quo facto, dicamus, ſi.q.p.ad.q.i.ſic ſe habet vt. κ. ad.q. o. itaque permutando, ſic ſe habebit.q.p.ad.κ. ſicut.q.i ad.q. o. hoc eſt. κ. u. ad. κ. ſicut. i. q. f.ad. q.f. (nam. κ.u.partes ſunt integrales totius.q.p. et. κ.u.ad. κ.eſt ſicut.i. q.f.ad. q. f. ex.18. quinti) Quare ita erit. i.q f.ad.q.f.ſicut.q.i. ad vnitatem.q.o. ex. 11. quinti Addatur deinde.q.i.ad.q.f. et.q.i.per. q.f.multiplicetur, cuius multiplicationis productum, ſit. x. f. quod probabo æquale eſſe ſummæ.f.q.cum.q.i. Secetur enim linea. q. x.in puncto. s.ita.vt. q.s. æqualis ſit. q. o. ſigneturque productum. s. f. quare eadẽ erit proportio quantitatis. x. f.ad.s.f.quæ eſt.q.x. ad.q.s.ex prima ſexti, aut. 18. vel 19. ſeptimi, hoc eſt, ſicut. q.i. ad. q. o. et ex. 11. quinti (vt dictum eſt) ſicut.i.q. f. ad.q. f. ſed numerus. s. f. ſuperficialis tantus eſt, quantus linearis. q. f. quare ex. 9. quinti tantus erit (ſuperficialiter) numerus. x. f. quantus (lineariter). f. q. i. quod erat propoſitum.



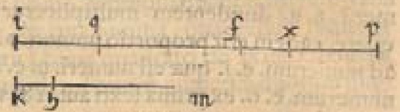

## THEOREMA. XXXII.

CVR numero aliquo in duas partes inæquales diuiſo, ſi rurſus diuidatur per ſingulas partes, ſumma duorum prouenientium per binarium, ſemper maior ſit ſumma prouenientium ex diuiſione vnius partis per alteram.

Exẽpli gratia, ſi proponeretur numerus. 24. qui in duas partes inæquales diuideretur.

retur. 20. scilicet et. 4. certè. 24. per singulas partes diuiso, daretur vnum prouenięns sex integra, & alterum vnum & quinta pars, quorum summa esset septem integra cum quinta parte, tum altera parte per alteram diuisa, daretur vnum proueniens quinque integrorum & alterum vnius quinti tantum, quorum summa esset quinque integra, & vna quinta pars, minor prima reliquorum duorum prouenientium per binarium.

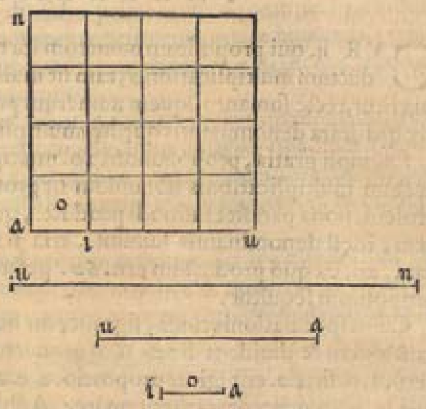
Cuius considerationis causa, propositus numerus linea. q. p. significetur, eius duę partes lineis. q.x. et. x. p. tū. q.f. sit proueniens ex diuisione totius. q.p. per. x. p. et. q.i. sit proueniens ex diuisione eiusdem. q.p. per. q.x. adhæc. h. m. sit proueniens, ex diuisione. q.x. per x. p. et. h. k. proueniens ex diuisione. p.x. per. q. x. patet igitur ex. 22. theoremate huius libri proueniēs. h. m. minus esse proueniente. q. f. per vnitatē, & proueniens. h.k. minus proueniente. q.i. per alteram vnitatem. Itaque. f.q.i. maior erit. m.h.k. per numerum binarium, quod erat propositum.




## THEOREMA. XXXIII.

QVILIBET numerus, medius est proportionalis inter numerum sui quadrati & vnitatem.

Detur enim numerus propositus, qui linea. a. u. significetur, cuius quadratum sit. u.n. vnitas linearis sit. i. a. et superficialis. o. patebit ex. 18. sexti aut 11. octaui proportionem. u. n. ad. o. futuram duplam proportioni. u. a. ad. i. a. sed. i. a. et. o. eadem (specie) res sūt, tanta scilicet. a. i. quanta. o. vnitas est, Itaque proportio numeri. u.n. ad. u.a. æqualis erit proportioni. u. a. ad. i. a. Quare numerus. u. a. inter numerum. u. n. & vnitatem, medius erit proportionalis.




## THEOREMA XXXIIII.

HOc ipsum quod diximus & alia ratione speculari licebit.

Propositus numerus, nunc etiam per. a. u. significetur, eius quadratum per. u. n. vnitas linearis per. a.i. productumq;. a. u. in. a. i. terminetur, sitq;. n. i. quare n. i. constabit numero superficiali æquali numero lineari. a. u. & ex prima sexti aut. 18. vel. 19. septimi, eadem erit proportio. u.n. ad. i.n. quæ est. a.u. ad. a. i. sed numerus. a.u. cum numero. n. i. idem specie est. Itaque medius est proportionalis inter. u.n. & vnitatem.

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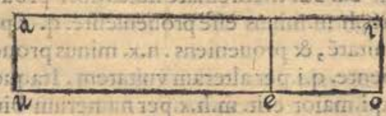
## THEOREMA XXXV.

QVIVIS numerus per alterum multiplicatus, & diuisus, medius est proportionalis inter productum multiplicationis, & proueniens diuisionis.

Exempli gratia, si. 20. multiplicetur per quinque & inde per quinque diuidantur productum erit. 100. proueniens. 4. inter quos numeros. 20. medius est proportionalis.

Hoc vt speculemur, proponatur numerus multiplicandus & diuidendus, qui significetur linea. u. e. multiplicans autem & diuidens linea. a. u. multiplicationis productum sit. e. a. proueniens ex diuisione sit. o. e. Nunc proueniens. e. o. per numerũ. a. u. diuidentem multiplicetur, cuius multiplicationis productum sit. e. i. quare, eadem erit proportio numeri. a. e. ad numerum. e. i. quæ est numeri. u. e. ad numerum. e. o. ex prima sexti aut. 18. vel 19. septimi. Sed cum numerus. u. e. ex. 11. theoremate præsentis libri, numero. e. i. æqualis sit. verum esse, quod propositum fuit consequetur.

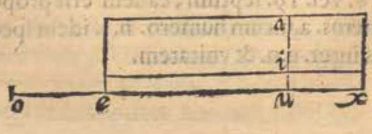



## THEOREMA XXXVI.

CVR ij, qui propositum numerum ita multiplicare & diuidere cupiunt, vt productum multiplicationis, tam sit multiplex prouenienti ex diuisione, quam quæritur, rectè sumant aliquem numerum pro multiplicante & diuidente, qui sit radix quadrata denominantis quęsitę multiplicitatis.

Exempli gratia, proponuntur. 20. multiplicanda atque diuidenda, ita vt productum multiplicationis nonuplum sit prouenienti ex diuisione, nempè, vt proueniens, nona pars sit eiusmodi producti, quare quadratam radicem ipsorum nouem, idest denominantis sumunt, tria scilicet, multiplicant igitur & diuidunt data. 20. ex quo productum erit. 60. proueniens autem. 6. cum duabus tertijs. & propositum sequitur.

Cuius speculationis causa, significetur numerus propositus linea. u. e. multiplicans autem & diuidens linea. u. a. productum sit. e. a. proueniens. e. o. quadratum verò. a. u. sit. x. a. erit igitur proportio. a. e. ad. e. o. dupla proportioni. a. e. ad numerum. u. e. ex præcedenti theoremate: Adhæc, cogitemus in linea. u. a. vnitatem. u. i. terminenturq; duo producta. e. i. et. x. i. quare eadem erit proportio. a. e. ad. e. i. quæ est. a. e. ad. u. e. numerus enim. e. i. (quamuis superficialis) idem est cum numero lineari. u. e. sed. a. e. ad. e. i. sic se habet sicut. a. u. ad. u. i. ex prima sexti aut. 18. vel. 19. septimi, (quod ipsum dico de. a. x. ad. x. i.) quare proportio. a. x. ad. x. i. hoc est. x. u. ęqualis erit ꝓportioni. a. e. ad. u. e. at trigesimotertio & trigesimoquarto theoremate probatum est proportionem numeri. a. x. ad vnitatem, duplam esse proportioni eiusdem numeri. a. x. ad. u. x. sequitur igitur cum dimidia sint æqualia, tota etiam æqualia esse: hoc est proportionem numeri. a. e. ad numerum. e. o. æqualem esse proportioni numeri. a. x. ad vnitatem. Itaque rectè sumitur numerus. a. u. eiusmodi vt quadratũ ipsius.



ipsius. a. x. tam sit multiplex ad vnitatem, quam cupimus numerum. a. e. numero. e. o. multiplicem esse.

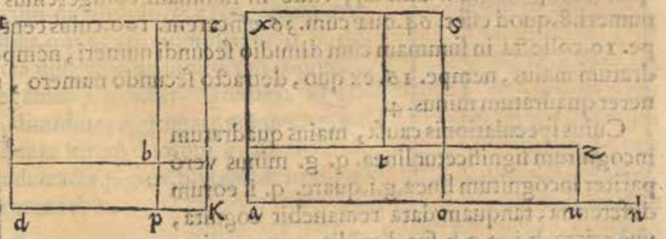
## THEOREMA XXXVII.

CVR inuenire cupientes duos numeros, quorum quadrata in summam collecta, æqualia sint numero proposito, & ijsdem numeris multiplicatis adinuicem, productum alteri numero proposito sit æquale, rectè sumant dimidium primi numeri propositi, cui summa quadratorum æquari debet, hocq; dimidium in seipsum multiplicent, vnà etiam alterum numerum propositum in seipsum multiplicent, quod quadratum detrahunt de primo, & residui quadratam radicem, dimidio primi numeri propositi coniungunt, ex qua summa, quadratam radicem eruũt, quæ duobus quæsitis numeris maior erit, cuius quadrato de primo numero detracto, & ex reliquo eruta radice quadrata, detur minor numerus, duorum quęsitorum.

Exempli gratia, si proponerentur. 34. pro primo numero cui æquari deberet summa duorum quadratorum, quorum radicum productum æquale esse deberet alteri numero, verbi gratia. 15. iubet antiquorum regula, dimidium primi numeri in seipsum multiplicari, cuius dimidij quadratum erit. 289. è quo si detrahas quadratum secundi numeri, nempe. 225. remanebit. 64. atq; huius si quadratam radicem sumas nempe. 8. quam dimidio primi numeri, nempe. 17. coniungas, dabitur duorum quadratorum numerorum quęsitorum maior numerus. 25. hac deinde radice è dimidio detracta, minus quadratum dabitur. 9. scilicet, quorum radices. 5. et. 3. essent ij numeri, qui quæruntur.

Cuius speculationis gratia, cogitemus primum numerum, cui quadratorum summa æquari debet, significari linea. a. n. tum concipiamus quæsita quadrata significari, coniungiq; modo subscripto. t. b. k. secundum porrò numerum propositum, significari producto. d. b. Iam nil superest aliud quam vt quantitates. d. p. et. b. p. quæramus.

Itaque cum in linea. a. n. summæ quadratorum numerus detur, quadratum dimidij. o. a. sit. s. a. quod nobis erit cognitum; sit etiam. a. u. numerus quadrati maioris, et. u. n. minoris, et. a. z. productum vnius in alterum; qui quidem numerus. a. z. æqualis erit quadrato numeri. d. b. ex. 19. theoremate huius libri. Itaq; a. z. cognitum erit, cum eius radix. d. b. sit secũdus numerus propositus, quæ

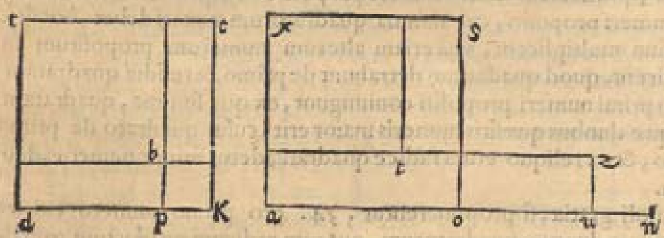


minor erit. a. s. ex quinta secundi, aut septima consequentia post. 16. noni Euclidis. Iam subtracta quantitate. z. a. è quadrato. a. s. cognoscetur quadratum. t. x. cuius radix æqualis erit. o. u. ex postremo adductis, Itaque cognoscemus. o. u. qui numerus coniunctus dimidio. o. a. cognito, dabit quadratum. a. u. cognitum, atque ita. u. n. pariter cognoscetur, & eorum radices consequenter.

Hoc

Hoc ipſum & alia ratione perfici poteſt, nempe, iuncta ſumma. k. b: b. d: et. b. t. alteri rectangulo æquali. b. d. quod ſit. b. c. ex quo totum quadratum lineæ. d. k. cognitum erit, atq; ita etiam conſequenter eius radicem. d. k. cognoſcemus, cuius ope ac producti. d. b. cognoſcemus. d. p. et. p. k. prout ex theoremate quadrageſimoquinto huius libri patebit.

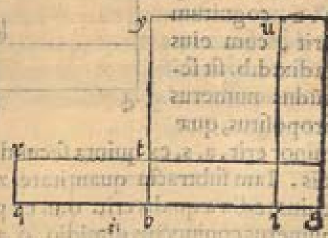
Michael Stifelius, vndecimo cap. tertij libri, problema eiuſmodi proponit, quod tamen ipſe via algebræ diſſoluit.




## THEOREMA XXXVIII.

CVR ij, qui duos numeros inuenire volunt, quorum productum alicui numero propoſito æquetur, & quadratorum eorundem differentia alteri numero propoſito æqualis ſit. Rectè dimidium ſecundi numeri propoſiti in ſeipſum multiplicent, cui quidem numero differentia quadratorum æquari debet; porrò huic quadrato primi propoſiti numeri, cui æquandum eſt productum numerorum quæſitorum, quadratum adiungant; tum radicem quadratam huius ſummæ copulet dimidio ſecundi numeri propoſiti, ei inquam, cui differentia quadratorum æqualis eſſe debet, ex quo quadratum maius conſurgit, à quo, detracto ſecundo numero, ſupereſt quadratum minus.

Exempli gratia, ſi proponeretur primo loco numerus. 8. cui æquandum eſt productum numerorum quæſitorum, tum proponeretur numerus. 12. cui, detracto minore à maiore, differentia quadratorum vtriuſque quæſiti numeri æqualis eſſe debet, oportet huius vltimi numeri. 12. dimidium in ſeipſum multiplicare, fieretque. 36. quadratum dimidij, vnde in ſummam colligeremus quadratum primi numeri. 8. quod eſſet. 64. quæ cum. 36. efficerent. 100. cuius centenarij radice, nempe. 10. collecta in ſummam cum dimidio ſecundi numeri, nempe. 6. daretur quadratum maius, nempe. 16. ex quo, detracto ſecundo numero, nempe. 12. remaneret quadratum minus. 4.




Cuius ſpeculationis cauſa, maius quadratum incognitum ſignificetur linea. q. g. minus verò pariter incognitum linea. g. i. quare. q. i. eorum differentia, tanquam data remanebit cognita, vnà etiam. b. i. et. q. b. ſua dimidia; tunc cogitetur quadratum. y. g. ſuper. b. g. et parallelogrãmum rectangulum. g. r. deſignatum, & ita etiam gnomon. u. g. t. prout ſexta ſecundi Euclidis proponitur, ex quo quadratum. b. i. nempe. u. t. cognitum erit, ſed gnomon æqualis eſt rectangulo. g. r. ex prædicta, aut ex. 8. poſt. 16. noni,

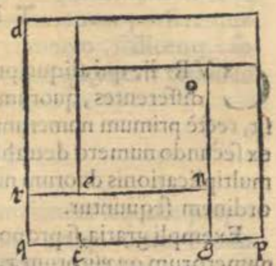
noni, hocq; rectangulum.g.r. quadratum est primi numeri propositi ex.19. theoremate huius libri, itaq; cognitum erit. vnà etiam gnomon. u. g. t. cognoscetur; quare totum quadratum. g. y. eiusq; radix. b. g.manifesta erit; cui coniuncta. q.b. data, maius quadratum. q.g.cognoscetur, ex qua.b. g.detracta. b.i.data, cognoscetur.i.g.quadratum minus consequenter, etiam eorum radices notæ erunt.

## THEOREMA XXXIX.

ALIA etiam ratione idipsum definiri potest, prætermissa antiquorum via, nempe multiplicatis in semetipsis primo & secundo, numeris propositis, quadruplicatoq; quadrato primi, qua summa coniuncta cum quadrato secundi numeri, & ex hac altera summa eruta radice quadrata, ex qua detracto secundo numero, & è reliquo sumpto dimidio, quod erit quadratũ minus, quo detracto ex radice postremo iuncta, supererit quadratum maius.

Exempli gratia, si proponeretur numerus.8. cui productum duorum numerorum quæsitorum æquandum est, proponeretur idem. 12. cui differentia quadratorum duorum numerorum æqualis esse debet. Iubeo primum numerum, nempe. 8.in se ipsum multiplicari, ex quo exurget. 64. pro numero sui quadrati, quod quadruplicari volo, eritq; productum. 256.quod censeo coniũgendum cum quadrato secundi numeri propositi, nempe. 144. eritq; summa.400.ex qua sumetur radix, scilicet. 20. & ex hac detrahetur secundus numerus. 12. residuiq; dimidium, nempe. 4. pro quadrato minore, quo in summam collecto cum. 12. dabit quadratum maius. 16.

Cuius speculationis causa, quadratum maius per lineam.q.g. minus per. g. p. significetur: super integram autem.q.p.erigatur quadratum integrum. d.p. diuisum, vt quadratum. f.g.vigesimiseptimi theorematis huius libri, (idipsum accideret diuiso quadrato modo octauæ secundi Euclidis) quæ quidem diuisio, est via quatuor productorum.q.g. in.g.p.è quibus vnum sit.g. r.quod erit cognitum ex. 19. theoremate cum sit quadratũ primi numeri ꝓpositi, ex quo illa quatuor cognita erũt. Iam verò si cogitemus.q.p.sectam in puncto. t. ita vt.q.t.æqualis sit. p.g.dabitur differentia.t.g.cognita, vt radix quadrati. e.o.cum ex præsupposito.r.n. æqualis sit.q.g. et.r.e:g.p.ex quo etiam. q.t. ita pariter.e.n.t.g.æqualis erit. Collecto itaq; quadrato. e. o. ipsius. t.g.cum quadruplo. g. r: cognitum erit quadratum. d.p.ipsius.q.p.quare cognoscetur. q. p. de quo numero detracta differẽtia quadratorum cognita. t.g.supererit aggregatum.p.g.et.q.t.cognitum. Quare ex consequenti, dimidium aggregati, nempe. g. p. cognoscetur, tanquam minus duorum quadratorum. cui iuncta. g.t. aut detracta.p.g.ex.p. q.quadratum. q. g.maius cognitum remanebit.



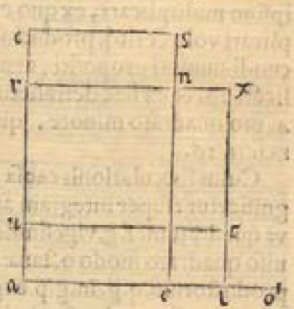

## THEOREMA XL.

CVR ijs, qui volunt duos eiusmodi numeros inuenire, vt eorum maior minorem, numero proposito superet, & productum vnius in alterum, alteri numero proposito adęquetur, consultissimum sit dimidium primi numeri propositi;

numerum inquam, cui differentia duorum quæſitorum æquanda eſt, in ſeipſum multiplicare, atque huic quadrato, ſecundum numerum propoſitum iungere, cui, productum numerorum quæſitorum æquale eſſe debet, & ex hac ſumma eruere quadratam radicem, quæ coniuncta dimidio primi numeri propoſiti, dabit maiorem duorum numerorum & ex eadem radice detracto dimidio primi numeri, minorem numerum duorum quæſitorum.

Exempli gratia, ſi proponeretur. 12. cui differentia vnius numeri ab altero æquari deberet, tum proponeretur. 64. cui productum multiplicationis duorum quæſitorum ſimul æquãdum eſſet. Dimidium primi numeri in ſeipſum multiplicaremus, proueniretq; quadratũ. 36. cui coniuncto ſecundo, nempe. 64. totum eſſet. 100. ex quo detracta quadrata radice. 10. et ipſi coniuncto ſenario, dimidio primi numeri, & ex eadem detracto eodem dimidio. 6. pro maiore numero proueniret. 16. & pro minore. 4.

Cuius rei ſpeculatio hæc eſt. Sit. e. o. differentia cognita duorum incognitorum numerorum. a. o. et. a. e. quorum productum datum ſiue cognitum ſit. a. s: conſideremus nunc. e. i. dimidium. e. o. datæ differentiæ, & ex compoſito. a. i. imaginetur quadratum. a. x. in quo protracta ſit. t. u. æquidiſtans lateri. a. i. & tam ab ipſa. a. i. remota, quam. x. i. ab. s. c. vnde. t. c. quadratum erit. e. i. dimidiæ ſcilicet differentiæ datæ, e. o. et. t. n. rectangulum æquale erit rectangulo. n. c. vt cuilibet licet per ſe conſiderare, vnde ſequitur gnomonem. e. r. t. æqualem eſſe producto. a. s. ideo cognitus, qui quidē gnomon, ſi coniunctus fuerit quadrato. e. t. cognito ex radice. e. i. cognita (vt dimidia totalis differentię. e. o. datæ) habebimus quadratum totale. a. x. cognitum, & ita eius radicem. a. i. cognitam & reliqua omnia conſequenter quæ quidem ſpeculatio eadem eſt quæ. 6. ſecundi ſeu. 8. noni Euclidis.




Poteris tamen ex modo & rationibus præcedenti theoremate allatis, hoc ipſum concludere.

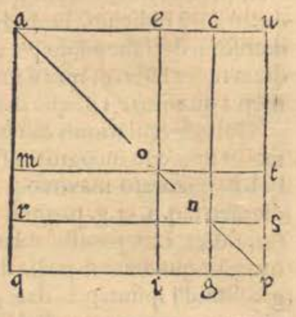
## THEOREMA XLI.

CVR ij, qui aliquo propoſito numero, inuenturi ſunt duos numeros inter ſe differentes, quorum quadratorum ſumma altero numero propoſito æqualis ſit, rectè primum numerum propoſitum in ſeipſum multiplicant, quod quadratum ex ſecundo numero detrahũt, & dimidium reſidui ſumunt, quod productum erit multiplicationis duorum numerorum inter ſe, in reliquis præcedentis theorematis ordinem ſequuntur.

Exempli gratia, ſi proponeretur. 12. tanquam numerus, cui differentia duorum numerorum quæſitorum æquanda eſt, proponerentur præterea. 272. quibus ſumma quadratorum duorum numerorum quæſitorum æquari deberet, oporteret ſanè primum numerum, nempe. 12. in ſeipſum multiplicare, cuius quadratũ hoc loco eſſet. 144. atque hoc detrahere ex ſecundo numero, ſupereſſet. 128. ſumpto deinde dimidio hujuſce numeri, nẽpe. 64. producto in quam duorum numerorum quæſitorũ. Cum hoc. 64. poſtea et duodenario primo propoſito numero, præcedentis theorematis ordinem ſequeremur.

Quod

Quod vt ſpeculemus, conſideremus ſubſcriptam figuram, vigeſiminoni theorematis figuræ ſimilem, in qua numeri quæſiti duabus lineis directè coniunctis. q.g. et.g.p.ſignificentur, horũ quadrata erũt.r.c.et.g.s.quorũ sũma iterũ proponitur, quare etiam cognita. Differẽtia autem duorũ numerorum primo propoſita ſit. q. i. eius verò quadratum.m. e. quod cognitum eſt ex ſua radice. q. i. quare gnomon.c.n.m. ſimul cum quadrato minori. g. s. cognitus erit, quæ ſumma æqualis eſt duplo.g.r. producto datorum numerorum. Itaque & ipſa. g. r.cognoſcetur, nunc ſi præcedentis theorematis ſpeculationem in reliquis conſuluerimus propoſitum conſequemur.




## THEOREMA XLII.

ADhuc etiam & alia ratione idipſum conſequi poſſemus, non conſulto quadrageſimo theoremate. Nam ſubtracto quadrato differentiæ, numeri primi (inquã) propoſiti, ex sũma duorum quadratorum, nempe ex ſecundo numero propoſito colligendum eſſet reſiduum in ſummam cum prædicto ſecundo numero, & ex ſumma hac deſumenda quadrata radix, quæ duorum numerorum ſumma erit, de qua detracto primo numero, remanebit duplum minoris numeri quæſiti, cuius dimidio addito primo numero propoſito, aut detracto minore inuento ex radice poſtremo inuenta, dabitur numerus maior, qui quæritur.

Exempli gratia, cum ſuperfuerint. 128. hæc ſi cum ſecundo numero nẽpe. 272. iunxerimus, dabunt. 400. quorum radix erit. 20. de quo numero detracto primo propoſito, nempe. 12. ſupererunt. 8. quorum dimidiũ erit. 4. quo ex. 20. detracto aut coniuncto. 12. maior numerus orietur.

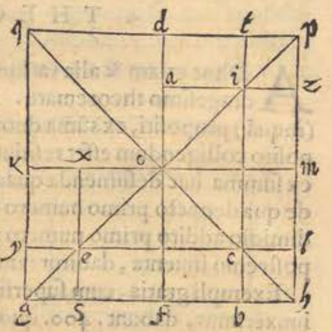
Cuius rei contemplatio, præcedenti figura aperitur. Nam reſiduum detractionis quadrati. m. e. ex ſumma duorũ quadratorum. r. c.et.g.s.numerum præbet æqualem duobus ſupplementis. q. n. et.n.u.ex. 8. ſecundi Euclidis. qui coniunctus duobus quadratis (quorum ſumma ſecundo propoſita fuit) cognitionem profert quadrati. q.u. & eius radicis.q.p.de qua, detracto primo dato numero, ſcilicet.q. i.ſupereſt. i.p.cuius dimidium nempe.g.p. minor eſt numerus qui quęritur; reſiduum verò totius.g.q.maior ſcilicet.

## THEOREMA XLIII.

CVR ij, qui volunt duos numeros inuenire, quorum ſumma æqualis propoſito alicui numero futura ſit, & ſumma quadratorum maior eorum producto per quantitatem alterius propoſiti numeri, rectè dimidium primi dati numeri in ſeipſum multiplicant, quod quadratum ex ſecũdo dato numero detrahunt, ſumuntque tertię partis reſidui quadratam radicem, quam dimidio primi numeri coniungunt, ex quo maior numerus duorũ quæſitorũ datur, quo ex toto primo detracto, ſupererit minor.

Exempli gratia, propoſito numero. 20. cui æquanda eſt ſumma duorum numerorum quæſitorum, datoq; ſecundo numero. 208. qui ſemper maior eſſe debet quadrato dimidij,prout ex ſpeculatione huiuſmodi operis cognoſcetur, cui æquãda eſt differẽtia inter ſummã quadratorũ duorũ qui quærũtur numerorũ, ſimul cũ pro ducto eorũ radicum. Dimidium numeri. 20. in ſeipſum multiplicandum eſſet,quadratumq; detrahendum ex.208. vt remanerent.108.quorum.108.tertiæ partis quadrata radix eſſet. 6. quæ ſi iuncta fuerit dimidio. 20.nempe. 10. daretur maior numerus quæſitus. 16. quo detracto è. 20.darentur. 4.

Cuius ſpeculationis cauſa, datus primus numerus ſignificetur linea. g. h. in qua maior numerus incognitus ſit. g.h.minor verò.b.h. quorum quadrata ſint. y. t. et. b. l. in quadrato maximo.g.p.tùm productum. g.b.in.b.h.ſit.g.c. cogitenturq; duo diametri. q.h. et.g. p. diuiſi per medium in puncto. o. per quod duę lineæ ducantur. f.d.et. k.m. parallelæ lateribus maximi quadrati. Hæ dictum quadratum in quatuor quadrata æqualia diuident, quorum vnumquodq;, æquale erit quadrato. g.f.dimidij ipſius. g.h. datę ,quare eorum vnumquodq; cognitum erit. Iterum cogitemus. s.x.per.e.parallelã.g.k.tantum diſtantem à.g.k. quantum.y. l.ab. g. h. diſtare inuenitur. Cogitetur pariter. z. i. a. per punctum. i. parallela.d.p. quare.a.t.æqualis erit.f.c. et. y. x. æqualis. f.e. et.y.s:b.l. æqualis. Ita ſubtractis è duobus quadratis ſuperius dictis. a. t.y.x.et.b.l. producto.y.b.æqualibus,ſupererunt. k.d.et.a.c. x. cognita, tanquam æqualia dato ſecundo numero, ſed.k.d. quadratum eſt medietatis. g. f. cognitæ,cognoſcetur igitur reſiduum. a.c.x.vnà etiam ſingulæ tertiæ partes nempe quadrata. o. i. o.c. et. o.e. & radix. b. f. vel. f. s. ſingularum, qua coniuncta dimidio.g.f.rurſusq; ab eodẽ detracta, propoſitum conſequemur.




## THEOREMA XLIIII.

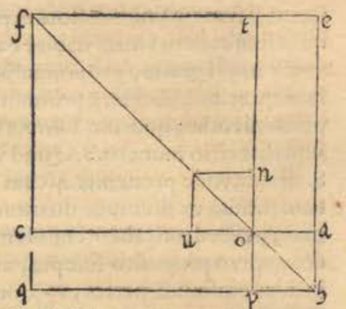
CVR ſi quis cupiat numerum propoſitum in duas eiuſmodi partes diuidere, vt quadratum maioris, quadratum minoris ſuperet quantitate alterius numeri propoſiti, rectè primum numerum in ſeipſum multiplicabit, & ab eodem ſecundum numerum detrahet, reſiduum verò per duplum primi diuidet, ex quo proueniens primi pars minor erit, quæ ex illo primo detracta, partem maiorem proferet.

Exempli gratia,ſi proponantur. 20. diuiſa in duas eiuſmodi partes, vt quadratũ maioris ſuperet quadratum minoris numero æquali ipſi. 240. oportebit primum numerum, qui quadratus cum fuerit, erit. 400. in ſeipſum multiplicare, & ex hoc quadrato ſecundum numerum nempe. 240.detrahere, tunc remanebunt. 160.quę diuiſa per. 40.numerũ duplũ primo, dabuntur quatuor pro minori numero, à reſiduo verò. 20.detractis quatuor, erunt. 16. pro maiori numero.

Quod vt exactè conſideremus, primus numerus propoſitus ſignificetur linea. q. h. diuidendus in duas partes. q.p.et.p.h.tales quales quærimus. Poſtmodum erigatur quadratum.q.e.diuiſum diametro. f. h. ductiſq;.p.o. t.et.a.o.c.parallelis lateribus quadrati, dabuntur imaginaria quadrata.c.t. et. p.a. duarum partium. q.p.et.p. h.incognitarum. Ad hæc cogitemus quadratum.u.n. æquale quadrato.p.a.è quadra

to

to maiore. c.t. extractum quare reſiduum quadrati.c.p.cognitum erit,quam quantitatem cognitam, cum ſit ſecundo loco data, cogitemus detrahi è toto quadrato cognito. q. c. ex quo ſumma duorum ſupplementorum. q. o. et. o. e. cognoſcetur, vnà cum quadratis. u.n.et.p.a.du plo ſcilicet.q. a.quo diuiſo per duplum.q.h.aut ſimplex.q.a.per. q. h. ſimplicem, dabitur. a. h. nempe.p.h. minor numerus quæſitus.

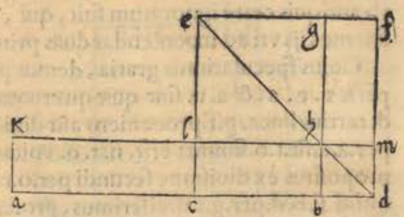



## THEOREMA XLV.

CVR volentes diuidere numerum propoſitum in duas eiuſmodi partes,vt pro ductum vnius in alteram, alteri numero propoſito æquetur,rectè dimidium primi dati numeri in ſeipſum multiplicant, ex quo quadrato ſecundum datum numerum detrahunt, reſiduiq́; radicem ſumunt, qua coniuncta vni dimidio primi numeri, pars maior datur,ex altero verò dimidio detracta, minorem manifeſtabit.

Exempli gratia,ſi numerus partiendus eſſet. 34.alter verò numerus eſſet.64. cui productum vnius partis in alteram æquale eſſe deberet.Dimidium primi numeri,in ſeipſum multiplicaremus, cuius quadratum eſſet. 289.de quo detracto ſecundo numero nempe. 64.remaneret.225.cuius quadrata radix nempe. 15. coniuncta. 17. dimidio.34.proferet.32.maiorem partem, detractoq́; ex. 17.ſupereſſet.2. pars inquam minor.

Cuius ſpeculationis cauſa, primus numerus propoſitus ſignificetur linea. a. d.cuius dimidium.c. d.cognitum erit, vnà etiam eius quadratum.c.f.quo diuiſo per dia metrum. e.d.ſupponantur partes ignotæ ipſius.a.d. eſſe.a.b.et.b.d. & à puncto.b. duci lineam.b.h.g.parallelam.d.f.et. m. h.k.parallelam.d.a. extructa figura ſimi li figuræ quintæ ſecundi Eucli. quare da bitur gnomó.l. d.g.æqualis producto.b. k.& proinde cognitus, quo detracto è quadrato,c.f.remanebit quadratum.g.l. cuius radice æquali. c.b. coniuncta. a. c. & detracta ex.c.d.partes. a.b. et.b.d.quæſitæ dabuntur.



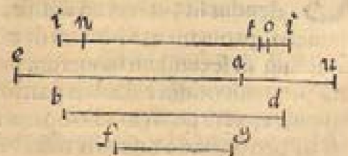

## THEOREMA XLVI.

CVR propoſitis tribus numeris, quorum prior in duas eiuſmodi partes diuidendus ſit, ut mutuò diuiſæ, & per ſummam prouenientium diuiſo ſecundo numero, proueniens vltimum ſit æquale tertio numerorum propoſitorum. Conſul tiſsimum ſit ſecundum numerum per tertium diuidere, ex quo proueniens ſit ſumma prouenientium è duabus partibus mutuò diuiſis, quam ſummam ſi quis velit diſtinguere,rectè poſſit medio operationis prçcedẽtis theorematis ſũpta vnitate ſuper ficiali pro ſecundo numero diſtinctis poſtmodum prouenientibus, rectè meo iudicio operabimur per regulã de tribus ( quod fuit ab antiquis prætermiſſum ) Si dixerimus,

rimus,ſi ſumma vnius dictorum prouenientium cum vnitate dat primum numerum, quid ipſa eadem vnitas dabit? ex quo propoſitum oriatur.

Exempli gratia, proponuntur tres numeri, primus. 20.ſecundus. 34. tertius. 8. Iam quærimus diuidere primum. 20.in duas partes quæ mutuò diuiſæ prębeant duo prouenientia, quorum ſumma tanta ſit vt per eam diuiſo. 34. proueniat numerus æqualis tertio numero.8.Quod vt præſtemus iubet regula ſecundum.34.per tertiũ. 8. diuidi,vnde proueniet.4.cum vna quarta parte,quod prouenìens erit ſumma prouenientium ex diuiſione duarum partium quæſitarum, quæ ſi diſtinguere voluerimus, præcedentis theorematis methodum ſequemur, vnitate ſuperficiali pro ſecundo numero propoſito ſumpta, ac ſi diceremus, diuidatur.4. cum vna quarta parte in duas eiuſmodi partes, vt productum vnius in alteram ſit vnitas ſuperficialis, certè fractis integris cum quarta parte coniungendis, darentur vnitatis decemſeptem quartæ lineares, verum cum neceſſe ſit, ex præcedenti theoremate, dimidium in ſeipſum multiplicare, eſſetq; dimidium. 8.quartarum partium cum octaua, commodius totum conſtituetur.34.octauarum, quarum dimidium,nempe decemſeptem octauæ, in ſeipſum multiplicatum erunt.289.ſexageſimæ quartæ vnius integri ſuperficialis, quandoquidem integrũ ſuperficiale, cuius vnitas linearis in.8. partes diuiditur eſt.64.vt ex primo theoremate huius libri depræhendi poteſt. Nunc vnitate hac ſuperficiali, nempe.64.ex.289. detracta, ſupererit. 225. cuius radix quadrata, ſcilicet. 15. coniuncta dimidio dictorum prouenientium, nempe. 17. dabit maius proueniens.32. detractaq; ex altero dimidio,dabit proueniens minus.2.hoc eſt pro maiore proueniente. 32. octauas, & pro minore duas, quatuor ſcilicet integros pro maiore, & quartam partem vnius integri pro minore. Nunc ſi ex regula de tribus dixerimus, ſi.4. iuncta vni, nempe. 5.dant.20.primum numerum, quid dabunt. 4.integra (proueniens inquam maius) dabũt certè. 16. partem maiorem. Tum ſi dixerimus, ſi quarta pars coniuncta vnitati dat. 20: quid dabit quarta illa pars(hoc eſt proueniens minus) dabit ꝑfectò quatuor ſcilicet minorẽ partem,quod ab antiquis certè ignoratum fuit, qui, inuentis prouenientibus quieuerunt, neſcientes ijs vti ad inueniendas duas primi numeri partes.

Cuius ſpeculationis gratia, demus primum numerum ſignificari linea. e.u.cuius partes. e. a. & a. u. ſint quæ quæruntur, alter verò numerus ſignificetur linea. b. d. tertius linea. g.f.proueniens aũt diuiſionis.e.a.per.a.u.ſit.n.t. diuiſionis aũt. a. u. per.a.e.ſit.t.o.ſumma erit. n. t. o. vnitas verò.n.i.et.o. i. Iam ſi numerus. f. g. tertiò propoſitus ex diuiſione ſecundi per.o.t.n.proferri debet.Ex. 13.theoremate patet, quòd ſi.b.d.per.g.f.diuiſerimus, proferetur.o.t.n. qui cum fuerit inuentus, ſummã eſſe oportet duorũ prouenientiũ, ex diuiſione mutua duorũ numerorum, nempe. a. e. per.a.u.et.a.u.per.a.e.deinde manifeſtum eſt ex. 24. aut.25.theoremate corũ productum (multiplicatis prouenientibus adinuicem) vnitatem ſuperficialem futuram eſſe. Hactenus igitur, totum. o.n. ex doctrina præcedentis theorematis diuiditur in puncto.t.ita vt productum.o.t.in.t.n. ſolam vnitatem ſuperficialem cõtineat, quo facto, ſi,vt antedictum eſt,cogitauerimus. n. t. proueniẽs eſſe ex diuiſione.e.a.per.a. u.et. t.o.proueniens ex diuiſione.a.u.per.a.e. patebit ex definitione diuiſionis, quod eadem erit proportio.a.e.ad.n.t.quæ eſt.a.u.ad vnitatem.n.i. et. a.u.ad.o.t. eadem quæ eſt.e.a.



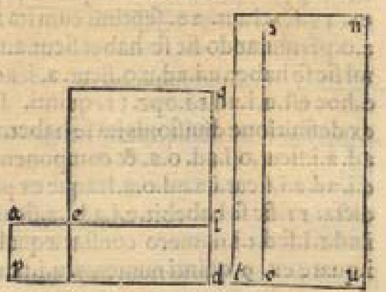
ad vnitatem.o.i.permutandoq;.e.a.ad.a.u.ficut.t.n. ad.n. i.& componendo. e. a.u. ad a.u. ficut.t.n.i.ad.n. i: & euerfim.e.a.u.ad.e.a.vt.t.n.i.ad.t.n. Quare, ex.20.fepti mi, recte vtimur regula de tribus. Idem & de altera parte dico,quamuis qui vnam teneat, alteram quoque habiturus fit. Non mirum tamen fi huiufmodi problema ab antiquis definitum non fuerit, qui hanc vltimam partem non cognouerunt.

## THEOREMA XLVII.

CVR duobus numeris mutuò diuifis, fi per fummam prouenientium, productum vnius in alterum multiplicetur, vltimum productum, fummæ quadratorum duorum numerorum æquale futurum fit.

Exempli gratia, propofitis. 16. et. 4.mutuò diuifis, fumma prouenientium erit. 4. integrorum cum quarta parte, qua fumma multiplicata cum producto primorū numerorum, nempe. 64.dabuntur.272.integri fuperficiales, qui fummæ quadratorum duorum numerorum æquantur.

Hoc vt confideremus,duo numeri partibus. a.e. et.e.i.in linea. a.i. fignificentur, quorum productum fit.e.d.& quadratū ipfius.a.e.fit.e.p:ipfius verò.e.i.fit.e. q.proueniens aūt ex diuifione.e.i.per.a.e.fit.o.u.proueniens aūt.a.e.per.e.i.fit.o.t. quorum fumma fit. o.u.t.tum productum. e.d: linea.u.n. fignificetur ad angulum rectū coniuncta in puncto. u. extremo ipfius. o.u.t.productum aūt.u.o.t.in.u.n.fit.n.t.Iam probandum nobis eft.n.t. æqualem effe fummæ duorum quadratorum.q.e.p.Quod fingillatim probo, & affero productum.o.n.æquale effe quadrato.q.e.& productū. s.t. quadrato.e. p. Nam ex. 35. theoremate patet numerum. e.i.medium effe proportionalē inter.e.d.et.o.u: cum numerus.e.i.ex præfuppofito ab. e.a.multiplicetur & diuidatur, cuius multiplicationis productum eft. d.e:nempe.u. n. & proueniens ex diuifione eft. o.u:quare ex dicto theoremate.e i.media proportionalis eft inter. u.n.et. u.o. Itaq; productum.o.n.æquale eft quadrato.e.q.ex. 16.fexti vel.20.feptimi.Idem dico de producto. s.t.nēpe æquale effe quadrato.e.p.quandoquidem numerus. a. e. ab e.i.multiplicatur ac diuiditur, cuius multiplicationis productum eft.d.e nempe o.s.& proueniens ex diuifione. o. t: inter quæ ex. 35. theoremate.a. e. media proportionalis eft.Quare ex allatis propofitionibus productū.s.t.æquale eft quadrato.e.p.fed totū productum.n.t.fumma eft duorum productorum.o.n.et.s.t. ex prima fecundi Eucli. Itaque verum effe quod dictum eft, confequitur.



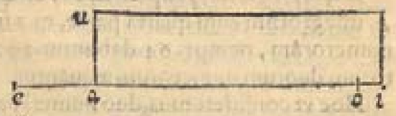
## THEOREMA XLVIII.

CVR fi quis maiorem duorum numerorum fola vnitate inter fe differentium, per minorem diuidat, maioremq; per proueniens multiplicet, productum, fūmæ ipfius maioris cum eodem proueniente æquale erit.

Exempli gratia.10.per. 9. diuifo, datur vnum cum nona parte, quo multiplicato per proueniens,ipfo nempe.10:datur productum.11.cum nona parte, tantum fcilicet

licet quarta ſumma eſt maioris cum proueniente.

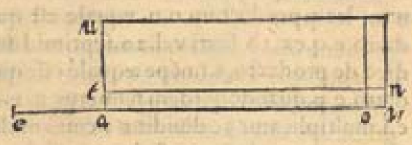
Cuius ſpeculationis cauſa, maior numerus ſignificetur. a.i. et minor linea. a.o. ex quo ex præſuppoſito. o.i. vnitas erit. Sit autem proueniens ex diuiſione. a.i. per. a.o. a.e: quod. e.a. directè coniungatur ipſi. a.i. et productum. a.i. in. a.e. ſit. u.i. Probabo numerum ſuperficialem. u.i. æqualem eſſe lineari. i.a.e. quare meminiſſe oportet, decimotertio theoremate probatum fuiſſe, quòd ſi numerus diuiſibilis per prouenięns diuidatur, proueniens futurus ſit numerus diuidens, quare. a.o. erit proueniens ex diuiſione. a.i. per. a.e. & ex definitione diuiſionis ita ſe habebit. e.a. ad. a.i. ſicut. o.i. ad. o.a. & componendo ita. e.i. ad. a.i. ſicut. i.a. ad. o.a. quare. a.i. erit media ꝓportionalis inter. e.i. et. a.o. ſed. a.i. non modò diuiſa nũc cogitatur ab. e.a. ex quo ſit proueniens. a.o. ſed etiam per eandem. e.a. multiplicata, ex quo productum oriatur. u.i. Itaq; ex. 25. theoremate. a.i. media eſt proportionalis inter. u.i. et. a.o. Quare. ex. 11. quinti. eadem erit proportio. u.i. ad. a.i. ſicut. e.i. ad eandem. a.i. Igitur ex. 9. prædicti numerus. u.i. æqualis erit numero. e.i. quod erat propoſitum.




## THEOREMA XLIX.

IDipſum etiam alia ratione conſiderari poteſt.

Linea. u.a. ſecetur in puncto. t. ita vt. a.t. æqualis ſit vnitati. o.i. & media parallela. t.n. terminetur productum. t.i. quod conſtabit æquali numero, quamuis ſuperficiali, numero. a.i. tametſi lineari. Tum parallela ducatur à puncto. o. ipſi. a.u. termineturq; productum. o.u. ex quo bina producta dabuntur. u.o. et. t.i. inter ſe æqualia ex. 15. ſexti aut. 20. ſeptimi cum ita ſe habeat. a.i. ad. a.u. ſicut. a.o. ad. a.t. ſed. a.i. ad. a.o. permutando ſic ſe habet ſicut. a.u. ad. a.t. & ex prima ſexti aut. 18. vel. 19. ſeptimi ſic ſe habet. u.i. ad. u.o. ſicut. a.i. ad. a.o. hoc eſt. u.i. ad. t.i. ope. 11. quinti. Iam ex definitione diuiſionis ita ſe habet. a.e. ad. a.i. ſicut. o.i. ad. o.a. & componendo. e.i. ad. a.i. ſicut. i.a. ad. o.a. Itaque ex prædicta. 11. ſic ſe habebit. e.i. ad. i.a. ſicut. u.i. ad. t.i. ſed. t.i. numero conſtat æquali. a.i. quare ex. 9. quinti numerus. u.i. numero. e.i. æqualis erit.



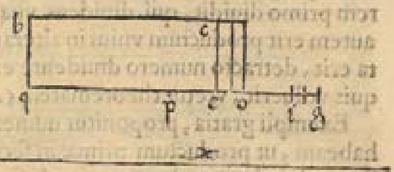

## THEOREMA L.

CVR diuidentes numerum propoſitum in duas eiuſmodi partes, vt productũ vnius in alteram cum ipſarum differentia in ſummam collectum, æquale ſit alicui alteri numero maiori primo. Rectè primum ex ſecundo detrahunt, reſiduum verò conſeruant, tum ex primo ſemper binarium deſumunt, dimidiumq; conſeruant, alterum verò dimidium in ſeipſo multiplicant, & ex quadrato numerum conſeruatum eruunt, reſiduiq; radicem ex dimidio conſeruato, quod vltimum reſiduum propoſiti numeri quæſita pars minor eſt.

Exempli gratia, ſi proponatur numerus. 20. ita diuidẽdus, vt productũ vnius partis in alteram, cum partium differentia collectum in ſummam, æquale ſit propoſito nume-

numerò, verbi gratia. 92. præcepit regula detrahi primum numerum ex ſecundo, nempe. 20. ex. 92. cuius reſiduum, ſcilicet. 72. conſeruetur, tum detrahi iubet binarium ex primo, ſic in propoſito exemplo remanebunt. 18. huius autem. 18. dimidium in ſeipſum multiplicari iubet, quod cum ſit. 9. datur numerus. 81. ex quo. 81. primum numerum conſeruatum, nempe. 72. vult regula detrahi, ſic remanebit. 9. tum huius. 9. quadrata radix detrahenda eſt ex dimidio ipſius. 18. quod fuit ante quadratum, ſic ſupererit. 6. hoc eſt. 9. excepta radice quadrata, qui. 6. erit minor pars quæſita, maior verò. 14. quarum productum. 84. coniunctum cum partium differentia præbet exactè. 92.

Cuius rei hæc eſt ſpeculatio. Primus numerus minor, qui proponitur diuiſibilis ſignificetur linea. q. g. maior vero linea. x. tum cogitemus. q. g. diuiſam, cuius maior pars ſit. q. o. minor. o. g. differentia. q. p. ex quo. p. o. æqualis erit. o. g. ſit autem productum. b. o. Oportet igitur, ut. b. o. ſimul cum differentia. q. p. æquale ſit numero. x. ſecundò propoſito, qui notus eſt, quare etiam ſumma producti. b. o. cum differentia q. p. cognita erit, ex qua detracto primo numero. q. g. reſiduum cognitum erit, nunc igitur quodnam erit hoc reſiduum? attendamus qua ratione ex ſumma. b. o. et. q. p. detrahenda ſit. q. g. In primis ſi ſubtraxerimus ex dicta ſumma. q. p. quę pars eſt. q. g. ſupererit detrahenda. p. g. ex. b o. pars inquam ipſius. q. g. quod fiet quotieſcunque cogitauerimus. q. o. duabus vnitatibus diminutam, et per. o. g. multiplicatam, ſit autem productum. b. e. nam cum. o. g. toties. b. o. ingrediatur, quot ſunt in. q. o. vnitates ex prima ſexti aut. 18. vel. 19. ſeptimi, detrahendaq́; ſit. p. g. ex. b. o. quæ. p. g. dupla eſt. o. g. patebit. o. c. æqualem eſſe. p. g. ſupererit itaque. b. e. productum. q. e. in. e. i. cognitum, erutis autem ex. q. g. ijſdem duabus vnitatibus, remanebit. q. i. nobis nota, ex quo. e. i. æqualis erit. e. c. Cum igitur productum. q. e. in. e. i. cognoſcamus ſimul cum. q. i: Si voluerimus partes. q. e. et. e. i. cognoſcere, vtemur. 45. theoremate huius libri, & propoſitum obtinebimus, nam cognoſcemus. e. i. & ex conſequenti. o. g. eius æqualem.




## THEOREMA L I.

DIVIDERE numerum in duas eiuſmodi partes, quæ pro medio proportionali alterum numerum propoſitum recipiant, primi dimidio minorem, aliud nihil eſt, quàm binas primi numeri partes inuenire, quæ inter ſe multiplicatæ quadrato ſecundi numeri numerum æqualem proferant, ex. 16. ſexti aut. 20. ſeptimi, quod tamen. 45. theoremate fuit à nobis ſpeculatum.

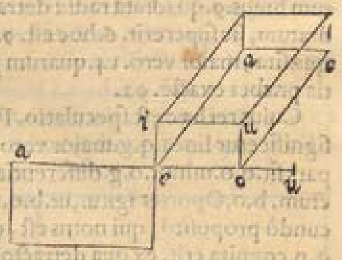
## THEOREMA L I I.

CVR propoſitis tribus numeris quibuſcunque, ſi productum primi in ſecundum per tertium multiplicetur, atque ſecundum hoc productum corporeū, per primum numerum diuidatur, proueniens erit numerus æqualis producto ſecundi in tertium.

Exempli cauſa, proponantur hi tres numeri. 10. 11. 12. multiplicenturq́;. 10. cū.

11. dabuntur. 110. quo producto multiplicato cum. 12. dabuntur. 1320. hoc pro ueniens per primum nempe. 10. diuisum dabit. 132. numerum æqualem producto secundi in tertium numerorum propositorum, scilicet. 132.

Hoc vt speculemur, primus numerus significetur linea. o. u. secundus. e. o. tertius. e. a. productum verò. o. u. in. o. e. sit. o. i. ipsius ve rò. o. i. per. e. a. productū corporeū sit. i. c. tum productū. e. o. in. e. a. sit. e. c. Dico nūc quod diuiso numero corporeo. i. c. per primū. o. u. proueniens æquale erit numero producti. e. c. Quare in primis cogitandum est, quod cum productum. i. c. ortum fuerit ex multiplicatione. o. i. in. e. a: dictum. o. i. toties ingreditur. i. c. quoties vnitas reperitur in. e. a. eadem ratione, toties. e. c. in. i. c. quot vnitates erunt in. o. u. Itaq; sequitur quòd diuiso. i. c. per o. u. proueniens sit e. c. corporeum, æquale nihilominus producto. e. c. superficiali.



## THEOREMA LIII.

CVR diuidens propositum numerum in tres partes sic se habentes vt productum primi in secundam, in tertia multiplicatū, præbeat numerum alteri numero proposito æqualem. Rectè secundum numerum per quemcunque alium mino rem primo diuidit, qui diuidens vna erit ex tribus partibus quæsitis, proueniens autem erit productum vnius in alteram reliquarum duarum, quarum summa cogni ta erit, detracto numero diuidente ex primo dato, quam quidem si distinguere quis voluerit, vtetur theoremate. 45.

Exempli gratia, proponitur numerus. 20. in tres partes diuidendus, quæ sic se habeant, ut productum primæ in secundam in tertia multiplicatum det. 90. itaque sumenda erit pro prima vna pars ipsius. 20. quæcunque illa sit, verbi gratia. 2. qua secundus numerus, nempe. 90. diuidatur, dabitur igitur. 45. quod erit productum cæterarum partium inter se, quarum summa est. 18. quam summam si distinguere volueris in cęteris duabus partibus separatis, vteris. 45. theoremate, vt quàm citissimè quod cupis exequaris, erunt autem partes. 3. et. 15.

In cuius speculationis gratiam nihil aliud occurrit, quàm quod præcedenti theoremate, & superiore. 45. allatum est.

## THEOREMA LIIII.

DIVIDERE numerum in. 3. eiusmodi partes, vt quadratum vnius sit æquale producto reliquarum duarum inter se, idem omnino est cum. 51. theoremate. Nam qui sumet quamlibet partem propositi numeri, quæ tertia parte maior tamen non sit, residuumq; in duas tales partes diuiserit, vt prima sumpta, media proportio nalis sit ex probatione. 51. theoremate allata, propositum consequetur.

## THEOREMA LV.

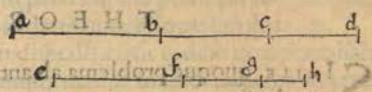
IDipsum alia ratione ab ea diuersa quā. 51. theoremate adduximus, pfici potest.

Suman-

Sumantur enim tres numeri continui proportionales, cuiuſcunque denique proportionalitatis, qui in ſummam colligantur, ac poſtmodum, regula de trib. dicamus. Si ſumma hæc primo numero propoſito in tres partes diuidendo reſpondet, cui reſpondebit vna ex tribus partibus huiuſcę ſūmæ? idem de reliquis duabus partibus dico.

Exempli gratia, ſi proponatur numerus. 57. diuidendus in tres continuas partes proportionales proportione ſeſquialtera, tres numeros in eiuſmodi proportionalitate diſtinctos ſumemus, vt potè. 4. 6. 9. qui in ſummam collecti dabunt ſummā. 19. dicemusq; ſi. 19. dant. 4. quid dabūt. 57? vnde proueniens vnius partis erit. 12. Tum ſi dicamus, ſi. 19. dat. 6. quid dabit. 57? nempe dabit. 18. Poſtremò, ſi. 19. dat. 9. quid dabit. 57? nempe. 26. atque ita dabitur. 18. cuius quadratum æquabitur producto reliquarum duarum partium inter ſe.

Quod vt ſciamus, numerus propoſitus in tres quaſlibet partes diuidendus ſignificetur linea. a. d. tres autem numeri dictæ proportionalitatis, lineis. e. f. f. g. et. g. h. directè inter ſe coniunctis denotentur. Cogitemus pariter lineam. d. a. in tres partes diuiſam. a. b: b. c. et. c. d. eadem cum cæteris proportionalitate, tunc eadem erit proportio. a. d. ad quamlibet ſuarum partium, quæ eſt. e. h. ad reſpondentem ipſius in. a. d: Verbi gratia reſpondentem. a. b. ipſi. e. f. et. b. c: f. g. et. c. d: g. h. Dico enim quòd ita ſe habebit. a. d. ad. c. d. ſicut. e. h. ad. g. h. Nam cum ſic ſe habeat. a. b. ad. b. c. ſicut. e. f. ad. f. g. ex præſuppoſito, permutando ſic ſe habebit. a. b. ad. e. f. ſicut. b. c. ad. f. g. & eadem ratione ſic ſe habebit. c. d. ad. g. h. ſicut. b. c. ad. f. g. & cōſequenter ſicut. a. b. ad. e. f. ex quo ex. 13. quinti ſic ſe habebit tota. a. d. ad totam. e. h. ſicut. c. d. ad. g. h. aut. b. c. ad. f. g. aut. a. b. ad. e. f. permutando itaque propoſitum manifeſtum erit, ipſum autem productum. a. b. in. c. b. æquale erit quadrato. b. c. ex. 15. ſexti aut. 20. ſeptimi.




## THEOREMA LVI.

VETERES aliud quoque problema indeterminatum propoſuerunt, quod ex more ratione à me definietur, eſt autem eiuſmodi.

Quomodo propoſitus numerus in tres eiuſmodi partes diuidatur, vt quadratū vnius æquale ſit ſummæ quadratorum reliquarum duarum partium.

Hoc vt efficiamus tria quadrata ſeparata ſumamus, quorū vnū æquale ſit reliquis duobus; eorū autē radices in ſummam ſimul colligantur, tum regulam de tribus ſequemur, ratione præcedenti theoremate demonſtrata, & rectè vt infra docebimus, quod autem dico de quadratis, etiam de cubis, & quibuſuis dignitatibus aſſero.

Exempli gratia, ſi numerus diuiſibilis proponatur. 30. in tres eiuſmodi partes diuidendus, vt quadratum vnius æquale ſit ſummæ quadratorum reliquarum duarum partium, in primis radices trium quadratorum ſumemus, ſic quomodocunque ſe habentes, vt maius ipſorum ęquale ſit ſummæ reliquorum duorum, verbi gratia. 25, 16. et. 9. nempe. 5. 4. et. 3. quæ ſi colligantur in ſummam efficiunt. 12. Tum ex regula de tribus dicemus, ſi. 12. reſpondet. 30: cui. 5. radix maior reſpondebit? nempe. 12. cum dimidio.

Deinde ſi dixerimus ſi. 12. valet. 30. quid valebit. 4. radix media? nempe valebit. 10. tertia autem minor, 7. cum dimidio. Itaque tota ſumma erit. 30. & quadra-

 tum.

tum. 12. cum dimidio erit. 155. quod æquale erit summæ quadratorum duarum partium, nempe. 100. cum. 55.

Hoc vt demóstremus, numerus diuisibilis propositus significetur linea. a. d. & summa radicum, nostro modo sumptarum, linea. e. h. quarum prima & maior sit. e. f. secunda. f. g. tertia. g. h. cogitemus etiam lineam. a. d. ea ratione diuisam esse qua. e. h. patebit enim ex modo præcedentis theorematis vnamquanque partium. a. d. ita se habituram ad suum totum sicut se habent singulæ. e. h. ad suum. Quod ideo dico, vt intelligamus rectè nos dicere. Si. e. h. dat. a. d. ergo. e. f. dabit. a. b. atq; ita de cæteris. Quare permutando sic se habebit. a. b. ad. b. c. sicut. e. f. ad. f. g. idem dico de reliquis. Igitur ex. 18. sexti aut. 11. octaui, eadem erit proportio quadrati. a. b. ad quadratú. b. c. quæ quadrati. e. f. ad quadratum. f. g. tota enim sunt æqualia, cum eorum partes similes inter se sunt æquales. Idem dico de proportione quadrati. a. b. nempe ita se habere ad. c. d. sicut quadratum. e. f. ad quadratum. g. h. ex quo ex. 24. quinti proportio quadrati. a. b. ad summam quadratorum duarum partium. b. c. et. c. d. sic se habebit ut quadrati. e. f. ad summam quadratorum. f. g. et. g. h. At quadratum. e. f. æquale est summæ quadratorum. f. g. et. g. h. igitur sic etiam se habebit quadratum. a. b. nempe æquale quadratis. b. c. et. c. g. Idipsum de cæteris dignitatibus dices, vteris q; 21. theoremate huius libri.


## THEOREMA LVII.

SIMILE quoque problema ab antiquis indeterminatum proponitur, quod eiusmodi est.

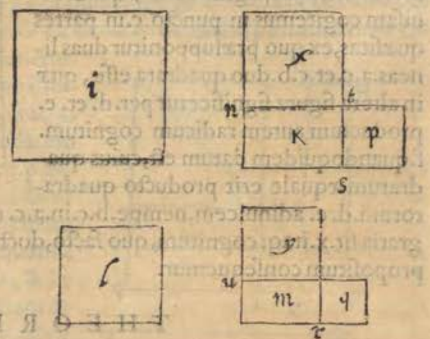
An numerus aliquis in tres eiusmodi partes diuidi possit, vt quadratum vnius æquale sit summæ quadratorum cæterarum duarum partium simul cum producto vnius in alteram.

Exempli gratia, si proponatur numerus. 50. vt iam dictum est diuidendus, repe riendus erit alius quilibet numerus, qui tamen summa sit trium radicum sic se habentium, vt quadratum vnius æquale sit summæ quadratorum duarum partium simul cum producto vnius in alteram, cum autem qui primò occurrit sumamus, vtpotè. 30. qui summa est numerorum. 6. 10. 14. partium sic se habentium, vt quadratum ipsius. 14. æquale sit summæ quadratorum cæterarum partium simul cum producto vnius in alteram, agamusque regula de tribus, ac dicamus, si. 30. valet. 50. quid valebit. 14. nempe. 23. cum tertia parte. Idem efficiemus in cæteris partibus, quarum vna erit. 16. cum duabus tertijs, altera verò. 10. absque fractis, ex quo quadratum primæ erit. 544. cum. 4. nonis, secundæ. 277. cum septem nonis, tertiæ. 100. & productum secundæ in tertiam. 166. cum. 6. nonis, quod productum, cum quadratis secundæ & tertiæ collectum erit. 544. cum. 4. nonis.

Huius rei speculatio eadem est, quę fuit præcedentis theorematis vsque quo noueris eandem proportionem esse quadrati. a. b. ad summam quadratorum. b. c. et. c. d. quæ quadrati. e. f. ad summam quadratorum. f. g. et. g. h. Sed cum hic non demus quadratum. e. f. æquale summæ quadratorum. f. g. et. g. h. sed maius ex producto. g. h. in. f. g. aut quod idem est, è contrario, subsequentes figuræ cogitandæ erunt, quarum. i. sit quadratum. a. b: l. sit quadratum. e. f: x. quadratum. b. c: y. quadratum. f. g: p. quadratum. c. d: q. quadratum. g. h: x. sit productum. b. c. in. c. d: m. sit productum. f.

g. in

g.in.g.h.Nunc ex ſpeculatione præcedentis theorematis, eadem erit proportio. n.t.ad.o.u.quæ eſt.n.s.ad.o. r. quare productum. κ. ex definitione ſimile erit producto. m. cum vtraque ſint rectangula, vnde proportio. κ. ad. m. ad proportionem.n.t.ad.o.u. ex. 18. ſexti dupla erit. Igitur proportio. κ. ad. m. æqualis erit proportioni. x. ad. y. et. p. ad.q. et.i.ad.l.& permutando ſic ſe habebit. κ.ad.i.ſicut.m.ad.l.ſed.x. p.ad.i. ſic ſe habere probatum eſt vt.y.q. ad.l. Quare ex eadem. 24.quinti ſic ſe habebit.x.p.κ.ad.i.ſicut.y.q.m.ad.l.ſed.y.q.m.æqualis eſt.l. Itaque.x.p.κ. pariter.i. æqualis erit.




## THEOREMA LVIII.

ALIVD quoque problema, nec tamen definitum, veteres propoſuerunt, nempe an aliquis numerus in.4.eiuſmodi partes diuidi poſſit, vt ſumma quadratorum duarum partium dupla ſit ſummæ quadratorum reliquarum duarum.

Verum huius effectio & ſpeculatio non erit difficilis,cũ ſit eadem quæ præmiſsis proximè duobus theorematibus allata fuit, ſumpta nempe ſumma radicum quarumcunque ſic ſe habentium, prout dictum fuit. Verbi gratia. 44. cuius partes erunt. 16 12. 14. 2. tũc progrediemur regula de tribus dicentes.Si 44 numerum propoſitum valet, quid. 16.pars maior? nempe valebit partem maiorem numeri propoſiti reſpondentem. 16. idem de cæteris dico.

Porrò ſpeculatio eadem eſt cum ſuperioribus.

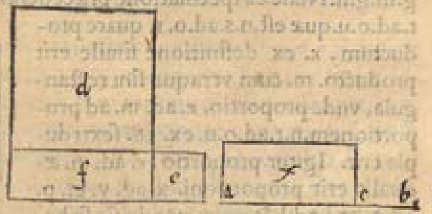
## THEOREMA LIX.

CVR diuidens propoſitum numerum in duas eiuſmodi partes, vt productum radicum quadratarum ipſarum partium æquale ſit alteri numero propoſito, cuius tamẽ quadratum maius nõ ſit quadrato dimidij primi numeri propoſiti. Rectè ſecundum numerum propoſitum in ſeipſum multiplicat, & eundẽ ex quadrato dimidij primi detrahit, reſiduiq; quadratam radicem ſubtrahit ex dimidio ipſius primi, ex quo datur minor pars quæſita, qua ipſi dimidio coniuncta, maior pars habetur.

Exempli gratia, ſi proponatur numerus. 20. propoſito modo, in duas partes eiuſmodi diuidendus, vt productum radicum æquale ſit (verbi gratia) 8. Dimidium primi numeri in ſeipſum multiplicabimus, cuius quadratum erit. 100. ex quo quadratum ſecundi numeri,nempe.64.detrahemus,remanebitq;.36.cuius radice quadrata coniuncta. 10. dimidio inquam primi numeri propoſiti,dabitur numerus.16.pars maior, & ſubtracta à dimidio, dabitur minor pars, nempe.4.

Hoc

Hoc vt demonstremus, primus numerus linea.a.b.significetur, quam diuisam cogitemus in puncto.c.in partes quæsitas,ex quo præsupponitur duas lineas.a.c.et.c.b.duo quadrata esse, quæ in altera figura significetur per.d. et. c. productum autem radicum cognitum. f.quandoquidem datum est,cuius quadratum æquale erit producto quadratorum.d. e. adinuicem,nempe.b.c.in.a.c. ex.19. theoremate huius. Quod verbi gratia sit.x.itaq; cognitum, quo facto,doctrinam.45.theorematis libri huius secuti, propositum consequemur.

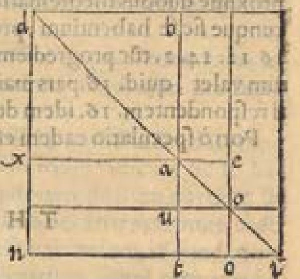



## THEOREMA LX.

CVR productum differentiæ duarum radicum in summam ipsarum, semper differentia sit quadratorum ipsarum radicum.

Exépli gratia, quoslibet duos numeros pro radicibus sumpserimus, vt potè. 3.et. 5. quorum differentia est.2. certè si differentiam hanc per summam radicum scilicet. 8.multiplicauerimus, dabitur numerus. 16. quod productum differentia est suorum quadratorum, nempe inter.9. et. 25.

Hoc vt speculemur, duæ radices in linea.n.i.significentur, quarum vna sit.n. c. & altera.c.i. ipsarum autem differentia.n.t. ex quo.t. c.æqualis erit.c.i.Tum cogitato toto quadrato.d.i. cum diametro. d. i. ductaq; parallela lateri. n. d. à puncto.c.& altera à puncto.t. & à puncto. o. tertia ipsi.n.i. & à puncto. a.quarta. x. a. e. parallela ipsi. o. inueniemus. b.n.productum esse differentiæ. n. t. in summa radicum.n. i. & cum. d. o. et. a. o. sint quadrata radicum prædictarum: b. e. æquale erit. n.u.cum vtrunque horum productorum æquale sit. x.u.ex quo gnomon.e. d. u. æqualis erit producto. b. n.quod scire cupiebamus.



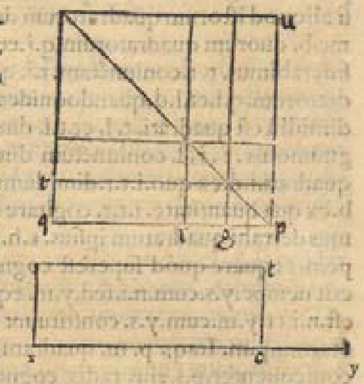

## THEOREMA LXI.

CVR propositum aliquem numerum diuisuri in duas eiusmodi partes, vt differentia radicum quadratarum æqualis sit alteri numero proposito, cuius tamen quadratum dimidij primi quadratum non excedat. Rectè secundum numerum in seipsum multiplicant, productum verò ex primo numero detrahunt, rursusq; dimidium residui quadrant, & quadratum hoc ex quadrato dimidij primi subtrahunt, atque ita radice quadrata residui, dimidio primi coniuncta, pars maior datur, qua ex ipso dimidio detracta, pars minor relinquitur.

Exempli gratia, proposito numero. 20. ita ut propositum est, diuidendo, nempe vt differentia radicum quadrararum dictarum partium æqualis sit binario, binarium hoc in seipsum multiplicabimus, cuius quadratum.4. è primo numero. 20. detrahemus,

trahemus, supereritq; numerus. 16.cuius dimidium scilicet.8. in seipsum multiplicabimus, dabiturq; numerus. 64.qui cum ex quadrato dimidij primi detractus fuerit, nempe ex. 100.& residuo. 36.radix quadrata nempe. 6.coniuncta denario, dimidio primi, dabit. 16.partem maiorem,& ex denario detracta, partem minorem.

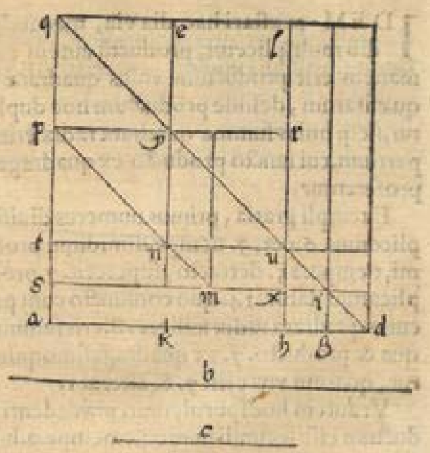
Cuius speculationis causa, primus numerus propositus significetur linea.x.y.pro voto diuisa in puncto. c. et. x.t. productum sit ipsius. x. c. in.c.y.pariter etiam. q. p.sit summa radicum quadratarum, nempe.q.g.ipsius. r.c.et.g.p.ipsius.c.y. Tum super.q.p. extruatur & diuidatur quadratum.q.u.ea ratione qua.41.theoremate aut. 29.diuisimus,in quo sanè quadrato,quadratum ipsius.q.i.cernemus datæ differentiæ, & in eo collocata quadrata. x. c.et.c. y. ita etiam & rationem,qua cognoscimus productum.g.r.(vsi modo.29. theorematis) cuius quidem.g.r.quadratum, ex. 19.theoremate æquale erit producto. x.t.ideo etiam cognitũ, ac proinde cum nouerimus. x. y.si rationem sequemur. 45.theoremate cognoscemus non solum ratione. 41. theoremate allata hoc rectè perfici, sed hac etiam alia ratione.




## THEOREMA LXII.

CVR propositum numerum diuisuri in duas eiusmodi partes, vt differentia suarũ radicũ quadratarũ æqualis sit alteri numero proposito. Cuius tamẽ quadratũ maius non sit quadrato medietatis ipsius primi propositi numeri. Rectè etiã quadratũ dimidij secundi numeri ex dimidio primi detrahũt, residuiq; radicem per secundum multiplicant, & productum ex dimidio primi detrahunt, vt residuum pars quæsita minor sit,& illud alterum totius residuum, pars maior.

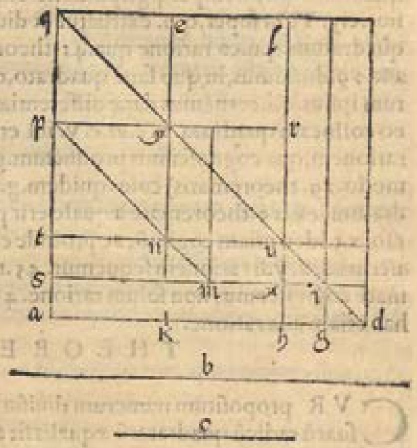
Exempli gratia, si numerus. 50. in prædictas duas partes diuidendus proponeretur, & alter etiam. 6.quadratum dimidij secundi numeri esset.9.eo detracto ex dimidio primi, remaneret. 16.cuius radix.4.scilicet per totum secundum nempe. 6. multiplicata, proferet. 24. quo producto ex dimidio primi detracto, nempe.25. dabitur. 1. pars minor, maior autẽ erit residuũ. 50. hoc est.49. radices autem erunt. 1.et. 7. differentes inter se, numero senario.




Hoc vt sciamus, duo numeri lineis significẽtur, primus linea.b: secũdus linea. c.duæ autem partes. b. duobus quadratis.q.i.et.i.d.notentur, eorum verò radices lineis.a.g.et.g.d.differentia porrò ipsi.c.æqualis & cognita sit. a.h.ex quo.h.

g.æqua-

g. æqualis erit.g.d.tum productum.a.g. in.g.d.ſit.a.i.et.t.i.æqualis. a.i.et.l. i.pariter ſecetur æqualis.t.i.quæ omnia ex diametro.q. d. cogitari poſſunt:erit igitur. u. i. æqualis. i. d. ſupereritq; quadratum. q.u.differentiæ.a. h.cognitum, hoc verò cogitemus diuiſum eſſe in.4. partes æquales medijs diametris.p.r.et.n.e.quare vnaquęq; partium cognoſcetur, & quadratũ erit ipſius. a. x.aut ipſius.x. h.dimidij.a.h. Quòd ſi aliquod iſtorum quadratorum detrahere voluerimus, nempe.n.r.ex dimidio ſumme.b. duorum quadratorum.q.i.et. i. d.cognitæ, hac via procedemus, primum conſiderabimus. t. r.coniunctam. t.i. quæ quantitates erunt ſumma dimidij duorũ quadratorum. q.i.et.i.d.quandoquidem.t.r. dimidiũ eſt quadrati. t. l. et.t.i. dimidiũ gnomonis. t. i. l. coniunctum dimidio quadrati.i.d.ex quo.i.t.r.dimidium erit. b.ex qua quantitate. i.t.r. cogitare debemus detrahi quadratum ipſius. x.h. nempe.n. r: quare quod ſupereſt cognitum erit nempe.y.s.cum.n.i.ſed.y.m.æqualis eſt.n.i.et.y.m.cum.y.s. conſtituunt quadratum.p.m. Itaq;. p. m. quadratum & conſequenter.p.s.eius radix cognoſcetur, ita etiam & productum huius.p.s.in. s.x. æqualis.c.nempe. p. x: eſtq; productum huiuſmodi ſemper minus quantitate.r.t.i:per.u.i.æquale quadrato minori. i. d. quare.i.d.cognoſcetur, conſequenter.i.q. tanquam reſiduum ex. b. & eorum radices quadratæ cognoſcentur. a. g. et. g. d.




## THEOREMA LXIII.

IDEM præſtari hac alia via, meo iudicio poteſt. Secundus numerus in ſuũ dimidiũ multiplicetur, productũ autem ex dimidio primi detrahatur, ex quo remanens erit productum vnius quadratæ radicis in alteram partium primi numeri quæſitarum, deinde productum hoc duplicetur, & primo numero dato coniungatur, ſicq; huius ſummæ quadrata radix erit ſumma radicum quadratarum dictarum partium, cui iuncto producto ex quadrageſimoquinto theoremate ſingulæ radices proferentur.

Exempli gratia, primus numerus diuiſibilis erat. 50. alter verò. 6. Iam ſi multiplicemus. 6.per. 3. nempe dimidium proferetur numerus. 18. quo ex dimidio primi, nempe.25. detracto, ſupererit. 7.productum vnius radicis in alteram, quod duplicatum dabit. 14. quo coniuncto cum primo numero. 50. dabitur numerus. 64. cuius quadrata radix ſcilicet. 8.erit ſumma radicum duarum partium quæſitarum, qua & producto. 7. ex quadrageſimoquinto theoremate dictæ radices diſtinguentur, quarum vna erit. 7. & altera. 1.

Vt autem hoc ſpeculemur, præcedenti figura vti poterimus, in qua patet.t.r. productum eſſe ſecundi numeri.c.nempe.a.h.hoc eſt. t.u.in dimidio.a.e.ſcilicet.p.t.reſiduum autem dimidij primi.b.eſſe.t.i.nempe.a.i.productum radicum, quod ſupplemenmentum eſt quadrati. q.d.totalis. Quarè duplicato.a.i. & coniuncto. b. cognoſcimus totum. q.d. & conſequenter.a.d. ſuam radicem, hoc eſt ſummam duarum radicum.a.g.et.g.d.quæ medio.a.i.cognito, & quadrageſimoquinto theoremate ſingulæ cognoſcuntur.

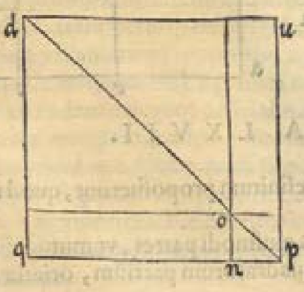
## THEOREMA LXIIII.

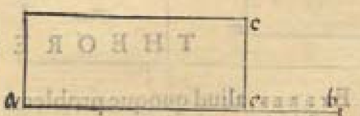
CVR propoſitum aliquem numerum in duas eiuſmodi partes diuiſuri, vt ſumma radicum dictarum partium æqualis ſit alteri numero propoſito. Rectè ſecundum numerum in ſeipſum multiplicant, ex quo quadrato, primum datum numerum detrahunt, rurſusq́; reſiduum in ſeipſum multiplicant, & ex eo quadrato quartam partem deſumunt, quã ex quadrato dimidij primi numeri detrahunt, radicemq́ue quadratam reſidui cum iunxerint, & ex dimidio primi numeri detraxerint, partes quæſitæ proferuntur.

Exempli gratia, ſi proponeretur primus numerus.20. diuidendus et. 6. ſecundus pro ſumma radicum, hunc ſecundum.6. in ſeipſum multiplicabimus, dabiturq́; numerus.36. ex quo quadrato primus numerus detrahetur, ſupereritq́; numerus. 16. qui quadratus dabit.256.cuius numeri quarta pars ſumetur, nempe.64.quæ ex quadrato dimidij primi numeri detrahetur, nempe.100.ſupereritq́;.36.cuius radix quadrata. 6. coniuncta & detracta ex.10.dabit.16. partem maiorem et.4.minorem.

Cuius rei hæc ſpeculatio, primus numerus diuiſibilis ſignificetur linea.a.b.diuiſa in puncto. e. in partes adhuc incognitas, et.a.c.ſit productum.a.e.in.e.b. item.q.p.ſecundum numerum ſignificet, æqualem ſummæ radicum, quæ puncto. n. diſtinguantur. Poſtmodum totum quadratum.p.d.erigatur (quod nobis eſt cognitum) in duo quadrata diuiſum.o.p.et.o.d.quorum ſumma.a.b.cum detur, cognita remanet ſumma duorũ ſupplementorũ.o.u.et.o.q. qua quadrata cũ fuerit dabit quadruplũ quadrati ſuppleméti.o.q.nẽpe quadruplũ producti.a.c.etenim.a.c.ex.19. theoremate huius libri quadratum eſt ipſius.q.o. ſicq́; poterant etiam veteres quadrare dimidium differentiæ. a.b.ab.p.d.nempe quadrato tantummodo ſupplemento. q.o. Tunc habito.a.c.eius ope tanquam producti.a.e.in. e.b.ex.45.theoremate ſingulæ partes cognoſcentur.

Quod alia etiam ratione præſtari poterat, nempe cognito ſupplemento. q.o.diſtinguendæ radices q.n.et.n.p.ex. 45. theoremate, quibus cognitis, eorum etiam quadrata cognoſcuntur.



## THEOREMA LXV.

CVR proposito numero in tres qualescunque partes diuiso, si prima in tertiam multiplicetur, & huic producto, secundæ in primam productum coniungatur, itemq; secundæ in tertiam, hæc summa duplicata æqualis sit summæ productorum singularum in cæteras duas.

Exempli gratia, si proponatur. 20. diuisus in tres partes nempe. 12.5.3. multiplicato primo. 12. per. 3. tertiam partem dabitur. 36. secunda verò multiplicata per reliquas duas, hoc est. 5. per. 12. et. 3. in primis dabitur. 60. postea. 15. quorũ triũ productorum summa erit. 111. quæ duplicata dabit. 222. qui numerus æqualis esse dicitur summæ productorum singularum partium in reliquas duas, nempe summæ. 60. 36. 60. 15. 36. 15. hoc est ipsis. 222.

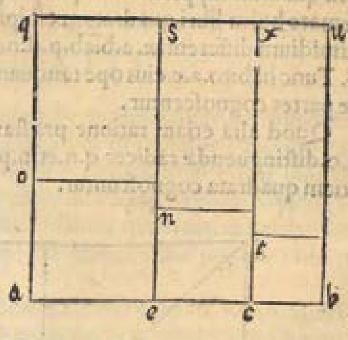
Cuius rei per se patet speculatio, cum in his sex vltimis productis, singula tria prima duplicentur.

## THEOREMA LXVI.

CVR proposito numero in. 3. qualescunque partes diuiso, si in reliquas duas singulæ multiplicentur, & hæc producta cum summa suorum quadratorum coniungantur, tota summa hæc vltima æqualis erit quadrato totali propositi numeri.

Exempli gratia, si fuerit idem numerus. 20. in. 3. partes diuisus. 12.5.3. Si. 12. in 5. et. 3. producatur, summa productorum erit. 96. at. 5. in. 12. et. 3. erit. 75. postmodum. 3. in. 12. et. 5. erit. 51. nempe in vniuersum. 222. quadratorum porrò summa erit. 178. quæ coniuncta. 222. dabit. 400. quadratum ipsius. 20.

Erit autem huiusce rei facillima speculatio, si sequentem figuram mente conceperimus, in qua. a. b. propositum numerum significet, cuius partes distinctæ sint medio. e. et. c. Ipsum autem. q. b. sit quadratum totale parallelis. e. s. et. c. x. diuisum, quæ quadratum in tria rectangula diuident, quorum primum erit. q. e. compositum ex producto. a. e. in semetipsam, nempe quadratum. o. e. & ex producto eiusdem. a. e. in. e. b. quod erit rectangulum. o. s. ex quo tria rectangula. o. s. et. n. x. et. t. u. tria producta erunt singularum partium in cæteras duas, et. e. o: c. n: b. t. tria quadrata erunt: quibus sex quantitatibus quadratum totale. q. b. completur.




## THEOREMA LXVII.

VETERES aliud quoque problema indefinitum proposuerunt, quod tamen à nobis determinabitur.

Cur diuisuri propositum numerum in duas eiusmodi partes, vt mutuò diuisis, & per summam prouenientium diuisa summa quadratorum partium, oriatur proueniens alter numerus propositus.

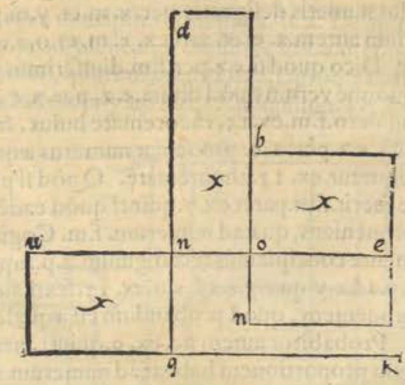
Proposito deinde tertio quolibet numero diuidendo per singulas partes primi, ita

ita vt ſimul prouenientibus in ſummam collectis huius ſummæ ad primum numerum propoſitum proportio futura ſit ea quæ eſt tertij ad ſecundum. Rectè dimidium primi numeri in ſeipſum multiplicant, ex quo quadrato ſecundum numerum detrahunt, tum reſidui radicem ſumunt, quam iungentes, & detrahentes ex dimidio primi, partes quæſitas habent, cætera ex neceſsitate ſubſequuntur, prout nunc à me docebitur.

Exempli gratia, proponitur numerus. 20. in duas partes diuidendus, quibus poſtea mutuò diuiſis, & per ſummam prouenientium diuiſa ſumma quadratorum, dent ſecundũ numerum propoſitum. 36. nam reliqua conſequuntur. Itaque. 10. dimidium primi in ſeipſum multiplicatur, & ex quadrato. 100. eruitur numerus. 36. nempe ſecundus propoſitus reſidui porrò. 64. quadrata radix. 8. ſumitur, quam coniungimus & detrahimus ex dimidio primi ſcilicet. 10. ex quo partes quæſitæ dabuntur. 18. et. 2. quæ mutuo diuiſæ dabunt ſuorum prouenientium ſummam. 9. cum nona parte, per quam diuidentes. 328. ſummam quadratorum ipſarum partium, exactè dabitur numerus. 36. qui fuit ſecundò propoſitus. Tum ſi per ſingulas iam inuentas partes quilibet numerus diuiſus fuerit, verbi gratia. 72. ſumma prouenientium erit. 40. qui numerus eandem proportionem cum primo nempe. 20. ſeruabit, quam tertius propoſitus. 72. cum ſecundo. 36.

Quod vt ſpeculemur, primus numerus ſignificetur linea. n. e. ita diuidendus à puncto. o. vt diuiſa parte. n. o. per. o. e. et. o. e. per. n. o. & per ſummam prouenientium diuiſa ſumma quadratorum. n. o. et. o. e. detur ſecundus numerus notatus linea. q. k. Porrò meminiſſe oportet quòd. 26. theoremate probatum fuit vltimum hoc proueniens æquale producto partium inter ſe futurum, nempe producto. n. o. in. o. e. quod ſignificetur rectangulo. n. e. Itaque datis. n. e. et. q. k. ſi. 45. theorema conſuluerimus, partes. n. o. et. o. e. cognoſcemus.

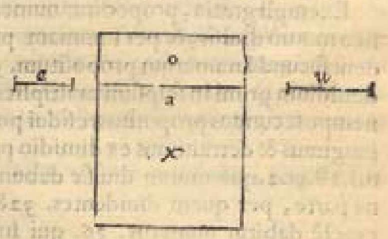
Proponitur deinde tertius quilibet numerus, verbi gratia. x. diuidendus per. o. e. et. o. n. qui ſi diuidatur per. o. e. dabit proueniens. b. o. Si verò per. n. o. proueniens erit. d. n. nunc aſſerimus ſummã duorum horum prouenientium, ſic primo numero. n. e. dato proportionatam eſſe, ſicut tertius. x. ſecũdo. q. k. Producatur enim linea. d. n. donec. n. q. æqualis ſit. o. b. ex quo. q. d. erit ſumma vltimò prouenientium: item producatur. e. n. donec. n. u. æqualis ſit. o. e. termineturq; rectangulum. q. u. quod tertio numero propoſito. x. vt patet, æquale erit, quare ex. 15. ſexti aut. 20. ſeptimi eadem erit proportio. d. n. ad n. q. quæ. u. n. nempe. o. e. ad. o. n. & componendo. d. q. ad. q. n. ſicut. e. n. ad. n. o. & permutando. d. q. ad. e. n. quę. q. n. hoc eſt. b. o. ad. o. n. nempe ſicut. b. e. ad. e. n. ſuperficialem, ex prima ſexti aut. 18. vel. 19. ſeptimi, ſed rectangulum. e. n. conſtitutum fuit æquale numero. q. k. itaque verum eſt propoſitum.



## THEOREMA LXVIII.

CVR numero per numerum diuiso, productoq; duorum numerorum per proueniens multiplicato, quod vltimò productum est, diuisi numeri semper quadratum existat.

Exempli gratia, si diuidamus. 10. per. 2. proueniens erit. 5. quo producto ex duobus numeris multiplicato, nempe. 20. habebimus. 100. quadratum numeri diuisi.




Cuius gratia duo numeri sint. a. et. e. porrò. a. per. e. diuiso detur. u. tum. o. productum. a. in. e. esse constituatur, quo per. u. multiplicato dabitur. x. quadratum. a. propterea quòd. a. medium est proportionale inter. o. et. u. ex. 35. theoremate. itaque ex. 16. sexti aut. 20. septimi, propositi veritas elucescet.

## THEOREMA LXIX.

CVR numero aliquo per duos alios multiplicato & diuiso, si per horum duorum productum, summa duorum primorum productorum diuisa fuerit, vltimum proueniens, summæ duorum primorum prouenientium æquale sit.

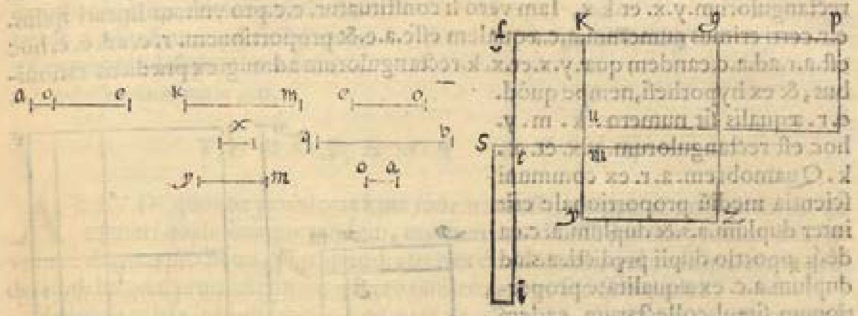
Exempli gratia, proponitur numerus. 24. per. 8. et. 6. multiplicandus & diuidendus summa productorum erit. 336. prouenientium autem. 7. si igitur summam. 336. productorum per productum duorum secundorum numerorum nempe. 48. diuiserimus, proueniens pariter erit. 7.

In cuius gratiā primus numerus significetur linea. q. b. multiplicandus & diuidendus numeris designatis per. κ. m. et. y. m. productorum summa sit. κ. z. prouenientium autem. a. e. et. a. o. ex. κ. m. et. o. e. ex. y. m. tum productum. κ. m. in. m. y. sit. f. m. Dico quòd si. κ. z. per. f. m. diuiserimus proueniet. a. e. Quod cum sic fuerit, erit quoque verum quòd diuisa. κ. z. per. a. e. proueniet. f. m. numerus scilicet æqualis numero. f. m. ex. 13. theoremate huius. Itaque quotiescunque probauero quòd diuisa. κ. z. per. a. e. proueniat numerus æqualis ipsi. f. m. propositum verum esse consequetur. ex. 13. theoremate. Quòd si proueniens ex diuisione. κ. z. per. a. e. æquale fuerit. f. m. patet ex. 7. quinti quòd eadē erit proportio numeri. κ. m. y. ad ipsum proueniens, quæ ad numerum. f. m. Cogitemus itaq;. κ. u. æqualem. a. e. super quam mente concipiamus rectangulum. u. p. æqualem. κ. z. ex quo eadem erit proportio. κ. p. ad. κ. y. quæ. g. κ. ad. κ. u. ex. 15. sexti, aut. 20. septimi, numerus autem. κ. p. erit proueniens, quod probandum est æquale esse. f. m.

Probabitur autem sic, ex. 9. quinti, nempe demonstrato quòd numerus. κ. p. eandem proportionem habeat ad numerum. κ. y. quam habet numerus. f. m. ad eundem κ. y. Sed probatum est sic se habere. κ. g. ad. κ. u. sicut. κ. p. ad. κ. y. sufficiet igitur probare sic se habere. κ. g. ad. κ. u. sicut. f. m. ad. κ. y. Sed. κ. g. dicitur æqualis esse. q. b. et. κ. u. a. e. satis erit igitur probare ita se habere. q. b. ad. a. e. sicut. f. m. ad. κ. y. Scimus autem quòd eadem est proportio. q. b. ad. a. o. quæ. m. κ. ad vnitatem, quæ sit. x. & quod proportio. o. e. ad. q. b. eadem est, quæ. x. ad. m. y. ex definitione diuisionis. Quare ex æqualitate proportionum eadem erit proportio. κ. m. ad. m. y. quæ. e. o. ad. o. a. &

componen-

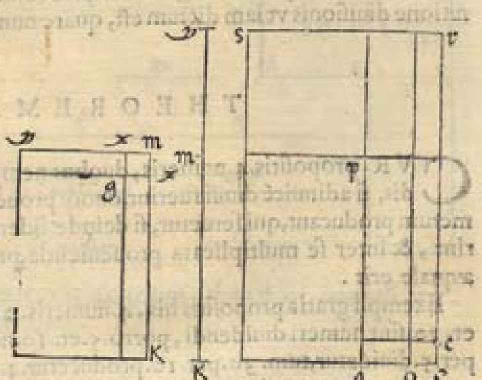
componendó sic se habebit. κ.y.ad.m.y.sicut.e.a.ad.o.a.& permutando.κ.y. ad.e. a.sicut.m.y.ad.o.a.& ex.19. quinti ita.κ. m.ad.e.o.sicut.κ.y.ad.e.a. & permutando. κ.m.ad.κ.y.sicut.e.o.ad.e.a. Nunc producatur.f.t.donec.t.i.æqualis sit. κ.y.productaq;.m.t. donec.t.s.æqualis sit vnitati.x. termineturq; rectangulum. s. i.ex quo dabitur proportio numeri. f. m. ad numerum.s.i.composita ex.m.t.ad.t.s.et.f.t.ad.t.i. ex. 24.sexti,aut quinta octaui, sed ita etiam proportio.q.b.ad. a. c. componitur ex eisdem proportionibus, nempe ex.q.b.ad.o.e.æquali.m.t.ad.t. s.& ex proportione. o.e.ad.a.e.æquali. f.t.ad.t.i.itaque proportio numeri.f.m.ad.s. i. hoc est ad numerũ ipsius.κ.y.equalis est proportioni numeri.q.b.ad.a.e.nẽpe.κ.g.ad.κ.u. hoc est.κ.p.ad κ.y.ex quo sequitur.κ.p.constare numero equali.f.m.proueniens igitur ex diuisione numeri.κ.z.per.f.m.æquale est numero ipsius.a.c.



## THEOREMA LXX.

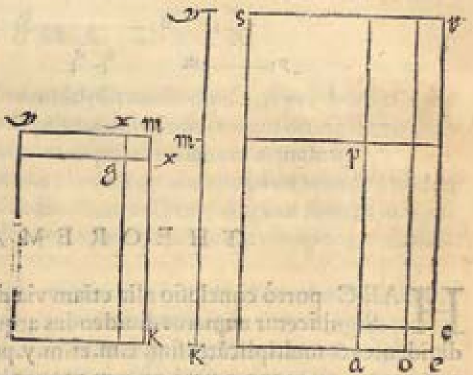
HAEC porrò conclusio alia etiam via demonstrari potest.

Significetur numerus diuidendus atque multiplicandus linea. b. a. Deinde diuidentes & multiplicãtes sint. κ.m. et.m.y.prouenientia ex diuisione sint.a.o.et.o. e.atque.a.o.ex.m.y:o.e.verò ex.κ.m.proueniat, quorum summa sit.a.e:productum autem.b.a.in.k.m.sit.b.p.et.p.s.productum.b.a.in.m.y.ad hæc rectangulum. k.y:sit productum.k.m.in. m . y : quo totum productum.a.s.diuidatur, prouenensq; sit.a.c.cui,a. c: productũ. a.s.eãdẽ proportionẽ seruabit,quã k. y. rectangulum ad vnitatem ex definitione diuisionis , hoc autem proueniens.a.c.cõstare numero æquali assero summæ. a. e .Primum enim ex dicta definitione diuisionis habemus eandem esse proportionem.b.a.ad. a. o. quæ. m. y . ad vnitatem, & quod sic se habet.b.a. ad.o.e.sicut. k.m.ad eandem vnitatem. Itaque vnitas hæc linearis significetur per.m.x.in singulis lateribus. k.m.et. m.y.producentibus rectangulum. k.y: superficialis autem vnitas sit.



g.m.

g.m.cogiteturq; rectangulum.y.x.& rectangulum.k. x. Itaque dabitur eadem proportio.k.m.ad.m.x.nempe.k.x.rectanguli ad.m.g.quæ est.b.a.ad.o.e.et.y.x.ad. m. g.quæ.b.a.ad.a.o.sed ex prima sexti aut.18.vel.19.septimi, sic se habet rectangulum. k.y.ad.x.y.sicut.k.m.ad.m. x quare sicut.b.a.ad.o.e.ex. 11.quinti,& eiusdem rectanguli. k. y.ad rectangulum. k.x.sicut.y.m ad.x.m.nempe.b.a. ad. a. o. Quare ex communi scientia, sic se habebit duplum rectanguli. k.y.ad summam. y.x. cum. k.x.rectangulorum, sicut duplum. b.a.ad summam.a.o.e. et proportio summæ rectangulorum. y.x.et.k.x.duplo.g.m.sicut duplum.b.a.ad.a o.e. Igitur summa duorum rectangulorum.y.x.et.x.k.media proportionalis erit inter duplum rectanguli. k.y.& duplum vnitatis superficialis. g.m. Nunc terminetur rectangulum.a.r.ex quo dabitur eadem proportio dupli.a. s.ad.a.r.sicut dupli.b. a.ad. a. e. ex propositionibus notatis, sexti aut septimi. Quare etiam sicut dupli rectanguli. k. y. ad summã rectangulorum.y.x. et.k.x, Iam verò si constituatur. e.c.pro vnitate lineari ipsius. e.r.certi erimus numerum.a.c.æqualem esse.a.e.& proportionem. r. e. ad. e. c. hoc est.a.r.ad.a.c.eandem quæ.y.x.et.x. k.rectangulorum ad.m.g.ex prædictis rationibus, & ex hypothesi, nempe quòd. e.r. æqualis sit numero. k. m. y. hoc est rectangulorum. y.x. et. x. k. Quamobrem. a. r. ex communi scientia mediũ proportionale erit inter duplum.a.s.& duplum.a. c.eadẽq; ꝓportio dupli prędicti. a.s.ad duplum.a.c. ex æqualitate proportionum simul collectarum, eadem erit quæ proportio dupli rectanguli.k.y.ad duplum.m. g. hoc est. a. s. simplicis ad simplicem.a.c. quæ simplicis rectanguli.k.y.ad simplicem vnitatem.g.m.sic enim se habet simplex ad simplex, sicut duplum ad duplum. Sed pariter ita se habet.a.s.ad.a.c. cogitato. a.c. tamquam proueniente ex diuisione. a.s.per rectangulum.k.y.vt constitutum est, sicut.k.y.ad.m.g.ex definitione diuisionis vt iam dictum est, quare numerus.a.c.æqualis erit numero. a.o.e.




## THEOREMA LXXI.

CVR propositis.4.numeris, duobus nempe diuidentibus ac duobus diuidendis, si adinuicẽ diuisi fuerint, duoq; proueniẽtia inuicẽ multiplicata quẽuis numerum producant, qui seruetur, si deinde ijdem numeri versa vice mutuo diuisi fuerint, & inter se multiplicata prouenientia, productũ hoc, primo seruato numero æquale erit.

Exempli gratia propositis his. 4.numeris. 20.30.5. 10. duo autem. 20. scilicet et. 30.sint numeri diuidendi, porrò.5.et. 10.numeri diuidentes, nẽpe vt primo.20 per.5.diuidatur, tum. 30.per. 10.producetur.4.et.3.qui simul multiplicati proferẽt. 12.tum. 20.per.10.diuiso et.30.per.5.prouenientia erunt.2.6.quæ inter se multiplicata producent etiam.12.

Cuius

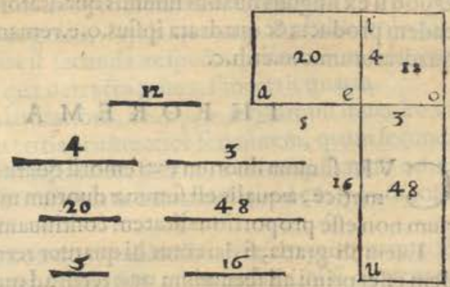
Cuius rationem si quæris, significentur.4.numeri lineis. a.e. o.'u. diuidaturq́;. a. per.o. & oriat.s.& per.u.oriat.y. et. e. diuiso per. o. oriatur. z. & per. u. proueniat.f.tum.n.sit productum.z. in.y. et.m. productum. s. in. f. Dico n. futurum æquale. m. Sit deinde. x.vnitas, quare ex definitione diuisionis eadem erit proportio. s.ad.a. et.z.ad.e.quæ.x.ad.o. Sed ita se habet.a.ad.y.et.e.ad.f.sicut.u.ad.x. ex quo sic se habebit.s. ad.a.sicut. z. ad e. et.a.ad.y,sicut.e.ad. f. Itaque ex æqualitate proportionum sic se habebit s.ad.y.sicut.z.ad.f. Igitur ex 15.sexti aut.20.septimi productum, n.producto.m.æquale erit.


## THEOREMA LXXII.

ALIVD quoque problema à me inuentum est, nempe vt proponantur. 4. numeri qualescunque tandem, quorum duo diuisibiles sint, tertius diuisor vnius è duobus pro libito, quæramusq́; alterius diuidentem, qui sic se habeat vt productum duorum prouenientium quarto numero proposito sit æquale.

Exempli gratia, proponuntur.4.numeri.20.48. 5. 12. porrò. 20. et. 48. numeri sint diuisibiles et. 5. diuidẽs vnius, ut potè.20. Quærẽdus nunc erit diuidens alterius nempe.48. eiusmodi vt productum prouenientium æquale sit.12. Diuidam itaque. 20.per.5.proueniet q́;. 4.quem per.48. multiplicabo, nempe per alterum diuisibilem, sicq́; proueniet.192.quod productum per quartum numerum nempe.12. diuisum dabit.16. qui erit diuidens quæsitus, quo diuiso. 48.proueniet.3. secundum scilicet proueniens, quo per alterum hoc est.4.multiplicato producetur quartus numerus.12.

Quod vt sciamus, primus numerus diuisibilis significetur rectãgulo. a. i.secundus rectangulo.o.u.primus diuidens latere.a.e. quartum numerum rectangulo.i.o. primum proueniens latere.e.i.secundus diuidens latere.e.u.(hic autem est quem quærimus) tum alterum proueniens significetur latere.e.o.Iam eadẽ erit proportio. e.i.ad.e.u.quæ.o.i. ad. o. u. Sed cum cognitæ sint tres quantitates.e.i:i.o:et.o.u.quarta quoque.e.u. ex regula de tribus immediatè cognoscetur, cætera in subscripta figura facillimè patebunt.

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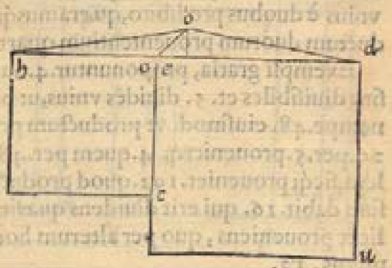
## THEOREMA LXXIII.

HOC etiam problema à me inuentum est, nempe si duæ radices quadratæ in summam collectæ fuerint, & ex dimidio eiusmodi summæ detracta fuerit minor radix, residuiq; quadratum duplicatum eiq; summæ coniungatur duplum producti ipsius residui in dimidium summæ radicum, atque huic summæ duplum producti eiusdem residui in radicem minorem coniunctum fuerit; vltima hæc summa differentia erit duorum quadratorum propositorum.

Exempli gratia duæ radices quadratæ sint.5.et.11.harum summa erit.16.& dimidium.8.differentia minoris ab ipso dimidio erit.3:duplum quadrati huius differentiæ erit.18:duplum producti huius differentię in dimidium summę radicum erit.48. item & huius differentiæ duplum in minorem radicem erit.30. quarum omnium summa erit. 96. tantaque erit differentia suorum quadratorum, quorum vnum erit.25.alterum verò.121.

Pro cuius rei scientia, duæ quadratæ radices sint.h.o.et.o.d.directæ inter se coniunctæ, quæ summa per medium in puncto.e.diuidatur, tum cogitetur.e.b.æqualis o.e.perpendicularis.h.d.ducanturq; lineæ.b.h:b.o.et.b.d.Iam ex.4.primi.b.h.æqualis erit. b.d.& quadratum.b.h.æquale quadrato. h.o.& quadrato.o.b.simul cum duplo producti.o.e.in.o.h.ex.12.secundi Eucli. Sed ex. 13. eiusdẽ quadratum. b.d. minus est quadrato. o.d.cum quadrato.o.b.ex duplo producti.o.e.in.o.d.at duplum eiusmodi producti æquale est duplo quadrati .o. e. & duplo producti.o.e.in.e.d. ex tertia eiusdem, itaque duo quadrata scilicet.o.b.et.o.d. maiora erunt duobus quadratis, nempe. o.b.et.o. h.collectis cum duplo producti.o.e.in.o.h. ex duplo quadrati o.e.vnà cũ duplo producti.o.e.in.e.d.Quare differẽtia summæ duorum quadratorum o.b.et.o.d.à summa duorum.o.b.et.o.h.duplum erit quadrati.o.e.cum duplo producti.o.e.in.e.d.& duplo producti.o.e.in.o.h.




Quòd si ex singulis duabus summis quadratorum demptum fuerit quadratum. o. b. eadem producta & quadrata ipsius.o.e.remanebunt, tanquam differentia duorum quadratorum.o.u.et.h.c.

## THEOREMA LXXIIII.

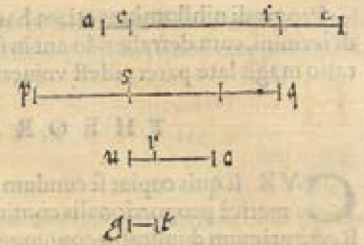
CVR summa duorum extremorũ quatuor terminorum proportionaliũ arithmeticè, æqualis est summæ duorum mediorum, vbi nota hac in re necessarium non esse proportionalitatem continuam existere.

Exempli gratia, si darentur hi quatuor termini.20.17.9.6.quorum proportio eadem esset primi ad secundum quæ tertij ad quartum, summa primi cum quarto esset 26.tantaq; secundi cum tertio.

Cuius speculationis causa, primus maiorq; numerus significetur linea.e. o. secundus.s.q.tertius.u.c.quartus.g.t.differentia porrò inter. e.o.et.s.q.sit.i.o. quæ æqualis erit differentiæ.r. c. qua quartus à tertio superatur ex hypothesi. Itaque assero summam.e.o.cum.g.t.nempe.a.o.æqualem esse summę.q.s.et.u.c.sitq;.q.p.Nam in.a.o.

Secundus

Secundus tertiusque terminus reperiuntur, est enim secundus.e.i.tertius. i.o. et.c. a. quandoquidem ex præsuppositо.e. i.æqualis est.s.q. et i.o. æqualis.r.c.et.a.e. cum sit æqualis. g. t. cui pariter æqualis est. r. u. ex quo. a. e. æqualis est.u.r. Itaque illud sequitur. a. o.ipsi. q. p. æqualem esse.

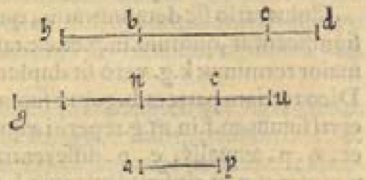


## THEOREMA LXXV.

CVR summa duorum terminorum extremorum imparium arithmeticæ proportionalitatis semper duplo medij termini æqualis est.

Exempli gratia, sunt hi tres termini proportionalitatis arithmeticæ. 20. 15. 10, summa duorum extremorum erit.30.quæ duplum est medij termini.15.

Quod vt speculemur, tres termini, tribus lineis.b.d:n.u. et.q.p.significetur. Dico nunc quòd summa.b.d.cum. q. p. nempe. h.d.semper duplo. n. u. scilicet. g. u. æqualis erit. Tum differentia. b.d. ad.n.u. sit. c.d.quæ æqualis erit. c.u.differentiæ inter n.u.et. q.p. patet enim in linea.h.d:b. c. æqualem esse. n. u. sed.n.u.ex.n.c.componitur æquali.q.p. et ex.c.u. æquali.c.d.cum itaq; in.h. d. partem. h.b.reperiamus æqualem.n.c. gratia. q. p. & partem.c.d. æquale m.c. u.manifestum erit h.d.æqualem esse. g.u.



## BINA PROBLEMATA EX DVOBVS PRAEDICTIS THEOREMATIBVS DEPENDENTIA.

EX duobus prædictis theorematibus duo problemata oriuntur, quorũ primum est. Datis tribus quantitatibus cognitis, si quis quartam inuenire voluerit, quæ eiusmodi sit respectu tertiæ,qualis est secunda respectu primæ, secunda cum tertia in summam colligenda erit, ex qua detracta prima, supererit quarta.

Exempli gratia, cognitis tribus quantitatibus. 20.17. 9. si quartam inuenire voluerimus eiusmodi proportionem cum tertia arithmeticè seruantem, quam secunda cum prima, secundam cum tertia in summam colligemus, dabiturq; summa.26.ex qua detracta prima quantitate, quarta relinquetur nempe.6. quod ex. 74. theoremate dependet.

Idipsum tamen proueniret si quis ex tertio termino differentiam primi atque secundi detraheret;hæc tamen via non tam vniuersalis estqu àm illa.N si quartus terminus incognitus tertio maior esse deberet, dictam differentiam cum tertio termino in summam colligere oporteret.

Alterum problema est, quod inuentis duobus terminis, si tertius requiratur, secundus duplicandus erit, ex qua summa detracto primo,statim tertius proferetur, quod problema ex præcedenti theoremate dependet.

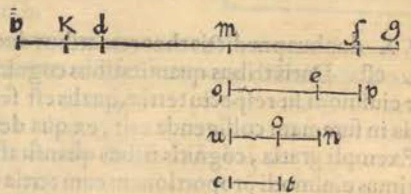
Progredi nihilominus etiam hac in re possemus per differentiam primi & secundi termini, eam detrahendo aut in summam cum secunda colligendo, attamen prior ratio magis latè patet, idest vniuersalior est.

## THEOREMA LXXVI.

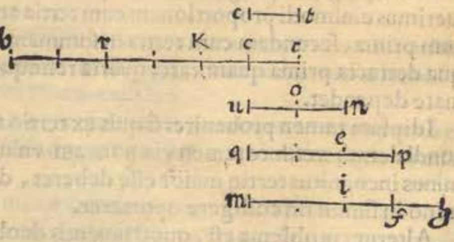
CVR si quis cupiat secundum terminum inuenire, quatuor terminorum arithmeticè proportionalis continuæ, quorum nobis duo extrema proponantur. Rectè primum duplicabit coniungetq́; vltimo termino, nempe quarto, ex qua summa tertiam partem desumet, quæ erit secundus terminus quęsitus.

Exempli gratia, si horum quatuor terminorum. 12. 9. 6. 3. duo nobis extrema proponantur. nempe. 12. et. 3. quorum secundus inueniendus sit, sumpto quolibet pro primo, sit autem. 3. primus numerus, quartus verò. 12. quare duplicato. 3. vtpote primo, & coniuncto. 12. quarto, summa erit. 18. cuius est tertia pars. 6. secundus numerus scilicet sumpto principio à minimo. Idipsum euenit sumpto principio à maximo. Nam si datur secundus à minimo aut à maximo, illico tertius datur differentia inter hunc & primum, secundo coniuncta, aut ex eodem detracta.

Cuius ratio sic demonstratur, quatuor termini quatuor lineis. m. g: q. p: u. n: c. t. significentur, quorum. m. g. et. c. t. tantummodo cognoscantur. sitq́;. m. g. primus ac maior terminus: k. g. verò sit duplum primi. m. g: cui coniungatur. b. k. æqualis. c. t. Dico tertiam partem. b. g. quæ summa totalis est, æqualem esse. q. p. In primis enim certi sumus. m. f. in. m. g. reperiri æqualem. q. p. superestq́;. f. g. differentia inter. m. g. et. q. p. æqualis. e. p. differentiæ inter. q. p. et. u. n. & æqualis. o. n. differentiæ inter. u. n. et. c. t: simul etiam in. k. m. habemus. d. m. æqualem. m. f. quare etiam. q. p. et. k. d. æqualem. f. g. nempe. e. p. aut. o. n: Hactenus in. k. g. reperimus duplum. q. p. simul cum. f. g. et. k. d. æqualibus. e. p. et. o. n. & quia. b. k. æqualis. c. t. fuit coniuncta. considerandum est an hæ tres quantitates. f. g: k. d. et. b. k. simul æquales sint. q. p. quod tamen per se manifestum est. nam. q. p. superat. u. n. per. e. p. et. u. n. excedit. c. t. per. o. n. æqualem. e. p. quare. q. p. per duplum differentię. f. g. superat. c. t. ita que. f. g: k. d. et. k. b. ipsi. q. p. sunt ęquales, ex quo sequitur. q. p. tertiã partem esse. b. g. Hæc quæ hactenus dicta fuerunt, in genere maioris inæqualitatis probata fuerunt.




At in genere minoris, sumpto ordinis principio à minimo terminorum, duplicetur. c. t. sitq́; duplum hoc. k. t. cui. k. b. æqualis. m. g. coniungatur, quæ summa sit. b. t. Dico. u. n. tertiam esse partem ipsius. Nam in primis in. b. t. datur terminus. b. k. æqualis vltimo. m. g. in quo semel reperitur. u. n. vnà cum duabus differentijs, nempe. i. g. in ipsa autem. b. t: u. n. significetur primo loco per. r. k. ex quo supererit. b. r. duabus differentijs prædictis æqualis, sed ex præsupposito. u. n. componitur ex. o. u. æquali. c. t. et. o. n. ęquali vni differentiæ. Itaq;




cum

cum in.b.t.præter.r.x.bis detur.c.t. nempe.x.t.et. b. r. duabus differentijs æquipollens, illud efficitur. u.n.pariter ipsius.b.t.esse tertiam partem, quod erat propositū.

## THEOREMA LXXVII.

CVR si quis velit secundum quinque continuorum proportionalium terminum inuenire, solis extremis cognitis. Rectè vltimū triplo primi coniunget, ex qua summa quartam partem detrahet, quæ erit secundus terminus quæsitus. Quod ipsum faciet qui inuenire vult secundum terminum senarij septenarij, octonarij aut alterius cuiuscunque, crescente tamen multiplicatione primi, vltimoq́; cōniuncto.

Exempli gratia, dantur duo extremi termini, horum quinque numerorum. 18. 16.14.12. 10. nempe.18.et.10.si.18.primus erit, hoc est, si à genere maioris inæqualitatis progrediemur, triplicabimus terminum.18.dabunturq́;. 54.cui numero coniuncto quinto termino. 10. dabitur numerus.64. cuius quarta pars erit.16.vtpote secundus terminus gratia, aut secundi sex terminorum, quadruplicandus esset primus.18.deinde adiuncto vltimo, quinta pars summæ esset secundus terminus, atq; ita deinceps.

Cuius speculationis gratia, dicti termini lineis.z.h:f.s:u.p:e.g.et. r. x. significētur. In primis ex genere maioris inæqualitatis, triplicabimus. z.h.sitq; triplum hoc. x. h.cui cōiungatur.b.x.ęqualis vltimo termino.r.x. Dico.f.s.quartā partem esse summę.b.h. Nam in.x.h.secundus terminus.f.s.ter cum tribus differentijs æqualibus.n.h. reperitur. Probandum nunc est tres has differentias. n.h:a.c.et.d. x. simul cum. b. x.ęquales esse.f.s. quod in dubiū reuocari nō potest, cum. f. s. superet. r.x.per.o.s:t.p.et. i. g. At in genere minoris inæqualitatis, triplum. r.x. sit. x. a. et. a. b.sit æqualis.z.h. & cū z.h. tribus differētijs.n.h:o.s:t.p.superet.e.g. quæ in. a. b. sint. b. x: x. d: d.c. ex quo. a. c. æqualis erit. e. g. et. a.x.cum.b. c.tripla.e. g. Itaque tota summa.b.x.quadrupla erit.e.g.

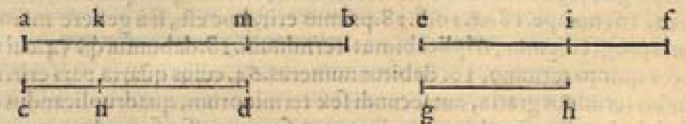
## THEOREMA LXXVIII.

QVantitates quæ fuerint inuicem in proportionalitate arithmetica proportionales, permutan do quoque proportionales erunt.

G 2 Sint

Sint exempli gratia.4.quantitates.a.b:c.d:e.f:et.g.h: inuicem proportionales in proportionalitate arithmetica. Hoc est vt quæ proportio (licet impropriè dicta) est ipsius.a.b.ad.c.d.eadē sit ipsius.e.f.ad.g. h. Tunc permutando dico eandem proportionem fore ipsius.a.b.ad.e.f.quæ ipsius. c.d.ad.g.h.

Nam, ex hypothesi, differentia qua.a.b. superat. c.d. (quæ sit.m.b.) æqualis est differentiæ qua.e.f.superat.g.h.(quæ sit.i.f.) vnde.a.m.residuum ex. a.b.æquale erit c.d.& residuum.e.i.æquale.g.h. Sit igitur exempli gratia. c. d. maior.g.h.per. c.n. vnde.n.d.æqualis erit.g.h.quare.a.m. maior erit.e.i.per. a. k.æqualem.c. n. ex communi scientia. Vnde.k.m.æqualis erit.n.d.hoc est ipsi.g.h.hoc est ipsi.e.i. Quare ex communi conceptu. b. k.æqualis erit ipsi.f.e.sed.n.d.æqualis est.g. h. vt dictum est. Cum ergo.b.k.æqualis sit.e.f.et.d.n.ipsi.g.h.et.a.b.maior sit ipsa. k.b.per.a.k.æqualem ipsi.c.n.per quam.c.n.d.c.maior est ipsa.d.n.sequitur verum esse propositū hoc est, quod eadem proportio sit ipsius.a.b.ad. e.f.quæ.c.d.ad.g.h.arithmetice scilicet.

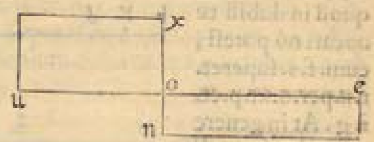



## THEOREMA LXXIX.

CVR prouenientia duorum numerorum diuidentium eiusdem numeri diuisibilis, geometricè eandē inter se proportionē seruant, quā ipsimet diuidētes.

Exempli gratia si per senarium & octonarium numerus vigintiquatuor diuidatur, prouenientia erunt.4.et. 3.eadem proportione, qua diuidentes.

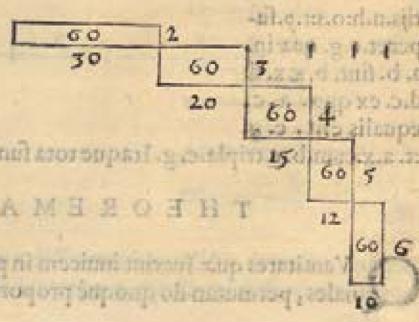
Cuius est ratio numerus diuisibilis significetur rectangulis.u.x. et.n.e.diuidentes autem sint.u.o.et.e.o.quare ex ijs, quæ. 10. theoremate dicta fuerunt.u.x.per.u.o.diuiso dabit. x.o.& diuisō.n.e.per.e.o.dabit.o. n. Dicimus itaque eandē esse proportionē o. x.ad.o.n.quæ.e.o.ad.o.u.quod patet sub scriptam figuram considerantibus, in qua, ex.15.sexti aut.20. septimi, eadem proportio cernitur. o. x. ad.o.n.quæ. o. e. ad.o.u.




## THEOREMA LXXX.

CVR quauis quantitate, tribus aut quatuor aut etiam pro libito pluribus diuidentibus numeris diuisa, prouenientia eandem prorsus inter se proportionem seruabunt, quam ipsi diuidentes habere comperiuntur.

Exempli gratia, proponitur numerus.60.quinque numeris diuidendis, vtpote.30.20.15.12.10. prouenientia erunt.2.3.4.5.6.eadem




pro-

proportione diuidentium, quamuis ex aduerso.

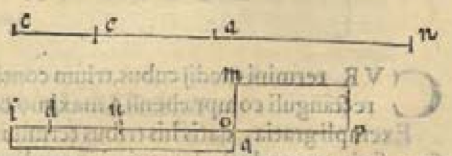
Cuius ratio ex.15. sexti aut.20.septimi dependet. prout in subscripto ordine facillimè deprehendi potest.

## THEOREMA LXXXI.

CVR quantitate in tres continuas partes proportionales secta, & per singulas ipsarum diuisa, summa trium prouenientium quadrato medij prouenientis æqualis est.

Exempli gratia, proponitur. 14. diuidendus in tres continuas partes proportionales, nempe.8.4.2.ipseq; numerus.14.per singulas diuiditur, ex quo tria prouenientia oriuntur, nempe ex prima parte. 8. proueniés erit. 1. cum tribus quartis partibus ex secunda.4. datur proueniens.3.cum dimidio vnius, & ex tertia. 2. prouenient. 7. integri, qui in summam collecti dant. 12. integros & vnam quartam partem tantumdem, videlicet quantum quadratum prouenientis medij, nempe. 3. cum dimidio.

Cuius speculationis gratia, totalis numerus significetur linea.n.c.qui in tres partes diuidatur.n.a: a.e.et. e.c.quæ sint continuæ proportionales, quarum singulis, numerum.n.c. diuisum esse cogitemus, proueniens autem ex diuisione.n.c. per. n. a.sit.i. d.quod verò prouenit ex diuisione.n.c.per.a.e.sit.d.u.proueniens quoque ex diuisione.n.c.per.e.c.sit.u.o.quorum summa sit.i.o.quæ asseritur esse numeri æqualis numero quadrati.d. u.Quod hac ratione probabo, producatur linea.i. o. donec. o.p.æqualis sit.o.u. erigaturq;.o.m.æqualis.d.i. perpendiculariter.o.p.in puncto. o. quæ producatur donec.o.q.vnitati sit æqualis, termineturq; duo rectangula. m. p. et.q.i.ex quo habebimus rectangulum, aut productum.m.p.æquale quadrato. d. u. ex. 16 sexti aut. 20. septimi, quandoquidem tria prouenientia.o.u.u. d. et. d. i. ex præcedenti theoremate sunt inter se continua proportionalia, proportionalitate qua partes. n.c.Iam verò si probauero.q.i.productum, producto.m.p.æquale esse, propositum quoque probatum erit.Numerus enim producti.q. i. æqualis est numero. summæ.i.o.Habemus autem ex definitione diuisionis ita se habere.n.c.ad.i.d. sicut. n.a.ad.o.q. Itaque permutando sic se habebit.n.c.ad.n. a.sicut.d.i.hoc est.m.o.ad. o.q.sed sicut se habet.n.c.ad.n. a. ita pariter se habet.i.o.ad. o.u.hoc est ad.o.p.Itaque.i.o.ad.o.p.sic se habebit sicut.m.o.ad.o. q.ex quo ex. 15. sexti aut. 20. septimi. q. i.æqualis erit.m.p.& consequenter quadrato.d.u. Vt autem lector minori labore cognoscere queat.i.o.ad.o.u.sic se habere, vt.n.c.ad.n.a. sciendum est quòd, sic se habet. i. d.ad.d.u.ut.c.e.ad.e.a.ex quo componendo sic se habebit.i.u.ad.d.u. sicut.c.a. ad.a.e.& permutando ita.i.u. ad.c.a.vt. d.u.ad.e.a.sed cum ex præcedéti theoremate sic se habeat. d. u. ad.u.o.sicut.e.a.ad.a. n. permutandó sic se habebit.d.u.ad.a.e.sicut.u.o.ad a.n.ex quo ex. 11.quinti sic se habebit.i.u.ad.c.a.prout. o.u.ad.a. n. permutandoque. i. u. ad.u.o. vt. c. a.ad.a. n. & componendo, ita.i. o.ad.u.o. sicut.c.n. ad.a. n.

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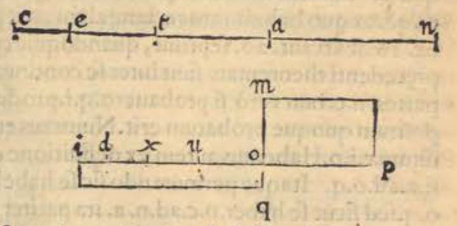
## THEOREMA LXXXII.

CVR quantitate aliqua in quatuor partes cõtinuas proportionales secta perque singulas diuisa, summa quatuor prouenientium æqualis sit producto secundi in tertium.

Exempli gratia, si triginta in quatuor partes proportionales secetur, hoc est. 16.8.4.2.perq; harum singulas idem numerus. 30. diuidatur, primum proueniens erit. 1.cum septem octauis partibus. Secundum.3. cum tribus quartis, tertium. 7. cum dimidio, quartum. 15. integri, quorum summa erit.28.cum octaua parte, tantumq; erit productum secundi prouenientis in tertium.

Quod vt sciamus, quantitas.n.c. in partes continuas proportionales quatuor secetur. n.a:a.t:t.e. et. e.c.rursusq; per singulas partes illa ipsa diuisa, prouenientia sint. i.d:d.x:x.u:u.o.quorũ summa sit.i.o.hanc summã dicimus æqualem esse numero producti.d.x.in.x.u.

Quod hac ratione probo, cogito productam esse lineam.i.o.quousq;.o.p.æqualis sit.o.u. erectamq;.m.o.æqualem.i.d.perpendiculariter.o.p.& productam donec. o.q.vnitati sit æqualis. Iam terminatis rectangulis.m.p.et.i.q. patebit ex. 15. sexti aut.20.septimi, productum.m.p.producto.d.x.in.x.u. æquale esse. Ita quòd si probauero productum.i.q.producto.m.p.æquale esse, facile patebit propositum. Cuius gratia, sequuti præcedentis theorematis ordinem, primum ex definitiône diuisionis, eadem proportio erit.n.c.ad.i.d.quæ.n.a.ad.o.q.ex quo permutando.n.c.ad.n.a.sic se habebit vt. i.d. hoc est.m.o.ad.o.q.& si progrediamur eodem ordine, quo præcedenti theoremate, sumpto principio ab.i.d.et.e.c.versus.d.x.et.e. t. gradatimque permutando ac coniungendo, inueniemus eandem proportionem esse c. n.ad.n.a.quæ.i.o.ad.o. u. nempe. o.p.ex quo ex.11 quinti, ita se habebit.i.o.ad.o.p.vt. m.o.ad.o.q.quare ex.15. sexti aut. 20. septimi productũ.i.q.erit producto.m. p. æquale, ex quo etiam æquale erit producto. d.x.in. x. u. Idem ordo in qualibet quantitate in quantasuis partes diuisa seruari poterit, cum huiusmodi sciẽtia in vniuersum pateat.




## THEOREMA LXXXIII.

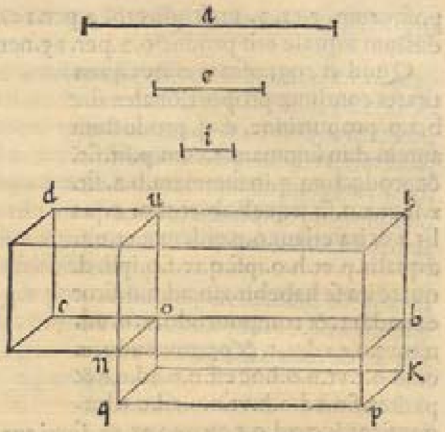
CVR termini medij cubus, trium continuè proportionalium, semper producto rectanguli compræhensi à maximo & medio in minimo termino æqualis sit.

Exempli gratia, datis his tribus terminis continuis proportionalibus. 9.6.4. si sumpserimus productum maximi in medium nempe.54.quod per minimũ.4.multiplicemus, dabitur numerus.216. cubo medij.6.æqualis.

In cuius gratiam tres numeri continui proportionales tribus lineis. a.e.i. significẽtur, cubus autem.e. significetur figura. d. n.productumq;.a.in.e.sit.b.n. ipsius autẽmet in.i.sit.p.o.ita quod.q.p.aut.b.o.cum sint eiusdẽ speciei, æqualis erit.a:et.o.n.

æqua-

æqualis.e:et.q.n. æqualis. i. Nunc cogitemus absolui corpus. n.h. ita ut. b. o.c.sit vnica recta linea,ex quo ex.25. vndecimi proportio. n.h.ad. n. k. eadem est quæ. o. h.ad o.k.sed sic se habet.o.h.ad.o.k.vt.h.b.ad.b.k. ex prima sexti aut.18.vel.19.septimi itaque.n.h.ad.n.k.ex.11. quinti sic se habebit.vt. h. b. ad. b. k. sed.n.h.ad.n.d.ex eisdem sic se habet ut.h.u.ad.d.u.et.h.u.ad.u. d. ita ut. h. b.ad.b.k. ex præsupposito. Itaque ex 11. prædicta. n.h.ad. n. k. eadem erit proportio quæ. n. h. ad. n. d. Quare ex.9.quinti. n. k. æqualis erit. n. d. Quod erat propositum.

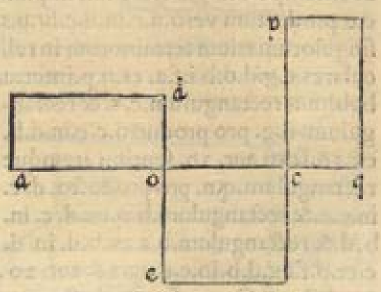


## THEOREMA LXXXIIII.

CVR quadrato vnius quantitatis radice proportionalis,per singulos tres terminos diuiso, prouenientia, singulis dictis terminis sint æqualia.

Exēpli gratia, datis tribus terminis continuis proportionalibus.9.6.4. quadratum medij erit. 36. quod per. 9. diuisum dabit. 4: per.6:6.per.4:9.

Cuius gratia,sint tres termini cōtinui ꝓportionales.a.o:o.c.et.c.q. quadratū autē medij sit.e.c. Iam si applicetur rectangulū. a. d. æquale quadrato.e.c.ipsi.a.o. & rectangulum.q.p. æquale eidem quadrato.e.c.ipsi.c.q. si quadratum.e.c.per.a. o.diuiserimus, proueniens erit.o.d.diuisoq; per.c.q.proueniens erit. c.p.quod si per suam radicem.o.c.diuidatur, proueniens erit. o. e.quod sine dubio æquale est. o.c. sed dico. o.d.æqualem esse.c.q. Nam ex.16.sexti aut 20.septimi eadem est proportio.a. o. ad. o. c. quę.o.e.ad.o.d.nempe.o.c.ad.o.d.itaque o.d.ex.9. quinti æqualis est.c.q. quandoquidem ex.11.sic se habet.o.c.ad. o. d. sicut.o. c.ad.c.q. Applicatis ijsdem rationibus ipsi. p.c.probabimus.c.p.æqualem esse.a.o. cum o.c.media sit proportionalis,tā inter. c.p. et c.q.quam inter.a.o.et.c.q.itaque.c.p.æqualis est. a.o.

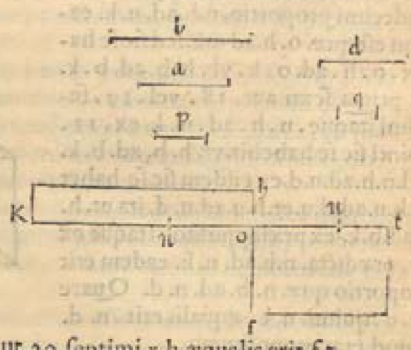


## THEOREMA LXXXXV.

CVR propositis tribus quantitatibus continuis proportionalibus proportione aliarum duarum nobis datarum, multiplicata maiori postremarum duarum in summam mediæ cum minima trium primarum,productum æquale sit producto minoris duarum in summam maximæ cum media trium.

Exempli gratia proponuntur quantitates. 9.6.4. proportione numerorum propositipositorum. 3.et. 2. multiplicato. 3.per.10.summã. 6.cum. 4.dantur: 30: quod productum æquale erit producto.2.per.15.nempe per summam.9.et. 6.

Quod vt cognoscamus,tres quantitates continuæ proportionales sint b.a.p. proportione. d. q. productum autem.d.in summam.a.cum.p.sit.f.t. & productum.q.in summam.b.a. sit. x.h.et.x.n.sit æqualis.b.et.n.o. æqualis.a.& ita etiam.o.u.eidem.a.et.u.t. æqualis.p. et.h.o.ipsi.q.et.f.o.ipsi. d. quare ita se habebit.x.n.ad.n.o.sicut o.u.ad.u.t. & componendo. x. o. ad. n.o.vt.o.t.ad.u. t. & permutando. x. o.ad.o.t.vt.n.o.hoc est.o.u.ad.u.t.& pariter.f.o.ad.o.h.vt.o.u.ad.u.t.Itaque sicut.k.o.ad.o.t.ex quo ex.15.sexti aut.20.septimi.x.h.æqualis erit.f.t.




## THEOREMA LXXXVI.

CVR multiplicatis singulis tribus quantitatibus continuis proportionalibus in reliquas duas, sex producta æqualia sint producto dupli summæ ipsarum trium in mediam proportionalem.

Exempli gratia, proponuntur hi tres termini continui proportionales.9.6.4.productum.9.in.6.erit.54.at.9.in.4.erit.36. et.6.in.9:54.et.6.in.4:24.et. 4.in.9:36.et. 4.in. 6:24.quæ producta simul collecta efficiunt numerum. 228 sed tantũ est productum dupli summæ trium terminorum in secundum nempe.38. in.6.

Cuius intelligẽtiæ causa,tres termini cõtinui proportionales significentur linea. b. e.nempe.b.d:d.c:c.e. cuius duplum sit.u.e.et. b.f.æqualis sit.b.d.et.f.n:d.c.et.n.u: c.e productum verò.u.e.in.d.c.sit.u.s.cui dico æqualem esse summam productorum singulorum trium terminorum in reliquos duos. Quamobrem ducantur perpendiculares.c.g:d.o:b.i:f.a. et.n.p.inter.u.e.et.q. s.ex quo pro producto. c.e.in. c. d. habebimus rectangulum.c. s. & rectangulum. d.g. pro producto.c.e.in.d.b. ex.16.sexti aut. 20. septimi itemque rectangulum.q.n. pro producto. d. c. in.c.e.& rectangulum.b.o.ex.d.c. in. b. d.& rectangulum.b.a.ex.b.d. in. d. c.et.p.f.ex.d.b.in.c.e.ex.16. aut. 20. prędictas. Quare sex producta æquantur inter se,replentq; productum.u.s.ex quo verum est propositum.


## THEOREMA LXXXVII.

QVA ratione cognosci possit verũ esse proportionem summæ quatuor quantitatum continuarum proportionalium ad summam secundæ & tertiæ, eandem esse, quæ summæ primæ & tertiæ ad secundam simplicem.

Exempli gratia, si inuenirentur hæ quatuor quantitates continuæ proportionales.16.8.4.2.earum summa erit.30.summa verò secundæ & tertiæ.12, tum summa

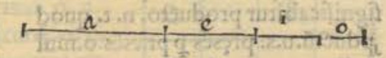
primæ

primæ cum tertia. 20. ex quo sic se habet. 20. ad. 8. nempe ad secundam, vt. 30. ad. 12.

Quod vt sciamus, quatuor prædictæ quantitates significentur linea. a. e. i. o. probabo ita se habere. a. e. i. o. ad. e. i. vt. a. i. ad. e. Nam cum sic se habeat. a. ad. e. ut. e. ad. i. & vt. i. ad. o: ex æqualitate proportionum vel permutando ita se habebit. a. ad. i. vt. e. ad. o. & è conuerso ita. o. ad. e. vt. i. ad. a. & cōponendo ita. o. e. ad e. vt. i. a. ad. a. permutandoq;. o. e. ad. i. a. vt. e. ad. a. nempe. i. ad. e. & componendo ita. o. i. e. a. ad. i. a. vt. i. e. ad. e. & permutando ita. o. i. e. a. ad. i. e. vt. i. a. ad. e. quod erat propositum. Ex quo patet error antiquorum qui idipsum, accidere arbitrati sunt in quantitatibus discretæ proportionalitatis, quod tamen falsum est.

Exempli gratia, si proponantur. 12.6.4.2. proportio. 12. ad. 6. eadem est quæ. 4. ad. 2. Sed à proportione. 6. ad. 4. frangitur, cum non sit eadem quæ. 12. ad. 6. harum autem summa erit. 24. & summa secundæ cum tertia. 10. sed primæ cum tertia erit 16. ex quo. 16. ad. 6. non sic se habebit vt. 24. ad. 10.

At in speculatione quatuor quantitatum. a. e. i. o. si proportio. e. ad. i. non esset eadem quæ. a. ad. e. minimè licuisset dicere ita se habere. i. ad. e. vt. e. ad. a.

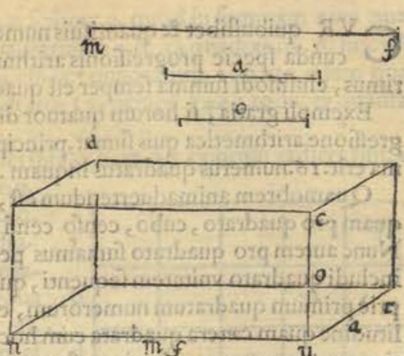



## THEOREMA LXXXVIII.

CVR ex tribus quantitatibus quibuslibet, productum duarum in tertiam, vna semper eademq; sit quantitas.

Exempli gratia, proponuntur. 15.8.2. si multiplicauerimus. 15. per. 8. tum productum per. 2. tantum erit quantum si quis multiplicaret. 8. per. 2. & hoc per. 15. et. 15. per. 2. rursusq; per. 8.

Quod ut pateat, tres quantitates tribus lineis significentur. m. f. a. et. o. Dico productum. m. f. in. a. multiplicatum. per. o. æquale esse producto. a. in. o. multiplicato per. m. f. aut producto. m. f. in. o. multiplicato per. a. Sit enim corpus. d. u. rectāgulum, cuius latus. n. u. sit æquale m. f. et. u. t: a: et. u. c: o. patebit manifestè n. t. esse productum. m. f. in. a. quod. n. t. multiplicatum in. u. c. æquali. o. producit corpus. d. u. sed idipsum corpus. d. u. ex multiplicatione producti. c. t. in latus. n. u. æquale. m. f. oritur, & idipsum. d. u. ex multiplicatione. n. c. in latus. u. t. æquale. a. profertur.




## THEOREMA LXXXIX.

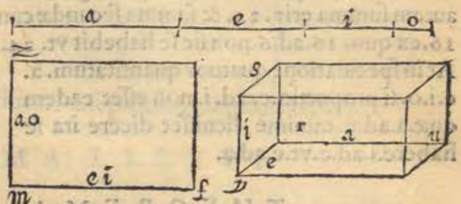
CVR quarumcunque quatuor quantitatum, si prima in secundam multiplicetur & hoc productum in tertiam, rursusq; hoc alterum in quartam, vltimum productum æquale sit producto producti secundæ in tertiam, in productum primæ in quartam.

H Exempli

Exempli gratia, casu sese offerunt hi quatuor numeri.8.5.3.2.multiplicato.8. per.5. & hoc. 40.per.3.rursus hoc.120. per.2.vltimum productum esset.240.æquale producto.15.( quod ex.5.in.3.oritur ) in productum. 16. quod ex. 8. in. 2. profertur.

Cuius speculationis gratia, cogitemus quatuor numeros quatuor lineis. a. e. i.o. significari, productum autem.e.in.i.esse.m.f.et.r.s.similiter & productum.a.in.o.esse. m.z:et.z. f.productum esse.m. f.in.m. z.cui productum.a.in.e. multiplicatum per i. & hoc tandem per.o.æquari debet.

Sit itaque.u.y.productum. a.in. e.quod.u. y. per. i.multiplicatum proferat.u. s. hocque. u.s. multiplicatum per.o.Dico quod dabit numerum æqualem numero.f.z. Quamobrem. r.s.aut. m. f.quod idem est, in figura præcedentis theorematis significetur linea.n.u. & linea. r. u. huius, nempe. a. significetur per. u. t. præcedentis, ex quo numerus producti. u.s. præsentis,in præcedenti significabitur producto. n. t. quod ꝓductū.u.s. pręsēs ꝑ præsēs.o.multiplicatum, quod erat in præcedenti.u.c.significabitur per. d.u. præcedentis, quod non modo ex multiplicatione.n.t.præcedentis, nempe.u.s.præsentis.in.u.c.præcedentis æquali. o. præsentis oritur, sed etiam ex.c.t.præcedentis æquali. m. z.præsentis in.n.u. præcedentis æquali.m.f.præsentis. Itaque verum est propositum.

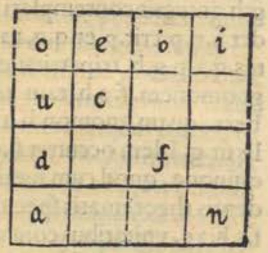



## THEOREMA XC.

CVR quibuslibet & quantisuis numeris in summam collectis,si ab vnitate in secunda specie progressionis arithmeticę imparium numerorum progressi fuerimus, eiusmodi summa semper est quadratus numerus.

Exempli gratia, si horum quatuor disparium numerorum summā, in dicta progressione arithmetica quis sumat,principio ab vnitate sumpto,nempe.1.3.5.7.summa erit.16.numerus quadratus inquam. Idem de cæteris.

Quamobrem animaduertendum est, vnitatem,tam sumi pro sui ipsius radicem, quam pro quadrato, cubo, censo censi, primo relato, & alia quauis dignitate. Nunc autem pro quadrato sumamus per. o.significato, cogitemusq; quadratum.o. includi quadrato vnitatem sequenti, quod, vt patet, est quatuor vnitatum, ac propriè primum quadratum numerorum, ex quo etiam nomen accepit, vnde ex similitudine quam cætera quadrata cum hoc primo retinent,ex quaternario denominationem acceperunt. Hoc itaq; sit. o.u.c.e.ita ex communi scientia quadrato.o.iungitur gnomon.e.c.u.constans tribus vnitatibus, quare primus gnomon,numero impari constat.Scimus etiam ex additione numeri binarij ad imparem, numeris disparibus summam excrescere,cum propius accedere quā binario nequeant, ex quo medio binario, sibi inuicem succedunt. Dico igitur quòd quinario ternarium sub sequente, coniuncto quadrato.o.u.c.e.profertur quadratum, quod in numeris, binarij quadratum sequitur,eritq; ternarij,quodq; significetur per.o.f. patet enim primo non differre ab.o.c.præter quam gnomone.b.f.d. qui coniungitur quadrato. o. c.quique duabus vnitatibus maior est.e. c.u. Iā scimus gnomonem.e. o. u. æqualem eſſe gnomoni.e.c.u.itemq; gnomonem.b.f.d.æqualem gnomoni.b.o.d.at hic gnomon.b.o.d.ex præſuppoſito, maior eſt gnomone. e.o.u. duabus vnitatibus. b. et. d. Itaque etiam gnomon.b.f.d. duabus vnitatibus gnomonem.e.c.u.ſuperabit. Quare. b.f.d.erit impar immediatè ſequens ternarium, qui coniunctus quadrato. o. c. quadratum ſubſequens componet. Eadem ratione probabitur de quadrato.o.n.ſequenti. o. f. & gnomone. i. n. a. cum hic ordo ſpeculationis ſit vniuerſalis. In quo cernitur quemlibet gnomonem ſibi contiguũ inferiorem ſemper duabus vnitatibus excedere, cumque quadrata non niſi gnomonibus ſibi inuicem ſuccedant. Sed cũ primus.e.c.u.diſpar fuerit, ꝓculdubio etiã neceſſarioq; cæteri diſpares erũt.



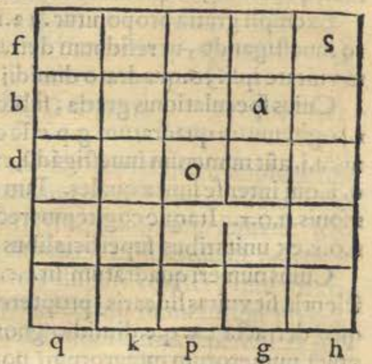

Ex qua ſpeculatione, oritur regula ab antiquis tradita inueniendi vltimi numeri diſparis cõcurrentis ad cõpoſitionem alicuius quadrati. Vt ſi quis ſcire deſideret numerum vltimum diſparem, quo mediante quadratum. o.n.conſtitutum fuit, quod aliud non eſt quam ſcire quantus ſit numerus vltimi gnomonis.i.n.a.æqualis gnomoni.i.o.a. Itaque vt ſciamus hunc gnomonem.i. o. a. patet duplicandam eſſe radicem.o.e.b.i.dabiturq;.o. e. b.i.et.o.u.d.a.vbi bis reperitur.o.nos autem tantummodo quærimus ſcire gnomonem.i.b.e. o. u. d. a. Itaque minor eſt vnitate duplo radicis, cum vnitas.o. bis repetatur, quæ tamen in gnomone ſemel tantum ſumebatur.

## THEOREMA XCI.

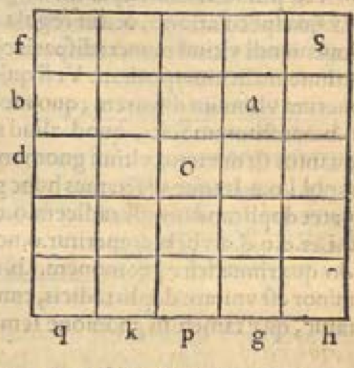
CVR ſumma quadratorum, quorum radices ſunt in proportione ſeſquitertia nempe.4.ad.3.quadrata ſit.

Exempli gratia, ſumemus quadratum.3. ſcilicet 9. quod in ſummam cum quadrato.4.colligemus, nempè. 16.eritq; quadratum.25.& ita quadratum. 6. hoc eſt. 36.collectum cum quadrato.8. nempè.64.efficiet quadratum. 100. ita etiam quadratum. 9. hoc eſt. 81. coniunctum quadrato. 12. nempè.144.producet quadratum.225.

In cuius gratiam ſint duo quadrata ſubſcripta.q.o. et.q.a.quorum radices ſint. q. g.et.q.p.hoc eſt.q.g.quatuor vnitatum, et.q. p.trium, ex quo.q.a.erit. 16.vnitatum et.q.o. nouem. Ad hæc cogitemus applicari quadrato.q.a. gnomonem.f.s.h.tam amplum ſiue latum quã gnomon.b.a.g. nempè vt. h. ſit æqualis.g:g. verò differentia ſit qua.q.g. maior eſt. q.p.huncq; gnomonem.f.s.h.dico ęqualem eſſe quadrato.q.o.nam ex pręſuppoſito.g.ter radicem.q.p.ingreditur, & quater.q.g. ex quo, tres partes.q k. p. inter ſe æquales ſunt vnde etiam quadratum.q.o.nouem partibus ſuperficialibus quadratis conſtabit,quarum ſingularum radix æqualis erit. g.cumque præcedenti theoremate didicerimus quemlibet gnomonem quadrati immediatè ſequentis æquę amplitudinis cum gnomone præcedentis,



per duab.vnitatibus ſuperficialibus creſcere,quarũ ſingularũ radix æqualis eſt.g. neceſſariò ſequitur gnomonem. b.a.g. duabus partibus aut vnitatibus gnomonem. d.o.p. ſuperare, ita vt gnomon. b.a.g.ſeptem vnitatibus,aut partibus ſuperficialibus quadratis conſtet. Quare eadem ratione gnomon.f.s.h.conſtabit nouem ſimilibus. Itaque æqualis erit quadrato.q.o. Quamobrem verum eſt, quòd quadrato. q. o. coniuncto quadrato.q.a.proueniet quadratum.q.s.cuius radix ita differet à. q. g.vt. q.g.à.q.p: ex quo tres radices arithmeticè inter ſe continuæ proportionales erunt. Idipſum dico ſi.q.p.fuerit. 6.et.q.g:8:tunc enim ſingulæ partes.q. k.p.g.h. æquipollebunt duabus vnitatibus, quæ cogitabuntur in ſummam collectæ, ut cum partibus.q. k. p. g.h. integris contemplari liceat. Idem accidet ſi.q.p.erit.9,et.q.g.12. ſingulæ enim partes.q.k.p.g.h. tripartitæ erunt. Idcircò dixi gnomonem. f. s,h. tam amplum cogitari debere, quam gnomon.b.a.g.nempè ut.h.æqualis ſit.g. Idem occurret ſi.q.g.erit. 12.et. q. p. quinque, quod cum fuerit patebit ex præcedentis theorematis ſpeculatione, gnomonem f.s.h:25. vnitatibus conſtare, cogitatum amplitudinis ſimplicis vnitatis denominatæ in.q. p.aut.q.g.non amplitudinis gnomonis. b.a. g. qui ſeptem vnitatibus latus eſſet. Cum igitur. q.p.quinque vnitatibus linearibus conſtet ſcimus.q.o:25.ſuperficialibus conſtare, collecto itaque in ſummam quadrato.q.o.cum quadrato.q.a. cognoſcetur quadratum.q.s.vnà etiam eius radix. Eadem ratione, aliæ multa quadrata ſimilia contemplari licebit.




## THEOREMA XCII.

CVR propoſito numero pari maiori binario, qui detrahi & in ſummam colligi debeat ex altero numero quærendo, vt tam reſiduum quam ſumma ſint quadrata numerorum integrorum. Rectè dimidium propoſiti numeri in ſeipſum multiplicamus, & quadrato huic addimus vnitatem, eritq; numerus quæſitus.

Exempli gratia proponitur. 12.numerus detrahendus, & coniungendus numero inueſtigando, ut reſiduum detractionis, & ſumma ſint quadrati numeri. Addita vnitate ipſi.36.quadrato dimidij, dabitur. 37. numerus quæſitus.

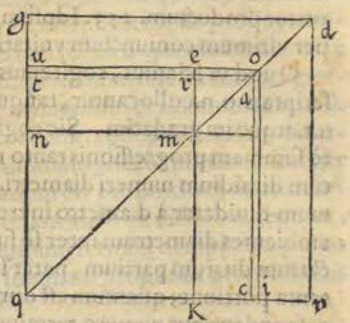
Cuius ſpeculationis gratia, ſubſcripta quatuor quadrata cogitemus.g.p:u.i:t.c:n.k.cogitemusq; quadratum.g.p.eſſe quadratum ſummæ, k. n.verò reſidui ſubtractionis:u.i. aũt numerum inueſtigãdũ,ex quo gnomon.u.d.i.cognoſcetur ita etiam et.n.o. k.qui inter ſe ſunt æquales. Iam certi erimus.c.i.eſſe plus quam dimidium gnomonis.n.o.k. Itaque cogitemus rectangulum.r.c. exactum dimidiũ eſſe gnomonis. n.o.k.ex unitatibus ſuperficialibus quarum una erit.m.a.

Cuius numeri quadratum ſit.t.c.vnde etiam cognitum & cum. k. c. ex communi ſcientia ſit vnitas linearis, propterea quod. m. a. eſt ſuperficialis hoc eſt quadrata, quæ detracta ex. q.c.dimidio gnomonis.n.o.k.(quamuis lineari) ſupererit. k.q.cognita,numerorum integrorum (nota q. k. i.ſemper minor erit duabus vnitatibus linearibus & maior vna ex dictis vnitatibus, ut ex te ipſo contemplari potes) quare.

n. k.

n.k.ipſius quadratum numerorum integrorum cognoſcetur, cui addito gnomone. n.o.ĸ.cognoſcemus numerum.u.i.quæſitum.

Sed cum nobis hæc via, tenenda propoſitum non fuit, hoc eſt primo loco inueniendi quadrati minoris.n.ĸ. ideo ſupereſt probandum gnomonem.t.o.c. vnitati ęqualem eſſe, nempe quadratulo.m.a.quod patebit, ſi conſideremus nos ſumpſiſſe rectangulum.r.c.pro dimidio gnomonis.n.o.ĸ.etenim ſi ſupplemento etiam.n.r.quadratulum æquale. m.a.adderetur, pateret gnomonem. n.a. ĸ.cum dicto quadratulo collectum, æqualem eſſe gnomoni.n.o.ĸ:cum duo ſupplementa.m.t.et.m.c.inter ſe ſint æqualia. Quamobrem inuento quadrato.t.c. ex dimidio gnomonis cognito, additur vnitas, gnomon ſcilicet.t.o.c.ex quo cognoſcitur numerus. u. i. quæſitus. Quod autem quadratum.g.p.numeris integris conſtet, hac ratione probatur viſum enim fuit ſupra quadratum.n. ĸ.verè quadratum eſſe, & numeris integris conſtare, pariter etiam.t.c.ſeq; mutuò conſequi ( nam.ĸ.c.eſt vnitas linearis) ex quo gnomon n. a.ĸ. numero diſpari conſtabit, ex ijs quæ.90. theoremate probata fuerunt. Itaq; ex eodem theoremate neceſſe eſt gnomonem.t.d.c.etiam numero diſpari conſtare, ita vt à numero.n.a.ĸ.non niſi duabus vnitatibus differat, nempe vt.c.p.ſit vnitas linearis, ſed ita reuera eſt, numerus enim.u.d.i. ex præſuppoſito par eſt, quare numerus. t.d. c. diſpar erit, cum alterum vnitate ſuperet, videlicet gnomone.t.o. c. vnitati æquali, tum.n.a. ĸ.minor eſt.n.o.ĸ. ex eodem gnomone.t.o.c. unitati æquali. Itaque.n.a.ĸ.minor erit.u.d.i.per vnitatem,& minor.t.d.c.per duas unitates,ex quo ſequitur.g.p.eſſe quadratum integrorũ ex dicto theoremate ac conſequens quadrato t.c.quare.c.p.vnitas erit, & radices. q.ĸ.et.q.p.horum quadratorum numero binario inter ſe different. Vnà etiam ſcienda eſt cauſa, cur numerus propoſitus neceſſariò binario maior eſſe debeat. Etenim cũ ipſe ſit futurus gnomon.n.o. ĸ. nec poſſit minor eſſe numero ternario, vt patet ex. 90.theoremate, idcirco ſequitur neceſſariò maiorem eſſe binario debere. Quòd ſi diſpar numerus proponeretur, nec forma operis nec ſpeculationis mutãda eſſet. Non erit tamen neceſſarium vt ipſa quadrata.n.ĸ.et. g. p. numeris integris conſtarent. Sæpius enim fractis cõponerentur, quod ex. 90. theoremate facile erit ſpeculari nihilominus fractis integris, ipſisq; collectis cum ſuis fractis ſummæ eſſent quadratæ.




## THEOREMA XCIII.

CVR propoſitis duobus numeris altero pari, altero verò diſpari, duplo primi minore per vnitatem, ſi alium inuenire numerum voluerimus, cui alterum iſtorum coniunctum proferat quadratum, & altero detracto, quadratum ſuperſit. Rectè datos numeros in ſummam colligemus, quam ſummam in duas quam maximas poterimus partes diuidemus, quarum vna pari, altera diſpari conſtet, tum vtranque in ſeipſam multiplicabimus, & quadrato minori, duorum numerorum propoſitorum quemuis ademus, ex quo cupimus nobis quadratum minus ſupereſſe, & prouenient nobis numerum quæſitum.

Exempli gratia, proponuntur numeri. 11.et. 6.quorum alter alicui numero addendendus, alter ex eodem detrahendus sit, ex quo proferri debeant bina quadrata. Itaq; numeri illi in summam collecti dabunt. 17. differentiam minoris quadrati & maioris. Iam si ex hoc. 17. binas partes fecerimus, altera erit. 8. altera. 9. quibus in seipsis multiplicatis alterum quadratum erit. 64. alterum. 81. addito itaq; ipsi. 64. 11. aut. 6. pro libito, propositum numerum consequemur. cui addito. 6. vel. 11. dabit nobis. 81. vel ex ipso detracto. 11. vel. 6. relinquet nobis 64. in pręsenti autem exemplo talis numerus erit, aut. 70. vel. 75. Huius autem theorematis speculatio ex. 90. dependet, quo demonstratum fuit gnomonem proximè quadratum sequentem, vnitate duplo radicis minorem esse.

## THEOREMA XCIIII.

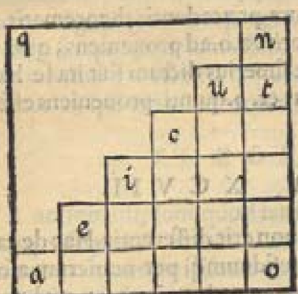
CVR si quis cupiat summam progressionis arithmeticæ quam citissimè cognoscere. Rectè coniunget vltimo termino vnitatem primum terminum, huius postea vltimi termini dimidium cum numero terminorum multiplicabit, ex quo multiplicationis productum, erit omnium propositorum terminorum summa, aut eundem vltimum terminum iunctum primo, per dimidium numeri terminorum multiplicabit. Nam idipsum eueniet.

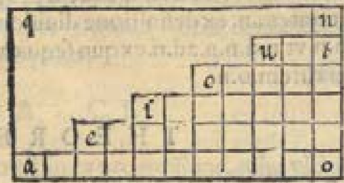
Exempli gratia, si proponerentur. 17. termini in prima progressione arithmetica naturali, vltimus esset. 17. cui coniuncta vnitate primo termino summa erit. 18. cuius dimidium cum numero terminorum, nempe. 17. multiplicatum cum fuerit, oritur productum. 153. Idpsum eueniet, multiplicato dimidio numeri terminorũ per vltimum coniunctum vnitati primo termino.

Quod vt sciamus, cogitemus terminos progressionis collocari, vt in figura subscripta. a. o. n. collocantur, tanquã per gradus, sumpto principio ab vnitate. n. tum. u. t. atque ita gradatim. Sic cogitato absoluto parallelogrammo. q. o. sciemus apertè summam progressionis tanto maiorem esse dimidio totius parallelogrãmi, quantum dimidium numeri diametri. a. e. i. c. u. n. requirit. Nam cum parallelogrammum diuidatur à diametro in tres partes, diameter vnam occupat, reliquæ verò duę ambientes diametrum inter se sunt æquales. Sumpto itaq; diametro cum altera dictarum duarum partium, patet sumi plusquam dimidiũ totius parallelogrãmi. pro tanta portione, quantum est dimidiam occupatam à diametro, qui cũ ex discretis respondentibus numero terminorum componatur, constat numero æquali esse dicto numero terminorum. o. n. Iam si quis multiplicet. a. o. per dimidium. o. n. procul dubio, ex prima sexti aut. 18. septimi, orietur dimidiũ numeri parallelogrãmi. q. o. quod minus erit summa progressionis dimidio numeri diametri, aut quod idem est dimidio. o. n. sed hoc dimidium. o. n. æquale est producto dimidij vnitatis. n. in. o. n. ex. 20. septimi, cum dimidium. o. n. sit eius productum in vnitatẽ. Itaque multiplicato. n. o. per dimidium. o. a. coniunctum dimidio vnitatis. n. oritur summa quæsita propositæ progressionis. Idipsum accidet multiplicata summa. o. a. & vnitate. n. p dimidium. o. n. ex. 20. septimi, cum proportio totius ad totum eadem sit, quæ dimidij ad dimidium, ex causa permutationalitatis. Patet etiam in progressionibus, quæ ab vnitate initium ducunt, si fiat ascensus per binarium summa vltimi termini cum primo semper duplam futuram esse numero terminorum, quod sequentes figu

ras

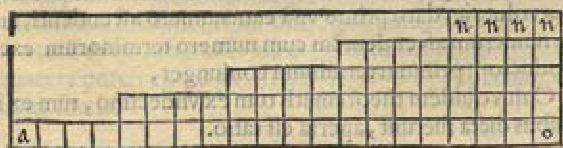
ras consideranti speculari licebit, Diametros harum figurarum notaui literis siue characteribus.a.e.i.c.u.n.




## THEOREMA XCV.

IN progressionibus, quæ ab alio termino quam vnitate incohantur, idipsum vt monuimus accidit, hoc tamen notato, quòd ex consequenti quælibet pars diametri parallelogrămi, minimo termino æqualis erit, prout in progressionibus quæ ab vnitate originem ducunt, singulæ partes diametri, vnitati sui primi termini æquales sunt. At in reliquis progressionibus, vt in figura patet, eadem est proportio totius diametri ad.o.n.quæ minimi termini ad vnitatem ex. 13. quinti, nempe. a.o.ad.o.n.vt.n.n.n.n.ad.n. In eiusmodi progressionibus accidit quoque parallelogrămum à diametro in tres partes diuidi, quarum vnam ipse occupat, reliquæ vero inter se æquales ipsum ambiunt. Ex quo illud etiam sequitur, productum.a.o. in dimidium.o.n.æquale esse dimidio parallelogrămi, quod minus est summa progressionis dimidio diametri, quod dimidum si inuenire voluerimus, minimum terminũ. n.n.n.n.per dimidium.o.n.multiplicabimus, & ex. 18.aut.19.septimi ipsum habebimus, quandoquidẽ minimo termino per totum.o.n.multiplicato profertur integer diameter ex.20.prædicti. Etenim vt diximus, eadem est proportio totius diametri ad.o.n.quæ minimi termini ad vnitatem. Ita etiam dico ex dicta. 20. septimi. idem dimidium diametri oriri, si quis dimidium minimi termini nempè.n.n. per totum.o.n.multiplicauerit. Quamobrem qui statim summam propositæ progressionis cognoscere voluerit, semper primum terminum.n.n.n.n.cum.a. o. coniunget, qua summa per dimidiũ.o.n.multiplicata, aut.o. n. per dimidium dictæ summæ, ex prædictis rationibus propositum consequemur.




## THEOREMA XCVI.

CVR si quis numerum terminorum inuenire velit, cognitis tantummodo prĩmo atque vltimo, rectè vltimum per primum diuidet, ex quo proueniens nume-

numerus quæsitus erit.

Quod intelligendum est tamen quoties primus terminus differentia terminorũ est, nempe ascendens ipsorum terminorum.

Cuius ratio manifestè speculari potest in figura præcedentis theorematis. Nam diuisa . a . o . per . n . n . n.n.eadem proportio erit . a.o.ad proueniens, quæ . n . n.n. n.ad vnitatem.n. ex definitione diuisionis. At superius dictum fuit ita se habere.a. o.ad.o.n.vt.n.n.n.n.ad.n.ex quo sequitur ex. 11. et. 9.quinti proueniens esse numerum quæsitum.o.n.

## THEOREMA XCVII.

VBI verò primus terminus, reliquorum non erit differentia. Hac de causa necesse est detrahere primum ex vltimo, residuumq; per numerum ascendentem differentiam scilicet, partiri, proueniensq; vnitati coniungere, quò numerum terminorum habere possimus. Scimus etenim tam multas vnitates esse in vltimo terminorum quot in omnibus interuallis aut differentijs in summam collectis simul cum vnitatibus primi termini, totq; sunt termini, quot interualla simul cum primo termino. Quare si minimus terminus interuallo æqualis fuerit. Vltimo per primum diuiso, ex adductis præcedenti theoremate propositum consequemur. Itaq; primo termino ex vltimo detracto residuoq; per interuallum, hoc est numerum differentiæ diuiso, proueniens erit numerus terminorum absque primo qui vnus est, coniuncto quoque dicto prouenienti propositum consequemur.

## THEOREMA XCVIII.

CVR si quis arithmeticæ progressionis dato primo & vltimo simul cum numero terminorum, ascendentem numerum cognoscere voluerit. Rectè primum ex vltimo detrahet, residuumq; per numerum terminorum excepto vno diuidet.

Huius theorematis speculatio ex. 13. theoremate manifesta erit, nam in præcedenti cap. numerus terminorum erat proueniens diuisionis residui subtractionis primi termini ex vltimo.

## THEOREMA XCIX.

CVR si quis maximum omnium terminorum dictæ progressionis cognoscere voluerit, dato primo vnà cum numero ascendenti, numeroq; terminorum. Rectè numerum ascendentem cum numero terminorum excepto vno multiplicabit, productoq; primum terminum coniunget.

Cuius quidem theorematis tum ex vndecimo, tum ex ijs quæ præcedentibus capitibus dicta fuerunt, aperta est ratio.

## THEOREMA C.

CVR veteres cupientes obtinere summam progressionis continuæ naturalis, quæ ab vnitate initium ducit, dato vltimo termino tantummodo. Dimidium vltimi termini cũ toto sequenti multiplicabant, productumq; summa quæsita erat.

Exempli gratia, si vltimus terminus eiusmodi progressionis fuerit. 7. multiplica-

to

to dimidio ipsius nempe. 3. & dimidio, cum numero ipsum terminum sequēti, nempè. 8. summa dictorum terminorum erit. 28.

Huius autem speculatio ex. 94. theoremate dependet, in quo facilè deprehendere licet ex figura continuæ progressionis naturalis, numerum terminorum maximo termino semper æqualem esse; ex quo tātum est dimidium numeri terminorum, quantum maximi dimidium, tantusq; est vltimus terminus vnitati coniunctus, quantus numerus is, qui vltimum terminum consequitur.

## THEOREMA CI.

CVR antiqui idipsum, quod iam dictum est, in ea progressione, cuius vltimus terminus dispar est scire cupientes, numerum integrorum proximè dimidium maximi sequentem sumebant, quem per maximum multiplicabant, ex quo summa quæsita oriebatur.

Exempli gratia, si dimidium maximi fuisset. 3. cum dimidio, sumebant quatuor, & per maximum. 7. multiplicabant, ex quo pariter proferebatur summa. 28.

Cuius ratio ex. 20. septimi Euclidis oritur, cum eadem sit proportio numeri sequentis maximum ad numerum dimidium maximi sequentem, quæ maximi ad suū dimidium, est enim dupla.

## THEOREMA CII.

TRaditum est à nonnullis, à veteribus obseruatam fuisse hanc regulam, qua scire possent summam alicuius progressionis arithmeticæ discontinuæ aut intercisæ, quæ numero pari terminetur. Multiplicabāt enim dimidiū vltimi termini per proximum numerum dimidio dicto maiorem, ex quo inquiebāt semper productum summæ quæsitæ æquale esse, subijciuntq; exemplum progressionis, quæ à binario inchoata crescit per binarium. In qua quidem progressione non per se, sed per accidens regula vera est. Hoc est, non quia ex se vnus ex producentibus numeris dimidium termini maioris futurus sit, alter uerò proximè sequens dimidium, sed quia vt dictum est. 95. theoremate, eadem est proportio maximi termini ad numerum terminorum, quæ minimi ad vnitatem. Cumque in præsenti exemplo minimum sit duplum vnitati in eiusmodi casu, numerus terminorum, dimidio maximi termini æqualis est, qui terminorum numerus ex se, vt patet, vnus est ex producentibus, alter verò producens numerus, est proximè dimidium sequens, non ex se, sed quia numerus sequens, dimidium est summæ maximi, & minimi, quæ per se alter esse debet producens numerus. In cæteris enim progressionibus, quæ binario non crescūt regula falsa est, prout facilè patere potest ei, qui ex scientiæ legibus ope speculationis. 95. theorematis speculatus fuerit.

## THEOREMA CIII.

ALIAM quoque tradunt regulam, qua veteres vsos fuisse dicunt, quo summam scire possent progressionis discontinuæ, quæ numero dispari absoluitur. Ea autem est eiusmodi. Vltimum terminum in duas quam maximè poterant maximas partes diuidebant, quarum vna semper altera maior erat, hanc autem maiorem in seipsam multiplicabant, atque quadratum hoc, summam progressionis esse affirmabant. Quæ ſanè regula, non ſemper, etſi interdum vera ſit.

Sumebant hi exemplum progreſſionis, quæ ab vnitate incohata creſcit per binarium, in qua per accidens euenit vt numerus dimidium vltimi termini proximè ſequens, nempe è duabus partibus vltimi termini maior, æqualis ſit numero terminorum, qui per ſe vnus è producentibus, ex ijs quę.94.theoremate diximus, eſſe debet; alter vero producens, qui per ſe dimidium ſummæ primi & vltimi eſſe debet, per accidens pars maior eſt duarum vltimi termini, & alteri producenti æqualis.

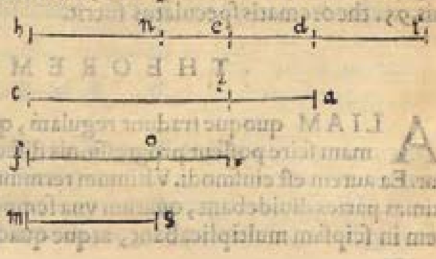
Aut alio modo ratiocinemur, dicentes, in huiuſmodi progreſſione dimidium ſummæ vltimi termini cum primo, ſemper medium proportionale eſt inter eam ſummam & dimidium numeri terminorum, etenim huiuſmodi ſumma numero terminorum ſemper dupla eſt, prout.94.theoremate tradimus. Itaque ex.20.ſeptimi, quadratum partis maioris, producto ſummæ dictæ in numerum dimidij terminorũ æquale erit, quod productum per ſe ſummæ progreſſionis eſt æquale. At in cæteris eiuſmodi progreſſionibus fallit regula, vt ex ſupradictis facilè demonſtratur.

## THEOREMA CIIII.

PErmultis terminis ad libitum propoſitis, diſpoſitis nihilominus progreſſione, aut proportionalitate geometrica continua, ſi minimus ex maximo & ex ſequenti minimum detrahatur, reſiduum maximi, eam proportionem ad ſummam reliquorum omnium terminorum retinebit, quam reſiduum ſecundi ad primum.

Proponuntur, exempli gratia, quatuor termini.3.12.48.192.continui geometricè proportionales, ſi primum, hoc eſt minimum, ex ſecundo, & maximo detrahas, ex ſecundo ſupererit.9.ex maximo.189.quod ſi minimum per reſiduum maximi multiplicaueris, hoc eſt.189.orietur.567.tum ſi huiuſmodi productum per.9.(reſiduum ſecundi) diuiſeris, proueniet.63.quod proueniens æquale erit ſummæ reliquorum omnium terminorum, maximo excepto. Ex quo inferre licet ex.20.ſeptimi eandem proportionem eſſe.189.ad.63.quæ.9.ad.3.aut ſi reſiduum ſecundi per ſummam dictorum terminorum multiplicaueris producetur idem.567.quare ex.20.ſeptimi & cætera.

Quod vt ſciẽtificè poſſimus, & in vniuerſum ſpeculari. Quatuor termini propoſiti, quatuor ſubſcriptis lineis ſignificẽtur.b.i:c.a:f.r:m.s.(quod aũt de his quatuor dico de centũmillibus, & eo amplius dicere poſſum.) Nunc minimus terminus.m.s.ex maximo.b.i.detrahatur, ſuperſitq;.n.i.idemq;.m.s.ex ſecundo termino.f.r.ſubtrahatur, ſuperſitq;.o.r.Dico proportionem.n.i.ad ſummam reliquorum omnium terminorum.c.a:f.r:m.s.eandem eſſe, quæ.o.r.ad.m.s. Quamobrem ex tertio & quarto ſecundus.f.r.detrahaſ, ex tertioq; ſuperſit.t.a.& ex quarto.e.i.ita etiam tertius.c.a.ex quarto.b.i.ſuperſitq;.d.i.ſanè ſic ſe habebit.c.a.ad.f.r.vt.c.t.ad.f.o.vt quiſq; per ſe ſcire poteſt. Quare ex 19.quinti ſic ſe habebit.a.t.ad.r.o.vt.c.a.ad.f.r.& permutando ita.a.t.ad.a.c.vt.o.r.ad.r.f.& ſeparando ſic.a.t.ad.a.c.(hoc eſt.f.r.) vt.r.o.ad.o.f.videlicet.m.s.Idẽ dico de.d.i.ad.a.c.nempe ſic ſe habebit.d.i.ad.a.c.vt.a.t.ad.



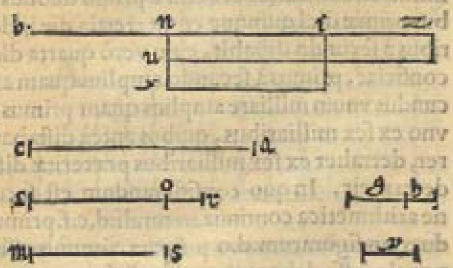
r.f. hoc est.o.r. ad. m.s.ex.11.quinti. Itaque ex communi scientia sic se habebit.d.i.ad.d.b.vt.e.d.ad.e.b: cum.e.d.æqualis sit.r.a. Ita etiam vt.e.n.ad.n.b:cum.n.e.æqualis sit.o.r.Iam si sic se habeat.d.i.ad.d.b.vt.d.e.ad.e.b.permutando quoq; sic se habebit.d.i.ad.d.e.vt.d.b.ad.b.e.& componendo ita.i.d.e.ad.e.d.vt.d.b.e.ad.e.b. & permutando sic.i.d.e.ad.d.b.e.vt.de.a.d.e.b.nempe vt.e.n.ad.n.b.& permutando ita.i.d.e.ad.e.n. vt.d.b.e.ad.b.n.& componendo ita.i.d.e.n.ad.n.e.vt.d.b.e.et.b.n.ad.b.n.& permutando sic.i.d.e.n.ad.d.b.e.et.b.n.nempe ad. a.c:f.r: m.s:vt.e.n.ad.n.b. hoc est.ut.o.r.ad. m.s.quod erat propositum.

## THEOREMA CV.

CVR desideranti summam quorumcunque terminorum progressionis continuæ geometricæ cognoscere.Rectè minimus terminus ex maximo detrahendus est, residuumq; per denominantem progressionis dempta vnitate diuidendum, prouenientiq; maximum terminum addendum, ex quo oritur summa quæsita.

Exempli gratia, si darentur quatuor termini continui proportionales. 8. 12. 18. 27. primum hoc est minimum. 8.ex vltimo.27.detraheremus: remaneretq;.19.qui per denominantem progressionis, dempta vnitate, diuideretur. Quo loco animaduertendum est, quamlibet denominationẽ cuiuscunque proportionis numerorum supra vnitatem fieri, nam de proportionibus multiplicibus dubitandum non est, & idipsum de superparticularibus, & superpartientibus est intelligendum, vt in præsenti proportio sesquialtera inter duos terminos cogitanda est, nempe inter vnum & dimidium, atque vnum. Sesquitertia autem inter vnum & tertiam partem, & vnum. Sesquiquinta inter vnum cum quintæ parte, & vnum. De superpartientibus idem assero quod de proportione superbipartiẽte tertias appellata, vt.5. ad.3.quæ cogitanda esset inter vnum duas tertias, & vnum, superbipartiens quartas inter vnum tres quartas, & vnum, ita vt minor terminus, numerans scilicet, semper sit vnitas, alter verò denominans. Idem de cæteris. Quare in præsenti exemplo, detracta vnitate ex denominante progressionis, supererit tantummodo dimidium, quo diuiso. 19. proueniet.38.qui numerus æqualis erit summæ reliquorũ omnium terminorum,cui coniuncto vltimo termino.27.dabitur summa quæsita.65

Pro cuius speculatione, quatuor termini significentur, quatuor lineis.m.s:f.r:c.a.b.i. primus autem terminus.m.s.ex vltimo.b.i.detrahatur, residuumq; sit.n.i. & ex secundo.f.r.cuius residuum sit. o.r. proportio verò progressionis ea sit,quæ. g. h. ad. y. quo vnitas repræsentatur ( ex quo sic se habebit. g.h.ad.y.vt.f.r.ad.m.s.)qua.y.detracta ex.g.h.supersit. h. Tum erecta cogitetur linea.n.u.x. indefinita perpendicularis.b.i.à puncto.n.quę diuidatur in puncto.x.ita vt.n.x. æqualis sit vnitati.y.& in puncto. u. ita. vt.n.u. æqualis sit. h. ex quo eadem erit proportio.n.u ad.n.x.vt.h.ad. y. nẽpe.o.r.ad.m.s.Nam cũ sic se habeat.f.r.ad.m s.hoc est ad.f.o.vt.g.h.ad.y hoc est ad.g. permutando quoq; sic se habebit.f.r.ad.g.h.vt.f.o.ad.g.Itaque ex. 19.quinti.o.r.ad.h.vt.f.r.ad.g.h.ex quo ex.11.eiusdem.o. r.ad.h.vt. f. o. ad



g.& permutando.o.r.ad.f.o.hoc eſt ad.m.s.vt.h.ad.g.hoc eſt . y. Quamobrem eadem erit proportio.o.r.ad. m.s.quæ.n.u.ad.n.x. Abſoluantur itaque duo rectangula.x.i.et.u.z.ita tamen vt rectangulũ. u. z.cogitetur ęquale rectangulo.x.i.cuius.x.i. ſuperficialis numerus ex communi conceptione lineari.n.i.æqualis erit, quare ex eadẽ communi conceptione, numerus ſuperficialis.u.z.lineari.n.i.æqualis erit,qui quidem numerus in figura rectangula ſuperficialis cogitandus erit, cum diuidendus ſit per.h.hoc eſt per.n.u. ex quo proueniens ex huiuſmodi diuiſione erit numerus . n. z. ex ijs quæ. 10.theoremate dicta fuerunt. Sed ex. 15. ſexti aut. 20. ſeptimi eadem eſt proportio.n.i.ad. n. z. quæ.n.u.ad.n.x.hoc eſt.o.r. ad. m. s. videlicet vt.n.i.ad aggregatum reliquorum omnium terminorum.c.a: f. r:m.s. ex præcedenti theoremate, & ex. 11. quinti Euclidis.Itaque ex.9. eiuſdem numerus.n.z.æqualis erit ſummæ trium terminorum.c.a:f.r:m.s.cui coniuncto quarto termino.b.i.propoſitum obtinetur.


## THEOREMA CVI.

PRopoſuere veteres quæſita nonnulla de itineribus interq́; hoc vnum fuit. Ponamus duos iter agere per eandem viam quorum alter quatuor milliaria ſingulis diebus conficiat,alter verò prima die milliare vnum, ſecunda duo, tertia tria, atque ita ſingulis diebus milliare addit; quærimus quot dierum ſpacio ſocium conſequetur.

Quamobrem numerus milliarium primi viatoris duplicatur, ſic ſunt.8.milliaria. ex quo ſemper vnitas detrahitur, quæ in præſenti exemplo erit. 7. totq́; dies erunt quibus ſocius ſocium conſequetur, & milliarium numerum æqualem abſoluerit.

Cuius rei facilis erit ſpeculatio, ſi ſubſcripta figura diligenter conſideretur, in qua primus viator, die prima, quatuor milliaria linea.q.d.ſignificata conficit, atque illa ipſa die alter vnum tantum deſignatum per.d.perficit, ita vt primus viator tribus milliaribus ſocium anteceſſerit, altera verò die ſecundus uiator cum duo milliaria cõficiat,excedetur à primo duobus milliaribus tantummodo, quę cum tribus primæ diei quinque erunt; tertia die ijſdem de cauſis primus ſex tantum milliaribus à ſecundo diſtabit, cum verò quarta die tot ſecundus quot primus milliaria conficiat, primus à ſecundo amplius quam antea non diſtabit; quinta verò cum ſecundus vnum milliare amplius quam primus conficiat. propius accedit ad primum vno ex ſex milliaribus, quibus anteà diſtabat, tum ſexta cum duobus primum ſuperet, detrahet ex ſex milliaribus præteritæ diſtantiæ tria, ſeptima tandem illa ſex detraxerit. In quo conſiderandum eſt ſecundum viatorem iter agere progreſſione arithmetica continua naturali.d.c.f.primum autem per rectangulum. q. f. quarũ duarum figurarum.d.o.p.f.pars cõmunis eſſe reperitur, quæ quantitates ſi inuicem æquales eſſe debent, neceſſe eſt ſeparatas partes.u.q.n.et.t.i.c.inter ſe æquales eſſe, & quoniam quarta die ( hoc eſt die ſic diſtante à primo, nempè numero milliarium

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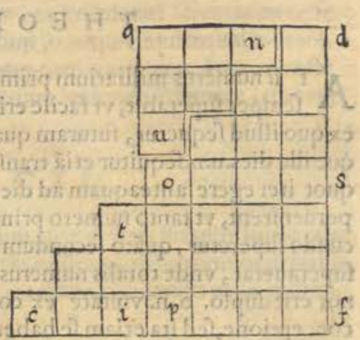
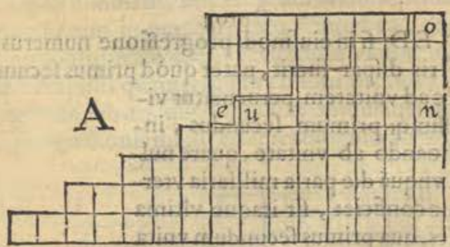
primi viatoris ) tot milliaria absoluat vnus quot alter absque vlla differentia, quæ significetur per.o.s.necesse est itaque ex communi conceptione tot dies esse post.o.s. quot antecesserant, vt excessus æqualis sit defectui, qui simul collecti, iuncta etiam.o.s.duplum erunt d.s.dempta vnitate, prout facilè in subscripta figura qui sque per se scientificè poterit speculari. Quamobrem consultum erit duplicare numerum.o. s. & ex duplo vnitatem detrahere, quandoquidem dies supra infraq;.o.s.cum die. o.s.minores sunt duplo numeri.d.s.aut.o. s. (quod idem est) vnitate.




## THEOREMA CVII.

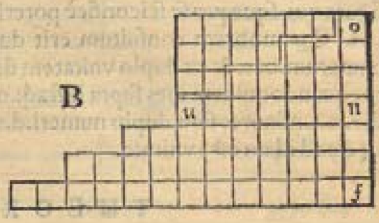
QVOD si secūdus viator ordinē secūdæ progressionis arithmeticæ seruās iter agat, nempe ea quæ ab vno per binarium ascendit, semper numerus dierum æqualis erit numero milliarium diurnorum primi viatoris.

In cuius gratiam animaduertendum est numerus ne milliarium diurnorum primi viatoris par an impar sit. Etenim si par est, primus viator in fine singulorum dierum primæ medietatis numeri omnium dierum secundum antecedet numero dispari milliarium; altero verò dimidio numero dierum, à secundo numero etiam dispari præteribitur, vt in sequenti figura patet. Nam prima die, secundus ex primo milliare vnum ex numero pari, qui à primo conficitur detrahit; secunda verò die idem secundus, duo subtrahit milliaria ex dispari, qui primo reliquus fuerat, sicq; perpetuò dispar remanet vsque ad vnitatem, ad quam cum peruenerint, nempe ad illius diei exitum, quo primus secundum vnitate tantummodò superat, manifestè deprærhendetur subsequente die secundum vnitate primum superaturum, altera verò tribus vnitatibus, prout penultima die secundus à primo tribus vnitatibus superabatur. Quare necesse erit, tot diebus secundum cum primo iter agere, inchoando ab ea die, qua secundus primam superabit, quot egerat dum à primo superaretur, vt ex communi conceptione, media figura. A. deprærhendi potest. Quod autem singula dimidia dierum, dimidia sint numeri milliarium diurnorum primi, patebit ex sequenti figura, cogitato termino. u. n. vltimo progressionis superatę à primo vsque ad vnitatem.e.qui terminus u.n.coniunctus primo.o.nempe. e. semper duplū est numeri terminorum.o.n. vt. 94. theoremate circa finem dictum fuit. Sed.u.n.cum.e. numero æquali constat numero milliarium diurnorum primi viatoris, ex quo sequitur totum numerum dierum, quorum. o. n. dimidium est, æqualem esse numero milliarium diuruorum primi viatoris.



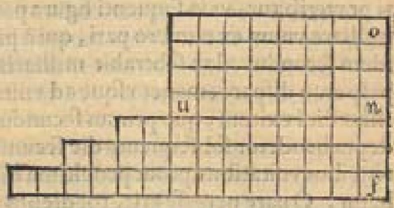
## THEOREMA CVIII.

AT si numerus milliarium primi viatoris dispar fuerit, secundum numero pari semper superabit, vt facile erit sequentem figuram consideranti intelligere, ex quo illud sequetur, futuram quandam diem, qua paria milliaria conficient. Sitque illa dies.u.n.sequitur etiã transacta ea die, tot diebus vtrique ambulandum esse quot iter egere anteaquam ad diem. u. n. peruenirent, vt tanto numero primus à secundo superetur, quãto secundum primus superauerat, vnde totalis numerus.o.f. minor erit duplo. o. n. vnitate ex communi conceptione, sed ita etiam se habet terminus.u.n.hoc est minor duplo.o.n.per. o. vt 94.theoremate dictum fuit, itaque.o.f. æqualis erit.u.n.quod erat propositum.



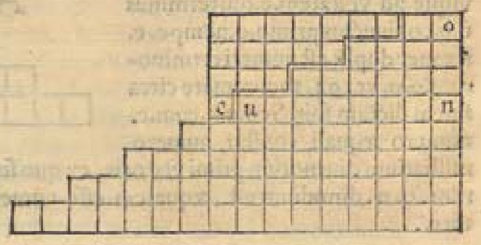

## THEOREMA CIX.

SIN verò progressio secundi viatoris, non ab vnitate sed à binario inchoata, per binarium quoque ascenderet, numerusq; milliarium diurnorum primi viatoris par esset, absque dubio quadam die paria milliaria vterq; conficeret, quę significetur.u.n.qua transacta, tot diebus vtrique ambulãdum erit, quot fuerũt dũ primus secundum superaret, vt totidem alijs primus à secundo superetur, in qua tamen progressione terminus.u.n.semper duplus est numero terminorum.o.n.ex.95.theoremate, totq; sunt infra.u.n.termini vsque ad.f.quot supra.ex quo illud sequitur omnes terminos aut dies.o.n.f.pauciores esse u.n.vnitate, atque ita præcipit regula detrahendam esse vnitatem ex numero milliarium diurnorum primi viatoris, si dierum numerum habere voluerimus.




## THEOREMA CX.

SED si in eiusmodi progressione numerus milliarium diurnorum primi viatoris dispar fuerit, patet quòd primus secundum numero dispari superabit, donec ad vnitatem perueniatur vicissimq; primum secundus, inchoando ab vnitate, quare nulla vnquã die paria milliaria vterque conficiet, sit itaque vltima dies, qua primus secundum vnitatem antecedit.u. n. qui terminus duplus est numero terminorum o.n.& cum illa die primus secundum milliario antecedat, sequen

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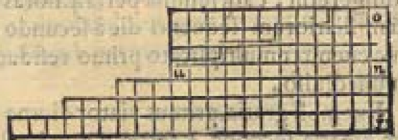
te verò à ſecundo milliario vno primus antecedatur , ex communi ſcientia neceſſe eſt ſecundum tot diebus cũ primo iter agere quot ſunt .o.n.qui ſimul æquales erunt. u.n.ſed.u.n.minor eſt numero milliarium diurnorum primi vnitate. c. Itaque rectè ſequemur regulam , quæ iubet ex numero milliarium vnitatem demere , quo nu merum dierum habere poſſimus.

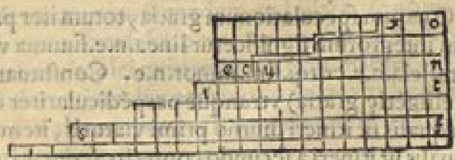
## THEOREMA CXI.

SI verò ſecundi viatoris progreſſio per ternarium aſcenderet, ſumpto initio ab ipſo ternario,animaduertendum eſt an numerus milliarium diurnorum primi, ternario menſuretur nec ne, etenim ſi menſuretur, tandem aliquando paria milliaria conficient,quæ dies ſit.u.n.quare ſub u.n.totidem quot ſupra termini erũt, & cũ. o.n.tertia ſit pars.u.n.ex.95.theoremate. Itaque tota. o. f. minor erit duabus tertijs. u. n. vnitate, vt iam rectè ſumendæ ſint duæ tertiæ partes.u.n. ex quibus vnitas detrahatur ſuperſitque numerus. o.f. dierum quæſitorum.




## THEOREMA CXII.

CVM verò milliarium numerus primi viatoris metiri non poterit à numero aſcendente ſecundi,patet nullam futuram diem qua pari milliaria conficient, quare illa vltima qua primus ſecundum antecedet, vno aut duobus milliaribus antecedet in præſenti caſu. Antecedat itaque duobus milliaribus, ſitq́; dies.u.n.& altera.t.i.ſecundus primum vno milliari ſuperabit, ita quod ſub.t.i. non poterunt plures integros dies iter agere, quam ambulauerunt ante diem. u.n. hoc eſt vſquequo ſecundus iunctus ſit primo, qui numerus dierum, tertia parte.o.n.ipſius.u.n.vnitate minor erit,cum ex. 95.theoremate.o.n.ſit tertia pars. u.n.ex quo numerus. o. f. terminorum aut dierum intergrorum cognitus erit, qui ſi cum numero aſcendente cognoſcetur, ſtatim ex. 99. theoremate deueniemus in cognitionem vltimi diei integri.s.f.atque ita etiam totius ſummæ progreſſionis ex. 95.theoremate. Iam verò cognito numero milliarium diurnorum primi,ſimul cum numero terminorum, aut dierum conſequenter nouerimus rectanguli ſummam, hoc eſt productum à primo viatore formatum, quarum duarum ſummarum in præſenti caſu ſemper ea, quæ huiuſmodi producti eſt, maior erit, cum conſtitutum fuerit ſecundum viatorem à primo ſuperari ipſa die.u.n.vno milliari amplius quam ſequente die.t.i.primus à ſecundo ſuperatur, tum pari gradu iter egerunt ſub.t.i. quo ſupra.u.n. ambulauerant. Hoc animaduertendo, quòd ſi ſumma progreſſionis maior eſſet rectangulo, ex ea ſumma neceſſe eſſet numerũ milliarium vltimi termini in ſumma incluſi detrahere, & reſiduo operari. Nunc verò ſummam progreſſionis ex ſumma rectanguli à primo viatore facti ſubtrahi debet,reſiduumq́; ſeruari voceturq́;

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primū residuū. Ad hæc numerū milliariorū, quæ secūdus viator die seqēnti.s.f.conficiet sumat, ex quo numerus milliariū diurnorū primi detrahatur, residuūq; pariter reseruetur, voceturq; secundum residuum, postmodum numerum milliarium primi vnius diei multiplicetur per primum residuum seruatum, productumq; per secundū residuum diuidatur.a.c.proueniēs (q erit iter primi in sequenti die) iungatur residuo primo, tot enim erunt milliaria conficienda a secundo sequenti die, vt sese consequantur.

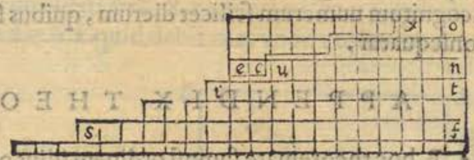
Vt autem sciamus quantam partem diei seqēntis, singulos itinere agere oporteat, proueniēs per.24.horas multiplicetur (supposito quod ambulātes nullā requiē nec die nec nocte capiāt), pductūq; p numerū milliariorum vnius diei primi viatoris diuidatur, ex quo dabitur quātitas horarū, & pars horæ, qua cuiq; illa die ambulandū est. Idem accideret si primum residuum reseruatum cum proueniente in summam colligeretur, eaq; summa per.24.horas multiplicaretur, productumq; per numerū milliariorum sequenti die à secundo conficiendorum diuideretur. Idipsum quoque eueniret multiplicato primo residuo per.24. & producto per secundum residuum diuiso.

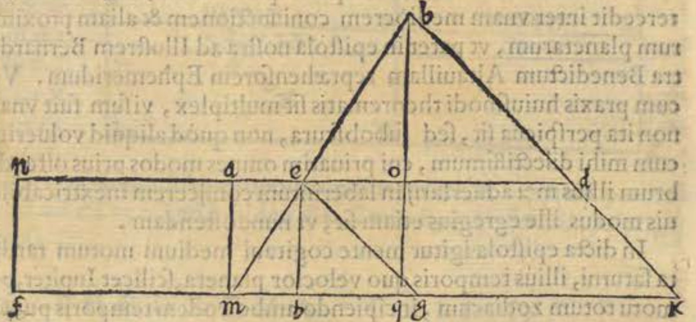
Exempli gratia, primus viator diurna milliaria vndecim conficit, secundus, prima die tria, secunda.6.tertia.9.atq; ita deinceps, diuidatur ergo.11.per.3.vnde pro numero.o.n.dabitur.3.supereritq;.2.quare.u.n.ab.e.n.duobus milliaribus superabitur, et.i.t.dictum.e.n.vno milliario, ex quo ante diem.e.u.n.duobus diebus iter egerunt, totq; diebus ambulandum erit post.t.i.hoc est.6.in vniuersum integris. Ad hęc multiplicato.o.f.hoc est.6.per.x.o.hoc est.3.habebimus.s.f.milliariorū 18.tū cōiūcto.x.o.primo termino hoc est.3.cū.s.f.hoc est.18.vltimo termino, habebimus.21.quo multiplicato cū dimidio.o.f.hoc est.3.habebimus totam summam progressionis.63.sex dierum integrorum ex.94.theoremate, cum multiplicato.11.nempe numero milliariorū diurnorum primi cum.6.hoc est cum.o.f.habebimus parallelogrammum à primo sex diebus integris confectum milliariorum.66.ex quo detracta.63.summa inquam progressionis, supererit pro primo residuo.3.sumptis postea milliaribus.21.pro itinere, quod secundus die sequenti.s.f.conficeret, & ex ijs detracto numero milliariorū diurnorum primi, nempe.11.secundum residuum erit.10.quod pro diuidenti seruabitur. Iam multiplicato.11.cum primo residuo.3.dabitur.33.qui diuisus per.10.secundum residuum profert.3.cum tribus decimis, eritq; iter à primo viatore sequenti die conficiendum, hoc etiam ipsum proueniens cum primo residuo.3.coniunctum, dat.6.cum tribus decimis, quod est iter secundi viatoris illa sequenti die. Ad inueniendam autem quantitatem diei, qua vtrique ambulandum est, perinde erit multiplicare proueniens.3.& tres decimas per.24.horas, & productum per.11.dimidium iter primi viatoris partiri, ac multiplicare summam.6.& tres decimas cum.24.horis, productumq; diuidere per.21.hoc est per iter secundi viatoris sequentis diei, vtrinque enim semper septem horæ cum.12.minutis prouenient. Idipsum accidet multiplicato per.24.horas primo residuo.3.productoq; diuiso per secundum residuum.10.

Quarum speculationum gratia, totum iter parallelogrammi primi viatoris dierum integrorum significetur linea.n.e.summa verò progressionis secundi linea.f.m.parallela.n.e.eritq;.f.m.minor.n.e. Constituamus deinde à termino.f.n.(maioris intelligētię gratia) vtranque perpēdiculariter duci, pducatur deinde.n.e.donec.e.d.æqualis sit itineri diurno primi viatoris, item etiam producatur.f.m.donec.m.k.æqualis sit itineri à secundo confecto sequenti die vltimum integrum progressionis,

nis, ex quo.m.k. prolixior erit.e.d. ex præſuppoſito. Poſtmodum.m.e.et.k.d.duabus lineis rectis coniungantur, quæ productæ concurrent in puncto.b. ducatur pariter.e.g.à puncto.e.parallela.b.k.et.m.a:e.h.et.b.q. parallelæ.f.n.ex quo.f.m.æqualis erit.n.a.et.m.h:a.e.et.h.q:e.o.et.g.k: e.d.et.f.q:n.o.ex.34.primi Eucli. vnde proportio.m.h.ad.h.q.erit vt.m.g.ad.g.k.quandoquidem vtraque æqualis eſt proportioni.m.e.ad.e.b.ex. 2.ſexti, ſed cum.m.k.et.g.k.notæ ſint, pariter cognoſcetur.m.g.ſecundum reſiduum, cum etiam notæ ſint.n.e.et.n.a. Itaque cognoſcemus.a.e.hoc eſt.m.h. cognitis verò.m.g:g.k.et.m.h.ex.15.ſexti aut.20.ſeptimi cognoſcetur.h.q.erit igitur.a.e.aut quod idem eſt.m.h primum reſiduum, et.m.g.ſecundum, et.h.q.aut.e.o.proueniens, et.n.o.et.f.q. itinera vtriuſque viatoris inter ſe æqualia.

Nec verò prætermittenda eſt ſpeculatio vltimæ rationis inueniendæ quantitatis diei, quæ conſtat ope diuiſionis producti.m.h.in.24.per.m.g. Ea autem eiuſmodi eſt. Probatum fuit ſic ſe habere.m.h ad.h.q.ut.m.g.ad.g.k. Itaque componendo ſic ſe habebit.m.q.ad.h.q.vt.m.k.ad.g.k.& permutando.m.q.ad.m.k.vt.h.q.ad.g.k. Sed cum ſic ſe habeat.m.h.ad.h.q.vt.m.g.ad.g.k.permutando ſic ſe habebit.m.h.ad.m.g.vt.h.q.ad.g.k. itaque ex.11.quinti ita.m.h.ad.m.g.vt.m.q.ad.m.k. ex quo permutando m.h.ad.m.q.vt.m.g.ad.m.k. ſed cũ.m.k. ſit motus toti diei reſpondens, ſecurè dicere poterimus, ſi m.g. talis eſt reſpectu horarum. 24.ſignificatarum per.m.k. qualis erit.m.h.& quotæ parti diei reſpondens: quæ poſtmodũ erit.m.q. quæ, vt dictũ fuit, talis eſt reſpectu.m.k. qualis.m.h.reſpectu.m.g. Reliquę duæ ſpeculationes priorum modorũ, vna & eadem eſt, faciliſq; per ſe mediocriter intelligenti. Eodem modo reliquæ omnes progreſſiones ſecundi viatoris cũ rectangulo primi conferri ex hoc theoremate poterunt.



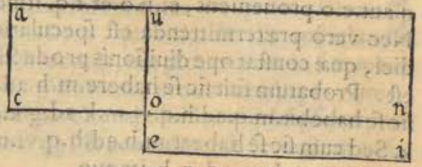




## THEOREMA CXIII.

PRoponitur & aliud, primum ſcilicet viatorem iter incipere diebus aliquot antè ſecundum, primum tamen lentius, quàm ſecundum ambulare, & utrunque eorum certa quædam milliaria conficere. Iam ſi ſcire voluerimus in quot diebus ſeſe conſequentur, uulgaris regula iubet, inſpici quot milliaria primus ſolus iter agens confecerit, tum animaduerti differentiam diurnam motus vnius ab altero, atq; milliarium numerum primi viatoris ſoli abundantis per hanc differentiã diuidi, proueniens autem erit numerus dierum quæſitus.

Exempli gratia, ſi primus octo diebus antequam ſecundus iter arripuiſſet, confeciſſetq; ſingulis diebus.20.milliaria, tum ſecundus.25.quotidie perfeciſſet, multiplicandus eſſet numerus.8.cum.20.ex quo darentur.160.milliaria à primo ſolo ambulāte confecta, quibus diuiſis per.5.differentiam motuum diurnorum, daretur.32.numerus quæſitus dierum.

Cuius ratio apertiſſima eſt. Sint enim duo rectanguli.a.n. et. u.i. æquales inter ſe, quibus motus itinerarium ſignificentur, quorum.a.n.ſit primi, et.u.i.ſecundi, prætereà.a.c.numerum milliarium diurnorum primi.et.u.e.ſecundi, ex quo. a. c. minor erit.u.e.per.o.e. atque ita.o.e.cognoſcetur. Tum.c.o.numerum dierū primi ſoli iter agentis denotet, cūq; conſtituamus.a.n.æqualem eſſe.u.i. o.i.ęqualis erit.o.a.atque.o.a. cognitus ex ſuis producentibus.a.c.et.c.o. itaque.o.i.etiam cognitus, qui diuiſus per latus cognitum.o.e. dabit. e. i.cognitum numerum ſcilicet dierum, quibus ſecundo ambulandum eſt, vt primum conſequatur.




## APPENDIX THEOREM. CXIII.

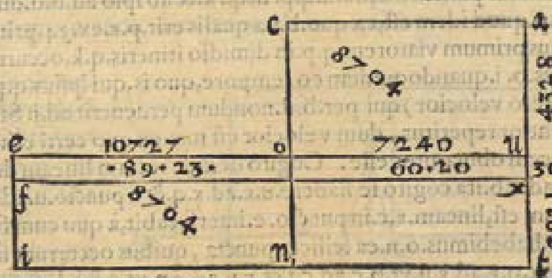
AB hoc theoremate ſumpſi ordinem illius operationis, numeris mediantibus, ad inueniendam exactam temporis quantitatem, ſeu interuallum, ꝙ tranſit, vel intercedit inter vnam mediocrem coniunctionem & aliam proximam ſequentē duorum planetarum, vt patet in epiſtola noſtra ad Illuſtrem Bernardum Trottum contra Benedictum Altauillam repræhenſorem Ephemeridum. Verum tamen eſt ꝙ cum praxis huiuſmodi theorematis ſit multiplex, viſum fuit vnam proponere, quę non ita perſpicua ſit, ſed ſubobſcura, non quòd aliquid voluerim latere illum amicum mihi dilectiſſimum, cui priuatim omnes modos prius oſtenderam, ſed vt cerebrum illius mei aduerſarij in laberintum conijcerem inextricabilem vt feci, quamuis modus ille egregius etiam ſit, vt nunc oſtendam.

In dicta epiſtola igitur mente cogitaui medium motum tardioris planetæ, puta ſaturni, illius temporis quo velocior planeta, ſcilicet Iupiter, percurrit ſuo medio motu totum zodiacum, incipiendo ambo eodem temporis puncto, nec non ab vna eorum media coniunctione, hoc eſt ab eodem zodiaci puncto, in quo coniunctę fuerunt eorum lineæ mediorum motuum, vbi inueni vi regulæ de tribus, quòd Saturnus ſpacio dierum vnius mediocris reuolutionis Iouis, qui ſunt.4328.progreditur medio motu gra.145.min.4.hoc eſt min.8704.poſito quòd ipſe Saturnus perficiat vnam mediam reuolutionem ſpacio dierum.10746.vt dixi. Incipiendo igitur iterum Iupiter aliam reuolutionem percurrere, reperto Saturno per min.8704.ante ipſum ſpacio.4328.dierum, certus eram hos dies ſignificatos eſſe à linea.a.u.vel.c.o.(æquales enim inuicem ſunt) in figura huiuſmodi theorematis, & quòd rectangulum.a.o.præbebat ſummam graduum.145.min.4.hoc eſt min.8704.et quòd.a.c. vel.o.u.ſignificabat iter vnius diei ipſius Saturni, et.u.e.iter vnius diei Iouis. Cogitemus nunc.u.x.ſignificari dies.30.& à puncto.x.productam eſſe.x.f.parallelam ipſi u.o.e.vnde certi erimus rectangulum.e.x.ſignificare iter Iouis ſpacio temporis dierum.30.rectangulum verò.o.x.iter Saturni eodem temporis interuallo, vnde rectangulum,

gulum.e.x.erit minutorum.149.& secundorum.43.et.o.x.minutorum.60.& secun. 20.vt in dicta epistola, vnde rectangulum.o.f.erit min.89.& secun. 23. & quia rectangulum.o.i. æquale est rectangulo.a.o.ergo.o.i.similiter continebit min. 8704. Nunc quia.a.c.vel.o.u. denotat iter vnius diei Saturni et.u.c.vnius diei Iouis vt diximus ergo.u.o. erit minutorum. 2. secun. o. & tertiarum. 40. videlicet tertiarum. 7240.supposito periodo totali ipsius Saturni dierum. 10740.et.u.c. erit minutorũ. 4. secun.59.& ter.27.vel circa hoc est tertiarum.17967.vnde. o.c. erit tertiarum. 10727. Nunc si dixerimus cum.o.c.tertiarum.10727.dat.o.u. vel.a.c. (nam tam vna quam altera est tertiarum.7240.) quid dabit.a.u.vel.o.c. (quia tam vna quam altera est partium.4328.) clarum erit quód dabit.o.n.vel.u.t.uel.e.i. quia tam vna quam altera erit partium.2921.quæ partes coniunctæ cum fuerint cum partibus ipsius.a.u.dabunt totam.a.t.partiũ.7249.quæ erunt tot dies, hoc est periodus quæsita.

Alia methodo similiter possumus idem cognoscere, scilicet dicendo si rectangulum.f.o. quod est minutorum.89. & secun.23.hoc est secundorum.5363.dat rectangulum.o.x. minutorum.60.& secun.20.hoc est secun.3620.quid dabit.a.u.partium 4328.vnde veniet.u.t.partium.2921.similiter, eo quod eadem proportio est rectanguli.f.o.ad.o.x.quæ.e.o.ad.o.u.ex prima sexti, vel.18.19.septimi seu.15. quinti.

Posset etiam aliquis dicere si. f.o.dat.o.x.quid dabit.o.a. vnde veniet.o.t.quo diuiso per.o.u. daret. u. t. quia ita se habet. a. o. ad.o.t.vt.a.u.ad. u. t. ex supra hic iam citatis.




Sed ego, in dicta epistola, aliam methodum obseruaui, quæ est multiplicando minuta. 8704. per. 30.productumq; diuisi per min.5363.quasi dicens. Si.o.f. dat. o.i. quid dabit.e.f. Vnde ex iam supradictis propositionibus veniet. e. i. & quia permutando ita se habet.o.f.ad.e.f.vt.o.i.ad.e.i.ideo dixi, si min.89.cum secun.23.dat.30 quid dabit min.8704.

## THEOREMA CXIIII.

PRoponunt veteres & quærunt aliud, nempe si duo iter agentes, eodem instanti diuersis è locis proficiscantur, ita vt vnus locum vnde alter profectus est petat, alterq; altero velocior sit, quo loco quáue die sibi inuicem occurrent.

Exempli gratia, Patauio profectus quidam Taurinum petit, eodem instanti alter Taurino Patauium, estq; iter.400.milliarium, ille tamen vndecim diebus, hic 9.motu regulari & vniformi appellit. Quærimus quot milliaria quisque confecerit, quotq; diebus iter egerit, priusquam sibi occurrant.

Iubent nos veteres dies vtriusque inuicem inter se multiplicare, eritq; productum.99.item etiam in summam colligere, eritq; summa.20.per quam productũ.99.diuiserimus dabuntur dies.4.cum.19. vigesimis vnius diei. At pro milliaribus vtriusque, pro eo qui.11.diebus iter conficit, multiplicatis.400.per.4.et.19.vigesimis, tum diuiso per.11.dabitur numerus.180.à Patauio Taurinum & è contra, qui

Taurino Patauium.220.quæ quisque confecerit.

Dum autem hæc specularer attentius,occurrit alius soluendi modus,quamuis prolixior. Is aũt est eiusmodi. Accipiatᷓ medietas minoris numeri dierũ, nẽpe.4.cũ dimidio,& per.400.multiplicetur,productũq; per maiorẽ numerum diuidemus scilicet 11.ex quo dabuntur.163.cum.7.vndecimis, quo proueniente è dimidio milliariorũ itineris.290.detracto, & p residuũ nẽpe.36.cũ.4.vndecimis multiplicato productoq; diuiso p summã dimidij itineris.200.cũ primo prouẽtu.163.et.7.vndecimis nẽpe p.363.et.7.vndecimas partes p ueniet.16.cũ.4.vndecimis,quo cõiuncto primo p ueniẽti, primus.180.milliaria cõfecerit, quæ è.400.detracta supererunt.220. pro itinere secundi, qui.9.diebus iter absoluit. Ad hæc si tempus scire velimus eius, qui.11.diebus appellit,multiplicabimus.11.cum.180.productumq; per.400. partiemur, proueniẽtq; paulominus, quam quinque dies, nempe.4.cum.22.horis et.48.minutis, quod tempus vtrique viatori inseruiet, quandoquidem idipsum prouenit multiplicato.220.per.9.productoq; per.400.diuiso.

Huius autem,qui à me præscribitur modi,speculatio talis est. Duo termini duabus rectis lineis æqualibus,& parallelis inter se.b.p.et.d.q.significentur, quæ alijs duabus.b.d.et.q.p.coniungantᷓ,quę parallelæ & æquales erunt ex.33.primi,quibus significentur duo itinera. Viator primus quidem lentior à.b.in.d. velocior à.q.in.p. Iam sumatur punctũ medium.q.p. sitq;.k.& ab ipso ad.b.d.ducatur.k.i.parallela.d.q.aũt b.p.quod idem est,ex quo.b.i.æqualis erit.p.k.ex.34.primi, hoc est.q.k. certiq; erimus primum viatorem.q.p.in dimidio itineris.q.k.occurrere non potuisse viatori ipsius.b.i.quandoquidem eo tempore,quo is,qui ipsius.q.p.mouetur per.q.k.(cum sit altero velocior) qui per.b.d.nondum peruenerit ad.i: Sit itaque punctum.c.in quo lentior reperitur, dum velocior est in.k.ex quo certi erimus eos inter.c. et.i.sibi inuicem obuiaturos esse. Cogito deinde rectam lineam ductam.k.c.& ut se habet.i.c.ad.c.b.ita cogito se habere.u.k.ad.k.q.& à puncto.u.ad.i.duco.u.i.quæ, vt manifestum est,lineam.k.c.in puncto.e.intersecabit,à quo cum fuerit ducta.e.o.n.parallela k.i.habebimus.o.n.ea scilicet puncta, quibus occurrunt sibijpsis, nam cum sic se habeat.q.k.ad.k.u.vt.b.c.ad.c.i.et.k.u.ad.k.n.vt.c.i.ad.c.o.ex similitudine manifesta triangulorum, ex æqualitate proportionum sic se habebit.q.k.ad.k.n.vt.b.c.ad.c.o. & permutando ita.k.q.ad.b.c.vt.k.n.ad.c.o.& cum.q.k.et.b.c.spatia sint temporibus æqualibus confecta, itaque spatia.k.n.et.c.o.ex communi scientia temporibus æqualibus conficientur.

Quare rectè dicimus, si tot diebus à.b.in.d.aliquis peruenit, quot milliaria in dimidio temporis alterius viatoris idem conficiet? ex quo ex regula de tribus quam primum iter.b.c.cognoscitur, quo ex dimidio itineris detracto, remanet.c.i.cognitus, sed cum probauerimus.q.k.ad.k.n.hoc est.i.o.(cum sint æquales inter se, ex.34. primi) ita se habere.vt.b.c.ad.c.o.permutando sic se habebit.q.k.ad.b.c.vt.i.o.ad. o.c.& cõponendo.q.k.et.b.c.ad.b.c.vt.i.c.ad.c.o.quare rectè dicimus si summa.q. k.cum.b.c.dat.b.c.quid dabit.i.c? nempe dabit.c.o.quo coniuncto cum.b.c.cognoscitur.b.o.quo.b.o.detracto ex.b.d.remanet cognitus.o.d.nempe.q.n.illi æqualis ex.34.prædicta. Gratia verò tẽporis patet nos rectè dicere si.b.d.tot diebus absoluitur, aut etiam.q.p:quo.b.o.aut.q.n.absoluetur.

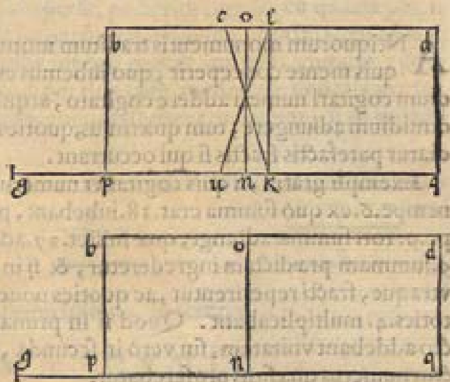
Vt autem ad speculationem regulæ antiquorum deueniamus, cogitemus primum viatorem ipsius.q.p.velociorem eo,qui per.b.d.iter agit, tanto tempore prætergredi.p.quanto alter.b.d.absoluit. Is autem ad.g.pertingat, ex quo eadem proportio spacij.q.g.ad.q.p.hoc est.b.d.dabitur, quæ temporis quo.b.d.absoluitur ab

co

eo qui per . b. d. ad tempus quo . q . p . ſolum , qui per . q . p . mouetur ( motus enim continui regulares & vniformes conſtituuntur ) eadem ratione itaque ea erit proportio . q . κ . ad . b . c . quæ . q . g . ad . q . p . & cum probatum fuerit ita ſe habere . κ. n. ad. c. o. vt. q. κ. ad b. c. itaque ſic ſe habebit . κ . n . ad . c. o. ut. q. g. ad. q. p. probatum etiam fuit ita ſe habere. q. κ. ad. κ. n. vt. b. c. ad. c. o. ex quo componendo ſic ſe habebit. q. n. ad. n. κ. vt. b. o. ad. o. c. & permutando ita. q. n. ad. b. o. vt. κ. n. ad. c. o. hoc eſt. q. g. ad. q. p. nempe vt tempus lenti ad tempus velocis itinerantis, & componendo ita. q. p. cum. o. b. hoc eſt. b. d. ad. b. o. vt ſumma dierũ vnius & alterius viatoris ad minorẽ numerũ dierũ velocioris. Breuiter itaq; obtineremus intentũ qñ diceremus ſi ſumma dierum, quibus iter agitur à viatoribus talis eſt (20) reſpectu numeri dierum velocioris (9) qualis & cui reſpõdebit totum ſpacium. b. d? vnde dabitur ſpacium. b. o. vnde reliqua omnia nobis cognita emergent.

Cum autem antiquorum regula iubeat numerum dierum vnius, cum numero dierum alterius multiplicari, at poſtmodum diuidi productum per ſummam omnium dierum, rectè id quidem fit. Nam cum ſic ſe habeat. b. d. ad. b. o. vt ſumma omnium dierum ad minorem quantitatem dierum velocioris ſcilicet. Ideo temporis proportio à mobili per. b. d. abſumpti ad tempus mobilis per. b. o. eadem erit, quæ ſummæ omnium dierum ad numerum dierum velocioris. Quare rectè dicemus, ſi eiuſmodi ſumma talem reſpectum habet ad minorem numerum dierum. quem numerum reſpiciet dies ipſius. b. d? ex quo proferentur dies reſpondentes ipſi. b . o . cætera iam dicta fuerunt.

Huiuſmodi verò ſpeculationis amplitudo ad pauciſſima verba reduci poteſt, in cuius gratiã ſit ſubſcripta figura pars inquã præcedentis, in qua cõſtituam⁹. o. n. locũ eũ eſſe quo ſibi viatores obuient, ex quo ſpacium. q. n. à ſuo viatore conficietur, eo ipſo tempore, quo à ſuo ſpacium. b. o. itaque eadem erit proportio. q. n. ad. b. o. quæ. q. g. ad. b. d. eadem erit inquã proportio. d. o. ad. o. b. quæ numeri dierum eius, qui à. b. pergit in. d. ad numerum dierum alterius qui à. q. in p. proficiſcitur, & componendo eadẽ erit proportio. d. b. ad. b. o. quæ ſummæ dierum ad minorem numerum ipſorum, & eadem quæ dierum. b. d. ad dies ipſius. b. o.




## THEOREMA CXV.

CIRCA hæc ipſa itinera aliud quæritur peruenuſtè, in quo quæſito illud conſtituitur cognitum eſſe, nempe interuallum inter duo diuerſa loca, è quibus duo viatores eodem inſtanti vt ſibi occurrant proficiſcuntur, certaq; milliaria ſingulis diebus conficiant, ita tamen, ut unus ordinatè plura altero ambulet, quæritur deinde quoto die ſibi occurrent. Hoc autem fit diuiſo toto interuallo locorum per ſummam milliariorum quam vterque quotidie abſoluit.

Exem-

Exempli gratia, distant loca. 100. milliaribus à se inuicem; vnus autem viator singulis diebus. 15 milliaria, alter. 10. conficit si itaque. 15. cum. 10. coniũgamus, summa erit. 25. per quã diuisis milliaribus. 100. totius interualli proferetur. 4. numerus quæsitus dierum quo viatoribus iter agendum erit prius quam sibi obuient.

In cuius speculationis gratiam totum iter significetur linea. a. u: primi autem viatoris iter diurnum sit. a. e. & alterius. u. o: terminus verò. i. sit occursus ita vt eodem tempore, alter spacium. a. i. alter. u. i. confecerit, spacij autem. a. e. tempus per. b. significetur & tempus spacij. u. o. per. c. quæ tempora erunt inter se æqualia, porrò spacij. a. i. tempus per. d. & spacij. u. i. per. f. denotetur, æqualibus inquam, ex quo eadem proportio erit. a. e. ad. a. i. quæ. b. ad. d. et. o. u. ad. u. i. quæ c. ad. f. vnde permutando eadem erit proportio itineris ipsius. b. ad iter ipsius. c. quæ itineris. d. ad iter ipsius. f. & componendo itinerum ipsius. b. c. ad iter. c. vt itinerum. d. f. ad iter. f. & permutando itinerum b. c. ad itinera. d. f. vt itineris. c. ad. iter ipsius. f. meritò itaque quæritur si itinera. b. c. dat itinera. d. f. quid dabit tempus. c. nempe dabit tempus. f. sed. c. signatum est pro vna die, quare in proposito exemplo. f. significabit 4. dies.


## THEOREMA CXVI.

ANtiquorum monumentis traditum motum reperimus diuinandi numeri quem quis mente conceperit, quo iubemus eum qui numerum cogitauerit, dimidium cogitari numeri addere cogitato, atque huic summæ, rursus eiusdem summę dimidium adiungere, tum quærimus, quoties nouenarius totam eam summam ingrediatur patefactis fractis si qui occurrant.

Exempli gratia, si quis cogitasset numerum. 12. iubebant huic dimidium addi, nempe. 6. ex quo summa erat. 18. iubebant, prætereà dimidium huius summæ nempe. 9. toti summæ adiungi, quæ fuisset. 27. adhæc quærebant sibi patefieri quoties. 9. summam prædictam ingrederetur, & si in prima aut secunda diuisione aut etiã vtraque, fracti reperirentur, ac quoties nouem vltimam summam ingrediebatur, toties. 4. multiplicabant. Quod si in prima diuisione fracti erant, vltimo producto addebant vnitatem; sin verò in secunda, binarium adiungebant, ex quo exactus numerus quæsitus proferebatur.

Pro cuius rei ratione sit. a. numerus cogitatione compræhensus et. e. ipsius. a. cum eiusdem medietate summa et. i. ipsius. e. cum eiusdem medietate itidem summa, vnde. i. e. a. tres numeri continui proportionales, in sesquialtera proportione euadent. Sumantur nunc tres numeri. 4. 6. 9. in eadem proportionalitate. Vnde ratione ęqualitatis proportionum ita se habebit. i. ad. a. quẽadmodum. 9. ad. 4. & permutando. i. ad. 9. quemadmodum. a. ad. 4. & ob id. 4. toties ingredietur. a. quoties. 9. ipsam. i. Sed quia sępe contingit, vt in secunda diuisione, aut in ambabus etiam diuisionibus reperiantur numeri fracti, animaduertendum est numerum animo compræhensum. a. scilicet aut parem aut imparem semper futurum. Si par est, aut multiplex erit ad. 4. aut non. Si priori modo se habebit in duabus diuisionibus, nullus numerus fractus admittetur; sed si ad. 4. multiplex non erit, à multiplicibus per duo semper differet, & si per medium diuidatur, eiusdem medietas impar semper erit, vnde prior

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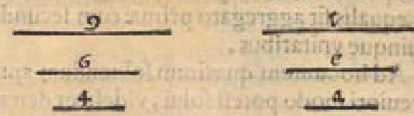
quoque summa par nunquam existet, cuius medietatem aliquod medium semper ingredietur, & hanc ob causam posterior summa cum fracto semper erit, & numerum desumptum maiorem esse multiplici ad quatuor per duo significabit.

At verò si inter impares reponatur, aut eorum erit qui superant multiplicem ipsius quatuor per vnum, seu per tria, quod hinc innotescet, nempe, quia si eorum erit qui dictum multiplicem per vnum tantum vincunt, sua medietate ipsi numero addita, & præter hanc medietatem medio etiam integro adiuncto, tota hæc prior summa in numerum parem semper euadet, vnde in posteriori summa nullus numerus fractus conspicietur, & hanc ob causã multiplici ipsius. 4. vnitas semper addetur.

Sed si numerus desumptus, in serie eorum, qui multiplicem ipsius. 4. per tria superant, collocabitur, hinc compræhendetur, quia primæ summæ numerus cum media vnitate semper impar erit, vnde secunda summa præter integras cum media vnitate nobis semper occurret.

Quod autem nobis prodere faciamus an in prima diuisione, & secunda numerus aliquis fractus consistat, eò tantum nobis inseruit, quò deueniamus in cognitionem an numerus animo conceptus multiplicem ipsius. 4. per vnum, per duo, aut tria superet. Quòd etiam medias eas vnitates ad integros reducere faciamus, eò tantum refertur, vt minori labore eum, qui numerum imaginatione compræhendit, oneremus, quia reuera numerus impar nunquam mente concipi potest, quin aliquis fractus in prima diuisione, aut in secunda sequatur: vnde à numeris imparibus, qui multiplicem ipsius. 4. unitatis tantum excessu superãt, posterior summa cũ quarta parte vnitatis, præter integros numeros, & ab imparibus qui dictum multiplicem ipsius. 4. per tria vincunt, cum tribus quartis vnius integri præter integras vnitates; & à numeris paribus, qui multiplicem ipsius. 4. per duo cum medietate vnitatis præter integros semper procedit. Ita cum is qui numerum secum considerat, si in numeris fractis versatus esset, qui eum interrogat prudenter se gereret, si sibi declarari curaret, quis nam ex fractis super integros secũdæ sũmæ remaneret, quia p quot quarta integros secũdæ summæ superaret, per totidē integros numerus mente conceptus multiplicem ipsius. 4. superaret.




## THEOREMA CXVII.

VNDE fiat, vt si aliquis quemuis numerum animo compræhendat, eique numero alium etiam quemlibet numerum propositum addat, & à tertia parte huius summæ tertiam partem numeri imaginati detrahet, residuum secundi numeri adiuncti, idest propositi, tertia pars erit.

Vt exempli gratia, si aliquis de numero denario cogitasset, huicq́; 24. adderet, vnde triginta quatuor efficerent, detrahendo nunc tertiam partem numeri denarij cogitatione concepti, idest. 3. cum tertia parte vnius, à tertia parte huius summæ idest ab vndecim & vna tertia parte remanerent. 8. idest tertia pars numeri additi. Id quod mihi inter iocos in honestorum hominum cætu in mentem venit.

Pro cuius ratione, prior numerus imaginatus mediante linea. a. b. et is, qui additus est intercedēte linea. b. d. è directo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coniunctis denotetur, et.b.c.sit tertia pars ipsius.a.b.prioris numeri imaginati,et.b c.tertia pars ipsius,b.d.secundi numeri propositi, vnde coniunctum vnius harum tertiarum partiũ cũ alia sit.e.c. quod quidem. e.c.esse tertiam partem summæ duorum primorum idest.a.d.assero. Iam manifestum est ipsius. d.b. ad.b.c.esse quemadmodum ipsius.a.b.ad.b.e.vnde vicissim ipsius. d.b.ad.b.a.erit quemadmodum ipsius.b.c.ad.b.e.& coniunctim ipsius.d.a.ad.a.b.quemadmodum ipsius.c.e.ad.e.b. & vicissim ipsius.d.a.ad.c.e.quemadmodum ipsius.b.a.ad.b.e.sed proportio ipsius.b.a. ad.b.e.est tripla,ergo ea quæ est ipsius.a.d.ad.e.c.erit quoque tripla; vnde sumendo. e.c.pro tertia parte ipsius.a.d.& ab ipsa .e.c.subtrahendo tertiam partem ipsius . a . b. tertia pars ipsius.b.d.remanebit.b.c.

Aut alio hoc modo, supponendo.e.c.tertiam partem ipsius. a.d.et.e.b. ipsius. a. b. exister. Dico.b.c.tertiam partem ipsius.b.d.futuram:quia si totius. a.d. ad totum e. c. ita se habet,quemadmodum.a.b.à toto.a.d. dissecti atque diuulsi ad.e.b.à toto. e.c.distractum, ergo ex. 19.lib.quinti Euclid.residui.b.d.totius.a.d.ad residuum.b.c. totius.e.c.erit,vt totius.a.d.ad totũ. e. c.atque hic quidem modus rem propositã speculandi mihi aptior & commodior esse videtur.

## THEOREMA CXVIII.

PErmulta ac varia problemata inuenerunt antiqui.longioribus verò vijs resoluta, propterea quòd nõ semper nobis succurrit breuissima in vnaquaque re explicatio. Vt exempli gratia, proponitur numerus. 50.diuidendus in tres tales partes,quod secunda dupla sit primę,& adhuc eam superet tribus vnitatibus, tertia verò æqualis sit aggregato primæ cum secunda,& amplius ipsum aggregatum superet quinque vnitatibus.

Ad hoc autem quæsitum soluendum antiqui vtebantur regula falsi, quod reuera breuiori modo potest solui, videlicet detrahendo illud secundum excessum, quinque scilicet ex.50.ita vt nobis.45. remaneret,cui medietati hoc est.22.cum dimidia vnitate,si addiderimus illud quinque habebimus. 27.cum dimidia vnitate pro tertia parte quæsita ipsius numeri. 50. deinde si ab eodem numero . 22 . cum dimidia vnitate detractum fuerit illud. 3. primus excessus datus, remanebit. 19. cum dimidia vnitate, cuius tertia pars,hoc est . 6 . cum dimidia vnitate, prima pars, ex tribus quæsita erit, quæ quidem si detraxerimus ex.19.cum dimidia vnitate, reliquum erit.13.cui cũ additus fuerit primus excessus idest. 3. Iam propositum resultabit nobis. 16. pro secunda parte quæsita.

Ratio verò huiusmodi operationis talis est, sit verbi gratia totalis numerus propositus significatus per lineam.a.b.cuius secundæ partis numerus datus significetur per lineam.g.& numerus tertiæ partis propositus per lineam.h. Nunc dempta. h. ex a.b.nobis cognita,remanebit.f.a.qua quidẽ per æqualia imaginatione diuisa in puncto. e. & ipsi. e. f. addita. f. b. tota. e.b.nobis cognita erit, quæ quidem tertia pars quæsita ipsius.a.b.erit,propterea quòd.a.e.( quæ æqualis est ipsi. e. f.) erit summa primæ, & secundæ partis. Detrahatur posteà.g.ex.e.a. & remanebit.d.a.cuius tertia pars sit.a.c.quæ quidem prima pars quæsita erit, & nunc cognita, & ita.c.d. cognita, cui cum addita fuerit . d.e. habebimus secundam partem quæsitam, quæ

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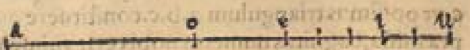
componitur ex.d.c.dupla. ad. a. c. primam partem, & ex.d. e.numero dato. tertia verò pars.e.b.compoſita eſt ex. e.f.æquali.a.c.hoc eſt æquali compoſito ex prima, & ſecunda parte, & ex.f. b. numero dato vt proponebatur.


## THEOREMA CXIX.

INter alia problemata ab antiquis inuenta, hoc etiam ponitur. Aliquis interrogat quot ſint horæ, alius verò reſpondit tot eſſe, quot duæ tertiæ præteriti temporis ſimul iuncta cum tribus quintis futuri temporis totius diei naturalis efficiunt. Nunc quæritur quot ſint horę.

Antiqui, hoc etiam problema ſoluebant mediante regula falſi, ſed mihi alio modo ſoluendum eſſe dictum problema videtur. Accipio enim ex quinque, tres vnitates, pro parte futuri temporis, quas quidem in tres vnitates præteriti temporis duco, vnde proueniunt mihi nouem vnitates, quod productum coniungo cũ quinque futuri temporis, vnde veniunt.14.vnitates, ex regula poſtea de tribus ita dico ſi ex.14. mihi prouenit. 9. quid reſultabit ex. 24. & prouenient mihi horæ. 15.cum tribus ſeptimis vnius horæ, hoc eſt minuta ferè. 26.

Pro cuius ratione, quinque vnitates, ſeu partes temporis futuri ſignificentur à linea.c.u.quarum trium ſignificentur à linea. e.i.ſumpta deinde ſit linea.c.o.æqualis lineæ.e.i. et.c.a.tripla ſit ad.o.e.vel ad.e.i.quod idem eſt, vnde.a.e. compoſita erit ex.a.o.(hoc eſt ex duabus tertijs ipſius.a.c.) & ex o.c.(hoc eſt ex tribus quintis ipſius.c.u.)vnde.a.u.ad.a.e.eandem rationem obtinebit, quæ.14.ad.9. propterea igitur poſſumus rectè ratiotinari ſi.14.nobis dat.9.quid dabit.24. qui quidem.24. nobis dabit.15. cum min.26. quod rectè factum erit ex.20.ſeptimi Euclidis.

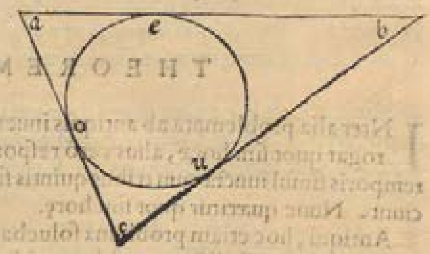



## THEOREMA CXX.

SVpponunt etiam antiqui tres ſocios nummos habere, quorum ſumma primi & ſecundi cognita ſit, item ſumma primi & tertij cognita & ſumma ſecundi & tertij item cognita, atque ex huiuſmodi tribus aggregatis veniunt in cognitionem particularem vniuſcuiuſque illorum.

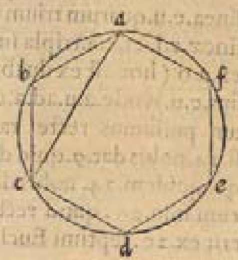
Gemafriſius ſoluit hoc problema ex regula falſi. At ego tali ordine progredior. Sit verbi gratia, ſumma primi cum ſecundo.50.& ſecundi cum tertio. 70. & primi cum tertio. 60.harum trium ſummarum accipiantur duæ quæuis, vt puta. 50. &.70 quæ coniunctæ ſimul dabunt. 120.à qua ſumma detrahatur reliqua, ideſt.60. & reſtabit nobis. 60. cuius medietas erit.30. hoc eſt numerus nummorum ſecundi ſocij quo numero detracto à.70.hoc eſt à ſumma ſecundi cum tertio remanebit.40. hoc eſt numerus tertij ſocij, & hic numerus deſumptus à.60.reſiduus erit numerus primi ſocij.

Pro cuius ratione conſideremus triangulum hic ſubnotatum .a. b. c. cuius unumquodque latus ſignificet ſummam duorum ſociorum, vtputa latus.a. b. ſignificet ſummam primi cum ſecundo, latus verò.b.c.ſummam ſecundi cum tertio, latus autem. a. c. ſummam primi cum tertio, et.a.e.ſeu.a.o.ſit numerus primi ſocij, et. e.b.vel.b.u.ſit ſecundi ſocij, et.c.u.ſeu.c.o.ſit tertij, cum autem.a. e. æqualis ſit. a. o. et.b.e.æqualis.b.u.et.c.u. æqualis. c. o. ex ſuppoſito ſi dēpta fuerit ſumma ſeu latus a.c.datum ex aggregato laterum. a.b.cum.b.c.reliquarum ſummarum, relinquet nobis cognitum aggregatum ex.b.e.cum.b.u. Quare & eius medietas.b.e.ſiue.b.u.nobis cognita erit, qua detracta ex ſumma.b.a. relinquetur nobis cognitus numerus.a.e. detracto verò numero.a.e.hoc eſt.a.o.ex.a.c. ſumma, ſeu latus, aut.b.u.ex.b.c.remanebit o. c. ſeu.c.u.cognitus.




## THEOREMA CXXI.

HAC etiam methodo hoc facere poſſumus non ſolũ de tribus ſocijs, ſed etiã de omnibus quotquot volueris, vt exempli gratia, ſint ſex ſocij.a.b.c.d.e.f. quorum ſumma per binos cognita, vtputà ſumma numeri.a.cum.b.cognita nobis ſit, & ſumma numeri.b.cum.c.& ſumma.c. cum. d. & ſumma.d. cum.e.& ſumma.e.cum. f. neceſſe eſt etiam ſcire ſummam duorum vno relicto, vtputa ſummam.a. cum c, vt poſſimus triangulum a.b.c.conſtituere, Vnde ex præmiſſa, cognitus numerus nobis erit vniuſcuiuſque.a. b. c. Quapropter dempto numero.c.ex ſumma.c. cum d.& numero.d.ex ſumma.d.cum.e.& numero.e.ex ſumma.e.cum.f.habebimus intentum.



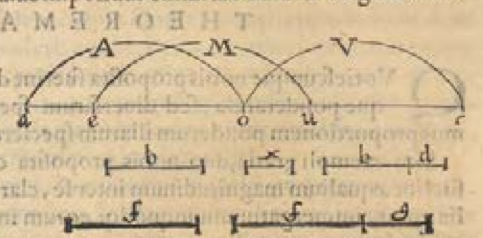

## THEOREMA CXXII.

CVM aliquando, illud quod Archimedes inuenit, vt furtum Regi ab aurifabro in regia corona factum, quemadmodum ſcribit Vitruuius, proderet, contemplarer, mihi etiam viſum eſt, vt aliquem modum ſcientificum inueſtigarem, quo proportio auri ad argentum, quod in aliquo propoſito corpore ex ipſis miſto cogniti ponderis cognoſci poſſet. Et cum multos diuerſis temporibus excogitarim officio meo deeſſe nolui in ijſdem literarum monumentis mandandis, quorum hic vnus erit: propoſita nobis ſint tria corpora. A. M. V. æqualia inter ſe, ſed diuerſarum ſpecierum materiei, vtputa quod. A. ſit argenteum, & omogeneum. V. verò aureum omogeneum, & M.mixtum ex auro, & argento, ideſt heterogeneum, cupimus ergo ſcire iuſtã quantitatem auri & argenti, quæ eſt in ipſo corpore. M. miſto. Ita igitur faciamus. Videamus primum quantum ſit pondus vniuſcuiuſque ipſorum corporum, ponamus autem pondus corporis. V.auri eſſe vt. 234. pondus

autem

autem corporis. M. misti.vt.216. argentei verò. A.vt. 156. detrahatur nunc pondus. A.ex pondere. V. Reliquum erit.78.quod vocetur prima differentia seruanda, dematur etiam pondus. M. ex pondere. V. reliquum erit. 18.pro secunda differentia, etiam seruanda, multiplicetur posteà pondus. A. per secundam differentiam, productum verò diuidatur per primam differentiam. Vnde in præsenti exemplo proueniet nobis. 36.quiquidem prouentus erit quantitas argenti ipsius corporis misti. M.quo etiam detracto ex pondere totali ipsius. M. reliquum erit quantitas auri eius corporis, hoc est. 180.

In cuius operationis speculatione, aliquid natura sua prius cognitum præcedere oportet hoc est, quod omnia corpora omogenea eandem proportionem obtinent inter quantitates, quam inter pondera. Quo supposito denotetur corpus. A. linea.o.a.corpus autem.V. linea. o.c.& corpus.M. linea.e.u.sed.e.o. significet partem argenti, et.o.u.partem auri in corpore misto. M. vnde ex communi conceptu habebimus.o.e.æqualem.u.c.cum ex hypothesi.e.u.æqualis sit.o.c.et. a.o. similiter. Significetur posteà pondus.a.o.ab.f.& pondus.e.u.ab.b.x.& pondus.o.c.ab.f.g.pondus verò. o.e. ab.b.pondus autem.o.u.ab.x.pondus enim.u.c.ab.b.d. et. g. sit differentia, qua. f.g. maior est. f.et.d. differentia qua. b. d. maior est. b. Vnde ex ratione omogeneitatis eadem proportio erit.a.o.ad.e.o.vt. f.ad.b.et.o.c.ad.u.c.quæ.x.b.d.seu f.g.(quod idem est)ad.b.d.Quare ex. 11.quinti eadē erit proportio. f.ad.b.vt.f.g.ad.b.d.& permutando ita erit.f.ad.f.g.vt.b.ad. b. d. &




separando ita. f. ad. g. vt. b. ad. d. Sed.g. cognita nobis est, vt differentia inter.f.g,et.f.cognita nobis est etiam.f. cognoscimus itidem.d. vt differentiam inter. x.b.d.et.b.x.quapropter cognoscemus.b. ex. 20. septimi Eucli. & sic. x. residuum. ex.b. x.

## THEOREMA CXXIII.

NVNC ex methodo præcedentis propositi deuenire possumus in cognitionem veræ quantitatis auri, & argenti confusi in corona Hieronis constituendo primum duo corpora simplicia æqualia inter se, & coronæ hoc modo videlicet, immergendo coronam, seu corpus mistum in aliquod vas aqua plenum, & diligenter colligere aquam, quæ ex eo effundetur, posteà verò oportet aliud vas inuenire præcisæ capax illius aquæ collectæ, in quod demum infundatur tantum auri, & posteà tantum argenti, quantum fieri potest, vnde vnumquodque horum duorum corporum simplicium æquale erit mixto, seu coronæ, & sic quod dictum est in præcedenti theoremate exequemur.

## THEOREMA CXXIIII.

SED vt breuiori methodo idem præstemus, quod in antecedenti proposito dictum est, quædam theoremata præmittenda sunt, videlicet quòd quotiescunque fuerint tria corpora, quorum duo inuicem æqualia sint in quantitate, sed diuersarum ſpecierum materiæ, tertium verò corpus maius, vel minus ſit in quantitate vtroque illorum, ſed eiuſdem materiæ vnius quod vis illorum, ponderis verò alterius, sẽper eadem proportio erit inter pondera æqualium corporum, quæ inter quãtitatem corporis inæqualis, & eam quæ vnius cuiuſuis æqualium.

Exempli gratia, ſit.b.corpus aliquod aureum æquale corpori.u. argenteo, ſit etiam corpus.a.argenteum maius corpore.b.vel.u.ſed ponderis eiuſdem, quod auri.b. Tunc dico eandem eſſe proportionem ponderis.b.ad pondus.u.quæ eſt magnitudinis.a. ad magnitudinem.u.Quod ratiocinemur hoc modo, nam cum proportio corporeitatis.a.ad corporeitatem.u. eadem ſit, quæ ponderis.a.ad pondus.u. ex ratione omogeneitatis, ponderis verò.b.ad pondus.u.ex.7. quinti, eadem quæ ponderis.a.ad pondus.u.ideo ex 11.eiuſdem proportio ponderis.b.ad pondus.u.eadem erit, quæ corporeitatis. a. ad corporeitatem.u.vel ad corporeitatem.b.quæ æqualis eſt alteri.


## THEOREMA CXXV.

QVotieſcunque nobis propoſita fuerint duo corpora cuiuſuis magnitudinis æque ponderantia, ſed diuerſarum ſpecierum materiæ, cum ſcire voluerimus proportionem ponderum illarum ſpecierum inter ipſas hoc modo faciemus.

Sint exempli gratia, duo nobis propoſita corpora.a.et.b.(vt dictum eſt) quæ ſi fuerint æqualium magnitudinum inter ſe, clarum erit quod quæritur, ſed inæqualia erunt, immergatur unumquodq; eorum in vas aqua plenum, & collecta ſit aqua effuſa ab vnoquoque illorum, tunc vnaquæq; iſtarum aquarum æqualis magnitudinis erit ſui corporis impellentis, & proportio ponderoſitatis illarum eadem erit, quæ earum magnitudinum ex omogeneitate, quapropter ſi vnamquamque illarum ponderabimus, habebimus propoſitum ex præcedenti theoremate.

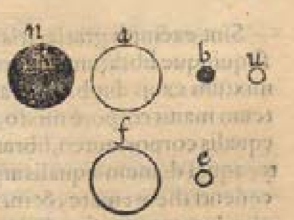
## THEOREMA CXXVI.

SED cum ſcire voluerimus pondus alicuius magnitudinis aquæ æqualis alicui corpori ponderoſo, breuiſſimus modus erit ponderando ipſum corpus tam in aere, quàm in aqua, & quia ſemper leuius erit in aqua, tunc differentia ponderum ipſius corporis, erit pondus quæſitum, hoc eſt vnius corporis aquei æqualis magnitudinis magnitudini corporis propoſiti ex. 7. propoſitione lib. Archimedis de inſidentibus aquæ.

Quare ex præmiſſis quotieſcunque immerſa fuerint in aquam dicti vaſis duo corpora æquè ponderantia, ſed diuerſarum ſpecierum, vt dictum eſt, proportio ponderis aquæ maioris ad pondus aquæ minoris magnitudinis eadem ſemper erit, quæ ponderis minoris corporis ad pondus alicuius corporis eidem æqualis, ſpeciei verò maioris, vel eadem proportio ponderis alicuius corporis æqualis maiori, ſpeciei verò minoris ad pondus ipſius maioris.

Vt puta ſit corpus.a.argenteum æqualis ponderis corpori.b.aurei, & corpus.u. argenteum æqualis magnitudinis corpori.b.aurei, corpus verò.n.aureum æqualis magnitudinis corpori.a.argentei, corpus verò.f.aqueum æqualis magnitudinis cor-

pori

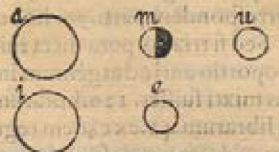


pori.a.argentei,corpus autem.e.aqueũ æqualis magnitudinis corpori.b.aurei. Tunc dico proportionem ponderis.f.ad pondus.e.eadem esse, quæ ponderis.b.ad pondus.u.vt in præcedenti theoremate iam dictum est,vel quæ ponderis.n.ad pondus.a.ex 11.quinti Euclidis. Propterea quòd ponderis.n.ad pondus.a.est vt poderis.b.ad pondus.u.eo quòd permutando ponderis.n.ad pondus.b.est vt ponderis.a.ad pondus.u.ex corporum omogeneitate, & ex æqualitate magnitudinum corporum antecedentium & consequentium.

## THEOREMA CXXVII.

SCire etiam nos oportet,quòd quotiescumque fuerint duo corpora aquea,quorum vnum æqualis magnitudinis sit alicui misto, quod quidem mistum graue sit tam in aere, quàm in aqua, alterum verò corpus aquem æqualis sit magnitudinis alicui corpoli simplici, quod quidem corpus simplex æqualis ponderis sit dicto corpori misto. Tunc proportio ponderis aquei, cuius magnitudo æquatur magnitudini corporis misti, ad pondus corporis aquei, cuius magnitudo æqualis est magnitudini corporis simplicis, eadem erit, quæ proportio ponderis alicuius corporis simplicis, cuius magnitudo æqualis sit magnitudini corporis misti superius dicti, sed speciei corporis simplicis iam dicti, ad pondus dicti misti.

Exempli gratia, sit corpus aqueum.e.magnitudinis æqualis corpori.m.mixto, corpus verò aqueum.i.æqualis magnitudinis sit corpori simplici.a.quod quidem corpus.a.æqualis ponderis sit cum corpore.m.& corpus.u.sit æqualis magnitudinis cum corpore.m.sed speciei corporis.a. Tunc dico proportionem ponderis.e.ad pondus.i.eãdem esse,quæ ponderis.u.ad pondus.m.primum nulli dubium est, quin eadem proportio sit magnitudinis.e.ad magnitudinem.i.quæ magnitudinis.m.ad a.sed.m.ad.a.est vt.u.ad.a.ex.7.quinti quare ex.11.eiusdem proportio.e.ad.i.erit vt.u.ad.a. de ipsius magnitudinibus loquendo,sed proportio ponderis.u.ad pondus.a.eadem est, quæ magnitudinis.u.ad magnitudinem.a.ex omogeneitate.Idem dico de pondere.e.ad pondus.i.Quare proportio ponderis.e.ad pondus.i.eadem erit quæ ponderis.u.ad pondus.a. Sed ponderis.u. ad pondus.m.eadem est quæ ponderis.u.ad pondus.a.ex.7.quinti, ergò ex.11. eiusdem proportio ponderis.e.ad pondus.i.eadem erit, quæ ponderis.u.ad pondus.m.quod est propositum.



## THEOREMA CXXVIII.

NVNC ad cognoscendam proportionem duarum diuersarum specierum in corpore misto proposito, tribus corporibus aqueis mediantibus, quæ quidẽ corpora æqualium magnitudinum sint alijs tribus corporibus vnius & eiusdem ponderis, quorum vnum sit mixtum, reliqua verò duo simplicia,sed specierum mixti, hoc ordine procedemus.

Sint

Sint exempli gratia, tria corpora æquè ponderantia, & vnumquodque illorum sitquinque librarum, quorum vnum sit aureum, aliud argenteum, reliquum vero mixtum ex ijs duobus metallis, vnde corpus aureum simplex minus erit, & argenteum maius corpore mixto, quod nulli dubium est, sit nunc pondus corporis aquei ęqualis corpori aureo, librarũ. 3. aquei verò ęqualis misto, sit librarũ 3. cũ quarta parte, aquei demum æqualis argenteo, librarum. 4. cum dimidia, vnde ex ijs, quæ in præcedenti theoremate, & in. 126. theoremate diximus, si imaginatione concipiemus alia duo corpora simplicia, auri, & argenti, sed æqualium magnitudinum mixto, habebimus proportionem ponderis aurei ad pondus corporis mixti vt triũ librarum cum quarta vnius ad. 3. libras, & proportio ponderis mixti ad pondus argentei erit, vt proportio librarum. 4. cum dimidia ad tres libras cum quarta parte vnius libræ, & proportio ponderis aurei ad pondus argentei vt librarum. 4. cum dimidia ad libras. 3: hoc est aurei ad mixtum, vt. 13. ad. 12. & mixti ad argenteum, vt. 18. ad. 13. & aurei ad argenteum, vt. 3. ad. 2. idest, vt. 18. ad. 12.

Nunc inueniantur duo numeri ita inter se proportionati, vt. 3. ad. 2. habentes tamen inter ipsos numerum ita proportionatum ad maximum, vt. 12. se habet ad. 13. & ita proportionatum ad minimum, vt se habet. 18. ad. 13. quod hoc modo inueniemus; multiplicabimus. 18. per. 12. & proueniet nobis. 216. pro numero medio, posteà multiplicabimus. 18. per. 13. & proueniet. 234. pro maximo, demũ multiplicando. 12. per. 13. proueniet. 156. pro minimo, ita quod. 234. correspondebit ponderi corporis aurei: 216. verò ponderi mixti, et. 156. ponderi argentei æqualium magnitudinum.

Cum autem proportiones horum trium corporum inuenerimus, si ordinem theorematis. 122. sequemur, habebimus quod quærebamus, & inueniemus in præsenti exemplo proportionem ponderis auri ad pondus argenti in corpore mixto esse, vt. 180. ad. 36. sed quia suppositum fuit corpus mixtum esse quinque librarum, propterea dicemus. Si. 216. hoc est toti corpori mixto correspondent quinque libræ: tunc parti. 180. hoc est auro in ipso corpore mixto, correspondent libræ. 4. cum duabus vncijs, ex règula de tribus, residuum verò quinque librarum, idest vnciæ decem, correspondent parti. 36. hoc est argento in dicto corpore mixto.

Sed si tria corpora dicta fuissent inuicem ita proportionata, vt. 40. 47. 60. tunc proportio auri ad argentum in corpore mixto esset vt. 13. ad. 7. quapropter cũ pondus mixti fuisset. 120. librarum, tunc aurum ipsius esset librarum. 78. argentum verò librarum. 42. ex eadem regula.

Pro quarum rerum speculatione nil aliud oportet nunc dicere cum satis dictum à nobis superius fuerit, vno excepto, hoc est rationem reddere, qua motus fui ad inueniendos illos. 3. numeros ita inter se dispositos, vt dictum est, quæ quidem ratio fuit, vt haberemus. 3. numeros ita inter ipsos ordinatè dispositos, vt sunt pondera trium illorum corporum æqualium magnitudinum. Propterea quòd quamuis inter primos. 3. numeros ponderum corporum aqueorum eædem fuerint proportiones ponderum corporum metallicorum, nihilominus medius numerus extra proprium locum, & inordinatè inueniebatur, respectu extremorum, vnde medius numerus in suo vero situ inter. 18. et. 12. fuissent. 16. cũ. 8. tertijs decimis, sed vt fractorũ incommoditatem euitemus, præcepi, vt multiplicarentur extrema per. 13. vnde producti fuerunt numeri. 234. et. 156. in eadẽ proportione, quæ est. 18. ad. 12. ex. 18. septimi, iussi etiam multiplicari. 18. per. 12. vt nobis prodiret. 216. ad quem numerum, numerus. 234. ita se haberet, ut. 13. ad. 12. ex. 19. septimi, quod autem ita sit propor

tionatus

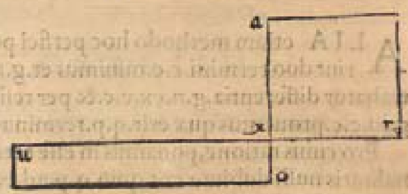
tionatus.216. ad.156. vt.18.ad.13.manifeſtum eſt ex ijſdem, nam tam. 18. quam. 13.multiplicatus fuit per. 12.

## THEOREMA CXXIX.

ALIVD proponitur problema hoc modo: ſupponitur obſidio alicuius loci, vbi alimento ad nutriendos. 10000.homines ſufficiunt pro quinque menſibus tantum, ſed quia eum locum obſidione non liberari putatur niſi.18. menſibus exactis, quæritur, quot homines eo tempore illis alimentis nutriri poſſint, hoc eſt. 18. menſibus.

Præcipit regula, vt multiplicetur primus numerus, hoc eſt hominum. 10000.cum ſecundo, hoc eſt menſium quinque, productum verò diuidatur per.18.hoc eſt menſium, tunc proueniet. 2777.cum.7. nonis.

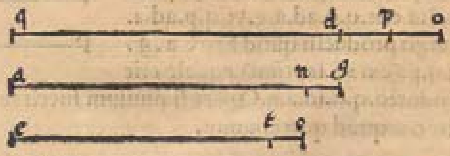
Cuius operationis ratio eſt hæc, ſint exempli gratia duo hic ſubſcripta producta ſuperficialia.a.n.et.o.u.inuicem æqualia, ſed tali figura delineata, vt proportio. u. x.ad.x.o.ſit, vt.10000.ad quinque, & proportio.a.x.ad.x.o.ſit vt. 18. ad quinque, et.x.n.ſit nobis ignota, quæ quidem eſt illa, quæ indagatur, ita ꝙ vnumquodque iſtorum productorum ſignificabit alimentum, et.u.x.ſignificabit numerum hominum.10000.qui quidem homines comederent totum alimentum. u. o. ſpacio temporis.x. o. quinque menſium, propterea quòd u.o.ſupponitur productum eſſe ab. u.x.in.x.o. Deinde ſupponẽdo.a.x. tempus eſſe.18.menſium, ergo.x. n. ſignificabit numerum hominum, qui eo temporis ſpacio ali poſſunt, hoc eſt. x. a. alimento.n.a.eo quòd.a. n. producitur ex. n. x.in.a.x.vnde ex.15. ſexti, ſeu ex. 20. ſeptimi proportio.x.u.ad.x.n.eadẽ erit, quę.a.x.ad.x.o.quapropter rectè factum erit accipere productũ. u. o. quod idem eſt in quantitate, quod productum. a. n. & ipſum diuidere per. a. x. vnde nobis proueniat.n.x.




## THEOREMA CXXX.

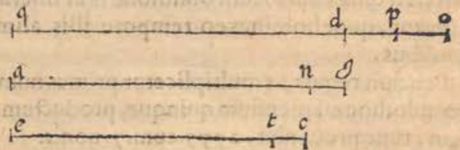
QVotieſcunque nobis propoſitum fuerit inuenire tertium terminum, trium terminorum continuè proportionalium armonicæ proportionalitatis, quorum duo nobis cogniti ſint, ita agemus.

Sint, exempli gratia, tres termini.q.p: a.g.et.e.c. continuæ proportionalium armonicæ proportionalitatis, quorum.q.p.maior et.a.g. medius ſint nobis cogniti, cum ergo voluerimus tertium. e. c.cognitum nobis eſſe:a.g. detrahatur ex. q.p. differentia verò. d. p.addatur. q. p. quorum ſumma erit.q.o.cognita, qua mediante diuidatur productum, quod ex.a. g.in.d.p.exurgit, & proueniet nobis. n. g.hoc eſt minor differentia, eo quòd productum.q.o.in.n. g. æquale eſt producto



ducto.a.g .in.d.p.ex.20. ſeptimi, propterea quòd proportio.q.o.ad.o.p.hoc eſt ad. d.p.eſt vt. a.g.ad.g.n.coniunctim cum diſiunctim ita ſit. q. p.ad.p.o.vt.a.n. ad . n.g. permutādo eo quòd.q.p.ad.a.n.(ideſt ad c.c.)ita ſe hēt ut.p.o.(hoc eſt.d.p.)ad.n.g. ex cōditionibus armonicæ proportionalitatis. Deinde ſi detraxerimus.n.g.ex.a. g. remanebit.c.c.minor terminus.

Sed ſi.e.c.tertius terminus nobis propoſitus eſſet ſimul cum.a.g.medio , & volue rimus maiorem inuenire.q.p.ſcilicet, oportebit.e.c.ex.a.g.detrahere , differentiam verò. n . g. ſimiliter demeremus ex.e.c.unde remaneret nobis.e.t. cognitum , quo reſiduo.e. t. mediante diuidemus productum , q̃ ſurgit ex.a.g.in.t.c. & prouentus. d.p.erit differentia maior , eo q̃ productū quod fit ex.e. t. in.d. p. æquale eſt producto quòd fit ex.a.g.in.t.c.per 20. ſeptimi Eucli.eo quòd. a.g. (ideſt.q.d.)ad.d.p.eſt ut.e.t.ad.t.c.diſiunctim, cum coniunctim ita ſit.q.p.ad.d.p.vt.e. c.ad.t.c.permutando , quia.q.p.ad.e.c.eſt vt.d.p.ad.t.c. hoc eſt ad. n. g. ex legibus dictis.



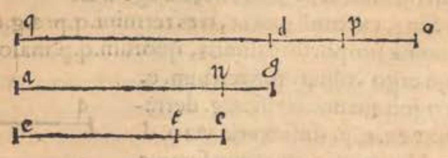

## THEOREMA CXXXI.

ALIA etiam methodo hoc perfici poſſe comperi.Propoſiti enim cum nobis fuerint duo termini.c.e.minimus et.g.a.medius,maximus verò quærendus ſit, detrahatur differentia.g.n.ex.e.c.& per reſiduum.e.t.diuidatur productum q̃ fit ex.a. g.in.e.c.prouentus quæ erit.q.p.terminus quæſitus .

Pro cuius ratione,ponamus in eſſe terminum.q.p.tunc ex forma huius proportionalitatis nulli dubium erit quin.q.p.ad.e.c.ſit vt.d.p.ad.n.g.hoc eſt ad. t. c. vnde ex 19.quinti vel. 12.ſeptimi ita eſſet. q.d.ad.e.t.vt.q.p.ad.e.c.quare ex.20.ſeptimi productum q̃ naſcitur ex.p.d.(hoc eſt.a.g. )in.e.c.æquale erit producto.e.t.in.q.p. quapropter ſi diuiſerimus id per.e.t.proueniet nobis.q.p.

Sed cū nobis propoſiti fuerint duo termini.q.p. maximus,et.a.g.medius,ſi minimū.c.c.voluerim⁹ inuenire. Termino.q.p.maximo,iūgat̄.p.o.ęqualis,p.d.differētię propoſitæ, diuidatur poſtea productum q̃ ex.q.p.in.a.g.generatur per.q.o. prouentus autem ſit.e.c. qui quidem erit terminus quæſitus.

Cuius operationis ſpeculatio hæc erit, ſupponatur terminum.c.c.inuentum eſſe vnde.n.g.differentia ſit inter. e. c. et.a.g. ex forma igitur armonicæ proportionalitis ita erit.q.p.;ad. a. n.vt.p.o.ad.n.g.vnde ex. 13.quinti . Ita erit.q.o.ad.a.g.vt.q.p.ad.a. n.ergo productū quòd fit ex. a . g . in.q.p.(ex.20.ſeptimi) æquale erit producto.q.o.in.a.n.Quare ſi diuiſum fuerit tale productum per.q.o.proueniet nobis. e.c.quòd quęrebamus.



## THEOREMA CXXXII.

SED quia aliquis posset in dubium reuocare, an possibile sit inuenire tertium terminum rationalem, seu communicantem duobus datis terminis inter se communicantibus in tali proportionalitate, hoc est harmonica. Vt hoc ostendatur.

Sint duo termini dati.a.o.et.a.e.inter se communicantes, tertius verò inuentus sit.a.c.qui maximus, primò, sit in ea proportionalitate, quem dico communicantem esse cum primis datis.

Nam ex conditionibus huiusmodi proportionalitatis, habebimus primum eandem proportionem esse. a.c.ad.a.o.quæ est.e.c.ad.e.o.vnde permutando ita erit. a.c.ad.e.c.vt.a.o.ad.o.e. & quia ex.9.decimi Euclid.a.o.communicat cum.o.e.quare ex.10.eiusdem.a.c.communicabit cum.e.c. & per.9.cum.a.c. et per. 8. cum. a. o. quod est propositum.

Sed si datus fuerit maximus.a.c.cum medio.a. e. inter se communicantes minimum verò. a.o.probabo cõmunicantem cum illis esse. Cogitemus ergo.c.f.æqualem esse differentiæ.c.e.cognitæ, vnde habebimus proportionem,a.c.ad.c.f.vt.a.o.ad.o.e.& componendo.a.f.ad.f.c.vt.a.e.ad.e.o.& quia (ex supposito).a.c.communicat cum. e. c. hoc est cum. c. f. quare ex eadem.9.dicti decimi.a.f.et.f.c. erũt inter se communicantes, & per.10.a.c. communicabit cum.o.e.& per.9.a.e.cõmunicabit cum. a.o.vnde per. 8.a.o.communicabit cum.a.c.similiter.

## THEOREMA CXXXIII.

SED si nobis duo extremi termini propositi fuerint, & medium inuenire desideremus in dicta proportionalitate, ita faciendum erit.

Sint, exempli gratia, duo termini dati.q.b. et.b.r.minor. b.r.ex maiori.b. q. detrahatur, residuum verò.q.x.multiplicetur per.b.r.productum posteà diuidatur per q.r.vnde proueniet nobis.x.l.pro differentia minori, quæ addita cum.b, x. minimo termino, dabit nobis. b.l.medium terminum harmonicum.

Pro cuius ratione cogitemus dictum medium terminum. b.l.iam inuentum esse, vnde ita erit proportio.q.l.ad.l.x.vt.q.b.ad.b.r.ex forma huius proportionalitatis, quare coniunctim ita erit.q.r.ad.r. b. vt q.x.ad.x.l.& propterea ex. 20. septimi productum.quod fit ex.q.r.in.x.l.æquale erit producto.q.x.in.b.r. Rectè igitur fit cum diuiditur hoc productum per.q.r.vt proueniat nobis. x. l. differentia minor.

## THEOREMA CXXXIIII.

POssumus etiam harmonicè diuidere vnam datam proportionem absque aliqua diuisione productorum, ne nobis fractiones proueniant, hoc modo videlicet.

Nobis propositum sit diuidere harmonicè sesquialteram proportionẽ inueniantur primo minimi termini huius proportionis ut purà. 3. et. 2. quarum summa, hoc est quinque, multiplicetur per minorem idest. 2. vnde proueniet nobis.10.qui quidem erit minor terminus trium quæsitorum, quorum maximus erit productum sum mæ iam dictæ in maiorem eorum, hoc est quod fit ex quinque in. 3. quod erit. 15. Vt autem medium terminum harmonicum inter istos habeamus, accipiatur duplũ producti, quod fit ex primis minimis terminis, quod erit. 12.

Cuius rei speculatio est ista: significentur duo termini datæ proportionis ab. q. b. et. b. r. quorum summa erit. q. r. cuius quadratum sit. q. o. sit etiam imaginata. b. e. parallela ad. o. r. Sitq;. b. x. æqualis. b. r. et. q. u. similiter, & ducatur. x. y. parallela ad r. o. et. u. l. ad. q. x. Tunc habebimus. b. o. æquale ei producto, quod fit ex. q. r. in. b. r. et. b. y. eidem etiam æquale, et. q. e. pro producto, quod fit ex. q. r. in. q. b. et. q. l. pro eo, quod fit ex. q. x. in. b. r. Vnde. q. l. cum. b. y. æquale fiet duplo ei, quod fit ex. q. b. in. b. r. Dico nunc. b. o. esse minimum terminum eorum, quos quærimus, et. y. b. cum. x. u. medium. q. e. verò maximum huiusmodi proportionalitatis.

Primum ergo certi scimus ex prima sexti vel. 18. septimi eandem existere proportionem. q. e. ad. b. o. seu ad. b. y. quæ. q. b. ad. b. r: sed. u. y. ad. u. x. est vt. y. l. ad. l. x. hoc est vt. q. b. ad. b. r. idest vt. q. e. ad. b. o. & summa. u. y. cum. u. x. idest. q. y. minor est quam. q. e. maximus terminus per. b. y. minimum terminum. & cõiunctim. q. y. ad. q. l. vt. y. x. ad. x. l. hoc est vt. q. r. ad. r. b. Vnde ex speculatione præcedentis theorematis, sequitur. u. y. esse differentiam inter maximũ & medium terminum, et. u. x. esse differentiam inter medium & minimum dictæ proportionalitatis. Nam eadem proportio est. q. e. maximi termini ad. b. o. minimi. quæ. u. y. (differentia inter. q. e. & gnomonem. u. b. y.) ad. u. x. (differentia inter dictum. u. b. y. et. b. y. minimum terminum, quia sunt ambæ ut. q. b. ad. b. r. vt diximus. Quare. b. y. coniunctũ cum. x. u. medius terminus erit, qui quidem (vt dictum est) duplus est ei quod fit ex. q. b. in. b. r.


## THEOREMA CXXXV.

ALIVM etiam modum ab antiquis traditum ad hoc problema perficiendum inueni, qui talis est. Inueniatur primo inter datos terminos extremos, medius terminus in arithmetica proportione, per quẽ multiplicetur vnusquisque dictorum extremorum, deinde multiplicentur ipsi extremi inter se, vnde habebimus tria producta eadem proportione inuicem existentia, vt quærebatur.

Exempli gratia, ponamus duos propositos terminos esse. 3. et. 2. quorum medius arithmeticè esset. 2. cum dimidia vnitate, per quem cum vnum quemque priorum multiplicauerimus, emergẽt nobis duo producta, quorum primum idest maius esset 7. cum dimidia vnitate, reliquum verò esset quinque, productum posteà quod ex ipsis extremis prouenit, erit. 6. quod quidem est harmonicè collocatum inter. 7. cum dimidia vnitate, & quinque.


Cuius rei speculatio omnis à præcedenti theoremate dependet. Sint exempli gratia, duo termini

pro

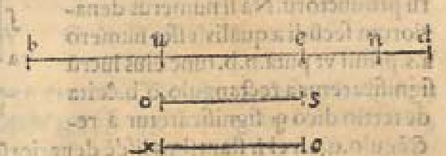
propositi.a.c.maior,et.c.o.minor, Sitq;.o.x.medius arithmeticus inter dictos, vnde clarè patebit.o.k.esse dimidium summæ dictorum terminorum ex. 75.theoremate huius libri. Sit ergo productum a.t.id quod fit ex.a.c.in.o.k.et.o.t.sit productũ quod fit ex.c.o.in.o.k.et.n.m. sit productum quod fit ex.a.c.in.c.o. quorum vnumquodque erit dimidium vniuscuiusque producti præcedentis theorematis, ex.18.et.19.septimi Eucli vnumquodque sui relatiui. Quare argumentando per mutando à conclusionibus præcedentis theorematis ad has præsentis, habebimus productum.

## THEOREMA CXXXVI.

MEDIVM autem contra harmonicũ inuenire cum quis voluerit inter duos propositos terminos, ita faciendum erit, hoc est per summam datorum extremorum diuidatur productum quod fit ex minimo termino in differẽtiam datorum, prouentus postea erit differentia inter maximum & medium quæsitum.

Vt exempli gratia, si nobis propositi fuerint hi duo termini. 3. et. 2. summa eorum erit quinque, per quam cum diuiserimus productum, quod nascitur ex minimo. 2. in differentiam eorum, quæ est vnum, quod quidem erit. 2. tunc duæ quintæ partes prouenient, quæ si demptæ fuerint ex maximo termino, reliquum erit. 2. cũ 3. quintis, hoc est medius terminus contra harmonicus.

Pro cuius ratione cogitemus.u.d.et.x.c.esse duos terminos nobis propositos, inter quos desideremus inuenire.o.s.medium ita illis relatũ, vt proportio excessus ipsius supra.x.c.(qui sit.c.n.) ad excessum.u.d. supra.o.s.(qui sit.n.d.) eadem sit quæ.u.d.ad.x.c.




Cogitemus igitur.x.c. coniunctum esse cum.u.d. & hæc summa vocetur.b.d.vnde habebimus proportionem.u.d.ad.u.b.vt.c.n.ad n.d. Quare cõponendo ita erit.d.b.ad.u.b.ut.c.d.ad.n.d.sed quia.d.b.u.b.et.c.d. quantitates nobis cognitę sunt, ideò.d.n.ex.20.septimi cognita nobis erit.

## THEOREMA CXXXVII.

SVpponunt antiqui aliquot mercatores dantes pecunias lucro in diuersis vnius anni temporibus, tunc in fine anni summa totius lucri datur cognita, sed quæritur quantum vnicuique illorum ex ipsa summa debeatur.

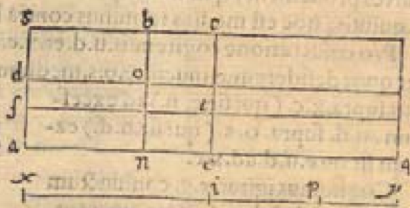
Exempli gratia, primus in principio anni posuit.100.aureos, secundus verò.100 diebus post primum posuit.50.aureos tertius autem.200.diebus post primum posuit.25.aureos summa lucri postea in fine anni fuit aureorum.60

Nunc vt sciamus quantum huius summæ vnicuique illorum proueniat, præcipit regula, vt faciamus tria producta, quorum primum sit ex numero dierum totius anni in numerum aureorum primi, vnde tale productum in præsenti casu erit.36500. secundum verò sit ex numero dierum à primo die in quo ipse secundus posuit usque ad finem anni, in numerum ipsorum nummorum, quod erit.13250.tertium autem productum ex diebus tertij in numerum suorum aureorum, quod quidẽ erit.4125. quæ producta simul collecta faciunt.53875.deinde multiplicetur vnumquodque

 ipso-

ipsorum productorum per summam lucri hoc est per. 60. vnde multiplicatio primi producti erit. 2190000. multiplicatio verò secundi producti erit. 795000. tertij postea erit. 247500. quarum multiplicationum vnaquæque diuidatur per summam 53875. productorũ, & proueniet ex prima diuisione. 40. cũ fractis. 35000. vnius integri diuisi in partes. 53875. quod erit lucrum primi, prouentus autem secundæ diuisionis erit. 14. cum fractis. 41050. vnius integri diuisi in partes. 53875. lucrum secũdi. prouentus verò quartæ diuisionis erit. 4. cum fractis. 32000. vnius integri, vt supra diuisi in partes. 53875. hoc est lucrum tertij.

Cuius rei speculatio ex se in sub scripta figura patet, vbi. a. q. significat numerum dierum totius anni pro primo mercatore. q. n. autem significat numerum dierum secundi mercatoris. e. q. posteà significat numerum dierum tertij sit etiam. s. a. pro numero denariorum primi, et. o. n. pro numero secundi, et. e. t. pro numero tertij, productum autem. q. s. significet valorem primi lucri, et. q. o. secundi, et. q. t. tertij. x. y. autem significet summam lucri omnium, et. x. i. significet partem primi, et. i. p. secundi, et. p. y. tertij. vnde clarè patebit ex communi scientia quòd eadem proportio erit. x. y. ad. x. i. quæ aggregati omnium productorum. q. s: q. o. et. q. t. ad. q. s. & ita. x. y. ad. i. p. vt aggregati dicti ad. q. o. et. x. y. ad. p. y. vt dicti aggregati ad. q. t. Rectè igitur ex regula de tribus multiplicatio. q. s. in. x. y. diuiditur per aggregatum omnium productorum, ita vt si aliquis diceret, si ex dicto aggregato, prouenit x. y. quid proueniet vnicuique illorũ productorũ. Nã si numerus denariorum secũdi æqualis esset numero a. s. primi vt purà. n. b. tunc eius lucrũ significaretur à rectangulo. q. b. & ita de tertio dico q significaretur à rectãgulo. q. c. vel si stantibus ijsdẽ denariorũ quantitatibus. n. o. et. e. t. omnes suas pecunias eodem tempore posuissent, tunc rectangula significantia eorum lucra essent q. s. q. d. et. q. f. sed cum nec eodem tempore, nec eandem quantitatem posuerunt rectè eorum lucra significantur à rectangulis. q. s. q. o. et. q. t. q ex prima. 6. vel. 18. aut. 19. septimi ratiocinando clarè patebit.

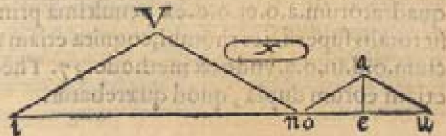



## THEOREMA CXXXVIII.

NIcolaus Tartalea in primo libro vltimæ partis numerorum ad. 35. quæsitum docet inuenire quantitatem laterum vnius propositi trianguli, cuius laterum proportio nobis data sit simul cum area superficiali ipsius trianguli; sed quia ipse Tartalea vtitur regula algebræ, mihi visum est breuiori methodo hoc idem facere, & etiam vniuersaliori via.

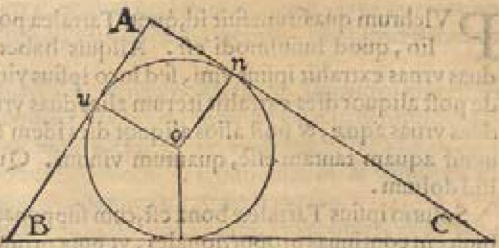
Supponamus igitur duo triangula, quorum vnum. u. n. i. sit nobis propositũ, & cognitæ superficiei, proportiones similiter laterum. i. n. ad. n. u: et. u. n. ad. u. i. sint nobis datæ, alterũ verò triangulũ sit. a. o. u. à nobis tamen ita confectũ, vt latera sint inter se proportionata eodem modo, quo latera prioris trianguli, sed hæc nobis etiã cognita sint, q facillimum est. Nunc verò si demptũ fuerit quadratũ. a. o. minimi lateris, ex quadrato. o. u. maximi, relinquet nobis duplum producti. o. u. in. u. e. per penultimã. 2. Eucli. supponẽdo. a. e. perpendicularem ad. o. u. vnde tale productum quòd fit ex. o. u. in. u. e. consequenter nobis cognitum erit, & quia. o. u. nobis cognitum est, tum eſt, ideo cognoſcemus.e.u.ſed cū.e.u.minor ſit.a.u.ex.18.& penultima primi, ſi demptū fuerit quadratum.e.u.ex quadrato.a.u.remanebit nobis cognitū quadratū.a.e.& ſic nota erit nobis perpendicularis.a.e.ex penultima primi, quæ quidem.a.e.ſi multiplicata fuerit in dimidium.o.u.dabit nobis ſuperficiē trianguli.a.o.u.ex 41.dicti libri. Et quia proportio trianguli.a.o.u.ad triangulum.u.i.n.(propter ſimilitudinem) eſt vt quadrati.o.u.ad quadratum.n.i.ex communi ſcientia cum vnaquæque iſtarum proportionum dupla ſit proportioni.o.u.ad.n.i.ex.17.et.18.ſexti, deinde cum nobis cognitæ ſint tres iſtarum quatuor quantitatum hoc eſt ſuperficies trianguli.a.o.u.ſuperficies trianguli.u.n.i.& quadrati.o.u.quare ex regula de tribus cognoſcemus etiam quadratum.n.i.& ſic.n.i.latus primi trianguli, vnde reliqua latera illicò nobis innoteſcent ex ipſa regula de tribus, cum dixerimus, ſi.o.u.dat nobis u.a.tunc.i.n.dabit.u.n.quòd etiam infero de.u.i.

Poſſemus etiam ita hoc perficere, ſcilicet inuenire.x.quantitatem mediam proportionalem inter duas ſuperficies triangulorum, vnde ſuperficies trianguli.i.a.u.o.ad.x.ſe haberet ut.o.u.ad.i.n.& ita ex regula de tribus cognoſcemus.i.n. Multo tēpore poſtquàm hoc theorema conſtruxi, ipſum conſcriptum inueni in decimo ſecundi libri Ioannis de monte Regio, ſatis tamen obſcurè expreſſum.




## THEOREMA CXXXIX.

IN eodem primo libro vltimæ partis numerorum, Tartalea probat, via algebrę quòd quælibet duo latera trianguli orthogonij, angulum rectum continentia, ſint tertio longiora per diametrum circuli inſcriptibilis in ipſo triangulo. ſed hoc breuius geometricè poteſt demōſtrari, quemadmodum in ſubſcripta hic figura videre eſt, propterea quòd cum anguli.A.o.u.et.n.omnes ſint recti et.A.u.æqualis o.n.et.A.n.ęqualis.u.o.ipſæ.A.u.et.A.n.æquales erunt diametro ipſius circuli. Sed eædem.A.u.et.A.n.ſunt ſuperfluum, quo.A.B.et.A.C.ſunt maiores.B.C.cum.B.u.et.C.n.ſint æquales.B.C.ex penultima tertij Eucli.




## THEO. SEQVENS THEO. CXXXIX.

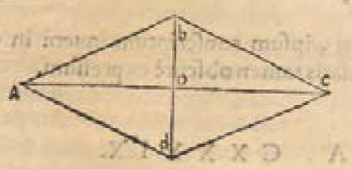
SImiliter in nono capite ſecundi libri nouæ ſcientiæ poterat ipſe Tartalea breuiori methodo abſque vlla operatione ipſius Algebræ inuenire.A.H.reſpectu.A.E.eſſe vt.4.cū vno ſeptimo ad vnū. Nā ipſe ſupponit.A.E.decimā partē eſſe ipſius

A.I.

A. I.vnde quadratum lineæ. A. I.erit.100.idem dico de quadrato lineæ. I. L.quare ex penultima primi.A.L.erit radix quadrata quadrati.200.ideſt.14.cum vno ſeptimo ferè.quare. A.L.iuncta.A.O.erit.28.cum duobus ſeptimis.ſed. L.O.ex ſuppoſito erit.20.eo quòd.L.I.ęquatur ipſi.A.I.ſimiliter et.I.O.vt ipſe etiam probauit.qua dempta ex. L.A.O.relinquetur.H.A.M.(nam.L.H.cum.O.M.æquatur ipſi.L.O.ex.35.tertij ipſius Eucli.partium.8.cũ duabus ſeptimis. cuius dimidiũ hoc eſt.A.H.erit 4.cum una ſeptima, quod eſt propoſitum. Reſpice figuram ipſius Tartaleæ.

## THEOREMA CXL.

QVadrageſimum nonum quæſitum ſimiliter poſſumus alio modo ſoluere, vt putà cum vnumquodque latus rhombi ſimul cum area cognitum, ſeu datum nobis ſit cognitũ ſimiliter nobis erit quadratum lateris.a.d.hoc eſt ſumma duorum quadratorum.a.o.et.o.d.ex penultima primi Euclid.cùmque nobis cognita etiam ſit totalis ſuperficies rhombi,cognita etiam nobis erit eius medietas,hoc eſt productum.o.d.in.o.a.vnde ex methodo.37. Theorematis cognoſcemus.a.o.et.o.d. & ſic etiam eorum duplà, quod quærebatur.





## THEOREMA CXLI.

PVlchrum quæſitum fuit id,quod Tartalea ponit pro.18.noni libri in quarto folio, quod huiuſmodi eſt. Aliquis habet dolium mero plenum, ex quo duas vrnas extrahit ipſius vini, ſed loco ipſius vini infundit duas vrnas aquæ. Deinde poſt aliquot dies extrahit iterum alias duas vrnas illius miſti, & iterum infundit duas vrnas aquæ, & poſt alios aliquot dies idem facit, & hac vltima tertia vice inuenit aquam tantam eſſe, quantum vinum. Quæritur nunc quot vrnas capiat illud dolium.

Solutio ipſius Tartaleæ bona eſt,cum ſupponat illas quatuor quantitates vini eſſe inuicem continuas proportionales, vt putà primò totum vinum merum, poſteà reſiduum pro ſecunda quantitate, deinde pro tertia in ſecunda, & pro quarta in tertia extractione, hoc eſt quòd proportio totius vini meri ad vinum in prima ſit,vt huius ad vinum in ſecunda, & vt huius ad vinum in tertia miſtione. Sed quia ipſe non probat hanc continuam proportionalitatem ex methodo ſcientifica, mihi viſũ eſt hoc loco illam deſcribere.

Cogitemus igitur.a.u.pro capacitate dolij, et.a.i.pro quantitate duarum vrnarum. Nunc uerò ſupponamus quamlibet partem huius miſti omogeneam eſſe ſuo toto, quapropter ſequetur eandem proportionem eſſe vini ad aquam in qualibet parte, quæ erit in toto, & ideò imaginemur.e.o.æqualem. a. i. Sed in puncto. i. tali modo diuiſam, vt proportio.i.e.ad. i. o. eadem ſit quæ.i.a.ad.i.u. Supponamus etiã

e.o.

e.o.esse duas primas vrnas vini misti hoc est primæ mistionis, vnde cum eadem proportio sit.a.i.ad.i.u.vt.e.i.ad.i.o.ita erit (ex.19.quinti).a.e.ad.o.u.ut.a.i.ad.i.u.& componẽdo ita erit.a.e.cum.o.u.hoc est.i.o.u. (propterea quòd.i.o.æqualis est.a.e. vt residua totorum æqualium) ad.o.u.quemadmodum.a.i.u.ad.i.u. Quare.i.u.erit media proportionalis inter.a.u.et.o.u. vnde proportio.a.u.ad.o.u.dupla erit proportioni.i.u.ad.o.u. Nunc autem cum extracta fuerit quantitas.e.o.ex primo misto, & postea infusa aqua vsque ad plenitudinem dolij, proportio ingredientium huius secundi misti erit ea, quæ est inter.o.u.et.o.a.eo quòd in prima mistione proproportio ingredientium erat ea, quæ est inter.o.u.et.a.e.vel inter.a.e.et.o.u. vt demonstrauimus. Accipiamus ergo.t.m.huiusmodi secundi misti, magnitudinis.a.i.vel.e.o.significantis duas vrnas, & permutemus eum in tantam aquam, sitq; punctum.o.quod nobis diuidat.t.m.in.o.m.et,o.t.partes simplices, tali proportione inuicem relatas, vt sunt.o.u.et.o.a.vnde habebimus ex supradictis rationibus eandem proportionem ipsius.a.t.ad.m.u.vt.a.o.ad.o.u.& componendo.a.t.cum.m.u.hoc est.i.m.u.(eo quòd cum.t.m.æqualis sit.a.i.per consequens.i.m.æqualis erit.a.t.)ad.m.u.vt.a.o.u.ad.o.u,sed proportio.a.o.u.ad.o.u.dupla erat proportioni.i.o.u.ad.o.u.quemadmodum supra diximus. Ergo proportio.i.m.u.ad.m.u.erit dupla similiter proportioni.i.o.u.ad.o.u.quapropter.o.u.erit media proportionalis inter.i.u.et.m.u. Ecce igitur quomodo eadem est proportio.a.u.ad.i.u.quæ.i.u.ad.o.u.& quæ.o.u.ad.m.u.qui quidem modus necessarius est vt intellectus acquiescat, id quod experientia non facit.


## THEOREMA CXLII.

PRæcedens Tartaleæ quæsitum elegans quidem est, sed pulchrum etiam videtur quærere proportionem ingredientium in ultima mistione, cum cognita fuerit nobis proportio continentiæ dolij ad capacitatis vrnæ simul cũ numero vitium extractionum & impletionum.

Exempli gratia, si proportio.a.u.ad.a.i.cognita nobis fuerit, cognoscemus etiam e.i.ex regula de tribus & per consequens etiam.i.o.residuum ex.e.o.& similiter aggregatum.a.i.cum.i.o.& sic.o.u.residuum totius, et.o.t.similiter, eo quòd.a.u.ad.a.o.est ut.t.m.ad.o.t.vnde cognoscemus etiam.o.m.vt residuum.t.m.& similiter aggregatum.a.o.cum.o.m.hoc est.a.m.& etiam.m.u.residuum totius.

Cognoscere autem proportionem totius dolij ad vrnam, vel è contrà, cum cognita nobis fuerit proportio ingredientium in vltima mistione simul cum numero vitium extractionum, & repletionum, quod scribit Tartalea, hoc etiam modo possumus.

Exempli gratia, si proportio.m.u.ad.m.a.cognita nobis fuerit, illicò sciemus proportionem.a.u.ad.m.u.& cum sciuerimus numerum vitium extractionum, & impletionum illicò cognoscimus multiplicitatem proportionis.a.u.ad.m.u.ad proportionem.o.u.ad.m.u.quapropter proportio.o.u.ad.m.u.nobis cognita erit hoc est.a.u.ad.i.u.& similiter ea, quæ est.a.u.ad.a.i.& è conuerso similiter.

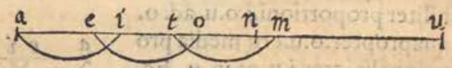

Vnde

Vnde cum aliquis diceret priori modo, dolium habeo vrnarum.400.vini, & per vices.25.extraxi & impleui ipſum, vt dictum eſt. Nunc verò velim ſcire proportionem vini ad aquam hac vltima vice. Nunc igitur ſi procedemus iuxta doctrinam primi exempli huius theorematis, obtinebimus quod quærebamus.

Sed ſi diceret iuxta Tartaleæ quæſitum, hoc eſt dolium habeo, quod ignoro quot nã urnas contineat, volo tamen per.25.vices extrahere, & implere vt ſupradictũ eſt, ita vt vltima vice proportio vini ad aquam ſit ſeſquialtera. Tunc ſi iuxta modum ſecundi exempli huius theorematis procedemus habebimus quod cupimus.

Alio etiam modo aliquis quærere poſſet, hoc eſt, habeo doliũ quod capit.400. vrnas. Habeo etiam vas trium vrnarum, quo mediante me oportet extrahere, & implere. Velim tamen ſcire quoties me hoc facere oporteat, ita vt poſtrema vice vinum ſe habeat ad aquam in proportione ſeſquialtera, vnde multoties accidet vltimam extractionem, & impletionem mutilatam, ſeu imperfectam, euadere.

Exempli gratia, ſi proportio vini ad aquam in vltima miſtione deberet eſſe vt.n.u.ad.n.a.ita vt extrema vice fuiſſet.t.m.quæ quidem.t.m.excederet terminum per.n.m. quæ.n.m.reuera eſſet nobis cognita, eò quòd ex priori modo hic ſupra dicto proportio.a.m.ad.m.u.nobis innoteſceret, & proportio.n.a.ad.n.u.nobis data eſt ſimul cum quãtitate.a.u.quare quantitas.n.u.& m.u.nobis cognita, remanebit, et




n.m.eorum differentia ſimiliter, etiam, et.t.n.reſiduum vaſis, quo metimur, vnde neceſſe erit, quod vltima vice vas contineret ſolum.t.n.reliqua uerò per ſe patent.

## THEOREMA CXLIII.

HIeronymus Cardanus in lib.ſuæ arithmeticæ cap.66.quæſtione.56.quam Cardanicam vocat, ita inquit.

Quidam perambulauit prima die certam quantitatem ſpatij, & ſecunda die, tãtò plus proportionaliter, quantò diameter eſt maior coſta, & tertia die tantò plus ſecunda, quantò proportionaliter portio lineæ diuiſæ ſecundum proportionem habentem medium, & duo extrema excedit minorem portionem, & quarta die in proportione ad tertiam vt ſecunda ad primam, & quinta die proportionaliter tantò plus quarta, quantò in tertia plus ſecunda, & ita alternatis vicibus in diebus nouem peregit nouem milliaria. Quæritur igitur quantum ambulauit die prima.

Hoc autem nihil aliud eſt, quàm ſi aliquis diceret, propono tibi, exempli gratia, lineam.a.l.nouem partibus inuicem non æqualibus ita diuiſam.a.c:c.d:d.e:& cæteris, quarum partium proportiones tibi etiam do, vt putà.a.c.ad.c.d.et.c.d.ad.d.e. et. d.e.ad.e.f.& ſic de cæteris vſque ad poſtremam. k.l.quæ quidem proportiones ſint etiam inuicem diſſimiles, ſeu inæquales, do tibi etiam proportionẽ totius lineæ.a.l. ad.a.b. ſuam partem, quæ vt in propoſito exemplo nonupla eſt.

Quæro nunc quam proportionem habebit.a.c.ad.a.b. & ſic de cæteris partibus eiuſdem ad eandem.a.b.

Quod quidem facillimum erit ſpeculari, nec non operari vnicuique, qui omnino practicæ numerorum ignarus non fuerit, dum ab ordine ſcientifico non diſcedat.

Cum enim cognoſcimus proportionem.a.c.ad.c.d.conſequenter cognoſcemus etiam proportionem aggregati.a.c.d.ad.c.d.cum autem cognouerimus proportionem.

nem.c.d.ad.d.e.ſi.c.d.accipiemus, vt medium inter.a.d.et.d.e.cognoſcemus etiam proportionem.a.d. ad. d. e. quare etiam eam quæ. a.c.ad.d. e. collocando poſteà. d.e.inter.e.f.et.a.e.innoteſcet ea,quæ eſt.a.e.ad.e.f. & ita gradatim accedemus ad perfectam cognitionem proportionis totius.a.l.ad.k.l. Nunc autem mediante.k.l. cognoſcemus proportionem totius.a.l.ad.i.k.& hac mediante, eam cognoſcemus, quæ totius.a.l.ad.g.h.& hac mediante eam quæ totius.a.l.ad.f.g.& ſic gradatim, cognita nobis erit proportio totius lineæ.a.l.ad ſuam partem.a.c. beneficio poſteà totius lineæ.a.l. cognoſcemus proportionem.a.c.ad a.b.& ſic aliarum reſpectu lineæ.a.b.vt quærebatur, quæ quidem propoſitio,etſi cardanica uocetur leuiſſima tamen eſt.

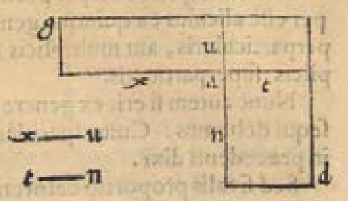

## THEOREMA CXLIIII.

QVamuis multi de modo in ſumma colligendi,ſubtrahendi, multiplicādi, & diuidendi proportiones ſcripſerint,nullus tamen ( quod ſciam ) perfectè, ac ſcientificè ſpeculatus eſt has operationes, quapropter hanc rem cum ſilentio tranſire nolui, quin aliquid de ipſa conſcribam à ſumma dictarum proportionum inchoando.

Quotieſcunque igitur volunt duas proportiones inuicem aggregare, ſimul earum antecedentia multiplicant, & ſimiliter earum conſequentia. Tunc proportio terminata ab illis productis euadit in ſummam illarum duarum propoſitarum proportionum.

Vt exempli gratia,ſi voluerimus colligere proportionem ſeſquialteram cum ſeſquitertia, multiplicando.3.cum.4.antecedentia ſcilicet,productum erit.12.poſteà multiplicando.2.cum.3.conſequentia, tunc productum erit. 6. Proportio igitur, quæ inter.12.et.6.reperitur. (quæ dupla eſt) eſt ſumma propoſitarum proportionū.

Cuius rei ſpeculatio erit huiuſmodi ſint.x.et.u. duo antecedentia quarumuis proportionum.t. verò et.n ſint eorum conſequentia, productum autem antecedentium ſit.a.g.illud verò quod cōſequentium ſit.d.a.vnde proportio.a.g. ad. a.d. compoſita erit ex proportione.x. ad.t. & ex ea, quæ eſt.u.ad.n.per.24.ſexti vel quintam octaui. Patet igitur ratio rectè faciendi,vt ſuprà dictum eſt.

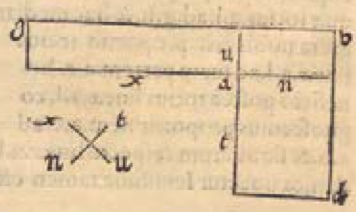



## THEOREMA CXLV.

QVotieſcunque deinde detrahere volunt vnam proportionem ex altera multiplicant antecedens vnius cum conſequenti alterius. Tunc proportio, quę inter talia duo producta incluſa reperitur, eſt reſiduum,ſeu differentia illarum duarum proportionum datarum.

Vt exempli gratia, ſi aliquis vellet ex proportione dupla detrahere ſeſquialteram, multiplicaret.2.antecedens duplæ cum.2.conſequenti ſeſquialteræ, quorum productum eſſet.4.pro antecedenti reſiduę proportionis. Deinde multiplicaret.3 antecedens ſeſquialteræ cum. 1. conſequenti duplæ, & productum eſſet.3.pro cō-ſequenti reſiduę proportionis; quæ quidem reſidua proportio eſſet vt.4. ad. 3. hoc eſt ſeſquitertia, & ſic de cæteris.

Pro cuius ratione, ſit proportio.x.ad.n.ea quæ ( exempli gratia ) maior ſit, à qua volumus demere proportionem.t. ad.u. minorem ſcilicet. Nunc autem productum. x. in. u. ſit. a. g. illud verò. t. in. n.ſit.a.d. Tunc dico proportionem.a.g. ad. a. d. eſſe reſiduam quæſitam. Sit.b.a.productum u.in.n.vnde eadem proportio erit producti.a. g.ad productum.a.b.quę.x.ad.n.et.a.d.ad.a.b. quæ.t.ad.u.ex prima ſexti, ſeu.18. vel. 19. ſeptimi, ſed proportio.a.g.ad.a.b.hoc eſt. x. ad. n.componitur ex ea, quæ eſt.a.g.ad.a.d.& ea, quæ eſt.a.d.ad.a.b.hoc eſt.t.ad.u.ergò ea, quę eſt.a.g.ad.a.d. erit quàm quærebamus.



## THEOREMA CXLVI.

RATIO verò, quòd rectè fiat, quotieſcunque aliquam proportionem duplicare volentes, quadramus terminos ipſius proportionis, vel ſi eam triplicare voluerimus, cubamus ipſos terminos, vel ſi eam quadruplicare voluerimus inuenimus cenſicos cenſicos terminorum ipſius proportionis, & ſic de ſingulis, in.17 Theo. huiuſmodi tractatus manifeſta eſt.

## THEOREMA CXLVII.

QVotieſcunque nobis propoſiti fuerint duo numeri ad libitum, deſideraremus que duas proportiones tali relatione inuicem refertas, quali ſunt hi duo propoſiti numeri inter ſe, ita faciendum erit.

Sciendum primo eſt proportionem maioris numeri propoſiti ad minorem ſemper eſſe alicuius ex quinque generum, hoc eſt aut erit generis multiplicis, aut ſuperparticularis, aut multiplicis ſuperparticularis, aut ſuper partientis, aut multiplicis ſuperpartientis.

Nunc autem ſi erit ex genere multiplici, iam ab antiquis traditus eſt modus, quē ſequi debemus. Cuius ſpeculatio à me inuenta patet.in. 17. Theo. huius libri, vt in præcedenti dixi.

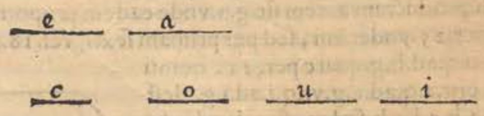
Sed ſi talis proportio datorum numerorum erit alicuius aliorum generum, ita agemus, ſi fuerit ſuperparticularis.

Sit exempli gratia, ſeſquialtera, tunc ſumantur duo numeri inuicem inæquales, quos à caſu volueris.o.et.c.qui quidem cubentur, & eorum cubi ſint.a.et.e.Inueniatur poſteà.u.ita proportionatus ad.o.vt.o.eſt ad.c.ex regula de tribus, hoc eſt diuidendo quadratum ipſius.o.per.c.vnde nobis proueniat.u. & quia proportio.a.ad.e. tripla eſt proportioni.o.ad.c.& proportio.u.ad.c.dupla eſt eidē, quæ.o.ad.c. ideo proportio.a.ad.e.ſeſquialtera erit proportioni.u.ad.e.

Sed ſi proportio numerorum propoſitorum fuerit ſeſquitertia, faciemus.a. et.e. eſſe cenſica cenſica ipſius.o.et.c.tunc ſumemus.u.conſequentem ad.o.vt dictum eſt, deinde inueniremus.i.conſequens ad.u.ita ut.u.conſequens ipſius. o. tunc habebimus proportionem.i.ad.c.triplam, & eam quæ eſt.a.ad.e. quadruplam proportioni.o.

ni.o.ad.c. Idem dico de reliquis proportionibus ſuperparticularibus.

Sed ſi data proportio numerorum fuerit ex ſuperpartientibus, vt exempli gratia de quinque ad tria, efficiemus, vt.a.et.e.ſint prima relata ipſius.o. et. c. vnde proportio.a.ad.e. ita ſe habebit ad proportionem. o. ad. c. vt quinque ad vnũ & proportio.i.ad.c.ut tria ad vnũ. Quare proportio. a. ad. e. ad proportionem.i.ad.c. ſe habebit, vt quinque ad tria,& ſic de reliquis.

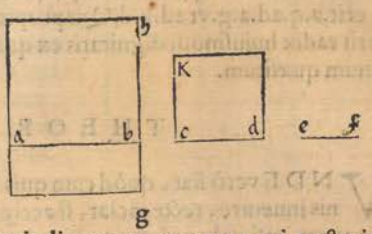



Pro alijs, eundem ordinem ſeruando, obtinebimus quod volumus.

## THEOREMA CXLVIII.

QVamuis in.16.ſexti et.20.ſeptimi manifeſtè pateat ratio, quare rectè fiat accipiendam radicem quadratam illius producti, quod ſit ex duobus datis terminis, vt medium proportionale geometricè inter ipſos habeamus: nihilominus, quia per aliam methodum hoc idem ſcire poſſumus, inconueniens non erit aliquid circa hoc dicere.

Cogitemus igitur exempli gratia, tres numeros continuè proportionales geometricè.a.b:c.d.et.e.f.quorum.a.b.et.e.f.tantummodo nobis cogniti ſint, imaginemur etiam.g.a.eſſe productum quod ſit ex.a.b.in.e.f.et.d.k.quadratum.c.d.et. a.h. id quod ſit ex.a.b.vnde eandem proportionem habebimus.a.h.ad.a.g.quæ eſt. h. b. ad.b.g.ex prima.6. aut.18.vel.19.ſeptimi, ſed per.11.octaui ita eſt quadrati. a. h.ad quadratum. k. d. vt. a.b.ad.e.f.hoc eſt vt.h.b.ad.b.g.ergo per.11. quinti ita erit.a.h.ad.a.g.vt ad.k.d.vnde.a.g. æquale erit.k.d.per.9.quinti. Rectè ergo erit accipere radicem quadratam.a. g. pro.c. d.quod etiam eſt diuidere vnam datam proportioné per æqualia, hoc eſt in duas æquales partes, non dubito quin poſſet aliquis dicere non oportere vti poſterioribus Theorematibus ad demonſtrandum priora illis, ſed hoc. 148. dictum ſit ludendi loco.



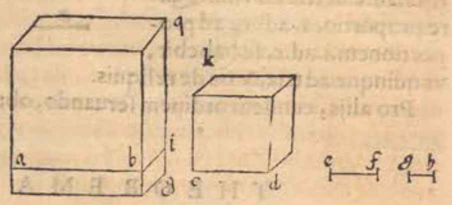

## THEOREMA CXLIX.

VNDE fiat ꝙ ſi quis inuenire voluerit ſecundum terminum ex quatuor numeris continuè, & geometricè proportionalibus,quorum duo extremi tantummodo nobis cogniti ſint, rectè factum ſit quadrare primum eorum, & hoc quadratum poſteà per alium terminum cognitum multiplicare,cuius producti demum accipere radicem cubam pro ſecundo termino quæſito, hoc loco videbimus.

Imaginemur quatuor terminos continuè proportionales, vt dictum eſt, eſſe.

 a.b:

a.b:c.d:e.f.et.g.h.quorum.a.b.et.g.h.nobis tantummodo cogniti ſint, ſitq; imaginatione deſcriptus cubus.a.q.primi termini, cubusq;.d.k.ſecundi termini, conſideremus etiam baſim.a.i.quadratam ipſius cubi.a.q. hoc eſt præcedentem dignitatem ipſius cubi eiuſdem radicis, quæ quidem baſis.a.i.multiplicetur per quartum terminũ g.h.productum autem ſit.g.a.vnde eadem proportio erit.a.q.ad.a.g.quæ.b.q.ad.b.g.per.25.vndecimi, ſed per primam ſexti, vel.18.aut.19.ſeptimi ita eſt.q.i.ad.i.g. vt.b.q.ad.b.g.quare per. 11. quinti ita erit.a.q.ad.a.g.vt.q.i.ad.i.g.ideſt vt.a.b.ad.g.h.ſed vt eſt. a. b ad.g.h. ſic eſt.a.q.ad.k.d.per.36. vndecimi, ſeu per.11.octaui,vnde per.11.quinti ſic erit.a.q.ad.a.g.vt ad.k. d.Quare per.9.eiuſdem.a.g.ęqualis erit.k.d.Vnde rectè erit accipere radicem cubam.a.g.pro ſecũdo termino. c. d.




id, quod nobis inſeruit ad inueniendam tertiam partem vnius propoſitæ proportionis.

## THEOREMA CL.

SED vt ſpeculatio iſta ita vniuerſalis fiat vt ad oẽs dignitates applicari poſſit; Supponamus.a.q.et.k.d.eſſe duas dignitates quas volueris vnius, ſed eiuſdem ſpeciei, et.a.i.dignitas præcedens dignitatem.a.q. à cuius multiplicatione in. a. b. eius radix producitur dignitas.a.q.& ab ipſius. a.i.multiplicatione in.g.h.reſultet.a.g.vnde ex.18.vel.19. ſeptimi eadem proportio erit. a.q.ad.a.g.quæ.a.b.ad.g.h. ſed eadem etiam eſt.a.q.ad.k.d.ex ijs, quæ in.17.theoremate dixi, vnde ex.11.quinti, ita erit.a.q.ad.a.g.vt ad.k.d.Quapropter.a.g.æqualis erit.k.d. & ideo cum inuenta fuerit radix huiuſmodi dignitatis ex quantitate.a.g.habebimus.c. d. ſecundum terminum quæſitum.

## THEOREMA CLI.

VNDE verò fiat, quòd cum quis voluerit dimidium alicuius datæ proportionis inuenire, rectè faciat, ſi accipiat radices quadratas illorum datorum terminorum, etſi voluerit tertiam partem, accipiat radices cubas: ſi autem quartam, accipere radices cenſicas cenſicas ipſorum,& ſic de ſingulis in.17. Theoremate omnia patent.

## THEOREMA CLII.

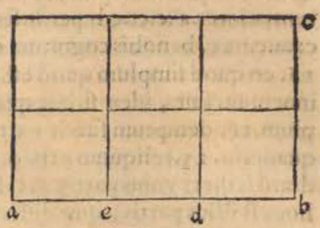
VNDE autem fiat, vt cum quis voluerit multiplicare aliquam proportionem per fractos, rectè faciat prius multiplicando eam per numeratorem, deinde productum diuiſerit per denominationem ipſorum fractorum.

Vt exempli gratia, cum aliquis voluerit multiplicare proportionem ſeſquiquartam per duo tertia, multiplicabit prius ipſam proportionem per numeratorem. 2. & productum, erit proportio.25.ad.16.qua poſtea diuiſa per. 3. denominatorem, prouentus erit proportio radicis cubæ.25.ad radicem cubam.16. vel vt proportio.

25.ad

25.ad radicem cubam.10000. quæ quidem proportiones æquales inuicem sunt, cũ tam vna, quàm alia, sit tertia pars totius.

Pro cuius ratione cogitemus.a.b.esse aliquod totum, quod multiplicare cupimus per duas tertias, quod quidẽ nihil aliud est, quàm accipere duas tertias partes vnius totius superficialis, imaginemur igitur hoc totum. a. b. lineare diuisum esse in tertias partes mediantibus.e.et.d.& tunc multiplicando ipsum per 2.tertias lineares productum erit.a.c.sex vnitatum superficialium, quod quidem productum posteà diuisum per.3.dabit. d. c. hoc est duas tertias superficiales (quæ est tertia pars ipsius. a. c.) & ęquales numero.c.b.duabus vnitatibus linearibus, idest duabus tertijs ipsius.a.b.No tandum etiam est, quòd cum ferè omnia reducantur ad regulam de tribus, propterea etiam multiplicatio alicuius quantitatis per aliam quantitatem, nihil aliud est quàm quædam operatio ipsius regulæ de tribus, vt exempli gratia volo multiplicare. 25. per 20.hoc nihil aliud est nisi quærere alium numerum ita proportionatum ad. 25. vt 20.se habet ad vnum, vnde multiplicando.25.cum. 20. & productum diuidendo per vnum ex regula de tribus, prouentus est idem numerus ipsius producti, & propte rea cum volumus multiplicare aliquem numerum per fractos hoc nihil aliud est quàm quærere aliquem numerum ita proportionatum ad ipsum numerum datum, vt se habet numerator ad denominatorem, exempli gratia si.24.aliquis voluerit mul tiplicare per duo tertia hoc idem est vt si quæreret numerum ad quem. 24.ita se habeat, vt.3.ad.2.& idem dico de proportionibus, hoc est quod aliud non est multiplicare aliquam proportionem per fractos, quàm aliam proportionem quærere ad quã data se habeat, vt denominator se hẽt ad numeratorẽ; & hoc ex regula de tribus perficitur, cõstituẽdo denominatorẽ in primo loco, qui locus est diuisoris, numerato rẽ verò in secũdo loco, multiplicãdo posteà pro portionem per numeratorẽ, & productũ diuidẽ do per denominatorem, prouentus demum erit proportio, ad quam data se habebit, vt denominator se hẽt ad numeratorem ex ratione ipsius re gulę de tribus. Ratio verò methodi diuidẽdi vnã datam proportionẽ per fractos, ex se satis patet, cum idem sit modus diuidendi quemlibet nume rum integrum per fractos. Quare, quæ vnius, & alterius est ratio.




## THEOREMA CLIII.

Nicolaus Tartalea in.3.lib. quintæ partis numerorum soluit. 24.quæsitum si bi propositum à Hieronymo Cardano, via particulari & non generali. Quæsitum autem tale est quamlibet propositam rectam lineam in duas partes ita diuide re via Euclidis, ut cubus totius lineæ ad cubos partium se habeat in proportione tripla.

Tartalea igitur inquit quòd vt satisfiat speculatiuis ingenijs soluendum sit huiusmodi quæsitum, secando lineam propositam.a.b.in tres æquales partes, quarum vna sit.c.b.vnde problema solutum erit.

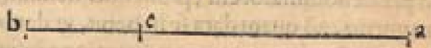
Verum dicit, sed hæc non est methodus generalis, propterea, quod cum tale problema alterius fuisset proportionis quàm triplæ, talis methodus nihil valeret.

lem

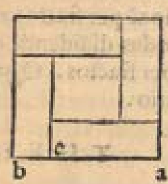
Quapropter non tacebo quod mihi in mentem venit circa hoc problema.

Sit ergo linea.a.b.diuisibilis in puncto.c.ita vt cubum totius dictæ. a. b. lineæ ad summam cuborum suarũ partium. a.c.et.c.b.oporteat eam proportionem habere, exempli gratia,vt.125.ad.65.vt vitemus fracta pro nunc, notantes talem proportionem quadrupla nunquam maiorem esse posse, vt quilibet ex se contemplari potest, constituendo punctum.c.in medio loco inter. a . et . b. vnde proportio totalis cubi ad summam partialium esset omnium maxima quæ possint esse, collocando. c. vbi volueris in dicta linea.a.b. & hæc esset quadrupla.

Sed vt ad propositum reuertamur, considerabimus cubum totalem ipsius . a . b . esse vt.125.& summam partialium vt.65.quam detrahemus ex cubo totali & nobis remanebit.60.pro summa trium solidorum inuicem æqualium, quorum longitudo vniuscuiusque erit tota linea.a.b.nobis cognita vt radix dati cubi totalis,quæ erit in hoc exemplo quinque partium, latitudo verò vniuscuiusque dictorum solidorũ erit.a.c.pars maior ipsius.a.b.quæ quidem.a.c.adhuc nobis ignota est, profunditas seu altitudo vniuscuiusque illorum solidorum,erit.c.b.pars reliqua ipsius.a.b.& etiã nobis incognita, sed quia summa horum trium solidorum nobis manifesta superius fuit ,quæ erat.60.propterà nobis cognita erit quantitas vniuscuiusque illorum solidorum, vt tertia pars totius summæ ipsorum quæ erit.20.in proposito exẽplo, deinde cum vnumquodque illorum solidorum producatur à superficie contenta seu producta ab.c.a.in.c.b.in tota linea.a.b.sequitur quòd si diuiserimus hoc solidum . 20 . per lineam.a.b. quinque partium proueniet nobis cognita superficies producta ab. a.c.in.c.b.quatuor partium, sed cum quadratum totius.a.b.nobis cognitum sit, eo quod.a.b. vt eius latus etiam cognitum est. Tunc dictum quadratum erit.25.quod quidem æquale est quadruplo illius quod fit ex.a.c.in.c.b.simul cum quadrato differentiæ inter.a.c.et.c.b.per.8.secundi Eucli. Vnde quia quadruplum illius quod fit ex.a.c.in. c. b. nobis cognitum est,vt




16. eo quod simplum quod est. 4. iã inuentum fuit , ideo si hoc quadruplum.16. demptum fuerit ex totali quadrato. 25.reliquum erit. 9. quadratũ scilicet vnius partis. a. c. ipsius hoc est illius partis, quæ differentia est inter a.c.et.c.b. quæ quidem erit. 3. partium quæ differentia cum subtracta fuerit ex.a.b.reliquum erit duplum ipsius.c.b.duo scilicet. Quare. c.b.erit vt.1.et.a.c.vt.4.& productum. a. c.in.c. b. erit. 4. vnitatum superficialium. &c.



# APPENDIX DE SPECVLATIONE REGVLAE FALSI.

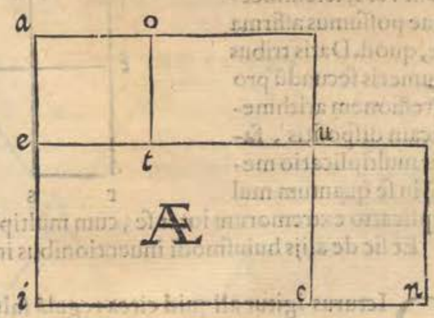
NVNC idem ferè mihi accidit, quod & Michaeli Stifelio, à quo cum Petreius Tipographus nuper totam suam Arithmeticam recepisset, mox posteà per literas petijt explicationẽ regulæ falsi. Similiter post incisas omnes superiorum Theorematum figuras, opereq; Typographo commisso, amicus quidam omnium scientiarum ornatissimus maxima necessitudine mecum coniunctus monuit me, vt aliquid de regula falsi scribere vellem, cuius suasu hæc, quæ sequuntur appendicis vice ponere libuit, ne lector, quidpiam quod ad hanc rem pertinet iure merito à nobis desiderare posset; vt autem ad ipsam regulã accedamus Ego sicut, & in alijs multis, ita & in huiusce regulę inuentione cum ipso Stifelio maximè conuenio, putans regulam falsi, seu falsarum positionum inuentam fuisse per paruos numeros in quæstionibus facillimis & cognitis, eodem ferme modo, quo ipse monstrat illis duobus exemplis, quæ quamuis ipse appellet theoremata, nihilominus theoremata ego illa non vocarem, nisi adiuncta fuerit speculatio ab ipso præterita, & non experientia tantummodo, vt ipse fecit. Primum eius exemplum est, quòd

Quorumcumque duorũ numerorum differentia, si fuerit multiplicata in aggregatum eorum, producit ipsam differentiam, quæ est inter quadrata eorum.

Secundum verò exemplum est, quod

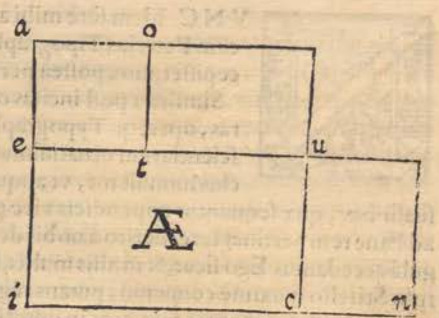
Datis tribus numeris secundum progressionem arithmeticam dispositis, facit multiplicatio medij in se, quãtum multiplicatio extremorum inter se cum multiplicatione differentiarum inter se.

Talia enim exempla ipse aliter non probat nisi experientia in aliquibus numeris, arbitratus ex eo inuentam esse regulam falsi, experientia tantummodo confirmatam, quod quidem etiam & ego credo. At experientia in philosophia mathematica, aut nullã prorsus facit scientiã, aut omnino superfluus fuit Euclides in multis suis propositionibus, & præcipuè in eius secundo libro, si sufficeret experientia. Idcirco quo magis ad euidentiam ipsius veritatis, quam profiteor, deuenire possim, accipiã primò primum exemplum ipsius Stifelij hic superius citatum, & pro numero maiori, in prima hic subscripta figura. AE. accipio. a.i. cuius quadratum sit. a.c: pro minori verò numero capio. a.e. partẽ ipsius a.i. cuius quadratum fit. a.t. differentia autem horum numerorum erit. e.i. reliqua pars ipsius. a. i: & differentia ipsorum quadratorum erit gnomon. e.c.o: Nunc autem protraho. i.c. latus quadrati maioris quousque c.n. æqualis sit. a.e. numero minori, perficioq; rectangulum. e. n. quod




produ-

producitur ex.i.e.differentia in.i.n.aggregatum amborum numerorum,ſed hoc productum excedit productum.e.c: partem gnomonis dicti per.u.n. quod quidem.u.n. æquatur ipſi.u.o.reliquæ ſcilicet parti ipſius gnomonis,nã.e.u.æqualis eſt. i.c.quare et. a.i.ſed.e.t.ęquatur.e.a.vnde.t.u.æqualis erit.e.i.quare et.u.c:at cum.c.n.æqualis ſit ipſi.a.e.erit etiam æqualis ipſi.o.t.quare. u. n. æqualis erit ipſi. u . o. & tunc intellectus quieſcit , & abſq; aliqua alia experientia verè ſcientificeq́; dicere poteſt, quòd




Quorumcumque duorum numerorum differentia, ſi fuerit multiplicata in aggregatum eorum, producit ipſam differentiã, quæ eſt inter quadrata eorum.

Hæc autem propoſitio à me ipſo etiam in. 60.Theoremate huius libri aliter demonſtrata fuit.

DE ſpeculatione autem, et ſcientia ſecundi exempli, in ſecunda hic ſubſcripta figura. ω. cogitemus lineam.u.a.tribusin partibus arithmeticè diuiſam,quarum maxima ſit.u.o.media.ſit.o.e.minima verò ſit.e.a.multiplicatio autem mediæ.o.e.in ſe ſit quadratum.o.t.abſcindatur deinde ex.o.e:e. i.æqualis.e.a.tunc. o. i. erit differentia inter.o.e.et.e.a.& æqualis differentiæ inter.o.e.et.o.u.ex hypoteſi, quæ quidem.o.i.in ſe ducta procreabit quadratum.o.c.quod erit productum ex differentijs ipſarum partium, & erit pars quadrati . o.t.ſuperius dicti, vt ex ſe patet. Nunc autem dico gnomonem.i.t.n.æqualem eſſe ei quod fit ex.a.e.in.o.u. Producatur igitur.e.t.quouſque.t.r.æqualis ſit ipſi.o.i.tunc.e.r.erit æqualis.o.u.quod etiam clarum eſt . Claudatur ergo rectangulum. i.r.quod erit æquale producto ipſius. e.a. in.o. u. Nam.e.i.ſumpta fuit æqualis.e.a. ſed ex rationibus in priori exéplo allatis, ꝓductum. i . r . æquale erit gnomoni.i.t.n. Nunc autem verè, ſcientificeq́ue poſſumus affirmare, quòd. Datis tribus numeris ſecundũ progreſſionem arithmeticam diſpoſitis , facit multiplicatio medij in ſe quantum multiplicatio extremorum inter ſe , cum multiplicatione differentiarum inter ſe.


Et ſic de alijs huiuſmodi inuentionibus infero.

DIcturus igitur aliquid circa regulã falſi, videtur mihi nullam oportere facere mentionem de origine huiuſcæ regulæ, cum in hoc Stifelius ſatisfecerit,ſed

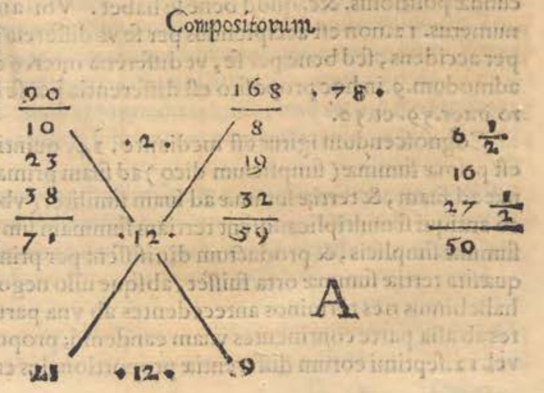
potius

potius veras rationes propriaq; fundamenta huiuſmodi operationis oſtendere, ſumendo eadem exempla propoſita ab ipſis practicis, & maximè à Nicolao Tartalea viro accuratiſſimo, qui vbicunque potuit ſpeculatus eſt cauſas ipſarũ operationum, etſi de huiuſmodi falſi regula circa finem cap.8.lib.17.promittat poſtea loqui, nullibi tamen loquutus eſt. Monendum etiam cenſeo, me nihil de rationibus regulæ falſi ſimplicis dicturum, cum ex ſeipſis ſatis appareant, quod non ita eſt de poſitionibus duplis. Incipiam ergo à primo problemate lib.17. ipſius Tartaleæ, quo etiã ipſe vtitur pro exemplo docendi gratia, ipſam regulam duplæ poſitionis, quod quidem problema aliter à me ſolutũ fuit in.118.Theoremate huius mei lib.quod ſimiliter ob hanc demum occaſionem mihi oblatam,alia etiam via,ſpeculatus ſum idem poſſe fieri, quæ quidem via ſeu methodus generalis erit, & ita ſe habet.

Accipio enim propoſitum numerum diuiſibilem, à quo detraho ſummam datorum numerorum, primo duplicato, eo quòd tam in ſecunda quam in tertia parte reperitur, vt in propoſito exemplo, datus numerus eſt, 50. à quo detraho ſummam dictorum numerorum, quæ eſt. 11. nam tres, & tres, & quinque ſunt vndecim, eo quòd primus ingreditur in ſecunda, & in tertia parte, dempto igitur hoc numero. 11. ex. 50. remanet. 39. qui quidem numerus intelligendus eſt pro ſumma trium partium ſimplicium adhuc incognitarum, à quo extrahenda eſt prima, eo modo quo nunc proponam ex regula de tribus, hoc eſt aggregando dictas partes ſimplices ſine aliqua additione vtcunque volueris (ſed commodius erit in minimis numeris) iuxta propoſitum, quod quidem propoſitum eſt, vt ſecunda pars dupla ſit primæ, tertia verò æqualis ſit primæ & ſecundæ, quæ partes in dictis minimis numeris, ita diſpoſitæ erunt. 1.2.3.quarum ſumma erit.6. Nunc ſi ex regula de tribus dixerimus, cum hæc ſumma proueniat nobis ab vno, à quo prouenict. 39.et veniet nobis.6.cum dimidio pro prima parte quæſita in propoſito numero. 39. cum ergo habuerimus primam partẽ, reliquas poſteà illicò cognoſcemus.

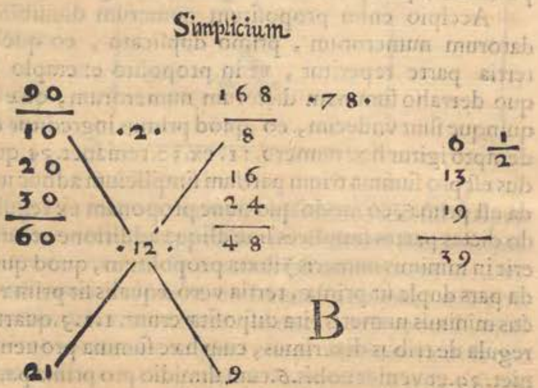
Huiuſmodi verò operationis ratio ex ſe manifeſta patet, eo quòd proportio ſummæ partium in minimis numeris ad primam eorum partem eadem eſſe debet, quæ ipſius.39.ad primam partem quæſitam huiuſmodi aggregati partium ſimpliciũ, ſed quia nemo adhuc, quod ſciam, ſatis animaduertit rationem modorum, qui ab antiquis obſeruati ſunt, qui quidem modi duo ſunt circa hoc Helcataym duplæ falſæ poſitionis, igitur non prætermittam aliquid de hac re ſpeculari, & primo de primo modo.

In primis igitur ſciendũ eſt, ꝙ veritas ita inueniri poterit eorum modo, mediantibus ſimplicibus partibus, vt etiã mediantib⁹ cõpoſitis, ut in prȩſenti exẽplo pro primis poſitionibus acceperunt. 10.et.8. pro ſecundis verò compoſitis cũ numero. 3. inuenerũt. 23.et. 19. pro tertijs aũt cõpoſitis cũ quinq;, notauerunt. 38.et. 32. vnde prima ſumma reſultauit. 71. ſecunda verò. 59. ita ꝙ prim⁹ error remanebat 21. ſecũd⁹ aũt. 9. vt in figura. A.



SED ijdem errores proueniunt ex summis partium simplicium.

Vt exempli gratia, in figura. B. summa proposita partium simplicium est.39. vt diximus, eo quòd ab ipso.50.detraxerimus.11.summa scilicet numerorum adij ciendorum ad efficiendas partes compositas, summa posteà simplicium partium primæ positionis, erit.60.eo quòd prima pars erat.10.secunda autem simplex.20. tertia verò simplex.30.iuxta ordinem propositi. Summa deinde simplicium partiū secundæ positionis esset.48.quia prima eius pars erat.8. secunda verò simplex. 16. tertia autem simplex.24.vnde prima summa excederet datam. 39.per.21. differentiæ, secunda verò per.9. vt supra vidimus de summis compositis à dato.50.composito, & hoc quidem mirandum non est, quod scilicet tres summæ simplicium partium sint inuicem inæquales,ijsdem differentijs mediantibus, quibus differūt dictæ tres summæ compositæ, cum ab vnaquaque cōpositarū ablatus sit numerus.11.æqualiter, vnde ex necessitate, permutando, earū differentiæ relinquēdæ erant æquales inuicem ex. 78. theoremate huius nostri lib.summæ enim compositæ erant.71.59.et 50.simplices verò.60.48. et.39.differentes à primis per.11. vt dictum est, quare veritas ita manabit à compositis, quemadmodum à simplicibus, sed à simplicibus per se, & à compositis per accidens vt iam iam videbimus.




ANtiquorum igitur primus modus vtitur regula de tribus, hoc ordine, multiplicando scilicet secundum errorem,qui est.9. cum differentia primarum partium positarum,quæ est.2.& productum diuidendo per differentiam errorum, quæ est. 12.proueniens postea quod est.1.cum dimidio additur hoc loco primæ parti secundæ positionis. &c. quòd benè se habet. Vbi animaduertendum est, quod ille numerus. 12.non est accipiendus per se vt differentia errorum hoc est. 21. et. 9. nisi per accidens, sed benè per se, vt differētia inter. 60.et.48.simplices summas,quemadmodum.9.in hoc proposito est differentia per se inter.48.et.39.per accidens verò inter.59. et.50.

Cognoscendum igitur est mediante. 24. quinti Eucli. quod eadem proportio est primæ summæ (simplicium dico) ad suam primam partem, quæ secundæ summæ ad suam, & tertiæ summæ ad suam similiter (vbi rectè etiam fecissent hoc in loco antiqui si multiplicauissent tertiam summam simplicem cum prima parte prioris summæ simplicis, & productum diuisissent per primam summam, vnde prima pars quæsita tertiæ summæ orta fuisset, absque ullo negotio ipsius plus vel minus) Quare habebimus tres terminos antecedentes ab vna parte, & tres terminos consequentes ab alia parte continentes vnam eandemq; proportionem, vnde ex. 19. quinti, vel.12.septimi eorum differentiæ proportionales erunt, hoc est, quòd eadem proportio

tio erit eius differentiæ, quæ eſt inter primam & ſecundam ſummam, ad differentiam quæ eſt inter primas earum partes, quæ illius differentiæ, quæ eſt inter ſecundam & tertiam ſummam, ad differentiam, quæ eſt inter primas illarum partes, ſed harum.4.differentiarum, tres nobis cognitæ ſunt, ideſt.12.2.et.9.ergo ex regula de tribus ab Eucli.in.20.ſeptimi ſpeculata inueniebatur quarta differentia, quæ eſt.1. cum dimidio.

A compoſitis ſummis idem etiam proueniet, ſed non vt ex proprijs cauſis, & per ſe, ſed per accidens. Nam quamuis eadem differentia ſit inter 71.et.59. quæ inter.60.et.48.& eadē inter.59.et.50.quæ inter.48.et.39.Nihilominus non eſt eadē proportio (propriè) ipſius.71.ad.59.quæ ipſius.60.ad.48.nec ea quæ ipſius.59.ad.50.eſt quæ ipſius.48.ad.39: Vnde non erit eadem proportio ipſius.71.ad.59.quæ ipſius.10.ad.8.nec ea quæ eſt ipſius.59.ad.50.quæ ipſius.8.ad.6 cum dimidio. Sed minores illis. Nam ex æqualibus additamentis diminuuntur proportiones maioris inęqualitatis.

A ſimplicibus igitur ſummis pendet ratio huiuſmodi effectus.
Si vero prima pars ſecundæ poſitionis eſſet.4.tunc ſecunda eius pars eſſet.8.& tertia.12.quarum ſumma eſſet.24.(harum ſimplicium partium ſcilicet)& minor vera (39.)per.15.& differens à ſumma primarum.(60.)per.36.& differentia primarum partium eſſet.6.differentia vero primæ partis ſecundæ poſitionis, a prima parte quęſita eſſet.2.cum dimidio. Vnde in huiuſmodi exemplo videre eſt quare colligantur errores inuicem, quando alter eorum eccedit, reliquus vero deficit à numero propoſito. Quod quidem ob aliam cauſam non fit, niſi vt cognoſcatur differentia.36. differentia ſcilicet ſimplicium ſummarum ipſarum poſitionum.

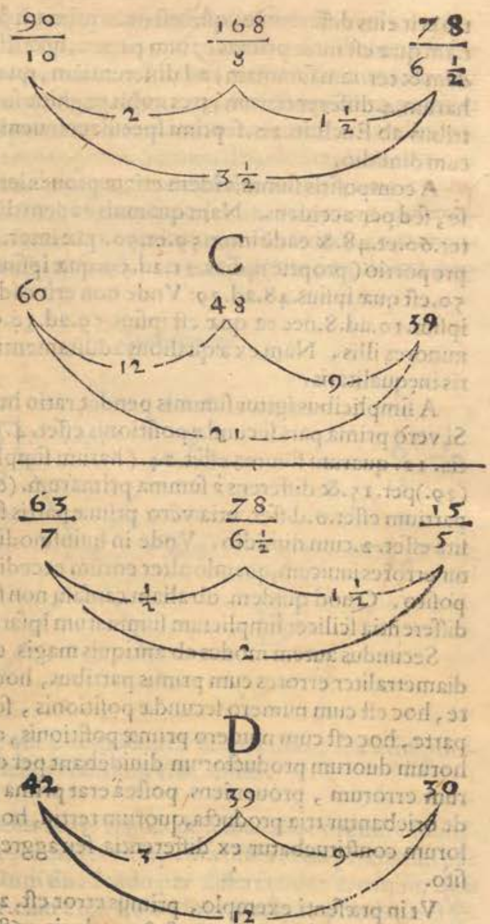
Secundus autem modus ab antiquis magis exercitatus eſt, quod multiplicabant diametraliter errores cum primis partibus, hoc eſt primum errorem cum prima parte, hoc eſt cum numero ſecundæ poſitionis, ſecundum vero errorem cum prima parte, hoc eſt cum numero primæ poſitionis, differentiam poſteà vel aggregatum horum duorum productorum diuidebant per differentiam vel aggregatum dictorum errorum, proueniens poſteà erat prima pars quæſita numeri propoſiti. Vnde oriebantur tria producta, quorum tertiū, hoc eſt differentia, ſeu aggregatum illorum conſtituebatur ex differentia ſeu aggregato errorum, & ex numero quæſito.

Vt in præſenti exemplo, primus error eſt.21.qui multiplicatus cum prima parte ſecundæ poſitionis, quæ eſt.8.producit.168.ſecūdus verò error eſt.9., qui multiplicatus cum prima parte primę poſitionis producit.90.differentia autem horum productorum eſt.78.quæ diuiſa per differentiam errorum, quæ eſt 12.dabit.6.cū dimidio, pro prima parte quæſita dati numeri diuiſibilis, qui erat.50.

Hæc omnia rectè ſe habent. Sed, vt ſupra dixi diuiſor non eſt per ſe differentia errorum, neque etiam differentia per ſe ſummarum compoſitarum, ſed bene ſimplicium.

Pro cuius rei ſpeculatione, accipiendæ ſunt ſummæ ſimplices, quarum differentiæ per ſe vtiles ſunt in huiuſmodi operatione; & quia etiam rationes veritatis ex iſtis, & non ex illis fluunt; quamuis tam vnæ, quam aliæ ſint eædem in quantitatē, ideſt æquales.

Diſponantur igitur huiuſmodi numeri tali ordine, vt ſimplex ſumma, quæ ab vna reliquarum ſuperatur, & aliam ſuperat, medium locum teneat; vt in propoſito exemplo ſumma mediocris eſt. 48. quę à ſumma. 60. ſuperatur, & ſuperat ſummam. 39. locata igitur ſit hęc. 48. inter illas, ſuæ verò primæ partes ſimiliter conſtitutæ ſint ſupra dictas ſummas, cum ſuis differētijs, & tria producta iam dicta, vt in figuris. C. et. D. arithmeticis clarè patet: figura enim. C. eſt pro exemplo ipſius plus ſimpliciter: figura verò. D. pro exemplo ipſius plus, & minus. Et ſic in figura. C. habebimus tres numeros conſequentes. 60. 48. 39. & tres antecedentes. 10. 8. 6. cum dimidio, vnam, & eandem proportionem terminantes, ex. 24. quinti, vt diximus; quare eorum differentiæ ſimiliter proportionales erunt, quod etiam vidimus. Supponamus nunc nos ignorare æqualitatem maximi producti cum reliquis duobus, accipiendo ſolum pro hypoteſi, quòd dicta producta oriantur ex lateribus iam dictis.

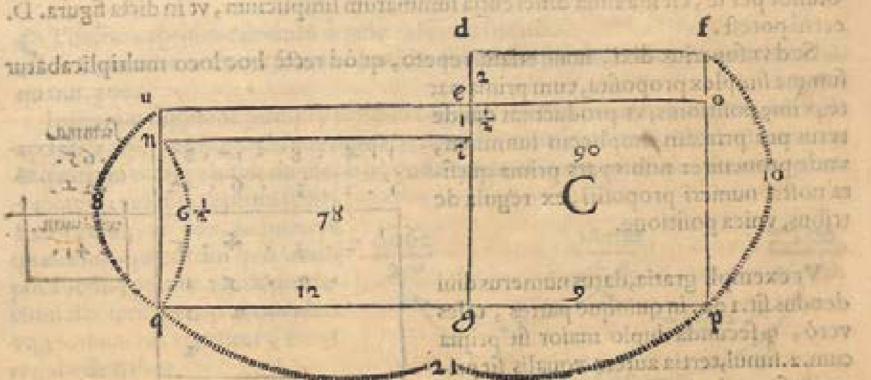



Demonſtrandum nobis nunc relinquetur, maximum productum æquale eſſe reliquis duobus; hoc eſt productum. 168. æquale eſſe productis. 90. et. 78. quorum duorum productorum alterum. 90. ſcilicet, generatur à differentia. 9. quæ eſt ſecundę, & tertię ſummæ, in primum numerum antecedentem, qui eſt. 10. alterum verò productum. 78. ſcilicet, generatur à differentia. 12. quę eſt primę, & ſecundę ſummę in tertium numerum antecedentem, qui eſt. 6. cum dimidio, maximum vero productum. 168. ſcilicet generatur à differentia maxima. 21. quę eſt primę, & tertię ſummę (& ſemper ęqualis prioribus duabus differentijs. 12. et. 9.) in ſecundum numerum antecedentem, qui eſt. 8.

Conſtituantur igitur duo producta ſimul iuncta ęqualia duobus. 90. et. 78. lateralibus ſupra vnam aliquam rectam lineam. q. p. ſitq; productum. f. g. ęquale. 90. productum verò. g. n. ęquale. 78. ſit etiam baſis. g. p. vt. 9. et. g. q. vt. 12. vnde. g. i. vel. q. n. erit vt. 6. cum dimidio. et. g. d. vel. p. f. vt. 10. & ideo. i. d. differentia erit. 3.

O 2 cum

cum dimidio, ut in figura.C.geometrica hic subscripta videre licet,et.q.p.erit . 21. Cogitemus nunc differentiam.d.i.diuisam esse in puncto.e. ita vt eadem proportio sit ipsius.d.e.ad.e. i. quæ ipsius.q.g.ad.g.p.hoc est vt.12.ad.9.quapropter.d. e. erit. 2.et.e.i.erit.1.cum dimidio, vt in dicta figura.C.arithmetica reperiuntur esse differentiæ ipsorum anteced.ntium numerorum,deinde à puncto.e.ducatur imaginatione.u.e.o.æquidistans ipsi.q.p. & producatur. q.n.vsque ad.u.vnde ita se habebit u.e.ad.e.o.ut.q.g.ad.g.p.quare vt.d.e.ad.e.i.ideo ex.15.sexti vel. 20. septimi. n. e. rectangulum æquale erit ipsi.e.f.quapropter rectangulum. q. o. æquale erit duobus rectangulis.f.g.et.g.n:sed cum.g.i.sit vt.6. cum dimidio, et.i.e.vt.1.cum dimidio,ergo.g.e.erit ut.8.qui quidem numerus multiplicatus cum.q. p.21.producit.168. verum est igitur quod dictum fuit, hoc est ℘ maximum productum æquale sit reliquis duobus.



Dempto

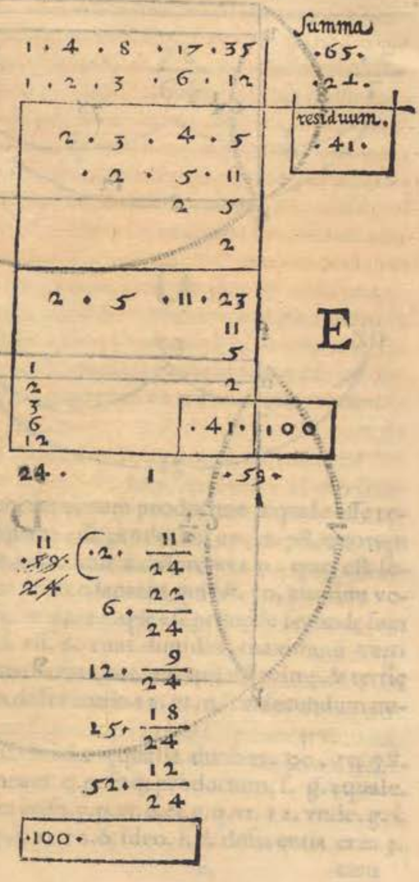
DEmpto poſteà quo volueris horum altero productorum ex maximo, diuiſoq́; reliquo per differentiam conſequentium, ipſi diametraliter oppoſitam, prouenie tibi numerus antecedens correſpondensq́; illi.

Animaduertendum tamen eſt, quòd ſi in figura à me ita ordinata, ſumma ſimplex propoſita medium locum occuparet, vt in figura. D. arithmetica videri poteſt; tunc vt habeatur eius productum, addenda ſimul erunt circunſtantia producta. eo ꝙ eius ſecundum latus eſſet antecedens medio loco conſtitutum, & prima pars quęſita numeri propoſiti: in qua figura. D. manifeſtè patet ratio, quare colligendi ſint tam errores, quam producta, dum eorum alterum eſt plus, reliquum verò minus.

Speculatio figurę. D. arithmeticę videbitur in figura. D. geometrica, eodem ferè modo quo fecimus in figuris. C. mutatis mutandis, reſpectu ipſius plus, & minus.

Collectio namque errorũ ſimiliter accidentalis eſt, eo quòd eſſentialis numerus diuiſor per ſe, eſt maxima differentia ſummarum ſimplicium, vt in dicta figura. D. cerni poteſt.

Sed vt ſuperius dixi, nunc etiam repeto, quòd rectè hoc loco multiplicabatur ſumma ſimplex propoſita, cum prima parte primę poſitionis, vt productum diuideretur per primam ſimplicem ſummam, vnde proueniret nobis pars prima quęſita noſtri numeri propoſiti, ex regula de tribus, vnica poſitione.




Vt exempli gratia, datus numerus diuidendus ſit. 100. in quinque partes, tales verò, ꝙ ſecunda duplo maior ſit prima cum. 2. ſimul, tertia autem æqualis ſit primæ & ſecundæ cum. 3. vnitatibus iunctis, quarta poſteà maior ſit prima ſecunda, & tertia per. 4. vnitates, quinta demum ſuperet reliquas omnes per quinque vnitates, vt in figura. E. videre eſt, quæ quidem partes compoſitæ (ſumpta vnitate pro prima) ita diſpoſitæ erunt, 1. 4. 8. 17. 35. quarum ſumma erit. 65. ſimplices autem cum diſpoſitæ fuerint erunt. 1. 2. 3. 6. 12. quarum ſumma erit. 24. dempta igitur cum fuerit hæc ſimplex ſumma. 24. à compoſita. 65. reſiduum erit. 41. hoc eſt ſumma numerorum propoſitorum cum ſuis iterationibus in ipſis partibus, quod cum per ſe clariſſimum ſit, ſuperfluum eſt ipsã ſummam annatomizare per ſingulas partes, niſi quis habuerit eius cerebrum à figura Omega terminatũ, cui tamen poſſemus dicere dictam ſummam. 41. in. 4. partes diuidi, cuius prima eſſet. 2. pro additione ad ſecũdam partem ſimplicium,

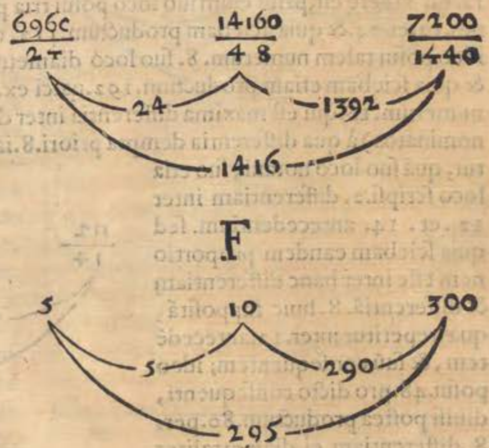
ſecun-

ſecunda verò eſſet.5.pro additione ad tertiam partem ſimplicium, tertia autem eſſet.11.pro additione ad quartam partem ſimplicium, quarta demum eſſet. 23.pro additione quintæ partis ſimplicium, quarum partium.2.5.11.23. ſumma eſt. 41. vt diximus. Hæc igitur ſumma.41.ſubducenda eſt à numero.100. propoſito, vnde relinquetur.59.pro ſumma partium ſimplicium numeri propoſiti, quarum prima erit 2.cum vndecim vigeſimisquartis ex diuiſione huiuſmodi.59.per.24. ſummam partium ſimplicium ex vi regulæ de tribus, dicendo ſi.24.prouenit nobis ab. 1. prima partium ſimplicium, à quo prouenıet nobis.59?vnde proueniet à. 2. cum vndecim vigeſimisquartis pro prima parte quæſita, ſecunda verò iuxta propoſitum, erit. 6. cum 22.vigeſimisquartis, tertia autem.12.cum nouem vigeſimisquartis, quarta poſteà.25.cum.18.vigeſimisquartis, quinta demum erit.52.cum.12.vigeſimisquartis, quarum omnium ſumma erit.100.

STifelius in primo exemplo regulæ falſi, ita inquit.

Quæratur numerus, à cuius dimidio ſubtractæ partes tertia, & quarta relinquatur.300.

Ipſe enim ſupponit.300.pro reſiduo cognito alterius numeri incogniti, deinde accipit.24.pro prima poſitione numeri cogniti, à cuius medietate abſcindit tertiam & quartam partem ipſius medietatis, vnde remanet.5.qui quidem numerus. 5. ex.22.quinti vel. 15. ſeptimi ſe habebit ad.24. vt. 300. ad numerū quæſitum, quare cum quis multiplicauerit.300.per.24. & productum diuiſerit per. 5. proueniet. 1440. numerus quæſitus, ex vi regulæ de tribus.



F

Conſideremus igitur meā diſpoſitionem numerorum huiuſmodi exempli, in figura hic ſuppoſita. F. in qua videre licebit quo pacto ipſe etiam Stifelius accipiat diuiſorem.5. vt differentiā errorum & non ut differentiam duorum conſequentium.5.et. 10 ſicuti eſt re uera, ut diuiſor dico, ex rationibus à me hic ſupra adductis, quamuis vna & eadem ſit quantitas neceſſariò ut patet.

ACcipiamus adhuc aliud exemplum à Tartalea propoſitione.9. datū, & oppoſitū priori; nam ſicut in illo numerus ſimplex habebatur per ſubtractionem ſummæ numerorum adijciendorum, in hoc fit è conuerſo, hoc eſt per additionem numerorum ſubtrahendorum.

Problema igitur ita ſe habet. Fuit quidam mercator qui habebat aliquot aureos, cuius quantitas poſteà quærenda erit, hic enim fecit duo itinera, ut aliquod dictis aureis mediantibus lucrum faceret, in primo autem itinere duplicauit numerum ſuorum aureorum, ex quibus poſteà conſumpſit.4.pro aliquibus expenſis, in

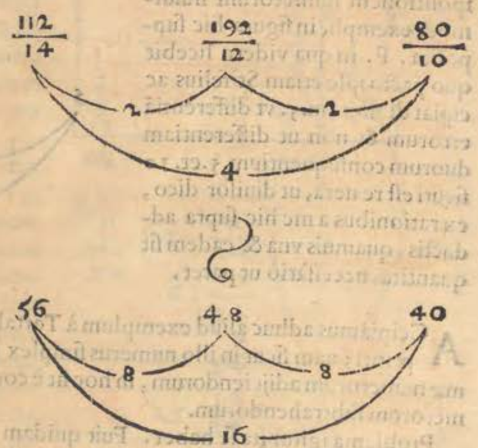
ſecun-

ſecundo itinere iterum duplicauit ſuos aureos, ex quibus etiam poſtea conſumpſit. 8.numeratis poſteà pecunijs reperit tantummodo.24.aureos in eius marſupio, quęritur nunc quot habebat aureos in principio primi itineris.

In tali caſu, cum ipſe quolibet itinere duplicabat eius pecuniam, nulli dubium eſt quòd in fine ſecundi itineris ipſe habuiſſet pecuniam ſuam quadruplicatam, ſi ex ipſa nihil detractum fuiſſet, ſed quia in fine primi itineris conſumpſit. 4. aureos, quibus alios.4. lucratus eſſet in ſecundo itinere, poſteà conſumpſit iterum. 8.aureos, ita ꝙ ex quadruplo ſuæ primæ pecuniæ, rectè dici poteſt, quod conſumpſerit. 16.aureos; qui quidem numerus ex communi conceptu erit differentia inter.24.& quadruplum prioris pecuniæ, cum qua profectus fuit in principio eius itineris; quapropter ſi addiderimus. 16.ipſi.24. habebimus.40. pro quadruplo eius prioris pecuniæ. Rectè igitur dici poteſt, ſi. 4.prouenit ab vno, à quo numero proueniet. 40.

Videamus igitur nunc quo pacto hoc reſpondeat cum methodo antiquorum. Ego enim inueni duas poſitiones ſcriptas à Tartalea pro prima pecunia hoc eſt.12. et.14.ſed à.12.pro primo errore reperi.8.more antiquo à.14.verò pro ſecundo errore proueniebat.16.producta autem horum numerorum diametraliter, ſunt.112. et.192.quorum differentia eſt. 80.pro tertio producto, quo diuiſo per differentiam errorũ.8. ſcilicet, præbet nobis.10.pro pecunia quæſita, vt etiam ego inueni.

Sed hoc mihi viſum eſt ſubtilius examinare mea methodo mediante, vt in figurà. G. videre eſt, prius enim ſuo loco poſui tria producta dicta, deinde duas poſitiones.12.et.14.& quia ſciebam productum.112. oriri à multiplicatione. 14. cum. 8. ideo poſui talem numerum. 8. ſuo loco diametraliter oppoſito ei producto. 112. & quia ſciebam etiam productum.192.naſci ex.12.et.16.ideo ſuo loco poſui hunc numerum.16.qui eſt maxima differentia inter duos conſequentes( ita à me ſupra nominatos) à qua differentia dempta priori.8.iam inuenta, reliqua. 8. mihi dabatur, quã ſuo loco notaui, ſuo etiã loco ſcripſi.2. differentiam inter 12.et. 14. antecedentium. ſed quia ſciebam eandem proportionem eſſe inter hanc differentiam & differentiã. 8. huic ſuppoſitã, quæ reperitur inter.12.antecedẽtem, & ſuũ conſequentem; ideo poſui.48.pro dicto conſequenti, diuiſi poſtea productum.80.per. 8. differentiam ei diametraliter oppoſitam, vnde prouenit mihi 10.cui ita proportionatus eſt ſuꝰ numerus conſequens. 40. vt. 48. ad.12.et.56.ad.14.ex ijſdem rationibus à me ſupra dictis.



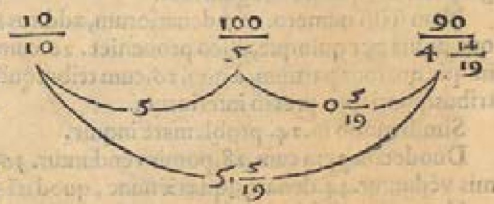

In tali igitur figura videntur numeri naturaliter correſpondẽtes ipſis poſitionibus,& hac methodo poſſumus inuenire tales numeros conſequentes in omnibus alijs exemplis à noſtris maioribus ſcriptis.

Pro-

PRoponitur etiam quoddam vas, cuius pes ſit quarta pars totius vaſis cum operculo, pars autem media ſine operculo, ſit quinta pars ipſius pedis, operculum verò.18.libras pendeat. quæritur nunc quantitas dicti pedis.

Ex methodo enim antiquorum inuentus eſt pes. 4. cum. 14. decimisnonis talium partium, ſeu librarum, qualium operculus eſt.18. Videamus igitur & nos ex noſtra figura, quo pacto hoc reſpondeat veritati.

Inuenta enim ſunt tria producta, iã orta ex dicta methodo.10.100. 90. quæ ſuis locis notaui, vt in figura. H. ſubſcripſi etiam duas illorum poſitiones.5.et. 10. cum ſua differentia.5.& cum productum. 10. orirerur ab vno latere. 10. reliquum erat. 1.quod ſuo loco notaui, ſimiliter quia.100.productum, pro vno eius laterum erat. 5.reliquum autem.20. ſuo loco poſui, & quia differentia inter.20.et.1.duo latera, quę eſt.19.æqualis eſt ei, quæ inter duo conſequentia duarum poſitionum, etiam ſuo loco ipſam conſtitui, ſed quia hæc differentia eſt vnum laterum producti.90. ergo reliquum latus quæſitũ erit. 4. cum. 14. decimisnonis, rectè igitur operatur. ſed cum eadem proportio ſit inter differentiam. 5. ſuperiorem, et. 19. inferiorem, quæ eſt vnius antecedẽtis ad ſuum conſequens, quare.10. antecedẽs habebit pro ſuo conſequenti. 38. et.5.habebit.19.et.4.cum.14.decimisnonis habebit. 18. rectè igitur dictum fuiſſet ſi.19.prouenit.à. 5. à quo proueniet.18 ?

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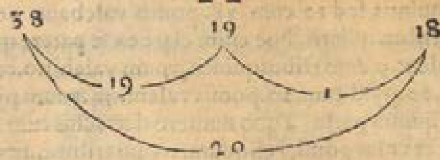



Huiuſmodi autem rei ratio ita ſe hẽt, eſto linea. a. e. u. cuius pars a.ſit quarta reliquarum.e.u.iunctarum, ſed.e.ſit quinta ipſius.a. Tunc clarum erit quod. e. erit vigeſima dictarum. e. u. quare erit decimanona ipſius.u.ſed cũ u. ſũpta ſit vt. 18.rectè igitur dici poteſt, ſi.u: ut. 19. prouenit ab. a. ut quinque, à quot ipſius.a.proueniet.u.ut.18.

Quis enim non uidet quod diuiſa cum fuerit.u.in partes. 19. quod quinque illarum æquabuntur ipſi. a.cum quælibet fuerit æqualis. e. quintæ parti ipſius.a.


HAc igitur mea numerorum diſpoſitione mediante reperiuntur ipſi numeri inferiores naturaliter conſequentes, correſpondentesque ipſis ſuperioribus antecedentibus; quamuis multoties cõtingere poſſit, ut generationes ſeu compoſitiones ipſorum ignorentur: & quia tam à differentijs errorum, quam ab illis, quę ſunt inter ueros conſequentes numeros (propter eorum æqualitatem) elicitur ipſa ueritas, propterea rectè antiqui illis vſi ſunt, quamuis ſint potius ſenſum ſequuti, uel experientiam, quam rationem: quæ quidem ratio pendet ab ipſis naturalibus numeris conſequentibus (ut ſupra uidimus) etſi incognitis ut plurimum, quod ſi ipſos inuenire primò nobis datum fuiſſet, unica tantũmodo poſitio ſufficeret

ret, mediante ipſa regula de tribus, vt iã ſępius dictũ eſt, quod etiã clarè patet ex diuerſis problematibus. 17. lib. ipſius Tartaleæ, vt ex primo, quod aſſumpſimus pro noſtro etiam primo exemplo, ex. 9. 15. 16. 17. 18. 19. 20. 27. 28. 29. 30. 33. & ex alijs multis, vbi facillimè inuenitur conſequens ipſius poſitionis, qui quidem numerus eſt diuiſor producti ipſius numeri propoſiti in numerum poſitionis, vnde poſteà prouenit ſecundũ latus huiuſmodi producti, hoc eſt numerus quæſitus, per regulã de tribus, vt dixi.

Alia verò multa problemata inueniuntur, pro quorum reſolutione poſſumus aliqua methodo vti, in qua manifeſtè pateant eorũ rationes abſque regula falſi, cuius regulæ rationes non ita promptè ipſi intellectui ſe offerunt, vt ſupra vidimus.

Accipiamus pro exemplo. 21. problema ipſius Tartalæ in dicto. 17. libr. vbi ſupponit vnum hædum diuiſum in. 4. partes, quarum quælibet vendebatur eodem precio, interiora vero. 6. denarijs minus quam quælibet dictarum partium, ſumma autem omnium iſtorum denariorum fuit. 127. quæritur nunc precium cuiuſque partis.

Tale enim problema hoc etiam alio breuiori modo poteſt ſolui, vt rationes magis pateant, quam ex regula falſi.

Nam ſi illi numero. 127. denariorum, additus fuerit numerus. 6. ſumma erit. 133. qua diuiſa per quinque, illico proueniet. 26. cum tribusquintis pro precio vniuſcuiuſque quatuor partium, à quo. 26. cum tribusquintis dempto. 6. remanebit. 20. cum tribusquintis pro precio interiorum.

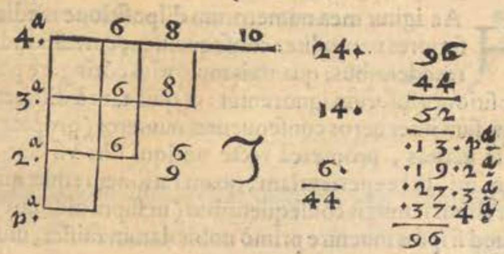
Simili modo in. 24. problemate inquit.

Duodecim pyra cum. 28. pomis venduntur. 36. denarijs, et. 20. pyra. cum. 200. pomis vẽduntur. 44. denarijs, quærit̃ nunc, quod nã fuerit preciũ vniuſcuiuſq; illorum.

Hoc etiam problema, hac alia methodo ſolui poteſt, dicendo ex regula de tribus, ſi ex. 20. vtrorunque qui ea vendit, vult. 44. quid volet ex. 12? manifeſtũ erit quod volet. 26. cum duobus quintis, quare. 12. pyra cum. 12. pomis valebunt. 26. cum duobus quintis, ſed 12. cum. 28. pomis valebant. 36. ergo. 16. poma ſola valebunt. 9. cum tribus quintis, hoc enim clarè ex ſe patet; quare cum dixerimus, ſi. 16. poma ſola valent. 9. cum tribusquintis, vnum valebit. o. cum tribusquintis, ſed quemadmodum. 20. pyra cum. 20. pomis valent. 44. vnum pyrum, cum vno pomo valebunt. 2. cum quinta parte, à quo numero detractus cum fuerit. o. cum tribus quintis, precio ſcilicet vnius pomi, reliquum. 1. cum tribusquintis, erit precium vnius pyri.

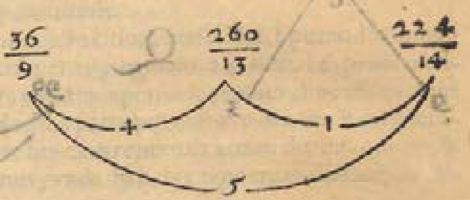
Idem etiam dico de. 28. problemate, vbi ſupponit quod quidam comparaſſet quatuor petias, vt vulgo dicitur, panni pro ducatis. 96. quarum primæ precium oblitus ſit, ſed memoria tenet pro ſecunda ſoluiſſe. 6. plusquam pro prima, & pro tertia ſoluiſſe. 8. plus quam pro ſecunda, & pro quarta ſoluiſſe. 10. plus quam pro tertia, quæritur nunc quantum fuerit precium vniuſcuiuſque illarum.

Quod quidẽ problema breuius eſſet ita ſolui, vt in ſubſcripta figura. I. videri poteſt, addẽdo ſimul omnes exceſſus. Nam exceſſus ſecũdæ ſupra primam eſt. 6. ſed cum exceſſus tertiæ ſupra ſecundam ſit. 8. ergo exceſſus tertiæ ſupra primam erit. 14 ſed



sed excessus quartæ supra tertiam est.10.vnde supra secundam erit.18.& supra primam erit.24.quæ omnia simul addita erunt.44.& in qualibet harum tri m remanebit una pars æqualis primæ quantitati, quare si ex.96.detractus fueri. n: merus.44. reliquus.52.erit quadruplus primæ, quare prima pars valebit.13. secunda.19. tertia.27.& quarta.37.quarum omnium summa est.96.

EX positionibus autem Tartaleæ in nostra figura. K. digestis, videre possumus quo pacto colligātur huius modi consequētes numeri simplices.36.et.52. more figuræ. E quia colliguntur primò partes compositæ.9.15.23.33.ex quarum summa 80. subtrahitur.36. summa simplex ex simplicibus partibus.9.9.9.9.& residuū quod est.44.subducitur ex.96.summa composita & proposita, vnde remanet.52.pro summa simplici, ex numero dato, cuius proportio ad.13. eadem est quæ.36 ad.9.& propterea superflua est secunda positio, quādo scimus inuenire tales duos numeros consequentes, vt in hoc exemplo sunt.36.et.52.quia ex regula de tribus posteà elicitur veritas quæsita. Idē dico de 33. problemate.

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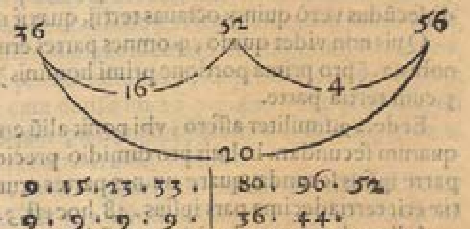
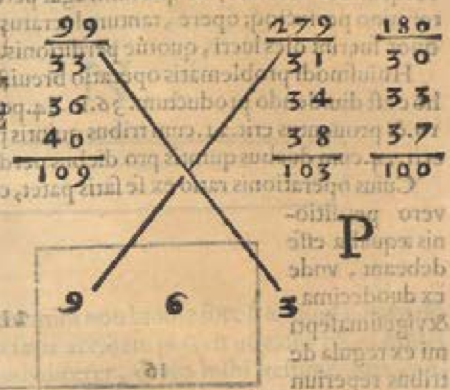



PRO quo.33.problemate accipiantur positiones primi exēpli Tonstalli hoc est.33.et.31.vt in figuris hic subiectis. P.Q.facile quis potest videre, vbi in figura P.videbit numeros compositos, in figura verò.Q.cernet numeros simplices, à quibus proueniunt rationes per se huiusmodi operationis, in figura autem.R. videbitur meus ordo, & istæ tres figuræ similes erūt tribus illis primis. A. B.C. ita quòd, cum quis illas intellexerit, illicò etiam istas cognoscet, vbi etiā videbit quam confusè ratiocinētur ij qui ignorant hunc meum ordinem simplicium numerorū, à quibus fluit tota ratio (vt supra dixi) huiuscemodi operationis.

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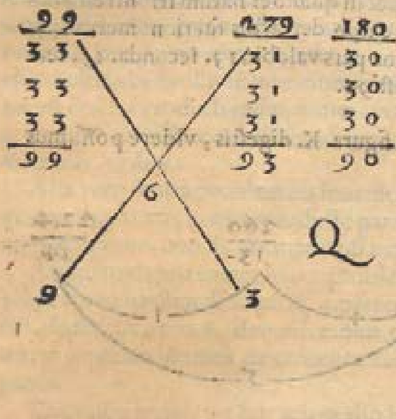




IDem etiam potest dici de.15.problemate (sicut de alijs multis.) vbi ponit tres homines habentes.40. aureos quorum primus habet duas quintas partes secundi, secũdus verò quinq; octauas tertij, quærit nũc quot ducatos habeat vnusquisque.

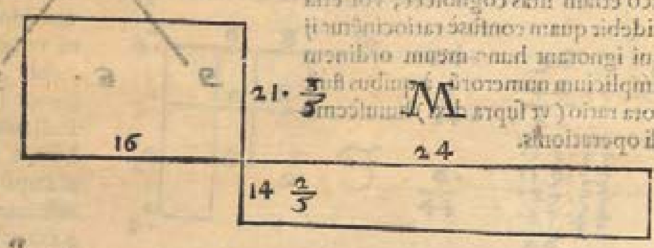
Quis non videt quæso, ꝙ omnes partes erunt.15.quare cum dixerimus si.15.dat nobis.2.(pro prima portione primi hominis) quid dabit.40? vnde nobis proueniet 5.cum tertia parte.

Et de.29.similiter assero, vbi ponit aliũ emisse tria frusta panni pro ducatis.48. quarum secundam habuit pro dimidio precio primæ, tertiam autem pro quarta parte ipsius secundæ, quare omnes partes erunt.13. quapropter precium tertiæ petiæ erit tertiadecima pars ipsius.48.hoc est.3.cum.9.tertijs decimis.

Adhuc duo exempla videtur mihi proponere, quorum primum est.58. eiusdem lib. vbi supponitur operarium quendam velle perficere opus quoddam spacio dierum.36 tali pacto, quod qualibet die, in qua ipse operaturus sit lucretur solidos. 16.qualibet verò die, in qua nihil agat perdat solidos.24. Tunc accidit, vt exacto termino perfectoq; opere, tantum lucratus sit, quantum perdiderit. Quæritur nũc quot fuerint dies lucri, quotúe perditionis.

Huiusmodi problematis operatio breuissima absque vlla falsa positione ita erit, hoc est diuidendo productum.36.in.24.per.40.idest per aggregatum ipsius.24.cũ 16.& prouentus erit.21.cum tribus quintis pro diebus lucri, vnde reliquum ex.36. erit.14.cum duabus quintis pro diebus perditionis.

Cuius operationis ratio ex se satis patet, cum duo producta, vnum lucri, alterum vero perditionis æqualia esse debeant, vnde ex duodecima, & vigesimasepti mi ex regula de tribus reperiuntur partes ipsius 36. eodem modo



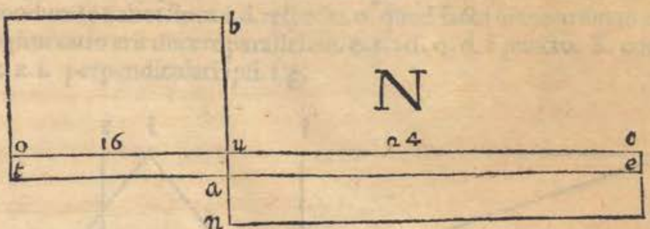
do se inuicem habentes., vt.24.et.16.quæ sunt.21.cum tribus quintis,et.14. cũ duabus quintis, ex quo sequitur, vt quod fit ex.21.cum tribus quintis,in.16.ęquale sit ei quod fit ex.14.cum duabus quintis,in.24.& ita reperiuntur duo producta æqualia, vnum lucri, reliquum vero perditionis, vt in figura.M.clarè videtur.

ALiud verò exemplum est.39.quod quidem à superiori non differt, nisi quod in fine operationis, operarius dictus lucratus est solidos.60: quęritur nũc vt supra, quot fuerunt dies lucri, & quot perditionis.

Hoc etiam absque vlla falsa positione dicto citius potest solui, hoc modo, diuidẽdo scilicet illos. 60 solidos per.40.idest per aggregatum. 24. cum. 16. proueniens autem, quod erit.1.cum dimidio,adde ad latus superius inuentum, hoc est.21.cum tribus quintis,& summa erit.23.cum decima parte pro numero dierum lucri, deinde idem prouentum deme ex alio latere superius reperto.14.cum duabus quintis,& residuum erit.12.cum nouem decimis, vnde habebis numerum dierum perditionis.

Pro cuius rei speculatione cogitemus in figura.N. duo dicta producta inuicem æqualia, o.b. et.n.c.existente latere.u.c.vt.24.u.o.ut.16:b.u.vt.21.cum tribus quintis, et.u.n.vt.14.cum duabus quintis. Nunc verò si mente concepta fuerit recta. e. a.t.æquidistans.o.c.ita vt rectangulum.o.e.sit.60.tunc rectangulum,seu productum b.t.superabit rectangulum seu productum.n.e.per idem.60.ex communi conceptu, eo quòd ex producto.n.c.sublatum est productum.a.c.24.& producto.o.b.additum est productum.e.a.16.rectè igitur feci cum diuiserim.60.per.40. vnde prouenit mihi.u.a.idest.1.cum dimidio, quod additum ipsi.b.u.composuit.b.a. & dempto ex.u.n.relinquit.a.n.pro lateribus duorum productorum.b.t.et.n.e.

Sed si idem operator perdidisset. 60. tunc cogitaremus parallelam dictam. e. a.t superius ductam esse ita vt secaret. b.u.& non.u.n.vnde adderet.24.ipsi producto.n.c.& demeret.16.à producto.b.o.




CIRCA verò talia quæsita videtur mihi non inutile fore si aliquid notatu dignum aduerterim, hoc est quod sæpe accidere poterit ut casus impossibiles proponantur. Quemadmodum si aliquis diceret, cupio mihi uestimentum conficere ex duobus pannis colore & pretio differentibus, quorum unus exempli gratia.

tia ſit albus, rubeus uerò alter, deinde albus ſit pretij. 40. ſolidorum uniuſcuiuſque cubiti, rubeus uerò precij. 50. uellemque omnes cubitos eſſe. 8. nec plus nec minus. Vellem etiam ſoluere ſolidos. 450. neque minus.

Hic igitur caſus impoſſibilis eſt, eo quòd. 8. cubiti totius rubei eſſent precij ſolidorum. 400. tantummodo, unde ex alio panno albo minoris precij ſumere aliquid non poſſumus.

Idem etiam eueniret ſi uoluiſſet ſoluere ſolidos. 320. neque plus, eo quòd. 8. cubiti illius minoris precij, hoc eſt. 40. ſolidorum, eſsent ualoris. 320. ſolidorum tantummodo, quare pro alio panno nullus eſset locus. Animaduertendum igitur erit quod numerus poſſibilis ad ſoluendum tale quæſitum erit inter. 400. et. 320. & non extra iſtos terminos, vt vnicuique patere poteſt.

Similiter idem in hoc alio caſu accidere poterit, ut ſi quis diceret.

Emi quinque petias panni pro aureis. 55. pretium tamen primæ oblitus ſum, ſed memoria teneo, quòd ſecunda altioris pretij erat quam ipſa prima per. 4. & tertia precioſior ſecunda per. 7. et quarta carior tertia per. 9. quinta verò ſuperabat quartam per. 2.

Hic etiam reperitur impoſſibilitas quædam, eo quòd aggregatum omnium harum rerum, dato etiam quòd pro prima nihil ſolutum eſſet, ſuperat aureos. 55. quòd quidem nullo pacto fieri poteſt, vt veri ſint ſupra dicti exceſsus, ſi verus eſt numerus totalis aureorum. 55. Nam. 4. cum. 7. faciunt. 11. qui quidem. 11. cum. 9. efficiunt. 20. & hic cum. 2. facit. 22. ſed. 22. cum. 20. et. 11. et. 4. dant. 57. qui numerus maior eſt quam. 55.

## FINIS THEOREM. ARIT.

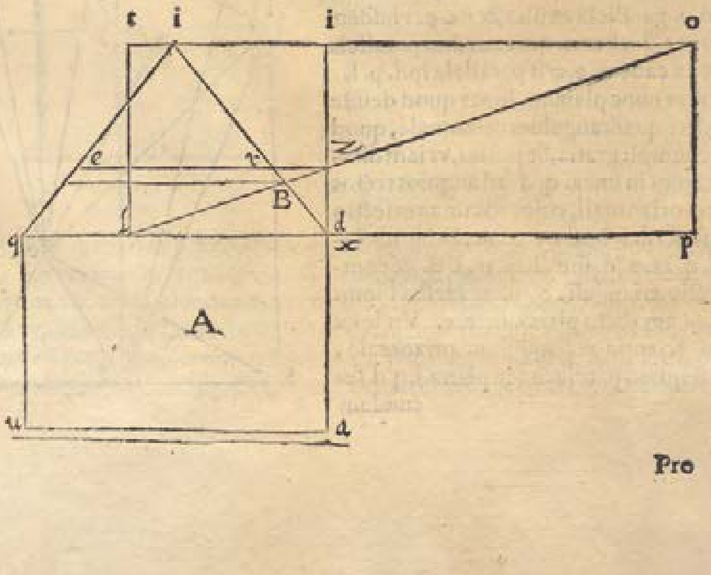
# DE RATIONIBVS OPERATIONVM PERSPECTIVAE.

## CAP. I.

CVM nullus adhuc (quod ſciam) veras internasq; cauſas operationis perſpectiuæ perfectè docuerit, operæprecium exiſtimaui aliquã de ijs diſputationem ſuſcipere.

Multi enim eorũ, qui huiuſmodi operationis regulas præſcribunt, cum eius effectuum veras cauſas ignorent, varios diuerſosq; errores committunt, vt exempli gratia in ſubſcripta figura ſuperficiali. A. volentes degradare (vt dicunt) rectangulum.q.a.in triangulo.i.d.q.ducunt parallelã ipſi.q.d. à puncto. B.interſecationis lineæ. o.l.cum latere. i. d.trianguli,& (idem)indifferenter, eandem quoque à puncto. Z.interſecationis ipſius.o.l.cum perpendiculari.x.i. ducunt. neſcientes hunc ſolum eſſe verum modum,n on item alium, quia ſi alius, talis eſſet, hic, verus non exiſteret,nam ſi vellent ſeſe excuſare, quòd ducendo dictam parallelam à puncto. B. hoc fiat præſupponendo planum ipſius.i.d.q. verſus rectangulum. q.a. orizontale inclinatum, ſecundum angulum. i. d. q. hæc excuſatio accipienda non eſſet, quia horum conſenſu,præſupponendo planum. i. d. q. inclinatum, anguli inferiores rectanguli degradati,non tam acuti, quam ſunt duo.i.d. q. et.i. q. d. eſſe deberent, quod facilè eorum ratione innoteſcet, quæ de figura corporea. A. hic ſubſcripta mox proponam, præter id, quòd volentes deinde aſpicere quadratum degradatum, oporteret huiuſmodi planum reſpectu oculi ita collocare, quemadmodum ſe habet linea.i.d.reſpectu.o. quod factu nimis arduum eſſet.

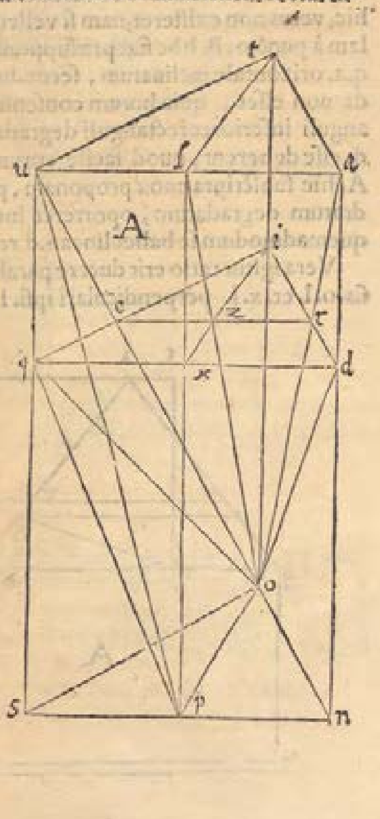
Vera igitur ratio erit ducere parallelam. e. r. ad. q. d. à puncto. Z. communi ipſis. o. l. et. x. i. perpendiculari ipſi. l. p.




Pro

Pro cuius rei ſpeculatione imaginemur in figura corporea. A: q. a. eſſe figuram rectangulam orizontalemq́; ad degradandam ſuper aliquod planum perpendiculare orizonti,& cum eo primum coniunctam in linea.q.d.cuius plani triangulum. i. q. d. pars erit, ſit autem oculus reſpicientis. o. cuius altitudo.o.p. ab orizonte, qui quidē conſpicit rectangulum dictum orizontale. q. a. in pyramide. o. q: o. u: o. a. et. o. d. terminata quatuor triangulis. o. q. u: o. u. a: o. a. d. et. o. d. q. ſit verò primum ita collocatus pes. p. eius qui reſpicit, vt linea. p. l. perpendicularis ipſi. u. a. lateri rectanguli, medio loco poſita ſit, inter. a. n. et. u. s. Idq́; primum nobis erit exemplum.

Imaginemur nunc lineas.u.q.et.a.d. indefinitè productas eſſe, quæ in ſuperficiebus duorum triangulorum.o. u. q. et. o. a. d.& rectanguli orizontalis. q. a. ex prima vndecimi Euclid. poſitæ erunt. Imaginemur etiam lineam. p. s. n. perpendicularem ipſi.p.l. quæ etiam cum duabus. u. q. s. et. a. d. n. ex. 34. primi Euclid. angulos rectos conſtituet,cum ex. 28.duæ.u.q. s. et. a.d.n.ſint parallelæ ipſi.p.l. et.s.n. ipſi.u. a. & quia ſupponitur.o.p.perpendicularis plano orizontali, Angulus ergò.o. p. l. rectus erit ex ſecunda definitione. 11. Euclid. Imaginemur quoque ductas eſſe duas.o.s.et.o.n. vnde.l.p. ei ſuperficiei, in qua ſunt duæ lineæ. o. p. et. s. n. ex. 4. 11.perpendicularis erit,& ſuperficies orizontalis.a.s.perpendicularis erit cum dicta o.s.n.ex.18.eiuſdem lib.vnde ex dicta definitione.o.s.u.et.o.n. a. erunt anguli recti et.o.s.et.o.n.ex communi ſcientia,in ſuperficiebus duorum triangulorum.o. u. q. et. o.a.d.erunt,ſi noluerimus cogere aduerſarium ad confitendum duas lineas rectas includere ſuperficiem, quemadmodum cogeretur facere, ſi opinaretur duas alias rectas per eadem puncta. o. s. n. tranſire, quæ ſunt in dictis ſuperficiebus. Vnde.o.s.et.o.n. communes erunt ſectiones duarum dictarum ſuperficierū cum ſuperficie. o. s. n. Imaginemur nunc has duas ſuperficies. o. u. et. o. a. quarum communis ſectio ſit.o.t.(quæ erit linea recta ex. 3. lib. 11.)quæ erunt perpendiculares ſuperficiei.o.s. n. ex. 4. et. 14. iam dictis. & ex. 19. eiuſdem o. t. perpendicularis eidem ſuperficiei. o. s. n. erit,& ex.6.eiuſdem hæc linea. o. t. duabus. u. q.s.et.a.d.n. parallela exiſtet,& ex. 9. eiuſdem hæc linea.o.t.duabus.u. q.s.et.a. d.n. parallela exiſtet,& ex eadem. 9.erit parallela ipſi. p. l. Imaginemur nunc planum, ſuper quod deſideremus videre quadrangulum orizontale, quod planum,exempli gratia,ſit primo,vt iam diximus, locatum in linea. q. d. ad angulos rectos cum plano orizontali, cuius communes ſectiones cum ſuperficiebus. s. t. et. n. t. viſionis laterum. u. q. et. a. d. ſint. i. q. et. i. d. & communis ſectio trianguli. o. u. a. ideſt viſionis lateris.a.u.cum dicto plano, ſit. r. e. Vnde ex communi ſcientia rectangulum orizontale, oculo.o.ſeipſum patefaciet in plano.i.q.d.ſecundum



cundum figuram quadrilateram.q. d. r. e. Communis autem ſectio ſuperficiei. p. t. cum dicto plano,ſit.i.x. quæ.i. x. perpendicularis erit. s. a. ſuperficiei orizontali ex 19.lib.11.quia.p.t.eſt etiam orizonti perpendicularis ex. 18.eiuſdem,cum. o. p. eidem perpendicularis exiſtat. Vnde.i.x.erit altitudo trianguli.i.q.d. & æqualis ipſi. o. p. ex. 34. primi. Sit deinde.o.l.cõmunis ſectio ſuperficiei triangularis.o. a. u. cũ ſuperficie.p.t.quæ.o.l.ſecando lineam.e.r. in puncto.Z.nobis oſtendet quantum diſtare ſeu eminens eſſe debeat latus.e.r. in plano ab.q. d. medio ipſius.z. x. Et quia præſuppoſuimus.p.l. in eodem medio,inter.u.s. et. a. n. ideo. x. q. ęqualis erit. x. d. & ex.4. lib.primi.i.q.ipſi.i.d.et.e.r.parallela ipſi.q.d.ex.6.lib. 11. cum ipſa quoque ſit perpendicularis ſuperficiei.p.t. ex. 19. eiuſdem. Hucuſque igitur in figura corporea. A. prodeunt in lucem omnes cauſæ effectuum figuræ ſuperficialis. A. ideſt vnde fiat,vt in ipſa figura ſuperficiali, triangulum.o.p.l.tale conſurgat,& quid ſignificet.o.et.o.p.et. p.l. et. o.l. & quam ob cauſam tale quoque formetur triangulum. i. q.d.atque in tantam altitudinem, quantam obtinet.o.p. & quid ſint latera.i. q. et.i. d.& quare erigatur.x.i.parallela ipſi.p.o.ab eadem. p. o. tanto ſpatio diſtans, & qua ratione producatur à puncto.Z.ipſa.Z.r.e.parallela ipſi. q.d.

Nunc obſeruandum eſt, quòd ſi planum ipſius. i. q.d. in figura corporea aliquantulum inclinatum eſſet orizontem verſus,anguli.i.q.d.et.i. d. q. maiores exiſterent, quàm cum idem eſt ipſi orizonti perpendiculare,quemadmodum clarè demonſtratum fuit in. 39.primi Vitelionis.

Non igitur rectè fit ſi in figura ſuperficiali ducatur à puncto.B. parallela ipſi. q. d. abſque maiori apertura angulorum.i.q.d.et.i.d.q.

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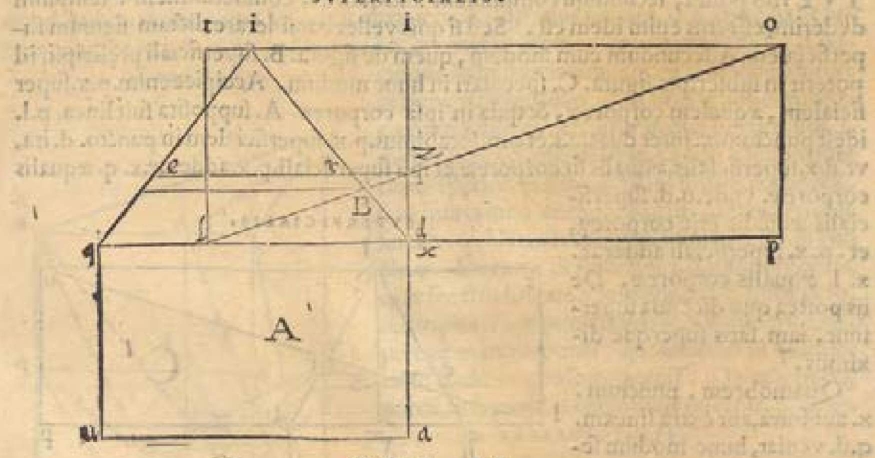


## CAP. II.

CVM verò duæ præcedentes figuræ intellectæ erunt, facilè quoque erit intelligere duas ſubſequentes.B.B. in corporea quarum.p.l. extra lineas.u.s.et.a.n. reperitur,vbi enim aduertendum erit oportere ſumere ſemper. p. x. figuræ ſuperficialis æqualem ei,quæ eſt corporeæ,& eidem ſuperficiali, adiungere. x. d. æqualem ei,quæ eſt corporeæ,& compoſito.p.d. ex dictis duabus lineis, in figura ſuperficiali, addere.d.q.æqualem ei,quæ eſt figuræ corporeæ, deinde accipere punctum. l. in ſu-

Q perficiali

perficiali,ita diſtans à. p.vt in corporea reperitur. Poſteà. x. i. erigetur æqualis lineæ. p. o. & ſuis terminis concludetur triangulum.i. q. d. & id quod remanet. Vnde ſi longius diſerendo progrediare, patebit ex. 4. primi. i. d. ſuperficialem, futuram æqualem. i. d. corporeæ. Idem dico de.i. q. & de reliquis.

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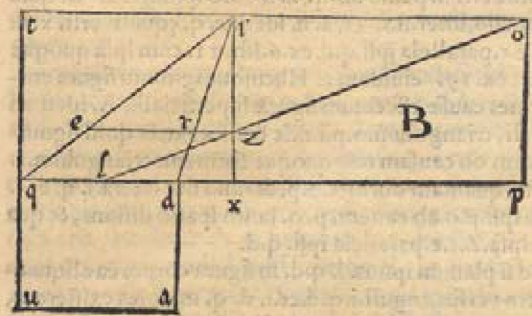


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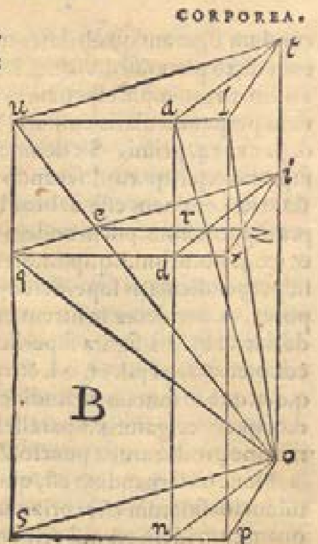


## CAP. III.

MODVS hic, proprius eſt, & vniuerſalis, licet in figura ſuperficiali. A. ſuperius poſita, ſecundum communem antiquorum conſuetudinem exemplum dederim, effectus enim idem eſt. Sed ſi quis vellet conſiderare dictam figuram ſuperficialem. A. ſecundum eum modum, quem de figura. B. ſuperficiali prȩſcripſi, id poterit in ſubſcripta figura. C. ſpeculari in hunc modum. Accipiet enim.p.x.ſuperficialem, æqualem corporeæ, & quia in ipſa corporea. A. ſuppoſita fuit linea. p.l. ideſt punctum.x. inter duas.u.s.et.a.n.ſecabimus.p.x. ſuperficialem in puncto. d. ita, vt.d.x.ſuperficialis,æqualis ſit corporeæ,& ipſi ſuperficiali.p.x.addetur.x. q. æqualis corporeæ. vnde.q.d. ſuperficialis æqualis erit corporeȩ, et. p. x. ſuperficiali addetur. x. l. æqualis corporeæ. De ijs poſtea quæ dicenda ſuperſunt, iam ſatis ſuperque diximus.

Quamobrem, punctum. x. aut intra, aut extra lineam. q.d. veniat, hunc modum ſequentes, in errorem non incidemus, imò efficietur quadrilaterum. q. r. ſuperficiale, ſimile, & æquale corporeo.

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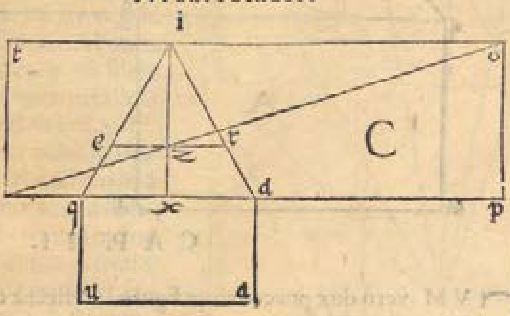



## CAP. IIII.

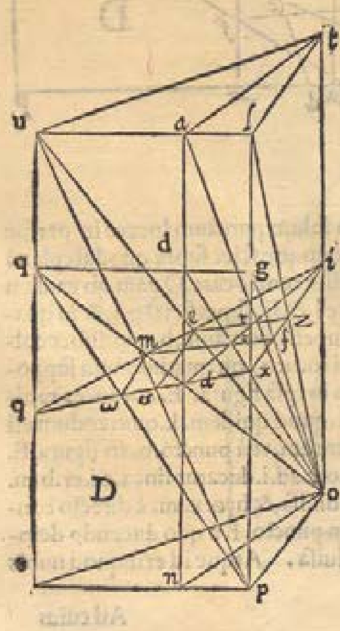
PVNCTVM verò.i. (quod verum eſt punctum perſpectiuæ, vt practici dicere ſolent) quid ſit, hac via & ratione ſub noſtram cognitionem cadit: quòd nihil

alIud

aliud eſt, quàm punctum, varijs ſectionibus commune, & huiuſmodi punctum, ocu lus non eſt, quemadmodum multi Pictores, Sculptores, Architecti, & Perſpectiui ignari, ipſum punctum, oculum appellando, falsò crediderunt, quaſi punctũ.i. perſpectiuæ oculus eſſet.

In ſupradictis igitur figuris manifeſte eluceſcit cauſa diminutionis obiectorum, & altitudinis trianguli æqualis ei, quæ eſt oculi à plano orizontali, vt etiam diſtantię. p.l.p.x. & cuiuſuis tandem rei. Sed vt huius effectus ſcientia magis in vniuerſum pateat. Volo duas hic ſubſcriptas figuras. D. corpoream, & . D. ſuperficialem à vobis conſiderari, in quarum corporea, linea. p.l. ſit extra duas. u.s. et. a. n. vt in figura. B. locata, ita tamen vt planum trianguli. i.q.d. diſiunctum ſit à rectangulo ſuperficiali, ideſt, vt ſeparatum exiſtat à linea. q.d. latere ipſius rectanguli, & ſit etiam obliquum, reſpectu ipſius rectanguli, ideſt vt communis ſectio dicti plani cum ſuperficie a.s. orizontalis ipſi. u.a. parallela non ſit, ſed ſit obliqua, ſi tamen idem planum perpendiculare dictæ ſuperficiei orizontali. a.s. erit: & dicta communis ſectio exprimatur characteribus. q. ω. α. d. x. nunc in figura corporea habebimus figuram. e. r. c. m. in plano, quod viſualem pyramidem ſecat, medio cuius figuræ. e. r. c. m. oculus poſitus in. o. rectangulum orizontale conſpicit. Volentes vero nunc in figura. D. ſuperficiali eam deſcribere, faciemus. p. x. ſuperficialem, æqualem corporeæ, eique addemus. x. l. æqualem corporeæ, aut ſumemus. p. l. eidem corporeæ ęqualem, quam ſecabimus in puncto. x. eodem planè modo, quo corporea reperitur diuiſa; erigemus deinde. p. o. et. x. i. æquales corporeis. Secabimus deinde. x. q. æqualem corporeæ, & ducemus. q. i. et. l. o. vnde habebimus triangulos. o. p. l. et. i. x. q. ſimiles & æquales corporeis ex. 4. primi Eucli. Secabimus deinde. q. x. in puncto. d. eadem ratione, qua ſecta fuit corporea, & ducemus lineam. d. i. vnde habebimus triangulos. i. d. q. et. i. d. x. ſimiles corporeis. & mediante triangulo. i. q. d. huc

CORPOREA.




uſque habebimus ſitus duorum laterum figurę rectanguli degradati, ideſt ſitus ipſius. e. m. et. r. c. etiam ſi adhuc neſciatur in qua parte ipſius. i. q. & ipſius. i. d. eſſe debeāt. Quod ſi ſcire voluerimus ſecabit. p. l. in pũcto. g. ſimilis corporeæ, ſi in ipſa tamen corporea prius protraxerimus lineam. q. d. latus rectanguli vſque ad. p. l. in puncto. g. Ducetur deinde linea. o. g. ſuperficialis, quæ ſecabit lineam. i. x. in puncto. f. linea vero. o. l. in puncto. z. punctis ſitis in. i. x. ſuperficiali, pręcisè vt in corporea, quemadmodũ quilibet ex ſe facilè cognoſcere poteſt. Deinde in corporea, in ſuperficie orizontali ducatur. p. q. et.

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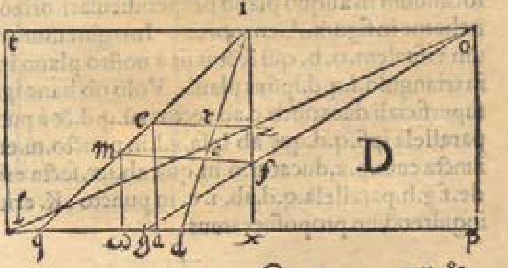
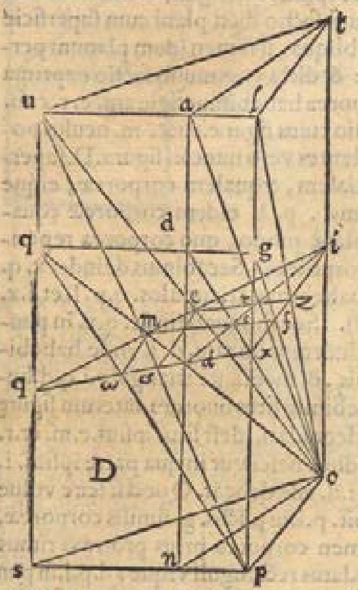


p.u.& imaginemur.o.q.in ſuperficie.t.s.vnde trianguli.o.p.q.et.o.p.u.erunt perpendiculares orizonti ex.18.lib.11.et.ω.m.et.α.e. communes ſectiones dictorum duorũ triangulorum cum plano trianguli.i.q.x.ipſi quoque plano ex.19. eiuſdem lib.erunt perpendiculares. Nunc autem ſecetur.q.x. ſuperficialis in punctis.ω.et.α. eadem ratione; qua corporea ſecta fuit à duabus.p.q.et.p.u.à quibus punctis.ω.et.α.ſuperficialibus ductæ ſint duæ.ω.m.et.α.e.perpendiculares vſque ad latus.i.q. in punctis. m.et. e. quę ſitum habebunt in. i. q. ſuperficiali prȩcisè, vt in corporea, ex. 26. primi, ducendo deinde in ſuperficiali duas.m.f.et.e. Z. eæ æquales erunt corporeis ex. 4. primi, & ſic anguli.i.e.z.et.i.m.f.& eę duę lineę.e.z.et.m.f.ſectę erunt à linea.i.d.in duobus punctis.r. et. c. vnde. e.r. et. m. c. æquales erunt corporeis ex. 26. primi, ſed ita quoque ſe habent duę.e.m.et.r.c.ſi verum eſt ꝙ differentię rerum æqualium ſint adinuicem etiam æquales. Hac ratione igitur habebimus figuram quadrilateram.m.e. r.c. ſuperficialem omninò ſimilem, & ęqualem corporeæ. Is tamen modus prolixus eſt, & arduus, quam ob cauſam neque ego vnquam eũ vſui accommodarem, neque alijs, vt eodem vterentur ſuaderem.

CORPOREA.

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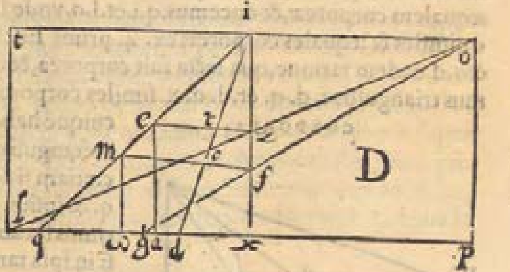


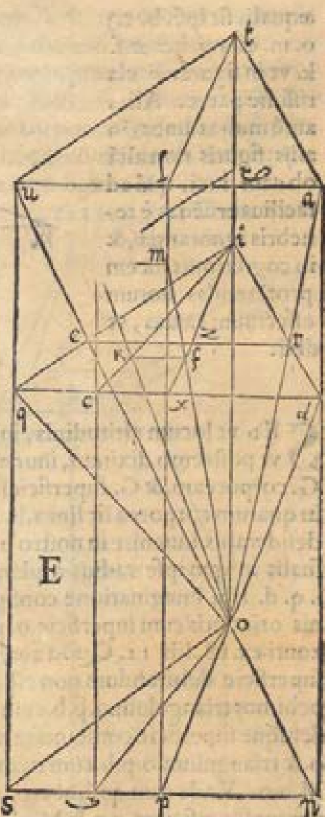

## CAP. V.

EST igitur ſciendum, quòd qui ſciuerit vnum ſolum punctum locare in perſpectiua, eo modo quem nunc proponam, facilè quoque ſciet ſupra quoduis planũ (quod tamen ſit perpendiculare orizonti) quamlibet rem locare. Quam ob cauſam imaginemur hic ſubſcriptas duas figuras. E. corporeã, & E. ſuperficialem, & in quadrilatero rectangulo orizontali.a.u.q.d.imaginemur eſſe punctum.b.quodlibet collocandum in aliquo plano perpendiculari orizonti locato, quemadmodum ſupponebatur in figura. A.corporea. Imaginemur ergo in ipſa figura. E. corporea radium viſualem. o. b. qui ſectus ſit à noſtro plano in. k.quod quidem. k.quærendum eſt in triangulo.i.q.d.ipſius plani. Volo ob hanc igitur rem, vt à puncto.b. in figura.E. ſuperficiali ducatur.b.c.ad rectos cũ.q.d.& à puncto.c.ad.i.ducatur linea.c.i.et.b.m. parallela ipſi.q.d.quę ab ipſa.x.l.in puncto.m.erit diuiſa, & hęc. x. m. è directo coniuncta cum.p.x.ducatur.o m.quæ ab.i.x.ſecta erit in puncto. f. à quo ducendo deinde.f.g.h.parallela.q.d.ab. i. c. in puncto. K. erit diuiſa. Atque id erit quod nobis inquirendum propoſueramus.

Ad cuius

Ad cuius rei ſpeculationẽ,imaginatione concipiamus lineam.b.c.corpoream,protractam eſſe vſque ad.y.lineæ.s.n.& imaginatione ſit compræhẽſa linea.y.o.et.b.℞. parallela eidem,ideo ob rationes iam dictas de figura. A. hæ tres lineæ.o.y:i.c: et. ℞.b. ſimul cum linea.o.b. erunt in vna eademque ſuperficie plana, quam characteribus . y. ℞. notemus . et . i .c. eius erit ſectio communis cum plano, in quo quæritur pũctum, et. f. k. ipſius plani cum triangulo.o.b.m. erit ſectio communis, & parallela ipſi . q. d. ex. 6. lib. 11. quia. k. f. perpendicularis eſt ſuperficici. p. t. ex. 19. eiuſdem cum triangulus. o. b. m. eidem ſuperficiei . p. t. ex. 18. eiuſdem perpendicularis exiſtat . Vnde perſpicuè patet ratio quare protracta ſit parallela . b. c: et quare ducta ſit . i. c. et coniuncta . x. m. cum. x. p. directè, & quare ducta ſit. o. m. et. f. k. Laudo igitur vt ſemper præſupponatur.p.x. perpendicularis baſi ipſius plani & præſupponatur, (vt rem totam vnò verbo complectar) ſuperficies. p. t. perpendicularis plano, & orizonti . Quod reliquum eſt, neceſſarium non eſt,niſi ad ſpeculandum. Neceſſariæ ergo non ſunt aliæ lineæ, quàm. p. x: p. o. x. i: b. c: et. x. m. è directo coniuncta cum . p. x. ( quæ. x. m. coniuncta æqualis

CORPOREA.

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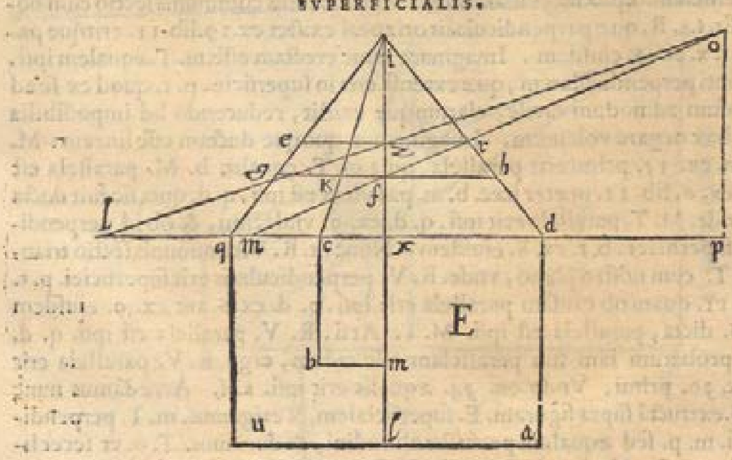
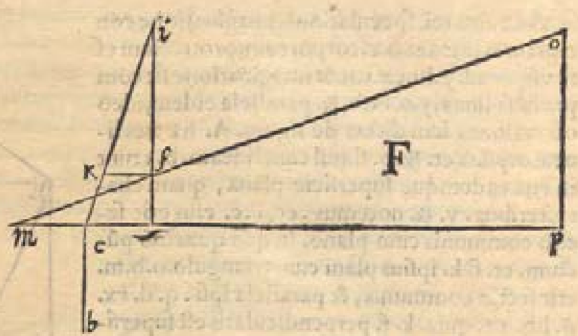


æqualis ſit ipſi. b. c.) o. m. etiam. i. c: et. f. k. vt in figura. F. clariſſimè patet. Alias autẽ multas lineas in alijs figuris non aliã ob cãm duxi, quã ad facilius eruẽdas è tenebris ignorantiæ, & in cognitionis lucem proferendas horum effectuum cauſas, vt dixi.



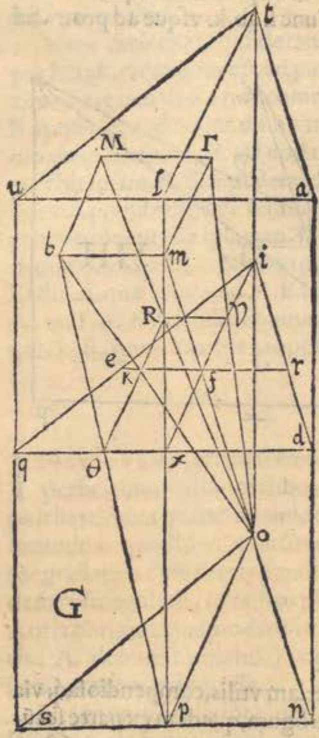

## CAP. VI.

SED vt locum altitudinis, in noſtro plano perpendiculari orizonti, & ita locatũ, vt poſtremo diximus, inueniamus; duas hic ſubſcriptas figuras conſiderabimus. G. corpoream, & G. ſuperficialem, ſimiles duabus. E. E. proximè præcedentibus, in quarum corporea ſit linea, b. M. altitudinis perpendicularis orizonti. Quare ſi deſiderabis inuenire in noſtro plano ſitum puncti. M. ideſt punctum radij. o. M. viſualis in quo ipſe radius à plano eſt diuiſus, quod ſit. R. quamuis extra triangulũ i. q. d. tibi imaginatione confige ductam eſſe lineam. p. b. quæ erit ſectio communis orizontis cum ſuperficie. o. p. b. M. quæ ſuperficies erit perpendicularis ipſi orizonti ex. 18. lib. 11. Quòd autem non minus. o. p. quàm. M. b. ſit in vna eademq́ue ſuperficie dubitandum non eſt, quia ſi imaginabimur ductam eſſe lineam. p. M. habebimus triangulum. o. p. b. cum triangulo. M. b. p. communibus partibus in vna eademq́ue ſuperficie conſtantem, vt triangulum quoque. o. p. M. cum triangulo. M. b. o. & triangulum. o. p. b. cum triangulo. o. p. M. & triangulum. M. b. p. cum triangulo. M. b. o. Vnde cum quilibet triangulus in vnica tantum ſuperficie ſit ex. 2. lib. 11. ſequetur ſuperficiem. o. p. b. M. planam eſſe, & vnicam, cuius communis ſectio cum noſtro plano ſit. 8. κ. R. quæ perpendicularis orizonti exiſtet ex. 19. lib. 11. eritq́ue parallela ipſi. i. x. ex. 6. eiuſdem. Imaginare nunc erectam eſſe. m. T. æqualem ipſi. b. M. orizonti perpendicularem, quæ extenſa erit in ſuperficie. p. t. quod ex ſe ad conſiderandum admodum facilè, clarumq́ue exiſtit, reducendo ad impoſſibilia quemlibet hæc negare volentem. Imaginemur quoque ductam eſſe lineam. M. T. quæ. b. m. ex. 33. primi erit parallela, quia. m. T. ęqualis. b. M. parallela eſt ipſi. b. M. ex. 6. lib. 11. præter hæc. b. m. parallela eſt ipſi. q. d. quia ſic fuit ducta ſuperius, vnde. M. T. parallela erit ipſi. q. d. ex. 9. vndecimi, & ob id perpendicularis erit ſuperficiei. b. t. ex. 8. eiuſdem. Nunc ſit. R. V. communis ſectio trianguli. o. M. T. cum noſtro plano, vnde. R. V. perpendicularis erit ſuperficiei. p. t. ex. 19. lib. 11. quam ob cauſam parallela erit ipſi. q. d. ex. 6. aut ex. 9. eiuſdem quia ex. 6. dicta, parallela eſt ipſi. M. T. At ſi. R. V. parallela eſt ipſi. q. d. etiam. f. κ. probatum iam fuit parallelam eſſe eidem, ergo. R. V. parallela erit ipſi. κ. f. ex. 30. primi, Vnde ex. 34. æqualis erit ipſi. κ. f. Accedamus nunc ad figurã. G. extructã ſupra figuram. E. ſuperficialem, & erigamus. m. T. perpendicularem ipſi. m. p. ſed æqualem perfectæ altitudini, & ducamus. T. o. vt ſecet lineam. i. x. in puncto. V. ab ipſo ducentes. V. R. parallelam ipſi. q. d. ducendo de-

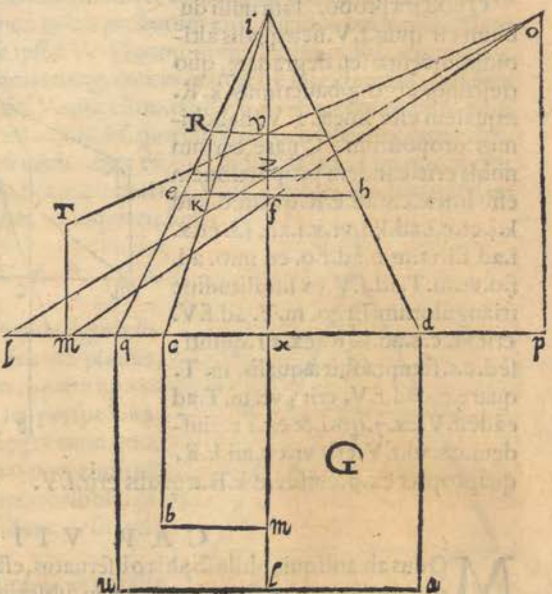
inde

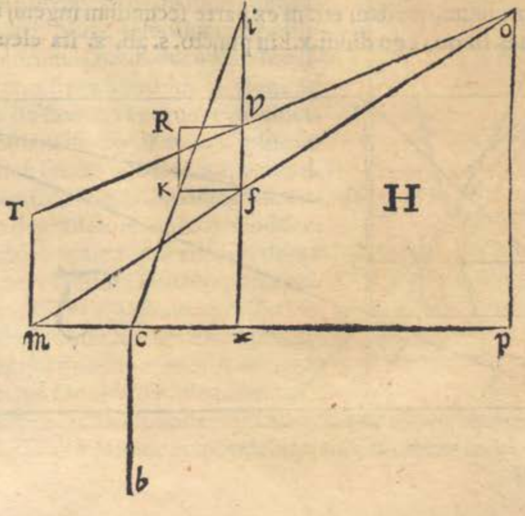
inde. k. R. parallelam ipsi. i. a. habebimus altitudinem. k. R. quam quærebamus in nostro plano. Quod cum sui natura clarum euadat, laborem ratiocinandi de eo, cuilibet vel mediocriter in præclarissima hac scientia erudito relinquo. idest, vt probetur. k. R. superficialē, æqualē esse corporeæ. Sed tollēdo superfluitatē linearum, & hoc accōmodantes vt in figura. F. diligenter consideretur figura. H.

CORPORBA.




SVPERFICIALIS. CAP. VII.

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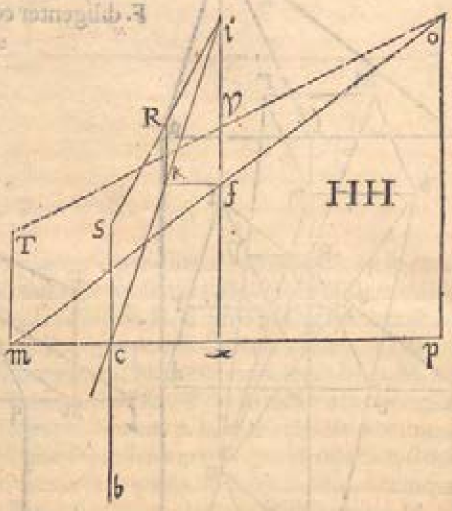
## CAP. VII.

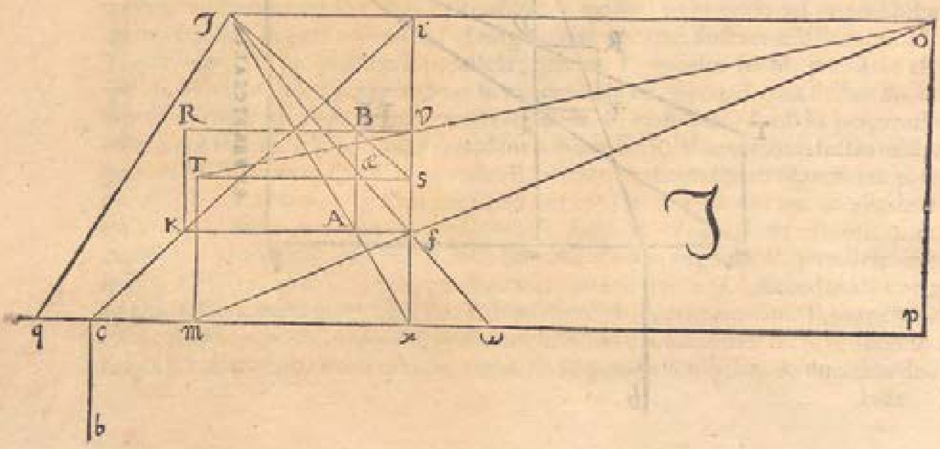
ALiam tamen inueni viam breuiorem vt in figura . H. H. in qua sit punctus . b. perfecti , & . k. degradati plani . Nunc ducatur. b. c. s. ad rectos cum . p. m. indefinitè , quæ quidem abſcindatur in puncto. s. ita quòd. c. s. æqualis sit altitudini perfectæ, deinde coniungatur rectà. s. cum. i. Tunc si ab. k. vſque ad protractã i. s. ducta fuerit. k. R. parallela lineę. c. s. hæc. R. k. erit altitudo quæsita ſeu degradata.




Quod ita probo. Iam nulli dubium eſt quin. f. V. ſit æqualis altitudini quęſitæ ſeu degradatę, quotieſcunq; ergo ꝓbauerimus. k. R. æqualem eſſe lineæ. f. V. habebimus propoſitum . Quare certum nobis erit eandem proportionem eſſe lineæ. c. s. ad. k. R. quam. c. i. ad k. i. et. c. i. ad. k. i. vt. x. i. ad. f. i. et. x. i. ad. f. i. vt. m. o. ad. f. o. et. m. o. ad. f. o. vt. m. T. ad. f. V. ex ſimilitudine triangulorum. Ergo. m. T. ad. f. V. erit vt. c. s. ad. k. R. ex. 11. quinti, ſed. c. s. ſumpta fuit æqualis. m. T. quare. c. s. ad. f. V. erit , vt. m. T. ad eãdẽ. f. V. ex. 7. qnti, & ex. 11. eiuſdem. c. s. ad. f. V. erit vt. c. s. ad. k. R. quapropter ex. 9. eiuſdem. k. R. æqualis erit. f. V.

## CAP. VIII.

MOdus ab antiquis philoſophis obſeruatus, eſt etiam vtilis, compendioſaq; via progreditur, cuius ſpeculationem, in ſubſcripta figura, quadam ex parte ſecũdum morem antiquum, quadam etiam ex parte ſecundum ingenij mei vires cõſtructa, cognoſcemus. In qua ego diuiſi. x. i. in puncto. s. ab. x. ita eleuato , quanta eſt vera

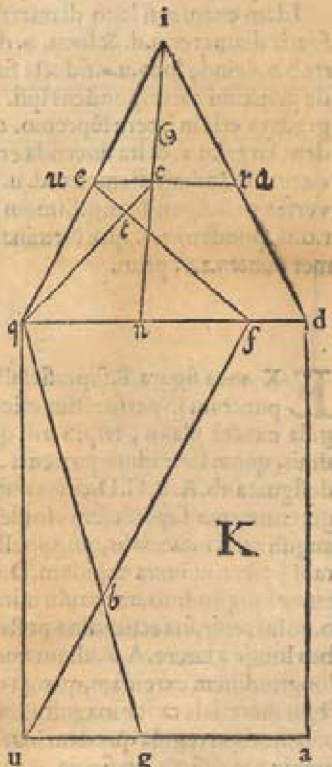


vera altitudo ipsius. M. T. et I.s.duxi supponendo esse.I.punctū pspectiuæ secundū antiquos,idest angulum supremum trianguli antiquorum à punctoque.k. meo duxi k.f.parallelam ipsi.c.m.p.vsque ad.i.x.in puncto.f.& à puncto à communi ipsis. k. f. et.i.x.vsque ad.I.s.duxi quoque.A.B.parallelam ipsi.i.x. atque hæc omnia ex more antiquo præstiti.

Nunc verò eum considerans modum, quem ego de figuris.G.H. antecedentibus præscripsi,videndum est,an punctum.B.tribus lineis.A.B.I.s.et.R.V.quarum hęc vltima à me iam ducta fuit,commune existat,idest vtrùm.A.B. æqualis existat ipsi. K. R.quam secundum modum à me adinuentum, reuera scimus esse desideratam altitudinem in perspectiua. Quod tunc à nobis probatum erit, quando rationibus clarè patebit ipsam.A.B.æqualem esse ipsi.f.V. Quamobrem ducamus. I. f. vsque ad.ω. lineæ.c.p.vnde ratione similitudinis triangulorum manifestè intelligemus, eandem proportionem esse ipsius.m.T. ad.f.V.quæ est.m.o.ad.f.o.& eius,quæ est.m.o. ad.f. o.quæ est.ω.I.ad.f.I. & eius,quæ est.ω.I.ad.f.I.quæ est.x.I.ad.A.I.& eius, quæ est. x. I.ad.A.I.quæ est.x.s.ad.A.B.idest vt eius,quæ est.m.T.ad.A.B.sed idem quoq; erat de. m.T.ad.f.V.Vnde sequitur.A.B.æqualem esse.f.V.ex.9.quinti Eucli.atq; etiam ipsi. k. R. quod à nobis propositum est inquirendum.

## CAP. IX.

INSTITVENS etiam sermonem de figuris superficialibus orizontalibus, seu de plantis, pulcherrimum quendam modum, quem ego ad locandum quodlibet punctum in perspectiua, (degradatum cum fuerit parallelogrāmum quoddam rectangulum, in nostro plano perpendiculari orizonti,quemadmodum in superioribus figuris. A. demonstrauimus ) consideraui, silentio haud prætereundum esse.

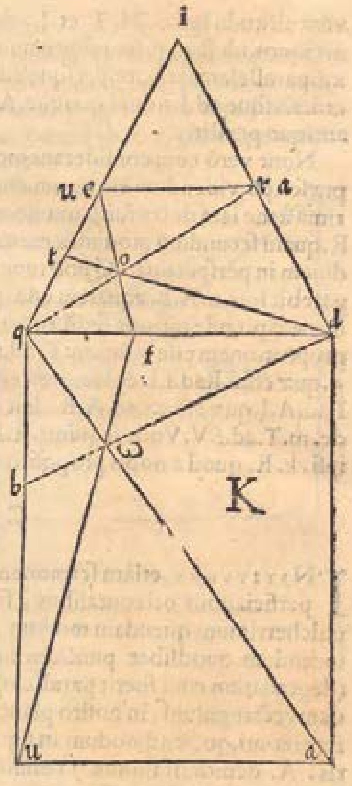



Sit igitur in subscripta hîc figura. K. in parallelogrāmo perfecto pūctum.b. quod locari debeat in degradato.e.q.d. r. Nunc à duobus quorumlibet quatuor angulorum.q.u.a.d.ducuntur duæ lineæ occultæ.q.g.et.u.f. per punctum. b. vsque ad latera.q.d.et.u. a. ita tamen vt eorum extremitates. g.et.f.intus cadant inter.q.d.et. u.a. ipsorum laterum,idest vt non secent duo latera.q.u.aut.d. a.Deinde punctum.f.inter.q.et.d.cōiungatur occultè cum angulo degradato.e. qui corresp̄ōdet. u.perfecti,mediante linea.e.f.quæ erit.u.f. degradata in nostro plano. Deinde sumatur punctum. n.in linea.q.d.tam distans à.q.quam. g. distat ab. u.ducaturque linea.i.n.quæ lineam.e.r.in puncto c.diuidet,quod ex ijs, quæ superius iam diximus ad ipsum.g.referetur. Ducendo postea lineam occultam.q.c.patebit eam correspondere lineæ. q.g.quæ secans lineam.e.f.in puncto. t. hoc, communi scientiæ ratione,respondebit ipsi.b.vt omnes cognoscent.

R Sed

Sed ſi fortè punctum . b. eſſet in aliquo laterum, puta. q. u. volo vt in rectangulo perfecto. q. d. a. u. ducta ſit vna diagonalis quam volueris puta. q. a. deinde à puncto. b. ad reliquum angulum oppoſiti lateris ducta ſit recta. b. d. ita quod à diagonali ſecetur in puncto. ω. per quod punctum demum à reliquo angulo lateris. q. u. ducta ſit. u. ω. vſque ad latus. q. d. in pũcto. f. quo facto, ita faciendum erit in rectangulo degradato, hoc eſt ducenda erit diagonalis. q. r. quę correſpondet diagonali. q. a. perfecti deinde. f. e. quæ correſpondet rectæ. f. u. perfecti, quæ etiam interſecabitur à diagonali. q. r. in puncto. o. correſpondens. ω. perfecti, per quẽ. o. à puncto. d. cum ducta fuerit. d. o. vſque ad. t. in latere. q. e. hoc punctũ. t. correſpondebit puncto. b. perfecti.

Idem eueniet ſi loco diametri. q. a. ſumpta fuerit diameter. u. d. & loco. b. d. protracta fuerit. b. a. deinde loco. u. ω. f. ducta fuerit. q. ω. f. vnde punctum correſpondens ipſi. f. in figura degradata erit in latere ſupremo. e. r. correſpondens lateri. u. a. & ita ducenda erit diameter. d. e. correſpondens diametro. d. u. et. q. f. ſurſum verſus correſpondens. q. f. imum verſus deinde. r. o. reſpondens. a. ω. quæ terminabitur ab eodẽmet puncto. t. vt prius.

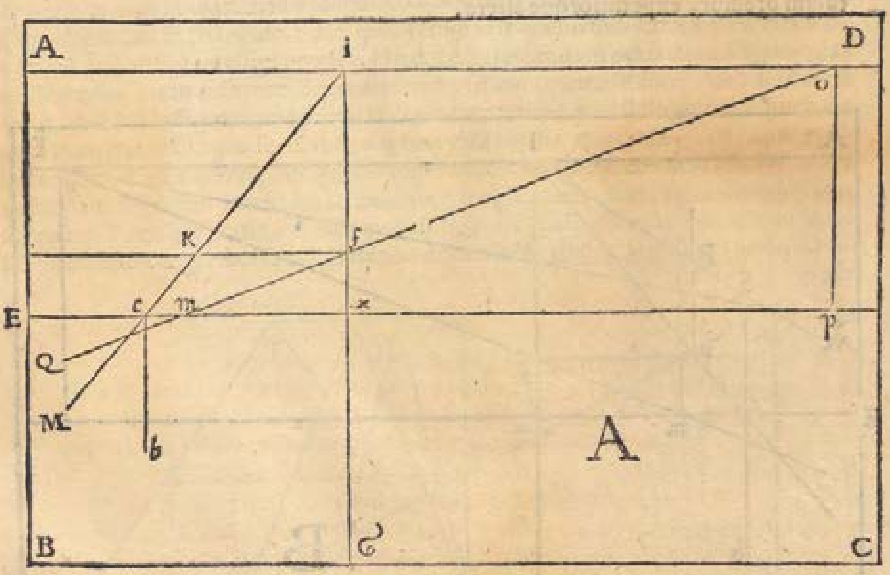



## CAP. X.

Ex mea figura. F. ſuperficiali perſpectiuæ facillimum modum locandi quoduis punctum in perſpectiua elicui. Iuſſi enim vt aptaretur tabula quædam rectangula exactè plana, triplo aut quadruplo, aut quanto volueris maioris longitudinis, quàm latitudinis protenſa, quæ quidem latitudo erat ad duos circiter pedes deſignata ab. A. B. C. D. cuius duobus lateribus. A. B. et. B. C. iuſſi, vt duæ regulæ affigerentur, quæ ſuperficiem eiuſdem tabulæ excederent, vt vnũ ex lateribus alicuius anguli recti materialis, qui appellaĩ norma (vt inferius dicam) ei adherere poſſit, curaui poſtea, ut iuxta angulum. D. in puncto. o. fixo mobilis regula. o. Q. affigeretur tantæ longitudinis, aut paulò minoris, quantam occupabat latus. D. A. quæ circum. o. volueretur, in rectitudine poſteà, o. i. parallela ipſi. D. A. in puncto. i. duobus pedibus longè à latere. A. B. aliam quoque mobilem appendere feci. i. M. in tantam ferè longitudinem extenſam, quanta conſtat. A. B. cõſtitui etiam, vt quoddã angulum rectum materiale tantæ magnitudinis, quanta nobis vſui eſſe poterat ſuper eadem tabula; necnon regula quædam materialis neceſſariæ longitudinis ſtatuerenĩ, atq; hæc omnia tenuiſſima, vt fierent curaui. Quandam deinde lineam ad. o. i. parallelam, ideſt. p. E. ſuper eadem tabula adeò diſtantem ab. o. i. vt inter. E. p. et. B. c. perfectæ res, quæ degradari debebant, locari poſſent, ſignaui. Hæc autem diſtantia, quæ

inter

inter.o.i.et.p.E.intercedebat,altitudinem oculi ab orizonte ſignificabat. Signaui etiam lineam.i.G. perpendicularem lineæ. E.p. cui affigi poſſet non nihil chartæ quotieſcunque volebam in perſpectiua aliquid delineare. Quod cum facere deſiderabam,ponebam perfectum optimè affixum in quadrangulo. E.G:& in quadrangulo.E.i.aliquod folium papyri affigebam. Ponamus nunc, me voluiſſe conſtituere punctũ.b.ſumebã angulũ rectũ materialẽ,ſeu normã,& eius vnum latus, iuxta latus. B.c.ponebam,atque aliud per punctum.b. tranſire faciebam, & vbi hoc latus lineã E.p.diuidebat, punctum.c ſignabam per quod efficiebatur,vt regula.i.M.tranſiret, quieſceretq́ue aliquantulum aliquo modo in huiuſmodi ſitu, opera deinde circini interuallum.b.c.ſumebam.& in.p.E.à puncto.x.verſus.E. punctum.m.ſignabã : per quod faciebam, vt tranſiret regula.o. Q. quæ lineam. x.i. in puncto. f. diuidebat. Angulum deinde rectum materialem accipiebam,cuius vnum latus.A.B.ponebam, aliud verò per punctum.f.tranſibat,quod quidem latus regulam. i. M. in puncto. k. (quod ſtatim ſuper folio papyri ſignabatur) interſecabat, atque hoc erat punctũ, quod quærebam,puncto.b. correſpondens.Huiuſmodi effectus rationes ab ijs, quæ ſuperius dixi eliciuntur. Atque hæc ad baſes rerum,vt in ſubſcripta figura eluceſcit, ſpectabant.




## CAP. XI.

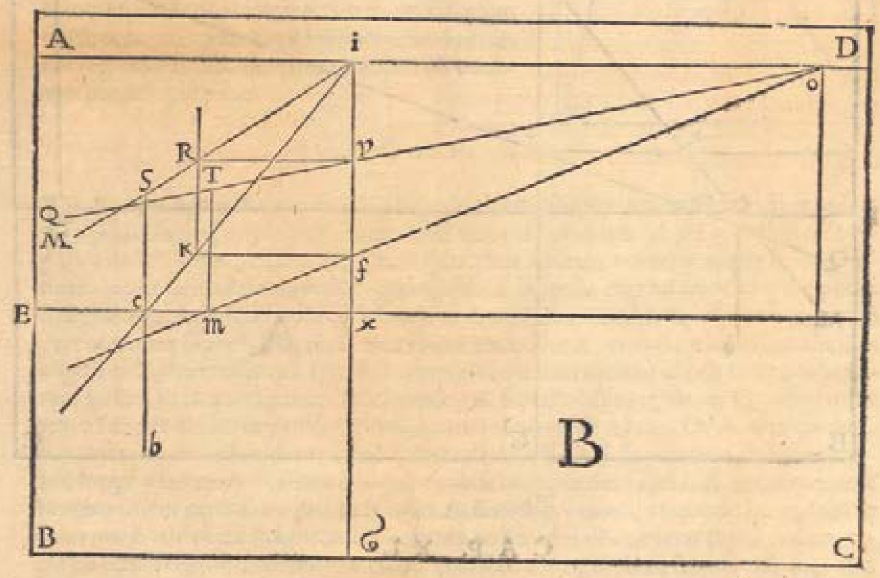
AD degradandas deinde altitudines, vſus ſum mea figura tam.H. quã ẽt. H.H. vt mihi ſeſe offerebat occaſio. In primis ratione modi figuræ.H.curabã, vt

R 2 vnum

vnum ex lateribus anguli recti, seu normæ regulæ. B. C. anniteretur, aliud verò per.m.in rectitudine cuius signabam.m. T. interuallum æquale altitudini perfecti, idest punctum.T.æqualiter distans ab.m.transire faciebam, deinde regulam. o. Q. per punctum.T. transire quoque faciebam: & notabam intersectionem ipsius cum linea.i.x.in puncto. V.efficiebam deinde vt vnum ex lateribus anguli recti,lateri tabulæ. B. C.anniteretur,aliudque per punctum.k.transire faciebam, & in huiusmodi rectitudine à puncto.k.signabam quandam mensuram æqualem lineæ.f.V. quę erat k.R.pro altitudine degradata.

## ALITER IDEM.

MEDIANTE deinde figura. H. H. vnum ex lateribus anguli recti, lateri tabulæ.B.C.vt anniteretur faciebam;aliud verò per punctum.b.perfecti,idest basis eiusdem perfecti transire faciebam. Et in huiusmodi rectitudine signabam. c. s. æquale interuallum altitudini perfecti,idest punctum.s.ita distans à.c. efficiendo deinde,vt latus anguli recti,lateri.B. C. tabulæ anniteretur, aliudque per punctum.k. transire faciens signabam. k.R.indeterminatam. Faciens deinde transire regulam i.M. per punctum.s.notabam punctum.R. intersectionis eiusdem cum linea. k. R. iã ducta. Itaque altitudinem. k. R. degradatam habebam. Hæc autem via aliquantulum breuior, expeditiorque altera.

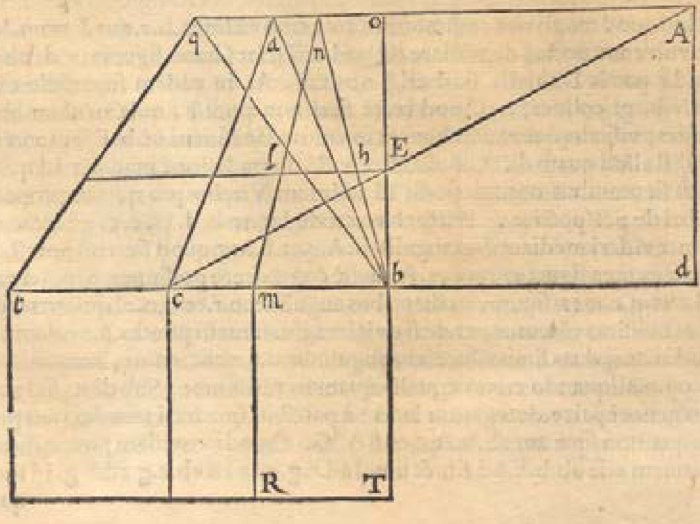



IACOBO

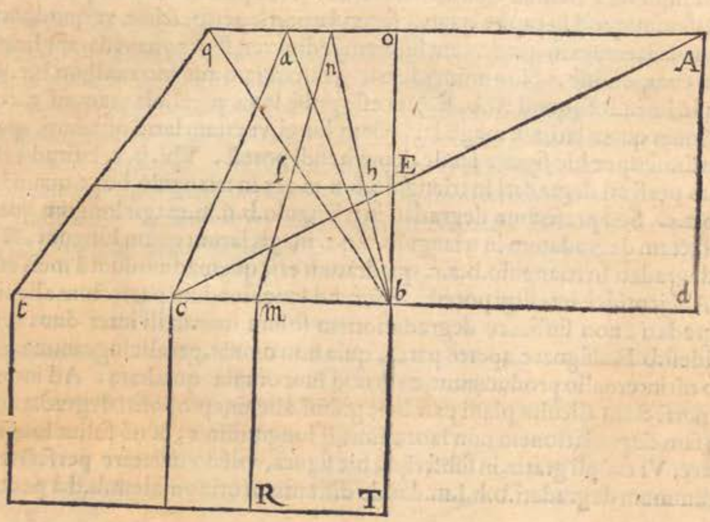
## *JACOBO SOLDATO MEDIOLANENSI Serenissimi Ducis Sabaudiæ Architecto peritissimo.*

### CAP. VII.

SVPERIORIBVS diebus non diu postquam de perspectiuis inter nos sermonem habuimus, dum animus totus adhuc in his esset. Illud in mentem venit quòd eximius ille vir, & profundissimæ doctrinæ, nec vnquam satis laudatus Daniel Barbarus se accepisse profitetur à Ioanne Zamberto patritio Veneto, qui ad verbum omnia desumpserat à Ioanne Cusino Parisiense. Nec parum mirabar peritissimum illum Cusinum, quod in capite quarto secundæ partis perspectiuæ, vt quodpiam planum quadrilaterum in quadratam figuram redigeret, super vnam datam lineã quadratam composuisse. Non animaduertens distantiam aut interuallum.b.c. degradatum in linea.b.f.(quod est.b. E.) ita esse posse latus parallelogrammi rectanguli magis longi quam lati, aut magis lati quam longi, vt etiam latus quadrati, quod beneficio subscriptæ hic figuræ facilè depræhendi potest. Vbi. b. c. latitudo esse potest, tam perfecti degradati in triangulo.b.n.m. aut in triangulo.b.q.t. quam in triangulo.b.a.c. Sed perfectum degradati in triangulo.b.n.m. magis longum quam latũ & perfectum degradatum in triangulo.b.q.t. magis latum quam longum, & perfectum degradati in triangulo.b.a.c. quadratum erit quemadmodum à meis etiam figuris.A.scientificè intelligi potest. Hinc, ad inueniendum perfectum alicuius plani degradati, non sufficere degradationem solum interualli inter duos terminos solos, idest.b.E. assignare, apertè patet, quia non omnia parallelogramma perfecta ab vno tñ interuallo producuntur, eo ꝙ non sunt omnia quadrata. Ad inquirendũ igitur perfectum alicuius plani parallelogrãmi, alicuius propositi degradati, oportet vniuersam degradationem tam latitudinis, q̃ longitudinis, & nõ solius longitudinis assignare; Vt exẽpli gratia, in subscripta hic figura, volẽdo inuenire perfectum parallelogrammum degradati.b.h.l.m. dando distantiam orizontalem.b.d. à pede.d. hominis



minis vſque ad plani ſitum in quo degradatio facta ſit : ſtatim altitudo.A. oculi à pede, quæ tanta ſemper eſſe debet quanta eſt altitudo trianguli.b.n.m. qui clauditur, protrahendo. m. l. et. b. h. vſque ad concurſum in. n. in lucem prodibit. Oportet deinde erigere lineam.b.o. perpendicularem lineæ.d.b.m. & vſque ad eandem producere lineam.l.h.in puncto.E.et à puncto.A.per.E. vſque ad.c.ipſius.d.b.m.productæ ducere.A.E.c.atque deinde protrahere lineam.o.b.vſque ad.T.ita vt.b T.ęqualis ſit ipſi.b.c.& ad ipſam à puncto.m.ducere parallelam.m.R.& à puncto.T.ducere. T.R.parallelam ipſi.b.m. Vnde ex.34.primi Eucli.m.R.æqualis erit ipſi.b. T.et.R. T. ipſi.m.b.& anguli in rectos euadent,atque hoc parallelogrammum rectangulum erit verum perfectum degradati. b.m.l.h.ob rationes à me circa figurã. A.adductas.




Sed eſt hic quod magis nos commoueat,quia cum ex linea.b.c.quadratum. b.g.produxerit,vult eum poſtea degradare. Quod vt faciat ( hanc figuram videbis in cap. 4. ſecundæ partis Danielis Barbari ) oculum. A. in eadem ſuperficie extenſa, quadrati. b. g. collocat. Quod rectè fieri non poteſt, quia oculum hoc modo locantes, viſualesque radios beneficio vnius plani ſituati in. b. f. ſecantes in ipſo plano,nihil aliud quam dictã lineã.b.f.& nullã degradationẽ inueniẽt.Id quod,& ſi natura ſua ſit omnibus notum, ponit tñ id ipſum Vitelio pro quinta propoſitione quarti libri de perſpectiua. Præter hæc,credit latera.b.d. et. c. e. quadrati degradati ſemper videri mediantibus angulis. b.A.c.et.f.A.g.quod fieri nõ poteſt,quemadmodum ex mea figura corporea.A.facilè cognoſcere poſſumus, propterea quòd latera.d.r.et.q.e.meæ figuræ,mediantibus angulis.d.o.r.et.q.o.e. qui extra ſuperficiem. s. a. exiſtunt videntur, vnde ſi quis imaginaretur in puncto.p.oculum eſſe, & ab ipſo ad.u.et.q.duas lineas duceret, angulus.q.o.u.nunc maior, nunc minor eſſet angulo.q.p.u.aliquando etiam æqualis,quamuis rariſſime; Sub diuerſis igitur angulis,pro maiori parte,deteguntur latera,à partibus quadrati tam degradati, quàm perfecti,quæ non ſunt anguli.b.A.c.et.f.A. G. Quod vero idem poſtea dicat eam proportionem eſſe ab.b.E.ad.f.h.& ſimul ad.c.g.quæ eſt ab.a.g.ad.h.g. id tuo relin

quam

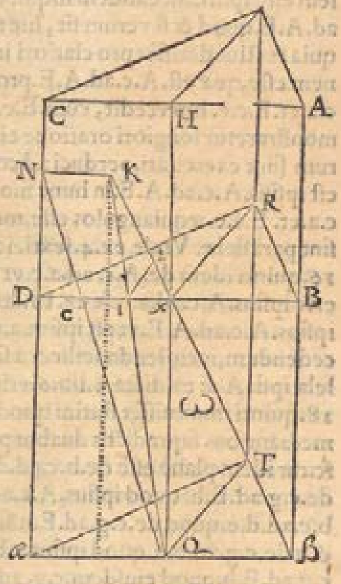
quam iudicio. Tibi quoque conſiderandum relinquo; cum rationabilis degrada-
tio eſſe debeat, qua ratione neceſſarium ſit, vt diſtantiæ reſque, in vna & eadẽ pro-
portione cum altitudine oculi ad rem degradatam exiſtant? Cum poſtea degrada-
uerit quadratũ, is ſcriptor, in figura. d. b. c. e. eum bene & ex perſpectiuæ optimis
legibus degradatum fuiſſe probare nititur; ſolum probans. d. e. æqualem eſſe ipſi.
E.h. ꝙ.E.h. ſecundũ ipſum eſt degradatio lateris.c. g. & cũ ſuperius dixerit, ſe tria
quadrati plana degradauiſſe, quia.b.E. degradat. b. c. et: E. h. degradat. c. g. et.f.
h. degradat. f. g. nec quidem de lateribus. b. d. et. c. e. loquitur, quia ſi. c. g.
perfecti, degradatum eſt in. E. h: et. d. e. rectè protracta exiſtit, cum ſit æqua-
lis ipſi. E. h. cum etiam. b. d. et. c. e. rectè protractæ eſſe debeant: qua de cau-
ſa ipſis.b.E. et.f.h.quæ, ex ipſo, ſunt degradationes.b.c.et. f. g. æquales eſſe non de-
bent? Poſſet is mihi quidem reſpondere, ꝙ hoc pacto nulla ſuperficies clauderetur.
Ergo tria latera.b.c: c.g.et.g.f.nõ benè ſunt degradata, eiuſq; ꝓportionalitates ma
lè intellectæ nil probant. quia ſi dictæ proportionalitates, nobis tutò promitterent
degradationes, ab eo primum effectas, in linea.b.f.eſſe bonas, ergo duæ.b.d.et.e.c.
falſæ exiſterent, quarum quælibet maior eſt.b.E.et.f.h.ex.18.primi Eucli. Omitta-
mus etiam quod vbi is ſcribit eam eſſe rationem, aut comparationem ab. A.d.ad.b.
E.quæ eſt ab.d.c.ad.b.c. eandemq́ue eſſe ab.E.h.ad.c.g.quæ eſt ab.A.E.ad.A.c.nil
probet; nec ſimilitudinem triangulorum, nec aliquam propoſitionem Eucli.citans.
In quo excuſari non poteſt, quòd non ſoleat Euclidem, aut alium quemuis autorem
citare, cum vel in ipſo operis principio capite.3. primæ partis, Apollonium Pergeũ
Euclidemq́;, & ſi etiam præter rem, citet. Deinde quũ idem probare vult.d.e.æqua
lem eſſe ipſi.E.h. eandem inquit eſſe proportionem.a.b.ad.a.d.quæ eſt ipſius. A. c.
ad.A.E.quod & ſi verum ſit, hic tamen modus ratiocinandi nullo ordine nititur,
quia rectius dixiſſet pro clariori intelligentia ipſius.a.c. ad.a.e. eandem proportio-
nem eſſe, quæ eſt.A.c.ad.A.E.propter ſimilitudinem, quæ inter duos triangulos. A.
c.a.et.E.c.e. intercedit, cum.E.c.ſupponatur parallela ipſi.A. a. quod etiam vt de-
monſtraretur longiori oratione ei opus fuiſſet ſi voluiſſet intellectum eorum, qui pa
rum ſunt exercitati, perduci ad cognoſcendũ idem planè futurum de.a.c.ad.a.e. vt
eſt ipſius.A.c.ad.A.E.in hunc modum, ideſt probando primùm duos triangulos.A.
c.a.et. E.c.e. æquiangulos eſſe, mediante.29.primi Eucli. cum.A.a. et.E.e.inuicem
ſint parallelæ. Vnde ex.4.ſexti. idem extitiſſet de.A.c. ad.E.c.vt.a.c. ad.e. c. et. ex
16.quinti idem de.A.c. ad.a.c.vt ipſius.E.c. ad.e.c.& ex.19. eiuſdem de.A.E. ad.a.
e.vt ipſius.A.c.ad.a.c.& ex.16.iam dicta de.A.E.ad.A.c. vt ipſius.a.e. ad. a.c. ideſt
ipſius.A.c.ad.A.E.vt eſt ipſius.a.c.ad.a.e: Aut hoc alio modo, qui breuior eſt pro-
cedendum, incipiendo ſcilicet à ſecunda ſexti Eucli. dicendo ꝙ exiſtente.E.e.paral
lela ipſi. A.a: ex dicta.2.lib.6.erit idem de.c.E.ad.E. A. vt de.c.e.ad. e.a. vnde ex.
18.quinti innotuiſſet ſtatim quod de.c.A.ad.E.A.vt de.c.a.ad.e.a. extitiſſet. Nunc
mediantibus ſupradictis duabus propoſitionibus ideſt.29. primi, &.4. ſexti, cogno-
ſcitur idem planè eſſe de.b.c.ad.d.e.quod ipſius.a.c. ad. a. e. & ex eiſdem idem eſſe
de.c.g.ad.E.h. quod ipſius.A.c.ad.A.E.vnde ex.11.quinti bis repetita idem erit de
b.c.ad.d.e.quod de.c.g.ad.E.h.ſed cum ex ſuppoſito.c.g. ſit æqualis ipſi. c. b. idem
erit de.c.g.ad.e.d.quod ipſius.c.b.ad eandem ex.7.quinti, vnde ex.11.idem erit de
c.g.ad.E.h.quod eiuſdem.c.g. ad.e.d.ex.9.igitur eiuſdem. d. e. æqualis erit ipſi. E.
h. atque hic verus eſt modus ducendi intellectum parum exercitatum in cognitio-
nis campum. quem quidem mihi obſeruandum proponerem ſi onus ſcribendi ſu-
ſciperem ijs, qui in ſcientijs parum verſati ſunt, quos tanquam puerulos manu du-
cere

cere oportet. Ratio verò ab ipſo adducta propter quam.E.repreſentatur oculo altius quam.b.nempe eo quod. A.ſuperſtet ipſi. E. nihil valet, quia ſi inferius eſſet, idem contingeret, ſed hoc euenit eo quod.E.altius eſt ipſo. b. Idem dico de. h. vbi ſimiliter decipitur. Idem etiam in.7.cap.fallitur in ſecundo modo,quem oſtendit pro ſecundo quadrato aliquo degradato à parallelogrammo degradato magis longo quàm lato,cum ducat parallelam.l.m.ad.b.c.à puncto.l. interſectionis ipſius. o.c.id,quod non rectè efficitur quemadmodum ex rationibus à me allegatis circa meas figuras. A. A. facilè innoteſcit.

Nono deinde cap. contrario planè ordine, quam oporteret proceſsit, quia cũ angulus. 2. trianguli perfecti magis diſtet à plano ſuper quod degradari debet triangulum,quàm latus. 1.3. oppoſitum dicto angulo. 2. & per conſequens longè remotior ſit ab oculo, ipſe in degradato, cũ magis propinquum eſſe facit, è contra cap. 10. rectè fecit contra id, quod capite. 9. tradiderat.

Quod autem deinceps in prima parte. 11.& vltimi capitis aſſerit eſt,admittendũ. Quod verò in ſecunda parte ab eo traditur, ideſt alius quidam modus quem de trãſferendis punctis à perfecto in degradato proponit, non eſt modus vniuerſalis; quia ſi altitudo. T.Q. oculi à plano orizontali, non eſſet æqualis medietati lateris. B. D. perfecti, interualla.a.b.c.d.e.lateris. B.D. admittenda non eſſent.

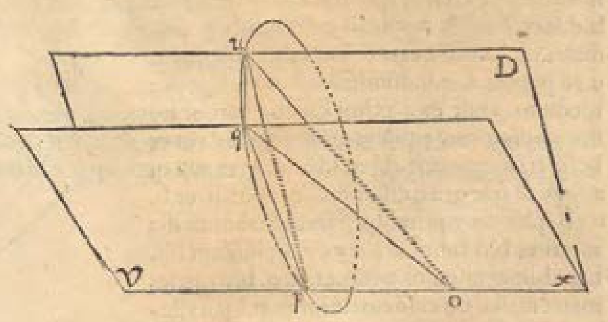
Pro cuius rei intelligentia ſit in ſubſcripta hic figura corporea. æ. parallelogrammum rectangulum A.B.C.D. in plano orizontali, & linea. Q.H. illud per medium diuidat,quæ ſit parallela duobus lateribus. A.B.et. C. D. in cuius quolibet puncto. Q. ſit infimus terminus altitudinis oculi, & in. T. ad perpendiculum ipſius. Q. ſit verus ſitus eiuſdem, tantum eleuatus à. Q. quanta eſt medietas ipſius. D. B. ſitque figura corporea finita ſimilis meæ. A. vnde. Q. T. æqualis erit ipſi. Q.æ.& planum perpendiculare orizõti, ſuper quod punctum. k. perfecti duci debet ſit. R. D. B. ſintque ductæ per imaginationem lineæ. T.K.Q.K. et ſit.K.N. perpendicularis lateri. C.D. à quo puncto. N. imaginatione ſit cõpræhenſa linea. N.Q. atque hæ tres lineæ ſectæ ſint à plano in punctis.c.i. et. 2. quorum punctũ. 2. erit quæſitum plani. Imaginemur nunc duos triangulos. K.T.Q. et. N.Q.æ. qui ſecti erũt à plano. R. B. D.quorum communes ſectiones erunt. 1.2. et D.c. & quia. N. K. D. i. et. æ. Q. inuicem ſunt parallelæ, ſequitur eandem proportionem futuram ipſius. Q.K.ad.K.i. quæ eſt ipſius. æ. N. ad. N. D. imaginatione concipiendo à puncto. K. vſque ad. æ. Q. quandam parallelam ipſi. N.æ.quemadmodum ex te ipſo intelligere potes. Sed ratione ſimilitudinis triangulorum ita ſe res habet de.æ.Q.ad.D.c.vt de. æ. N. ad. N. D. vt quoque de. T.Q. ad. 2. 1. quemadmodum ipſius. Q.K.ad.K.i. vnde ex. 11. quinti, idem erit de. Q.T.ad. 1.2. quod de. Q.æ.ad.c.D.& ex. 16. eiuſdem de. Q.T.ad.Q.æ.quod de. 1.2.ad.c.D.& exiſtente.æ. Q. ex ſuppoſito æquali ipſi



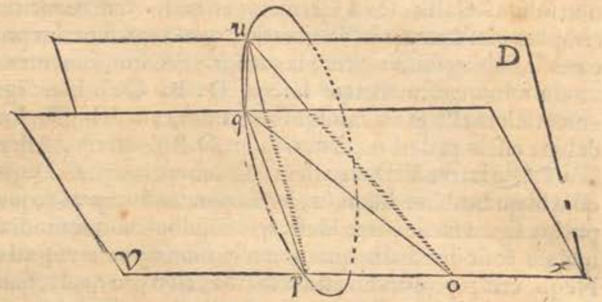
.1.2. Vnde huiusmodi regula tunc bona redditur, quando T. Q. æqualis est ipsi. œ. Q. idest medietati ipsius. D. B. at verò si æqualis non esset hoc minime sequeretur, vt facilè patet. Quòd verò.2.℞.2. &.sint benè disposita, dubitandum non est, quia punctum. i. meæ hic subscriptæ figuræ, quod corespondet K. eius figuræ adeò distat à medio. R. X. trianguli. R. B. D. vt.2.cum.1.2. dicto medio. R. X. ex.6. Vndecimi fit parallela. Idem de reliquis dico. quod manifestè cognosci potest, ab eo, quod in superius positis figuris corporeis dixi. Huiusmodi modus ducendi res in perspectiua, non solum à Gallis, sed à Germanis etiam in vsum reducitur. Sed quia ad hæc vsq; tempora eiusdem perfectionis ratio, quam ego superius proposui, nōdum in lucem emersit, factum fuit, vt errorū laqueis irretirentur, sumentes. T. Q. modo maiorem, modo minorem medietate lateris. D. B. Cum hunc igitur modum hic Autor vniuersalem esse putet, labitur in errorem, cum debuisset longitudinem ipsius. T. Q. debere esse æqualem medietati ipsius. D. B. proferre. Asserit deinde distantiam ipsius. T. Q. à latere. B. D. æqualem esse debere lateri. C. D. quod necessarium non est, quia in quibuslibet distantijs, iusta operatio fieri potest, quemadmodum in subscripta hîc figura facile patet, idest, quòd quibuscunque modis. c. D. æqualis remaneat ipsi. 1.2. & sic interualla, quæ p transuersum aguntur vsq; ad mediū trianguli. D. R. B. Neque etiam probandus est auctor ille, cum pro oculo, suum. T. loco. Q. à me positi, ponit, cum is locus sit verus situs pedis eius qui respicit, & non oculi. Quòd autē Auctor iste, modo vniuersali intelligat, vt iam diximus, cōsideretur figura tertij modi primi cap. tertiæ partis, in qua suum oculum (vt ita dicam) ponit in. o. altius seu distans à rectitudine lateris. c. d. plus quam sit totum latus. d. b.

## AD EVNDEM IACOBVM.
### CAP. XIII.

TVAS accepi literas omnis humanitatis & officij plenas, in quibus requiris causam, quæ me in alijs meis literis impulit ad dicendū, angulū. q. o. u. modo maiorem, modo verò minorem futurum angulo. q. p. u. meæ figuræ corporeæ. A. hanc igitur ob causam imagineris in subscripta hîc figura duo triangula. q. o. u. et. q. p. u. quorum. q. p. u. perpendiculariter sit super superficie trianguli. q. o. p. collocatum, præcisè vt in mea figura corporea. A. superficies verò trianguli. q. o. p. sit exempli gratia. V. M. & trianguli. u. o. p. sit. V. D. quarum cōmunis sectio sit. V. p. o. x. non est enim dubitādum quin triangulum. q. p. u. sit perpendiculare triangulo. q. o. p. cū hoc ex. 18. lib. 11. Eucli. perpendiculare sit superficiei. a. s. in qua reperitur triāgulum. q. p. u. & hoc

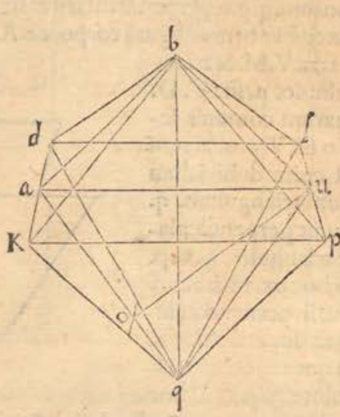



ex linea. o. p. perpendiculari dictæ superficiei. a. s. Nunc dico angulum. q. o. u. modo maiorem, modo minorem esse angulo. q. p. u. Notissimum igitur primum nobis eſt angulum.p.q.u.obtuſum eſſe; Imaginemur ergo circa triangulum.p.q.u.circunſcriptum eſſe circulum, cuius portio.p.q.u. minor erit medietate eiuſdem medij circuli,vt iam ex 30.Eucli.lib.tertij nouiſti.nunc imaginemur dictum circulum circum lineam.q.u.loco axis verſus.x.moueri,vnde girus eiuſdem,per quem tranſibat linea V.x.remouebitur ab eadem linea non nihil cum motus erit à primo ſitu vſquequò ad ſecandam dictam lineam.V.x.in alio quodam puncto inter.p.et.x.redibit; quod quidem punctum ſi erit inter.o.et.x.angulus. q. o. u. maior erit angulo. q. p. u. Sed ſi idem punctū erit inter.p.et. o. dictus angulus. q. o. u. minor erit. q. p. u. de qua q-dē re tu ipſe mediante.20.lib.3.et.16.lib. primi certior fieri potes . Valde miror ꝙ hæc Ioannis Cuſini dicta ad hæc vſque tempora tanto in prætio ſint habita , vt ab excellentibus ſcriptoribus quaſi ſi proprij eorum ingenij partus eſſent,de verbo ad verbum vt theſauros,in ſuis ipſorūmet libris reſcripta fuerint,quemadmodum iam omnes admonui in mea gnomonica Orontium, Munſterum, alioſq́; permultos feciſſe.



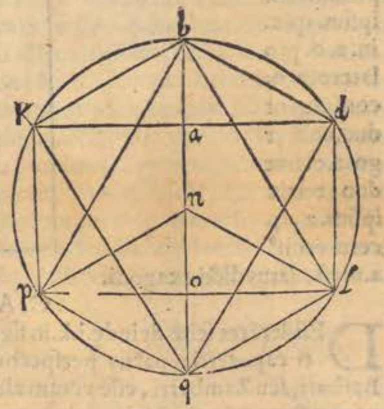

## CAP. XIIII.

EX ijs, quæ de nonnullis effectibus ducendo in perſpectiua tertium corpus regulare, ꝙ octo triangulis æquilateribus eſt terminatum,ſcire deſideras,hoc vnū eſt caput: vnde fiat, aut quomodo probetur quaſlibet duas facies oppoſitas eiuſdem corporis octoaedri inuicē æquidiſtantes eſſe. Quamobrem ſit hic ſubſcriptū octoaedrū, cuius diameter vna ſit.b.q.et.b.p.l. vna ex faciebus, cui opponatur facies.q. k. d.quas adinuicē æquidiſtantes eſſe contendo ſint aliæ duæ facies, quæ inter has ponuntur. b.d.k.et.q.p.l.& à punctis extremis.b.q. diametri.ductæ ſint quatuor lineæ.b.a:b.u:q.a:q. u.ad puncta.a.et.u.diuidentia.k.d.et. l. p. per medium, vnde ex 4.primi Eucli. quatuor hæ lineæ adinuicem ęquales erunt ſumēdo eas vt baſes triangulorū.a.d.b:u.l.b:a.d. q. et.u.l. q. adinuicē quoq; æqdiſtabūt.a.b. ab.u. q. et.b. u.ab.q.a.ex.27.primi;qa ſi imaginabimur diametrum.b.q.tunc ex.4.aut ex.8.eiuſdem lib. habebimus angulos.a.b.q.et. u. q. b. æquales inuicem; ſed ob eaſdem rationes.p.l.parallela eſt ipſi.d.k.vnde ex 15.lib.11.facies. b.p.l. parallela ſit,aut æquidiſtans ipſi. q.d. k. ideſt primum propoſitum.



Ad habendam deinde quantitatem diſtantiæ, aut interualli ſimul cum ſitu, in facie.q.d.k.quem latus.p.l.perpendiculariter reſpicit. Imaginemur à puncto. u. ſuper q.a. cadere lineam perpendicularem. u. o. quæ illico reperitur cum triangulum. a. u.q.ex lateribus datis & cognitis conſtet, quodquidẽ triangulum,medietas eſt quadrilateri,ſeu.rumbi.q.a.b. u. cui vnaquæque dictarum quatuor facierum perpendicularis exiſtit ex.4.et.18.lib.11.& ob id linea.u.o.extenſa in ſuperficie dicti quadrilateri, & perpendicularis lineæ. q. a. perpendicularis erit faciei. q. d. k. & ex. 29. primi, angulus.b.u.o.rectus erit, ut etiã angulus.o.u.l.ex.2.definitione lib.11.vnde ex.4.eiuſdem lib.o.u.perpendicularis erit faciei.b.p.l. Habebimus ergo ſitum in facie.q.d.k.qui reſpicietur ad angulos rectos à linea.p.l.quiquidem erit in perpendiculari à puncto.o.ad.q.a.ducta.

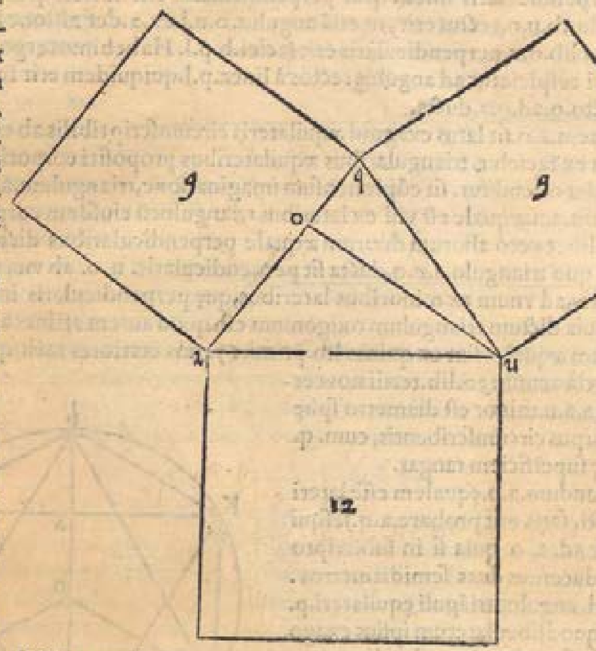
Quòd autem.a.o.ſit latus exagoni æquilateris circumſcriptibilis ab eodem circulo, qui vnam ex faciebus triangularibus æquilateribus propoſiti corporis circunſcribere poteſt,ita oſtenditur. ſit cóprehenſum imaginatione,triangulum.a.q. u.ſeparatim,cuius latus.a.u.æquale eſt vni ex lateribus triangulorũ eiuſdem corporis ex. 33. primi,quodlibet verò aliorum duorum æquale perpendicularibus dictorum triangulorum, in quo triangulo.a.u.q.ducta ſit perpendicularis. u. o. ab vna extremitatũ lateris maioris,ad vnum ex minoribus lateribus,quę perpendicularis intra triangulum cadet,quia dictum triangulum oxigonium eſt. quod autem attinet ad duos angulos. a.et.u.cum æquales ſint ex quinta lib.primi;17. nos certiores facit;quod verò angulus.q.ſit etiã acutus:30.lib.tertii nos certos reddit,q.a.a.u.minor eſt diametro ſphęræ datum corpus circumſcribentis, cum. q. dictæ ſphęrę ſuperficiem tangat.




Ad probandum.a.o.ęqualem eſſe laterî exagoni dicti, ſatis erit probare.a.q.ſeſquialteram eſſe ad. a. o. quia ſi in ſubſcripto hîc circulo ducemus duas ſemidiametros. n.p.et.n.l.ad. angulos triãguli ęquilateri.p. et.l.& cum quodlibet laterum ipſius exagoni, ęquale ſit ſemidiametro circuli ex. 15. lib. 4. habebimus ex. 8.primi,angulum.n. p.l.æqualem angulo.q.p.l. Vnde ex.4.eiuſdem.o.n. ęqualiserit ipſi .o.q.ideſt.q.a. ſeſquialtera erit ad.a. o.

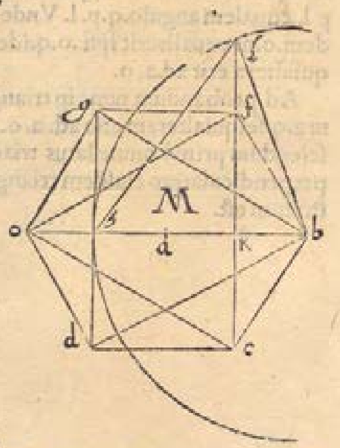
Ad probandum nunc in triangulo. a.q. u: a.q.ſeſquialteram eſſe ad. a. o. eſt quoq; ſciendum primò omne latus trianguli ęquilateri in potentia ſeſquitertium eſſe ad perpendicularem eiuſdem trianguli, quod vndecima lib.14. Eucli. breuiter demonſtratum eſt.

Ponamus nunc quadratum lateris.a.u.eſſe.12.clarum erit quodlibet quadratum aliorum duorum laterum.a.q.et.u.q.futurum nouem,ex ijs quæ poſteriore loco diximus, & quia quadratum ipſius. q. a. eſt tantò minus aliorum duorum quadratorum ſumma,quantum eſt duplum producti ipſius.q.a.in.a.o.ex.13. ſecundi, ſed alia duo quadrata ſimul collecta faciunt.21.à quo numero ſubtrahendo quadratum ipſius.a.q.ideſt nouem,remanebit numerus.12.pro duplo producti ipſius.q.a.in. a. o. cuius dupli media pars,ideſt ſimplex productum ipſius . q. a. ĩ a.o.erit 6. Sed qa quadratum ipſius.q.a. eſt nouem, eius radix. q.a. erit. 3. per quã diuidendo. 6. productum ipſius. q. a. in. a. o. pro latere. a. o. conſurgent duo,cum ergo .a.o. ſint duo tertia ipſius. a. q. certi erim⁹ a.o.eſſe latus dicti exagoni.




## CAP. XV.

DEſiderãtes ſcire deinde.l.k.in figura.M. quarti cap. tertiæ partis perſpectiuę Danielis Barbari, ſeu Zamberti , eſſe veram altitudinẽ corporis octoaedri,primũ ſcire debemus ꝙ exiſtẽte.b.h.vt etiã.b.l.tripla ad.b.k.vt ex ijs, quę ſuperius iã diximus,facile percipi poteſt;ex penultima primi. b.l.in potentia,ſeſquioctaua erit ad. k. l. ipſa ẽt. k. l.dupla in potẽtia ad.h.k.& ob id ducta cũ eſſet. h. l.exiſteret in potentia tripla ad.h.k. & ſeſquialtera ad.l.k.& ſeſquitertia ad.l.b.& ſic ad.h.b. vnde.l.h. æqualis eſſet vni ex lateribus triãguli ęquilateri dicti corporis. Ex rationibus igitur ſuperius hic poſitis.l.k. erit altitudo dicta,id eſt diſtantia inter duas facies inuicem oppoſitas, octoaedri.




Neq; volo te ignorare aliũ nõ paruũ fuiſſe errorẽ illius Zamberti : cum eodẽ capite affirmet angulos octoaedri rectos eſſe cũ ſint acuti,nã vnuſquiſq; minor eſt angulo cubi ſolido. DE

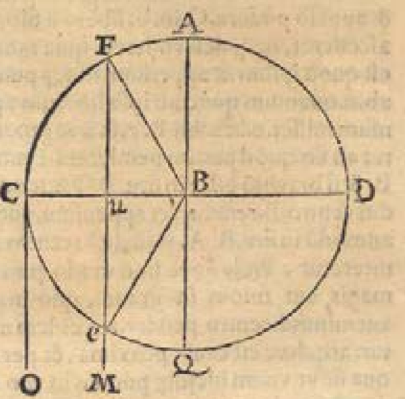
# DE MECHANICIS.

*SCRIPSERVNT multi multa, & quidem scitissimè, de mechanicis, at cum natura vsusq; aliquid semper vel nouum, vel latens in apertum emittere soleant, nec ingenui aut grati sit animi, posteris inuidere, si quid ei contigerit comperuisse prius tenebris inuolutum: cum tam multa ipse ex aliorum diligentia sit consequutus. Paucula quadā futura, vt reor, non ingrata his qui in hisce mechanicis versantur, nusquam ante hac tentata, aut satis exactè explicata in medium proferre volui: quo vel iuuandi desiderium, vel saltem non ociosi ingenioli argumentum aliquod exhiberem: atque vel hoc vno modo me inter humanos vixisse testatum relinquerem.*

## *De differentia situs brachiorum libra.*

### CAP. I.

OMNE pondus positum in extremitate alicuius brachij libræ maiorem, aut minorem grauitatem habet, pro diuersa ratione situs ipsius brachij. sit exempli gratia. B. centrum, aut, quod diuidit brachia alicuius libræ, &. A. B. Q. verticalis linea, aut, vt rectius dicam, axis orizontis, &. B. C. vnum brachium dictæ libræ, & in. C. sit pondus, &. C. O. linea inclinationis, seu itineris. C. versus centrum mundi, cum qua. B. C. angulum rectum constituat in puncto. C. Existente igitur in huiusmodi situ brachio. B. C. dico pondus. C. grauius futurum, quam in alio quolibet situ. quia supra centrum. B. omninò non quiescet, quemadmodum in quouis alio situ faceret. Ad quod intelligendum, sit dictum brachium, in situ. B. F. cum eodem pondere in puncto. F. & linea itineris seu inclinationis dicti ponderis sit. F. u. M. per quam lineam dictum pondus progredi non potest, nisi brachium. B. F. breuius redderetur. Vnde clarum erit quòd pondus. F. aliquantulum supra centrum. B. mediante brachio. B. F. nititur. Est quidem verum, quòd pondus. C. nec ipsum etiam per lineam. C. O. proficiscetur, quia iter extremitatis brachij est circularis, &. C. O. in vno quodā puncto est contingens. Sit hoc iter. A. C. Q. Oportet nunc præsupponere pondus extremitatis brachij debere tanto magis cētro. B. inniti, quanto magis linea suæ inclinationis (ponamus. F. u. M.) propinqua erit dicto centro. B. quod sequenti cap. probabo, vt exempli gratia, sit. F. super. u. punctum medij ex æquo inter. C. et. B. quapropter. u. B. æqualis erit. u. C. vnde se-

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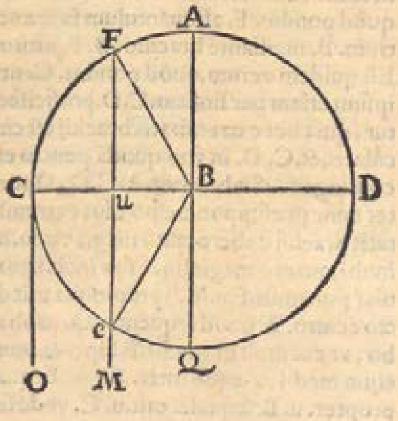
quetur dictum pondus grauius futurum pro parte.F.C.quam pro ea,quæ est.A.F. & minus supra centrum. B. pro dicta parte. F. C. quam pro parte. A. F.quieturum; & dictum brachium quanto magis orizontale erit à situ.B.F.tantò minus supra dictum centrum.B.quiescet,& hac ratione grauius quoque erit, & quanto magis vicinum erit ipsi.A.à dicto.F.tantò magis super centrum.B.quoque quiescet,vnde tātò quoque leuius existet. Idem dico de omni situ brachij per girum inferiorem.C. Q. vbi pondus pendebit à centro.B.dictum centrum attrahendo, quemadmodum superius illud impellebat. Hæc verò omnia cap. sequenti melius percipientur.

## *De proportione ponderis extremitatis brachij libræ in diuerso situ ab orizontali.*

### CAP. II.

PROPORTIO ponderis in. C. ad idem pondus in F. erit quemadmodum totius brachij.B.C.ad partem.B.u.positam inter centrum & lineam.F.u.M.inclinationis, quam pondus ab extremitate.F.liberum versus mundi centrū conficeret.Quod vt facilius intelligamus imaginemur alterū brachium libræ.B. D. & in extremo. D. locatum aliquod pondus minus pondere. C. vt. B. u. pars.B.C.minor est.B.D.clarè cognoscetur ex.6.lib.primi de ponderibus Archimedis, quòd si in puncto.u.collocatum erit pondus ipsius.C.libra nihil penitus à situ orizontali dimouebitur. Sed perinde est quòd pondus.F. æquale.C.sit in extremo.F.in situ brachij.B.F.quā vt sit in puncto.u.in situ ipsius.B.u.orizontali. Ad cuius rei euidentiam imaginemur filū. F.u.perpendiculare,& in cuius extremo.u.pendere pondus,quod erat in.F.vnde clarum erit quòd eundem effectum gignet, ac si fuisset in.F.quod, vt iam diximus remanens affixum puncto.u.brachij.B.u.tantò minus graue est situ ipsius.C. quantò.u.B.minus est ipso.B.C.Idem assero si brachium esset in situ. e. B. quod facilē cognoscere poterimus, si imaginemur filum appensum ipsi. u.brachij. B. C. & vsque ad. e. perpendicularē,in quo extremo appensū esset pondus æquale ponderi. C. & liberū ab.e.brachij.B.e.vnde libra orizontalis manebit. Sed si brachium.B. e. consolidatum fuisset in tali situ cum orizontali.B.D. & appēso pōdere.C.in. e. libero à filo,nec ascēderet, neq; descenderet. quia tantum est quod ipsum sit appensum filo,ꝙ pendet ab.u.quantum quòd ab ipso liberum appē nsum fuisset.e.brachij.B. e.& hoc procederet ab eo quòd partim penderet a centro. B. & si brachiū esset in situ. B.Q.totum pōdus centro.B.remaneret appensum,quemadmodū in situ.B. A. totū dicto centro anniteretur. vnde fit vt hoc modo pondus magis aut minus sit graue, quò magis aut minus à centro pendet,aut eidem nititur: atq; hæc est causa proxima, & per se, qua fit vt vnum idemq; pondus in vno eodemq; medio magis aut minus graue existat.

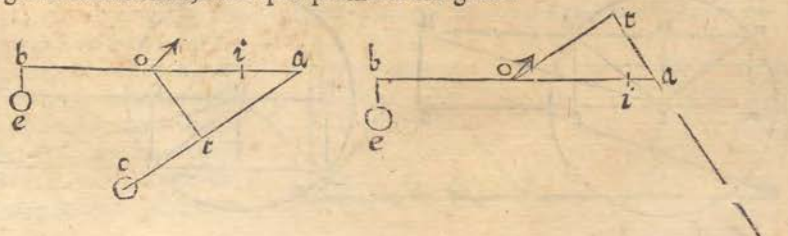


ſtat. Et quamuis appellem latus.B.C.orizontale, ſupponens illud angulum rectum cum. C.O. facere, vnde angulus.C.B.Q.ſit vt minor ſit recto, ob quantitatem vnius anguli ęqualis ei, quem duæ.C.O.et.B.Q.in centro regionis elemẽtaris conſtituũt, hoc tamen nihil refert, cum dictus angulus inſenſibilis ſit magnitudinis. Ab iſtis autem rationibus elicere poſſumus, quòd ſi punctus.u.erit ex æquo medius inter centrum.B.& extremum.C.pondus.F.aut.M.pendebit, aut nitetur pro medietate dicto centro.B.& ſi dictum.u.erit propius.B.quam puncto.C.pendebit ab ipſo, aut nitetur ipſi amplius quã ex medietate, & ſi magis verſus. C. minus quã ex medietate nitet.

## *Quòd quantitas cuiuſlibet ponderis, aut uirtus mouens reſpectu alterius quantitatis cognoſcatur beneficio perpendicularium ductarum à centro libræ ad lineam inclinationis.*

## CAP. III.

EX ijs, quæ à nobis hucuſque ſunt dicta, facilè intelligi poteſt, ꝙ quantitas. B. u. quæ ferè perpendicularis eſt à centro. B.ad lineam. F.u. inclinationis, ea eſt, quæ nos ducit in cognitionem quantitatis virtutis ipſius. F.in huiuſmodi ſitu, conſtituens videlicet linea. F.u.cum brachio. F.B.angulum acutum.B.F.u. Vt hoc tamen melius intelligamus, imaginemur libram.b.o.a.fixam in centro.o. ad. cuius etrema ſint appenſa duo pondera, aut duæ virtutes mouentes.e.et.c.ita tamen ꝙ linea inclinationis.e.ideſt.b.e.faciat angulum rectum cum.o.b.in puncto.b. linea verò inclinationis.c.ideſt.a.c.faciat angulum acutum, aut obtuſum cum.o.a.in puncto.a.Imaginemur ergo lineam.o.t.perpendicularem lineæ.c.a. inclinationis, vnde. o. t. minor erit.o.a.ex.18. primi Euclidis. ſecetur deinde imaginatione o.a. in puncto. i. ita ut o.i.æqualis.ſit.o.t.& puncto.i.appenſum ſit pondus æquale ipſi.c.cuius inclinationis linea parallela ſit lineæ inclinationis ponderis.e.ſupponendo tamen pondus aut virtutem.c.ea ratione maiorem eſſe ea, quæ eſt.e.qua.b.o.maior eſt.o. t. abſque dubio ex.6.lib.primi Archi.de ponderibus.b.o.i.non mouebitur ſitu, ſed ſi loco.o.i. imaginabimur.o.t.conſolidatam cum.o.b.& per lineam.t.c.attractam virtute. c. ſimiliter quoque continget ut b.ó.t; communi quadam ſcientia, non moueatur ſitu. Eſt ergo quod propoſuimus verum quantitatem alicuius ponderis reſpectu ad eam, quæ eſt alterius debere deprændi à perpendicularibus, quæ à centro libræ ad lineas inclinationis exiliunt. Hinc autem innoteſcit facillimè, quantum vigoris, & vis pondus, aut virtus.c.ad angulum rectum cum.o.a.minimè trahens, amittat. Hinc quoque corollarium quoddam ſequetur, quò d quantò propinquius erit centrum.o.libræ centro regionis elementaris, tantò quo que minus erit graue.

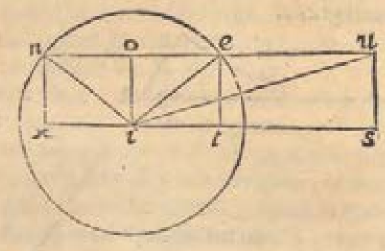


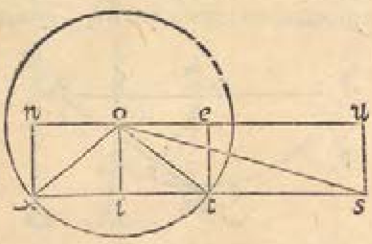

C A P.

## *Quemadmodum ex ſupradictis cauſis omnes ſtaterarum & uectium cauſæ dependeant.*

### CAP. IIII.

VIs brachij longioris alicuius ſtateræ, aut vectis, maior breuioris, ab ijs, quæ in ſuperioribus capitibus diximus, ideſt ꝙ nitatur pendeatuè magis aut minus à centro pondus in extremitate brachij maioris poſitum, oboritur. Quamobrem illud à nobis primò eſt cognoſcendum, ſtateras, aut vectes, puras mathematicas lineas non eſſe, ſed naturales, hincque exiſtere corpora cum materia coniuncta. Nunc igitur imaginemur.n.s.eam ſuperficiem eſſe, quæ ſecundum longitudinem axem ſtateræ ſcindit.& ſupponamus ipſius centrum eſſe primum in.i.& maius brachium eſſe .i. u: minus autem. i. n. & lineam verticalem. i. o. quæ tanta ſit, quanta eſt ſpiſſitudo, aut craſſities ipſius ſtateræ à ſuperiori latere ad inferius, ad faciliorem intelligentiam, ſupponendo.n.s.parallelogrāmam. Poſitis igitur duobus ponderibus æqualibus in extremitatibus brachiorum, experientia innoteſcit, ꝙ pondus ad. u. s. appenſum, viol entiam faciet ponderi appenſo ad.n.x.ſed nos volumus inueſtigare cauſā huius effectus, quæ à nemine vnquam literarum monumentis, ꝙ ſciam, conſignata fuit. Iam diximus ſtateram, aut vectem materialem eſſe &.n.s.eius ſuperficiem mediam, ſupponendo.i.eſſe centrum quo nititur dicta ſtatera aut vectis; Cum hoc ergo ita ſe habeat, ſint.u.s.et.n.x.lineæ inclinationum ponderum, & imaginemur, ꝙ dicta pondera pendeant à punctis.u.et.n.vt reuera pendent, etiam ſi appenſa eſſent ſub.s.et. x. quia punctum.u.& punctum.n.ita coniuncta ſunt cum.s. et.x.ut qui vnū trahit alterum quoque trahat. Imaginemur quoque duas lineas. i. u: i.n. et.i.e.quę i.e.faciat angulum.o.i.e.æqualem angulo.o.i.n.Hinc clarè nobis patebit, ſi quis ipſi e.pondus ipſius.u. (ꝙ æquale eſt ponderi.n.) appenderet, id eandem planè vim haberet, quam pondus ipſius.n.habet, & ſtateram neque ſurſum, neque deorſum moueret, quia ambo pondera ad centrum.i.mediantibus lineis.e.i.et.n.i. ex ęquo anniterentur, ſed dicto pondere poſito in.u:linea.u. i. per quam pondus centro annititur, magis orizontalis quam.e.i.ſit, & linea.u.s.inclinationis longius diſtans à centro. i. quam linea.e.t.vnde huiuſmodi pondus magis quoque liberum à centro.i.reſultat. magiſque ponderoſum, quam cum erat in.e.ratione eorum, quæ primo & ſecundo capitibus diximus, & ob hanc cauſam ſuperat pondus poſitum in.n. Sed ſi centrum fuerit.in.o. imaginabimur duas lineas.o.s.et.o.x.& ſupponemus quòd pondera poſita ſint in. s. et. x.vnde exiſtente magis orizontali linea.o.s.quam erit.o.x.& linea u.s.inclinationis longius diſtante à centro.o.quàm linea.e.t.eius pondus erit quoq;



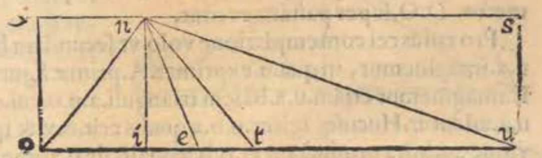

grauius

grauius, quia tantò minus pendebit à centro. o.& ratiocinando, vt superius diximus, inueniemus eundem effectum verum esse. In stateris, rectè & propriè appellari potest.x.i.s.aut.n.o.u.orizontalis, sed in omni vectium specie, hoc tãtum per quandam similitudinem dicetur. Idem contemplari licet supponendo centrum in medio inter.o. et.i.quod vnusquisque ex se absque alterius auxilio facile præstare poterit.

## *De quibusdam rebus animaduersione dignis.*

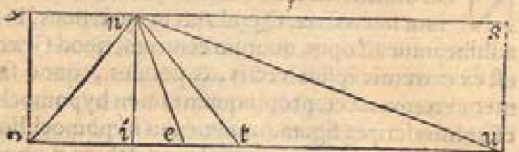
### CAP. V.

NOn omittenda mihi videtur quædam, quæ ad tractationẽ vectium admodum sunt necessaria. Quod autem quærimus, in eo consistit, quòd aliqui vectes adhibeantur ad opus, quorum centrum, quod Græci hypomochliõ appellant vnum est ex extremis ipsius vectis, & pondus, quod sursum eleuari debet, inter ipsamet extrema iacet, propinquum tamen hypomochlio, vt exempli gratia, si vectis esset infrascripta figura.o.s.u.x.cuius hypomochlion esset in puncto.o: & pondus in puncto.n.clarum erit, ꝙ cum eleuari debeat.n.oportebit quoque opera manus eleuari.u. Nunc considerandum est quomodo pondus.n.annitatur ad.u. Hanc ob causam imaginabimur rectas lineas.n.o:n.i:n.e:n.t.et.n.u.quarum.n.i.versus mundi centrum sit posita, et.n.t.faciat angulum.i.n.t.æqualem angulo.i.n.o.Nunc ponendo aliquam virtutem in.i.æquali inclinatione ad superius constante, vt.n. ad inferius (remota tamen grauitate materiæ vectis) huiusmodi virtus, totum pondus ipsius.n.communi quadam scientiæ notione sustinebit. & si põdus ipsius.n.esset in.x. è directo super.o.totum pondus super hypomochlio se haberet, & tanta virtus ipsius hypomochlij sufficeret ad resistendum pro sustinendo, quanta est grauitas ipsius ponderis, sed ipsum iterum ponamus in.n.ibi clarum erit, quòd si alia virtus à parte inferiori ad superiorem vectis non opponitur, excepto tamen hypomochlio, oportebit virtute cuiusdam partis ponderis.n. (absque consideratione tamen, vt iam dixi, ponderis materiæ vectis) vt vectis à parte.s.u.deprimatur, & dixi vnius cuiusdam partis ponderis.n.quia alia eiusdẽ ponderis pars annititur ipsi hypomochlio. o. mediãte linea o.n.quæ angulos rectos cum.o.x.non facit. Si autem à puncto.t.opponet sese huiusmodi resistentia, vt vectis non deprimatur, clarum erit communi scientia, ꝙ virtus ponderis.n.diuisa erit per medium æqualiter, cuius vna medietas super. o.quiescet, & alia super.t.mediantibus duabus lineis.n.o.et.n.t. Imaginemur nunc resistentiam t.ablatam esse, positamq; in.e.clarum quoque erit, ꝙ maior pars ponderis. n. ipsi. e. annitetur beneficio lineæ.n.e.quàm ipsi. o.cum linea.n.i. inclinationis ipsi.e. sit propinquior quam.o.quia omnis resistentia aut in. i.aut in. e. aut in. t. aut in. u. est loco centri, quemadmodum est.o.& alter alterius opera iuuatur. Si verò eadem resistentia posita erit in.u.clarum quoque erit, ꝙ minor pars ponderis.n.annitetur ipsi.u.quã ipsi.o.cum dicta.n.i.à centro.u.longius quam à centro.o.distet, & proportio partis ponderis.n.in.o.ad proportionem partis ponderis.n.in u.non erit secũdum proportionem angulorum.u.n.i.et o.n.i.sed secundum proportionem. u.i.ad.i.o.quod clarè compræhendi potest ab




T huius

huius effectus conuerſo, ideſt, vt quemadmodum nunc ſupponuntur. o. et. u. eſſe duo centra quibus ſuſtinet̃ pondus. e. ipſius. n. imaginemur. n. eſſe quoddam centrum à quo pendeant duo pondera. o. et. u. ſic inuicem proportionata, ut ſunt. u. i. et. i. o. certe horum ponderum cauſa ſtatera. o. s. quam vectem appellabamus à nulla parte inclinabitur. Redeuntes nunc ad propoſitum, dicemus ꝙ annitente pondere ipſius. n. minus ad. u. quam ad. o. ideſt ad. t. minori vi opus erit in. u. quàm in. t. ad attollendum pondus ipſius. n. & ſic per conſequens quantò longius erit punctum. u. ab. t. tantò minori quoque vi egebit, & conſequenter quando vis, aut reſiſtentia in. u. ita proportionata erit illi, quæ eſt ipſius. o. vt eſt. o. i. ad. i. u. vectis non mouebitur. Sed quando erit proportio maior, reſiſtentiæ ipſius. u. ad eam, quæ eſt ipſius. o. ea, quæ eſt. o. i. ad. i. u. tunc vectis à parte ipſius. u. s. eleuabitur, ſi vero proportio minor eſſet quàm. o. i. ad. i. u. tunc vectis ab eadem parte deprimetur.




## *De ratione cuiuſdam uis adaucta.*

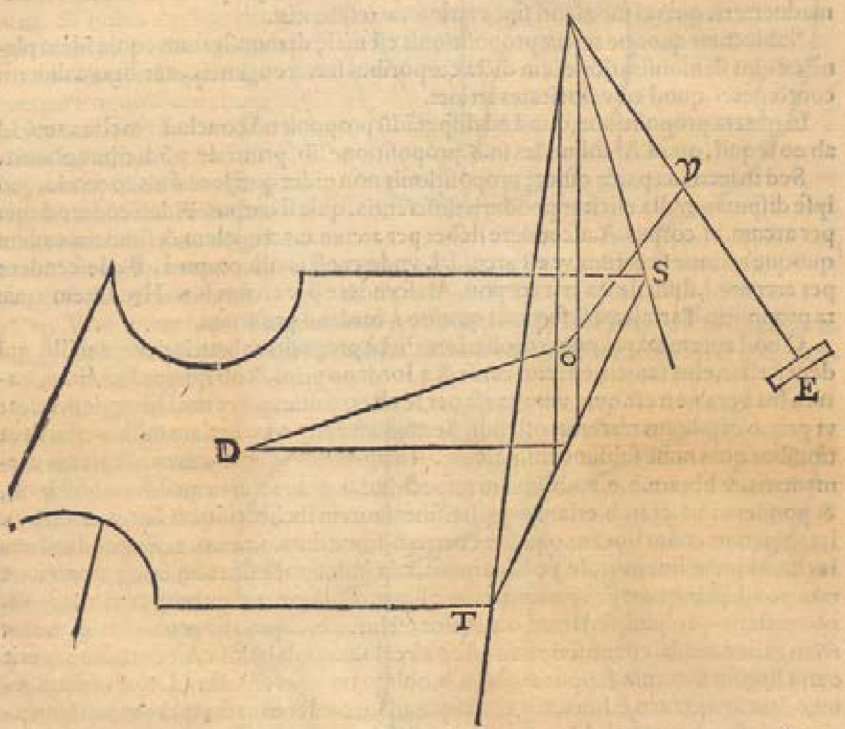
### CAP. VI.

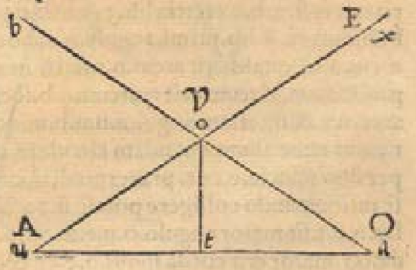
QVibuſdam in locis vtuntur quidã quodã inſtruméto piſtorio ad ſubigédã paſtam, vnius tantum hominis ui adhibita, quæ quidem machina cum mihi digna contemplatione eſſe videatur, eius aliquam rationem proponere volui, pro cuius deſcriptione imaginemur planum, in quo ſedet ille, qui voluit paſtam, & in quo ipſa paſta eſt repoſita. T. S. D. & triangulum. T. A. S. immobile perpendiculareque ſuperficiei dicti plani, angulo autem. A. coniunctum lignum. A. E. vt ſemidiametrum mobilem, & æqualem perpendiculari ipſius trianguli, unde. A. loco centri erit et. D. O. ſit ſemidiameter, qui paſtam contundit, & ab eius extremo. O. (quod. O. quando. D. O. orizontalis eſt, in baſi dicti trianguli reperitur) veniat lignum. O. V. quod cum. A. V. ſit æquale perpendiculari imaginatæ ab angulo. A. baſi. T. S. denodatũ tñ utvulgo dicit̃ ſeu flexile in. O. & in. V. vt elleuare atq; deprimere ſemidiametrum. D. O. poſſit, et. V. O. ſit æqualis. A. V. et. V. medium ſit inter. A. et. E. vnde. A. V. cum. O. V. æquales erunt. A. E. ſunt deinde duo ligna perpẽdicularia ab. A. ad baſim fixa, & immobilia inter ſe adeò diſtantia, vt inter ipſa pertrãſeãt. O. V. et. D. O. ſupra & infra, ne deuiet ſemidiametrum. D. O. In extremitate deinde ipſius. E. ſit lignum quoddam tenue, vt digitus polex, ad angulos rectos cum. A. E. quod ab aliquo, qui antedictam machinam ſtet, manibus teneatur, qui quidem homo idipſum lignum, ideſt ſemidiametrum. A. E. à ſuperficie trianguli dicti, ad ſe trahendo, & deinde verſus eundem triangulum impellendo, vim quandam maximam mediante ſemidiametro. D. O. ſuper paſtam excitat.

Pro cuius rei contemplatione volo vt ſecundam hanc ſubſcriptam figuram. b. a. u. x. imaginemur, in qua. u. exprimat. A. primæ figuræ, & . a. denotet. O. & . o. V. & . x. E. imaginemur etiam. u. a. baſem trianguli. a. u. o. cui. o. t. perpendicularis dictæ baſi. u. a. addatur. Hucuſq; igitur. u. o. æqualis erit. o. x. & ipſi. o. a. imaginemur etiam. a. o. vſque ad. b. ita productam vt. o. b. æqualis ſit. o. a. ponamus etiam pondus in. a. impellere

lere

lere verſus.u.vnde linea eius inclinationis ſit ſemper.a.u.ſupponamus etiam. a. o. b. eſſe librã, aut ſtateram, aut vectem, &c. o. eius centrum, vnde vis, aut virtus ipſius.a. proportionalis erit ipſi. o. t. reſpectu virtutis, aut vis imaginatæ in. b. inclinationis perpendicularis ipſi.b.a.quæ quidem virtus, aut vis in.b. proportionalis erit ipſi. b. o.ex tertio capite huius tractatus; Si ergo fuiſſet poſita in. b. virtus quædam ad angulum rectum, trahens lineam.b.o.tam proportionatam virtuti perpendiculari ipſius.a.quam eſt.o.t.proportionata ipſi.o.b.ſtatera.b.o.a.non moueretur, ſed quæuis portio maior in.b.ſuperaret.a.cum autem fuerit.o.x.æqualis ipſi.o.b.idẽ planè eue-




niet, communi quadam ſcientia, ponendo virtutem.b.in.x.Quantitas ergo virtutis in.x. quæ ſuperare debet reſiſtentiam in.a.quæ ipſi.u.contraponitur, debet habere aliquantulum maioris proportionis ad reſiſtentiam, quæ in. a. angulum rectum efficeret cum. a.o. ea, quæ eſt. o. t. ad.o.x.

## *De quibusdam erroribus Nicolai Tartaleæ circa pondera corporum & eorum motus, quorum aliqui desumpti fuerunt à Iordano scriptore quodam antiquo.*

CAP. VII.

CVm magis amici veritatis esse debeamus quàm cuiusquam hominis, quemadmodum Aristo. scribit, detegam hoc loco quosdam errores Nicolai Tartaleç de ponderibus corporum, & velocitatibus motuum localium. Et primum decipitur is in. 8. lib. suarum diuersarum inuentionum in secunda propositione, cum non animaduerterit quanti momenti sint extrinsecæ resistentiæ.

Subiectum quoque tertiæ propositionis est malè demonstratum, quia idem planè ex eius demonstratione iam dicta corporibus hætereogeneis, aut figura diuersis contingeret, quod ad velocitates attinet.

In quarta propositione, quod ad disputādū proponit nō concludit melius. autē id ab eo sequit̃, quod Archimedes in. 6. propositione lib. primi de pōderibus, pbauit.

Sed in secunda parte quintę propositionis non uidet ꝙ uigore situs eo modo, quo ipse disputat, nulla elicitur ponderis differentia. quia si corpus. B. descendere debet per arcum. i. l. corpus. A. ascendere debet per arcum. u. s. æqualem, & similem. eadem quoque ratione situatum, vt est arcus. i. l. vnde vt est facilè corpori. B. descendere per arcum. i. l. difficile ita erit corpori. A. ascendere per arcum. u. s. Hęc autem qnin ta propositio Tartaleæ est secuuda quæstio à Iordano proposita.

Quòd autem ad primum corollarium dictæ propositionis attinet, verum ille quidem scribit, eius tamen effectus causa & à Iordano prius, & ab ipso postea citata, natura sua vera non est. quia vera causa per se ab eo oritur, ꝙ à centro libræ dependeat vt primo cap. huius tractatus ostendi. Secundum verò corollarium falsum esse, ijs rationibus quas nunc subiungam, patebit. Imaginemur. u. pro centro regionis elementaris, & libram. b. o. a. obliquam respectu ad. u. & brachiis æqualibus constātem, & pondera in. a. et in. b. etiam æqualia. lineæ autem inclinationum sint. a. u. et. b. u. imaginemur etiam lineam. o. u. & à centro. o. libræ duas. o. t. et. o. e. perpendiculares inclinationum lineis; vnde pondus ipsius. a. in huiusmodi situ tam erit proportionatum ponderi. b. quam proportionata erit linea. o. t. lineæ. o. e. ex eo ꝙ tertio cap. huius tractatus probaui, sed linea. o. t. maior est linea. o. e. quod sic probo. Imaginemur triangulum. u. a. b. circunscriptum esse à circulo. u. a. n. b. cuius. c. sit centrum, ꝙ erit extra lineam. u. o. cum supponatur. a. o. b. obliquam esse respectu ad. u. o. Imaginemur deinde à centro. c. lineam. c. o. s. vsque ad circunferentiam, quæ perpendicularis erit ipsi. a. b. ex tertia lib. 3. Eucli. si posteà imaginemur duas lineas. c. a. et. c. b. habebimus ex. 8. lib. primi, angulum. a. c. o. æqualem angulo. b. c. o. Vnde ex. 25. lib. 3. arcus. a. s. æqualis erit arcui. b. s. sed si imaginabimur. u. o. ad circunferentiam vsque productam, clarum erit ꝙ arcum. s. b. secaret in puncto. n. vnde arcus. n. b. minor erit arcu. n. a. & sic etiam angulus. n. u. b. minor erit angulo. n. u. a. ex ultima lib. 6. Imaginemur nunc alium quendam circulum, cuius. o. u. sit diameter, cuius circunferentia per duo puncta. e. et. t. prætergradiat̃, cum in ipsis sint anguli recti, quod quilibet ex se ratiocinando colligere potest, si. 30. lib. 3. in mentem reuocauerit. Sed cum angulus. o. u. t. sit maior angulo. o. u. e. arcus. o. t. maior erit arcu. o. e. ex vltima. 6. vnde corda. o. t. maior erit corda ipsius. o. e. ex conuerso. 27. lib. 3. quod est propositum. Pondus igitur ipsius. a. in huiusmodi situ, pondere ipsius. b. grauius erit. Quod è directo ijs repugnat quæ Tartalea in 2. parte quinræ propositionis ediserit, & per consequens 2. corollarij falsitatem ostendit, vt eam quoqne, quæ in 6. propositione latet. quia cū

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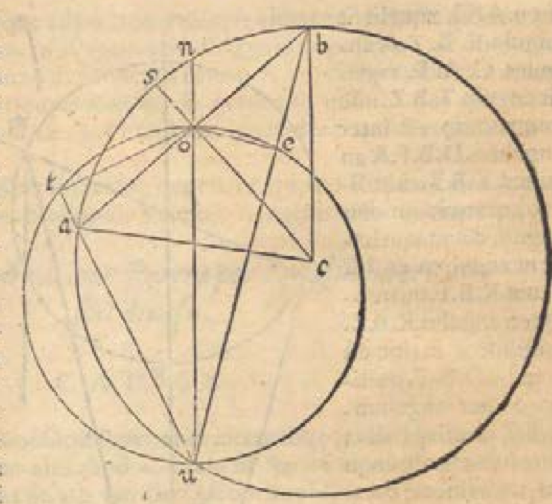
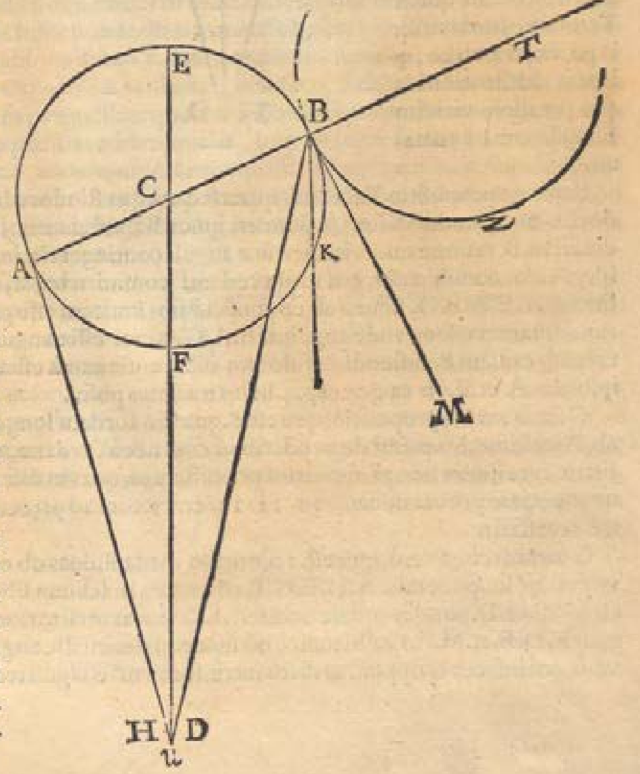


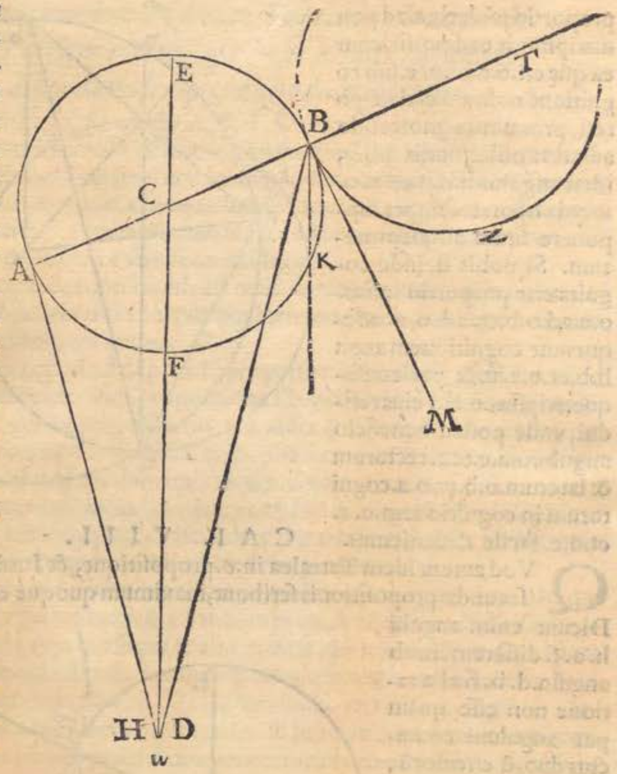

proportio pōderis.a.ad pondus ipsius.b.eadem sit cum ea quę est.o.t.ad.o. e.sub cognitioné nostram cadere potest, primum cognoscendo angulos obliquitatis librę, idest angulos.b.o.u.et. a. o. u.quia oportet semper supponere situm aliquem notum. Si nobis deinde cognita erit proportio ipsius. o.u.ad.o.b.et. ad. o. a. assequemur cognitionem anguli.b.et.o.a. u. & per consequens ipsius.o.a.t. eius residui, vnde postea beneficio angulorum.e.et.t. rectorum & laterum.o.b.et.o.a.cognitorum in cognitionem.o. t. et.o.e. facile deueniemus.

## CAP. VIII.

QVod autem idem Tartalea in.6.propositione, & Iordanus in secunda parte. secundæ propositionis scribunt, maximum quoque errorem in se continet. Dicunt enim angulū h. a. f. differentem ab angulo.d. b. f. alia ratione non esse quàm per angulum contactus duorū circulorū, vt in sua figura scribit Tartalea; id quod falsissimum est. Quā ob causam in subscripta figura sit libra. B . A. & eius centrum.C. et. u. centrū regionis elementaris,et. A.u.et.B. u.lineæ inclinationū. Imaginemur deinde lineam.B.K. parallelā ipsi.A. u. quæ gyrum. B.F. A. in puncto . K. communi scientiæ prę cepto scindet,& habebimus angulum.K. B . Z . æqualem angulo . H. A. F. idest.u. A. F. ( quia.H. u.et.D. unū sunt ) cum ex.29. libr. primi Euclidis angulus.



lus u.A. C. æqualis ſit angulo.K. B. T. & angulus. C. A. F. æqualis angulo.T.B.Z. nūc comparatio eſt inter angulum.D.B.F.& angulum.K.B.Z. miſtilineos,qui quidem duo anguli, cōmunem habent angulum miſtilineum.K.B.F.quapropter ſi angulus.K.B.Z. miſtilineus maior eſt angulo.D.B. F. miſtilineo per angulum. K.B.Z.contingentiæ, circulorum ergo angulus miſtilineus communis.K.B.F. æqualis erit miſtilineo, angulo. D.B. F. pars videlicet ſui toto. Omnis autem error in quem Tartalea, Iordanusq; lapſi fuerunt ab eo, ꝙ lineas inclinationum pro parallelis viciſſim ſumpſerunt, emanauit.




Septima propoſitio Tartaleæ, quæ eſt q̄nta quæſtio Iordani mihi videt̄ excipienda riſu,cum pondus ipſius. A.ponderi ipſius.B.exiſtens æquale, grauius ſit pondere eiuſdem.B. ratione minoris aperturæ anguli contingentiæ in.A.quam in. B. in quo idem error committitur, qui in præcedenti committebatur, cum ſcilicet ipſe putet lineas. A.E.et.B.D. figuræ ab eo confictæ ſibi inuicem eſſe parallelas, quæ etiam ſi æquidiſtantes eſſent(vnde angulus. E.A.G.minor eſſet angulo.D.B.F.) non eam tamen ob cauſam huiuſmodi angulorum differentia cauſa eſſet differentiæ grauitatū ipſorum.A.et.B.ob ea quæ cap.4.huius tractatus poſui.

Octaua autem propoſitio,quæ eſt.6.quæſtio Iordani longè melius demonſtratur ab Archi.in.6.lib.primi de ponderibus, cum nec à Iordano,nec à Tartalæa probata fuerit,cum ijdem non probauerint præcedentes, quas in dicta.8.Tartalęa citat, qui neque etiam probat nonam. 10. 11. 12. et.13.cum ad pręcedentes probandas mini mè acceſſerit.

Quartadecima verò,quæ eſt. 10.quęſtio Iordani,duas ob cauſas eſt falſa, quarum vna eſt,ꝙ( ſupponendo.A.D.E.G.B.eſſe vnum brachium librę,et.A.punctum cētri eiuſdem,et.D.pondus ęquale ponderi.E.& lineas inclinationum.D.K.et.E. M.) anguli. K.D.E.et.M.E.G.ſibi inuicē nō ſunt ęquales;cū ille angulus ſit intrinſecus,hic verò extrinſecus & oppoſitus dicto intrinſeco vni⁹ triāguli terminati à.D.E. à.D.K. et. E.

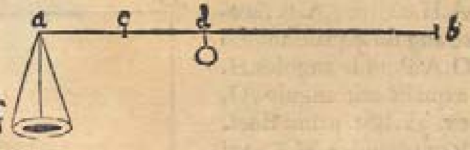
et. E. M. lineis productis vsque ad centrum regionis elementaris, vnde dictus angulus. M.E.G. maior est alio, ex. 16 lib. primi Eucli. Qua ratione fit, vt hanc ob causam E. grauius sit ipso. D. cum minus dependeat à centro. A. vt primo cap. huius tractatus iam dixi. Alia quoque est ratio, qua dictum. E. grauius sit ipso. D. quę quidem est maior distantia à centro. A. libræ, per similes rationes capit. 4. huius tractatus citatas.

Decimaquinta quoq; nil penitus valet, quę est. 11. quęstio Iordani, cuius Authoris opusculum opera Traiani Bibliopolę Venetijs è tenebris in lucem emersit.

## *Quòd summa ratione ſtateræ per æqualia interualla ſint diuiſæ.*

## CAP. IX.

Magna cum ratione diuidūtur ſtateræ per interualla ęqualia, in libras, aut in vncias, aut quoquo alio modo. Nam sit statera exempli gratia. a. b. & punctum, ꝙ eam sustinet sit. c. & vas illud, ꝙ continet id, quod ponderari debet f. Imaginemur nunc quod pondus brachij. c. b. ab una parte, & pondus brachij. c. a. cū eo, ꝙ est dicti vasis. f. ab altera parte, sint causę, quibus statera. a. b. c. stet orizontalis. cui sic orizontali manenti imaginemur ad punctum. a. adiunctum esse pondus, veluti vnius librę. & ad punctum. d. tam distanti à. c. ut est. a. ab ipso. c. aliud quoque pondus vnius libræ additū esse, vnde cōi quadā scientia statera, non mouebitur situ. qa existentibus duobus hisce ponderibus æqualibus, altero in. d. & altero in. a. remota cum essent. d. b. et. f. absque dubio. a. d. non mutaret situm, sed. d. b. et, f. in situ, in quo reperiuntur, à centro paribus viribus prędita sunt. Addendo igitur. d. b. ipsi. d. et. f. ipsi. a: summa earum, æqualibus quoque viribus constabunt. ex communi sententia, quæ habet si ęqualibus addas ęqualia, tota quoque fient ęqualia. Si verò ponderi ipsius. a. aliud adderetur eidem ęquale, haberemus in. a. duplum pondus ei ꝙ est ipsius. d. sed volentes vt solum cum pondere ipsius. d. statera stet orizontalis, si dictum pondus ipsius. d. longè distabit à centro. c. per duplum ipsius. c. a. idest ipsius. c. d. id ꝙ volumus assequemur, beneficio supradictarum rationum, adiuti opera sextę lib. primi de pōderibus Archimedis. Et si quis aliud quoq; pondus adiungeret ipsi. a. æquale illi priori, ad efficiēdum, vt statera semper orizontalis maneret, oporteret, vt pōdus ipsius. d. ab. c. longè distaret, ita vt huiusmodi distantia tripla esset primæ, & sic per quosdam quasi gradus interualla redderentur æqualia.




Quòd

## *Quòd linea circularis non habeat concauum cum conuexo coniunctum, & quod Ariſto. circa proportiones motuum aberrauerit.*

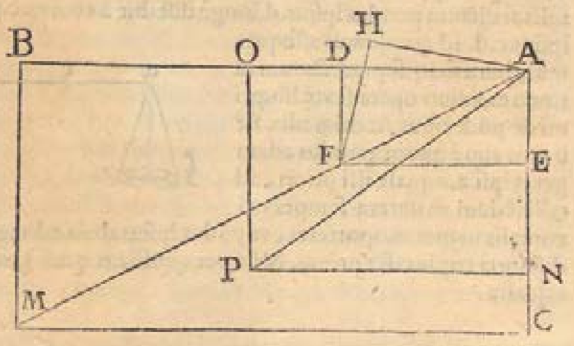
### CAP. X.

ARiſtoteles in principio quæſtionum Mechanicarum ait lineam, quæ terminat circulum videtur conuexum habere coniunctum cum concauo, quod falſum eſt: quia huiuſmodi linea partes nullas ſecundum latitudinem habet, (vt ipſe etiam confirmat) ſed eſt idem conuexum circuli: linea verò quæ terminus eſt ſuperficiei ambientis, & amplectentis circulum eſt eadem concauitas dictæ ſuperficiei eundem circulum ambientis, quæ nullam conuexitatem habet. & hæ duæ ſunt lineæ, quarum vna diuerſa eſt ab alia, neque altera alterius, quod ad conuexum, & ad concauum attinet.

Sed illud, quod Ariſtoteles ſcribit de duplici reſpectu motus vnius puncti ſecundum vnam datam proportionem, non ſufficit, ille enim ſic ait.

Sit proportio ſecundum quam latum fertur, quam habet. A.B.ad. A. C. et. A. quidem feratur verſus. B: A.B. verò ſubterferatur verſus. M.C. latum autem ſit. A, quidē ad. D. vbi autem eſt. A. B. verſus. E. Quoniam igitur lationis erat proportio, quam. A.B. habet ad. A. C. neceſſe eſt &. A.D. ad. A.E. hanc habere rationem. Simile igitur eſt proportione paruum quadr ilaterum maiori. Quamobrem etc.

Cui reſpondeo, punctum. A. quod mouetur in linea. A.M. ab. A. verſus. M. vſque ad. F. non moueri ab aliqua proportione determinata magis quàm ab alia: vnde nō ſolum poſſumus imaginari dictum punctum. A. moueri ab. A. vſque ad. F. eiuſdem velocitatis ſub alia quadam proportione, ſed etiam ſub alia, quæ iam datæ contraria ſit, vt eſt proportio ipſius. A.C. ad. A.B. imaginātes moueri. A. verſus. C. et. A. C. verſus. B.M. delatam. Dico etiam idem. A. moueri vſque ad. F. ſecundum proportionem ipſius. A.O. ad. A.N. Quamobrem imaginemur à puncto. F. lineam. F. H. cum linea. F. A. efficere angulum æqualem angulo. O. P. A. & à puncto. A. lineā A.H. cū linea. A. F. facere angulū æqualē angulo O.A.P. unde angulus. H. æqualis erit angulo. O. ex. 32. libr. primi Eucl. & triangulū. A.H.F. ęquiangulum erit triangulo. A.O.P. Quam ob causā eadē proportio erit ipſiꝰ A.H. ad. F.H. quę ē ipſius A. O. ad. O.P. punctum

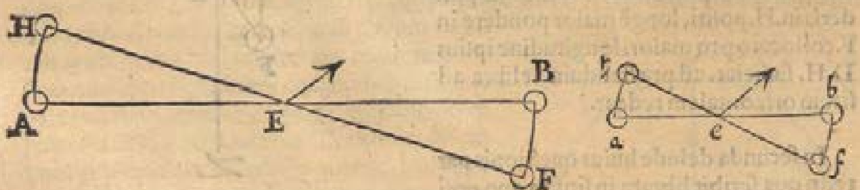



igitur. A. vſque ad. F. mouetur ſecundum proportionem etiam ipſius. A.O. ad. O. P. Huiuſmodi igitur conſideratio, ab Ariſtotele facta, nullius eſt momenti.

## *Quod Ariſto. in prima mechanicarum quæſtionum eius quod inquirit, ueram cauſam non attulerit.*

### CAP. XI.

QVærens Ariſtoteles vnde fiat, vt eæ libræ, quæ brachia habent alijs longiora, ſint exactiores cæteris, ait hoc euenire ratione maioris velocitatis extremorum earundem. Quod verum non eſt; quia hic effectus nil aliud eſt, quam clarius proponere ob omnium oculos obliquitatem brachiorum à linea orizontali, & oſtendere etiam facilius à dicto orizontali ſitu exire brachia iam dicta. Quæ quidem per ſe neque à velocitate, neque à tarditate motus, ſed à ratione vectis, & à maiori interuallo inter ſecundum ſitum extremorum à primo proficiſcuntur. Vt exempli gratia, imaginemur magnam libram. A. B. orizontalem, cuius centrum ſit. E. et pondus. B. maius ſit pondere ipſius. A. vnde conceditur, quòd ob hanc rationem dicta libra ſitum mutabit, qui ſecundus ſitus ſit in. H. F. Imaginemur etiam paruã quãdam libram. a. c. b. orizontalem, quæ pondera habeat. a. et. b. æqualia duobus ponderibus alterius libræ & ſecundus ſitus ſit in. h. f. ita tamen vt anguli circa. e. æquales ſint ijs, qui ſunt circa. E. ideſt. b. e. f. ſit ęqualis. B. E. F. Nunc dico ſitum. H. F. exactiorẽ futurum & clariorem ſitu. h. e. f. ratione interualli. B. F. maioris, interuallo. b. f. quod. B. F. in eadem proportione maior eſt ipſo. b. f. in qua. B. E. maius eſt. b. e. quod autem interuallum. B. F. breuiori, aut longiori temporis ſpacio quam. b. f. ſit factum, nil planè refert. Ratione vectis deinde, dico ꝙ ſi ſupponemus duas libras pares æqualesq; in omni alio reſpectu, præter quàm in brachiorum longitudine, pondus. B. maiorem vim habebit ad deprimendum brachium. E. B. quàm pondus. b. quia libræ materiales, cum ſuſtineantur ab. E. c. & non à puncto mathematico, ſed à linea, aut ſuperficie naturali in materia exiſtente, vnde aliqua reſiſtentia ipſi motui brachiorum oritur, & hanc ob cauſam, ſupponendo hanc reſiſtentiam æqualem tam in. E. quàm in. c. clarum erit ob ea, quæ in cap. 4. huius tractatus oſtendi. B. cum minus dependeat ab. E. aut minus quoque eidem. E. annitatur, ponderoſum magis futurum, quam. b. & hac de cauſa mouebit ad partem inferiorem, maiori cum agilitate, brachium. E. B. multo magis etiam illud ipſum deprimet, ideſt maiorem etiam angulum. B. E. F. quàm erit angulus. b. e. f. faciet.



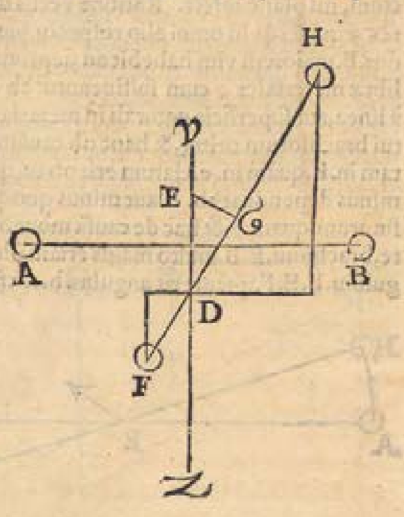
## *De uera causa secundæ, & tertiæ quæstionis mechanicæ ab Aristotele non perspecta.*

### CAP. XII.

ARistoteles in secunda quæstionum mechanicarum quærens illius rationem sic scribit.

Cur siquidem sursum fuerit spartum quando deorsum lato pondere quispiam id admouet rursus ascendit libra: si autem deorsum constitutum fuerit non ascendit, sed manet? an quia sursum quidem sparto existente plus libræ extra perpendiculum fit (spartum enim est perpendiculum) quare necesse est deorsum ferri id, quod plus est, quare & cætera.

Sed vera causa, vnde fiat, vt si spartum fuerit sursum, & brachium vnum ipsius libræ deprimendo, & idem liberum deinde permittendo, ad situm orizontalem redeat, non solum est maior quantitas ponderis brachiorum quæ iam prætergressa est vltra verticalem lineam, sed etiam est longitudo brachij eleuati, quæ vltra verticalem lineam reperitur, vnde eius extremi pondus redditur grauius in proportione, quam in hoc exemplo proponam, sit. A. B. libra in situ orizontali, cuius spartum sit. E. super ipsam. & deprimentes brachium ipsius. A. vsque ad. F. eius situs fit in. F. H. vnde medium pũctum. G. prætergressum erit lineam verticalem. V. Z. versus. B. quæ. V. Z. secabit brachium. F. G. in puncto. D. vnde. D. H. longius erit ipso. F. D. Nunc nobis supponendum est id, quod verissimum existit, dictam scilicet libram in situ. F. H. etiã si sustineatur à puncto. E. idem tamen futurum ac si sustentaretur in puncto. D. vnde sequitur, quod pondus appensum ex ipsa. H. ita grauius reddatur, ipso. F. in eadem proportione, quæ maior est. D. H. ipso. D. F. ob rationes quas in primis huius tractatus capitibus posui, vt etiam si. D. H. quod materiale esse supponitur, nullam planè grauitatem haberet, solus tamẽ excessus vis ponderis in. H. positi, longè maior pondere in F. collocato pro maiori longitudine ipsius D. H. sufficiat. ad præstandum vt libra ad situm orizontalem redeat.




In secunda deinde huius quęstionis parte, in qua scribit libram in situ, in quo posita est, firmam manere, toto cęlo aberrat, quia necessariũ est, vt omninò cadat, eòusq; quò spartum sursum remaneat: ablato tamen omni impedimento, quod nulla eget probatione, cum natura sua clarissimè pateat.

Causa, deinde, vera tertiæ quæstionis non est ea, quam Aristoteles ponit, sed huiusmodi effectus ab eo, quod capitibus. 4. et. 5. huius tractatus proposui originem habęt.

Quod

## *Quòd Aristotelis ratio in 6. quæstione posita non sit admittenda.*

### CAP. XIII.

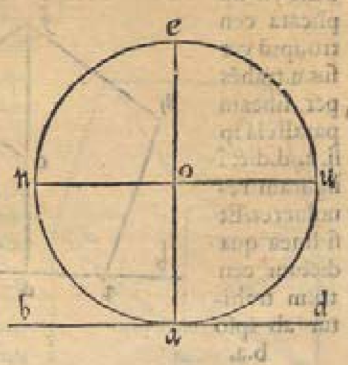
VOlens Aristoteles rationem proponere, vnde fiat, vt nauis velocius moueatur cum antennam altiorem quàm cum depræssiorem habet, id ad vectis rationem refert, quod verum nõ est. Huiusmodi enim ratione nauis tardius potius, quàm velocius ferri deberet, quia quantò altius est velum, vi venti impulsum, tãtò magis proram ipsius nauis in aquam demergit. Sed huiusmodi effectus à maiori potius quantitate venti quam recipit, quàm ab alia aliqua causa oritur, quia ventus liberius vehementiusq; in altiore parte, quàm in depræssione vagatur & perflat.

## *Quòd rationes ab Aristotele de octaua quæstione confictæ sufficientes non sint.*

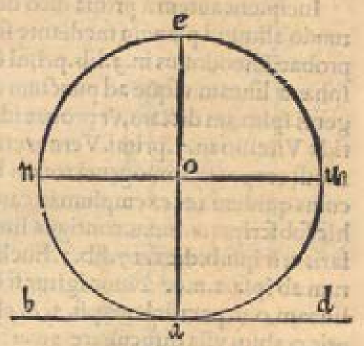
### CAP. XIIII.

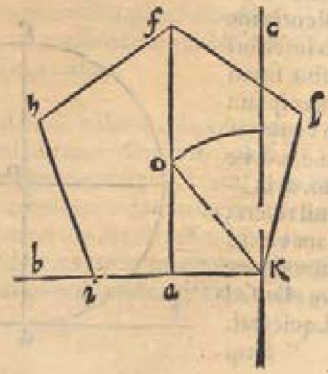
RAtiones etiam ab Aristotele propositæ pro indaganda octauæ quæstionis veritate, in qua quærit vnde fiat, vt corpora rotundæ figuræ, ad voluendũ sint faciliora reliquis, quarum reuolutionum corporum tres species assignat, quarũ vna est, vt rotarum curruũ; altera vt rotarum puteorum, aut trochlearum, quibus hauritur aqua; & tertia, vt paruorum vasorum a figulis fabricatorum, sufficiẽtes nõ sunt.

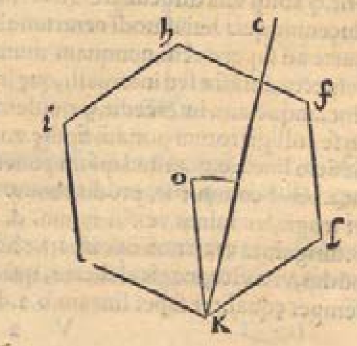
Incipiens autem à prima dico dubium non esse, quin tangente corpore aliquo rotundo aliquod planum mediante solo quodam puncto contingat, quemadmodum probat Theodosius in. 3. lib. primi & Vitellio in. 71. lib. primi, & ducẽdo per centrũ sphæræ lineam vsque ad punctum contactus, ipsa erit perpendicularis plano contingenti spheram dictam, vt probat idẽ Theodosius in. 4. lib. primi Alhazẽ in. 25. quarti, & Vitellio in. 7. primi. Verum etiam est omnem inclinationem ponderosam huiusmodi corporis homogenei totam hanc lineam æqualiter omni ex parte circundare; cuius quidem rei exemplum in carta describere possumus mediante figura circulari hic subscripta. a. n. e. u. contigua lineæ rectæ. b. d. in puncto. a. vnde. e. o. a. perpendicularis erit ipsi. b. d. ex. 17. lib. 3. Eucli. & tantũ ponderis habebimus à parte. a. u. e. quantum ab ipsa. a. n. e. Nunc igitur si imaginabimur ductum esse centrum versus. u. per lineam. o. u. parallelam ipsi. a. d. clarum nobis erit, ꝙ absq; vlla difficultate aut resistentia idẽ ducemus, quia huiusmodi centrum ab inferiori parte ad superiorem, nunquam mutabit situm respectu distãtiæ seu interualli, quę inter ipsum lineamque. a. d. intercedit, ꝙ quidem centrum in se colligit totum pondus figurę. a. n. e. u. & beneficio lineæ. e. o. a. illud ipsum puncto. a. in linea. b. a. d. committit, productum. a. nil refert, vt magis, aut minus versus ipsum. d. aut versus b. dirigat; ita vt cũ non oporteat vt huius figuræ põdus, vna vice, magis eleuetur, quàm alia, sed semper ęqualiter super lineam. b. a. d. quiescat.



Sitq; ſemper diuiſum à linea.a.o.e.per medium, ſequitur communi quodam conceptu, nullam nobis difficultatem oborituram, dictum centrum ad quam uoluerimus partem ducendo, quemadmodum à qualibet alia figura, quæ perfectè rotunda non eſſet, emergeret; Vt exēpli gratia, ſi imaginabimur pentagonum.K.i. h.f.l.quie ſcere ſup eandē lineā.a.b. K.ita ut primū totū latus. i.K.in linea.b.K.extēdat, ducēdo poſteà centrum.o. (ponamus.) verſus.l.dubium non eſt, quin oporteat, vt dictum centrum.o.à linea.b.d.eleuetur, ab eademq; magis diſtet, voluens ſe per arcū vnum circuli, q ꝓ ſuo ſemidiametro habeat.o.K.quę maior eſt ipſa.o.a.ex.18. li. primi Eucli. vnde ſi a puncto.K.imaginabimur lineam. K. c. reſpicientem centrum regionis elementaris, dubium non eſt, quin ſi velimus transferre cētrum hoc à priori ſitu vſq; ad dictam lineam, oporteat addere pondus parti ipſius.l.quæ à linea.K. c.fuit ſecta, aut aliquid de ipſo pondere partis centri detrahere. quod quibuſuis modis fiat, arduum certè eſt ad efficiendum; neque hoc etiam accidit figuræ perfectè rotundæ, cum cētrum ꝙ perfectè in medio ipſius ponderis eſt, reperiatur ſemper in linea perpendiculari ipſi plano, in quo animaduertendum eſt, ꝙ etiam ſi ipſum planum appellem; pro plano tamen perfecto intelligi nolo, ſed pro ſuperficie perfectè ſphęrica circa centrum à corporibus grauibus expetitum; nam ratione magnæ amplitudinis huiuſmodi ſuperficiei, nullam differentiam notatu dignam à perfecto aliquo plano exigui interualli ad curuitatem eiuſdem ſuperficiei imaginari poterimus. Sed ut redeamus ad ſermonem de reuolutione figuræ rotundæ ſuſceptum, clarū igitur erit quamlibet minimam vim (vt ita dicam) quę trahat, aut impellat centrum.o.verſus.u. huiuſmodi figuram reuoluturam, cuius media pars ad trahendum, aut impellendum punctum.e.ſufficiere; Imaginemur autem ꝙ linea.n.o. u. eſſet libra quędā in figura perfectè rotunda.a.n.e.u.poſita, & vis, quę trahere centrum deberet, diuiſa eſſet per medium, cuius medietas appenſa eſſet extremitati. u. diametri.n.o.u.clarū erit, ꝙ abſque vlla difficultate reuolueret figuram ſuper lineam.b.a.d.verſus. d. quia huius vis, aut pondus nullū contra pondus haberet vltra centrum.o.uerſus. n. ꝙ centrum. o.perpetuo quieſcit ſup. a.in linea. e. o. a. per medium diuidente ſemper totum pondus figurę ſuppoſitę. Tantò facilius ergo tota dicta vis applicata centro, ipſū verſus.u.trahēs per lineam parallelā ipſi. a. d.dictā figuram reuolueret. Et ſi linea qua dictum centrum trahitur ab ipſo b.a.

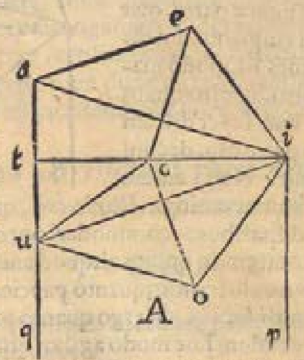


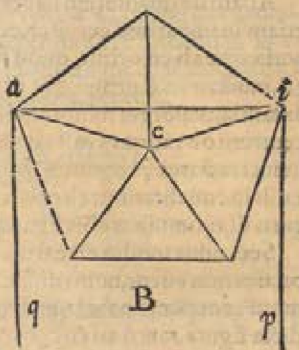



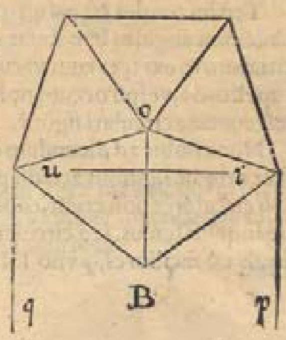
b.a.d.non æquediſtaret,ſed ſurſum traheret ſuper.u.aut ſubter, aliquid de ſua vi vir
tuteq; amitteret,& tantò plus, quantò inclinata magis eſſet verſus.a.o.e. & tandem
cum eſſet vnita cum.a.o.e.aut ad ſuperius,aut ad inferius quantalibet ui, etiam ſi in-
finita,figuram extra ſitum primæ lineæ.a.o.e.non moueret,ſed ſi ſurſum traheret ſe
iungeret eam à linea.b.a.d.non ob id tamen efficeret,ut centrum.o. exiret extra pri
mam lineam.a.o.e.

Secunda verò ſpecies,tribus reuolutionum modis,abſque axis mutatione conſta
re poteſt,ideſt modo,quo reuoluuntur trochleæ mediante fune, & quo reuoluuntur
aliquæ rotæ,in quibus aliquod animal incedit; & quo reuoluuntur illæ,quæ in homi
nis manu circunuoluuntur medio alicuius manubrij inflexi. Hi omnes modi cum
circulari figura magis,quã cum alia quauis,faciliores euadunt. Et primò ſi priorem
modum conſiderabimus,vt mediante fune quælibet figura, quæ circularis non ſit,
voluatur,ſupponamus exemplo debere reuolui pentagonum æquiangulum.a.e.i.o.
u.circa centrum.c.mediante fune.q.u.a.e.i.p.neceſſariò occurrent(in hac figura an-
gulorum,laterumq; diſparium) plures inæqualitates,quæ reuolutionem eiuſdem fi-
guræ irregularem efficient; quarum vna erit,quod duæ partes funis,ideſt.u.q. et.i.p.
non erunt in vna eademq; inter ſe diſtantia ſemper,quod facile intellectu erit,ſi ima
ginabimur ductas eſſe lineas.a.i: u. i: et. i. c. t. ſi funis duo pondera habebit alterum
altero maius,ſuis extremis appenſa,vnde debeat figura virtute ponderis maioris cir
cunuolui: dictæ duæ partes. u.q.et.i.p.eiuſdem funis,mũdi centrum,dum firmæ ma
nebunt,reſpicient;ſed permittentes pondera libera;maius,efficiens vt circunuolua-
tur figura; efficiet,vt aliquando vnum ex lateribus,eiuſdem figuræ mundi quoq; cen
trum reſpiciet, vt in
figura.A.ſicq; etiam
linea.i. c.t.(pro exé-
plo)erit menſura di-
ſtantiæ funium inter
ipſas, & deinde circũ
uoluendo etiam di-
ſtabunt inter ſe per li
neã.i.a.aut. i. u. vt in
figura.B. ĩnotuit exé
plo, & ſic etiam ali-
quando erunt magis
diſtãtes, quàm linea
t. i. & minus quàm.i.

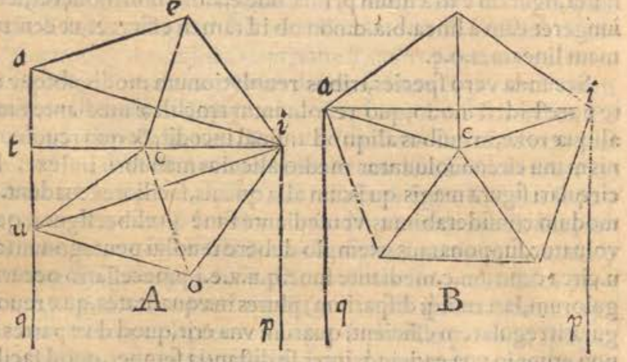


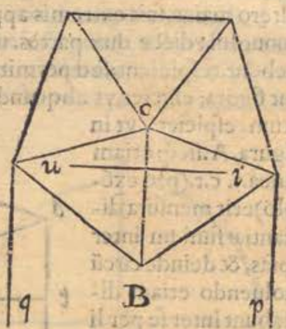

a:nunquam tamen minus quam.t.i.neque magis quã
i.a.aut.i.u. quæ ſunt æquales; Quæ quidem varietas,
in hanc,& in illam partem impellet partes penden-
tes funis, vnde æqualiter non trahent. Idem dico, ſi
extrema.q.et.p.eſſent quoque ſemper in vna eadẽq;
diſtantia; neque à corpore pôderoſo eſſent attracta,
quia aliæ partes ipſius.u.q.et.i.p.ex ſupradictis ratio-
nibus vnam eademq; diſtantiam nõ ſemper ſeruarẽt.
vnde fieret vt cum diuerſis angulis tam.i. p. quã.u.q.
traherẽt ſemidiametros.c.i:c.e:c.a:c.u.et.c.o.quia nõ
ſemper traherent ope ſeu virtute anguli æqualis ipſi.
c.i.p.Hæc autem inęqualitas communis eſt omnibus
figu-



figuris rectilineis tam paris, quàm disparis numeri. Sed aliam quandam maiorem inęqualitatem habent hæ figuræ numeri disparis, quæ est, quòd quãdo linea. t.i. tam .u.q. quàm ipsi. i.p. ppẽdicularis fuerit, idest quãdo. t.i. cum dictis partibus funis angulos rectos constituerit, tũc ratione lõgitudinis ipsius. c. i. maioris quam. t. c. (quia cum sit. c.i. ęqualis ipsi. c.a. et. c.a. maior ipsa. c.t: c.i. etiam maior fit ipsa. c.t.) pondus aut vis ipsius. p. superabit eã quæ est ipsius. q. sed quando. t. erit in opposita parte, et. i. in ea, quæ est ipsius. t: q. eãdem ob causam superabit. p. & sic motum faciet irregularem, & nõ vniformem; & ob id etiam perarduum, præter ictus, quos infligunt anguli in partem pendentem ascendẽtem funis, quãdo vnum ex lateribus vnitur cum fune.




Aliam inęqualitatem habent figuræ pares, quæ etiam in imparibus cernitur, etsi aliquantulum diuersa; quæ ab eo oritur, quod funes sit modò magis, modo minus propinquę centro; quæ inæqualis distantia, maiorem minoremq; vim super dictum centrum ob rationes in secunda parte cap. decimi huius tractatus propositas, gignit. Nulla autem ex ijs inæqualitatibus circulari figuræ contingit. Illud verò, quod de pentagonis figuris dixi, omnibus aliis figuris disparibus accommodari potest.




Secundus modus est earum rotarum, in quibus aliquod animal incedit, quæ si circulares non essent, tantò difficilius voluerentur, quantò pauciores angulos haberent. quod cum per se pateat, non demonstrabo. Si ergo quantò plures angulos habebit dicta figura, tantò ad circunuoluendum hoc modo agilior erit. Circularis igitur figura, quæ ex infinitis angulis efficitur, omnium agillima erit.

Tertius modus est earum rotarum, quæ manubrium habent, quæ etiam quantò pauciores angulos habebunt, tantò quoq; difficiliores reddentur, tam ratione inimicitiæ: quam exercet cum vacuo natura, quàm violẽtię, quam anguli aëri faciunt, eum expellendo, vt ipsi occupent locum, quem ipse aër implebat. Quod nullo modo potest euenire circulari figuræ.

Nunc nobis ad dicendum restat de specie reuolutionis rotarum, quæ parallelæ sunt orizonti, quibus accidit posse volui primo tertioq; modo secundę speciei, & ob id si circulares non erunt, eadem subibunt incommoda, de quibus in secunda illa specie loquuti sumus. sed circulares rotæ huius tertiæ speciei ad reuoluendum erunt reliquis eò faciliores, quod vno solũ polo nituntur; Quod alijs nequaquam conceditur.

Super

Super hac tertia ſpecie formari poteſt problema, vnde fiat, vt quieſcens huiuſmodi rota parallela orizonti ſuper vnum punctum, & quantò fieri poteſt exiſtens ęqualis, ſi eam circunuoluamus maiore qua poterimus ui, & eādem poſtea dimittentes non perpetuò circunuoluatur.

Hoc quidem, quatuor fit ob cauſas. quarum prima eſt, quia huiuſmodi motus, eius rotæ non ſit naturalis. ſecunda eſt, quia etiamſi rota ſuper punctum mathematicum quieſceret, oporteret tamen vt ſuperius alterū haberet polum, qui ipſam orizontalē teneret, qui quidem munimento aliquo corporeo indigeret; vnde fricatio quędam conſequeretur, ex qua reſiſtentia prodiret.

Tertia eſt, quia aër contiguus eam perpetuò aſtringit, hocq; modo eius motui reſiſtit.

Quarta eſt, quia quęlibet pars corporea, quę à ſe mouetur, impetu eidem à quali-bet extrinſeca virtute mouente imprę́ſſo, habet naturalem inclinationem ad rectum iter, non autem curuum, vnde ſi à dicta rota particula aliqua ſuę circunferentiæ diſiū geretur, abſque dubio per aliquod temporis ſpatium pars ſeparata recto itinere ferretur per aërem, vt exemplo à fundis, quibus iaciuntur lapides, ſumpto, cognoſcere poſsumus, in quibus, impetus motus imprę́ſſus naturali quadam propenſione rectum iter peragit, cum euibratus lapis, per lineam rectam contiguam giro, quem primo faciebat, in puncto, in quo dimiſſus fuit, rectum iter inſtituat, vt rationi conſentaneum eſt.

Eadem, quoque ratiône fit, vt quantò maior eſt aliqua rota, tantò maiorem quoque impetum, & impreſſionem motus eius circunferentiæ partes recipiant, vnde ſępe euenit, vt dum eam ſiſtere volumus, id cū labore & cum difficultate agamus; quia quantò maior eſt diameter vnius circuli, tantò minus curua eſt eiuſdem circunferentia, & tantò propius accedit angulum eiuſdem circunferentiæ ad quantitatem duorum angulorum rectorum rectilineorum, ideſt circunferentia ad rectitudinem linearem. Vnde earundem partium dictæ circunferentiæ motus ad inclinationem ſibi à natura tributam, quæ eſt incedendi per lineam rectam, magis accedit.

## *Quod Ariſtotelis ratio nonæ quæſtionis admittenda non ſit.*

## CAP. XV.

Vera ratio nonæ quęſtionis à ſecunda parte decimi cap. huius tractatus, & non aliunde, accerſiri debet.

## *Quod Ariſtotelis rationes de decima quæſtione ſint reÿciendæ.*

## CAP. XVI.

Ariſtotelis rationes, vnde fiat, vt facilius moueantur libræ vacuæ, quàm plenè ad propoſitam diſputationem non pertinent; quia ſemper ineunda eſt ratio proportionis virtutis mouentis ſuper mobile; quod ipſe non fecit.

Sit

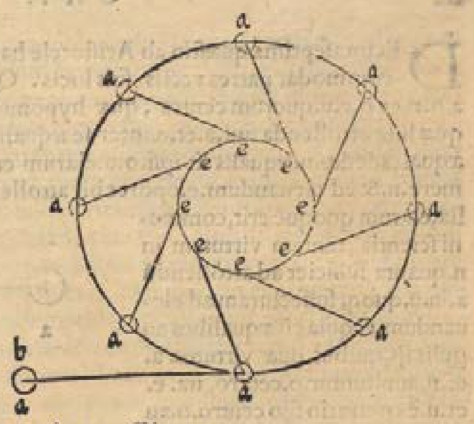
Sit exempli gratia libra. a. i. e. quæ in vtraque extremitate vnciam vnam solum ponderis obtineat, & sit libra. n. i. u. æqualis priori, quæ pro singula extremitate vnã ponderis libram habeat. Aristoteles admiratur, quòd addendo ipsi. e. mediam ponderis vnciam, brachium. i. e. velocius cadat, quàm adijciẽdo ipsã mediã vnciã ipsi. u. brachij. i. u. Quod à duabus causis proficiscitur, quarum prior est, magna differentia proportionis vnius libræ ad medietatem vnius vnciæ, ad proportionem vnius vnciæ ad ipsam medietatem, quia si pondus adiectum extremo. u. dimidiæ esset libræ, & cum eadem tarditate brachium moueret, optimo iure in admirationem posset Aristoteles duci. Sed hoc fieri non posset, quia ipsum deprimeret cum eadem quasi velocitate, qua media vncia brachium. i. e. Dixi autem quasi, quia nonnihil discriminis intercederet, quod proficiscitur à secunda ratione. Et hæc, resistentia est, quæ oritur à sparto, quia quantò maius pondus continet libra, tantò magis præmit spartum in loco, in quo sustinetur; vnde maior resistentia in circunuolutione eiusdẽ sparti, in loco, in quo quiescit, exoritur, quia ipsum est corpus materiale. Si quis autem vellet, vt brachium. i. u. eadem agilitate, qua. i. e. descenderet, oporteret, vt proportio dimidiæ libræ adiectæ ponderi ipsius. u. quod est vnius libræ, vim suam haberet, quæ excederet resistentiam sui sparti (medio brachiorum maiorum ijs qui sunt. a. i. e.) ita proportionatam, vt proportionata est vis dimidiæ vnciæ ipsi e. iunctæ, resistentiæ sui sparti. Huiusmodi rationes cum rotis grauioribus leuioribusq;, & ijs, quę à corporibus quibuslibet grauibus impelluntur, accommodatæ fuerint, titubantem intellectum confirmabunt.


## *De uera causa. 12. quæstionis mechanicæ.*

## CAP. XVII.

VEra ratio, cur multò longius corpus aliquod graue impellatur funda, quam manu, inde oritur, quòd circunuoluendo fundam, maior impræssio impetus motus fit in corpore graui, quàm fieret manu, quod corpus liberatum deinde cum fuerit à funda, natura duce, iter suũ à puncto, à quo prosilijt, per lineam contiguam giro, quem postremò faciebat, suscipit. Dubitandumq; non est, quin dicta funda maior impetus motus dicto corpori imprimi possit, cũ ex multis circumactibus, maior semper impetus dicto corpori accedat. Manus autem eiusdem corporis motus, dum illud ipsum circunuoluitur (pace Aristotelis dixerim) centrum non est, neque funis est semidiameter. Immo manus quam maximè fieri potest in orbem cietur; qui quidem motus in orbem, vt circumagatur etiam ipsum corpus, cogit, quod quidem corpus, naturali quadam inclinatione, exiguo quodam impetu iam incępto, vellet recta iter peragere, vt in subscripta figura patet, in qua. e. significat manum. a. corpus. a. b. lineam rectam tangentem girum. a. a. a. a. quando corpus liberum remanet. Verum quidem est, imprę ssum illum impetum, continuò paulatim decrescere vnde statim inclinatio grauitatis eiusdem corporis subingreditur, quæ sese miscens cum imprę ssione facta per vim, non permittit, vt linea. a. b. longo tempore recta permaneat,

maneat, ſed citò fiat curua, cum dictum corpus.a. duabus virtutibus moueatur, quarum vna eſt, violentia impreſſa, & alia natura, contra opinionem Tartaleæ, qui negat corpus aliquod motibus violento & naturali ſimul & ſemel moueri poſſe. Neq; eſt ſilétio prætereũdus hac in re quẽdã notatu dign⁹ effectus, qui eiuſmodi eſt, ꝙ quanto magis creſcit impetus in corpore.a.cauſatus ab augumento velocitatis giri ipſius.e.tãtò magis oportet, vt ſentiat ſe trahi manus à dicto corpore a.mediante fune, quia quantò maior impetus motus ipſi.a.eſt impreſſus, tantò magis dictum corpus. a. ad rectum iter peragendum inclinatur, vnde vt recta incedat, tantò maiore quoque vi trahit.




## *De decimatertia quæſtione.*

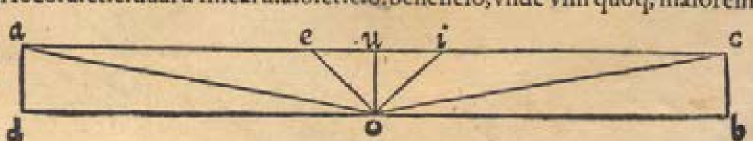
## CAP. XVIII.

Decimatertia quæſtio ad vectem omnino eſt referenda. Imaginari debemus axem cylindrici iugi, hypomochlion eſſe. Quod reſtat, illud ipſum totum dependet à.4.quintoq; cap.huius tractatus. Vna tamen differentia inter hanc machinam, vectemq; reperitur, quæ eſt, ꝙ iugum aliquam reſiſtentiam pro coniunctione calcata in loco, in quo voluitur, magis quàm hypomochlion vecti efficiat.

## *De decimaquarta quæſtione.*

## CAP. XIX.

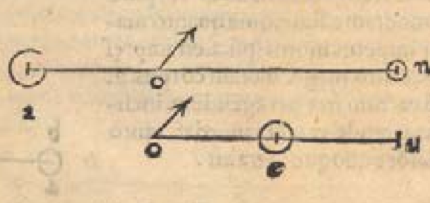
Rationes etiam decimæquartæ quæſtionis dependent ab ijs, quæ ſunt vectis, vt exempli gratia ſit lignum.a.b.c.d. frangendum in medio, annitendo genibus in punctum.o. clariſſimè tunc videbimus, ꝙ tenentes manus longè à medio, in locis a. et. c. facilius minoriq; cum labore illum frangemus, quàm ſi eaſdem vicinas medio eiuſdem ligni in locis.e.et.i.poneremus. Cuius rei rationes eædẽ ſunt cũ ijs, quæ primis huius tractatus capitibus propoſitæ fuerunt. Imaginemur lineas rectas ductas à puncto.o.ad loca.a.e.i.et.c.hinc manifeſtè perſpiciemus eorum, quæ iam diximus ratione, ꝙ loca.e.et.i.mediantibus duabus lineis.e.o.et.i.o.magis annitentur.o. centro, quàm loco.a.et.c.duarũ linearũ.a.o.et.c.o.beneficio; vnde vim quoq; maiorem habebũt in.a.et.c. quàm in e.et.i.



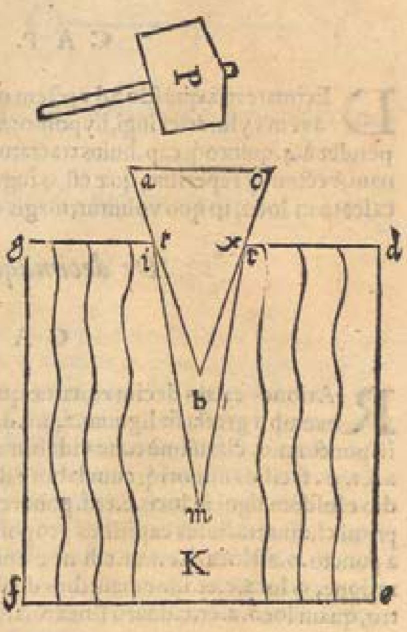
## *De uera ratione. 17. quaestionis.*

## CAP. XX.

DEcimaſeptima quæſtio ab Ariſtotele haud benè percepta fuit, quia is non accommodat partes vectis ſuis locis. Quamobrem imaginemur duos vectes. a.o.n.et.o.e.u.quorum centra, quæ hypomochlia appellantur ſint. o. & pondera, quæ ſunt attollenda ſint.a.et.e.inter ſe æqualia,& diſtantię ſint.a.o.et.e.o.ſibi inuicē æquales,ſed.o.n.æqualis ſit ipſi.o.u:clarum erit,ꝙ ad eleuandum.a.oportebit deprimere.n.& ad eleuandum.e.oportebit attollere.u.Et quia omnia ſupponuntur æqualia,clarum quoque erit,communi ſcientia, tantam virtutem in n.quanta ſufficiet ad attollendū a.in.u.quoq; ſuffecturam ad eleuandum.e. quia cū æqualibus angulis ijs,quibus duæ virtutes. a. et.n.annituntur.o.centro, ita. e. et.u.è contrario ſuo centro.o.annituntur. & omnes rationes pro vecte.a.o.n.quarto quintoq́; huius tractatus capitibus citatæ,vecti.o.e. u. vt ſatis ſuperq́; dixi in dicto capit.5.conuenire poſſunt.




Nunc ſit aliqua pars ligni cindenda ſecundum venulas ſuas.d.e.f.g. & ſit cuneus a.b.c.qui vi mallei. P. vſque ad.t.x.penetrarit. Hinc clarum erit, quòd apertura i.m.r.ligni, poſtquam infigitur cuneus ſecundum venas,longior erit parte.x.b.t.cunei, quæ ingreſſa eſt. Oportet nunc imaginari duos vectes ſimiles ſupradictæ. u. e. o. in hunc modum, vt puncta i.r. ligni ſint loco.u.extremi ipſi⁹ vectis, et.t.x.loco virtutis applicatæ ipſi. u. & reſiſtentia circa punctum.m.loco ponderis.e.vectis.o.e. u. dicti, & pars. K. quaſi immediata poſt.m. verſus extremitatem.f.e.ligni,ſit loco hypomochlij.o. Hinc fiet vt quanto longiores erunt lineæ.i.m.K.et.r.m.K.tantò quoque facilius virtutes.t.x.impellent.i.r.

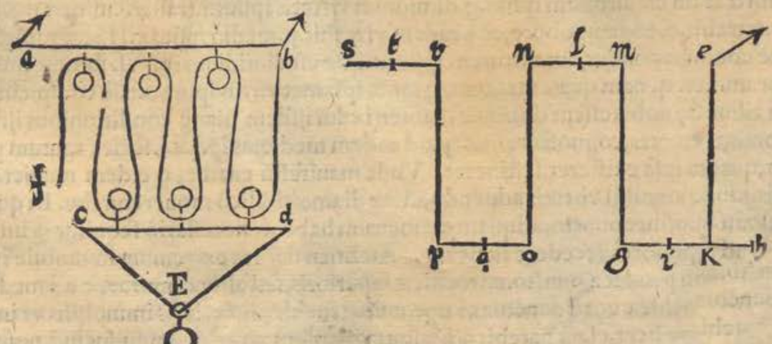



De

## *De uera & intrinseca causa trochlearum.*

## CAP. XXI.

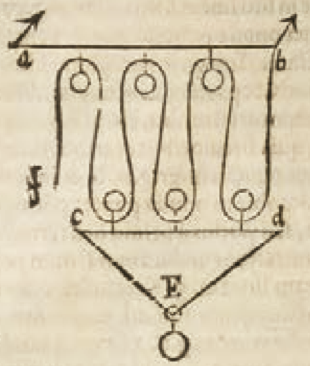
PRo intelligenda vera,& intrinseca ratione, vnde fiat ut multitudo rotularum in trochleis causa sit,ut exigua vis sursum moueat,aut attollat pódera magna. Imaginemur duas hic subscriptas trochlæas explicatas transuersaliter in hunc modum, idest sit paruū tignū.a.b.fixum & parallelū orizonti. cui sint rotulæ appensę ab inferiori parte ad superiorem huicq; è regione opposit⁹ sit aliud tignū.c.d.quod moueri possit ab imo ad summum,super quod totidem sint rotulæ aut radij, cū annexa postea fuerit funis puncto. b. fixo, eam faciendo pertransire per rotulas tam à parte superiore,quam ab inferiore; & appensum deinde cum erit paruo illi tigno. c. d. mobili pondus.E.ducendo postmodum extremum.f.funis transeuntis per rotulas,idem plenè fiet quod à trochlęis simul unitis fieri solet. Cuius quidem effectus ratio sub nostram cognitionem cadet facilius in huiusmodi figura. Imaginemur separatim stateram.g.h.cuius cētrum sit.K.ita situm,ut brachium.g.k.sit duplum ad brachium. K. h.supponendo igitur in puncto.g.pondus,aut virtutem mouentem unius libræ, & in h.duarum librarum,absq; dubio hæ duæ uirtutes in huiusmodi distantijs à centro ęquales inuicē erūt,ob rationes prioribus capitibus iam allatas,& statera orizontalis manebit. Vnde clarum erit,ꝙ quæuis etiam exigua virtus adiuncta ipsi. g. mouebit stateram extra orizontalem situm. Nunc si puncto.i.ex æquo medio inter.g. et. K. applicata erit virtus ipsius.h.non amplius considerato brachio.K.h.inclinante uirtute ipsius.i.eandem partem versus, in quam inclinabat,quando erat in.h.sed uirtus ipsius.g.inclinet contrario modo, diuersoq: ab eo,quo inclinabat prius;clarum quoq; erit,communi conceptu,& ob ea,quæ cap.5.huius tractatus sunt dicta. g. h. semper in eodem situ absque motu mansuram,hancq; stateram appellabimus mobilem, & primam. Imaginemur nunc à puncto.e.fixo descendere funem.e.K. quæ fulciat punctum.K.extremum diametri.g.K.quam intelligo pro diametro vnius ex rotulis inferioribus trochleæ; & sit.n.l.m.diameter vnius ex rotulis superioribus alterius parui tigni defixi à parte inclinationis ipsius.g.& parallela diametro. g. K. cuius diametri centrum fixum sit.l.& sit coniunctum.g.punctum, à fune cum puncto.m.quæ tā perpendicularis sit primo diametro.g.i.K.quàm secundo.n.m.idest ita vt anguli.n.m.g.

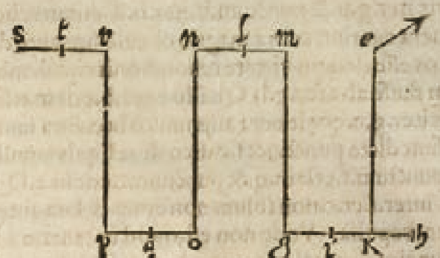


et.m.g.k.ſint recti. Imaginemur quoq; virtutem ipſius.g.applicatam eſſe extremo.
n.cum inclinatione tamen contraria,ideſt ad inferiorem partem,quæ quidem virtus
communi quodam conceptu eandem poſſidebit vim ſuſtentandi immobilem diame
trum.g.i.k. quam habebat, qñ erat in.g.cum inclinatione ad ſuperiorem partem,
& ſic etiam diameter.n.l.m.non magis ab una,quàm ab alia parte declinabit, quia
cum quædam virtus in.n.reperiatur æqualis medietati uirtutis ipſius.i. quæ uirtus ip
ſius.i.uim habet deprimendi ipſum.g.ideſt.m.pro dimidia ſui ipſius parte, ſequitur.
n.m.debere immobilem permanere. Nunc ſi alia diameter rotulæ mobilis erit de-
ſumpta,quæ ſit.p.q.o.cuius centrum ſit.q.in ſitu parallelo ipſi.n.l.m.& ſic collocata,
vt coniungendo.o.cum.n.anguli.m.n.o.et.n.o.p.ſint recti: ſi imaginati fuerimus trãſ
latum eſſe pondus ipſi⁹.n.in.o.cũ eadẽ inclinatione ad depræſſiorem partem,illud ip
ſum,ac ſi eſſet in.n.communi conceptu,ſine alicuius diametri mutatione præſtabit.
Et ſi centrum.q.fixum eſſet,& extremo.p.appoſitum fuiſſet pondus ipſius.o.cum in
clinatione ad ſuperiorem partem,idem etiam planè pręſtaret,etiam ſi nullum ullius
diametri ſitum,communi ſcientia,mutaret, cum extremum. m. deorſum ſit ductum
à.g.uirtute dimidiæ partis ipſius.i.& ab alia huic ſimili. m. quoque deorſum ſit tra-
ctum ab . o : quod quidem.o.deorſum eſt alteratum,ob inclinationem ad ſuperius
à uirtute poſita in.p.ſupponendo centrum.q.fixum. Sed ſi loco centri fixi,imagina
bimur in.q.pondus aliquod æquale ipſi. i. quod duplum erit in uirtute ad eam, quæ
eſt ipſius.p.& ipſius quoque. g: ſequetur etiã eadem immobilitas horum trium dia-
metrorum. Quia cum ſit huiuſmodi pondus ſeu virtus in.q. cum inclinatione con-
traria virtuti in.p.quæ æquipollet dimidiæ parti ipſius.q.& ſic ei quæ eſt ipſius.o. ſi-
militer quia.o.tractum eſt ſupra ab.n.virtute ipſius.g.quod.m.deorſum trudit; idcir
co quanta erit vis quam habebit virtus in.q.ferendi deorſum diametrum.p. o. tanta
quoque virtutes ipſorum.p.et.o.æquales,& æqualiter diſtantes à.q.ipſum ad ſupe-
riorem partem inclinabunt. Quamobrem nec aſcendet, nec deſcendet, nec locum
mutabit. Supponamus nunc quartum diametrum rotulæ.s.t.r. quæ ſit ſecunda rotu
larum fixarum,parallela ipſi.p.o.& in eo ſitu,quo coniungendo extrema.r.p.anguli
o.p.r.et.p.r.s.ſint recti,& imaginemur virtutem ipſius.p.reperiri in.s.cum inclinatio
ne tamen contraria,ideſt deorſum verſus,ex his idẽ quoque planè ſequetur, ideſt ꝙ
nulla harũ quatuor diametrorum mouebitur. quia eundem effectũ cũ inclinatione
deorſum verſus efficeret dicta virtus in.s.quem in.p.cum inclinatione ſurſum verſus.
et iam dictum eſt virtutem ipſius.g.dimidium virtutis ipſius.i.trahere.m.quæ mediã
te.n.attrahit.o.eodem robore,et.s.eadem vi trahit.p.medio ipſius.r.Hucuſque ſciẽ-
tificè nouimus pondus,aut virtutem ipſius.s.quæ eſt dimidium ipſi⁹. i. ſuſtinere uim
ipſorum.i.et. q. nam quater tantum, quanta ipſamet virtus ipſius.s.eſſe conſpicitur.
Et ſi adiunctę nobis eſſent duæ aliæ diametri cum ijſdem planè conditionibus ijſdẽ
rationibus vtentes,cognoſceremus quod eadem medietas ipſius.i.ſexies tantum põ
deris,quanta ipſa exiſteret,ſeſtineret. Vnde manifeſtũ euadit, ꝙ eidem medietati
ipſius.i.in.s.nonnihil virtutis addendo,dictæ diametri,illicò mouerẽtur ſitu. Et quia
rotulæ in quolibet puncto,aliquam diametrum habent, neceſſariò ſequitur ꝙ infe-
riores ad ſuperiores accedere debeant. Attamen ſi forte extremum immobile ip-
ſius funis non pendet à puncto.e.trochleæ ſuperioris,ſed alligatum fuerit ad mediũ
inferioris trochleæ ut ad punctum.i.ope unius trochleę ſuperioris immobilis vt in fi
gura.A.videre licet,clarè patebit ꝙ à tribus virtutibus æqualibus pondus in.i.poſitũ
ſuſtinebitur:hoc eſt à.g.ab.i.& ab.k. quarũ vnaquęque tertia pars erit ipſius.i.in con
contrariam partẽ,hoc eſt tertia pars reſiſtentiæ.propterea ꝙ ex æquo inter ſe diſtãt.

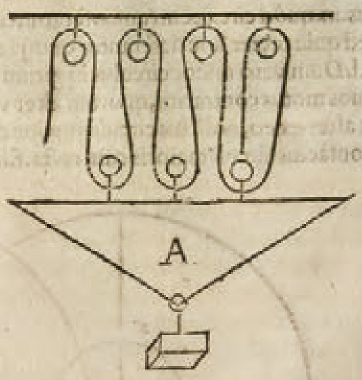
g.i.

g. i.et. K: Quà propter augebitur virtus per numeros impares, hoc modo; Nam. g. eſſet tertia pars reſiſtentię, quemadmodum prius media erat. Idem infero de.m.n. o.p.r.et.s.Sed cum oporteat pondus.q.tantum eſſe vt ſuffiĉiāt reſiſtentiæ in. o. et.p. ipſum ſuſtinere, idcirco ipſum pondus.q.ſubſeſquialter erit pōderi in.i.poſiti.Quapropter.s.quinta pars erit ponderum.i.et.q.Deinde ſi adhuc. duo diametri vnus inferior,alter verò ſuperior additi fuerint cum pondere æquali.q. ad medium diametri inferioris,tunc pondus.s.erit ſeptima pars trium ponderum.i.q.& tertij additi,ex




ſupradictis rationibus. Et quia virtus ſuſtinens totale pondus trochleæ inferiori appenſum in tot diuiditur partes æquales, quot ſunt diametri orbiculorum trochleæ inferioris,quando extremum immobile funis alligatum fuerit trochleę ſuperiori, vt puta in puncto.e.cum verò alligatum fuerit trochleæ inferiori,virtus primi diametri.g.i.K.trochleæ inferioris ſemper ſeſquialtera erit vnicuique aliorum diametrorū; ideò virtus reſiſtentię alterius extremi mobilis funis,puta.s.ſubmultiplex erit totalis ponderis,eo modo quo diximus, cuius virtus,ſeu grauitas diuiditur ſeu diſtrubuitur diametris inferioris trochleæ vt dictum eſt.

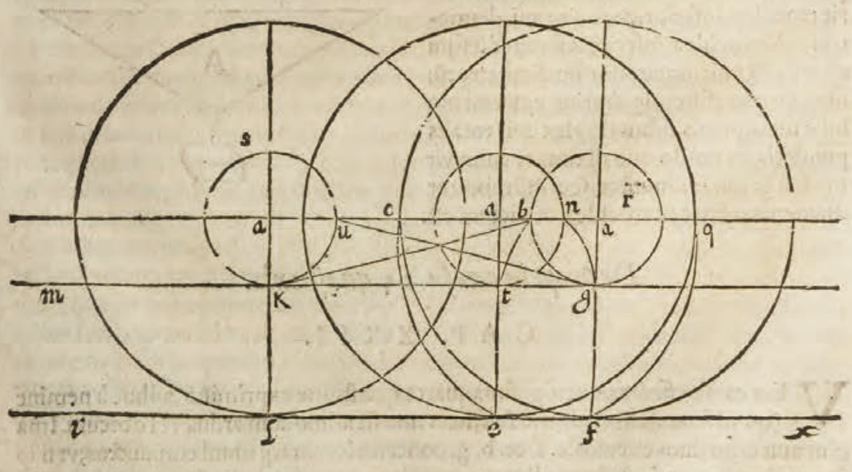



## *De propria cauſa. 24. quæſtionis.*

## CAP. XXII.

VEra cauſa effectus,qui vigeſimaquarta quæſtione exprimitur,adhuc à nemine (quod ſciam)animaduerſa fuit,licet non ſit admodum ardua vel obſcura.Imaginemur ergo duos circulos.c. f. et. b. g. concentricos,itaq; ſimul coniunctos,vt ſi ipſorum vnus feratur in orbem,alius quoque circumagatur,eo modo,quo curruum rotæ voluuntur. Et imaginemur primò ſuper lineam.f.i.reuolui maiorem,& quando idèm circulus erit in.l.dictam lineam.f.i.tangere circunferentiam eiuſdem in puncto.c.

cto.c.vnde linea.g.m.mediante.K.continget circunferentiam circuli minoris in puncto.b:et.K.g.ex.34.primi Eucli.æqualis erit ipſi.f.l.quia ex.17.tertii, anguli.f. et. g. ſunt æquales,vnde ex.28.primi.f.l.et.g.K.ſunt parallelæ.& ſic erunt.k.l.cum. f.g. ex eadem ſupradicta.Ratio autem,qua arcus.g.b.tranſierit lineam.g. K. maiorem ipſa, eſt, quia dum mouetur,quodlibet punctum ipſius.g.b.virtute reuolutionis ipſius.f.c. omne punctum eiuſdem arcus.g.b.vlterius verſus.K.quam ſi moueretur virtute reuolutionis ipſius.g.b.ſuper lineam.g.m.defertur. vt exempli gratia, quando virtute reuolutionis maioris circuli, centrum.a.reperitur in ſitu lineæ.l.K.punctum.g.conſecerit iter.g.u.& punctum.b.iter.b.K.etiam reliqua omnia puncta inter.g. b. magna itinera egerint, cum à magno circulo ſint ante delata. Imaginemur quoque hos circulos eſſe delatos virtute reuolutionis circuli minoris,& partē.g.t.rectè.g.m.dimenſam fuiſſe ab arcu.g.b Quādo ergo.b.erit in.t.factum erit iter.b.t.ab ipſo.b.et.g.faciet iter.g.n.quę itinera alijs multò breuiora ſunt, quia breuioribus cruribus reuoluta ſunt dicta puncta;& ſic dico de reliquis omnibus punctis inter.g.et.b. & in hoc caſu punctum.f.erit in.q.& punctum.c.erit in.e.Quamobrem omnia puncta cōtingentiæ inter.f.et.c.non ſolum nòn erunt delata anteà, ſed potius à primo ſitu retrorſum erunt repulſa. Vnde non eſt,quòd in tantam admirationem ducamur ſi dum reuoluitur circulus maior,arcus.g.b.circuli minoris,totam lineam. g. K. tranſire videtur, & dum reuoluitur minor,apparet arcum.f.c:maius iter quam ab.f.ad. e. non facere, cum maiore ſeſe in orbem ferente,quodlibet punctum arcus.g.b. ad vnam eandéq; partem duos motus obtineat.vt exempli gratia punctum.b.non ſolum mouetur verſus.m.quòd circa centrum.a.feratur,cum ipſum etiam centrum moueatur verſus.m. ſed quia pręter hoc deferantur quoque à circulo maiori verſus.m. vſque ad lineam. k.l.Dum verò minor circulus in girum ducitur,habet quodlibet punctum arcus. f.c. duos motus contrarios,quorum alter verſus.i. virtute reuolutionis circuli minoris, & alter ex eo, ꝙ dictus circulus maior circa centrum.a. voluatur, vnde omne punctū contactus circuli maioris cum recta.f.i.retrorſum pellitur verſus.x.




De

*De uera causa 30. quaestionis.*

CAP. XXIIII.

VEra ratio, cur homo dum sedet ( non tamen Turcarum mor[illegible] velit sese in pedes erigere, calcaneos retrahit, vt efficiat angulum acutum, c[illegible]n fę-moribus coxis à parte inferiori, & ventrem inclinat, ad constituendum etiam angulum acutum in superiori parte, ea est; vt totius corporis pondus, ex ęquo, idest ab oppositis partibus circundet lineam rectam, quæ transit per locum, in quo conquiescunt pedes versus mundi centrum. idest, ut edatur ęquilibrium ponderis ipsius corporis circum lineam illam, quę sub pedibus inseruit pro sparto. Vnde aperiendo, deinde dictos duos angulos circa dictam lineã, absque vlla difficultate erigitur corpus, & absque periculo in alterutram partem cadendi.

*De ratione. 35. & ultima quaestionis.*

CAP. XXV.

VEra ratio, quare, quę reperiuntur in vorticibus aquarum, semper versus medium ipsarum vertiginum vniuntur, inde promanat, quod media vertiginum semper depressiora sunt. vnde quòd dicta corpora ad medium accedant, nihil aliud est, quàm ipsa corpora suo pondere grauitateque descendere, figura enim vorticibus est quasi conica, & concaua cum angulo deorsum, & gyro basis sursum. Atque hæc vera est huius effectus causa, & non ea quam Aristoteles ponit, à quo aliarum omnium quæstionum, quas ego omisi rationes sunt benè propositæ.

DISPV-

# DISPVTATIONES DE QVIBVSDAM PLACITIS ARISTOTELIS.

*TANTA est certè Aristotelis amplitudo atque authoritas, vt difficillimum ac periculosum sit quidpiam scribere contra quam ipse docuerit, & mihi præsertim, cui semper visa est viri illius sapientia admirabilis. Veruntamen studio veritatis impulsus, cuius ipse amore in seipsum si viueret excitaretur, in medium quædã proferre non dubitaui, in quibus me inconcussa mathematicæ philosophiæ basis, cui semper insisto, ab eo dissentire coegit.*

## *Qualiter & ubi Aristoteles de uelocitate motuum naturalium localium aliter tractauerit quam nos sentiamus.*

### CAP. I.

VOlens Aristoteles probare vacuum non esse in rerum natura. 8. cap. lib. 4. physicorum ait, idem corpus per varia diuersaq; media, vt per aerẽ, & per aquã si moueretur, proportionem velocitatis eiusdem corporis per aerem, ei, quæ per aquam fit, vnam eandemq; futuram cum ea, quæ est subtilitatis aereę ad subtilitatem aquæ. In postrema autem parte eiusdem capitis sic scribit: Nam cum ea quę maiorem vel ponderis vel leuitatis pręstantiam habent, si simili figura sint, spaciũ par, & æquale, maiore celeritate conficere cernamus, ea quam magnitudines inter se habent, proportione: profectò idem etiam per inane fieret. Aliam quoque rationem proponit phylosophus. 2. cap. sexti physicorum scribens eademmet proportione, qua tempus diuiditur, magnitudinem etiam diuidi. Sexto autem cap. primi de cœlo scribit, tempora eandem proportionem habere, quam habent è conuerso pondera; vt si media pars vnius ponderis, vnius horæ spatio moueretur, vniuersum pondus in media hora moueretur. Secundo cap. lib. 3. de cœlo duobus in locis apertè commonstrat velocitatem corporis minoris, maiori corpori comparatam, in eadem existere proportione, in qua dicta corpora adinuicem relata existunt. Quinto cap. eiusdem lib. idem affirmat, exemplo ab igne desumpto. Ex alijs etiam plurimis locis cognosci potest, sensisse Aristotelem duo corpora eadem specie, & figura prædita eandem planè proportionem in suorum motuum velocitatibus, quam in suis magnitudinibus habent, retinere. Alij quoque permulti eandem opinionem retinuerunt, & omniũ postremus Nicolaus Tartalea, secunda propositione vigesiminoni quæsiti octaui libri, vbi profitetur se demonstratiuè probare hanc propositionem veram existere; neq; videt quàm magna resistentiarum sit differentia, quæ tam ex diuersitate figurarum, quàm ex magnitudinum varietate exoriri potest; quas quidem diuersitates ne considerat quidem.

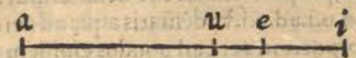
Quędam

## *Quædam ſupponenda ut conſtet cur circa uelocitatem motuum naturalium localium ab Ariſtotelis placitis recedamus.*

### CAP. II.

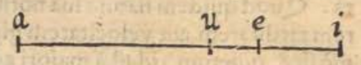
CVM ſuſceperimus prouinciam probandi quod Ariſtoteles circa motus locales naturales deceptus fuerit, ſunt quædam primo veriſſima & obiecta intellectus per ſe cognita pręſuppónenda, ac primum quælibet duo corpora, grauia, aut leuia, area æquali, ſimiliq́; figura, ſed ex materia diuerſa conſtantia, eodēque modo ſitum habentia, eandem proportionem velocitatis inter ſuos motus locales naturales, ut inter ſuamet pondera aut leuitates in vno eodemq́; medio, ſeruatura. Quod quidem natura ſua notiſſimum eſt ſi conſiderabimus non aliunde maiorem tarditatem, aut velocitatem gigni, quàm à. 4. cauſis (dummodo medium vniformè ſit & quietum) ideſt à maiori aut minori pondere aut leuitate; à diuerſa figura; à ſitu eiuſdem figuræ diuerſo, reſpectu lineę directionis, quæ recta inter mundi centrum, & circunferentiam extenditur; & ab inæquali magnitudine. Vnde patębit, quòd figuram non variando, nec in qualitate nec in quantitate, neque eiuſdem figuræ ſitum, motum fore proportionatum virtuti mouenti, quæ erit pondus aut leuitas. Quod autem de qualitatē, de quantitate & ſitu eiuſdem figuræ dico, reſpectu reſiſtentiæ ipſius medii dico: Quia diſſimilitudo aut inęqualitas figurarum, aut ſitus diuerſus non parū alterat dictorum corporum motus, cum figura parua facilius diuidat continuitatem medij, quam magna; vt etiam cęlerius idem facit acuta, quàm obtuſa; & illa quæ cum angulo, qui antecedat mouebitur velocius quàm illa, quæ ſecus. Quotieſcunque igitur duo corpora vnam eandemq́; reſiſtentiam ipſorum ſuperficiebus, aut habebunt aut recipient, eorum motus inter ſeipſos eodem planè modo proportionati conſurgent, quo erunt ipſorum virtutes mouentes: & è conuerſo, quotieſcunque duo corpora vnam eandemq́; grauitatem, aut leuitatem, & diuerſas reſiſtentias habebunt, eorum motus inter ſeipſos eandem proportionē ſortientur, quā habebunt eorum reſiſtentiæ conuerſo modo; quæ quidem reſiſtentiæ inter ſeipſas, eandem proportionem quàm ipſarum ſuperficies habebunt, aut in qualitate ſola figuræ, aut in quantitate ſola, aut in ſitu, aut in aliquibus ex dictis rebus, eo tamen modo, qui ſuperius poſitus fuit, vt ſcilicet corpus illud quod alteri comparatum, æqualis erit ponderis, aut leuitatis, ſed minoris reſiſtentiæ, exiſtet velocius altero, in eadē proportione, cuius ſuperficies reſiſtentiam ſuſcipit minorem ea quæ alterius eſt corporis, ratione facilioris diuiſionis continuitatis aeris, aut aquæ; Vt exempli gratia, ſi proportio ſuperficiei corporis maioris ſuperficiei minoris ſeſquitertia eſſet, proportio velocitatis dicti corporis maioris, velocitati corporis minoris, eſſet ſubſeſquitertia; vnde velocitas minoris corporis, maior eſſet velocitate corporis maioris, quēadmodum quaternarius numerus ternario maior exiſtit.

Aliud quoque ſuppønendum eſt, velocitatem ſcilicet motus naturalis alicuius corporis grauis, in diuerſis medijs, proportionatam eſſe ponderi eiuſdem corporis in ijſdem medijs; Vt exempli gratia, ſi pondus totale alicuius corporis grauis ſignificatum erit ab. a. i. quo corpore poſito in aliquo me-




Y dio

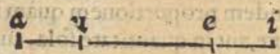
dio minus denſo, quàm ipſum ſit,(quia in medio ſe denſiore ſi poneretur,non graue eſſet,ſed leue,quemadmodum Archimedes oſtendit ) illud medium ſubtrahat partem.e.i.vnde pars.a.e.eiuſdem ponderis libera maneat;& poſito deinde eodem corpore in aliquo alio medio denſiore, minus tamen denſo quam ipſum ſit corpus, hoc medium ſubtrahat partem. u.i. dicti ponderis, vnde pars. a. u. eiuſdem ponderis remanebit. Dico proportionem velocitatis eiuſdem corporis per mediũ minus denſum,ad velocitatem eiuſdem per medium magis denſum futuram vt.a.e. ad.a.u.vt eſt etiam rationi conſonum,magis quàm ſi dicamus huiuſmodi velocitates eſſe,vt.u.i.ad.e.i.cum velocitates à virtutibus mouentibus ſolum ( cum figura vna , eademq; in qualitate,quantitate ſituq; erit) proportionentur. Quæ nunc diximus, planè ſimilia ſunt ijs,quæ ſupra ſcripſimus,quia idem eſt dicere,proportionem velocitatum,duorum corporum hetereogeneorum,ſed ſimilium figura, & magnitudine æqualium, in vno ſolo medio , æqualem eſſe proportioni ponderum ipſorum , vt ſi dicam' proportionem velocitatum vnius ſolum corporis per diuerſa media eandem eſſe cum ea , quæ eſt ponderũ dicti corporis in iſidem medijs.




## *Poſſe uelocitatem alicuius corporis proportionem contrariam in diuerſis medijs habere cum denſitate eorum.*

### CAP. III.

POſſibile eſt in rerum natura corpus aliquod huiuſmodi denſitate præditum reperiri, vt velocitas eius motus naturalis per aerem, velocitati per aquam ita proportionata exiſtat, vt eſt dẽſitas aquæ denſitati aeris. Denſitas aquæ notetur(exempli gratia) per.u.i.& ea,quæ aeris eſt per.e. i.& pondus alicuius corporis in aere per e.a.& pondus eiuſdem corporis in aqua per.u.a.ita tamen , quod eadem proportio ſit.e.a.ad.u.a.vt.u.i.ad.e.i.vnde per vltimam ſuppoſitionem præcedẽtis capitis, proportio velocitatis prædicti corporis per aerem, proportioni eiuſdem corporis per aquam erit , vt e.a.ad.u.a.ergo per.11.quinti,vt.u.i.ad.e.i.



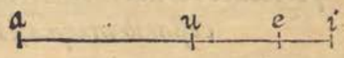

## *Oſcitanter ab Ariſtotele nonnihil prolatum cap. 8. lib. 4. Phyſicorum.*

### CAP. IIII.

EX ſupradictis patet in vniuerſum non eſſe verum quod Ariſto.8.cap.4.lib.phyſicorum ſcribit, velocitates ſcilicet motuum alicuius corporis per diuerſa media,proportionatas eſſe denſitatibus eorundem mediorum. Quapropter, ſit proportio.u.i.ad.e.i.vt dẽſitatis aquę ad aereã dẽſitatem.et.e.a.ad.u.a.vt ponderis alicuius corporis in aere ad pondus eiuſdem in aqua, ita tamen vt maior aut minor proportio ſit.e.a.ad.u.a.quam.u.i.ad.e.i.vnde exiſtente proportione velocitatis per aerẽ

ad

ad velocitatem per aquam vt.e.a.ad.a.u.non erit ergo vt.u.i.ad.e.i. Ob hanc igitur cauſam nimis diſſentaneum eſt rationi, opinari proportionem velocitatis omnium corporum grauium per aerem vnam eandemq́; eſſe cum velocitate eorundem per aquam, quemadmodum Ariſtoteles ſenſit.




## *Exempla dictorum.*

### CAP. V.

POnamus,exempli gratia, aquam eſſe in denſitate dupla ad aerem, & aliquod graue corpus in aqua duplum ad denſitatem ipſius aquæ, vnde dictum corpus in denſitate ad aerem quadruplum erit; quam ob cauſam,mediam ſui ponderis totalis partem in aqua,& in aere quartam partem, ex.7.lib.de inſidentibus aquæ ab Archimede conſcripto,amitteret. Moueretur igitur in aqua virtute illius mediæ partis pōderis ſui,in aere aūt uirtute triū quartarū;vnde proportio facultatis mouētis dicti corporis in aere ad facultatem mouentem eiuſdem in aqua ſeſquialtera erit. hocq́; corpus appelletur. A. Sit aliud quoque corpus,quod.B.nominetur,ſimile figura, & magnitudine corporea corpori. A.ſed dēſitate,in proportione ſeſquialtera ad aquā, & denſius erit aere in proportione tripla. quamobrem corpus. A.grauius erit corpore.B.in aere in proportione ſeſquialtera,vnde etiam velocius erit ipſo. B.in aere in eadem proportione, ſed corpus.B.in aere, duplo maius pondus habebit, quā in aqua, cum in aere remaneant ei duæ ponderis tertiæ partes, & in aqua vna tantum, ita vt Ariſtoteli concedam corpus.B.in aere,quam in aqua velocius futurum in eadem proportione,in qua,aqua eſt dēſior aere, ex Euclidis vndecima propoſitione lib.quinti. Sed præter hæc omnia,ſi corpus. A. eſſet etiam velocius in aere, quā in aqua,in eadem proportione,ſequeretur ex.16.dicti lib.quinti proportionem velocitatis.A.in aqua ad velocitatē ipſius.B. in aqua etiam ſeſquialteram eſſe. Sed cum corpus.A.in denſitate ad aquam duplū ſit, & corpus.B.ſeſquialterū ad ipſam aquā, ſequetur proportionē ponderis ipſius. A. ad pōdus ipſius. B.in aqua eſſe in proportione dupla; Vnde ex primo ſuppoſito capitis ſecundi proportio velocitatis. A. ad velocitatem.B.in aqua dupla erit,non ſeſquialtera. Si ergo proportio velocitatis. A.ad eam quæ eſt.B.in aqua dupla eſt,& ea,quæ eſt.B.in aere,ad eam,quæ eſt ipſius per aquam eſt etiam dupla( vnde ea quę eſt.A.per aquam ęqualis erit ei,quæ eſt.B. per aerem,ex.9.lib.quinti ) & cum ea, quæ eſt.A.ſit ei,quæ eſt.B.per aerem ſeſquialtera,erit ergo ea,quæ eſt. A.per aerem, ei,quæ eſt ſuimet ipſius per aquam ſeſquialtera,non autem dupla,ex. 7. eiuſdem libr.quinti. Hiſce rationibus accedimus ad confirmandam veritatem vltimi ſuppoſiti cap. 2.proportionem videlicet velocitatis motus naturalis in diuerſis medijs alicuius corporis pōderoſi in ipſis medijs eſſe eandem cum ea, quæ eſt inter pondera dicti corporis in dictis medijs. de ijs tamen medijs intelligendo, quæ unctuoſa, aut pinguia non ſunt, ut ſunt oleum,lac,aut huiuſmodi alia, quæ à qualibet minima qualitate frigoris aut caloris alterantur, & impermeabiles fiunt.

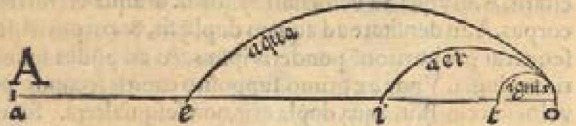
p aerem . A . 6 .
p aerem . B . 4 .
p aquam B . 2 .

p aquam . A . 4 .
p aquam . B . 2 .

## *Quod proportiones ponderum eiusdem corporis in diuersis medijs proportiones eorum mediorum densitatum non seruant. Unde necessariò inæquales proportiones uelocitatum producuntur.*

### CAP. VI.

OMne corpus graue variat proportionem ponderis per diuersa media, vnde proportiones velocitatum inæquales existunt. Vt exempli gratia, si fuerit corpus. A. cuius pondus totale sit. o. a. quod in aqua diminutum sit ratione partis. e. o. ita vt ei solum relinquatur pondus. a. e. & in aere adempta sit ei pars. i. o. vnde solum remaneat pondus. a. i. Supponamus aliud quoq; medium in eadem proportione minus densum, quàm aer, quemadmodum aer minus densus est, aqua, in quo, corpus. A. ammittat partem. t. o. ponderis sui, vnde ex. 7. lib. de insidentibus aquæ Archimedis, eadem proportio erit. e. o. ad. i. o. quæ est. i. o. ad. t. o. Supponamus quoq; eandem proportionem esse. a. i. ad. a. e. est. e. o. ad. i. o. tunc dico non futuram eandem proportionem. t. a. ad. a. i. quæ est. i. o. ad. t. o. Cum sit ergo proportio. a. i. ad. a. e. ut. e. o. ad. i. o. erit disiunctim. e. i. ad. e. a. vt. e. i. ad. i. o. Quare ex. 9. libr. quinti erit. a. e. æqualis. i. o. sed cum ita se habeat. e. o. ad. i. o. vt. i. o. ad. t. o. ita quoque se habebit, ex vndecima quinti. a. i. ad. e. a. ut. i. o. ad. t. o. Cum autem (vt vidimus). a. e. ęqualis sit ipsi. i. o. non poterit esse proportio. t. a. ad. i. a. vt est. o. i. ad. t. o. quia si hoc esset, esset etiam disiunctim proportio. i. t. ad. i. a. vt est. i. t. ad. t. o. & ex supradicta 9. lib. quinti. a. i. æqualis esset. t. o. Maximum autem inconueniens esset. t. o. minorem o. i. idest minorem. a. e. æqualem esse. a. i. quæ maior est. a. e. Ostensiuè tamen idem hoc modo probari potest, vt existente. i. o. ęquali ipsi. a. e. per consequens quoq; erit minor ipsa. a. i. cum. a. e. pars sit ipsius a. i. Per eãdem tamen rationem. o. t. minor est. o. i. Tanto magis igitur minor erit. t. o. ipsa. i. a. Vnde ex. 8. libri quinti maiorem proportionem habebit. i. t. ad. t. o. quam ad. i. a. & ex. 28. eiusdẽ lib. i. o. ad t. o. maiorẽ proportionẽ habebit, quam. t. a. ad. i. a. ex. 12. igitur dicti quinti maiorem proportionem habebit. i. a. ad. e. a. quàm. t. a. ad. i. a. ita ergo se habebunt ipsorum velocitates.



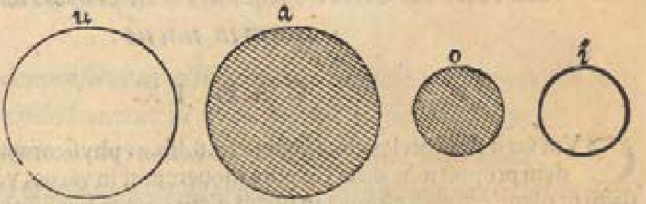

## *Corpora grauia aut leuia eiusdem figuræ et materiæ sed inæqualis magnitudinis, in suis motibus naturalibus uelocitatis, in eodem medio, proportionem longè diuersam seruatura esse quam Aristoteli uisum fuerit.*

### CAP. VII.

EST mihi nunc probandum q; in uno eodemq; medio duo corpora inæqualia, sed simili figura & materia, mouebuntur naturali motu, diuersa tamen ratione ab ea, ea, quàm Aristoteles præscripsit.

Sint igitur corpora.a.et.o.inæqualia ,eadē tamen figura & materia prædita,quorum.a.maius sit,& per consequens in eadem quoque proportione grauius ipso.o. in qua est maius,communi omnium sententia.

Scribit ergo Aristoteles proportionem velocitatis corporis. a. ad eam , quæ est corporis.o.(naturaliter se vnoquoque mouente) eandem futuram, quæ est magnitudinis,aut grauitatis corporis.a.ad magnitudinem, aut grauitatem corporis.o. Imaginemur igitur corpus u.eadem magnitudine & figura,qua corpus.a.præditum est , sed eandem grauitatem obtinere,quæ communicata est corpori. o. quod ex quauis materia constet. Hinc ex primo supposito secundi capitis certi erimus proportionem velocitatis corporis.a.si compararetur cum velocitate corporis.u.futuram, vt eā, quæ est ponderis corporis.a.ad pondus ipsius corporis.u. Ex.9.igitur lib.quinti Eucli.cogitur fateri Aristoteles velocitatem corporis. o. esse vnam eandemq; in specie,quæ est corporis.u.Quod primo supposito cap.secundi huius lib.planè repugnaret. Igitur hæc Aristotelis opinio falsa est. Idem quoque probaretur mediante corpore.i.æquali magnitudine, similiq; figura cum corpore. o. prædito , sed, quòd ad quantitatem attinet,æquali corpori.a.vnde ex primo supposito cap.secundi huius libri in eadem proportione veloci* esset corpore . o. in qua grauius est. ex. 9. igitur quinti cogitur Aristoteles affirmare tā velox esse corpus a . quā est corpus i. vnde idem planè inconueniens emergit ex secundo supposito cap.secundi huius lib.

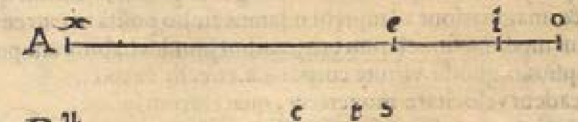



*Quod duo corpora inæqualia eiusdem materiæ in diuersis medijs eandem uelocitatis proportionem retinebunt.*

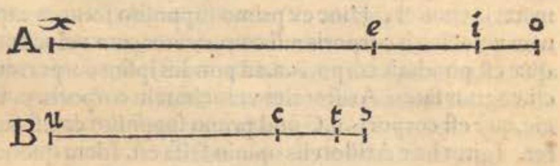
## CAP. VIII.

QVælibet duo corpora inæqualia simili tamen figura & eadem materia constantia, naturaliter se per diuersa media mouentia , vnam eandemq; semper proportionem velocitatum seruant.

Sint duo corpora.A.et.B.sibi inuicem inæqualia quorum.A. sit maius , sed simile figura & idem materia, cuius pondus totale sit. x.o.& pondus totale ipsius.B.sit. u.s. Imaginemur quoque corpus.A. positum in aqua amittere partē.o. e. ponderis. o.x.



o.x.et.B.quoque in eodem loco amittere.c.s.et.A.in aëre partem.i.o.et. B.partem. .t.s. Nunc quia corpus aqueum, cui correſpondet.e.o.æquale eſt ipſi. A. & corpus aqueum,cui correſpondet.c.s.æquale eſt i pſi. B.vt eſt ab Archimede probatũ: com muni quadam ſcientiæ ratione,ſequitur eandem proportionem futuram.o.x.ad.e.o. quæ eſt.u.s.ad.c.s.ob eaſdemq; rationes idem erit de. x. o.ad.i.o.ut.u.s.ad. t.s.& idẽ etiam erit de.o.x.ad.s.u.vt de.e.o.ad.c.s.vt etiam de. o.i.ad.s.t. Vnde ex. 19 . lib. quinti erit de.x.i.ad.u.t.quemadmodum de.x.o.ad.u.s.idem dico de.x.e.ad.u.c.Ex 11.igitur dicti lib.erit.de.x.i.ad.u.t.quemadmodum de.x.e.ad.u.c.ex quibus quidẽ proportionibus, ſi ſubtrahantur proportiones reſiſtẽtiarum extrinſecus aduenẽtium, proportiones quæ remanebunt, ex tertio communi axiomate ab Eucli.in principio primi lib.poſito,ad inuicem




erunt æquales,ſecundum quas eorundem corporum ſunt velocitates.

## *An rectè Ariſtoteles diſeruerit de proportionibus motuum in uacuo.*

### CAP. IX.

CVm verò Ariſtoteles circa finem cap.8.lib.4. phyſicorum ſubiungit quod eadem proportione dicta corpora mouerentur in vacuo, vt in pleno,id pace eiꝰ dictũ ſit planè erroneũ eſt.quia in pleno dictis corporibus ſubtrahitur proportio reſiſtentiarum extrinſecarum à proportione ponderum, vt velocitatum proportio remaneat, quę nulla eſſet, ſi dictarum reſiſtentiarum proportio, ponderum proportioni æqualis eſſet,& hanc ob cauſam diuerſam velocitatum proportionem in vacuo haberent ab ea,quæ eſt in pleno.

## *Quòd in uacuo corpora eiuſdem materiæ æquali uelocitate mouerentur.*

### CAP. X.

QVòd ſupradicta corpora in vacuo naturaliter pari velocitate mouerentur, hac ratione aſſero.

Sint enim duo corpora.o. et.g.omogenea, et. g.ſit dimidia pars ipſius.o.ſint alia quoque duo corpora.a.et.e.omogenea primis, quorum quodlibet æquale ſit ipſi.g. & imaginatione comprẹhendamus ambo poſita in extremitatibus alicuius lineæ, cuius medium ſit.i.clarum erit,tantum pondus habiturum,punctum.i.quantum centrũ ipſius.o.quod.i.virtute corporis.a.et.e. in vacuo, eadem velocitate moueretur, qua centrum ipſius. o.cum autem diſiuncta eſſent dicta corpora.a.et.e. à dicta linea,non ideo aliquo modo ſuam velocita tem

tem mutarent, quorum quodlibet esset quoque tam velox, quam est. g: igitur.g. tam velox esset quam.o.

*Corpora licet inæqualia eiusdem materiæ & figuræ, si resistentias habuerint ponderibus proportionales æqualiter mouebuntur.*

## CAP. XI.

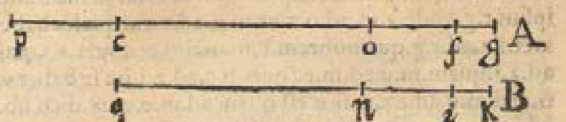
EAdem ratione, quam cap. antecedente præscripsimus, posset ostendi, si duo corpora.o.et.g. suas resistentias, ita ad inuicem proportionatas haberent, ut sunt eorum pondera, in pleno pari velocitate prædita esse, quod in fine capitis noni leuiter attigi, quia punctum.i. tam velox esset, ut centrum ipsius.o. cum à tanto pondere i. motum esset; quanto centrum ipsius.o. atque tantam resistentiam duo corpora.a.et. e. quãta ipsum o. solum haberet ex hypothesi, dicta tamen corpora.a.et.c. tam separata, quam coniuncta, eandem velocitatem retinerent. g. igitur tam velox esset, quam.o.


*Maior hic demonstratur esse proportio ponderis corporis densioris ad pondus minus densi in medijs dẽsioribus, quam sit eorundem corporum in medio minus denso, nec corporum pondera seruare proportionem densitatis mediorum.*

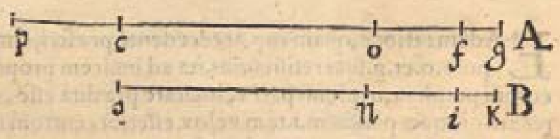
## CAP. XII.

PRoposita nobis cum fuerint duo corpora. A.et.B. area corporea æqualia, quorum. A. densius sit ipso. B. probabo in medio magis denso, maiorem proportionem futuram ponderis ipsius. A. ad pondus. B. quàm in medio minus denso.

Sit igitur.p.g. pondus totale ipsius corporis. A.et.q.k. ipsius corporis. B. vnde. p.g. maius erit ipso.q.k. Sit quoque.o.g. pondus, quod medium magis densum subtrahit à pondere.p.g.et.n.k. sit pondus, quod idem medium subtrahit à pondere.q.k.et f.g. sit pondus, quod medium minus densum subtrahit à.p.g.et.i.k. illud, quod idem mediũ subtrahit ab.q.k. vnde.o.g. æquale erit.n.k.et.f.g. ipsi.i. k. quia quod ad areã attinet, corpora supponuntur æqualia, vnde proportio.p.f.ad.q.i. maior erit ea, quæ est.o.f.ad.n. i. communi scientiæ notione, quia si scinderet aliqs.p.f. in puncto.c. ita. vt.c.f. æquale esset ipsi.q.i. proportio. c.f. ad.q.i. esset vt ea, quæ est. o. f.ad.n.i. (hoc est nulla) sed



ſed proportio.p.f.ad.q.i.maior eſſet ea, quæ eſt.c.f.ad.q.i.ex.octaua lib.quinti, vnde ex.11.eiuſdem lib.maior eſſet.p.f.ad.q.i.quàm.o.f.ad.n.i.ex.33.igitur eiuſdem, maior erit proportio.p.o.ad.q n.quàm.p.f.ad.q.i. Sic quoque ſe habebunt ad inuicem velocitates,quod eſt propoſitum. Cum autem proportio.p.o.ad.q.n.maior ſit, quàm.p.f.ad.q.i.permutando igitur maior erit propor.tio.p.o.ad. p.f.quam.q.n. ad. q.i.aut euerſim maior erit proportio.q.i.ad.q.n.quam.p f.ad.p. o.vnde ſi proportio p.f.ad.p.o. eſſet ac ea, quæ eſt.o.g.ad.f.g.non eſſet.q.i.ad.q.n.ut eſt.o.g.ad. f. g. aut vt.n.k.ad.i. k. quod idem eſt, de quibus quidem rebus, exemplis propoſitis quinto capite mētionem feci.




Velocitatibus autem ſequentibus pondera, ſequitur proportionem velocitatum duorum corporum hetereogeneorum eandem non eſſe per diuerſa media, contra id,quod ſequeretur ſi Ariſtotelis opinionem.8.cap. lib. 4. phyſicorum reciperemus.

*Longe aliter ueritatem ſe habere quam Ariſtoteles doceat in fine libri ſeptimi phyſicorum.*

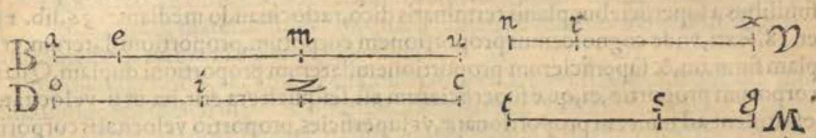
## CAP. XIII.

NOn tam facile eſt aſſignare proportionem velocitatum duorum corporum naturalium,quam Ariſtoteles vltimo cap.lib.7.phyſicorum putauit.

Quamobrem ſint duo corpora.B.et.D.materia magnitudineq; diuerſa,pondere tamen,& figura ſimilia,& proportio reſiſtentiarum,quas recipiunt à medio dū mouentur,ſit.ut.o.i.ad.a .c.denotentur deinde velocitates totales abſque vlla reſiſtentia ab.a.u.et.o.c.quæ æquales erunt ad inuicem per communem ſcientiam ex ſuppoſito,ſint alia deinde duo corpora.V.et.M.eodem modo ſe habentia ut prima.B. et.D.in eodem medio,ſed ex diuerſa materia ab ea, quæ eſt illorum duorum corporum, magnitudine tamen & figura ijſdem ſimilia : ſignificentur quoque eorundem reſiſtentiæ per. t.s.et.n.r.& eorundem velocitates à nulla ex reſiſtentijs diminutæ,per.n.x. et.t.g.vnde.n.r.æqualis erit.a.e.et.t.s.ipſi.o.i.et.n.x.ipſi.t.g:n.x. tamen et.t.g non erunt ęqualia.a.u.et.o.c. Sed exempli gratia,ponamus ea eſſe minora. Supponamus nunc.e.u.velocitatem eſſe quæ remanet ipſi. B. cum applicata erit reſiſtentia.a.e.dicto corpori.B.quæ diminutam facit totam.a.u.per.a.e.ſitq;.i.c. ea,quę remanet ipſi.o.c.corporis.D.et.r.x.ea,quæ remanet.n. x.corporis.V. et. s. g. ea,quæ eſt ex.t.g.corporis.M.Vnde communi omnium cōſenſu aſſequemur.e.u. maiorem futuram.r.x.et.i.c.ipſa.s.g.Scindatur deinde.a.m.ad ęqualitatem.n.x.et. o.z. ipſius,t.g.vnde.a.m.ad.o.z et.m.u.ad.z.c.æquales habebimus,ut quoque.e. m.ad.r. x.et.i.z.ad.s.g.quamobrem.e.m.maior erit ipſa.z.i.maior igitur erit proportio. z.c, ad.z.i.quàm.m.u.ad.m.e.(quia.z.c,ad.z.i.ita ſe habet vt.m.u.ad.i.z.ex.7. lib. quinti, ſed.m.u.ad.i.z.maior eſt quam ad.m.e.ex.8.dicti lib.vnde ex.12.eiuſdem.z.c.ad ad.z.i.maior erit, quàm.m.u.ad.m.e. Ergo ex.28. maior proportio erit. c.i. ad. z.i.

quam

quam.u.ad.m.e.& ex.27.maior erit proportio.c.i.ad.u.e.quam.z.i.ad.e.m.ideſt.s.g ad.r.x.quod Ariſtoteli in mentem non venerat. Alijs quoque modis idem probari poteſt, vt ſi diceret aliquis,maiorem proportionem eſſe.e.m.ad.m.u.quam.i.z.ad z.c.(quia.e.m.ad.m.u.eadem eſt ratio vt ad.z.c.ex.7.quinti, ſed proportio.e.m.ad. z.c.maior eſt quam.i.z.ad.z.c.ex.8.eiuſdem,ergo ea,quæ eſt.e.m.ad.m.u.ex.12.maior erit,quam.i.z.ad.z.c.)vnde componendo, ea quæ eſt.e.u.ad.m.u.maior erit illa, quæ eſt.i.c.ad.z.c.& permutãdo,quam ea,quæ eſt.e.u.ad.i.c.ea,quæ eſt.m.u.ad.z.c. & ex.33.quinti,ea,quæ eſt.e.m.ad.i.z.maior erit ea,quæ eſt.e.u.ad.i.c.




## *Quid ſequatur ex ſupradictis.*

### CAP. XIIII.

EX præcedenti capite manifeſtè depræhenditur, in vniuerſum Ariſtotelis opinionem veram non eſſe in prima parte vltimi capitis.lib.7.phyſicorum; quia in eo loco ſupponens ipſe corpus.B.pręcedentis capitis eſſe dimidiam partem ipſius D.quantum ad aream corpoream ſpectat(ſunt tamen pondere ad inuicem æqualia) ait.B.futurum duplo velocius ipſo.D.Ego verò præcedenti capite accepi. e. u. pro velocitate reſidua corporis.B.( ſubtracta ea tamen parte,quam ei reſiſtentia adimit, quæ erat.e.a.)et.i.c.pro ea,quæ eſt corporis.D.et.r.x.pro ea, quæ eſt corporis. V.et. s.g.pro ea,quæ eſt corporis.M.Dicat nunc Ariſtoteles,quę nam harum duarum proportionum dupla erit? quia ſi earum aliqua talis erit,alia nullo modo eſſe poterit, vt iam oſtendi,etiamſi duo corpora.V.et.M.eaſdem conditiones habeant, quas.B. et. D.Ratio autem,quæ Ariſtotelem induxerit ad illud credendum, nulla alia eſſe potuit,quàm quod putarit reſiſtentias proportionatas eſſe magnitudinibus corporeis,ideſt quemadmodum.B.erat corporaliter dimidia pars ipſius.D. ſic etiam haberet medietatem eius reſiſtentiæ,quam habuiſſet corpus.D. Quod etſi verum eſſet, non tamen ſequeretur neceſſariò in quibuſlibet corporibus futuram velocitatum proportionem eandem,quæ reſiſtentiarum eſt,vt ſuperiore capite oſtendimus.

## *Num rectè ſenſerit Philoſophus reſiſtentias proportionales eſſe cum corporibus mobilibus.*

### CAP. XV.

QVòd Ariſtoteles crediderit reſiſtentias proportionatas eſſe corporibus,errauit. Si ſuperficies ijſdem proportionatæ eſſent, dubium non eſt, quin reſiſtentiæ quoque ipſæ,ijſdem proportionatæ exiſterent, ſupponendo eas ſimiles ſitu,dum eadem corpora mouerentur. Sed eadem proportio non eſt inter ſuperficies,& quæ inter corpora reperit: Ariſtoteles igitur in eo defecit. Quòd autẽ inter ſuperficies non eadem ſit proportio, quæ inter corpora extat, ſi primo ad ſphęricas mentem verterimus, intelligemus proportionem eam,quæ inter duas ſphæras repe ritur triplam ſemper exiſtere ei, quæ eſt inter ipſarum diametros ex vltima. 12. libr. Euclid. Eſt autem proportio, quæ eſt inter ſuperficies ſphęricas ęqualis ei, quæ eſt ipſorum circulorum maiorum ex. 16. lib. quinti, cum ex. 31. primi de ſphæra & cylindro Archimedis, omnis ſphærica ſuperficies quadrupla, ſit maiori circulo ipſius ſphęræ, ſed proportio, quæ eſt inter dictos circulos, eſt dupla ei, quæ eſt inter eorũdẽ diametros ex. 2. lib. 12. Euc. ergo ꝓportio, quæ eſt inter corpora, ſeſquialtera erit ei, quæ eſt ſuperficierum, & non æqualis, ut Ariſtoteles putauit. Idem de corporibus ſimilibus à ſuperficiebus planis terminatis dico, ratiocinando mediante. 36. lib. 11. et. 18. ſexti, vnde cognoſcemus proportionem corporum, proportioni laterum, triplam futuram, & ſuperficierum proportionem, laterum proportioni duplam. Quare corporum proportio, ei, quæ ſuperficierum eſt, ſeſquialtera erit, ita ut ſi velocitates extitiſſent ad inuicem proportionatæ, vt ſuperficies, proportio velocitatis corporis. B. ei, quæ eſt corporis. C. fuiſſet ſubſeſquialtera proportioni corporum, & non æqualis eidem.

## *Idipſum aliter demonſtratur.*

## CAP. XVI.

ALio quoque modo probari poteſt non eſſe in vniuerſum verum id, quod Ariſtoteles in prima parte capitis vltimi lib. 7. phyſicorum ait, ſic ſcribens.

Si. A. quidem ſit id quod mouet. B. verò id quod mouetur, et. C. ſit longitudo per quam, et. D. tempus in quo eſt motum, in tempore nimirum ęquali, potentia æqualis. A. dimidium ipſius. B. per duplum mouebit ipſius. C. per ipſum autem. C. in dimi dio temporis. D. ſic enim erit rationis ſimilitudo.

Sit ergo corpus. c. ſeptimi capitis pondere æquali corpori. u. eiuſdem capitis, ſed area corporea minus ipſo. u. pro medietate. Simile tamen figura. Imaginemur nũc tertium aliud corpus omogeneum ipſi. u. quod ſit. i. magnitudine & figura ſimile ipſi o. vnde minor erit ipſo. u. pro media parte, & hanc ob cauſam ipſum. u. erit duplo mà gis graue, quàm ipſum. i. & per conſequens ipſum quoque. o. duplo grauius erit quã ſit ipſum. i. ex. 7. libr. quinti Euclidis. Ipſum ergo corpus. o. duplo velocius erit, quàm ipſum. i. ex primo ſuppoſito cap. 2. huius lib. Vnde ex. 9. quinti, velocitas ipſius i. æqualis eſſet ei, quæ eſt ipſius u. cum Ariſtoteles ſcribat. o. quoque futurum duplo velocius ipſo. u. ꝙ cap. 7. huius lib. falſum eſſe demonſtraui.

## *De alio Ariſto. lapſu.*

## CAP. XVII.

SCribit Ariſtoteles in ultimo cap. lib. 7. phyſicorum in hunc modum.

Si duo quædam ſeorſum per tantum ſpatium tanto tempore duo ſeorſum pon dera mouent, & compoſita per longitudinem æqualem, ęqualiuẽ in tempore, compoſitum ex ponderibus vtriſq; mouebunt, eſt enim in eis eadem ratio.

Quod

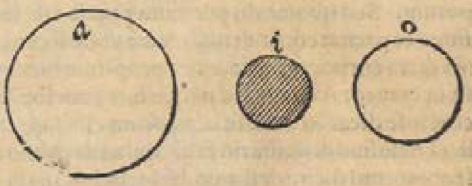
Quod in vniuersum nec etiam potest esse verum in pleno,quia cap.14. iam probaui,non eandem proportionem esse inter superficies corporum, & ipsa corpora.

## *Quomodo dignoscatur proportio uelocitatis duorum similium corporum omogeniorum inæqualium.*

### CAP. XVIII.

ETiam si reperire in qua proportione motus naturaliter moueantur duo corpora, figura & materia similia,inęqualia tamen ad inuicem,non facile sit,ostendam tamen qua ratione id consequi possimus.

Proponantur nobis,exempli gratia,duo corpora.a.et. o.sphęrica,inęqualia inuicem,omogenea tamen materia,quorum.a.maius sit; si voluerimus inuenire in qua nam velocitatis proportione naturaliter mouerentur. Volo vt inquiratur corpus.i. sphęricum,alia tamen & diuersa materia constans,sed pondere ęquale corpori.o. & superficie tam proportionata superficiei corporis.a.quàm est ea,quæ est sui ponderis ad pondus ipsius.a. Hoc facto, indagetur, quænam erit proportio inter superficies corporum.i.et.o.quę semper dupla est, vel subdupla ei quæ est diametrorum;ut iam cap.15.dixi,& hęc proportio superficierum sphęricarum ipsi⁹.o.et.i.subtrahatur ab æqualitate,quod igitur remanebit, erit proportio velocitatū inter duo corpora.o.et.i.idest inter.o.et.a.vt exempli gratia,si proportio superficiei.o.superficiei ipsius.i.sesquitertia esset,subtrahendo eam ab ęqualitate, remaneret ꝓportio subsesquitertia,vnde velocitas corporis maioris( quod in pręsenti loco supponitur esse.o.)ei, quę est corporis minoris, quale est corpus.i. subsesquitertia esset; aut dicamus quòd.i. esset velocius ipso o.in proportione sesquitertia ex secundo supposito secundi capitis huius libri. Sed.i.tam velox est quam ipsum. a.ex.11.cap.ergo proportio velocitatis ipsius.a.sesquitertia erit ei.quæ est ipsius.o.




## *Quam sit inanis ab Aristotele suscepta demonstratio quod uacuum non detur.*

### CAP. XIX.

EX ijs, quæ superius demōstrauimus facilè cognosci potest irritam esse eam rationem, quam Aristoteles.8.cap.lib.4.physicorum ad destruendum vacuum, cō finxit. Vt igitur idem facilius ostendamus,compræhendamus imaginatione infinita media corporea,quorum vnum altero rarius sit,in qua placuerit nobis ex proportionibus,incipiendo ab uno,imaginemur etiam corpus.Q.densius primo medio,cuius corporis,totalis grauitas sit.a.b.& positum in ipso medio,amittat partem.e.b.ipsius grauitatis,& in secundo medio amittat.i.b.& sic per gradus vnde nobis patebit dicto corpori.Q.Nunquam remansuram suam totalem grauitatem.a.b.in quolibet ex dictis medijs. Nunc si quærat à me Aristoteles proportionem velocitatis corporis.Q.per vacuum ad velocitatem dicti corporis per plenum,ego ei proponam proportionem ipsius.a.b.ad.a.e.exempli gratia, dicens,ꝙ quēadmodum.a. b. maius est ipso.a.e.sic etiam corpus.Q.velocius erit in vacuo,quàm in pleno, dicti autem pleni densitatem appellabimus.e.b.Aristoteles dicet nunc, ꝙ aliud quoddam medium in eadem proportione subtilius ipso.e.b.desumatur; quemadmodum.a.e. minus est ipso.a.b.sit ergo istud.i. b.in quo Aristoteles credit corpus Q.futurum tam velox ut in vacuo, in quo aberrat,q͞a proportio velocitatis corporis.Q.in medio.i.b. ad velocitatem eiusdem in medio e.b. ita se habebit, ut.i.a.ad e. a. ex ultimo supposito capit.2. huius libr.quæ minor esset ea, quæ est.a.b.ad.a.e.ex.8.lib.quinti Eucli.


*Non satis dilucidè Aristotelem de loco ratiocinatum fuisse.*

## CAP. XX.

QVæ Aristoteles de loco scribit multas in se continent difficultates. Primum, cap.4.lib.4.physicorum ait, omne corpus esse in suo proprio loco,supponendo vnum centrum pro loco grauium, et unam circunferentiam pro loco leuium corporum. Sed quomodo punctum potest esse locus ipsius corporis, cum omni dimensione capacitateq; sit denudatum? vnde si centrū locus esset corporum grauium,omnia dicta corpora grauia,extra proprium locum existerent, quia nullum ex iis est, ꝙ sit in centro. Adde quod neque hoc cum loci definitione ab ipso posita consentiret cum ipse dicat in eodem cap.locum esse superficiem quandam,& non interuallum, licet huiusmodi definitio falsa appareat primo ex incōuenienti falso,quod ipse hinc sequuturum dicit,idest,quod si locus interuallum esset,infinita loca existerent,quod reuera nec ob hanc causam inconueniens existit, quia eodem planè modo quo aliquod corpus potest esse infinita corpora,(quod ipse diceret in potentia) sic etiam interuallum aliquod posset esse infinita interualla. Cum autem dicat superficies corporis ambientis esse locum eius corporis,quod continetur, cogitur dicere lineam, quæ circundat superficiem,superficiei locum esse, & puncta ipsius lineæ, quod reuera absurdum est. Locus corporis est interuallum illud eadem magnitudine & figura,qua corpus ipsum pręditum est,quod si non esset, sed esset superficies, quemadmodum Aristoteles voluit,maximum inconueniens sequeretur, scilicet æquales locos capere inęqualia corpora,aut corpora æqualia,locos inęquales occupare, quod scitu facillimum est,cum Theon super Ptolomęi Almagestum iam probarit sphæricam superficiem maius interuallum corporeum continere, quàm aliam quāuis superficiem dictæ sphęricæ æqualem,vnde possent facilè reperiri duo loci,quorum alter millies altero maior esset,capaces tamen corporum æqualium, aut reperiri duo corpora,quorum alterum millies maius esset altero, quę tamen corpora apta essent ad occupandos locos ęquales,quamuis Aristoteles dicat,locum, neque maiorem neque minorem esse debere locato. Sed interualla corporea ęqualia à quauis figura terminata,continebunt semper corpora ęqualia. Corporeum igitur interuallum est

reuera

reuera locus corpori adęquatus, cum corpus in interuallum superficiale non intret, quamuis interuallum corporeum ingrediatur. Et hoc modo nullũ est corpus, quod in mundo aut extra mundum (dicat autem Aristoteles quicquid voluerit) locum suum non habeat.

## *Utrum bene Aristoteles senserit de infinito.*

### CAP. XXI.

TRactans Aristoteles in fine quinti cap. lib. 3. physicorum de infinito ait, impossibile cum sit inuenire locum infinitum, & omne corpus in loco cum sit, impossibile quoque esse in rerum natura aliquod infinitum corpus reperiri. Omittamus quòd cum Aristoteles debuerit beneficio loci destruere infinitum, ordine peruerso de infinito prius, quàm de loco disputationem instituat; sed dicamus ipsum intelligere de infinito corporeo, & cum probauerimus corporis locum esse corporeum interuallum, non autem superficiem, neque opus sit in definitione interualli mentionem aliquam facere terminorum, vnde ipsum infinitum esse potest, neque aliqua ratione de hac re dubitari potest; hoc modo nullum inconueniens sequeretur, quòd extra cęlum reperiri possit corpus aliquod infinitum, quamuis, id ipse nulla euidenti ratione inductus perneget. Sensit quoque, absque eo, ꝙ aliquam rationem proponat, aliquid extra cœlum reperiri quemadmodum apparet ex fine cap. 9. lib. primi de cœlo, cum etiam ait cap. 8. lib. 8. physicorum, infinitas partes alicuius continui esse solum in potentia, non item in actu, hoc non est illico concedendum, quia si omne totum continuum, & re ipsa existens, in actu est, omnis quoque eius pars erit in actu, quia stultum esset credere, ea quæ actu sunt, ex ijs, quæ potentia existunt, componi. Neque etiam dicendum est continuationem earundem partium efficere, vt potentia sint ipsæ partes, & omni actu priuatæ; Sit exempli gratia linea recta. a. u. continua quæ deinde diuidatur in puncto. e. per æqualia, dubium non est, quin ante diuisioné, medietas. a. e. tam in actu (licet coniuncta cum alia. e. u.) reperiretur, quàm totum. a. u. licet à sensu distincta non esset. Idem affirmo de medietate. a. e. idest de quarta parte totius. a. u. & pariter de octaua, de millesima, & de quauis, ita vt essentia actualis infiniti hoc modo tutò concedi possit, cũ ita sit in natura. Sed peius etiam sensit Aristoteles eodem loco capitis quinti lib. 3. physicorum, negando infinitum posse connumerari inter quantitates, dicens vnam aliquam quantitatem intelligi vt cubitum, tricubitum, & cætera; vbi non considerat eadem etiam ratione intelligi posse aliquam quantitatem infinitorũ cubitorum, & in quantitatis definitione nullam esse necessitatem terminorum, vt exempli gratia in definitione numeri, non est necessitas alicuius determinati numeri, quia multitudo, non minus infinita, quàm finita, intelligi potest. Vbi posteà cap. 8. libr. 4. physicorum ait nullam esse differentiam inter infinitum, & vacuum, reuera nihil absurdius hoc dicere fingereue poterat.

Exagi-

## *Exagitatur ab Ariſtotele adducta temporis definitio.*

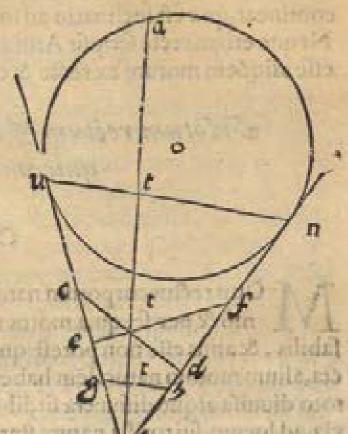
### CAP. XXII.

CVM ſenſerit Ariſtoteles tēpus abſque motu eſſe nō poſſe, ea tamen ab inuicē ſeparans, volēs definire tēpus ait, ipsū eſſe mot⁹ menſurā numerūq́;. Quæ quidem definitio, natura ſua non eſt bona, quia tempus, neque numerus eſt, neque etiā eſt mēſura motus ꝑ ſe, ſed tm̄ ꝑ accidēs, quia nihil eſt, qđ numeret aut menſuret aliud, quod non ſit eiuſdē ſpeciei cū illo quod mēſuratur, aut numero circunſcribiť, vt exēpli gratia, nulla vnquā ſuperficies ꝑ ſe numerabit aut mēſurabit lineā, aut corpus; nec linea ſuperficiem aliquā, aut corpus: nec corpus lineā aliquā aut ſuperficiem; Sed linea lineam menſurabit; ſuperficies ſuperficiem; & corpus corpus; etiāſi tam vna ex iis quantitatibus quàm altera ſit continua. Cum verò motus non ſit tempus, neque tempus ſit motus, ſed inter ſe maximè differant, ſequetur ex iis, alterum nullo modo per ſe eſſe menſuram alterius, niſi per accidens. Et ſi alicui videtur, ꝙ ad ſignificandam aliquam quantitatem motus, dicere huiuſmodi operationem duarum horarum, aut duorum dierum, aut duorum annorum ſpatio completam eſſe, ſit ponere tantum tempus: animaduertere debet hoc ſimpliciter non eſſe verum, quia horarum, dierum. & annorum interualla, imaginatione concipiūtur vt motus corporum cęleſtium, ſine quibus, neque anni, neque dies, neque horę exiſterent, etiā ſi omnis motus ſit (vt ita dicam) locatus in tempore, ut corpus in loco, vnde motus motu, & tempus tempore, non autem aliud ab alio menſuratur. Tempus ex neceſſitate (phyloſophicè tamen loquendo) res eſt æterna, motus non item, quia diuerſis modis terminari poteſt & ceſſare, & interim dum ceſſabit quieſcet corpus, quod primo mouebatur. nihilominus tamen, tempus continuabit curſum ſuum. Tempus igitur potius locus motus erit dicendum, quàm numerus aut menſura eius, & tale eſt, vt conſumatum uideatur à continuò quodam fluxu vnius inſtantis, quemadmodum iam dixi in. 38. capite meę gnomonicæ, & cum dico ab vno inſtanti, vnum in ſpecie, & non in numero intelligo, quod à ſenſibus noſtris percipi non poteſt, neq; etiam notari, quia nouum ſemper inſtans nobis occurrit. & ſi aliquis aliquod exemplū (largo modo) incompræhenſibilitatis ipſius inſtantis deſideraret, imaginetur rotam aliquam albam, in qua ſit nigrum aliquod punctum ſenſibile, aut è contra rotam nigrā imaginetur, in qua ſit punctum album, quæ rota velociſſimè moueatur; huiuſmodi punctum, nullo modo aſſignari poterit, magis ab una parte quàm ab altera; immo ſe ſe nobis offeret ſemper in forma lineæ circularis. poſſumus aliquo modo etiam ſumere exemplum à ſono, quia omnis chorda cuiuſlibet inſtrumenti muſici, dum ſonus editur, tremit, unde huiuſmodi ſonus, appellari poteſt aggregatum aliquod ex innumerabilibus ſonis. eodem modo ſe habet ſonus, quem ędunt campanę, & omnia inſtrumenta tam naturalia, quàm artificialia, quæ quantò velocius tremūt, tantò acutiorē generant ſonum, & quantò tardius, tantò grauiorem. Neque eſt quòd in admirationem ducamur, quòd ſenſui unum aliquod continuum appareat id, quod diſcretorum eſt multitudo (non putet tamen aliquis me negare continuitatem ſucceſſiuam ipſius temporis) quod clare cognoſci poteſt à niue, aut à chryſtallo, aut à vitro, aut à ſaccaro in minutiſſimas partes redacto, quæ continuam aliquam albedinē nobis ad inſpiciendum offerunt, quod nihil aliud eſt, quàm innumerabilis quædam multitudo minutorum reflexorum. Idē dico de ſputo, & qualibet ſpuma, & quanto to minutiora sunt corpuscula à quibus vt à speculis reflectitur lumen,tantò magis ag gregatũ illud albũ apparet. Hæc autẽ exempla cũ sint,nec non largo modo sumpta, mirũ non erit si claudicare videbũt. Sed ut ad motũ, & tẽpus reuertamur (quæ sunt cõtinua successiua) Aristoteles in definiendo tempore,non reduxit in mentem,quod scribit decimo metaphysicę et.4.cap. secundo.libr.de cęlo,omnia videlicet, ab eo, quod minimum est in suo genere,mensurari,& ex seipso in physicorum libris, tempus non est de genere motus;ergo eius ipsius rationum ui,tempus non erit mensura motus,sed motus quidem potest mensurare motum,videlicet velocior minus velocem,& breuior longiorem;& numer⁹ mensurat numero,& tempus tempore in quantum longum est,aut breue,non in quantum velox, aut tardum; Nullum autem inconueniens sequetur sumendo tempus tam proportionale motui, quam locus corpori,quia motus decem milliarium,quæ aliquis vnius horæ spatio conficiat, erit proportionalis corpori denso,& motus vnius milliaris eadem hora peracti, proportionalis erit corpori raro; & quemadmodum corpus densum occupat minus interuallum loci,contra quam fiat in corpore raro:sic etiam motus velox breuiori temporis spatio peragetur,quam tardus.

## *Motum rectum esse continuum,uel dißentiente Aristotele.*

### CAP. XXIII.

ARistoteles.8.capi.8. physicorum ait impossibile esse aliquid per lineã rectam nunc vno modo,nunc altero,idest eundo, & redeundo per dictam lineam in extremis absque quiete moueri. Id quod contrà possibile esse dico. Pro speculatione cuius rei imaginemur circulum.u.a.n. motu continuo circa centrum.o.in quã libet partem,aut dexrrã,aut sinistrã ferri; & imaginemur pũctum.b.extra ipsum,ubi magis nobis videbitur, à quo ducantur duæ lineæ rectę.b.u.et.b.n.contiguæ ipsi circulo in punctis.u.et.n. Imaginatione quoque inter has duas lineas, alteram quæ sit. u.n.aut.c.d.aut.e.f.aut.g.h.constituamus in quali bet parte, sumemus etiam punctum. a. circunferentiæ dicti circuli, à quo vsque ad.b. lineam. b.a.imaginemur fixã in.b. sed quod remanat mobile,secundum quod mouebitur punctum.a. vnde aliquãdo hæc linea erit eadem cum.b.u. & aliquando cum.b.n.& aliquando ab.b. u.versus. b. n.proficiscetur, & aliquando ab.b.n.versus. b.u. vt accidit lineæ directionis, & retrogradationis planetarum,vnde circulus.u.a.n.erit vt epiciclus et. b.vt terræ centrum. Clarum nunc erit,quòd quando linea.b. a.eadem erit cum. b. u. aut cum b.n.non quiescet,quia in instanti reuertetur,quia b.u.et.b.n.in puncto, dictũ circulum tangunt, & dicta.b. a. intersecabit semper aliquam ex dictis u.n.aut.c.d.aut.e.f.aut. g. h. quod intersectionis punctum sit.t. Imaginemur nunc quod secũdũ punctum. t. aliquid per aliquam ex dictis lineis mouea-



moueatur,clarum erit quod tale aliquid,nunquam quiescet,etiam si sit in quouis extremo. Aristotelis igitur opinio, tuta non est.

## *Idem uir grauissimus an bene senserit de motibus corporum uiolentis & naturalibus.*

## CAP. XXIIII.

Aristoteles in fine.8.physicorum sentit corpus per vim motum, & separatum à primo mouente,moueri,aut motum esse per aliquod tempus ab aere, aut ab aqua,quæ ipsum sequũtur.quod fieri non potest; quia imo aër, qui in locum desertum à corpore subintrat ad fugandum vacuum,non solum hoc corpus non impellit, sed potius id cohibet à motu,quia aer per vim à corpore ducitur retrò, & diuisus à parte anteriori à dicto corpore,resistit similiter,& quantum dictus aer in dicta parte condensatur,tantum in posteriori raresit,vnde per vim sese rarefaciens non permittit,vt dictum corpus cum ea velocitate fugiat,cum qua aufugeret, quia omne agens in agendo patitur.Quamobrem cum aer à dicto corpore rapiatur, corpus quoque ipsum ab aere rapitur. Huiusmodi autem rarefactio aeris,naturalis non est, sed violenta;& hanc ob causam resistit,& ad se trahit,sed non sufferente natura,vt inter vnũ & aliud ex dictis corporibus reperiatur vacuum;iccirco sunt hæc semper contigua , & mobile corpus aerem deserere cum nequeat,eius velocitas impeditur. Huiusmodi igitur corporis separatim à primo mouente velocitas oritur à quadam naturali impręssione,ex impetuositate recepta à dicto mobili, quæ imprẹssio & impetuositas, in motibus rectis naturalibus continuò crescit, cum perpetuò in se causam mouẽtẽ, idest propensionem eundi ad locum ei à natura assignatum habeat. Aristo. 8. capi primi lib.de cœlo,dicere non deberet ꝙ quantò propius accedit corpus ad terminũ ad quem,tantò magis sit velox;sed potius, ꝙ quantò longius distat à termino à quò tantò velocius existit.quia tantò maior fit semper impræssio, quantò magis mouetur naturaliter corpus,& continuò nouum impetum recipit,cum in se motus causam contineat,quæ est inclinatio ad locum suum eundi, extra quem per vim consistit. Neque etiam rectè scripsit Aristo.9.cap.lib.8.physicorum et. 2. lib. primi de cœlo esse aliquem motum ex recto & circulari mixtum,ꝙ omninò impossibile est.

## *Motum rectum & naturalem non esse primo & per se quicquid Aristoteli uisum sit.*

## CAP. XXV.

Motus rectus corporum naturalium sursum, aut deorsum, non est naturalis primò & per se, quia motus naturalis perpetuus est,aut vt melius dicam,incessabilis , & alius esse non potest quàm circularis, nullaq; pars cum suo toto coniuncta,alium motum naturalem habere potest, quàm eum,qui est totius. si autem à suo toto diuulsa atque disiuncta sit,libereq; vagetur,spontè , & quàm breuissima potest via,ad locum,sui toti⁹ à natura statutum proficiscitur. hic motus primò, & per se dicti corporis,naturalis non est,cum à causa naturæ suæ contraria sit generatus, idest,

ab eo,

ab eo quod sit extra suum locum, vbi contra naturam suam reperitur. Vnde huiusmodi motus, partim & non omninò, naturalis est. Is autem proprius est & naturalis motus, qui dicti corporis essentiam conseruat. hoc autem non præstat hic rectus, cum destruat, ergò hic motus primò & per se naturalis non est.

## *Omne corpus esse in loco proprio graue, ut Aristoteli placuit, non est admittendum.*

### CAP. XXVI.

ARist.4.cap.lib.4.de cęlo sic scribit.
Suo enim in loco grauitatem habent omnia præter ignem, signum cuius est vtrem inflatum plus ponderis, quam vacuum habere, &c.

Quo in loco, manifestè indicat se causam nec grauitatis, nec leuitatis corporum naturalium nosce, quæ est densitas aut raritas corporis grauis, aut leuis, maior densitate, aut raritate medij permeabilis, in quo reperitur.

Exemplum ꝙ ipse de vtre inflato proponit, debuisset saltem ei oculos ad veritatem, quæ clarissimè fulget, inspiciendum aperire. Verissimum est, vtrem inflatum plus ponderis habere quàm vacuum, aut quando aer in eo non est per vim inclusus.

Ratio autem huius rei est, quia quando inflatus est, ea quantitas aeris, in eum per vim iniecti, minorem occupat locum, quàm si eidem liberè vagari permitteretur, vnde violenter, quodam modo, condensata est, & quia corpus densum in minus denso, semper descendit, & minus densum in magis denso ascendit. Hanc ob causam vter inflatus plenus corpore magis denso, quàm est medium quod eum circundat, descendit, non quia aer in aere, aut aqua in aqua sit grauis.

## *Haud admittendam opinionem Principis Peripateticorum de circulo, & sphæra.*

### CAP. XXVII.

CVm Aristoteles senserit circulum esse figurarum superficialium primã, & sphęrã esse primã corporearũ ꝑꝑ earũ periferias, decipitur. Sunt enim vltimæ, non primæ. Sunt quidem (in quò rectè sentit) perfectè, licet rationem huius rei non nouerit. Nam centrũ cuiuslibet rei, eiusdem rei principiũ est, & eę figurę, quæ ipsum æqualiter circundant, possunt appellari perfectę, siue sint superficiales, siue corporeæ, &ècontrà illæ, quæ contrario modo se habent, imperfectæ. Quòd autem perfectum est, licet natura sit primum, est tamen vltimum generatione. Sed quando Aristoteles duas dictas figuras pronuntiauit primas, vt perfectas, prioritate scilicet ea, quæ oritur à perfectione, verum dixit; sed quando de figuris superficialibus loquens, vult circulum esse primum, quia ab vna tãtum linea terminetur; nõ minus pro circulo, quam pro oxigonia seu elipsi, aut cucurbitali, aut aliis multis figuris ab vna tantum linea terminatis concludit. Neque etiam hæc ratio perfectionem circuli mõstrat, quia aliæ figuræ, à lineis curuis terminatę, eandem conditionem sortiuntur. Circulus sphęraq́;, non ex vno solo angulo recto constant, vt idem Aristoteles putat cap.4.lib.4.de cęlo,etiam ſi triangulus ex duobus angulis rectis conſurgat, ſed ſunt figurę infinitorum angulorum rectorum,& hanc ob cauſam à me dicuntur vltimæ & perfectę, quia infinito nihil addi poteſt. Numerus angulorum rectorum circuli, eſt minor duplo infinito per duo infinita angulorum contingentiæ, quæ duo infinita mi nora ſunt quouis angulo acuto rectilineo,& numerus angulorum rectorum ſolidorũ ſphęræ,minor eſt quadruplo infinito per.4.infinita angulorum ſolidorum cõtingentiæ,quæ.4.infinita,minora ſunt quouis angulo ſolido acuto terminato à tribus planis. Triangulus inter figuras planas ſuperficiales eſt primus,& circulus vltimus; & pyramis quadrilatera, inter corpora eſt prima,& ſphęra vltima.

## *Occultam fuiße grauisſimo Stagiritæ cauſam ſcintillationis ſtellarum.*

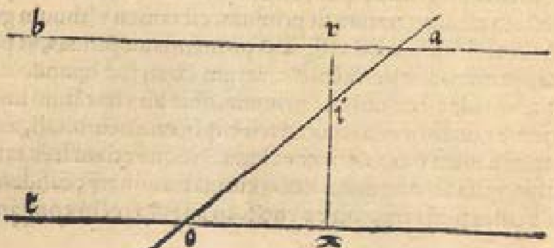
### CAP. XXVIII.

VBi Ariſtoteles ait ſcintillationem ſtellarum fieri ratione aſpectus noſtri ob,maximam diſtantiam,maximum errorem committit, vt etiam facit quum putat viſionem fieri extramittendo,contra id,quod alio loco, immo contra veritatem ipſam aſſeruit. Scintillatio ergo ſtellarum, neque aſpectus noſtri ratione, neque alicuius mutationis earundem ſtellarum,ſed ab inæqualitate motus corporum diaphanorum mediorum naſcitur,quẽadmodum clarè cernitur,quòd ſi inter aliquod obiectum,& nos,aliquis fumus,qui aſcendat,intercesſerit,videbimus obiectum illud quaſi tremere. Hoc autem tantò magis fiet, quantò magis diſtabit obiectum ab ipſo fumo; vnde admirationi locus non erit,ſi ſtellas fixas magis ſcintillare,quam errantes cernamus. Lumen ſtellæ ad oculum noſtrum accedens,perpetuò per diuerſas diaphaneitates penetrat, medio continuorum motuum corporum mediorum, vnde continuò eorum lumen variatur,& hoc in lõginquis magis, quàm in propinquis ſtellis apparet, quemadmodum ab exemplo de fumo allato, & etiam ab aliquibus vitris ex ſuperficie non plana,ſed irregulari conſtantibus,quilibet cognoſcere poteſt.

## *Dari continuum infinitum motum ſuper rectam atque finitam lineam.*

### CAP. XXIX.

OMnes hactenus ſenſerunt impoſsibile eſſe dari per imaginationẽ motum continuum & perpetuũ ſuper vnam lineam rectam finit:in quo tñ decipiuntur. Imaginemur iõ duas lineas parallelas. a.b. et.t.x.quarũ b.a.ſit ĩfinita à qualibet parte,& in ea imaginemur punctum.a.moueri continuò ad quam voluerimus partem, &



& in linea.r.x.imaginemur punctum fixum,quod ſit.c.imaginemur etiam inter.c.et a.vnam lineam rectam.c.a.& inter duas parallelas dictas.r.x.fixam, & motus puncti ſit ab.b.verſus.a.ita ut.c.a.ſecet.r.x.in puncto. i. quod interſectionis punctum mouebitur ab.r.verſus.x.continuò,in tempore infinito, neque vnquam idem erit cum puncto. x.

## *Non eſſe ſolis calorem à motu locali ipſius corporis ſolaris, ut Ariſtoteli placuit.*

## CAP. XXX.

ID nullo planè modo eſt admittendum quod Ariſtoteles credidit calorem ſolis à motu locali ipſiuſmet corporis ſolaris,& non à lumine,prouenire, quemadmodum manifeſtè aſſerit primo metheororum cap.3.circa finem ſic ſcribens.

Vt igitur tepor gignatur atque calor, ſolis latio duntaxat,ſatis eſt efficere,&c.ſed cap.7.lib.2.de cœlo ſic ſcribit,Caliditas autem ab ipſis, lumenq́; ideo fit, quia aer ab illorum motione fricatur.

Vbi non ſolum oſtendit ſe opinari,quòd motus corporum cœleſtium ſit cauſa caloris, ſed etiam luminis, paulò autem poſt dicit,ſuperiorum autem corporum vnum quodque fertur in ſphæra, vt ipſa quidem non igniantur. Opinio profecto abſurda. Nam cùm corpus ſolare fixum ſit in ſpiſſitudine ſui orbis deferentis, ſecundum communem opinionem, non mouetur per ſe, ſed accidentaliter,cum ſcilicet fertur à dicto ſuo orbe, vnde fieri poteſt, vt in motu ſui orbis, nullum ex orbibus ſuorum deferentium augis fricet,ſed ſi fricaret,id faceret mediante vno ſolo puncto, vt cuilibet,aliquantulum in mathematicis verſato patet. Quam ob cauſam,rationi cõſentaneum non eſſet credere,quòd tantùm caloris gigneretur. Quod tamen ſi poſſibile eſſet, quid ergo fricatio ſuperficierum orbis ſui,cum iis, quæ ſunt deferentium augis efficeret? Quãdo tamen hoc fieret,vt ſcilicet à fricatione ſuperficierum procederet calor,nil planè diſcriminis inter hyemen, & æſtatem intercederet, nec inter calorem diei,& noctis,nec inter unam horam, aut alteram; ſed ſecundum Ariſtotelis rationes,Venus, Mercuriusq́;, magis calefacere quam ſol deberẽt, cum ita ſint veloces vt ipſe Sol, & eodem magis propinqua terræ. Verum Ariſtotelis tẽporibus,nullũ aliũ planetã quam ſolem putabãt philoſophi ſupra Lunã eſſe. Atque etiam cõtigeret menſe Decembri,quam Iunio,magis inualeſceret calor, cum huiuſmodi menſe ſol ad nos propius accedat, quam menſe Iunii. per differentiam maiorem diametro regionis elementaris,(nam ſolaris eccentricitas maior eſt ſemidiametro elemẽtaris regionis) non conſiderans Ariſtoteles differentiam caloris, quæ naſcitur ex eo, ꝙ Sol aut altius ſupra orizontem excurrat, aut infra eundẽ feratur;neque eam,quę à longitudine,aut breuitate diei proficiſcitur. Sed quia Ariſtoteles eodem cap.tertio Metheororum intelligit de motu rapto, ideſt diurno, ſiue dicamus vniuerſali,hinc ſequi deberet,ꝙ Sol maiorem caloris uim menſe Martij & Septembris, quàm aliis menſibus,profunderet.quia in iiſdem temporibus, ſol virtute huiuſmodi motus velocior exiſtat, quàm alio quolibet tempore anni, cum tunc per æquatorem circũoluatur. Multa quoque alia incommoda ſequerentur ſi Ariſtotelis rationes admitteremus. Sed clarè uidemus,mediante reflexione aut refractione radiorum ſolarium,ꝙ vniente ſeſe lumine, unitur quoque, & augetur calor, atque omnis res ad comburendum apta accenditur,& inflammatur. In lumine igitur continetur calor,& non in motu ipſius ſolis,& ita in lumine ſedem habet,ut ſi ſol quieſceret,neque in orbe ſuo circumageretur, infęliciſſima eſſet ea regio , in cuius Zenith ipſe reperiretur.

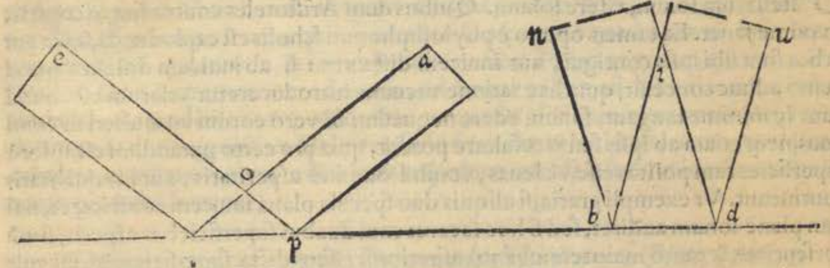
## *Vnde caloris ſolis prodeat incrementum æſtate,et hyeme decrementum.*

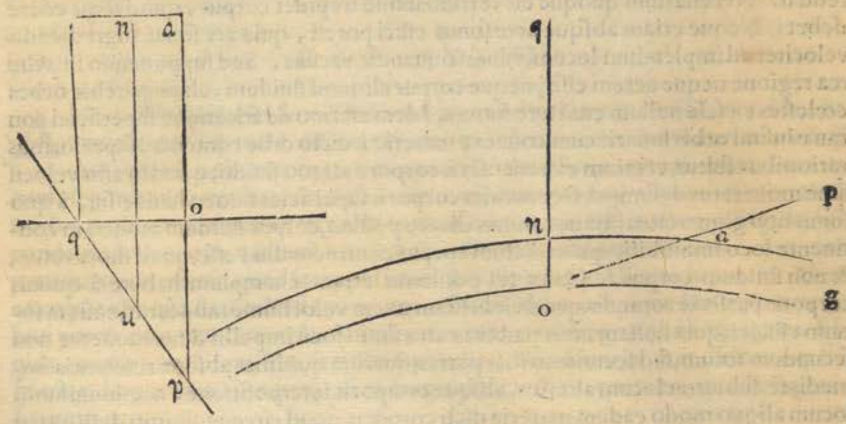
### CAP. XXXI.

CVm capite ſuperiore oſtenderim calorem ſolis non aliunde, quàm à lumine prouenire, oſtendam nunc ex ordine, ex quot,quibusq; cauſis oriatur magna differentia eius caloris æſtatis ad hyemem, quarum nonnullæ ab antiquis obſeruatæ fuerunt,aliæ autem à nemine, quod ſciam. Sunt autem quinque ad minus eæ cauſæ,quarum vna eſt diuturna ſolis mora,tempore æſtatis ſupra orizontem , quæ cauſa ab antiquis poſita,& citata fuit. Aliam quoque huius rei cauſam iidem antiqui dicebant eſſe propinquitatem ſolis noſtro Zenith, ſed hæc cauſa immediata non eſt, quia ab ea tres cauſæ immediatæ naſcuntur;quarum vna eſt maior unio radij reflexi cum radio incidenti; ſecunda maior quantitas luminis in ſuperficie terrę; tertia ,minor reſiſtẽtia vaporum ipſi radio luminoſo facta;quarta verò eſt impreſſio caloris facta in terra,quæ cum aliis cauſis coniuncta calorem adauget. quæ quidem cauſæ nemini adhuc,quod ſciam, in mentẽ venerunt. Quòd autem attinet ad maiorem coniunctionem radii reflexi cum incidente,quiſque, uel ſaltem mediocriter in cathoptricę cognitione verſatus hoc verum eſſe cognoſcet. Vt hoc tamen innoteſcat facilius. Imaginemur. q.p.et.b.d.eſſe duas particulas ęquales ſuperficiei ipſius terræ,ſuper quas cadant duo radii luminoſi ſolis.e.q.et.n.d.quorum.e.q.ſit ad modum obliquus,et.n.d.quaſi perpendicularis, vnde radii reflexi.p.a.et.b.u. aſcendent cum angulis æqualibus eis, qui ſunt ſuorum cadentium,cum omnis angulus reflexionis ſemper æqualis ſit angulo ſuæ incidentię,vt cuilibet in cathoptrica,vel mediocriter verſato patet.Mixtio autem primorum obliquorum erit.q.o.p.& ea , quæ eſt minus obliquorum.b.i.d.quorum duorum triangulorum nullus unquam erit,qui dubitare poſſit.q.o.p.non eſſe minorem.b.i.d.cum anguli.q.et.p.trianguli.q.o.p.acutiores ſint angulis.b.et.d.trianguli.b.i.d.ex ſuppoſito. Quòd uero attinet ad maiorem quantitatem luminis ſuper terræ ſuperficiem; Imaginemur radium.a.q.cuius reſpectu etiam imaginemur duos ſuperficiei terræ ſitus, quorum vnus ſit.q.o:cui dictus radius ſit perpendicularis,& alter.q.p.cui radius.a.q.ex obliquo incidat. Imaginemur ergo triangulum. q.o.p.cuius angulus.o.rectus eſt ex ſuppoſito,unde.q.o. minor erit.q.p.ex.18.primi Euclidis.hinc fit,vt ſuper.q.o.cadat vniuerſum lumen, quod ſuper.q.p.diffunditur. Sit.q.u.æqualis.q.o.& ſit imaginatione protracta.u.n. æquidiſtans.p.o.a.vnde.q.u.illuminata erit à radio.n.q.minore radio.a. q. ergo minus calida erit ſuperficies.q.u.ipſius terræ,quàm.q.o. quia maius lumen in ſe maiorem calorem includit: quod manifeſtè apparet in radiorum vnione mediante reflexione, aut refractione. Sed quod attinet ad minorem reſiſtentiam vaporum ad ipſum radium luminoſum, etſi primo capite meæ Gnomonicæ leuiter id attigerim, nihilominus tamen,& idem ipſum hoc loco proponam. Denotetur,exempli gratia, ſuperficies terræ ab.o.g.et ea,quæ eſt vaporum ab.n. a. ſupponatur etiam ſol in ſitu.

q.qui

q. qui ſit Zenith pũcti.o.& etiã in.p.ipſi orizõti propinquus,aut extra Zenith,cuius duos radios.q.o.et.p.o.Imaginemur, quorum duæ partes.a.o. et. n. o. erunt aliquo modo ab ipſis vaporibus offuſcatæ,ſed.o.n.breuior eſt.o.a.ex.7.lib.3. Eucli. minorem ergo reſiſtentiam habebit à vaporibus ſol in Zenith,quàm extra eundem commorans, & quantò longius erit idem ab ipſo Zenith, tanto maiorem reſiſtentiam à dictis vaporibus inferri ex eadem.7. lib.3.Eucli. dicemus.




## *Nullum corpus ſenſus expers à ſono offendi, præterquam Ariſtoteles crediderit.*

## CAP. XXXII.

POſſe ſonum corpus aliquod, quod ſenſu ſit deſtitutum , vt Ariſtoteles.9.cap.libr.2.de cęlo putauit,offendere,eſt falſum.

Corpus enim non niſi à corpore poteſt lædi, non ergo à ſono, cum ſonus corpus non ſit. Sed aer,& ignis,cum è contra ſint corpora, hoc facilè præſtare poſſunt implendo aliquem locum velociter ad excludendum vacuum; vnde generatur ſonus. Quod hucuſque à nemine animaduerſum fuiſſe comperio.

Pyta-

## *Pytagoreorum opinionem de ſonitu corporum cœleſtium non fuiſſe ab Ariſtotele ſublatam.*

### CAP. XXXIII.

SEnſerunt Pythagorici orbes cęleſtes dum circunuoluuntur,non autem corpora ſtellarum ſolum,ædere ſonum. Quibus dum Ariſtoteles contradicere cogitat, maximè fauet. Ea tamen opinio è phyloſophorum ſcholis eſt explodenda, quia aut orbes ſunt ſibi ipſis contigui, aut inuicem diſtantes : ſi ab inuicem diſtant ( quod nemo adhuc conceſſit, quia hac ratione vacuum introduceretur ) clarum eſt, quod cum ſe minime tangant, ſonum edere nequeunt: Si verò eorum vnus alteri ſit contiguus,neq; etiam ab ipſis ſonus reſultare poterit, quia pro certo putandum eſt,ipſorū ſuperficies tam politas eſſe,ac lenas, vt nihil omnino aſperitatis, aut inæqualitatis contineant. Vt exempli gratia,ſi aliquis duo ſpecula plana inuicem confricaret, nullum planè ſonum audiret, ſed ſi hoc faceret cum duabus ſuperficiebus aſperis, ſonū perſentiret,& tanto maiorem;quantò aſperiores eſſent dictæ ſuperficies, & qui vult vt arcus lirę,ex corda ſonum eliciat, colophonia dictum arcum illinet, vt aſperiorem reddat. Neceſſarium quoque eſt vt tremat ſiue trepidet corpus, quod ſonū edere debet; Neque etiam abſque aere ſonus effici poteſt, quia aer ſonat ingrediendo velociter ad implendum locum, vt non remaneat vacuus. Sed ſupponendo in ætherea regione neque aerem eſſe, neque corpus aliquod fluidum, clarè patebit orbes cœleſtes ex ſeſe nullum emittere ſonum. Idem affirmo de fricatione ſuperficiei concauæ infimi orbis lunaris cum conuexa materiæ à dicto orbe contentæ, ſuperioribus rationibus fultus, vt etiam experientia à corpore aliquo fluido, quod in alio velociſſimè moueretur deſumpta fretus, cuius corporis ſuperficies tamen lenis eſſet, à quo ſonus non gigneretur. Et non minus dicere poſſum, corpus fluidum moueri in continente loco immobili, quam dictum corpus continens illud eſſe, quod moueretur, & non fluidum corpus. Cuius rei poſſumus etiam exemplum habere à quouis corpore perfectè rotundo, quod circa ſuum axem velociſſimè moueatur, nullum ſonum efficiet, quia nullam aeris partem extra ſuum locū impellit dum mouetur non ſecundum totum, ſed ſecundum ſuas partes, quarum quælibet abſque reſiſtentia immediatè ſubintrat locum alterius, abſque temporis interpoſitione. nec huiuſmodi locum aliquo modo eadem materia dicti corporis, quod circunuoluitur: deſtitutum dimittat. Sed ſi Pythagorici de alia quadam harmoniæ ſpecie ab ea, quæ eſt ſonorum, vt à diuerſis velocitatibus motuum, aut à diuerſis magnitudinibus aut diſtantiis, aut ſtellarum influxibus intellexiſſent, rectè ſenſiſſent ex parte, non autem omnino, quia ea harmoniam efficere nequeunt, quæ ad inuicē ſecundum interualla harmonica proportionata non ſunt, vt ſunt dupla, ſeſquialtera, ſeſquitertia, ſeſquiquarta, ſeſquiquinta, ſupertripartientia quintas, ſuperbipartiētia tertias, & quę ab ijs dependēt ideſt coniuncta ſunt cum duplis; de conſonantijs loquendo. de diſſonantiis idem dico, quæ harmonicis inſeruiunt modulationibus, vt ſeſquioctauū, ſeſquinonū, ſeſquiquintūdecimū, ſeſquiuigeſimūquartū, ſeſquioctuogeſimū, & ſuperbipartiens vigeſimasquintas. Verū quidem eſt nonnulla harmonica interualla in aſpectibus cōperta fuiſſe, vt Ptolomeus oſtendit, & alii quoque aſſerunt. ineſt tamen huic rei nonnihil difficultatis. vt exempli gratia, ſi ſubtrahamus diateſſaron extra diapaſon, remanet diapente, & ſi à diapente ſubtrahamus ſemiditonum, remanet ditonum ( quæ duæ

conſo-

consonantiæ, cum habent respectum ad inuicem, quem habent diatessaron, & diapente, quia quemadmodum semiditonum, & ditonum simul coniuncta, componunt diapente, sic diatessaron, & diapente simul vnita componunt diapason; & quemadmodum terminus, qui diuidit diapason in diatessaró, & diapente, est mediator harmonicus inter extrema diapason diuisi, sic etiam terminus, qui diuidit diapēte in semiditonum, & ditonum, mediator est harmonicus inter extrema ipsius diapente diuisi) subtrahendo deinde à diapason semiditonum remanet exachordum maius, & ab eodem diapason subtrahendo ditonum remanet exachordum minus, quę quidē nō accidunt aspectuum circulo, quia subtrahendo aspectum quadratum ab opposito, remanet aliud quadratum, & subtrahendo sextilem à trino remanet quoque alius sextilis. Quòd autem attinet ad motus, ad magnitudines, ad distantias, & ad influxus, nihil est, quod hisce proportionibus conueniat, sed quia hæc omnia depēdent ab īfinita, & diuina ꝓuidētia Dei, necessariò fit vt istæ velocitates, eæ magnitudines, distantiæ, & influxus, talem ordinem, & respectum inter seipsa, & vniuersum habeāt, qualis perfectissimus sit.

## *De raro et denso nonnulla, minus diligenter à Peripateticis perpensa.*

### CAP. XXXIIII.

ANtiqui Peripatetici de videndo in hyeme animalium halitu. Id, quod in æstate non euenit, malè disputauerunt, quia hoc nascitur à condensatione halitus, quę ab ambiente frigore fit. quia halitus is ab ore, aut naso animalis exiēs non est purus aer attractus primò, sed mixtus est cum quodam vapore excrementitio, & subtili, quo semper ab ea parte euacuatꝰ corpus, qui statim ab aere frigido circundatur, & densatur, quam ob causam ab ipso ea luminis pars reflectitur, quæ eum penetrare non potest, quod in hypocaustis, huiusmodiq; calidis locis non fit. Idem exemplo ab aqua statim à cisternis, aut profundis puteis in hyeme extracta comprobari potest, quia tunc temporis, huiusmodi aqua, cum magis calida sit, quàm frigida, emittit vaporem, qui facillimè videtur, ob rationem iam dictam, quod in æstate non cernitur in aqua, etsi ea magis calida esset, quam ea, quæ in hyeme hauritur.

Ratio autem, quam ab antiperistasi desumptam citarunt iidem ad inquirendum, cur aqua subterranea magis calida, aut minus frigida, hyberno tempore, quàm ea, quæ est supra terram sit, vana est, quia hoc non aliunde fit, quàm ab eo, ꝙ terræ pori à frigoris siccitate sint clausi, vnde vapores & exalationes non tam facilè exire possunt. quamobrem calefiunt subterraneæ partes. Fimum, fœnum, frumentum hac in re sunt nobis exemplo, in quibus sępissimè visum est ignem accendi.

Priore illa quoque ratione de antiperistasi dicta, volunt philosophi maiorem caliditatem hyęme, quàm ęstate in animalium stomacho contineri, non animaduertentes siccitatem, frigiditatis partes superficiales corporis, restringētem, sanguinem versus originem suam impellere, qui in eo loco copiosior cum sit, eas partes tunc temporis calefacit magis.

Neque etiam ijdem nouerunt causam, vnde fiat, ut in æstate impleto vase vitreo, aut argenteo, aut ex materia non porosa constante, aqua frigida, vas sudet, quod tempo-

tempore hyemis,non niſi in calidis locis euenit,quem ſudorem , dicebant ipſi, eſſe eandem aquam,quæ per porros vaſis exiret, quod falſiſſimum eſt,quia ſi per porros aqua frigida exiret multò magis exiret calida, cum ſubtilior ſit,& ad penetrandum aptior. Sed hoc non aliunde oritur,quàm à condenſatione aeris vas circundantis, cauſata à frigiditate vaſis refrigerati ab aqua, quemadmodum tempore hyberno clarè videmus mane ſuperficies interiores vitri feneſtrarum ſudare,quia extrinſecũ frigus refrigerando vitrum, intrinſecum aerem ſibi contiguum congelat.

Neque ſilentio inuoluendum eſt,nec Ariſtotelem,neque alium ex ſuis fautoribus animaduertiſſe denſum,& rarum eſſe cauſam ventorum. Rarũ autem & denſum, mediante calore & frigore fit, & ſi à partibus, in omogeneis, licet argumẽtari, de toto deducat conſequentiam qui velit,obſeruans in calidis æſtatis diebus,dum aliqua nubecula ad Solem cooperiendum incedit,ibi ſtatim agitationem aeris ſentiri;ea verò nubecula prætergreſſa cum fuerit,& in ea parte, aer ad priſtinam raritatem cauſatam à calore Solis redierit, quieſcit;huiuſmodi autem aeris agitatio,à nulla certè exhalatione proficiſcitur,ſed à motu ſolum locali, quem dum condenſatur, facit. Omne denſum natura ſua frigidum eſt; omne rarum calidum, & è conuerſo. Et frigida aura,quæ à flabellis cauſatur,non ſolum à nouo aere qui nos tangit, ſed etiam a denſo, quod in agitatione eiuſdem aeris fit,naſcitur.

Cum autem de raritate & denſitate diſputationem ſuſceperim, non ſine ratione mihi videt̃ illorũ opinionẽ explodẽdã eſſe,qui Lunę maculas nõ aliud eſſe dixerunt, quàm aliquas partes rariores aliis eiuſdem Lunæ partibus,non obſeruantes rarum,& denſum,proportionabilia lumini,quod ab huiuſmodi corporibus reflectitur, non eſſe. quia corpus aliquod rarum aliquando aptum erit ad reflectendum maius lumen, quàm corpus minus rarum ut manifeſtè apparet à nubibus reflecti lumen: quod ab aere non fit. Non defuerunt qui contrarium dixerunt,ideſt,eas Lunę partes, denſiores eſſe;neque unquam aliquis fuit qui de diaphano,aut opaco mentionem fecerit,quia melius eſt credere,eas partes diaphanas,ſiue perſpicuas magis eſſe,quàm alias,quę per aliquod ſpatium, ſolis radio ingreſſum permittant, & alię partes cũ ſint opacæ ipſum à ſuperficie reflectant. diuerſa tamen ratione à ſpeculo, cum in plenilunio tota ferè Lunę pars illuminata cernatur,quamuis dictum lumen extenſiuè & intenſiuè ſit minus eo, quod ipſa in nouilunio recipit. Indignum autem mihi videtur ijs reſpondere,qui dixerunt huiuſmodi maculas,terræ vmbras exiſtere,cum craſſiſſimæ ignorantiæ tenebris ſint circunfuſi,vt etiã fuit Cornelius Agrippa,qui primo de occulta philoſophia dicens ſe noſſe modum quendã naturalem à Pythagora inuentum,quo in Luna id totum,quod ipſe ſuper ſpeculum ſcripſiſſet,videretur. oſtendit manifeſtè ſe ignorare luminum vmbrarumq; naturam.quia nulla vnquam vmbra generari poteſt a corpore,quod aut opacum non ſit, aut officio opaci non fungatur, vt nunc dicemus de diaphaneitate aquæ. Neque corpus opacum illuminatum adũbrare poteſt,niſi opacum illud in linea recta ſitum obtineat, quæ inter lucidum & illuminatum extenditur. Neque etiam reſpondebimus ijs,qui ſentiunt quotieſcunque nulla eſſet terra,ſed totus hic globus eſſet aqua,toties non futuram eclipſim lunarem,ratione diaphaneitatis aquæ. Quod falſiſſimũ eſt, quia omne corpus ſphęricum quantumuis diaphanum ſit,dummodo ſit denſius aere, luminoſos radios refrangit,& eos ad inuicem interſecare facit,qui deinde vltra interſectionis punctũ diſgregantur, ita vt amittant illuminationis actum. Adde ꝙ etſi huiuſmodi corpus aqueum,ſphęricum non eſſet,ſed cubicum,illud ſuper aliquã ex eius ſuperficiebus ad angulos rectos radius ſolaris percuteret,non cum tamen penetraret, quia dictus radius perpetuò debilitatur,& eò magis,quo maiorem profunditatem in diaphano

eius

eius corporis,quod sit densius aere acquirit, nec totus radius vnquam dictum corpus ingreditur,cum ab eius superficie magna pars reflectatur. Resistit ergo huiusmodi corpus lumini,& quantò magis spissum aut profundum existit, tantò validius resistit. Habemus huius rei testes, piscatores vnionum, in ijs mundi partibus,quæ paucis ab hinc annis Hispanorum opera nobis innotuerunt, qui affirmant ad maris vsq; fundum lumen Solis non peruenire.

Immediata ratio, cur nebulę in ijs locis in quibus cōspiciuntur permaneant,& nūc altiores,nunc verò depręssiores cernantur,non ea est,quam Aristoteles cap.3.lib.1. metheororum proponit, sed inde oritur, quòd sint eædem densiores ea parte aeris, quæ ipsis supereminet,& rariores e a,quæ ijsdem subiacet.Quòd autem alicuius corporis densitas maior ea, quæ est medij, in quo reperitur,causa sit,vt ipsum corpus descendat,& maior raritas eiusdem corporis,ea,quæ est quoque medij,efficiat, vt dictum corpus ascendat,iam Archimedes in lib.de insidentibus aquæ docuit.

Rectissimè instituit natura, vt corpora densiora versus loca angustiora, & minora (intelligendo ea loca orbicularis figuræ)quæ ad centrum propius accedunt, & rariora ad ampliora loca,& maius spatium occupantia,sese reciperent. tum quia eadem quantitas materiæ condensatæ,eget minori loco quam rarefacta, cū etiam, quia cū corpus densum non ita ad velocitatem motus localis, vt rarum, idoneum sit, ad eas partes accedat,quæ motibus tardioribus magis sunt aptæ,corpora autē rara ad eas, quæ velocioribus motibus sunt aptiores sese transferant. præterquam ꝙ reuera appareat pro maiori parte, corpus magis densum,minus diaphanum; aut magis opacū futurum,quàm rarum, licet sæpissimè videamus contrarium, vt superius innuimus. est tamen naturale proportionatumq; magis opacum denso, & diaphanum raro, quàm è contra. Quamobrem summa ratione inducta natura voluit, vt corpora magis opaca,aut minus diaphana,magis vicina centro colligantur,vt spatium, quòd remanet,absque vllo impedimento à radijs solaribus penetrari possit. Tres autem eæ causæ,quas hoc loco posui,propriæ sunt,immediatæ,& per se,ex quibus fit,vt corpora densiora descendant,& rariora ascendant in mediis minus densis, aut minus raris dictorum corporum, quæ à nemine, ꝙ sciam,hucusque propositæ fuerunt.

Qui autem asserunt cucurbitæ, quam apponunt chirurgi,effectum ex eo nasci, ꝙ calidi sit attrahere, valdè aberrant à vero quia hoc,non nisi à raro,& à denso immediatè,à calido & frigido causatis efficitur, quia aer in cucurbita rarefactus à calore & per consequens dilatatus,statim vt à dicto calore deseritur,iterum condensatur & tantò citius, quantò aer ambiens frigidior existet, & quia eadem materia cum condensata fuerit minorem semper occupat locum, restringens igitur sese in cucurbita aer dum condensatur,necessariò fit,ne ulla, scilicet pars vacua remaneat, ꝙ cum alius aer ingredi cucurbitam nequeat aliud corpus ingrediatur. Idem cum amphora in qua nullum aliud, quam aèreum sit corpus experiri possumus, si eā ad ignem primò calefactam,deinde cū ore in amplo aliquo cyatho,aut alio vase. vino, aut aqua pleno vbi videbimus huiusmodi liquorem statim sursum ferri,quia dum calefit amphora, rarefit quoque aer qui in ea continetur, & quia rarescit dilatatur, & quia dilatatur,eget maiore loco; & ideo magna pars eius foras exit; Cum verò ea aeris portio, quæ intus remanserit,iterum condensatur ob defectum caloris, restringitur, minoriq; indiget loco;Quod cum ita se habeat,necessarium est,ne aliquis locus vacuus remaneat,vt aliud quoddam corpus ingrediatur, cum ad ingrediēdum aeri non patuerit aditus. quod si corpus admodum non erit fluxile,aut humidum, ita vt ingredi amphoram possit ita amphorę hærebit,vt non cito diuelli possit,& eo modo sępe cũ admiratione videm⁹ fragile vas vitreũ magnũ, & graue lapideũ corpus eleuare. Sed vt ad densum & ad rarum redeamus, mihi videtur frigidum esse consequentem qualitatem densi,& calidum rari, quia quæuis res dum calefit, rarefit, & quælibet materia dum refrigeratur, simul condensatur. Qua ratione fit, vt terra frigidior sit aqua, & ignis calidior sit aere.

Nec propriè locutus est Aristoteles.9.&.10.capite primi lib.& secundo secundi metheororum cum dixerit calorē Solis eum esse,qui sursum humores,vaporesq́; eue hat,quia Sol nil aliud facit,quam calefacere,cuius caloris ratione,ea materia rarefit, & ob rarefactionem leuior facta ascendit,non quia sursum à Sole feratur.

Quę subsequuntur,cum raro ac denso simbolum habere videntur.cum raro,scilicet calidum,humidum,leue,sublime,diaphanum,lumen,clarum,lux,albũ,dies, motus, velox, simplex, disgregatum, molle, lene, acutum,subtile,coctum,spaciosũ, dulce,voluptas,audacia,lætitia,liberalitas,veritas,industria,amor, misericordia,humanitas,sanitas,vita,& iis similia. Cum denso verò frigidum,siccum,graue, imum, opacum,vmbra,obscurum, tenebræ,nigrum,nox,quies, tardum,mixtum,congregatum,durum, asperum,ob tusum,crassum,crudum,angustum,amarum,dolor, timor, melãcholia,auaritia,mendacium,inertia,odium,crudelitas,feritas,infirmitas,mors, & ijs similia.

Verum est quod ea ratio,qua Aristoteles ait aerem humidum esse, parui est momenti,quia similiter de igne inferri posset,qui facilius à termino alieno, quã aer,aut aqua terminari potest.

## *Motum rectum curuo posse comparari etiam disentiente Aristotele.*

### CAP. XXXV.

SEd vt ad Aristotelē redeamus,rectè dicere non potest motum rectum ad curuũ comparabilem non esse.4.cap.lib.7.physicorum,vbi errat quoque dicens reperiri non posse lineam aliquam rectam alicuius circuli circunferentiæ æqualem. quia Archimedes iam probauit in lib.de quadratura circuli, triangulum illum orthogonium,cuius vnum ex lateribus circundantibus angulum rectum æquale esset semipiametro alicuius circuli,& aliud circunferentiæ,æqualem futurum dicto circulo. Illud igitur triangulum orthogonium,quod æquale erit alicui circulo, & habebit aliquod ex suis lateribus circundantibus angulum rectum æquale semidiametro dicti circuli, aliud quoque latus ipsum angulum rectum circundans,ex necessitate,circũferentiæ dicti circuli habebit æquale. Potest igitur dari vna quædam recta linea ęqualis circulari contra Aristotelis opinionem,qui non benè reuocauit in mentem, quod scripsit de relatiuis,cum dixit quadraturam circuli posse quidem dari,etsi tũc tpis de ea nõ haberet̃ scientia. Si igit̃ dicta quadratura dari põt,potest etiã dari vna recta linea ęqualis circunferentiæ eiusdē circuli,ob rationes iã dictas. Sed si Arist. dixisset,circularem corporum cęlestium motum, comparabilem non esse recto corporum elementarium, verum dixisset, non quia eorum alter circularis, alter verò sit rectus, sed quia cœlestis regularis sit, neque modò tardus, modò velox, sed vnam semper & eandem velocitatem retinens,mot⁹ aũt,qui est corporũ elementarium

tarium è contrà ſe habeat, præter id, ꝙ nunquam fuit neque ſit futurus aliquis horũ rectorum, qui naturales dicuntur, qui tam velociter moueatur, ut motus cœli, quia ſi voluerimus conſiderare motum diurnum. 24. horarum, ſecundum opinionem communem, reperiemus calculando, Lunam in quadraturis cum Sole, dum inuenitur in æquatore, ſingulis horarum minutis moueri per. 500. milliaria Italica vel circa, & in coniunctionibus, & oppoſitionibus ipſius Solis. 1000. vel circa, & Solem tempore ęquinoctiorũ. 18000. & Saturnũ circa æquatoris ſitũ. 260000. & ampliꝰ de ſtellis aũt fixis circa æquatorem poſitis quiuis cogitet; quod reuera difficillimum quibuſdam videbitur, quod quidem non occurrit ſecũdum pulcherrimam Ariſtarchi ſamij opinionem, diuinitus à Nicolao Copernico expreſſam, contra quam nil planè valent rationes ab Ariſtotele; neque etiam à Ptolomeo propoſitę. Motu verò proprio, quo libet horę minuto, Sol mouetͬ per milliaria circa. 48. Luna quãdo cõiuncta eſt, aut oppoſita reperitur Soli. 36. milliaria, & in quadraturis. 18. Saturnus. 24. Iupiter. 40: Mars. 100: Venus. 26: Mercur. 5. Sed Saturnus motu rapido, vno horæ minuto mouetͬ circa. 260000. milliaria, vt diximus Iupiter circa. 170000. Mars. 75000. Venus. 10000. Mercurius. 2000. corpus autẽ elementare, & ſi moueretͬ motu recto hoc mõ, & velocius etiam corpore cęleſti, non obſeruans tamẽ uniformitatem, ut dictum cœleſte facit, cum eodem nullo modo comparari poſſet, quia rectus dictus naturalis, ſuam ſemper velocitatem adauget, ob continuam impreſſionem, quam recipit à cauſa perpetuò coniuncta cum ipſo corpore, quę eſt propenſio illa naturalis eundi breuiori quadam via ad locum ſuum, ita vt etiam ſi dictum corpus elementare à motu tardiore ad velociorem, ſuperare poſſet motũ alicuius corporis cęleſtis, ij duo motus interſecarent ſeſe in vno ſolo pũcto, quod diuidi diſtribuiq; in partes nequiret, ideſt non niſi in vno ſolo temporis inſtanti redderentur æquales, vt ita dicam. Neq; ſolũ loquor de circulari cœleſti cum recto elementari, ſed de qualibet alia motuum ſpecie, ſiue ſint ambo recti, ſiue ambo curui, quando aliquis eorum irregularis erit.

## *Minus ſufficienter exploſam fuiſſe ab Ariſtotele opinionem credentium plures mundos exiſtere.*

### CAP. XXXVI.

MAior ratio, qua Ariſtoteles eorum opinionem, qui plures eſſe mundos dixerunt, refutare nititur, in eo conſiſtit, quod is credat partes terræ, quæ alijs mundis aſſignarentur, ad huius mundi centrum inclinationem habere, & ſic ignem illorum, propenſionem habiturum ad circunferentiam huius.

Quæ certè ratio tam debilis eſt, vt per ſe cadat, non conſiderans ipſe, quòd ſi eſſent dicti mundi, eorum quilibet ſuum proprium centrum, ſuamq; propriam circunferentiam haberet, terrasq; & ignes haberent inclinationem ad centra circunferentiasq; ſuorum mundorum, abſque eo, ꝙ vna terra, alterius centrum appeteret; vt exempli gratia, ſi doctiſſimi Ariſtarchi opinio eſt vera, rationi quoq; conſentaneum erit maximè, vt quod Lunæ contingit, cuilibet etiã ex aliis quinque planetis eueniat, ideſt, vt quemadmodum Luna ſuorum epicyclorum ope circũ terram voluitur, quaſi per circunferentiam alterius cuiuſdam epicycli, in quo terra ſit inſtar centri naturalis (ideſt ſit in medio) delati ab orbe annuo circa Solem; Sic etiam Saturnus, Iupiter, Mars, Venus, atque Mercurius, circum aliquod corpus in medio ſui epicicli maioris,ſitum habens, voluantur; quod quidem corpus, & aliquem quoque habeat motum circa ſuum axem, ſit opacum, ijs conditionibus, quæ terræ ſunt ſimiles,præditum exiſtat,& in dicto epyciclo ſint res ſimiles iſtis lunaribus.

## *An rectè loquutus ſit Phyloſophus de extenſione luminis per uacuum.*

### CAP. XXXVII.

ARriſtoteles ſecundo lib.de anima ſentit ꝙ per vacuum non extenderetur lumē, quod procederet à corpore lucido.Quod veriſimile nō eſt; ꝗa quēadmodum quantò rarius eſt aliquod corpus, tanto aptius eſt vt diaphanum exiſtat;& quātò rarius eſt dictum corpus,tantò minorem quantitatem materiæ contineat; ſic quātò magis diaphanum eſt, cum ex perexigua materia conſtet,tantò magis liber tranſitus luminis patet; Vnde quantò minor quantitas materiæ erit in dicto ſpatio,tantò nitidius pertranſibit lumen. Sequitur ergo, quòd vbi nulla eſſet materia, totum lumen libere tranſiret. Color cęruleus quem videmus in profunditate aquæ,& aeris,color eſt aquæ & aeris,qui denotat reſiſtentiam factam ab aere & ab aqua ipſi lumini; Quod quidem lumen ubi corpus aliquod non eſſet, minime reflecteretur,ſed abſque vllo impedimento rectà tranſiret.

## *An rectè phyloſophiæ penus Ariſtoteles ſenſerit de loco impellendo à pyramide.*

### CAP. XXXVIII.

ARiſtoteles.8.cap.lib.3. de cœlo,diſputans contra antiquos de elementorum figuris, ait pyramidem implere poſſe locum corporeum. quod verum non eſt. Cubus quidem id facit ab. 8. enim cubis perfectè impletur locus, ſed non item. 12.pyramides, ut Ariſtoteles ſenſit ( ideſt ſex ſuper aliquam exagonam figuram ſuperficialem & ſex ſub eadem) id præſtant,cum potius maius vacuum remaneat ad quamlibet partium ſupra, & infra, quam plenum. Rectius Ariſtoteles egiſſet, ſi probaſſet ratione immobilitatis conuenire pyramidem terræ, quam cubum. quamuis,de horum corporum altero, ſit ſtultum hoc credere. decepti tamen fuerunt antiqui, credentes cubum ad motum minus idoneum eſſe, quam reliqua quatuor corpora regularia (loquor autem habita volubilitatis ratione) quia pyramidale eſt illud, quod ita ſe habet, vt multis rationibus probari poteſt, quarum vna hæc nobis ſufficiet. Scimus iam ex communi conceptu corpus ſphęricum eſſe magis volubile, inſtabileq́;,quàm alia ſint. Illud ergo corpus,cuius figura ad ſphæricam magis accedet,ad uoluendum,& ad mouendum facilius erit quouis alio, quod æqualis ſit quantitatis,& ſibi omogeneum materia,vt exempli gratia corpus. 20.baſium ad voluendum, & ad mouendum promptius erit eo, quod ex.12.conſtat,& id, quod eſt. 12. eo, quod eſt.8.& id, quod eſt. 8.eo, quod eſt.6.& id, quod eſt. 6. vt cubus eſt, eo, quod eſt.4. cuiuſmodi eſt pyramidale. Huc accedit, quòd pyramidale corpus aliam conditionem habet, quàm cubicum, cum in quauis facie inalte-

ra-

rabile sit, cubicum autem econtrà sit alterabile vndequaque, suaq; quadrata in rhũbos mutare possit, iisdem existentibus lateribus.

## *Examinatur quam ualida sit ratio Aristotelis de inalterabilitate Cœli.*

## CAP. XXXIX.

ARistoteles textu. 22. primi lib. de Cœlo ita inquit.

Accidit autem, & hoc per sensum sufficienter, quo ad humanam dixisse fidem, & omni pręterito tempore secundum traditam inuicem memoriam, nihil videtur transmutatum neque secundum totum vltimum cęlum, neque secundum partem ipsius propriam vllam.

Hoc autem in loco Aristo. non considerauit, ꝙ similiter de terra dici posset, quando ipsa ita eminus prospiceretur, imo absque dubio putandum est, ꝙ si terra luce Solis prædita esset, & aliquis ipsam ab octauo orbe vellet videre, nullo pacto cerneret, cum sidera illa quæ primæ magnitudinis vocantur, & quæ plusquam centies maiora ipsa terra putantur non nisi vt puncta videantur.

IN

# IN QVINTVM EVCLIDIS LIBRVM.

*Quamuis omnia libri quinti Euclid. uerißima ſint. Animaduertimus tamen permultos ſumma cũ difficultate eorũ demonſtrationes percipere. Præcipuè ubi quinta, aut ſeptima deffinitiones eiuſdem libri neceſſariæ ſunt. Illæ enim adeo obſcuræ uidentur, ut longè facilius admißuri ſint hæc noſtra poſtulata tanquam clariora. Atque etiam tanquam intellectui commodiora, quam ſit illud quintum idemq; ultimum poſtulatum eiuſdem in primo libro poſitum, de linea duas alias ſecante. Quandoquidem ijs noſtris poſtulatis admißis, ſequentia Theoremata per facillima reddentur.*

## *Horum autem primum eſt.*

QVOD tota compoſita ex æquali numero partium æqualium, ſunt inuicem æqualia.

Vt ſi quis diceret omnes proportiones quæ cõpoſitæ ſunt ex æquali numero aliarum proportionum inuicem æqualium, ſunt etiam inuicem æquales, quod Euclides conatur demonſtrare in. 22. et. 23. quinti libri.

## SECVNDVM.

QVOD ſi à totis æqualibus detractæ fuerint æquales partes, quæ remanent erunt partes inuicem æquales.

Et è conuerſo ſi æqualibus æqualia addas compoſita erunt inuicem æqualia.

Quod in ipſis proportionibus hoc loco ſemper intelligendum eſt.

## TERTIVM.

### *Quę eſt Euclidis ſeptima propoſitio.*

QVOD ſi fuerint plures termini æquales inuicem, ratio ſeu proportio vnius ipſorum ad alium tertium terminum maiorem, minoremúe, ſed eiuſdem generis, erit eadem quæ cuiuſuis alterius termini ad eundem tertium. Et è conuerſo, quæ fuerit proportio tertij termini ad vnum prædictorum æqualium, eadem erit, ſpecie, cum alio eorundem terminorum.

QVAR-

## QVARTVM.

### *Euclidis uerò nona propositio.*

QVOTIESCVNQVE proportio vnius plurium terminorum collatorum cum aliquo tertio eiusdem generis, eadem fuerit cum ea quæ est cuiusuis alterius dictorum terminorum cum eodem tertio, aut proportio dicti tertij, cum aliquo dictorum, eadem fuerit cum ea quæ ipsius est ad aliquem alium eorundem terminorũ, tunc eiusmodi termini, æquales erunt inter se.

## QVINTVM.

### *Euclidis uerò octaua propositio.*

QVOTIES plures erunt termini, quorum vnus fuerit maior altero, si comparentur alicui tertio eiusdem generis, proportio maioris ad tertium illum, maior erit ea, quæ est minoris ad prædictum tertium, & proportio illius tertij ad maiorem, minor erit ea quæ eiusdem tertij ad minorem terminum comparati.

## SEXTVM.

### *Euclidis uerò decima propositio.*

QVOTIES proportio vnius, ex pluribus terminis comparatis ad aliquem tertium, maior fuerit proportione alicuius alterius dictorum cum eodem tertio, primus ille terminus, altero maior erit. Et quoties proportio tertij termini ad vnum quã ad alterum terminum maior fuerit, eiusmodi terminus altero minor erit.

## SEPTIMVM.

### *Euclidis uerò undecima propositio.*

PROPORTIONES, quarum vnaquęque cum aliqua tertia æqualis est, ipsæ quoque inter se sunt æquales. Vt illud, Quæ vni & eidem sunt æqualia, sibi inuicem sunt æqualia.

## OCTAVVM.

### *Euclidis uerò duodecima propositio.*

QVOTIESCVNQVE proportio vnius ex pluribus antecedentibus cum suo ex pluribus consequentibus, æqualis fuerit ei cuiusuis alterius dictorum antecedentiũ, cum suo plurium cõsequentium, proportio totius aggregati antecedentium cum toto aggregato consequentium, dictæ primę proportioni ęqualis erit, nempe illius antecedentis ad suum consequens.

NONVM

## NONVM.

*Euclidis uero tertiadecima propositio.*

Qvotiescvnqve aliqua proportio plurium proportionum inuicem æqualium, tertia aliqua proportione, maior aut minor fuerit, quælibet prædictarum æqualium inter se, tertia illa proportione maior aut minor pariter erit.

## DECIMVM.

Qvotiescvnqve fuerint ex vna parte plures termini (siue coniuncti siue disiuncti sint) æquales singuli vni tertio termino; ex altera verò parte totidem fuerint alteri tertio termino æquales, proportio aggregati priorum terminorum ad suũ tertium, æqualis erit proportioni aggregati reliquorum terminorum ad suum tertium, & è conuerso, ita se habebit primus tertius terminus ad suos multos terminos, sicut se habet secundus tertius terminus ad suos simul sumptos.

## VNDECIMVM.

Aggregatum ex partibus proportionalitatis continuæ, quod inter maximum, & minimum terminum omnium terminorum proportionalium compræhenditur, semper multiplex est ad singulas partiales proportiones, ex quibus ipsum componitur.

## DVODECIMVM.

Quæuis proportio quocunque modo diuisa fuerit, ex iis partibus componitur, in quas diuiditur.

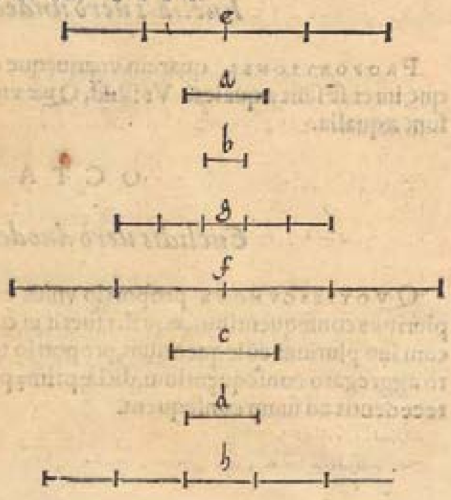
*Cum enim hæ præpositiones sint ita conspicuæ ipsi intellectui, ut absq; dubio inter obiecta ipsius intellectus connumerari possint, nullus sanæ mentis eas negabit.*

## THEOR. I. II. ET III.

PRimum, secundum, & tertium theorema quinti Euclidis ab ipso satis exactè demonstratur, studiosus itaque autorem consulat.

## THEOREM. IIII.

QVartum vero Theorema Euclidis ego sic demonstrarẽ. sit, verbi gratia, proportio . a. ad. b. quæ est. c. ad. d. sumptis multiplicibus. e. et. f. ad. a. et. c. æqualiter, item multiplicibus. g. et. h. ad. b. et. d. dico proportionem. e. ad. g. esse eandem quæ est. f. ad. h. Habemus enim ex. 10 postulato præmisso, eandem futuram proportionem. e. ad. a. quæ est. f. ad. c. & ita. b. ad. g. quæ est. d. ad. h. ex præsupposito verò cũ sic se habeat. a. ad b. sicut. c. ad. d. erit ex primo postulato eadẽ proportio. e. ad. g. quæ est. f. ad. h. Nam proportio. e. ad. g. componitur ex eis quæ sunt. e. ad. a: et. a. ad. b.



b.et.b.ad.g.& similiter proportio.f.ad.h.cōponitur ex eis quæ sunt.f.ad.c.et.c.ad.d. et.d.ad.h.

THEOR. V. ET VI.

Circa 5.et.6.theorema nihil notandum occurrit.

THEOR. VII. VIII. IX. X. XI. XII. XIII.

Theoremata à. 6.in. 13. cum sint de obiectis intelligibilibus, sine vllo medio, ab intellectu cognitis, inter axiomata à me relata fuerunt. 7. inquam quinti Euclid. fecimus tertium Postulatum,.8.quintum,.9.quartum,.10.sextum,.11.septimum,.12. octauum,.13.nonum.

THEOREM. XIIII.

Quartumdecimum Theorema ex Euclide demonstrabitur, mutatis tantum theorematibus ab interprete notatis, ita vt loco.7.8.noni, & decimi citetur tertium.5.4.et.6.postulatum à me propositum.

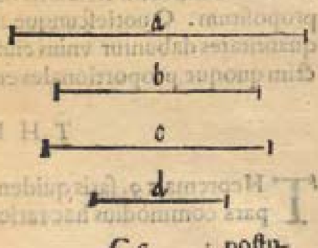
THEOR. XV.

Quintumdecimum Theorema sic demonstrabo; Sit, exempli gratia, a. terminus antecedens.et.b.consequens, quibus duo multiplices sumantur. c. et. d. Dico eandem proportionem habiturum. c. ad. d. quam.a.ad.b. habet. In primis enim manifestè patet quamlibet partem ipsius. c. habituram eandem proportionem cum qualibet parte.d.quam habet. a.ad. b.quare ex. 7.et. 8.postulato propositum elucescet.

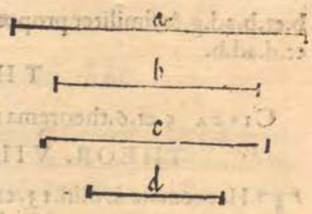

THEOREM. XVI.

Sextumdecimum theorema sic demonstrabitur. Sit, exempli causa, eadem proportio.a.ad.b.quæ est.c.ad.d. Dico q ita se habebit.a.ad.c.sicut.b.ad.d. Cogitemus itaque alterum istorum terminorum.c.aut.b.medium inter.a.et.d.quare primum intelligamus.b.inter.a.et.d proportio ipsius.a.ad.d.componetur ex ea quę est.a. ad.b.& ea quæ est.b.ad.d.ex.12. postulato. Et ex eodem, illa ipsa proportio. a.ad.d.pariter componetur ex ea quæ est.a.ad.c.& ea quæ est.c.ad.d.sumpto. c.pro medio termino. Ex quo sequitur, aggregatum duarum proportionum, videlicet.a. ad.b.et.b.ad.d.æquale esse aggregato.a.ad.c.et.c.ad.d.ex quibus aggregatis æqualibus si duas proportiones æquales subtraxerimus, eam videlicet quæ est.a.ad.b.& illam quæ est.c.ad.d.supererunt duæ proportiones inter se æquales. erit enim proportio.a.ad.c.æqualis proportioni.b.ad.d.ex prima parte secundi postulari diuisim.

Alia etiam ratione idipsum demōstrari potest, sumpto. b. pro medio termino inter.a.et. c: et. c. pro termino medio inter. b. et. d. quare proportio.a.ad.c.componetur ex.a.ad.b. et.b. ad. c. illa verò quæ est.b.ad.d.ex.b.ad.c.et.c, ad. d. ex. 12.

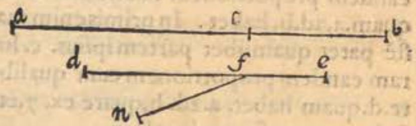


postulato. Sed cum proportio.a.ad.b.ęqualis sit proportioni.c.ad.d.communis autem.b.c:proportio. itaque.a.ad.c.æqualis erit.b.ad.d. ex secunda parte.2.postulati compositè,& sic habebimus propositum,ita quòd quotiescunque dabũtur.4. quãtitates ex unà parte proportionales, illæ ipsæ ex altera proportionales erunt.



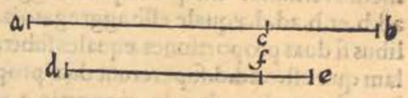

## THEOR. XVII.

DEcimiseptimi theorematis hæc est demonstratio. Ita se habeat.a.c.b.ad.c.b.sicut se habet.d.f.e.ad.f.e.Probo ita se habere.a.c.ad.c.b.sicut se habet.d.f.ad.f.e. Cogitemus itaque alterum terminum scilicet.n.f.qui sic se habeat.ad.f.e.sicut se habet.a.c.ad.c.b.Quare ex præcedenti theoremate ita se habebit.a.c.ad.n.f.sicut se habet.c.b.ad.f.e.& ex.8 postulato ita se habebit.a.c.b.ad.n.f.e.sicut se habet.c.b.ad.f.e. Sed cum ex præsupposito ita se habeat.a.c.b.ad.c. b. sicut se habet.d.f.e.ad.f.e.ideo ex præcedenti theoremate ita se habebit.a.c.b.ad.d.f.e.sicut se habet.c.b.ad.f.e.demonstratum autem est ita se habere.c.b.ad.f.e.sicut se habet.a.c.b.ad.n.f.e.Quare ex.7.postulato proportio.a.c.b.ad.d.f.e,æqualis erit proportioni.a.c.b.ad.n.f.e.& ex.4.postulato.d.f.e.æqualis erit.n.f.e. Itaque ex 3. postulato primi Euclidis.f.d.æqualis erit.n. f. Quamobrem proportio.a.c. ad. d. f. ęqualis erit proportioni.a.c.ad.n.f.ex secunda parte tertij axiomatis præmissi. Igitur ita se habebit.a.c.ad.d.f.sicut.c.b.ad.f.e.ex 7.postulato. & sic ex præcedenti theoremate ita se habebit.a.c.ad.c. b. sicut.d.f.ad.f.e. quod erat propositum: Quotiescunque igitur dabuntur.4.quantitates coniunctim proportionales, diuisim quoque proportionales erunt.




## THEOREM. XVIII.

THeorema.18.hac ratione demonstrari potest. Detur proportio.a.c.ad.c.b. similis ei quæ est.d.f.ad.f.e.probo ita se habere.a.c.b.ad.c.b.sicut se habet. d.f.e.ad.f.e. In primis notum est ex.16.theoremate ita se habiturum,a.c.ad.d.f.sicut.c,b.ad.f. e. Quare ex.8.postulato ita se habebit.a.c.b.ad.d.f.e.sicut.c.b.ad.f.e. Itaque ex.16.theoremate ita se habebit.a.c.b.ad.c.b.sicut.d.f.e.ad.f.e.Quod erat propositum. Quotiescunque igitur. 4. quantitates dabuntur vnius eiusdemq; generis disiunctim proportionales, coniunctim quoque proportionales erunt.




## THEOREM. XIX.

THeorema.19. satis quidem apud Euclidem demonstratur: eius tamen tertia pars commodius hac ratione demonstrari poterit (nempe) quod cum sit pro-

portio

portio.a.ad.b.quæ eſt.c.ad.d.probabo ita ſe habituram proportionem. b.ad.a.ſicut ſe habet.d.ad.c.hoc argumento: ſi.a.ad. b. ita ſe habet ſicut.c.ad.d.ex. 16. theoremate ita ſe habebit.a.ad.c,ſicut.b.ad.d. Quare ſic ſe habebit b.ad.d.ſicut.a.ad.c.Itaque ex eodem. 16. ita ſe ſe habebit.b.ad.a.ſicut.d.ad.c.


## THEOREM. XX.

QVamuis.20.theorema apud Eucli.perfectè demonſtratum fuerit, poteſt nihilominus & hac via demonſtrari. Sic ſe habeat proportio.a. ad. b. ſicut ſe habet.c.ad.d.& proportio.b.ad.e.ſicut. d. ad. f. Dico q ſi. a. maius fuerit. e. pariter.c.maius erit.f.& ſi.a.minus fuerit.e:c.quoq; minus erit f.ſin verò equale,ét æquale erit. Nam ex primo poſtulato certi ſumus ita ſe habere proportioné.a.ad.e. ſicut ſe habet proportio.c.ad p. Quare ex.12.theor ꝓpoſitū manifeſtū erit.


## THEOREM. XXI.

VIgeſimumprimum theorema,ſatis apud Eucli. probatum , nihilominus præſcripto nunc modo demonſtrari poterit.

## THEOREM. XXII. XXIII.

DVO hæc theoremata in primum poſtulatum collegimus.

Sequentia verò cum exactè apud Eucli.demonſtrentur non eſt cur nos in ijs immoremur.

# PHYSICA, ET MATHEMATICA RESPONSA.

*IO. BAPTISTAE BENEDICTI PATRITII Veneti, Philosophi Mathematici.*

## Ad Lectorem.

*VT Nil magis virtutis est proprium, quàm agitari, & incessabili motu prodesse. Ac veluti fulgidum sydus ante oculos spectantiũ comicare. Ita mihi mathematicis ijsq; maximè philosophicis speculationibus dedito, sapissimè, ut in principium summorum aulis, & amplissimis ciuitatibus degenti, ubi multa semper Nobilium mira curiositate, sciendi desiderio, & conferendi cupiditate referta, uersantur ingenia, contigit, modo ab his, modo ab illis, aut uerbis tentari, aut literis prouocari ad differendum, de his, in quorum studijs uersamur. Quarum concertationum & responsionum, quoniam non omnino indigna existimaui, quæ memoriæ comendarentur, partem aliquam apud me conseruaui. Vbi uerò per ocium licuit, collegi, relegi, ac tandem de manu mittere decreui. Tum ut scientia ipsa quo magis diffundetur, crescat; & quicquid ualeo, sine inuidia in communem utilitatem conferam Tum ut uirorum præstantissimorum, qui me suis interrogationibus excitauerunt, quantum in me erit, gratitudinis ergo, nomina reddam immortalia, & eorum exemplo alios, ocio sordidiore abiecto, quod solet aurialium præcipuè excelsa ingenia corrumpere, ad sciscitandum conferendum, & differendum, de rebus serijs, & quæ usui aliquando esse possint, & quandoq; euulgari mereantur, alliciam. Tu interim nostris laboribus fruere, & nostram diligentiam boni, & æqui consule, & Vale.*

DE

# DE TEMPORVM EMENDATIONE

## IO. BAPTISTAE BENEDICTI Patritij Veneti, Philosophi Mathematici.

*AD SERENISS. EMANVELEM PHILIB. Allobrogum & subalpinarum gentium Ducem Inuictißimum.*

## EPISTOLA.

MIRVM, Quàm lectione epistolæ seu (vt vocant) Breuis. S. D. N. Gregorij XIII. Pont. Max. quod ad me nuper tua Celsitudo misit ex Nicea, vt meam de ea re sententiam proferrem, delectatus sim; ex quo, non tantum recta illius mens ac verè sancta cogitatio, sed etiam aperta maximaq́; , si ad exitum perducatr, imo summè necessaria vniuerso orbi vtilitas percipi potest; qua de re memini cum Celsitudine tua aliquando sermonem habuisse. Vidi præterea cum ipso breui transmissum compendium Domini Aloisij Lilij: cuius mihi sententia perplacet, de correctione eius diei, qui 134. quoque anno præter, necessitatem, gignitur. qui sanè dies perpetuæ retrogradationis ingressus Solis in Zodiaci signa, causa fuit. quod ita perspicuè patebit. Cum Numa Pompilius anni cursum correxit emendauitq́; , ea sanè mente id videtur præstitisse, vt principium Ianuarij primi mensis anni, præcisè in ipso hyemalis soltitij puncto collocaretur. quod hac tempestate, dictam ob causam adeò retrocessit, vt circa vndecimam diem Decembris esse reperiatur. quod si centesimo trigesimo quarto quoque anno detractus dies vnus fuisset, nihil erroris prorsus accidisset. Atq; dies hic (vt alias Celsit. tuæ significaui) inde generatur, quod quarto quoque anno addentes nos ad quarti anni dies. 365. diem horarum. 24. ob errorem annuum horarum quinque minutorum. 49. secundorum ferè. 16. (anni æqualis siue medij) fallimur quarto quoque anno in minutis. 42. secũdis propè. 56. amplius quàm par sit minutis scilicet. 10. secundis ferè. 44. singulis annis; qui numerus. 134. multiplicatus, diem penè horarum. 24. constituit; penè inquam, quia minutum vnũ deesset tãmodo, & secunda. 44. si decẽ illa minuta, &. 44. secunda annua, exquisita essent atque perfecta; quæ tamen differentia nullius adeo esset momenti, aut certè perexigui, vt vix exactis. 11086. annis, diem vnum afferret. Itaq; planè necessaria eiusmodi esset emendatio, aptaq́; eius ratio à D. Lilio ostenditur, prout etiam Petrus Pitatus Veronensis tradidit, in eo, quem de vera anni quantitate tractatu conscripsit, nempe vt tribus primis centesimis annis, centesimus quisque annus communis sit, quartus subsequens centesimus intercalaris: quod sanè fieri necesse est. Nam

cùm

cùm tribus centesimis cōmunibus, tres quartas diei partes plus æquo detraxerimus, non enim centesimo quoque anno, sed centesimo trigesimo quarto, dictus dies detrahi debet, postquam tres integros dies, qui quadringentis detrahendi erant, trecentorum annorum spacio detraxerimus; sitque 134. penè tertia pars. 400. quarto annorum centenario, tres quartæ diei partes recuperabuntur; atque ita in fine quadringentorum annorum omnia exactè suo loco restituta erunt. Idcirco dictus iam quadringentesimus annus intercalaris & non communis constituendus erit, non alia de causa, quam vt bissexti ordinem sequamur.

Is verò modus, qui à D. Lilio traditus est, de ratione inueniendi singulis mensibus Nouilunij diem, interdum fallere nos posset vno die; prout Ianuario proximè lapso accidit; quo ex præscripto modo nouilunij, dies nonus illius mensis esse debuisset, qui fuit tamen dies septimus, sexta decima hora cum dimidia post meridiem. Neque etiam tutum est, via integrorum dierum, nulla habita horarum aut minutorum ratione, nec minus ea, quæ à Pitato tradita est, mediorum seu æqualium motuū progredi: At censerem potius veros motus sequendos esse ex calculis exactarum tabularum, quales Prutenicas esse iudico; Et cum solius Paschæ causa laboremus hac in re, pleniluniorum verorum, in multos annos tabulas formarem, quæ æquinoctia vernalia sequuntur, cum assignatione diei Paschatis præcisè, prout fecit Pitatus; non via tamen æqualium pleniluniorum sed verorum. Porrò quod ad Paschatis celebrationem attinet, rationi consentaneum est, concilij Niceni decretum ea de re seruari, prima scilicet dominica die post primum plenilunium, quod æquinoctium vernale sequitur; hoc tamen animaduerso, si dictum plenilunium primum post æquinoctium contingens, diē dominicum sortiretur; nulla ratione tali die Pascha celebrandum esse; verum subsequenti, ne cum Hębreis consentiat Ecclesia Christi: quæ fuit causa, vt in decreto concilij Niceni statutum sit, à quartadecima, in vigesimam primam celebrari debere: Quod mihi Petrus Pitatus non animaduertisse videtur, cum ex ei⁹ sentētia in suis tabulis die Paschate declarata, huiusce anni Pasca celebrandū fuerit. 23. Martij, ipsomet de plenilunij non tantum æqualis, sed veri.

Dies autem Paschatum elapsorum, quos hactenus examinaui, reperi omnes concordare cum ea regula, quam nonnulli de die carnis priuij tradiderunt. nempe primum diem martis post nouilunium Februarij, carnis priuij diem esse; non autem cū sāctione Patrum concilij Niceni, qua statuerunt à vigesima prima Martii dirigendum esse Paschatis diem, vt porè qui sibi persuaserunt, circa eum diem æquinoctiū perpetuò esse debere; prout tunc temporis erat. Non itaq; error accidit, quod Pascha ex huiusmodi suppositione concilij, post vigesimam primam lunę celebretur, cum seruata regula concilij non fuerit. Prout manifestū est de Paschate anni. 1566. celebrato. 14. Aprilis (quę fuit. 24. lunę) quod. 7. dicti mensis celebrādum erat. Tum anno. 1569. 10. Aprilis solenne fuit Pasca, quod tertia eiusdem esse debuerat. Anno deinde 1572. 6. Aprilis, dies fuit Paschatis, quæ. 30. Martij futura erat, anno verò 1575. in tertiam Aprilis Pascha incidit, casurum in. 27. Martii.

Cum igitur (vt ex diplomate ad Celsit. tuam misso patet) S. D. N. mens sit atq; voluntas, ut quisque liberè in medium proferat quid hac de re sentiat: quædam mihi non omnino præmittenda occurrunt, quæ tantis cœptis non nihil adiumenti fortasse adferre queant.

Atque illud in primis non tantum ut corrigatur Calendarium ob Pascha cæteraque festa mobilia ab illo manantia, vt decreto concilij Niceni sancitum est, scilicet vt ipsum Pascha celebretur prima dominica post primum plenilunium, quod æquinoctium

noctium vernale proximè ſequitur; verum etiam quò anni principium emendetur, ſcilicet vt ad ſuum verum principium reuocetur annus. nempè ad diem hyemalis ſolſtitij, quæ prima Ianuarij dies eſſe debet.

Deinde, tot dierum menſes conſtituantur, quot hac noſtra tempeſtate, ſol in ipſis Zodiaci ſignis verſatur. Poſtremò, quædam feſta immobilia in alios dies transferantur, celebrenturq; aptis temporibus: quod à S.D.N. mente diſſentire minimè videtur. cùm non magis de feſtis mobilibus quam immobilibus agat, imo etiam planè æquum ſit, vt habeatur vtrorunque ratio, quò ſtatutis temporibus celebrentur.

Vt autem ad primam Ianuarij diē verum principium anni reuocetur; cenſerem ex eo anno, quem corrigere voluerimus, non modò dies. 10. eſſe detrahendos, verū etiam vnum & uiginti, illo ipſo anno; idq; duplici via; aut partiendo menſes, atque ex illis demendo eos dies, qui minus ad rem hanc facere videbuntur, ac tum remaneat annus trecentorum quadraginta quatuor dierum ita vt decem menſes ſint dierum duorum ſpatio ſolito breuiores, alter menſis vno deficiat: aut conſtituto Decēbri dicti anni dierum decem, dies autem ille, qui decimum proximè ſequitur, ſit & primus Ianuarij, & dies ſolſtitij. ob quam cauſam exiſtimarem conſultiſſimum eiuſmodi annum eſſe mileſimum quingenteſimum ſeptuageſimum nonum. Quò quam primum. S.D.N. Pontifex max. ſuis temporibus huius correctionis manifeſtos effectus experiri & perpendere, atque diſpoſitionem anni non ſolum principio, ſed cęteris partibus ſuis in vniuerſum tam concinnè apteq; reſpondere, & aſtrorum motibus, & Eccleſiæ ſacroſanctæ ſanctionibus, ſe authore lætari poſſit.

Omnino itaq; iudico detrahendos eſſe vnum & viginti dies elapſi erroris: non decem tantum, quo hyemmalis conuerſio ad initium Ianuarij reuocetur; idque ne à communi opinione de ipſo anni principio veritas diſcrepet, quæ principium Ianuarij, anni principium arbitratur. etenim cum credant omnes annū à Ianuario inchoari, veritas autem ipſa ſic ſe habeat, vt nobis ſeptentrionalibus tunc inchoet annus, cum ad nos Sol accedere incipit, aut dies augetur; non conuenit principia eiuſmodi ſeparata & diſcrepantia eſſe. Et hanc fuiſſe Numæ Pompilio mentem credibile eſt, qui ad annum Romuli decem menſium, Ianuarium & Februarium addidit, vt principium Ianuarij principium eſſet anni: cuius rei argumentum eſſe poteſt, quod C. Iulij Cæſaris temporibus (qui multis annis poſt Numam fuit) atq; vti Pont. Max. corrigendorum feſtorum curam ſuſcepit hyemale ſolſtitium per aliquot dies retroceſſerat; nec mirum tamen eſſet, ſi Numæ temporibus, exactè prima Ianuarij die non fuiſſet hyemale ſolſtitium, adhuc pubeſcente in Italia Aſtronomia.

Huiuſmodi autem correctio dierum. 21. poſt. 2300. annos à Numa, quæ ſit perpetuo ſeruitura, media emendatione ea, quæ de tribus centeſimis annis communibus, & quarto intercalari, ſuperius propoſita fuit, non repudianda ei videatur, qui ſciet, qua ratione Numæ Pompilij annus corrigeretur, octauo quoque anno, intercalando annum vltimum medijs diebus. 90. quo prima dies Ianuarij ad verum principium anni, hoc eſt hyemale ſolſtitium, reduceretur.

Alio item argumento cuique patere poteſt, priſcos Romanos ſtatuiſſe annum ab hyemali ſolſtitio initium ſumere, vt inquit Ouidius primo Faſtorum.

Bruma noui prima eſt, veteriſq; nouiſſima Solis.
Principium capiunt Phębus, & annus idem.

eo quod diem naturalem à medio noctis inchoarent, ab eo puncto ſcilicet, quo Sol ad noſtrum hemiſpherium accedere incipit.

Tribuebant igitur veteres diei, atque anno principium ab eo puncto, quo Sol

ad nos accedit:cum punctum Zodiaci, quod tropicum hyemalem Capricorni nobis
producit,respondeat puncto meridiani sub terra, in quo Sol semel in die reperitur:
Quòd apertè norunt hi, qui sub polo boreali constituti sunt. Atq; facilè discerne
re possumus, diem scilicet & annum, quasi sibi ad inuicem medio suarum partium
respondere; solstitium inquam hyemale, mediæ nocti, æstiuum meridiei, æquino-
ctium vernale ortui Solis,autumnale occasui. Quam tamen similitudinem, multò
quam nos manifestius deprehendunt,hi qui(ut diximus)sub polo boreali versantur.

Quod si quis dubitet hac ratione correcto anno, quo nam pacto ad calculos cœ-
lestes motus medijs tabulis astronomicis hactenus in lucem æditis redigi possint, id
facilimum sanè erit,exempli gratia,aliquis planetę situm,aut alicuius stellę fixæ,quo
cunque die mensis anni correcti inuenire cupit,detrahat ex huiusmodi tẽpore dies.
21.ab Aera Christi, cum residuo supputet stellam, cuius situm scire desiderat;sum-
pta quacunque tabula, supputatio erit exacta: Cuius ratio cuilibet manifesta erit,
qui sciet annum vt potè. 1579.dierum.344.tantummodo constitutum fuisse. Nam
in ijsdem locis cœli prima die Ianuarij correcti,erunt stellæ quibus esse solebant.21
Decembris præcedentis anni ex supputatione tabularum: atque ita deinceps. Alia
præterea via idem perfici posset inuentione omnium motuum cęlestium ipso princi
pio anni.1580.correcti: hoc statuto,vt hi motus radices essent Aeræ S.D.N.Grego
rij XIII. quod si alio tẽpore quispiam motus cęlestes ad calculos redigere voluerit,
supputabit ab Aera huiusmodi, quæ anno.1580.principium habuerit: Quæ vt nobi
lius nomen sortiatur,idq; merito ex nomine Gregorij.XIII.Pont.Max.appelletur;
exemplo antiquarum, quæ ex Principum nominibus sunt appellatæ:vt tanto Pontifi
ci, cùm ex alijs multis,tum etiam ex hac non infima re, inter mortales immortale no
men comparetur. Ei verò summæ, quæ ex huiusmodi Aera Gregoriana ex tabu-
lis colligetur, ipsiusmet Aeræ radices addantur, vt exactus calculus habeatur. Et
hæc sit primæ sententiæ nostræ explicatio.

Altera erit numerum dierum mensium anni alia ratione quam nunc se habeat,or
dinandum esse:nempe vt Ianuarius, Nouember atq; December dies.29.singuli con
tineant, Februarius,Martius,& October.30.Aprilis,Maius,Augustus,& September
dies.31.Iunius,ac Iulius.32.atque id hac potissimum de causa, vt Sol unum quodq;
signum calendis mensium ingredi possit. Nam detractis (ut dictum est) diebus.21.
& reuocato ingressu Solis in principium Capricorni ad principium Ianuarij, in quo
signo hac nostra tempestate,Sol, dies propè.29.& quartam vnam versatur: si Ianua
rius.29.dies continebit, exactis hisce diebus, ingredietur Aquarium circa princi-
pium Februarii; hæret autem hoc nostro sæculo in Aquario Sol dies propè.29.cum
dimidio;quare si Februarius erit.30.dierum,elapsis ipsis diebus, Sol ingredietur pi-
sces circa principium Martii:& sic de cæteris.

Quamobrem si generali correctione annus emendandus erit, pulcherrimè acci-
det,si menses anni cum duodecim partibus cœlestibus, itineris annui Solis,concor-
dauerint;eisq; aptè responderint.Qua ex re,varię vtilitates promanabunt,pręsertim
Nautis, Agricolis, Medicis, & alijs qui vera principia, & interualla temporum per
specta habebunt:terminos item & interualla incrementi & diminutionis dierum &
noctium, & eorundem æqualitatis. Exempli causa, scient omnes principium Ianua-
rij,esse non modo anni principium,verum etiam hyemis,esse minimam anni diem,
& eius noctem maximam; principium incrementi diei,& diminutionis noctis;atque
etiam omnia illa,quæ ex huiusmodi conuersione Solis ad nos dependent. pariter
scient omnes primam diem Iulij, non tantum æqualiter annum diuidere, sed prin

Dd cipium

cipium quoque esse æstatis, maximam diem, noctem minimam totius anni; principium diminutionis diei & incrementi noctis, vnà etiã ea, quę Solis conuersionem ad australes sequuntur.

Necnon intelliget vnusquisque primam diem Aprilis, primamq; Octobr. æquinoctiorum dies esse; primam autem diem Aprilis, initium veris; Octobris Autumni; Item Aprilis diem esse eum, quo dies noctis prolixitatem vincere incipit: Octobris, quo nox diei longitudinem superat, & alia huiusmodi, quæ ab æquinoctijs depēdēt.

Si vero quispiam obijciat, modum hunc nostrum & ordinem perpetuum esse non posse, ob motum augis Solis; quod punctum cum fuerit in principio Capricorni, tūc Sol hærebit in signo Sagittarij. 32. diebus, totidem in Capricorno, in Geminis vero 29. totidem in Cancro; ex quo sequetur prioribus cōtrarius effectus; huic ego respondebo, tale quidpiã non euenturum, nisi exactis ab hoc anno annis. 24000. quod si mundus posthac totidem annis, quot fuit antehac, perdurauerit; punctus augis non amplius à situ præsenti, quàm. 45. gradibus distabit. Verum demus modũ nostrum & regulam in annos ter, aut quater mille subseruire posse, nec amplius, certè hoc toto tempore nullius momenti penè erit, quæ accidere poterit mutatio, tametsi elapsis quatuor millibus annorum Februarius esse debebit. 29. dierum. Aprilis & Nouember. 30. Iunius & October. 31. Augustus. 32. in aliis verò mensibus nihil mutandum erit. Ecce quam sit nullius momenti mutatio.

Quæ si Iulij Cæsaris temporibus fuissent animaduersa nunquam omissa fuissent, sed scientiæ Astronomicæ nondum (vt ita dicam) confirmata ætas, cum alibi, tũ maximè in Italia, quo minus hæc aut scirentur aut statuerentur impediebat.

Tertia ratio est, vt non solũ festa mobilia, verum ēt immobilia ad meliorem regulam (ut dictum est) reuocentur, si suis temporibus celebranda erunt. Quorum primũ est Natiuitas Domini, & quæ ab ea pendent; nempe Circuncisio, Epiphania, Purificatio, Annunciatio; & Natiuitas Io. Baptistæ. ita vt dies Natalis Domini celebretur prima die anni, cum Dei filius nasci voluerit circa verum principium anni, quod à solstitio hyemali initium ducit, & in ipso principio diei naturalis ex Romanorũ sententia, media scilicet nocte, tanquam qui summæ lætitiæ principium, post longos & graues filiorum Adæ mærores, esset allaturus. Nec forsan Ianuarij nomini, à veteribus Iano bifronti dicati hæc mutatio non conueniret, cum in ipso seruatore, duæ veluti frontes & formæ vnitæ sint, duæ scilicet naturæ diuina & humana. Hac ratio ne abusus tolletur, natus ex diuersis moribus Tabulariorum, quorum alij monumenta, seu quæ uocant Instrumenta, à die Natiuitatis Domini incōhant, alij à Circuncisione, alij à Calendis Martij, nonnulli à Paschate; quæ varietas innumerabiles lites affert & abusus propè infinitos, ob dubiam & ancipitem scripturam. Indictionum præterea ordini, hic noster modus nihil officiet; celebrato Natali celebrabitur Circuncisio octaua Ianuarij. Epiphania. 13. eiusdem. Purificatio. 11. Februarij quæ erit 40: dies à Natiuitate seruatoris. Prima Aprilis Annunciatio Virginis solennis erit, ipso nempè die æquinoctij, natiuitas Diui Io: Baptistę celebrabitur Prima Iulij die quæ erit solstitij æstiui, cum illa diminutionem capit. vt rectè Diuus Augustinus illa verba Io: Baptistę interpretatus fuerit. Illum oportet crescere, me autem minui: in quibus sic tantus Doctor philosophatur, vt tempus etiam natiuitatis serui & domini præclare notet dicens, natus est seruus cum decrescunt dies, natus est Dominus cùm crescere incipiunt.

Insignes etiam Theologi admonuerunt habendam rationem esse nonnullorum festorum, vt Diui Antonij, diuorum Fabiani & Sebastiani, & aliorum sanctorum,

Dd si forte

ſi forte in octauam Epiphaniæ inciderint: Verum hęc.S.D. N.curæ erunt, ut in aptiſſima tempora transferantur.

Admonuerunt præterea transferendos eſſe dies feſtos Beati Stephani, Ioannis, & Innocentium, vt quemadmodum factum eſt hactenus, diem natalis proximè ſequantur, ob multorum Doctorum, non recentium modo, ſed etiam antiquorum obſeruantiam; qui ſuis omelijs & concionibus multa piè, de myſteriis ſucceſſionis Feſtorum huiuſmodi tradiderunt.

Cuperent etiam præclari Theologi diem Aſſumptionis Beatæ virginis incidere in primam Septembris, Natiuitatem autem in.25.vt quemadmodum toto illo menſe in ſigno Virginis ſol verſabitur, ita Eccleſia Dei in cęlebrandi tantæ Virginis matris Dei laudibus occupetur.

Atque hęc ſunt Sereniſſime Princeps, quę longa & attenta cogitatione à me examinata, atque perpenſa fuerunt; quæ ſi tam diligenter & accuratè expendentur ab his, quorum intereſt, quam mihi apta & rationi conſentanea, ac vera penitus, imo (quod me magis afficit) etiam tibi viſa fuerunt; non dubito quin placitura ſint; & votis ſummi Pont. aliqua ex parte ſatisfactura. eò magis quòd te iubente, & cogitata à me, & ſcripta fuerint. Vale Princeps Sereniſſime, & qua ſoles hylaritate cętera noſtra, etiam has breues vigilias ſuſcipe & foue. Dat. Auguſtæ Taurinorum Kal. Aprilis. MDLXXVIII.

T. Celſitudinis.

Deditiſſimus Mathematicus.

Io. Bap. Benedictus.

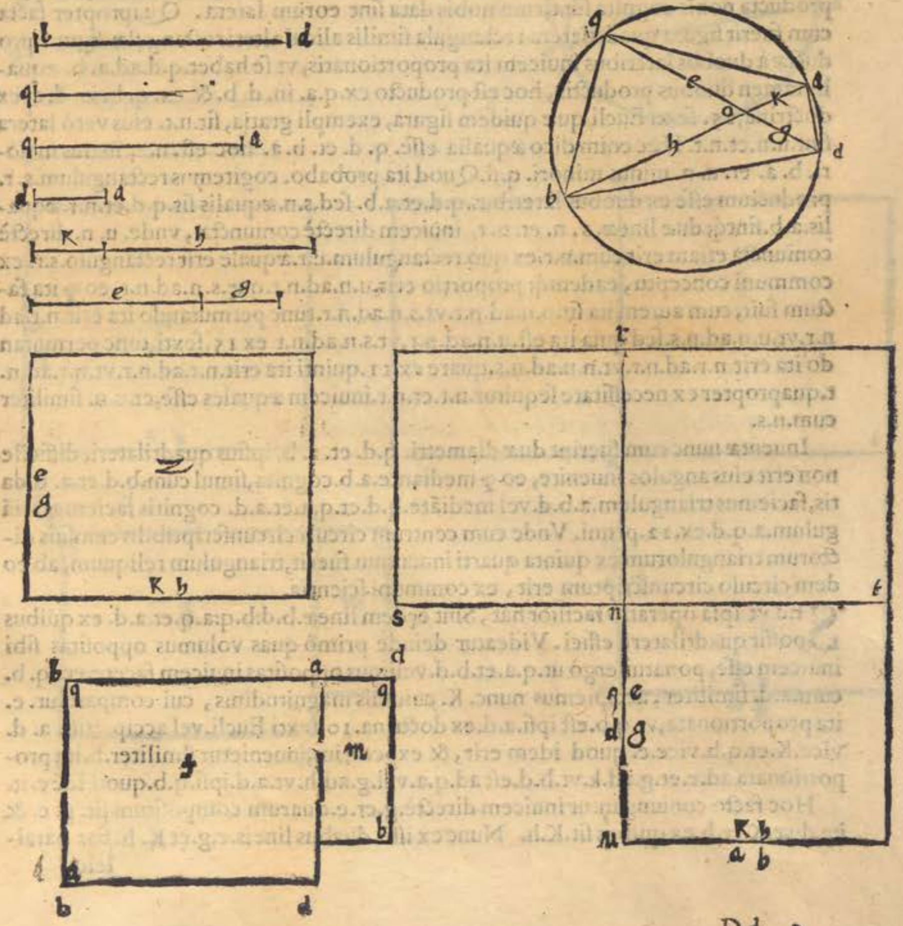
D E

# DE CIRCVLO AMBIENTE QVADRILATERVM.

## AD SERENISS. CAROLVM EMANVELEM Pedemontis Principem.

PRoblema quod à celſitudine tua nobis proponitur non ſolum poſſibile eſt,ſed facile etiam ad ſoluendum,hoc eſt quod circulus talis inueniatur,qui poſſit circunſcribere,ſeu capere quadrilaterum ex quatuor datis rectis lineis terminatum,vel ſic,datis quatuor rectis lineis ex quibus quadrilaterum poſſit effici, tale efficiatur vt circa ipſum, circulus poſſit circunſcribi.

Sint igitur.4.lineæ propoſitæ.b.d:q.b:a.q:et.a.d.ex quib⁹ poſſibile ſit quadrilaterum conſtitui,tale verò conſtituatur, vt aliquis circulus poſſit ipſum circunſcribere. imaginemur autem hoc factum eſſe,quod quidem quadrilaterum ſit.a.d.q. b. cuius



diametri ſint.q.d.et.a.b.quæ ſe inuicem interſecent in puncto. o. vnde cum anguli contra ſe poſiti circa.o.æquales inuicem ſint ex. 15.primi Eucli.& angulus.a.q.d.æqualis angulo.a.b.d.& angulus.q.b.a.æqualis angulo.q.d.a.et.b.q.d.angulo. b.a. d. ex.20.tertij tunc triangulus.a.o.q.ſimilis erit triangulo.d.o.b.et.q.o.b.ſimilis triangulo.a.o.d.ex definitione. Vnde eadem proportio erit ipſius.q.o.ad.b.o.quæ ipſius q.a.ad.b.d.& ipſius.b.o.ad.o.d.eadem quæ.q.b.ad.a.d.& ipſius.q.o.ad. o. a. eadem quæ.q.b.ad.a.d.proportio igitur.q.o.ad. o. d. cognita nobis erit, vt compoſita ex ea quæ eſt.q. o. ad. o. b. ex. o. b. ad. o. d. quæ nobis cognitę ſunt, mediante proportione ipſius. q. a. ad. b. d. & ipſius.q.b.ad.a.d.proportio ſimiliter ipſius.b.o. ad. o. a. nobis cognita erit, vt compoſita ex proportione ipſius. b. o. ad. o. q. & ipſius.o.q.ad.o.a.cognitis,mediante proportione ipſius.b.d.ad. q.a.& ipſius.q.b.ad a.d.cum autẽ proportio ipſius.q.o.ad.o.b.nobis cognita ſit, tunc nobis cognita erit proportio ipſius.q.d.ad.a.b.Nam ut.q.o.ad.o.b.eſt vt.a.o.ad.o. d. ex ſimilitudine, quare proportio compoſiti ex primo,& quarto terminorum ad compoſitum ex.2.& tertio,cognita erit.ſed quod fit ex.q.d.in.a.b.cognitum nobis eſt, vt æquale duobus productis,hoc eſt ex.q.a.in.d.b.& ex.q.b.in.d. a. ex ſecunda primi Almageſti.quæ producta nobis cognita ſunt,cum nobis data ſint eorum latera. Quapropter facta cum fuerit figura quadrilatera rectangula ſimilis alicui alteri rectangulæ figuræ producta à duobus lateribus inuicem ita proportionatis, vt ſe habet.q.d.ad.a. b. æqualis tamen duobus productis, hoc eſt producto ex.q.a. in. d. b. & ex. q. b. in. d. a. ex doctrina,25. ſexti Eucli.quæ quidem figura, exempli gratia, ſit.u.t. eius verò latera ſint.u.n.et.n.t. Hæc enim dico æqualia eſſe. q. d. et. b. a. hoc eſt. n. t. maius maiori. b. a. et. u. n. minus minori. q.d.Quod ita probabo. cogitemus rectangulum.s. r. productum eſſe ex duobus lateribus.q.d.et.a.b. ſed,s.n.æqualis ſit.q.d.et.n.r.æqualis.a.b.ſintq; duæ lineæ. s. n. et. n. r. inuicem directè coniunctæ, vnde. u. n. directè coniuncta etiam erit cum.n.r.ex quo rectangulum.u.r.æquale erit rectangulo.s.r. ex communi conceptu, eademq; proportio erit.u.n.ad.n.t.quę.s.n.ad.n.r. eo ꝙ ita factum fuit, cum autem ita ſit.u.n.ad.n.t.vt.s.n.ad.n.r.tunc permutando ita erit.n.t.ad n.r.vt.u.n.ad.n.s.ſed quia ita eſt.u.n.ad.n.r.vt.s.n.ad.n.t ex 15.ſexti,tunc permutando ita erit.n.r.ad.n.t.vt.n.u.ad.n.s.quare ex 11.quinti ita erit.n.t.ad.n.r.vt.n.r.ad.n. t.quapropter ex neceſſitate ſequitur.n.t.et.n.r.inuicem æquales eſſe,et.u.n. ſimiliter cum.n.s.

Inuentæ nunc cum fuerint duæ diametri. q.d. et. a. b. ipſius quadrilateri, difficile non erit eius angulos inuenire, eo ꝙ mediante.a.b.cognita,ſimul cum.b.d.et.a. d.datis,faciemus triangulum.a.b.d.vel mediãte.q.d.et.q.a.et.a.d. cognitis faciemus triãgulum.a.q.d.ex.22.primi. Vnde cum centrum circuli circunſcriptibilis cuiuſuis dictorum triangulorum ex quinta quarti inuentum fuerit,triangulum reliquum, ab eodem circulo circunſcriptum erit, ex communi ſcientia.

SEd vt ipſa operatio facilior fiat, Sint eędem lineæ.b.d:b.q:a.q.et.a.d. ex quibus poſſit quadrilaterũ effici. Videatur deinde primò quas volumus oppoſitas ſibi inuicem eſſe, ponatur ergò ut.q.a.et.b.d.velimus oppoſitas inuicem facere,et. q. b. cum.a.d. ſimiliter, accipiemus nunc. K. cuiuſuis magnitudinis, cui comparetur. e. ita proportionata,vt.q.b.eſt ipſi.a.d.ex doctrina.10.ſexti Eucli.vel accipiatur. a. d. vice.K.et.q.b.vice.e. quod idem erit, & expeditius,inuenietur ſimiliter.h.ita proportionata ad.e.et.g.ad.k.vt.b.d.eſt ad.q.a.vel.g.ad.h.vt.a.d.ipſi.q.b.quod idẽ erit.

Hoc facto coniungantur inuicem directè.g.et.e.quarum compoſitum ſit. g. e. & ita duæ.K.et.h.ex quibus ſit.K.h. Nunc ex iſtis duabus lineis.e.g.et K. h. fiat paral-

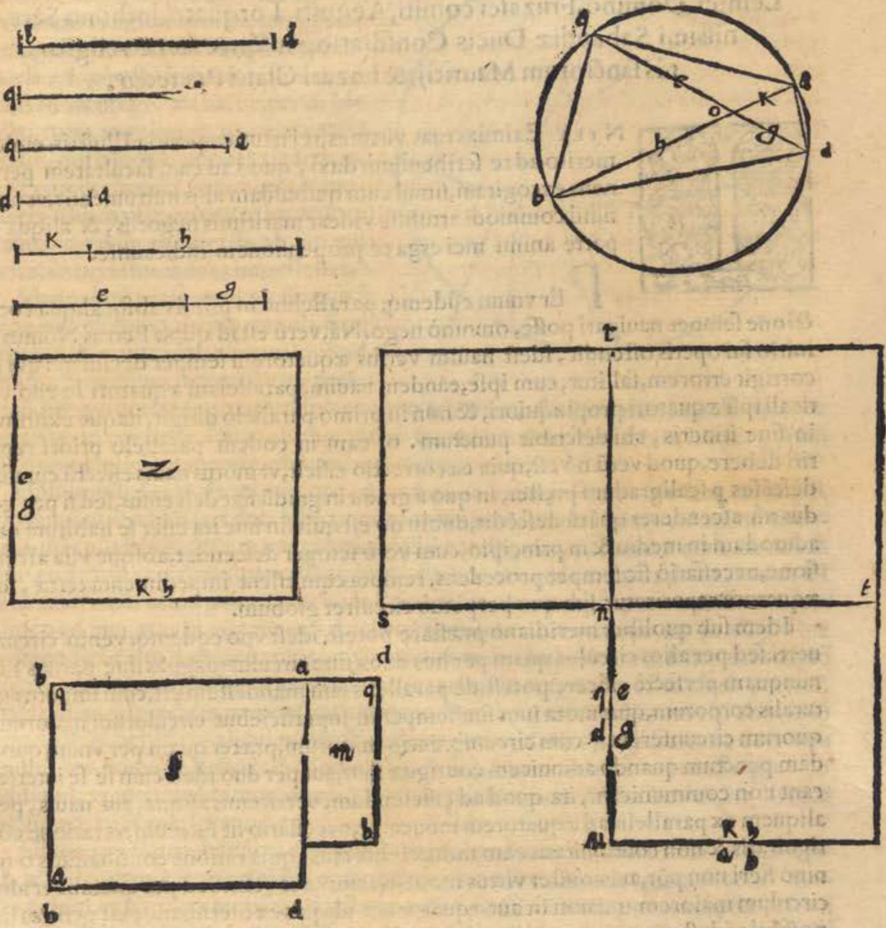
lelo-

lelogrammum.Z.deinde fiant alia duo parallelogramma rectangula quorum vnum ſit ex.q.a.in.b.d.reliquum verò ſit ex.q.b.in.a.d.quæ quidem ſint.f.m.

Quo facto deſignetur rectangulum.u.t.ex.25.ſexti, quod æquale ſit duobus rectangulis.f.et.m.ſimile tamen.Z.cuius rectanguli vnum latus correſpondet.e.g.reliquum verò.K.h.in proportione,ſed in æqualitate,vnum correſpondet.q.d. reliquũ vero.a.b.diametris ipſius quadrilateri.

Accipiatur nunc latus illud quod correſpondet.K.h.hoc eſt ipſi. a. b. maius ſcilicet,& ſimul cum.b.d.et.a.d.formetur triãgulũ. a.b.d.ex.22.primi Eucli. circa quod circunſcribatur circulus ex.5.quarti.& inuentum erit quod quęrebamus.

PER



# PER EVNDEM PARALLELVM abſque correctione ſemper nauigari non poſſe.

*Vbi notantur Petri Nonij lapſus in correctione erroris nauis. Et alij Petri Medinæ errores.*

ILLVSTRISSIMO ANDREAE PROVANAE Leinici Dominò, Fruzaſci comiti, Aequiti Torquato, inthimo Sereniſsimi Sabaudiæ Ducis Conſiliario, eiuſq́; , & ſacræ religionis ſanctorum Mauritij, & Lazari Claſsi Præfecto.

INTER Eximias tuas virtutes, rei nauticæ peritia Illuſtris emicat merito ad te ſcribendum duxi, quod ad eam facultatem pertinens excogitaui, ſimul cum quibuſdam alijs inſtrumentis, vt nonnihil commodi attuliſſe videar maritimis negotijs, & aliqua ex parte animi mei erga te propenſionem indicauiſſe.

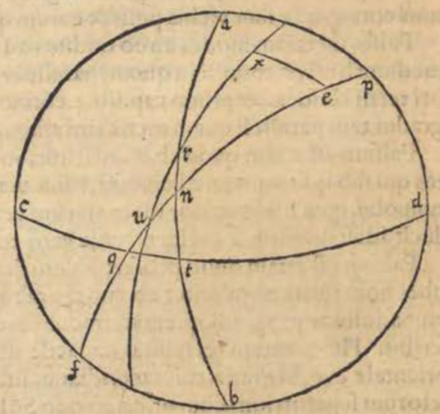
PEr vnum eūdemq́; parallelum in primis abſq; aliqua correctione ſemper nauigari poſſe, omninò nego. Nā, verū eſt id quod Petrus Nonius in initio ſui operis oſtendit, ideſt nauim verſus æquatorem ſemper declinare: qui cū corrigit errorem, fallitur, cum ipſe, eandem nauim, parallelam æquatori in vno verticali ipſi æquatori propinquiori, & non in primo parallelo dirigit, itaque exiſtimat in fine itineris, vbi deſcribit punctum. o. eam in eodem parallelo priori reperiri debere, quod verū nō eſt, quia ea correctio efficit, vt motus nauis effectū cuiuſdā deſcēſus p ſcaligradum pręſtet, in quo à gradu in gradū fiat deſcenſus, ſed ſi per gradus tm̄ aſcenderet quātū deſcēdit, dubiū nō eſt quin in fine ita eſſet ſe habitura quē admodum in medio & in principio, cum verò ſemper deſcendat, abſque vlla aſcenſione, neceſſariò ſic ſemper procedens, remota cum eſſent impedimenta terræ, ſub æquatore reperiretur, ſub quo perpetuò circuiret globum.

Idem ſub quolibet meridiano præſtare poteſt, ideſt vno eodemq́; vento circunuerti: ſed per alios circulos quam per hos duos (ſiue circulus magnus ſiue paruus) id nunquam perfectè efficere poteſt, de parallelis iam manifeſtum eſt, cum impetus naturalis corporum, quæ mota ſunt ſint ſemper in ſuperficiebus circulorum maiorum, quorum circunferentię cum circunferentijs minorum, præter quam per vnum quoddam punctum quando adinuicem contiguæ ſunt, aut per duo ideſt cum ſe ſe interſecant non communicant, ita quod ad efficiendum, vt triremis aliqua, aut nauis, per aliquem ex parallelis ad æquatorem moueatur, neceſſario ſit futurum, vt ratione cōtiguitatis & non continuitatis eam moueri curemus. quia ratione continuitatis omninò fieri non pōt, aut conſtet virtus mouens remis, aut velis. Sed per quemlibet aliū circulum maiorem, qui non ſit aut æquator aut aliquis ex meridianis, eſt penitus impoſſibile. ideſt vt vnius venti vi nauis impellatur. Quod vt clarè pateat, ſit orizon. a.c.b.d. & æquator. c.q.t.d. & vnus meridianorum ſit. a.r.n.t.b. in quo. n. ſit Zenit ſub quo primum nauis reperiatur et. r. ſit polus ſeptentrionalis. Ponamus etiam quod

azimut.

azimut.f.q.u.n.p.conſtituat angulum.a.n.p.ſeu.f.n.b.cum meridiano graduum.45. vnde tot graduum erunt arcus.a.p.et.b.f.orizontis, quapropter punctum.f.commune ipſius orizontis cum azimut,erit medio in loco inter. b.et.c.& ideo quarta.n.f.ipſius azimut ſecabit quartam.c.t.ipſius æquatoris in puncto.q.& habebimus triangulum.q.t.n.cuius angulus.t.rectus erit,& angulus.n.cognitus ſimul cum latere.n.t. latitudinis loci,quibus rebus mediantibus deueniemus in cognitionem lateris.q.n.lateris.q.t.& anguli.q.ex.4.primi Copernici ſi voluerimus.

Ponamus nunc nauem à puncto.n.diſcedere ſeu iter facere verſus.u.punctum, & in ipſo.u.reperiri, iam in hoc ſitu habebimus angulum huius ſecundi meridiani.r.u.p.qui quidem in hoc caſu minor eſſet angulo. r. n. p. extrinſeco trianguli.r. u. n. ex conuerſo ſecundæ partis.48.ꝓpoſitionis tertij lib.de triãgulis Monteregij,ſeu ex.13.primi Menelai,cuius anguli.u.arcus orizontalis ſit.x.e. qui quidem minor erit arcu. a. p.vt patet ratione anguli.r.u.e. minoris,ergo alius ventus nauem impellet à puncto.u.verſus.q.diuerſus ab illo qui prius ab.n.verſus.u.eam impellebat.




Vnde clarè patet verum eſſe quod dico,hoc eſt quod aliquo modo fieri non poteſt,vt nauis ab aliquo loco ad alium, breuiſſimo interuallo ire poſſit id eſt per gyrum circuli maioris ſphæræ vno tantummodo vẽto eam impellente,præter quam in ęquatore,ſeu in aliquo quouis meridianorum, nos autem ire per gyrum alicuius paralleli dementia eſſet,niſi neceſſitas cogeret.

Huiuſmodi demonſtrationis ope, quantum decipiatur Petrus Medina cap.6.lib.3.cognoſcitur,vbi ſic ſcribit; Vbicunque locorum reperiatur homo, aliquem circulum qui vniuerſum ambiat imaginatione ſibi confingens, per totum eum circulum vno eodemq́; vento nauigatio ſuſcipitur. Ex hac etiam demonſtratione, quàm falſa ſit charta maritima patet,cuius beneficio exiſtimant nautę ſe per breuiſſimum iter a loco ad locum vehi etiamſi dicti loci non ſint ambo in æquatore, aut in aliquo meridiano,ſed extra dictos circulos vnico tantum vento impellente & ſi in paruis æquoribus hic error parum depræhenditur,forte tamen in magno Oceano clarè pateret. In ſuperius igitur dicta demonſtratione iam oſtendi, quod ſi velimus vehi ab vno loco ad alium beneficio alicuius circuli maioris,præter duos iam dictos,hoc fieri non poteſt vno eodemq́; vẽto impellente. Vnde ſequitur,omnia ea interualla quæ vno eodemq́; vento tranſibimus futura longiora,præterquam in duobus dictis circulis æquinoctiali & meridiano.

Cum verò Petrus Medina cap.7.volens probare chartam maritimam bonam eſſe, planiſphęrium Ptolomei & Iordani citat, non animaduertit quam diuerſo modo à charta maritima huiuſmodi inſtrumentum ſit fabricatum,cum exceptis orizonte recto,& meridiano in dicto inſtrumento quilibet alius circulus ſit circulus, ſiue ſit almicantarat,ſiue azimut,ſiue æquator,ſiue tropicus,ſiue zodiacus, ſiue alius quiuis circulus,cum in charta maritima ne vna quidem ſit linea, quę non ſit recta, quolibet nomine vocetur. Superius

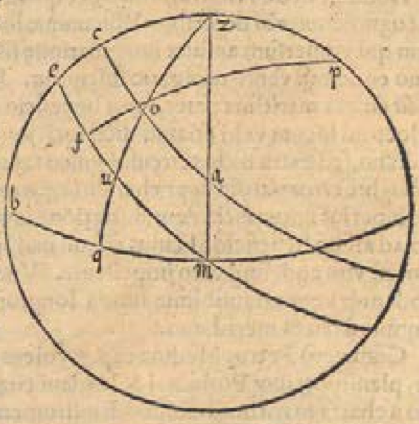
Superius poſitæ meæ demonſtrationis ope, deuenimus in cognitionem magnitudinis arcus.n.q.cognoſcimus etiam angulum.n.q.t.vnde nobis manifeſtũ eſſet quo vento oporteret iter facere.cum à puncto.q.nauis aliqua diſceſſura eſſet, in eodem azimut propoſito. Idem etiam dico de puncto.u.cum cogniti eſſent arcus.n.u.et.n.r.vt ſupponitur,ſimul cum angulo.r.n.u.vnde cognitus eſſet nobis angulus.n.u.r.ex 11.primi lib.Copernici, ex quo ventus nobis cognitus foret.

Modus autem quem idem Medina cap.9.lib.tertij ad cognoſcendam diſtantiam vnius meridiani ab alio præſcribit,in genere eſt falſus,etiam ſi is ab antiquis eum deſumat,qui,hic non viderunt quam magna inter meridianos differentia ſit interuallorum eorum quæ ſunt vicina polis & eorum quæ ſunt circa æquatorem.

Falſus eſt etiam modus ab eo traditus ad cognoſcendos gradus longitudinis per medium itineris cogniti;in quouis parallelo extra æquatorem facti,& hoc cap. 14.libri tertij eiuſdem,& primo cap.lib.4.cõtinetur,vbi.17.leucas cum dimidia cuilibet gradui tam paralleli quàm meridiani aſſignat.

Falſum eſt etiam quod ab eo aſſeritur,Solem,cum reperiretur in æquatore, circa eos qui ſub ipſo æquatore habitant,vnius diei noctiſq; ſpatio per omnes uentos circunuolui.quia illis æquator idem eſt cum verticali, qui duos tantum rhumbos producit,ideſt orientis,& occidentis:hic verò error,in ſecundo cap.lib.6.habetur.

Falſum eſt etiam quod profert Solem ijs qui habitant ſphæram obliquam, qualibet hora tertia,regulariter ab vno rhumbo ad alium ex præcipuis ideſt ab vno azimut ad alium progredi,quemadmodum eadem cap.2.lib.6.et.7.cap. ſeptimi libr. ſcribit. Huius autem rei falſitas ita facile depræhendetur, ponamus hemiſphęrium orientale, verbi gratia,cuius meridianus ſit.p.z.b.æquator aũt.e.m.vnus verò parallelorum ſeptentrionalium ſit.c.a.in quo Solem exiſtere ponamus,orizon autem ſit.b.m.zenit vero.z.polus arcticus.p.ſit poſtea azimut.z.q.à meridiano diſtans per gradus.45.qui quidem azimut in hoc hemiſphęrio erit rhumbus illius venti,quem uulgo Itali Sirocum dicunt,et.z.m.ſit azimut verticalis qui in hoc hemiſphærio erit rõbus venti orientalis, ita ꝙ ſecundum Medinam à rhumbo.z.m.ad.z.q. Sol abſoluet ſpatium tẽporis trium horarum, & aliud æquale temporis ſpatium abſoluet à rhũbo.z.q.ad.z.b.ex ipſo Medina, vnde arcus.a.o. paralleli eſſet graduum. 45. & item arcus.o.c.




Ponamus nũc Solem reperiri in ęquatore,vbi per ipſum Medinam arcus.u.m. ſimiliter eſſet graduum.45.& ſic.u.e:protracto ergo arcu. p. o. f. palam erit arcum.f.e.fore graduum.45. ſed cum arcus. e. u. ſit graduum. 45. ex ſuppoſito ipſius Medinæ, ſequeretur arcum. e. f. æqualem eſſe arcui.e.u.pars igitur æqualis erit ſuo toto.

Id etiam quod Petrus Nonius pagina 124.et.125.lib.de arte nauigandi contra nautas de diſtantijs Solis à meridiano ſcribit,hanc opinionem Petri à Medina & eorum qui idem ei perſuaſerunt falſam eſſe demonſtrat.

Fal-

Falſum eſt etiam id quod cap.3.lib.6. pronuntiat,ita dicens.

Quod cum verum eſſet à parte oriētali inſularum quæ azore dicuntur, pyxidem verſus eum ventum qui vulgò Græcus dicitur,& ab occidentali verſus eum qui Magiſter dicitur,vergere, huius rei nulla eſt ratio.

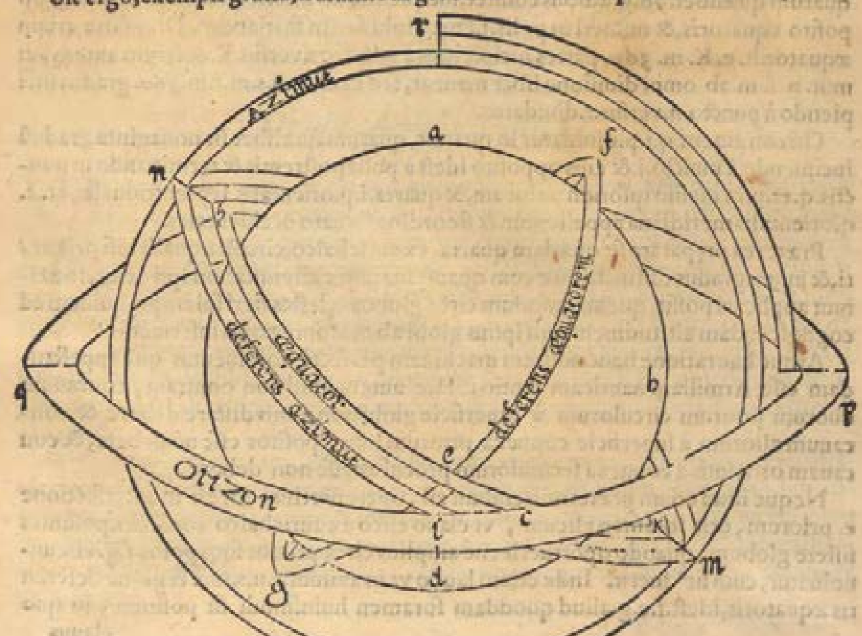
Ego enim huiuſmodi rationem reperiri poſſe contendo, quæ talis eſt, quia pars roſæ(ut vocant) à magnete tacta, ad aliquod punctum,aut ſitum globi terrę, in eodem meridiano inſularum,quæ Azore dicuntur,vltra ſitum poli arctici in terra dirigeretur,ita vt ſitus dicti poli in terra eſſet in dicto meridiano, inter locum qui ab indice roſæ aut pyxidis reſpiceretur, & dictas inſulas,id quod ſuperius ſcripto meridiano facile cognoſci poteſt,ſumendo pro inſulis ſitum.e.in meridiano et.z. pro polo, et.p.pro loco qui à pyxide ſit viſus, imaginādo deinde pyxidem in.f. magis orientali quam eſt.e.clarum eſt lineam quæ reſpicit(ponamus)f.p.verſus Græcum & ab alia parte verſus Magiſtrum declinare.

## *De Armilla Nautica.*

## AD EVNDEM.

CVm ſæpe viderim quam in magnis æquoribus nos fallant,atq; decipiant maritimæ,ſeu nauigatoriæ chartę,quemadmodum aliquoties inter nos ſermonem habuimus: in id totus incubui vt aliquam machinam excogitarem,quæ difficilis non eſſet, efficeretq; vt nauis ſuper aquę globum, beneficio circulorum maiorum,quam optimè poſſet, ideſt breuiſſimo itinere ab uno loco ad alium ferretur. Id ꝙ mihi ex animi voto ſucceſſurum putaui,beneficio quinq; circulorum circundantium aliquē globum terreſtrē & maritimum,quales ij ſunt qui in inferiori Germania à Gerardo Mercatore ſtruuntur,qui vno pede cum dimidio diametri conſtet,ideſt ſeſquipedē.

Sit ergo,exempli gratia,huiuſmodi globus.a.b.d.circa quem duo circuli, aut cir




Ec ulaculares lineæ ex aurichalco applicentur inuicem coniuncti per medium ad angulos rectos,quorum prior.f.e.g.in se globi polos mediantibus extremitatibus axis mundi contineat, qui quidem poli à punctis suarum intersectionum per quartã ex æquo in punctis.f.et.g.ita distent, vt globus circa eosdem, in situ longitudinis mundi volui possit. Huiusmodi autem circulus,æquatoris deferens appelletur.

Secundus autem circulus sit.h.e.K. cum primo ad angulos rectos in puncto. e. & in suo opposito connexus,& is appellabitur æquator,& poli.f.g.primi poli dicentur.

Circa huiusmodi duos circulos, alios etiã duos existere vellẽ simul cõiũctos medio ad angulos rectos. In quibus quidẽ intersectionis punctis sint duo poli,qui hos duos circulos cum secundo priorum idest cum æquatore in duobus punctis inuicem oppositis connectant; quæ æquatoris puncta à punctis intersectionis eiusdem cum suo deferente, ratione vna quarta distent, quorum duorum circulorum primus sit. n.i. m.quem deferentem azimut appellabimus; secundus. r. n.s. m. azimut locorum nominabimus.eorundem intersectionis rectæ,puncta sint.n.et.m. à quibus duo poli ex aurichalco confecti similes primis.n.h.et.m.K. vsque ad puncta. h. et. K. æquatoris perueniant, qui spissitudinem æquatoris distantem à puncto.e.vna quarta penetrẽt, ita vt æquator circum circa.n.h. et. m. K. in situ latitudinis mundi verti queat. Et hos, secundos polos nominabimus.

Alius deinde circulus.q.i.p. duos posteriores circulos ambiat, cum deferente tamen azimut mediantibus duobus polis in puncto.i.& in suo opposito ex ęquo distantibus à secundis polis vnius quartæ spatio iungatur. Ita vt dictum deferens azimut circa hos tertios polos volui possit, atque hunc circulum.q.i. p.orizontem vniuersalem vocabimus. Hic vero orizon super quatuor quartas circuli, aut super quatuor paruis columnis,ut fieri solet innixis suæ basi, ita ponatur, vt moueri non possit.

Primus autem circulus.f.e.g.deferens æquatoris in. 4. partes æquales diuidatur, quarum quælibet.90.gradibus constet,incipiendo ab intersectionibus. e. & eius opposito æquatoris,& numeri in polis.f.et.g.globi finem sortiantur. Diuidatur etiam æquator.h. e. K. in. 360. partes incipientes à puncto.e.versus.K.deferens autem azimut. n. i. m.ab omni diuisione liber maneat, sed azimut.n.s.m.r.in.360. gradus incipiendo à puncto.n.versus.r.diuidatur.

Orizon autem.q.i.p.diuidatur in quartas, quarum quælibet sit nonaginta graduũ incipiendo à puncto.i.& eius opposito idest à polis postremis & terminando in punctis.q.et.p.in medio ipsorum polorum,& quarta.i.p.orientalis septentrionalis, et. i. q.orientalis meridiana appellentur.& sic ordine seruato occidentales.

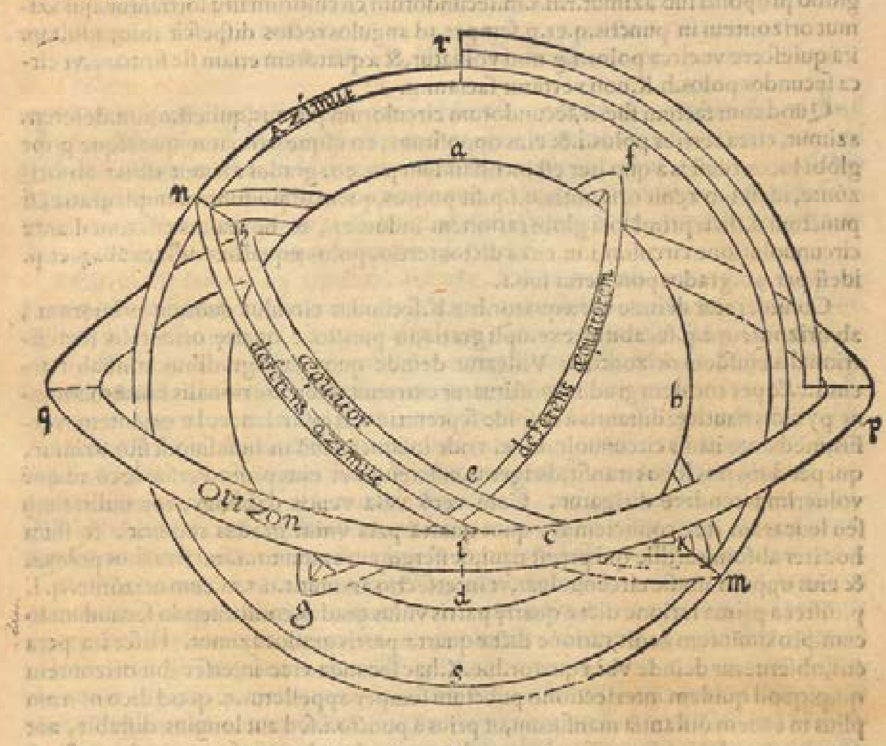
Præterea pręparata sit quædam quarta, ex aurichalco,circuli æqualis ipsi orizonti,& in.90.gradus distincta quæ cum quauis suarum extremitatum ipsi zenit, in azimut applicari possit, quemadmodum circa globos cęlestes fieri solet;quę quidem ad cognoscendam altitudinem poli ipsius globi ab orizonte nobis inseruiet.

Atque hac ratione hanc nostram machinam perfectè absoluemus quã appellandam esse Armillam nauticam sentio. Hic autem illud non omittam, concauum duorum priorum circulorum à superficie globi non nimis distare debere & concauum aliorum à superficie conuexa priorum longe positos esse nõ debere, & concauam orizontis à conuexa secundorum procul abesse non debere.

Neque illud etiam prætermittendum est, operepretium fore si in intersectione e. priorum, erit foramen elicum, vt clauo elico ex aurichalco confecto,possimus sistere globum, quando oportuerit, ne amplius circa primos suos polos.f.g. circunuoluatur, cum sit[v] fuerit. Inde etiam laudo vt in azimut.r.n.s.m.è regione deferentis æquatoris,idest.f.e.g.aliud quoddam foramen huiusmodi sit positum, in quo

clauus

clauus elicus vſque ad circulum.f.e.g.perueniens, æquatorem ſiſtere poſſit,ne circa ſecundos polos.h.et.K.amplius moueatur quum noluerimus eum mutare ſitum.




## *De uſu Armillæ nauticæ.*

VT autem noſtra Armilla nautica vti poſſimus pyxidem nos prius oportebit habere, diuerſam tamen ab ijs, quibus nautæ hactenus vſi fuere: nolo enim vt tã craſſa minerua beneficio vẽtorum communium circa hanc rem nos geramus, ſed ratione graduum orizontis in.360.partes diſtincti, atque ob hanc cauſam ſentio,vt ima pars pyxidis penitus detecta videatur,& in.360.partes diuidatur,nilq́; aliud quam quandam lanceolam ſupra eius acum eſſe volo, quæ dum mouebitur nauis, per gradus quamlibet orizontis partem oſtendet; hos autem.360.gradus, ita ſe habere volo,vt quęlibet quarta.90.contineat,ſupputatioq́; à linea meridiana incipiat,& in verticali deſinat, vt huiuſmodi diuiſio cum ea, quæ eſt orizontis Armillæ eadem ſit.

Pręſupponãtur nunc in globo duo loci extra æquatorem, & in diuerſis meridianis quomodolibet adinuicem diſtantes,à quorum vno ad alium ſit nauigandum itinere quo ad fieri poterit breuiori,ideſt per gyrum circuli maioris, dixi autem extra æquatorem,ideſt vt ambo,nec in æquatore,nec in uno eodemq́; meridiano exiſtãt, quia vt aliàs dixi in huiuſmodi locis, vnico tantum vento comite, iter conficere poſſumus.

Volo primum vt mediante circũuolutione globi circa primos polos.f.g. & æquatoris circa ſecundos.h.K.hoc eſt per longitudinem, & latitudinem, hi duo loci in globo propoſiti ſub azimut.r.n.s.m.ſecundorum circulorum ſitũ ſortiantur,qui azimut orizontem in punctis.q.et.p.ſemper ad angulos rectos diſpeſcit ibiq; globum ita quieſcere vt circa polos.f.g.non voluatur,& æquatorem etiam ſic firmare,vt circa ſecundos polos.h.K.non vertatur faciamus.

Quod cum factum fuerit,ſecundorum circulorum primus,qui eſt.n.i.m.deferens azimut, circa tertios polos.i.& eius oppoſitum, eo uſque voluatur quouſque prior globi locus,ideſt is à quo iter eſt incohandum per.90. gradus azimut diſtet ab orizonte, ideſt ſub zenit orizontis.q.i.p.ſit poſitus,quemadmodum,exempli gratia, ſi punctum.a.dicti primi loci globi rationem indueret, & borealius eſſet,mediante circunuolutione circuli.n.i.m.circa dictos tertios polos æqualiter diſtans ab.q. et.p. ideſt per.90.gradus poneretur ſub.r.

Conſideretur deinde vbi æquator.h.e.K.ſecundus circulus duorum primorum, ab orizonte.q.i.p.ſecabitur,exempli gratia,in puncto. c. quartę orientalis ſeptentrionalis eiuſdem orizontis. Videatur deinde quot nam gradibus conſtabit arcus.i.c.& per totidem gradus conſtituatur extremitas ſeptentrionalis lineæ meridianę pyxidis nauticę, diſtantis à cuſpide ſeptentrionali ipſius lanceolæ orientem verſus,mediante nauis circunuolutione. vnde ipſamet nauis in huiuſmodi ſitu azimut, qui per duos hos locos tranſit,dirigetur,efficiendo vt eius prora verſus locũ ad quẽ voluerimus tendere dirigatur. Cum verò vela ventis dabimus, tot milliarium ſeu leucarum iter conficiemus, quot quarta pars vnius gradus requirit. & dum hoc iter abſoluitur,ille qui pręeſt naui,defferentem azimut.n.i.m.circa ſuos polos.i. & eius oppoſitum,ſic circunuoluat,vt interſectio azimut.r.n.s.m.cum orizonte. q. i. p.diſtet à prima ratione dictæ quartę partis vnius gradus,conſtituendo ſecundum locum,proximiorem zenit,ratione dictæ quartæ partis gradus azimut. Hiſce ita peractis,obſeruetur deinde vbi æquator.h.c.K.hac ſecunda vice interſecabit orizontem q.i.p.quod quidem interſectionis punctum ſemper appelletur.c. quod dico non amplius in eadem diſtantia manſurum,ut prius à puncto.i.ſed aut longius diſtabit, aut propius accedet, vt in præſenti exemplo.quemadmodum ex ſe manifeſtum eſt, cũ poli globi,ideſt ęquatoris ſint extra azimut,vt pręſupponitur,quia loci ſunt in diuerſis meridianis.

Pro huiuſmodi autem diſtantiæ ratione denuo dirigatur nauis prout æquator.h.e.K.in orizonte.q.i.p.nobis oſtendet, atque hoc modo omnium iter quaſi breuiſſimum fiet. dico autem,quaſi,quia omnibus modis neceſſariò conficitur iter contortum & in formam ſerpigineæ lineæ. Applicantes deinde per vices extremitatem quartæ appoſitæ(de qua ſuperius mentionem fecimus)ipſi zenit.r.efficientes ut per ſitum poli globi pertranſeat, deueniemus in cognitionem altitudinis eiuſdem ab orizonte,& per conſequens quantum itineris per latitudinem eiuſdem globi peregerit. mediante deinde interſectione orizontis.q.i.p.cum æquatore, cognoſcemus quãtum itineris per longitudinem eiuſdem globi,in ipſo ęquatore fuerit peractum,

In-

## *Instrumentum ad ortum, & occasum Lunæ cognoscendum qualibet anni die.*

### AD EVNDEM.

ECce tibi vir Illustris. modũ conficiẽdi instrumenti nuper à me inuẽti, vt tibi significaui, quo, scire possis fermè in dies, qua hora (de astronomicis loquor) ad determinatum parallelum & absque multa supputatione, etiam absque Astrolabio Luna oriatur occidatq; . In quo instrumento poteris etiam videre quo in signo Sol, & sæpius itidem Luna permeat, & huiusce aspectus cum Sole, atque longitudinem diei noctisq; toto anni tempore exactè discernere.

Circularis lamina ex argento, aut ære, aliaúe materia paranda est, in cujus superficie ambarum facierum Zodiacus delineabitur, modo inferius depicto, deinde pro anno quinque circuli sibi inuicem cõcentrici, at respectu Zodiaci excentrici cęlabuntur in ea, adeo vt vtriusque centri distantia sit pro. 32. parte semidiametri concauitatis Zodiaci è regione locis augis, temporis qui nostra ætate circa finem secundi gradus cancri inuenitur, eandem viam, in hoc, sequuti, quam Stoflerus in dorso Astrolabij docet. At nomina mensium media ponantur inter duos maiores circulos, postea inter secundum, & tertium ab vna facierum laminæ, arcus semidiurni, ab altera vero arcus seminocturni, per quinos quosque dies collocentur, ita exactè, vt hic subtus videbis. adeo vt numeri dierum & ipsorum dierum signa sint in interuallis vicinioribus centro communi dictorum quinque circulorum.

Posteaquam ab vna & altera facierũ laminæ hæc insculpta fuerint, aliæ duæ circulares laminæ, magnitudinis semidiametri minimi quinque circulorum accipiantur: quarum vna pro ortu, & altera pro occasu Lunæ deseruiet. In qualibet ipsarum constituctur circuli quatuor, eo modo qui paulo inferius cernitur, quos omnes diuidemus in triginta spacia æqualia: & in interuallo q̃d inter duos primos circulos positum est, triginta dies annotabim⁹ qui ipsos Lunæ triginta dies pręscribẽt, vt in figura.

Postmodum in lamina quæ ortus Lunæ indicabit, ac duorum maiorum circulorũ interuallo è regione numeri. 1. videlicet primi diei, ponemus horas. 12. & minuta. 48. ex aduerso diei secundi ho. 13. et min. 36. ex opposito tertij ho. 14. min. 24. & sic successiuè augendo per min. 48. & indicem è diuerso diei. 30. statuẽdo, qui coitus Lunæ cum Sole significabit: atque lineas aspectuum, vt inferius videre est facilè inueniemus.

Altera in lamina quæ occasum Lunæ indicabit, postquam distincta fuerit, vt altera. 30. dies ac cęteræ lineæ, eo modo quo in superiori collocabuntur, at numeri interualli maioris, aliter disponentur, vt potè ex aduerso diei primi solum. 48. minuta describi debent, è directo secundi diei ponenda erit hora vna cum minutis. 36. & è regione tertij inscribentur. 2. horæ, & min. 24. & sic ex ordine per. 48. minuta augendo.

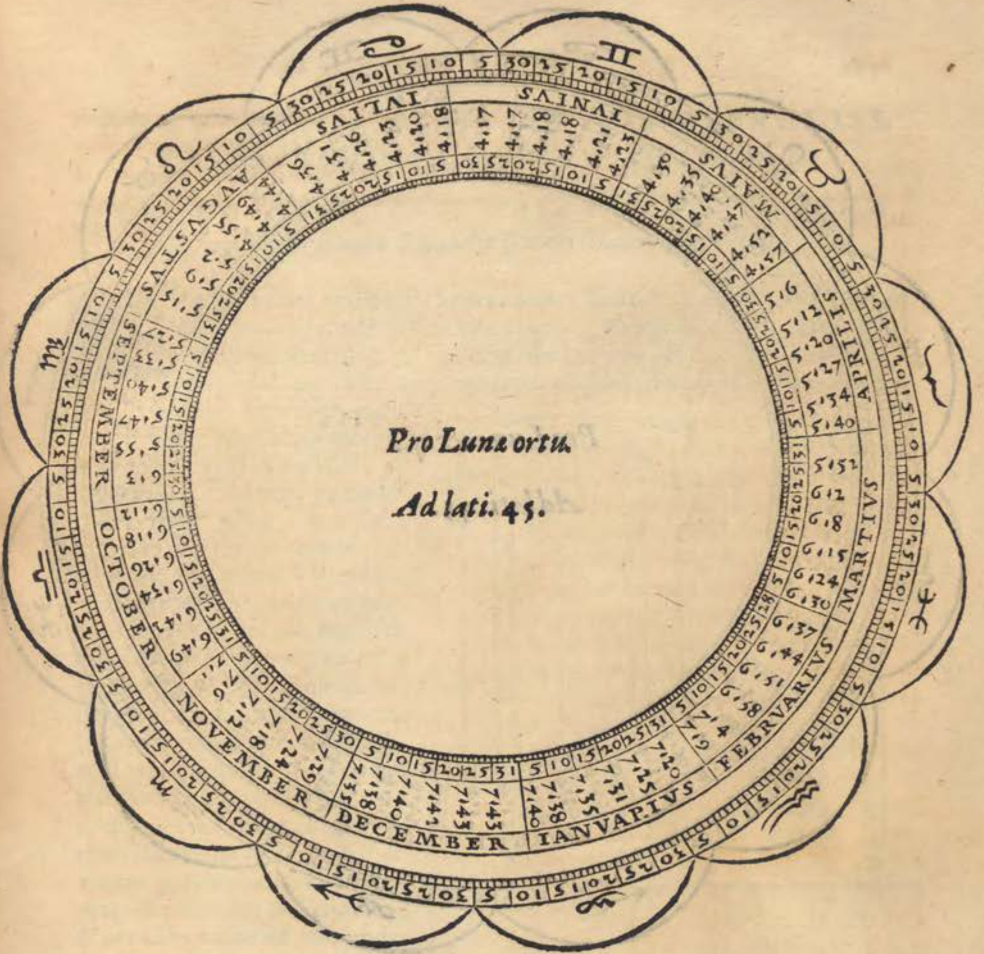
Nunc lamina ortus Lunę, cum anno arcuum seminocturnorum, & illa occasus cum anno arcuum semidiurnorum concẽtrari debet, & ita nostrum instrumentum perfectum erit & absolutum.

Quoties igitur voluerimus medio instrumento dignoscere fermè in tali orizonte qua hora Luna oriatur, ita necesse erit volubilem rotam ortus flectere, ut index veniat è regione diei mensis in quo talis operatio fit & tali rota firma manente perspicere,

cere ex aduerso diei Lunæ, numerum horarum & minutorum in maiori interuallo ipsius rotæ notatorum, qui cum arcu seminocturno anni, quo cum in ipsa rectitudine centri conueniet colligetur, & summa quæ ex tali supputatione proueniet apertas faciet horas astronomicas, quibus ferè etsi non exactè in die proposito Luna orietur. Idipsum fiet pro occasu Lunæ.

DIEM ætatis Lunæ iam totus orbis scit inuenire, media supputatione numeri Epactę currentis cum numero mensium, sumpto principio à Martio, adiunctis diebus mensis currentis, & detracto numero. 30. à summa prędicta, si ab ipsa dictus numerus. 30. superatur.

Sed ne aliquis putet sufficere tantummodo additionem quatuor quintarum horę qualibet die. à nouilunio inchoando, sciendum est huiusmodi recessum Lunæ (quamuis non ita exactæ fiat) non computandū esse ab orizonte aliquo, sed à recto, seu à meridiano quod idem est, quemadmodum vnicuiq; mediocriter erudito patere potest. At propositum nobis non est scire qua hora Luna in meridiano reperiatur, sed in nostro obliquo orizonte, in parte orientali seu occidentali, propterea igitur addendus est, ei summæ temporis, qua Luna distat à meridiano, arcus semidiurnus, vel seminocturnus illius loci Zodiaci, in quo Luna reperitur illa die in proposito parallelo, vt sciatur proximę, qua hora (ex astronomicis) Luna erit in orizonte oriētali, vel occidentali dicti paralleli. supra dicta enim additio quatuor quintarum horæ tantummodo, sufficiens erit temporibus æquinoctij, sed aliis anni temporibus falli ratione iam dicta.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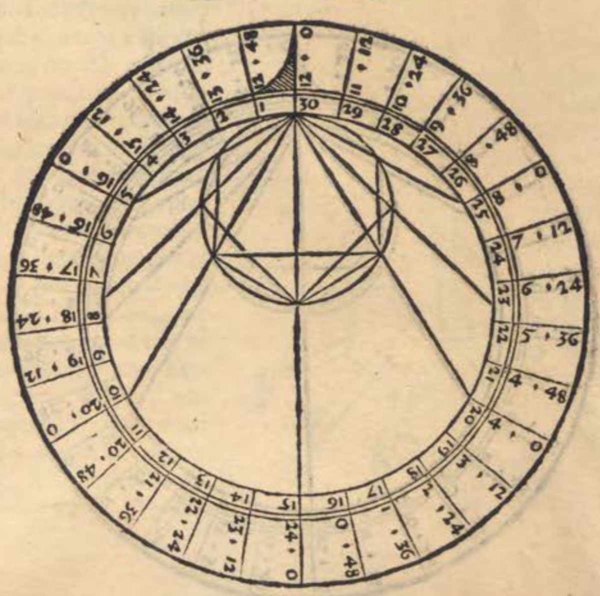

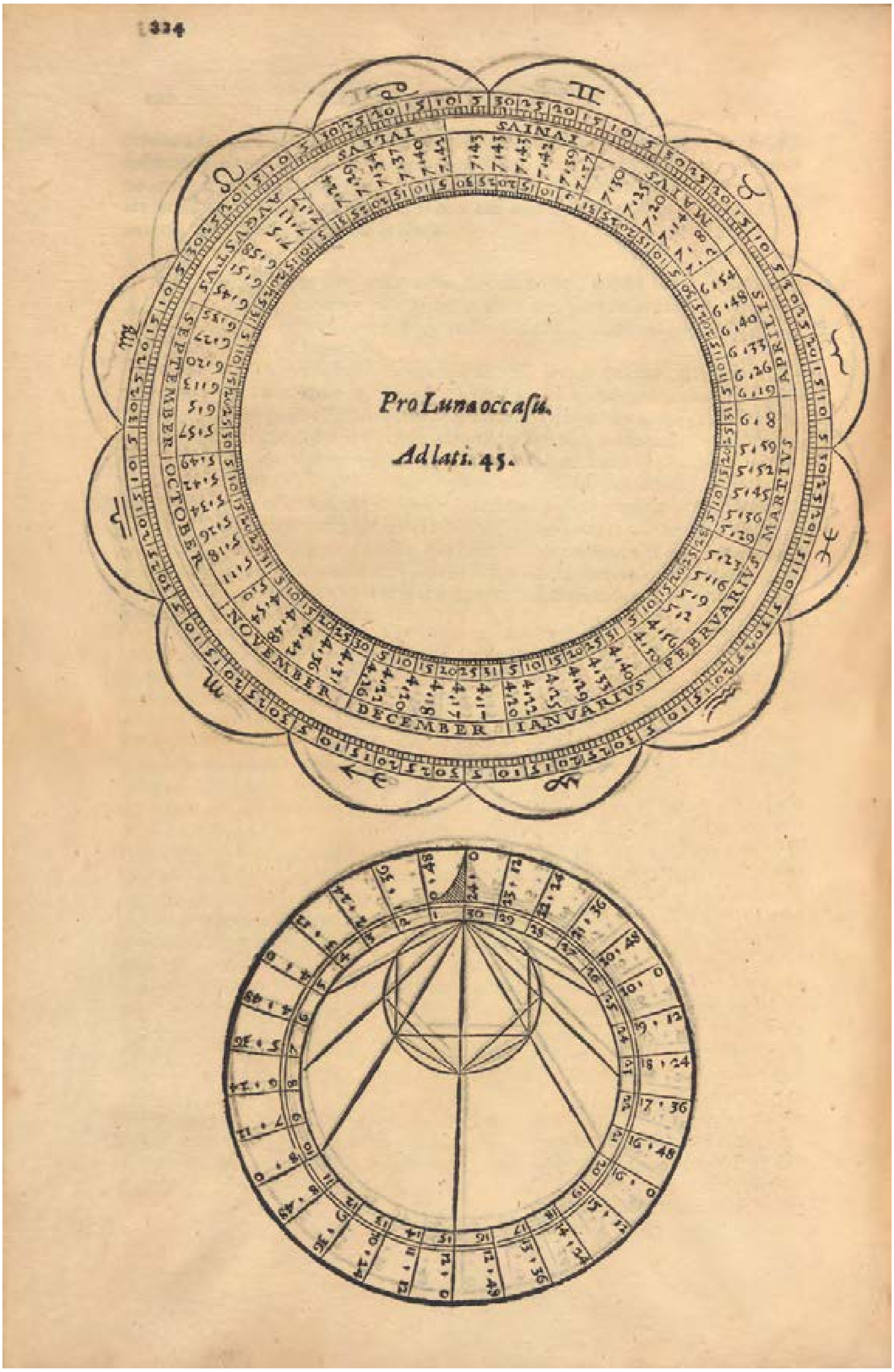
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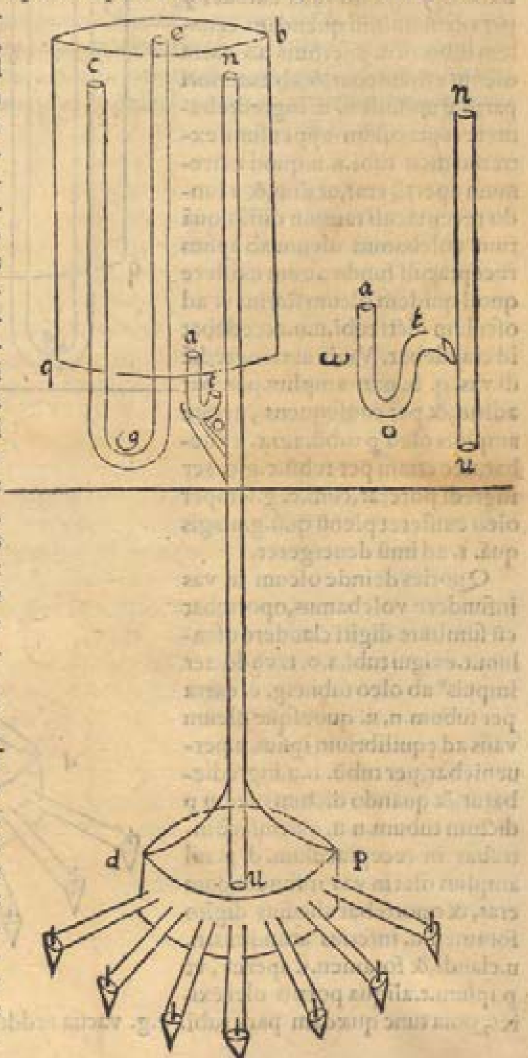
DECEMBER

## DE LVCERNA SPIRITALI QVAM SERENISS. Sabaudiæ Duce.D.meo collendiſs.anno.1570.conſtruxi.

### *CLARISS. FRANCISCO BARBARO VENETORVM apud Sereniſſimum Sabaudiæ Ducem Oratori Illuſtriſſimo.*

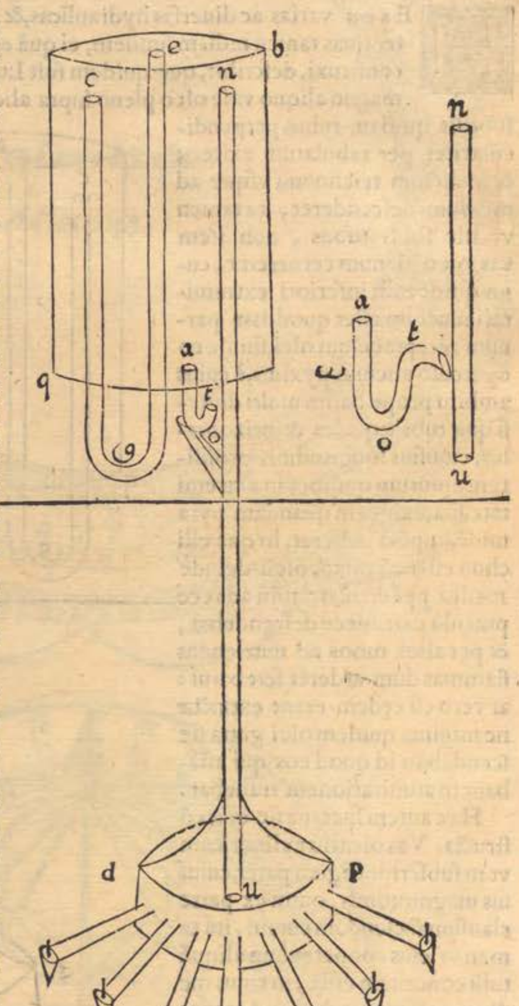
HERON varias ac diuerſas hydraulicas,& ſpiritales machinas propoſuit,inter quas tamen nullam ſimilem, ei quã ego Sereniſſimo Sabaudiæ Duci conſtruxi, deſcribit, quę quidem fuit Lucerna, & erat huiuſmodi, vt à magno aliquo vaſe oleo pleno ſupra alicuius triclinij tabulatum poſito, ſubtilis quidam tubus perpendicu/ariter per tabulatum exiret, & in dictum triclinium vſque ad medium deſcenderet, ita tamen vt hic ſolus tubus, non item vas oleo plenum cerneretur, cuius quidẽ tubi inferiori extremitati iunctum eſſet quoddam paruum receptaculum olei,ſimile co operculo alicuius pyxidis,è cuius ambitu prope baſim multi diuerſi quæ tubi æquales & orizontales, cuiuſuis longitudinis proſilirent,quorum quilibet in extremitate ſua,exiguam quandam pyramidẽ,appẽſã haberet, in qua ellichnũ eſſet cũ mixo. oleũ deinde medio ꝑpẽdicularis tubi ad receptaculũ extrinſece deſcendebat, & per alios tubos ad nutriendas flammas dum arderẽt ferebatur: at vero cũ eędem erant extinctæ ne minima quidem olei gutta deſcendebat: id quod eos qui aſtabant in admirationem trahebat.




Hæc autem lucerna ſic erat cõſtructa. Vas oleariũ cylindricum vt in ſubſcripta figura patet,cuiuſuis magnitudinis, omni ex parte clauſum faciendum curaui, ita tamen vt eius coopereulum aliquãtulũ concauum eſſet, in cuius medio erat foramen. e. quod erat os tubi.e.g.qui ſub eiuſdẽ vaſis fundo vſque ad.g.tranſibat, ſed poſtea ſurſum, quaſi vſq; ad cooperculum in ſitu.c. ab inferiori parte reflectebatur, & ibi terminabať.

Ff Vnde

Vnde oleum quod in vas infundebatur per foramen.c. dictum vas postea ingrediebatur per foramen.c.Habebat deinde tubum.n.u.rectum,qui à situ.n. propinquo co operculo ad libellam extremi.c.incipiebat, & per fundum contignationemq; vsque ad centrum supradicti receptaculi(circa quod tuborũ ope appensę erant ellychniorum pyramides)transibat, atque huiuiusmodi tubi. n. u. extremitates tam superius quam inferius erant apertæ,& hic tubus aeris erat. Præterea aliud quoddam foramen in vasis fundo feceram, cui paruum tubũ.a.o.t. reflexum, ita tamen, vt. o. altius esset quam. g. aptauerã,atq; ꝑ hunc reflexum tubum.a.o.t.oleum vasis exibat, ꝙ per osculum.t.in quendam canalem tubo.n.u. insertum, ab extra oleum effundebat,& ab exteriori parte arundinis.n. u. ingrediebatur receptaculum appensum extremo dicti tubi.n.u.quod extremum apertũ erat,ut dixi,& à fundo receptaculi tantum distãs,quã tum volebamus oleum ab ipsius receptaculi fundo altum existere quod quidem oleum statim vt ad osculum dicti tubi.n.u.accedebat id claudebat. Vnde aeri ingrediẽdi vas. q. b. non amplius patebat aditus,& per consequens, neque amplius oleũ ꝑ tubũ.a.o.t. effluebat,nec etiam per tubũ.e.g.c. aer ingredi poterat,cum.c. g.semper oleo existeret plenũ quũ.g.magis quã. t. ad imũ deuergeret.




Quoties deinde oleum in vas infundere volebamus,oportebat cũ sumitate digiti claudere osculum.t.exigui tubi.a.o. t. vnde aer impulsꝰ ab oleo tubi.e.g. c. extra per tubum n. u. quousque oleum vasis ad ęquilibrium ipsius.n.perueniebat,per tubũ. n.u.ingrediebatur.& quando dictum oleum ꝑ dictum tubum.n.u. extrinsecè intrabat in receptaculum. d. p. nil amplius olei in vas infundendum erat, & oportebat alicuius digito foramen.u. inferius arundinis. n. u.claudi,& foramen.t. aperiri, vt ꝑ ipsum.t.aliqua portio olei exiret, quia tunc quædam pars tubi. e.g. vacua reddebatur, &cum per. t.nil amplius

olei

olei egrediebatur, aperiebatur.u.& per ipſum.t.denuo tantum olei exire permittebamus, quantum in receptaculo ad claudendum foramen.u.idoneum exiſteret. Ratio verò,quę me mouit,ut punctum.g.inferius ipſius.t.conſtituam, eſt, quia cũ clauſum erit.u.per dictum.t.oleum non amplius egredietur,quia pondus olei in tubo. c. g.maius euadet oleo quod vſque ad.t.progrederetur,tubum autem.e.g.c. reflexum facio,ne cogamur claudere foramen.e.quia hoc difficile præſtaretur, tubum etiam a.o.t. ſurſum verſus reflexum conſtitui,vt aerem ab ingreſſu per foramen.t. arcerẽ, quia huiuſmodi aer nunquam deſcendit ſi corpus magis denſum non deſcendat.

Verum eſt, ꝙ melius erit, vt maiores difficultates euitemus, ſtatuere dictum tubũ a.o.t.ita curuum vt eſt.ω. qui cum ſuo extremo inferiori ipſi.n.u.ſit contiguus ita tamen ut dictum extremum inferius nõ ſit inferius quam.o.quia totum oleum exiret.

Volui etiam vt ſuperior extremitas.n.tubi.n.u.ſit in aere vaſis & non in oleo, ne per eam oleum exeat,quia cum extremitas.u.inferior ſit. g. totum oleum quod ſuperaret oſculum. n. per dictum tubum.n. u. ratione maioris ponderis egrederetur, quẽadmodum cuilibet, vel mediocriter in philoſophicis rebus verſato innoteſcet.

# DEFENSIO EPHEMERIDVM.

## *AD ILLVST. D. BERNARDVM Trottum.*

EDITA erant ſcripta quædam, quorum titulus ANIMADVERSIONES IN EPHEMERIDAS. & breuis alia diſputatio de erroribus calculorum Aſtronomicorum.ac demũ Theſes quædam typis datæ.11.Auguſti.1581.quę omnia cum ad manus meas perueniſſent, non potui,non eis animum admouere, cũ ibi de his ſtudijs ageret, in quibus partem non exiguam annorum meorum conſumpſi.nec tamen ſcribere aliquid ſtatueram; tum quod exiſtimarem viros Aſtronomiæ peritos facile,quanti facienda eſſent ea quæ edita erant iudicaturos,alijs verò haud gratam futuram harum rerum tractationem. Tum quod ſi ingenue meam ſententiam proferre voluiſſem, non poteram,ſine maxima authoris moleſtia ferè omnia reprobare. Quandoquidem ſolet vnuſquiſque indignationem concipere ex his, quæ ſuæ opinioni repugnant, id omne maleuolentiæ potius quam veritatis ſtudio tribuens. Quinimo cum nec deeſſent qui dicerent in meipſum directa ea tela fuiſſe,nullam fidem eis adhibendam duxi, nec enim qui in ephemeri das inuehitur, me arguere poteſt, qui nullas ephemeridas ſcripſi,nec tabulas compoſui. Nec ſi author quidpiam ex his quæ à nobis edita fuiſſent impugnare voluiſſet,ægrè ferre debuiſſem,modo à veritate nuſquam deuiaſſet. Liberum enim eſt cuique ſcribere quodlibet. nec Ariſtotelem afficit iniuria, quicunque illi fidem ſuam non accommodat,& ſi valdè iniquus ſit, quiſquis maiorum opiniones veras,& ab omnibus merito comprobatas nó admittit.Hinc mihi ſatis oĩbus feciſſe videbar, cum his qui de ſcriptis illis me interrogauerãt reſpondiſſem,ea non ſatis firmis eſſe innixa fundamẽtis, & quod ad talia tractanda opus fuiſſet exercitatiore iudicio. Verumtamen cum tu vnus maxime omnium deſideres tibi clarius,quæ nam de his mea ſit ſententia explicari, non tam tuis precibus deuictus quam mea ipſius cupiditate de te benemerendi impulſus, non ſuſtineo diutius animum tuum hęſitantem relinquere. Atque vt tibi adeo honeſta cupienti morẽ geram paucis hiſce ſcriptis incultis quidem, vt ab homine omni prorſus facundia deſtituto exaratis,ſed ex quibus nihilominus facile, atque perſpicue,vt ſpero,conceptum animi noſtri percipere poſſis, ſi tamen eam præſtantis ingenij tui aciem adhibueris, qua ſoles intima quæque ſcientiarum penetrare, noſtræ opinionis ſummam perſtrinxi,quę ad te mittere decreui.& quamuis ipſa res de qua agitur, quæ exactiorem deſiderat expoſitionem,prolixiorem me eſſe cogerit quam voluiſſem. multa tamen me obmiſiſſe intelliges, non admodum neceſſaria his quibus Aſtrologiæ noti ſunt termini, vt tuarum occupationum rationem me etiam habere intelligeres, atque vt ſummam oblectationem cõcipiam animo ſi me tibi aliqua ex parte ſatisfeciſſe intellexero ita humanitati tuæ gratiam habebo, quę mihi occaſionẽ pbuit,imò verò me impulit,ad ea proferẽda:quæ grata eſſe poſſint tui ſimilibus, ideſt pclaro & cãdido ingenio præditis,atq; ad euellẽdã ex eorũ animis falſam opinionem

nionem, ſi quam fortaſſe ex illorũ ſcriptorum lectione cõceperunt circa ea, de quibus nunc ſum acturus.

Quemadmodũ igitur ab hoc authore ter ſcriptum fuit de cõtradictionibus, ſiue erroribus Ephemeridum,& earum calculos ſequentium,& de ratione qua cognoſci poteſt ſitus & locus alicuius ſuperioris planetæ, diuerſus ab eo, qui ab ipſis Ephemeridibus aſſignatus eſt, ita diſputationem hanc meam diuidam in tres partes, quo ſcilicet minus confuſè,& magis diſtinctè à me ſcribatur, ꝓſupponendo, vt animaduertere potes,huius ſcriptoris intentionem,aliam non fuiſſe,quã oſtẽdere,quod ſcriptores Ephemeridum diuerſimode eiuſdem temporis locum planetæ aſſignauere ; & quod eum faciant modo nimium velociter currere,modo nimium in vno ſigno morari, vt(exempli gratia)Martem interdum faciunt morari ſex, aut ſeptem menſibus in vno ſigno. Idq; poſtea in cauſa eſſe ait, vt Aſtrologi iudiciarij fallãtur, & ſimul careant certis fundamentis rationum quibus futura iudicent, & prædicant. Primum ergo videndum eſt, quam rectè hic vſus ſit arte,& ſcientia, vt aliorum opiniones, & ſcripta redarguere poſſet. Deinde videbimus quomodo verum ſit, & poſſibile id quod ab Aſtrologis hactenus creditum,atque traditum eſt,& qua ratione poſſint fieri veri calculi à peritis regularum ſcientiæ.

In primo igitur tractatu inſcripto Animadnerſiones, præſupponit Author profeſſores huius ſcientiæ neſcire inuenire vera loca planetarum, quia vtuntur Ephemeridibus, in quibus eorum loca non rectè ſunt notata. Quod ſecundum ipſum oritur, ex errore calculatorum, ſeu computiſtarum, potius quam ex varietate tabularum, à quibus Ephemerides ſumptæ ſunt, hoc tamen verum non eſt, Ephemeridas, ſcilicet,ita inter ſe differre, ratione errorum computiſtarum tantummodo, ſed potius ratione ipſarum tabularum,& ſi interdum contingere poſſit error aliquorum minutorum, nec non graduum, non propterea Ephemerides ita ſpernendæ ſunt. In multis enim calculis, tales errores excuſabiles ſunt, cum ab innumerabilibus propè accidentibus oriri poſſint, præſertim in calculis prutenicis.

Videatur deinde vbi is profert quinquageſimum enuntiatum centiloquij Ptolemęi, ſatis mendoſe. Ptolemęus enim ibi ſic ait.

Non obliuiſcaris eſſe centum viginti coniunctiones,quæ ſunt in ſtellis erraticis,
» in illis enim eſt maior ſcientia eorum quæ fiunt in hunc mũdum ſuſcipiendi incre-
» mentum, & decrementum.
» Nam, neque eo in loco, neque alibi, Ptolemęus quidquam eius dicere voluit quod ab hoc profertur.

Pergatur poſtea in pag. 2.& videbitur hunc exiſtimare abſurdum quod Saturni, & Iouis coniunctio vera anni. 1563. potuerit eſſe in Leone ſigno igneæ triplicitatis cum eorum coniunctio vera anni. 1544. fuerit in Scorpione, ſigno triplicitatis aqueæ,& cum coitus eorum anni. 1583. futurus ſit in Piſcibus, ſigno pariter triplicitatis aqueæ. Ita enim ait.

Nam poſtquam duæ ſtellæ coiuerint,non prius ſub alio alterius triplicitatis ſigno
» inter ſe ſunt conuenturæ,quam per omnia ſigna quæ eiuſdem ternarij cum primo ex
» titerint prius coniungãtur. Ita ſentit Ptolemęus, cæteriq; non aſpernendi nominis
» Aſtronomi.
» Et tamẽ Ptolemęus nunquam quidquã huius rei attigit, & quamuis Albumaſar & Alchibitius de eo loquãtur,is tamen eos non intellexit, cum illi ibi nõ agant de

pe-

230 IO. BAPT. BENED.

periodis apparentibus, aut veris, ſed de mediocribus aut æqualibus, & quidem rectè dicunt, quia lineæ eorum mediorum motuum non coeunt in aliquo ſigno alterius triplicitatis, prius quam pertranſiuerint omnia ſigna illius, in qua inçperunt. Itaq; nullum inconueniens ſequitur, ſi in veris cõiunctionibus non reperitur hæc regula. Fieri enim poteſt, vt lineæ mediorum motuum coniungantur in vno ſigno, corpora verò eorum planetarum coeant in alio, cum rarò eueniat, vt linea medij motus, eadem ſit cum linea veri.

Nunc quidem tamẽ non affirmauerim, nec negauerim eorum coniunctionem anni. 1563. fuiſſe potius in Cancro, quam in Leone. Sed tantum dicam vanũ eſſe credere id eueniſſe propter ſimilem naturam, aut qualitatem ſignorum. Hunc enim reſpectum non habent illi planetæ in veris ſuis coniunctionibus. Exempli autem cauſa ponamus, quod rectè ſupputatæ fuerint coniunctiones annorum. 1484. 1504. & 1524. quod attinet ad differentiam duodecatemorij, ſcilicet prima in. 24. gradu Scorpij, ſecunda in. 20. Cancri, tertia in. 10. Piſcium. Cum ſecunda anticipauerit trigonum perfectum cum prima, gradibus. 4. & tertia anticipauerit trigonum perfectum cũ ſecunda gradibus. 10. ſi forte prima vt facta fuit in. 24. gradu Scorpij facta fuiſſet in. 2. gradu eiuſdem, planũ eſt ꝙ ſecũda facta fuiſſet in. 28. gradu geminorum & tertia in. 18. Aquarij, quæ ſigna ſunt diuerſæ triplicitatis ab illa Cancri. Inſuper ſi coniunctio anni. 1544. quæ fuit in. 28. gradu Scorpij fuerit recta correſpondẽs prçcedenti, anni. 1524. per gra. 18. ſine dubio ſi coniunctio anni. 1524. facta fuiſſet in 18. gradu Aquarij, illa anni. 1544. fuiſſet in. 6. Scorpij ſigni alterius triplicitatis quã ſint Gemini. Præterea, vt anno. 1544. cõiunctio facta eſt in. 28. gra. Scorpij, & 1563 in. 29. Cancri, ponendo eas eſſe rectas, quod attinet ad ſuperandum trigonum vno gradu, ſi anno. 1544. facta fuiſſet in. 30. Scorpij, anno. 1563. proculdubio facta fuiſſet in primo gradu Leonis. Et ſuppoſitis ijs interuallis, quæ ſuperſunt, aut deſunt perfectis trigonis, ſi coniunctio anni. 1524. fuiſſet in. 20. gradu Piſcium, anno. 1544. fuiſſet in. 8. Sagittarij. Quæ quidem omnia aduerſantur opinioni huius ſcriptoris.

Quod autem opinatur coniunctionem anni. 1583. fore in Ariete, ſic dicens pagina ſecunda.

„ Non erit ab re ſi & eandem Saturni, & Iouis coniunctionem in primo igneæ tri-
„ plicitatis ſigno, quod eſt Aries futuram afferamus anno. 1583. ſi ab accidentibus no-
„ bis licet, vt ab omnibus paſſim conceditur, planetarum loca diſcernere.

In eo fallitur, nã neq; Saturnus, neq; Iupiter, errãt à vero per. 9: nec. 8. gra. ac ne p 4. quidẽ in ꝗbuſuis Ephemeridibus aut tabulis. Itaq; videbit eiuſmodi cõiunctionẽ contra ſententiã ſuã fieri in Piſcibus, nõ aũt in Ariete. (vt poſtea res ipſa nos docuit ſub mẽſe Aprili. poſt ꝗdẽ ſcriptã hãc epiſtolã, vulgariq; ſermone trãſmiſsã, ſed antequã in latinũ trãſlata, & huic volumini inſerta cũ alijs Typographo cõmitteretur.)

Vbi poſtea meminit magnæ periodi annorum. 960. non tantum ei cogitandum erat hãc fuiſſe opinionem antiquorũ, vt videri põt apud Albumaſarẽ & Alchibiciũ, ſed etiam perpendendum an eſſet vera, priuſquã ei adhæreret. Hic enim fuit vnus ex erroribus illius ætatis, quæ nondum penetrauerat intima huius ſcientiæ. Sunt tamen illi antiqui excuſatione aliqua digni. Ponebant enim vigeſimo quoque anno præciſè fieri mediam coniunctionem Saturni cũ Ioue, & in quolibet ſigno eiuſdem triplicitatis cõiungi quater. Itaque in qualibet triplicitate dicebant eos coire duodecies.

Quod

Quod ſecundum primum ſuppoſitum finiebatur ſpacio annorum.240. qui nume
rus fit.ex.20.duodecies multiplicatis. Et quia triplicitates ſunt.4. ideo credebant
in ſpacio annorum.960.qui numerus fit ex.240.quater multiplicato,perfici.48.cō-
iunctiones,priuſquam redirent ad ſe coniungendos in eodem loco, ubi prius iun-
cti fuiſſent. Primum autem ſuppoſitum, quod vigeſimo quoque anno iūgerentur,
colligebant ſic ratiocinantes. Si Saturnus annis.30.peragit ſuum curſum per om-
nia ſigna Zodiaci, Iupiter autem peragit eum annis.12. Saturnus ambulauerit.4.ſi
gna,et.4.quintas partes ſigni,ſiue gra.24. dum Iupiter peragit integrum ambitum
ideſt annis.12.Itaque deſunt ei anni.8.ad perueniendum ad.20.quibus.8.annis Sa-
turnus pambulat ſigna tria & quintā partē unius ſigni.i.gradus.6.qui iuncti dictis ſi
gnis.4.& gra. 24.faciunt ſigna.8.quæ Iupiter item percurrit in annis. 8.atque ita in
annis.20.Iupiter percurrit.20.ſigna antequā perueniat ad Saturnū,cum Saturnus eo
dem tempore perfecerit curſum ſignorum.8. Eandem concluſionem etiam fortaſ
ſe collegerant ex dictis ſuppoſitis, dicentes, ſi Saturnus annis.30.ambulat.12.ſigna
proculdubio annis.20.ambulat.8.ſigna,quo tempore Iupiter perambulat.20.ad ra
tionem.12.ſignorum in annis.12.

Verum hoc ſuppoſitum non eſt bonum,quoniam,ſi ita eſſet, coniunctiones horū
duorum planetarum nunquam exirent ex vna triplicitate, & non modo.960.quoq;
anno,ſed etiam ſexageſimo rurſus coniungerentur in eodem puncto.nec coniunctio
nes eorum(ſemper autem intelligo de medijs) unquam egrederentur ex illis tribus
ſignis Zodiaci.

Sed periodus æqualis Saturni,eſt dierum circiter.10740.atque ita minor an. 30.
atque etiam.29.cum dimidio.periodus autem æqualis Iouis, eſt circiter.4328. vt
ego eam comperio,quidquid alij dicāt, vtq; planius alias oſtendam. Itaq; hæc per
iodus Iouis,etiam minor eſt ann.12.prætermittendo in ſupputatione tā Saturni quā
Iouis quaſdam minutias horarum & earum partium, quæ hac in re pro nihilo habe-
ri poſſunt. Atque his duabus periodis eccentricorum duorum planetarum poſſu-
mus cognoſcere interuallum quod erit inter vtramque mediam coniunctionem,hoc
modo agendo,& ratiocinando.

Si Saturnus diebus.10740.circuit gradus.360.diebus. 4328. qui ſunt periodus
Iouis,conficiet gradus.145.& min.4.ideſt min. 8704. & eadem regula inueniemus
ꝗ Saturnus. 30.quibuſq; diebus,conficiet min.60.& ſecunda. 20. Iupiter autem ſin-
gulis.30.diebus,conficiet min.149.& ſecunda.43.vnde ſubtrahendo minuta Satur-
ni à minutis Iouis,ſupererunt min.89.cum ſecun. 23. Itaq; Iupiter.30.quibuſq; die-
bus velocitate curſus,ſuperabit Saturnum minutis.89.cum ſecundis.23. Atq; dicen-
do,ſi minuta.89.cum ſecundis.23 dant nobis dies.30.ſupradicta,minuta. 8704. da-
bunt nobis dies.2921.quibus iunctis cum diebus.4328.periodi Iouis,efficientur dies
7249.ideſt anni Aegiptij.19.cum diebus.314.& hæc erit æqua periodus temporis
inter vtranque coniunctionem horum duorum Altiorum planetarum. Vt autem pla
nius oſtendatur hanc operationem rectam eſſe (nam demonſtrationem ſpeculatiuā
huius operationis in.113.Theoremate noſtræ Arithmeticę cuiq; videre licet) fieri
poteſt his alijs calculis.

Si Saturnus diebus.10740.tranſit per gra.360.in ſpacio dierum. 2921. tranſibit
per gradus.97.min.54.quibus iunctis cum gra.145.min.4. ſupra notatis, efficientur
gra.242.min.58.Deinde,ſi Iupiter ſpatio dierum.4328.tranſit per gra. 360. igitur
ſpatio.2921.per eandem regulam inueniemus eum tranſire gradus.242.mi.58. qui
numerus par eſt illi Saturni. Cum ergo Iupiter confecerit vnum ambitum poſt con

iun-

iunctionem cum Saturno,vt rurſus aſſequatur Saturnum,tranſeundum ei erit ꝑ gra. 242.min. 58.iter confectum à Saturno toto tempore annorum.19.& dierum. 314. ad rationem graduum.360.diebus.10740.(poſſumus etiã dicere gra.243.quia præ termiſimus quaſdam exiguas particulas periodorum perfectorum cuiuſque planetę in ſuperioribus ſupputationibus.) Illos verò gradus.243.Iupiter conficiet diebus. 2921.ad rationem graduum.360.diebus.4328.Atq; ita vt diximus,ab vna coniunctione ad aliam intererunt anni.19.Aegiptij cum diebus.314. vel circa.

Nunc autem vt videatur an tabulæ Alfonſi conueniant cum hoc nr̃o calculo, cõſiderabimus, ꝙ Era(vt vocant)dicti temporis annorum.19.cum diebus.314.eſt duarum tertiarum ſexagenarum,ſecundæ nullius,& 53.primarũ ſiue dierum. Et per hãc Eram colligendo motum mediocrem,tum Saturni,tum Iouis,omiſſis radicibus,& incipiendo ab Ariete,comperiemus ꝙ vtriuſq; planetæ lineæ eiuſmodi motus tranſibunt per min.56.tertij gradus Sagittarij, ideſt coniunctæ erunt.

In fine poſtea ſecundæ periodi, cuius era erit.4.tertiarum,ſecundæ.1.et. 47.primarum ſexagenarum,locus mediocris vtriuſq; erit in min.56. gra. ſexti Leonis. In fine verò tertiæ periodi,cuius era erit.6.tertiarum.2.ſecundarum,et.41. primæ, locus eorum mediocris inuenietur in.56.minuto gradus.9.Arietis. Atq; ita deinceps in fine cuiusq; periodi,locus eorũ mediocris coniunctim ſemper diſtabit à loco mediocri præcedentis coniunctionis gradibus.117.ideſt in trigono antecedenti, minus gra.3.Vnde apparet has coniunctiones procedere in contrariam partem reſpectu ordinis ſignorũ Zodiaci,ſed reſpectu ordinis graduum ſignorũ,ſemper progrediunt̃ ordine per ternos gradus nunquam retrogradientes. Hinc ſequitur,vt non duodecies in omni triplicitate coniungantur hi duo planetę, vt antiqui putauerunt, ſed decies tantum.& ad ſummum ter in ſingulo ſigno,ſpatio annorum.198.& dierum.220.aut circiter,non autem.240.nec.242. Atque decem vices comprehendunt gra. 27. & vltima vice inueniuntur in ſigno ſequenti alterius triplicitatis. Exempli gratia, ponamus ꝙ prima vice cõiungant̃ in gra.2.Arietis,ſecunda coniunctio erit in. 5.Sagittarij,tertia.in.8.Leonis.quarta in.11.Arietis,quinta in. 14. Sagit. 6. in. 17. Leonis. ſeptima in.20.Arietis,octaua in.23.Sagittarij,nona in, 26. Leonis, decima in. 29. Arietis,et vndecima erit in gra.2.Capricorni ſigni ſequẽtis triplicitatis. Decem igitur interualla ſingula annorum. 19.& dierum.314. faciunt annos.198.& dies. 220. Immo per tabulas Alfonſi,eiuſmodi periodus non modo non reperitur annorũ.242 nec.240.vt antiqui credidere,ſed tribus diebus minor annis.198.& diebus.220.ideſt per dictas tabulas inuenitur eſſe annorum.198.& dierum.217.tantum,qui numerus multiplicatus per.4.triplicitates,efficiet periodum maiorem, quæ erit annorum 794.& dierum.138.quo tempore dicti planetæ redeunt ad eundem locum vbi primum ſe coniunxere.

Vt exempli gratia,locus mediocris Saturni & Iouis in fine annorum.198.dierum 217.reperitur in gradu.30.Sagittarij. Si quæſiuerimus hunc locum per aggregatũ annorum.794.& dierum.138.cum annis.198.& diebus.217. quorum ſumma eſt. 992.& dies.355.inuenietur locus mediocris ipſorum planetarũ in dicto vltimo gradu Sagittarij. Sed ſi quęſiuerimus eorum locum mediocrem per aggregatum annorum.198.& dierum.217.cum annis.960.quod erit ſumma annorum.1158.& dierũ 217.reperiemus Iouem in gradu.18.Sagittarij & Saturnum in.16. Leonis diſtantibus inter ſe duabus eorum lineis motuum mediocrium gra.circiter.122.Atq; Iupiter præcedet,& oportebit ꝙ coniunctio eorum mediocris fuerit multis annis ante omittendo(vt dixi)radices,quia ſatis eſt inuenire interuallum inter lineas eorum mediorum motuum.

Debe-

Debebat igitur author animaduerſionum non quaſi cæcus cæcos ſequi, ſed prius laborare, vt certior fieret, an interuallum annorum. 960. Verum eſſet.

Sed peius eſt, ꝙ idem author paulo inferius citat coniunctiones horum duorum planetarum anni. 1493. et. 1512. quas neſcio vnde ſumpſerit.

Nam, etſi inter hos annos eſt interuallum annorum. 19. tamen tantum abeſt, vt coiuerint dictis annis, vt Saturnus anno. 1493. ante finem Auguſti fuerit in. 28. gradu Aquarij, Iupiter verò in. 28. Leonis ex diametro oppoſiti. Et anno. 1512. per totum menſem Iunium & Auguſtum, Saturnus fuerit in Libra, Iupiter verò in Ariete, itaque inter ſe ſimiliter oppoſiti, & ſi perfecta oppoſitio non fuit poſtea niſi ad finē Iunij ann. 1513. & locus Monteregij ab eo citatus, vbi ait eum ponere coniunctionem anni. 1484. in gra. 23. min. 4. Scorpij, eſt mendoſus. Nam ipſe Monteregius ponit dictam coniunctionem in mi. 42. gra. 24. non autem in min. 4. ipſius gradus. Sed hic error nullius eſt momenti, fortaſſe qui impræſſorum incuria irrepſit.

Pergatur poſtea obſecro ad paginam. 3. ipſarum Animaduerſionum, vbi hic conatur oſtendere calculatores non obſeruaſſe verum modum, ſic dicens.

Anno. 1484. Nouembris. 25. Saturno locum conſtituit Monteregius in grad. 23.
» min. 4. Scorpij. Anno poſtmodum ſubſequenti qui eſt. 1485. eundem in min. 7. Sa-
» gitarij collocat. 21. Februarij die. Interq; tempora duo interſunt menſes dies. 26.
» At cum ex motus ſui natura Saturnus hoc temporis ſpacio gradus. 4. non debeat trā-
» ſcendere, ſit tamen inter vtrunq; tempus differentia graduum. 7. minutorum. 3. quæ
» ratione ſui motus requirunt menſes. 6. vt eos perficiat, conſtat pluſquam tribus men
» ſibus fallere nos Saturnum.

» Hic videre licet quam veram viam hic ſecutus ſit ad aperiendos errores Ephemeridum, & miſeri Monteregij, qui Saturnum claudum facit tantum itineris conficere tribus mēſibus, quātū vix confeciſſet mēſib. ſex. Sed fortaſſe ratiocinaē hoc modo.

Si motus naturalis Saturni facit vt circumeat totum cęlum annis. 30. igitur menſibus. 30. conficiet duodecimam partem circuitus, cum menſes. 30. ſint duodecima pars annorum. 30. & quia duodecima pars circuitus cęli intelligitur conſtare ex. 30. gradibus, igitur quilibet menſis poſtulabit gradum vnum. Ideo illi. 6. aut. 7. gradus poſtulant tempus, amplius menſium ſex.

Atque eiuſmodi mira ratiocinatio poteſt in. 2. exemplo eius, inſcripto.

» De eodem ex eodem

Vbi miratur, ꝙ Monteregius faciat Saturnum ambulare gra. 9. min. 10. in menſibus. 7. & diebus. 6. Ad quod iter Saturno ſeni opus eſſet ſaltem menſibus. 9. eius iudicio.

Sed ſi hoc miratur, quid dicturus fuiſſet, ſi animaduertiſſet, quod idem calculator Monteregius facit Saturnum ambulare immo volare gra. 9. min. 48. non in 7. ſed in 2. menſibus cum dimidio, videlicet à. 10. die Iunij vſque ad. 26. Auguſti eiuſdē anni. 1504.

Quid ſi etiam animaduertiſſet ꝙ à. 10. die Iunij ſupradicti vſque ad. 16. Ianuarij anni ſequentis, faciunt Saturnum, ſurſum, deorſum curſitare amplius gra. 17. mi. 54. Immo ſi animaduertiſſet, quod anno. 1524. Stoflerinus ab initio anni, vſque ad medium Maium, ideſt menſib. 4. cum dimidio, facit Saturnum ambulare gra. 15. Profectò ob has velocitates, eius iudicio, tam abſurdas, obſtupuiſſet.

Vbi autem in tergo eiuſdem paginę ait, quod gradibus. 13. min. 42. reſpondent menſes. 19. errauit in calculo, nam ex eiuſmodi tempore ſecundum eius regulam efficerentnr singuli ambitus Saturni ad rationem annorum amplius. 40.

Videamus nunc vbi agit de Ioue,& reperiemus q; in primo exemplo circa annũ. 1484. reprehendit Monteregium, quia facit Iouem ambulare gradus. 14. cum min. 6. in mensibus. 2. diebus. 4. ad quod iter, vt ipse ait, opus esset saltem mẽsibus. 11. atq; ita secundum ipsum, Ioui opus esset anno vno pro singulo medio signo. Vbi bonus hic vir pariter cæcutit.

Idem in secundo exemplo sumpto à Stoflerino ait, q; Ioui ad cursum vnius gradus, & min. 5. opus est diebus. 30. non autem mensibus. 7. & diebus. 28. vbi ostendit, se paruum discrimen facere inter Iouem, & Saturnum.

Miratur postea q; Stoflerinus faciat laborare generosum Iouem ferè mensibus sex in vno gradu. Sed multo magis, vt puto, miratus esset, si vidisset, quod idem Stoflerus in eodem anno facit, quod Iupiter die. 4. Ianuarij sit in eodem puncto, in quo postea reperitur die vltima Augusti. At fortasse dici posset, quod Iupiter propter prudentiam, & bonitatem suam factus est Rex omnium Deorum, vt ait Homerus, & ideo expulit è sede Saturnum, & ascendit in altiori cœlo. Vnde euenit vt factus fuerit lentior in cursu, Saturnus autem velocior. Aut iam tot annos esse natum Iouem, vt iure credi possit eum iam factum esse senem, & pariter tardiorem in se mouẽdo. aut tũc temporis illum detentum fuisse in sibi dilecta Arcadia cũ Calisto. Aut fortasse erat in alta specula intentus audiendo ingenti certamini Timoclis & Damidis, vnde pendebat exitium aut gloria familiæ suæ, nam alioquin Stoflerus non depræhendisset eum tam otiosum & morantem. Sed iam relinquamus Saturnum & Iouem, & ad Martem veniamus.

Ferox & inquietus Mars, qui semper bella & ignes spirare solet, etiam, & ipse ab Astrologis factus est piger, & languidus, vt velint eum nonnunquam commorari in vno signo sex aut septem mensibus; quod nullo pacto placet authori Animaduersionum, cum pag. 4. ita scribat.

„ Quod citra notam, ab omnibus creditur posse obseruari, quamuis à nobis non ac
„ cipiatur.

Itaque ei videtur impossibile. Quia Mars peragit suum circuitum minus. 2. annis. Sed audacior fuisse videtur, qui voluerit arguere tot egregios viros antiquos, & recentiores, qui vti diligentes rerum cœlestium obseruatores, ipsis oculis certi facti sunt tam de his effectibus Martis, quam aliorum, vnde coacti sunt fingere tantam magnitudinem eius epicycli, cum ipse nunquam obseruauerit motum, nec huius nec alterius planetæ, sed tantum viderit eius moram in Ephemeride scriptam. Si enim saltem diceret, se aliquo tempore obseruasse iter Martis, & comperuisse aliorum opinionem falsam, attulisset aliquem colorem sententiæ suæ. Sed si obseruasset, non scripsisset postea contra, vt puto. Res enim ita se habet, quod Mars in omni circuitu sui epicycli transiens per inferiorem partem ipsius epicycli, semper commoratur multis mensibus in vno duodecatemorio Zodiaci, scilicet. 6. et. 7. mensibus, atque etiam amplius, quod quidem ego sæpe obseruaui, præsertim anno. 1565. et. 1566. hoc ordine. Primum inspiciens Ephemeridas stadij, reperi q; Mars secundum eum egrediebatur retrogradationem circa diem. 12. Ianuarij anni 1566. in. 16. grad. Geminorum. Et similiter quod anno. 1565. die vltima Augusti Mars futurus erat in eodem supradicto loco, priusquam retrogradi inciperet. Postea inueni, quod post retrogradationem die. 11. Aprilis. 1566. Idem Mars futurus erat in gra. 16. Cancri, itaq; in his. 30. gradibus à. 16. Geminorum ad. 16. Cancri consumebatur spatium mensium. 7. & dierum. 11.

Quo

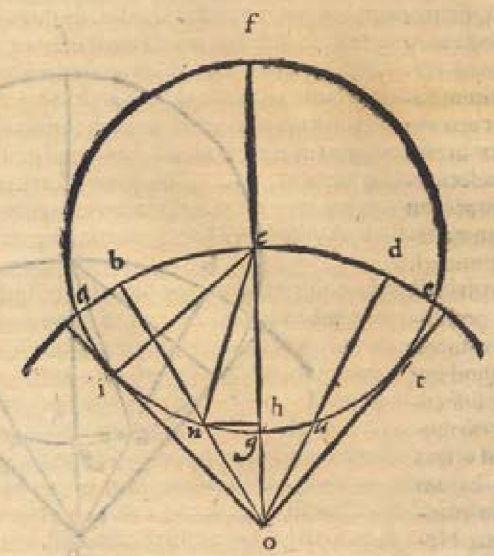
Quo ſupputato, ſumpſi inſtrumenta, & ad experimentum me paraui, & vltima nocte menſis Auguſti anni. 1565. reperi Martem eſſe in dicto gradu geminorum vt ſcribebat Stadius. Deinde ſingulis ebdomadibus obſeruans retrogradationem, vidi circa finem Octobris quod retrogradi incipiebat, & ea retrogradatio perſeuerauit vſque ad medium menſem Ianuarium, aut circiter, anni. 1566. obſeruaui poſtea etiam ſitum eiuſdem planetæ die. 11. Aprilis ſequentis eumq́; inueni in gradu. 16. Cancri, vti eum poſuerat Stadius. Atque ita experimentum meum conuenit cum calculo Stadij, comperiq́; eum non erraſſe: Et ſic quiſque binis quibusq́; annis poterit certior fieri de veritate. Si autem delectationis cauſa id experiri volueris, expectato primam retrogradationem Martis, cuius initium ſecundum Stadium futurum eſt circa diem. 20. Nouembris anni. 1582. & finis circa diem. 10. Februar. 1583. circa grad. 9. Cancri, & animaduerte quando Mars erit circa dictum gra. 9. Cancri prius quam retrogradi incipiat, quod erit circa diem. 19. Septem. 1582. Deinde aſpice quum erit in grad. 9. Leonis, quod erit circa diem. 7. Mai. 1583. & videbis ꝙ ipſe Mars in his gra. 30. morabiť ꝑ menſes. 7. & dies. 18. atq; vt eius rei periculum facias, obſerua noctem præcedentem diei. 19. Septem. 1582. locum lõgitudinis eius ſtellæ, & idem poſtea obſerua nocte præcedente diei. 7. Mai, aut nocte ſequenti. 1583. & inter duos hoſce terminos obſerua aliqua alia nocte ſtatum eius. Manifeſtoq́; videbis Martem conſumere totum dictum tempus in hoc duodecatemorio. Et quicunque aliquid intelligit in hac facultate quamuis non viderit Ptolomęi Almageſtum, minori labore poſſet per calculos ſcientificos colligere veritatem, ſuppoſitis tamen terminis ſcriptis in theoricis planetarum. Qui enim vidit Almageſtum vel reuolutiones orbium cœleſtium Nicolai Copernici, non poteſt de hoc vllo pacto dubitare. Sed qui nondum tantopere progreſſus eſt, ſaltẽ capiat huiꝰ rei notitiam vniuerſalem, hoc modo. Supponat primum eccentricitatem deferentis epicycli Martis, eſſe. 6. partium taliũ, quales ſunt ſexageſimæ ſemidiametri ipſius deferentis, & ſemidiametrum epicycli eſſe, partium ſupradictarũ. 39. cum dimidia, & quod argumenta vera, in temporibus primarum ſtationum (cum epicyclus eſt in auge, aut in eius oppoſito, aut in lũgitudinib. mediocribus) iã ab antiquis rectè ſupputata ſint, ſicuti ſunt. Et præſupponat motum diurnum centri epicycli. min. 31. cum dimidio, quamuis reuera ſit min. 31. & ſecundorum. 27. aut circiter, nunc quidẽ prætermittens, quod vnus habeat reſpectum ad augem mediam epicycli, & alter ad centrum æquantis. Atque his præſuppoſitis fingat (exempli gratia) quod centrum epicycli ſit in quauis longitudinum mediarum, & Mars in prima maxima æquatione argumenti, ſcilicet in prima linea, quæ attingens epicyclum, à centro mundi pergat ad circunferentiam Zodiaci, quæ erit illa linea cõtingentiæ à qua proficiſcẽs Mars perget ad lineam primæ ſtationis, vt poſtea retrogradiatur, veluti ſi in infrapoſita figura maiori, cẽtrũ mũdi eſſet. o. & vnus arcus eccẽtrici eſſet. a. b. c. d. & vna ex lineis mediocribus longitudinum eſſet. o. c. f. & centrum epicycli. c. qui notabitur per a. f. e. g. & lineæ contingentes epicyclum in punctis. i. et. t. ſint notatæ. o. i. et. o. t. & linea primæ ſtationis. o. n. b. & linea ſecundæ. o. u. d. ſi igitur Mars eſſet in puncto. i. angulus. i. o. e. maximæ æquationis argumenti eſſet gra. 40. minut. 55. quãuis talis maxima æquatio argumenti in longitudinibus mediocribus Alfonſi ponatur eſſe gra. 41. minut. 10. quod euenit quia calculatores ipſarum tabularum interuallum. o. c. quod in eo ſitu epicycli interponitur inter centrum mundi, & centrum dicti epicycli, acceperunt partium ſexaginta præciſè, nihili facientes minuta illa. 18. aut circiter, quę verè ſunt præter dictas partes. 60. quandoquidem euenit vt dictum interuallum in

tali ſitu epicycli ſit baſis vnius trianguli orthogonij,cuius vnum ex illis duobus late-
ribus eſt ſemidiameter eccentrici partium.60.pręcisè,aliud eſt interual lum eccen-
tricitatis partium.6.eiuſmodi. Angulus ergo.i.o.c.vt dixi, erit partium. 40. minu.
55. qui angulus continuò variatur ſecundum ſitum epicycli. & cum centrum
eius eſt in auge eccentrici. eſt minimus quã eſſe poſſit. eſtq́; tantum grad.36. min.
46. & in oppoſito ipſius augis eſt grad.47. min.1. maximus quam alibi vnquam ſit,
& ſic continuò variatur,ſecundum ſitum,quem habet epicyclus in eccentrico. Qui
quidem angulus inuenitur per doctrinam.27. et.28.libri primi Monteregij de trian
gulis. Nam triangulus.c.i.o.eſt ſemper rectangulus in puncto.i.& latus.c.i. reſpectu
ſemidiametri eſt datum. Quod.c.i.erit veluti partium.39.cum dimidia, et dictum
interuallum.o.c.veluti partium.60.min.18.& quia datur nobis etiam eccentricitas
veluti partium.60.talium, & cum.c.o.ſit linea veri motus epicycli, & latus ſimiliter
vnius trianguli, cuius duo latera ſunt ſupradicta,ſcilicet ſemidiameter eccentrici, &
eccentricitas, inter ſe compræhendentes angulum datum. Nam ſemper præſuppo
nitur datus locus centri ipſius epicycli, cum ipſe eſt extra augem aut oppoſitum eius
quia in auge linea.o.c.conſtat ex ſemidiametro eccentrici & interualli eccentricita-
tis. & in eius oppoſito, ipſa linea.o.c.eſt minor dicto ſemidiametro eccentrici per in
teruallum dictæ eccentricitatis. Vnde etiam poſſumus extra augem, vel oppoſitum
eius cognoſcere.o. c. tanquam latus dicti trianguli duorum laterum cũ angulo cogni
torum. Idq́; per.49.propoſitionem libri primi eiuſdẽ Monteregij cum ſcilicet dictus
angulus nõ fuerit rectus. Nam ſi fuerit rectus videbitur per.27.et.28.ſupra citatas.

Cum igitur habeamus angulum.c.o.i.gra.40.mi.55.angulus.o.c.i.tanquam reli-
quus ex recto,erit grad.49.mi.5.cui reſpondet arcus.i.g.epicycli confectus à Marte
in diebus circiter.105.ad rationem min.28.aut circiter in ſingulos dies, prætermiſ-
ſis nunc quidem minutijs cum exigui momenti ſit error.15.aut.20.dierum ad verifi
cationem longæ morę Martis in vno duodecatemorio, atque per hoc tempus cen-
trum epicycli conficit gradus.55.min.7.aut circiter,ad rationem minutorum.31. cũ
dimidio in ſingulos dies. qui numerus graduum.55.min.7:differt à numero graduũ.
40.min.55.maximæ æquationis argumenti gradibus.14.mi.12.nec refert quod gra.
55.min.7.habeant reſpectum ad centrum æquantis, magis quam ad centrum mũdi,
quia differentia non eſt tanta, vt poſſit inducere errorem menſium. Hinc ſequitur
quod in fine dictorum dierum.105.Mars erit in linea.o. c. veri motus epicycli, ſed
gradibus.14.min. 12.vlterius quam in primo loco,in quo erat in Zodiaco, & erit in
medio ſuæ retrogradationis. Sed quoniam Mars manifeſtè retrogradi non incipit
in puncto.i.contingentiæ,imo ab illo puncto vſque ad terminum primæ ſtationis li
neæ.o.n. interponitur arcus.i.n.epicycli,qui eſt graduum.32.minu.14.Idq́; cogno-
ſcitur ſubtrahendo arcum.f.i.n.graduum.163.mi.9.qui eſt inter augem, & primam
ſtationem,à gradibus. 180. (qui arcus. f. i. n. erit verum argumentum, quod ſi-
militer variatur ſecundum ſitum epicycli,etſi eiuſmodi varietas, nobis nõ eſt magni
momenti,vnde poſſumus præſupponere, quod.c.centrum epicycli non alteret in-
teruallũ.c.o.à centro mũdi, cũ non poſſit intercedere,error mẽſiũ reliquum verò.g.
n.graduum.16.min.51.ſubtrahendo ex arcu.g.i.graduum.49.minuti. 5. vnde reli-
quus nobis erit arcus.n.i.graduum.32.min.14.in eiuſmodi tamen ſitu mediocrium
longitudinum. Nunc hic arcus epicycli graduum.32.mi.14.fit à ſtella Martis die-
bus.69.ad rationem ſupradictam, omittendo quod ipſa ſtella habeat reſpectum ad
augem mediocrem epicycli, & quod dicta aux mediocris mutet diſtantiam à vera
propter motum epicycli,quod nunc quidem parui refert, in quibus diebus.69.cen-

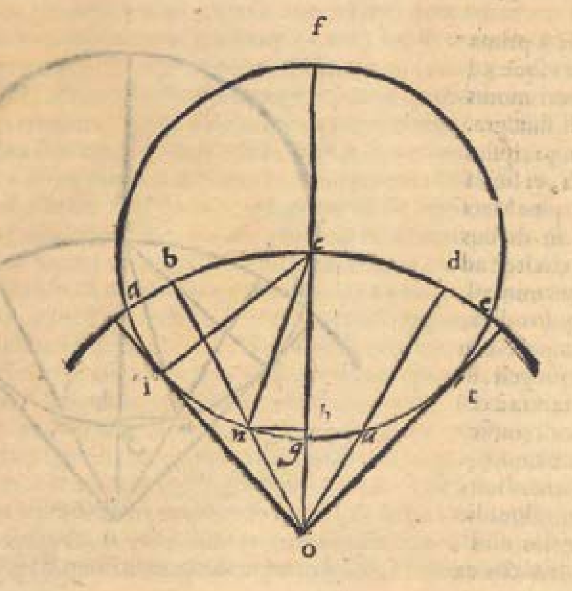
trum

trum epicycli conficit gra.36.min.13.ad rationem supradictam. Restat nunc nobis inuenire angulum.b.o.c.in centro mūdi, inter duas lineas, b.o.et.c.o.quarū prior est primæ stationis, altera est veri motus epicycli, quod facilè intelligemus per dictam.49.lib.1.Monteregij.cum duo latera.n.c.et.c.o.& angulus.n.c.o.sint nobis nota. Hoc autem fiet fingendo lineam.n.h.perpendiculerem ad.o.c.quæ tanquam sinus anguli.n.c.h.erit partium.28986.talium qualium.n.c.esset partium.100000.& c.h.tanquam sinus anguli.c.n.h.restantis ex vno recto, erit partium.95706.dicendo postea si.n.c.tanquam sinus totalis partium.100000.dat nobis.n.h.partium.28986 quid dabit nobis diameter.n.c.tanquam partium.39.mi.30.inueniemus.n.h.venire nobis ex partibus 11.mi.27.& idem faciendo de.c.h.inueniemus quod veniet nobis partium.37.mi.48.quibus subtractis ex tota.c.o.quę est partiū.60.mi.18.reliqua erit nobis.h.o.partium.22.min.30.capiendo postea radicem quadratā summæ quadrati.n.h.cum quadrato.h.o.veniet nobis.n.o.partium.25.min.12.talium qualis.n.h.est partium.11.min.27.si igitur ad.o.n.tanquam partium.25.min.12.respondet.n.h.partium.11.minuti.27.linea.n.h.ad.o.n.tanquam partium 100000.respondebit part.45436.tanquam sinus anguli.n.o.h.qui angulus erit gra.27.minut.1.subtracto postea hoc angulo ab angulo.c.o.i.graduum.40.minut.55.remanebit angulus.n.o.i.graduum.13.minut.54.inter lineam contingentiæ, & lineam primæ stationis in eiusmodi situ. Et ideo Mars accesserit ad lineam.o.c.veri motus epicycli. Sed quia linea.o.i.contingentiæ, propter motum centri epicycli, in dictis diebus.69.confecerit gradus.36.minut.13.(præsuppošita semper eadem distantia.o c.quamuis nonnulla sit differentia, quam nunc prætermittemus) & Mars in dicto tempore retrogressus fuerit per dictum angulum gra.13.mi.54.quibus deductis, ex.36.& min.13.restabunt gra.22.min.19.itaque in diebus.69. Mars promotus fuerit à primo situ gra.22.min.19.aut circiter, prius quam retrogradatio eius incipiat esse apparens.




Nunc à prima statione vsque ad lineā veri motus epicycli sunt gra.16.min.51.ipsius epicycli, vt supra vidim⁹ quos Mars transit in diebus 36.aut circiter ad rationem min.28.in singulos dies, quo tempore centrum epicycli, in tali distantia à cētro mundi conficeret gra.18.mi.54.ad rationem min.31.cum dimidio in singulos dies, quib⁹ deductis ex gra.27.min.1.anguli

guli.c.o.n.remanebunt gra.8. min. 7. pro numero dimidiæ retrogradationis quum Mars erit in linea.o.c.veri motus epicycli. Quibus gradibus.8. min. 7. subtractis à grad. 22. minu. 19. per quos Mars progressus erat, supererunt grad.14.minut.12. quibus ipse in media retrogradatione existens in linea o.c.veri motus epicycli promotus erit à principio primi situs. Quod cum eo concordat quod supra diximus circa hos gradus.14.min.12.vltra primum situm in spatio dierum.105.vt supra, ad tãtum enim ascendunt.69.et.36.nunc fingendo q Mars pergat in suo motu composito qui constat ex his duobus circulis, vi eccentrici, & epicycli ( quanquam vt dixi omittimus illam summam subtilitatem seu scrupulositatem cõtinuæ inæqualitatis distantiæ centri epicycli à centro mundi, & præterimus etiam irregularitatẽ eius circa centrum mundi, propter regularitatem eius circa centrum æquantis, atque etiam missum facimus motum epicycli recti à sua media auge ) fingendo inquam, quod Mars dicto motu suo pergat, vsque ad punctum secundæ stationis, præteribunt alij dies.36. vt prius, quibus iunctis cum. 105. fiet summa dierum. 141. & Mars retrogressus erit per alios grad.8. minut. 7. quibus subtractis a gradibus.14.min.12.per quos progressus erat supererunt.gra.6.min.5. quibus ipse Mars in fine suæ retrogradationis promotus erit à primo loco vnde moueri cœpit. Inter hanc igitur secundam stationem lineæ.o.u.d.& lineam.o.t.secundæ contingentiæ,Mars diebus.69.vt prius,confecerit gradus.32.min.14.sui epicycli,& eodem tempore linea contingens.o.t.ambulauerit gra.36.min.13.vt prius,à quo itinere subtracto angulo secundo.d.o.t.graduum. 13.min.54.supererunt gra.22.min.19 vti prius,quos Mars ambulauerit directè & apparenter, quibus additis ad gra.6.minut. 5. quibus Mars progressus erat à principio motus, fient gra.28.min.circiter.24. quibus processerit à priori loco in diebus.177.idest in.141.et.36.qui sunt ferè menses.6.Itaque Mars partim sursum partim, deorsum ambulans detentus erit mẽsibus 6. in grad.28.Zodiaci, atq; si finxerimus quod epicyclus moueatᷓ versus oppositum augis,lõgior erit mora planetæ in eiusmodi duodecatemorio,propter augumentum æquationis argumenti. Itaque probata à nobis est possibilitas huius moræ Martis. Quod quidem mihi sufficere videtur nõ modo tibi, sed etiam cuiuis, qui harum scientiarum principia teneat. Neque enim nũc docere volo eos qui




in

in ijs ſunt conſumati, nec curam mihi ſuſcipere erudiendi imperitos. Satis igitur ſit oſtendiſſe, quod qui ſcripſit Martem commorari poſſe tam multos menſes in vno ſigno, non impoſſibilem rem tradidit. Immo per obſeruationes huius veritatis millies factas, Aſtrologi fecere ſupradictas ſuppoſitiones neceſſarias ad reducẽdum in ſuas cauſas, & ad regulam, eiuſmodi veriſſimos effectus.

Non oportebat autem ſcriptorem harum animaduerſionum tantopere eiuſmodi mora commoueri, ſed cogitare ꝙ fortaſſe calculi facti fuerunt eo tempore quo miſer Mars à Vulcano rete vinctus erat. Vnde cum non ita celeriter ſe expedire poſſet iter eius ſegnius peractum fuit. Aut ꝙ quũ vulneratus fuit in bello Troiano, vis eius & agilitas per aliquantulum temporis imminuta fuit. Atque ſi hic etiam intellexiſſet eum aliquãdo fuiſſe in poteſtate Othi, & Ephialtis vinctũ & carceri incluſum mẽſes tredecim, dum ab Eribea ſolutus fuit, vt tu, antiquos ſequens, eleganter ſcribis in illis tuis pulcherimis dialogis. non exiſtimaſſet, credo, tam abſurdum quod alius eũ detinuiſſet ſex aut ſeptem menſibus, ſed operam dediſſet vt a te intelligeret quid ſibi vellet tam longa captiuitas.

Sed vt ad rem redeamus. Idem pag.4. ait, quod verus motus Martis diſtat à medio circiter dies.8. ſupponens medium motum eſſe dierum.683. ꝙ etiam falſum eſt. Sed vtcunque ſit, fallitur. Solet enim periodus veri motus Martis eſſe dierum circiter.708. modo paulo plus, modo paulo minus, & interdum poteſt etiam eſſe multo breuior, ſicuti erit à die.3. Decembris anni. 1593. vſq; ad initium Iunij. 1595. Tunc enim erit tantum dierum.545. & non quidem ſine ratione, nam dicto initio Decembris Mars paulo ante cæperit eſſe directus, cum centrum epicycli erit circa medium Tauri, & eius ſtella in principio Arietis & initio Iunij. 1595. Mars parum diſtabit ab initio retrogradationis, regreſſus tamen ad initium ipſius Arietis, & centrum epicycli erit circa medium Aquarij, in cuius ſigni medio, hac ætate reperitur oppoſitum augis, & in quo ſitu, æquationes argumẽti ſunt, quam maximę eſſe poſſint, quum centrum epicycli circuiuerit ſolum circiter tres quartas totius ambitus, & Mars circuiuerit per partem ſuperiorem epicycli circiter gradus.252. Hoc autem dico, vt oſtendam poſſibilitatem huius eius extraordinariæ velocitatis. Nam quicunq; voluerit poterit certior fieri, per calculum partium motus Martis.

Vbi autem poſtea idem author miratur interualla, quæ ponũtur inter coniunctiones Iouis, & Martis in eodem ſigno, eaq́; vocat errores maximos, oſtendit ſe non rectè conſideraſſe motus eorum. Et præcipuè primum miratur ꝙ inter annum. 1528. et. 1553. Iupiter & Mars nunquam coeant in Leone, cum hæ duæ coniunctiones inter ſe diſtent ann. 25. afferens pro ratione, quod hæc duo ſydera, altero quoque anno coniunguntur, ſic dicens.

» Qui ſciet has duas ſtellas ſecundo quoque anno inter ſe coniungendas, mirabitur
» quomodo non poterunt numeratores, huiuſmodi animaduertere errores.

Et præter hanc rationẽ fortaſſe ẽt conſideraui t, ꝙ in dicto temporis interuallo Iupiter ſẽper fuit in Leone, vt ann. 1540. et. 1541. Mars aũt in eo ſæpe fuit. Vnde impoſſibile ei videtur eos non conueniſſe in dicto ſigno. Idemq́; dici poteſt de alijs coniunctionibus eorundem planetarum, atque has differentias temporum inter dictas coniunctiones ipſe tribuit erroribus calculorum Ephemeridum, non autem tabularum, vt ſupra dixit. ſed neſcio quare vellet dictos planetas coire in Leone, ſi quum Iupiter in eo erat anno. 1540. et. 1541. & in eo deambulabat, Mars interea erat mõ in Libra, modo in Scorpione, Sagittario, Capricorno, & alijs ſignis vſq; ad Cancrum, in quo cum repertus fuit anno. 1541. cogitans congredi cum Ioue in Leo

ne,

ne,comperit eum inde aufugiſſe. Idq́; fortaſſe, Iupiter data opera fecit, vt huiuſmodi Aſtrologos in admirationem induceret.

Idem dico de alijs coniunctionibus horum duorum.

Quod poſtea ait, eos ſecundo quoque anno coniungi, animaduertendum eſt, qa (vt iã dixi) duæ ſunt ſpecies coniunctionum, quarum vna eſt linearum eorum mediorum motuum, altera corporum eorum, ſaltem in longitudine, cum ambo inueniuntur in eodem circulo, qui tranſit per polos eccliptica, nam eos inueniri in eadẽ linea recta trãſeũte per centrum mundi, rariſſimum eſt. Atque coniunctio ſupra dictarum linearum vocatur media, & inter Iouem & Martem fieri ſolet ſpatio dierũ. 816. cum dimidio, aut circiter. Altera dicit vera, ſiue apparens, & irregulariſſima, quæ quidem non ſeruat tempus determinatum. Quare quamuis altero quoq; anno coniungantur, & Iupiter duodenis annis tranſeat per totum Zodiacum, non ideo neceſſe eſt, vt in ſpatio. 24. annorum coniungantur in ſingulis ſignis, nunquam in eo deficientes, vt ipſe credit loquens de veris coniunctionibus apparentibus, eo quod ſint irregulariſſimæ, vt dixi.

Atque ſi quis velit inuenire periodum coniunctionum mediocrium horum duorum planetarum, ita faciendum erit. Sumat periodum motus mediocris Iouis, quę eſt dierum. 4328. & Martis, quæ eſt dierum. 687. in quo tempore Martis, Iupiter ambulat gra. 57. min. 8. & diebus. 30. conficit. grad. 2. minut. 29. & ſecun. 23. ad rationem gra. 360. in diebus. 4328. Mars verò ad rationem graduũ. 360. in diebus. 687. ſingulis. 30. diebus conficit. gra. 15. mi. 43. ſecũ. 14. vnde differentia inter eos eſt graduum. 13. mi. 15. ſecũ. 51. per quam diuidendo productum graduum. 57. min. 8. in dies. 30. obueniẽt dies. 129. & duæ tertiæ. quibus addendo periodum Martis fient. 816. cum dimidio, aut circiter. Atque hęc eſt periodus infallibilis mediarum coniunctionum Iouis cum Marte.

Nunc venientes ad tabulas Animaduerſionum, videbimus hæc mirabilia eius, in quo conſiſtant & vbi ſint tam multi inſignes errores.

Primum igitur neminem latet quod calculus Saturni, à Leouitio editus, difert à calculo Stadij circiter gra. 2. aut. 3. cum Leouitius faciat eum progredi per tãtum interuallum, modo plus, modo minus, & ſimiliter Iouem. ſed longe minori differentia, & ſępe gra. 1. minus, atque in alijs planetis differunt, modo plus, modo minus. Huic igitur mirũ videtur, quod vnus ex his calculatoribus detineat Saturnum pluribus menſibus in vno ſigno, & alter in alio, non animaduertens dictam differentiã eſſe eius rei cauſam. Miratur item, quod vnus ex is faciat Saturnum morari paucis menſibus in vno ſigno, alter vero eum ibi detineat integris annis. Vt exempli gratia, verſus finem ſuæ tabulæ Saturni, dicit quod Leouitius eum carceri includit in geminis annis. 2. menſe vno, & diebus. 9. Stadius vero clementior eum liberat intra menſes. 3. & dies. 14. Sed hic non cogitat, quod Stadius facit eum ingredi in geminos anno. 1559. die. 10. Iunii, & ambulare directum vſque ad diem. 6. Septembris, eiuſdem anni gra. 6. min. 34. eumq́; poſtea retrogradum inde exire die. 22. Decem. eiuſdem anni, cum ingreditur in Taurum, vbi partim retrogradus, & partim directus manet vſque ad diem. 20. Februa. 1560. rediens poſtea in geminos, in quibus manet vſque ad diem Iunii. 1561. & inde ingreditur in Cancrum, ambulatq́; directus. gra. 4. min. 59. vſque ad diem. 4. Octob. Vnde retrogradiens rurſum intrat in Geminos die. 28. Decemb. eiuſdem anni, atque ibi partim retrogradus partim directus manet vſque ad diem. 12. Apr. 1562. itaque in pluribus vicibus facit eum morari in Geminis dies circiter. 816. ideſt circiter menſes. 27. ſumpſit autem hic ſcriptor bre

uiſſimam

uissimam moram causa comparationis cum calculo Leouitij, vt faceret differentiam apparere maiorem. Tamen in quouis dictorum temporum nunquam inuenietur Leouitius differre à Stadio plus gradibus tribus integris. Idem fecit in multis alijs locis dictorum virorum eos conferens tum in Saturno tum in Ioue, & Marte, putãs magnum esse errorem, ꝗ planeta non perambulet totum signum, in quod est ingressus vel directus vel totum retrogradus. Atque hæc opinio similis est superiori de coniunctionibus veris Saturni, & Iouis, vbi dicit quod nunquam coniunguntur in vno signo alterius triplicitatis, nisi perfecerit coniunctionem in omnibus signis primæ triplicitatis. Verum vt supersedeam vlterius disputare, mihi videtur, quod hactenus dixi, posse tibi satisfacere, quod attinet ad sciendam sententiam meam super dictis Animaduersionibus latinè scriptis. Hoc tamen non prætermittam, ꝗ hic non animaduertit, népe ꝗ differẽtiæ locorum planetarũ quæ sũt inter ephemeridas Leouitij & Stadij, euenere, quia vnus supputat cũ radicibus, & fundamẽtis Alfonsi alter verò Reinoldi ex Copernico recentius obseruatis, ita idem euenire poterit futuris temporibus, si supputati fuerint dicti motus, & loci cum recentioribus obseruationibus cum impossibile sit tam subtiliter, tanq́ue perfectè supputare loca & mot⁹ eorum, vt lungo interuallo temporis non comperiantur in eis aliquæ differentię, cuius rei remedium est semper sequi recentiores obseruationes & tabulas.

Atque vt tibi satisfaciam etiam circa alia scripta vulgari lingua edita mensibus. 4 post latina, etsi intelligere potes, qualia possint esse alia eius scripta, ex ijs quæ supra dicta sunt, atque etiam ex eo, quod dicit se misisse multa exempla suarum Animaduersionum in varias terras, illis qui profitentur has scientias, aut earum studiosi sunt, nec quenquã inuenisse qui ad tã laudabilem prouinciam motus sit, nec vidisse, ꝗ aliquis responderit eius rationibus; laudabilem prouinciam, autem puto, ꝗ intelligat correctionem ephemeridum, verens, ne culpa calculatorum, qui eas sumpsere e tabulis, tam differentes sint, vt quibusdã locis cap. 1. Videtur, & præcipuè vbi sic ait.

» Perche essendo impossibile alli studiosi di dette scientiæ di non seruirsi delle
» ephemeridi, maggiormente a quelli che non sanno seruirsi delle tauole, e cono-
» scendo d'incorrere in errori senza hauerui altro rimedio, sarebbono forzati di ab
» bandonare i studij loro.

Quanquam circa finem dicti capitis redeat in meliorem viam & aduersetur sibijpsi vbi sic ait.

» Che poi essi possessori della scienza, &c.

Etiam aperiam tibi, quæ mea sit de ijs sententia.

Hic igitur in scriptis Italicis, vt morderet aliquem ex ijs, qui eius superiora scripta non laudauerant, occasionem capit aperiendi aliquos illius errores, per editionem collationis quorundam calculorum a se collectorum illius, atque etiam aliorum, cuius calculi sunt in secunda, & septima figura. Sed prius quam veniamus ad defensionem harum duarum figurarum vide obsecro quam alienum ei videatur, quod alij dixerint differentiam ephemeridum non esse magni momenti, non afferens respectum vllum, qui enim dixerunt eiusmodi differentiam non esse magni momenti id dixerunt habito respectu ad signum in quo est planeta, vt (exempli gratia) quamuis in ponendo loco Saturni Leouitius interdum differat à Stadio gradibus. 3. quum vterque eum ponat in eodem signo, tunc id nullius momenti est, & sic in coniunctionibus aut alijs aspectibus duo, aut. 3. gradus non faciunt alterationem sensibilem, cum virtus coniunctionum, & aspectuum insit, & duret per multos gradus ante aut post ipsum punctum. Nec quicquam tamẽ est qui dubitet quin præstaret scire subtiliter ipsum punctum. Nec vnquam fuit aliquis qui negauerit re-

Hh fer-

ferre vt anni directionum correſpondeant gradibus æquatoris. Et præterea in ephe
meridibus videntur certè motus & aſpectus luminarium,quamuis inſit differẽtia mi
nutorum. Nam non differunt gradibus, præter ſitum parum diſtantem à vero om-
nium planetarum , quorum cognitio in cœlo, quamuis circa eorum locum error eſ-
ſet gra. 1 0.tamen in hoc prodeſſet,& tempus aſpectus eorum,etſi non diei præcisè ,
quia influentia eiuſmodi aſpectuum,præterquam Lunæ durat multis diebus, & non
vno tantum.præterquam quod ipſæ ephemerides oſtendũt nobis tempus ecclipſiũ,
in quo certè non differunt nec diebus nec multis horis, &itidem multa alia.

Non ſunt igitur contemnendæ ephemerides, nec habendæ pro re nullius pretij ,
vt hic ait.

Quod attinet ad illa alia,quæ hic vocat errores ephemeridum,tam de apparenti
coniunctione Saturni cum Ioue in ſignis alterius triplicitatis prius quam perẹgerit
præcedentem, quam de faciendo currere Saturnũ , & de retinendo Ioue, de detinẽ
do Marte.6.aut.7.menſibus in vno ſigno , de Marte,& Ioue non coeuntibus ſingu-
lis.24.annis in quolibet ſigno , & eius generis alia , minime verum eſt quod ſint er-
rores,quamuis huic præbuerint occaſionem toties errandi .

Comparatio poſtea inter eius calculos ſumptos partim ex tabulis Iunctini,& par
tim ex ephemeridibus Stadij tanquam calculis Copernici,& calculos figurarum ſu-
per eis poſitarum ſupputatarum à diuerſis per ephemeridas Alfonſinas, etiam pro-
poſita ab eo eſt ad oſtendendum magnam & monſtruoſam differentiam , vt ait cap.
2 .vbi miratur,quod cum ex communi ſententia calculi Copernici meliores ſint,cal
culatores dictarum figurarum potius eos ſumpſerint à tabulis Alfonſi, quam Coper
nici. Quæ admiratio quam aliena ſit, conſiderandum permittam cuiuis intelligen-
ti harum facultatum ,cum ſæpe accidere poſſit . vt cum aliquis velit ſcire ſolum vni
uerſalia alicuius geneſis , ſiue natiuitatis,cum non inueniantur ephemerides Coper-
nici,ſed tantum Alfonſi, calculator vtatur tantum ephemeridibus, quas inuenit , tũ
cauſa vitandi tædij calculi tabularum, qui magni laboris eſt, pręcipuè in tabulis Pru
tenicis Reinoldi.tum quia ſuperflua ei eſt ſumma ſubtilitas,cum non curet laborare
circa directiones vt factũ eſt pro ſecũda figura ab hoc propoſita,quæ erat anni.1 5 5 1
quo non inueniebãtur ephemerides Copernicæ , quæ non editæ ſunt ante annum
1 5 5 4.præter quam quod ille nobilis vir pro quo ſupputata fuit dicta ſecunda natiui
tas dubitabat de anno, vt hic ſimiliter ſcit. quare potuiſſet perdi tempus ,& labor,ſi
ſupputata fuiſſet per tabulas Reinoldi , nam Iunctini tabulæ nondum editæ fuerant.
Calculus poſtea ſeptimæ figuræ,qui erat reuolutio dictæ ſecundæ natiuitatis,duab⁹
de cauſis non factus eſt per tabulas prutenicas, primum,quia eius anni. 1 5 8 0 . non
inueniebantur amplius ephemerides Copernicæ . Ephemerides enim Stadij in-
cipiẽtes ab ann. 1 5 5 4. definũt ann. 1 5 7 6.& cõtinuatæ poſtea quæ perueniunt vſque
ad annum. 1 6 0 0.non peruenere ad manus calculatoris ante hunc annum. 1 5 8 1. Al-
tera ratio eſt, quia in reuolutionibus, quoniam in eis non fiunt directiones, non po-
nuntur à doctis,ne minuta quidem.quare non ſolum non curant eas ſupputare per
tabulas,ſed nec exquiſitè quidem per ephemeridas. Calculi poſtea ab hoc ſumpti
ex tabulis Iunctini,& poſiti ſub dicta ſecunda figura,adeò rectè facti ſunt , vt cum ſe
cundum ipſas tabulas oporteat Saturnum eſſe circa. 3 2 .minutum gradus. 2 3 .Aqua-
rij,ipſe eum ſcribat in gra. 1 1 .mi. 3 . dicti ſigni.Iupiter ſimiliter qui ſecundum dictas
tabulas inuenitur circa finem gradus. 5 .Cancri,ab eo ponitur in min. 2 8.gra. 1 9.eiuſ
dem. ex quibus planetis Saturnus in figura poſitus eſt in min. 2 7.grad. 2 3. Arietis,Iu
piter autem in min. 3.gra. 6.Cancri, Vnde ſecundum verum , inter calculum Alfon

ſi, &

ſi & Iunctini in Saturno non erat differentia plus quam minu.5.& in Ioue min.4.ſed ſecundum calculum huius in Saturno fuiſſet differentia gra.11.minu. 54. & in Ioue grad.13.min.35. Atque hæ ſunt quidem differentiæ magnæ, & monſtruoſæ vt ipſe eas vocat, vt etiam eſt illa Veneris,& Mercurij inter tertiam figuram, & eius calculum ſumptum,non quidem à tabulis laborioſis,ſed à ſimplicibus ephemeridibus Stadij, quæ differentia nõ eſt quidem paucorum graduum,cum ſit tertiæ partis cœli in quolibet dictorum planetarum. Huiuſmodiq; monſtra certè non ſunt orta à tabulis ſiue ephemeridibus diuerſis,ſed ſunt partus huius authoris.

Pergens poſtea aſſiduè bonus hic vir hominibus dare ſpecimen doctrinæ ſuæ aperiendo (vt conatur) aliorum errores, proponit duas differentias inter primam figuram,& ſuum calculum ſuppoſitum Saturni,& Iouis. Primum de Saturno ait, ꝙ cum differentia ſit gra.1.min.30.oſtendit in directione, ꝙ accidens ſit euenturum anno vno,& menſibus ſex ante,aut poſt, quaſi eiuſmodi differentia eſſet partium æquatoris,ſicuti eſt partium Zodiaci. Idem dico de differentia Iouis. Quod quidem, manifeſtum eſt inditium ſcientiæ ſuæ, & quantum ea intelligat de quibus loquitur.

Quod poſtea attinet ad differentiam inter Copernicum & Alfonſum, circa Solé, nullus eſt harum ſcientiarum peritus,qui id neſciat,& ſimiliter de differentia ſitus cęli in reuolutionibus annuis.

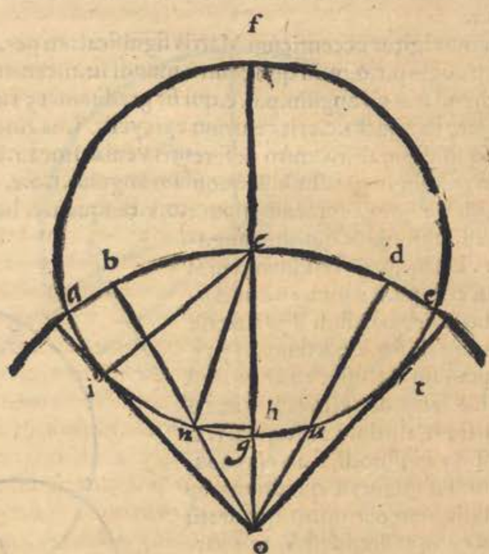
Quod vero ait ſeptimam figuram malè ſupputatã fuiſſe,ſi non eſt maximus certè non eſt minimus monſtruoſorum eius errorum. Vbi itidem videri poteſt, quam alienus hic ſit ab hac ſcientia. Nam ſi ſaltem curaſſet ſibi ab aliquo ſupputandum locum Solis per tabulas Alfonſi in inſtanti minutorum.36.pomeridianorum,certior factus eſſet quod in illo puncto Sol inueniebatur in minu 54.grad.11.Geminorum, ideſt prætererat gra.10.cum min.54. vel ſi curaſſet ſibi inueniendum tempus, per dictas tabulas cum grad.10.min. 54. Geminorum vt faciendum eſt, ſequendo tamé Alfonſum, & non per calculum Solis poſitum in ephemeridibus, vt parum periti facere ſolent,vidiſſet ꝙ inuenta eſſent min.36.pomeridiana. Leuis tamen occaſio huic fuit ſuſpicandi eiuſmodi tempus eſſe falſum, quod viderit in illa figura Solé poſitum eſſe cum gra.11.& non cum gra.10.min.54.non animaduertens ita notatum fuiſſe Solem vt omnes alios planetas, ſcilicet ſine minutis, quum, vt dixi,in reuolutionibus non adhibeatur tanta ſcruploſitas.

Quod deinde ait, in illa figura Solem poſitum eſſe in decima domo,& non in. 9. id relinquam iudicio eorum qui ſciunt numerare domos,ſaltem poſuiſſet authoritate ſua Solem in dicta decima diuersè ab exemplo ei dato ab amico, vt oſtenderet ſe dicere verum,vt in ſecunda figura diſcrepat ab ipſo exemplo in collocando Leone, Virgine,& Libra,& Scorpio, quos malè locauit, & ſi alii bene ſe habent.

Atque quod hactenus à me dictum eſt, ſatis ſit ad intelligẽdum quale ſit reliquũ dictæ eius diſputationis. Si enim velim pergere notare omnia eius errorum loca,eſſet mihi inanis labor,& tibi nimia moleſtia. Et quamuis non defuerint præſtantiſſimi viri,qui viſis eius ſcriptis familiariter eum monuere,& tu ipſe,vt audiui,cum inſtrumẽto theoricę in manibꝰ ei oſtẽderis quo mõ Mars poſſit morari amplius ſex mẽſibus in vno ſigno.& præterea cum iam ab initio Taurinum aduenit, mecum communicauerit illa ſua prima ſcripta, egoq; cum monuerim,quod in varijs rebꝰ fallebatur,diſſuaſerimq; ne ea imprimenda curaret, quia nullum honorem inde referret, eum hortans,vt potius alijs rebus operam daret,atque ei dixerim quod ad animaduerſiones differentiarum ephemeridum attinet,quod id iam oẽs animaduerterant. Mihi reſpondit ſe decreuiſſe illa edere, vt poſtea fecit, & tot admonitionibus non

acquiescens, die.11.Augusti edidit chartam illam impressam inuitans ad disputa-
tionem quotquot adhęrerent contrariæ sententiæ, volens sustinere Marté non pos
se commorari in vno signo amplius duobus mensibus, supponens partem princi-
piorum ab omnibus admissorum,& in fine paginæ exponens modum, quo vtitur ad
probationem suæ intentionis. Puto autem quod secum ratiocinabatur de Marte,
vt fecit de Saturno in scripto latino, hoc modo. Si Mars in duobus annis ambulat
per omnia.12.signa,necesse est igitur,vt in mensibus duobus ambulet per vnum si-
gum, cum menses.2.sint duodecima pars annorum duorum. Sed ibi statim in ipso
initio committit errorem graduum ferè.7.dicens, quod medius motus Martis inue-
niebatur signorum.4.& gra.17.cum eo tempore dictus medius motus non esset reue
ra plus quam sign.4.grad.10.mi.36.verum hoc ad ea,quæ sequuntur exigui est mo
menti.Is postea particulatim colligit medium motum Martis ad diem. 29. Mai an-
ni.1514.quem ait esse signorum.9.gra.27.min.53.& tamen reuera erat tantũ signo
rum.9.gra.21.mi.29.sed missum faciamus etiam hunc errorem tanquã à primo pen
dentem. Cum deinde ibidem ponit centrum epicycli, similiter errat, nam
centrum epicycli nunquam poni debet vbi est linea medij motus,nisi sit in auge,aut
in opposito augis eccentrici,quia debebat collocare ipsum centrum tãto post lineã
medij motus,quanta erat æquatio centri,quia medium centrum Martis tunc erat mi
nus signis sex,& aux eccentrici eius erat in sexto minuto grad. 16. Leonis. Tamen
hoc etiam leue est. Præsupponamus igitur quod centrum epicycli esset in grad.28
Capricorni, vt ipse credidit, idest gradibus.7.vlterius quam erat reuera. Ait postea
se comperisse Martem ambulasse signa.4.& grad.22.eius epicycli,sed non explicat
an intelligat de argumento medio, an de vero,quod vocatur æquatum,nam si intel
ligatur de medio,hoc esse non potest,cum mediũ esset signorum.4.gra.24. mi.35.
sed si intelligatur de vero, vt iure credendum est (alioquin etiam errasset) cer-
tè falsum est. Ná, verum,erat signorum.4.grad.29.minu.39. Itaque Mars non di-
stabat à linea veri motus epicycli amplius gradibus. 30.& minu.21. ipsius epicycli,
& æquatio argumenti secundo correcta erat gra.44.minu.2.à quo subtracta æquatio
ne centri, quæ erat gr.5.minu.4. (cum centrum epicycli deberet tanto spacio esse
post lineam medij motus quantum supra dixi) supererant gra.38. minu.58. adden-
di gradibus, & minu.medij motus, qui cum reuera essent grad.21.& minu.29.Ca-
pricorni, perueniebant ad minu.27. grad.1. Piscium. Sed præsupposito secundũ
ipsum, quod medius motus esset grad.28. Capricorni, & quod Mars esset non solũ
vbi hic ait, sed etiam in prima linea contingentiæ epicycli,idest in prima linea ma
ximæ æquationis argumenti,& præsupposito etiam quod dicta æquatio esset æqua-
lis illi, quam haberet ad medium Aquarij scilicet grad.47.quum centrum epicycli
est in opposito augis,manifestum est, quod eiusmodi linea contingentiæ non transi
ret vltra grad.15.Piscium, & tamen hic ait, quod linea veri motus Martis vadit ad
grad.16. Arietis. vnde oporteret, quod æquatio argumenti esset plus quam grad.
78. Quod si verum esset, &.o.c.etiam esset partium.54.secundum distantiam pro-
ximiorem centro mundi, semidiameter epicycli esset eiusmodi partium. 52. minut.
49.& quum Mars esset in. g. idest in opposito veræ augis epicycli,dum centrum epi
cycli esset in eiusmodi distantia à terra, distantia. o.g.idest à terra ad Martem non
esset plus, quam vna sola pars ex dictis, cum minut.11. cum partes. 52. minu. 49.
ad.54.sint vt sinus anguli gra.78.qui est partium.97814. ad sinum totalem partium
100000. Nam iam supra dixi, quod triangulus.o.c.i.est rectangulus. Hinc seque-

retur




retur, quod in interuallo.o.g. vnius partis, & min. 11. respectu.o.c. partium.54. Collocaretur semidiameter terrę cum spissitudine aeris, ignis, cęlorum Lunæ, Mercurij, Veneris, & Solis, pręterquam quod vt inter Solem, & terram sunt circa 605. diametri ipsius terræ, inter terram, & Martem cum esset in auge sui epicycli, & epicyclus in auge eccentrici, inuenirentur circa. 60000. diametri eiusdem terræ, & tamen ea distantia siue interuallum non potest continere. 5000. diametri terræ. Et quod plus est, hic tam vastum facit hunc suum epicyclum, vt ambiente Marte per inferiorem eius partem, necesse ei esset manere in vno duodecatemorio multo plus quam.7.aut.8.mens. vnde hic multo magis miraretur quam prius. Hinc cernere licet quam rectè facti sint hi eius calculi.

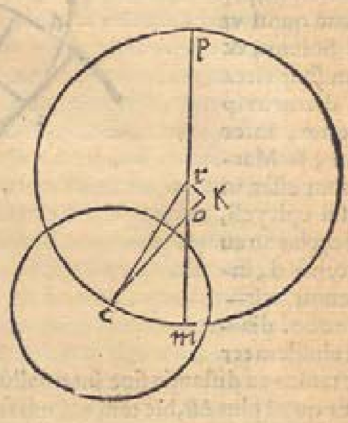
Vt autem etiam hinc aliqua vtilitas capiatur(prætermissis inconuenientibus vna cum falsis suppositis huius) Videamus ordine scientifico vbi poterat esse verus locus Martis, aut vero proximus, die.29. Mai anni. 1514. quem hic exempli causa sumit. Idq; tam ad defensionem tabularum Alfonsi, quam ephemeridum ex eis collectarum. quæ quidem exactæ sunt, vt quisque peritus facile videre poterit, non autem calculatæ à tam stupidis hominibus, vt à vero aberrent etiam gradibus. 46. vt hic ait se depræhendisse.

Primum igitur supponemus eosdem illos terminos, quos ipse nec debet, nec potest negare, præter ea quæ supra supposita sunt, nempe quod semidiameter epicycli sit partium.39.minu.30.& eccentricitas partium.6.talium qualium est semidiameter deferentis diuisus in.60.& quod dicto tempore aux eccentrici Martis esset circa minutum. 5.grad.16.Leonis, scilicet graduum.135.min. 5. & quod linea motus medioeris esset circa minu. 30 .gradus.22.Capricorni, & quod verum centrũ Martis esset grad.151 minut. 20.& quod argumentum verum esset grad.149.minu.39. atq; ita ostendam, neque tabulas, neque ephemerides errare, ne quidem vno gradu, ac ne quidem multis minutis, non modò tam monstruosa differentia, vt ipse ait.

Quare primum nobis scientificè inueniendum est, quanta esset distantia. o. c. præ-

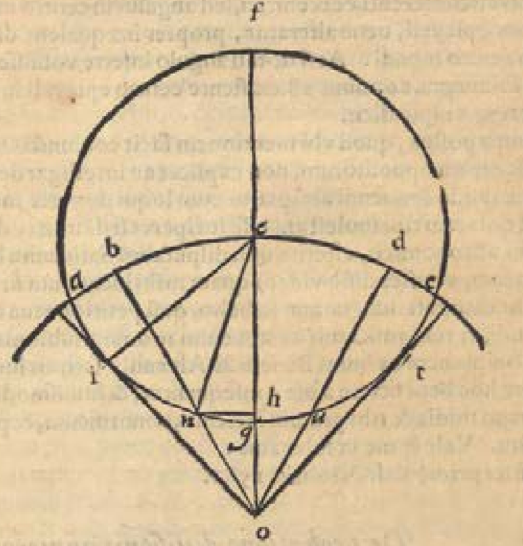
præciſe ideſt interuallum inter centrum mundi,& centrum epicycli Martis in huiuſmodi ſitu.

Fingemus igitur eccenticum Martis ſignificatum per.p.c. m. cuius centrum ſit. r. & lineam augis.p.r.o.m.in qua centrũ mundi ſit.o.centrum autem verum epicycli, comprehendatur ab angulo.p.o. c.qui ſit graduum.151.min.30.ſecundum ſuppoſitum. Quare in puncto.c.erit centrum epicycli. Imaginemur ergo.c.o.productam à parte.o.quouſque ab.r.centro deferentis veniat linea.r.k.perpendiculariter,faciens angulum rectum in puncto.k.& quoniam angulus.r.o.c. datur nobis graduum. 151. min.30.ideo cognoſcemus angulum.r.o.k.tanquam reliquum ex duobus rectis, qui erit gra.28.min.30. & ſimiliter angulum. o. r. k. tanquam reſiduum vnius recti, qui erit gra.61.min.30.cuius ſinus ideſt.o.k.erit partium 87881.et.k r.vt ſinus anguli.r.o k.partium.47715 talium qualium.o.r. eſſet 100000. ſed vt.o.r.eſt. 6.latus.o.k.erit.5.& min.16 et.r k.partium.2.min. 52. & quia. r. c. eſt partiũ.60.eiuſmodi, ſi ab eius quadrato ſubtractum fuerit quadratum ipſius.r.k.reliquum erit nobis quadratũ ipſius.k.c.cuius radix,ideſt.k. erit partium.59.min.56.à qua.c.k.ſubtrahendo poſtea.k.o.partium.5.minu.16.remanebit.o.c.partium.54.min.40. pro diſtantia quæſita.




Fingamus poſtea epicyclum. f. n.g. in quo argumentum verum graduum. 149.minu.39. ſit arcus.f.n.vbi Mars inueniatur in.n.per quem punctum tranſeat linea.o.n.veri motus Martis. Deinde inueniamus angulum.c.o.n. æquationis argumẽti,modo iam dicto,ideſt ducendo ſinum.n. h.arcus.n. g. qui arcus tanquam reliquus argumenti veri, iam præſuppoſiti, ex dimidio circulo,erit graduum.30.minu.21.& n.h.eius ſinus partium.50528.ſinus ſimiliter anguli.n.c.h.et.c. h. tanquam ſinus anguli.c.n.h.reſtantis ex uno recto grad.59. minu.39.erit partium.85295.taliũ qualium.c.n.ſinus totus eſſet partium.100000.ſed vt partium.39.& min. 30. ſinus. c. h. erit partium.34.min.5.et.n.h.partium.19.mi.57.reliquum poſtea.h.o.ex. o.c. partium.20.min.35.quia iam ſupra inuenimus.o.c.eſſe partium eiuſmodi.54.minu.40. vnde.o.n.vt radix quadrata ſummæ duorum.n.h.et.h.o.erit partium.28. minu. 41. talium qualium.n.h.inuenta fuit partium.19.min.57.quæ.n.h. erit poſtea partium, 69552.talium qualium.n.o.partium.100000.& ſumpta dicta.n.h.vt ſinus dictarum partium,dabit nobis angulum.n.o.h.quæſitum gra.44.min.4.qui per tabulas Alfonſi inuentus eſt gra.44.min.2.par huic, vt dici poteſt. Qui angulus gra.44. minu.4. collectus cum angulo veri centri iam ſuppoſito graduum.151.minu. 20. & cum angulo augis eccentrici Martis, ſimiliter ſuppoſitæ grad.135.min.5.dabit nobis ſummam veræ diſtantiæ Martis à principio Arietis grad.330. min. 29. quod aliud non ſignificat, niſi quod Mars inuenietur in minu.29.primi gradus Piſcium. Et Stoflerus in ſuis ephemeridibus ponit eum in.22. minuto dicti primi gradus, cuius differenrentia à tabulis est minut. 5. tantũ, & à meo calculo min. 7. videlicet minima.

Nunc autem nolui sumere ipsum angulum æquationis à tabulis propter duas rationes, primũ quia ne hic quidem repræhensor in hoc voluit credere dictis tabulis. Sed id voluit videre proprijs oculis i sua theorica Martis. Vbi iuenit quod linea.o.n. transit per gra.16.Arietis. Secunda ratio est, vt videatur quod dictæ tabulæ rectè supputatæ sunt, super dictis suppositis.




Sed vt videat quantus sit medius motus Martis die.29. Mai colligit frustatim, q̃ eleganter colligere poterat, vna opera in columellis ipsius medij motus eiusmodi stellæ per eram eiusdem temporis, quæ erat.2.primarum sexagenarum.33. secundarum.32. tertiarum, et.52.quartarum.

Primum deinde suppositum quod scribit, scilicet, quod diameter epicycli sumptus in longitudine media sit signorum.2.& grad.19.vti superfluum est, ita etiam falsum, nam eiusmodi diameter in dicto loco, non occupat ad centrum mundi plus quã gra.66.min.28. idest signa.2.gra.6.min.28.quia proportio.o.c. ad semidiametrum epicycli in eiusmodi loco est ut partium.60.minu.18.ad partes.39.min.30.quę duę lineæ intellectę, vt latera vnius trianguli rectanguli, habebunt pro basi aliam lineam partium similium.72.mi.5. Quæ intellecta vt sinus totus dabit semidiametrum epicycli partium.54798.tanquam sinum subiectum angulo gra.33.min.14. pro medietate illius, quod quæritur.

Nec prætermittenda mihi videtur ratio, qua credere possumus, hunc cogitasse, quod diameter epicycli compleat spatium duorum signorum cum gradibus. 19. quæ quidem ratio alia esse non potest, nisi quod cum iste inuenerit, in comẽtarijs Theoricarum, semidiametrum huiusmodi epicycli esse partium.39.min.30.talium qualium sunt.60.illæ quæ sunt semidiametri huius eccentrici, dictas igitur partes.39 mi.30.hic putauit esse gradus Zodiaci, & propterea dixit diametrũ huiusmodi epicycli esse signorũ duorũ, & gra.19.qui numerus.79.duplus est numero. 39. cũ dimidio hoc autem dixit accidere in longitudinibus medijs, quia si hic intellexisset de proportione horum duorum diametrorum, quæ est ut.120.ad.79. non specificasset locum

cum epicycli,cum ipsa proportio nullo modo alteratur existente epicyclo vbi volueris ipsius circunferentiæ eccentrici,sed angulus in centro mundi, cui subiacet dictus diameter epicycli, bene alteratur, propter inæqualem distantiam centri epicycli ab ipso centro mundi. At si de tali angulo inferre voluisset, iam probaui ipsum cõtinere solum gra.66.minu.28.existente centro epicycli in longitudinibus medijs & non gra.79.vt ipse dicit.

Omitto postea, quod vbi mentionem facit coniunctionum Solis cum Marte augium & earum oppositorum, non explicat an intelligat de veris an de medijs. Nam si ex eius modo loquendi accipiatur eum loqui de veris multum erraret.

Sed quia iam tibi molestum esse inciperet si diutius te detinerem in his contentionibus astronomicis, vlterius non disputabo. Satis enim hactenus explicaui sentẽtiam meam, vt ostendisse videor quam mihi iucundum sit tibi morẽ gerere. In quo etiam hnmanitati tuæ gratiam habebo, quũ petitione tua occasionem mihi dederis efficiendi, vt tum amici tui (amant enim te omnia sublimia ingenia) tum alij, si quã falsam opinionem ex huius Benedicti Altæuillæ scriptis sumpsissent, eã relinquant, & per te hoc beneficium à me consequantur, & huiusmodi occasionem, & iuuandi hominum studia & tibi gratum faciendi,honorificum, & per gratum mihi fuisse intelligant. Vale & me vt soles ama.

Taurini pridie Kal.Octobris. 1581.

## *De probatione diuisionis numerorum.*

### AD EVNDEM.

INter alia quæ à me scire cupis, vir doctissime, hoc vnum est, vt ex literis tuis accepi, vnde sit vt prisci nostri probatione numeri nouenarij potius quam septenarij vsi fuerint, & qua ratione non idem proueniat ex probatione numerorum octonarij,senarij, vel quinarij, aut cuiuslibet alterius: Vnde pariter oriatur quod in partitionis probatione necessum sit probationum euentus multiplicare cum probatione diuisoris, ac eam quæ est producti postea cum probatione fractionis in summam colligere,&c. Ad hæc in primis respondeo, cum aliquoties accidere possit tales probationes nos fallere posse, idq; si in tali summa similis numerus, ut puta septem,aut nouem,plus vel minus æquo iustouè positus fuerit,attamen per raro eueni re potest, vt quis per nouenarium potius quam per septenarium decipiatur. Exempli gratia,ponamus summam esse.100. quam numerus nouenarius vndecies solum ingreditur,at septenarius quatuordecies,vnde quis sępi⁹ ex septenario, hac ratione, quam ex nouenario numero se posse errare facile depræhendet, etsi ex probatione nouenarij magis quam septenarij,vt practici scribũt,duabus de causis errare possimus. Alia tamen ratio mihi suppetit,ob quam credibile est ipsos potius nouenario adiutos fuisse, quam septenario,quæ est ob sui cum velocitatem tum facilitatẽ, neq; enim in septenario est adeo facilis.Nã quamuis, tam vna quam altera aliud non sit, quam numerorum ordines diuidere (si de summis primo loquamur) aut è summa superfluum ordinum colligere, & videre an idem met superfluum ex eadem summa emanet; attamen cum modus, qui in hoc adhiberi potest in nouenario quã in septenario velocior sit, & ob id probationem nouenarij seligunt potius quam septenarij.

Verum

Verum nolo te in ea, quæ falſa eſt, opinione conſiſtere, non idem, & cum octonario, ſenario, vel quinario, aut quouis alio numero poſſe efficere, cum eademmet ratio, quæ in ſeptenario, aut nouenario, ẽt in cæteris perhibeatur. Ponamus exemplũ hos tres ordinum numeros velle ſupputare, quorum primus ſit. 679. ſecundus. 846. & tertius. 935. & illorum ſummã. 2460. nunc maiorem numerum primi ordinis ab octonario menſi, proijciendo, remanebit. 7. deinde maiorem numerum demendo à ſecundo ordine, reſiduum erit. 6. ac ſi idem in tertio ordine fecerimus, erit nobis reliquum. 7. Demum tria hæc reſidua in vnum collecta. 20. efficient, à quibus ſi numerum maiorem ab octonario menſum dempſeris, ſupererunt. 4. & totidem à ſumma. 2460. remanebunt, reiecto maiori numero ab octonario menſo. Atque idem medio quouis alio numero, euenire poteſt.

Cujus ratio tam per ſe clara atque euidens eſt, quod ſi ſummam trium reliquorũ, quæ eſt. 20. à ſumma. 2460. ſubduxeris, remanebunt. 2440. pro ſumma trium numerorum dictorum trium ordinum ab octonario menſorum, cui numero addito. 16. pro maiori numero ſummę reliquorũ, qui ab octonario menſus ſit, ſupererunt. 4. At ſi per ſenariũ experimẽtũ feceris, remanebit. 0. & ſic de reliquis per ordinem procedẽdo.

Verum poſſes ſciſcitari, quare velocius, exceſſus ordinum, potius per nouenariũ, quam per cæteros numeros, prout docẽt practici, inueniri queat, videlicet aggregando prius duas figuras numerorum primæ ſummæ, deinde alias duas. Exemplum ſit primus ordo. 679. colligendo. 6. et. 7. faciunt 13. & cum hæc ſumma ſit duarum figurarum, ſupputantur & ipſæ, è quibus prodeunt. 4. & conſimilis erit probatio numeri. 67. facta per. 9. quod idem eſt, ac ſi quis diuidat. 67. per. 9. ex quo reliqui erunt ſemper. 4.

At quo ratio huiuſce rei perſpicuè dignoſci poſſit, in primis ſciendum eſt, cuique ex ſe cognitum, atque exploratum eſſe, denarium numerum vnitate nouenarium ſuperare, & ex hoc ſequitur, ſex denarios continere in ſe ſex nouenarios, & ſex vnitates.

At ſex vnitates, vna cum. 7. faciunt. 13. & quia in. 13. eſt denarius, igitur in illo erit vnitas ſupra. 9. Quæ vnitas addita ternario, præbet nobis ſuperfluum, per quod. 67. ſuperat. 54. iunctum cum. 9. ſcilicet ſummam. 63.

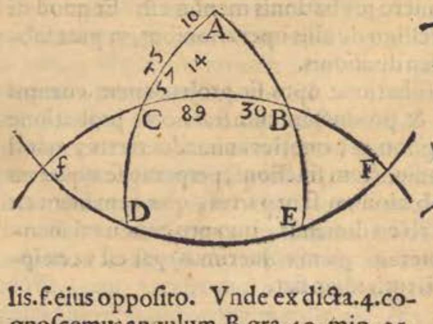
Idem dici non poteſt de octonario, ſeptenario, vel ſenario, & de reliquis, quoniam numerus denariorum, in cæteris minoribus nouenario non præbet illico numerum exceſſus maioris numeri, qui à numero probationis menſus eſt. Et quod dico de probatione aggregationis, idem intelligo de alijs operationibus, vt puta ſubtractionis, multiplicationis, & partitionis ſeu diuiſionis.

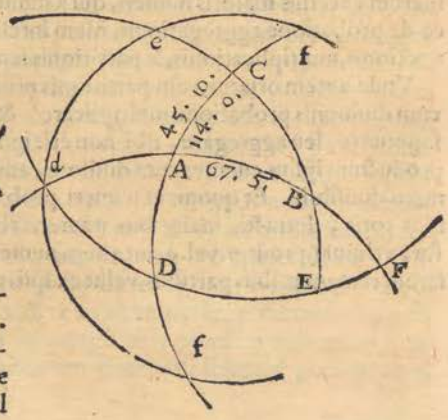
Vnde autem oriatur, vt in partitionis probatione opus ſit probationem euentus cum diuiſionis probatione multiplicare, & productum cum fractionis probatione ſupputare, ſeu aggregare, tibi non erit ignotum, quoties animaduerteris, quod productum ipſius euentus cum diuiſore, adiunctum fractioni, perpetuo ſe æquat numero diuiſibili. Et quoniam numeri probationum ſunt partes, quæ remanent ex ipſis totis, detractis maioribus numeris ab eo dimenſis, quo pro communi menſura vtimur (prout. 7. vel. 9. aut alium numerum, quem voluerimus) par eſt vt ex ipſarum remanentibus partibus, velut ex ipſis totis idem fiat.

## *De falacia operationis triangulorum sphericorum.*

### AD EVNDEM.

QVod diebus præteris tibi significaui, idem nunc confirmo, scilicet sphæricorum triangulorum operationem sæpe nos fallere, vt exempli gratia, si propositus nobis fuisset triangulus. A.B.C. cuius angulus. A. nobis datus esset graduum. 114. mi. 0. & eius latus. A.B, graduum. 67. min. 5. & latus. A.C. graduum. 45. mi. 10. si reliquos angulos cum tertio latere etiam cognoscere voluerimus, ex methodo. 11 primi Copernici propositum obtinebimus. vnde latus. B.C. esset graduum. 89. min. 30. angulus vero. C. graduum. 57. min. 14. angulus autem. B. grad. 48. min. 38. Quare vltimus hic angulus. B. falsus esset, eo quod operatio paruorum triangulorum in causa est, quotiescunque eorum latera tam breuia sint, ut non eccedant vnum gradum, quare ipsorum angulorum veram quantitatem non tribuunt. propterea igitur cum voluerimus veram quãtitatem ipsius anguli. B. oportet post quam inuenerimus angulum. C. mediante arcu. D.E. supponere alium polum in. B. deinde producere. B A. vsque ad. d. et. B.C. vsque ad. e. imaginando. B.d. et. B.e. duas quartas esse magnorum circulorum, extendendo postea. d.e. vsque ad intersectionem cum. A.C. & eũdem ordinem prosequendo, tunc. e.d. nobis ostendet angulum. B. esse gra. 40. mi. 22 quæ erit eius vera quantitas. Cuius quidem rei experientiam possumus etiam facere, hoc modo, esto, exempli gratia, quod nobis datus sit angulus. C. graduum. 57. min. 14. cum latere. A.C. gra. 45. min. 10. & latus. B.C. gra. 89. min. 30. Tunc si ordinem. 11. dicti lib. sequemur, obtinebimus intentum, hoc modo scilicet supponendo in. A. polum, & non in. B. ducendo etiam. A.B. et. A.C. sed. A.B. vsq; ad gra. 90. ducendo postea. D.E. ita quod ab omni parte concurrat cum latere. B.C. producto, vnde tam. f.C.B.F quam. f.D.E.F. erunt semicirculi magnorum circulorum. quare. C. D. nobis cognitus erit gra. 44. min. 50. & sic etiam angulus. D.C.f. gra. 57. min. 14. ex 4. dicti lib. postea habebimus. F.l. gra. 60. min. 54. & angulum. f. gra. 53. mi. 24. aggregatum postea. f.C, cum. C.B. habebimus. f.B. gra. 150. min. 24. qui si a semicirculo dẽptus fuerit, nobis remanebit. B.F. gra. 29. mi. 36. cum angulo. F. cognito cũ sit æqua-




lis. f. eius opposito. Vnde ex dicta. 4. cognoscemus angulum. B. gra. 40. min. 31. qui ferè æqualis est superiori iam inuento, nec ab ipso differt nisi per min. 9. quæ quidem differentia parua est respectu al

terius

terius differentiæ quam supra inuenerimus.

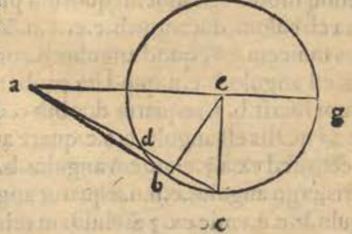
Superius enim dixi non esse ponendum polum in.B.eo quod. B.C.fit gra.89. mi. 30.vnde nobis prodijsset triangulus.f.C.D.trium valde paruorum laterum, quorum latus.C.D.esset gra. o.mi.30.& latus.f.l.gra.o.mi.55.& latus.F.D.gra.•.mi.47.vnde angulus.f.gra.32.min.40.falsus esset, qui quidē postea nobis daret. D.E.gra.45 minu.16.falsum similiter.

## *De paßione circuli hactenus incognita.*

## AD EVNDEM.

DVbitandum quidem nō est quin passiones circuli innumerabiles penè sint,quę quidem omnes ferè casu inueniuntur, vt mihi nunc accidit,quam tibi mitto, hæc autem est, quòd quadratum lineæ. a. g. in figura hic subscripta semper æquale est ei producto,quod fit ex.a.e.in diametro circuli.g.c.b.simul sumpto cum quadrato inscriptibili in dicto circulo,& simul cum quadrato lineæ.a. b. contingētis ipsum circulum,supponendo.a.g.per centrum ipsius circuli transire.

Pro cuius demonstratione à centro.e.duco semidiametrum.e. c. perpendicularē ipsi.g.a.& à puncto.c.ad.a.duco.c.a. quæ secabit circunferentiam ipsius circuli in pūcto.d.eo, quod angulus.c.acutus est. Nunc ex.35.tertij,productum.c.a.in.a.d. æquale est quadrato.a.b.productum autem.a.c.in.d.c.æquale est quadrato inscriptibili in circulo.g.c.b.ex.130.primi Vitellionis, ī qua propositione ipse Vitellio supplet pro eo,quod in quinta propositione libri de lineis spirabilibus Archimedis desideratur, sed quadratum.a.c.æquale est ijs duobus productis.per.2. secundi Eucli. ergo quadratum.a.c.æquale erit quadrato inscriptibili in circulo.d.c.g.& quadrato. a. b. sed quadratum lineæ.a.c.æquale est duobus quadratis,hoc est lineæ.a.e. & lineæ.e.c. ex pitagorica,quare ex communi conceptu duo quadrata lineæ.a.e.& lineę.e.c.hoc est lineæ. e. g. quod idem est, æqualia erunt duobus iam dictis, hoc est inscriptibili, & ei, quod fit ex.a.b.sed quadratum lineæ.a.g.æquale est quadrato lineæ.a.e.& quadrato quod fit ex.e.g.& duplo illius quod fit ex.a.e.in.e.g. hoc est producto. a. e. in diametrum. Quare quadratum lineæ a.g.æquale est quadrato circunscriptibili, & quadrato lineæ.a.b.& producto lineæ.a.e.in diametrum circuli.d.c.g.




Breuiori etiam methodo demonstrare possumus quadrata lineæ . a. e. et. e. g. æqualia esse quadrato circunscriptibili,& quadrato lineæ. a.b.ducendo lineam.e.b.quæ æqualis est lineæ. e.g.tali methodo,hoc est, considerando , quod quadratum inscriptibile semper duplum est quadrato semidiametri, vel medietati circumscriptibili,quod quidem nihil aliud est, nisi æquale esse ijs duobus quadratis,hoc est lineæ.e.b.& lineæ.e. g.sed quadratum lineæ.a.e.æquale est iis duobus quadratis, hoc est lineæ.a.b.& lineæ. b. e. vnde quadratum lineæ. a. e. cum quadrato lineæ.e.g.æquale est quadrato circunscriptibili,simul collecto cum quadrato lineæ. a.b.

## *Demonſtrationes quarundam propoſitionum de quibus agit Cardanus capite primo.libro. 16. de ſubtilitate.*

## AD EVNDEM.

EA quæ Cardanus in primo cap.lib.16.de ſubtilitate ita ſcribit,quod ſi diametros producatur extra quantumlibet, alia verò diametro in centro ſecetur ad rectos, ex huius fine &c. quæ quidem ſecundum illum eſt vndecima proprietas circuli, quoniam te id non intelligere ſcribis,idemq; dicis etiam de duodecima, & ſimiliter de tribus illis paſſionibus, quas ipſæ communes facit circulo, defectioni,ſeu ellipſi, & hyperboli, tibi breuiter reſpondebo.

Circa vndecimam proprietatem circuli verum dicit. Imaginemur circulum.p.d.q.à duabus diametris,inuicem ad angulos rectos coniunctis,diuiſum.p.d.et.d.g.diuidatur enim quarta.q.d.per quot partes æquales volueris,mediantibus punctis.b.a.o.ducanturq; ab ijſdem punctis tot perpendiculares diametro.d.g.quæ ſint.b.m.a.n.et.o.s.quæ quidem erunt parallelæ diametro.q.p.coniungatur deinde extremitas.d.diametri.d.g.cum primo puncto.b.& protrahatur.d.b.vſque ad concurſum cum diametro.p.q.protracto in puncto,h.Nunc dico.q.h.quæ adiacet diametro.q.p.æqualem eſſe omnibus dictis perpendicularibus, quapropter coniungantur puncta.m.a:n.o.et.s.q.& producantur vſque ad adiacentem diametro.q.p. in punctis.c.et. e. vnde habebimus angulos. b.a.o.q. inuicem æquales ex.26.tertij,cum verò.o.s. a. n. et b.m.parallelæ ſint ipſi.p.h.tunc anguli.b.h.c:a.c.e:et.o.e.q.æquales erunt angulis.d.b.m:m.a.n.et.n.o.s.ex.29.primi: quare anguli.h.c.e.q.erunt inuicem æquales, vnde ex.28.eiuſdem.b.h:m.c:n.e.et.s.q.erunt inuicē parallelę,& ex.34.e.q. æqualis erit.o.s.et.e.c.æqualis.n.a.et.m.b.æqualis.c.h.verum eſt igitur propoſitum.

Duodecima vero ꝓprietas eſt,ut ſi fuerit circulus.a.b. e.q. cuius duo diametri ad rectos coniuncti ſint.a.e.et.q.b.& diameter.a.e.protractus indeterminatè ad partem e.tunc ſi ab extremo.b.diametri.q.b.ducta fuerit.b.n.u.extra circulum,ſeu.b.u. n. intra circulum, vt in ſubiecta figura patet, ita vt ſecta ſit à circunferentia circuli in pũcto.n.vel à diametro in puncto.u.ſemper id quod fit ex.u.b.in.b.n. æquale erit quadrato inſcriptibili in dicto circulo, hoc autem diuerſimodè cognoſci poteſt, tribus enim modis ego inueni, quorum primus ita ſe habet. Nam ſi punctus. u. fuerit extra circulum, ducantur.b.e.et.e.n.& habebimus duos triangulos.b.n.e.et.b. e.u.ſimiles inuicem,eo, quod angulus.b.communis ambobus exiſtit, & angulus.b.n.e.æqualis eſt angulo.b.c.u.quod ita probatur,nam angulus.b.n.e.cum angulo.b.a.e.(ducta cum fuerit.b.a.)æquatur duobus rectis ex.21.tertij, ſed ex quinta primi angulus. b.e.a.ęqualis eſt angulo.b.a.e:quare angulus.b.n.e.cum angulo.b.e.a.ęquatur duobus rectis,ſed ex.13.eiuſdem angulus.b.n.e.cum angulo etiam.e.n.u.æquatur duobus rectis,ergo angulus.e.n.u.æquatur angulo.b.e.a.quare angulus.b.n.e.æquatur etiã angulo.b.e.u.vnde ex.32.eiuſdem reliquus angulus.b.u.e. æqualis erit reliquo angulo b.e. n. latera igitur erunt proportionalia ex. 4. ſexti, vnde ita ſe habebit.u.b.ad.b.e.vt.b.e.ad.b.n.ex.16.ſexti igitur verũ erit propoſitum.

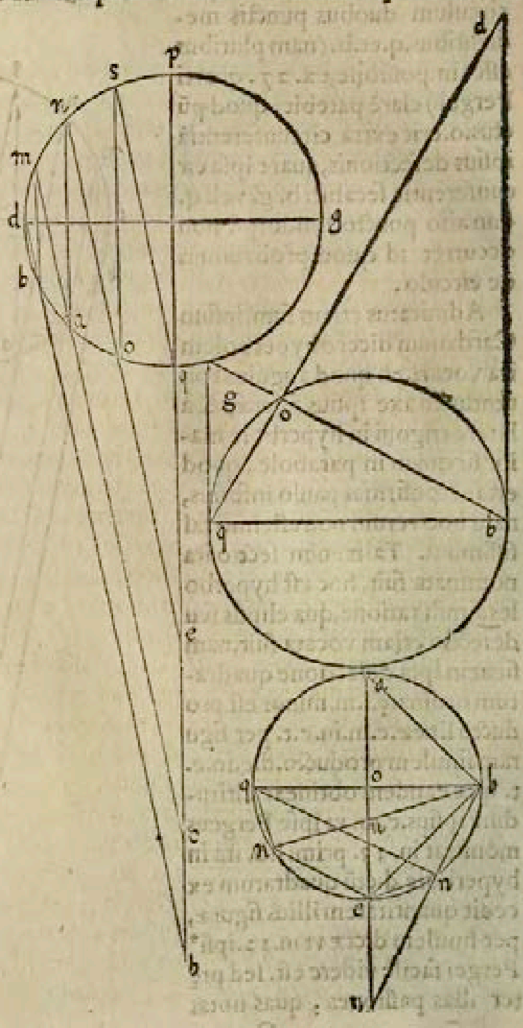
Sed ſi punctus.u.intra circulum fuerit, triangulus.b.e.n.ſimilis erit triangulo.b.u.e.nam angulus.b.ambobus communis erit. Angulus vero.b.n.e. ęqualis eſt angulo.b.e.u.ex.26.tertij, quare ex.32.primi reliquus angulus. b. e. n. æqualis erit reliquo

angulo

angulo.b.u.c. vnde ex.4. sexti eadem proportio erit ipsius.b.n.ad.b.c.quæ. b. c. ad b.u.quare ex.16.eiusdem patebit propositum.

Secundus autem modus ita se habet,ducta.q.n.habebimus duo triangula orthogonia similia inuicem . b.q.n.et.b.u.o.eo quod angulus.b.communis ambobus existit , quare ex.4.sexti ita se habebit.u.b.ad.b.o.vt.q.b.ad.b.n.vnde ex. 15. eiusdem quod fit ex.u.b.in.b.n.æquale erit ei,quod fit ex.q.b.in.b.o.Sed ex. 16. eiusdem, ꝙ fit ex.q.b.in.b.o.çquatur quadrato.b.e.quia.b.e.media proportionalis est inter diametrum & semidiametrum eiusdem circuli.ex.4.eiusdem,quare quod fit ex.u. b. in b.n.æquale erit quadrato ipsius.b.e.

Tertius modus adiungitur,& est quod cum quadratum.u.b.existente.u.extra circulum æquale sit ei , quod fit ex.u.b.in.b.n.simul sumpto cum eo,ꝙ fit ex.u.b.in.u.n. ex secunda secundi,& idem quadratum.u.b.æquale duobus quadratis.u.o.et.o.b.ex penultima primi, ideo duo dicta producta æqualia erunt dictis duobus quadratis.o. u.scilicet et.o.b.sed quadratum o u.æquatur ei,quod fit ex. a. u. in.e.u.& ei quod fit.ex.o.e.in se ipsam ex.6.secundi , quare duo iã dicta producta æqualia erunt duobus dictis quadratis,o.b. scilicet.et.o. e. & ei quod fit ex. a. u.in.u.e.sed quod fit ex b.u.in.u n.æquale est ei quod fit ex. a. u. in.u.e.ex. 35. 3. relinquitur ergo vt id qđ fit ex.u.b.in.b.n. æquale sit duob⁹ quadratis.o.b. et. o. e.quare & quadrato ipsius. b. e. ex Pitagorica.



Si autem pũctũ.u.fuisset intra circulum idem eueniret. Nam quadrato.b.e.æquãtur duo quadrata.o.b.et. o. e. sed vice quadrati.o.e.dicemus quadratũ. o. u.cum eo quod fit ex.a.u.in.u.e. ex.5.secundi,id est quadratum. o.u.cũ eo quod fit ex. b.u. in. u. n.ex.34.tertij,vnde quadratum b.e.æquale erit quadrato. o. b. & quadrato.o.u. idest quadrato b.u.ex Pitagorica simul cũ producto.b.u.in.u.n.idest producto n.b.in.b.u. quod æquale est quadrato.b.u.cum producto.b. u.in u.n.ex.3.secundi.

Circa tres passiones communes postea circulo hyperboli, & defectioni notandum est primã patere ex.36. primi Pergei , secun-

cun-

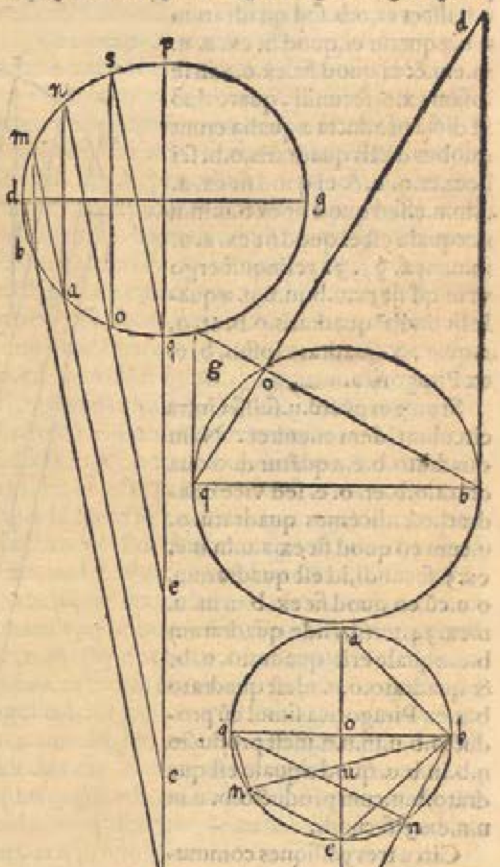
cundam verò ex.37.et.38.eiusdem,propterea quod in.37.probat mediante maiori diametro ipsius hyperbolis & defectionis.In.38.autem mediante minori diametro ordinatè ad maiorem.

Tertia autem passio,non nisi circulo conuenit; pace ipsius Cardani dictum sit.

Quapropter sit circulus.q.o.b.cuius diameter sit.q.b.contingentes vero ab extremitate diametri sint.d.b.et.q.g.per punctum autem.o.quoduis,ipsius circũferentiæ, transeant.b.o.g.et.q.o.d.tunc dico productum.q.o.in.q.d.vel.b.o.in.b.g.ęquale esse quadrato.q.b,quod ita probo.

Nam angulus.q.b.d.seu.b.q.g.rectus est ex. 17.tertij Eucli.et.b.o. q. similiter rectus ex.30.ipsius lib.angulus verò.b.q.d.seu.q.b.g.communis est. quare.b.q. media proportionalis erit inter dictas lineas.q.d.et. q.o.& inter.b.g.et.b.o.Vnde sequetur propositum ex.16.6.Eucli.

Sed si circa diametrum.q.b.mente fingamus aliquam elipsim, quæ tangat ipsum circulum duobus punctis mediantibus.q.et.b.(nam pluribus esset impossibile,ex. 27. quarti Pergei) clarè patebit, quod pũctus.o.erit extra circunferentiã ipsius defectionis,quare ipsa circunferentia secabit. b. g. vel. q. d.in alio puncto, vnde ipsi non occurret id quod probauimus de circulo,



Admiratus etiam sum,ipsum Cardanum dicere hyperbolem ita vocari,eo quod angulus contentus ab axe ipsius figuræ, & à latere trigoni in hyperbole maior sit quam in parabole, quod etiam confirmat paulo inferius, nam hoc verum non est, imo falsissimum. Talis enim sectio ita nominata fuit, hoc est hyperboles,simili ratione,qua elipsis seu defectio etiam vocata fuit,nam sicut in ipsa defectione quadratum ordinatę.l.m.minor est producto lineæ.e.m.in.e.t. per figuram similem producto.d.e.in.e.t. quæ eandem obtineat altitudinẽ ipsius.e.m. vt ipse Pergeus monstrat in. 13. primi lib. ita in hyperbole dictũ quadratum excedit quantitatem illius figuræ, per similem dictæ vt in.12.ipsi' Pergei facilè videre est. sed pręter illas passiones, quas notat Car-

Cardanus in ſupradicto capite, multæ aliæ ſunt, cum corollarium primæ tertij Eucli. ſit paſſio propria ipſius circuli, & idem dico de propoſitione. 3. 4. 7. 8. 9. 11. 12. 13. 14. 15. 17. 18. 19. 20. 30. 31. ipſius tertij lib. nec non de. 8. 9. et. 10. tertijdecimi, & de prima. 3. 4. 5. 6. et. 7. quartidecimi eiuſdem. Idem infero de ea quod ſcripſi Mario Nizzolio, Franciſco Vimercato, Franciſco Contareno, Angelo Agrimenſori, & de alijs nonnullis à me excogitatis.

## DE FINE CORPORVM COELESTIVM, & eorum motu.

### *Illuſtri viro, Philiberto Pingonio Sabaudo Cuſiacenſium Baroni.*

CVm antea meo nomine Sebaſtianus noſter omnia ferè tibi retuliſſet, inter alia, quæ relinquebantur tibi dicēda, hoc vnum erat, quod ſi abſque lumine ſuperiori, in quem finem facta fuerint corpora cœleſtia ſcire deſideras, & humanam rationem ſequi volueris, putandum tibi non erit ea ſolum effecta eſſe, vt tam vile corpus, vt eſt terra aquis irrigata, animalia, & plantas regant, cum ea corpora ſint diuina, in numero incomprehenſibilia, maximis magnitudinibus, & motibus velociſſimis, prædita, id etiam minus putabunt hij, qui opinionem Ariſtarchi Samij, & Nicolai Copernici ſequuntur, quorum ratione fieri non poteſt, vt credant. eius, quod ex vniuerſo reliquum eſt, alium finem non habere, quam regimen huius centri epicycli Lunaris, vt illorum more loquar. Quā enim turpe eſſet ſi centra aliorum epicyclorum planetarum tali regimine priuarentur, id quod nullo modo cum ratione conſentit, ſi tam vera eſt ea opinio, quemadmodum rationabiliorē eam exiſtimāt. Neq; quidquam valet opinio Ariſtotelis, qui corpora cœleſtia, ab ortu, & interitu libera eſſe ſentit. dicens ſuperioribus ſeculis, à noſtris antiquis nullam vnquam animaduerſam fuiſſe alterationem in cœlo, cum non videat ſi quis eſſet in cœlo, neq; etiam obſeruare poſſet alterationes quæ in terra, & circa terram fiunt, quæ in partibus, & non in toto ſpectantur: vnde etiam fieri poteſt, vt in cœlo ſint particulares alterationes, quæ à nobis tamen, qui ab illis longè diſtamus, non comprehendantur, terra, mareq; (quamuis minimum reſpectu ipſius terræ) ratione totius ita ſe ſemper habuerūt quēadmodum ſeſe habere corpora cœleſtia videmus, ſed alteratio, ratione tantum aliquarum minimarum partium quaſi inſenſibilium, ſi cum toto comparentur fit. Quis enim ſcit, vt iam tibi dixi, quin, quemadmodum Luna circa terram voluitur, ipſaq; terra ſit veluti centrum epicycli maioris eiuſdem, vt Ariſtarchus Samius, & Nicolaus Copernicus cenſuerunt; ſic etiam Saturnus, Iupiter, Mars, Venus, atq; Mercurius circa alia huiuſmodi corpora, huic terræ ſimilia, in orbem agantur, quaſi ſpecula, lumen Solis ſuo centro ex reflexione, deferentia (ſuppoſita dico vera illorum opinione) Nollē tamen tibi è mente excidere, vt aliàs te monui, ꝙ ſi communis opinio vera eſt, neceſſario fatendū ſit corpus ſolare, dū in æquatore reperitur motu diurno quolibet horę minuto, magis quā decē & ſeptē mille milliaria ꝑagere, ideſt paulo minus quam. 18000. milliaria, Saturnum verò cum ſimiliter eſt in æquatore, eodem tēporis ſpatio, quaſi tercentamille milliaria Italica conficere, & ſic per gradus alia corpora velociora alijs moueri; quæ quidem omnia, cū ſimplici gyro terræ circa ſuum

axem

axem (vt dicunt) tolluntur, quod ſufficit ad recipiendum lumen, & influentias illorum corporum. Et ita, veluti princeps corporum vniuerſi, intra vnum annum circa eam vertitur. Ita etiam ſufficeret, vt ipſa terra circa dictum diuinum corpus ſolare, interſecando axem diurnum cum axe annuali (cum ab eo lumen, calorem, & influentiam ſuſcipere debeat) circunuolueret. Rationes autem a Ptolomeo in contrarium adductæ apud ipſos, nullę ſunt, quia quęlibet pars (vt inquiunt) retinet naturam totius, præterquam ꝙ aer, & aqua, quæ ipſam terram circundãt, planè eundem naturalem impetum motus obtineant, qui tanto lentior eſt, quanto longius diſtat aer ab ipſa terra, ſecundum etiam talem opinionem, nulla neceſſitas foret, vt locus fixarum terminaretur aliquibus ſuperficiebus, conuexa ſcilicet, & deuexa.

## *De Luce, Lumine, & Colore, De obiectu oculi, De lumine Luna, & Rubedine nubium.*

### AD EVNDEM.

QVod proximè quærebas, an ſit lux aliqua, quæ à corpore lucido non proueniat, mihi facilè ad conſiderandum videtur. hic enim oportet, vt nos ad id quod perpetuò videmus referamus, exiſtimo autem te velle dicere lumen, non lucem, quia propriè lux, qualitas ea viſibilis appellatur, quæ eſt in corpore lucido, à quo quidem corpore lumen effunditur; lumen verò, ea qualitas eſſe dicitur, quæ extra ipſum corpus reperitur, à luce, quæ in dicto corpore manet emanans. vnde patet, nullam lucem abſque corpore ſubiecto eſſe poſſe, id quod cum fieri quiret, idē de quolibet alio accidente dici poſſet, id eſt quod ex ſe, & abſque aliquo ſubiecto ſubſiſteret.

Lumen deinde à luce proficiſci patet, ꝙ penetrat diaphanum, neque aliquo modo ſuum actum oſtendit, niſi, aut per incidentiam, aut ratione opaci, ex reflexione, cuius ſuperficiei colorem induit. Atque hæc eſt cauſa, vt inter crepuſculum matutinum, aut veſpertinum, nox etiam ſi ſit ſerena, adeo obſcura nobis appareat, quamuis totum vniuerſum diaphanum, extra conum vmbræ, quæ ex terra prouenit ſit vndiq; radijs luminoſis Solis colluſtratum; qui quidem radij, non niſi à ſuamet reflexione à Luna, & ab alijs ſtellis (vt corporibus opacis, quæ reſiſtunt lumini, ne vlterius penetrare poſſit, vnde retrò redit) comprehenduntur.

Ais etiam propria viſus obiecta plura eſſe, nominans pro vno, colorem, & lucem pro alio. Ego autem reſpondeo, obiectum oculi eſſe vnicum tantum, ideſt lumen. Quod ad lucem ſpectat, iam tibi dixi, eam eſſe quandam qualitatem in corpore lucido, & non extra ipſum poſitam, à quo quidem corpore, cum non exeat, oculi obiectum eſſe nequit, ſed lumen quidem ab ipſa luce productum. Color etiam, qui eſt in corpore colorato, obiectum oculi eſſe non poteſt, cum dictum corpus non deſerat, ſed lumen quidem ab eodem corpore reflexum, & huiuſmodi corporis colore tinctum: vnde tam lumen incidens, quam reflexum colore eſt ſemper imbutum.

Illud quidem coloratum eſt qualitate lucis corporis lucidi, aut medij, per quod tranſit, ſed hoc colore corporis, à quo reflectitur.

Neque etiam te ignorare volo, lumina reflexa colorata, non reflecti à puris propriiſq; ſuperficiebus corporum coloratorum, eo qđ pauca corpora tam opaca reperiuntur, ut immediatè lumen à ſuperficie propriè reflectãt, ſed lumen penetrat aliquanquantulum dicta corpora, & ita illorum colore afficitur, vbi verò non penetrat, non coloratur colore corporis illius.

Sed vt ad propoſitum redeamus, dico lumen tantum eſſe viſus obiectum, quod ſi colore eſt imbutum, aut tale eſt rationę coloris lucis, quæ cum mittit, aut ratione medij per quem tranſit, aut ratione corporis, vnde reflectitur. etſi ſuperficies corporis vnde lumen reflectitur eſſet omnino priuata colore, ſub aſpectum non caderet, vt etiam cum huiuſmodi ſuperficies læuigata, & polita eſt ſecundum continuitatē ſuarum partium, videlicet, vt ſpeculi radio tamen non profundante, & ideo perfectiſſimorum quorundam ſpeculorum ſuperficies non cernuntur, ſed lumen tantum reflexum, colore alicuius alterius ſuperficiei, aut à luce, corporis lucidi, aut à medio per quem tranſit, conſpicitur. Ego verò non aſſero colorem non eſſe quid diuerſum à lumine, ſed imagineris lumen eſſe veluti animam, aut ſubſtantiam & colorem corporis formam accidentalem, cum nullum lumen à ſenſu viſus percipi poſſit, quod aliquo modo colore non ſit imbutum: & eundem reſpectum quem ſonus ad auditum, lumen ad oculum habet, quia vt ſonus ſecundum eam velocitatem, quæ à motione aeris, aut aquæ, ex colliſione aliorũ corporum producitur ad euitandum vacuum, acutus, vel grauis ſentitur, ita lumen originem ducens à corpore lucido per medium diaphanum aeris, aut aquæ, aut alterius huiuſmodi corporis ad oculum tranſit colorem lucis, aut medij per quod tranſit, aut vnde reflectitur induit.

Quod verò Luna nullum ex ſe habeat lumen, ſufficiens inditium eſt nos ipſam tantò magis obſcuram videre, quantò magis in cono vmbræ terræ immergitur, & ſi eo tempore ipſam videmus rubeo colore affectam, hoc enim accidit, quia radij ſolares vndequaque refranguntur à vaporibus ipſam terram circundantibus, quæ quidem refractio fit verſus axem coni vmbræ terræ, & propterea vmbra dicti coni non eſt æqualiter obſcura, ſeu tenebroſa, circa vero axē ipſius coni, magis quam circa eius circũferentiã, obſcura vĩ, & quia corpus lunare tale eſt, vt facillimè recipiat qualecũque lumen, quod etiam manifeſtè videtur dum ipſa Luna reperitur ſecundum longitudinem inter Solem, & Venerem, quod pars Lunæ lumine Solis deſtitura, à lumine Veneris aliquantulum illuſtratur, quod ego ſæpè vidi, & multis oſtendi. Propterea dum ipſa Luna in cono vmbræ terræ reperitur adhuc videtur. Rubedo etiam illa nubium poſt Solis occaſum, vel ante ortum, aliunde non prouenit, niſi à qualitate vaporũ, per quos ſolares radij tranſeunt, à quibus vaporibus, tali colore ipſi radij afficiuntur, eomet modo quo radius, cuiuſuis corporis lucidi, trãſiens per vitrum, ſeu aliud diaphanum coloratum.

# DE ICTV BOMBARDAE SECVNDVM diuerſas eleuationes. Et de quibuſdam erroribus Nicolai Tartaleæ, circa idem.

*Illuſtri D. Ioſepho Cambiano ex Ruffiæ Dominis, æquiti ſtrenuo, & tormentis bellicis Sereniſſimi Ducis Sabaudiæ Præfecto.*

Excogitaui quędam dum ocio frui licuit per abſentiam Ducis Sereniſſimi, quæ ad te ſcribere placuit, vt ſi probaueris in lucem quandoque proferre non dubitem, ſi deſpexeris, ocius ſupprimam, ſunt autem huiuſmodi. Vnde fiat vt tormentum bellicum vehementi⁹ feriat ictu ſuperius delato quam orizontali, vt Tartalea ſcribit, quæſito ſecundo libr. primi quæſitorum, à nemine adhuc (quod ſciam) traditum eſt.

Rationes verò Tartaleæ nullius ſunt momenti, quia ſi validæ eſſent, ſequeretur vt inclinata bombarda, adeo vt angulus ſub orizonte factus æqualis eſſet ei, qui ſupra orizontem eſt, ictum bombardę in vtroque huiuſmodi ſitu eundem eſſe futurū. & ſi aliqua differentia oriretur ratione grauitatis pilæ ab ipſa bombarda emiſſæ, hoc fieret, vt ſcilicet velocior eſſet in motu inclinato quam in eleuato cum pondus, motui adeq non opponatur. Id quod non ita se habet, vera enim cauſa vnde fiat, vt bombarda eleuata vehementius feriat, quàm ea quæ eſt minus alta, eadem eſt ferè, in genere, cum ea, qua aliquod corpus materia magis denſa, ſed ſimile & ęquale alteri corpori materiæ minus denſæ velocius mouetur ab vna eademque, aut æquali vi compulſum. Eſt eadem etiam in ſpecie ei, qua maiorem effectum producit puluis, qui in locis ſubterraneis ponitur quum vaſis optimè colligatis ferro includitur. Eſt etiam ſimilis ei, qua longius impellitur pila, qua ludimus, ab aliquo inſtrumento ligneo, quando percutitur contra, quam cum ſecundum ſuum motum proijcitur. Id quod inde fit, quia virtus mouens maiori vi, & intenſiori huiuſmodi corpus percutit, quia corpus quod moueri debet, quanto magis reſiſtit virtuti mouenti (certum tamen terminum præſcribendo) in exiguo eo temporis ſpatio, tanto maiorem virtutem colligit, quæ ipſum deinde tanto cum impetu mouet, & tanto magis impellens concomitatur, vt maiorem effectum efficiat, quam ſi ad mouendum ſeſe facilè reddidiſſet. Atque hoc ſupradictis ictibus eleuatis accidit, quia grauitas pilæ, ea eſt quæ reſiſtens virtuti mouenti, dat ei commoditatem colligendi dictam virtutem, multo magis quam eſſet ea, quæ ad depreſſiorem eleuationem eam impelleret. Et quia huiuſmodi multiplicatio virtutis, nullam proportionem cum pondere pilæ gerit, volo inferre quod dum colligitur tanta virtus, colligitur multo plus eo, quod ad impellendam dictam pilam ſufficeret, ratione magnæ velocitatis augumenti, quia quanto plus temporis ei conceditur ad commutandam puluerem in ignem, tanto maior quantitas ignis progignitur, vnde fit, vt tanto maiori loco indigeat, quamobrem tanto magis impellit, ſed vt dixi, tanta cum velocitate adauget, vt huiuſmodi virtus longè ſuperet reſiſtētiā pōderis pilæ, & ſic eſt cauſa, vt effectus, quod experiētia innoteſcit producat. Sed ea ratio, qua ſeſe idē author in tertio quæſito ad aliquod impoſſibile, circa iter ipſius pilæ Legatum Hiſpanum redu-

reducere putat, nullo fundamento nititur, quia non est semper dicendum, quod quã to velocior sit quædam pila, tanto rectius moueatur, quia ei dici posset, vsque ad certum quendam terminum velocitatis, per tantum spatij eam aptam esse, vt recta perfectè moueatur, sed si velocius iret, non tamen futurum, vt per idem spatium rectius moueretur, sed quod per longius spatium recta motum perageret, & sic nihil haberet quod replicaret, præter quam quod ipse supponit id quod in 18. quæsito negat, in quo ait pilam uicinam orificio, non adeo uelocem esse, quam cum aliquantulum ab eodem est remota, ratione resistentiæ sui cyllindri aerei. Sed quod pila, recta eat quanto altior, aut depressior bõbarda erit, iõ fit, quia linea inclinationis naturalis cum linea inclinationis uiolentæ angulum rectum non facit, unde quanto longius distat à recto huiusmodi angul⁹, siue sit acutus siue obtusus, tanto minorem uim habet, eodem planè ferè modo quem tertio capite mei tractatus de rebus mechanicis descripsi. Quia in ictibus eleuatis, iter inclinationis violentæ ipsius pilæ versus terminum ad quem, incipiendo à loco ipsius pilæ cum itinere inclinationis naturalis, angulum obtusum, & in ictibus inclinatis acutum constituit. Neque etiam hic prætermittam notatu dignum errorem, quem Tartalea eodem loco committit, cũ putet indifferenter aliquod corpus impellere, aut percutere maiori cũ impetu cum est in itinere recto. Quia sequeretur quod aliquod corpus graue perpendiculariter sursum versus proiectum, in qualibet parte sui itineris, semper fortius percuteret, quam in qualibet parte itineris alterius cuiusuis eleuationis obliquæ, quod quã sit falsum, tibi considerandum relinquo.

Est etiam falsa ea ratio, quam in quarto quæsito idem adducit, quia aer in motu non tantum durat, quantum ipse putat, imò huiusmodi violenta agitatio, citò cessat & citius etiam, quam si extra aliquam bombardam cum tanta violentia impullisset saccum plumis plenum.

Ratio etiam quam in. 18. quæsito de eo, quod pila pertranseat illud corpus cyllindricum aereum adducit, est planè vana, quia statim aer, qui prius in bõbarda erat inclusus, extra ipsam erũpit, cedit, à pilaq; diuiditur, vt si nunquam eam figuram induisset, neque aer ambiens ei resistit. Sed quod velocior sit in certa quadam distantia, quam in principio erat, si hoc verũ esset, ab alia causa dependeret, quæ partim similis esset ei, quæ efficit, vt corpora in motibus naturalibus, cum longius distant à termino vnde naturaliter sese mouerunt, sint velociora, quia per aliquod spatium huiusmodi corpus moueretur quemadmodum motu naturali cietur.

Ratio autem eius quare pila, aut globus bombardæ sibilet ab eodem in septimo quæsito nil valet, quia hoc fit cum pila aliquam paruam concauitatem habet.

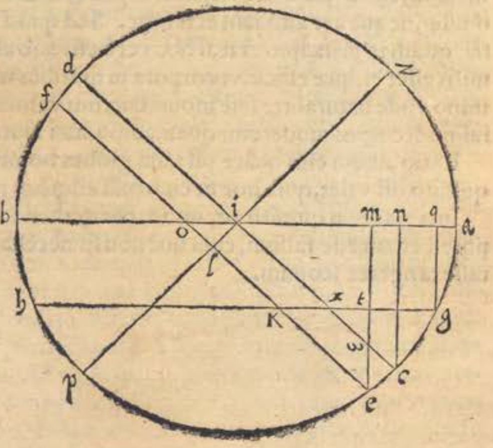
In. 27. autem quæsito ait, quod retrotrahendo signum, ictus altius tenderet, quod potest etiam esse falsum, cum hoc non sit necessarium, quia pila dum descendit, fortasse tangeret scopum.

*De erroribus Ioannis Stadij.*

## AD EVNDEM.

FIguram quam ponit Ioannes Stadius pag.147.in lib.ſuarum tabularum Prutenicarum, à Nicolao Copernico ſumpſit pag.64.à tergo in libr. reuolutionum cœleſtium, ſed ipſe Stadius eam non intellexit,omitto, quod mutauerit characteres ipſius figurę, vt illa ſua videatur,quod nihil refert,alterat etiam demonſtrationē, ſed ipſum putare.i.K.perpendicularem à centro circuli ſemper dependere, eſt intollerabilis error; nec vnquam verificatur hoc, niſi quando punctum. K. interſectionis diametrorum parallelorum, forte reperitur in axe mundi. Reliqua verò ſuæ demonſtrationis,ſi non intelligis, minimè miror, eo quod ipſemet Stadius ſeipſum confundit. Veram autem demonſtrationem huiuſmodi figuræ in dicto libr.Copernici clarè videbis. Quod verò diuersè cogitaui nunc accipito.

Cum nobis cognita ſit maxima ecclipticæ declinatio,vt puta. a. c. ſi latitudo etiā ſtellæ nobis data fuerit, vt puta.c.e.cognitus nobis erit totalis arcus.a.e.& eius ſinus. e.m.& quia notus etiam nobis eſt ſinus arcus.a.c.hoc eſt.c.n.& corda.e.f.medio eius arcus.e.p.f.minoris media circunferentia,per duplum latitudinis datæ,vnde.e.l.eius dimidium nobis cognitum erit,vel vt ſinus arcus.e.p.cognitus etiam nobis eſt ſinus. q.g.declinationis.a.g.datæ,cui æqualis eſt. m.t.ex.34.primi Euclid.vnde. e. t. nobis cognita remanet,cum verò duo trianguli.i.c.n.et.t.e.K.æquiāguli ſint, propter duas parallelas.e.m.et.n.c.ex.28.primi, & propter duas.a.b.et.g.h.& propter duas. c. d. et.e.f. eo quod ex communi ſcientia anguli.c.et.e.ſunt æquales,cum ex.29.dicti lib. vnuſquiſq; æqualis ſit angulo.m.ω.i.ita etiam infero de angulis.e.K.t.et.c.i.n. quorū vnuſquiſque æqualis eſt angulo.ω.x.t.& ſic de alijs dico, eo quod vnuſquiſque eorū æqualis eſt angulo.m.vnde cum cognitum nobis ſit latus.n.c.et.c.i.et.t.e notū etiam nobis erit.e.K.ex.19.ſeptimi, eo ꝙ ex.4.ſexti ſunt inuicem proportionalia, detrahendo poſtea.e.K. ab. e.l.cognito, vel ècontra, hoc ab illo,nobis innoteſcet.K.l.ſinus longitudinis ſtellæ.




Valde etiam miror id,quod dictus Stadius pag.9.illius libr. ſcribit, hoc eſt, Solem maiorem eſſe Luna, ſolum. 1644. vicibus, propterea ꝙ cum affirmet Solem maiorem eſſe terra (vt etiam in Almageſto videre eſt) 166.vicibus cum tribus quartis, terram vero maiorem Luna.39. vicibus cum quarta parte, tunc Solem oporteret maiorem eſſe Luna.6545.vicibus,& non.1644.

De

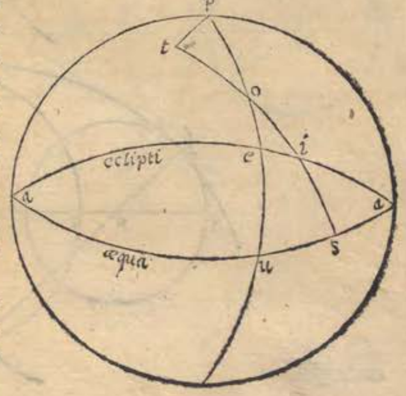
## *De cognitione latitudinum stellarum.*

## AD EVNDEM.

AD cognoscendam latitudinem stellæ, eiusq; declinationem, Monteregius in 10. propositione. 8. li. Almagesti methodũ satis docuit, sed si alia aliqua metho do hoc idem cognoscere voluerimus, oportebit nos prius altitudinem poli cognoscere, deinde altitudinem meridianam ipsius stellæ, nec non horam, quãdo ipsa stella in meridiano supra terram reperitur, qua hora mediante, illicò cognoscemus punctum ecclipticæ à meridiano intersecto, eo tempore, quo stella cœlum mediat supra terram. Et quia ex cognita altitudine poli, illico cognoscitur altitudo æquatoris, cuius altitudinis differentia ab altitudine stellæ est declinatio ipsius stellæ, habebimus ideo eius declinationem cognitam; qua mediante ad cognoscendũ etiam latitudinem ita faciemus.

Sit exempli gratia. p. o. u. meridianus. u. a. verò æquator. e. a. autem ecclíptica, & o. centrum astri. u. o. verò eius declinatio ab æquatore, et. e. a. arcus ęcclipticæ inter æquatorem, & meridianum, hoc est minor quarta, et. a. u. ascensio recta ipsius arcus, et. u. e. sit declinatio puncti. e. ęcclipticę ab æquatore, residuũ vero declinationis stellę sit. o. e. quæ oĩa nobis cognita erunt, sitq;. t. polus ęcclipticus, à quo per. o. vsque ad ęcclipticam transeat quarta. t. i. in qua quęrendus erit arcus. o. i. hoc modo.

Primum arcus. o. u: e. u: e. o: a. e: et. a. u. nobis cogniti sunt, cum angulo. a. declinationis ęclipticę, & cum angulo. u. recto, vnde ex. 4. primi Copernici, cognoscemus angulum. a. e. u. collateralem, & eius. o. e. i. quare in triangulo. o. e. i. cognoscemus angulũ e. et deinde. i. vt rectũ, & latus. o. e. ergò ex eadẽ. 4. cognoscemus arcũ. o. i. quæsitum, & similiter arcum. e. i. qui coniunctus vel dẽptus ab. a. e. tribuet nobis longitudinem stellę, sed quia huiusmodi operatio in paruis triangulis valde fallit. Ideo tibi suadeo alia methodo, hoc facere, hoc est inuenire angulum. o. trianguli. t. p. o. cuius duo latera. t. p. et. p. o. cognita nobis sunt, cum angulo. p. Nam. o. p. est complementum declinationis stellæ, et. p. t. est arcus coluri solstitiorum inter duos polos, & angulus. p. residuum ex recto. t. p. a. duorum colurum dempto angulo. a, p. u. cognito ascensionis rectæ, vnde angulus. u. o. s. vt contrapositus cognitus remanet. angulus verò. u. rectus est, & arcus. o. u. cognitus, quare cognitus nobis erit arcus. u. s. & angulus. u. s. o. vnde arcus. a. s. nobis cognitus remanebit cũ angulo. a. s. i. residuo ex duobus rectis. Et quia etiam angulus. s. a. i. cognitus est, cum sit angulus maximę declinationis Zodiaci ab æquatore. Ideo in triangulo. a. s. i. cuius duo anguli. a. et. s. cum latere. a. s. dantur, facilè inueniemus arcum. s. i. cũ arcu. a. i. sed a. i. erit longitudinis stellæ dempto postea. s. i. ex. s. o. iam inuento habebimus arcum. i. o. latitudinis ipsius stellæ.




Hæc autem tibi scribo non vt ipsis vtaris, sed potius vt tibi morem gerã, cum breuissima methodus sit illa, quã Monteregius scripsit ĩ. 10. ꝓpositione. 8. li. in Almagest.

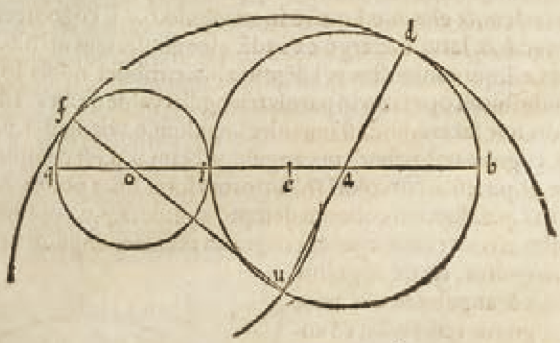
## *Qualiter circulus designari poßit alios duos circulos propositos includens.*

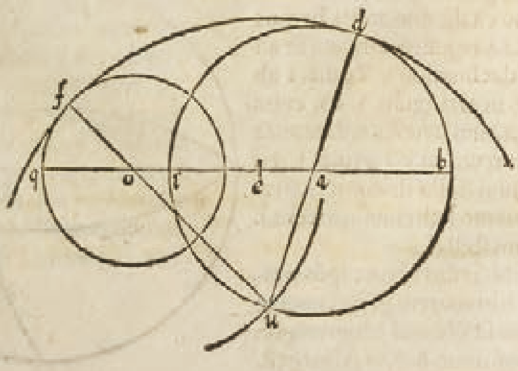
### CLARISS. PETRO PIZZAMANO.

SVperioribus diebus per tuas literas à me quæsiuisti, vt modum tibi scribere vellem, quo circulus designari possit circunscribens alios duos propositos circulos. Qua in re vt tibi satisfaciam quod maximè cupio ita rem accipe.

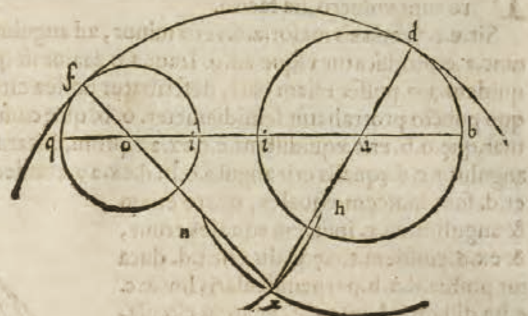
Propositi circuli sint, aut inter se contigui, aut intersecantes vel separati. Esto primũ contiguos esse, qui sint. d. b. et. f. q. quorũ. d. b. maior sit et. f. q. minor, corũ vero centra sint. a et. o. punctũ autem cõtingentię sit. i. Nũc ꝓtrahatur. b. a. o. q. per cẽtra eorum ab vna circunferentia ad aliam, quę quidem linea transibit per punctum. i. ex 11. tertij Eucli. deinde à diametro maiori abscindatur. i. e. ad æqualitatem minoris semidiametri, quo facto sumatur distantia inter. e. et. b. circino mediante factoq; centro. o. scindatur, alio circini pede, circunferentia maioris circuli in puncto. u. à quo si mente concipiemus duas lineas. u. a. d. et. u. o. f. transeuntes per eorum centra. a. et. o. vsque ad circunferentias in punctis. d. et. f. ipsę erũt inuicem ęquales, eo quod. e. i. sũpta fuit æqualis. o. f. et. o. u. æqualis. e. b. quare. u. f. æqualis erit. b. i. sed. u. d. etiã æqualis. b. i. ergo. u. d. æqualis erit. u. f. & circulus, cuius. u. d. vel. u. f. erit semidiameter, contiguus erit ipsis propositis circulis ex conuerso. 11. iam dictæ. Idem dico pro circulis se inuicem secantibus.

Sed



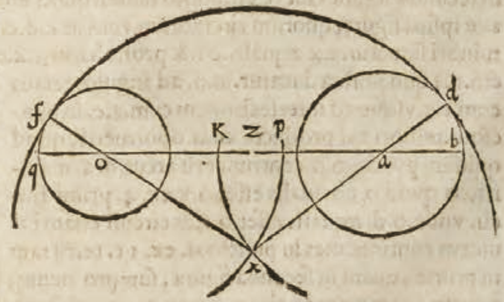



Sed ſi circuli propoſiti ſeiuncti fuerint,ſumatur.b.i.diameter maioris, qui fiat ſemidiameter vnius circuli circa centrum. o.& hic circulus vocetur. h. x. coniungatur deinde ſemidiameter.o. i. minoris circuli cum ſemidiametro.a.i. circuli maioris,& ex huiuſmodi compoſita linea, fiat vnus ſemidiameter.a.x.circuli.x.n.concentrici cum maiori,& à puncto.x.interſectionis horum circulorum ( poſito quod ſe inuicem interſecent) ducantur per eorum centra.x.a.et.x.o.vſque ad ipſorum circunferencias in punctis.d.et.f.duę lineæ, vnde habebimus. x. d. æqualem.x. f. eo quod tam in x.d.quam in. x. f. reperiuntur diametri,& ſemidiametri amborum circulorum,facto denique centro.x. vnius circuli,cuius ſemidiameter ęqualis ſit vni earum.x. d. vel. x. f. ſolutum erit problema, dicta ratione.




Si verò diſtantia duorum propoſitorum circulorum tanta fuerit, quod ſecundi circuli nequeant ſe inuicem tangere, vel ſecare,tunc alia via incedendum erit, quę talis eſt & generalis. Diuidatur tota.q.b.per æqualia in puncto.z.circa quod ſignētur duo puncta ab ipſo ęquidiſtantia.K.et.p.diſtantia vero.a.K.facta ſit ſemidiameter eſſe vnius circuli. K.x. circa centrum. a . diſtantia autem . o . p . ſemidiameter alterius circuli . p . x. circa centrum . o . qui quidem circuli ſe inuicem ſecent in puncto. x. à quo cum ductę fuerint. x.a.d.et.x.o.f.per centra dictorum circulorum,ipſe erunt inuicē ęquales, eo qđ cum.b.K.æqualis ſit.q.p.igitur.x.d.et.q.p.erunt inuicem ęquales, ſed.f.x.æqualis eſt q.p.quare.x.f.æqualis erit.x.d.tunc ſi.x.centrum fuerit vnius circuli, cuius ſemidiameter ſit vna dictarum, problema ſolutum erit.

Talis etiam ſolutio commoda erit ad inueniendum dictum circulum cuiuſuis magnitudinis, dato tamen ꝙ eius diameter, maior ſit.b.z. cum in noſtra poteſtate ſit accipere puncta.K.et.p.proxima vel remota ab ipſo.z.ad libitum. Vnde abſque vlla diuiſione ipſius. q. b. per medium, ſatis erit ſignare puncta.K. et. p. duabus diſtantijs mediantibus. b . K. et. q. p. inuicem æqualibus, & etiam propoſitis.




In omnibus casibus, diuisa bq bifariam, quod quidem punctum fiat centrum, circulus descriptus per bq transiens semper datos circulos in punctis bq contingeret.

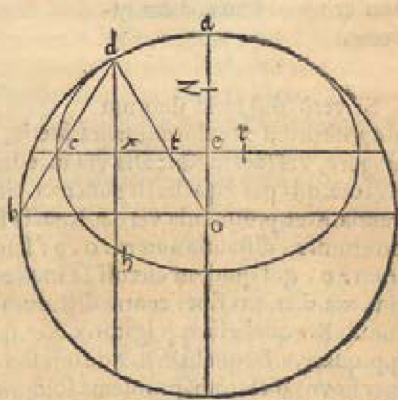
Figuram

## *Figuram ſuperficialem ellipſi ſimilem , ex datis axibus circino mediante delineari poſſe.*

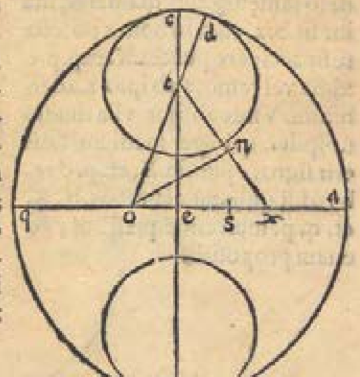
### AD EVNDEM.

FIguram ſuperficialem ellipſi ſimilem,ex datis axibus,circino mediante delineare cum volueris,ita facito.

Sit.e.c. ſemiaxis maior.a.e.verò minor , ad angulum rectum inuicem coniuncti , tunc.a.e.producatur vſque ad.o. Itaq;.a.o.maior ſit quam diſtantia inter.o.et.c.quę quidem.a.o.poſſet etiam dari , deſcribatur poſtea circulus.a.d.b.circa centrum.o. à quo puncto protrahatur ſemidiameter. o. b. quæ cum.a. o. angulum rectum conſtituat,quę.o.b.erit æquidiſtans.e.c.ex.28.primi,ducatur poſtea.b.c.d.et.o.t. d. vnde angulus.t.c.d.ęqualis erit angulo.o.b. d.ex.29.eiuſdem.ex quinta autem anguli. b. et.d. ſunt inuicem æquales, quare etiam & anguli.d.et. c. inuicem ęquales erunt, & ex.6.eiuſdem.t.c.ęqualis erit.t.d. ducatur poſtea.d.x.h.perpendicularis lineæ.c.e.ita diſtans ſub ipſa.c.e. vt arcus circularis circa.t. delineatus ex ſemidiametro. t. d. aptus ſit eam ſecare,.ſumpto poſtea. r. tam diſtante ab.e. vt. t. reperitur ab ipſo e.et.z.ab.e.vt.o.ab eodem, ducendo poſtea duos alios arcus magnitudinis priorũ circa centra.r. et. z. habebimus propoſitum.




Sed cum quis voluerit prius arcus minorum circulorum delineare circa maiorem axem,fiant cuiuſuis magnitudinis,vt in ſecunda figura videre eſt,poſito tamen quod eorum diameter , minor ſit minore axe ipſius figurę, quorum circulorum vnus ſit.c.d.circa.t.eius centrum,deinde in axe minori ſumatur.a.x.æqualis.c.t.& protrahatur.t.x.quę per ęqualia diuidatur in puncto.n.à quo poſtea ducatur. n.o. ad angulos rectos cum.t.x.vſque ad interſectionem cum.a.c. in puncto.o.minori axi producta cum oportuerit, quod quidem punctum.o.centrum erit arcus.d. a. maioris,eo quod.o.t.æqualis eſſet.o.x.ex. 4. primi Eucli. vnde.o.d. æqualis eſſet.o.a.& circuli etiam inuicem contingentes in puncto.d. ex. 11. tertij tam in prima , quam in ſecunda figura , ſumpto deniq; puncto.s.tam remoto ab. e. quam. o . reperitur ab eodem,ipſum, centrum erit alterius arcus oppoſiti,poſſemus etiam abſq; diuiſione ipſius,t.x.conſtituere angulum.x.t.o.æqualẽ angulo.t.x.o.vnde ex 6.primi haberemus.o.t.æqualem.o.x.




De

## *De inuentione axis proposita portionis data sphara.*

### AD EVNDEM.

VT axem propositæ alicuius datæ ſphæræ inuenire poſſis ita tibi operandum eſt vt gratia exempli. Propoſita nobis eſt ſphæra.c.i.e.t.diametri cognitæ. propoſita etiam eſt nobis eius portio.n.e.u.axis.e.a.cognitæ minoris ſemidiametro, data etiam nobis eſt proportio alterius portionis minoris hemiſphærio. i. c. t. ad portionem.n.e.u.quæritur nunc quantus ſit axis.e.x.ſecundæ portionis hoc eſt deſideramus cognoſcere proportionem.e.x.ad.e.a.vel ad diametrum ipſius ſpheræ.

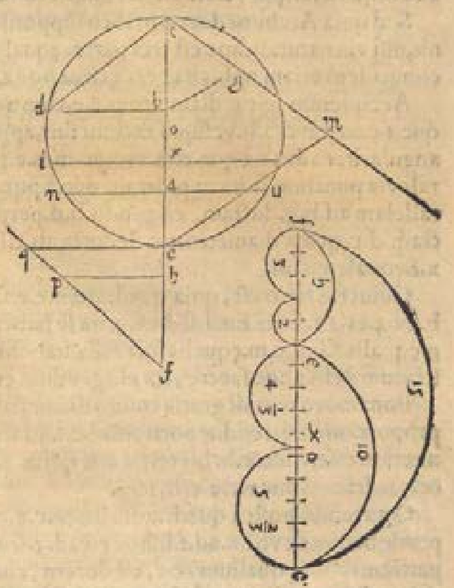
Cuius gratia reperiatur primò proportio circũferentiæ maioris circuli ipſius ſphęræ ad eius diametrum,quæ ferè eſt vt.22.ad.7.ex Archimede.

Quo facto,inueniatur quantitas ſuperficialis huiuſmodi maioris circuli,quæ ſemper æqualis eſt producto quod fit ex ſemidiametro in dimidium circunferentiæ ipſius circuli,ex eodem Archimede. Et ſic cognoſcemus quartam partem ſuperficiei ſphæricæ ſphærę propoſitę ex.31.primi lib.de ſphæra,& cyllindro Archimedis.

Deinde ſumatur tertia pars producti, quod fit ex ſemidiametro in ſuperficiem maioris circuli, & habebimus conum,cuius baſis erit circulus maior, altitudo verò ſemidiameter propoſitæ ſphæræ ex.9. duodecimi Eucli.

Quadruplum poſtea huiuſmodi coni,erit quantitas ſoliditatis,ſeu corporeitas totius ſphærę ex.32.dicti lib. Archimedis.

Imaginemur poſtea ĩ ſphærica portione.n.e.u.lineã.e.u.à sũmitate ad extremitatẽ baſis,cuius.e.u.quantitatem cognoſcemus, hoc modo ſcilicet,ſumendo radicẽ quadratam producti.c.e.in.e.a. eo quod quadratum.e.u.æquale eſt quadrato a.u. & quadrato. a. e. ex penultima primi Eucli. ho[illegible] eſt producto quod fit ex.c.a.in.a.e.ex. 34. tertij eiuſdẽ, & quadrato. a. e. hoc eſt producto, quod fit ex.c.e.in.e.a. ex. 3. ſecundi eiuſdem.



Inuenta poſtea.e.u.ponamus eam vnius circuli ſemidiametrum eſſe,cuius ſuperficialis quantitas etiam inueniatur, vt ſupra dictum eſt, quæ quidẽ æqualis erit ſuperficiei portionis n.e.u.ex.40.primi li. Archimedis de ſphæra, & cyllindro.

Hæc autem quantitas vltimo inuẽta multiplicetur cum tertia parte ſemidiametri datæ ſphæræ,& habebimus ſoliditatem vnius coni æqualis aggregato ſoliditatis portionis. n. e. u.ſimul ſumptę,cũ ſoliditate vnius coni,cuius axis ſit.a.o.reſiduũ ſemidiametri noſtræ ſphæræ dempta.a.e.ba ſis verò eadem quæ eſt portionis,cuius diameter eſt.n.u.ex.9.12.Eucli. & ex.42.id-eſt vltima primi Archimedis de ſphæra, & cyllindro.

Nunc autem ex hoc aggregato iam vltimo dicto detrahatur conus,cuius.o.a. eſt axis et.n.u.diameter baſis, qui quidem conus nobis cognitus eſt,cum.a.n. ſemidiameter eius baſis,nobis cognita ſit ex.34.3.Eucli.& ſic quantitas eius baſis, & ita tertia pars.a.o.eius axis, quę multiplicata cum dicta baſi,cuius.n.u.eſt diameter, producit dictum conum, qui quidem conus, vt diximus, demptus cum fuerit ex dicto aggregato,relinquet nobis ſoliditatem portionis.n.e.u. vnde cognoſcemus proportionem iſtius portionis ad totam ſphæram propoſitam.

Sed cum nobis propoſita ſit proportio portionis.n.e.u.ad portionem.i.c.t.cognoſcemus etiam ſoliditatem huius ſecundę portionis.i.c.t.& ſimiliter proportionē huius ad totam ſphęram,& ad reſiduū etiā ipſius ſphęrę hoc eſt portioni.i.c.t.

Protrahatur nunc diameter.c.e.à parte.e. vſq; quo.e.f.æqualis ſit.e.o.ſemidiametro ſphęrę, quæ quidem.f.c.diuidatur in puncto.h.ita vt proportio.f.h.ad.h.e.æqualis ſit proportioni portionis.i.c.t.ad portionem.i.e.t.quod quidem hoc modo efficietur.applicabimus lineam.f.q.(indeterminatam)cum.f.c.ad quemuis angulum in pūcto.f.in qua accipiemus duas lineas.f.p.et p.q.inuicem ita relatas,vt ſe habent in proportione duæ iam dictæ portiones,hoc eſt, vt.i.c.t.portio ad portionem.i.e.t. ducendo poſtea.q.c.et.p.h.parallelam ad ipſam.q.e.diuiſam habebimus.f.c.in eadem proportione vt dictum eſt ex.2.ſexti,&.11 quinti Euclidis, vnde.c.e:e.f.et.f.h.nobis cognitę erunt.

Oportebit nos nunc cognoſcere quantitatem.c.x.hoc modo,videlicet, quęramus quadratum,cuius.c.x.eius ſit radix,cui quadratum lineę.c.e.cognitum,ita ſit proportionatum,vt eſt linea.x.f.ad lineam.f.h.quę nobis cognita eſt,quod rectè factum erit ex eo,quod ſcripſit Archimedes in.4.ſecundi de ſphęra, & cyllindro.

Sed quia Archimedes eo in loco ſupponit id, quod nec ipſe, nec alius adhuc inuenit,niſi via naturali,hoc eſt tres partes ęquales ex proportione data effici,non erit inconueniens etiam nobis hac via , circa hoc aliquid dicere.

Accipiemus igitur diametrum.c.e.cum addita.e.f.eius ſemidiametro, diuidemus que.f.c.in puncto.h.vt ſupra factum fuit,applicabimus poſtea.c.m.indeterminatam angulariter ad.c.e.à qua.c.m.accipiemus.c.g.æqualem. f. h. quęremus deinde naturali via punctum.b.ita ut protrahendo à puncto.e.(altero extremo diametri)e.m.parallelam ad.b.g.ductam, erigendo.b.d.perpendicularem ad.c.e.in puncto.b.protractaq;.d.c.quæ à diametro.e.c.deducta ab.c.incohando vſque ad. x. relinquat nobis. x.f.ęqualem.c.m.

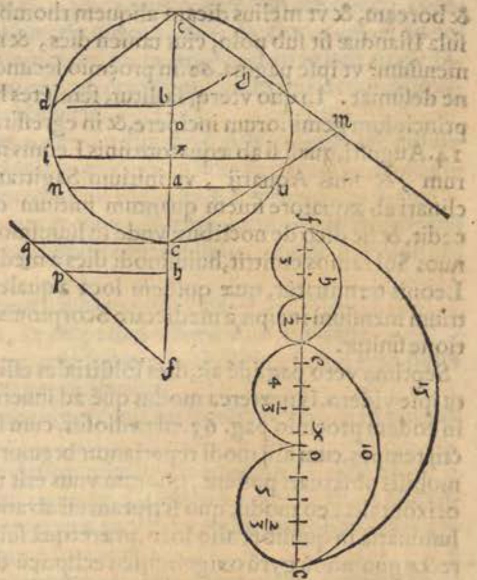
Cuius rei ratio eſt , quia quadratum.c.e.ſe habet ad quadratum.c.d.vt.c. e. ad. c. b.ex.4.et.18.ſexti Eucl.ſed ex.4.ita ſe habet.m.c.ad.c.g.vt.e.c.ad. b.c. & cum ſit.c. g.ęqualis.f.h.ſi.c.m.ęqualis fuerit.f.x.habebimus propoſitum. Quod ſi quis per diſcretum vellet hoc facere , ita ei agendum erit .

Ponamus exempli gratia totum diametrum.c.e.propoſitæ ſphæræ eſſe ut decem, proportionemq; reſiduę portionis.i.c.t.ad ſecundam.i.e.t.hoc eſt.f.h.ad.h.e.ſeſquialteram eſſe,vnde.e.h. bis tertia erit ipſius. f. h. totaq; linea.c.f.erit.15.et.f.h. erit.3. & quadratum lineæ.c.e.erit.100.

Quærendo poſtea quadratum lineæ.c.x.cui quadratum.c.e.hoc eſt.100. ita proportionatum ſit vt.f.x.ad.f.h.hoc eſt ad.3.ſi autem cogitauerimus. c. x. eſſe nouem partium talium qualium . c.e. eſt decem , eius quadratum erit.81.et.x.f.erit. 6. partium talium qualium.c.f.eſt. 15.dicendo poſtea ſi.100.dat.81.( ex regula de tribus)

x.f.

x.f.hoc eſt.6.dabit.4. integra cum. 86. centeſimis,ſed nos vellemus nobis prouenire tria, eo ꝙ ita eſt. f. h. qua propter deſcendere nos oportebit à nouem ad.8.& ab.8.ad. 7. & à. 7.ad.6.tunc inueniemus.c.x.oportere eſſe circiter quinque cum duabus tertijs, operãdo poſtea ex regula de tribus,ſi dixerimus quando.100.nobis dat. 32. cum nona parte integri, tunc nouem cum tertia parte integri dabit.2.cũ.296.de. 300. hoc eſt. 2. cum circa.49. quinquageſimis, quæ quidem quantitas,cum propinquiſſima ſit lineæ.f.h.trium integrorum dicemus.c.x. eſſe quinque integrorum cum duabus tertijs partibus vnius integri,et.e.x.reſiduum, hoc eſt axem quæſitum portionis.i.e.t.eſſe circa.4 integra cum tertia parte vnius integri.




## DE ERRORIBVS THOMAE PORCACHII & Benedicti Bordonij in eorum inſularijs.

*Excellentißimo D.Io.Baptiſta Famello Ciui Decurioniq́; Taurinenſi Philoſopho, Medico , & in Accademia eius Ciuitatis Medicinæ Practicæ Ordinario,Primarioq́; profeßori celeberrimo.*

DIj perdant tuas adeo moleſtas, & aſſiduas curas, quæ te nimis à ſuauioribus ſtudijs diſtrahunt, & à nobis longius abducunt. Nam, ut tibi quietẽ, ita mihi ingentem adimunt voluptatem. Sed ne in aliquo erga te deficere videar, quæ tibi olim promiſi,nunc mitto.

Negari quidem non poteſt,quin fuerit laborioſum opus Porcachij, & Benedicti Bordonij,hoc eſt inſularium,qui rectè etiam feciſſent, cum loqui eos oportebat de terminis ſphæræ ratione ſitus locorum,ſi ſeipſos alicuius excellentis Coſmographi conſilio ſubmiſiſſent. Conſidera quæſo, quomodo admitti poſſit, id quod ait Porcachius initio ſui operis, ideſt Iſlandiam ſub Polo arctico iacere, inter auſtrum, & boream:omittamus etiam quod idem in Proęmio lib. ſecundi, vbi ait Biarmiam, (& non Iſlandiam) eſſe ſub dicto polo arctico: in eodemq; principio repetit ipſam Iſlandiam inter auſtrum, & boream per centum leucas Germanicas extendi, deinde verſus occidentem,ea duo ſtupenda miracula conſpici. Vide quæſo, quomodo incolę ſub aliquo ex polis,habere poſſint occidentem,orientem,magiſtrum,auſtrũ,

& boream, & vt melius dicam aliquem rhombum. Sed quomodo fieri poteſt, vt inſula Iſlandiæ ſit ſub polo, eius tamen dies, & nox maior non ſit longior ſpatio triũ menſium? vt ipſe pagina.62.in proęmio ſecundi lib.affirmat, quamuis hoc à Bordone deſumat. In quo vterq; fallitur, ſentiẽtes huiuſmodi diem ab ingreſſu Solis, in principium geminorum incipere,& in egreſſu à Leone terminari,ideſt à.12.Maij ad 14.Auguſti,quaſi ſi ab æquatore finis Leonis ita declinaret, vt principium geminorum , & finis Aquarij , vt initium Sagittarij, nam ratio poſtulat, tantum declinari ab æquatore finem quantum initium diei, vbi maximus dies. 24. horas excedit, & ſic dico de noctibus: vnde in huiuſmodi regione, vbi per tres menſes continuos Sol radios emittit,huiuſmodi dies à medietate Tauri incipit, & in medietate Leonis terminatur, quæ quidem loca æqualem declinationem habent, & ſic nox trium menſium incipit à medietate Scorpionis, & in medietate Aquarij, eadem ratione finitur.

Septima verò pag.idẽ ait,dies ſolſtitiales eſſe circa.24.Iunij.Qđ,an tũc eſſet verũ, tu ipſe videto.Is præterea modus quẽ ad inueniẽdũ orientẽ,& occidẽtem præſcribit in eodem proęmio pag. *63*.eſt tædioſus, cum ſemper expectare nos cogat æquinoctij tempus,cum alij modi reperiantur breuiores, qui in qualibet reuolutione primi mobilis obſeruari poſſunt, quorum vnus erit mediante inuentione lineæ meridiane orizontalis, eo modo, quo ſcriptum eſt ab antiquis mediante Sole, aut Luna, quæ luminaria in quolibet alio loco,prǽterquã ſub polo efficiunt,vt extremitas vmbræ rectæ gnomonũ gyrũ oxigoniũ,ſeu eclipticũ ducat, ideſt in ijs locis,quorũ zenit eſt inter polum,& circulum arcticum,quemadmodum facit, vt alijs, exiſtentibus ipſis luminaribus extra æquatorem, & circulos arcticos gyrum hyperbolicum reddant. Sed id quod eidem Porcachio impoſſibile eſſe apud eos, qui habitant ſub polo videtur, ideſt vt multis rationibus, vt ipſe dicit,fieri non poſſit, ut fiat immediata quędam, & ſubita mutatio à continuo die ad continuam noctem abſque eo quod ijs, ſaltem ſemel conceſſa ſint dies, & nox terminata duodecim horarum, eſt magis admirandum impoſſibile, quod imaginari poſſimus, nam neceſſarium eſſet,ut orizonhabitatorum ſub polo ſecaret æquatorem contra id, quod ſuperius admiſerat, ideſt orizõtẽ Biarmiæ, eſſe eũdẽ cũ circulo æquinoctiali. Vide etiam quid is ab antiquis colligat, loquens de iis, quæ in inſula Taprobana ad finem pag.186.admirabilia ſunt, ſcribens eiuſdem inſulę habitatoribus, Lunam ſuper terram non apparere ab octauo uſque ad decimumſextum diem: pręter quam , quod etiam ſcribit, in eadem inſula, tramuntanam non uideri, quod falſum eſt, quia hæc à polo arctico circiter quatuor gradibus diſtat noſtris temporibus. unde ab ijs qui ſunt ſub æquatore, cum ea ſupra orizontem eſt, conſpici poteſt, cum ijſdem ſingulis diebus oriatur, & occidat. Idem etiam pro re admirabili ſcribit, uideri Canopum, qui à polo antarctico plus quam quadraginta gradibus diſtat.

## *De erroribus Lucilli Philalthæi.*

## AD EVNDEM.

QVod Lucillus Philalthęus tam eximius Mathematicus ſit, ut ipſum Antonius Berga facit, ego quidem non uideo. In ſuis enim commentariis de Cœlo, dicit primum, Pyramidem, quę inter corpora regularia primum locum tenet,

ſex

sex basibus constare, pag.15.583.632.et.647. Omitto errorem ab eodem commissum in fine pag.39. ubi oleum grauius esse quam aquam fatetur, cum id ad res mathematicas non spectet: Omitto etiam quod idem neget astrologiam pag. 74. 79.& quod etiam dicat pag.89. Deum esse ad orientem, non considerans aliquibus populis nostrum orientem esse occidentem.

Quod idem ait pag.241. Astrologiam esse antiquiorem Astronomia est falsissimum, quia iudiciaria semper præsupponit cognitionem situs stellarum, quæ ab Astronomia petitur. Mouebit tibi risum quod ait pag.307.his verbis.

» Verum propriè media dicitur illa, quæ rectam sphæram omninò habet, quæ eun» dem polum orizontis & mundi obtinet, quæ orizontem habet diuidentem sphærã » æquè secundum angulos rectè.

Paulo inferius continuans sermonem de sphæra recta, ait.

» Et nisi tumor terræ, & gibum esset, ijs perpetuus esset dies sine nocte.

Linea verò.56.ait habitatores sphęrę rectè habere. 4. solstitia, sese ipsum huius rei planè ignarum prodens.310.autem pag.sic scribit.

» Quoniam repercutiuntur radij, & per idem centrum transeunt, ob id stupam ap» positam centro radius accendit.

Quem quidem errorem ab Euclide desumit, et.15.linea pag.636. repetit.

Si vis ridere,legito.16.primas lineas.357.pag. Quod idem deinde dicat circa finem 396.pag. lucem esse substantiam corporis lucidi & corpoream, subijciam tuo iudicio,vt etiam quod ait.397.pag. his verbis vtens.

» Idcirco animalia illa, quæ nocte vagantur perpolita, dum volant, aerem terunt » nocturnum, & fulgent.

Et pag.398.

» Multitudo radiorum non admodum facit ad excitandum calorem si solum inci» dat sine repercussu, nec recta incidere iuuerit.

Quod falsum est cum radius incidens longè magis quam reflexus calefaciat. In fine autem.405.sic scribit.

» Sol in ortu & in occasu longius apparet, iccircò reuolui creditur. Hinc etiam in » abside stare putatur, & in opposito absidis, vnde solstitia vocant, sed nobis in Can» cro, antipodibus verò in Capricorno tum Sol abesse longius apparet vtrisque.

An hoc quid peius dici potest? Circa vero.40.lineam pag.459.sic scribit.

» Si enim alij planetæ,& stellæ fixę reciperent à Sole lumen,dum accederent ad So» lem,vel recederent,aut contra, Sol ad eas appropinquaret, & abscederet, easdem » lucis vicissitudinis subiret, quas Luna.

» Hoc autem nondum depręhensum est, quin etiam Mercurius,Venus,suo interpo» situ,Solem occultarent nobis,vt Luna.

» Paulo inferius sic ait. Rursus æquè Saturnus, Iupiter, Mars, subire deliquium, » more Lunæ,aut saltem obiectu terræ inter Solem & ipsos, quia tum ob interpositam » terram non possent haurire lumen à Sole.

Hæc verò omnia, talia sunt, qualia ab ijs qui incipiunt intelligere sphæram non proferrentur. Omittamus, quod ait deinde.

» Accedit quod si astra lumen à Sole acciperent eiusdem caloris essent. Itaque om» nia siccarent, & nulla essent frigidæ constitutionis contra Astrologos.

Quia hac ratione,Luna,quæ negari non potest, quin ab ipso Sole lumẽ accipiat, eiusdem caloris esset cum eodem Sole. Sunt ea etiam ridenda, quæ idem ait pag. 460.lineis.18.19.23.26.27.29.quasi ea lux infinita (vt ita dicam)magni Solis, non

in

in alium finem sit effecta quam ad illuminandam superficiem huius excrementi ipsius vniuersi ad vtilitatem hominum, imò, vt rectius dicam, animaliũ. vide etiam pag.632. et.633.vbi Aristotelem de implendo loco non intellexit, cum citet sphæram, loco pyramidis, & inter.46. et.47.lineas dicat quadratũ esse quid multiplex, cum sit vnicum tantum in specie, quia species est quadrilateri, & quadranguli, sed vbi in.6.linea pag.633.ait,

Item hexagonus.

Magnum errorem committit, vt etiam cum.12.linea.636.pag.scribens.

Pyramis, siue planum, siue solidum, habet acutissimum, & in. 2. libr. de anima pag.215.dicat de die posse videri stellas in speculo posito in vase aqua pleno, quod reuera est valde absurdum. Alios eiusdem errores tibi non patefacio, quia iam nihil amplius otij mihi est, sed eos tu ipse perspicere, & cognoscere facilè poteris, & multò plures quidem, quam putas.

## *Cur maius lumen extenuet minus.*

### PIRRO DE ARZONIS.

EX tuis literis intellexi id, quod etiam sine ijs exploratum mihi erat. Sed concedo tantum esse dicere vbi est maius lumen, minus non discerni, quantum inter diu stellas non videri: immo est etiam magis vniuersale, quia idem multis aliis luminibus, præter ea quæ sunt stellarum, ea ratione contingit, quia ingrediente per pupillam, tam lumine maiori, quam minori, reflexum ipsius maioris in oculo, in situ minoris, efficit, vt ipsum minus confundatur, & distingui nequeat, quemadmodum aperte cognosci potest in aliquo cubiculo, cuius parietes dealbati sint, in quo, vnicum tantum sit exiguum foramen, per quod aliqua lumina reflexa ab obiectis extrinsecis intra ipsum cubiculum ingredi possint, vnde imagines obiectorũ in parietibus conspiciuntur, sed si per idem foramen ingrederetur etiam primarius radius Solis, reflexus huiusmodi radij efficeret, vt dictæ imagines, magis aut minus euanescerent, prout dictus reflexus radij solaris, maiori, minoriue vi polleret.

Ad hoc tamen propositum, nolo tibi silentio inuolui mirabilem quendam effectum eiusmodi rei. Hoc est vt fiat foramen illud rotundum, magnitudinis tamen vnius specilli, quod foramen obturetur mediante vno illorum specillorum, quæ pro senibus (non breuis visionis) conficiuntur, hoc est quorum ambæ superficies conuexæ sunt, non autem concauæ. Deinde opponatur folium album papiri, adeo distans à foramine, vt extrinseca obiecta in eo appareant. Quæ quidem obiecta si à Sole illustrata fuerint, tam clara, & distincta videbuntur, vt nihil pulchrius delectabiliusq; videri poterit, inuersa tamen. Sed si ea directa videre voluerimus. hoc optimè faciemus, mediante reflexione alicuius speculi plani.

De

### *Cur hyems valde frigida sequatur æstatem in qua calor viguerit.*

## NOBILISSIMO, NECNON INGENIOSISSIMO Gabrieli Buschæ, Mediolanensi.

QVod dixi hyemem valde frigidam sequi æstatẽ, in qua calor viguerit, inde nascitur, quia calor terrę, aquæ, & aeris, non est naturalis horum corporum, vt est frigus, cum calor à Sole procedat, qui ea calefacit suo lumine, vnde quod æstate Sol præter modum calefaciat terrã, ideo cõtingit, quod minora impedimẽta contraria sortiatur, & cum eandem postea deserit, ad aliam partem æquatoris transmigrãs terra ad suam qualitatem reddit, maiori cum impetu, eo modo, quo res in motibus localibus naturalibus, qui etiam terminos sibi prę fixos, & constitutos excedunt, hinc etiam hyeme fit glacies, ex calefacta prius aqua, quæ durior postea est atque frigidior alia. Aestas etiàm quæ sequitur hyemem valde frigidam, non erit admodum calida, quia Sol inueniens contrarium naturale valde potens, non tam facile illud pellere potest, vnde etiam si in Geminis, Cancro, & Leone, moram trahat, non sufficit tamen ut magnum calorem imprimere possit. Vnde sequitur duas æstates quarum una sequatur aliam, in eodem loco, uehementi calore præditas esse non posse, quemadmodum nec duas hyemes excessiuo frigore, remotis tamen accidentibus uentorum, pluuiarum, & niuium.

## QVOD MALE SENSERIT NICOLAVS TARTAlea circa attractionem machinæ tormentalis.

## AD EVNDEM.

EFfectus, quem scribit Tartalea quęsito quinto primi lib. necnon quæsito 21. et. 24. maxima cum ratione esse uidetur, non tamen ea quam ipse in quinto profert, quia uerum non est, vt quanto aliquid sit calidius, tãto uehementius attrahat, eo quod si etiam huiusmodi res, in eodem calore, in quo semel reperitur, firma maneret; neque attraheret, neque aliquid impelleret. Nam dum aliquod corpus calefit, dilatatur, & per consequens circumcirca undiq; trudit, & partes uasis debiliores cedunt. dum uerò dictum corpus refrigeratur, restringitur, & dum in unum cogitur, si reperiatur in uase, quod aer, aqua, aut aliud aliquod corpus ingredi nequeat, dictum uas à quo circundatur frangit, ne aliqua pars loci uacua remaneat, sed si aliquod corpus ingredi potest, illud ipsum ad se attrahit, quemadmodum uidere licet in cucurbitulis. Vnde sequitur eam propositionem, qua dicitur, calidi est attrahere, ueram non esse, quia si hoc fieret, quanto aliquid calidus efficeretur, tanto magis attraheret, & ècontra, cum tamen planè contrarium appareat, cum quanto magis aliquid calefit, tanto uehementius impellat, & quanto magis frigefit, tanto plus attrahat. Quapropter uerius dicemus, frigidi esse attrahere, calidi uerò expellere, quamuis per accidens. Ex quo sequitur, ut quanto calidior facta fuerit materia aliqua, aliquo loco determinata, redeundo postea ad suam priorem frigiditatem, tanto minori loco indigeat, similiter etiam è conuerso accidit, ut quanto frigidior repitur talis materia, tanto maiori loco, po-

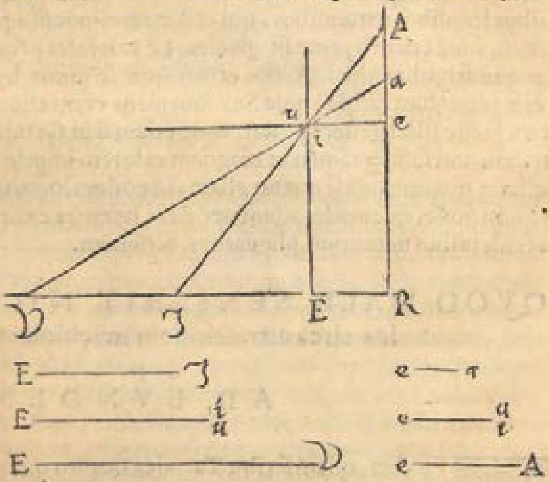
stea

ſtea egeat ipſa ualde calefacta. Quod Tartalea in quinto quęſito nòn animaduerterat.

## *Solutiones aliquæ, circa altimetriam.*

## AD EVNDEM.

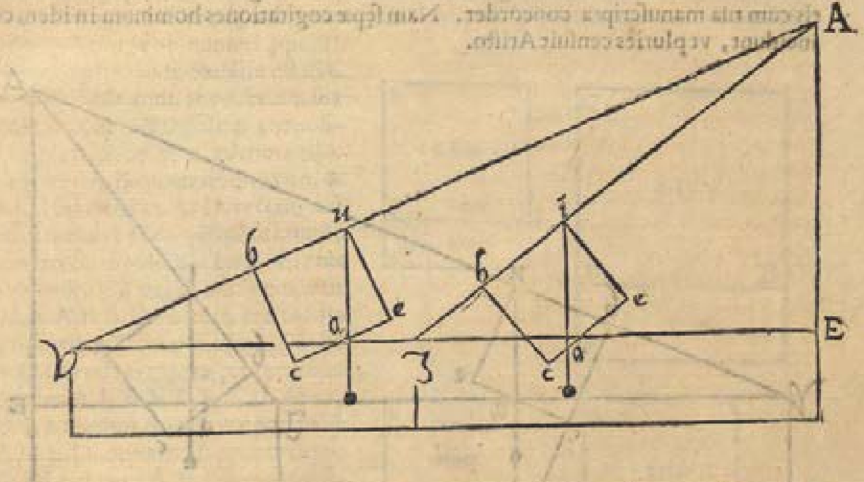
TVas literas accepi, tuasq; dubitationes conſideraui, quas quidem non inutiles inueni, quo uerò ad primam, dico te oportere illud Theorema ſpeculari ordine huiuſmodi methodi, uidelicet quod quotieſcunq; habuerimus angulũ aliquẽ cuiuſuis amplitudinis, puta. A.R.V. cuius duo latera. R. A. et. R. V. indeterminata intelligantur, ſi ab aliquo puncto inter ipſas poſito, puta. u. quod etiam uocetur. i. ductę fuerint. 4. lineę ipſis dictis lateribus, hac ſcilicet cõditione, qđ duę ex dictis. 4. ſint parallelę ipſis laterib9, puta u.e.et.u.E. reliquę uero duę ſecent ipſa latera, ut V. u. a.et.I.u.A. Dico nunc proportionem. e.A.ad.e.a. eandem eſſe, quę. E.V. ad. E. I. Nam ſcimus proportionem E.i.ad.E.i. eandem eſſe quę e.i. ad. e. A. ex ſimilitudine triangulorũ, ſimiliter ꝓportionẽ. E.u.ad.E.V. eãdẽ quę e.a.ad.e.u. quare aggregata ex iſtis erunt inuicem ęqualia, uel ſi mauis ex ęqua proportionalitate, quod idem eſt, ita ſe habebit. E.I.ad. E.V. ut.e.a.ad.e.A.



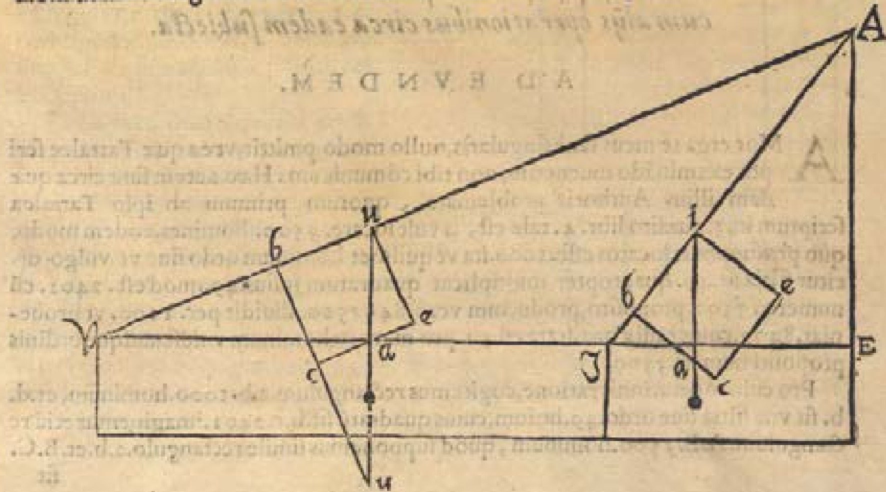

Suppoſito nunc plano orizontali. V.E. Altitudineq; inacceſſibili. A. E. Duę uerò ſtationes oculorum ſint. V.et.I. lineę autem uiſuales ſint. V.A.et.I.A. Et quadratum geometricum ſit. b.e. Supponatur nunc pro prima dubitatione, quod in ambabus ſtationibus filum perpendiculare ſecet latus.e.c.non autem.b.c.(nam quando in ambabus ſtationibus filum ſecat latus.b.c.nullum tibi dubium oritur, imo manifeſtè patent partes lateris.b.c.terminatas à.b.& à filo proportionales eſſe.V. E. & I. E. ſumpto. E.pro.b.et.I.V.pro punctis ſecatis à filo, ex euidẽti ſimilitudine triangulorum quadrati cum triangulis. A.E.V.et.A.E.I.) Sed cum in pręſenti caſu reperiatur triangulum.u.e.a.minus, in ſtatione remotiori, ſimile triangulo maiori.V.E. A.& triangulum maius.i.e.a. proximioris ſtationis, ſimile triangulo minori.I.E.A. (quod in alio iam dicto, caſu non accidit, ut unum triangulorum, minus ſcilicet, ſimile ſit uno triangulorum, maiori ſcilicet & è conuerſo) Non omnino abſque ratione dubitas quo pacto fieri poſſit ut.a.e.remotioris ſtationis ad.a.e.propinquioris ita ſe habeat quemadmodum.I.E.ad.E.V. Quapropter ſi pręcedentem figuram dili-

gen-

genter inſpexeris, omnis tua dubitatio euaneſcet. in qua figura apertè vi debis correſpondentiam talium triangulorum inter ſe, nec magis, nec minus quam in infraſcripta hic figura cernere licet, quamuis in hac, triangula quadrati, ſeparata ſint ab imaginarijs. A.E.V. et. A.E.I. in ſupradicta vero coniuncta, & inuicem communicantia in puncto.u.i. quod quidem nihil refert. Dempta igitur.a.c. minori ex.a.c. maiori, reliquum ita ſe habebit ad.a.c. minorem, vt, V.I. ad. I.E. quod nunc tibi clarè patebit. Vnde ex te poteris ordinem operationis proſequi, vt in cognitionem peruenias ipſius.I.E. ipſius.A.E.& ipſius.I.A. vel.V.A.

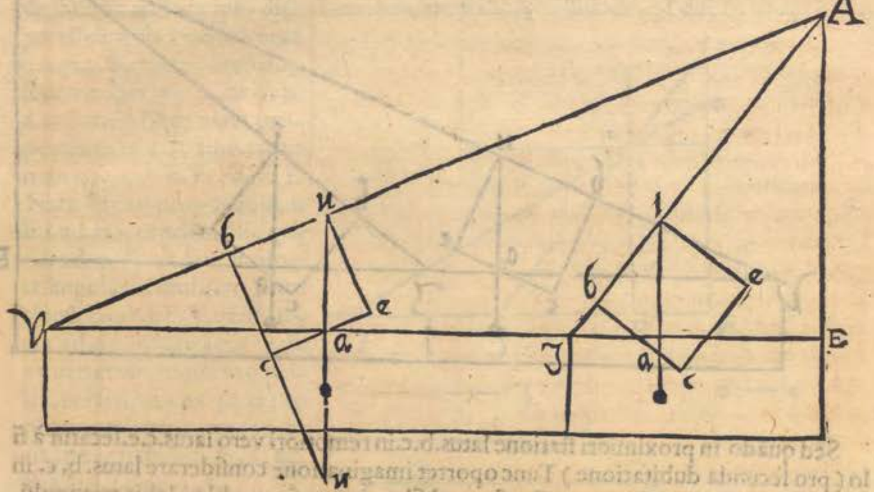



Sed quādo in proximiori ſtatione latus.b.c. in remotiori vero latus.c.e. ſecatur à ſilo (pro ſecunda dubitatione) Tunc oportet imaginatione conſiderare latus. b. c. in remotiori ſtatione diſtentum eſſe vſque ad filum in puncto.n. vbi videbis triangulū. u.b.n. ſimile triangulo. A.E.V. ita vt.i.b.a. ſimile ſuo. A.E.I. reperitur, vbi tam in vno



quam in altero.i.b.et u.b.correſpondebit ipſi.A.E.et.b.n.ipſi.E.V. et. b.a. ipſi. E. I. quapropter iubeo, vt quæras quantum ſit latus.b.n.ex regula de tribus, dicens ſi.a.e. tribuit mihi.e.u.quid mihi dabit.u.b?eo quod.a.e.u.ſimile eſt.u.b.n. reperto autem latere.b.n.ex quo dempto.b.a.breuioris diſtantię,reſiduum reſpondebit ipſi.I. V.vt ſcis, vnde proſequendo operationem tibi cognitam, obtinebis intentum, hoc eſt cognoſces reliqua interualla. Nihil enim miror demonſtrationem Tartaleæ circa huiuſmodi operationem te minime ſatisfeciſſe.

Quod autem quarta propoſitio illius ſcriptoris, de quo nuper mecū locutus es, vt mihi dixiſti, tua ſit, hoc enim ego, nec affirmare, nec negare audeo, quamuis in multis cum tua manuſcripta concordet. Nam ſępæ cogitationes hominum in idem coincidunt, vt pluries cenſuit Ariſto.




*Demonſtrationes quorundam problematum Nicolai Tartaleæ cum alijs operationibus circa eadem ſubiecta.*

## AD EVNDEM.

AMor erga te meus ſanè ſingularis, nullo modo ꝑmittit, vt ea quæ Tartaleę ſcripta examinādo inuenerim, non tibi cōmunicem. Hæc autem ſunt circa quædam illius Authoris problemata, quorum primum ab ipſo Tartalea ſcriptum in.3.quæſito libr. 4. tale eſt, is vult locare. 3500. homines, eodem modo, quo præſupponit locatos eſſe. 1000. ita vt quilibet hominum ordo ſiue vt vulgo dicitur filtia ſit.49. quapropter multiplicat quadratum ipſius.49.quod eſt. 2401. cū numero. 3500.propoſito, productum verò.8403500. diuidit per. 1000. vt proueniat.8403.cuius radix quadrata eſt.91.pro numero hominum vniuſcuiuſque ordinis propoſiti numeri.3500.

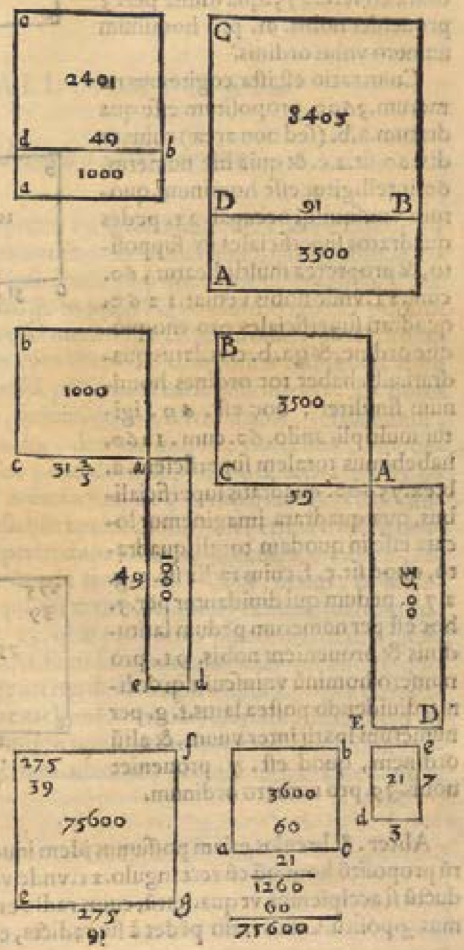
Pro cuius operationis ratione, cogitemus rectangulum.a.b.1000.hominum, et.d.b. ſit vna filtia ſiue ordo.49.hoīum, cuius quadratū ſit.b.c.2401.imaginemur etiā rectangulum.A.B.3500.hominum, quod ſupponemus ſimile rectangulo.a.b.et.B.C.

ſit

ſit quadratum ipſius. D.B. Nunc ſupponendo. A.B.ſimile.a.b.clarum erit ex diffinitione ſimilium figurarum, quod eadem proportio erit. A.D.ad.D.B.quę.a. d. ad. d. b.hoc eſt. A.D.ad.D.C.vt.a.d.ad.d.c.hoc eſt.A.B.ad.B.c.vt.a. b. ad. b. c. ex prima ſexti,vel.18.ſeu.19.ſeptimi,tunc cum dixerimus ſi.a.b.ita reſpondet ad.b.c.ergo.A. B. correſpondet etiam ita ad.B.C.quare ex regula de tribus rectè fit multiplicando. A.B.per.b.c.productum verò diuidendo per.a.b.ex.15.ſexti vel.20. ſeptimi, cuius prouentus radix quadrata erit quod quærebatur.

Sed aliter idem poſſe fieri ſpeculatus ſum, hoc eſt multiplicando numerum. 49. ordinis. 1000.hominum cũ radice quadrata numeri .3500.propoſiti,productum verò diuidere per radicem quadratam ipſius.1000. vnde prouentus. 91. erit numerus vnius ordinis.3500.numeri ꝓpoſiti.

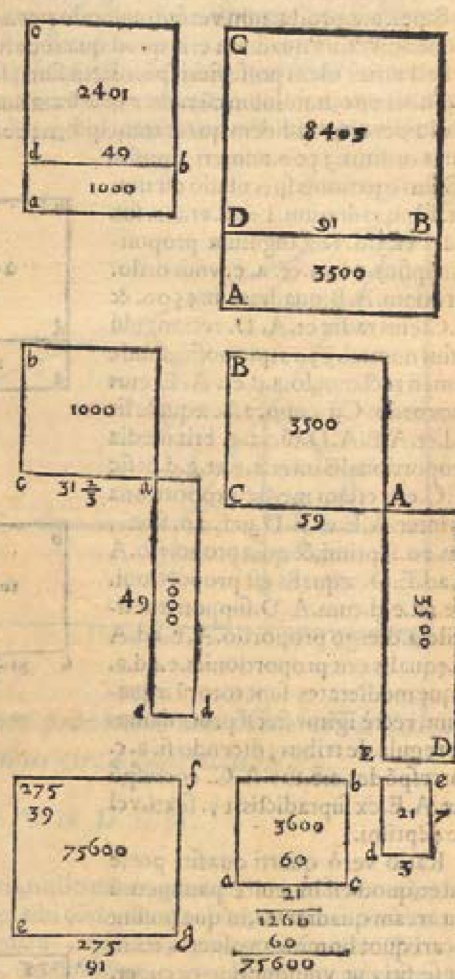
Cuius opationis ſpeculatio eſt iſta. Sit.a.b.quadratum.1000.et.a.c.ſua radix et.a.d. rectangulum propoſitum ipſius. 1000.et. a. c. vnus ordo. Sit etiam. A.B.quadratum.3500. & A.C.eius radix et. A. D. rectangulũ ipſius numeri.3500.propoſiti,ſimile tamen rectangulo.a.d.et. A. E. eius vnus ordo Cũ enim. a.b. æquale ſit a.d.et.A.B:A.D.tũc.a. c. erit media proportionalis inter.a.e.et.e.d.& ſic A.C.erit etiam media proportionalis inter.A.E.et.E.D.per. 16. ſexti, ſeu.20.ſeptimi,& quia proportio. A E.ad.E.D. æqualis eſt proportioni. a.e.ad.e.d.cum. A. D.ſupponatur ſimile.a.d.ergo proportio. A.E.ad. A C.ęqualis erit proportioni.a.e.ad.a. c.quę medietates ſunt totorũ æqualium, rectè igitur fiet ſi procedamus ex regula de tribus, dicendo ſi. a. c. correſpõdet.a.e.tũc.A.C. correſpõdet.A.E.ex ſupradictis.15. ſexti.vel 20.ſeptimi.




Ratio verò quarti quæſiti per ſe patet, quod eſt inuenire pauimentũ ſeu aream quadratam, in qua poſſint locari quot homines volueris, ita inter ſe ſiti, ut vnuſquiſque occupet. 7.pedes ipſius areę in longitudinem et.3.per latitudinem à lateribus.

Seu ex propoſito hominum numero inuenire numerum ipſorum locabilem in aliqua area quadrata, ita, vt vnuſquiſque occupet. 21. pedes quadratos ipſius areæ.

Sed aliter idem fieri posse inueni, hoc est multiplicãdo radicem quadratam propositi numeri hominum per.21.& productum item multiplicando per eandem radicem,& huiusmodi producti radicem diuidendo per.3.vnde prouentus esset numerus hominum vnius ordinis. Exẽpli gratia proponuntur. 3600.homines,multiplicabimus huiusmodi numeri radicem quadratam hoc est.60. per. 21. hoc est per productum quod fit ex.7. cũ 3.& resultabit nobis. 1260. quod si multiplicabitur, per.60.hoc est per eandem radicem, resultabit nobis. 75600. cuius producti radix quadrata est ferè.275.qua diuisa per. 3 proueniet nobis. 91. pro hominum numero vnius ordinis.




Cuius ratio est ista,cogitemus numerum.3600. propositum esse quadratum.a.b. (sed non areæ) cuius radix.60.sit.a.c. & quia hic numerus. 60.intelligitur esse hominum,quorum vnusquisq; occupat. 21. pedes quadratos superficiales ex supposito,& propterea multiplicatur, 60. cum.21.vnde nobis veniat.1260. quadrati superficiales pro vnoquoque ordine, & qa. b. c.vt.latus quadrati.a.b. habet tot ordines hominum similiter, hoc est. 60. igitur multiplicando. 60. cum. 1260. habebimus totalem superficiem. a. b.ex.75600. quadratis superficialibus, quæ quadrata imaginemur locata esse in quodam totali quadrato, quod sit.e. f. cuius radix sit. e. g. 275. pedum qui diuidantur per.3. hoc est per numerum pedum latitudinis & prouenient nobis. 91. pro numero hominũ vniuscuiusq; ordinis, diuidendo postea latus.f. g. per numerum spatij inter vnum, & aliũ ordinem, quod est. 7. proueniet nobis.39.pro numero ordinum.

Aliter. & breuius etiam possumus idem inuenire, hoc est multiplicando numerũ propositũ hominũ cũ rectangulo.21.vnde veniet nobis ꝓductũ.75600quod productũ si accipiemus vt quadratũ, cuius radix erit.275. quæ diuidatur p. 3. habebimus ꝓpositũ.Cuius ratio pẽdet à supradicta, eo qđ loco multiplicãdi.a.c. (hoc est. 60.)per

60.)per.21.deinde productũ etiam multiplicare per.b.c. (hoc eſt. 60.) breuius erit multiplicare totum numerum.3600.per.21.cętera verò facere, vt diximus.

Sed vnaquæq; iſtarum operationum, aliquid imperfectionis patitur, eo quod cũ aliquis cuperet quadratum perfectum ſuperficiale habere, abſq; aliquo defectu, vel exceſſu, aliquid aliud adhuc facere oporteret, hoc eſt, inuentum cum fuerit quadratum.e.f.cum ſuis radicibus.e.g.et.g.f.pedum.275.vnaquaque, vt in dicto exemplo factum eſt, oportebit numerũ quærere minorem ipſo.275. ſed proximiorem menſurabilem ab.3.& ab.7.quod facilè fiet ſi diuiſerimus.275.per. 21. detrahendo fracta diuiſionis ab ipſo.275.quæ quidem fracta in hoc exemplo ſunt.2. vnde remanebit.273.pro numero laterum quadrati ſuperficialis, in quo poſſent locari.3549.homines, eo ordine quo ſupra dictum eſt, quorum ſcilicet vnuſquiſque obtineat. 21. pedes ſuperficiales.

## DE INTERVALLIS MVSICIS.

### *Cypriano Rorè Muſico celeberrimo.*

OPINIO Hectoris Euſonij Cypriane mi dilectiſſime, vera non eſt, quod aliquis rectè poſſit intelligere rationes conſonantiarum muſicæ, abſque cognitione illarum mediante ipſo ſenſu, imo nemo pôt calere theoriã muſices, niſi aliquo mõ verſatus ſit in praxi. Qũo enim cognoſci poterũt quid nam ſint diapaſon, diapente, diateſſeron, ditonus, ſemiditonus, hexacordum maius, aut minus, & conſonantiæ ex ijs cum diapaſon compoſitæ, abſque earum praxi? vnde ſequetur neq; etiam cognoſci poſſe interualla diſſonantia. Et purus practicus non intelliget quid ſit octaua, quinta, quarta, tertia maior, tertia minor, ſexta maior, ſexta minor, decima maior, decima minor, vndecima, duodecima, decimatertia maior, aut minor, aut decimaquinta, & aliæ, ita vt ad comparandam perfectionem muſicæ neceſſarium ſit, & thęoriam & praxim addiſcere. Cum pręterea Ludouicus Folianus apertè monſtrarit (etiam ſi id a diatonico ſintono Ptolomei deſumpſerit) reperiri duos tonos, maiorem, & minorem, ideſt ſeſquioctauum, & ſeſquinonum, & tria ſemitonia, maius, minus, & minimum, ideſt ſeſquiquintumdecimum, qui eſt maius, ſeſquiuigeſimumquartum ideſt minimum, & mediocre, vt. 27. ad. 25. quæ proportio ſuperbipartiens vigeſimaſquintas appellatur, & cum cognouerit ſemiditonum conſonantem eſſe ſeſquiquintum, ditonum ſeſquiquartum, & hexachordum minus, vt.8.ad.5. quæ proportio dicitur ſupertripartiens quintas, & hexachordum maius, vt.5.ad.3.hęc autem vocatur ſuperbipartienstertias; omnium ſimplicium conſonantiarum cognitioni, extremam impoſuit manum. Et quia tibi etiam oſtendere promiſi in modulationibus

hæc

hæc omnia interualla ſeruari, ideo ad te mitto ſeptem hic ſubſcripta exempla, in quorum primo, & ſecundo, inter dieſim, et. b. in ſuperiori, agnoſces interuallum minimi ſemitonij, & ſi ibi ſit dieſis, tanquam terminus ad quem, et. b. tanquam terminus à quo: quod autem inter dieſim et. b. ſit ſemitonium minimum, facilè agnoſces ſi ſubtraxeris decimā minorē à maiori, quā facit ſuperius cū inferiori, ideſt cū baſſu.

Qua quidem modulatione tu etiam vſus es in cantilena illa, quæ Galica lingua incipit. Hellas comment. Eadem, ego quoque in meis cantilenis latino ſermone compoſitis, quæ Moreta vocantur aliquando vſus ſum.

Sed in tertio exemplo inuenies ſemitonium maius, neceſſariò genitum in ſuperiori, ſi ſextam maiorem cum baſſu efficere volueris, quia tenor, à ditono cum ſuperiori ad diapentem, & ad vniſonum cum baſſu procedit, vbi quieſcit, progrediendo poſtea baſſus ad ſemiditonum cum tenore, tunc ſi à proportione huius ſeptimæ, quæ eſt vt. 9. ad. 5. hoc eſt ſuperquadripartienſquintas demptum fuerit hexachordum maius, ſeu ſexta maior, quæ eſt vt. 5. ad. 3. remanebit proportio. 27. ad. 25. quæ maior eſt quam. 32. ad. 30.

In quarto exēplo habebis ſemitonium minus in ſuperiori, quod quidem remanet ex ſubtractione ditoni cōſonātis ab diateſſaron cōprehenſa à ſuperiori cum tenore.

In quinto exemplo videbis tonum minorem, & tonum maiorem ſucceſſiuè vnum poſt alium in tenore, detrahendo primo ſemiditonū à diateſſaron, quod ſuperius facit cum tenore, vel detrahendo diapente ab hexachordo maiori, quod facit tenor cum baſſu, vnde remanet tonus minor ſeſquinonus, detrahendo poſtea diateſſaron à diapente, quod ſuperius facit cum tenore, remanebit tonus maior ſeſquioctauus.

In ſexto exemplo deinde videbis tenorem aſcendere per duos tonos minores ſucceſſiuè vnum poſt alium in tenore, ſi dēpſeris ſemiditonū à diateſſaron cū ſuperiori.

In. 7. exēplo demum videbis ſuperiorē aſcendere per duos tonos maiores ſucceſſiuè vnū poſt aliū, ſi dempſeris diateſſaron à diapente, quod facit tenor cū ſuperiori.

De

## *De eodem ſubiecto.*

## AD EVNDEM.

QVod aliàs tibi dixi, verum eſt, quod neceſſarium nullo modo ſit, vt modulando, deſinat cantilena in eodem tono (quod Græci phthongum appellant) à quo incępit. immo neceſſariò ſemper ferè, altius, aut depræſſius terminatur, per differentiam alicuius interualli æqualis, vel multiplicis ipſi commati ſeſquioctuageſimæ, quod quidem comma, quamuis cantabile non ſit, inſenſibiliter tamen generatur, & toties ab aliqua parte ipſius cantilenæ poſſet dictũ comma generari, verſus acutum, vel graue, quod in fine ipſius cantilenę, vocis phtongus reperiatur diſtans à primo per interuallum alicuius toni ſeſquinoni, ſeu ſeſquioctaui plus, minúsue, vt in ſubſcripto exemplo clarè videre potes in prima figura, vbi ſuperius à.g.primę cellulæ ad.g.ſecundæ, intereſt vnum cõma, eo quod progrediens ſuperius in prima cellula ipſius cantilenæ à quarta ad quintam cum tenore, aſcendit per tonum ſeſquioctauum, à prima cellula deinde ad ſecundam, tenor aſcendit ſimiliter per tonum ſeſquioctauum cum tranſeat à quinta ad quartam, quod facit cum ſuperiori, in ſecunda cellula poſtea, cum ſuperius deſcendat à maiori ſexta ad quintam, quod facit cum baſſũ, ſeu à quarta ad tertiam minorem, quod facit cum tenore, tunc deſcendit per tonum ſeſquinonum, ita quod non reuertitur ad eundẽ phthõgum, vbi prius erat in prima cellula, ſed reperitur per vnũ cõma altius, qđ quidẽ cõma eſt differentia inter tonũ ſeſquioctauũ & ſeſquinonũ, vt alias tibi demõſtraui.

Progrediendo igitur hoc modo, videbis quod cum tenor à ſecunda cellula ad tertiam tranſeat à tertia minori ad quartam, quod facit cum ſuperiori, deſcendit per tonum ſeſquinonum, vnde in tertia cellula altius remanet quam in prima per vnũ comma, in qua tertia cellula, cum iterum tranſeat ſuperius à quarta ad quintam, ꝙ facit cum tenore, eleuatur per tonum ſeſquioctauum, proſequendo deinde tali ordine, videbis in quarta cellula cantilenam auctam per duo commata, in ſexta, aũt cellula per tria commata, in octaua verò per.4.commata, vnde hac methodo, ſi cantilena prolixior debito eſſet, vel ſi talia interualla frequentiora reperirentur, poſſet cantilena à principio ad finem differre per. 9. commata, & plus etiam, quæ quidem inter-

inter-

interualla ſuperant tonum ſeſquinonum, & ſi eſſent. 10.commata ſuperarent tonum ſeſquioctauum, eo quod aggregatum ex.9.commatibus continetur ſub iſtis duobus terminis hoc eſt. 150094635296999121.et. 134217728000000000. quæ quidem proportio maior eſt proportione ſeſquinona, ſumma verò.10.commatum continetur ſub.12157665459056928801.et. 10737418240000000000. quæ proportio maior eſt tono ſeſquioctauo, quod autem dico de aſcenſu cantilenæ, idem aſſero de eiuſdem deſcenſu, & hoc non tantum per interuallum illius commatis, quod eſt differentia toni maioris à minori, ſed etiam per illud quod eſt differentia ſemitonij maioris à minori, vt in ſecundo exemplo hic ſubſcripto videre eſt in deſcenſu cantilenæ per comma & comma, vt differentia inter ſemitonia maiora & minora, vbi in prima cellula diſcedens baſſus à quintà cum ſuperiori, & ab vniſono cum tenore deſcendens ad tertiam minorem cum ipſo tenore, facit cum ſuperiori ſeptimã maiorem, quæ eſt vt.9.ad. 5. ſuperquadripartiensquintas ſcilicet, à qua diſcedens poſtea ſuperius, vt faciat cum baſſu ſextam maiorem, deſcendit per ſemitonium maius, à qua ſexta maiori deſcendens baſſus, & aſcendens per quartam, efficit cum dicto ſuperiori tertiã maiorem, à qua diſcedens ſuperius, vt efficiat quartam cum ipſo baſſu (qui quidem baſſus tranſit in tenorem) aſcendit per ſemitonium minus, differens à ſemitonio maiori per vnum comma, vnde cantilena remanet depreſſa per vnum comma. cum deinde idem faciat inter tertiam, & quartam cellulam, per aliud comma deſcendit; & ſic toties facere poſſet, vt poſtremo valde deprimatur cantilena à primo phthongo.

Quod autem hic ſupradictum eſt, circa inſtrumenta artificialia non accidit, quapropter organa, & clauicimbula concordantur certo quodam ordine, ita vt omnes conſonantiæ, excepta diapaſon, ſeu octaua, ſint imperfectæ, hoc eſt, aut diminutę, aut ſuperantes à iuſto, vt exempli gratia, omnes quintæ ſunt diminutæ, quartæ verò ſunt exceſſiuę, quod quidem fit, vt tertiæ, & ſextæ, non multum auribus diſſonent, eo quod ſi quintæ omnes, & quartæ, perfectæ eſſent, tunc omnes ſextę, & tertiæ intollerabiles eſſent, & à perfectis differrent per vnum comma, quod manifeſtum nobis erit hoc modo, accipiamus tres diapentes, ſeu quintas, conſequenter ſucceſſiuas vnam poſt aliam, hoc eſt tres proportiones ſeſquialteras, quarum aggregatum erit vt.27.ad.8.quæ proportio, dicitur tripla ſupertripartiensoctauas, & quæ à practicis

appel-

appellaretur tertiadecima maior, vt exempli gratia, eſſet Gamaut cum ſecundo ela mi, tunc talis tertiadecima valde odioſa eſſet ſenſui auditus, à qua, ſi dempta fuerit diapaſon, ſeu octaua, remaneret quoddam hexachordum maius, ſeu ſexta maior, auribus valde inimica, ſub proportione. 13. ad 8. ſed hæc proportio differret à proportione ſuperbipartientetertias perfecti hexachordi maioris, hoc eſt ſextæ maioris conſonantis, per proportionem ſeſquioctuageſimam, hoc eſt per vnum comma, quod quidem eſt etiam differentia aggregati trium ſeſquialterarum, à tertiadecima maiori conſonanti, hoc eſt exceſſus proportionis triplæ ſupertripartientis octauas, ſupra triplam ſeſquitertiam, quæ eſt ſumma ipſius duplæ cum ſuperbi partientetertias.

A tali ſumma igitur trium ſeſquialterarum efficitur tertiadecima maior diſſonans excedens conſonantem per vnum comma (cuius proportio eſt. 81. ad. 80.) quæ conſonans continetur in proportione. 10. ad. 3. vt ſupra dixi.

Hæc igitur eſt vera ratio, propter quam debemus comma diſtribuere in organis & clauicymbalis, cum ab aggregato trium quintarum producatur talis exceſſus ſupra perfectam, ſeu conſonantem tertiamdecimam maiorem, quod quidem aggregatum, cum demptum fuerit à quintadecima, relinquet nobis tertiam minorem diſſonantem, & mancam, per eundem exceſſum à conſonanti. quæ quidem tertia minor diſſonans ſubtracta à diapente ſeu quinta perfecta, relinquet nobis tertiam maiorem diſſonantem, quæ conſonantem excedit per eundem exceſſum commatis, & hæc demum tertia maior diſſonans, dempta ex diapaſon, ſeu octaua, relinquet nobis hexachordum minus, hoc eſt ſextam minorem diſſonantem, & mutilam à conſonanti per eundem exceſſum commatis. De huiuſmodi verò commatis diſtributione doctiſſimè ſcripſit Excellentiſſimus Zarlinus in ſecunda parte Inſtitutionum Harmonicarum.

Sed quia ſenſus auditus non poteſt exactè cognoſcere debitam quantitatem exceſſus, vel defectus, intendendo vel remittendo chordas inſtrumentorum, ideo hanc viam ſequutus ſum.

Sit exempli gratia, hic ſubſcriptus ordo lignorum tangentium ſeu pinarum incipiens ab. G. deſinens ad. g. ita quod inter ipſos terminos ſit ea conſonantia quæ vocatur vigeſimaſecunda, quæro primum. b. inter. D. E. quod eſt nigrum ipſius Elami grauiſſimum, quod groſſo modo facio conſonans cum. G. grauiſſimo per ſextam minorē, deinde cū ipſo primo. b. ipſius elami concordo ſuum octauum & quintumdecimum, quo perfectius poſſum, deinde accipio. b. molle ſecundum ipſius. b fabmi quod concordo cum. b. primo ipſius Elami per quintam imperfectam, deinde cum hoc. b. ſecundo ipſius bfabmi concordo ſecundum. f. per quintam ſimiliter imperfectam, cum quo. f. poſtea concordo tertium. c. per ſimilem quintam, quem tertium. c. poſtea confero cū ſecundo. b. ipſius elami, ita quod inter ſe conſonent per ſextam maiorem tolerabilem, & ſi ſic inuenio, tunc nihil muto has treschordas hoc.

G A B b C * D b E F * g * a b b c * d b e f * g * a b b c * d b e f * g *
1 1 1 1 1 1 1 1 1 1 1 1 2 2 2 2 2 2 2 2 2 2 2 2 2 3 3 3 3 3 3 3 3 3 3 3 3 3 3 4 4.

Na est

eſt. b. ſecundum ipſius bfabmi,f.ſecundum , et. c. tertium , ſed ſi dictum tertium.c. valde diſſonans eſſet cum. b. ſecundo ipſius elami, tunc ipſum . c. intendo , aut remitto, quouſque aliquo modo ſit conſonans per ſextam maiorem aliquantulum excesſiuam cum.b.ſecundo ipſius elami,cum quo poſtea.c. conſonare aliquantulum facio.f.ſecundum per quintam defectiuam , & cum hoc demum. b. ſecundum ipſius bfabmi,quo facto concordo ſecundum.c.cum tertio per octauam , cum quo ſecundo.c.poſtea concordo tertium.g.per talem quintam, quod ipſum tertium.g.cum ſecundo.b.ipſius bfabmi conſonet tolerabiliter per ſextam maiorem aliquãtulum excesſiuam,deinde cum iſto tertio.g.concordo tertium.d.per talem quintam,ita quod ipſum.3.d.concordet tolerabiliter cum.2.f.per ſextam maiorem exceſſiuam, poſtea cum hoc.3.d.concordo.2.d.per octauam perfecte,cum quo. 2. d. poſtea concordo. 3.a.per quintam,vt in alijs factũ eſt,ita vt cũ.2.c.conſonet talis ſexta maior,vt ſupra dictum eſt,cum quo.3.a.poſtea concordo.3.e.per quintam, vt dictum eſt , ita quod cum.3.g.faciat ſextam maiorem vt ſupra,poſtea cum hoc.e.concordo.2.e. per octauam,cum quo concordo.b.quadrum tertium per quintam,vt dictum eſt,ita quod cũ 2.d.faciat ſextam maiorem ſimilem alijs ſuperius dictis,cum quo.b.quadrato tertio concordo tertium nigrum ipſius.f.per quintam,ita quod cum.3.a.faciat ſextam maiorem,vt ſupra,deinde cum hoc concordo.2. f. nigrum per octauam , cum quo , per quintam concordo 3.c.nigrum ita quod cum.2.e. faciat ſextam dictam , demum cũ hoc concordo.4.g.nigrum per quintam,ita quod faciat cum. 3. b. quadrato ſextam dictam,& ſic ad vltimam quintam peruenio,ſupra quod.g.nigrum nulla quinta amplius reperitur; poſtea cum iſtis chordis concordo per octauas omnes alias ab acutis ad graues.

G A B b C * D b E F * g * a b b c * d b e f * g * a b b c * d b e f * g *
1 1 1 1 1 1 1 1 1 1 1 2 2 2 2 2 2 2 2 2 2 2 3 3 3 3 3 3 3 3 3 3 3 4 4.

Valde etiam admiratione dignum eſt , quod perfectiores quæque conſonantiæ,ita in harmonica diuiſione ſibi inuicem conueniant , vt diapaſon cum diapente, cum diapaſondiapente,cum ditono , cum hexachordo maiori cum bisdiapaſon , cũ decimaſeptima maiori . Nam in ipſa diapaſon,harmonicè locãtur diapente in parte grauiori,& diateſſaron in acutiori.In diapente verò harmonicè locantur ditonus in parte grauiori , & ſemiditonus in acutiori . In ditono harmonicè locantur tonus maior in parte grauiori,& tonus minor in acutiori . In hexachordo maiori, harmonicè locantur diateſſaron in parte grauiori , & ditonus in acutiori . In diapaſondiapente , harmonicè locantur diapaſon in parte grauiori , & diapente in acutiori . In bisdiapaſon , harmonicè locantur decima maior in parte grauiori & hexachordum minus in acutiori.In decimaſeptima maiori,harmonicè locantur diapaſondiapente in parte grauiori , & hexachordum maius in parte acutiori . Ita quod tonus ſeſquioctauus in ditono,proportionalis eſt ipſi ditono in diapente. Tonus verò ſeſquinonus in ipſo ditono,proportionalis eſt triemitonio, vel ſeſquitonio ſeu ſemiditono (quod idem eſt ) in diapente . Ditonus autem in diapente , proportionalis eſt ipſi diapente in diapaſon.Seſquitonus verò in diapente , proportionalis eſt diateſſaron in diapaſon. Et ſic de ſingulis . Ita quod tonus ſeſquioctauus in ditono,ditonus in diapente,diateſſaron in hexachordo maiori,diapente in diapaſon , diapaſon in diapaſondiapente , decimamaior in bisdiapaſon , diapaſondiapente in decimaſeptima

ſeptima maiori, omnia ſibi inuicem ſunt proportionalia, idem etiam dico de reliquis partibus, cum relatæ fuerint ad ſua tota.

Nec alienum mihi videtur à propoſito inſtituto, ſpeculari modum generationis ipſarum ſimplicium conſonantiarũ; qui quidem modus fit ex quadam æquatione percuſſionum, ſeu æquali concurſu vndarum aeris, vel conterminatione earum.

Nam, nulli dubium eſt, quin vniſonus ſit prima principalis audituq; amiciſſima, nec non magis propria conſonantia; & ſi intelligatur, vt punctus in linea, vel vnitas in numero, quam immediate ſequitur diapaſon, ei ſimillima, poſt hanc verò diapente, cæteræq;. Videamus igitur ordinem concurſus percuſſionum terminorum, ſeu vndarum aeris, vnde ſonus generatur.

Concipiatur igitur mente monochordus, hoc eſt chorda diſtenta, quæ cum diuiſa fuerit in duas æquales partes à ponticulo, tunc vnaquæq; pars eundem ſonum proferet, & ambæ formabunt vniſonum, quia eodem tempore, tot percuſſiones in aerę faciet vna partium illius chordæ, quot & altera: ita vt vndæ aeris ſimul eant, & æqualiter concurrant, abſque ulla interſectione, vel fractione illarum inuicem.

Sed cum ponticulus ita diuiſerit chordam, vt relicta ſit eius tertia pars ab vno latere, ab alio vero, duę tertię, tunc maior pars, dupla erit minori, & ſonabũt ipſam diapaſon conſonantiam, percuſſiones verò terminorum ipſius, tali proportione ſe inuicem habebunt, ut in qualibet ſecunda percuſſione minoris portionis ipſius chordæ, maior percutiet, ſeu concurret cum minori, eodem temporis inſtanti, cum nemo ſit qui neſciat, quod quo longior eſt chorda, etiam tardius moueatur, quare cum longior dupla ſit breuiori, & eiuſdem intenſionis tam vna quam altera, tunc eo tempore, quo longior vnum interuallum tremoris perfecerit, breuior duo interualla conficiet.

Cum autem ponticulus ita diuiſerit chordam, ut ab uno latere relinquantur duæ quintæ partes, ab alio verò tres quintæ, ex quibus partibus generatur conſonantia diapente; tunc clarè patet, quod eadem proportione tardius erit vnum interuallum tremoris maioris portionis, vno interuallo tremoris minoris portionis, quam maior portio habet ad minorem; hoc eſt tempus maioris interualli ad tempus minoris erit ſeſquialterũ; quare non cõuenient ſimul, niſi perfectis tribus interuallis minoris portionis, & duobus maioris; ita quod eadem proportio erit numeri interuallorum minoris portionis ad interualla maioris, quæ longitudinis maioris portionis ad longitudinem minoris; vnde productum numeri portionis minoris ipſius chordæ in numerum interuallorum motus ipſius portionis, æquale erit producto numeri portionis maioris in numerum interuallorum ipſius maioris portionis; quæ quidem producta ita ſe habebunt, vt in diapaſon, ſit binarius numerus; in diapente verò ſenarius; in diateſſaron duodenarius, in hexachordo maiori quindenarius; in ditono vicenarius, in ſemiditono tricenarius, demum in hexachordo minori quadragenarius: qui quidem numeri non abſque mirabili analogia conueniunt inuicem.

Voluptas autem, quam auditui afferunt conſonantiæ ſit, quia leniuntur ſenſus, quemadmodum cõtra, dolor qui à diſſonantijs oritur, ab aſperitate naſcitur, id quod facilè videre poteris cum conchordantur organorum fiſtulæ.

# DE IVSTITIA COMMVTATIVA.

*Francisco Ferrario Ancisæ Iurisconsulto senatoriq; apud subalpinos grauißimo.*

SAEPIVS inter nos dum oportunitas vicinarum ædium, & amoris mutui vis, ad familiaria trahunt colloquia ego de meis mathematicis, tu de tuis legibus, in quibus tractandis magnum tibi nomen comparasti loquuti fumus. Cum vero nonnunquam de mirabili iustitiæ commutatiuæ instituto non ingratus incidisset sermo, dixi modum, quo formam suam à proportionalitate arithmetica disiuncta, & non a coniuncta desumat, à nemine literis proditum esse, libet autem nunc per otium latius explicare. dixi enim à disiuncta, & non coniuncta proportionalitate, quia in coniuncta, seu continua nullo pacto fieri potest talis commutatio, cum semper quatuor terminos ad minus transeat, vt nunc videbimus.

Exempli gratia, Petrus ex suis bonis tribuat Ioanni aliquid valoris quinquaginta aureorum.

Vnde priusquam Ioannes aliquid ex suis bonis retribuat Petro, bona ipsius Petri diminuta erunt per quinquaginta aureos, bona verò ipsius Ioannis, aucta totidem aureis.

Ecce nunc quo pacto constituti sunt. 4. termini in proportionalitate aritmetica, per quos fit talis permutatio, sed nondum æquata, nisi fiat æqualis retributio à Ioanne ad Petrum, vt videbimus.

Cogitentur itaque. 4. termini aritmeticè proportionales. C. A. B. D. Ita quod. A. mediante significentur bona Ioannis. B. vero Petri, prius quam Petrus aliquid ex bonis suis tribuat Ioanni. Tunc Petrus secat partem vnam ex. B. eamq; dat ipsi Ioanni, vnde ipsi Petro remanet. D. Ioanni autem. C. quatuor igitur termini constituti sunt. B. D. C. A. quorum. B. primus. A. quartus. C. uero tertius. D. aût secundus, sed B. et. A. sunt in sua naturali mediocritate absque defectu vel excessu sui ipsius. Non ita tamen se habet. C. et. D. quia. D. deficit. C. autem excedit à sua priori quantitate. Nihilominus isti. 4. termini constituti sunt in ipsa aritmetica proportionalitate, nam eadem quantitate qua. D. diminuta est à. B. eadem. C. aucta est supra. A.

Sed quia. B. et. A. tantummodo iusti sunt termini. C. uerò et. D. iniusti, vt ad suam priorem æqualitatem reuertantur, oportebit ex. C. secare aliquam partem æqualis valoris ei, qua. C. superat. A. vel qua. D. minor est. B. & ipsam partem addere ipsi. D. vt bona Petri reuertantur ad priorem suam quantitatem ipsius. B. & bona Ioannis remaneant æqualia. A. vt prius.

Quare necessarium non est, vt talis proportionalitas sit coniuncta (vt inquit Eustratius seu Michael Ephesius, super quinto capite libr. quinti Ethicorum) tribus terminis contenta, imò oportet ut ipsa disiuncta sit, ut diximus, vbi non est necesse quod. A. æqualis sit. B. aliquo modo.


DE

## DE MOTV MOLAE, ET TROCHI, DE AMPVLlis aquæ, de claritate aeris, & Lunæ noctu fulgentis, de æternitate temporis, & infinito ſpacio extra Cœlum, Cœliq; figura.

*Illuſt. Ioanni Paulo Capræ Nouarienſi Sabaudiæ Ducis hoſpicij Magiſtro, viro ingenij præſtantia, & morum candore, non minus quam familiæ nobilitate conſpicuo.*

SI vera eſſet animorum illa tranſmigratio quam ſibi Italicæ ſapientiæ Pater Pythagoras effinxerat, tuam, meamq; exiſtimarem animam canis, quandoque venatici fuiſſe.

Quæris à me literis tuis, an motus circularis alicuius molæ molendinariæ, ſi ſuper aliquod punctum, quaſi mathematicũ, quieſceret, poſſet eſſe perpetuus, cum aliquando eſſet mota, ſupponendo etiam eandem eſſe perfectè rotundam, & leuigatam. Reſpondeo huiuſmodi motum nullo modo futurum perpetuum, nec etiam multum duraturum, quia præterquam quod ab aere qui ei circumcirca aliquã reſiſtentiam facit ſtringitur, eſt etiam reſiſtentia partium illius corporis moti, quæ cum motæ ſunt, natura, impetum habent efficiendi iter directum, vnde cum ſimul iunctæ ſint, & earum vna continuata cum alia. dum circulariter mouentur patiuntur violentiam, & in huiuſmodi motu per vim vnitæ manent, quia quanto magis mouentur, tanto magis in ijs creſcit naturalis inclinatio recta eundi, vnde tanto magis contra ſuammet naturam voluuntur, ita vt ſecundum naturam quieſcant, quia cum eis proprium ſit, quando ſunt motæ, eundi recta, quanto violentius voluuntur, tanto magis vna reſiſtit alteri, & quaſi retrò reuocat eam, quam antea reperitur habere.

Ab eiuſmodi inclinatione rectitudinis motus partium alicuius corporis rotundi fit, vt per aliquod temporis ſpacium, trochus cum magna violentia ſeipſum circunagens, omnino rectus quieſcat ſuper illam cuſpidem ferri quam habet, non inclinans ſe verſus mundi centrum, magis ad vnam partẽ, quam ad aliam, cum quælibet ſuarum partium in huiuſmodi motu non inclinet omnino verſus mũdi centrum, ſed multo magis per tranſuerſum ad angulos rectos cum linea directionis, aut verticali, aut orizontis axe, ita vt neceſſariò huiuſmodi corpus rectum ſtare debeat. Et quod dico ipſas partes non omninò inclinare verſus mundi centrum, id ea ratione dico, quia non abſolutè ſunt unquam priuatæ huiuſmodi inclinatione, quę efficit vt ipſum corpus eo puncto nitatur. Verum tamen eſt, quod quanto magis eſt velox, tanto minus premit ipſum punctum, imò ipſum corpus tãto magis leue remanet. Id qđ apertè patet ſumẽdo exẽplũ pilę alicuius arcus, aut alicuiꝰ alterius inſtrumẽti, ſeu machinæ miſſilis, quæ pila quanto eſt velocior, in motu violento, tanto maiorem propenſionem habet rectius eundi, vnde verſus mundi centrum tanto minus inclinat, & hanc ob cauſam leuior redditur. Sed ſi clarius, hanc veritatem videre cupis, cogita illud corpus, Trochum ſcilicet, dum velociſſime circunducitur ſecari, ſeu diuidi in multas partes, vnde uidebis illas omnes, non illico uerſus mundi centrum

deſcen-

deſcendere, ſed recta orizontaliter, vt ita dicam, moueri. Id quod à nemine adhuc (quod ſciam) in trocho eſt obſeruatum. Ab huiuſmodi motu trochi, aut huius generis corporis, clarè perſpicitur, quàm errent peripatetici circa motum uiolentum alicuius corporis, qui exiſtimant aerem qui ſubintrat ad occupandum locum à corpore relictum, ipſum corpus impellere, cum ab hoc, magis effectus contrarius naſcatur.

Quod deinde ampullæ iungantur in aqua, non fit ratione ſimpathiæ, de qua loquitur Fracaſtorus, nam per accidens iunguntur, quia cum alia ad aliam accedit, quęlibet earum tentat aſcendere ab ea parte, à qua inuicem hærent, quemadmodum efficiunt iuxta labrum vaſis, ea enim ſuperficies aquæ vicina circunferentiæ vaſis aliquantulum aſcendit in vaſe, qui non eſt omnino plenus.

Ad id deinde quod de claritate noctium ſcribis, miror cur non videas, quod quãto magis obſcura nox apparet, non dico ratione nubium, ſed diſtantiæ Solis ſub orizonte ab eodem orizonte, tanto magis claram, & luminoſam ſeſe nobis oſtendit Luna in quintadecima, quia cum Sol eſt in Sagittario, & Capricorno, Luna eſt in Geminis, & in Cancro, vnde in media nocte, eius radius per valde exiguam quãtitatem vaporum tranſit, quia tunc ipſa eſt valde propinqua axi orizontis, & prætereain huiuſmodi tempore anni & noctis, aer eſt magis purgatus, quàm in qualibet alia temporis parte, quia hieme Sol non poteſt excitare multos vapores, & ij, qui attolluntur, nocte à frigore ſtatim congellati ratione grauitatis decidũt, unde remanet aer multo clarior, qua ratione apparent ſtellæ minutæ, & Cœlum ijſdem magis ornatum, quàm in quolibet alio anni tempore.

Dicere deinde, quemadmodum hic mundus eſt ætatis ſeptem, aut octomillium annorum, ita nunc potuiſſet eſſe (ſi Deus voluiſſet) ætatis quinquagintamillium; ergo erat tempus; ita ſe habet, ac ſi diceremus, quemadmodum hic mundus eſt tantæ magnitudinis, ita etiam quinquagies maior eſſe potuiſſet, ergo eſt ſpatium, aut interuallum corporeum, quod eum capere potuiſſet.

Illud, nihil, Ariſtotelis extra Cęlum, nullo modo nobis inſeruit pro eiuſdem Cœli ſphęrica rotunditate, cum cuiuſque alterius ex infinitis figuris Cęlum ipſum eſſe poſſit, ſecundum ſuam ſuperficiem conuexam. Nam Cœlum ea ratione ſphęricum non eſt, quod magis ſit capax, quia ei innumerahiles alias figuras adeo magnas poterat concedere cauſa diuina: ſed ſphæricum eſt effectum, ne partem aliquam haberet ſui termini ſuperfluam, quia nullum corpus à breuiori termino quam à ſphærico terminari poteſt.

## *De reuolutione rotæ putealis & alijs problematibus.*

### AD EVNDEM.

Funis cui appenſa eſt ſitula, longè facilius axi inuoluitur, ſi ipſi axi affixa ſit rota. atque item commodius eò fiet, quo amplior rota erit, & axis exilior.

Commodiſſimè autem, ſi ipſa rotæ extrema circunferentia, ex materia minori, & denſiori, ac proinde grauiori conſtabit. Cuius rei ratio multiplex eſt. Nempe quia omne corpus graue, aut ſui natura, aut vi motum, in ſe recipit impreſſionem

nem

nem & impetum motus, ita vt feparatum à virtute mouente per aliquod temporis fpatium ex feipfo moueatur. nam fi fecundum naturam motu cieatur, fuam velocitatem femper augebit, cum in eo, impetus & impreffio femper augeantur, quia coniunctam habet perpetuò virtutem mouentem. Vnde manu mouendo rotam, ab eaq; eam remouendo rota ftatim non quiefcet, fed per aliquod temporis fpatium circunuertetur.

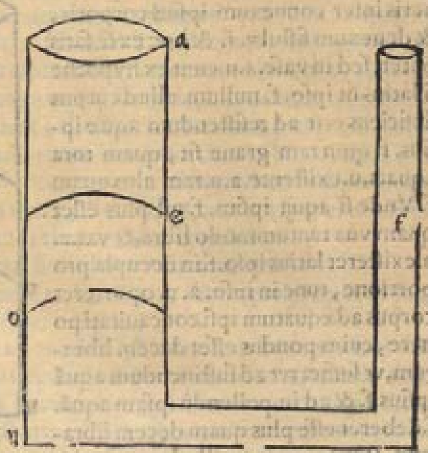
Secunda caufa eft, quia quoduis graue corpus, aut per naturam, aut per vim motum, rectitudinem itineris naturaliter appetat, quod clarè cognofcere poffumus, proijciendo lapides funda, & circunducentes brachium, nam funes tanto maius pondus acquirunt, & manum tanto magis onerant, quanto velocius voluitur funda, & incitatur motus, quod ab appetitu naturali infito ei corpori per lineã rectam progrediendi procedit. Vnde fit, vt pondus circunferentiæ ipfius rotæ, tanto facilius circunuoluatur, & ex feipfo tanto longiori tempore moueatur, quanto longius diftat à centro, cum eius iter tanto minus fit curuum. Hanc igitur ob caufam, rota, quanto maior erit, eiufq; pondus tanto magis vicinum circunferentiæ, tanto magis durabit impetus motus affumptus.

Tertia caufa eft, quod funis dum circunuoluitur, vicinius axi mathematico reuolutionis, quam corpus graue circunferentiæ rotæ, ratione vectis, cum rota eft in motu, eius impetus non obtinet refiftentiam æqualem à contrario pondere aquæ in fitu la pofitæ.

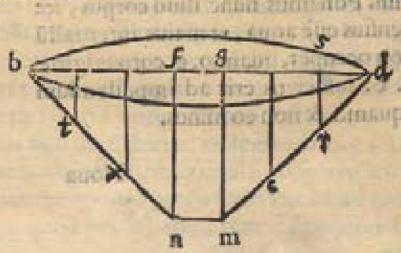
## *De machina, quæ aquam impellit & fubleuat.*

### AD EVNDEM.

VNde fit vt in fonte mandauerim, vas feu mortarium in quod ingreditur inftrumentum, quod aquam impellit, diametrum fuæ concauitatis, habere non oportere maiorem diametro fiftulæ, per quam debet afcendere aqua, ratio eft, quia fi maius effet, neceffarium effet aliquod inftrumentum quo aqua impelleretur multo grauius toto corpore aqueo, quod aptum effet implere aliquam fiftulam adeo altam, vt eft fons, quæ tamen effet adeo lata vt eft mortarium.




Sit exempli gratia, tota fiftula, feu hirundo, per quam afcendit aqua. f. mortarium verò fit. a. u. quod tam altũ fit vt. f. fed. f. anguftior ipfo. a. u. Nunc cum repleta fuerint hæc duo vafa, manifeftum erit, quod aqua ipfius. f. fufficiens erit ad refiftẽdum toti aquæ ipfi° a. u. & aqua. a. u. refiftet aquæ. f. quamuis aqua. a. u. maioris quantitatis fit, & ponderis ipfa. f. hoc autem euenit ex eo quod aqua. a. u. nõ impellit aquam

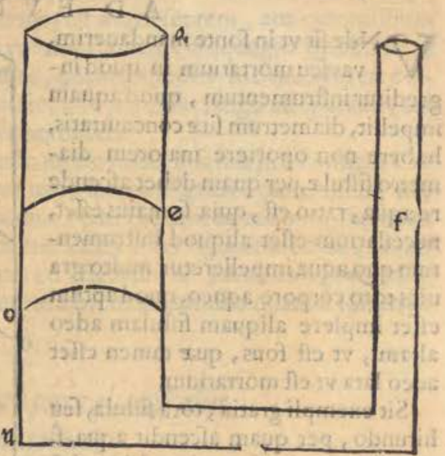


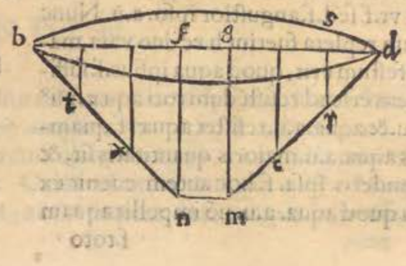

f. toto

f.toto ſuo pondere, propterea quod pondus diuiditur proportionaliter ſupra baſim vaſis.

Sit exempli gřa vas aliquod.b.d.n.m.conicæ figuræ,ſeu trũcus coni concaui aqua plenus, cuius orificij diameter ſit.b.d.& multiplex diametro. m. n. infimæ baſis. cogitemus etiam.b.d.diuiſum in tot partes,quarum vnaquæq; æqualis ſit. m. n. imaginemurq́; tot lineas perpendiculares deſcendere verſus mundi centrum ad puncta.r. c.m.et.t.x.m.vt in ſubſcripta hic figura videre eſt, per quas cogitemus tot ſuperficies curuas conicasq́;,inter quas,mente concipienda eſt aqua,quę pondere ſuo quieſcet ſupra maiorem ſuperficiem illa,quæ æque diſtans eſſet mundi centro, ſeu quam ſupra baſim.m.n.vt exempli gratia conſideretur aqua inter.g.m.et.s.r.cuius pondus diſtribuitur ſecundum latitudinem.m.r.quæ maior eſt.g. s.cogitemus igitur.m.c.æqualem eſſe.g.s.manifeſtum erit, quod.m.c.non ſuſtinebit totum pondus aquæ,quæ inter.g.m.et.s.r.reperitur,eo quod omnis pars aquæ ad perpendiculum inclinat verſus mundi centrum,quapropter fundus ſeu baſis.m.n.non ſuſtinet aliud pondus quã aquæ.f.m. ſed ſi quis hoc in dubium reuocaret dicens, quod aqua circunſcribens ſitum corporis aquei.f.m.impellit lateraliter dictum corpus aqueum,reſpondendum eſt, quod ex æquo huius corporis. f. m. aqua impellit etiam aquam circunſtantem, eo, quod ſunt corpora homogenea,cum in corporibus homogeneis æquales partes habeant æquales vires.

Sed redeundo ad vaſa.a.u.et.f.dico quod ſicut aqua. f. ſufficit ad reſiſtendũ aquæ a.u.ita quodlibet aliud pondus ęquale.f.cuiuſuis materiæ,in fiſtula.f.poſitum, ſufficiens erit, dummodo illud corpus ita ſit adæquatum concauitati fiſtulæ.f. quod non permittat tranſitum aliquem aquæ vel aeris inter conuexum ipſius corporis, & deuexum fiſtulæ. f. & hoc ex ſe ſatis patet, ſed in vaſe.a.u.cum ex hypotheſi latius ſit ipſo. f. nullum aliud corpus ſufficiens erit ad reſiſtendum aquæ ipſius. f. quin tam graue ſit, quam tota aqua.a.u.exiſtente.a.u.tam alto quam f. Vnde ſi aqua ipſius. f. nil plus eſſet quam vna tantummodo libra,& vas.a. u.exiſteret latius ipſo.f.in decupla proportione, tunc in ipſo. a. u. oporteret corpus adæquatum ipſi concauitati ponere, cuius pondus eſſet decem librarum,vt ſufficeret ad ſuſtinendum aquã ipſius.f. & ad impellendũ ipſam aquã. f.deberet eſſe plus quam decem librarum. Ponamus nunc illud corpus, ita denſius eſſe aqua, vt maius interuallũ non occupet, quam.o. e. corpus igitur o. e. ſufficiens erit ad impellendum aquam.f.& non eo minus.




Noua

# NOVA SOLVTIO PROBLEMATIS DE VASE pleno liquoris.

*Nicolao Caluxio Serenißimi Ducis Sabaudiæ à ſecretis.*

QVOD à me poſtulas eſt problema ab alijs iam ſcriptum, ſed illud tibi alio medio ſoluam.

Proponitur vas plenũ liquore aliquo, puta aqua, ꝙ tres habeat fiſtulas ad baſim, quarum vnaquæque poſſit euacuare ipſum vas, inæquales tamen, ita quod prima tam lata ſit, vt ſpatio vnius horæ poſſit ipſum euacuare totum; ſecunda vero ſpatio duarum horarum, tertia autem ſpatio trium horarum. Tunc quæritur quanto tempore omnes tres fiſtulæ ſimul apertæ euacuabunt ipſum vas. Ad hoc volo vt quæratur primo quanta pars aquæ vnaquęquę fiſtula euacuabit in aliquo dato tempore, quod facilè eſt, vt puta, prima fiſtula, ſpatio dimidiæ horæ euacuabit dimidium vas, eo quod ſpatio integræ horæ poteſt totum euacuare, ſecunda fiſtula, eodem temporis ſpatio, euacuabit quartam partem ipſius vaſis, tertia verò fiſtula, eodemmet ſpatio temporis dimidiæ horæ, euacuabit ſextam partem ipſius vaſis, quæ omnia fracta ſimul collecta faciunt vndecim duodecimas partes totius vaſis, vnde manifeſtum erit, quod omnes fiſtulæ pariter apertæ, ſpatio dimidię horæ euacuabunt vndecim duodecimas partes totius aquæ, ſed nos cupimus ſcire, quanto tempore, totum vas euacuabitur, apertis omnibus fiſtulis, quapropter dicemus ita; Si vndecim duodecimæ partes conſumunt minuta.30. temporis, quantum conſument omnes partes aquæ? quæ ſunt.12.quare ex regula de tribus prouenient nobis minuta.32. cum.8. vndecimis vnius minuti, hoc eſt cum.43. ſecundis horæ ferè, vel ſi accipiemus tres quartas vnius horæ, tunc prima fiſtula emittet tres quartas partes totius aquæ, ſecunda, tres octauas eiuſdẽ aquę, tertia verò, quarta pars, tunc omnia, hæc collecta, faciunt vnum integrum cum tribus octauis. Si dixerimus igitur quando vnum integrum cum tribus octauis abſumit.45.minuta temporis, ergo illud ſolum integrum abſumet idem vt ſupra hoc eſt min.32.cum.8.vndecimis vnius minuti vel.43.ſecundis. Cuius rei ſpeculatio tã coniuncta eſt operationi, quòd vna cognita, reliqua ſtatim cognoſcitur.

Idem eueniet de implendo vaſe tribus ſimilibus fiſtulis mediantibus.

Secundum quæſitum ab alijs traditum, tuum etiam, aliter quoque poteſt ſolui, propterea non prętermittam tibi ſatisfacere.

Problema itaque tale eſt, vt ſit vas aliquod in qđ infunditur aqua per tres fiſtulas, ſed dum infunditur aqua, eadem egreditur per duas alias fiſtulas in fundo vaſis poſitas, ſed tres ſuperiores ſint inuicem proportionatæ, vt ſupradictum eſt, primaque inferiorum talis ſit, vt ſpatio.4. horarum poſſit totum vas euacuare, ſecunda autem poſſit ſpatio.6.horarum idem facere, vnde ex ſupradictis, vas implebitur à tribus fiſtulis ſuperioribus, clauſis exiſtentibus inferioribus, ſpatio temporis minutorũ.32.cũ.8.vndecimis hoc eſt min.32.cum.43.ſecundis, deinde per duas fiſtulas inferiores poſſet euacuari ſpatio tẽporis horarum.2.et mi.24.ex ſupradictis.

Supponamus igitur omnes fiſtulas operari ſpatio temporis minutorum.32. cum ſecundis.43.tunc manifeſtum eſt quod vas non implebitur, eo ſpatio min.32. cum ſecundis.43.ſed tanta aqua deficiet, quanta ab inferioribus fiſtulis eo ſpatio temporis min.32.ſecun.43.poteſt euacuari, quare proportio partis vaſis vacuæ, ad totum vas, erit vt min.33.ferè ad horas.2.min.24.quod per ſe patet, tunc ſi demptum fuerit tempus.33.minutorum ex horis.2.min.24. reliquum erit hora.1. min. 51. vnde proportio aquæ, quæ in vase reperitur,ad eam, quæ totum vas implet, erit vt. 111. ad.144. Quare nunc possumus rectè dicere ex regula de tribus si.111.indigent minuta.33.temporis,ergo.144.indigent min.43.horæ, in quo tempore implebitur totum vas omnibus fistulis operantibus.

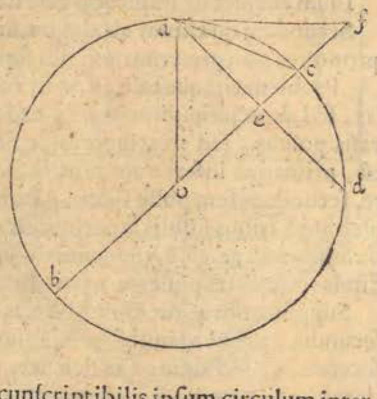
## *Aliæ circuli nouæ passiones.*

### AD EVNDEM.

VT ad ascendendum ignis,& ad descendendũ quicquid graue natum est, ita ad speculandum humanus intellectus. nec quiescit, dum potest, est enim versatile, agitandoq́; sese causis rerum immiscere, & abditum aliquid rimari, conatur,& est in nobis, quasi Diogenes quidam in Dolio.

Tibi igitur mitto quod vltimò inueni, alias scilicet nouas circuli passiones, quæ ita se habẽt. Sit circulus.a.b.c.in quo sit.a.d.latus quadrati inscriptibilis in ipso circulo,et.b.c.sit diameter ad rectos cum.a.d.in puncto.e. quod medium erit inter a.et.d.ex.3.tertij Eucli.sit similiter.a.f.contingens ipsum circulum in puncto.a. quæ protracta sit vsque ad punctum.f. intersectionis cum diametro protracto, quod ita eueniet cum anguli.a.e.f.et.f.a.e.minores sint duobus rectis, eo quod angulus.f.a.e. acutus sit, cum.a.d.transeat inter centrum et.f.

Dico nunc quod productum diametri.b.c.in parte.c.e.ipsius,æqualis erit producto ipsius.c.f.in.a.d. Protrahatur imaginatione.b.a.et.a.c.vnde ex.26.tertij Euclid. habebimus angulum.d.a.c.æqualem angulo.a.b.c.sed ex. 31. eiusdem angulus. f. a. c.æqualis est angulo.b.quare æqualis erit angulo.d.a.c.& ita habebimus per.3.sexti eandem proportionem.f.c.ad.c.e.quæ.f.a.ad.a.e.sed.a.f. est æqualis semidiametro circuli propositi, propterea quod si producta fuerit à puncto.a.ad centrum.o.semi diameter.a.o.hæc cum.o.e.faciet dimidium anguli recti, cum ex supposito.a. d. latus sit quadrati inscriptibilis in ipso circulo.& cum.a.f.rectum ex.17.tertij, vnde angulus.f.erit similiter medietas recti ex.32.primi, quare ex.6. eiusdem. a. f. æqualis erit.a.o. Ergo cum proportio.f.c.ad.c.e.sit.vt.f.a.ad. a.e.erit similiter vt.b.c.ad.a.d. hoc est ut dupli ad duplum, vnde ex.15. sexti manifestum erit propositum, ex quo alia passio oritur, hoc est, quod productum.f.c.in.a.d.æquale sit quadrato ipsius.a.c.ratio est,quia quadratum.a.c.æquale est producto.b. c. in.c.e.eo quod.a.c.media proportionalis est inter.b.c.et.c.e.ex similitudine triangulorum.a.b.c.et.e.a.c.nam anguli.b.a.c.et.a. e. c. recti sunt et.c.cõmunis,vnde.b.erit æqualis.e.a.c.ex.32 primi, sequitur etiam, quod.a.c.sit media proportionalis inter.a.d.et. f. c. & hæc etiam erit alia circuli passio, & quia.a.c.est latus octagoni igitur tale latus mediũ proportionale erit inter latus quadrati.et.f.c. eiusdẽ circuli, quę quidem.f.c.est una portio diametri quadrati circunscriptibilis ipsum circulum inter circulum & angulum ipsius quadrati.




Quod

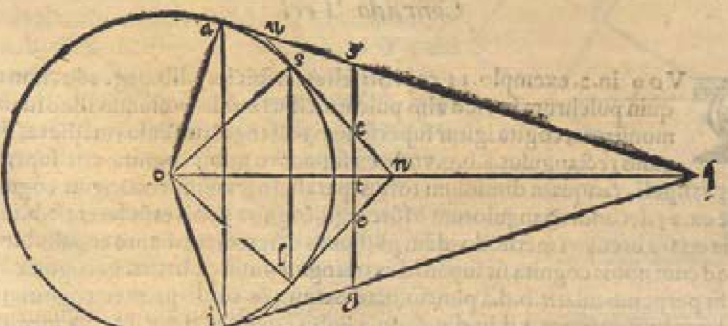
## *Quod incendium, ex reflexione radiorum solarium, non fiat in centro speculi sphærici, & aliquid contra Cardanum, & de motu radiorum solarium.*

### AD EVNDEM.

ITerum tibi dico, quod radij illi solares, qui à diuersis punctis ipsius solaris corporis veniunt, transeuntes per centrum speculi sphærici concaui, quamuis à superficie speculi ad centrum ipsum reflectantur, vt aliàs tibi dixi, nihilominus nullo modo possunt aliquod obiectum incendere duabus ex causis, quarum vna est, quia cum Sol valde remotus sit à nobis, valde etiam acutus generatur angulus coni radiorum in centro speculi, vnde à parua superficie ipsius speculi reflectuntur, quare paucissimi radij sunt qui reflectantur in ipso centro, & propterea non sufficiunt ad combustionem alicuius obiecti. Alia verò causa est, quod quamuis multi, & sufficientes radij fuissent ad cōburendū velociter quoduis obiectum. impossibile tamen omnino esset, vt aliquod obiectum comburerent, propterea quod cum radij incidentes debeant per centrum transire, obiectum combustibile, vt opacum, obstaret ipsis radijs, ne vlterius transirent, vnde nulla fieret reflexio, sed etiam si dicti radij in centro reflexi, sufficerent ad combustionem, incidentes hoc magis efficerent. & ita absque vllo speculo, omnia & in quolibet loco comburerentur, quod manifestè falsum est. Desine igitur mihi citare Lucillum Philalteum, qui in philosophia mathematica fuit omnium imperitissimus. Verum speculum vstorium illud est quod ab Alhazem Deinde à Vitellione describitur.

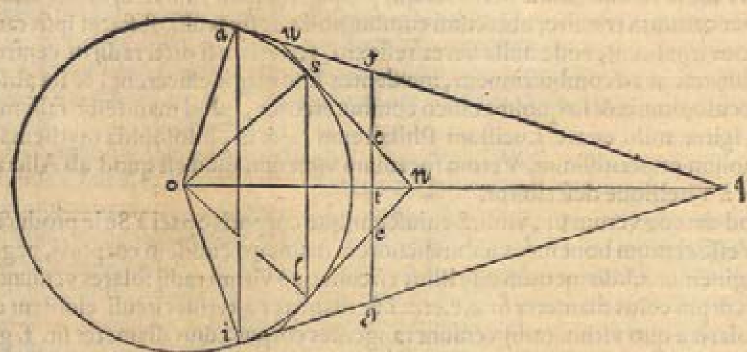
Quod deinde verum sit, vmbrā vniuscuiusque corporis opaci à Sole productam semper esse centum nouemq; vicibus maiorem diametro eiusdem corporis, nego.

Imaginemur.s.l.diametrum esse illius circuli, quo vltimi radij solares veniunt tangentes corpus cuius diameter sit.c.e.et.a.i.sit diameter alterius circuli eiusdem corporis solaris à quo vltimi radij veniunt tangentes corpus, cuius diameter sit. f.g. in eadem distantia, & eodem situ prioris corporis. Tunc conus vmbræ ipsius.f.g.sit.f.g.q. & ipsius.c.e.sit.c.n.e.centrum autem solare sit.o.conorum verò axes sint. t.n.q. tunc ex supposito.q.f.a:n.c.s: n.e.l: et.q.g.i.erunt omnes contigui corpori solari, vnde ex.17. tertij Eucli.anguli.o.a.q.et.o.s.n.erunt recti. protracta deinde cum fuerit a.s.habebimus angulos.u.a.s.et.u.s.a.minores duobus rectis. Quare.n.s.concurret



cum.a.q.in puncto.u. Nunc verò si vmbra.t.q.tanto maior est.f.g. quanto. 109. est vno et.t.n.etiam tãto maior.c.e.ergò eadem proportio erit.q.t.ad.t.f.quę.n.t. ad. t. c.sed cum angulus.t.communis sit ambobus triangulis.q.t.f.et.n. t. c. sequitur ex. 6. sexti dictos triangulos æquiangulos esse. Vnde si anguli.t.n.c.et.t.q.f.æquales inuicem sunt, ergo.q.f.æquidistans erit.n.c.quod est impossibile, quia nunc demonstranimus ipsas concurrere in puncto.u. Quare non est eadem proportio.q.t.ad.t.f.quæ n.t.ad.t.c.decipitur ergo Cardanus in.4.lib.de subtilitate.

Circa illud deinde quod à me quæris, hoc est, quæ sit causa, quod nos videmus radium solarem tardissimè moueri, cum alias tibi dixerim ipsum qualibet hora circa terram quindecim gradus perficere, respondeo, quod radius ille quem videmus, exempli gratia, in aliquo cubiculo, nunquam est idem numero, sed quia ipsi radij nullo modo differunt inter se, nisi in numero, propterea putamus eundem semper esse, cum semper alius, atque alius sit, quorum vnusquisque (de illis loquor, qui ad hunc terræ globum perueniunt) circa terram reuoluitur spatio.24.horarum, & cum quilibet circulus diuidatur in.360.gradus, quorum vigesimaquarta pars est. 15. verum est igitur, quod tibi iam dixeram.



## OPERATIONES DIVERSAE AB ALIIS Michaelis Stifelij.

### *Conrado Terl.*

QVOD in.2.exemplo.11.cap. Stifelius scribit in.3.lib.pag. 282. non nego quin pulchrum sit, sed alijs pulchrioribus modis possumus illud idem demonstrare; cogita igitur superficiem rectangulam, cuius medietas sit triãgulus rectangulus.a.b.g.vnde ex supposito nobis cognita erit superficies ipsius trianguli, tanquam dimidium totius parallelogrammi rectanguli cogniti. Quare ex.25.secundi triangulorum Mõteregij, cognita nobis erũt latera.a.b.et.b.g.

Alia etiam breuiori methodo idem possumus efficere, mediante angulo.b.recto, eo quod cum nobis cognita sit superficies trianguli simul cũ basi.a. g. cognita etiam nobis sit perpendicularis.b.d.à puncto.b.ad basim, & consequenter cognitum nobis erit productum ipsius.a.d.in.d.g.& quia nobis cognita est.a.g. & eius medietas,

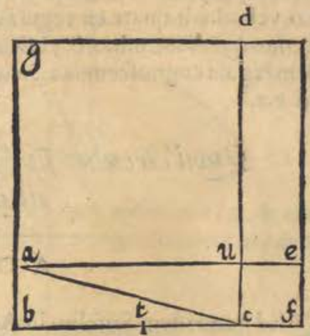
ideo

ideo vnaquæque eius pars.a.d.et.d.g. ſimiliter nobis cognita erit ex quinta ſecundi Eucl. vnde ex penultima primi habebimus propoſitum.

Poſſumus item circulum mente concipere cuius.a.g. ſit diameter, & ab eius centro. e. protracta cum fuerit.e.b.quæ nobis cognita erit, vt medietas ipſius.a.g. de cuius potentia,dempta cũ fuerit potentia ipſi⁹ b.o.remanebit nobis potentia ipſius. d. e. & ita eius longitudo, quæ addita medietati.e.g. & detracta à dimidio.e. d. erunt nobis cognitæ.a.d.et. d. g. vnde.b.g.et.b.d.remanebunt nobis cognitæ ex dicta penultima primi Eucli.huiuſmodi figuram videbis in dicto.25.problemate.2.li.Montisregij.

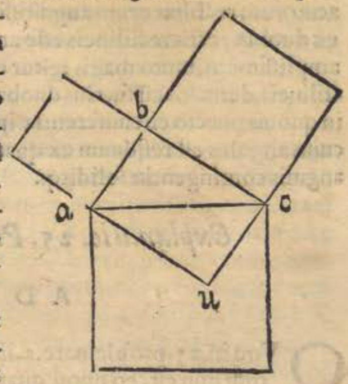
Aliter etiam poſſumus hoc idem efficere.

Sit rectangulus hic ſubſcriptus.a.b.c.u. ſuperficiei cognitę ſimul cum diametro.a. c.extendatur imaginatione.b.c.vſque ad,f. ita quod.c.f.æqualis ſit.c. u. intelliganturq; quadrata.g.f:g.u.et.u.f.vnde sũma quadratorũ.g.u:u. f. cognita nobis erit ex penultima primi. nam.a.c.data nobis fuit, quare ſummã.g.u:u.b: et. u. f. cognoſcemus, cui sũmæ addito ſuplemento.d.e. æquali. u. b. dabit nobis cognitũ quadratum.g.f.totale,quare cognoſcetur eius radix.b. f.,cognita igitur. b. f. cum producto.b.u.illico ex.5. ſecundi cognoſcetur.b.c. et. c. f.forte cognita.b.f. diuiſa p æqualia in puncto.t.& per inæqualia in pũcto. c.Nam quadratũ ipſius.t.f.cognitum, ęquatur rectãgulo.b.u. cũ quadrato ipſius.t.c.dẽpto igitur rectangulo,b. u.ex quadrato ipſius. t. f. relinquetur quadratum ipſi⁹.t.c.cognitum & eius radix.t.c.qua addita ipſi medietati.b.t.& dẽpta ex medietate.f.t.relinquetur propoſitum.




Similiter de tertio exemplo eiuſdem Stifelij infero.

Sit rectangulus.a.b.c.u.cuius diametri.a.c.quantitas,ſimul cum proportione laterum.b.c.et.b.a.nobis data ſit.cum autem ſcire voluerimus eius ſuperficiem. b. u.clarum eſt,quod cum nobis data ſit proportio.b.c.ad.b.a.illico cognoſcemus etiã proportionem quadrati ipſius. b.c.ad quadratum ipſius.b.a.cum dupla ſit ei quæ.b.c.ad.b. a. ita etiam & aggregati dictorum quadratorum ad quadratum ipſius. b. a. hoc eſt nota erit nobis proportio quadrati ipſius.a. c. diagonalis ad quadratum ipſius. a. b. idem dico de quadrato. b. c. ideſt quod proportio quadrati ipſius. a. c. ad quadratum.b.c. cognita nobis erit, ſed. a. c. data nobis fuit, quare cognoſcemus etiam omnia dicta quadrata eorumq; radices.a.b.et.b.c. quare & ſuperficiem rectanguli quæſitam.




Quartum exemplum etiam faciliori via poteſt ſolui, propterea, quod cum nobis cognita ſit baſis trianguli cum ſumma reliquorum laterum, & cũ angulo oppoſito baſi ipſius reliqua cognita nobis emergunt ex.15.problemate ſecundi lib.de Triangulis ipſius Monteregii.

Vel

Vel ſi tibi placet,accipe hanc aliam methodum à me excogitatum.

Duplicetur triangulũ.a.b.c.orthogoniũ, & fiat rectangulũ.b.u. vt in mea figura ſecundi exempli hic vides.producaturq́;.b.c.quouſque.c.f.æqualis ſit.c.u.vnde. b.f. cognita nobis erit ex hypotheſi, quare cognoſcemus etiam quadratum. g. f. à quo demptũ cum fuerit aggregatũ quadratorum.g.u.et.u.f.nobis cognitũ ( nam quadrata.g.u.et.u.f.æqualia ſunt quadrato ipſius.a.c.diagonalis datę ) remanebit aggregatum ſupplemẽtorũ cognitum, quare eius medietas cognoſcetur ideſt. b. u. vndæ ex. 5. ſecundi Eucli. vt ſuperius diximus cognoſcetur etiam.b.c.et.c.f.diſtinctæ.

Idem aſſero de exẽplo Gemmæ Friſij à Stifelio citato in Appendice regulæ falſi.

Sit gratia exempli rectangulum hic ſubſcriptum.a.b.datæ ſuperficiei data etiam nobis ſit proportio.a.e.ad.e.b.laterum producentium, cogitemusq́;. a. e. producta vſque ad.o.ita vt.e.o.æqualis ſit ipſi.e.b. imaginemus etiã perfectum eſſe quadratum.b.o. vnde ex prima ſexti ſeu.18.vel.19. ſeptimi vel. 15. quinti eadem proportio erit ipſius.a.b.ad.b.o.vt.a.e. ad e.o.vel ad.e.b.quare ex regula de tribus, cognoſcemus quadratum.b.o.& eius radicẽ.e.o.& ex eadem regula cognoſcemus.a.e.cum cognita nobis ſit.e.o.ſimul cum proportione.e.o. ad. e.a.


## *Quod circulus ſit figura infinitorum angulorum hoc eſt ultima poligoniarum.*

### AD EVNDEM.

SEd quod idem Stifelius in Appendice ſecundi libri dicat circulum eſſe figuram poligoniam, non eſt ita mirandum, nam & alij multi doctiſſimi viri hanc veritatem cognouerunt, de Leone Baptiſta Alberto nihil dicam, cum ipſe fateatur hoc accepiſſe à philoſophis, vt etiam refert Ariſt.de ſphæra tertio de cœlo. conſidera quæſo in circulo, quod cum angulus contingentiæ ſit angulus, quamuis omniũ acutorum rectilineorum anguſtiſſimus,vnde ex communi ratione ſequitur reliquum ex duobus rectis rectilineis eſſe angulum, & ſi omnium obtuſorum rectilineorum ſit ampliſſimum, tanto magis igitur erit angulus, id quod remanet ex duobus rectis rectilineis,detractis cũ fuerint duobus angulis contingentiæ, qui quidem angulus erit in quouis puncto circunferentiæ ipſius circuli, idem intelligendum eſt de ſphæra, cuius angulus eſt reſiduum ex quatuor rectis ſolidis, detractis cum fuerint quatuor angulis contingentiæ ſolidiſq́;.

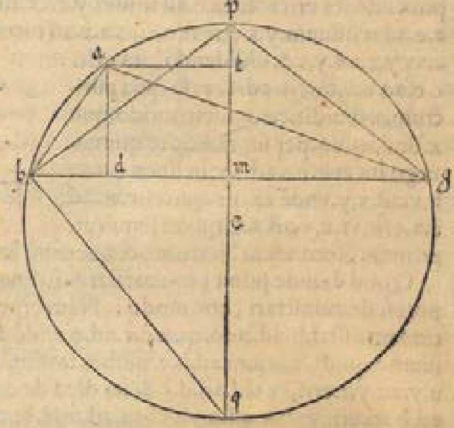
## *Explanatio. 25. Problematis lib. 2. Monteregij.*

### AD EVNDEM.

QVod in. 25. problemate. 2. lib. de triangulis Monteregium non intelligas, mirum non eſt, eo quod quandoque bonus dormitat Homerus. Puto enim illud problema ab ipſo Monteregio non fuiſſe viſitatum. Sed ne me aliquo modo culpes, accipe hanc aliã methodũ à me aliter etiã excogitatã in eadem ipſius figura.

Propo-

Propositũ sit nobis triangulum.a.b.g.cuius basis data sit cum area, seu perpendiculari.a.d.cum angulo etiam.a.ad cognoscendum autem.a.b.et.b.g.cogitemus circulum.a. b. q. g. circunscribere ipsum triangulum cuius diameter.p. q. ad rectos secet basim. b. g. in puncto. m. cogitemus etiam. b. p. et. p. g. vnde ex. 20. tertij Euclid.angulus. b. p. g. æqualis erit angulo.a.& angulus.m.p.b.erit eius dimidium,quod ex te ipso cognosces,& angul⁹.p. b. m. similiter cognoscetur, quare ex.29. primi eiusdem Monteregij cognoscemus.p.m.et.p.b.(nam.b.m.datum fuit, vt dimidium totius basis.b.g.) ducta postea.b.q.ex eadẽ.29. cognoscemus.p.q.cum.p.b.iam cognita fuerit, à qua.p.q. (diametro) dẽpta p.m.remanebit.q.m.cognita, cũ qua iuncta cùm fuerit.m.t.æquali.a. d. perpendiculari,dabitur.q.t. et. t. p. inter quas. a. t. media proportionalis locatur, quare cognoscemus.a.t.quæ sinus est arcus.a.p. vnde cognitus erit arcus a.p.sed arcus.p.g.cognitus est mediante angulo.p.b.g.cognito, qui quidem arcus.p.g.si coniunctus fuerit cum arcu.p.a.cognoscemus compositum. a. g. & eius chorda similiter (hoc est secundũ latus) qua cognita, illico cognoscemus chordam a.b.hoc est tertium latus trianguli propositi.



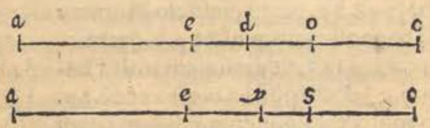

## *Quædam notanda in Federicum Comandinum.*

## AD EVNDEM.

PVtabas enim me ioco dixisse Federicum Comandinum non omnino irrepræhensibilem esse, vide igitur, quod scribit in quinto lemmate in decimam propositionem libr. 2. de insidentibus aquæ Archimedis, volens demonstrare eandem esse proportionem.l.b.ad.b.m.quæ.c.e.ad.e.a.vbi est aliquo modo prolixum, mediante linea.c.p.cum suis partibus, citans etiam antecedens lemma extra propositum, eo quod nec in antecedẽte lemmate, nec in alio, ipse vnquam probauerit proportionem.c.d.ad.d.q.esse,vt.l.b.ad.b.m.sed ne putes me falli, tibi demonstrabo non esse necessarium ducere lineam.c.m.p. vel. q. p.eo quod cũ per quintam lib. de quadratura parabolę Archimedis, ita sit.c.d.ad.d.e.vt.l.b.ad.b. m. existente a.c.dupla ipsi.d.e.et.e.c.dupla ipsi.g.c. et.l.d.dupla ipsi.l.b: erit, primo componendo.c.e.ad.e.d.vt.l.d.ad.d.m.& per æqualitatem proportionum, ita erit.e.g. ad.e.d. vt.b.d.ad.d.m.& per.19.quinti Eucli.ita erit.e.g.idest.g.c.ad.g.d.vt.b.d. idest. l. b. ad.b.m.sed.c.g.ad.g.d.est vt.c.e.ad.e.a.ratio est, quia componendo ita est.c.d.ad.d. g.vt.c.a.ad.a.e.& hoc est,quia permutando, ita est.a.c.ad.d.c.vt.a.e.ad. d. g. & hoc verum est ex.19.quinti eo quod totius.a.c.ad totum.d.c.est vt abscissi.e.c. ad abscissum.g.c.vt supradixi.

Sed

Sed etiam alio vniuerſaliori modo potes probare, quod ita ſit.u.x.ad.x.y.vt.c.e. ad.e.a.cogitando in linea.c.a .punctum quoddam quod vocabimus ſimiliter . y . in tali ſitu locatum , quod diuidat.c.a.eadem proportione qua.y.diuidit.u.s.vnde cum e.s.diuiſa eodem modo etiam ſit à puncto.s.ex ſupradicta quinta lib. de quadratura parabolæ, erit igitur proportio.a.y. ad.y.c.vt.e.s.ad.s.c.per.11.quinti Eucli.& com ponendo ita erit toti⁹.a.c.ad totum.y.c.vt abſcisſi.s.c.ad abſciſsum.s.c.quare reſidui a.e.ad reſiduum.y.s.erit vt totius.a.c.ad totum.y.c.& permutando, ita erit.a.c.ad.a. e.vt.y.c.ad.y.s.& diuidendo , ita erit. c.e.ad.e.a.ut.c.s. ad. s. y. & quia punctum.s. diuidit.c.a.eodem modo,quo x.diuidit.u.s.per ſupradictam quintã, ergo ita erit.c.s.ad.s.y.in linea.c.a.vt u.x.ad.x.y.vnde ex.11.quinti.c. e. ad e.a.erit,vt.u.x.ad.x,y.quare ſequitur, primum , ſecundum , tertium, & quartum lemma ſuperflua eſſe.




Quod deinde ponit pro corellario in fine.6.lemmatis, aliter quam per.6.lemma poteſt demonſtrari , hoc modo . Nam ſuperius demonſtrauimus eandem proportionem eſſe.l.b.ad.b.m. quæ.c.e.ad.e.a.idẽ dico de proportione.u.x.ad.x.y.& omnium æquidiſtantium ad.h.e.quibus rationibus mediantibus eodem modo ſcies, ꝙ u.y.ad.y.r.erit,vt.c.d.ad.d.c & ita dico de omnibus æquidiſtãtibus.ad.h.e.vnde.l.b. ad.b.m.erit vt.u.x.ad.x.y et.l.m.ad.m.d.vt.u.y.ad.y.r.per.11.quinti , ſed cum ſit. l. b.ad.b.m.vt.u.x.ad.x.y.componendo erit. l. m. ad.b.m.vt.u.x. ad.x.y.& euerſim.b. m.ad.m.b.erit,vt.x.y.ad.y.u.& per æquam proportionalitatem erit.b.m.ad.m.d.vt x.y.ad.y.r.quod eſt propoſitum.

Non video etiam, quare ipſe ducat lineam.s.r.cum in ipſo contextu nihil faciat de dicta.s.r.

Comentum poſtea contextus.P. pulchrius eſſet,ſi diceret,quod cum ita ſit totius, l.a.ad totum.a.d.ſic ſe habebit abſciſſum.a.i.ad abſciſſum.a.z.eo quod ita eſt,vt ſcis, hoc eſt in proportione dupla, ergo reſidui.i.l.ad reſiduum.d.z.erit vt totius. a . l. ad totum.a.d.hoc eſt in proportione dupla.

*De Viſu.*

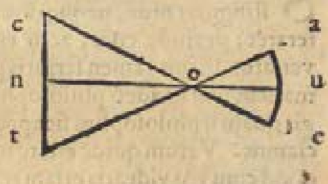
## AD EVNDEM.

Ratio vnde fiat, vt videamus diſtinctè omnes colores,cum in qualibet aeris parte, quo lumina reflexa poſſunt peruenire mixta ſint, & non diſtincta,oritur à paruitate ipſius pupillæ oculorum , & à magna expanſione virtutis viſiuæ in ſuperficie concaua orbis continentis humores diaphanos oculorum per ramuſculos nerui optici remotè ab ipſa pupilla. & quamuis radii luminoſi frangantur ab vnoquoque humore diuerſimodè, hoc nihilominus maximè iuuat ad diſtinctionem radiorum , ſed & ſi directè procederent, idem ferè eueniret, non tamen ſuis locis,cogita exempli gratia lineam.a.u.e.vt communis ſectio cuiuſdam plani ſecantis ſphæram oculi , per centrum ipſius, & pupillæ , et. o. punctum ſit proximum centro ipſius pupillæ , ſed interius aliquantulum,extra autẽ oculũ,ſint varij colores,vt.c.n.t.in dicto plano.

Iam nulli dubium eſt quod lumina quæ producuntur ab.c.n.t.ad.o. in ipſo.o.mi-

xta,

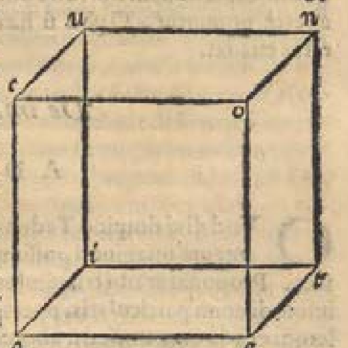
xta, & non distincta, procedendo igitur vlterius ipsi radij citra.o.tunc disgregātur, & separantur abinuicem, & cū perueniunt ad lineam. a.u.e. sentiuntur distincti alij ab alijs. Cuius quidem rei, exemplum manifestum accipere possumus à quouis cubiculo ex omni parte clauso, quod transitum nullū permittat radijs luminosis, nisi per aliquod paruum foramen, in quo foramine, & extra ipsum cubiculum, omnes radij mixti erunt, sed in obiecto pariete ipsius cubiculi videbuntur distincti, vnde sequitur, quòd quo remotius erit obiectum.c.n.t.ab.o.tanto acutior erit angulus.c.o.t.& suus contrapositus similiter, & perconsequens linea. e.u.a.breuior erit, & punctū. o. propinquius etiam erit ipsi lineæ.a.u.e. quæ omnia efficiunt, vt nobis obiectum.c.t.paruū, & minus distinctum, seu magis confusum appareat.




## DE APPARENTI DISTANTIA PARTIVM hæmisphærij.

*Anselmo Fucaro.*

GRatæ mihi tuæ literæ fuerūt, quibus ostēdis non paruū desideriū sciēdi vnde fiat, quod cum dies illucescit, & est serena pars Cœli, circa axem orizontis demissior appareat, quam aliæ partes, cū ab alijs (quod sciā) satis expressum nō fuerit, sed quia de eo à me aliquid scire desideras dicam quod mihi vr̄. Scias non solū multitudinē obiectorū oppositorū efficere, vt aliqua res alia longius distare videat, vt alij putarūt, sed etiam diuersitates colorum, quamobrem cum decipiamur, credentes Cœlum esse præditum colore cęruleo, cum is color, aeri, non Cœlo conueniat, & videntes huiusmodi colorem circa axem orizōtis magis densum, quā versus ipsum orizontem, ratione exiguæ reflexionis, à pauca quantitate vaporum inter nostrum situm, & reflexionis locum, iudicamus Cœlum proximiorem esse circa dictum axem, quam sint aliæ partes; præterquam, quod is color, qui videtur terminare, aut impedire radium visualem (aduertas tamen me hac in re platonicum non esse) eo semper propinquior esse videtur, qui ei locum dat, & hanc ob causam videntes nos dēsitatē cęrulei circa axem orizontis, & cernentes amplitudinem gyri aliarum partium, adducimur, vt putemus eā partē viciniorem esse. Neq; illud etiā omittā hoc etiā fieri ratione imaginationis, vnde etiā multis contrariū euenire potest, idest vt eis magis profundum videatur Cœlū, circa axem orizontis, quam vicinum gyro eiusdē orizōtis, iudicantibus eā partē lōginquiorē esse, quæ sese magis obscurā oculo demōstrat, & eā propinquiorē quę sese clariorē ostendit, vt ei &c contingere potest, qui subscriptā figurā cubicā non quidē ductā secundū ordinē opticè, sed ita, vt omnia latera opposita inuicē sint parallela, prospiciet, idest. a.i.ad.e.t.et.c.u.ad.o.n.et.a.i.ad.c.u.et.e.t.ad o.n. vnde sequitur, vt aliquando quadratum.a.o. videbitur citra, et.i.n.vltra dictū cubum aliquando verò èconuerso.




Pp De

## DE PHILOSOPHIA MATHEMATICA.

### *Dominico Pisano.*

SI omnia vno colore constarent, & corporum vmbræ à luminibus non distinguerentur, neque diuersitas situs, lumina, quæ veniunt ad oculum non alteraret; perinde esset, ac si essemus cœci. Miror quod cum in Aristotele sis versatus, in tuis tamen scriptis philosophum à Mathematico separes, quasi mathematicus non sit adeò philosophus, vt est naturalis, & metaphysicus, cum multo magis quam ij philosophus sit appellandus, si ad veritatem suarum conclusionum respiciamus. Verum quidē est, te in huiusmodi errore solū non versari; sed grauius est, quod cum vos videatis etiam res morales sub philosophię appellationē cadere, non animaduertatis diuinas scientias mathematicas etiam philosophiæ nomine ornandas esse. Quod si eiusdem nomen penitius considerare velimus, inueniemus apertè, mathematico magis illud ipsum quàm cuilibet alio conuenire, cum nullus ex alijs tam certo sciat id quod affirmat quam mathematicus, neque aliquis sit, qui in cognitionis, & scientiæ cupiditatem magis ducatur, vt apertè patet, cum nec etiam ipsi sensui det locum, neque aliquid præsupponat, quod non sit ita verum & intellectui notum, vt nulla quæuis potentia, illud esse falsum ostendere queat. Sed quia Græci, qui ad placitum nomina rebus imposuerunt, voluerunt etiam, non solum mathematica, sed etiam naturalia, metaphysica, & moralia, sub communi philosophiæ nomine contineri. Vt aūt tibi satisfaciam authoritate Aristotelis, quem tantopere colis, primum considera, nunquam eum de philosopho mētionem facere quin prius aperiat de quo philosopho loquatur, atque hoc semper præstat, exceptis quibusdam locis, vt cap. 2. lib. 4. Metaphysicorū, vbi de philosopho in genere loquēs, ait, proprium esse philosophi. vt res omnes speculetur atque hoc in principio quinti textus asserit, cum in quarto iam ostenderit mathematicum esse philosophum: omitto quod in. 2. textu secundi physicorum idem affirmet, æquum esse appellare philosophiam scientiam veritatis, & finem speculatiuæ existere veritatem. An non idem in primo cap. 6. metaphisicæ philosophiam speculatiuam, mathematicis physicis & supernaturalibus rebus contineri? An non idem paulo inferius scribit physicam primam futuram, si aliæ substantiæ quam naturales non reperirentur? considera deinde quid dicat in fine tertij cap. lib. 11. quo loco nil clarius esse potest, lege etiam quæ. 6. cap. eiusdem libri ab eodem adducuntur, & quæ in. 8. cap. 12. libri textu. 44. apertè ponuntur. Quod si hæc tibi non sufficiunt, vereor ne tuus morbus desperatus euadat.

### *De imaginatione specierum.*

### AD EVNDEM.

QVod dixi domino Tadeo est, quod aliquas particularium species, perfectè & integrè imaginari possumus, alias non item, id tibi melius exemplo innotescet. Proponatur tibi triangulus æquilaterus datæ magnitudinis, datiq; coloris, huiusmodi enim particularis, potes imaginatione tibi fingere integram speciem, totalemq; ei adæquatam, sed si aliquam speciem aliquando vniuersaliorem imaginatione ne concipere velles, quemadmodum vnius trianguli ęquilateri, tali magnitudine, sed non præfinito colore constantis, hoc minime præstare posses. quia nullam rem visibilem priuatam colore imaginari possumus. nec etiam potes imaginari speciē alicuius trianguli æquilateri, indeterminatæ magnitudinis, & indefiniti coloris, quæ cuilibet particulari cuiusuis magnitudinis,& coloris postea applicari queat. Species deinde alicuius trianguli ęquicruri, aut vnius trianguli laterum inęqualium, aut triāguli in genere, aut tandem figuræ, considerato tu ipse, an possit sub imaginationem cadere. Possumus quidem huiusmodi speciem (ratione mediante) intelligere, vnde quamlibet speciem rei particularis visibilis, compositæ, ex figura,magnitudine, & colore, perfectè imaginari possimus, & huiusmodi conceptus erit specialissima species, quia in infinito suorum indiuiduorum,nunquam fiet, vt aliquod eorum, aliquo modo ab alijs differre possit; admonens te, nil reperiri, quod differat, aut in se partem aliquam habeat, quod aliquid aliud non obtineat, quin dicta differentia sit specifica, eius tamen solum partis quæ differt ab alia duorum indiuiduorum, vnius, eiusdemq; speciei. quia si est in magnitudine nulla planè magnitudo reperitur, quę sua specie non sit dotata, quod si non esset, inter res omnes nulla æqualitas eluceret: & si in figura, & colore, idem affirmo, aliter nulla res similis esset alteri, neque aliqua similitudo reperiretur. Idem de quolibet alio obiecto sensibili dico. Ratio autem eorum omnium quæ dixi est, quia imaginatiua nihil aliud intellectui ostendere potest, quam id quod recipit à sensu, & cum sensus, alio modo moueri non possit quam supradicto, hanc ob causam verum est, quęquid scripsi. Vnde triangulum ęquilaterum datę magnitudinis,erit genus triangulorum ęquilaterū eiusdē datæ magnitudinis,sed diuersorū colorū,erit etiā species trianguli æquilateri indeterminatæ magnitudinis, & hic deinde erit species trianguli, & hic postea species figuræ. Idem de alijs omnibus rebus per gradus dico, quę sicut à sensu,ita etiam ab imaginatione longè recedunt,adeo vt has species specialissimas tantum,idest eas solùm, quas hic superius descripsi, integrè capere possit: at verò genera, quanto vniuersaliora sunt, ab eadem imaginatione,tanto longius distant.

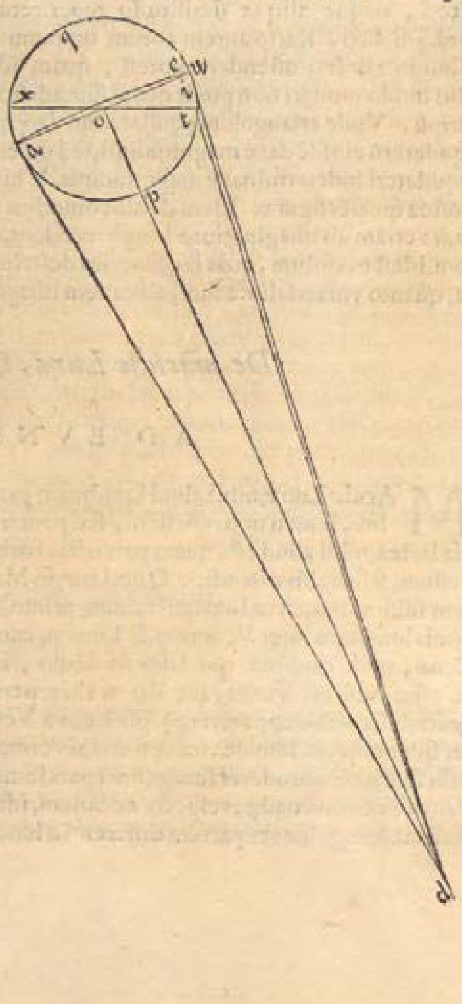
## *De maculis Lunæ, & eius lumine.*

### AD EVNDEM.

MAculæ Lunæ,nihil aliud sunt,quàm partes ipsius Lunę magis perspicuæ,à quibus, lumen non reflessum, sed penetrans, nobis occultatur; quemadmodū via lactea,nihil aliud est, quam pars octaui orbis magis opaca,à qua lumen Solis reflessum, sese nobis ostendit. Quod autem Maurolicus scribit folio.64.cap.de astrorum fulsionibus,circa Lunā,est falsum; primo,quia non considerat differentiā intensionis luminum inter Venerem,& Lunam, cum lumē illius sit magis intensum,quam Lunæ, quia quilibet qui sano sit oculo, facile potest compræhendere, si Luna esset, vbi est Venus, aut Venus vbi reperitur Luna (quibus in locis eiusdem magnitudinis nobis apparerent) ipsa Luna à Venere longè superaretur, & excederetur splendore, & lumine, ita vt si etiam verum esset, quod per tres gradus interualli sese nobis proderet sexagesima pars luminis (quod in quadraturis nec in vllo alio situ verum euadit, respectu ad Solem, idest vt tres gradus differentiæ situs, constituant sexagesimam partem differentiæ luminis respectu nostri) non ideo tamen dictum lumen conspiceretur, quia non sufficit extensio luminis, cum eiusdem intensio sit etiam necessaria. Sed id quoque tibi dico, quod etiam si dicta sexagesima pars totius luminis lunaris, eadem intensione splendoris, & luminis Veneris, in tali distantia trium graduum à Sole prædita esset, non eam tamē videremus, ratione obliquitatis curuę, & sphæricę superficiei Lunæ, respectu nostri, in huiusmodi situ: id q̃ tibi ita demonstratum volo.

Pars superficialis lunaris globi, quæ nos respicit sit.a.p. u. quam accipere possumus pro medietate ipsius superficiei totalis, eo quod respectu nostri visus, insensibiliter, ab ipsa medietate differat, pars autem à Sole visa sit.u.q.a. cogitemus etiam circulum.a.p.u.q. vnum esse ex maioribus ipsius globi, cuius superficies trāseat per oculum videntis, vnde pars eius.a.p.u. diuidet vmbram per æqualia, reliqua verò pars. a.q.u. diuidet per æqualia lumen ipsius Lunæ à Sole receptum, ita quod pars illuminata, erit medietas.u.q.a. excessus verò, cum nostro visui incompræhensibilis sit, pro nihilo reputetur, cuius causa est, maxima illa distantia, quæ inter Solem, & Lunam reperitur, quamuis Sol maior sit Luna multis millibus vicium, eo quod tunc inter Solem, & Lunam reperiantur plus quam. 570. diametri terræ.

Supponamus nunc Lunam remotam esse à loco ipsius cōiunctionis cum Sole per 3. gradus. vnde quēadmodum prius lumen erat in gyro. a. q. u. nunc reperiatur in gyro.x.q.t. ita quod.t.u. erit sexagesima pars ipsius.a.p. u. qđ à vero sensibiliter non discedit. Imaginentur nunc duæ rectæ lineæ ductæ ab oculo.d.ad puncta. t. et. u. verum tamen est quod linea.d.u. secabit arcū.t. u. sed ita propinqua pūcto.u. quod erit ei ferè contingens, vnde absque sensibili errore possumus arcum.t.u.intelligere inter duas lineas.d.t.et.d.u. quapropter tale lumen compræhendetur, ferè, sub angulo.t.d. u. quem quidem angulum oportet nos videre, cuius magnitudinis existat, respectu totalis anguli a.d.u. protracta cum fuerit.d.a.




Producatur primo. d. t. vsque ad diametrum in puncto. i. deinde per puncta.a.et.u.ducatur arcus.a.e.u. circa.d.cētrum, ad quem ducatur linea. d.t.i.in puncto.e. sed quia, cum diameter.a.u.tam breuis sit respectu distantiæ à terra, tempore interlunij, vnde minor cētesima parte ipsius distantiæ existit, sequitr nos posse absq; sensibili errore cogitare, à puncto.d. ad quoduis punctum ipsius diametri omnes lineas ad angulos rectos cum ipso diametro, & insensibilis inæqua

litatis

litatis à linea.d.o.Accipiemus igitur.t.i.pro ſinu arcus.t.u. qui eſt graduum. 3 . hoc eſt ſexageſima pars ſemicirculi graduum.180.quapropter.t.i.erit partium.5233.talium qualium.o.u.eſt.100000.cuius. t.i.quadratum demptum cum fuerit à quadrato ſemidiametri.o.t.relinquet nobis quadratum ipſius.o.i.quæ quidem.o.i. vt radix quadrata,erit partium.99862.talium qualium ſemidiameter eſt. 100000. vnde.i.u. reſiduum diametri, remanebit partium.138. Vel ſic, cum cognitus ſit nobis arcus. t.u.illicò cognoſcemus ſinum arcus.p.t.complementū vnius quartæ,qui ſinus æqualis erit ferè arcui.o.i.partium.99862.vnde.i.u.erit, vt dictum eſt, partium.138.quę quidem.i.u.æqualis eſt ferè ſinui arcus.u.e.& ita etiam.u.e. quare ſi diuiſa fuerit tota.a.u.partiū.200000.per.138.proueniet nobis.1449.& ſic angulus.t.d. u. erit vna partium.1449.anguli.a.d.u. Conſideremus igitur quomodo fieri poteſt, vt oculo comprætendatur hæc tam parua particula luminis lunaris.

## SOLVTIONES ALIQVAE.

### *Paulo Aemilio Raiſeſtaim.*

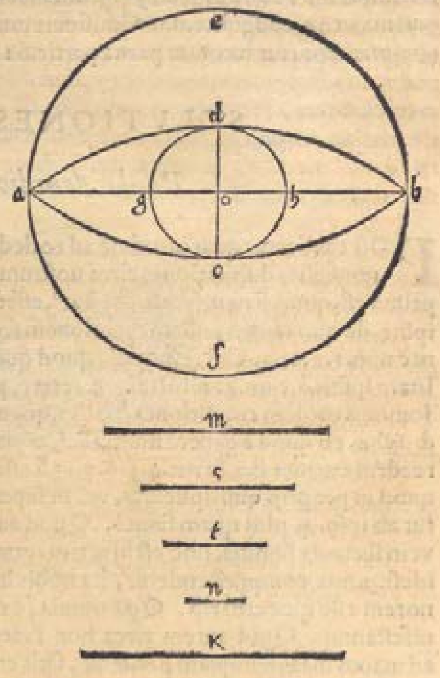
POſt eas literas quas proximè ad te dedi, Franciſcus Monardus mihi retulit tuas nonnullas dubitationes circa noſtrum Theorema Arithmeticum.116.quarum prima eſt,quod ſi numerus.a.cogitat⁹,eſſet æqualis.4.tunc ipſe non eſſet multiplex ipſi.4.de quo tamen nullam mentionem feci. Idem etiam inquis,ſi.a.fuiſſet.5.6.7. nec non.1.2.et.3. Cui reſpondi, quod quāuis nullam fecerim mentionem de æqualitate ipſius.a.cum.4. nihil tamen refert, propterea quod quando ita fuiſſet, nihilominus eaſdem conditiones ſubiret,quemadmodū ſi fuiſſet duplus,triplus,aut quadruplus. eò quod à genere multiplici, æqualitas, formam diuerſam non induat.Quare idem eueniet ſi.a.fuerit.4.5.6.7.vt ſi eſſet.8.9.10.et.11.& ſic de cæteris, excepto quod in proprijs multiplicibus, vel in ſuperantibus ipſis multiplicibus.a. menſurare tur ab ipſo. 4. plus quam ſemel. Quod autem dicis.de. 1. 2. et. 3 . nihil eſt, quia, vt in ſecunda ſumma, hoc eſt in tertio termino maximo, reliquus tertius terminus, ideſt.9.non comprætendetur, ita nobis indicabit primum numerum ſumptum mi norem eſſe quaternario. Quæ omnia, ex ipſa noſtra thęoria ibidem expreſſa manifeſtantur. Quid autem circa hoc Frater Lucas dicat, neſcio, quia ipſius opus ad manus meas nunquam peruenit, ſatis enim mihi fuit, in Tartalea hanc praxim vidiſſe, ratio vero nullibi à me reperta fuit. Tartalea enim multos citat authores, quorum ſcripta ego nunquam vidi, vt Leonardi Piſani, Proſdocimi, Petri Borghi, Fratris Lucæ,Ioannis Sfortunati, cæterorumq; ſimilium.

ELI-

## ELIPSIM PROPOSITAM QVALITER quadrare valeamus.

### *Illustri Viro Francisco Mendozza.*

QVOD antea tuo nomine fecerat Marcus Antonius amicus noster sufficiebat. Sed quia, quæ nunc à me petis, talia sunt, vt sine tripartita ęqualiter aliqua data proportione non possit aliquis exactè intentum perficere, nihilominus, supposita dicta diuisione, reliqua facilia erũt. Primũ enim est. Propositam Ellipsim quadrare.




Sit igitũ Ellipsis proposita.a.b.d.c. cuius axes sint.a.b et.d.c.dati, seu repti ex 47.secũdi Pergei, sintq; duo circuli.a.e.b.f.et.g.d.h.c.circa easdem diametros, tũc proportio.a.b.ad. d.c.dimidiũ erit proportionis circulorum ex.2.12. Euclid. sed proportio.a.b.ad.d.c.æqualis est proportioni maioris circuli ad Ellipsim.ex.5. Archimedis in lib.de conoidalibus, quapropter proportio Ellipsis ad minorem circulum altera medietas erit totius proportionis circulorum, hoc est maioris ad minorem, quare Ellipsis media proportionalis erit inter eos circulos. Nunc verò cum ex Archimede repertę fuerint duæ figuræ rectilineæ æquales duobus circulis iam dictis, & inter has, reperta fuerit alia media proportionalis propositum obtinebimus.

### *Sphæroidem propositam cubare.*

### AD EVNDEM.

PRoposita sphæroides erit, aut prolata, aut oblonga, sit prius prolata, sitq;.a.b. diameter circuli, qui eam per æqualia secat, circa quam.a. b. vt circa axem intelligatur sphæroides oblonga, cuius spissitudo sit.d.c.axis prolatæ, cogitemus nũc duas sphæras.a.e.b.f.et.g.d.h.c.circa dictos axes. Vnde quatuor corpora habebimus, hoc est duas sphæras, & duas sphæroides, quas probabo continuas proportionales inuicem esse.

Consideremus igitur duos conos rectos, quorum.a.b. diameter sit eorum basium, altitudo autem maioris, æqualis sit semidiametro maiori, hoc est medietati.a.b.altitudo

titudo verò minoris, æqualis sit semidiametro minori, hoc est medietati.d. c. vnde habebimus proportionem coni maioris ad conum minorem, eãdem quæ est diametri maioris ad diametrum minorem, quod ex.2.parte.11. duodecimi Eucli.nec non ex.9.eiusdem manifestum est, sed conus minor, est quarta pars sphæroidis prolatæ ex.29. Archimedis in lib.de conoidalibus, & conus maior, est etiam quarta pars sphæræ, ex.32.primi lib.de sphæra, & cyllindro, quare ex communi scientia, eadẽ proportio erit sphæræ maioris ad sphæroidem prolatam, quæ.a.b.ad. d. c. sed proportio.a.b.ad.d.c.est tertia pars proportionis maioris sphæræ ad minorẽ. Consideremus nũc alios duos conos rectos, vnius & eiusdẽ basis, cui⁹ diameter sit.d.c. sed altitudo maioris, æqualis sit semidiametro sphęrę maioris, altitudo verò minoris, sit æqualis semidiametro minoris sphæræ, vnde ex dictis rationibus habebimus proportionẽ maioris coni ad minorẽ, vt quæ est.o.b.ad.o.d.hoc est vt.a.b.ad.d.c.& ex dictis ꝓpositionibus ita se habebit sphæroides oblonga ad sphęram minorem vt.a.b. ad. d. c. hoc est tertia pars proportionis sphæræ maioris ad minorem. Quare proportio sphæroidis prolatæ ad oblongam, erit reliqua tertia pars proportionis maioris sphęræ ad minorẽm. Quapropter hæc quatuor corpora continua proportionalia inuicem erunt.

Nunc verò quærenda est inter.a.b.& suas duas tertias partes vna media proportionalis, quæ sit. K.& ex Archimede, inuentum sit quadratum ęquale circulo, cuius sit.K. diameter. Vnde proportio circuli (cuius.a.b.est diameter) ad circulum cuius.K.est diameter, sesquialtera erit ex.2.12.Eucli.

Ducatur deinde quadratum lineæ. K. in lineam. a. b. & proueniet nobis corpus quoddam, quod æquale erit sphærę maiori, ex corellario.32. primi de sphęra & cyllindro, cuius corporis, latus cubus sit.m.

Idem facere oportebit mediante.d.c.minoris sphærę, cuius corporis cubica radix sit.n.

Nunc verò inter.m.et.n.inueniantur duę medię proportionales.s. t.& ex. s. producatur cubus, qui ęqualis erit sphęroidi prolatæ propositi, cubus vero. t. æqualis erit sphęroidi oblongę, cuius axis esset.a.b.

Si autem sphęroides oblonga nobis proposita fuisset, eodem methodo soluetur problema.

## *Quadratum circulis mediantibus designare.*

## AD EVNDEM.

Modus autem conficiendi quadratum ex circulis supra datam lineam, vt Dominum Gasparem docui, facillimus est.

Sit enim linea.b.a.46.propositionis primi Euclidis, positoq; pede immobli circini in puncto.a.secundum quantitatem lineæ.a.b.propositę fiat circulus, similiter circa punctum.b.alius circulus eiusdem magnitudinis, erecta deinde sola.a.c. perpendiculari ipsi.a.b.ex puncto.a.ipsa secabitur à circunferentia circuli. cuius centrum est. a.in puncto.c.vnde. a.c.æqualis erit.a.b. posito demum pede immobili ipsius circini in puncto.c.secundum longitudinem ipsius.c.a.fiat alius circulus, qui æqualis erit reliquis duobus circulis cum eorum semidiametri æquales sint, & hic vltimo factus secabit circulum, cuius centrũ est.b.in pũcto.d.à quo cum ductæ fuerint.d.c.et.d.b. rectè

rectè habebimus quod volumus. nam omnia latera ſunt inuicem ęqualia ex conditionibus circuli, angulus autem.a.rectus effectus fuit, tunc ſi imaginatione cogitata fuerit diameter.b.c.ex.8.primi, concludemus angulum.d.eſſe rectum deinde ex.5 et.32.eiuſdem concludemus etiam reliquos angulos rectos eſſe.

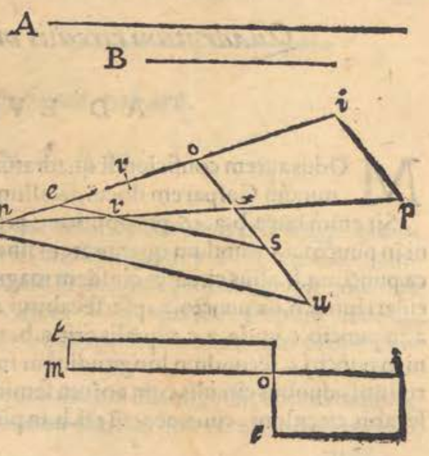
Circa verò id quod mihi ſcripſiſti de igne perpetuo putans nugas eſſe, quod Romæ inuentæ fuerint lucernę ardentes in ſepulchris antiquorum. Egò quidem minimè puto eas nugas eſſe, propterea quod tales lucernas non vnus tantum aut duo viderint, ſed multi homines fide digniſſimi. Prętera cum ais id nulla ratione poſſe fieri. Reſpondeo quod maxima ratione poſſibile eſſe puto, quam quidem rationem ita eſſe oportet, quod primum lucerna ſit perfectè circunclusa, vt materia in ea conſtituta nullo modo exire poſſit, deinde quod materia inflamabilis talis ſit, vt excrementum fuliginoſum ex flamma tranſmiſſum, tangendo ſuperficiem deuexam ipſius lucernæ, aptum ſit in priſtinũ humorẽ congelari, ſiue transformari, vnde materia prima per tres formas perpetuò tranſibit, hoc eſt per humorem, ſiue oleum tale, vt diximus, per ignem, ſeu flammam, & per vaporem, ſeu exhalationem fuliginoſam aptam condenſari, atque in priorem humorem illicò reuerti.

## DE DIVISIONE TRIANGVLI SECVNDVM propoſitam proportionem.

### *Michaeli Angelo Muciaſco.*

QVOD mihi proponis, tale eſt, vt ſcilicet tibi modum ſcribam diuidendi triangulum propoſitum ſecundum datam proportionem à linea tranſeunte per punctum notatum extra triangulum.

Triangulũ igitᷣ à te mihi propoſitum ſit.n.o.u. conſidero primũ quod ſi quis ipſum diuiſerit in duas partes mediante.e.s. parallela ad. n. u. ea proportione, quam mihi proponis. deinde inuenerit in dicta. e. s. punctum. r. per quod tranſiens alia linea à puncto. p. propoſito, ita quod efficiat duo triangula.m.r.e.et.r.s. x. inuicem æqualia, problema ſolutum erit. eo quod triangulum.m. o. x. æquale eſſet triangulo.e. o. s. & quadrilaterum reſiduum.m.n.u.x.etiam ęquale eſſet quadrilatero.e.n.u. s.




Sed dum punctum.r.uenarer, alia via mihi in mentem venit, cognoui igitur quod quum propoſitum expeditum fuiſſet, hoc eſt, ꝙ ſi à puncto p. protracta eſſet linea.p. m. quę triangulum.n.o.u.in duas partes inuicem ita proportionatas diuiſiſſet, vt ſe habet.A. et.B. ita ſe haberet productũ n.o.in.o.u.ad productum.m. o. in.o. x. vt trianguli.n. o. u. ad triangulum m. o. x. quod quidem non eſt difficile ſpęculari, ex methodo.24.ſexti,

eo quod

eo quod tam proportio producti.n.o.in.o.u.ad productum.m.o. in. o. x. quam proportio trianguli.n.o.u.ad triangulum.m.o.x.componitur ex proportione.u.o.ad.o.x. & ex proportione.n.o.ad.m.o.vnde proportio dictorum productorum nobis cognita erit, eo quod cum nobis cognita sit proportio. A.ad.B.vt data, cognita etiam nobis erit coniuncta, hoc est. A.B.ad.B.& propterea ea quæ trianguli.n.o.u. ad triãgulum.m.o.x.& similiter productorum. Quæsiui postea modum inueniendi duas dictas lineas. m. o. et. o. x. & cognoui quod si producta fuerit.p.i.æquidistans lineæ.o.x.producendoq;.o.n.quousque cum.p.i.se intersecarent in puncto.i. inueniendo postea lineam quandam, quæ ducta cum.p.i.efficeret rectangulum æquale rectangulo cognito quod ex.m.o.in.o.x.potest fieri, quod cognitum dico, eo quod nobis cognita est proportio data, & rectangulum etiam.n.o.in.o.u. deinde secando ab.o.n.partem æqualem lineæ iam inuentæ, quæ sit.o.t. Inueniendo postea,ex.28.sexti lineam.o.m.cuius productum in.m.t.æquale sit producto.t.o.in.o.i. vnde ex.15.eiusdem proportio.o.i.ad.m.o.eadem esset,quæ.m. t. ad. o.t.& componendo,ita se haberet.m.i.ad.m.o.vt.m.o.ad. o. t. sed ex. 4. sexti,ita esset p.i.ad.o.x.vt.m.i.ad.m.o. quare ex.11.quinti,ita esset.p.i.ad.o.x.vt.m.o.ad.o.t.vnde ex.15. sexti productum. o.x.in.m.o.æquale esset producto.p,i.in. o.t.& sic haberemus intentum.

Sed si punctum.m.caderet in punctum.n.idem esset, si verò punctum.m.transiret n.oporteret nos facere hoc in latere.n.u.ipsum quærendo in linea.n.u. ducendo primum lineam.p.i.æquidistantẽ.u.x.& producendo.u.n.ad partem.u.prosequendo, ꝙ superius iam dictum est.

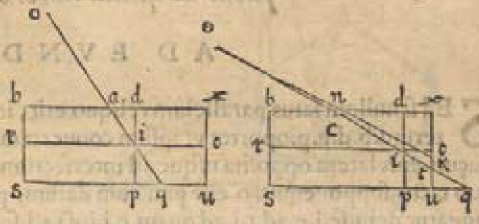
## *Idem facere de parallelogrammo.*

## AD EVNDEM.

DAtum parallelogrammum in duas partes diuidere, secundum aliquam datam proportionem à linea transeunte per punctum propositum.

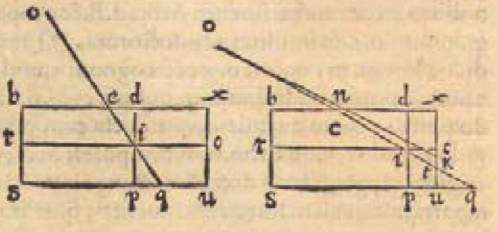
Sit exempli gratia, datum parallelogrammum.b.u.datum verò punctum.o.extra figuram, proportio autem ea sit, quæ.A.ad.B.vt supra. Nunc diuidatur primò rectangulum datum per æqualia, mediante linea.r.c.parallela ambobus lateribus.b.x. et.s.u.quæ quidem linea diuidatur in puncto.i. ita quod eadem proportio sit.r.i. ad. i.c.vt.A. ad.B.protrahatur deinde à puncto.o.linea.o.i.q.quæ secabit ambo duo latera.b.x.vel.s.u. intra terminos eorum, vel tantum.b.x.reliquum verò extra terminos. s. u.

Nunc autem si intra dictos terminos transibit, vt in prima figura videre potes, problema solutum erit, eo quod si à puncto.i. protracta fuerit. p. d.parallela ad. u. x. habebimus ex prima sexti eandem proportionem.s .d.ad.p.x. ut.r. i.ad.i.c. hoc est vt.A. ad.B. sed triãgulus i.e.d.æqualis est triangulo.i.q.p. vt tibi facilè parebit, vnde quadrilaterum.e.q u. x. æquale erit quadrilatero. d. u'. ex communi



ſcientia. Quare ex.9.quinti,ita erit.s.d.ad dictum.d.u. vt ad quadrilaterum.e.q.u. x.hoc eſt vt.A.ad.B.ex.11.eiuſdem.

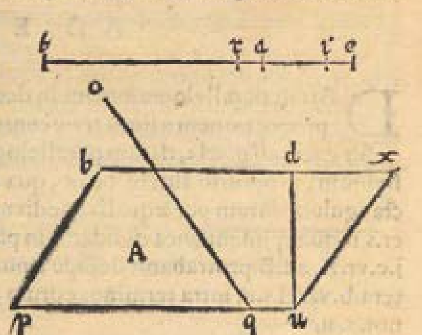
Sed ſi punctum.q.fuerit extra ut in.2. figura videre eſt. tunc manifeſtum erit, ꝙ triangulus. e. x. t.maior erit parallelogrammo. d.u.per triangulum.q.t.u.cum triangulus.q.i.p. æqualis triangulo.d.i.e. excedat quadrilaterum.i.t.u.p. per triangulum dictũ. q. t. u. quapropter cum diuiſus fuerit triangulus. e. x.t.mediante linea.o. n. K. ita ꝙ quadrilaterũ.e.n.K.t. ſit æquale triangulo.q.t.u. ex doctrina præcedenti, habebimus propoſitum.



*Idem de fruſto trianguli.*

## AD EVNDEM.

SEd ſi quadrilaterum dictum eſſet fruſtum alicuius triãguli ut in figura. A.hic ſubſcripta videre eſt,ſuppoſita,b.d.parallela ad.u.p.ita faciendum eſſet, ducendo ſcilicet parallelam.u.x.ad.b.p.quæ producatur vſque ad concurſum cum. b. d. in puncto.x.ſitq; proportio data inter.t.a.et.a.e.quas duas lineas cogitemus inuicem directè coniunctas, tunc diuidatur tota. t.e. in puncto.i. ita vt.t.i.ad.i.e.ſit vt quadrilateri.p.d.ad trigonum.u.d.x. deinde diuidatur t.i.in puncto.r.tali modo vt.t.r.ad.r.i.ſe habeat vt.t.a.ad.a.e. quo facto ex doctrina præcedenti diuidatur totum parallelogrammum.p. x. mediante linea. o. q. ſecundum quod ſe habet.t.r. ad.r. e. Atque ita ſolutum erit problema, vt ex te ipſo ratiotinari facile p otes.



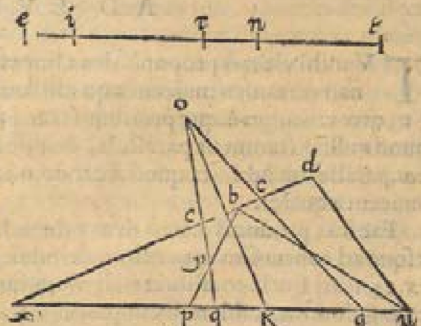
*Idem de quadrilatero in genere.*

## AD EVNDEM.

SEd ſi nullum latus parallelum reliquo erit, ita faciendum erit.ſi ſit tale quadrilaterum.b.d.u.p.oportet vt ipſum conuertamus in triangulum, producendo duo quęuis eius latera oppoſita uſque ad interſectionem ut pote.u.p.et.d.b.in puncto.x. quo facto,ſupponemus. o.eſſe punctum datum, proportio verò data ſit.t.r.ad.r..i.ad iungatur deinde.i.e.ad.t.i.ad quam.e.i.ipſa.t.i.ſe habeat vt quadrilaterum. b.d.u.p.

ſe

ſe habet ad triangulum.b.p.x.ducatur poſtea.o.q. quæ diuidat totale triangulum.d. u.x.in duas partes inuicem ita proportionatas, ut ſe habent.t.r.et. r. e. quæ quidem partes ſint.c.d.u.q.et.c.q.x.ut in primo problemate tibi monſtraui, & habebis propoſitum, dato quod punctum.c.ſit inter b. et d.



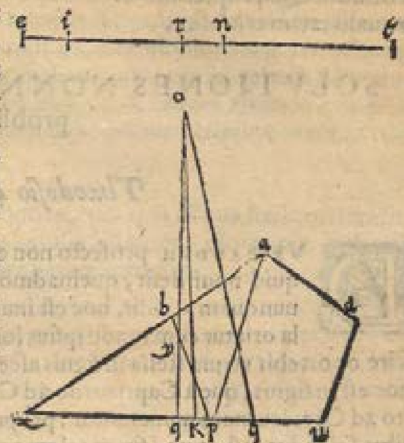
Sed ſi forte linea.o.q.ſecabit.b.x.hoc eſt ſi punctum.c.eſſet inter.b.et. x. manifeſtum eſt, quod.c.q.ſecaret.b.p.in puncto. y.vnde in tali caſu, alio modo operandum eſſet, hoc eſt ducendo.b.u. quæ diuideret quadrilaterum in duo triangula, & ut ſe haberet triangulum.b.d. u. ad triangulum.b.p.u.vellem vt ita ſecaretur t.i.in puncto.n.vt ita ſe haberet.t.n.ad.n. i.ut dictum eſt de iſtis duobus triangulis, deinde prout ſe habet.n.r.ad.r.i.ita ſecares triangulum.b.p.u. mediante linea. o. K.ex doctrina primi problematis, & ita haberes propoſitum.

## *Idem de Pentagono, Exagono, & de reliquis.*

## AD EVNDEM.

PEntagonum, ſeu hexagonum, vel alias quaſuis multilateras figuras propoſitas ita diuidere, vt dictum eſt de trilateris, & quadrilateris.

Sit exempli gratia pentagonus.a.d.u.p.b.quem ſecare volumus mediãte linea. o. q. in duas partes inuicem ſe habentes, vt ſe habent.t.r.et. r.i.oportet igitur ut ipſum pentagonum reducas ad quadrilaterum.x.a.d.u. quod diuidatur ſecundum præcedentem doctrinam, vt ſe habet.t.r. ad. r. e. vnde ſi punctum.q.incidit inter.p.et.u.tunc habebis propoſitum, ſi verò incidet inter. p.et.x. clarum erit quod linea.o. q. ſecabit latus.p.b. trianguli.b.x.p.in puncto.y.quapropter duces lineam.a.p. vt claudat triangulum.a.b.p.diuidaturq;.t.i.in puncto.n.ita vt.t.n.ad.n.i.ſe habeat, vt quadrilaterum.a. d.u.p.ad triãgulum.a.b.p. deinde hũc triangulum.a.b.p.diuidas mediante linea. o. K. vt.n.r.ad.r.i.ex doctrina primi problematis & habebis propoſitum. Idem dico de hexagono, reducendo ipſum ad pentagonum, & item de eptagono, ipſum reducendo ad exagonum, & idem infero de infinito ipſarum ſuperficialium figurarum rectilinearum.

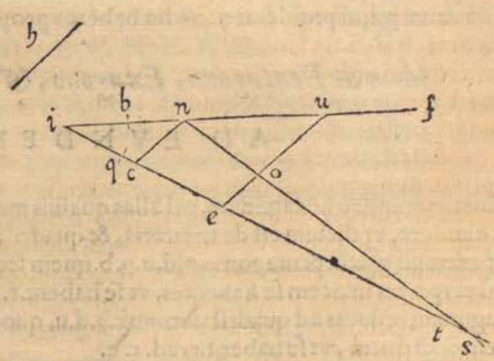


## *De duobus triangulis æqualibus inter lineas inuicem inclinatas.*

### AD EVNDEM.

TV mihi vltimò proponis duas lineas rectas.b.f.et.q.s.in eadem ſuperficie plana,non tamen inuicem æqu idiſtantes, proponis etiam.n.t.in eadem ſuperficie, quæ vnamquamque priorum ſecat,proponis etiam lineam. h. tali conditione, quod nulli dictarum ſit parallela, deinde ſcire cupis qua arte aliquis poſſet ducere. e.u.parallelam ad.h.ita quod ſecando.n.t.conſtituat duos triangulos.n.o.u.et.t.o.e. inuicem æquales.

Fac ita, producas primò duas primas lineas à parte, in qua inuicem inclinantur, vſque ad concurſum in puncto.i. deinde à puncto.n.duces.n.c.parallelā ad.h.poſtea ex.25.ſexti Eucli.conſtitues triāgulum.i.u.e.ſimile triangulo. i. c. n. æquale tamen triangulo.i.t.n.& ſolutum erit problema.




Vel ſic,inuenies.i.e. mediam proportionalem inter.i.c. & i.t. duces poſtea.e.u.parallelam lineç.h.vel.c.n.quod idē erit ex.30.primi Eucli.& ſolutum erit problema.

Nam ex. 17. ſexti eadem proportio erit trianguli. i. c. n.ad triangulum.i.e.u. ut.i.c. ad.i. t. Quare ut trianguli. i. c.n.ad triangulū.i.t.n.ex prima ſexti, et.11.quinti. Vnde ex.9.eiuſdem.i. e. u. æqualis erit.i.t.n. Quapropter.o.n.u. æqualis etiam erit.o. e.t.

## SOLVTIONES NONNVLLAE QVORVNDAM problematum.

### *Thæodoſio à Raiſeſtaim.*

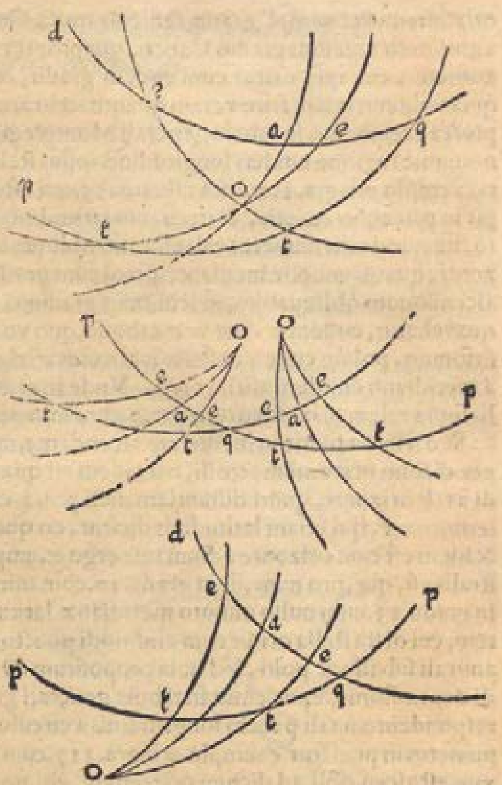
DVBITANDVM profectò non eſt, quin quotidie hominibus ſtudioſis aliquid noui deſit, quemadmodum, quod tibi nunc occurrit, mihi nonnunquam accidit, hoc eſt inuenire orizontem, cui aliqua propoſita ſtella oriatur cum gradu ipſius longitudinis. pro cui⁹ rei operatione te prius ſcire oportebit vtrum ſtella in ſignis aſcendentibus, vel deſcendentibus reperiatur, hoc eſt in ſignis, quç à Capricorno ad Cancrum procedunt, vel in illis, quç à Cancro ad Capricornum numerantur, propterea quod ſi in ſignis aſcendentibus inuenitur,ſciendum eſt, quod ſupra talem orizontem polus mundi auſtralis attollitur, ſed ſi in ſignis deſcendentibus reperitur, tunc polus borealis eleuatur ſupra dictum orizontem

orizontem, vt exempli gratia, canicula quæ à Græcis Prochyō vocatur, reperitur in 24. minuto vigeſimi gradus Cancri, quapropter polus borealis eleuatur ſupra orizontem, cui ipſa oritur cum eodem gradu, & minuto ecliptice illius ſigni. ſed quia volumus etiam ſcire veram quantitatem arcus eleuationis huiuſmodi poli, propterea accipiemus in tabula generali Monteregij numerum qui vocatur radix aſcenſionum, è regione numeri longitudinis ipſius ſtellæ, qui quidem numerus in præſenti exemplo erit gra. 107. cum minutis. 53. qui eſt cuiuſdã arcus æquatoris, qui incipit in principio Arietis, & in circulo latitudinis deſinit, hoc eſt ab orizonte quæſito, ita quod talis numerus erit aſcenſio obliqua huiuſmodi puncti eclipticæ illi orizonti, qua aſcenſione mediante, ſimul cum gradu, & minuto longitudinis in tabulis aſcenſionum obliquarum, inueniemus gradum, & minutum altitudinis pollaris, qđ quærebatur, eodem ordine ac methodo, quo vtimur ad inueniendum in tabulis poſitionum, polum circuli poſitionis alicuius aſtri, mediante declinatione & diſtantia à meridiano eiuſdem aſtri, vt ſcis. Vnde in præſenti exemplo eleuatio poli borealis ſupra talem orizontem erit gra. 7. cum minutis. 45.

Sed ſi ſtella fuerit in medietate aſcendente, tunc certi erimus polum auſtralem ſuper dictum orizontem attolli, nam idem eſt quærere altitudinem vnius polorũ mundi à tali orizonte, quod diſtantiam dicti poli à circulo ſecundum quem longitudo terminatur, qui etiam latitudinis dicitur, eo quod tunc temporis talis circulus vnus & idem eſt cum orizonte. Sumatur ergo exempli gratia ſtella, quæ in ore piſcis auſtralis eſt, quę, pro nunc, ſit in gradu. 20. cum minutis. 14. Aquarij longitudinis, & in gradu. 23. cum nullo minuto meridianæ latitudinis. Tunc certi erimus orizontem, cui dicta ſtella oritur cum eiuſmodi puncto eclipticæ, depreſſum eſſe à parte auſtrali ſub illoq; polo, ſed quia propoſitum eſt ſcire etiam quantitatem huiuſmodi depræſſionis, reperiemus in tabula generali gradum, & minutum æquatoris, correſpondentem tali puncto longitudinis à circulo latitudinis terminato, qui quidem numerus in præſenti exemplo erit gra. 317. cum minutis. 46. & hic numerus, vt diximus eſt aſcen. obli. ad dictum orizontem, vbi polus auſtralis attollitur, & deſcenſio obliqua, vbi polus borealis eleuatur. Quapropter ſi à. 317. gradibus cum minutis 46. demptus fuerit dimidius circulus gra. 180. remanebunt gra. 137. cum minutis. 46 & punctus oppoſitus gradibus. 20. cum. 14. minutis Aquarij eſt in eodem numero Leonis, & mediantibus iſtis gradibus. 137. min. 46. aſcenſionis, cum grad. 20. min. 14. Leonis inueniemus eleuationem poli borealis ab orizonte in tabulis aſcenſionum obliquarum Monteregij, hoc eſt gra. 17. min. 53. & eadem altitudo erit poli auſtralis ſupra orizontem à quo Fomahant cum dicto puncto eclipticæ oritur, in qua longitudine dicta ſtella reperitur.

Sed ſi propoſitus nobis fuerit punctus eclipticæ, cum quo aliqua ſtella oritura ſit, & oporteat inuenire vbi, hoc eſt orizontem huiuſmodi ortus, eleuatione poli artici, ſeu antarctici ſupra talem orizontem, ita operandum eſſet.

Sit




Sit exempli gratia ſtella.o. ecli ptica verò.d. q. æquator autem. p.q.punctus verò eclipticæ, cum quo ſtella oritura ſit.e.orizon de mũ.o. e . vbi ſtella oriri poſſit cũ puncto.e. Nam cum ſtella proponitur, datur etiam eius longitudo,nec non latitudo, quare arcus.a.q.& arcus.a.o. nobis cognitus erit, cum ſupponatur arcus.a. o.eſſe circuli latitudinis, et. a . o. latitudo ipſius ſtellæ, & angulus a.rectus erit, & quia punctum.e. datur,ergo arcus.a.o. & arcus. a. e.ſimul cũ angulo.a.recto cogniti ſunt, vnde ex.11.primi lib. copernici,angulus.a.e.o. cognoſcetur,& angulus.q.e.o. ſimiliter, vt reſiduum ex duobus rectis quo.e mediante cum angulo.q.declinationis ab æquatore, medianteq́; latere.q.e.cognito,cognitus quoque nobis erit angnlus.e. t. q. ex 12.eiuſdem.qui quidem angulus erit altitudinis æquatoris ab orizonte quæſito, qui demptus à 90. gradibus, dabit altitudinem poli ab orizonte quæſito.

Inuenire poſtea gradum eclipticę,cum quo ſtella data oriatur ad orizontem propoſitum, nullius eſt difficultatis.

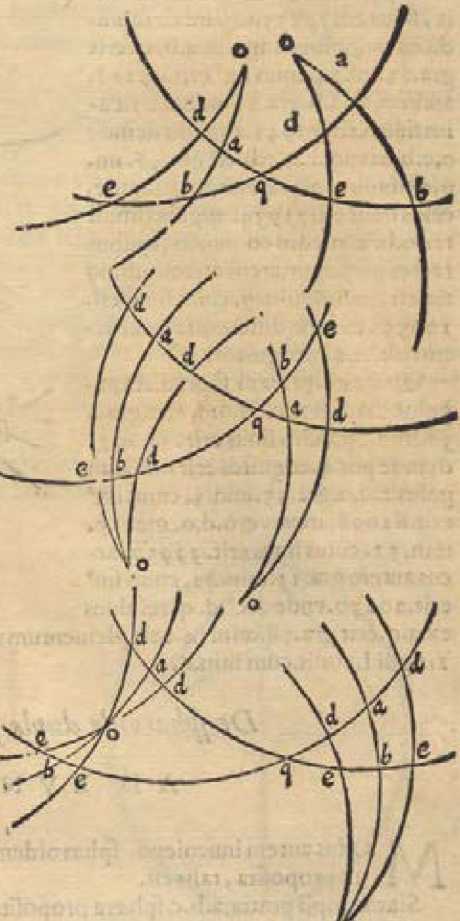
Ponamus exempli gratia,aliquem ſcire velle gradum eclipticæ, cum quo canicula oritur ad orizontem, cui polus boreus eleuatur per gradus.44.quæ canicula ſupponatur habere gradus.19.cum min.24.Cancri longitudinis,& gra.16.min.10.latitudinis meridianæ, quærere primum oportet eius declinationem ex doctrina.2.problematis tabularum directionum Monteregij, quæ erit graduum.6.cum minutis.5. ſeptentrionalis, deinde inuenire eius aſcenſionem rectam ex doctrina.4.problematis eiuſdem Monteregij, quæ erit gra.108.mi.42.deinde mediãte declinatione iam inuenta in tabulis differentiarum aſcenſionalium ſub polo.44.accipiemus differentiam aſcenſionum, qua differt recta ab obliqua, quæ in præſenti exemplo erit gra. 5.min.55.quæ dempta ab aſcenſione recta ſtellæ, vt præſens exemplum exigit,relinquet nobis aſcenſionem obliquam ſtellæ propoſitæ ad polum.gra.44. quæ erit gra. 102.minu.47.qua mediante,in tabulis aſcenſionum obliquarum poli. 44. habebimus gradum & minutum eclipticę cum quo ſtella oritur. quod in caſu noſtro erit gra.1.min.8.Leonis, ſed ſi tecum non fuerint tabulæ dictæ, potes eleganter omnia hæc perficere via triangulorum ſphæricorum.

Via

Via triangulorum idem facere.

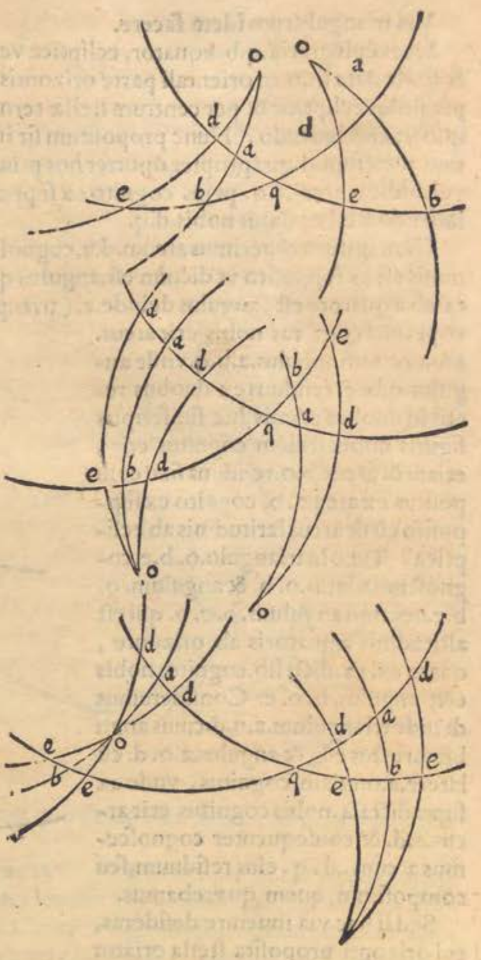
Sit exépli gratia.q.b.æquator, ecliptica verò.q.a.propositus aũt orizon sit.o.c.d. & stella data sit.o.in orientali parte orizontis, circulus verò.a.o.ille sit, qui transiés per polos eclipticæ & per centrum stellæ terminat longitudinem ipsius stellæ, & in ipso sit eius latitudo. Nunc propositum sit inuenire arcum.d.q. eo quod illicò scie mus punctum.d qua propter oportet nos prius cognoscere arcum.d.a.qui demptus, vel additus arcui.a.q. prius cognito ex supposito (nam data nobis est longitudo, & latitudo stellæ) dabit nobis.d.q.

Cum igitur voluerimus arcum.d.a.cognoscere, ita faciemus. nam. q. a. cognitus nobis est ex supposito vt dictum est.angulus quoque.a.q. b.qui declinationis eclipticæ ab æquatore est, angulus deinde.a. (trianguli.a.b.q.) rectus est, ergo ex.4.primi copernici cognitus nobis erit arcus. a.b.nec non angulus.a.b.q.vnde angulus.o.b. e. residuus ex duobus rectis in duobus primis hic subscriptis figuris nobis itidem cognitus erit, etiam & arcus.b.o.residuus siue compositus ex arcu.a. o. cognito ex supposito cũ sit arcus latitudinis ab ecliptica. Tunc in triangulo.o. b.e.cognoscimus latus.o. b. & angulum. o. b.e.nec non angulum. b. e. o. qui est altitudinis æquatoris ab orizonte, quare ex.12.dicti lib.cognitus nobis erit angulus. b. o. e. Consideremus deinde triangulum.a.o.d.cuius angulus.a.rectus est, & angulus.a.o. d. cũ latere.a.o.etiam cognitus, vnde ex supradicta.4.nobis cognitus erit arcus.a.d. & consequenter cognoscemus arcum. d. q. eius residuum, seu compositum, quem quærebamus.



Sed si hac via inuenire desideras, cui orizonti proposita stella oriatur cum eodem eclipticę puncto. a. longitudinis, hoc aliud nihil esset, quam cognoscere amplitudinem anguli.a. b.q.eo quod talis orizon, idem circulus esset.a.b.o.vnde cum quis sciret vnum illorum angulorum quem æquator efficit cum orizonte, reliqua illicò ei innotescent, sed dictus angulus. b. iam diximus quomodo cognoscatur.

Pona-




Ponamus nos ſcire velle pũctum eclipticę, cum quo Procyon oritur polo.44.o.dato, quod ſtella in gra. 19.cum min. 24. Cancri, reperiatur diſtans ab ecliptica per gra. 16.min. 10.meridiem verſus.vnde arc⁹. a.q. erit gra.70.min.36. eiusq́; ſinus partium.94321.talium qualium totalis eſt.100000.arcus verò.a.o.gra. 16. minut. 10. ſinus erit 27845. angulus autem.a.q.e.declinationis zodiaci ab ęquatore grad.23.min.30. cuius ſinus eſt.39875.Quare ex ſupradictis rationibus angulus.a.b. q. erit gra.82.mi.24.cuius ſin⁹ erit.99122. arcus vero.a.b.gra.22.minu. 17.cuius ſinus erit.37945.angulus deinde o.e.b.trianguli.o.c.b.eſt gra.46. mi. o.altitudinis æquatoris ab orizonte, cuius ſinus eſt.71934. angulus ſimiliter.o.b. e. medio coniuncti, quibus rectus perficitur,arcus etiam.o.b.notus eſt grad.6.min. 7. cuius ſinus eſt. 10655. cum ſit differentia inter arcus.a.b.et.a.o.cognitos.

Quare ex.12. iam ſupradicta angulus.e.o.b.hoc eſt.a.o.d. erit.grad. 36.min.39.cuius ſinus erit.59693. deinde per.4.cognitus erit nobis angulus.a.d.o.gra.55.min.5. cuius ſin⁹ erit.81998.arcus verò.d.o. gra. 19. min. 51. cuius ſinus erit. 33957. arcus autem gra.11.min.42. cuius ſin⁹ erit.20270.vnde arc⁹.d. q. reſiduus ex.a.q.erit gra.58.min.54.complementum aũt quartæ erit gra.31.mi.6.hoc eſt gra. 1. ſigni Leonis.cum min.6.

### *De ſphæroide duplæ ſphæræ propoſitæ.*

### AD EVNDEM.

MOdus autem inueniendi ſphæroidem ex dato axe, quod duplum ſit ſphæra propoſita, talis eſt.

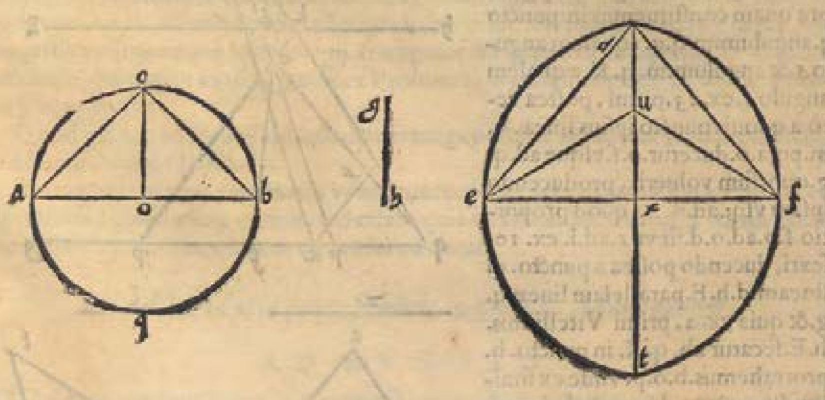
Sit exempli gratia.a.b.c.ſphæra propoſita. cuius ſemidiameter ſit. o. c. ſemiaxis vero ſphæroidis ſit.d.x.cuius dimidium ſit.u.x.tunc ex doctrina.9.ſexti Euclid. inueniatur.g.h.media proportionalis inter.u.x.et.c.o.deinde ſicut ſe habet.u.x.ad. g. h. facie-

faciemus, quod diameter.a. b. dictæ ſphæræ ita ſe habeat ad.e.f.ex.10. ſexti, quæ e.f.erit reliqua axis quæſita. Vnde conſtituta cum fuerit ellipſis.d.f.t.e.ex dictis axibus, deinde circumuertendo ellipſim circa maiorem axem, conſtituemus ſphæroidem oblongam, ſi autem circumuertemus ipſam circa minorem axim conſtituemus ſphæroidem prolatam.

Quod autem talis operatio rationalis ſit, nulli dubium erit, quotieſcunque cognoſcet conum rectum.e.u.f.æqualem eſſe cono recto.a.c.b ex.2.parte.12.duodecimi Euclid.& quod cum conus.e.d. f. duplus ſit cono. e. u. f. ex lemmate collecto ab 11.duodecimi, conus.e.d.f.duplus exiſtit etiam cono.a.c.b.ex.7.quinti. Cum deinde ex. 32. primi lib. de ſphæra, & cyllindro ſphæra.a.c.b.q.quadrupla ſit cono.a.c.b.ipſa conſequenter dupla erit cono.e.d.f.ſed ex.29.primi de conoidalibus, dimidium ſphæroidis.e.d.f.t.hoc eſt. e. d. f. dupla eſt cono.e.d. f. Quare talis medietas æqualis eſt ſphæræ propoſitæ, totaque ſphæroides dupla erit ſphærę datæ. Quod autem dico de proportione dupla, idem infero de qualibet alia, ſumendo.u.x.ita proportionatam ad.d.x. vt proponitur.

Sphęram autem inuenire quæ dimidia ſit ſphæroidis propoſitæ nullius erit negotij, quotieſcunque inuentus fuerit modus diuidendi vnam datam proportionem in tres æquales partes.

Sit propoſita ſphæroides.e.f.d.t.cuius axes ex conſequentia dantur.e.f.et.d. t.quę quidem ſphæroides ſit primo oblonga, et. u.x. ſit dimidium axis maioris. imaginetur etiam conus.e.u.f.vt ſupra. Imaginetur etiam factum eſſe, quod proponitur, hoc eſt, vt ſphæra.a.b.c.q.ſit dimidium ipſius ſphæroidis, vnde conus.a. c. b. æqualis erit cono.e.u.x.vt ſupra demonſtratum eſt; & ſit.g.h.media proportionalis inter.u.x.et o.c.Iam viſum ſuperius fuit, quod eadem proportio erat ipſius.u.x. ad.g h.quæ.a.b. ad. e.f.quare eadem quæ.o.b.ad.e.x.ſed.u.x.et.e.x.dantur.inter quas.g.h.et.o.b.vel o.c.(nam.o.c.æqualis eſt.o.b.) medię proportionales ſunt, eo quod cum.g. h. media proportionalis ſit inter.u.x.et.o.c.& proportio.o.b.ad.e. x. æqualis ſit ei, quæ. u. x. ad.g.h.hoc eſt ei quæ.g.h.ad.o.c.vel.ad.o.b. quare quotieſcunque inuentæ fuerint. g.h.et.o.c.vel.o.b.mediæ proportionales inter.d.x.et.x.e.ipſa.o.c.vel. o.b.erit ſemidiameter ſphæræ quæſitę.eodem modo faciendum erit ſi ſphęroides fuerit prolata.

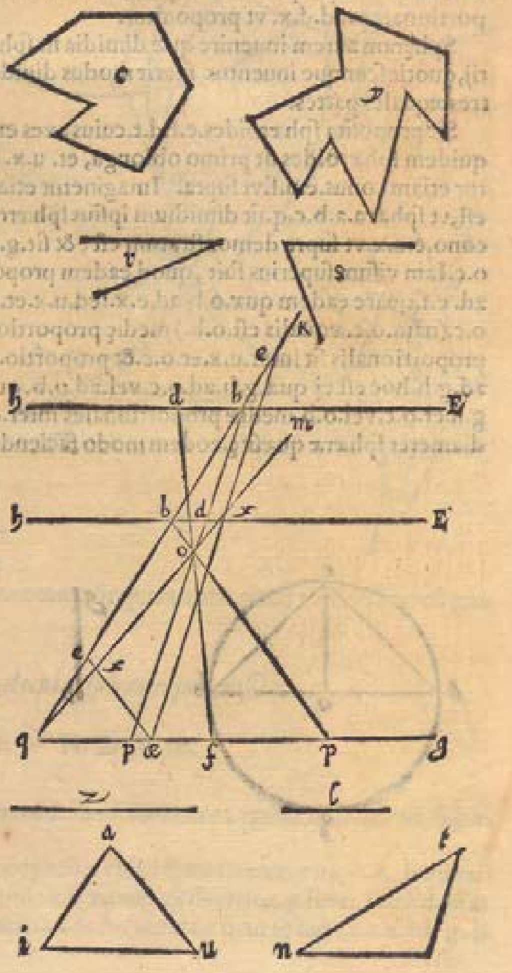


## *Modus inueniendi duo triangula varijs conditionibus affecta.*

## AD EVNDEM.

QVod etiam quæris ita se habet, duo scilicet triangula inuenire, æqualia duabus superficiebus rectilineis propositis, quę quidem triangula sint eiusdem altitudinis, & quod vnũquodque habeat angulum æqualem angulo proposito, & ꝙ alius angulus vnius, cum alio alterius, æquetur duobus rectis.

Sint exempli gratia duæ propositæ superficies.c.y.duo verò anguli dati sint.r.s. cum voluerimus inuenire duo triangula ( quæ sint. a. i. u. et.n.t.x. ) tali conditione prædita, quod angulus, a. æqualis sit angulo.s.& angulus. t. angulo. r. & quod angulus.x.simul cum angulo. u. æquetur duobus rectis, & quod triã gulũ. a. i. u. æquale sit superficiei. c. reliquum verò superficiei. y. Ex duabus superficiebus.c. et. y. constituemus duo quadrata, per vltimam secundi Eucli.accipiemus, deinde duo latera tetragonica ipsorum quadratorum, & inueniemus tertiam lineam in continua proportionalitate cum illis lateribus ex. 10. sexti, seruabimus postea extremas illarum, quæ sint. z. et.l.quarum proportio, eadẽ erit, quæ inter duas propositas superficies reperitur ex. 18. sexti, accipiẽmus, deinde lineam aliquam cuiusuis longitudinis, quæ sit.q.g. supra quam constituemus in puncto q.angulum.m q.g. ęqualem angulo.s.& angulum.m. q. K. æqualem angulo.r.ex. 23. primi, postea verò à quouis puncto ipsius lineæ. q. m.puta.o.ducetur.o.f.vsque ad.q. g.quorsum volueris, producendo ipsam vsq;.ad.d. ita quod proportio.f.o.ad.o.d.sit vt.z.ad.l. ex. 10. sexti, ducendo postea à puncto. d. lineam.d.h.E.parallelam lineæ.q. g.& quia ex. 2. primi Vitellionis. h.E.secatur ab. q. K. in puncto. b. protrahemus.b.o.p. vnde ex similitudine triangulorum habebimus proportionem.p.o.ad.o. b. vt.f.o. ad.o.d.



ad.o.d.hoc est vt.z.ad.l. hoc est vt.c.ad.y.quare triangulũ. p.q. o.ita erit proportio natũ triangulo.o. q.b. vt.c.ad.y.constituo deinde ex. 25. sexti duo triangula simi-lia duobus.p.q.o.et.o.q b.æqualiaq;.c.et.y.quę sint.a.i.u.et.n.t.x.secetur postea . q. g.in puncto.æ.ita, quod.q.æ.æqualis sit.i.a.duco postea.æ.e.æquidistantem.ad.p.b. & sic habebimus duo triangula. q. x. æ.et.q.x.e,vt quærebantur,quamuis duo trian gula.a.i.u.et.t.n.x.easdem habeant conditiones.

## DE IMPERFECTA SOLVTIONE PROBLE-matis Nicolai Tartaleæ ad Cardanum. De animad-uersione in Ptolomeum. De incendio carbo-num à vento.

### *Clarißimo Dominico Moresino.*

SCio propositam tibi quæstionem te diu agitauisse, nec tamen solutio-nem assequi potuisse, aduerte igitur ipsam falsam, idest impossibilem esse, quemadmodum etiam decimumoctauum quæsitum propositum à Cardano Tartaleæ, ab ipso Tartalea solutum minimè fuit. Quiquidem Tartalea vult circulum describi circa triangulum per quintam libri quarti Euclidis, vt in fine ferè quintæ partis suarum mensurarum affirmat, neque videt in quinta quarti Euclidem vti vndecima primi, & in vndecima primi,quarta aut octaua eius-dem, quas,ipse Euclides ostensiuè non demonstrauit. Quapropter oportebat Tar-taleam demonstrasse omnes propositiones ad hoc necessarias ostensiuè vsq; ad pri-mas indemonstrabiles, quia ad demonstrandam scientificè aliquã propositionem, aut à propositione in propositionem vsque ad prima principia vniuersalia ( vt ali-quando ego feci ) est retrogradandum, aut ab ipsis principijs incipiendum successi-uè eousque progrediendo donec ad propositionem quam demonstrare volumus perueniamus.

Quod ad Ptolomeum in geographia attinet, dico eum mihi non satisfacere,cum sumit portionem arcus circuli maioris inter vnam ciuitatem, & aliam, ea ratione quam describit. Quod si vsus fuisset modo Menelai, ab ipsomet deinde in suũ Al-magestum vsurpato,aut Monteregij triangulorum sphęricorum,quem Copernicus adhibuit ( qui tamen modus, tempore Ptolomei, nondum fortasse in lucem vene-rat ) bene egisset.

Quod deinde ad suum illud instrumentum geometricum attinet, est imperfectũ, vt ostendi domino Pandulfo.

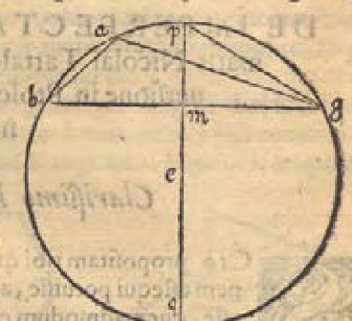
Motum autem aeris, aut mauis ventum,accendere ignem,non solum ratione an tiperistasis, quam affers euenit, sed etiam quia à carbonibus accensis totam excre mentitiam materiam, quæ eos circundat, auferat.

### *Alia dilucidatio propositionis. 25. lib. 2. Monteregij.*

### AD EVNDEM.

SCribis te non intelligere. 25. propositionem lib. 2. Monteregij. cum nec scias reperire diametrum circuli circunscriptibilis circa propositum triangulum, cuius data sit basis tantummodo simul cum angulo, qui ipsi basi opponitur.

Imagineris igitur triangulum datum esse obtusiangulum. a. b. g. cuius basi. b. g. sit nobis data simul cum angulo. a. ei opposito, obtusoque; Considera etiam circulum. a. b. g. q. ipsum triangulum circunscribentem, cuius diameter. q. e. p. transeat per. m. punctum medium ipsius. b. g. tūc protractis imaginatione. e. g. et. g. p. certi erimus angulos circa. m. rectos esse ex. 3. tertij Eucli. angulumq́;. q. e. g. duplum esse angulo. q. p. g. ex. 19. eiusdem, vnde æqualem angulo. a. qui etiam duplus est angulo. q. p. g. quapropter proportio arcus. q. g. ad arcum g. p. tibi cognita erit; & proportio etiam chordę. p. g. ad sinum. m. g. arcus. g. p. & quia. m. g. vt dimidium ipsius. b. g. tibi data est, cognosces etiam. p. g. vt. m. g. & sic tertium latus. m. p. trianguli orthogonij. p. m. g. & qa ex. 34. tertij quod fit ex. p. m. in. m. q. est æquale ei quod fit ex. b. m. in. m. g. ideo cum diuisum fuerit productum. b. m. in. m. g. per. p. m. proueniet. m. q. quapropter habebis totum. q. p.

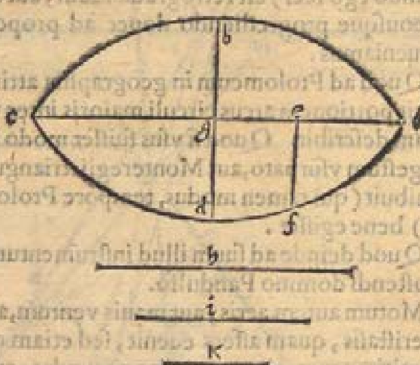



Idem efficies, si cū angulus. a. acutus fuisset.

## *Modus inueniendi puncta elliptica via Pergei.*

### AD EVNDEM.

MOdus inueniendi puncta elliptica, via. 21. primi lib. Pergei ex datis axibus, vt vbi alias significati, talis est. Sit exempli gratia maior axis propositus. a. c. minor autem. b. d. cum ergo volueris inuenire punctum circunferentiæ correspondentem puncto. e. maioris axis, inueniemus primo latus tetragonicum producti. a. g. in. g. c. quod sit. h. latusq́; tetragonicū producti. a. e. in. e. c. quod sit. i. deinde inueniemus lineam. K. tertiam in continua proportionalitate cum. h. et. i. vnde. i. erit media proportionalis inter. h. et. K. & vt. h. proportionalis erit ad. K. inueniemus. e. f. cui. g. d. medietas secundi axis ita se habeat, quæ postea iuncta axi maiori, ad angulos rectos in puncto. e. dabit situm puncti. f. quæsiti ex dicta. 21. primi lib. Pergei, sed talis modus prolixus est.



Accipe igitur hunc alium.

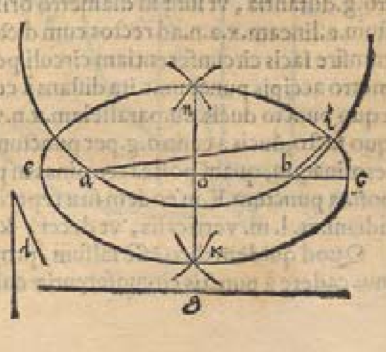
Sit propositus maior axis. q. p. minor verò. e. c. ad angulos rectos se inuicem secantes in puncto.o. describatur circulus.q.n.p.a. cuius diameter sit axis maior, in quo accipiatur punctum, quod volueris, vt puta. ú. à quo protrahatur.u.b. parallela ad.o.c.n.designetur postea separatim circulus.u.b.n.cuius diameter æqualis sit semidiametro prioris circuli, ita etiam fiat circulus.u.i.c.contingens circulum. u. b. n, in puncto.u.cuius diameter sit.u.c.æqualis dimidio axi minori. accipiatur deinde in circulo maximo longitudo.u.b.quæ collocetur in circulo mediocri à puncto.u. quæ secabitur à minimo circulo in puncto. i. cum itaque longitudo.u. i. mensurata fuerit in.u.b.maximi circuli à puncto.u.habebimus propositum.

Cuius rei ratio est, quia.u.b.mediocris circuli diuiditur à gyro minimi in puncto i.eadem proportione, qua diuisa est.u.n.in puncto.c.quod manifestum est ex similitudine triangulorum.u.b.n.et.u.i.c.imaginatæ cum fuerint duæ.b.n.et. i. c. sed ita esse oportet parallelas maximi circuli, quotiescunque circunferentia ipsius ellipsis transitura sit per.c.vt in.51.cap.meæ gnomonicæ ostensum fuit.

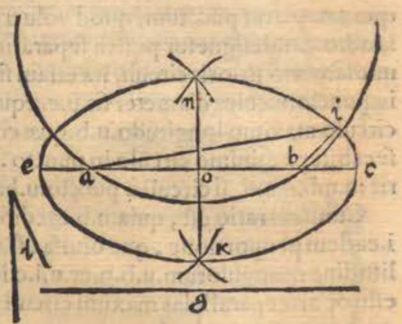
*Modus designandi angulum, certo modo conditionatum.*

## AD EVNDEM.

NVllius reuera difficultatis mihi videtur esse, quotiescunque nobis proposita fuerint duo puncta.a.et.b. simul cum angulo.d.necnó linea.g.ducere duas lineas à dictis punctis terminatas, quæ constituãt angulum æqualem dato, & ipsæ directè cõiunctæ constituant lineam æqualem datæ. Nam ducatur linea indefinita per puncta proposita, cuius lineæ, pars illa, quę intercepta fuerit inter dicta puncta, diuidatur per æqualia in puncto. o. etiam & linea data, quarum medietates accipio in linea indefinitè protracta à puncto. o. medio



dio, vt vna earum sit.o. c.reliqua verò sit. o. e. deinde aperiatur circinus quantum. o.c.positoq; vno pede in.b.designētur cum altero duo arcus.n.K.posito iterum vno pede in.a.designentur alij duo arcus intersecantes primos in punctis. n. K. Deinde à puncto.n.ad.K.ducetur linea.n.K.quæ per punctum.o.transibit,quam.n.K.mente cōcipio,vt axis minor vni⁹ ellipsis,cuius.e.c. sit axis maior, quibus axibus mediantibus designetur ellipsis.n.c.K.e. considero dein de.a.b.vt chordā vnius circuli, seu portionis circularis, quæ capax sit vnius anguli æqualis angulo.d. proposito,ex. 3 2. tertij Euclid. cuius circunferentia, circunferentiam ipsius ellipsis secabit in duob⁹ punctis quorum vnū sit.i.à quo protractæ cum fue




rint duæ lineæ.a.i.et.i.b.habebis propositum,cum.a.i. iuncta cum.i.b.æquetur.e. c. ex.5 2.tertij Pergei.

## EMENDATIO CVIVSDAM FALSI MODI delineandi horologia Italica orizontalia.

### *Joanni Paulo Dardano.*

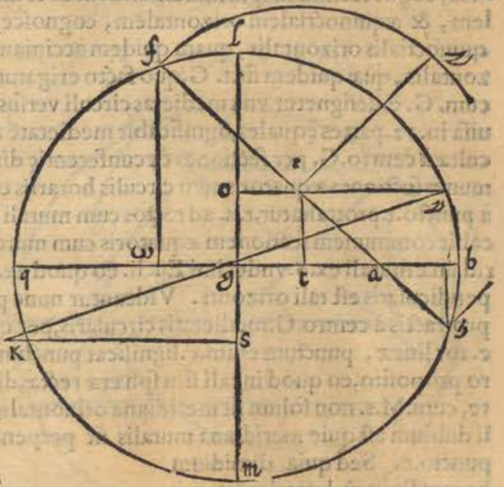
MODVS delineandi horologia Italica orizontalia, quem tibi monstrauit nescio quis, ni fallor,talis est. Designato meridiano.l.b.m.q.ductisq; duo bus diametris.l.m.et.b.q,inuicem ad rectos in centro.g. quorum. l. m. sit verticalis.b.q.vero orizōtalis, ductoq; diametro. f.h. tropici Cancri secundum altitudinem poli datam,descriptoq; dimidio circulo.f.z. h. ipsius paralleli, inuentoq; puncto.z. horæ propositę, & ab eo ducta per pendiculari. z.r.ad.f. h. & à puncto.r.ducta.r. o.y. parallela ad diametrum.q. b.orizontalem,ducis postea.f.ω.et. r.t.vsque ad orizontalem.q.b. parallelas ad diametrum.l.m.verticalem. Determinato postea gnomone.g.s. in orizontis axe, ductaq; vmbrarum linea. s. K. parallela orizontali, ductaq;.y.g.K.ad terminandam.s.K.delineas deinde separatim circulum q.x.b.n.magnitudinis prioris, qui quidem circulus significet orizontem ipsum, in quo ductis diametris.q.g.b.et.l.g.m.accipis in diametro.q.g.b.puncta.a.et.ω. ita à cē tro.g.distantia, vt sunt in diametro orizontali prioris circuli, ducis postea per punctum.a.lineam.x.a.n.ad rectos cum dicto diametro, deinde per tria puncta. n. ω. x. transire facis circunferentiam circuli per quintā quarti Euclidis,postea in dicto diametro accipis punctum.t.ita distans à centro, & ex eadem parte, vt in priori circulo, à quo puncto ducis.t.u.parallelam.x.n. vsque ad circunferentiam.x.ω.n.in puncto. u. quo facto,ducis à centro.g.per punctum.u.ipsius circularis circunferentiæ.g.u. indeterminatam, quam postea terminas in puncto.K.ita quod.g.K.æqualis sit.s.K. Dicis postea punctum.K.in eodem situ reperiri, respectu duorum diametrorum. q. b.meridiani.et. l. m. verticalis, vt decet, & oportet punctum horæ propositę existere.

Quod quidem dico esse falsum, propterea quod perpendiculares quas cogitamus cadere à punctis circunferentiæ cuiusuis paralleli supra quemuis orizontem ob

liquum

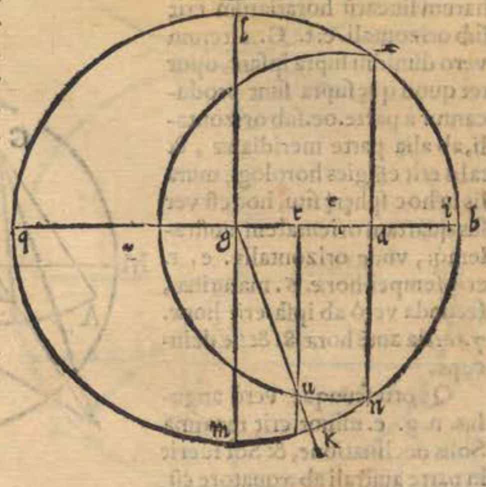
liquum fecantem æquatorem, omnes cadunt in gyro elliptico, oxygonio, feu defectionali, & non circulari. Vnde per fupradicta tria puncta.n.ω.x. oporteret transire talem circunferentiam, & non circularẽ, quæ circunferentia effet vnius ellipfis, cuius minor axis in diametro.b.q.effet.ab.ω.vfque ad. i.terminum fini h.i.arcus.h.b. in analemate,maior verò axis effet magnitudinis.f.h.diametri paralleli, quæ trãfiffet per punctum.c.medium inter.ω.et.i.quę quidem circunferentia tota effet intra circulum.q.n.b.x.cõtiguaq; gyro.q.n.b.x.in punctis.n. x.

Si ergò circunferentia.n.ω.x.effet elliptica tunc punctum.u.in orizonte illud effet vbi caderet finus altitudinis horę, et.t.u.æqualis effet.r.z. communi fectioni paralleli cum almicantarat ex.34.primi Euclid.et.u.g.æqualis effet.o.y. communi fectioni almicantarat cum meridiano, vel cum azimut illius horæ ex.4.primi,cum. g.t.æqualis fit ipfi.o.r.et.t.u.ipfi.r.z.& angulus.t trianguli.g.t.u.rectus, quemadmodum.r. qui compræhenditur ab.z.r.et.r.o.vnde anguli. K. g. m. et.K.g.b.rectè fe haberent, diftantia verò inter.K.et. g.iã rectè fumpta fuit.




Sed quia punctum. u.vt plurimũ (in gyro circulari fumptum) extra puncta interfectionum ipfius circularis gyri cum elliptico repetitur, propterea efficit angulos. K. g. m. et.K.g.b.falfos, & non æquales illis, qui fiunt ab azimut horæ cum verticali, & cum meridiano, quæ omnia ex cap.52.meæ gnomonicę facilè videre potes.

Nec tacere volo quod punctum u.verum,hoc eft ellipticum, inueniri poffet ea via quam fcripfi in eodem.52.cap.qua mediante docui demum inuenire punctum.ω. orizontis, quamuis in præfenti cafu.ω.λ. perpendicularis effet fupra minorem axem ipfius ellipfis,quãuis fupra maiorem axem,quod tamen minimè mutat ordinem, imò rationes eędem funt, tam in vna, quam in alia operatione, fed vt illicò idipsũ habeas,fac vt.t.u. æqualis.fit.r. z. & tunc punctum. K. erit quæfitũ,quod ego in.52. cap. meę gnomonicę, ijs verbis fignificaui.




„ Itaq; mediis binis triangulis ijs,
„ medioq; azimut Solis pariter ho-
„ rologia fabricari poterunt.

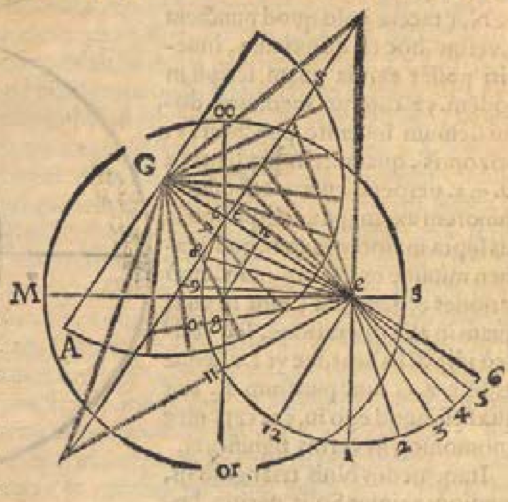
De

*De Horologio perpendiculari ad orizontem rectum.*

## AD EVNDEM.

MOdus quem tibi ſcribere promiſi delineandi lineas horarias communes in pariete perpendiculariter ad orizontem rectum, declinantem à meridiano, ſumendus eſt ex.46.cap.meæ gnomonicæ, hoc ſcilicet ordine.

Sit exempli gratia, orizon hic ſubſcriptus. or . oc. M . S. diuiſus à meridiana. M. S.et verticali ſeu æquinoctiali. or. oc . Sitque . e. t.communis ſectio muri cum orizonte,et.g.n.ſit gnomon perpendicularis ipſi muro , vnde ex dictis in mea gnomonica, cognoſcemus in ipſa murali orizontali totam.e.t. inter meridianam orizontalem , & æquinoctialem orizontalem, cognoſcemus etiam partem. g . t. ipſius æquinoctialis orizontalis , quam quidem accipiamus in rectitudine ipſius muralis orizontalis , quæ quidem ſit.t. G.quo facto erigatur. G. A.ad rectos cum.G.t.e.& circum. G. e.deſignetur vna medietas circuli verſus. e . cuiuſuis magnitudinis, quæ diuiſa in.12.partes ęquales,ſignificabit medietatẽ æquatoris,protrahanturq́; lineæ occultæ à centro.G. per ſectiones circunferentię dimidij circuli , quæ ſignificabunt cõmunes ſectiones æquatoris cum circulis horarijs communibus , quo facto oportet, vt à puncto.t.protrahatur.t. s. ad rectos cum murali orizontali,quæ quidem.t.s. ſignificabit communem ſectionem æquatoris cum muro propoſito, & erit ęquędiſtans meridianæ murali ex.6.vndecimi Eucli. eo quod ex.19.eiuſdem vnaquæq; illarum,perpendicularis eſt tali orizonti . Videantur nunc puncta communia iſti.t.s.& occultis protractis à centro.G.medietatis circularis,per quæ puncta protrahantur à puncto. e. tot lineæ , punctum enim.e.ſignificat punctum axis mundi , & meridianæ in muro propoſito,eo quod in tali ſitu ſphæræ rectæ,dictum punctum reperitur in orizonte, cum.M.s. non ſolum ſit meridiana orizontalis,ſed etiam axis mundi , deinde nulli dubium eſt quin meridiana muralis ſit perpendicularis orizontali murali . e . t . à puncto.e. Sed quia dimidium harum linearũ horariarum erit ſub orizontali. e. t. G. alterum vero dimidiũ ſupra ipſam, oportet quod quę ſupra ſunt producantur à parte.oc.ſub orizontali,ab alia parte meridianæ , & talis erit effigies horologij muralis in hoc ſphęrę ſitu, hoc eſt verſus quartam orientalem auſtralemq́; , vnde orizontalis . e . t. erit ſemper horæ. 6 . matutinæ, ſecunda verò ab ipſa erit horæ. 7.tertia autẽ horæ.8. & ſic deinceps.




Quotieſcunque verò angulus. n. g. e. minor erit maxima Solis declinatione, & Sol fuerit in parte auſtrali ab æquatore cũ

maiori

maiori numero declinationis quam fuerit angulus.n.g.e.tunc talis paries illuminabitur ab ipſo Sole à mane vſque ad veſperam.

Huius quidem rei ſpeculatio, vnicuique manifeſta erit, qui rationes.46.cap.noſtræ gnomonicæ prius intellexerit, vbi manifeſtè apparet proportionem ſemidiametri horologij (ſi ita eam appellare licet) ad ſemidiametrum æquatoris horarij ſemper eſſe, vt.e.t.ad.t.g. hoc eſt proportio maioris inæqualitatis. nolo etiam prætermittere.quin te admoneam, vt nullo pacto confidas in longioribus vmbris, eo quod valde nos decipiant, cum ſemper iuſto breuiores ſint.

## *Declaratio quorundam verborum noſtræ Gnomonicæ.*
## *Defenſioq; noſtra contra Chriſtophorum Clauium.*

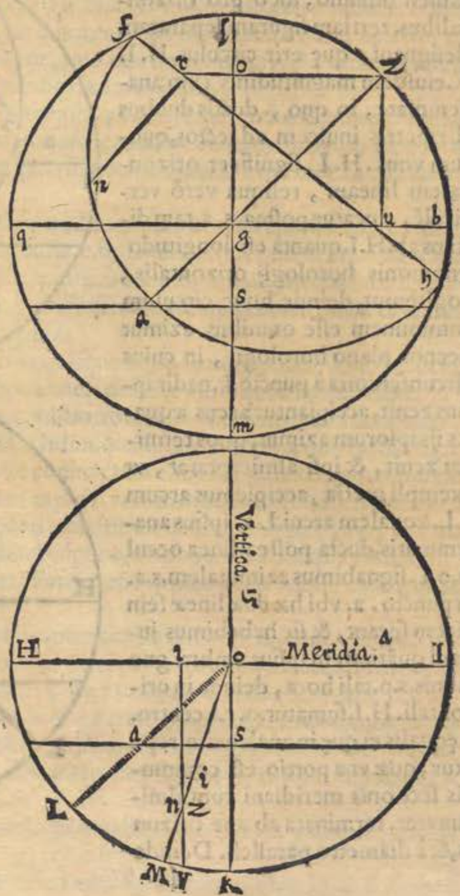
### AD EVNDEM DARDANVM.

TVas demum accepi literas, quibus mihi ſignificas te totum. 52. caput meæ gnomonicæ intellexiſſe, præter illa verba, quæ etiam ſuperioribus diebus ad te ſcripſi, hoc eſt.

» Itaq; medijs binis triangulis ijs,
» medioq; azimut Solis, pariter ho-
» rologia fabricari poterunt.

Quapropter ne aliquid tibi deſit, ſcire debes, me nihil aliud, eo in loco inferre voluiſſe, quàm qđ punctum horæ propoſitæ in plano horologij orizontali reperiri poteſt, ope longitudinis vmbræ gnomonis, & eius declinationis à verticali linea, ſeu à meridiana orizontali, iam in ipſo horologij plano ductis.

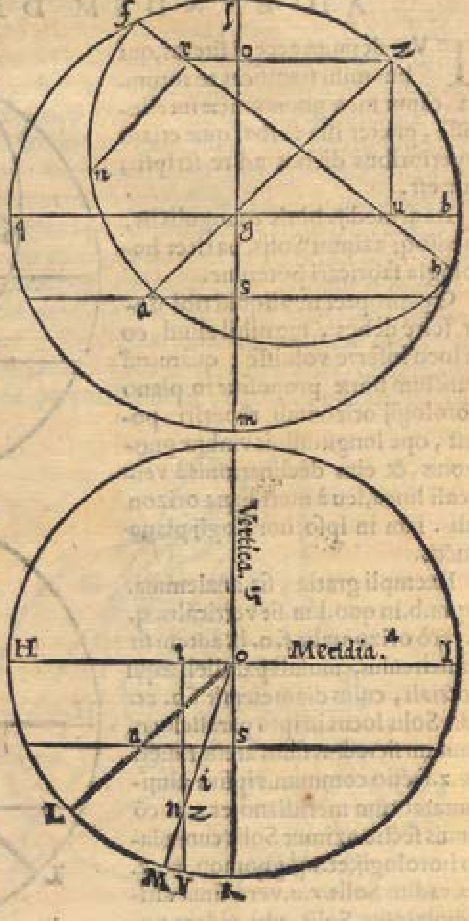
Exempli gratia, ſit analemma. l.q.m.b.in quo.l.m.ſit verticalis. q. b.verò orizontalis.f. n. h. autem ſit ſemicirculus, cuiuſuis paralleli æquinoctiali, cuius diameter ſit.f.h. et. n.ſit Solis locus in ipſo parallelo: n. r.autem ſit rectus ſinus arcus.f.n:et. r.o.z.ſectio communis ipſius almicantarat cum meridiano, et.s. a. cõmunis ſectio azimut Solis cum plano horologij, et.s.g.gnomon, et. z. g.a.radius Solis.z.u.verò ſinus altitudinis ipſius Solis, vbi videre potes duo triangula dicta eſſe. z. u.g.



et. g. s. a. quibus mediantibus cognoſcitur longitudo vmbræ gnomonis hoc eſt .s.a.

Cum autem dico, medioq; azimut Solis, nihil aliud ſignificare volo, niſi angulum, quem terminat linea azimutalis horologij, hoc eſt vmbra gnomonis cum linea meridiana, ſeu cum verticali in ipſo plano horologij. qui quidem anguli, æquales ſunt ijs, qui in triangulo conſtituto ex.n.r.ex.r.o.& ex.o.z.reperiuntur, cuius quidem trianguli, angulus puncti.r.rectus eſt, angulus verò terminatus ab.n.r.et.o.z.ille eſt quem conſtituit azimut cum verticali, vel ipſi æqualis, vt coalternus, reliquus verò in pũcto.o.ille eſt quẽ azimut facit cũ meridiano, vel ipſi ęqualis vt coalternus.

Vnde quotieſcunque volueris in aliquo plano, orizonti parallelo, lineas horarias ducere, iudico optimum fore ſi ſeparatim deſignatæ fuerint hæ tres figuræ, hoc eſt analemma meridianum, vel azimutale, vt ita dicam, deinde parallelus inſeruiẽs pro tropicis, vt ego feci cap. 51. meæ gnomonicæ, quæ duæ figuræ, ſufficiẽtes erũt pro omnibus horologijs, tam orizontalibus quam muralibus, non tamen omninò, ideo pro orizontalibus, tertiam figuram ſeparatam deſignaui, quę erit circulus. H. I. K. eiuſdem magnitudinis cum analemmate, in quo, ductis duobus diametris inuicem ad rectos, quorum vnus. H. I. ſignificet orizontalem lineam, reliqua verò verticalẽ, ducatur poſtea. s. a. tam diſtans ab. H. I. quanta eſt longitudo gnomonis horologij orizontalis, cogitemus, deinde hunc circulum communem eſſe omnibus azimut necnon plano horologij, in cuius circunferentia à puncto. k. nadir ipſius zenit, accipiantur arcus æquales ijs ipſorum azimut, quos terminat zenit, & ipſi almicantarat, vt exempli gratia, accipiemus arcum k.L. æqualem arcui. L. z. ipſius analemmatis, ducta poſtea linea occulta. o. L. ſignabimus azimutalem. s. a. in puncto. a. vbi hæ duæ lineæ ſe inuicem ſecant, & ſic habebimus iuſtam quãtitatem ipſius vmbræ gnomonis. s. o. tali hora, deinde in orizontali. H. I. ſumatur. o. r. à centro. o. ęqualis ei quę in analemate reperitur, quæ vna portio eſt communis ſectionis meridiani cum almicantarat, terminata ab axe orizontis, & à diametro paralleli. Deinde




du-

ducat.r.V.ad rectos cum.H.I.vſque ad circunferentiam,in qua accipiatur.r.n.æqualis ei quæ eſt in parallelo,ducatur poſtea.o.n.M.& habebimus triangulum.o.r n.ſimilem æqualemq́; triangulo iam ſupradicto. Vnde angulus.H.o.M.ei æqualis erit, quem azimut facit cum meridiano, & angulus.M.o.k.ei ęqualis, quem azimut conſtituit cum verticali, ita quod ſi talis circulus.H.k.I.eſſet planum horologij orizontalis, ſuppoſito.o.pro pede gnomonis, ſecando poſtea.o. M. in puncto.i. ita vt.o.i. æqualis eſſet.s.a.dato quod.o.M.ducta ſit ad partem ſibi conuenientem,reſpectu.o.k.ipſa pro verticali ſuppoſita, quod tibi relinquo, cum hoc facillimum ſit, tunc punctum.i.eſſet quod quærebamus. Quod verò de vno puncto dico, idem de omnibus infero.

Vbi verò mihi ſignificas Chriſtophorum Clauium, me duobus in locis meæ gnomonicæ redarguere, iam vidi. Circa primum locum igitur, qui eſt in pagin. 161. ita inquit.

„ Non enim deſunt, qui vel omninò negent, inter quos eſt Ioannes Baptiſta Be-
„ nedictus in ſua gnomonica cap.70.et.71.vbi alia, & multo longiore ratione cona-
„ tur arcus ſignorum deſcribere,vel certe dubitent, hoc modo rectè poſſe deſcribi ar-
„ cus ſignorum,cum rationem non videant, qua hæc noſtra deſcriptio quam quidem
„ omnes ſcriptores ſine vlla demonſtratione tradunt nitatur.

Abſque dubio raptim tranſcurrit illa capita.70.71.Reuerendus Clauius alioquin non ſcripſiſſet, quòd ego alia & multo longiore ratione conatus ſim arcus ſignorum deſcribere &c. præſertim cum eadem prorſus ratio, quæ ibi à me tradita eſt,illa ſit, quam ipſe ſuis ſcriptis inſeruit.

Meus igitur modus in dictis capitibus traditus, minime diſcrepat ab eo, ſed ab illorum modo, quorum opinio eſt interualla.e.h:h.u:u.n;n.m.et.m. d. meæ figuræ in pagi.75.poſitæ, æqualia eſſe interuallis.e.h:h.u:u.n:n.m.et.m.d.præcedentis figuræ, qui etiam ſupponunt.t.e.meæ figuræ.75.eſſe directè coniuncta cum linea. e. h. u.n. m.d.& propterea verſus finem.73.pag.dixi.

„ Aduertat autem quam diligentiſſime quiſque ne ſe decipi patiatur à ſubſcripta ſi
„ gura ſemicirculi. Q. æ. m. cum reliquis lineis ductis,ex antiquorum more,&c.

Eo quod non defuerunt aliqui, ex vetuſtioribus(quorum ſcripta ad meas manus peruenerunt) qui ſumentes interualla.e.h:h.u.&c.figuræ.pag.75.æqualia illis figuræ pag.74.putauerũt lineam.t.e.directè coniunctam eſſe cum.e.h.&c.quod quidem maximi erroris cauſa erat, & propterea cap.71.verum modum oſtendi, ſeruando illam eandem ſuppoſitionem,hoc eſt quod interſtitia.e.h:h.u:&c.figurę pag.75.æqualia ſint interſtitijs.e.h: h.u.&c.præcedentis figuræ, & ideò in dicto cap.71.dixi.

„ Suppoſito deinde.f.e.b.lineam meridianam eſſe in plano orizontali, cęterę linee
„ horarię erunt prędictę.

Stantibus igitur his ſuppoſitis, vt habeantur omnia ſcientificè,volui, vt intelligeretur pyramis quadrilatera,eo modo quo dixi,cap.71.vbi clarè patet eandem pyramidem eſſe, quam Pater Clauius(tacitè) poſuit in figura horologij, vt ipſe docuit propoſitione ſecunda,lib.ſecundi, cuius baſis eſt triangulum.H.I.F.ſuæ figurę(exempli gratia pro quinta hora poſt meridiana) Alterum verò triangulum à me cogitatum, terminatum ab.t.e:e.d:et.ab.t.d.eleuata in mea figura, eſt in ſua triãgulum. D. I. F.& propterea dixi.

„ Nam.t.e.et.e.d.vtræq; in plano horologii non ſunt, quamuis in plano æquatoris
„ tres ſint,&c.

Angulus verò.e.quem dico rectum eſſe, in ſua figura eſt angulus. D. I. F. & mea t.d.imaginata, eſt ſua.D.F. Tertium deinde triangulum, quod in mea figura terminatur ab.t.d.ab.f.d.& ab f.t.in ſua eſt triangulum. D.F.H. vnde mea. f. t. reſpondet ſuę.H.D & mea.f.d.ſuæ.H.F.& mea.t.d.ſuę.D.F. Quartum autem triangulum f.t.e.in mea figura, reſpondet ſuo.H.D.I.& meum punctum.t.ſuo.D, Nunc triangulum rectangulum, quod dico ſeparatim conſtituere, eſt illud tertium dictum correſpondens ſuo.D.F.H.vt ipſe facit in ſequēti figura, quod ipſe vocat.D.C.H. & meus radius.t.x.in ſua figura, ille eſt qui terminatur ab.D.& ab initio Tauri, & Virginis.

Et quamuis ego non ſcripſerim talem figuram, vt ipſe fecit, nihilominus ipſam verbis deſcripſi eomet modo, & propterea dixi.

» Quam diuiſionē, ſi in triangulo ſeorſum deſcripto inuenire voluerimus, res erit
» inuentu facillima, cum rectum angulum.f.t.d. (reſpondentem ſuo.H.D.C.)prędicti
» trianguli tertij ea ratione diuiſerimus,&c.

Quapropter Reuerendus Clauius non animaduertit meam rationem aliam non eſſe, nec puncto longiorem ſua, cum eadem met ipſa ſit.

Citaui etiam Munſterum cap.30.eo quod in ea impreſſione, quam tunc prę manibus habui, vidi in ea figura, quam ipſe vocat fundamentum horologiorum, literam c.poſitam eſſe loco.f.et.f.loco.c.quod cauſæ fuit, vt omnia mendoſa viderentur; recentiores autem impreſſiones correctæ ſunt.

Rurſus alio in loco mihi accidit vt repręhenderim Alexandrum Piccolomineum in libris de ſphęra, qui quidem dicebat eas figuras ſuperficiales, quæ paucioribus angulis circunſcriberentur, capaciores eſſe alijs, dummodo earum periphæriæ eſſent æquales.

Nunc autem correctę ſunt eo in loco impreſſiones, & qui non viderit primas, putabit me immeritò ipſum repræhendere.

Idem etiam dico de eo capite ipſius Piccolominei, in ijſdem libris, vbi tractat de modo, quo vſi ſunt antiqui ad diuidendum zodiacum in. 12.ſigna, quod erat circa finem quarti libri.

Nunc verò, in recentioribus impreſſionibus, illud caput poſitum non eſt. Impreſſiones autem illæ, vbi talia dixit, duæ fuerunt, quarum prima erat anni.1540. ſecunda verò. 1552.Venetijs apud Andream Puteum.

Alius verò locus ipſius Reuerendi Clauij, contra meas repręhenſiones, eſt circa finem pag.298.& circa.299.vbi ita ſcribit.

» Ex his liquido conſtat, non rectè à Ioan.Baptiſta Benedicto in ſua gnomonica ca
» pit.49.repręhendi hanc rationem deſcribendi horologij declinantis, qua omnes fe-
» rè alij ſcriptores vtuntur, quoniam, vt ex demonſtratione à nobis allata conſtat, re-
» ctè per eam lineæ horarię in plano, quod à verticali declinat ducuntur. Modus au
» tem quem eo loco pręſcribit differentem ab eo, quem nos tradidimus certus etiam
» eſt, ſed nulla ratione noſtro contrarius, quia nos conſtituimus.D.E.F. angulum de-
» clinationis plani à verticali circulo propriè dicto, ipſe autem loco huius anguli aſſu-
» mit angulum declinationis eiuſdem plani à Meridiano circulo, vnde mirum non eſt
» modum ipſius à noſtro diſcrepare. Quod ſi cōſtitueremus.D. E. F. angulum decli-
» nationis plani à Meridiano, ut ipſe (quemadmodum forſitan ab alijs putauit fieri)
» & in reliqua deſcriptione progrederemur, vt tradidimus, proculdubio horologiū
» declinans perperam deſcriberetur, vt rectè docet.

Optimè ſcripſiſſet Reuerendus Clauius, ſi verum fuiſſet, quod antiqui ſumerent declinationem ſuperius dictā à verticali propriè dicto, & non à meridiano. Sed ego dico, authores à me citatos. capit. 49. meę gnomonicę ſumere dictam declinatio-

nem

nem plani à meridiano, & non à dicto verticali.

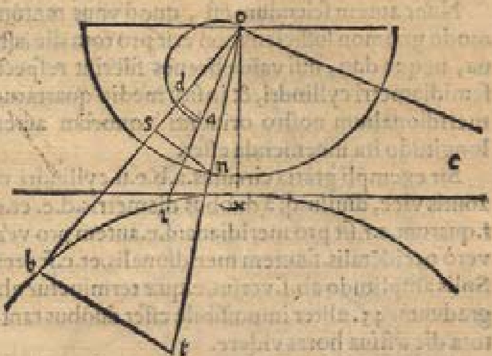
Con ſidera primum in Munſtero cap.16.ſuæ horologiographiæ, vbi clarè docet accipere angulum compræhenſum inter meridianum, & planum propoſitum, vbi etiam ponit quandam figuram ædificij cum pariete ſuper quo deſignatum eſt quod dam horologium, & vbi ſe manifeſtè declarat, ita dicens.

Nam ipſarum partium complementum. propoſitum indicabit angulum, quan-
" tus videlicet fuerit arcus eiuſdem circuli.d.c.f.g.à puncto.g.vſque ad productam li-
" neam meridianam interceptus, qui vnà cum ipſo.f.g.quadrantem integrare videtur,
" vt in ſequenti figura: quoniam arcus.f.g.eſt ſexaginta partium, qualium.e.f. quadrás
" nonaginta, vnde concluditur reliquam partem hoc eſt, datum inclinationis angulũ,
" fore partium triginta ſimilium.
" Orontius verò cap.13.ijſdẽ vtitur verbis, cum figura ſimili ad reliqua autem ipſius R. Clauij, videnda nondum mihi otium fuit. quod ſi dabitur, tibi libenter dicam quid ſentiam.

## DE MODO DVCENDI LINEAS HORARIAS ſuper cyllindro immobili.

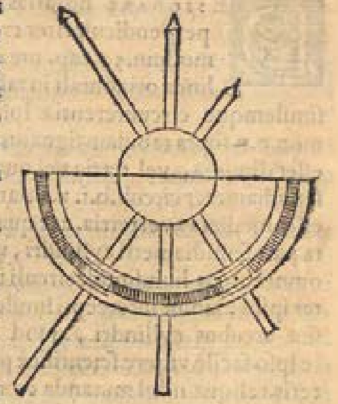
### *Hieronymo Ferrerio artium & Medicinæ Doctori peritiſſimo.*

DESIGNARE horarias lineas ſuper cyllindro immobili, ad orizontemq́ue perpendiculariter erecto difficile tibi non erit, (quod à me poſtulaſti) ſi modum.53.cap. meæ gnomonicæ obſeruaueris, accipiendo tamen pro linea orizontali in tabula non aliquam rectam lineam, ſed circularem, ſimilemque circunferentiæ ipſius cyllindri, dico autem ſimilem, eo quod ſi gnomon.o.x.ſupra tabulam ſignatus, & perpendicularis ipſi orizontali circulari. b. i. x. eſſet dimidia, vel tertia vel quarta pars gnomonis cyllindro infixi, oporteret, vt ſemidiameter circuli.b.i. x. etiam eſſet medietas, vel tertia, aut quar ta pars ſemidiametri cyllindri, vt omnes arcus huiuſmodi circuli in ter ipſos azimut intercepti ſimiles ſint arcubus cyllindri, quod à te ipſo facilè videre ſcientificè po teris. reliqua nihil mutanda erunt ab eo, quod ſcripſi circa figuram. 53. cap. vt dixi. Vnde inuenta cum fuerit diſtantia orizontalis puncti.b. à pede gnomonis.x.nec non quantitas azimutalis muralis b.t.quæ ſemper ab orizontali per pendiculariter deſcendit, illicò punctum.t. horæ propoſitæ in cy- lindro inuenietur.




Nunc

Nunc verò cum duo puncta alicuius horarię lineæ inuenta fuerint, quæ à Solis ſitu in diuerſis parallelis efficiuntur, ſi voluerimus ipſam lineam horariã ducere, ſciendum primò eſt ipſam lineam horariam eſſe communem ſectionem circuli horarij, illius horæ cum ſuperficie cyllindrica, & propterea ellipticam, vt oſtendit Serenus in. 19. primi lib. quod etiam ellicere poſſumus ab eo, quod Archimedes in. 10. propoſitione libr. de conoidalibus, ſcribit. Quapropter oportet nos inſtrumentum prius componere, modo circini, ſed trium crurum, quæ omnia in eadem plana ſuperficie ſint, ea tamen arte factum, vt quodlibet illorum poſſimus prolongare, necnon contrahere, ut cum duo extrema firmata fuerint, media poſſit circunduci circa centrum, ſeu punctum commune illarum interſectionum ſimulq́; poſſit produci, necnon abbreuiari vel augeri, & diminui, vt mediante ſua extremitate inferiori poſſimus delineare gyrum ellipticum horarium, dum cẽtrum ipſorum crurum adhæreat extremitati gnomonis, reliquæ vero extremitates ipſorum crurũ ſint ſupra puncta inuenta ipſius horæ. oportet etiam vt hoc inſtrumentum à tergo ipſorum crurum habeat in ſuperiori parte ſuperficiem quandam ſemicircularé, quę ſit vice vnius partis illius ſuperficiei, in qua ſupponuntur omnia crura inſtrumenti, & hoc quantum fieri poteſt, quod quidem fieri debet, ne crus medium, hoc eſt mobile, exeat à tali ſuperficie, ſeu declinet ab ea, quæ ſemper ſupponitur in ſitu circuli horarij talis horæ. oportet etiam, vt iuxta circunferentiam dimidij circuli ſint duo gyri eiuſdem materiæ inter ſe parum diſtantes, ita ut crura poſſint moueri, intra hos gyros, & dimidium circulum, & quod inter hos gyros locatæ ſint duæ cochleæ, ſeu duo helices, vt quando voluerimus, poſſimus firmare ipſa crura extrema, dum eorum extremitates fuerint ſupra puncta inuenta illius horæ, deinde in dorſo iſtius inſtrumenti, circa centrum coniunctionis, rectè factum erit ſi aliqua concauitas fuerit, in qua, extremitas gnomonis poſſit locari, dum ducere voluerimus aliquam horariam lineam.



Tale inſtrumentum excogitaui ad fugiendum tædium inueniendi dictam ellipticam ex punctis

Nunc autem ſciendum eſt, quod vnus tantummodo gnomon ſufficiens non erit pro tota die æſtiua, neque duo, niſi valde breues fuerint reſpectu ſemidiametri cyllindri, & in ſitu medio quartarum meridionalium noſtro orizonti, quorum autem longitudo ita inuenienda eſſet.

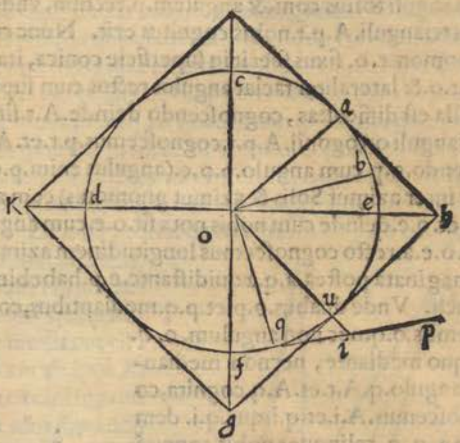
Sit exempli gratia circulus. a. b. e. u. cyllindri orizontis vice, diuiſusq́; à duobus diametris. d. e. et. c. f. quarum. c. f. ſit pro meridiana: d. e. autem pro verticali, ſitq́;. e. punctus orientalis: d. verò occidẽtalis. f. autem meridionalis. et. c. ſeptentrionalis, computeturq́; maxima Solis amplitudo ab. f. verſus. e. quæ terminetur ab. q. ita quod arc⁹. f. q. minor ſit quã graduum. 45. aliter impoſſibile eſſet duobus tantũmodo gnomonibus mediantibus tota die æſtiua horas videre.

Quo facto ducatur ab. q: q. p. contingens circulum & à centro circuli. o. per punctum. u. medium quartæ ducatur. o. u. i. vſque ad contingentem. q. p. vnde. u. i. longitudo erit vniuſcuiuſque gnomonis, qui gnomones infixi erunt in medio dictarum quartarum.

Huiuſmodi

Huiusmodi rei ratio per se nota erit quotiescunque cogitauerimus verum arcum e.b.amplitudinis æstiuæ, protractaq;.o.b.quę parallela erit.q.p. vnde cum Sol tempore æstiuo orietur, tunc radios suos emittet via istarum æquidistantium linearum.

Sed si longiores gnomones cuperes, oportebit eos tres esse, quorum vnus erit orientalis in puncto.e. alter occidentalis in puncto.d. reliquus verò meridionalis in puncto.f.quorum vnusquisq; potest esse maior tertia parte semidiametri cyllindri, sed si voluerimus scire quantū ad plus possit esse longus vnusquisque illorum, ita faciendum erit.

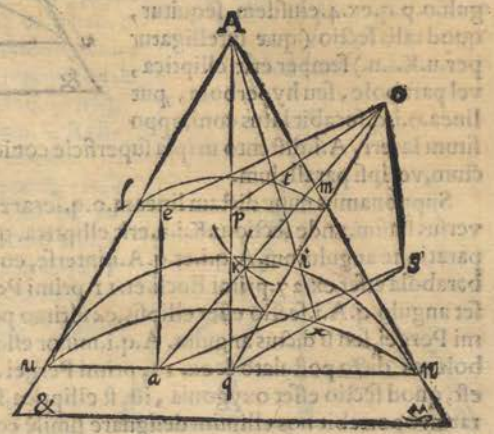



Faciemus quadratū.o.a.h.u. ex semidiametro dicti circuli, à diametro postea.o.h.huiusmodi quadrati subtrahatur semidiameter. o.e.circuli,residuum verò.e.h.ipsius diametri.o.h.quadrati,erit lõgitudo gnomonis, vbi simul appa ret huiusmodi rei ratio, eo quod cum gnomon.e.h.orientalis desinet operari, illico meridionalis. f.g.subintrabit, post hunc verò occidentalis.d.K.monstrabit reliquum diei.

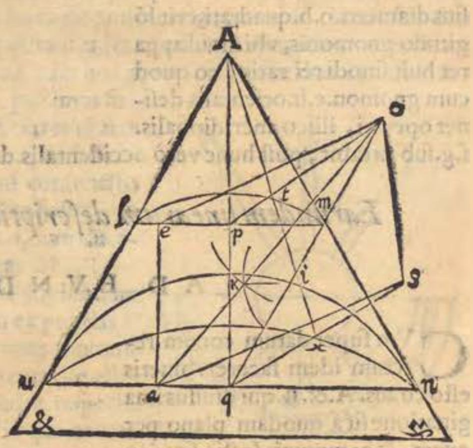
## *Earundem linearum descriptio super conum rectum.*

### AD EVNDEM.

CVm super datum conum rectum idem facere volueris esto conus. A.&.℞.qui diuisus imaginatione sit à quodam plano per axem, & communis sectio sit triangulus. A.&.℞.in quo plano cogitemus gnomonem infixum ad rectos vbi volueris,qui sit.p.t.o cogitem⁹ etiam.l.t.m.aliud esse planum (in quo sit gnomon) quod conum secet, quæ quidem sectio, circularis erit, ex. 4. primi Pergei.imaginemur etiam superficiem.p.s.esse azimut in quo gnomon reperirur, superficiemq;.e.s. azimut propositæ horę, angulumq;.e.o.a. contraposi tū angulo altitudinis Solis ab orizonte;



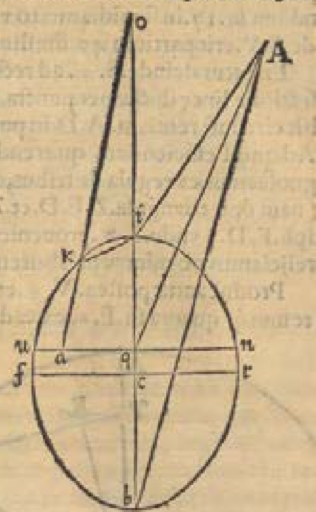
zonte; cogitemus etiam lineam. A.t.i.x. illud coni latus esse, qu od à summitate versus basim transit per medium latitudinis ipsius gnomonis, concipiamus etiam mente e.a. communem sectionem esse trianguli supradicti cum azimut horæ, necnon punctum. K. esse commune radio Solis. o.a. & superficiei conicæ, quod quidem est illud quod quæritur, hoc scilicet modo. Primum cognoscimus angulum. p. A.t. vt medietas anguli totius coni, & angulum. p. rectum, vnde. t. tam intrinsecus, quam extrinsecus trianguli. A.p.t. nobis cognitus erit. Nunc cum angulus. A.t.o. cognoscatur, si gnomon. t. o. fixus fuerit in superficie conica, ita qđ cum latere. A.t. efficiat angulũ A.t.o. & lateraliter faciat angulos rectos cum superficie conica, ad quod efficiendum nulla est difficultas, cognoscendo deinde. A.t. simul cum angulis. A. et. t. intrinsecis trianguli ortogonij. A.p.t. cognoscemus. p.t. et. A.p. vnde etiam totã. o. p. sed cognoscendo. o.p. cum angulo. p.o.e. (angulus enim. p.o.e. cognoscitur ex hypothesi cum sit inter azimut Solis & azimut gnomonis) cum angulo. o.p.e. recto cognoscemus. p.e. et. o.e. deinde cum nobis nota sit. o.e. cum angulo altitudinis Solis. e.o.a. & angulo. o.e.a. recto cognoscemus longitudinem azimutalis. e.a. necnon quantitatem. a.o. Imaginata postea. a.q. æquidistante. e.p. habebimus. p.q. æqualem. a.e. ex. 34. primi Eucli. Vnde duabus. o.p. et. p.q. mediantibus, cognitisq; cum angulo recto. p. cognoscemus. o.q. nec non angulum. o.q.p. quo mediante, necnon mediante angulo. q.A.t. et. A.q. cognita, cognoscemus. A.i. et. q.i. quę. q.i. dempta à. q.o. relinquet nobis cognitã i.o. Et quia. o.i.q. et. o.K.a. semper sunt in eadem superficie secante conum, quæ etiam secat superficiem trianguli. A.q.x. ad rectos ex. 18. vndecimi, cum linea. u.n. perpendicularis sit superficiei trianguli. A.q.i. ex. 8. dicti, quia parallela est. l.p. quę perpendicularis est superficiei triãguli. o.p.q. ex. 4. eiusdem, sequitur, quod talis sectio (quæ intelligatur per. u.K.i.n.) semper erit elliptica, vel parabole, seu hyperbole, ꝑut linea. o.i.q. secabit latus coni, oppositum lateri. A.i. distento in ipsa superficie conica, seu ad superiorem partem productum, vel ipsi parallelum.




Supponamus nunc dictam lineam. o.q. secare dictum oppositum latus lateri. A.i. versus basim, vnde sectio. u.K.i.n. erit elliptica. quod facile cognitu est mediãtę comparatione angulorum. A.q.i. et. q.A.i. inter se, eo quod si essent ęquales, dicta sectio barabola esset ex. 27. primi Eucli. et. 11. primi Pergei, sed si angulus. A.q.i. maior esset angulo. q.A.i. sectio esset ellipsis, ex ultimo postulato primi Euclid. & ex. 13. primi Pergei, sed si dictus angulus. A.q.i. minor esset angulo. A. tunc sectio esset hyperbole ex dicto postulato & ex. 12. primi Pergei. Sit ergo primum vt dictũ est, hoc est, quod sectio esset oxygonia, idest elliptica, seu defectio (quod idem est,) separatim oportebit nos ellipsim designare similẽ ęqualẽq; ei, quæ est. u.K.i.n. qđ quidẽ difficile non erit, quotiescunque suos axes inuenerimus, maiorem scilicet, & mino-

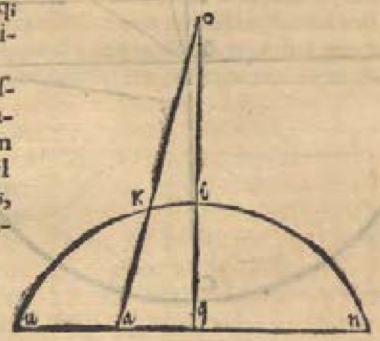
rem,

rem, quæ ita reperientur, efficiemus primo angulum coni, qui ſit.i.A.b.quem diuidemus per æqualia mediante.A.q.conſtituendo.A.i.huius anguli æqualem.A.'i.ſuperficiei conicæ et.A.q.diuidentem, æqualem parti.A.q. axis coni, ducendo poſtea ab.i.per.q.lineam vnam quouſque concurrat.A.b.in puncto.b.habebimus. i. b. pro maiori axi ipſi ellipſis, quod per ſe clarum eſt, cuius medietas ſit.i.c.ſed.i.q.ipſius.i.b.æqualis eſt ipſi.q.i.ipſius coni, ex quarta primi Eucli.et.q.b.ipſius.i. b. æqualis alteri parti inuiſibili. Reliquum eſt, vt reperiamus minorem axem, quem vocabimus. f.r.ducatur ergo primum.q.a.u.n.ad rectos cum.i.b.æqualisq; ei quæ eſt coni, & diuiſa ſimiliter in.a.quæ.u.n.ipſius coni nobis cognita eſt ex lateribus.A.u.et.A. n. & ex angulo coni, et.a.q.æqualis eſt.c.p.ex.34. primi. Nunc certi erimus ex. 21. primi Pergei, quod eadem proportio erit quadrati.u.q.ad quadratum ipſius.f.c. quæ producti ipſius.i.q.in.q b.ad productum ipſius. i. c. in. c. b. & cum cognita nobis ſint hæc tria producta hoc eſt.i.q.in.q.b.et.i.c.in.c.b.et.u.q.in ſeipſa, cognoſcemus etiã quartum ipſius.f.c.& ſic.f.c.eiuſq; duplum.f.r. cogniti nobis itaque cum ſint hi duo axes.i.b.et.f.r.formabimus ellipſim. Deinde producemus axim.b.i.à parte.i. quouſque.i.o.æqualis ſit ei quæ extra conum eſt, deinde ducemus.o.a.quæ circunferentiam ellipticam ſecabit in puncto. K. vnde habebimus quantitatem ipſius.o.K.et.K.i.rectam. inde mediante circino ſi acceperimus rectam diſtantiam ab.i.ad.K. in ellipſi, deinde firmando pedem circini in puncto.i.in ſuperficie conica, & cum alio ſignando lineam vnam curuam ad partem. K. in ſuperficie conica, ſumendo poſtea interuallum.o.K.extra ellipſim, deinde firmando vnum pedem circini in extremitate gnomonis, cum alio poſtea ſignando aliam lineam curuam in ſuperficie ipſius coni, quæ primam ſecet in puncto.K. hoc erit punctum quæſitum horę propoſitæ in ſuperficie conica propoſita.




Sed ſi talis ſectio fuerit parabole, vel hyperbole, tunc mediante ſuo diametro.i. q. cum baſi. u. q.n.cognita, deſignabimus ipſam ſectionem.u.i.n ope mei inſtrumẽti in calce meę gnomonicæ deſcripti, deinde diuiſa.u.q.in.a.ꝑductaq;q.i.vſq; ad.o.ductaq;.o.a.habebimus punctum.K. Reliqua facienda ſunt, vt dictum eſt de ellipſi.

Inuenta modo cum fuerint duo puncta eiuſdem horæ propoſitę, ducemus ab vno ad aliud, lineam horariam, mediante circino trium crurum, quem tibi ſcripſi nudius tertius pro cylindro, quæ quidẽ linea erit portio gyri ellipſis, ſeu hyperbolę, vel parabolę, vt à te ipſo cogitare potes.



## QVAEDAM NOTATV DIGNA IN Ptolomeum.

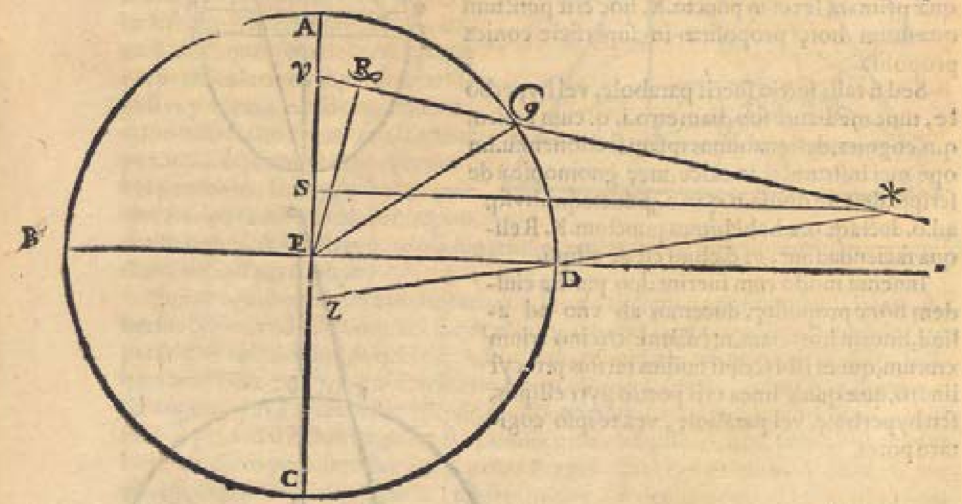
*Bartolomeo Christino Serenißimi Sabaudiæ Ducis apparitore.*

EX tuis literis cognoui quo erga me animo esses, qualiq; voluntate, sed ne tua pulcherrima studia aliquo modo imperfecta relinquant, vel ego tibi deesse videar, dum Problemata geographica Magni Ptolomei consideras, aduerte, quod si putares in figura.6.cap.libr. 7. geographiæ eiusdem (vt multi credunt) lineam.V.*.secare circunferentiam. A. D.in puncto.G.ita vt punctus. G.sit tropici æstiui, idest arcum.D. G.esse graduum. 24. cum illis incideres in maximum errorem. Quapropter considera quæ nunc tibi scribo.

Sit circulus.A.B. C.D.huius centrum. E. supponaturq; semidiameter. E.D. esse partium. 120.quarum.E.Z.in alio semidiametro.C.E.ei orthogonaliter coniuncto, talium sit.17.in semidiametro vero.E.A.accipiatur.E.S. talium. 24. et.E.V.64.vnde.S.V.erit partium 40.similium.

Erigatur deinde.S.*. ad rectos cum.E.A.in puncto.S. quæ terminetur ab intersectione lineæ ductæ per puncta.Z.D.in puncto. *.ducatur demum.V.*. quæ secabit circunferentiam. A.D.in puncto. G. Quæratur nunc quantitas ipsius. G. D. Ad quod efficiendum quærenda primum est quantitas ipsius. S. *. quam illico cognoscemus ex regula de tribus, cum dixerimus, si.17.dat nobis.120. quid dabit. 41. (nam duo triangula.Z.E.D.et.Z.S.*.sunt inuicem similia, cum. S. *. parallela sit ipsi.E.D.) vnde.S.*.proueniet nobis ex similibus partibus. 289.cum fracto, quod reijciamus ob minorem laborem.

Producantur postea. V. *.et.E.D.vsque ad eorum concursum in puncto.∞. quæremusq; quanta sit.E.∞. ex eadem regula, cum dixerimus, si.40.dat nobis.289.quid




dabit

dabit.64. (nam duo triangula.V.S. *.et.V.E.∞.ſunt inuicem ſimilia eadem ratione) vnde. E. ∞.veniet nobis ex talibus partibus.462.

Coniungatur nunc quadratum ipſius.E.V.quod eſt. 4096. cum quadrato ipſius. E.∞. quod eſt.213444.& habebimus quadratum ipſius.V.∞.talium partiũ. 217540.

Dicemus poſtea ſi.217540.dat nobis.4096.quid dabit quadratum ipſius.V. ∞.vt ſinus totus quod eſt. 10000000000.vnde veniet pro quadrato ipſius. V. E. talium partium, ſuperficialium ſcilicet.188272 11.cuius radix erit.13721. & erit ſinus anguli.V. ∞.E.qui erit grad.7.min.53.vnde angulus. ∞.V.E. erit grad. 82. min. 7. eius vero ſinus erit partium.99054.

Nunc autem quia angulus.E V.∞. eſt acutus,imaginemur.E.℞. ductam eſſe ad rectos ipſi.V. ∞. ſitq; etiam ducta ipſa. E. G. Vnde habebimus angulum.℞. E. V. graduum.7.min.53.eius vero ſinus.℞.V.partium.13721.( propter ſimilitudinem triangulorum.E.℞.V.et.∞. E.V.) talium ſcilicet,qualium.E.V.fuerit. 100000. Sed qualium.E.V.eſt.64.talium erit.8. cum tribus quartis, cuius. ℞.V. quadratum erit partium.76.cum dimidio ſimilium ſed ſuperficialium, quo quidem quadrato dempto ex quadrato ipſius.64.quod eſt.4096.remanebit quadratum ipſius. E. ℞. partium. 2871.quo etiam quadrato.E.℞.dempto ex quadrato.E.G.partium.14400.remanebit quadratum ipſius.℞.G partium. 11529.cuius radix. ℞.G.erit partium.107. taliũ qualium.E.G.eſt.120.ſed qualium.E. G. erit. 100000. talium. ℞. G. erit partium. 89166.quæ vt ſinus anguli.℞.E.G.habebit pro ipſo angulo,gra.63.min. 5. qui collecti cum gra. 7.min. 53. anguli. V.E. ℞. dabunt totum angulum. A. E. G. grad.70. min.58.cuius complementum ex grad.90.erit.G.D.graduum.19 .min.2.& non.24. vt omnes ferè putant.

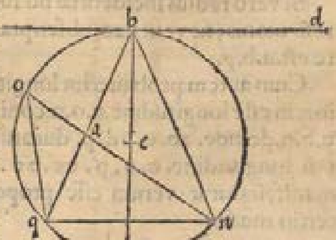
## DE REFLEXIONIBVS RADIORVM.

### *Excellentiſsimo Philoſopho Franciſco Vimercato.*

QVONIAM non videbatur quieſcere animus tuus,cum paucis ab hinc diebus tibi ſiſcitanti reſpondiſſem, nec tamen rationem omnium, quæ dixeram exactè explicare per temporis anguſtiam potuiſſem, cogitaui ad te per hanc occaſionem ſcribens,& iam dicta repetere , & omnium tibi rationem ſubiungere, & vt mihi plenius ſatisfaciam, & tibi commodè perlegenti facilius ſit veritatem intueri. Scripſiſti enim in tuis diſputationibus,vir doctiſſime,quod omnis res viſa per ſpeculũ quodcũque , ſub breuiſſimis lineis cõpræhendatur à viſu.

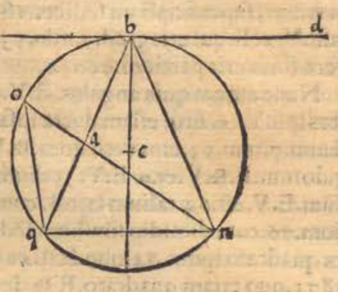
Propoſitio hæc non eſt vniuerſaliter vera(quamuis etiam ab alijs omnibus pro tali poſita ſit)cum in ſpeculis concauis non ſemper verificetur , vt nunc tibi demonſtrabo.

Eſto quod linea recta. b. d. tangat circulum b.o.q.n.qui ſit communis ſectionis ſuperficiei reflexionis, & ſphæricę alicuius ſpeculi ſphærici concaui, & punctum contingentiæ ſit. b. à quo exeant duæ lineæ.b.q.et.b.n.efficientes duos angulos inuicem æquales circa perpendicularem. b.c.res autem viſa primò ſit in ipſa circunferentia huiuſmodi circuli in puncto. n. oculus vero in puncto.q.ipſius circunferentię. Dico nunc duas



lineas.b.q.et.b.n. ſimul ſumptas longiores eſſe omnibus alijs lineis exeuntibus ab ipſis punctis.q.n.quæ in aliquo puncto dictæ circunferentiæ ſimul concurrant.

Sint igitur aliæ duæ.q.o.et.n.o.quas probare volo ſimul ſumptas, eſſe minores duabus ſimul ſumptis.q.b.et.n.b. Nam ex.20.tertij Eucli.cognoſcimus angulos.q.b.n. et.q.o.n.inuicem æquales eſſe, & ſimiliter angulos.b.n.o.et.b.q.o.deinde ex.15.primi eiuſdem habemus angulos contra ſe poſitos, circa. a.eſſe etiam inuicem ęquales. Vnde ex.4. ſexti, habebimus proportionem. a. b. ad. a. o. eandem eſſe, quæ. a. n. ad. a. q. & ſic. b. n.ad.o. q. Quare ita erit. a.b.n.ad.a.o. q.vt.a.n. ad.a.q.ſed cum.a.n.maior ſit.q.a.ex. 18. primi, eo quod angulus.b.q.n.(qui æqualis eſt angulo. b.n.q.ex.5.eiuſdem ) maior eſt angulo. a. n. q. qui pars eſt ipſius.b. n. q. ergo latera ſimul ſumpta.a.b.n.maiora erunt lateribus. a. o q. ſed ex. 20.primi.a.b.n.etiã maior erit.a.n.vnde ex. 25. quinti.q.a.b.n.maior erit.n.a. o. q. quare ſequitur verum eſſe propoſitum.



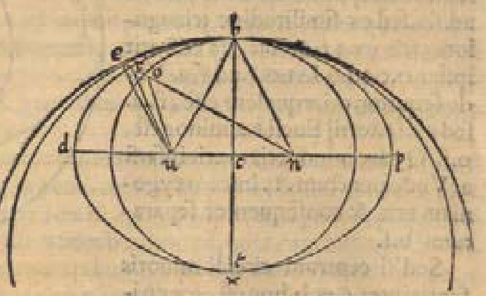

Sed ſi oculus eſſet in.u.quemadmodum in ſubſcripta hic ſecũda figura videre eſt, res autem viſibilis in.n.ambo extra dictum circulum, eſto etiam primum.b.u.æqualis.b.n.probabo ſimiliter.u.b.n.maiores eſſe.u.o.n. Nam angulus.o. maior eſt angulo.b,eo quód ſi circulum.u.b.n.cogitemus circunſcribere triangulum.u.b. n. ducendo vſque ad ſuam circunferentiam.o.n.in puncto.s.deinde ducendo.u.s.habebimus ex.20.tertij angulum.u.s.n.æqualẽ angulo.u.b.n.ſed cũ angulus.u.o.n.exterior trianguli.u.o.s.exiſtat, ipſe maior erit angulo.s.ex.16.primi.duco poſtea.o.q.parallelam ad.u.s.quæ ſecabit.a.u.in puncto.q.& habebimus angulum.a. o. q. ęqualem angulo. n.s.u.ex.29. eiuſdem,hoc eſt angulo. n. b. u. ſed ex ſupradictis rationibus, lineæ.q.b. n. ſimul ſumptæ maiorem efficient longitudinem, quam.q.o. n. Nunc cum ipſi.q.b.addita fuerit.u.q.& vice.q.o.ſumpta fuerit aliqua linea minor ipſa.u.q.o.eo amplius.u.q. b. n. maior erit, quod quidem hoc modo faciendum. Accipiatur.o.u. vt comes.o. n. quæ minor eſt ambabus. o. q. et.q.u.ex.20. primi, ita enim habebimus propoſitũ. ſed breuiori modo hoc ipſum videbis ex pręcedenti, & ex.21.primi Euclid. Nam ex præcedenti.u.b.n longior eſt ipſa.u.s.n.ex.21.autem primi.u.s.n.longior eſt ipſa.u.o.n.ergo verum eſt propoſitum.


Si verò radius incidentiæ nõ fuerit æqualis radio reflexionis, ſit vt in hac ſubſcripta tertia figura videre eſt.u.b.p.

Cum autem probauerim longitudinem.u.b.n.maiorem eſſe longitudine.u.o.n.coniungatur.n. p. cum u.b.n.deinde. ab. o. ad. p. ducatur. o. p. quæ minor erit longitudine. o. n. p. ex. 20. primi, & illicò manifeſtabitur verum eſſe propoſitum, etiam hoc tertio modo.

Si autẽ res visibilis oculusq́; ambo fuerint intra circulum, tũc possibile esset quod lõgitudo.u.b.n.modo maior,modo minor,modo verò æqualis esset ipsa.u.o.n. nũc. Quod etiam affirmo de.u. b.p.similiter etiam eueniet si vnus terminorum.u.vel. n. fuerit intra circunferentiam, reliquus vero extra ipsam.

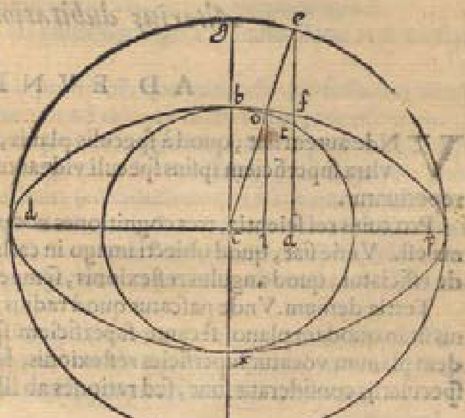
Consideremus nunc hic infrascriptam.4.figuram vbi.d.b.p.sit circunferentia oxygonia seu elliptica (quod idem est) cuius maior axis sit.d.p.in quo,duo termini. u. n. sint centra eius generationis:b.x.verò sit minor axis. Imaginemur etiam circulum. b.o.x.cuius semidiameter sit.c.b.non maior medietate minoris axis, ne circunferentia huiusmodi circuli secet circunferentiam oxygoniam. Cogitemus etiam circulum. b.e. cuius semidiameter, minor non sit minori axe.b.x. ipsius oxygoniæ, ne se inuicem secent huiusmodi circunferentiæ,sint etiam ambo eorum centra in linea.b. x.minoris axis, & punctum.b.sit commune vnicuique earum periphæriarum, vnde minor circulus, totus intra,maior autem, totus extra ipsam figurã oxygoniam erit. Nunc ad partem.o.r.e.vbi non communicant inuicem ipsæ circunferentiæ ducantur.n.o.r.e:u.o:u.r:et.u.e.& per. b.et.r.cogitetur transire alium circulum, cuius centrum in axe.b.x. sit.t. omnesq́; isti circuli imaginentur trium diuersorum sphæricorum speculorum, vnde pro generatione ipsi⁹ oxygonię,seu ex.52. tertij Pergei, habebis longitudinem. u.r.n.ęqualem esse longitudini.u.b. n.& ei, quæ est.u.o.n. ( vt minor ipsa.u.r.n.ex.21.primi Euclidis) minor ipsa.u.b. n. & longitudinem. u. e.n. ( vt maior ipsa.u.r.n.ex eadem. 21.primi Eucli. ) maior ipsa. u.b.n. Sed si quis vellet hoc demonstrare ope circuli, vni⁹ tãtũmodo speculi, multiplicãdo ipsas oxygonias quẽadmodum de ipsis circulis fecimus, obtineret similiter propositum.




*Solutio dubitationis.*

AD EVNDEM.

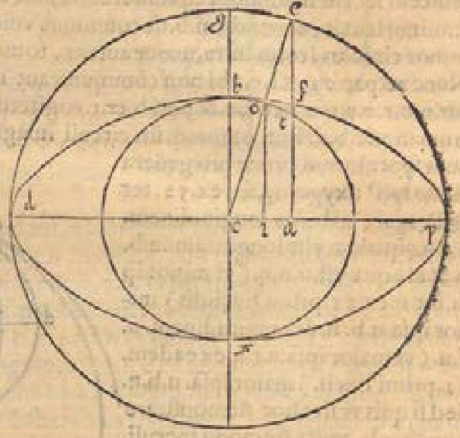
RAtionalis est dubitatio tua, vtrum ( cũ circulus minor hoc est.b.o.habeat suum centrum in minori axe inter centrum oxygoniæ, et.b: existente. b. extremo axis minoris,communeq́; ambobus circunferentijs circuli scilicet & oxigonię) dictus circulus minor, plura puncta communia habeat cum ipsis circunferentijs.




Cui dubitationi respõdeo quod quotiescunque centrum alicuius circuli fuerit idem cum.c. centro oxygoniæ, vel inter.c.et.b. in interuallo scilicet minoris axis, existente.b. sua extremitate communi ambabus cir-

circunferentijs, ipſas circunferentias inuicem contiguas eſſe oportebit in puncto.b. tantummodo.

Eſto primum quod centrum.c.commune exiſtat, vt dictum eſt. ſit etiam centrum vnius circuli, cuius diameter ſit idẽ cũ maiori axe. d. p.& in gyro oxygoniæ accipiatur punctum.f.proximum.b.quantum fieri poterit, tunc protrahatur.f.a.e.parallela ipſi.g.c. vſque ad gyrum maioris circuli in puncto.e.quæ cum. d. p. rectos efficiet angulos.ex.29.primi Eucli.ſecabitq; gyrum circuli.b.o.minoris in puncto.t.quod dico eſſe intra oxygoniam,ſeparatumq; ab.f. Quapropter duco.c. e. quæ ſecabit circunferentiam circuli minoris in pũcto.o.à quo puncto duco etiam.o.i.parallelam ad e.a. Deinde conſidero, quod ex rationibus ab Archimede adductis in quinta propoſitione libri de conoidalibus, & ſphæroidibus, eadem proportio erit ipſi⁹.g.c.ad.b. c. quę ipſius.e.a.ad.f.a. vnde permutando ita erit ipſius.g.c.ad.e.a.vel.b. c. ad f.a.hoc eſt ipſius.e.c.ad.e.a. vt. o.c. ad.f.a.ſed ex ſimilitudine triangulorum,& ex.11.quinti, ita etiã erit ipſius.o.c.ad.o.i.vt.o.c.ad.f. a. Vnde ſequitur.o.i.æqualem eſſe . f . a. ſed ex.14.tertij Eucli.t.a.minor eſt. o.i. Quare minor etiam erit ipſa.f. a. Vnde punctum . t . intra oxygoniam erit, & conſequenter ſeparatum.ab.f.




Sed ſi centrum circuli minoris fuerit inter.c.et.b.hoc eſt eccentricum ipſius oxygoniæ, ipſe tanget concentricum in puncto.b.tantummodò, vt in.3. Euclidis libro probatur. Vnde tanto magis diſtans erit punctum.t.à puncto.f. quod erit propoſitum.

## *Alterius dubitationis ſolutio.*

## AD EVNDEM.

VNde autem fiat, quod à ſpeculis planis, obiectorum imagines, ita diſtantes vltra ſuperficiem ipſius ſpeculi videantur, vt obiecta citra ipſam ſuperficiem reperiuntur.

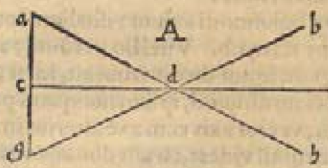
Pro cuius rei ſcientia, tres cognitiones nos primum habere oportet, quarum prima eſt. Vnde fiat, quod obiecti imago in catheto incidentiæ videatur. Secũda. vnde efficiatur, quod angulus reflexionis, ſemper æqualis ſit angulo incidentiæ.

Tertia demum. Vnde naſcatur quod radius incidentiæ ſimul cum radio reflexionis ſit in quodam plano ſecante ſuperficiem ſpeculi ſemper ad rectos, quod quidem planum vocatur ſuperficies reflexionis. Huiuſmodi tres paſſiones, ab omnibus ſpecularijs conſideratæ ſunt, ſed rationes ab illis traditæ, mihi non ſatisfaciunt.

Nam

Nam circa æqualitatem angulorum reflexionis & incidentiæ, iam tibi probaui illud non vniuerſaliter euenire à breuitate aggregati radiorum incidentiæ reflexionisq́; . Sed hoc naſcitur potius ab eo, quod cum radius incidentiæ non poſſit ſuperficiem corporis opaci penetrare, reflectit, vt citra ipſam cũ angulo æquali ei, quem faceret cum eadem ſuperficie vltra ipſam ſi tranſiuiſſet.

Exempli gratia ſit.a.obiectum.b.autẽ oculus in figura.A. et.c.e. ſuperficies ipſius ſpeculi.d.verò ſit punctum ipſius ſuperficiei, à quo ad oculum reflectitur imago ipſius.a. Nunc ſi radius.a.d.incidentiæ, recta incederet ſub.c.e.efficeret angulum.e. d.h. æqualem angulo.c. d. a. eius contrapoſito, ſed quia impeditur ipſæ radius ab opacitate ipſius ſpeculi.c.e.ne vlterius incedat, propterea reflectitur ab ipſa ſuperficie ſpeculi, conſtituens cum ipſa angulum.e. d. b. æqualem angulo.e.d.h.ſed quia angulus.c.d.a.eſt etiã




ęqualis ipſi angulo.e.d.h.propterea angulus.e.d.b.ęqualis exiſtit angulo. c. d. a; per accidens igitur ſequitur.a.d.et.d.b. ſimul ſumptas, breuiorem facere longiludinem omni alia, quæ ab ipſa ſuperficie.c.e.ad eadem puncta.a. b.ducta eſſet, quare naturæ intentio eſt efficere angulum.e.d.b.æqualem angulo.e.d.h.vnde ex accidenti poſtea ſequitur, ipſum æqualem eſſe angulo.c.d.a.& deinde qđ lineæ.a.d.et.d.b. conſtituant longitudinem breuiorem. Quare illud quod omnes putabant eſſe primum & per ſe, vltimum eſt, & ex accidenti.

Quare vero ſuperficies, quæ vocatur reflexionis, in qua ſunt duæ lineę, hoc eſt incidentię, reflexionisq́;, ſemper ſit perpendicularis ſuperficiei ipſius ſpeculi: Hæc eſt ratio, quia cum quilibet radius incidentiæ, perpendicularis ipſi ſuperficiei ſpeculi, in ſeipſo reflectit, ex ijſdem dictis rationibus, hoc eſt, quia cum tali angulo vult reflecti, cum quali tranſiret, ita etiam putandum eſt, quod radius incidens obliquus, cum in ſeipſum non poſſit redire, quia non eſt perpendicularis ſuperficiei ſpeculi, reflectitur tamen per planum erectum ipſi ſuperficiei ſpeculi, vt in eo, cui magis reſiſtit ſuperficies corporis opaci, quàm alicui alij plano ipſius infiniti inclinatorum planorum, ab vtraque parte ipſius plani perpendicularis, quod vnum etiam tantummodo eſt, & in quo, radius maiorem vim obtinet reflectendi, ſeu in eo, in quo radius ipſe cum maiori reſiſtentia repercutitur à ſuperficie corporis opaci.

Poſtremo ſciẽdũ vnde oriatur, ꝙ rei viſibilis imago, à ſpeculo plano reflexa, ſemper in catheto incidentiæ videatur.

Pro cuius rei ratione cognoſcendum primò eſt, quo modo ſit perfecta ſimplexq́; viſio, & non reflexa, deinde proſequemur ad reliqua huius tertiæ propoſitionis.

Animaduertendum igitur eſt, quod quotieſcunq; obiectum aliquod viſibile aſpicimus, nos nunquam perfectè illud comprehendere poſſumus, niſi in puncto concurſus, ſeu interſectionis axium viſualium, ſeu radialium (vt ita loquar) quã interſectionẽ, nos efficimus ope reuolutionis oculorum adinuicẽ, hoc eſt voluendo vnum verſus alium, ita vt in ſitu ipſius obiecti, ſeinuicem ſecent axes iam dicti, tunc enim vtroque oculo mediante, exacte rem perſpicimus, cęteris. 8. circunſtantijs non obſtantibus.

Vnde ſtantibus oculis in tali ſitu, altero reſpectu alterius, ſi eorum alter tectus; ſeu velatus fuerit, tunc alio tantummodo oculo mediante, videbimus obiectum, in ea diſtantia, exactius, quam in quauis alia propinquiori, & remotiori.

Animal

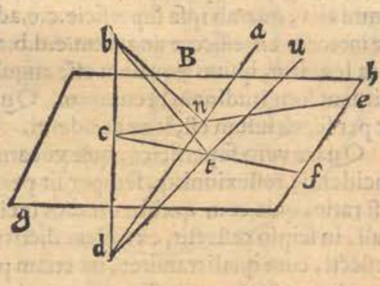
Animal igitur, ſecundum diſtantiam obiecti, oculum accommodat ad recipiendum quam exactiſſimè ſpeciem ipſius obiecti, & hoc voluendo ambos oculos, vnum verſus alium, ita quod interſectio axium ſit in ſitu ſeu loco dicti obiecti, nam tunc videant ambo vel aliquis eorum ſolus, in tali diſtantia exactè obiectum videbit.

Vnde ſequitur obiectum viſibile, compræhenſibile non eſſe ab vno tantummodo oculo in quolibet ſitu axis ipſius oculi, ſed in eo, vbi alius axis interſecatur à dicto. Quæ quidem interſectio poteſt fieri propinqua, vel remota à viſu, ad certos tamen terminos vſque.

De huiuſmodi axium viſualium interſectione ſcribit Alhazem in.2.et.15.propoſitione tertij lib. Vitellio verò in.32.et.45.eiuſdem.

Quod igitur dico, verum eſt, ideſt, quod ſi vno tantummodo oculo aſpiciemus obiectum aliquod, ipſum nunquam perfectè proſpicietur, niſi cum oculus ita ſitus fuerit, vt eius axis cum axe alterius in loco obiecti ſe inuicem ſecent, quamuis alter oculus nihil videat, cũ aũt duobus oculis in tali ſitu cõſtitutis obiectũ videmus, vnum tantummodo nobis cernere videbimur, & ſi extra talem punctum interſectionis ipſum obiectum poſitum fuerit, tunc duo talia obiecta nobis apparebunt, ſed huiuſmodi rei cauſam alias tibi manifeſtabo.

His igitur cognitis, ponamus aliquam ſpeculi ſuperficiem eſſe. g. h. in figura. B. obiectum autem viſibile. b. oculos vero. a. et. u. punctum autem. n. in ſuperficie ſpeculi, à quo imago ipſius. b. reflectit ad. a. & punctum. t. à quo reflectitur ad. u. et. c. e. ſit cõmunis ſectio ſuperficiei reflexionis radiorum. b. n. a. et. c. f. ſit communis ſectio ſuperficiei reflexionis radiorum. b. t. u. quarum vnaquæq; ſuperficies reflexionis, erecta eſt ad ſuperficiem ſpeculi. g. h. vt ſupra diximus. Nunc ex. 19. vndecimi Eucl. ſequitur communem ſectionem harum duarum ſuperficierum. (b. c. d. ſcilicet) ad rectos etiam eſſe ſupra ſuperficiem ſpeculi. g. h. tum qua. b. c. quælibet linearum. a. n. vel. u. t. reflexarum (productę cum fuerint) ſeinuicem interſecabunt eo quod duo anguli. d. c. n. et. d. n. c. ſimul collecti minores ſunt duobus rectis, & ita. d. c. t. cum. d. t. c. cum anguli. a. n. e. et. u. t. f. reflexi, ipſis contrapoſiti, æquales ſint angulis. b. n. c. et. b. t. c. incidentiæ, quorum vnuſquiſq; ex. 32. primi, minor eſt recto.




Dico etiam quod in eodem puncto huiuſmodi catheti. b. c. d. in quo interſecabitur à linea. a. n. in eodem ſecabitur à linea. u. t. & quod punctum dicti concurſus, tantum depreſſum erit ſub ſuperficie ſpeculi. g. h. quantum. b. ſupra ipſam reperietur. Nam anguli. b. n. c. et. d. n. c. ſunt inuicem æquales, anguliq;. b. c. n. et. d. c. n. recti. c. n. verò communis ambobus triangulis. b. c. n. et. d. c. n. vnde ex. 26. primi Eucli. latus. d. c. commune, vt trianguli. d. c. n. æquale erit lateri communi. b. c. vt trianguli. b. c. n. Idem etiam dico de latere. d. c. vt ipſius trianguli. d. c. t. quod æquatur lateri. b. c. vt trianguli. b. c. t. Vnde cum. b. c. vnum, & idem ſit: d. c. igitur etiam erit, & ipſum vnũ & idem, quod erit propoſitum.

Nunc autem cum hi duo radij ſeinuicem ſecent in puncto. d. ergo in ipſo puncto. d. videbimur nobis videre imaginẽ obiecti. b: cũ ope duorũ iſtorũ radiorũ. n. a. et. t. u. ita inuicem ſitorũ, videamur nobis imaginẽ proſpicere. Vnde ſi in tali caſu, vnus

ocu-

oculorum clauderetur, nihilominus cum reliquo obiectum vidissemus in codē ipso loco.d. & non in alio ex superius dictis rationibus.

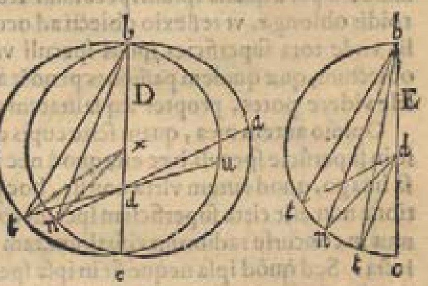
Et si stantibus ijs terminis voluceremus pupillam oculi.u. versus aliam.a. ad aspiciendum punctum.n.in superficie.g.h. ipsius speculi, hoc est si fecerimus quod axes visuales seinuicem secarent in ipso puncto. n. tunc videremur nobis videre duas imagines ipsius obiecti. b. intra speculum, eo quod obiectum, propter hoc non cessaret reflectere ad oculos ab ipsis punctis .n. et. t. quapropter recipiendo radium.t.u.in situ axis oculi.u.& radium.n.a.in situ axis oculi.a. hi axes ex necessitate (vt probauimus) seinuicem secant in puncto.d. vnde vnam tantummodo imaginem ipsius obiecti nobis apparebit.

Ex his igitur omnibus potes facilè videre omnem imaginem, cuiusuis obiecti, reflexam à speculo, reperiri in ipso catheto incidentiæ, cum ipse semper sit communis sectio duarum superficierum reflexionis, in quo catheto concurrunt ipsæ axes visuales.

Ex ijsdem etiam dictis rationibus facile compræhendere poteris, vnde fiat, vt videamus imaginem reflexam à speculis sphęricis concauis citra ipsorū superficiem, & non vltra. Quod nunquā euenit, nisi quando punctū.d. intersectionis ipsorū radiorū visualium (quod alio in loco non fit, nisi in catheto incidentiæ hoc est in communi sectione duarum superficierum reflexionis. Dato quod obiectum non sit in vna eademque superficie, in qua reperti fuerint axes visuales, hoc est dato, ꝙ ambo axes visuales non sint in vna eademq; superficie reflexionis) reperitur citra & non vltra superficiem ipsius speculi.

Ad cuius rei euidentiam non prætermittā dicere, quod cum debeant semper superficies reflexionum perpendiculares esse, vel ad rectos secare superficiem ipsius speculi, ipsarum communes sectiones cum superficie speculi sphęrici, semper erunt circunferentiæ magnorum circulorum illius sphæræ, cuius portio est speculum propositum, vt etiam Vitellio affirmat in prima sexti libri. Vnde vnusquisque cathetus incidentiæ transibit per centrum speculi, cum ipse sit communis sectio duarum superficierum reflexionis, quare in ipso catheto erit punctum intersectionis ipsorum axium visualium ex necessitate, vt videbimus, si vnam tantummodo imaginē obiecti nobis videremur videre.

Exempli gratia, sint duæ superficies reflexionis speculi sphærici concaui.b.n.c.a. et.b.t.c.u. obiectumq; sit.b. oculi autem sint.a.u. punctum verò superficiei speculi, à quo obiectum emittit reflexionem suę imaginis ad oculum.a. sit.n. pūctum autem à quo eandem reflectit oculo.u. sit t. communis autem sectio harum duarum superficierum sit.b.c. sed.x. centrū sit speculi, radius verò incidentię superficiei.b.n.c. erit.b.n. cuius reflexus sit.n.a. radij autem alterius superficiei erunt b.t.et.t.u. Imaginemur nunc duos semidiametros.x.n.et.x.t. quæ angulos.b.n.a.et.b.t.u. per æqualia diuidant ex supposito.




Nunc ijs suppositis, si vnam tantummodo obiecti imaginem videbimus,

Vu clarum

clarum erit ex rationibus ſupradictis nos ipſam videre in cõmuni concurſu ipſorum axium viſualium, qui axes cum reperiantur vnà cum ipſis radijs reflexis.n.a. et. t. u. ex neceſſitate ſeinuicem ſecabũt in catheto.b.c.cum extendantur in ipſis ſuperficiebus reflexionum, quæ ſuperficies nihil aliud commune inuicem habent,quam cathetum dictum.b.c.ſit igitur in puncto.d.

Ex his dictis alia oritur neceſſitas, hoc eſt, quod quotieſcunque vnam tantummodo imaginem obiecti.b.videmus, dato quod duæ ſuperficies reflexionis ſint, & non vna tantum, tunc angulos.n.et.t.ſemper inuicem æquales eſſe oportebit. Vnde arcus.n.c.et.t.c.ex neceſſitate inuicem æquales erunt.

Scimus enim ex. 3. ſexti Euclid. quod eadem proportio erit ipſius. b. n. ad.n. d. quę ipſius. b. x. ad. x. d. & ipſius. b. t. ad. t. d. ſimiliter, quare ipſius b. n. ad. n. d. erit vt ipſius. b. t. ad. t. d. Vnde ſequitur. b. n. æqualem eſſe ipſi. b. t. et. n. d. ipſi.t.d.vt à medio circulo.E. potes videre, quamuis etiam. b. non eſſet extremum diametri, ſed vbicunque volueris in ipſo diametro,vel etiã protracta, eo quod punctum.n.& punctum.t.in eodem ſemicirculo,vel in æqualibus ſemicirculis,non poſſet aliter in ipſa circunferentia locari,eãdem ſeruando proportionem.b.n.ad.n.d.vt. b. t.ad.t.d.propterea quod in omni alio ſitu exiſtente puncto.t.ipſa.b.t.eſſet aut maior aut minor ipſa.b.n.et.t.d.aut minor, aut maior ipſa.t.d.ex.7.& 14.tertij Eucli.vnde aut maior,aut minor proportio eſſet ipſius.b.t.ad.t.d.quam ipſius.b.n.ad.n.d.& non eadem.

Nunc è conuerſo ſi.b.n.et.b.t.ſunt ſibi inuicem æquales, & ſic.n.d.cum.t.d.ſequitur ex.8.primi Eucli.angulos.n.et.t. inuicem æquales eſſe.

Ab ijſdem ſpeculationibus potes etiam videre vnde accidat quod partes ſuperiores alicuius obiecti reflexæ à tali ſpeculo concauo videntur nobis inferiores eſſe, & inferiores appareant ſuperiores, & dextræ ſiniſtræ, & ſiniſtræ dextræ. quod autem hucuſque demonſtraui de ſpeculis planis,& ſphæricis concauis,ratiocinare tu ijſdem medijs circa ſphærica conuexa,vbi clarè videbis puncta huiuſmodi ſpeculi conuexi, à quibus reflectitur imago obiecti ad ambos oculos, ſemper oportere æquidiſtantia eſſe à pũcto communi ipſius ſuperficiei ſpeculi,& catheto incidentiæ, dum unam tantummodo imaginem ipſius obiecti videmus,& à diuerſis ſuperficiebus reflexionum.

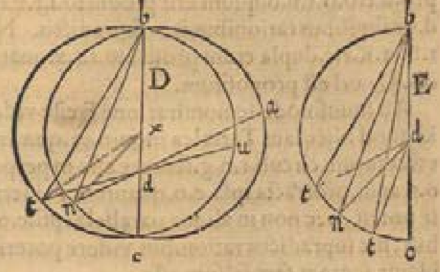
Nolo etiam prætermittere, quod nunc mihi ſuccurrit, hoc eſt quod poſſet aliquis duos ſitus inuenire, vnum pro oculo,alterum verò pro obiecto,reſpectu alicuius ſpeculi concaui, ſphęroidis prolatæ, vt reflexio ipſius obiecti videretur, vt linea diuidens per æqualia ipſum ſpeculum. Reſpectu verò alicuius ſpeculi concaui ſphæroidis oblongæ, vt reflexio obiecti ad oculum veniret à tota ſuperficie ipſius ſpeculi, vnde tota ſuperficies ipſius ſpeculi videretur colorata illo colore cuius eſſet obiectum, quæ quidem paſſiones pendẽt à.48.tertij lib.ipſius Pergei,vt ex te ipſo facile videre potes, propter æqualitatem angulorum reflexionis, & incidentiæ.

Opinio autem mea, quam ſcire cupis de imagine obiecti reflexa,quam putas eſſe in ſuperficie ſpeculi,hæc eſt, quod nec in ſuperficie,nec ultra, nec citra eam eſt ipſa imago, quod autem vltra non ſit, hoc puto nulli dubium eſſe. eadem etiam ratione non erit citra ſuperficiem ſpeculi concaui, quamuis ipſam nos compræhendamus in concurſu radiorum viſualium,tam ab vno ſpeculo quam ab alio reflexione facta. Sed quòd ipſa neque ſit in ipſa ſpeculi ſuperficie, manifeſtum erit ex hoc, ꝙ duo ſpectantes in eodem ſpeculo,duas diuerſas imagines vident,tres, aũt tres, quatuor, quatuor, & ſic deinceps, vnde tot eſſent imagines ſupra ſuperficiem ſpeculi, quot obiecta, ꝙ tamen ita non eſt, nec plus eſt in vno loco ipſa imago,quam in alio,

nisi

nisi in obiecto ipso, lumen enim ab ipso obiecto reflexum, seipsum diffundit vndique,& radij ipsius luminis reflexi, vt plurimum se inuicem secant. Vnde in ipso aere sunt omnes misti. Quapropter natura sagacissima pupillam oculi animalibus tam paruam construxit ad superficiem tam amplæ sphæræ ipsius oculi, vt distinctæ viderentur omnia obiecta.

Nolo etiam tibi tacere, quod quotiescunq; oculorum pupillæ positæ fuerint inter cathetum incidentiæ, & superficiem speculi sphærici concaui. vt puta in lineis.d.t.et.t.n.in figura.D. tunc nullo pacto possemus videre vnam imaginem obiecti, sed duas nec non confusè, propterea ꝙ nullo pacto radij.t.d. et.n.t.reflexi poterint ambo vniri cũ ambobus axibus visualibus, eo quod axes visuales nunquam possunt inuicem intersecari post visum, sed semper ante ipsum, vnde nec inuicem paralleli possunt esse.

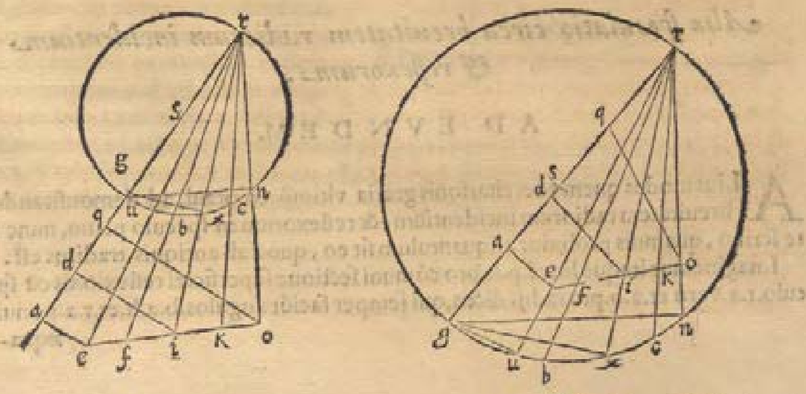



Dico etiam, quod si obiectum inciderit in eadem superficie, in qua duo axes visuales, vel radij reflexi reperiũtur, hoc est in vna eademq; superficie reflexionis, tunc locus imaginis non erit in catheto incidentiæ, eo quod intersectio axium uisualium non erit in ipso catheto sed extra, in qua intersectione fit visio vnius tantummodo imaginis, quod antiqui non animaduerterunt. Hoc autem dico de speculo sphærico concauo.

## *Speculatio cuiusdam propositionis arithmeticæ.*

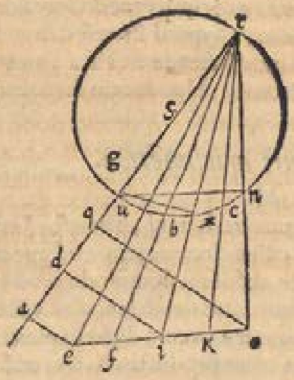
### AD EVNDEM.

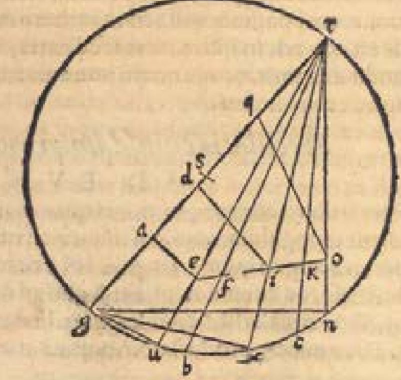
SPeculatio vltimæ propositionis quam numerorum via inueni, hæc est. Imaginemur triangulum.r.e.o.abscisum à circulo, in cuius circunferentia sit punctum r. superioris anguli ipsius trianguli, vel etiam non sit abscisum dummodo protrahantur lineæ vsq; ad circunferentiam, à quo ad oppositum latus descẽdant duæ.r.K.et.r.f.ita ꝙ.K. o. æqualis sit.f.e.vnde hæc.4.lineæ secabuntur à circulo dicto in punctis.n.c.b.u. Dico nunc ꝙ producta.o.r.n.et.e.r.u.æqualia erunt productis. K. r. c. et.f.r.b.



quapropter cogitemus.r.a.indeterminatam transire per centrum.s. ipsius circuli, similiter etiam.r.i.ad punctum medium lateris.c.o.deinde à tribus punctis,e.i.o. imaginemur tres perpendiculares ad.r.a.hoc est.e.a:i.d.et.o. q. & vbi circulus secat. r. a. sit punctum.g.protractis deinde.g.n:g.x:et.g.u.habebimus triangulum. a. e. r. similem triangulo.g.u.r.vnde clarum erit productum.g.r.a.æquale esse producto. e. r. u. productumq;.g.r.q.æquale esse producto.o.r.n.nam trianguli.g.r.n.et.o.r.q.sunt inuicem similes,sed productum.g.r.a.simul cum producto.g.r.q. duplum est producto. g.r.d.ex prima sexti, eo quod.a.r.q.dupla est.d.r.& ideo productum.e.r.u. simul cũ producto.o.r.n.duplum erit producto.i.r.x.quod quidem æquale est producto. g. r. d.ex similibus rationibus iam supradictis. Nunc ex similibus rationibus producta.f. r.b.et.K.r.c.dupla erunt producto.i.r.x.quare prima producta æqualia erunt secundis. Quod est propositum.

Ab huiusmodi demonstratione facilè videre poteris non esse generaliter verum, id quod Nicolaus Tartalea inquit.43.quæsito vltimæ partis suorum tractatuum,hoc est centrum circuli.r.n.g.semper esse in perpendiculari,quæ à puncto.r.ad lineam. e. o.transit,protracta ipsa.e.o.quantum volueris,imò in quacunque alia linea ipsum esse potest, nec non in aliqua parallela ipsi.e.o.quemadmodum ex te ipso, mediantibus, hic supradictis rationibus videre poteris,vnde ex necessitate sequitur illud problema semper ferè falsum esse.




## *Alia speculatio circa breuitatem radiorum incidentium & reflexorum.*

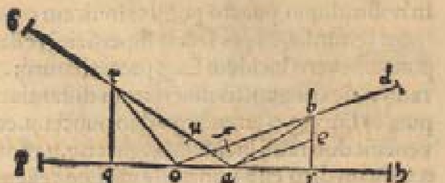
## AD EVNDEM.

Alius modus quem exercitationis gratia vltimò cogitaui, ad demonstrandum breuitatem radiorum incidentium, & reflexorum in speculo plano, nunc ad te scribo, quamuis prolixior aliquantulum sit eo, quod ab antiquis traditus est.

Imaginemur itaque lineã.p.h.pro cõmuni sectione superficiei reflexionis cũ speculo.r.a.verò et.a.b.pro radijs dictis,qui semper faciũt angulos.b.a.h.et.r.a.p.inuicẽ

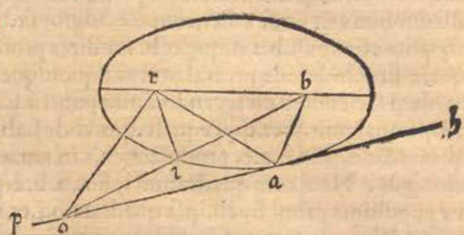
æqua-

æquales. Nunc protrahantur duæ.r.o.et.b.o.ab iisdem punctis.b.r.ad aliud punctum, quod volueris ipsius lineæ.p.h.quas probabo longiores (simul sumptas) esse prioribus. Imaginemur igitur duas perpendiculares, seu cathetos.b.i.et.q.r.à punctis.b.r.ad.p.h.abscissaq; sit linea.o.b.in puncto.x. ita quod. b.x.æqualis sit ipsi.b.a. quod nulli dubium erit posse effici, cum.o.b.lōgior sit.b.a.eo quod opponatur angulo obtuso ipsius trianguli.b.a.o.quę.o.b. similiter protrahatur vsque ad.d. ita quod. b. d. æqualis sit.x.b.deinde protrahatur.o.i.quousque.i.h.æqualis sit.a.i. In alia parte postea idem faciendum est secando.a.r.in puncto.u.ita quod.u.r.æqualis sit.r.o.efficiendo.r.s.æqualem.r.u.et.q.p.æquale.q.o.vnde habebimus productū.o.d.in.o.x. æquale producto.o.h.in.o.a.& productum.a.s.in.a.u.æquale producto.a.p.in.a. o. ex istis rationibus. Nam cum quadratum ipsius.o. b.æquale sit duobus quadratis.o.i.et. i. b.ex penultima primi Eucli.ipsa quadrata.o.i.et.i.b.æqualia erunt producto.o. d.in o.x.simul sumpto cum quadrato.b.x.ex.6. secundi, hoc est ipsi producto simul sumpto cum quadrato.b.a.hoc est ipsi producto simul sumpto cum duobus quadratis.a. i.et.i.b.sed quia productum.o.h.in.o.a.simul sumpto cum quadrato.a.i.ęquatur quadrato.o i.ideo productum.o.h.in.o.a.simul sumptum cum quadrato.a.i.& cum quadrato.i.b.æquale erit producto.o.d.in.o. x. simul sumpto cū duobus quadratis dictis hoc est ipsius.a.i.et.i.b.quę quadrata dempta cum fuerint ab vtraque parte, tunc certi erimus producta esse inuicem æqualia. Idem dico de alijs ex altera parte. Nunc imaginemur protractam esse.a.e.parallelam ipsi. o. b. & habebimus proportionem ipsius.a.b.ad.a.i.maiorem esse ea quæ est ipsius.a.e.ad eandem.a.i. cum. a. b. maior sit ipsa.a.e.vt opposita angulo obtuso, quapropter proportio.x.b.ad. a. i. maior erit ea quæ est.o.b.ad.o.i. Iam enim scis proportionem.o.b.ad.o.i.esse, vt.a.e.ad.a.i. ex similitudine triangulorum. quare proportio.b.d.ad.i.h.maior erit proportione.o.b. ad.o.i.tūc ex.27.quinti ꝑmutādo ꝓportio. b.d.ad.b.o.maior erit proportione.i.h. ad.i.o.& ex.26.eiusdē cōponēdo maior ꝓportio erit.o.d.ad.o.b.ea quę est.o.h.ad.o i. & permutādo maior ipsius.o.d.ad.o.h.ea quæ.o.b.ad.o. i.& ex.33.maior ipsius.b. d.ad.i.h.ea quæ.o.d.ad.o.h. Sed vt.b.a.ad.a.i.ita est.a.r.ad.a.q.ex similitudine triā gulorum. Erit igitur.a.r.ad.a.q.maior proportio, ea quæ est.o.b.ad.o.i. & ex ijsdem supradictis rationibus maior erit proportio ipsius.s.a.ad.p.a.ea quæ est. a. r. ad. a. q. sed cum iam probatum fuit proportionem.b.d.ad.i.h.hoc est.a.b.ad.a.i. maiorem esse. o.d.ad. o. h. ergo eo magis maior erit proportio ipsius.a. s.ad a.p.ea quæ.o.d.ad.o.h.sed cum ex.15 sexti, eadem sit proportio.o.d.ad.o.a. quæ.o.h.ad.o.x.et.s.a.ad.o.a. quę a.p. ad. a. u. tunc erit permutādo eadem proportio ipsius.o.d. ad.o.h.quæ.o.a. ad.o.x.& ipsius.a.o. ad. a. u. quemadmodum ipsius.a.s.ad.a.p. Quare maior proportio erit ipsius.a.o.ad.a.u.quam. a. o. ad.o.x. Vnde sequitur.o.x.maiorem esse.a.u.ex.8.quinti, ergo.b.x.o.r. longior erit ipsa.b.a.u.r. Quod est propositum.



Alia

Alia etiam via possumus idem concludere. Imaginemur maiorem axem alicuius ellipsis transire per duo puncta.r.et.b.supponendo ipsa puncta, ea esse, quæ ita axem diuidunt, vt singula producta sectionum sint, vt inquit Pergeus.imaginemur,etiam.p.h.contiguam esse ipsi ellipsi in pũcto. a. vnde si protractæ fuerint duæ.r.a. et.b.a.habebimus ex.48. tertij ipsius Pergei angulos.b.a.h. et. r. a. p. inuicem æquales. Ducendo postea ad quoduis punctum ipsius p.h.duas.b.o.et.r.o. certi erimus, quod secabuntur à gyro oxygonio, quarum vna secta sit in puncto.i.ducta postea.i.r.clarum erit ex.52.dicti, quod longitudo.b.i.r.æqualis erit longitudini.b.a.r.& minor ipsa.b.o.r.ex.21.primi Euclid.




## *De errore Euclidis circa speculum vstorium.*

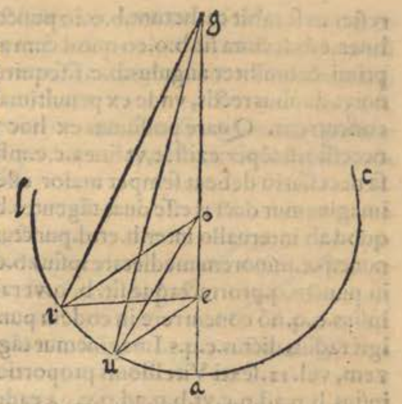
## AD EVNDEM.

Verum speculum vstorium, illud non est, quod ab Euclide traditum fuit,& q̃ tu etiam putas, Nam Euclides errat, cum credat radios reflexos à superficie sphærica concaua seinuicem in centro speculi intersecare. Nam cum omnes lineę rectę à centro, & circunferentia alicuius sphæræ terminatæ, sint eidem circunferentiæ perpendiculares, sequeretur ex necessitate radios incidentiæ etiam perpendiculares eidem superficiei esse, cum anguli incidentiæ semper æquales sint angulis reflexionis, vnde etiam ex necessitate sequeretur punctum corporis lucidi, à quo radij luminosi exeunt, in centro speculi reperiri. quod quidem falsissimum est.

Alia etiam via possum hanc ostendere impossibilitatem, & tibi probabo, quod in nullo aliquo puncto possunt inuicem conuenire ipsi radij reflexi omnes.

Sit igitur.l.a.c.cõis sectio superficiei reflexionis cum speculo,cuius centrum sit.o. punctum verò lucidum sit.g.protrahaturq́;. g. o. a.Nunc autem primum dico, quod radij reflexi à punctis diuersarum distantiarũ ab.a.non coincidẽt inuicem in aliquo puncto lineę.g.o a:sint ergo duo puncta.u.et.r.diuersarum distantiarũ ab.a. à quibus veniant duo radij incidentiæ.g.r.et.g.u.radius verò reflexus ab.r. sit.r.e.protrahatur u.e.quam dico esse non posse radium reflexum ab. u. quotiescunque eius incidens descendat ab.g. Protrahantur ergo duæ lineæ.o.r.et.o.u. vnde cum dixerit aliquis u.e.reflexũ esse ipsius. g.u.igitur anguli.g.u.o.et.o.u.e. erunt inuicem æquales, & sic etiam erunt duo.g.r.o.et.o.r.e.vnde ex tertia sexti &.11.quinti Eucli.proportio.g. u.ad.u.e.æqualis esset ei, quæ.g.r.ad.r.e.quod quidem impossibile esse demonstrabo, eo quod cum.g.u.maior sit.g.r.ex.8.tertij,erit ex.8.quinti proportio ipsius.g.u. ad.r.e.maior proportione ipsius.g.r.ad.r.e.sed ex.7.tertij.u.e.minor est.r.e. erit igitur ex dicta.8.quinti maior proportio ipsi⁹.g.u.ad.u.e.quam.g.u.ad.r.e. vnde eo ma

gis




gis erit maior proportio ipſius.g.u.ad.u. c. quam ipſius. g. r. ad.r.c. ergo non æqualis, quapropter impoſſibile eſt.u.e. eſſe radium reflexum incidentis radij.g.u. Vnde ſequitur concurſum radiorum reflexorum à ſpeculo ſphærico concauo non eſſe in vno, & eodem puncto ipſius catheti incidentiæ, quando à ſitu non æquidiſtanti ab ipſo catheto reflectũtur, ex hac ſpeculatione etiã videre licet, verum eſſe id quod in.3. Epiſtola tibi ſcripſi nempe, quod quotieſcunque axes viſuales, vel radij reflexi, in vna eademq; ſuperficie reflexionis fuerint, tunc imago obiecti nullo modo videbitur in catheto incidentiæ, in ſpeculo ſphærico concauo.

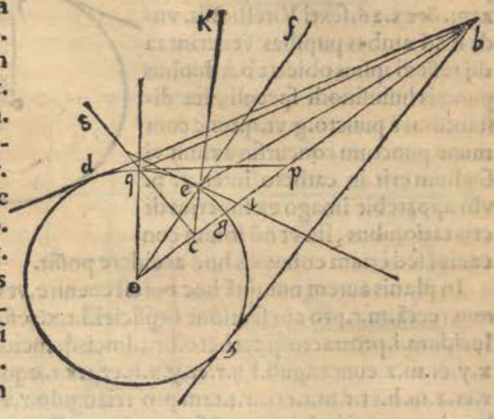
## *Alterius dubitationis ſolutio.*

## AD EVNDEM.

NOn abſque ratione dubitas, vtrum etiam in ſphæricis ſpeculis conuexis idem accidat, hoc eſt, an radij reflexi à punctis inęqualis diſtantiæ à catheto incidentiæ conueniant inuicem in eodem catheto.

Ad quod reſpondeo, non concurrere in dicto catheto, ſed extra ipſum, & ſimiliter extra ipſum videbitur imago.

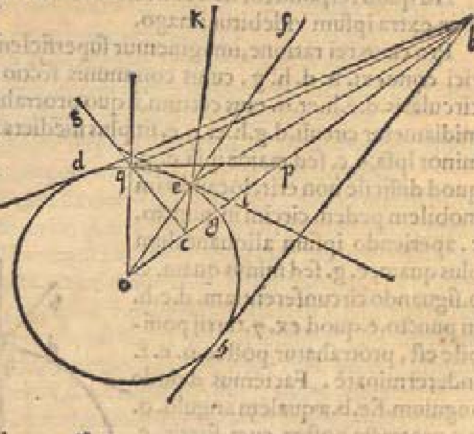
Pro cuius rei ratione, imaginemur ſuperficiem reflexionis alicuius ſpeculi ſphærici conuexi. b. d. h. g. cuius communis ſectio cum ſuperficie ſphærica ſit linea circularis.d.e.h.et.o. eius cẽtrum, à quo protrahatur.g.b. indeterminata, et.o.g. ſit ſemidiameter circuli.d.g.h.et.o.c. ſit plus medietate ipſius.o.g. accipiaturq; linea.e.c. minor ipſa.o.c. ſed maior ipſa.c. g. quod difficile non erit, locando im mobilem pedem circini in puncto. c. aperiendo ipſum aliquantulum plus quam. c. g. ſed minus quam. c. o. ſignando circunferentiam. d.e.h. in puncto.e. quod ex. 7. tertij poſſibile eſt, protrahatur poſtea.o. e. f. indeterminatè. Faciemus deinde angulum.f.e.b. æqualem angulo. o. e.c. protracta poſtea cum fuerit. c. e.K. indeterminatè, habebim' duos angulos.b.e.f.et.f.e. K. æquales inuicem mediante. 15. primi, ita qđ ſi radius incidens veniet à puncto. b. ad.e. reflexus erit.e. K. qui quidem




reflexus

reflexus ſecabit cathetum.b.o.in puncto.c.intra ſpeculum, nec dubitandum eſt quin linea.c.b.ſectura ſit.b.o.eo quod cum angulus.o.e.c.ſit maior angulo.e. o. c. ex. 19. primi, & ſimiliter angulus.b.e.f.ſequitur ex.13.dicti,angulos.b.e.o.et.e.o.b. eſſe mi nores duobus rectis, vnde ex penultima petitione primi,duæ lineæ. b.e.et.o.b.inuicẽ concurrent. Quare poſſumus ex hoc , quoddam corollarium extrahere , hoc eſt neceſſariũ sẽper exiſtat,vt linea.c.e.minor eſſe linea.c.o. Sed vnde eueniat quod ip ſa neceſſario debeat ſemper maior eſſe ipſa.c.g.clarum eſt ex.7.tertij Eucli. Nunc imaginemur ductas eſſe duas tãgentes.b.d.et.b.h.& ab.e.ipſã.e.i.vnde certi erimus, quod ab interuallo inter.h.et.d.punctum.b. poſſibile ſit vt reflectatur. Accipiamus nunc.p.c.minorem medietate ipſius.b.c.& à puncto.p.imaginemur tangentem.p. q. in puncto.q.protractaque ſit. b. q.vt radius incidentiæ, tunc dico, radium reflexum ipſius.b.q.nõ concurrere in eodem puncto.c.ipſius catheti, ſi vero dixeris q ſic.Eſto igiť radius dictus.c.q.s.Imaginemur tãgentẽ.e.i.in puncto.e.vnde ex.18.quinti Alha zem, vel.12.ſexti Vitellionis proportio.b.i.ad.i.c.erit,vt.b.o.ad.o.c.& ſimiliter erit ipſius.b.p.ad.p.c.vt.b.o.ad.o.c. ex eadem. Quare ex.11.quinti Eucli.proportio ip ſius. b. p. ad. p. c.erit vt ipſius.b.i.ad.i.c.ſed quia.p.b.vt pars ipſius.b. i. minor eſt ip- ſa,ergo ex.14.dicti.p.c.minor erit ipſa. c. i. hoc eſt totum minus ſua parte, quod eſt impoſſibile, quare non in ipſo catheto videbitur imago ipſius obiecti.

Aliud notandum etiam cernere potes ex ipſis ſpeculis ſphæricis conuexis,hoc eſt quod poſſibile ſit aliquoties , radium reflexum concurrere cum catheto incidentiæ extra ſpeculum inter puncta. g. et. p. vt exempli gratia. ſi punctus. p. eſſet exactè in medio inter.b.et.g.tunc punctum.c.ipſius concurſus cum catheto incidentiæ eſſet inter.g.et.p.eo quod cũ linea.p.q.debeat diuidere angulũ.b.q.c.p equalia,oportebit c.poſitum eſſe inter.g.et.p.quia angulus.g.q.p.maior eſt angulo.p.q.b.vt per te faci le potes ratiotinari,imaginando cir culum circa triãgulum.g.q.b.& dia metrum perpendicularem. ad. g. b. in puncto.p.producendo poſtea. q. p.vſq; ad alterã partẽ circunferentiæ ipſius circuli. argumẽtãdo deinde mediante vltima ſexti,illud idẽ potes etiam ſcire ex.22.quinti Alha zeni. & ex.26.ſexti Vitellionis. vnde ſi ad ambas pupillas venerint ra dij reflexi ipſius obiecti.b.à duobus punctis huiuſmodi ſpeculi , ita diſtantibus à puncto.g.vt.q.tunc com mune punctum concurſus axium vi ſualium erit in catheto inter. g. p. vbi apparebit imago ex ſuperius di ctis rationibus , ita vt nõ ſolum con cauis , ſed etiam conuexis hoc accidere poſſit.

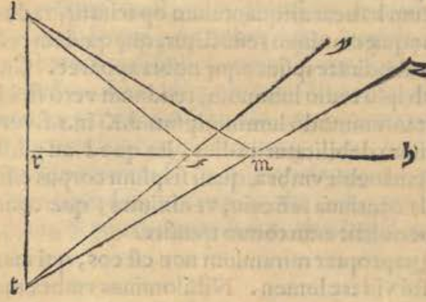



In planis autem nunquã hoc poteſt euenire,vt tibi alias dixi, eo quod ſi accéperimus rectã.m.r.pro cõi ſectione ſupficiei.l.t.x.reflexionis & ſupficiei ſpeculi,pũctũq; lucidum.l.protractoq; catheto.l.r.t.lineiſq; incidentiæ.l.x.et.l.m. reflexionis etiam x.y.et.m.z.cum anguli.l.x.r.et.y.x.h.et.r.x.t.æquales inuicem ſint ,& ſic anguli.l. m. r.et.z.m.h.et.r.m.t.erit.r.t.tam pro triangulo.r.x.t.quam pro triangulo.r.m.t. æqua lis.r.l.ex.26.primi, ita quod ſemper in puncto.t.conueniẽt omnes radij reflexi ipſius

puncti

puncti.l.clarum igitur nunc habes, quod in ſphærico concauo, ſeu conuexo,non omnes radij reflexi conueniunt in vno, eodemq́; puncto catheti incidentiæ, quemadmodum in planis accidit, in quibus ſemper vnum, & idem punctum eſt ipſis commune in ipſo incidentiæ catheto.

Non prætermittam etiam hunc alium breuiorem modum ſpeculandi æqualitatē depreſſionis imaginis ſub ſpeculo plano, ei quæ ſupra reperitur ipſius obiecti,in catheto incidentiæ, quemadmodum nunc vltimò diximus,hoc eſt quod cum imago obiecti.l. reflexa à puncto. x.reperiatur in linea.y.x.t. & imago eiuſdem obiecti reflexa à puncto.m.reperiatur in linea.z.m.t. & iſtæ duæ lineę ſeinuicem ſecent in puncto.t. ipſius catheti, exiſtente. r.t.æquali.r.l.vt nunc vidimus, ergo ſemper imago reflexa à ſpeculo plano, nobis apparebit ī ipſo catheto,tam vltra ſpeculum,quam citra ipſum,reptū fuerit ipsū obiectū quod nec Alhazem, nec Vitellio,



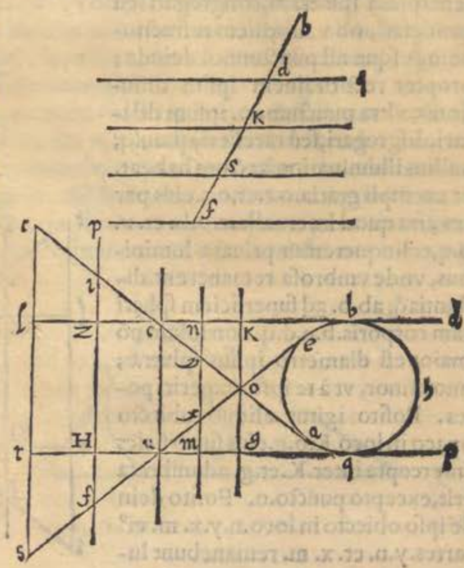

nec alius aliquis (quod ſciam) adhuc ſcientificè demonſtrauit. exempla enim vel experientia non faciunt ſcire. Credo etiam te non dubitare quin duæ lineæ.y.x. et. z. m.inuicem concurrant,cum anguli.t.x. m. et. t. m. x. minores ſint duobus rectis cum æquales ſint angulis.l.x.m.et.l.m.x.

## *De rotunditate vmbræ terræ in ecclipſibus Lunaribus.*

### AD EVNDEM.

ROtunditas vmbræ in ecclipſibus lunaribus oritur tā à rotunditate maris, quā terræ,& ſi terra eſſet etiā cuiuſuis alterius figurę, quā ſphæricę,dummodo aqua impleret locū ſphęriceitatis à terra derelictū, nihilominus vmbra eſſet rotunda, quę quidem ab aqua produceretur, quāuis Alexander Piccolhomineus aliter ſentiat in libro de magnitudine terrę,& aquæ.

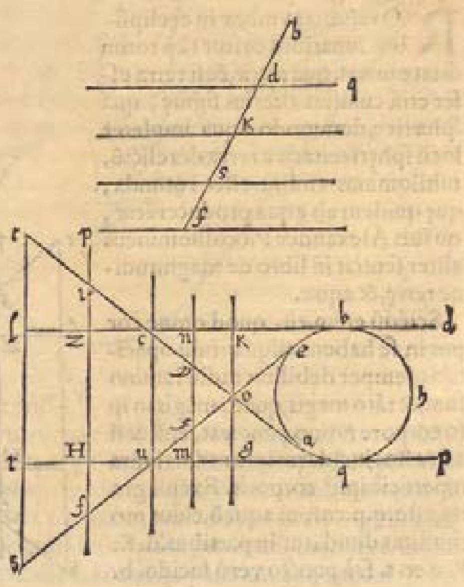
Sciédū enim eſt, quod omne corpus in ſe habens aliquantulū opacitatis,ſemper debilitat radiū lumino ſum,& tāto magis,quāto magis in ipſo corpore radius penetrat, etiā & ſi ad rectos incideret ipſe radius ſupra ſuperficiē ipſi⁹ corporis. Exēpli gratia,eſto.q.p.corpus aqueū,cuius profunditas diuidatur in partibus.d. K: K.s: et. s. f. à puncto verò lucido. b.




Xx deſcen-

descendat radius.b.d.K.s.f. ad libitũ hoc est rectè vel obliquè, cuius pars.b.d.in ipso aere existat. Nunc manifestum erit partem.b.d.ipsius radij clariorem seu minus im peditã esse quam.d.K.quod ex eo etiam cognoscere possumus quia.b.d.reflectitur à puncto.d.superficiei corporis aquei, quapropter minus luminosa remanebit pars. d. K.cum non tota claritas.b.d.descendat in corpore aqueo, sed vna eius pars reflectatur,reliqua verò tantummodò descendat, deinde pars. K. s. ex necessitate debilior erit ipsa.d.K.eo quod succedit post ipsam. d. K. propter hoc etiam,quia cum corpus aqueum habeat aliquantulum opacitatis, radius.d.K.ab omni puncto ipsius spissitudinis aquæ continuo reflectitur, quę quidem reflexio est illud lumen cęruleum,quod in profunditate ipsius aquę nobis apparet. Cum igitur reflexio ipsa semper detrahat ab ipso radio luminoso, residuum verò sit id quod penetrat, ideo. K.s. erit vna pars tantummodò luminis ipsius.d.K: in.s.f.verò aliqua pars luminis ipsius.K.s.& sic continuò debilitatur radius, ita quod ad nihilum vsque deuenit, & vltra tale corpus remanebit vmbra, quasi si ipsum corpus esset perfectè opacum, cuius rei causa, est illa continua reflexio, vt diximus, quæ continuò adimit aliquid ex ipso radio, nec permittit eum totum transire.

Quapropter mirandum non est eos, qui margaritas quærunt in fundo maris nullum ibi videre lumen. Nihilominus vmbra maris, quam dico nos posse videre in superficie corporis lunaris,ab alia etiam ratione prouenire posset. Imaginemur enim aggregatum terrę,marisq; esse tantummodò aqueum, quod quidem esset perfectè sphæricum ratione centri grauitatis, supponamusq; ipsũ esse valde diaphanum, ita quod radij solares ipsum penetrassent. Tunc dico quod in superficie corporis lunaris produceret vmbram. Pro cuius intelligentia cogitemus subscriptam hic figuram b.h.q.a.e.esse sphęram aliquam crystallinam,& ad partem.b.h.q. sit radius luminosus solaris qui ipsam illuminet,cuius radij extremitates sint.d.b.l.et.p.q.r.supponendo.d.l.et.p.r.terminos esse vnius plani secantis ipsum radium per axem, tunc videbis ipsum radium.b.p.q. d. transeũtem ipsam sphæram,congregari seu condẽsari, ob vniformem refractionem, vsque ad punctum.o. deinde; propter rectitudinem ipsius diffusionis, vltra punctum. o. ipsum dilatari,disgregari,seu rarefieri,quousq; nullius illuminationis actum habeat. vt exempli gratia.o.t.et.o.s.eius partes, ita quod interualla.c.o.b. et. u. o.q.relinquerentur priuata luminibus, vnde vmbrosa remanerent.distantiaq; ab.o. ad superficiem sphęricam corporis.b.e.d.q. non solum nõ maior est diametro ipsius sphæræ; imo minor, vt à te ipso experiri potes. Posito igitur aliquo obiecto opaco in loco.K.o.g.eius superficies intercepta inter.K.et.g. adumbrata erit,excepto puncto.o. Posito deinde ipso obiecto in loco.n.y.x. m. ei⁹ partes.y.n. et. x. m. remanebunt lu-



mine

mine destitutæ interuallumq; tantummodò inter. y. x. illuminatum erit, sed si in loco. c.u. positum fuerit, tunc totum.c.u. illuminatum erit, sed debili modo propter detractionem factam à reflexione in superficie corporis sphærici, vt supra diximus.

Posito deinde obiecto in loco.i.z.H.f. tunc partes.z.i. et. H. f. rectos Solis radios habebunt cum aliquibus refractis, sed. z. H. paucissimum habebit lumen, propter disgregationem radiorum. Posito postea ipso obiecto in loco. t.l.r.s. tanto minus lumen habebit pars.l. r. propter dictam disgregationẽ, seu dissipationẽ radiorum, & sic successiuè quanto remotius positum fuerit ipsum obiectum, tanto minus illuminabitur. vnde ita remotum poterit locari, ut nullus actus luminis in eo videatur, de radijs scilicet, qui per sphæram chrystallinam transibunt, sed videbitur vmbra ipsius sphęrę in obiecto proposito, cum nullum actum illuminationis in eo loco obiecti habeant radij transeuntes per dictam sphęram. quapropter partes. t. l.et. r. s. illuminatæ erunt à Sole, et.l.r. omnino lumine destitura.

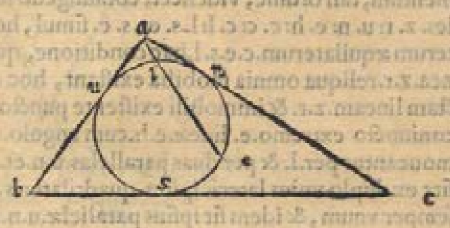
Quòd vero tolerabilior sit oculis radius reflexus Solis à superficie aquæ, quàm à superficie alicuius speculi, oritur ab eo, quod supra diximus, hoc est, quod magna pars ipsius luminis penetrat in aquam, & non totum reflectit, quod quidem non accidit speculis opacis.

## DE LONGITVDINE DVORVM LATERVM cuiusuis trianguli supra tertium.

*Hieronymo Fenarolo.*

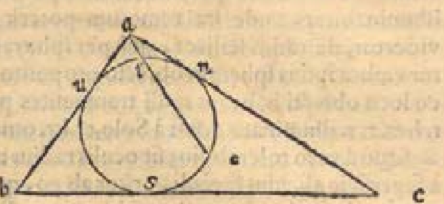
QVÒD quælibet duo latera continentia rectum angulum cuiusuis trianguli orthogonij, longiora sint tertio latere, per diametrum circuli in eo inscripti, ab alijs iam demonstratum fuit. Sed quòd quælibet duo latera cuiusuis trianguli longiora sint tertio per latus tetragonicum, quadrupli producti cuiusuis lineæ descendentis ab angulo contento à dictis duobus lateribus ad oppositam partem circuli inscripti, in partem extrinsecam ipsius lineæ, nullus (quod sciam) vnquam scripsit, vel animaduertit.

Sit exempli gratia triangulus.a. b. c. quem volueris, in quo describatur circulus. u.s.n.& puncta contingentiæ sint eadem. u. s. n. à puncto vero.a. descendat linea. a. i. e. quæ terminetur à circunferentia in puncto.e. ipsius circunferentiæ, vbi volueris. Dico nunc latera.a.b. et.a.c. longiora esse latere.b.c. per latus tetragonicũ quadrupli producti ipsius.a.e.in.a.i. Nam certi sumus ex vltima parte penultimæ tertij Eucli.n.c.et.s.c. æquales inuicem esse, & similiter.b.s.et.b. u. vnde ex communi concepttu dicta latera maiora erunt ipso.b.c. per. a. u. et. a. n. quæ duæ partes sunt inuicem æquales dicta ratione, & quadratum lineæ æqualis aggregato earum, esset quadruplum quadrato cuiusuis earum ex.4. secundi, sed ex penultima tertij, productum.a.e.in.a.i. æquale est quadrato ipsius.a.u. vel ipsius.a.n.



Verum est igitur quod. a.b.cum.a.c. longiores sint ipsa. b. c. per latus tetragonicum quadrupli eius quod fit.ex.a.e.in.a.i.quod fuit propositum.

Illud etiam non est spernendum,quod quotiescunque data fuerint omnia latera alicuius trianguli, illicò possumus cognoscere puncta.u.n. s. contingentiæ circuli inscripti, ope vltimæ partis penultimæ tertij, eo quod ex illa iam scimus, quod detrahendo.b.c.ex aggregato aliorum duorum laterum, remanebit.u.a. et. a. n. quarum vnaquęque nota erit, cum illarum quælibet, medietas sit residui cogniti, detrahendo postea vnam illarū ab altero duorum laterum.a.b. vel. a.c.remanebit.u.b.vel.c.n.ęqualis.b.s.vel.c.s. vnde similiter nobis innotescet punctum.s. cum duobus punctis. u. et. n. à quibus duobus punctis, si duę perpendiculares ad talia latera ductæ fuerint, vbi hæc perpendiculares seinuicem secabunt, ibi centrū circuli inscriptibilis erit in triangulo proposito.



Inter alia, quæ tibi dixi de Iride, quod memoria non tenes, nihil aliud est nisi quod cum Iris videtur, non eodem loco ab omnibus videtur, quia reflexio est, & vt reflexio luminis à speculo non omnibus ab eodem puncto fit, ita etiam tibi dixi de Iride.

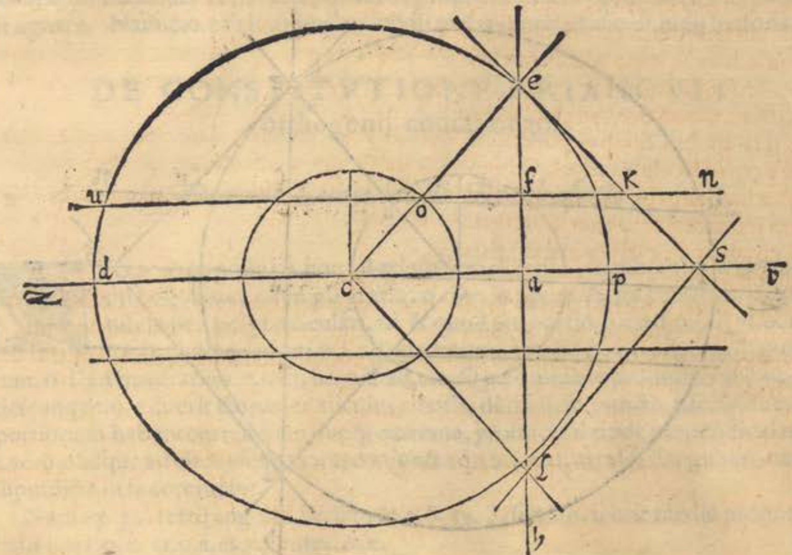
## *De Instrumento oxygonio, seu elliptico.*

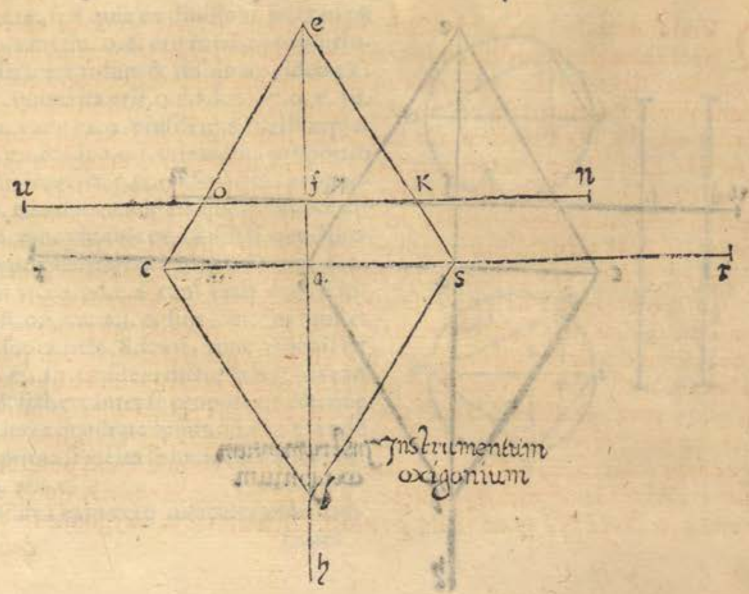
## AD EVNDEM.

Quod aliquando à me audiuisti falsum non est, scilicet possibile esse (vt speculatus sum) particulare instrumentum fabricari ad designandum oxygoniam, seu ellipticam sectionem,quæ à Pergeo defectio appellatur,quod quidem instrumentum valde diuersum est ab alijs, quę aliàs inueni, pro ipsis conicis sectionibus delineandis. Occasionem aūt huiusmodi instrumenti inueniendi mihi præbuit secūda dubij solutio quā feci ann. 1568.grauiss. philosopho Francisco Vimercato, nā cū viderim in ea figura.f. a. semper æqualē esse.o.i. suæ parallelæ scilicet, vnde cum recta linea fuerit protracta per.o.et.f.ipsa foret semper ęquidistās.d.p.ex 33.primi Eucli. Venit mihi in mentem modus construendi hoc subscriptum instrumentum, tali ordine, videlicet, coniungēdo septem hic subnotatas lineas materiales. z. r: u. n: e. h: e. c: c. l: l. s. et. s. e. simul, hoc modo, scilicet fabricādo quadrilaterum æquilaterum.c.e.s.l.hac conditione, quod immobili existente puncto.c.in linea.z.r.reliqua omnia mobilia existant, hoc est quod punctū. s. moueatur per dictam lineam.z.r. & immobili existente puncto.e. vt extremum lineæ. e. h. hoc est coniuncto extremo.e. lineæ.e.h.cum angulo.c.e.s. reliqua puncta lineæ ipsius. e. h. moueantur per.l. & per duas parallelas.u.n.et. z. r. longitudo vero.e. h. sit composita ex duplo vnius lateris ipsius quadrilateris. Oportet deinde quod punctum. f. semper vnum, & idem sit ipsius parallelæ.u.n.moueatur tamen per.e.h. quod quidem punctum illud erit, quod vnam portionē circunferentiæ oxygoniæ sectonis desi-

designabit, puncta verò . o . et . K . vt puncta laterum. c. e. et. s. e. æquædistantia à punctis. c. et. s. eadẽ semper sint, ita tamen vt puncta lineæ.u.n.semper diuersa exi stãt, & quodlibet ipsius quadrilateri latus, æquale sit medietati maioris axis ipsius oxygoniæ sectionis delineandæ, et. c.o. seu.s. K (quod idem est) sit æqualis medie tati axis minoris dictæ sectionis,et.z.r.æqualis duplo.e.h. vnde,quando puncta.e.et. l.coniuncta simul erunt, similiter coniunctæ simul erunt.c.e.et.e. s. cum. c. l. et. l. s.

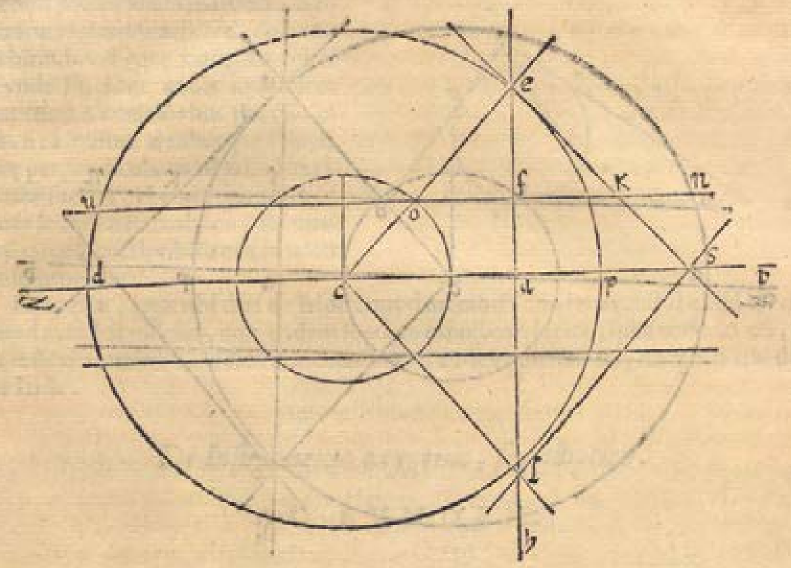
Quapropter

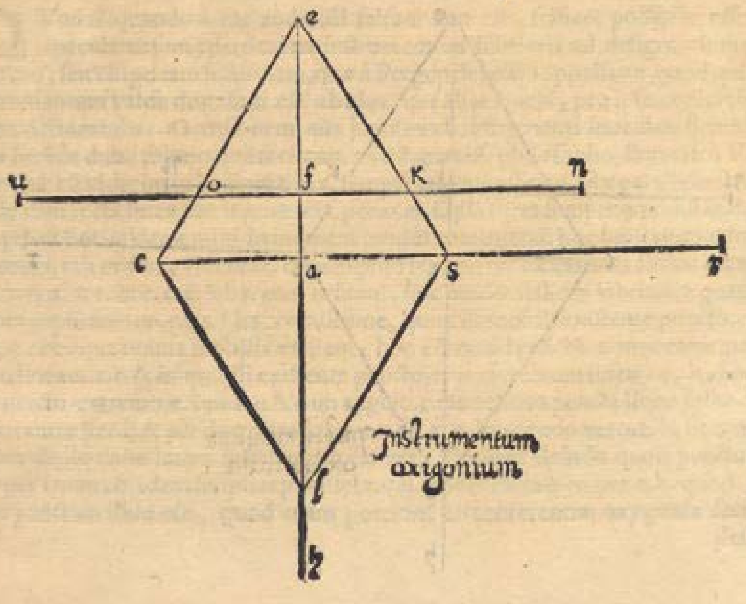


Quapropter puncta.e.l.f.et.p.extremum axis maioris, in eodem met loco erunt, hoc est in aliquo extremorum maioris axis, & cum punctus.s.coniunctus fuerit cum centro.c.punctus.f. parallelę.u.n. in extremo axis minoris erit, & in eodem loco erit cum.o. & cum. K. In extremitatibus verò lineæ. z.r.necesse est, vt sint duo pũcta ferrea, ad firmandum ipsam.z.r.super subiectam lineam significantem maiorem axem propositę sectionis.

Volo



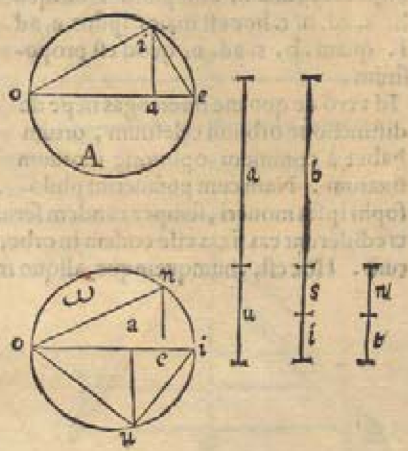
Volo etiam quod ad partem.c. l. s. quadrilateri constituta sit alia parallela ad. z. r. & in æquali distantia ab ipsa quemadmodum.u.n.distat ad eademmet. z. r.ad eandem operationem faciendam. Vnde in vno tantummodo itinere puncti.s.ab. r. vsq; ad.c. designabimus quartam partem sectionis,conuerso postea instrumento,hoc est posito puncto.r. vbi prius erat. z. et. z.vbi erat.r. aliam delineabimus quartam, & sic ad oppositam partem ipsius.z. r. faciendum erit. Hoc instrumentum possumus etiam ita construere,vt puncta.o.et.K.possint collocari in laterihus. c. e.et.e.s.vbi nobis magis libuerit,ita vt licebit in qualibet proportione axiũ proposita, oxygoniam designare. Nam.c.o.erit longitudo dimidij axis minoris,et.c.e.dimidij maioris.

## DE CONSTITVTIONE TRIANGVLI orthogonij conditionati.

### *Domino Ludouico de Rocchaforte.*

QVOD à me postulas,non est admodum difficile, cupis enim triangulum orthogonium, exempli gratia . o. i. e. in figura. A. ita constituere, vt diuisum sit à perpendiculari.a.i. & quod proportio.o.e.ad.o.i.sit vt.o. i. ad i. e.& quod quadrati.o.i. ad quadratum.o.a. sit vt. e.i. ad. e. a. & quadratum. o. i. ad quadratum. e.i. sit. ut. o. a.ad.e.a. Quæ omnia in promptu veniunt,quotiescunque.o. e.fuerit diameter alicuius circuli, diuisaq; in puncto.a.secundum proportionem habentem medium duoq; extrema, protracta deinde perpendiculari. a. i.ad.o.e.usque ad circunferentiam, coniunctæq;. o. i.et.i. e: tale triangulum, omnia supradicta in se continebit.

Nam ex. 30. tertij angulus.i.rectus erit, & ex. 8. sexti.o. i. erit media proportionalis inter.o. e. et.o.a.et.e.i. inter. o. e. et. a. e. sed quia ex diuisione facta in pũcto.a.etiam. o. a. erit media proportionalis inter totum & residuum, ideo ex. 11. quinti ita erit.o.e.ad.e.i.vt.o. e. ad. o.a. vnde ex.9. eiusdem.a.o.erit æqualis. e.i.& ideo.o.i. erit media proportionalis inter. o. e.et.e.i.Sed quia proportio. e.i.ad.a.e.eadẽ est,quę ipsius.o.e.ad o. a. tunc videbis ex.18.sexti,quod proportio quadrati.o.i.ad quadratum.o. a. erit vt.e.i. ad.e.a. cum vero duo trianguli.o.i.a.et.a.i. e. sint inuicem similes ex supradicta. 8.sexti, tunc videbis ex 18..et. 17.eiusdem dictos triãgulos eandem habere inter se proportionem, quę est inter quadrata ipsius.o.i.et.i.e. vnde ex prima sexti ita se inuicem habebunt. a.o. et.a.e.



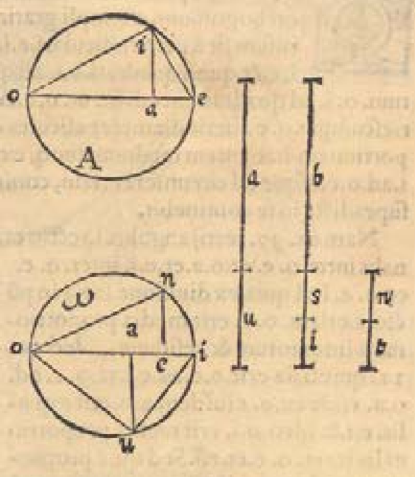

Circa eam verò difficultatem quam

habes

habes in circulo. ω. vbi fateris te non videre qua ratione eadem proportio sit quadrati.u.o. ad quadratum.o.n.vt lineæ.o.a.ad lineam.o.e. partes diametri. o. i. ipsius circuli, terminatæ à perpendicularibus.u.a.et.n.e.

Hoc necessario contingit, propterea quod cum fuerint protractæ.u.i. et.n.i.tūc habebimus ad partem.o.u.i.triangulum.o.u.i.diuisum in duo triangula similia ipsi totali triangulo. Idem etiam dico ad partem.o.n.i. vnde ex tali similitudine habebimus.o.u.mediam proportionalem inter.o.i.et.o.a.et sic.o.n.erit media proportionalis inter.o.i.et.o.e.quare ex. 16.sexti,quadratum.o.u.æquale erit producto ipsius o.i.in.o a.& quadratum.o.n. æquale producto.o.i.in.o.e. sed ex prima eiusdem, eadem proportio est ipsius.o.a.ad.o.e.quæ producti ipsius.o.i.in.o.a.ad productum.o.i.in.o.e.quare,ex cōmuni conceptu,ita erit quadrati.o.u.ad quadratum.o. n. Et hęc est alia circuli passio.

Reliqua verò difficultas quam te habere scribis, est, quare cum duæ lineę a.u.et. b.s.i. sint inuicem ęquales, diuisæ verò non æquali modo, sed tali,quod. a. maior sit quam.u. et. b.s. maior quam.i. quomodo potest fieri, quod si.u.maior fuerit.i.proportio.a.ad.i.maior sit quam ipsius.b.s.ad.u.




Hoc etiam ex necessitate euenit, eo quod si accepta fuerit.t.n.æqualis. u. ab ipsaq; abscisa fuerit. t. æqualis. i. & ab. b.s.abscisa. s.æqualis.n.habebimus.a.et b. inuicem æquales, vnde habebis maiorem proportionem ipsius.b.ad.t. quā s.ad.n.quod cum clarum per se sit, tibi relinquo. sed ex.27. quinti, proportio b. ad. s, maior erit quam. t. ad. n. & ex 28. eiusdē ꝓportio.b.s.ad.s.maior erit, quam. t.n.ad. n.& ex.27.maior proportio erit ipsius. b.s.ad.n.t.quam. s. ad. n. ergo ex.33.maior erit ipsius.b.ad.t.quā b. s. ad. n. t. hoc est maior ipsius. a. ad i. quam. b. s. ad. u. quod est propositum.

Id verò de quo me interrogas nēpe de distinctione orbium cęlestium, ortum habet à communi opinione motuum fixarum. Nam cum putauerint philosophi ipsas moueri, semper eandem seruādo inuicem distantiam, non sine ratione crediderunt eas fixas esse eodem in orbe, idem etiam postea de planetis opinauerunt. Hoc est, vnumquemque, aliquo in orbe, fixo existere.

DE

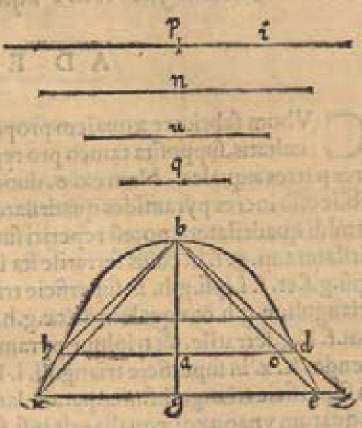
# DE MODO DIVIDENDI PARABOLAM propositam secundum datam proportionem.

## *Pamphilo Gothfrid.*

QVOD à me quæris, est quidem possibile, non tamen adhuc inuentum, quoniam nemo ad hũc vsque diem diuisit vnam datam proportionem in tres æquales partes, sed si hoc pro facto concesseris, nunc tibi morem geram. Nam proponis mihi parabolem.x.b.e. cum proportione.p.ad.q. cupisq; scire modum diuidendi ipsam parabolem vna mediante linea parallela ipsi basi, ita vt eandem habeat proportionem tota parabola ad partem abscissam, quæ est inter. p.et. q. Ad quod faciendum, supponendum primò datam proportionem inter. p.et.q. diuisam esse in tres partes æquales, duabus lineis mediantibus.n.et.u. quæ mediæ proportionales vocabuntur inter. p. et.q. deinde à quouis puncto circunferentię ipsius figuræ ducatur parallela basi. x. e. postea verò per puncta media harum duarum æquidistantiũ protrahatur.g.b. quæ diameter erit sectionis, ex 28. secundi Pergei, diuidatur deinde hæc diameter in puncto.a.ita quod eadem proportio sit ipsius b. g. ad.b.a.quæ ipsius.p.ad.u.quod tibi facile erit, secando à linea.p.partem.i.æqualem ipsi.u. tali modo postea diuidendo.b.g.ex.12.sexti, ducatur a puncto. a. ipsa.d.h.parallelam ipsi.x.e.& habebitur propositum.

Pro cuius rei ratione, scies primum quod.h.d.diuisa erit à diametro.b.g.per æqualia ex. 7.primi Pergei, vel si cogitabimus aliquam lineam tangentem ipsam parabolam in puncto.b.tunc ex quinta secundi ipsius Pergei habebimus ipsam esse parallelam.e.x.& ex.30.primi Eucli.erit similiter æquidistans.d.h.vnde ex.46. primi eiusdem Pergei.h.a.æqualis erit.d.a. Protrahatur deinde.e.b:d.b:x.b.et.h.b.vnde ex.17 lib. de quadratura parabolæ Archimedis, habebimus eandem proportionem superficiei totalis parabolæ.x.b. e. ad trigonum.x.b.e.quæ portionis.h.b.d. ad suum trigonũ, eo quod tã vna quàm alia erit sesquitertia, eiꝰ etiã medietates sic se habebũt.

Vnde permutando, proportio medietatis totalis parabolę ad medietatem partialem ipsius, æqualis erit proportioni trianguli g.b.e.ad triangulum. a. b. d. sed ex. 20. primi Pergei, eadem est proportio quadrati ipsius. g.e.ad quadratum ipsius.a.d.quæ.b.g. ad. b.a. hoc est, vt.g.e.ad.a.o.ex similitudine triangulorum, & quia.b.g.ad.b.a.est sicut.p. ad. u. ita igitur erit quadrati ipsius. g. e. ad quadratum ipsius.a. d. quare. g. e. ad.a. d. erit ut p. ad. n. ex. 18. sexti Euclid. sed cum ex.24. eiusdem proportio trianguli.b. g. e. ad triangulum. b. a.d. composita sit ex proportione. g. e. ad. a. d. et. ex. g. b ad. b. a. hoc est. g. e. ad. a. o. & quia proportio.g.e.ad. a.o.æqualis est ei quæ.p. ad.u ex. 11. quinti Euclid. & proportio.g.e. ad.a.d.æqualis est ei quæ.p.ad.n.hoc est vt. u. ad.q.ergo proportio trianguli.b.g.e.ad triangulum.b.a.d.composita erit ex ea quę.p.ad.u. & ex ea quæ.u.ad.q.æqualis ergo erit ei, quæ p. ad.q. & ita medietates parabolarum, & eorum dupla.

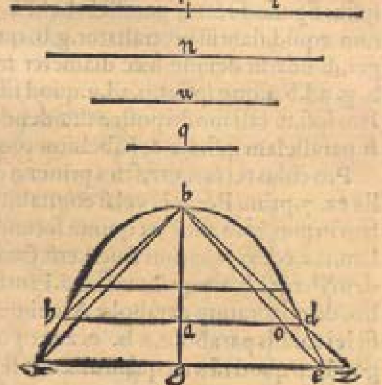


COROLLARIVM.

Proportio maioris portionis ad minorem semper erit sesquialtera proportioni ipsius.b.g.ad.a.b. eo quod cum sit proportio totalis portionis ad partialem vt trianguli.b.g.e.ad.b.a.d. & hæc sesquialtera proportioni ipsius.g.e.ad. a. o. hoc est vt ipsius. b.g.ad. b.a. ideo proportio ipsarum portionum erit similiter sesquialtera proportioni diametrorum.

Deinde si protractæ fuerint.b.d.et. g.e. quousque conueniant in puncto. z. habebis inter.g.z.et.a.o.duas.g.e.et.a.d.medias proportionales in proportionalitate continua, eo quod cum ( ex ijs quæ supra diximus. ).a.d. media proportionalis sit inter. g.e.et.a.o.& proportio. g.z.ad.g.e. vt ipsius.a.d.ad.a.o. eo quod ipsius.g.z. ad. a. d. & ipsius.g.e.ad.a.o.est vt ipsius. b. g. ad. b.a.ex similitudine triangulorum, ideo dictæ ꝓportiones erunt inuicẽ æquales. Vnde permutatim ita erit ipsius.g. z.ad.g. e. vt ipsius.a.d.ad.a.o.& ut ipsius.g.e.ad.a.d.

Amplius etiam dico, quod proportio parabolæ totalis ad partialem, eadem est,quę cubi ipsius.g.e.ad cubum ipsius.a.d.& ex cõsequenti, vt cuborum earundem basium,eo quod cum sit, ex. 36.vndecimi Euclid.proportio cubi ipsius. g.e.ad cubum ipsius. a.d. tripla ei quæ ipsius.g.e.ad.a.d. ideo æqualis erit ei quę trianguli.b.g.e.ad triangulum.b. a.d.cum proportio horum duorum triangulorum composita sit ( vt supra vidimus ) ex ea quæ.g.e.ad.a.o.& ex ea quæ.g.e.ad.a. d. & hæc medietas illius, sed trianguli ita se inuicem habent,vt parabolę,quare ipsæ parabolæ seinuicem habebunt, vt cubi ipsarum basium.




## *Cubum fabricare æqualem pyramidi propositæ.*

## AD EVNDEM.

CVbum fabricare æqualem propositæ pyramidi quadrilateræ,nullius erit difficultatis,supposita tamen pro reperta diuisione cuiusuis datæ proportionis in tres partes æquales. Nam ex. 6. duodecimi Eucli.patet omne corpus serratile diuisibile esse in tres pyramides quadrilateras æquales, scimus etiam quod cuilibet pyramidi quadrilateræ potest reperiri suum serratile. Sit igitur proposita pyramis quadrilatera.m. g. f. h. cuius serratile ita inueniemus, ducendo primum.h.i.parallelam ipsi.g.f.et. f. i.ipsi.g.h. in superficie trianguli. f. g. h. et. m. K.ipsi. g. h. in superficie trianguli.m.g.h.& æqualem dictæ.g.h. ducetur postea.K.h.et. K.i.& habebimus corpus.f. K.g.serratile, & triplum pyramidi propositæ. Nunc duplicemus ipsum,ducendo. K. x. in superficie trianguli. i. k. h. parallelam,æqualemq; ipsi. i. h. et. m. y. in superficie trianguli.f.m.g.parallelam,ęqualemq; ipsi.f.g.ducatur postea.g.y.et.h. x.quarum vnaquæq; æqualis erit ipsi.f.m. vnde habebimus corpus.f.x. parallelepipidum, & sexcuplum ipsi pyramidi propositæ.

Inue-

Inueniatur nunc quadratum.u.n.æquale sextæ parti superficiei.f.i. g. h. quod per se facile erit, deinde accipiatur altitudo corporis.f.x. ducendo vnam perpendicularem à puncto. m.ad basim.f.g.h.quę sit. n.e.qua mediante,cum quadrato. u.n. fabricetur solidum parallelepepidum.u.e.quod erit æquale dictæ pyramidi ex.33.vndecimi Euclid.

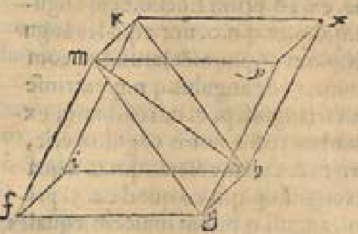
Repertæ nunc sint duæ mediæ proportionales.r,s. inter. n.e.et.n.p. quarum.s. sit proximior ipsi.n.p. ex qua. s. si constitutus fuerit cubus, habebimus propositum.

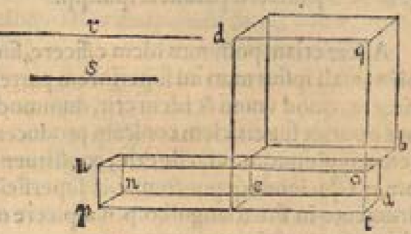
Pro cuius rei ratione,cogitemus corpus.u.e.productum esse vsque ad.a.o.per longitudem.s. latus dicti cubi, qui quidem cubus sit.d.b.vnde proportio corporis. u.e. ad corpus.e.o.erit, vt superficiei.p.e.ad superficiem.t.e.ex. 33. undecimi,ipsæ verò superficies sibi inuicem erunt vt.n. e.ad.e.a. ex prima sexti, quare proportio corporis.u.e.ad corpus.e.o.dupla erit proportioni ipsius.s.ad.n.p. sed cum ex. 33. vndecimi,proportio cubi.d.b.ad corpus.e.o.sit vt quadratū.q.b.ad quadratum.o.a. & cum proportio.q.b.ad.o.a.dupla sit ei quæ.q.o.ad.o.t.ex. 18.sexti, erit igitur proportio cubi.d.b.ad corpus.e.o.dupla ei quæ.q.o.ad.o.t.hoc est ei quæ.s.ad. n. p. sed ita erat corporis.u.e.ad corpus.e.o.quare ex.9.quinti, cubus.d.b.æqualis erit corpori. u.e. hoc est pyramidi propositæ.

Sed si oportebit cubum maiorem vel minorem ipsa pyramide reperire, in qua proportione tibi placuerit, tunc opus erit aliud quadratum inuenire, quod in ea proportione se habeat ad quadratum.u. n.quam volueris, quo mediante simul cum altitudine pyramidis consequemur propositum.

Aduertendum tamen quod fabricare ipsum corpus serratile.k.f.h.& solidum.f.x.necessarium non est,nisi pro demonstratione. idemq; dico de alijs solidis, nam pro simplici operatione huiusmodi problematis, absque aliqua re necessaria ad speculandum, ita faciendum erit.

Data pyramide.m.f.g.h.accipe ei9 alitudinem à pūcto.m. vsque ad superficiem basis.f.g.h. quæ sit. n. e. accipe deinde latus tetragonicum quadrati. u.n.æqualis tertiæ partis ipsius basis.f. g.h. quod latus sit. n. p. inter quod,et. n.e. inuentæ cum fuerint duæ lineæ mediæ proportionales. s. et.r. quarū. s.proximior sit. n. p. quæ quidē. s. erit latus cubi quæsiti.



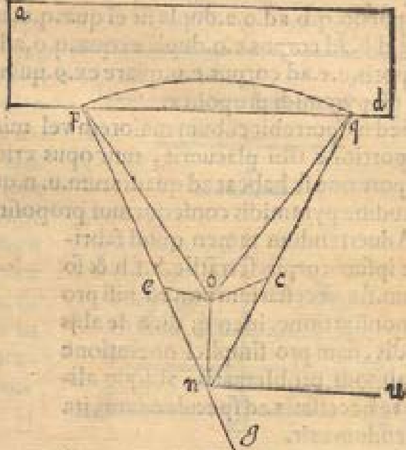


## *Duplex modus parallelam orizontalem alicui muro proposito una tantummodo statione ducendi.*

### AD EVNDEM.

DVcere parallelam orizontalem alicui muro recto proposito vna tantummodò statione, non solum possibile est sed etiam facile.

Sit exempli gratia murus rectus. a. d. situs verò. o. n. Si cupimus ducere. n. u. parallelam dicto muro, accipiatur quadratum geometricum, seu scala altimetra vel aliquod simile instrumentum, quo mediante à situ. o. videbimus punctum. q. quod volueris ipsius muri, dexterā versus, inferius tamen. ipso.o. vnde formatum habebimus triangulum. n.o.q. Quo facto ad partem sinistrā cum eodem angulo.n.o. q. oportebit nos inuenire punctum aliquod. p. in dicta superficie muri, & tunc habebimus angulum.n. o. p. æqualem angulo.n.o.q. vnde angulus. q. n.p. nobis cognitus erit, duoq; latera.n.q.et.n.p. erunt inuicem æqualia, ex. 26. primi Euclid. cum anguli.q.o.n.et.q.n.o. sint æquales angulis.p.o.n et.p.n.o. & latus.o.n. commune, vnde angulus.q n.g. extrinsecus trianguli. p. q. n. residuusq; ex duobus rectis nobis cognitus erit, etiam & eius medietas.q.n.u. æqualis angulo.p.q.n. eo quod ex. 5. primi, anguli.q.p. sunt inuicem æquales, & ex. 32. eiusdem, æquales sunt extrinseco.q.n. g. & ex 27.n.u. erit parallela ipsi.q.p.

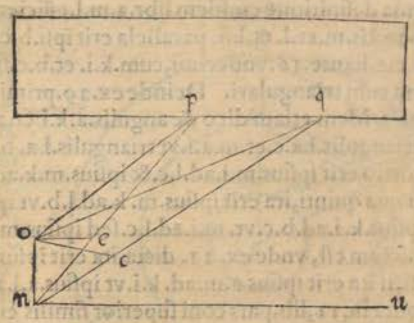



Aliter etiam possumus idem efficere, sumendo duo illa puncta in suprema linea orizontali ipsius muri ad superiorem partem aspiciendo, quemadmodum ad inferiorem, quod vnum & idem erit, dummodò non aspiciamus orizontaliter, eo quod nos oportet superficiem conicam producere, linea visuali mediante. cognoscere autem angulum.q.n.p. facile erit, constituendo primò instrumentum in situ trianguli. o.n.q. aspiciendoq; punctum.c. in superficie.n.q. o. & sic in alia parte, existente instrumento in situ trianguli.o.p. n. aspicere oportet punctum. e. proximum puncto. n. vbi possit metiri angulum.c.n.e.

Sed si situs puncti.n. talis esset, vt ab eo non posset aliquis murum videre ad rectos angulos, aspiceremus punctum.q. sub orizontali ab oculis nostris, in orizontali tamen puncti.n. ita quod angulus.o.n.q. rectus existat, quo facto obseruando angulum.n.o.q. eo mediante, medianteque. n. o. cum angulo. o. n.q. cognoscemus quantitatem distantiæ. n. q. idem etiam faciendum est cum alio puncto. p. quod volueris, & mediantibus duobus punctis inuicem proximis. c. e. cognoscatur an-

gulus

gulus.p.n.q.vnde ex methodo.56. primi triangulorum Monteregij, cognoscemus reliqua trianguli. q.p.n. Constituendo postea angulum.q.n.u.æqualem angulo.n.q.p. propositum habebimus.




Si etiam puncta.q.p.lineæ. q. p. orizontali in eodem plano non existerent cum puncto. n. nihil referret, dummodo in pauimento notẽtur pũcta.c.e.proxima.n. in ijsdem superficiebus triangulorum.n.o.p. et.n.o.q.vnde.n.c.et.n.e. erunt cõmunes sectiones dictarum superficierum cum superficie pauimenti supra quam fit statio.

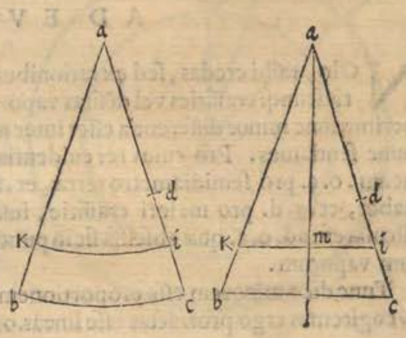
## CONI RECTI DIVISIO A PLANO parallelo basi secundum datam proportionem.

### *Raphaeli de Auria.*

QVOTIESCVNQVE volueris conum rectum diuidere à plano parallelo basi secundum vnam datam proportionem, nullius tibi erit difficultatis, concessa tamẽ pro inuenta diuisione cuiusuis propositę proportionis per tres æquales partes.

Sit exempli gratia conus rectus. a. b. c. secandus vt dictum est, accipiatur latus ipsius, quod sit. a. c. ipsumq́; diuidatur in puncto. d. secundum illam proportionem quam desideras, hoc est ipsius.a.c.ad.a. d. quo facto, inter totum.a. c.et.a.d.inueniantur duæ lineæ proportionales, quarum maior sit. a. i. tunc si conus. a.b. c. sectus fuerit à plano per punctum.i.parallelo basi, habebimus quod quærebamus.

Cuius rei ratio, primò est, quia quotiescunque conus aliquis sectus fuerit ab aliquo plano parallelo basi ipsius, pars superior similis semper erit totali cono, quod ita probo, cogitemus conum sectum esse à plano per axem. a. l. vnde ex. 3. primi Pergei, talis sectio triangularis erit, quæ sit.a.b.c.et.b.c.diameter erit basis.



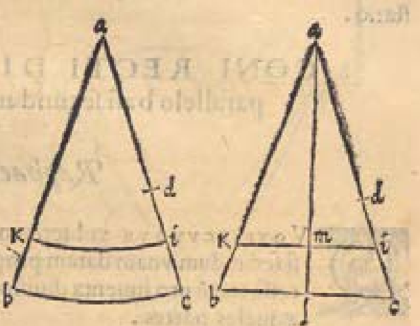

Imaginemur deinde.K. i. communem esse sectionem huiusmodi trianguli cum plano parallelo ipsi basi, tunc tale planũ, circulare erit ex.4.primi ipsius Pergei.K. i.verò, eius diameter erit, et.a.m.suꝰ axis.

Cum verò.a.l. sit perpendicularis ipsi basi coni totalis, eo quod rectus supponitur, ideo eadem.a.m.l.erit perpendicularis etiam ipsi secundo plano circulari, ex conuersa. 14. vndecimi Euclid. vnde ex

secunda

ſecunda definitione eiuſdem libr.a.m.l.efficiet angulos rectos cum duabus.b.c.et.K. i.in punctis.m.et.l. et.k.i. parallela erit ipſi.b.c.ex.28. primi, quod etiam poteſt con cludi mediante.16.vndecimi,cum.k.i. et.b.c.ſint communes ſectiones duorum pla norum cum triangulari. Deinde ex.29.primi anguli.a.i.m.et.a. c.l. erunt inuicem æquales, idem etiam dico de angulis.a.k.i.et.a.b.c. anguli poſtea ad.a. communes ſunt triangulis.l.a.c.et.m.a.i.vt triangulis.l.a. b. et. m. a. k. Vnde ex. 4. ſexti, eadem proportio erit ipſius.m.i.ad.l.c.& ipſius.m.k.ad.l.b.vt ipſius.a.m. ad.a. l. Quare ex vndecima quinti,ita erit ipſius.m. k.ad.l.b.vt ipſius.m.i.ad.l.c.& ex. 13. eiuſdem,ita erit ipſius.k. i.ad.b.c.vt. m.i.ad.l.c.ſed ipſius.m.i.ad.l.c.eſt vt ipſius.a.m.ad.a.l.quod iam dictum eſt, vnde ex. 11. dicta,ita erit ipſius.k.i.ad.b.c.vt ipſius.a.m.ad.a.l. & ex 16. dicti ita erit ipſius.a.m.ad. k.i.vt ipſius.a.l.ad.b.c. Quare ex definitione ab Eucli.poſita in.11,lib.pars coni ſuperior ſimilis erit cono totali.

Deinde ſciendum eſt illud quod Euclid.ſcribit in. 10. duodecimi lib.hoc eſt, ꝙ proportio duarum pyramidum inuicem ſimilium, triplicata eſt ei diametrorum ſuarum baſium, hoc eſt, quod proportio. b.c.ad.k.i. tertia pars erit proportionis totius pyramidis.a.b.c.partiali pyramidi.a. k.i.ſed ita eſt ipſius.a c.ad.a.i. vt ipſius.b. c.ad.k. i. ex. 4. ſexti cum trianguli.a. b.c. et.a. k.i.ſint æquianguli, quod ex ijs, quę ſuperius diximus facile compręhenditur. Quare ꝓportio.a.c.ad.a.i.tertia pars erit proportionis totius coni.a.b.c.ad eius partem abſciſſam.a.k.i.ſed eadem proportio ipſius.a.c.ad.a.i.erat etiam tertia pars proportionis ipſius.a.c.ad.a.d.Quare ex communi conceptu,proportio totius pyramidis, ad partem abſciſſam, æqualis erit proportioni ipſius. a.c.ad.a.d.




## *De differentia caloris Solis propter vaporum altitudinem.*

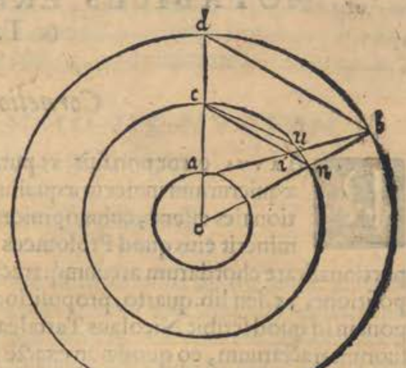
### AD EVNDEM.

NOlo, mihi credas, ſed ex rationibus, quas tibi ſcribo conſidera, quod quotieſcunq; craſſities vel dēſitas vaporū,ſeu altitudo,maior eſſet ea,quę nunc reperitur,tunc minor differentia eſſet inter maiorem minoremq́; calorem Solis,quam nunc ſentiamus. Pro cuius rei euidentia,imaginemur in hac ſubſcripta figura, lineam. o. a. pro ſemidiametro terræ, et. a. c. pro craſſitie vaporum, vt nunc ſe habet, et. a. d. pro maiori craſſitie, imaginemurq́ue lineam. a. b. quaſi perpendicularem ad. o. a. quæ abſciſſa ſit in puncto u. à circunferentia. c. u. inferiori priorum vaporum.

Tunc dico minorem eſſe proportionem ipſius.a. b. ad. a.d. quam ipſius. a.u.ad.a. c. cogitemus ergo protractas eſſe lineas.o.b: d. b: c.u. et. c.n.quæ.c.n.ſecabit.a.u. in

puncto

punĉto. i. ex communi conceptu, & parallela erit ipſi. d. b. ex. ſecunda parte ſecundæ ſexti, vnde ex prima parte eiuſdem, ita erit ipſius. b. i. ad. i. a. vt. d. c. ad. c. a. & coniunĉtim ita erit ipſius. b. a. ad. a. i. vt ipſius. d. a. ad. a. c. & permutatim ipſius. a. b. ad. a. d erit, vt. a. i. ad. a. c. ſed cum. a. u. maior ſit ipſa. a. i. vt omne totum maius eſt ſua parte. maior proportio erit ipſius. a. u. ad. a. c. quam ipſius. a. i. ad. a. c. hoc eſt quam ipſius. a. b. ad. a. d. Verum igitur eſt propoſitum.

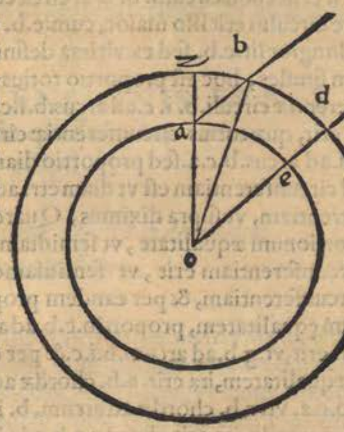



## *De differentia caloris Solis reſpeĉtu altitudinis ipſius.*

### AD EVNDEM.

QVod à me poſtulas deinde, ita ſe habet. Inquis enim, quod cum differentia inter maiorem, minoremq́; calorem, oriatur etiam ex differentia maioris quantitatis vaporum ad minorem, per quam quantitatem vaporum tranſit lumen Solis (vt alias etiam tibi dixi) velles nunc ſcire quantitatem ipſius differentię, quæ inter duas Solis datas altitudines ſupra orizontem reperitur.

Quapropter imaginemur circulum. a. e. pro magno terræ, et. z. b. d. pro magno vaporum, ſupponatur etiam quod angulus. z. o. d. vel. z. a. b. qui ſunt inuicem ferè æquales, ſit angulus diſtantiæ Solis à zenit, z. a. verò ſit ſpiſſitudo vaporum, et. a. b. radius tranſiens per vapores diĉtos. nunc quæratur proportio, quæ eſt inter. a. b. et. a. z. qua inuenta, angulo. z. a. b. mediante, quæremus eandem mediante angulo. z. a. b. maiore priori, vel ipſo minore, vnde cognoſcemus differentiam duarum. a. b. quæ quidem inæquales inuicem erunt, eo quod ſupponatur. a. z. immutabilis, & hoc ita faciemus. Imaginabimur. o. b. quæ claudat triangulum. a. b. o. & quia. a. z. cognita eſt quam Alhazem docet inuenire, cognoſcimus etiã o. a. vt ſemidiametrum terræ, vnde. o. b. et. o. a. duo latera trianguli. a. o. b. cognita erũt ſimul cum angulo. o. a. b. reſiduo duorum reĉtorum, eo quod reliquus. z. a. b. datus eſt. Quare. a. b. cognita erit reſpeĉtu. o. a. et. o. b. et. a. z. quæ eſt eorum differentia. Nunc ſi idem faciemus cum alia. a. b. ſub diuerſo angulo, habebimus propoſitum.

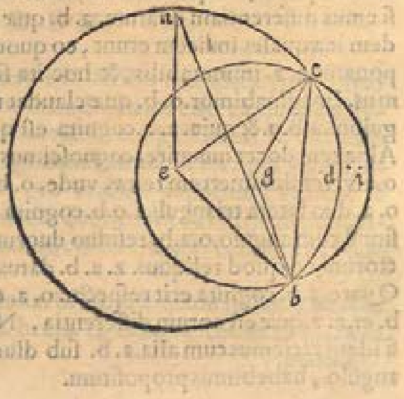



NOTA-

# NOTABILES ERRORES ORONTII & Tartaleæ.

## *Cornelio Bitonto.*

PARVVS error non fuit, vt putabat Orontius, quod anguli triangulorum æquicrurium inuicem æqualium, baſibus oppoſiti, iiſdem baſibus proportionales eſſent, cuius opinionis cauſa fuit quod nunquam viderit vel meminerit eius quod Ptolomeus ſcripſit lib. primo Almagesti, vbi de diſproportionalitate chordarum arcuumq; tractat, vel quod ſcribit Vitellio lib. primo propoſitione. 35. ſeu lib. quarto, propoſitione. 21. quod idem eſt. Sed nec ego tibi proponam id quod ſcribit Nicolaus Tartalea diuiſioni. 28. quinti capitis quartæ partis ſuorum tractatuum, eo quod non exactè ſcientificè ſcripſerit, nec vniuerſaliter, quãuis talis propoſitio poſſit ſcientificè ſcribi, accipiendo. b. c. in eius figura, pro latere octagoni, vnde angulus. a. e. b. duplum foret angulo. b. e. c. collocato poſtea. b. c. in arcu. a. b. punctum. c. medium fuiſſet dicti arcus, et. e. c. diuideret. a. b. per æqualia, ex quinta primi, nec non ad rectos ex. 3. tertij, vnde ex. 18. primi, clare vidiſſemus non eſſe proportionem. a. b. ad. b. c. vt anguli ad angulum. Sed vniuerſaliori modo poſſumus hoc ſpeculari. Nam manifeſtè ſcimus, eandem eſſe proportionem circunferentiæ ad diametrum in omnibus circulis tam maioribus, quam minoribus. Sint igitur duo anguli. a. e. b. et. c. e. b. cuiuſuis amplitudinis, quorum latera. e. a: e. b: et. e. c. ſint inuicem æqualia, protrahatur. b. a. et. b. c. Tunc dico maiorem proportionem eſſe anguli. a. e. b. ad angulum. b. e. c. quam. a. b. ad. c. b. ducatur enim. b. g. ita q faciat angulum. g. b. c. æqualem angulo. e. b. a. protracta poſtea. c. g. quæ idem faciat in puncto. c. vnde. g. b. et. g. c. æquales inuicem erunt ex. 6. primi, & quia angulus. a. æqualis eſt angulo. e. b. a. ex quinta eiuſdem, ideo ex. 32. dicti, et. 4. ſexti, horum duorum triangulorum latera, erunt inuicem proportionalia. Conſtituto deinde. g. centro, & ſecundum ſemidiametrum. g. b. vel. g. c. quod idem eſt, deſcripto circulo. b. i. c. necnon circulo. b. c. a. circa centrum. e. ope ſemidiametri. e. b. et. e. a. vnde iſte circulus erit illo maior, cum. c. b. maior ſit. g. b. ex. 14. quinti. cum ex. 14. tertij a. b. longior ſit. c. b. ſed ex vltima definitione tertij, arcus. b. i. c. et. b. c. a. erunt inuicem ſimiles, hoc eſt proportio totius circunferentiæ circuli. b. i. c. ad arcus. b. i. c. eadem erit, quæ totius circunferentiæ circuli b. c. a. ad arcus. b. c. a. ſed proportio diametri ad circunferentiam eſt vt diametri ad circunferentiam, vt ſupra diximus; Quare ex proportionum æqualitate, vt ſemidiametri ad circunferentiam erit, vt ſemidiametri ad circunferentiam, & per eandem proportionum ęqualitatem, proportio. c. b. ad arcũ b. c. a. erit, vt. g. b. ad arcum. b. i. c. & per eandem æqualitatem, ita erit. a. b. chordæ ad arcum. b. c. a. vt. c. b. chordæ ad arcum. b. i. c. & permutando, ita erit chordæ. a. b. ad chordam. c. b. vt arcus. b. c. a. ad arcum. b. i. c. ſed arcus. b. i. c. maior eſt arcu. b. d. c. ex commu




ni

ni ſcientia. Quare maior proportio erit acus.b.c.a.ad arcum.b.d.c.quam ad arcum b.i.c. ex. 8. quinti. Vnde ex vltima ſexti et.12.quinti,proportio anguli.a. e.b.ad angulum.c. e.b.maior erit quam chordæ, ſiue baſis.a. b.ad chordam ſiue baſim.c. b.

## DE CAVSA SVSPENSIONIS NVBIVM in aere contra Antonium Bergam.

### *Clariſsimo Franciſco Venerio.*

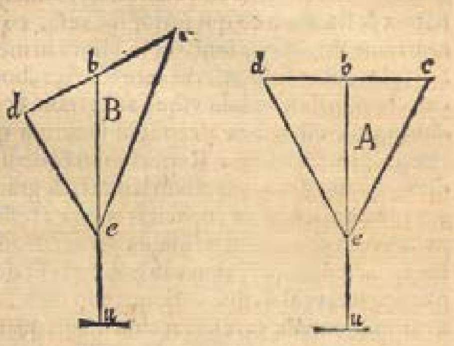
EGo enim non tantum miror ea quæ mihi ſcripſiſti de opinione Orſenſij quantum quod Antonius Berga putat nubes à Sole ſupenſas teneri,id planè falſum eſt, vera cauſa huiuſmodi effectus, alia nulla eſt, niſi earundem raritas hoc eſt, cum rariores ſint ipſo aere ſubiecto, propterea ſupra ipsū natant & ſtant ſub eo qui rarior ipſis eſt, eo quod corpora rariora poſita in medio non tam raro, aſcendunt, & denſiora in medio minus denſo deſcendunt. Nam ſi Sol ipſas nubes ſuſpenſas in aere teneret, hoc interdiu tantummodo fieret, ſed noctu, cur non deſcendunt vſque ad terram, & in eodem loco ſemper manent? Sciendum igitur eſt nubes aſcendere in altum quouſque inueniant aerem eiuſdem raritatis cuius ipſæ ſunt. Raritas enim & denſitas non ſunt res viſibiles niſi per accidens, quemadmodum etiam leuitas, & grauitas, opacitas verò & diaphaneitas magis compræhendūtur, opacitas enim ex reflexione radiorum luminoſorum, diaphaneitas verò compræhenditur ex penetratione ipſorum radiorum,opacitas autem nubis non eſt denſitas, cum valde diuerſa ſit denſitas ab opacitate, ſicut raritas ab diaphaneitate, vt aliàs dixi. Et quando dicit, quod Sol calefaciendo aerem ipſam nubem ambientem, rarefaciat eum magis quam ipſam nubem reſpondeo, hoc verum non eſſe, propterea quod radius Solis non multum calefacit ea corpora,quæ ipſi permittunt liberum tranſitum. vnde corpora quanto magis diaphana ſunt tanto minus ab ipſo radio luminoſo calefiunt,ſed ea quæ magis opaca ſunt,magis etiam calefiunt & per conſequens magis rarefiunt, cum calidi ſit per ſe rarefacere,& non attrahere, vt ipſe & ferè omnes alij putant.

## DE RATIONE EXTENSIONIS FVNIS cuiuſdam libramenti, & de quadam ſimboleitate circuli cum ellipſi.

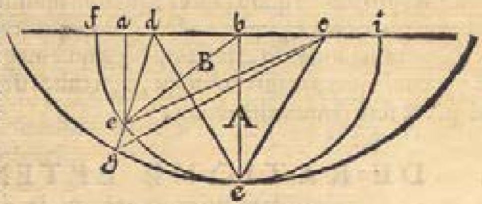
### *Angelo Ferrario Sereniſsimi Ducis Sabaudiæ Agrimenſori expertiſsimo.*

TIBI in mentem veniet, quod cum ſuperioribus diebus in villa lucenti, in qua degebat Sereniſſimus Dux noſter, dum viridarium ad æquilibrium reducebas,eſſemus, à te quæſiui an ſcires vnde fieret, vt ſtante libramento ad angulos rectos ſupra ſuum pedem, funis quæ extrema eiuſdem libramenti cum pede in formam trianguli æquicruris coniungit, magis diſtentus exiſteret,quam cum dictum libramentum cum pede obliquum remanet, ita vt huiuſmodi funis cum libramento triangulum ſcalenum conſtitueret.

Exempli gratia, ponamus lineam.d.b.c.eſſe libramentum.et.b. e. u. eius pedem; funem autem, qui aliquando cum libramento facit triangulum iſocellum, & aliquando ſcalenum, eſſe.d.e.c.eſto etiam quod in figura. A. dictus triangulus.d. e. c. ſit iſocellus, & in figura. B. ſcalenus. Tunc quæſiui à te an ſcires rationem, quare funis.d.e.c.in figura. A. eſſet diſtenſus, & in figura. B. laxus quemadmodum videbamus. cum mihi reſponderis, neſcio quid, quod nunc memoria nō teneo, ſed quia pollicitus ſum me tibi eam afferre, propterea nunc ad te mitto. Scias ergo huiuſmodi rationem nihil aliud eſſe niſi quod in figura. A. duæ lineæ. c. e. et. d. e. ſimul è directo iunctæ longiores ſint illis, quę reperiuntur in figura. B. ſed quia funis tam in figura. B. quam in figura. A. vnus, & idem eſt, ideo in figura. B. laxatus eſt, & non intenſus, ut in figura. A. Sed vt huiuſmodi veritatis certam notitiam habeas, infraſcriptum circulum mente concipe. f. e.i. cuius ſemidiameter, æqualis ſit. b. e. & diameter ſit. f. i. in quo imaginare eſſe tuum libramentum.d.b. c.& figuras. A.et.B. & probabo lineas.d.e. c. figurę. A.longiores eſſe lineis.d.e.c. figuræ. B.




Imaginemur igitur lineam. b. e. eſſe dimidium minoris axis alicui' ellipſis cuius quidem figuræ ponamus. d.et.c. centra ipſius circunſcriptionis eſſe, cuius circunferētia, nulli dubium eſt, quin extra propoſitum circulum tranſitura, & in vno tantummodo puncto ipſum circulum tactura ſit, qui exiſtat. e. figuræ. A. ſeparatum tamen à puncto e. figuræ. B. Tunc ſi protracta fuerit linea.d. e. figuræ. B. vſque ad girum ellipticum in puncto. g. à quo ad punctum.c. ducta etiam ſit linea g. c.tunc manifeſtū erit duas lineas d.e.et. e.c.figuræ. A. ſimul iunctas, æquales eſſe duabus.d.g.et. g. c. ſimul poſitis, vt etiam ex. 52. tertij Pergei facilè videre eſt, ſed ex. 21. primi Euclid.iam certò ſcimus.d.g.c. longiores eſſe.d.e.c.figuræ. B.ergo.d.e.c.figuræ. A.longiores ſunt.d.e.c.figuræ.B. quod eſt propoſitum.



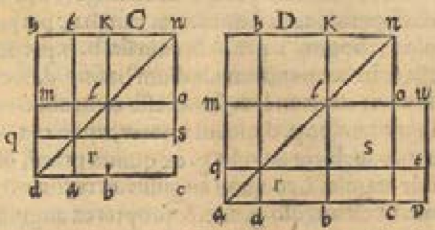

Quod etiam mihi nunc circa hoc ſuccurrit, tibi libenter ſignifico, hoc eſt, quod ſicut in ellipſi duæ lineæ.d.e.e.c.figuræ. A. ſimul iunctæ, ſunt ſemper æquales duabus lineis.d.g.g.c.in longitudine, ita in circulo duæ. d.e.e. c.figuræ. A. æquales ſunt in potentia duabus.d.e.e.c.figurę. B.

Manifeſtum enim primum eſt ex penultima primi in figura. A. quadratum. e. c. æquale eſſe duobus quadratis ſcilicet.e.b.et.b.c.& quadratum. e. d.æquale duobus. e.b. et.b.d. Quare quadrata.e. c.et.e.d.æqualia ſunt quadratis.e. b. figuræ. A.et. e. b. figurę. B.et.b.c.et.b. d. hoc eſt duplo quadrati.e.a.(ducta cum fuerit.e.a.perpendicularis ad.c.b.d.a.) duplo quadrati.a.b. ex penultima primi, & duplo quadrati.b. c. Sed quadrata.d.e.et.e.c.figurę.B.æqualia ſunt duplo quadrati.a.c.& quadrato a.d.

&

& quadrato.a.c. ex eadẽ. Nunc videndum eſt vtrũ duplũ quadrati.a.e.cũ duplo quadrati.b.a.cũ duplo quadrati.b.c. ſit æquale duplo quadrati.a. e. cũ quadrato.a. d. & cum quadrato. a. c. Sed quia tam ex vna parte quàm ex alia habemus duplum quadrati.a.c. Videndum igitur erit vtrum duplum quadrati.a.b.ſimul cum duplo quadrati.b. c. ęquale ſit quadrato.a.c.cum quadrato.a. d.ſed hoc manifeſtum eſt.ex. 10. ſecundi Euclidis, dato quod punctũ. a. ſit inter.f.et.d. ſed ſi fuerit inter.d.et. b. hoc manifeſtum erit ex. 9.ſecundi dicti, nihilominus accipe hunc alium modum.

Sit hic ſubſcriptum quadratum. D. ex. a. c.in ſeipſa producta, cuius diameter ſit a.n. protrahanturq; parallelę.d.h: b. K:l.m.o.et.r.q.s. eiq; addatur.c.p.ad.a.c.æqualis tamen.d.a. ſitq; protracta.p.u. vſque ad.m.o.u.vnde habebimus. a. n. pro totali quadrato,et.p.s.pro partiali,& æquali quadrato lineæ.a. d. Videndum nunc eſt, vtrũ hęc duo quadrata æqualia ſint duobus quadratis lineæ.a. b.& duobus lineæ.b. c. Nã duo quadrata lineæ. b. c.ſint. K. o.et. h.l.videndum nunc eſt utrum reſiduum ęquale ſit duobus quadratis lineę. a.b. quorum vnum ſit.m.b.alterum verò.l.p. quod ſuperat.l. c.et.s.p.figuræ. D.per ſupplementum.o.t.cui æquale eſt parallelogrammum.h. m.figuræ. D. ſed ſi punctus.a. poſitus fuerit inter.d. et.b.conſtituto quadrato.d.u.cũ omnibus parallelis,vt in figura. C. videre licet, in qua figura videbimus quadrata. r. n.et.d.r.ęquari duplo quadratorum.l.n.et.r.l.nam in quadrato.r.n. ipſa duo quadrata.l.n.et.r.l.capiuntur, reliquum eſt igitur vt videamus an duo ſupplementa.l.t.et.l. s.cum quadrato.d.r.ſint æqualia dictis quadratis.l.n.et.r.l.ſed quadratum.d.l. æquatur quadrato.l.n. videndum igitur eſt, an duo ſupplementa.l.t.et.l.s.cum quadrato.d.r.ſint æqualia duobus quadratis.d.l.et.r.l.ſed quadratum.d.l.æquatur quadrato.d. r.& ſupplemento. l. t. mediante.q.l.& ſupplemento.r.b. ſupplementum verò.l.s.ſuperat ſupplemẽtum.r.b.per quantitatem æqualẽ quadrato.r.l.quare duo ſupplementa. l. t. et.l.s.cum quadrato.d.r.æquantur quadrato.d.l. cũ quadrato.l. r. verum igitur eſt duas.d.e. e. c. figuræ. A. æquales eſſe in potentia duabus d.e.c.c.figurę. D.quæ quidem affectio circuli,à nemine fuit adhuc (quod ſciam) detecta.

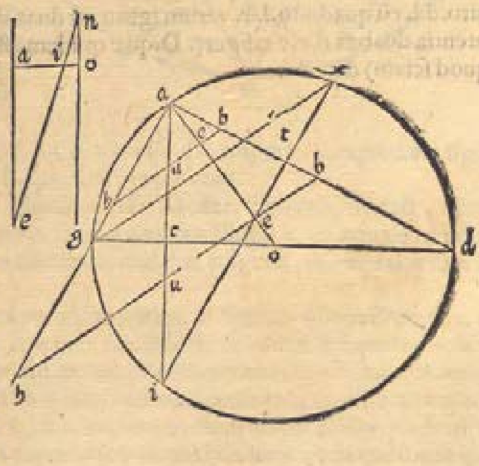


## DE AVGMENTO PONDERIS CORPORIS ad ſtateram appenſi, & quadam alia demonſtratione, & quibuſdam erroribus Tartaleæ.

### *Mutio Groto.*

SI ea quæ à me audiuiſti non credis, conſidera quæſo libram ſeu ſtateram o.a.cuius centrum non longitudinis ſed ponderum ſit.i.quę ſtatera, vt orizontaliter conſiſtat, oportebit pondus extremitatis. o. ita ſe habere ad pondus extremitatis.a.ut.a.i.ſe habet ad. o. i. quod te ſcire puto, imaginemur nunc duas lineas.a.e.et. o.n.parallelas infinitasq́; & à puncto.n. immobili, & fixo extra ſtateram, tranſeat per.i. linea.n.i.e. Cogitemus etiam punctum.e. interſectionis ipſius.n. i.e.cum.a.e.progredi vniformiter continuòq́; ab.a.per lineam.a.e. vnde punctum.i.interſectionis ipſius.n.i.e.cum.a.i.o. ſemper vicinius fiet puncto. o. nec unquam cum illo vnum erit, quamuis moueatur tempore infinito. Nunc autem dico, quod cum ſtateram.o.i.a.oporteat ſemper orizontalem eſſe virtute ponderis, o.oportebit pundus.o.in infinitum etiam augeri, quotieſcunq; pondus.a. nunquam diminui voluerimus vel econtra hoc in infinitum diminui, ſi illud nunquam augeri voluerimus.

Sed re vera non putabam te indigere aliqua demonſtratione, quod linea.b.h.diuiſa ſit per æqualia à linea.c. a. cum hæc perpendicularis ſit ab.a.ad baſim.g.d.in triãgulo orthogonio. g. a.d.& cum ſit. b. h.perpendicularis ad.a.o.ex ſuppoſito quæ. a. o. in ſe habet punctum medium baſis.g.d.nec nõ illud anguli recti.a. quod per ſe clariſſimum eſt, cum iam ſcis.o.eſſe centrum circuli circundantis triangulum.g.a. d.orthogonium, et.g. d.eius diameter, vnde.o.a.æquabitur ipſi.o.g.quapropter angulus o.a.g.æquabitur angulo.g. ex quinta primi, deinde ex.32.eiuſdem, angulus. h.æquabitur angulo.d.eo quod angulus.e. rectus eſt, quemadmodum et. a. ſed angulus. d. æqualis eſt angulo.g. a. c.& propterea angulus.h.erit etiam æqualis angulo. h. a. u. vnde. h. u. æqualis erit ipſi. u. a.ex.6.primi, cum poſtea angulus. o.a.d.æqualis ſit angulo.d.ex quinta primi erit angulus. a.b.c. æqualis angulo.g.ex.32.dicta, eo quod c. rectus eſt, & ex eadem æqualis erit angulo. d. a. c. vnde.u.b. erit æqualis ipſi.u.a.ex.6. dicti, & ideo æqualis erit ipſi. u. h. Reliqua verò illius propoſitionis credo ex te omnia poſſe ĩtelligere, excepto, q̃ vt tibi ſignificaui ſi à pũcto.i.communi ipſi.a.c.u. & circunferentiæ, ducta fuerit.i.x.ad pũctum.x. commune vni parallelæ à pũcto.g. ipſi h. b. & circunferentiæ, quod dicta.i.x. ad rectos erit ipſi.a.b.d.eo quod cum angulus.a.g. x. æqualis ſit



ſit angulo.a.h.b. propter æquidiſtantiam dictam, æqualis etiam erit angulo.d.& ar-
cus.a.x.æqualis arcui.a.g.vnde angulus.a.i. x. æqualis erit.d.ſed angulus.i.a.d.com-
munis eſt triangulis.c.a.d.et.i.a.t.quare angulus.a.t.i.rectus erit,vt.c.hoc eſt.i.x.per
pendicularis erit ipſi.a.d.

Sed vbi tibi ſcripſi circa finem illius epiſtolæ, Tartaleam erraſſe in quinta propo-
ſitione primi lib.ſuæ nouæ ſcientiæ,non ſine ratione illud ſcripſi. Nam, inquit ipſe,
nullum corpus æquè graue poteſt in aliquo temporis ſpatio moueri motu naturali,
violentoq; ſimul miſtis. Vbi decipitur,eo quod non animaduertit incrementum ve
locitatis vnius motus, ſimul eſſe cum decremento velocitatis alterius, eodemq; tem
pore, vt manifeſtè patet in itinere corporis, ab ipſo pro exemplo aſſumpto, hoc eſt
quod velocitas motus in ſpatio.c.d. creſcit vt naturalis, & decreſcit vt violenta. nã
creſcit orizontem verſus & decreſcit in remotione à linea. a. b. ſed ſi à puncto. c. ad
punctum.d.motus eſſet purè violentus, vt putat Tartalea, corpus illud minimè de-
ſcenderet, eo quod uirtus mouens,in.a. poſita,nullo pacto poteſt talem effectum ef-
ficere, vnde ab ipſa natura prouenit deſcenſio illius corporis propter grauitatẽ, quã
dictum corpus habet in tali medio, aeris ſcilicet, & non ex violentia aliqua. Sed ſi
dixiſſet ipſe, illum motum eſſe purum naturalem, hoc eſſet falſum, eo quod purus
naturalis motus alicuius corporis non impediti, extra locum ſuum,fit per lineam re
ctam, & non per curuam, vt videre eſt inter.c.et.d.

In vltima propoſitione deinde eiuſdem lib.quæ.6.eſt decipitur ſimiliter, & hæc
deceptio oritur ab ignoratione quintæ,& à putando motum naturalem non eſſe cau
ſam ipſius deſcenſus per ſpatium.c.d. Sed quia tibi ſignificaui expeditiorem viam
reperiri ad cognoſcendã proportionem inter.a.h.et.a. c. in vltima propoſitione ſe-
cundi lib.ipſius Tartaleæ, ipſam nunc tibi ſcribo. Nã iam ſcis angulum.h. l. i. diui-
ſum eſſe per æqualia ab.P. l.& quod.a.h.et.h.p.ęquales inuicem ſunt ex.6.primi Eu-
cli.vnde.p.i.et.a.h.æquales erunt inuicem ſimiliter, ſed ex.3.ſexti ita eſt ipſius. a. l.
ad.l.i.vt ipſius.a.p.ad.p.i.& coniunctim ita erit.a.l.i.ad.l.i.vt.a.i.ad.p.i.ſed.a.l.cogni
ta eſt ex eius quadrato,et.l.i.etiam, cum æqualis ſit ipſi. a.i.vnde ex regula de tribus
notam habebimus.p.i.reſpectu.a.i.& ita reſpectu.a. c. ſi hypotheſes ipſius Tartaleæ
veræ ſunt.

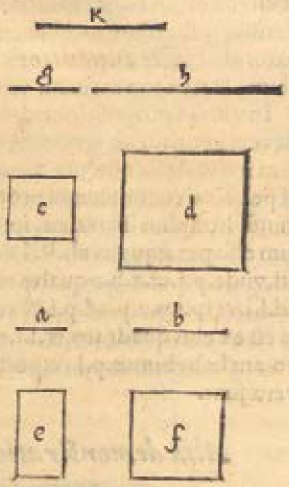
### *Alia demonſtratio impoßibilitatis diuidendi per æqualia proportionem ſuperparticularem in diſcretis.*

### AD EVNDEM.

QVod à me poſtulas, hoc eſt ſcientiam impoſſibilitatis diuidendi per æqualia
proportionem ſuperparticularem in numeris ſatis à Campano in. 8. octaui
potes habere, Iacobus Faber Stapulenſis etiam idem tractat in libello ſuę muſicæ
demonſtratæ. Sed ſi etiam alia via idem deſideras, quamuis longiori, nihilomi-
nus vniuerſaliori, conſidera duos numeros.g. et.h.inuicem relatos ſecundum pro-
portionem ſuperparticularem, quam volueris. Tunc dico impoſſibile eſſe, vt per
æqualia diuidatur, quod ſi dixeris poſſibile eſſe, ſit per te. K. medius numerus
propor-

proportionalis inter.g.et.h.quare.g.et.h.non erunt minimi in ea proportione, quia vnitas diuisibilis esset si.g.h.minimi fuissent, quod non conceditur, sint igitur minimi in dicta proportione.a.et.b.quorum differentia erit vnitas, vt scis, sitq;.c.quadratum ipsius.g.et.d.quadratum ipsius.K.tunc clarum erit ex.11.octaui, quod proportio ipsius c.ad.d.eadem erit quæ.g.ad.h.hoc est vt ipsius.a.ad. b. vnde si vnus terminorum.a.vel.b.esset quadratus, reliquus etiam quadratus esset ex.22. octaui, & ex 16.eiusdem,inter.a.et.b.reperiretur aliquis medius numerus proportionalis, quod fieri non potest ex hypothesi, cum inter.a.et. b. nullus sit numerus, quia differunt inter se per vnitatem tantummodo. Nunc autem cum nullus numerorum.a.vel.b.quadratus sit, ponatur quod.f. quadratus sit ipsius.b.et.e.sit productum ipsius. a.in.b.vnde ex.18.septimi,proportio ipsius.e. ad.f.erit vt. ipsius.a.ad.b.hoc est vt ipsius. c.ad d. quapropter.e. erit quadratus ex.22.octaui, cuius latus tetragonicum esset mediũ proportionale inter.a.et.b.ex.20.septimi, quod est impossibile, vt iam dixi, cum.a. et.b.sint inuicem consequentes, vnus post alium immediatè.

Superius enim dixi hunc modum esse vniuersalem, hoc est quod hac methodo possumus in cognitionem venire, quod non solum in duas æquales partes diuidi non possit, sed nec in tres, nec quatuor nec quot volueris. Primum enim quod non in tres diuidatur à te ipso cognosces ope cuborũ vice quadratorũ,ope vero censuũ cẽsuũ, vel qui cognouerit eam proportionẽ esse indiuisibilem per æqualia,illicò etiam cognoscet indiuisibilem esse per quatuor partes, ope verò primorum relatorum,cognosceet non esse diuisibilem per quinq; partes, & sic de cęteris, sed mediantibus ijs quas scripsi de istis dignitatibus in libro Thęorematũ arithmeticorum.




Id autem quod Illustrissimus Daniel Barbarus scribit in quinta parte suæ perspectiuæ, si supra aliquo immobili, atque magno pariete facere volueris, te oportebit hoc ex reflexione radij solaris à speculo plano perficere.

## DE INVENTIONE DIAMETRI circuli circunscribentis triangulum.

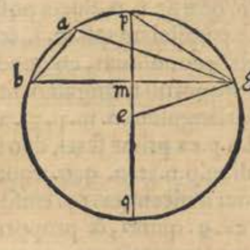
### *Franchino Triuultio.*

QVOD mihi nunc proponis est triangulum, cuius basis cum angulo sibi opposito dantur. Vellesq; diametrum circuli apti eum triangulum circunscribere inuenire in discreto.

Sit igitur triangulum.a.b.g. cuius basis.b.g.simul cum angulo. a. ei opposito data sit in numeris. Imaginetur ergo circulus circunscribens ipsum triangulum.b.p.g.q.cuius diameter sit.q.p.perpendicularis eius basi. b. g. vnde. b. g. diuisa erit per æqualia ab ipso diametro in puncto.m. per tertiam tertij, protrahatur etiam

e.g.

e.g.vnde angulus.g.e.q.æqualis erit angulo.b.a.g. portionis, cum duplus ſit angulo q.p.g. medietati anguli ipſius portionis ex.19. tertij, ita quod angulus.q.e.g. nobis cognitus erit,& ſimiliter arcus.g.q. & conſequenter arcus.p.g.reſiduum medij circuli, & ſic.m. g. eius ſinus rectus, & etiam chorda.p.g.vt dupla ſinus dimidij arcus. p.g.& ſic.p.m.eius ſinus verſus,vel vt tertium latus trianguli orthogonij.p.g.m.vnde nobis cognita erit proportio ipſius.b.g.( quæ dupla eſt ipſi.m.g.)ad. m.p. & quia productum.p.m.in.m. q.æquale eſt ei, quod fit ex.b.m. in.m.g. ex.34.tertij,quapropter nobis cognita erit pars q.m.quæ cum.p. m.complet totum diametrum.q.p. vnde nobis cognita erit proportio ipſius.b.g.ad.q. p. qua mediante cognoſcemus diametrum ſecundum partes illas quibus propoſita fuerit. b.g.

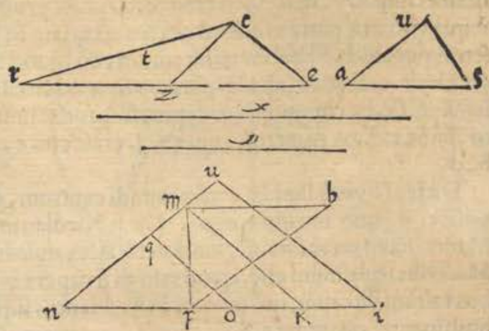



Hoc autem problema non in numeris ſed in continuo ab Euclid. ponitur in. 32. tertij.

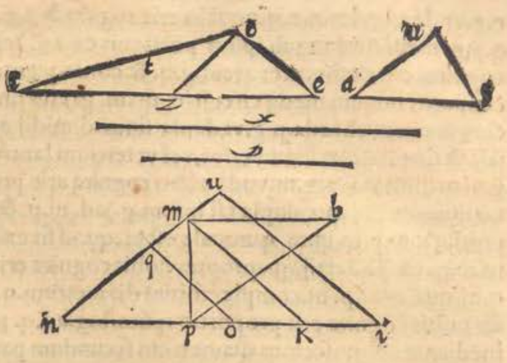
## *De inuentione alterius trianguli conditionati.*

## AD EVNDEM.

Quotieſcunque etiam inuenire voluerimus triangulum aliquem, puta. n. q. o. æqualem triangulo.t. (exempli gratia) propoſito,qui habeat angulum.n. æqualem angulo.a.dato, latera vero continentia ipſum angulum.n.ſint inuicem proportionata vt.x.et.y.ita faciemus, accipiemus lineam.n.m. cuius volueris magnitudinis, ſupra quam conſtituemus triangulum.m.n.p.æqualem triangulo.t.hac methodo, hoc eſt prolungando latus.r.z.trianguli.t.quod ſit.r.e. ita vt duplum ſit ipſi. r. z. ducendo poſtea.c.e. habebimus ex. 38. primi triangulum. t. eſſe dimidium totius trianguli.r. c. e. deſignabimus deinde ex.44.dicti ſuperficiem.p. n. m. b.parallelogrammam æqualemq; triangulo. r. c. e. habentem angulum. n. æqualem angulo. a. ducatur poſtea.p. m.& habebimus triangulum. m.n.p.æqualem.t.cum angulo.n.æquali angulo.a.producatur poſtea.n.p. ita vt.n. K. ſe habeat.ad.n. m. quemadmodum.x. ad.y. quod erit facillimum producendo. n. m. et. n. K.indeterminatè ſi oportuerit, deinde eas ad æqualitatem ſecando ipſis. x. et. y. efficiendo exempli gratia quod. n. i. ſit æqualis ipſi.x.et.n.u. ipſi. y.ducendo poſtea.u. i. deinde à puncto.m. ducendo.m. K. æquidiſtanter. u. i. ex. 31. primi. & ſic habebimus ex. 4. ſexti proportionem.x. ad. y. eſſe inter. n. K. et. n. m.inue-



m. inuenies poſtea ex. 9. eiuſdem lineam aliquam mediam proportionalem inter.n. K. et. n..p. quæ ſit. n. o. duces poſtea o.q.parallelam ipſi.m. K.& habebis propoſitum, eo quod cũ ſit proportio trianguli.n. m.K. ad triangulum.n. m. p. vt.n.K. ad.n.p.ex prima ſexti, duo triãguli.m.p.n.et. n. q. o. æquales erunt inuicem,ex.17. eiuſdem & ex. 9. quinti, & proportio. o.n.ad.n.q. erit, vt.x.ad.y. ex. 11.dicti, cum ex. 4. ſexti ſit vt. n. k. ad.n. m.

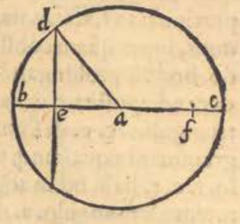


## *De producto conditionato.*

## AD EVNDEM.

PRoponis deinde mihi duas rectas lineas, vni quarum, vis vt aliam quandam directè coniungam, ita quod productum huius aggregati in lineam adiunctam æquale ſit quadrato alterius.

Vt exempli gratia ſi fuerint duæ lineæ.e. d. et. e.f.oporteretq; nos ad lineam.e. f.aliam lineam puta.f.c.vel.e. b.iũgere, ita longam, vt productum totius compoſiti.e. c. vel. f.b. in.f.c. vel. e. b.eſſet æquale quadrato ipſius. e.d.



Hoc enim nullius eſſet difficultatis, eo quod quotieſcũque.e.d.coniuncta erit cum. e.f. ad rectos, diuiſaq; per medium à puncto. a. à quo ducta.a.d. deinde ſecundum ſemidiametrum.a. d. deſignato circulo.b.d.c. & protracta. e. f. à qua volueris parte vſque ad circunferentiam in pũcto. c. ſeu in puncto.b. habebimus intentum, eò quod ſi producta fuerit. e. f. etiam ab alia parte, vſque ad circunferentiam, habebimus.b.e.æqualem ipſi.f.c.ex communi conceptu, & productum. e. c. in. e. b. æqualem quadrato ipſius.e.d.ex.34.tertij, cum ex. 3. eiuſdem. e. d.medietas ſit chordæ arcus dupli b. d.

De lapſu verò lapidis verſus mundi centrum, dum ipſum attingere, ac præterire poſſet, de quo me interrogas. Dico Nicolaum Tartaleam, nec non Franciſcum Maurolicum rectè ſenſiſſe, malè verò Alexandrum Piccolhomineum, & exemplum Maurolici optimum eſſe, quod tamen ſi capere non potes, crede ſaltem authoritatibus talium virorum, qui tantum in ijs ſcientijs ſuperant ipſum Alexandrum Piccolhomineum, quantum à Sole cætera ſuperantur aſtra.

Lapis igitur ille tranſiret centrum, redderetq;,cum diminutione tamen motus impreſſi, eo fermè modo vt ſcribunt iudicioſiſſimi illi viri, donec poſt multas redditiones ſurſum,deorſumq; quieſceret circa centrum mundi. Lucidioris tamen intelligentiæ

gentiæ gratia cogita filum illum (exempli adducti ab illis doctissimis viris) cui pon dus appensum est, æqualem esse axi orizontis, hoc est eius extremitatem immobilem esse in primo mobili, & in ipso zenit tui orizontis, tunc arcus motionis ipsius lapidis per tantum interuallum, quantum est diameter terræ, insensibiliter differret à linea recta, & cum lapis distans à centro mundi per semidiametrum terræ, iret rediretq;, vt scis, ergo idem faceret si filum longius esset per dictum terræ semidiametrum, ita vt posset ipsum centrum attingere, nam differentia illa semidiametri terræ, ferè nulla est respectu semidiametri ipsius primi mobilis.

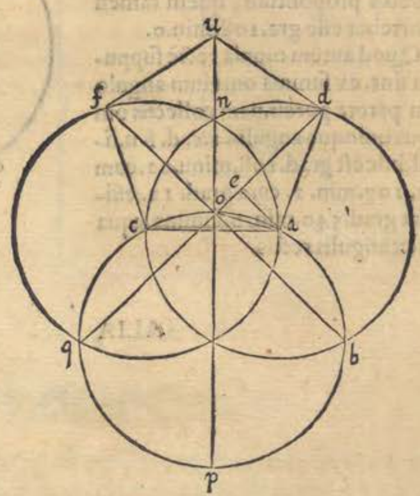
## AN PENTAGONVS AB ALBERTO DVRERO descriptus æquiangulus sit.

### *Conrado Neubart.*

SI non credis Pentagonum ab Alberto Durero super datam lineam designatum, æquiangulum non esse. Fingamus hic subiectam figuram similem ei quæ à Durero ponitur, in qua primò, ducta sit linea. o. a. & habebimus angulum.a.o.b.graduum.60.talium qualium duo recti fuerint gra. 360. vel.30.talium qualium duo recti fuerint. 180.nam ex supposito,arcus. a. b. est sexta pars totius circunferentiæ,angulus vero.b.o.d.rectus est,eo quod.b.o.q.rectus etiam sit,quare angulus.d.o.a.residuus ex recto erit graduum.60.talium, ut rectus est. 90.angulus verò.o.a.c.erit gra.15.eorundem.

Ducatur deinde perpendicularis.a.e.ad.o.d.quæ vt sinus anguli.a. o. e. erit partium.86602.talium qualium.a.o.erit.100000.quæ quidem.o.a.vt chorda arcus. a. o.est partium.51762.talium qualium.a.d.vel.a.c.semidiameter est.100000.

Nam sinus dimidij arcus.a.o. (existente.a.o.graduum. 30.) est partiũ. 25881. ex quo. a. e. erit partium. 44827.talium qualium. a. d. erit 100000.vnde angulus.a.d.o.cuius sinus est.a.e.erit graduum.26.min.38 qui quidem angulus, sumptus cum angulo.a.o.d.erit gra.86.min.38.Dempta denique hac summa ex duobus rectis gra.180.reliquum erit gra.93. min.22.idest angulus.o.a.d. cui additus cum fuerit angulus.o.a.c.gra. 15. talium, habebimus angulum.c. a. d. graduum.108.min.22.exuperantem verum angulum pentagoni per min. 22.vel sic, cum inuentus fuerit angulus.a.d.o.gra.26.min.38. si ex vno recto demptus fuerit, relinquetur angulus.d.a.e.gra.63.min. 22. qui quidem collectus cum fuerit cum angulo.e.a.o.residuo ex recto dempto angulo.a.o.e.grad.60.qui.e. a. o. est grad. 30. &



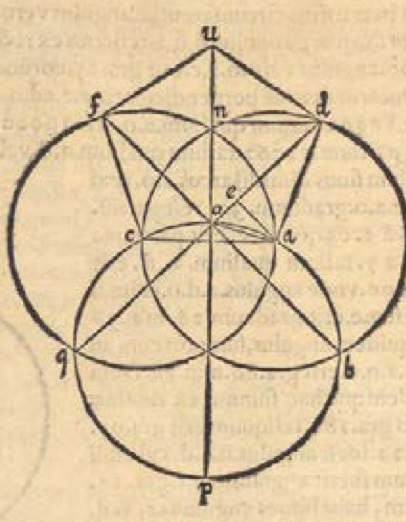

Aaa etiam

etiam collectus cum angulo.o.a.c.grad.15.hi tres anguli efficient angulum. d. a. c. dictum grad.108.min.22.

## *Examinatio anguli . u .*

Ducatur.d.n. quam quidem.d.n.cognoscemus vt sinus anguli.d.o.n.gra.45.nam angulus ei contrapositus.q. o.p. est dimidium recti, quare.d.n.erit partium. 70710. talium qualium.d.o. fuerit.100000.sed.d.o.est partium.115270.qualium. a. d. est. 100000.nam.c.d.vt sinus anguli.c.a.d.gra.63.min. 22.est partium.89389. o. c. vero est partium.50000.talium qualium.a.o.est.100000.vt sinus anguli.c.a.o.gra.30. sed vt.a.o. est partiũ. 51762.hoc est vt.a.d.est.100000.ipsa.o.c.erit partiũ.25881 quæ iuncta cum fuerit cum.c.d.efficiet.d.o.partium.115270.vt dictum est, quapropter cum. d.n. sit partium.70710.talium qualium.d.o.fuerit.100000, ipsa. d. n. erit partium.81507.talium qualium.d.o.erit.115270. idest qualium.d. a. vel. d. u. erit 100000.quæ quidem.d.n.est sinus anguli.d.u.n. graduum scilicet.54. 36. cuius duplum erit gra.109.mi.12.debebat tamen esse.108.m.o.

## *Examinatio anguli . d .*




Accipe angulum. a. d. o. gra.26. min. 38. vt supra, cui applica angulum.o.d.n.gra.45.min. o. simul cum angulo. u. d. n. residuo ex recto graduum.35.minu. 24. & conficies angulum.a.d.u.grad. 107. minu. 2. & habebis propositum, quem tamen oportebat esse gra.108.min.o.

Quod autem omnia rectè supputata sint, ex summa omnium angulorum patere potest.nam collectis omnibus quinque angulis. a.c. d. f. u.simul,hoc est grad.108.minu.22.cum gra. 107.min. 2. cum grad. 12. efficient grad.540.min. o.summa æqualis sex angulis rectis.

ALIA

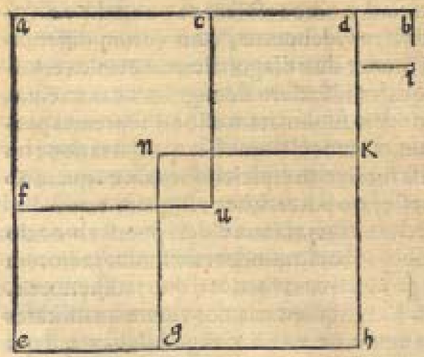
# ALIA DEMONSTRATIO NONÆ, ET DECIMÆ ſecundi Euclidis.

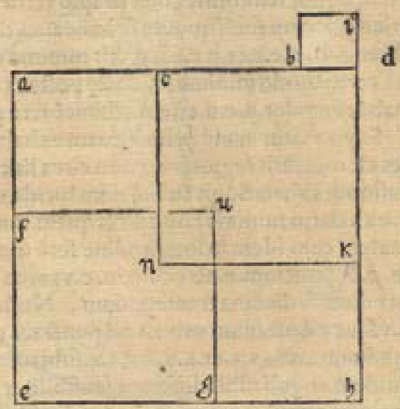
## *Petro Catenæ.*

QVAMVIS nona ac decima ſecundi Euclid. aliter à Comandino & Maurolico demonſtratæ fuerint, nihilominus mihi etiam viſum eſt non nihil meo moræ in eas tibi ſcribere, vt ſenſibiliter quoque cognoſcas illas veras eſſe.

Eſto linea.a.b. pro nona propoſitione, diuiſa per æqualia in. c. per inæqualia verò in.d. quadratum autem. a. d. ſit.d.e. quadratum verò. d.b. ſit.d. i. quadratum. a. c. ſit. c. f. & quadratum.c. d. ſit.c. K.clarum enim erit. K. h. æqualem exiſtere ipſi.a.c. ſeccetur igitur. e. h. in. g. ita vt. h. g. ęqualis exiſtat ipſi. K. h.vnde.g. e.æqualis erit c. d. perficiatur etiam quadratum.h. n. vnde in totali quadrato.a. h. habebis duplũ quadrati partis. c. d. nempe. c. K. et. f.g. & quadratum.a. u. cum gnomone.u.g.h.k. cui deficit quadratum æquale. d.i.quadrato, vt ſint etiam duo quadrata partis.a.c.

In decima aũt propoſitione, quadratũ totalis lineæ.a.d.ſit.d.e.& lineæ.b.d.ſit.b.i.et c.d.ſit. d.n. et.a.c.ſit.c.f.et.f.e.ſit.e.u.vnde.n.u.æquale erit quadrato.b.i.vnde in quadrato totali.a.h.videbis duo quadrata æqualia.f.c.et.g.k.partis.a.c.& quadratum.c. K.cum gnomone.n. f.e.g.cui addito quadrato.b.i.habebis duplum quadrati partis. c. d.




# DE STELLA CASSIOPEIÆ.

## *Annibali Raymundo Aſtrologo Peritiſsimo.*

POSTQVAM tua doctiſſima ſcripta perlegi, conſideraui, quod ſi à multitudine exhalationum in regione elementari acciderit anno. 1572. & 1573. vt totos ſex menſes ab omnibus per vniuerſum terrarum orbem viſa fuerit ſtella illa, quæ eſt in angulo ſeptentrionali quadrilateri Caſſio peiæ tam lucida, vt ipſo lucifero videretur rutilantior atq; cæterarum (abſque vlla aſpectus diuerſitate) magis ſcintillans. Quî fieri poterat, vt ſtellæ quæ ab illa parum diſtant, alioqui multo maiores, non etiam illa clariores apparuerint? ſed ſi aliquis diceret eam exhalationem non ita fortaſſe dilatari, vt inter nos, & aliam aliquam ſtellam interponeretur. Tunc ego reſponderem neceſſariò ſequi debere talem exhalationem, tantam latitudinem occupare, quod aliquibus populis aliam aliquã ſtellã circunuicinã hac ipſa de qua mentionem fecimus redderet lucidiorem. Sed cum hoc perſpectum fuerit nulli, ſequebatur lucem illam ab ipſis exhalationibus elementaribus haud poſſe oriri: quod nobis ſcintillatio illa maxima permagno fuit inditio, ſi phas eſt credere, nã quo magis aliquod cœleſte corpus ſcintillat, eo longius à nobis diſtare.

Verum quoniam efflagitaſti à me vt aliquid circa huiuſce rei ſpeculationem tibi ſcribam, idcirco tibi morem gerere volens paucis ſubiungam.

Conſidera primo hanc ſubſcriptam primam figuram, in qua. c. a. e. ſignatur pro Globo terreſtri cuius.i.centrum ſit et.u.o.n.pro conuexo ignis, ſed.K. x. s. pro orbe octauo.x.autem pro ſtella iam ſuperius dicta, quæ ſemper fuit, eſt, & erit, quamuis cæteris tribus nunc obſcurior ſit. Accipiantur deinde duo loca in ſuperficie terræ, quę ſint.c.et.e.diametraliter inuicem oppoſita, ita quod circa eorum orizontes poſſibile ſit ſtellam.x.videre, radijs ipſius ſtellæ mediantibus.x.n.e.et.x.u.c.quorũ partes.n.c.et.u.e. ita breues ſint, reſpectu eorum totorũ, vt vix ſexcenteſima pars ſit vnaquæq; illarum, nec non.c.e.ita breuis reſpectu ſemidiametri octauæ ſphæræ, quod vix ſit vna ex partibus decemmillibus, vt ſcis, ſequitur quod recta terminata ab. u. et. n. minor ſenſibiliter non ſit ipſo terræ diametro. c. e. cum duo hæc interualla ex triangulorum ſimilitudine ſe habeant vt. x.i.ad.x.o.hoc eſt ferè vt.602.ad.601.vnde anguli. n.e.c.et.u.c.e.à rectis minime differre videbuntur, cum eorum differentia certo modo minima ſit. ductę poſtea cũ fuerint duæ diagonales.e.u.et.n.c.terminabũt angulos.n.e.u.et.c.n.c.inuicẽ ferè ęquales, idẽ aſſero de angulis.u.c.n.et.e.u.c.

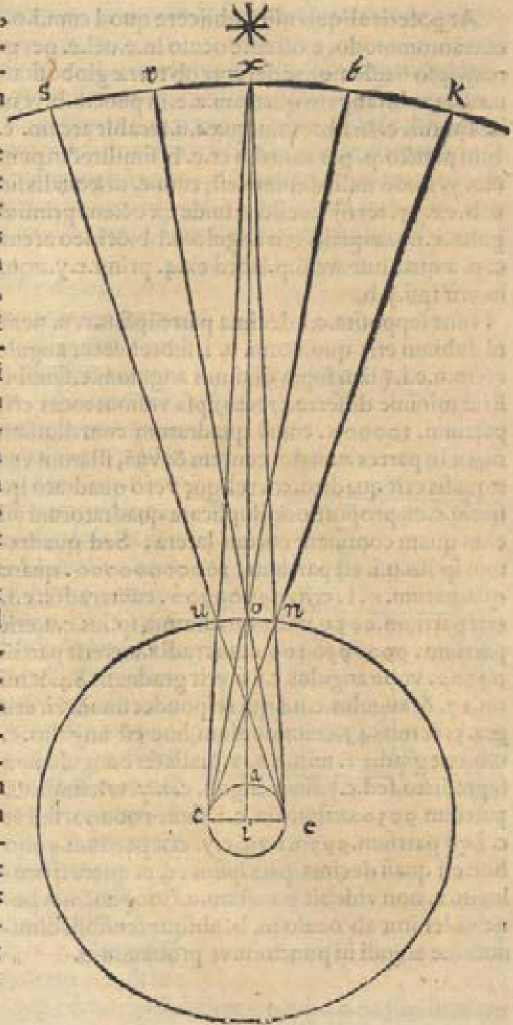
Supponatur nunc primò tuam exhalationem ſublimatam eſſe ad ſupremas partes elementaris regionis circum circa lineam.o.i:tunc clarum eſſet quod ſi ratione huiuſmodi exhalationis ſtella. x.ita lucida viſa fuerit tam aſpicientibus ab. e. quam ab c.exhalatio minoris latitudinis quam.u.n.eſſe non poterat, hoc eſt, quam terræ diameter, cum idem in longitudine ferè ſit, ſed punctum.u.ſatis videri poteſt ab oculo in. c.& punctum.n.ab oculo in. c.vt alias tibi probaui, ratione refractionis radiorum per diuerſa diafana tranſeuntium. Nunc producti cum fuerint ij duo radij.e.u.et.c. n.vſque ad octauum orbem ad puncta.s. et. K. reliquum erit nos videre quantitates graduum arcus.s.x.et.k.x, ſed.s.x.ſubiacet angulo.s.e.x.et.k.x.angulo.k.c. x. qui qui quidem anguli nihil differunt ſenſibiliter ac ſi eſſent in centro.i.Et cum ſuperius dixerimus angulos.s.e. x. et. k. c. x. ſenſibiliter minime differre ab angulis.c.n.e.et.e. u.c. ſi cognouerimus quantitatem iſtorum, cognita etiam nobis erit quantitas illorum.

Cum igitur ſemidiameter elementaris regionis maior ſit ſemidiametro terræ, vt 33.ad vnum, & cogitata.c.n.vt dicta ſemidiameter, quia ſenſibiliter ab ea minime differt, nunc ſi ſupponatur dicta.n.c.vt baſis triãguli orthogonij eſſe partiũ.100000 & dixerimus ſi.c.n.vt partium.33. præbet nobis. c. e.duarum partium, quid nobis pręſtabit eadem.c.n.vt partium.100000.vnde proueniet nobis.c.e.vt partiũ.6060. cuius angulus.c.n.e.erit graduum.3.& min.29.ita etiam erit angulus.k.c.x.cuius ar-

cus

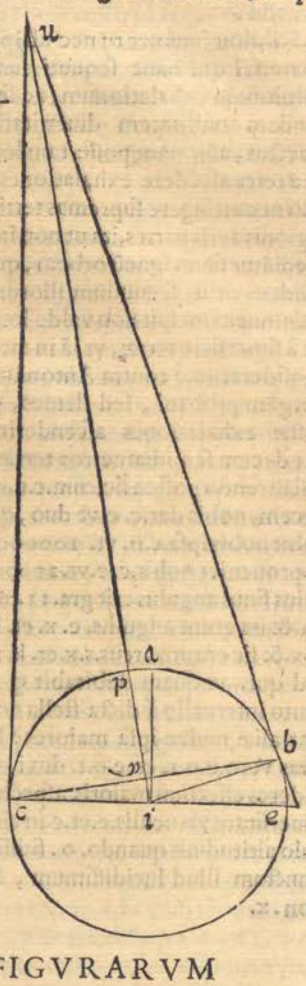
cus.s.x. eorundem graduum erit, & minutorum.idem dico de arcu. x.k. Sed circa dictam stellam omnes aliæ non distant huiusmodi interuallo.

Nihilominus nec tu nec alij peripatetici qui hanc sequuti sunt opinionem exhalationum, ad seruandam nullitatem diuersitatis aspectus, affirmant posse tam longe à terra ascẽdere exhalationes, imo nec attingere supremas tertię regionis aeris partes, ita ut non ingrediãtur suum igneũ orbem, qui quidem orbis secundum illorum opinionem incipit non valde longè à superficie terræ, vt iã in mea consideratione contra Antonium Bergam probaui, sed demus, ꝙ dictæ exhalationes ascenderint per decem semidiametros terræ, discurrendo postea sic, cum.c.n.ut decem, nobis dat.c. e.vt duo, qd dabit nobis ipsa.c.n. vt. 100000. & proueniet nobis.c. e.vt. 20000. cuius sinus angulus erit gra. 11.mi. 32.& ita erunt anguli.s. e. x. et. k. c. x.& sic eorum arcus.s.x.et. k. x. sed quis vnquam dubitabit ꝙ in tanto interuallo à dicta stella non sint aliæ multæ ipsa maiores? lineas vero.e.o.r. et. c.o.t. duxi, vt videres effectum maioris aspectus diuersitatis ab oculis.e.et.c.in circulo altitudinis quando. o. fuisset punctum illud lucidissimum, & non.x.




Ac

At poterit aliquis mihi obijcere quod cum.i.o. fuisset longior.i.e. per decem vices tantummodo, existente oculo in.e.uel.c. per gradus.90.ab.a.tunc punctus.u.vel n.ab ipso oculo non videretur ob terræ globositatem. Imaginemur igitur à puncto u.recta.u.b.tangens quartam.a.e.in puncto.b.vt in secunda figura videre est, in qua ducantur. c. b: i.b: et.i.u.quæ.i.u.secabit arcum. c. b.in puncto.p. per æqualia et.c. b. similiter in puncto. y. quod nulli dubium est, cum.c. u. æqualis sit. u. b.ex.35. tertij Euclidis, unde ex octaua primi angulus.c.i.u. æqualis erit angulo.u.i.b.& ideo arcus. c. p. æquabitur arcui.p.b.sed ex.4. primi.c.y.æqualis erit ipsi.y.b.




Nunc supposita.c.i.decima parte ipsius.c. u. nemini dubium erit quod cum. u. i. subtendatur angulo recto.u.c.i. ( iam supra diximus angulum.c.sensibiliter minime differre à recto)ipsa vt sinus totus erit partium. 100000. cuius quadratum cum diuisum fuerit in partes æquales centum & vnã, illarum vna æqualis erit quadrto.c.i. reliquę vero quadrato ipsius.u.c.ex proportione duplicata quadratorum ad eam quam continent eorum latera. Sed quadratum ipsius.u.i. est partium. 10000000000. quare quadratum. c. i. erit. 99009900. cuius radix. c.i. erit partium.9950.vnde quadratum ipsius.c.u.erit partium. 9900990100.cuius radix.u.c.erit partiũ 99500. vnde angulus.c.i.u.erit graduum.84.& minu.17. & angulus.c.u.i.qui respondet sinui.c. i. erit gra.5. & min.43.cuius duplum,hoc est angulus. c. u.b.erit grad.11. min. 26. æqualis ferè angulo iam supradicto.sed.c.y.sinus anguli. c. i. y. erit similiter partium.99500.talium vt.c. i. sunt. 100000. sed vt c. i.est partium.9950. tunc.c.y. erit partium.9900 hoc est quasi decima pars ipsius. c. u. quare si oculus in. e. non videbit punctum.u. hoc punctum bene videbitur ab oculo in. b. absque sensibili diminutione anguli in puncto.u.vt probauimus.

## DE MAGNITVDINIBVS FIGVRARVM isoperimetrarum.

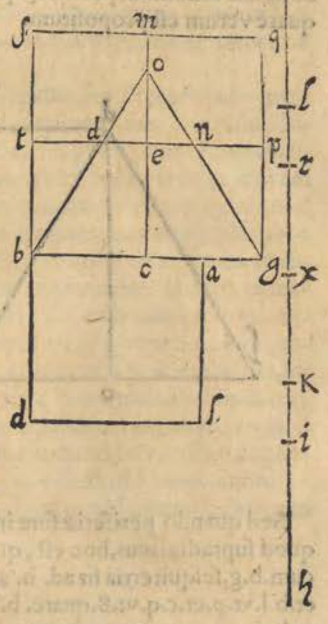
### *Domino Ioanni Mariæ Agatio.*

QVAMVIS à Theone supra Ptolomei Almagestum sufficienter traditum sit de magnitudinibus figurarum isoperimetrarum, nihilominus vt tibi morem geram, ea nunc scribo, quæ mihi in mentem venerunt contra Alexãdrum Piccolhomineum, antequã aliquid ipsius Theonis vidissem Alexan-

Alexander Piccolhomineus in libro primo de mundi ſphæra vbi tractat de cęli rotunditate, ita inquit.

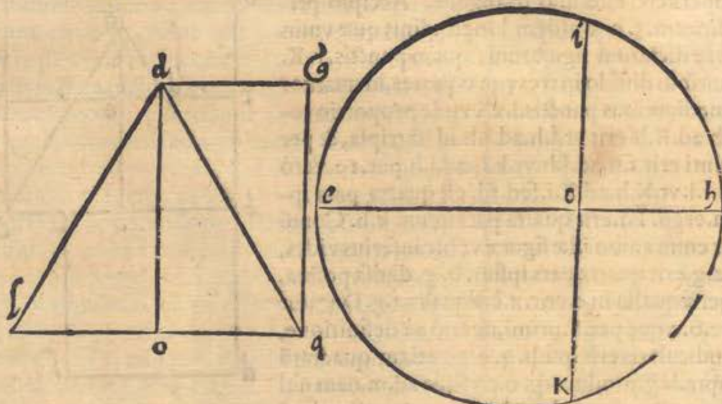
» Oltre di queſto, douendo il decimo cielo contenere & in ſe chiudere tutte le co-
» ſe, è conueneuol coſa il penſare, che foſſe fatto di quella più capace figura che eſ-
» ſer poſſa, la qual è la figura rotunda, però che ſi può trar da molti luoghi d' Euclide
» che ſi come ſe noi ci immagineremo più figure ſuperficiali talmente che tutte le li-
» nee de l'vna congionte inſieme, ſieno vguali à tutte le linee pur inſiememente com
» poſte di qual ſi voglia de l'altre figure, ne ſeguirà, che quella figura ſarà più capa-
» ce la qual haurà manco angoli, & quella capaciſſima che ſarà ſenza alcuno come è
» la figura circolare, &c.

Cogitemus igitur primò de triangulo æquilatero & quadrato iſoperimetris, ſit enim triangulus æquilaterus.o.b.g. quadratum verò.b.l. quorum periferiæ inuicem æquales ſint. Dico quadratum maioris ſuperficiei eſſe ipſo triangulo. Accipio primum lineam. f. h. eiuſdem longitudinis quæ vnius periferiæ dictarum figurarum, quam punctis. r. K. mediantibus diuido in tres ęquas partes, in quatuor verò mediantibus punctis.l.x.i. vnde proportio totius.f.h.ad.K.h.erit vt.l.h.ad.i.h.ideſt tripla, & per 16.quinti erit.f.h.ad.l.h.vt.k.h.ad.i.h.per.19.verò f. h.ad.f.l.vt.K.h.ad.K.i.ſed.f.l. eſt quarta pars ipſius.f.h.ergo. k.i.erit quarta pars ipſius. k.h. Coniũgantur enim ambo iſtæ figuræ vt hic inferius vides, vnde.a.g.erit quarta pars ipſius. b. g. diuiſa poſtea. b. g. per æqualia in.c.erit. a.c.æqualis.a.g. Ducatur deinde.o.c.quę per.8.primi, nec nõ ex definitione, perpendicularis erit ipſi.b. g. ergo etiam quadratũ b q.ſupra.b.g.producoq;. o.c.vſque ad.m.nam nulli dubium eſt quin.o.c.breuior ſit.o.g.ex.18.vel.48 primi cui æquatur.q.g.diuido etiam.c. m.per æqualia in puncto. e. ducoq;.t.e. p. æquidiſtantem. b.g. vnde habebimus duo quadrata. e.g.et.e.b. ſed quadratum. b.l. æquatur quadrato ipſius.c.a. cum duplo illius quod fit ex. b. c. in. c. g. vt patet ex.9. ſecundi, hoc eſt æquatur quadrato.c. a. & rectangulo.t. g. Deinde vt ſe habet.p.g.ad.o.e.ita ſe habet.u.p.ad.u.e.ex ſimilitudine triangulorum. Sed.p.g.maior eſt ipſa.o.e.cum.p.g.æqualis ſit. e. m. quare triangulus.u.g. p. maior erit triangulo.o.e.u.ex.17.ſexti. Similiter dico maiorem eſſe triangulum.b.d.t.triangulo.e.o.d.vnde ſequitur rectangulum.t.g.maiorem eſſe triangulo.b.o.g. ſed quadratum. b.l.eſt etiam maior ipſo rectangulo.t.g.ex quadrato ipſius c.a.vt diximus, tanto igitur maior erit triangulo.b.o.g.




Poſſumus

Poſſumus etiam probare quod periferia quadrati æqualis triangulo æquilatero minor ſit periferia ipſius trianguli æquilateri. Cogita triangulum æquilaterum hic ſubſcriptum.d.l.q. cuius baſis. l.q. diuiſa ſit per æqualia à perpendiculari. d. o.deſcriptũq; ſit rectangulum.o.g.quod æquale erit triangulo.d.l.q. ſed periferia trianguli maior eſt periferia rectanguli,nam.l.q.æqualis eſt.o.q.cum.d.g.ſed.q.d.maior eſt.o.d. ex. 18. primi,vnde.l. d.maior etiam.q.g.cum ex.34.dicti latera oppoſita ipſius rectanguli ſint inuicem æqualia, accipiamus poſtea. e.c.æqualem.o.d. et. c. h. indirectum æqualem.o.q.circa quem diametrum.e.h.intelligatur circulus.e.i.h.k.et.à puncto.c. dirigatur perpendicularis.k.i.ad.e.h.vnde ex.3.tertij.c.i.æqualis erit.c.k.& ex 34.quod fit ex.c.i.in.c.k.hoc eſt quadratum ipſius.c.i.æquale erit ei quod fit.ex. e.c. in.c.h.hoc eſt rectangulo.g.o.hoc eſt triangulo.d.l.q. ſed.e. h. eſt dimidium periferię ipſius rectanguli.g.o.quæ minor eſt di midio periferiæ trianguli.d.l.q.vt vidimus et.i.k.eſt dimidium periferię quadrati ipſius. i.c.&minor etiam ipſa.e.h.ex. 14.tertij quare verum eſt propoſitum.




Sed quando periferiæ ſunt inuicem æquales,poſſumus etiam breuiter videre id quod ſupradiximus,hoc eſt, quod quadratum,maius ſit triangulo æquilatero. Nam cum.b.g.ſeſquitertia ſit ad. b. a. ergo. b. g. erit vt.4.et.b.a.ut.3.vnde.b.q.erit vt.16 et.b.l.vt.9.et.c.q.vt.8.quare. b.l.maius erit ipſo rectãgulo.c.q.ſed. c.q.maius eſt triãgulo.b.o.g.cum.q.g.quæ æqualis eſt.o.g.maior ſit.o.c.ex.18.vel penultima primi, nam ſi.q.g.æqualis eſſet.o.c.tunc.c.q.æqualis eſſet triangulo.b.o.g.ex.41. primi.

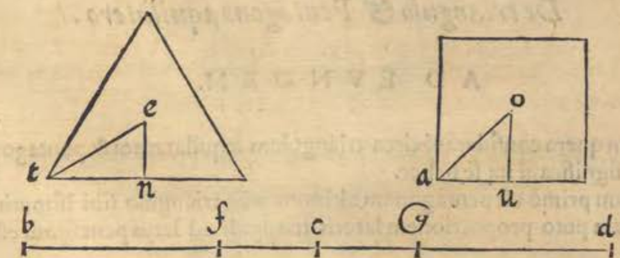
Alia etiam via maiores noſtri vſi ſunt quæ generalis eſt vt in Theone ſupra Almageſtum videre eſt, medijs perpendicularibus à centris ad latera figurarum, ſed quia differẽtia longitudinum ipſarum perpendicularium alio medio inueniri poteſt, eo quo ipſi vſi ſunt, prætermittere nolo quin tibi ſcribam.

Ego enim ita diſcurro.

Sint duæ figuræ iſoperimetrę æquilaterę & æquiangulæ, puta primò triangulum & quadratum quorum centra ſint. e. et. o. à quibus centris ad latera ſint perpendiculares.e.n.et.o.u.vnde.n.et.u. diuident latera per æqualia vt ſcis, ducantur poſtea.e.t.et.o.a.ad angulos dictorum laterum,vnde habebimus angulum.o.a. u. dimidiũ recti, et.e.t.n.tertia pars vnius recti, vt ex te ipſo videre potes, quare angulus a.ſeſquia.ſeſquialter erit angulo.t.quod vt clarius videas cogita lineam.b. d. cuius medietas ſit.c.d.tertia verò pars illius ſit.g.d.tunc dico.c.d.ſeſquialteram eſſe ipſi.g d.ſit enim f.d.duplum ipſius.g.d.quare.f.d.erunt duæ tertiæ totius lineę.b.d.& quia eadem proportio eſt totius. b.d.ad.c.d.quæ.f.d.ad.g.d.ergo permutando eadem erit totius . b. d.ad.f.d.quæ.c.d.ad.g.d. Sed.b.d.ad.f.d.ſeſquialtera eſt,verum igitur erit quod angulus.a.ſeſquialter ſit ipſi.t.deinde.t.n.eſt ſeſquitertia ipſi.a.u. vt ſuperius vidimus. in eorum duplis . ſcimus etiam.n.e.eſſe dimidium ipſius.t.e. eo quod cum. e. t. n. ſit tertia pars vnius recti,angulus.t.e.n.erit duo tertia vnius recti, vnde. e. n . erit latus . exagoni æquilateris inſcriptibilis circulo cuius diameter ſit.e.t.quare.e.t. dupla erit ipſi.e.n.in longitudine, ſed quadrupla in potentia:t.n. vero tripla in potentia ipſi.n. e.ex penultima primi , quæ omnia etiam ex.8.tertijdecimi.Eucli. elicere potes , ſed t.n.erat ſexquitertia ipſi.a.u.in longitudine, hoc eſt ipſi.o.u.nam.o.u.æqualis eſt ipſi a.u.quare.n.t.erit minus quam dupla in potentia ipſi.o.u.hoc eſt, vt. 16. ad. 9. ergo maior proportio erit ipſius.t.n.in potentia ad.n.e.quam ad.o.u. quare etiam in longitudine , maior proportio erit ipſius.t.n.ad.n.e.quam ad.o.u.vnde.o.u. longior erit ipſa.n.e.quod eſt propoſitum .

Sed ſi.o.a.u.eſſet pentagonus æquilaterus & æquiangulus,ſimiliter probabo perpendicularem.o.u.longiorem eſſe.n.e.ipſius trianguli æquilateri,dummodo ſint iſoperimetrę. Sit enim.a.u.dimidium lateris pentagoni ex ſuppoſito,cuius centrum ſit o.tunc proportio.t.n.ad.a.u.erit ſuperbipartienstertias,vt ex ordine iam hic ſupradicto à te facillimè elicere potes, hoc eſt,vt.5.ad.3. et. a. u. minor erit. o. u. eo quod angulus.o.minor erit angulo.a.nam angulus.o.erit quinta pars duorū rectorum,hoc eſt duæ quintæ vnius recti , vnde angulus. a . reſiduum vnius recti erit tres quintæ vnius recti, quare angulus.a.maior erit angulo.o. & conſequenter latus.o. u. maius latere.a.u. ſed.t.n.minor eſt quam tripla in potentia ad.a.u.eo quod erit vt. 25. ad.9.cum in longitudine ſit vt.5 .ad.3.ſed dicta.t.n.tripla eſt in potentia ad.e.n.quare.a.u.maior erit ipſa.e.n.ſed.o.u.maior eſt ipſa.a.u.vt diximus, igitur multo magis. o.u.maior eſt ipſa.a.u.vt dixim⁹ & cōſequēter multo magis. o.u.maior erit ipſa.n.e.

Quotieſcunque enim cognoſcimus proportionem anguli.o.ad angulum. a. quod quidem facillimum eſt, nec non proportionem.t.n.ad.a.u.quod,etiam illico cognoſcitur , tunc ex ſcientia cordarum & arcuum omnia etiam facillimè innueniuntur. Verum circa triangulū æquilaterum,& pentagonum,alium modū inueni,ſed aliquantulum prolixiorem.

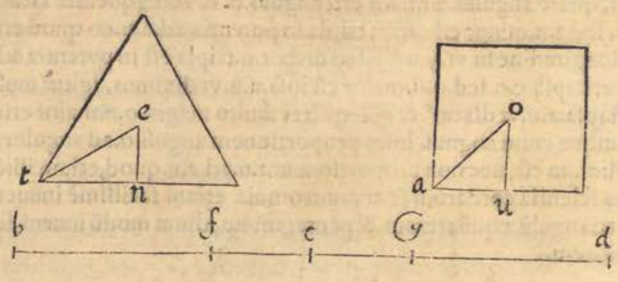


## *De incommmensurabilitate, in longitudine perpendicularis trianguli æquilateri cum eiusdem latere.*

### AD EVNDEM.

ID quod à me postulas est omnino impossibile, velles enim duos numeros inueniri inter se ita se habentes, vt se habent perpendicularis in triangulo æquilatero cum vno eius laterum, quod vero hoc fieri non possit, considera in figura præcedenti triangulum æquilaterum. d. l. q. cuius perpendicularis sit. d.o.quæ diuidit. l. q. per æqualia in.o.vnde ex.4.secundi Euclidis,quadratum.l.q. ( idest. d. q. ) quadruplum erit quadrato.o.q.& ex penultima primi ęquale quadratis.d.o.et.o.q.quare erit sesquitertium quadrato ipsius.d. o. & ita quadratum.d.o.erit triplum quadrato ipsius. o. q. hæc autem proportiones non sunt vt numeri quadrati ad numerum quadratum quod si ita fuissent,sequeretur ternarium numerum esse quadratum ex . 22 . octaui . Cum igitur non sint vt numeri quadrati ad numerum quadratum , sequitur ex septima decimi.d.o.esse incommensurabilem ipsi.l.q.seu. d.q.in longitudine.

Vel dicamus ita , proportio quadrati ipsius.l.q.ad quadratum ipsius.o.d.est in genere superparticulari , cum sit sesquitertia , vnde quadratum ipsius.d.o.numeris dari non potest, eo quod si dabilis fuisset , sequeretur , quod inter quadratum ipsius. l. q.& ipsius.d.o. esset aliquis numerus medius proportionalis ex.16. octaui, vnde ex octaua eiusdem vnitas diuisibilis esset, quod fieri non potest.




## *De triangulo & Pentagono æquilatero.*

### AD EVNDEM.

MOdum quem consideraui circa triangulum æquilaterum & pentagonum , vt tibi significaui ita se habet.

Probandum primò est pentagonum altiorem esse triangulo sibi isoperimetro. Iam tibi notam puto proportionem lateris trianguli ad latus pentagoni esse vt . 5 . ad. 3.

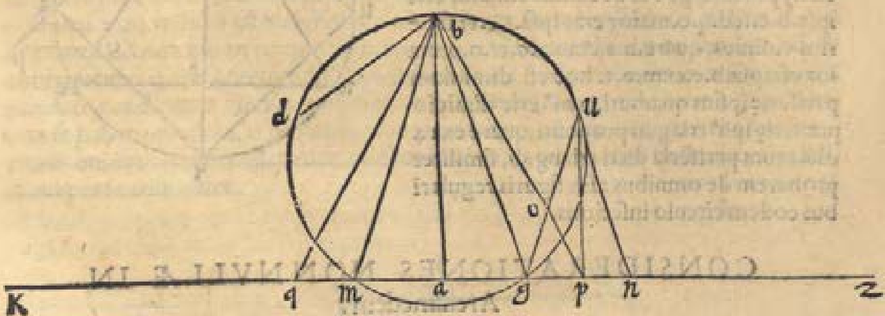
Sit igitur pentagonus.b.d m. g.v. cuius periferia distenta sit. K.z. basis autem. m. g. bifariam diuisa sit in puncto.a.ductaq;.a.b: b.g.et.b.m. clarum erit.a. b. perdicularem esse ad.m.g.ex.8. primi Eucli.cum.b.m.et.b.g. (bases triangulorum. b.d. m.

et.b.u.

et. b.u.g.) sint inuicem æquales ex. 4. eiusdem.

Accipiatur deinde vel intelligatur. g.p. æqualis duabus tértijs ipsius.a.g.ducatur que.b.p.quam probabo maiorem esse duplo ipsius.a.p. vnde maior erit latere ipsius trigoni æquilateri, cuius dimidium est.a.p. scimus enim ipsum latus se habere ad.m.g. vt quinque ad.3. ita etiam.a.p.ad.a.g. vt diximus.

Cum auté angulus. a.b.g.sit quarta pars anguli.b.g.a.ex.10.quarti & quinta pars vnius recti ex.32.primi, dictus angulus erit graduum.18.et.a.g.erit partium.30902. et. a. b. partium.95015.et.a.p.51503.vnde ex penultima primi latus.b.p. erit partium.108075.duplum vero ipsius.a.p. erit. 103006, latus igitur dicti trigoni, quod ab.p.erigitur, secabit perpendicularem.a.b.sub.b. hoc est inter.b.et.a. ex penultima primi. Finiatur enim triangulus æquicrurus.b.q.p.quem probaui maiorem esse æquilatero isoperimetro pentagono proposito, ducaturq́;.u.p.ducatur etiam. u. n. parallela ipsi.b.g.quæ concludet triangulum.g.u.n. similem triangulo.m.b.g.eo quod cum angulus.m.b.g. æqualis sit angulo.b.g.u.ex.16.tertij, per.27.primi.m.b.et. g.u. erunt inuicem æquidistãtes, vnde angulus.b.m.g.æqualis erit angulo.u.g.n.et.ex.29. angulus.g.u.n. æqualis erit angulo.u.g.b.quare etiam angulo.g.b.m.& angulus. u.n. g.angulo.b.g.m.ex.32.eiusdem, vnde ex. 4. sexti proportio.g.n.ad.g.m.erit.vt. g.u. ad.m.b.sed cum.g.u.maior sit dimidio ipsius.b.g.ex.20. primi, hoc est maior dimidio ipsius.b.m.ergo.g.n.etiam maior erit ipsa.g.a. quapropter maior erit ipsa. g. p. cum.g.p.minor sit ipsa.g.a.ex hypothesi, ducta deinde cum fuerit. b. n. habebimus triangulum.b.n.g.æqualẽ triangulo.b.u.g.& maiorẽ triãgulo. b.p.g. ex prima sexti vel quia totum maius est sua parte. Triangulus igitur.b.u.g.maior est triangulo.b.p.g.quare triangulus.b.u.o.maior erit triangulo.g.o.p.ex communi conceptu, idem infero ab alia parte dictarum figurarum. Quare pentagonus.b.d.m.g.u.maior erit triangulo.b.q.p.quem probauimus maiorem esse triangulo æquilatero sibi isoperimetro.




## *Comparatio periferiarum quadrati & trianguli æquilateri circunscriptorum ab eodem circulo.*

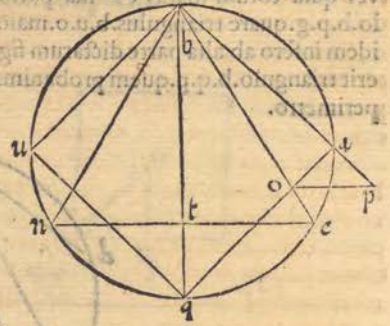
### AD EVNDEM.

QVod autem periferia quadrati in eodem circulo inscripti, in quo sit triangulus æquilaterus, longior sit periferia ipsius trianguli æquilateri, absque vllo negotio cordarum & arcuum possumus geometricè demonstrare quod valde desideras.

Quapropter sit circulus.b.a.e.q.in quo sit triangulũ æquilaterum.b.e.n.& quadratum. b.a. q.u. cuius periferiam probabo longiorem esse periferia trianguli. Sit enim diameter circuli.b.q.qui etiam erit diameter quadrati, vt à te scire potes. Sit etiam punctũ.b.commune tam anguli quadrati quam trianguli.vnde sequitur quod dictus diameter secabit latus.n.e.trianguli ad rectos & per æqualia in.t. Nam cum arcus.b.e.æqualis sit arcui.b.n.ex.27.tertij, remanet vt arcus.q.e.equalis sit arcui. q.n.vnde angulus.q.b.e.æqualis erit angulo.q.b.n.ex. 26. eiusdem. quare ex. 4. primi anguli ad.t.erunt recti,et.n.t.æqualis erit ipsi.t.e. vt diximus.

Deinde.b.e.et.q.a.seinuicem secãt in puncto. o.vt ex se clarum patet, ducatur postea.q.e.vnde habebimus angulum.b.e.q.rectum ex.30.tertij, quare ex.18.primi.q.o. longior erit ipsa.q.e.et.q.e.longior erit ipsa.e.t. quare.q.o. longior erit ipsa.t.e.

Vt probemus postea.b.a.o.longiorem esse ipsa. b.e. producatur.b.a. ita quod. a.p.æqualis sit ipsi.a.o.ducaturq́; o.p et.a.e.cum autem ex iam dicta.30. tertij angulus b.a.o.rect⁹ sit, erit angulus.o.a.p.similiter rect⁹ ex.13. primi, vnde ex.5.et.32. eiusdẽ angulus.a.p.o.erit dimidium recti,& similiter,ex ijsdem,angulus.b.q.a.est dimidium recti quare angulus.a.p.o.æqualis erit angulo.a.q.b.sed angulus.a.e.b.æqualis est angulo.a.q.b.ex.20.tertij, ergo angulus.b.p.o.æqualis erit angulo.b,e.a.angulus vero a.b. e.communis est ambobus triangulis.a.b.e.et.o.b.p.quare ex. 32. primi anguli. b.a.e.et.b.o.p. reliqui ex duobus rectis æquales inuicem erunt. Quare ex quarta sexti, et.18. quinti proportio.b.o.ad.b.p.erit, vt b.a.ad.b.e.sed ex.18. primi.b. o. maior est ipsa.b.a.quare ex.14.quinti.b.p.maior erit ipsa.b.e.sed.b.p.æquatur ipsis.b.a.cum.a.o ex hypotesi,ergo . b. a . cum.a.o.maior erit ipsa.b.e.sed.q.o.maior erat ipsa.t.e.vt superius vidimus, quare.b.a.cum.a.o.et.o.q maior est ipsa.b.e.cum.e.t. hoc est dimidium periferię ipsius quadrati, mai⁹ erit dimidio periferię ipsi⁹ triãguli propositi,quare ex 14 dicta tota periferia dicti trianguli, similiter probarem de omnibus alijs figuris regularibus eodem circulo inscriptis.




## CONSIDERATIONES NONNVLLÆ IN Archimedem.

*Doctißimo atque Reuerendo Domino Vincentio Mercato.*

QVOD tibi aliàs dixi verum est.intellectum scilicet non omninò quiescere circa illas duas Archimedis propositiones, quæ in translatione Tartaleæ sunt sub numeris. 4. et. 5 . & in impressione Basileæ sub numeris. 6 . et. 7. vbi tractat de centris libræ,ſeu ſtateræ: Aſpice igitur in.4. ſupradicta, quod cum appenſæ fuerint omnes illæ partes ponderum, partibus longitudinis ipſius.l.K.in qua volo vt à punctis.e.et.d.imagineris duas lineas.e.o.et.d.u.inuicem æquales, & ferè perpendiculares ipſi.l. K. hoc eſt reſpicientes mundi centrum; imagineris etiam. o. u. quæ ſit parallela ipſi.l.k. quæ diuiſa ſit in puncto.i. ſupra.g. Hinc nulli dubium erit, cum.g.fuerit centrum totius ponderis appenſi ipſi.l.K.quod.i.ſimiliter erit centrum cum directe locatum ſit ſupra.g.hoc eſt in eadem directionis linea, quod quidem non indiget aliqua demonſtratione, cum per ſe ſatis pateat. Vnde ex communi conceptu.o.erit centrum ponderis appenſi ipſi.l.h. et.u. erit centrum ponderis appenſi ipſi h.K. Scimus igit. i. eſſe cẽtrum duorum, hoc eſt ipſius.l.h. & ipſius.h.k.continuatorum per totam.l.k. Nunc ergo ſi conſideremus.l.k.diuiſam eſſe, hoc eſt diſiunctam in puncto.h.inueniemus nihilominus.i.centrum eſſe dictorum ponderum, & quod tantum eſt,ipſam eſſe continuã, quantum diuiſam in dicto pũcto.h. neque ex hoc, punctum.i.erit magis vel minus centrum duorum ponderum.l.h.et.h. k.quorum vnum pendet totum ab.o.aliud verò totum ab. u. & hoc modo in longitudine. o.u.diuiſa vt dictum eſt, habebimus propoſitum.

Reliquam propoſitionem tibi relinquo.

Illa verò propoſitio, quam tibi dixi Archimedem tacuiſſe in huiuſmodi materia eſt, quod ſi duo pondera æquilibrant ab extremis alicuius ſtateræ, in certis præfixis diſtantijs à centro. Tunc dico ſi eorum vno manente alterum moueatur remotius ab ipſo centro quod illud deſcendet, & ſi vicinius ipſi centro appenſum fuerit aſcendet. Hæc enim propoſitio quotidie omnibus in locis videtur, ipſam verò puto Archimedem prætermiſiſſe ob facilitatem, cum ab antedicta ferè dependeat.

Sit exempli gratia ſtatera.a.u.cuius centrum ſit.i.& pondera.u. a. appenſa, ſe inuicem habeant vt.i.u.et.i.a.ſe inuicem habent. Nunc dico quod ſi pondus ipſius.u. poſitum fuerit vicinius centro vt puta in.o. inmoto exiſtente pondere, a. quod brachium.i.o.u.aſcendet, & è conuerſo, ſi remotius poſitum fuerit, deſcendet.

Ponat ergo vt dictũ eſt in.o.vicinius cẽtro, quapropter brachium. i. o. breui⁹ erit brachio.i.u.vnde minor proportio erit ipſius.i.o.ad.i.a.quàm.i.u.ad eundem.a.i. & conſequenter quam ponderis ipſius.a.(quod ſit.n.e.) ad pondus ipſius.u.Quare ſi ex pondere.n.e.dempta fuerit.e.pars eius, ita quod reliqua pars.n.ſe habeat ad pondus o.vt ſe habet.i.o.ad.i.a. tunc ſtatera non mouebitur; addita verò parte. e. ex communi conceptu,a.deſcendet vnde.o.aſcenderet conuerſum verò ex ſimilibus rationibus per te concludes.

Ineo

In eo quod à me petis, mittendo te ad Eutotium, tibi non satisfacerem, cum Eutotius citet sextum librum Pergei, quem nunquam vidimus, supponatq; ea, quæ nec ipse nec alius vnquam quod scimus probauit.

Desideras enim demonstrationem illius quod Archimedes dicit inter primam, & secundam propositionem secundi libri, vbi tractat de centris grauium, propterea quod illud supponit pro manifesto.

Sit enim figura hic subscripta, ferè similis parabolæ positæ in. 2. propositione dicti libri, vt in impressione Basileensi habetur, sintq; diuisæ duæ.a.b.et.b.c. per æqualia à punctis.x.et.u. protractisq;.f.x.et.u.i.ad.b.d. quæ inuicem etiam erunt parallelę ex.30. primi Eucli. vnde ipsæ etiam, diametri erunt ipsarum portionum: vt ex eo colligere est, quod in.49. primi lib. Pergei probatur. Imaginando postea ad puncta.b.f.et.i. tres contingentes, manifestum erit punctum.b. illud esse quod terminat altitudinem huiusmodi portionis, et.f.et.i. terminantia altitudines partialium, ex.5. secundi ipsius Pergei, eo quod dictæ contingentes paralellæ erunt ipsis basibus, vnde trianguli inscripti, easdem habebunt altitudines, quas portiones ipsæ, quod erit ex mente Archimedis. Et sic deinceps poteris multiplicare angulos figuræ rectilineæ in parabola, quæ designata erit vt desiderat Archimedes, qui quidem dicit, quod protractæ cum fuerint aliæ deinceps post.f.i. ipsæ inuicem ęquidistantes erũt, diuisęque per æqualia ab.d.b. quod quãuis verũ sit, tñ ab Eutotio non satis demõstratũ est, cum supponat.a.f.b. æqualem esse ipsi.b.i.c. probare volens eius diametros æquales esse absque aliqua citata ratione, quæ quidem ratio esset conuersum.4. propositionis libri de conoidalibus. Sed oporteret nos etiã videre.6. librum ipsius Pergei, & propterea tibi non satisfacerem.

Esto igitur, ut inuenta sit linea. K. cuius productum in.u.i. æquale sit quadrato ipsius.u.c. inuenta etiam sit linea.h. cuius productum cum.f.x. æquale sit quadrato ipsius.a.x. vnde ex conuerso.49. primi ipsius Pergei, proportio ipsius.K.ad.b.c. erit ut ipsius.b.c.ad.b.d.& ipsius.h.ad.a.b, vt ipsius.a.b.ad.b.d. Erit igitur ex.1 . sexti Eucli. quadratum.b.c. æquale producto ipsius.K.in.b.d.& quadratum.a.b. æquale producto ipsius.h.in.b.d.& ex prima sexti, ita erit ipsius.K.ad.h. vt producti quod fit ex.K. in.b.d. ad productum ipsius.h.in.b.d. hoc est vt quadrati ipsius.b.c. ad quadratum ipsius.b.a. ex.16.et.11. quinti, hoc est vt quadrati ipsius.u.c. ad quadratum ipsius.a.x. hoc est ut productum ipsius.k.in.u.i. ad productum ipsius.h.in.x.f. Nunc si ipsius.k. ad.h. est vt producti ipsius.K.in.u.i. ad productum ipsius.h.in.f.x. ergo ex.24. sexti, & communi conceptu, proportio ipsius.k.ad.h. composita erit ex ea quæ ipsius.u.i. ad.f.x.& ex ea quæ ipsius.k.ad.h. Cum ergo dempta fuerit proportio ipsius.k.ad.h. (vt simplex) à proportione ipsius.k.ad.h. (vt composita) reliquum nihil erit. Quare.f.x. æqualis erit ipsi.u.i.

Sed quod.f.m. æqualis sit ipsi.m.i. Videto in Eutotio, quia hoc satis sui natura facile est.

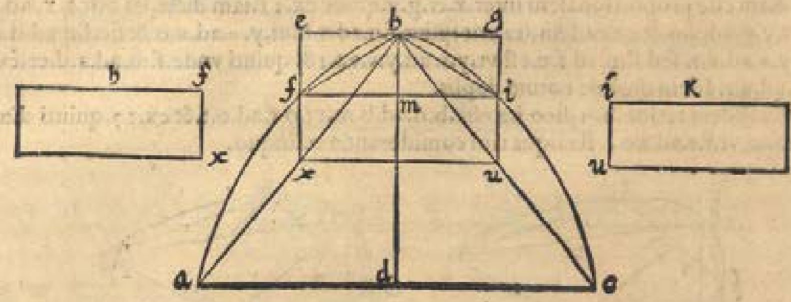
Sed accipe alium modum breuiorem ad probandum.f.x. esse æqualem ipsi.u.i.

Finge lineam.e.b.g. contingentem in puncto.b. prolungatisque diametris.f.x.et.u.i. vsque ad contingentem ipsam, habebis.f.e. æqualem ipsi.f.x et.g.i. ipsi.u.i. Ex.35. primi Pergei, producta postea.x.u. habebis ex.2. sexti Eucli.x.u. parallelam ipsi.a.c. sed.e.g. parallela est ipsimet.a.c. ex quinta secundi ipsius Pergei, quare ex.30 primi Euclid.e.g. parallela erit ipsi.u.x & ex.34. eiusdem æqualis erit.e.x. ipsi.u.g. vnde.f.x. etiam æqualis erit.u.i. ex communi conceptu.

Sed ne quid desideres probabo.f.m. æqualem esse ipsi.m.i. Iam igitur scis quod

cum

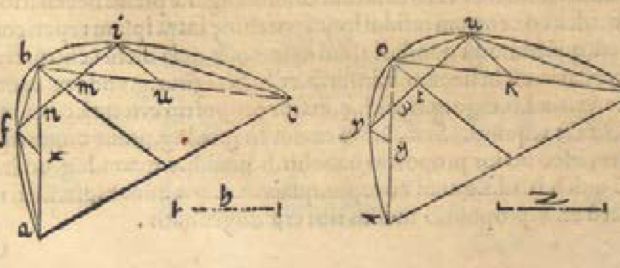
cum sit.f.x.æqualis ipsi.u.i. vt tibi probaui, & inuicem parallelæ ideo.f. i. parallela erit ipsi.x.u. ex.33. primi Euclidis. Vnde ex.30. eiusdem, parallela erit etiam ipsi.a.c.sed cum.x.u. diuisa sit ab. d.b. per æqualia, eo quod diuidit.a.c.eodem modo, quę ipsi parallela est ex. 2.sexti. Reliqua tibi consideranda relinquo. cum verò ambæ. f.x.et.u.i. parallelæ sint ipsi.b.d.sequitur quod cum ex.34.primi vnaquæq;.f.m.et. m.i. æqualis sit medietati ipsius.x.u. erunt inuicem æquales.



Minime dubitabam tibi non satisfacere Eutocium in. 3. propositione secundi lib.de centris Grauium Archimedis, cum citet.6. librum de elementis conicis, adde quod si aliud in ipso.6. libro ab eo citato non esset magis ad propositum, quàm eà quæ ab ipso citata sunt, nihilominus adhuc irresolutus maneres.

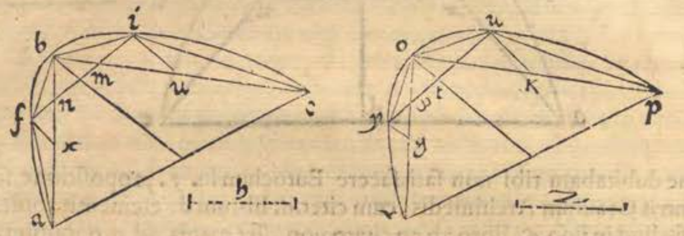
Considera igitur eandem ipsam figuram præcedentem; pro alia verò parabola simili dictæ, accipe secundam figuram ipsius tertiæ dictæ propositionis. Deinde imaginabis duo latera.o.x.et.o.p.diuisa esse per æqualia in punctis.g. et. K. protractisq; diametris.g.y.et.K.u.quæ, vt in præcedenti probaui, sunt inuicem æquales, scire debes quod similes parabolæ inuicem aliæ non possunt esse, nisi eæ quæ diametros proportionales suis basibus habeant, similiterq; positæ, hoc est, ut proportio ipsius b.d.ad.a.c. sit eadem quæ ipsius.o.r.ad.x.p.& quod anguli ad.r.sint æquales angulis circa.d. Notentur ergo primum puncta communia ipsius.o.g.cum.y.t.& ipsius.b.x cum.f.m.characteribus.*.et.n. Nunc igitur scimus.f.m.æqualem esse.m.i.totamq;.f.i.parallelam esse ipsi.a.c. Idem dico de.y.t.u.trianguliq;.x.f.n.et.g.y.*. esse similes triangulis.n.m.b.et.*.t.o.quod ita probatur, nam ex.15.primi Euclid. anguli ad. n. sunt inuicem æquales, ex. 29.verò eiusdem anguli.f.x. n.et.n.b.m. similiter æquales ita etiam.n.f.x.et.n.m.b.

Idem dico in secunda figura, vnde ex.4.sexti Eucli.proportio.n.f.ad.m.n.erit eadem quę.f.x.ad.b.m.& ipsius.n.f.ad.x.f.vt.n.m.ad.m.b.ex. 16.quinti. Quare ex.11.
eiusdem



eiuſdem erit vt.a.d.ad.d.b. Idem etiam dico in ſecunda parabola, ſed ipſius.x.o.ad o.r.eſt vt.a.b. ad.b.d.ex.6.ſexti Eucli. vnde ex.11.quinti.n.f.ad.f.x. erit vt.ω.y. ad.y.g. Sed in precedenti iam tibi dixi.a.b.mediam proportionalem eſſe inter.h. et.b.d. Sit nunc.z. pro ſecunda parabola, ita ut.h.eſt pro prima, vnde.o.x.erit media proportionalis inter.z.et.o.r.& ex.11.quinti ita erit.h.ad.a.b.vt.z.ad.x.o.& ex.22. h.ad.a.x.ut.z.ad.x.g.& quia ex.16. ſexti. a.x.media proportionalis eſt inter.h.et.f. x. cum ſupponatur productum.h.in.f.x.æquale eſſe quadrato. a.x. Idem dico. x.g. mediam eſſe proportionalem inter.z.et.g.y.quare ex.11.iam dicta, ita erit.a.x. ad.f. x.vt.y.g.ad.x.o.& ex eadem, ita erit ipſius.f.n.ad.a.b.ut.y.ω.ad.x.o.& ſic.f.n.ad.d.a. vt.y.ω.ad.x.r. ſed.f.m.ad f.n.eſt vt.y.t. ad.y.ω.ex.18. quinti vnde.f.m.ad.a.d.erit vt y.t.ad.x.r. Idem dico de eorum duplis.

Ex ijſdem rationibus dico ita eſſe.b.d.ad.b.m.vt.o r.ad.o.t.& ex.17.quinti.d.m. ad.b.m.vt.r.t.ad.t.o. Reliqua tibi conſideranda relinquo.



In reliquis verò propoſitionibus illius lib. nullo pacto poteris dubitare: Verum ne in. 4. aliquid tibi noui exurgat, te ſcire volo corollarium. 20. in libr. de quadratura parabolę docere poſſibile eſſe inſcriptionem rectilineæ, ea tamen conditione quã dicit Archimedes.

In quinta poſtea animaduertendum eſt, quod prima pars, probat tantummodo de centro trianguli, et.2.pars probat de centro pentagoni, à te ipſo deinde potes probare de centro nonanguli:& ſic de cæteris: eo quod cum probatum fuerit de centro figuræ in medio locatæ ſi conſtitutæ poſtea fuerint ſimiles figuræ in portionibus lateralibus habebitur propoſitum in infinitum.

Idem intelligendum eſt in.3. propoſitione quamuis exemplum vlterius non extendatur quam ad pentagonos.

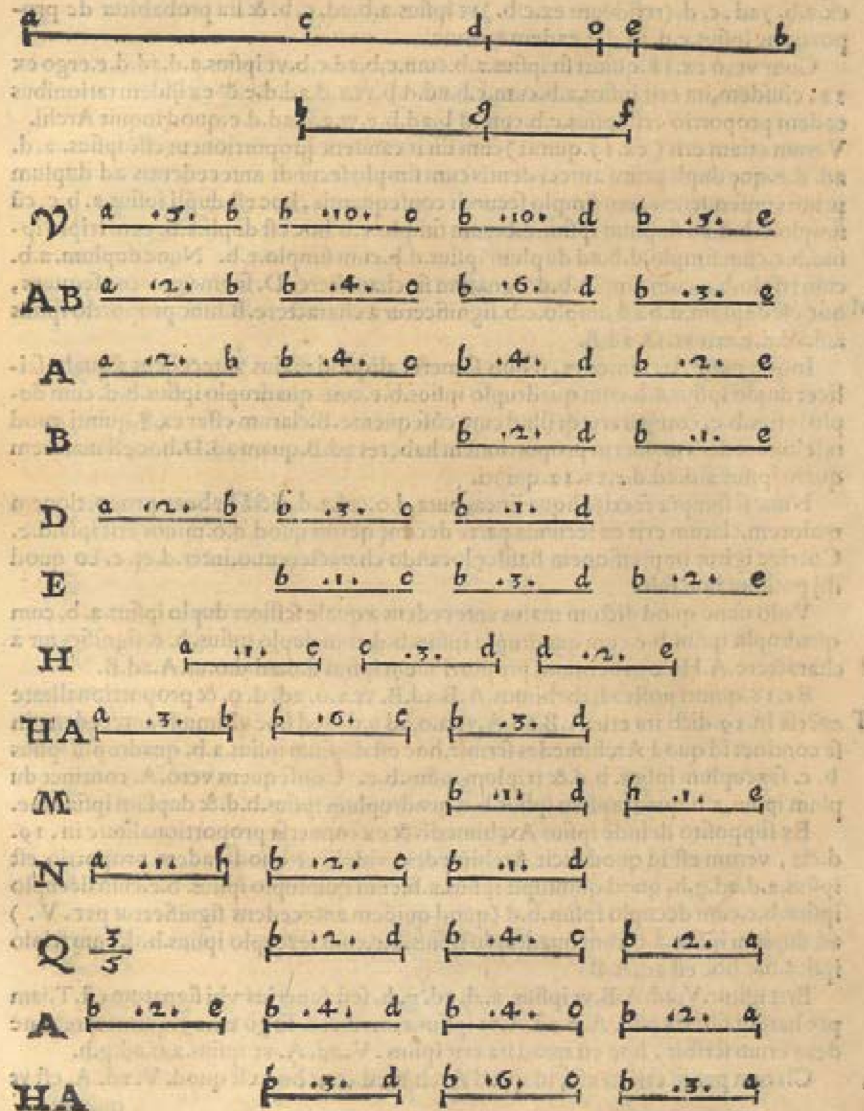
Sexta verò ꝓpoſitio tibi facilis erit, quæ nihilominus põt demõſtrari hoc mõ ſcilicet. Sint.4.quãtitates.a.b.c.d. ipſius Archimedis ſupponẽdo.a.pro figura rectilinea inſcripta in parabola, et.b.pro reſiduo ipſius parabolę et.c.pro triangulo.a.b.c.in medio ipſius parabolę et.d.pro triangulo. r. Nunc cum. a. maior ſit.c.prout totum maius eſt ſua parte, ideo ex.8.quinti maior proportio habebit.a.ad.b. quam. c. ad. b. Cum autem.b.minor ſit.d.ex ſuppoſito, ideo ex eadem dicta, maior proportio habebit.a.ad.b.quam.c.ad.d.cum verò centrum cuiuſuis figuræ plenæ neceſſariò ſit intra ipſam figuram, idcirco centrum reſidui ipſius parabolę intra ipſam reperietur. quod ita clarũ ꝑ ſe eſt, quẽadmodũ quoduis aliud axioma,& quia dictũ centrũ, ex.8.primi de centris, neceſſariò eſt in linea.b.h.inter.b.et.h. Sit igitur.g.vnde ex eadem.8.ita erit.g.h.ad.h.e.vt.a.ad.b.ergo.g.h.ad.h.e.maior proportio erit quã.c. ad. d. hoc eſt quam.b.h.ad.f.ex.12.quinti. Sed cũ.h.b.maior ſit ipſa.h.g.prout omne totum maius eſt ſua parte, ideo maior proportio habebit.h.b.ad.h.e.quam.h.g. ad.h. e. vnde multo maiorẽ quã.h.b.ad.f.ex cõi cõceptu, quare.h.e.erit minor ipſa.f.ex.10.quĩti.

Septima verò et. 8.propoſitio nullius tibi erit difficultatis.

Quam-

Quamuis Eutotius scribat super duas vltimas lib. secundi de centris grauiũ, nihil miror ipsum tibi non satisfacere. Accipe igitur quod ego nunc tibi mitto.

Archimedes eo in loco primũ supponit in penultima dicti libri quatuor lineas proportionales.a.b: c.b: d.b: et. e.b: supponit etiam quod proportio quæ est ipsius. e. b.ad.c.a.eadẽ sit quæ ipsius.f. g.ad tres quintas ipsius.a.d.& quod proportio com positi dupli ipsius.a.b.cum quadruplo ipsius.b.c.cum sexcuplo ipsius.b.d. cum triplo



ipſius.b.e.ad compoſitum quintupli ipſius. a.b.cum decuplo ipſius.c.b.cum decuplo ipſius.b.d.cum quintuplo ipſius. b.e.eadem ſit quæ ipſius.g.h.ad.a.d. & vult probare.f.h.eſſe duas quintas ipſius.a.b.

Cum autem dicit proportionem ipſius.a.c.ad.c.d.& ipſius.c.d.ad.d.e.eſſe vt ipſius a.b.ad.b.c.& cętera, verum dicit ex.19.quinti Eucli.eo quod cum ex hypotheſi ſit ipſius,a.b totalis ad.c.b.totalem vt ipſius.c.b. partialis (ſumptæ vt pars abſciſa ab.a.b.pro nunc) ad.d.b. partialem (abſciſam ab.c.b.) erit ex.19.dicta ipſius.a.c.(reſidui ex.a.b.) ad.c.d.(reſiduum ex.c.b.) vt ipſius.a.b.ad.c.b.& ita probabitur de proportione ipſius.c.d.ad.d.e.eadem ratione.

Cum verò ex.18.quinti ſit ipſius.a.b.cum.c.b.ad.c.b.vt ipſius.a.d.ad.d.e.ergo ex 22.eiuſdem,ita erit ipſius.a.b.cum.c.b.ad.d.b.vt.a.d.ad.d.e.& ex ijſdem rationibus eadem proportio erit ipſius.c.b.cum.d b.ad.b.e.vt.a.d.ad.d.e.quod inquit Archi. Verum etiam erit (ex.13.quinti) cum dicit eandem proportionem eſſe ipſius. a.d. ad.d.e.quę dupli primi antecedentis cum ſimplo ſecundi antecedentis ad duplum primi conſequentis cum ſimplo ſecundi conſequentis, hoc eſt dupli ipſius.a.b.c.cũ ſimplo.c.b.d.ad duplum ipſius.d.b.cum ſimplo.e.b.hoc eſt dupli.a.b. cum triplo ipſius.b.c.cum ſimplo.d.b.ad duplum ipſius.d.b.cum ſimplo.e.b. Nunc duplum.a.b. cum triplo.b.c.cum ſimplo.b.d.ſignatum ſit charactere.D.ſuum verò conſequens,
M hoc eſt duplum.d.b.cũ ſimplo.e.b.ſignificetur à charactere.B.hinc proportio ipſius a.d.ad.d.e.erit vt.D.ad.B.

Inquit nunc Archimedes, ſi quis ſumeret aliquod maius antecedens æquale ſcilicet duplo ipſius.a.b.cum quadruplo ipſius.b.c.cum quadruplo ipſius.b.d. cum duplo ipſius.b.e. compararetq; illud cum cõſequente.B.clarum eſſet ex.8.quinti quod tale antecedens maiorem proportionem haberet ad.B.quam ad.D.hoc eſt maiorem quàm ipſius.a.d.ad.d.e.ex.12.quinti.

Nunc ſi ſumpta fuerit aliqua linea, puta.d o.cui.a.d.dictã habeat proportionem maiorem,clarum erit ex ſecunda parte decimę quinti quod.d.o.minor erit ipſa.d.e. Corrige igitur impreſſionem Baſileę locando characterem.o.inter.d.et.e.eo quod ibi poſitum non fuit.

Volo nunc quod dictum maius antecedens æquale ſcilicet duplo ipſius.a.b.cum quadruplo ipſius.b.c.cum quadruplo ipſius.b.d.cum duplo ipſius.b.c.ſignificetur à
β charactere.A.Hinc habebimus proportionem ipſius a.d.ad.d.o.ut.A.ad.B.

Ex.18.quinti poſtea habebimus.A.B.ad.B.vt.a.o.ad.d.o.& proportionalitate
T euerſa in.19.dicti ita erit.A.B.ad.A.vt.a.o.ad.a.d. Sed hoc vltimum antecedens in ſe continet id quod Archimedes ſcribit,hoc eſt duplum ipſius.a.b. quadruplũ ipſius b.c.ſexcuplum ipſius.b.d.& triplum ipſius.b.e. Conſequens verò.A.continet duplum ipſius.a.b.quadruplum ipſius.b.c.quadruplum ipſius.b.d.& duplum ipſius.b.e.

Ex ſuppoſito deinde ipſius Archimedis & ex conuerſa proportionalitate in.19.dicta, verum eſt id quod dicit Archimedes, videlicet quod eadem proportio eſt ipſius.a.d.ad.g.h. quod quintupli ipſius.a.b.cum quintuplo ipſius.b.e.cum decuplo ipſius.b.c.cum decuplo ipſius.b.d.(quod quidem antecedens ſignificetur per.V.) ad duplum ipſius.a b.cum quadruplo ipſius.b.c.cum ſexcuplo ipſius.b.d.cum triplo ipſius.b.e.hoc eſt ad.A.B.

Erit igitur.V.ad.A.B.vt ipſius,a.d.ad.g.h.ſed ſuperius vbi ſignatum eſt.T.iam probatum fuit ita eſſe.A.B ad.A.vt ipſius.a.o.ad.a.d. Ergò ex.23.quinti Archimedes verum ſcribit, hoc eſt quod ita erit ipſius.V.ad.A.vt ipſius.a.o.ad.g.h.

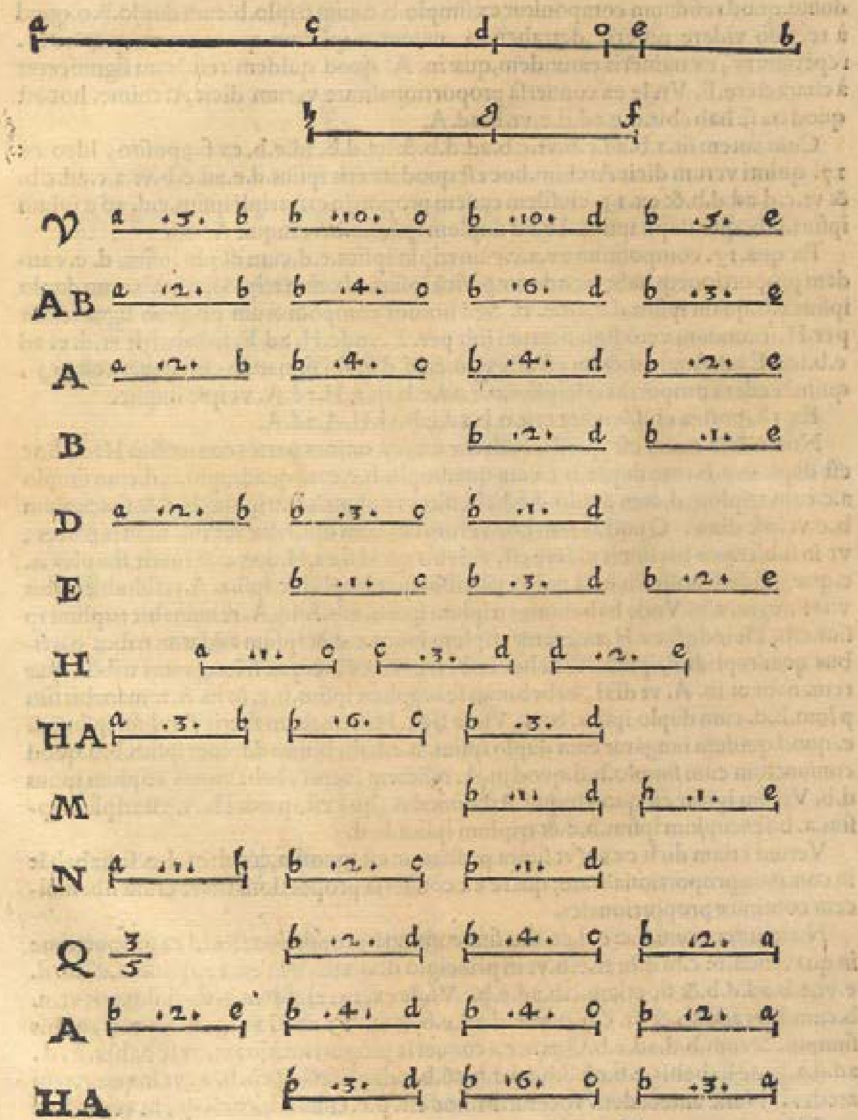
Clarum per ſe etiam eſt, id quod Archimed.dicit hoc eſt quod.V.ad.A.eſt vt

quinque

quinque ad duo, cum quodlibet ingredientium in composito. V. ad quodlibet ingredientium in composito. A. sit vt quinque ad duo. Quare ex.13. quinti verum dicit. Vnde.a.o.ad.g.h.erit vt quinq; ad duo ex.11. eiusdẽ vt inquit Archimedes.

Corrige impressionem vbi scriptum est, rursus quoniam.o.a.quia oportet dicere Rursus quoniam.o.d.

Archimedes igitur verum dicit, quod ipsius.o.d.ad.d.a.est vt ipsius.B.ad. A. ex



conuerſa proportionalitate in. 19. quinti, cum.a.d.ad. d. o. iam probatum fuit ( vbi B. ) ita eſſe ut. A.ad.B.

Sed in principio huius ſpeculationis probatum iam fuit ita eſſe ipſius. d.a.ad.d.e. vt ipſius. D.ad.B. vbi notatum eſt. M. quare ex. 23. quinti, Archimedes verum dicit, qu od.d.o.ad.d.e.erit vt. D.ad. A.

Sed cum.d.o.ad.d.e.ſe habeat ut. D.ad. A. erit ex conuerſa proportionalitate iam
A dicta.d.e.ad.d.o. vt. A.ad. D. per euerſam vero erit. d.e.ad.a.o.vt. A. ad ſuum reſiduum. quod reſiduum componitur ex ſimplo.b.c.cum triplo.b.cum duplo.b.o.quod à te ipſo videre poteris detrahendo numeros ipſarum quantitatum quæ in. D. reperiuntur, ex numeris earundem, quæ in. A. quod quidem reſiduum ſignificetur à charactere. E. Vnde ex conuerſa proportionalitate verum dicit Archime. hoc eſt quod ita ſe hab ebit.o.e.ad.d.e.vt. E.ad. A.

Cum autem ſit.a.b.ad.c.b.vt.c.b.ad.d.b.& ita.d.b.ad.e.b. ex ſuppoſito, ideo ex 17. quinti verum dicit Archim. hoc eſt quod ita erit ipſius.d.e.ad.e.b.vt.a.c.ad.c.b. & vt.c.d.ad.d.b.& ex. 13. eiuſdem eadem proportio erit tripli ipſius.c.d. ad triplum ipſius.d.b. quæ dupli ipſius.d.e.ad duplum ipſius.e.b.vt inquit Archi.

Ex qua. 13. compoſitum ex.a.c.cum triplo ipſius.c.d.cum duplo ipſius. d. e. eandem proportionem habebit ad compoſitũ ipſius.e.b.cum triplo ipſi⁹.d.b.cum duplo ipſius.e.b.quam ipſius.d.e.ad.e. b. Sed horum compoſitorum primum ſignificetur per. H. ſecundum verò ſignificatum fuit per. E. vnde. H. ad. E. ſe habebit vt.d. e. ad e.b.ſed. E.ad. A. iam dictum eſt eſſe vt.o.e.ad.d.e.vbi ſignatum eſt. x. quare ex. 23. quinti eadem proportio erit ipſius.o.e.ad.e.b.quæ. H.ad. A. vt ipſe inquit.

X Ex. 18. poſtea eiuſdem ita erit.o.b.ad.e.b.vt. H. A.ad. A.

Notandum etiam eſt quod ſi collectæ fuerint omnes partes compoſiti. H. A. hoc eſt duplum.a.b.cum duplo.b.c.cum quadruplo.b.c.cum quadruplo.b.d.cum ſimplo a.c.cum triplo.c.d.cum duplo.d.e.habebitur triplum.a.b.triplum.b.d.& ſexcuplum b.c.vt ipſe dixit. Quod autem hoc verum ſit, cum diſtinctæ fuerint omnes partes, vt in ſubſcriptis his lineis videre eſt, videbis quod ſi ex. H. detracta fuerit ſimplex.a. c.quæ quidem poſtea iuncta vni ex partibus quadrupli.b.c.ipſius. A. reſultabit nobis vna integra. a.b. Vnde habebimus triplum ipſius. a.b.& in. A. remanebit triplum ipſius.c.b. Deinde ſi ex. H. auferatur triplum ipſius.c.d.& ipſum addatur tribus partibus quadrupli.b.d. ipſius. A. habebimus tres vices.b.c.quæ ſi iungantur tribus, quæ remanebant in. A. vt dixi, habebimus ſexcuplum ipſius.b.c. & in. A. remanebit ſimp lum.b.d. cum duplo ipſius. b. e. Vnde ſi ex. H. demptum fuerit duplum ipſius. d. e. quod quidem iungatur cum duplo ipſius. b.e.habebimus duplum ipſius.b.d.quod coniunctum cum ſimplo.b.d.quod in. A. relictum fuerat, habebimus triplum ipſius d.b. Verum igitur eſt quod inquit Archimedes, hoc eſt, quod. H. A. eſt triplum ipſius.a. b.ſexcuplum ipſius.b.c.& triplum ipſius.b. d.

Verum etiam dicit ex eo ( vt ſupra probatum eſt ) quod.a.c: c.d: et.d.e. ſe habebãt in continua proportionalitate, quare ex conuerſa proportionalitate erunt ſibi inuicem continuæ proportionales.

Nunc autem cum.a.c: c.d.et. d.e.ſint continuæ proportionales in ea proportione in qua ſunt.a.b: c.b: d.b: et.e.b.vt in principio diximus, erit ex. 22. quinti.a.c. ad. d. e.vt.a.b.ad.d.b.& ſic etiam.c.b.ad.e.b. Vnde ex. 24. eiuſdem.a. d.ad.d.e.erit vt. a. b.cum.b.c. ad.d.b.& vt. c.b.cum.b.d.ad.e.b.& ex. 13. dicti vt. a. b. cum. b. c. bis ſumpto, & cum.b.d.ad.e.b. Quare ex conuerſa proportionalitate, vt ſe habet. e. d. ad.d.a.ita ſe habebit.e.b.cũ.d.b.ad d.b.cũ.b.c.duplicato & cũ. b. a. vt inquit Archimedes. Nunc antecedens vocetur. M. hoc eſt. b.c. cum.d.b.conſequens verò, hoc

eſt

est.d.b.cum duplo.b.c.cum simplo.b.a.vocetur. N.

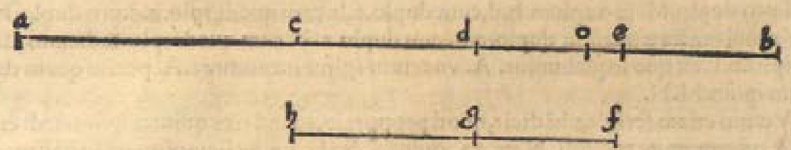
Animaduertendum tamen est quod impressio mendosa est ubi dicit.

„ vnaquæque.c.b: b.d. & cætera,

propterea quod dicendum est ita

vnaquæq;.c.b:b.d.

Nunc ex.18.quinti, quemadmodum se habet.a.c.ad.d.a.ita se habebit.M.N.ad.N.




| | | | | |
|---|---|---|---|---|
| ♈ | a .5. b | b .10. c | b .10. d | b .5. e |
| AB | a .2. b | b .4. c | b .6. d | b .3. e |
| A | a .2. b | b .4. c | b .4. d | b .2. e |
| B | | | b .2. d | b .1. e |
| D | a .2. b | b .3. c | b .1. d | |
| E | | b .1. c | b .3. d | b .2. e |
| H | a .1. c | c .3. d | d .2. e | |
| HA | a .3. b | b .6. c | b .3. d | |
| M | | | b .1. d | b .1. e |
| N | a .1. b | b .2. c | b .1. d | |
| Q $\frac{3}{5}$ | | b .2. d | b .4. c | b .2. a |
| A | b .2. e | b .4. d | b .4. c | b .2. a |
| HA | | b .3. d | b .6. c | b .3. a |

Vbi

Vbi autem ſcriptum eſt

„ ad vtrunque ſimul.b.d; d.a.cum dupla.b.c.

dicendum eſt ita,

ad vtranque ſimul.b.d.b.a.cum dupla.b.c.

Inquit deinde Archi.quod ſicut ſe habet.e.a.ad.d.a.ita ſe habebit duplum.M.N. ad duplum.N. Quod quidem verum eſt ex.13.quinti, huiuſmodi vero antecedens & conſequens, Archi.manifeſtat ex ſuis partibus, ſumendo duplum.e.b.cum duplo b.d.pro duplo.M. & duplum.b.d.cum duplo.a.b.cum quadruplo.b.c.pro duplo.N. quę ſimul iuncta æquantur duplo.e.b.cum duplo.a. b. cum quadruplo.b.d.cum quadruplo.b.c.ex quo æquabuntur. A. vocentur igitur hæc omnia.A. potius quàm duplum ipſius.M.N.

Verum etiam ſcribit,vbi dicit,quod proportio.e.a.ad tres quintas ipſius.a. d. erit vt. A.ad tres quintas dupli. N.ex.22. quinti. Sed cum ex ſuppoſito ita ſe habeat.f. g.ad tres quintas ipſius.a.d.quemadmodum.b.e.ad.e.a. erit ex. 16. quinti verum ꝙ dicit Archimed. hoc eſt, ita ſe habere. b. e.ad.f.g.vt.e.a.ad tres quintas ipſius.a.d.

Et per. 11. eiuſdem verum etiam erit quod ſicut ſe habet.e.b.ad.f.g. ita ſe habebit.A.ad tres quintas dupli. N. quod quidem duplum.N.ſignificetur per . Q.

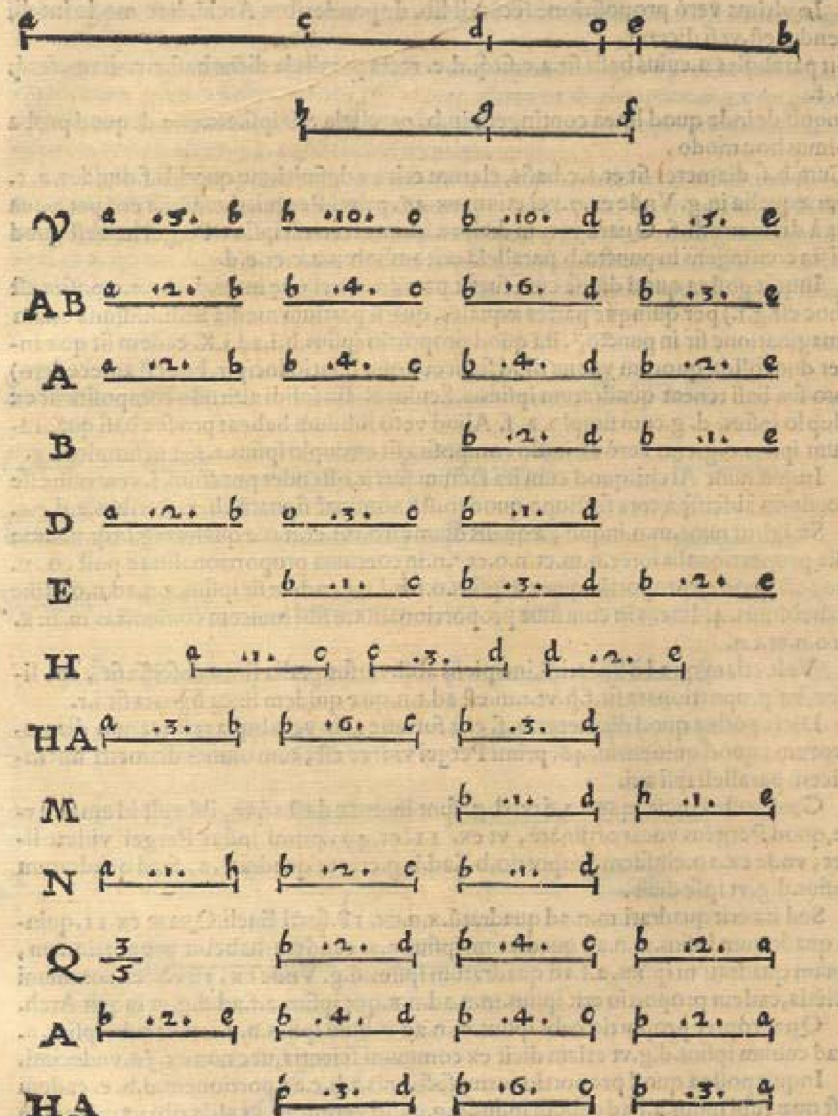
Sed ſuperius iam demonſtratum fuit ( vbi.X. ) quod.o.b.ad.b.e.ita ſe habebat vt H.A. ad. A.& nũc demum probatum fuit ita eſſe. A.ad tres quintas ipſius.Q.vt.e.b.

Y ad.f.g.Quare ex.22.quinti ita erit. H.A.ad tres quintas ipſius.Q.vt.o. b. ad. f. g. vt idem inquit.

Sed. H.A.ad.Q. ( vt ex ſuis partibus videre eſt) ita ſe habet vt tres ad duo ex.13. quinti, vt inquit Archimedes.

Ipſe etiam dicit proportionem. H.A.ad tres quintas ipſius. Q. eſſe vt quinque ad duo. Pro cuius rei euidentia imaginemur tam.H.A.quam.Q. diuiſa per quinq; partes æquales, vnde ex.16.quinti habebimus quamlibet quintam partẽ ipſius. Q. æqualẽ eſſe duabus tertijs vniuſcuiuſque quintæ partis.H.A. vnde tres quintæ ipſius Q. erunt, ex communi conceptu, ſex tertiæ vnius quintæ ipſius. H. A. hoc eſt duæ quintæ.ipſius. H. A. Quare.o.b.ita ſe habebit ad.f.g. vt quinque ad duo ex communi cõceptu,cum.o.b.ad.f.g.probatum fuerit ſe habere vt.H.A. ad tres quintas ipſius Q. ( vbi . Y. ) ſed iam probatum fuit ( vbi.*. ) quod. o.a.ad.h.g. erat etiam vt quinque ad duo, hoc eſt quod.f.h.erit duæ quintę ipſius.a. b. Quod eſt propoſitum.

In vltima


a c d o e b
h g f
V a .5. b b .10. c b .10. d b .5. e
AB a .12. b b .4. c b .6. d b .3. e
A a .12. b b .4. c b .4. d b .2. e
B b .2. d b .1. e
D a .2. b o .3. c b .1. d
E b .1. c b .3. d b .2. e
H a .1. c c .3. d d .2. e
HA a .3. b b .6. c b .3. d
M b .1. d h .1. e
N a .1. h b .2. c b .1. d
Q 3/5 b .2. d b .4. c b .2. a
A b .2. e b .4. d b .4. c b .2. a
HA e .3. d b .6. c b .3. a

In

In vltima verò propoſitione ſecundi lib. de ponderibus Archi. hoc modo intelligendus eſt, vt ſi diceret,

Sit paraboles.a.cuius baſis ſit.a.c.ſitq;.d.c. recta parallela dictæ baſi.a.c. diameterq; b. f.

Inquit deinde quod linea contingens in.b. parallela erit ipſi.a.c.et.e.d. quod probabimus hoc modo.

Cum.b.f. diameter ſit et.a.c.baſis, clarum erit ex definitione quod.b.f.diuidet. a. c. per æqualia in.g. Vnde ex.7.vel etiam ex.46. primi Pergei. d.e.diuiſa erit per æqualia à diametro.b.f. Quare verum dicit ex quinta ſecundi ipſius Pergei hoc eſt quod dicta contingens in puncto.b parallela erit ambobus.a.c.et.e.d.

Inquit poſtea quod diuiſa cum fuerit pars diametri quę inter.d.e.et.a.c.poſita eſt (hoc eſt.g.f.) per quinque partes æquales, quarũ partium media ſit.h.k.diuiſa etiam imaginatione ſit in puncto.i. ita quod proportio ipſius.h.i.ad.i.K. eadem ſit quæ inter duo ſolida quorum vnum(illud ſcilicet à quo relatio incipit, hoc eſt antecedens)

R pro ſua baſi teneat quadratum ipſius.a.f.cuius etiam ſolidi altitudo compoſita ſit ex duplo ipſius. d. g.cum ſimplo. a. f. Aliud verò ſolidum habeat pro ſua baſi quadratum ipſius.d.g.eius verò altitudo compoſita ſit ex duplo ipſius.a.f.cum ſimplo.d.g.

Inquit nunc Archi.quod cum ita factum fuerit, oſtendet punctum. i. centrum eſſe portionis abſciſſę à tota ſectione, quod fruſtũ nominat ſignatũ characteribus.a.d.e.c.

Sit igitur nunc.m.n.inquit, æqualis diametro.b.f.et.n.o.æqualis.b.g.ſitq;.x.n.media proportionalis inter.n.m.et.n.o.et.t.n.in continua proportionalitate poſt. o. n. hoc eſt quod ea proportio quæ eſt ipſius.o.n.ad.n.t.eadem ſit ipſius.x.n.ad.n.o. Hinc habebimus. 4. lineas in continua proportionalitate ſibi inuicem coniunctas.m. n: x. n: o.n.et.t.n.

A Vult etiam quod à linea.i.b.incipiens ab.i.verſus.g.alia linea abſciſſa ſit, cui lineæ, ita proportionata ſit.f.h.vt.t.m.eſt ad.t.n.quæ quidem linea ſignata ſit.i.r.

Dicit poſtea quod diameter.b. f. erit fortaſſe axis vel aliqua reliquarum diametrorum, quod quidem in. 46. primi Pergei videre eſt, cum omnes diametri ſint inuicem paralleli ipſi axi.

Cum poſtea dicit, quod.a.f.et. d.g. ſunt intentæ ductæque, ibi vult idem inferre, quod Pergeus vocat ordinatè, vt ex. 11. et. 49. primi ipſius Pergei videre licet, vnde ex.20.eiuſdem proportio.b.f.ad.b.g.erit vt quadrati. a. f. ad quadratum ipſius.d.g.vt ipſe dicit.

Sed ita erit quadrati.m.n.ad quadratũ.x.n.ex. 18.ſexti Eucli. Quare ex.11.quinti quadratum ipſius.m.n.ad quadratum ipſius.n.x. eandem habebit proportionem, quam quadratum ipſius. a.f.ad quadratum ipſius.d.g. Vnde ex. 18. & ex communi ſciẽtia, eadem proportio erit ipſius.m.n.ad.n.x.quę ipſius.a.f.ad.d.g.vt inquit Arch.

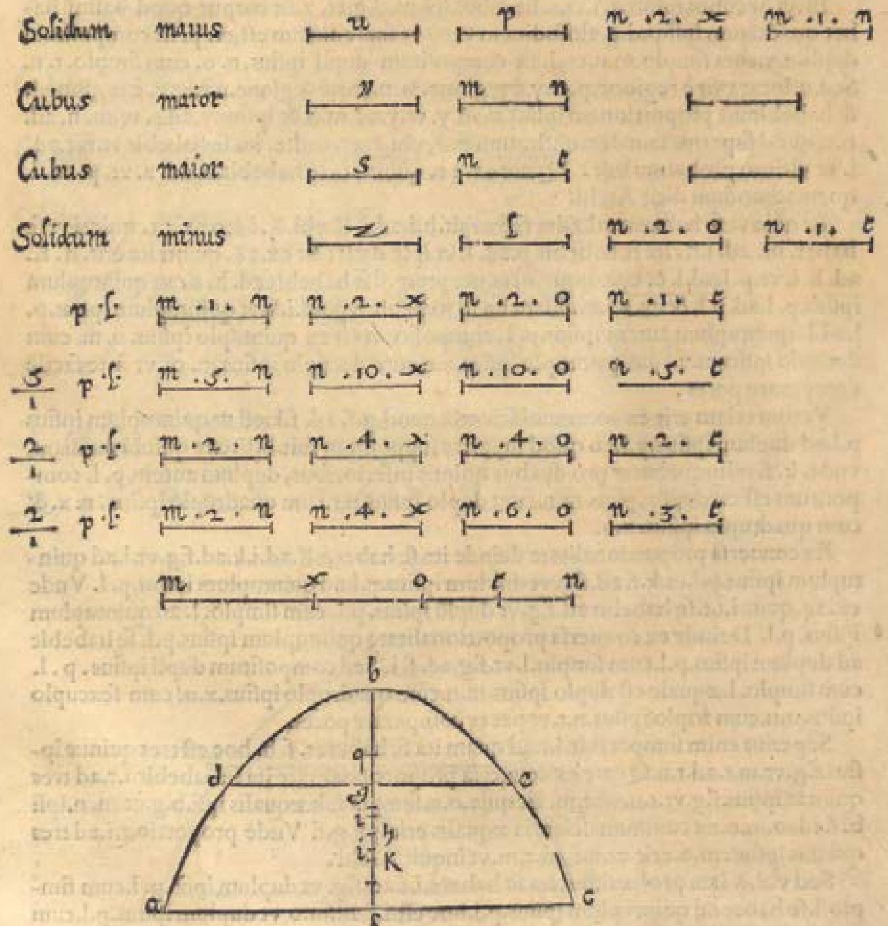
Quapropter proportio cubi ipſius.m.n.ad cubum ipſius.n.x.erit vt cubi ipſius. a. f.ad cubum ipſius.d.g.vt etiam dicit ex communi ſcientia, nec non ex.36.vndecimi.

Inquit poſtea quod proportio totius ſectionis.a.b.c.ad portionem.d.b. e. eadem eſt quæ cubi ipſius.a.f.ad cubum ipſius.d.g.quod verum eſt, vt aliàs tibi monſtraui in diuiſione parabolæ ſecundum aliquam propoſitam proportionem.

Quando autem dicit quod proportio cubi ipſius.m.n.ad cubum ipſius.n.x.eadem eſt quæ ipſius.m.n.ad.n.t. verum dicit ex. 36. vndecimi. Vnde ex. 11. quinti ita ſe habebit totalis ſectio.a.b.c.ad portionem.d.b.c.vt.m.n.ad.n.t.& ex.17. eiuſdem ita erit ipſius.m.t.ad.t.n.vt fruſti.a.d.e.c.ad ſectionem.d.b. e. quemadmodum ipſe dicit. Sed quia ſuperius, vbi.A.ipſa.f.h.(quæ eſt tres quintæ ipſius.f.g.)ad.i.r.ita relata fuit ta fuit vt.m.t.ad.t.n. idcirco ex.11. quinti ita erit ipsius frusti.a.e. ad sectionem.d.b. e. vt tres quintę ipsius.f.g.ad.i.r.

Inquit deinde quod proportio corporis iam supradicti, quod pro sua basi habeat quadratum ipsius.a.f.altitudinem verò compositam ex duplo ipsius.d.g.cum simplo a.f.ad cubum ipsius.a.f.eadem erit quæ dupli ipsius.d.g.cum simplo.a.f.ad.a.f.Quod quidem verum est ex.33. vndecimi & ex prima sexti.

Sed superius( vbi.a.) iam probauimus eandem proportio nem esse inter.m.n.& n.x. quæ inter.a.f.et.d.g.ideo ex conuersa proportionalitate ita erit ipsius.x.n.ad.n.m. vt ipsius.d.g.ad.a.f.sed dupli.x.n.ad simplum.x.n.est vt dupli.d.g.ad.d.g.Quare ex.22. quinti dupli.x.n.ad.m.n.erit vt dupli.d.g.ad.a.f.& ex.18.eiusdem ita erit dupli.x.n.cum simplo.m.n.ad.m.n.vt dupli.d.g.cum simplo.a.f.ad.a.f.Quare solidi



δ iam dicti ad cubum ipſius.a.f.ex. 11. quinti erit vt dupli.x.n.cũ ſimplo.m.n.ad.m.n.

Superius autem vbi. β. demonſtratum fuit ita eſſe ipſius.m.n.ad.n.t.vt cubi.m.n. ad cubum. x. n. & inter. α. et. β. probatum fuit ita eſſe cubi.a.f. ad cubum. d. g. vt cubi.m.n.ad cubum. x. n. Vnde ex. 11. quinti.m.n.ad.n.t.erit vt cubi.a. f. ad cubum d. g.

Dicit poſtea quod eadem proportio erit inter cubum. d. g. & corpus illud quod pro baſi habeat quadratum ipſius.d.g. altitudinem verò vt dictum eſt, quæ eſt inter d.g. & compoſitum ex duplo.a.f.cum ſimplo. d. g. quod compoſitum eſt altitudo dicta,& verũ dicit ex ratione ſuperius allegata pro reliquo corpore & cubo ipſius.a.f. Quare etiam quemadmodum. t. n. ſe habet ad duplum ipſius.o. n.cum ſimplo.t.n. ex ijſdem rationibus ſupradictis, vbi loquuti ſumus de.x.n.cum.m.n.

Diſponantur nũc omnia tali ordine, ita vt. u. primum ſit corpus quod pro ſua baſi habeat quadratum ipſius.a.f.&c.

Et.y. ſit cubus ipſius.a. f.et.s. ſit cubus ipſius.d.g.et. z.ſit corpus quod baſim habet quadratum ipſius.d.g.altitudinem verò vt ſupradictum eſt,et. p. ſit compoſitum dupli.n.x.cum ſimplo.m.n.et. l. ſit compoſitum dupli ipſius. n. o. cum ſimplo. t. n. Sed.u.locata ſit è regione.p.et.y.è regione.m. n.et.s.è regione.n.t.et.z. è regione.l. & habebimus proportionem ipſius.u.ad.y. vt.y.ad.m.n.& ipſius.y.ad.s. vt.m. n. ad. n. t. quod ſuperius iam demonſtratum fuit, vbi,δ. et.s.ad.z. ita ſe habebit vt.n.t.ad. l. vt vltimò probatum fuit. Quare ex.22. quinti ita ſe habebit.u. ad. z. vt. p. ad. l. quemadmodum dicit Archi.

Et quia vt ſe habet.u.ad.z.ita facta fuit.h.i.ad.i. K.vbi.R. ideo ex.11. quinti vt ſe habet. h.i.ad. i.K. ita ſe habebit.p.ad. l. vt ipſe dicit: Et ex.18. quinti ita erit. h. K. ad. K.i. vt.p.l.ad.l. & ex communi conceptu.g.f.ſe habebit ad. h. K. vt quintuplum ipſius.p. l.ad.p.l. & ex.22.eiuſdem ita ſe habebit.f. g.ad.i.k.vt quintuplum ipſius.p. l.ad.l. quintuplum autem ipſius.p. l.compoſitum eſt ex quintuplo ipſius. n. m. cum decuplo ipſius.n.x.cum quintuplo ipſius.n.t. cum decuplo ipſius. n. o. vt à te facilè computare potes.

Verum etiam erit ex communi ſcientia quod.g.f. ad. f.k.eſt ut quintuplum ipſius p.l.ad duplum ipſius.p.l.eo quod ſuperius ſuppoſitum fuit.h.K.eſſe quintã mediam, vnde. k. f. relinquebatur pro duabus quintis inferioribus, duplum autem.p. l. compoſitum eſt ex duplo ipſius.m.n.cum duplo ipſius.n.t.cum quadruplo ipſius. n. x. & cum quadruplo ipſius. x.o.

Ex conuerſa proportionalitate deinde ita ſe habet,i.K.ad.i.k.ad.f.g.vt.l.ad quintuplum ipſius.p.l.et.k.f. ad.f.g. vt duplum ipſius.p.l.ad quintuplum ipſius.p. l. Vnde ex.24.quinti.i.f.ſe habebit ad.f.g.vt duplũ ipſius. p.l. cum ſimplo. l. ad quintuplum
θ ipſius. p.l. Deinde ex conuerſa proportionalitate quintuplum ipſius.p.l. ſe habebit ad duplum ipſius.p.l.cum ſimplo.l.vt.f.g.ad. f. i. Sed compoſitum dupli ipſius. p. l. cum ſimplo. l.æquale eſt duplo ipſius.m.n.cum quadruplo ipſius.x.n. cum ſexcuplo ipſius.o.n.cum triplo ipſius.n.t.vt per te computare potes.

Superius enim ſumpta fuit.i.r.ad quam ita ſe haberet. f. h. hoc eſt tres quintæ ipſius.f.g.vt.m.t.ad.t.n.Quare ex conuerſa proportionalitate ita ſe habebit.i.r.ad tres quintas ipſius.f.g.vt.t.n.ad.t.m. Et quia.o.n.ſumpta fuit æqualis ipſi.b.g.et.m. n.ipſi b. f. ideo.m.o.ex communi ſcientia æqualis erit ipſi.g.f. Vnde proportio.r.i.ad tres quintas ipſius. m.o.erit vt.n.t.ad.t.m.vt inquit Archi.

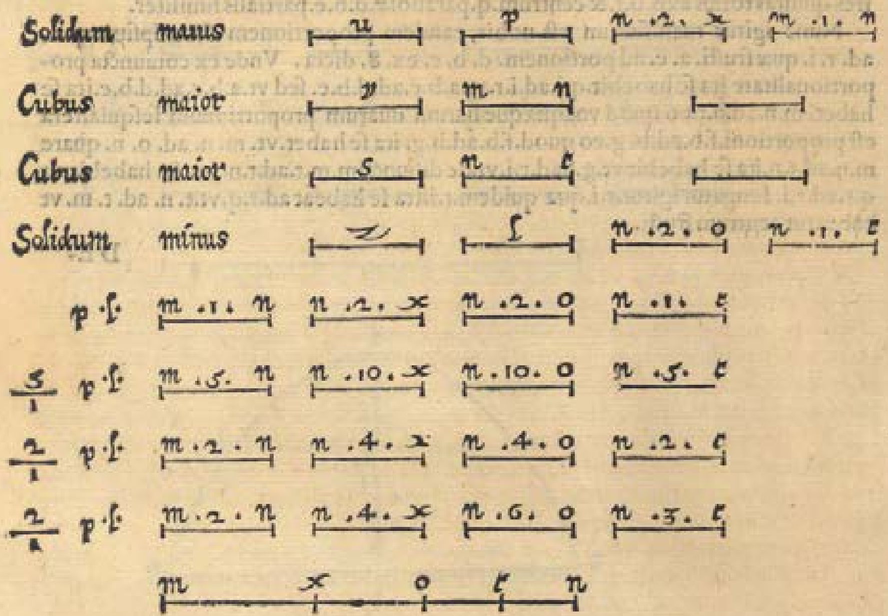
Sed vbi. θ. iam probauimus ita ſe habere.i.f.ad.f.g. vt duplum ipſiꝰ.p. l.cum ſimplo.l.ſe habet ad quintuplum ipſius.p.l.hoc eſt.i.f.ad.m.o.vt duplum ipſius.p.l.cum ſimplo,l.ad quintuplum ipſius.p.l.

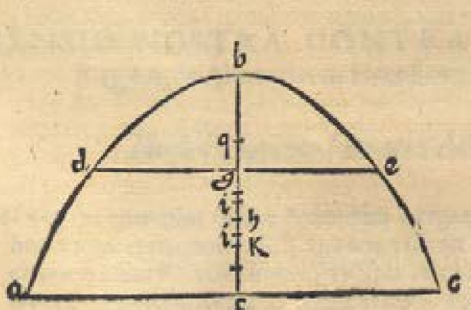
Habemus

Habemus igitur nunc omnes illas conditiones quas Archimedes in præcedenti propositione supponit. Vnde ex rationibus ibi allegatis sequitur.f.r.esse duas quintas ipsius.m.n. hoc est ipsius.f.b. Quapropter punctum.r.centrum erit ponderis totius sectionis parabolæ ex.8. secundi lib.de ponderibus eiusdem Archimedis.

Inquit nunc Archimedes, quod existente.q. centro ponderis ipsius parabolæ.d.b.e.partialis, centrum frusti erit in linea recta.q.r.f. ita remotum à centro.r. quod proportio.q.r.ad partem illam ipsius.r.f.quæ reperitur inter centrum.r. & centrum huius frusti æqualis est proportioni totius parabolæ ad partialem. Quod quidem verum est ex.8.primi libri eiusdem.

Inquit etiam punctum.i.illud esse, eo quod cum probatum sit.f.r.duas quintas esse ipsius.f.b.ideo.b.r.tres quintas erit ipsius.b.f. vt ipse dicit.



Sed.q.b. ſimiliter tres quintæ eſt ipſius.d.b.ex.8.prædicta. Quare.q.r.tres quintæ erit ipſius.f.g.ex. 19. quinti.

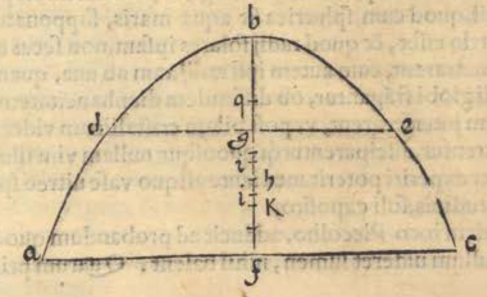
Dicamus igitur hoc modo cum. f.b.totum ad totum.b. r. ita ſe habeat vt abſciſſum.b.g.ad abſciſſum.q.b.ex.7.et.8. dicti primi libri eiuſdem ideo reſiduum.f.g.ex f.b.ad reſiduum.r.q.ex.r.b.erit vt totum.f.b.ad.totum.r.b.ex. 19.quinti Eucli.

Sed iam ſub. A. probauimus ita ſe habere fruſtum.a.d.e.c.ad parabolam.d.b.e. vt m.t.ad.t.n.ſed vt.m.t.ad.t.n.ita aſſumpta fuit (vbi. A. ).i.r.ad quam ſic ſe haberet. f. h.hoc eſt tres quintæ ipſius.f. g.hoc eſt.q.r, quare ex.11. quinti proportio fruſti. a. d.e.c.ad parabolam partialem erit vt.q.r.ad.r.i. Exiſtente igitur. r. centro totius parabolæ et.q. centro partialis, ergo, i.centrum erit fruſti propoſiti.

Sed ſi nullo ſolido intercedente, voluerimus centrum.i.fruſti.a.c. citius inuenire, inueniemus primò centrum.r.totius figuræ ex.8.ſecundi eiuſdem conſtituendo.b.r. tres quintas totius axis.b.f.& centrum.q.parabolæ.d.b.e.partialis ſimiliter.

Nunc igitur manifeſtum eſt nobis, eandem proportionem fore ipſius. q.r. ad. r. i. quæ fruſti. a. c. ad portionem. d. b. e. ex. 8. dicta. Vnde ex coniuncta proportionalitate ita ſe habebit.q.i.ad.i.r.vt.a.b.c.ad.d.b.e. ſed vt.a.b.c.ad.d.b.e.ita ſe habet. m.n.ad.n.t.eo quod vnaquæque harum duarum proportionum ſeſquialtera eſt proportioni.f.b.ad.b. g.eo quod.f.b.ad.b.g. ita ſe habet.vt. m. n. ad. o. n. quare m.n.ad.t.n.ita ſe habebit vt.g.i.ad.r.i.vnde diſiunctim.m.t.ad.t.n. ita ſe habebit vt q.r.ad.r.i. Iungatur igitur.r.i.quæ quidem.r.i.ita ſe habeat ad.r.q.vt.t. n. ad. t. m. vt habeatur centrum fruſti.

DE-

## DEFENSIO NOSTRA CONTRA ANTONIVM Bergam, & Alexandrum Piccolhomineum.

### *Illustri Domino Horatio Muto.*

INTER ea quæ olim contra Antonium Bergam, sermone Italico scripsi, hoc vnum erat, quod ipse Berga non viderat quendam notatu dignum errorem ipsius Piccolhominei, vbi ipse Alexander arguit quendam auctorem in tractatu de magnitudine terræ & aquæ pag. 37. linea. 26. ita di- „ cens, & erit maior aqua.

Quo

Quo in loco clare videtur ipſum putare eandem proportionem inter diametros, quæ inter ſphæras ipſas eſſe, nec amplius recordari eius quod ſcripſerat pag. 24.

Piccolhom. igitur ibi ſupponēs centrum. D. eſſe magnitudinis aquæ, & intra ſphæram terreſtrem, putat omnino cauſam eſſe vt terra ſuperet aquam magnitudine, quaſi quod ſi punctum. D. vt centrum ſphæræ aquæ, vnum idemq́; eſſet cum puncto. E. extremo diametri ipſius terræ, ſphæra. A. G. H. ſphæræ. A. B. E. dupla eſſe deberet, quod quidem nullo pacto fieri poteſt, quamuis etiam proportio. A. H. ad diametrū A. E. ſuperbipartiens ſeptimas exiſteret, quæ minor eſſet quam ſeſquitertia, ita quod quando etiam. D. E. maior medietate ipſius. D. H. fuiſſet, nihilominus tamen terra minor eſſet aqua, eò quod proportio dupla minor eſt, quam tripla ad proportionē ſuperbipartientenſeptimas, & maior quā tripla ad proportionem ſeſquiquartam. Vnde ſi Piccolhom. ſuppoſuiſſet proportionem ipſius. D. H. ad. C. E. eſſe ſeſquiquartam, rectè profectò dixiſſet, ſed dicere quod ubicunque exiſtat punctū. D. intra ſphæram terreſtrem, ſequitur ipſam eſſe maiorem aquea, verum non eſt.

Scripſi etiam quòd Piccoloho. decipiebatur vbi loquitur de diaphaneitate aquæ pag. 40. ita dicens.

» Et cum rationabiliter aliquis exiſtimare non poteſt, quod vmbra quæ facit ori-
» ri e eclipſes Lunæ, producta ſit à terra, & ab aqua ſimul, vt ab vno corpore aggre-
» gato ex ijs duobus elementis, & ad vnam communem ſphæreccitatem reductis, pro
» pterea quod cum vmbra produci debeat à corporibus opacis, quorum opacitas effi-
» cit illa corpora vmbroſa, aqua autem, ſit corpus diaphanum, & tranſparens, nullam
» vmbram poterit à ſe eminus producere.

Hic enim decipitur Piccolhom. duabus rationibus, quarum prima eſt, quod radius luminoſus non poteſt multum in profundum mergi, vt probaui in. 8. epiſtola ad Vimercatum, altera verò eſt, quod cum ſphærica ſit aqua maris, ſupponatur etiam quod ſub ea nulla terræ portio eſſet, & quod radij ſolares ipſam, non ſecus ac pilam ex criſtallo fabrefactam penetrarent, cum autem ipſi radij, tam ab una, quam ab alia parte ſuperficiei huiuſmodi globi frāgantur, ob diſſimilem diaphaneitatem inter aerem & aquam, ipſi ſeinuicem interſecarent, vt poſt pilam criſtallinam videre eſt, deinde procedentes, diſgregarentur, diſciparenturq́; quouſque nullam vim illuminationis haberent, quod quilibet experiri poterit mediante aliquo vaſe uitreo ſphærico, aqua pleno, cuiuſuis magnitudinis, ſoli expoſito.

Rationes etiam quas eodem loco Piccolho. adducit ad probandum quod ſi quis in fundo maris exiſteret, nullum uideret lumen, nihil ualent. Quarum prima eſt, ubi ita dicit.

» Ille qui ſe in aquam mergit, cum maiorem lucem, quæ ſupra aquam eſt, relin-
» quat, iudicat pro magno temporis ſpatio locum illum obſcurum, quemadmodum
» accidit quando per multum temporis ſpatium fixis oculis in corpore Solis intuiti ſu
» mus, ab eodem poſtea eoſdem amouentes, omnia obſcura nobis videntur.

Ipſe autem non conſiderat quod talis obſcuritas quæ ſequitur viſionem maioris luminis, parum durat, immo cito euaneſcit, ſed in aqua nunquam reuertimur ad videndum, neque veſtigium aliquod luminis ibi videtur, in fundo maris dico, quemadmodum nobis nuntiauerunt hi qui margaritas expiſcantur in imis partibus ingentium æquorum indicorum.

Secunda uerò ratio ipſius Piccolhom. eſt ubi ita dicit.

» Altera cauſa quod nobis obſcurus appareat locus ſub aqua, eſſe poteſt obſtacu-
» lum quod aquæ habent ab opacitate terræ ſub eorum fundo, etenim ſicut chriſtallū

quamuis

quamuis perſpicuũ ſiue tranſparẽs ſit, nihilominus propter obſtaculum plumbi ſub ipſo poſiti, efficit vt radij viſuales repercuſſi reuertantur. ita etiam quamuis aqua ſit corpus tranſparens, nihilominus propter obſtaculum terræ opacæ, quæ ſubſidet in fundo maris efficere poteſt obſcuras partes illas ſub aqua, illis hominibus qui in ipſa aqua merguntur.

In hac ſecunda ratione decipitur Piccolhom. Primum quia ſi vſque ad imam partem maris, Solis radius ferri poſſet, ille qui ibi eſſet, attollens oculos ſurſum Solem cerneret, deinde aſpiciendo ipſum fundum Maris, videret illum, ratione reflexionis luminis ab ipſo fundo, & ex eadem ratione ſpeculi ab ipſo adducta, quæ contra ipſum eſt.

Decipitur etiam cum dicat radios viſuales à ſpeculo ſeu plumbo repercuti, eo ꝙ non radij viſuales ſunt hi qui reflectuntur, ſed ſunt radij luminoſi primarij, ſeu ſecundarij qui non ab oculis exeunt ſed à corpore lucido.

Scripſi etiam quod ſi verum eſſet proportionalitatem continuam quãtitatum elementorum ex proportione decupla conſtare, ignem pro maximo, terram verò pro minimo terminorum ſumentes, totum aggregatum ex terra, aqua, aere, & igne, ita eſſet maius terra, quemadmodum mille centum & vndecim ad vnum, vnde ſemidiameter regionis elementaris eſſet quaſi aut paulo maior decuplo ſolum ſemidiametro terræ, vnde inter conuexum ignis, & concauum minimi, ſeu inferioris orbis lunaris, relinqueretur quidam orbis vacuus ſpiſſitudinis vnius interualli plus quam viginti terræ ſemidiametrorum, quod ſpatium vacuum orbiculariter, maius exiſteret ipſa totali regione elementari plus quam trigeſies millies, immo ſi ſemidiameter dicti primi orbis lunaris maior eſſet terreſtri vt trigintanouem ad unum, dict⁹ orbis vacuus maior eſſet elementari regione plus quam. 58208. ad vnum, proportionalitatem igitur continuam quæ ex decupla proportionalitate reſultat in elementis eſſe putare eſt maximus error.

Subdit deinde Berga, hoc voluiſſe Platonem neceſſario requiri, vt extrema elementa, nempe ignis & terra cum duobus medijs aere, & aqua coniungerentur, cum in corporibus ſolidis (quaſi Bergę ſint quædam corpora quæ ſolida non extent) poſſit dari medium æquale in geometrica proportione.

Sed vbi Plato ad ſermonem de numero elementorum ſe confert, poſtquam ratione creationis ignis, & terrę ſe propoſuiſſe putat, vt idẽ de alijs duobus corporibus medijs probet, comparatione proportionalitatis continuæ geometricæ in tribus terminis, ratione rerum ſuperficialium primò, deinde in quatuor, ratione corporearum vtitur, ita dicens.

Vinculorum verò ideſt aptiſſimum atque pulcherrimum quod ex ſe, & ex ijs quę aſtringunt, quam maximè vnum efficit, &c.

Quo in loco Plato inferre vult de proportionalitate geometrica trium terminorum, in qua ijdem ita ſe habent, vt medius, primi, vltimiq; vice fungatur, ita vt vtriuſque ipſorum extremorum particeps fiat, cum productum quod à medio termino in ſeipſo progignitur idem ſit ei quod ab extremis fuit, vnde medius, potentia idem eſt quod productum ab extremis.

Subdit deinde Plato dicens.

Quando enim in tribus numeris, aut molibus, aut viribus, medium ita ſe habet ad poſtremum vt primum ad medium, viciſſimq; vt poſtremum cum medio, ita medium cum primo congruit, tunc quod medium eſt, & primum fit & poſtremũ, poſtremum quoque, & primum & media fiunt.

Hic

Hię animaduertendũ eſt omnes interpretes falli, qui hoc loco Platonem de omnibus proportionalitatibus continuis quæ ternario numero ( alia enim Arithmetica, alia geometrica, alia harmonica dicitur ) continentur, intelligendum eſſe cenſent, quia de numeris, magnitudinibus, viribusq́;, aut ut dici ſolet, virtutibus mentionem fecerit. Plato enim nihil aliud inferre voluit, quam eandem paſſionem ( ut ipſe recitat ) inter medium extremaq́; vnius proportionalitatis continuæ geometricæ, tam in quantitate, quam in qualitate reſultaturã, cum tres termini eiuſdem eſſent ſpeciei, & quia quantitas in duas principes primariasq́; partes, ideſt in continuam, & diſcretam diuiditur, hanc ob cauſam Plato hoc præcipuè ſignificat numerorum magnitudinisq́; vocabulis vtens, quibus vniuerſum quantitatis genus complectitur.

Cum verò ait vires, uniuerſum qualitatis genus inferre uult. Quia proportio & proportionalitas tam continua quam diſcreta, non ſolum inter terminos quanti, ſed inter eos etiam qui quali attribuuntur elucet.

Sed quod eo loco de harmonica proportionalitate quæ cũ geometrica magis ſimbola eſt quam cum Arithmetica Plato minime intelligat, ex eiuſdem uerbis cum ita ſcribit manifeſtè patet.

„ Quando enim medium ita ſe habet ad poſtremum ut primum ad medium, uiciſ-
„ ſimq́; ut poſtremum cum medio ita medium cum primo congruit.

Id enim in harmonica proportionalitate non cernitur in qua primus terminus ad poſtremum, & non ad medium, ita ſe habet geometricè ut differentia inter primum & medium ad differentiam inter medium & ultimum.

Quod ſi clarum eſt ipſum de harmonica proportionalitate nullo modo intelligere, quanto minus de Arithmetica, quæ cum geometrica nihil habet commune.

Cum uerò Plato ait.

„ Tunc quod medium eſt & primum fit & poſtremum, poſtremum quoque, & pri-
„ mum media fiunt, &c.

Nihil aliud oſtendere uult, quam ſimilitudinem quæ inter huiuſmodi medium & extrema intercedit, cum ipſum medium ad poſtremum, quem primus ad ſeipſum, eundem reſpectum habeat, in quo eſt ſimilis primo, & contra ad primum terminũ, eundem reſpectum, quem poſtremum ad ſeipſum habet, unde hac ratione ultimum repręſentat, uolens Plato inferre de conuenientia quę inter media elementa, & extrema intercedit, ut aquæ inter aerem, & terram, cum aqua, ratione ſuæ frigiditatis, terrę, ratione uero ſuæ humiditatis aeri ſimilis euadat. Aer uero qui inter ignem, aquamq́; ponitur quod ad caliditatem attinet cum igne, quod uero ad humiditatem ſpectat cum aqua communicet.

Sed quia Plato multis in rebus doctrinam Pythagoricam ſequutus eſt, Pythagorici autem omnia numeris metiebantur, & de omni re ſecundum numerorum rationem diſſerebant, uidensq́; Plato quod inter duos numeros ſuperficiales, inuicemq́; ſimiles exiſtentes, unum tantum numerum medium in proportionalitate continua geometrica cadere poteſt, ideo ſubiungit.

„ Quod ſi uniuerſi corpus latitudinem habere debuiſſet, nullam uerò profundita-
„ tem, unum ſanè, tum ad ſeipſum, tum ad extrema uincienda interiectum medium
„ ſuffeciſſet.

Sequitur poſtea ſic.

„ Sed cum ſoliditatem mundus requireret, ſolida uerò non uno, ſed duobus ſem-
„ per modis copulentur, inter ignem, & terram, Deus, Aerem, Aquamque loca-
„ uit, &c.

Volens

Volens inferre, quod quemadmodum inter duos numeros ſolidos, & inuicem ſimiles, vn⁹ tãtũ medius proportionalis intercedere nõ poteſt, ſed duo neceſſariò requiruntur (vt ex ijs quæ Euclid. 8. lib. 16. 17. 18. et. 19. propoſitione proponit videntur) ita dictãte ratione inter igneum, terreumq́; corpus duo corpora interiecta eſsẽt, non ratione proportionalitatis continuæ in quantitate eorũdem corporum, ſed propter ſimilitudinem connexionis, cum productum ex duobus medijs proportionalibus æquale ſit producto ab extremis, & idem reſpectus, quem primum ipſorum quatuor ad ſecundum habet, ſecundi ad tertium extet, vnde ſecundum primo ſimile euadit, & contra, reſpectus qui eſt quarti ad tertium, ſit etiam tertij ad ſecundum, vnde ipſum tertium, ratione vltimi ſubit, & eius imaginem induit, & hanc ob cauſam ſic ſcribit Plato.

» Propterea ex huiuſmodi rebus numero quaternario concluſis, mundi corpus con
» flatum eſt, ea connexum comparatione qua dixi. Ex quo ſeipſum amicitia concor-
» di complectitur, &c.

Vbi Platonem, elementa maiora, minoráue in proportionalitate continua, nec geometrica, nec alterius cuiuſuis generis eſſe noluiſſe, clarè perſpicitur, ſed huiuſmodi ſimilitudine, in eo quod media elementa cum extremis conueniunt eſt vſus, quæ quidem conuenientia, nullibi maior, quam in proportionalitate continua geometrica reperitur. Sed etiam ſi Plato de huiuſmodi corporea elementorum magnitudine ſeipſum intelligi voluiſſet, ſi ſemidiameter regionis elementaris ex ęquo vt. 39 ad vnum, reſpectu ſemidiametri terræ fuiſſet, aqua, ipſam terram, magis quam trigeſies, & octies, non ſolum decies, & aer quoque eandem magis quam. 1500. & ignis magis quam. 55000. partibus magnitudine ſuperaret.

Subſtantia vero rerum quas ſcripſeram circa finem illius conſiderationis talis fuit.

Nunc autem tempus eſſe videtur, vt ego etiam, ne tantum deſtruxiſſe, ſed etiam conſtruxiſſe videar aliquid pro veritate differam.

Non eſt igitur dubium, ſolidæ doctrinæ viris, quin præſtantiſſimus Piccolo. ſecutus ſit tutam viam ad explorandum, quod terra maior ſit quam aqua, metiendo vtriuſque horum corporum ſuperficiem detectam. Omittamus autem compenſationem illam curuitatis, & concauitatis vallium, & montium, &c. quam ipſe Piccolo. propè finem ſexti cap. vellet dare fluminibus, ſtagnis, fontibus, & eiuſmodi aquis. eo enim in loco labitur Piccolo. vbi non conſiderat, quod eiuſmodi obliquis ſuperficiebus non reſpondent anguli ſolidi centri ſphæræ, qui reſpiciunt eorum baſim ad rectos angulos. Sed poſtquam Piccolo. comperit ſuperficiem terrę detectam, eſſe maiorem apparente ſuperficie ſphærica aquæ, proculdubio poterat concludere terram eſſe maiorem aqua, ſicuti fecit, etiã ſi aqua profunda eſſet pyramidaliter vſq; ad mundi centrum, ideſt. 3500. milliaria, ſupponendo tantum eſſe huius globi ſemidiametrum.

Verum quia poſſet aliquis dubitare circa diligentiam Piccolo. in hiſcæ duabus ſuperficiebus dimetiendis, viſum eſt mihi non alienum ſequi aliam viam pro hac veritate probanda, ſupponendo verum eſſe, quod non vnus ſolus metitus fuerit, ſed multi, ideſt ſupponendo verũ eſſe quod maris profunditas menſurari poſſit, & præterea, quod non modo ipſius maris maxima profunditas non perueniat ad quingentos paſſus, ſicuti refert Piccolo. in fine ſui tractatus, & mihi aſſeruerunt Hiſpani multi, & Luſitani præſtantiſſimi nautæ, tum Venetijs, tum Parmæ, in Aula Sereniſſimæ quondam Principis, inter quos, Venetijs fuit Illuſtris Rodericus Guzmanus, Dominus Franciſcus Lopes, Dominus Garzias de Seuilia, multiq́; alij. Parmæ autem varij quos omnes recensere molestum esset. Sed etiam supponendo quod maxima pelagi profunditas sit, non modo. 500.passuum, sed etiam. 500.millium passuum, vt dixi, & quod mare sit huius profunditatis, non vno in loco tantum, aut multis, sed quod supra totam etiam faciem terræ, mare tantę profunditatis ipsam terram vndique operiret, idest, quod vbicunque nunc terra detecta est, esset aqua, spissitudinis.500.millium passuum. Atque vt planius intelligar supponendo quod sicuti totus huius globi semidiameter est milliariũ. 3500. Terrestris partis semidiameter esset tm̃. 3000. & reliquum semidiametri, id est quingenta milliaria esset crassitudo siue profunditas orbis aquei, in quo nihil necesse esset laborare in dimetiendis fontibus, fluminibus, lacubus, stagnis, paludibus, & huiusmodi particulis nullius momenti apud peritos, nec curare subterraneas aquas cauernarum, aut aliorum terræ cauorum, seu terræ porrositatum, quæ omnia sunt circa ipsius terræ superficiem. Quia verisimile non est naturam eiusmodi caua siue spongositates produxisse demissius libramenti maris. Supponendo igitur ea quæ nunc dicta sunt, terra tamen esset ferè duplo maior aqua, hoc est, vt.12.ad.7. Quod quidem, cuiuis mathematicæ philosophiæ mediocriter perito, supputatu facillimum est. Cum proportio diametrorum, seu semidiametrorum, tertia pars existat proportionis eorundem sphærarum. Sed vt parum periti minore labore supputare possint.

Primum sciendum est, quod supponendo diametrum globi, ex terra, & aqua compositi, esse.3500.milliarium, & semidiametrum puræ terrestris partis esse.3000.tantum, eiusmodi proportio erit ut.7.ad. 6.quia communis maior numerator horum duum semidiametrorum erit. 500. qui in maiorem ingredietur septies, in minorem autem sexies. Et eiusmodi proportio superparticularis, vocatur sesquisexta, cuius triplum erit vt.57.cum sexta parte ad.36.& idem erit inter dictum globum compositum, & partem terrestrem simplicem. Quare subtrahendo puram, seu simplicem partem terrestrem, ex composito, reliqua pars erit, vt. 21. cum sexta, pro quantitate aquei orbis, ad quam, terrestris quantitas.36.erit ferè in eadẽ proportione, quæ.12. ad.7.

Nunc fortasse alienum non erit videre quanto ferè maior esset terra, quam tota aqua, non dico autẽ solum de parte illa maximæ eius profunditatis, quæ nusquam ad quingentos passus peruenit, sed de ficto illo orbe aqueo, profunditatis.500. passuum, qui totum terrestrem orbem circundaret, & tegeret, supponendo quod per quingentos passus profunditatis, quidquid est terra, esset aqua, idest supposito quod ex totius orbis compositi semidiametro existente.3500.milliarium, purę terræ semidiameter esset milliarium. 3499. cum dimidio. Supponendo igitur, vt supradixi. Comperietur quod terra esset maior aqua amplius quam.2333. vicibus. Sed quia partes terræ detectæ rumpunt eiusmodi fictum orbem aqueum, quæ quidem partes, sunt ampliores superficię aquæ, vt obseruauit Piccolo.atque alij præstãtes viri, ideo sequetur, vt terra sit maior aqua amplius.4666.vicibus imo amplius quinquies millecuplo. Si autem quis diceret, in quantitate aquæ computari etiam illam, quæ gignatur ex vaporibus, qui globum hunc compositum circundant: respondeo quod non modò ei concedo computari eiusmodi aquam, sed supponendo etiam quod totus locus à vaporibus occupatus, qui attolluntur. 52. milliaria supra superficiem huius globi, vt iam supradictum est, totus esset aqueus, & amplius, supponendo quod orbis hic aqueus esset spissitudinis, siue altitudinis quingentorum milliarium supra totum ipsum globum compositum. Tamen terra esset maior ipsa aqua ferè duplo; qua de re, quisque eiusmodi supputationum peritus certior fieri poterit. Vnde itidem

dem

dem affirmare possemus, terram non solum maiorem esse aqua, sed aqua & præterea aere, si aer non tam altè pertingit, quam multi alij præter Piccolo. sentiunt, qui dicunt inde euenire quod aerea humiditas non tam altè ascendere potest, quoniam humiditas ipsa grauitatem secum affert, præterquam quod nubium situs ostendit supra eas materiam esse rariorem quam sint ipsę nubes, infra vero densiorem. Corpora enim eousque ascendunt donec inueniunt constitutionem mediam formæ æqualis (vt ita dicam) suis. Quare materia illa quæ impropriè ignis vocatur (non enim est ignis) incipit carere humiditate (qua mediante aer definitur) circa quinquagesimum secundum milliarium supra superficiem terræ, vt iam supradixi à Vitellione demonstratum fuisse. Aristo. autem affert rationē quare nubes altius nō transcēdāt. Vnde apparet tertiam aeris regionem impropriè aerem appellari, si humiditate caret, vt ait Arist. qua mediante aer definitur, immo potius retinet ignis naturam, vt etiam asserunt interpretes Aristotelis in primum Meteororum. Qui Aristo. in locis supra citatis itidem ostendit se etiam huius modi esse opinionis.

Quod autem attinet ad probandum quod superficies terrę detecta sit altior quam superficies detecta aquæ, id tam clarum est sua sponte philosophis, qui sciunt quid sit altum, quidue demissum, quod superfluum esset quidquid super hoc dicerem præterquam, quòd constat ex demonstratione ab Aristo. facta textu 31. li. 2. de cœlo, in quo agit de corporibus in aqua positis, vnde eiusmodi veritas planissimè aperitur. Omittimus etiam quod præstantes Moderni omnes, eam pro manifestissima ponūt, sicuti apud omnes sani iudicij homines reuera existimatur.

Hæc enim sunt quæ in fine illius considerationis scripseram.

Anno autem præterito editus in lucem fuit tractatus quidam Pulcherrimus, ab Excellentissimo, nec non Doctissimo viro Augustino Michele, Patritio Veneto, ad corroborandam opinionem antiquorum, vbi tot authoritates, totq; rationes adducit, vt nil amplius dici possit. At ego sensum, rationemq;, & non authoritatem aliquā sequutus sum: cum verò dico sensum, de sensu illorum intelligo, qui profunditatem maris metiti sunt, vt non mihi solum, sed, & Piccolo. & alijs permultis retulerunt, de ratione vero à me adducta, aliorum sit iudicium.

Sed iste mirabilis & Excellentissimus vir, verba mea non accepit in eo sensu, vt ego scripsi, ita vt omnino alienas consequentias sibi confingat, quemadmodū pag.
3. sui tractatus inquit, me non concedere naturam produxisse in magna quantitate,
» atque immensa, id totum, quod bonum, & necessarium est. Hanc enim consequen
» tiam ipse colligit ex eo, quod ego pag. 19. meæ considerationis contra Antonium
» Bergam scripseram, quod videntur multa corpora alijs nobiliora, nihilominus mi-
» nora, eo quod quantitas non sequitur nobilitatem, neque ab ea pendet, ita vt res
» illa quæ nobilior est, necessarium sit vt etiam maior existat. Sed Excellentissimus
» istæ vir scribit ita me dixisse.
» Multa immo infinita corpora sunt nobilia, & necessaria, nihilominus sunt paruę
» molis.
» Vide igitur quantum hoc distat ab illo.

Præterea cap. 12. aliam consequentiam facit, quam ego non tam amplam facio. Ipse enim me inferre vult in alijs terrę partibus cauernas non reperiri, eo quod Mon
» tes sint cauernosi. Aspice quæso. pag. 29. meæ considerationis, & clarè videbis me
» nullo modo negare illas concauitates seu porrositates terræ extra montana loca, circa superficiem terræ, vsque ad æquilibrium, orbiculariter, infimæ profunditatis maris.

Sed putare inferius has porrositates reperiri, cum nulla ratio nobis persuasibilis adhuc ab aliquo prodita sit, idoneum nullo pacto esset. Rationes autem ab ipso Excellentissimo Augustino adductas circa huiusmodi rem, alij dijudicent, de authoritatibus verò, nihil dicam, quia ab illis petendæ sunt, qui profitentur tales facultates, quorum vnius tantummodo authoritas præualere deberet, contra omnes alias corũ qui nunquam attigerunt summis labris orificia harum scientiarũ. Vt si exempli gratia non solum authoritas illorum virorum, quos ipse recensuit, sufficiens esset vt puta Pioccolo. Naibodæ, Bordini, Clauij, reliquorumq; fautorum verę opinionis, sed Francisci Maurolici tantummodo, qui in primo Dialogo suæ cosmographiæ ita inquit.

> „ Existimo autem totum terræ corpus rigidum esse saxum, nam si arena esset, aut gleba fragilis, ita humorem imbiberet, vt cum eo quasi confunderetur; huc accedit, quod si mineræ, ac rupes, quæ sunt grauissimæ partes in ipsa plerunque superficie comperiuntur, multo magis apud centrum esse debent. Videtur ita ratio exigere, vt grauiora centro quoque sint propinquiora.

Hæc igitur sola authoritas, instar reliquarum omnium sufficere posset. Verum de authoritatibus minime curandum est, vbi sensus, ratioq; vera illis opponuntur.

Quod autem numerus animalium aquatilium maior existat numero terrestrium, satis respondimus pag. 41. nostræ considerationis.

Sed in cap. 14. Excellentissimus Augustinus ita inquit (vt etiam superius dixerat)

> „ quod certiorem cognitionem homo non habet illa, quæ à sensu prouenit. Et quod nemo est qui aspiciat terram, & aquam, quod hanc maiorem illa non iudicet, & nõ existimet.

Quod autem certiorem cognitionem homo non habeat illa, quæ à sensu prouenit, concedendum non censeo. Nam omnis cognitio mathematica (cum primum gradum certitudinis obtineat) ab ipso sensu fieret, quod omnino alienum est à veritate. Sensus enim nunquam vidit incommensurabilitates magnitudinum, vel incoincidentias linearum non tangentium cum curuitate hyperbolica, aut angulum contingentiæ aliquem, nec (vt vno verbo dicam) aliquam conclusionem mathematicam, quam volueris. Neque per sensum est scire, inquit *Aristoteles*. Cognitio igitur sensitiua, certior non est illa, quæ per habitum scientificum acquiritur.

Ad reliqua verò, supponamus nos tunc fuisse in Arca Noe, cũ aquæ cooperiebant omnia cacumina montium, vbi nullum terræ vestigium videbatur, quare proculdubio aquam iudicaremus, atque existimaremus maiorem terra, dũ nulla alia re vteremur nisi sensu absque alio discursu intellectuali, ut reliqua illa animalia irrationalia, quæ nobiscum erant in dicta arca. Nõ sufficit igitur superficiem aquæ tantummodo aspicere, quia neque tunc temporis, aqua erat maior terra, etiam si non solum tot cubitis attolleretur supra cacumina montium, sed quingenta milliaria, vt supradiximus.

Ratio autem illa, ex infinitis, ab ipso, eo in loco adducta, talis est.

> „ Aqua est eccentrica ad terram, & pro cẽtro habet centrum grauitatis terræ, aqua igitur maioris est amplitudinis ipsa terra.
> „ Hanc etiam consequentiam alijs relinquo Philosophis dijudicandam.

Subsequitur postea dicens.

> „ Præterea proprius locus terræ, est superficies aquæ, igitur terram oportet ab aqua tegi.

Ad hoc etiam aliquis posset quærere, quis nam erit locus illius partis terræ detectæ ab aqua? nulli dubium erit quin superficies aeris, & non aquæ existet.

Nunc

Nunc autem si locus terræ est sub aqua, ergo locus aquæ proprius est sub aere, & non sub terra, vnde non erit rationabile putare maiorem copiam aquarum existere in cauernis subterraneis, quam supra superficiem terræ. Adde quod locus illarum aquarum non esset superficies aeris, sed terræ, vnde non minus locus aquę esset terra, quam locus terræ, aqua. Sed missa faciamus hæc.

Cap. verò. 20. ita inquit.

Materia elementorum æqualis est. Ergo aqua maior est terra.

32 Hæc enim consequentia verissima esset. Sed nullus vnquam Philosophus (vt Philosophus dico) concedet totam materiam elementarem, in quatuor æquales partes esse diuisam.

Cap. verò. 21. inquit me dixisse non suffecturam paucam spissitudinem. Eo enim in loco pag. 26. mei tractatus contradicens ipsi Bergæ, dixi, quod secundum ipsum Bergam non sufficeret pauca spissitudo.

Similiter etiam dixi, quod secundum ipsum, quanto remotius diffunditur lumen fortasse tantò magis illuminat. Putans ipse Berga quod in propinquo debilius existeret dictum lumen. Et propterea dixi, quod apud ipsum fortasse nihil valet illa propositio, quæ dicit. Agens in propinquo, fortius agit quam in remoto.

Cap. autem. 22. vbi Excellentissimus Augustinus inquit, vnum tantummodo elementum non sufficere ad generationem mistorum. Hoc enim concedo, sed hoc nihil ad me spectat, eo quod meum responsum ad Bergam, erat circa transitum luminis, & non circa generationem elementorum.

Cap. demum. 23. pag. 20. linea. 10. vbi scribit me dixisse, iudicare, oportebat scribere, dubitare.

Puto tamen hoc vocabulum esse errorem Thypographi, quamuis in correctione illud non inuenerim, quia vt ego multoties expertus sum, difficillimum omnes Thypographi errores corrigere, neque (vt fertur) Argi oculi sufficerent.

Hactenus enim in mei defensionem hæc subiungere volui.

Ad defensionem autem Piccolo. aliorumq; virorum meæ opinionis, nec non de proportione duplicata profunditatis maris ad suam amplitudinem, ex consequentia pyramidali: alijsq; similibus rationibus, prodeant alij. Huiusmodi tamen Doctissimi viri ingenium, memoriam, nec non doctrinam valde admiror, atque obseruo.

## DE METHODO PRODVCTIONIS FRACTORVM qua vtuntur Pedemontani Agrimensores.

*Anselmo Rosemburg Agrimensori Cæsareo.*

METHODVS quàm mihi scribis in Prouincia tua maximè in vsu esse, nimis longa atque prolixa est, Pedemontani verò Agrimensores in productione fractorum, valde breui methodo vti solent, quam libenter tibi scribo, eo maxime, vt videas quam rationabiliter operentur.

Scire igitur primum te oportet illos, maximam eorum communem mensuram vocare Trabucum, cuius sextam partem vocant Pedem, duodecimam verò pedis, Vnciam, duodecimã autẽ vnciæ punctũ, duodecimã demum puncti; Attomum.

Quotiescunque igitur multiplicant trabucum, per trabucum nulli dubium est quin producant trabucum superficialem scilicet.

Similiter

Similiter multiplicando pedes, vncias, puncta, & attoma per trabucum, producunt pedes, vncias, puncta, & attoma superficialia rectangula oblonga, quorum longitudo est ipsius trabuchi, latitudo vero lineæ dictarum specierum.

Dum vero multiplicant pedem per pedem, nulli dubium est quin producant pedem quadratum, sed apud ipsos non vocatur quadratum, quamuis reuera ita sit, sed illud vocant duas vncias, quæ quidem sunt rectangula oblonga iam hic supradicta, quarum vniuscuiusque longitudo sit vnius trabuchi, latitudo vero vnius duodecimæ partis ipsius pedis linearis.

Productum autem pedis per vnciam, vocant duo puncta, quæ etiam sunt duo rectangula oblonga, vt supra.

Productum deinde vnciæ per vnciam, vocant duos attomos, qui etiã sunt duo rectangula oblonga, vt dictum est, quæ omnia scientificè videbimus.

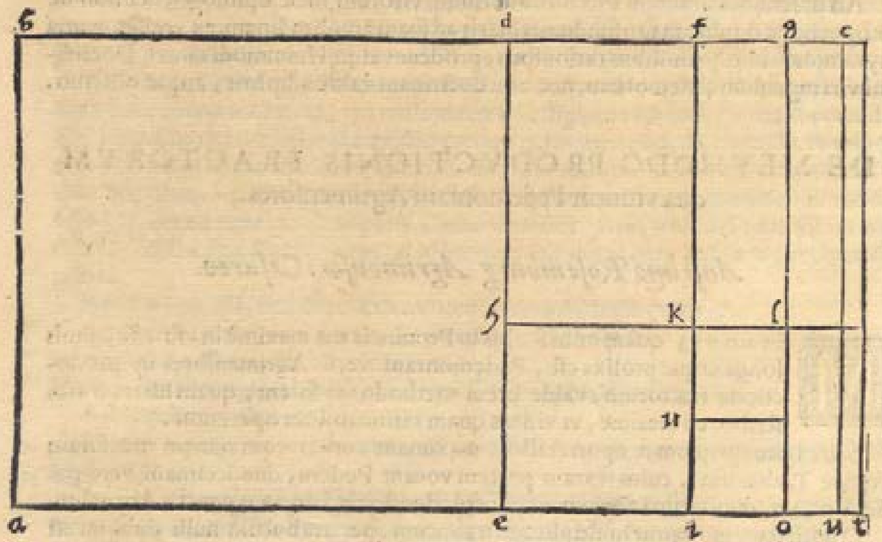
Pro cuius rei cognitione, sit, exempli gratia. a. e. vnus Trabuchus linearis. e. i. vero vnus pes. i. o. autem vna vncia, o. u. postea vnum punctum, et. u. t. vnus Attomus.

Vnde. e. i. erit sexta pars ipsius. a. e. et. i. o. duodecima ipsius. e. i. et. o. u. duodecima ipsius. i. o. et. u. t. duodecima ipsius. o. u. Sit etiam. a. b. æqualis. a. e. lineæ & sic. e. d: i. f: o. g: o. n. &c. terminenturq; parallelogramma. b. e: d. i: f. o: g. u. et. c. t. vnde. b: e. erit trabuchum quadratum, et. d. i. pes rectangulus oblongus vt supra, et. f. o. vncia rectangula oblonga, et. g. u. punctum rectangulum oblongum, et. c. t. attomus rectangulus oblongus.

De producto igitur trabuchi per trabuchũ, nulli dubium est quin sit quadratum. a. d. vt superius diximus.

Productum autem trabuchi cum pede erit. d. i. sexta pars ipsius. a. d. cum. e. i. sit sexta ipsius. a. e. ex prima sexti vel. 18. aut. 19. septimi, siue etiam ex. 15. quinti Eucli.

Productum autem pedis cum pede erit. e. K. quadratum, quod probandum est duplum



duplum eſſe rectangulo. f.o. Nã.K.i.ſexta pars eſt ipſius.f.i.ex ſuppoſito,et.i.o.duodecima ipſius.e.i.proportio igitur.e.i.ad.o.i.dupla eſt proportioni ipſius.f.i.ad.K. i. quare.K.e.duplo maius eſt ipſius.f.o. eo quod ſi.i.o.vel.f. g. (quod idem eſt) duplo maius eſſet ipſo latere prǽſenti.o.i.vel.f.g.tunc.f.o.æquale eſſet ipſi.K.e.ex. 15.ſexti vel.20.ſeptimi quod quidem.f.o.duplo maius eſſet ipſo præſenti. f. o. Rectè igitur inquiunt dicentes productum pedis cum pede eſſe duas vncias,vel ſi mauis,ita dicas e.K.ſexta pars eſt ipſius.d.i.ex iam dictis propoſitionibus.f.o. autem eſt duodecima ipſius.d.i.ex ijſdem,cum ex ſuppoſito.i.o.duodecima ſit ipſius.e.i.quare.e.K. duplũ erit ipſius.f.o.ex communi notione.

Productum verò pedis cum vncia.ſit.K.o.quod probabimus ex ijſdem rationibus duplum eſſe ipſius.g.u.puncti rectanguli oblongi. Nam.l.o. ſexta pars ſimiliter eſt ipſius.g.o.et.o.u.duodecima ipſius.o.i.quare proportio.i.o.ad.o.n.dupla eſt proportioni.g.o.ad.o.l. ſequitur ergo ex prædictis rationibus.k.o.duplum eſſe ipſius.g .u. vel ſic,vt in præcedenti,cum.K.o.ſit ſexta pars ipſius.f.o. ex dictis propoſitionibus. g.u.verò duodecima eiuſdem.f.o.ex ijſdem,nam.o.u.duodecima eſt ipſius.o.i.ergo K.o.duplo maius eſt ipſo.g.u.

Ex ijſdemmet rationibus productum.l.u.pedis cum puncto duplum eſt ipſius.c.t. attomi rectanguli oblongi.

Probandum nunc relinquitur productum.o.n.vnciæ cum vncia,quod eſt quadratum, duplum eſſe ipſius.c.t.attomi rectanguli oblongi. Nam.i.n.eſt pars vna ex.72. ipſius.c.u.et.u.t.pars vna ex.144.ipſius.o.i.ex ſuppoſito,quare proportio.i.o.ad.u.t. dupla eſt proportioni ipſius.c.u..ad.n.i.ex dictis igitur rationibus.o. n. duplo maius eſt ipſo.c.t. Vel ſi placet dicas.n. o.eſt vna pars ex.72.ipſius.f.o. ex ſupradictis, eo quod.n.i.ita ſe habet ad.f.i. vt vnitas ad.72.ſed ex ijſdem rationibus.c.t.pars vna ex 144.eſt ipſius.f.o.eo quod ita ſe habet.u.t.ad.o.i.quare.o. n. duplo maius erit ipſo. c.t.

Propoſitum ſit nobis nunc, exercitij gratia, quærere ſuperficiem alicuius rectanguli,cuius vnum latus ſit trabuchorũ. 3. pedum.2.& vnciarum.3.aliud vero latus ſit trabuchorum.2.pedum.3. vnciarum vero.2.

Huiuſmodi autem methodo mediante, multiplicando primum latus dictũ.3.2.3. per numerum trabucorum ſecundi lateris.2.ſcilicet producentur nobis primò trabucha ſuperficialia.6.pedes.4.& vnciæ.6.omnia rectagula, vt dictum eſt. Multiplicando deinde idem primum latus.3.2.3.per pedes.3. ſecundi lateris. Ex trabuchis. 3. primi lateris cum. 3. pedibus ſecundi , producentur. 9. pedes rectanguli , hoc eſt vnus trabuchus cum tribus pedibus rectangulis. Ex pedibus autem huius. 2. cum ijſdem alterius lateris.3.producentur.12.vnciæ rectangulæ ideſt vnus pes rectangulus. Ex ijſdem pedibus.3.ſecundi lateris,cum.3.vncijs primi lateris producentur.

| Trabucha. | pedes. | vnciæ. | |
|---|---|---|---|
| 3. | 2. | 3. | |
| 2. | 3. | 2. | |
| 6. | 4. | 6. | |
| 1. | 3. | 1. | 6. |
| | 1. | 6. | 8. |
| | | | 1. |
| 8. | 3. | 2. | 3. |

18.

18. puncta rectangula, hoc eſt vna vncia cum. 6. punctis rectangulis. Deinde ex multiplicatione vnciarum. 2. ſecundi lateris, cum. 3. trabuchis primi lateris, producentur. 6. vnciæ. Ex multiplicatione poſtea dictarum. 2. vnciarum ſecundi lateris cum. 2. pedibus primi, producentur. 8. puncta.

Demum ex ijſdem. 2. vncijs ſecundi lateris cum. 3. primi, producentur. 12. attomi, ideſt vnum punctum. Quæ omnia collecta facient trabucha. 8. pedes. 3. uncias 2. & attomi. 3. omnes rectanguli oblongi. Pulcherrima profecto operatio.

| Trabucha. | pedes. | vnciæ. | |
|---|---|---|---|
| 3. | 2. | 3. | |
| 2. | 3. | 2. | |
| ——— | ——— | ——— | |
| 6. | 4. | 6. | |
| 1. | 3. | 1. | 6. |
| | 1. | 6. | 8. |
| | | | 1. |
| ——— | ——— | ——— | ——— |
| 8. | 3. | 2. | 3. |

Videamus nunc exercitij cauſa, vt dixi, quomodo conueniat calculus iſte cum calculo ordinario communi?

Nam quotieſcunque dicta latera, fracta fuerint in vncias, primum latus erit vnciarum. 243. ſecundum autem. 182. productum vero vnius in alterum erit vnciarum quadratarum. 44226. quod quidem productum cum diuiſum fuerit per. 5184. vncias quadratas vnius trabuchi quadrati, prouentus erit. 8. trabuchorum, reliquus verò numerus, ſiue fractus, erit vnciarum quadratarum. 2754. qui cum diuiſus fuerit per numerum. 144. vnciarum vnius pedis quadrati, prouenient pedes. 19. quadrati cum vncijs. 18. ſuperabundantibus, dicti autem pedes. 19. ſignificant tres pedes rectangulos oblongos cum vno pede quadrato, hoc eſt cum duabus vncijs rectangulis oblongis, vt ſupra.

Videndum nunc eſt, vtrum illæ. 18. vnciæ æquipolleant tribus punctis rectangulis oblongis: ſed hoc manifeſtè videre eſt, ex hoc, quia quęlibet vncia rectangula oblonga componitur ex. 72. quadratis, punctum autem rectangulum oblongum, cū ſit duodecima pars ipſius vnciæ rectangulæ oblongæ, ipſum componetur ex. 6. vncijs quadratis. 18. igitur vncijs quadratis, triplum erit ipſius puncti rectanguli dicti. Vnde clarè patet, quod, quotieſcunque voluerimus ſcire proportionem ipſarum vnciarum quadratarum ſuperabundantium, ad punctum rectangulum oblongum, ſi dixerimus ex regula de tribus, ſi. 72. (vncia rectangula oblonga) dat. 18. quid dabūt 12? puncta rectangula oblonga, quarum vnaquæque eſt duodecima pars ipſius vnciæ rectangulæ oblongæ, in præſenti autem caſu prouenient. 3. pro quarto termino quæſito, & habebimus propſitum.

SO-

## SOLVTIO CVIVSDAM QVÆSITI.

### *Magnifico Ludouico Fauzzoni amico carißimo.*

TVI quæſiti ſolutio quam neſcio quis te docuit, valde diuerſa eſt à vera. quæſitum enim tale fuit.

Reperiuntur quatuor ſocij, Ludouicus, Hieronymus, Franciſcus, & Laurentius quorum primus, Ludouicus ſcilicet, poſuit aureos. 6000. Hieronymus verò aureos. 5000. Franciſcus autem. 2000. & Laurentius. 1000. quorum ſumma faciebat aureos. 14000. interim tamen de tali ſumma Ludouicus recepit aureos 2000. Hieronymus verò. 1000. Franciſcus autem. 900. & Laurentius. 800. quapropter in ſumma reſidua Ludouicus non habebat niſi aureos. 4000. Hieronymus etiã 4000. Franciſcus. 1100. & Laurentius. 200. quorum ſumma erat. 9300. Nunc autem iſti ſocij cupiunt augere hanc ſummam per aureos. 20000. tali tamen conditione quod quilibet tantum tribuat vt in totali ſumma, tantam partem unus habeat, quantam alter.

Hoc autem problema tam facile eſt, & cum ſuo theoremate ita coniunctum, quod miror amicum noſtrum illud illico non vidiſſe.

Accipe igitur illos aureos. 20000. & eos collige cum ſumma. 9300. vnde habebis aureos. 29300. pro sũma totali, cuius quarta pars erit. 7325. quã vnuſquisq; poſtea habebit in dicta ſumma. Sed ut reperias quantitatem aureorum quam quilibet prius debet contribuere, vt poſtea habeat aureos. 7325. in dicta ſocietate. Iubeo, vt Ludouicus demat illos aureos. 4000. quos demum habebat, ex. 7325. reliquum autem erit. 3325. qui quidem numerus erit aureorum nunc contribuendorum ipſius Ludouici. Demptis ſimiliter aureis. 4000. ex dictis. 7325. remanebũt. 3325. pro contributione ipſius Hieronymi. Deinde ſi ex. 7325. extracti fuerint aurei. 1100. relinquent. 6225. pro contributione Franciſci. Demptis demum. 200. ex. 7325. reſidui erunt. 7125. pro contributione Laurentij, & ſic quilibet habebit æqualem portionem in totali ſumma.

### *Speculatio cuiuſdam Methodi reductionis numiſmatum unius ſpeciei in aliam.*

## AD EVNDEM.

MIrum tibi videtur quo pacto verum ſit, quod ſumma medietatis cuiuſuis numeri illorum numiſmatum, quæ hic vocantur Blanci, cum ſexta parte eiuſdem medietatis, ſemper ſit numerus florenorum huius prouinciæ. Vt exempli gratia, quotieſcunque reducere voluerimus. 48. Blancos in Florenos, ſi medietati ipſius. 48. hoc eſt. 24. adiecta fuerit ſexta pars ipſius medietatis, quæ eſt. 4. tunc habebimus. 28. & ita dicemus quod. 48. Blanci conſtituunt Florenos. 28. quod quidem verum eſt.

Huiuſmodi autem rei ſpeculatio ita ſe habet. Nam vnuſquiſque Blancus diuiditur in. 7. æquales partes, quarum. 12. conſtituunt vnum Florenum, horum verò numiſmatum communis menſura, vocatur Groſſus, vt ſcis, ex quo ſequitur, quod ſi

28.Floreni æquantur Blancis.48.tot Groſſi erunt in.28. Florenis quot in.48. Blancis. Fingamus igitur, mente,noſtram figuram.79. Theorematis Arithmetici.x.u.o.e.n.ſupponendo ambo producta.u.x.et.n.e.inuicem ęqualia exiſtere,& vnumquodque eſſe groſſorum.336. ſit etiam.o.x.vnus Florenus.12.groſſorum.o.n. verò Blancus. 7.eorundem groſſorum. o. e. autem Blancorum. 48. Nunc certi erimus ex. 15. ſexti vel.20.ſeptimi Euclidis eandem fore proportionem.o.u.ad.o.e.quæ.o.n.ad.o.x. ſed.o.n.eſt ſumma medietatis ipſius.o.x.cum ſexta parte dictæ medietatis,ita igitur erit.o.u.ipſius.o.e.hoc eſt ſumma medietatis.o.e.cũ ſexta parte medietatis eiuſdem, quæ ſumma in præſenti exemplo erit.28.

Hac enim ſpeculatione mediante, poteris methodum inuenire conuertendi Florenos in Blancos.Vt ſi nobis propoſiti fuerint Floreni. 28. Voluerimusq́ue inuenire quot Blancos faciant, ſuppoſita menſura communi, iam ſupradicta. Nam duplicabimus numerum Florenorum, à quo duplo detrahemus ſeptimam partem,reliquũ verò erit numerus quæſitus.

Huiuſmodi autem rei ratio eſt, quia, cum in ſupradicta figura, proportio.o.e. ad o.u.ęqualis exiſtat ei, quæ.o.x.ad.o.n.atque etiam.o.x. ſit minor duplo ipſius. o. n. per ſeptimam partem ipſius dupli.o.n.minor erit.o.e.duplo ipſius. o. u. per ſeptimã partem eiuſdem dupli ipſius.o.u.

Idem affirmo de quauis conuerſione aliorum numiſmatum, quorum ſemper.o.x. maior ſit.o.n.verò minor. Vt ſi.o.x.æquiualeret.7: et.o.n.valeret.4. et. o. e. valeret 42. quæ quidem.o.e.menſuraretur ab.o.n.

Si cuperemus ſcire quot.o.x.ſint in.o.n. Primo dicemus in. o.n.reperiri ſummam medietatis ſex ſeptimorum ipſius.o.x.collectæ cum vna ſeptima parte ipſius.o.x.ſeu (vt ita dicam) cum tertia ipſius medietatis. Vnde dempta ſeptima parte ipſius.42. quæ eſt.6. collectaq́; cum medietate reſidui, quæ eſt.18.habebimus.24.res,quarum vnaquæque æqualis erit ipſi.o.x.

Sed ſi quis cupiat reperire.o.e.dato.o.u.duplicet.o.u.à quo demat quartam partẽ ipſius.o.u.& habebit propoſitum. Nam ita ſe habere oportet.o.e.ad.o.u.quemadmodum.o.x. ad.o. n.

## *De lucro mercantili.*

## AD EVNDEM.

QVOD demum ſcire à me deſideras,eſt, quod cum vendideris libram vnam mercis pro.4. ſolidis,& lucratus fueris.2. cum quarta parte vnius pro ſingulis decem libris, ſcire velles quantum lucri facturus eſſes in libris decẽ dando ſingulam libram pro. 6. ſolidis.

Nulli dubium eſt quin decima pars de.2.cum quarta vnius ſit lucrum libræ vnius. Quæ decima pars ſunt nouẽ quadrageſimæ partes, & hæc ſubducta à ſolidis.4. reliqui erunt ſolidi.3. cum.31.quadrageſimis partibus pro ſorte vnius libræ. Quę ſors ſubtracta à ſolidis.6.remanebunt ſol. 2. cum.9. quadrageſimis lucri pro libra, quod multiplicatum per.10.proueniunt ſol.22. cum quarta parte vnius, & tantum aſcenderet lucrum, quod fieri poſſet in libris decem ſi quamlibet, ſol. 3.cum.31.quadrageſimis nobis conſtaret.

Vel ſic multiplicemus ſortem vnius libræ per.10.productum erit.37.cum tribus quartis

quartis, iterum multiplicemus per. 10. fortem cum lucro vnius libre quod est. 4. productum erit. 40. differens à primo sol. 2. cum quarta parte, multiplicemus pariter per. 10. precium. 6. solidorum proueniens erit. 60. à quo deducendo productum fortis librarum. 10. quod erat sol. 37. cum tribus quartis superérunt sol. 22. cum quarta parte, vt supra.

## DE DIGNITATIBVS PLANETARVM.

### *Adriano Panetio.*

QVOD eam distinctionem orbium, quæ iam inualuit, non teneas, sed putes totum esse quoddam continuum excipiens corpora stellarum, nouum nõ est, nam nonnulli solidæ doctrinæ Philosophi idem censuerunt. Sed quod attinet ad dignitates planetarum in signis zodiaci, scias huiusmodi ordinem me comprehendere esse desumptum ab ordine antiquo orbium ipsorũ planetarum, quiquidem ordo erat, vt statim post Lunam succederet Sol, post Solem Mercurius, tum Venus deinde Mars, postea Iupiter, & tandem Saturnus per eosdemq; orbes, retro redibant, atque hoc cognoscitur constituendo Cancrum domicilium Lunæ, Leonem, Solis, Virginem, Mercurij, Libram, Veneris, Scorpionem, Martis, Sagittarium, Iouis, Capricornum, Saturni, Incipientes deinde ab Aquario, qui ad nos propius accedit eundemq; tribuentes Saturno, Pisces, Ioui, Arietem, Marti, Taurum, Veneri, & Gemellos, Mercurio, septem Planetas cum duodecim signis zodiaci concordes reddebant.

Quod deinde Aristoteles in libris de sensu & ijs quæ sensibus percipiuntur, dicit pupillam oculi esse nigram, non ita se habet, nam idem est, ac si quis diceret nigrũ esse illud medium, quod permitteret transitum lumini per suam diaphaneitatem, nullum lumen à seipso reflectens, & etiam ac si quis diceret nigrum esse aerem alicuius cubiculi vndequaque clausi tenebrosi.

Quod etiam idem Aristoteles volens adducere causam, cur oculus magis materiam aquæ, quam aeris participet, dicens id ea ratione fieri, quod aqua magis quam aer seruari possit, eodem libro scribit, est reuera admirandum. Ibi enim clarè demonstrat se planè ignorare, & constructionem oculi, & causam diuersitatis eorum humorum tam in substantia, quam in figura, quæ non aliunde dependet quam quod diuersam refractionem radiorum luminosorum producat, qui per pupillam ingrediuntur, vt ad proprios sibiq; destinatos locos dirigantur radij, vt à virtute visiua perfectius sentiantur.

## *De ratione Frigiditatis locorum umbrosorum.*

### AD EVNDEM.

VEra ratio vnde fiat, vt quanto magis sentitur calor in locis expositis Soli, tanto minus sentiatur in vmbra, vbi Solis radius non reflectitur, est quia cum rarefactus est aer à vehementi calore radij solaris, seipsum colligit, & condensatur in locis, à quibus à calore, ratione rarefactionis, non expellitur, & quia naturaliter calor sequitur rarum, rarum calorem, & frigidum densũ, & densũ frigidum, vt vnicuique sanę mentis patet, hanc ob causam sequitur rem ita se habere vt diximus. Possumus etiam absque dubio credere huiusmodi ratione fieri, vt frigus matutini temporis, in crepusculo maius esse eo, quod noctu viguit. Nam materia consistens in cono vmbræ terræ, semper densior est ea, quæ extra reperitur, imò noua materia continuo condensatur, propter motum vmbrę, quæ semper corpori solari opponitur. hęc

Fff 2 autem

autem noua condensatio dico semper fit in crepusculo matutino, hoc est in parte coni à Sole pulsa, in parte vero contrari a ipsius coni hoc est in parte crepusculi vespertini, contrarium accidit, quia potius aliquantulum in hac parte materia coni rarificatur, quia extrinseca condensatur, in parte vero matutina extrinseca rarificatur; & propterea intrinseca condensatur.

## QVOD RECTE ARIST. SENSERIT COELVM casu non esse productum.

### *Hieronymo Condrumerio.*

FErunt Aristippũ tempestate maris ad incognita littora delatum, cum in arena vidisset quasdã figuras geometricas delineatas exultantẽ lętitia dixisse: Hæc sunt hominum vestigia. Nam consonum rationi non erat, vt huiusmodi figuræ casu essent impressæ: neque etiam credendum est ingentem hanc machinam tanto ordine constantem fortuitò esse productam, cum nulla quantumuis minima eiusdem particula, dummodo nitatur ordine, aliquo modo casu effecta fuerit; cum casus nihil producat, quod regulam & ordinem seruet. Non est igitur producta casu admiranda correspondentia, quæ est obiectorum cum potentijs, luminis cum oculo, soni cum auditu, saporis cum gustatu, odoris cum odoratu, qualitatum tangibilium cũ tactu. Si diligenter deinde cuiuslibet rei naturalis operationem considerabimus, eas tanta arte constructas videbimus, vt cogamur fateri aliquam prudentissimam, & sagacissimam mentem eas formasse, si ergo quælibet mũdi pars tanta cum ratione & ordine est constructa: quomodo fieri poterit, vt de toto ipso mundo id in dubium vocemus, certissimeq; non credamus diuinissimam aliquam mẽtem esse à qua exquisitissima huius vniuersi harmonia, quæ ex tot tantisq; partibus, maximo ordine nitentibus conficitur, non dependeat?

## VARIA RESPONSA.

### *Nicolao Petreio.*

AD ea quæ mihi scribis dico, quod excrementa quæ ex corpore sano prodeunt in sua ipsorũ qualitate sensibili ita se habẽt ad facultatẽ illius partis eiusdem corporis, ut eã non lędant, queẽadmodũ efficeret sputũ, si esset amarũ, aut quod ex cernitur naso fętidũ esset. Imagineris igitur queẽadmodũ possit esse verũ id quod idẽ amic⁹ noster ait. Pręterea si aliquid tibi in oculũ inciderit, an nescis quomodo statim affatim affluat humor, vt id foras ꝓpellat, vel abducat? (mirabile opus naturæ.) Dic etiã eidem non absque mysterio naturam in tot miserijs senectutem posuisse, cum sæpissimè senex mori desideret, ut huius vitæ calamitatibus liberetur, vnde fit, vt cum eius aduentum sentiat, minus affligatur. Dicito etiam eidem, naturam non fuisse tam solicitam de quibusdam partibus quemadmodum est de toto, vnde magis rotunda, & polita poterat esse superficies terræ, quam nunc est, quia natura magis respicit totum, quam partes, & magis maiores, quam minores.

Dum tuas legerem, me continere non potui quin riserim, id quod scribis te interrogasse eum Philosophum naturalem, vnde fit, vt ventus sit frigidus, eumq; tibi respondisse, quod à remotissimis partibus veniat, genereturq; à vaporibus terræ frigidis. (cum ipsa sit frigida.) Cæterum miror quod ab eo non quæsieris, vnde oriatur frigiditas, quæ percipitur ab agitatione aeris, qui quidem à vaporibus terræ non prosilit, nec à remotissimis partibus ad nos accedit. Sed quia de eadem re me inter-

terro-

terrogas, ſcito naturã coniunxiſſe frigiditatẽ cũ denſitate, & caliditatẽ cũ raritate, vt ſup. diximus, ita vt cum aliquod corpus denſat̃, frigidũ reddat̃, & dum rarefit maiorem caliditatem acquirat, & ſic econtra fit, vt quanto magis aliquod corpus refrigeratur, tanto denſius reddatur, & quanto calidius fit tanto rarius efficiatur. Quoties igitur agitabitur aer, aut aliud corpus, quod ratione ſuæ ſubtilitatis, velociter condenſari, & rarefieri poſſit, eius partes denſiores ſemper erunt frigidæ, & hanc obrem quilibet ventus, qui per calida loca non tranſeat, natura ſua frigidus, calidus autem per accidens erit. Hinc fit vt vaſa vitrea, & terrea tam in vehementi frigore, quam in magno æſtu frangantur, quia horum vnum fit, ne aliquis locus vacuus remaneat, & aliud ob loci neceſſitatem, ſed hoc non ſequeretur, ſi in materia, qua huiuſmodi vas conſtat, aliqua aeris portio non contineretur.

## DE LVMINE LVNÆ, DE FINE LVMINIS, de fine motus corporum cęleſtium, de albedine, de ſphæra.

*Clariſſimo Antonio Nauaiero.*

LVMEN Lunæ etiam ſi ſit lumen reflexum Solis ab ipſa Luna, ab ea tamen non ita reflectitur, vt à ſuperficie polita ſpeculi, cũ eius luminis tantã quãtitatem ſuper ipſum corpus lunare videamus, & eo modo terminatã quo conſpicimus. per ſe lumen, cauſa oculi eſt effectum, per accidens autem puta quod vis. Terra deinde nunquam lunari lumine (quãuis ſolaris reflexio exiſtat) omnino deſtituta eſt, dico etiam, neque in ipſis ecclipſibus ſolaribus vel lunaribus, in ſolaribus enim cum Sol tot millia vices maior ſit Luna, Luna verò minor terra, ſequitur, vt terra non omnino priuata remaneat lumine Lunæ, in ecclipſibus verò lunaribus Luna ſemper videtur, gratia luminis ſolaris, quamuis refracti. Motus corporum cœleſtium fit ratione ſitus, & varietatis virtutis ſtellæ in diuerſis locis, hæc autem varietas abſque diuerſo ſitu eiuſdem ſtellæ, nec diuerſus hic ſitus abſque motu fieri poſſet, ita vt motus ſtellarum ſit ratione diuerſitatis ſituum ipſarum, ergo motus, & diuerſitas ſituum, fit, ob diuerſam influentiam. Quæ autem de albedine fratri tuo dixeram, erant, quod inter oẽs colores albedo, certo quodam modò, maiorẽ ſimilitudinem habet cum lumine. Primò quia magis coniungitur cum lumine. Secundò quia magis afficit ſenſum. Tertiò quia abſque reſiſtentia magis recipit qualitatem aliorum colorum, quam alij colores. Quartò quia maximus eſt omnium colorum. Quintò quia ſimplicior eſt reliquis. Sextò quia diſgregat viſum. Septimò quia qualitas quæ in niue alba eſſe videtur, nihil aliud eſt quam multitudo quædam luminum reflexorum, & non albedo, ſimilis ei, quæ eſt lactis, aut panni, quæ quidem ſeptima cauſa effecit, vt ipſam albedinem, magis quam alium quemuis colorem cum ipſo lumine compararem, cum nihil ſit, quod eſſe ſuum trãſmutans, aut apparenter, aut eſſentialiter, illud ipſum prius non tranſmutet in formam ſibi propinquiorem, vt manifeſtè patet. Eſt etiam huius rei octaua ratio, magni ponderis, quia ſcilicet nullus ſit color, qui magis reſiſtat lumini, aut in quem lumen minorem impreſſionem faciat, quam albedo. Vnde ſequitur, obiecta alba, minus eſſe combuſtibilia quam alia, cum quælibet res in ſuum contrarium quam in

ſibi

ſibi ſimile valentius agat, vt rectè vidit Ariſtoteles cum dixit, omne contrarium à ſuo contrario pati natum eſt.

Inter corpora, multum ſimplicitatis retinet ſphæra.

Circa quod, præter rationes adductas ab Ariſtotele in libris de Cœlo, poſſumus etiam ratiocinari à facilitate motus vndiq; ab eo quod violentiæ non reſiſtat, ab eo, quod apta nataq; ſit quieſcere ſupra quoduis punctum ſuę ſuperficiei, ab eo quod ab aliqua ſuperficie alterius corporis ſeſe tangi non permittat, quæ curuitate concaua non adæquetur, niſi medio vnius puncti. Verum eſt, quòd licet hæc vltima ratio nō ſit propria ſphæræ, eſt tamen cauſa ſimplicitatis in eo, in quo reperitur, ſed proprię paſſiones ſphæræ ſunt ſupradictæ, præter quam quod alia eiuſdem ſphæræ eſt propriiſſima, quæ eſt diſtantia eius termini ab vno tantummodo puncto ideſt ab eiuſdē centro, & etiam poſſe diuidere corpus aliquod medium, cum æquali reſiſtentia circa punctum, quod prius in motu reperitur.

Aequalitas autem rerum, eſt etiam valde ſimilis ſimplicitati, & vnitati.

## *Comparatio uiſus, & auditus.*

### AD EVNDEM.

QVod ad viſum & auditum attinet, magis neceſſarium eſſe viſum, & nobiliorem quam auditum exiſtimo, primò quia ſi quis viſu orbatus eſſet, contra frigus, & calorem, contra famen, & ſitim nil prouidere poſſet, neque aliud quicquam hoc vocabulum prouidere ſignificat, neque abſque periculo vitæ ab vno loco ad alium ferri poſſet, neque aliquid arte facere.

Sed ſi quis deſtitutus eſſet facultate audiendi, ſupradictas tamen operationes pręſtare poſſet, neque modo careret, quo animi ſui ſenſa abſque beneficio ſoni, ſed ope figurarum & characterum alteri aperiret: neque etiam munere ſpeculandi ſcientias (excepta muſica) deſtitueretur. Ad ſcientiam comparandam, longè magis neceſſarius eſt viſus, quam auditus præterquam, quod viſus maiorem numerum obiectorum, & differentiarum rerum percipit, & inter reliquos ſenſus velociſſimè imò in inſtanti operatur, magis remotè quam alij, & exactius ſentit, minusq; quam reliqui afficitur, præterquam quod ſemper agit, dummodò non dormiat animal. Præterea ſeſe magis patefacit, & prodit anima per oculos, quam per aliud, cuiuslibet ſenſus, inſtrumentum. Oculo magis quam alia corporis parte, hominis natura cognoſcitur: & ſi aliquid ſpeculari volumus, quod ſine imaginatiua fieri non poteſt; ſtatim imaginamur nos videre huiuſmodi rem, ac ſi oculo fuiſſet compræhenſa, & ab imagine quæ eſt vnum ex obiectis oculi, imaginatiua nuncupatur. Beneficio oculorum omnes ferè ſcientiæ ſunt adinuentæ. Auditus nil aliud quam ſonum capit, auditus nunquam detulit intellectui figuram, corpus ſuperficiem, aut lineam, materiam, formam, locum, dimenſionem, plenum inane, nec innumera alia accidentia, quæ ab oculo compræhenduntur. Quæ verò viſui, & auditui ſunt communia, ſunt etiam tactui communia, vt numerus, motus, maius, & minus, ſunt tamen aliqua oculo & tactui communia, quæ auditus non poteſt capere, vt durum, molle, acutum, obtuſum, aſperum, lene, planum, curuum, concauum, conuexum, magnum, paruum, & ſupradicta, ideſt figura corpus & cętera, vt etiam rectum, obliquum, & ſimilia.

Ariſto-

Ariſtoteles circa finem primi capitis libri de ſenſu ait mediante viſu, magis quã quolibet alio ſenſu, nos percipere ſenſibilia communia. Vbi eundem per ſe, & non per accidens magis neceſſarium eſſe quam auditum, tam in ijs quæ ad victum, quam in ijs quæ ad ſcientiam pertinent eſſe aſſerit, quia auditus intellectui confert per accidens. Vide etiam quod idem ſcribit primo metaphyſicorum. Et ſi ad aliquid perfectè cognoſcendum, oculo ſeſe nobis offerrent ea omnia obiecta, quorum ſpecies in imaginatiua formamus, ipſa imaginatiua non egeremus. Sed quia hoc fieri non poteſt, hunc theſaurũ imaginatiuè, ſeu memoriæ ad conſeruandam imaginẽm omnium obiectorum ſenſibilium nobis dedit natura, vt ope diſcurſus intellect⁹ circa dictas imagines, rerum veritatem venari poſſimus. Sed vt ad propoſitum redeamus, beneficio oculi animal liberum eſt, cum ſine ipſo locum mutare nequeat, vt ſit tutum. tenebræ, priuatioq; viſus ſunt ferè vnum, & idem. Neque vllus eſt ſenſus, qui ſit magis ſimilis intellectui quam viſus: neque alij ſenſus habent obiecta viciſſim communia, quæ non ſint etiam oculo communia, ſed inter oculum, & quem libet alium ex ſenſibus, inuenientur quidem obiecta communia, quæ cum alijs non communicabunt, vt inter oculum & tactum, figura, acutum, obtuſum, & ſimilia, quæ alijs ſenſibus non percipiuntur. Mediante viſu, & auditu etiam, compræhendũtur variæ diſtantiæ, ſituſq; obiectorum, nec non proportiones, & alia quę ab alijs ſenſibus non compræhenduntur. Multa obiecta deinde ſunt ſubiecta guſtatui, quę alijs accidentibus prędita ſunt, vnde cum fuerint ſemel deguſtata, talia, qualia ſunt ab oculo percipiuntur, quod nullus ex alijs ſenſibus præſtabit. Idem de obiectis odoratus dico. Senſuum nullus eſt qui maiorem ſimilitudinem gerat cum vigilia & cum vita, quam viſus, neque aliquid eſt, quod magis repræſentet imaginem ſomni, & mortis, quàm cęcitas.

Qui ſibi oculos eruit vt melius ſpecularetur maxima ſtultitia prius obcęcatus fuit quia ſoni magis impediunt ſpeculationem quàm lumina, imò qui commodè vult contemplari, quantum plus poteſt nititur longius eſſe ab omni ſtrepitu, magis quàm à locis luminoſis, & animal magis lætatur lumine quam ſono: & ad ſpeculationem nos magis inuitat harmonia luminum variorum colorum & figurarum, quàm harmonia ſonorum, præterquam quod inſtrumentum viſus totius corporis partium eſt pulcherrima, & in eminentiori loco locata, ſi de inſtrumentis ſenſuum loquamur, & veluti fineſtræ animæ. Et ſi Ariſtoteles dicat oculos & aures in vno eodemq; orbe exiſtere, volens inferre quod in eodem æquilibrio ſint æqualiter alta non ita ſe habet, quia (ſi de homine loquamur) oculus eſt altior aure. Beneficio huius ſenſus, eorum quæ abſunt, & longo iam tempore ſunt mortui, animi ſenſa, & conceptus intelligimus, neque alia ratione rerum omnium memoria ſeruari poteſt. Si cabala unquam vera fuit, nulla alia ratione eſt deleta, quam quia alicuius ſigni viſibilis medio conſeruata non fuerit, & quæcunque non ſcribuntur, ideſt oculo non cõmendantur parum durant cito obliuioni tradunt. In maiori ſemper pretio fuit pictura quã muſica: obiectis viſibilibus magis quam ijs quæ ſub auditu cadunt, affectus animi, atq; alia quælibet res naturalis exprimi poſſunt. Aegyptij volentes ſignificare Deum, oculi medio id præſtabant.

Oculus, reſpectu aliorum inſtrumentorum ſenſuum, eſt quaſi epicyclus animæ, neque defuit qui crederet oculum eſſe principem animi partem.

Oculus à Sole, & à Luna ita dependet, vt qui tempore defectus cuiuslibet luminaris naſcitur, ſtatim cæcus euadat, neque aliqua eſt corporis pars in qua magis ap-

pareat

pareat differentia vitæ à morte; quam in oculo. Ariſtoteles ad finém cap.15.lib.primi poſteriorum ait, clarum eſſe quod ſi aliquis ſenſus deficiat, futurum vt aliqua quoque ſcientia deſit. Conſidera, quot ſcientijs careret homo, ſi viſu orbaretur.

Et in tertio de anima ait, eum qui non ſentit, nihil intelligere poſſe; id quod inde confirmat, quia nihil ſit in intellectu, quod prius non fuerit in ſenſu. Plato in timeo ait, oculos nobis attuliſſe rerum optimarum notitiam, & ſi oculus non fuiſſet nihil eorum, quæ ad cœlum ſpectant inueniri potuiſſe, & cognitioné diei ac noctis ab oculis ortum duxiſſe, vt reuolutiones menſium, & annorum metiri, & tempus cognoſcere, & inueſtigare ordinem naturæ vniuerſalis poſſemus; quibus philoſophiã nobis comparauimus, ut alia multa omittam, quæ ibi à Platone dicuntur. Addam hic & aliam ſpecialem differentiam inter auditum & viſum, quæ eſt, vt obiectum viſus ſit permanens, & obiectum auditus tranſitorium ſiue ſucceſſiuum aut, vt alio modo idem dicamus, obiectum viſus particpes ſit æternitatis, illud autem quod eſt auditus non item, nam auditus tempori ſubiectus eſt, viſus autem minimè. Vel ſi dicamus operationem auditus abſque tempore fieri non poſſe cum ſit motio, operatio verò viſus, nullo indiget tempore, cum ipſa ſit momentanea, & propterea inſtantanea. Nam momentum non eſt motus, nec inſtans tempus.

## QVARE HYEME VIDEATVR HALITVS animalium non autem æſtate, & de vento.

### *Pancratio Mellano.*

VNDE fiat vt hyeme halitum noſtrum videamus, & non æſtate, ratio eſt ab eiuſdem halitus congelatione, quæ ab extrinſeco frigore fit. Prius enim ſcire debes aerem attractũ in pulmone, foras deinde erumpere cum alio vapore aliquantulum craſſiore humido, & excrementitio expulſo à natura, quæ continuò noſtrum corpus euaporare facit, vnde ſequitur dum aer foras à pulmone pellitur, maiorem ſemper materiæ portionem, ea quæ intus attracta eſt exire: vnde ſtatim vt dicta materia foras expulſa, frigidum æerem offendit, cum conſtet ex partibus craſſis, & obnoxiis congelationi, condenſatur in formam vaporis, ad differentiam aeris ambientis qui in ſe eas partes craſſas non habet, à quibus quidé partibus condenſatis, & redditis opacis reflectitur lumen, atque hanc ob cauſam æſtate hoc non fit, quia calor vim condenſandi non habet.

Ventus nihil aliud eſt quam quidam aeris motus, cum condenſatur, ob defectum caloris, neque (pace Ariſtotelis dicam) eſt exhalatio ſicca. Exemplum à Vitruuio allatum nil planè valet, quantum ſpectat ad venti naturam, cuius rationem à me requiris. Exemplum etiam ventilabri quo tempore æſtate vtimur negligendum penitus non eſt, quia eius beneficio, non ſolum arcemus à nobis aerem ambientem calidum, ſed alium etiam aerem circa nos condenſamus: & quia ordo naturæ eſt huiuſmodi quod quemadmodum calor ſequitur raritatem corporũ, ſic etiam frigus eorundem denſitatem ſequatur. Quod ſi vis vt exemplo illuſtrem, diligenter obſeruato tempore æſtatis cum aliqua nubes nobis Solem adimit, vbi aer qui in eius

vmbra

vmbra reperitur, tantum quantum defectus caloris radij solaris fert, qui per vim, dictum aerem rarefactum conseruabat, statim dictum aerem condensari cognosces: & quia ea condensatio homogenea non est, ob diuersas rationes, hanc ob causam percipimus eamaeris impulsionem, & inæqualiter, dum verò eadem vmbra discedit, ventus, ferè, statim cessat, & sæpe ante quam dicta vmbra discedat; cuius rei causa est longa mora quam trahit vmbra, ita vt prius absoluatur reditus aeris ad formã, quæ ei conuenit in huiusmodi vmbra, quam faciet nubes dum Sol degitur.

Vera non sunt ea, quæ tibi Arnoldus dixit, vt mihi tuis literis significasti. Nam ego ita dixi, videlicet, quod quotiescunque aliquis aspexerit aliquod punctum in superficie speculi, tunc imaginem ipsius post dictam superficiem videbit duplicatam, si verò aspexerit imaginem intra speculum, tunc illud punctum videbit duplicatum, huiusmodi autem rei ratio pendet ab hijs quę ad Franciscum Vimercatum scripsi, quæ si memoria tenes, nullum tibi dubium remanebit. Nam ea tibi omnia ostendi.

Dum verò dicis omnem proportionem rationalem diuidi posse duobus numeris mediantibus in tres æquas partes, mihi ad memoriam reuocas id quod quidam Vitruuij commentator asserit super primum cap. noni lib. eiusdem Authoris, ita dicens.

» Benè esse potest vt diagonalis (quadrati scilicet) numerorum via reperiatur, sed
» fortasse intercedent fracta.

Miror te non memoria tenere quid sint numeri rationales quidúe surdi, neq; considera, non solum non esse diuisibilem in tres æquas partes omnem proportionem rationabilem, sed neque in duas, vt sunt superparticulares proportiones, necnon aliæ innumeræ, sed cum talia scribis te nimis parum versatum in istis rebus ostendis.

Id verò quod tibi dicere volebam nudiustertius de Mercurio erat, quod nullo pacto confidendum est calculis qui fiunt de cursu Mercurij, eo quod eius situs nullo modo obseruabilis est, nam ipse nunquam nec vbiuis locorum orbis terrarum visibilis est altior. 18. gradibus supra orizontem, sed neque confidendum esset si etiã ipsum videremus altum. 20. gradibus, propterea quod magna refractio radiorũ infra hos gradus nos valde fallit, quæ quidem refractio, nec vbiq;, nec omni tempore vniformis est, propter diformem seu inæqualem crassiciem vaporum quæ continuò mutatur. Imo multoties eum videre putabimus supra orizontem, existente ipso sub orizonte.

## *Quod Ouidius transcurrit à motu diurno, ad motum annuum præter rem.*

### AD EVNDEM.

TVus etiam Ouidius cespitauit, cum pro itinere vnius diei efficiat, vt Phaeton à patre edoctus sit etiam de itinere annuali.

Nam, quod Phaeton petat pro cursu vnius diei, clarè patet ex diuersis locis, & primò vbi ita scribit Ouidius.

» Currus petit ille paternos.
» Inq; diem alipedum ius & moderamen equorum.

Deinde vbi Pater ita loquitur.

» Ardua prima via est, & qua vix manè recentes.

„ Enituntur equi medio est altissima cælo.
„ Vnde mare, & terras ipsi mihi sæpe videre.
„ Fit timor & pauida trepidat formidine pectus.
„ Vltima prona via est & eget moderamine certo.
Etiam vbi dicit.
„ Dumq; ea magnanimus Phaëton miratur, opusq;
„ Perspicit, ecce vigil nitido patefecit ab ortu.
„ Purpureas aurora fores, & plena rosarum.
„ Atria, diffugiunt stellæ, quarum agmina cogit.
„ Lucifer, & cœli statione nouissimus exit.
Nec non vbi ita inquit.
„ Et si (modo credimus) vnum
„ Isse diem sine Sole ferunt, incendia lumen
„ Præbebant.
Quod autem à Patre instruatur etiam de cursu annuali, videbitur vbi ita dicit.
„ Nitor in aduersum, nec me, qui cætera vincit.
„ Impetus, & rapido contrarius euehor orbi.
Et vbi ita loquitur.
„ Forsitan & lucos illic, vrbesq; Deorum.
„ Concipias animo delubraq; ditia donis
„ Esse per insidias iter est, formasq; ferarum.
„ Vtq; viam teneas, nulloq; errore traharis.
„ Per tamen aduersi, gradieris cornua Tauri.
„ Aemoniosq; arcus, violentiq; ora Leonis.
„ Sæuaq; circuitu curuantem brachia longo.
„ Scorpion atque aliter curuantem brachia cancrum.
„ Nec tibi quadrupedes animosos ignibus illis.
„ Quos in pectore habent quos ore & naribus efflant, &c.
Sed lucidius etiam hoc videre est inferius vbi ita loquitur.
„ Nec tibi directos placeat via quinque per arcus.
„ Sectus in obliquum est lato curuamine limes.
„ Zonarumq; trium contentus fine, polumq;
„ Effugit australem iunctamq; aquilonibus arcton.
„ Hac sit iter, manifesta rotæ vestigia cernes.
Et vbi etiam dicit.
„ Neu te dexterior tortum declinet ad anguem.
„ Ne ve sinisterior pressam rota ducat ad aram.

## *De supputatione quinque corporum regularium. De aliquibus etiam eorum sympathijs.*

### AD EVNDEM.

ID quod à me desideras, ab alijs etiam factum est, sed ne me putes laborem euitare, non prætermittam aliquid tibi scribere, earum rerum quæ ab Euclide collegi, gi, methodo etiam qua vtebar dum in iſtis rebus me aliquo modo exercebam.

Quotieſcunque igitur ſcire volueris quantitatem corpulentiæ cuiuſq; quinq; corporum regularium ab vna eademq; ſphæra terminatorum ſeu circunſcriptibiliũ curabis primum, cognoſcere quantitatem lateris cuiusq; eorum, talium partium, qualium ſemidiameter dictæ ſphæræ ſit. 100000. ex tabulis ſinuum Nicolai Copernici. Propone igitur tibi ante oculos figuram ſemicircularem vltimæ propoſitionis. 13. lib. Eucli. & inuenies. c. d. tertiam partem ſemidiametri. d. b. eſſe partium. 33333. æqualem ſinui arcus. f. e. graduum. 19. mi. 28. qui quidem arcus dẽptus cũ fuerit à tota quarta. b. f. remanebit arcus. e. b. gra. 70. mi. 32. cuius corda erit latus exaedri, quod latus ita cognoſces, ſumendo ſcilicet ſinum medietatis. b. e. hoc eſt ſinum gra. 35. mi. 16. qui erit partium. 57738. cuius duplum erit partium. 115476. pro latere cubi.

Dempto poſtea quadrato lateris exaedri, & quadrato totius diametri. a. b. reſidui radix quadrata, erit. a. c. latus Tetraedri. Vel ſi duplicaueris ſinum dimidij arcus. a. c. qui quidem arcus, componitur ex quarta. a. f. & ex arcu. f. e. iam inuento, ſiue, vt reſiduus totius dimidij circuli, dempto. b. e. iam ſupra inuento, habebimus idem latus. a. c. partium. 163294.

Pro latere verò Octaedri accipere potes radicem quadratam dupli quadrati ipſius. d. b. & habebis. f. b. latus quæſitum. Vel ſi malis accipe duplum ſinus medietatis arcus. b. f. quod duplum erit. f. b. partium. 14142.

Pro latere verò Duodecaedri, diuide latus Exaedri ex methodo. 11. ſecundi Eucli. cuius maior pars erit latus quæſitum, partium. 71368.

Sed pro latere Icoſaedri, te primum oportebit inuenire quantitatem anguli g. d. a. hoc eſt ipſius arcus. b. n. qui tali angulo ſubiacet, quod cum pluribus modis inueniri poſſit, nihilominus, hunc ſeruabis, inuenies primò quantitatem. d. g. quæ eſt radix quadrata ſummæ duorum quadratorum hoc eſt. d. a. et. a. g. quæ. a. g. æqualis eſt diametro. a. b. vt ſcis, dices poſtea, ſi. d. g. correſpondet ipſi. g. a. cui correſpondet. d. h. ſemidiametro ſphæræ? tibi veniet. h. k. ſinus arcus. a. h. hoc eſt. b. n. graduum. 63. min. 26. cuius medietas gra. 31. mi. 43. pro ſinu ſuo habet partes. 52571. cuius ſinus duplum eſt partium. 105142. pro latere Icoſaedri.

Incipiendo nunc à Tetraedro, ſcire debes, quod pars. a. c. totius diametri. a. b. æqualis eſt axi ipſius Tetraedri, quæ quidem. a. c. vt ſubſeſquialtera ipſius. a. b. erit partium. 13333.

Quæres poſtea quantitatem ſuperficialem vnius faciei ipſius Tetraedri, hac methodo, inueniendo primum radicem quadratam trium quartarum quadrati ipſius. a. c. lateris Tetraedri, eo quod latus hoc, ſeſquitertium in potentia eſt ipſi perpendiculari terminatę ab vno angulorum trianguli æquilateris & à latere ei oppoſito ex. 11. tertijdecimi ipſius Eucli. quę quidem perpendicularis, erit partiũ. 141416. & hæc multiplicata cum medietate lateris trianguli, hoc eſt cum. 81647. tibi dabit ſuperficiem quæſitam, hoc eſt baſim Tetraedri partiũ ſuperficialiũ. 11546192152. Hác demum baſim multiplicando cum tertia parte axis Tetraedri habebis corpulentiam totius Tetraedri, quæ erit. 513158964003488.

Neque tibi hoc loco occultare volo quandam meam animaduerſionem, quæ eſt, quod diameter ſeu perpendicularis (ſupradicta) faciei ipſius Tetraedri, ſemper æqualis eſt lateri ipſius Octaedri circunſcriptibilis ab eadem ſphæra, hoc eſt ipſi. b. f. quapropter quotieſcunque ipſam perpendicularem habere voluerimus accipiendo b. f. habebimus intentum. Et quod hoc verum ſit poſſumus ita demonſtrare.

Primum, notum nobis eſt, ipſam perpendicularem, triplam eſſe eius parti, quæ à centro circuli, ipsum triangulum circunscribentis, terminatur, & à basi, vt in tertio proposito decimæseptimæ quartidecimi Eucli. probatur, ex quo sequitur proportionem huiusmodi perpendicularis ad axem Tetraedri, hoc est ad.a.c. sesquioctauam esse in potentia, ex penultima primi Eucli. Sed cum.d.c. tertia pars sit ipsius.d.a. vt etiam ex.2. proposito, seu corollario decimæseptimæ.14.lib. discurrere licet, cum ex dicto corollario.d.c. sit sexta pars ipsius.a.b. Quare.d.c. quarta pars erit ipsius.a.c. vnde.a.c. sesquitertia erit ipsi.a.d. in longitudine, ideoq; quadratum ipsius.a.d. ad quadratum ipsius.a.c. erit vt.9.ad.16: & ita duplum quadrati ipsius.a.d. hoc est quadratum ipsius.b.f. ad quadratum ipsius.a.c. erit, vt.18.ad.16. hoc est sesquioctauum, ergo.b.f. æqualis erit dictæ perpendiculari, ex.9. quinti.

Cubus postea ipsius.b.c. erit partium.1539838576570176.

Pro Octaedro deinde, accipies productum diametri in semidiametrum, quod productum, æquale erit quadrato diuidenti per æqualia Octaedron, hoc igitur productum, multiplicando per.100000. semidiametrum sphæræ, tibi dabit columnam quadrilateram cuius tertia pars, erit partium.666666666666666. cuius duplum erit ipsum Octaedron partium.1333333333333.

Pro Icosaedro autem, oportet prius quantitatem perpendicularis inuenire, quæ perpendicularis, per æqualia diuidit basim ipsius Icosaedri, quæ vt radix quadrata trium quartarum quadrati lateris ipsius basis, erit partium.91055. talium, qualium dictum latus erit partium.105142. cuius medietas est.52571. quæ medietas si multiplicata fuerit cum dicta perpendiculari, dabit totam basim superficialem, hoc est superficiem vnius trianguli æquilateris partium superficialium.4786852405. quo facto, accipe quadratum duarum tertiarum ipsius, hic supra dictæ perpendicularis, ipsumq; deme ex quadrato semidiametri sphæræ, hoc est, ex quadrato ipsiꝰ.100000 radix postea quadrata residui, erit partium.79468. & hæc erit perpendicularis à centro sphærę ad vnam basim ipsius Icosaedri, quam volueris, quam perpendicularem si multiplicaueris cum quantitate superficiali, hic superius reperta, vnius basis, consequeris columnam trilateram partium.380401586920540. cuius tertia pars, erit partium.126800528973513. pro vna ex.20. Pyramidibus ipsum corpus componentibus. Breuius tamen hoc efficiens, si multiplicaueris basim dictam, cum tertia parte ipsius perpendicularis, hanc postea pyramidem multiplicando per.20. habebis totam corpulentiam ipsius Icosaedri partium.2536010579470260.

Pro Duodecaedro demum, accipe sinum gra.36. qui gradꝰ sunt pro dimidio quintæ partis totius gyri circularis, q quidẽ sinus, erit partium.58778. cuius quadratum si dẽpseris ex quadrato ipsiꝰ.100000. semidiametri circuli circũscribentis aliquẽ pẽtagonum æquilaterum, & æquiangulum, tunc radix residui, erit perpendicularis ducta à centro dicti circuli ad medium vnius lateris ipsius pentagoni, quæ perpendicularis, erit partium.80902. talium qualium medietas lateris dicti fuerit.58778. Nunc verò dicendo si.58778. dat.80902. quid nobis dabit.35684? medietas lateris ipsius Duodecaedri, vnde dabit.49116. pro perpendiculari, à centro ipsius pentagoni, ad latus ipsius Duodecaedri, quæ multiplicata cum medietate supradicta ipsius lateris, hoc est cum.35684. producet vnum ex quinque triangulis componentibus vnum pentagonum, seu vnam basim ipsius Duodecaedri, quod quidem triangulum, erit partium.1752655344. superficialium, quas si per quinque multiplicaueris habebis vnam basim pentagonam dicti corporis partium.8763276720. Dicendum postea est, si ad.80901. conuenit semidiameter circularis partium.100000. quid cõueniet partibus.49116. dabit.60711. pro tali semidiametro circulari, cuius quadratum,

tum,si dempseris ex quadrato ipsius.100000.semidiametro sphęræ, tunc radix quadrata residui, erit perpendicularis à centro sphæræ ad centrum pentagoni partium. 79461.cuius tertia pars, si multiplicata fuerit cum pentagono supra reperto dicti corporis producet vnam ex.12. pyramidibus componentibus dictum Duodecaedron, quæ pyramis, demum, multiplicata per.12.dabit totam corpulentiam ipsius Duodecaedri partium.27853549 25791680.

Nunc verò si experiri voluerimus vtrum isti calculi duorum corporum maiorum sint rectè supputati,dicem⁹ si ad corp⁹.12.basiũ,qđ est partiũ.278535492579168o conuenit numerus partium.25360105794702б0. ipsius Icosaedri, quid conueniet lateri cubi partium.115476.& inueniemus conuenire latus ipsius Icosaedri partium 105138.eo quod probatum sit in.10.propositione.14.li.Eucl.eandem proportionẽ esse corpulentiæ ipsius Duodecaedri ad corpulentiam ipsius Icosaedri, quæ lateris cubi ad latus Icosaedri.

Hæc autem corpora, ita sibi inuicem,& cum eorum sphæra harmonicè conueniũt quemadmodum antiqui philosophi inuenerunt, vt mirandũ non sit, ipsos credidisse omnia quæ natura constant, aliquo pacto ex istis corporibus fieri.Considera quæso quomodo conueniant inuicem Tetraedron, Octaedron,& Icosaedron,cum uniuscuiusque bases sint triangulares æquilateræ intelligendo semper hæc corpora ab eadem sphæra circunscriptibilia.

Octaedron, cum Tetraedro etiam in hoc conuenit, quod latus Octaedri æquale sit ei perpendiculari, quæ diuidit basim Tetraedri per æqualia, vt supra demonstrauimus.

Harmonicis etiam interua llis hæc duo corpora inuicem concordantur, cum basis Tetraedri ad basim Octaedri seruet proportionem sesquitertiam, consonantiæ diatessaron. Et proportio omnium superficierum siue basium Octaedri simul sumptarum,ad omnes bases ipsius Tetraedri simul sumptas sit sesquialtera, consonantiæ diapentis. Neque omittendum est, quod proportio Octaedri ad triplum Tetraedri sit, vt latus Octaedri ad latus Tetraedri.

Proportio verò lateris Octaedri,ad axem Tetraedri, potentia est sesquioctaua, vt supra vidimus interuallum scilicet harmonicum toni maioris.

Harmonia verò Tetraedri,& Exaedri cũ eorum sphæra, talis est,vt proportio diametri sphæræ, potentia, tripla sit lateri Exaedri, & sesquialtera lateri Tetraedri, ex quo sequitur latus Tetraedri potentia duplum existere lateri Exaedri. Interuallum enim triplum in harmonicis, componitur ex diapason,& diapente, & sonat speciem diapentis. Duplum verò est diapason, sesquialterum autem est di apente, quę consonantiæ perfectissimæ sunt.

Proportio verò diametri sphæræ, potentia dupla est lateri Octaedri,consonantię diapason. Ex quo sequitur proportionem lateris Tetraedri ad latus Octaedri, potentia,sesquitertiam esse,hoc est consonantiæ diatessaron, & proportionem lateris Octaedri ad latus Exaedri, potentia,sesquialteram esse, ita quod quatuor istæ potentiæ,idest diametri sphæræ, lateris Tetraedri, lateris Octaedri,& lateris Exaedri constituunt harmoniam ferè perfectissimam,ijs terminis comprehensam.6.4.3.2.(dixi ferè, quia ditonus supra terminum.3.vel semiditonus sub termino.2.hoc loco non reperitur, cuius quidem terminus esset.2.cum duabus quintis.)

Adde quod diameter sphæræ triplus est longitudine ad perpendicularẽ ductam à centro sphæræ ad basim Octaedri, quæ proportio, vt supra dictum est, dicitur diapasondiapente, practici verò eam vocant duodecimam.

Diameter verò ſphæræ ſeſquialter eſt longitudine axi Tetraedri, conſonantiæ diapentis. Axis autem Tetraedri ſeſquitertius eſt longitudinis ſemidiametro ſphæræ conſonantiæ diateſſaron. Ita quod iſti tres termini, qui ſunt, diameter ſphæræ, axis Tetraedri,& ſemidiameter ſphæræ conſtituunt etiam valde perfectam harmoniam huiuſmodi numeris contentam. 6. 4. 3. corpulentia verò Exaedri ad corpulentiam Tetraedri tripla eſt, conſonantiæ iam ſupradictæ diapaſondiapente. Si verò de vniſono aliquid videre deſideras, conſidera æqualitatem dupli quadrati diametri ipſius ſphæræ, cum omnibus baſibus Exaedri, vel potentia diametri ſphæræ cum duabus potentijs ſimul ſumptis, quarum vna eſt lateris Tetraedri, reliqua verò lateris Exaedri, vel æqualitatem numerorum laterum Tetraedri, cum baſibus Exaedri. Nec mihi videtur ſilentio inuoluendum eſſe, antequam vlterius progrediar notabilem ſympatiam inter triangulum æquilaterum, & Tetraedron (quãuis triangulũ corpus non ſit) non ſolum ob inalterabilitatẽ harum duarum figurarum. (nam omnes aliæ alterabiles eſſe poſſunt, ijſdem lateribns exiſtentibus, cum ex quadrato rombus, vel ex pentagono ęquiangulo, pentagonum non æquiangulum &c. efficiatur) ſed quod quemadmodum latus trianguli æquilateri ſeſquitertium potentia eſt perpendiculari ipſum per æqualia diuidenti, ita latus Tetraedri, ſeſquialterum eſt potentia axi ipſius Tetraedri, vnde cum dempta fuerit illa proportio ſeſquitertia, ex hac ſeſquialtera relinquetur nobis proportio ſeſquioctaua, inter perpendicularem trianguli, & axem Tetraedri (quod etiam ſupra demonſtrauimus.) Tranſeamus nunc hęc, nec omittamus tamen ſympatias quaſdam inter Exaedron, Octaedron, & Tetraedron, hoc eſt quod eadem proportio ſit inter corpulentias Exaedri, & Octaedri, quæ inter eorum ſuperficies, nec non, vt latus Exaedri ad ſemidiametrum ſphæræ. Proportio verò baſis Exaedri ad baſim Tetraedri, vt latus Tetraedri ad perpendicularem diuidentem per æqualia eius baſim.

Hactenus ſatis dictum ſit de Tetraedro, Exaedro, & Octaedro cum ſphæra. Dicẽdum nunc cenſeo aliquid de reliquis duobus mirabilibus corporibus, quamuis ferè omnia hæc ab antiquis philoſophis inuenta ſint, quorum primum eſt, quod tam baſis Duodecaedri, quàm Icoſaedri, ab vno eodemq; circulo circunſcriptibiles ſunt, verùm, talis paſſio accidit etiam baſibus Exaedri & Octaedri. Prætεrea quemadmodum in Duodecaedro, quilibet angulus ſolidus terminatur tribus angulis pentagonorum æquiangulorum ita in Icoſaedro, quilibet angulus ſolidus viceuerſa terminatur quinque angulis triangulorum æquiangulorum. Et tam vnum, quam alterum horum corporum, triginta lateribus continetur. Et tot ſolidos angulos triangulares, habet Duodecaedron, quot baſes triangulares continet Icoſaedron.

Et Icoſaedron, tot ſolidos angulos pẽtagonos, quot baſes pẽtagonas habet Duodecaedron. Et tam vnum quam alterum habet. 60. angulos ſuperficiales. Eadẽq; proportio eſt omnium baſium ſimul ſumptarũ Duodecaedri ad omnes baſes ſimul ſumptas ipſius Icoſaedri, quæ corpulentiæ ipſius Duodecaedri ad corpulentiam Icoſaedri (quamuis hęc paſſio accidat Exaedro cum Octaedro, vt ſpra diximus) quę quidem proportio, eadem etiam eſt, quę lateris Exaedri ad latus Icoſaedri, vt ſupra iam dictum fuit.

NOVA

# NOVA INVENTIO COMPONENDI ASTROLABIA cum Horologijs artificialibus.

## *Jacobo Mayeto Ingeniosissimo Horologiorum Serenissimi Sabaudiæ Ducis Artifici.*

NONNVNQVAM consideraui mirabilem pulchritudinem, simul cum vtili-
tate coniunctam, illorum horologiorum, quæ in Germania construuntur
cũ mobili Rete, seu Aranea Astrolabij sup Tabulã regionis, in qbus cõti
nuo videnẽ oriri, occidereq́; cælestia signa, cælum mediare supra orizon
tẽ, necnon sub eo, & vt vno verbo dicam, continuo erecta videtur tota cœli figura.
Sed quia talia horologia omnia eorum limbum distinctum habent in.24.horas, qua
propter diametrum limbi, minorem duobus palmis, seu semipede esse non oportet
ne interstitia horarum iustò breuiora seu angustiora efficiantur, etiam ne interualla
dentium rotæ indicis nimis angusta sint. Sed quia talis magnitudo vt plurimum in-
commoda existit. Ideo non inutile fore cogitaui, si modus aliquis inuentus fuerit,
vt ea omnia efficiantur in limbo diuiso tantummodo in.12.horas æquales, ipsumq́;
inueni, qui quidem erit, efficiendo vt Tabula (in qua designantur cęlestes domus,
cum almicantarat, atque azimut) Reti subiectæ, mobilis sit, tardior tamen ipso Re-
te cum indice, pro duplo temporis, hoc est, quod eo tempore, quo Aranea cum in
dice circunuoluetur spacio.12. horarum vno gyro perfecto, ipsa Tabula efficiat tan
tummodo sex interstitia horarum. Idest dum Tabula dicta efficit vnam integram re
uolutionem, Aranea, seu Zodiacus cum indice, duas efficiat reuolutiones. Ita quod
Aranea cum indice perficiet vnam reuolutionem spaci o temporis.12.horarum, Ta-
bula verò perficiet eam spacio temporis. 24.horarum. Vnde sequetur quod Ara-
nea seu Zodiacus cum indice, spacio.24.horarum perfectè circunuoluetur supra Ta-
bulam, & ita huiusmodi horologia, in hoc nihil differrent ab illis supradictis. Vt au
tem facias dictam tabulam tardiorem duplo temporis Araneæ cum indice, quamuis
diuersis modis hoc fieri possit, pręstantiorem tamen iudico, si cum Rota indicis, aliã
Rotam concentricã coniunxeris, ita tamen, vt vnaquęq; liberè possit volui, similiter
si cum ea horologii particula (quę circũagit Rotam indicis, quæ Italicè ROCHETTO.
Germanicè verò TRIB vocatur, Latinè aũt ipsum vocabo, COLINVM, qui sub ro-
ta fusi reperitur) coniunxeris alium colinum quem, secundum vocabo, concentricũ
verò cum primo, cum eoq́; consolidato, numerum verò dentium, tam Rotę adiunctę
quam secundi colini, varijs modis poteris inuenire, quorum primus erit, vt numerus
dentium secundæ Rotę duplus existat numero dentium primę, efficiendo secundum
colinum eiusdem numeri dentium quo primum, sed quia interualla dentium huius-
modi Rotę, nimis angusta fortasse resultabunt, propterea alios etiam modos inue-
ni, quorum vnus erit (dum numerus dentium primi colini par fuerit) efficiẽdo secun
dam Rotã eiusdẽ numeri dentiũ cuius est prima. secũdũ vero colinum, medietatis
numeri dentium cuius erit primus. Attamen si primus colinus esset.4.dentium, secun
dum oporteret esse duorum dentium, vnde motus secundę Rotę non esset ita conti-
nuus. Quapropter alium etiam modum excogitaui, hoc est, cupiendo vt secundus
colinus, ex tribus dentibus existat, si primus ex.4.repertus fuerit, oportebit prius ex
regula de tribus, numerum quendam inuenire quo inuento ipsum duplicare, & hunc
duplicatum numerum conueniet secundam Rotam habere, vt ipsa possit ab illo co-
lino triũ dentiũ circunuolui in duplo temporis, quo prima à suo colino quatuor den
tium.

tium. Exempli gratia, ſi prima Rota conſtaret ex.36. dentibus, dicendum eſſet, ſi 4.conuenit cum.36. cum quibus conuenient. 3.& inueniemus.27.cum quo numero dicta ſecunda Rota circunuolueretur eodem tempore à ſuo colino trium dentium, quo prima à ſuo quatuor dentium, quare duplicando.27.haberemus.54. pro numero dentium dictę ſecundæ Rotæ, vt duplo temporis circunuoluatur quo prima. Sed ſi primus colinus conſtaret ex.6. dentibus, exiſtente ſua Rota ex. 36. vellemusq́; ꝙ ſecundus exiſteret ex.4.tunc ſuam Rotam oporteret habere dentes. 48. ex dicta regula. Si autem primus colinus conſtaret ex numero impari, nihil referret, dummodo huiuſmodi numerus impar, ſeu par, exiſteret pars propria numeri dentium, vel ipſius dupli primæ Rotę, hoc eſt, eſſet pars aliquota numeri dentium ipſius primæ Rotæ vel ipſius dupli. In ijs verò horologiis in quibus duplum numeri dentium dictę primę Rotę non erit multiplex numero dentium primi colini, hoc fieri non poterit. Ratio enim tam clarè, tibi conſideranti, patebit, vt nullis verbis indigeat cum ſemper numerus dentium ſecundę Rotę multiplex eſſe debeat numero dentium ſecundi colini. Idem autem non dico de prima Rota cum ſuo colino, hoc eſt; vt numerus primę multiplex ſit numero ſui colini, nam hoc neceſſarium non eſt. Ponamus exempli gratia primum colinum conſtare ſex dentibus, ſuam vero Rotam dentibus.21.cuius quidem numeri, 6.non eſt pars aliquota, ſed dupli ipſius. 21.ipſe.6. eſt pars aliquota. Nunc verò ſi voluerimus numerum dentium ſecundæ Rotę inuenire, cuius colinus ex quinque dentibus exiſtat (ſuppoſito primo ex.6.conſtare) tunc ex regula de tribus, diuiſo producto, quod fit ex.21.in.5.per. 6. exibit. 17. cum dimidio, cuius duplum eſſet.35.qui multiplex eſt ipſi quinque. Reperto igitur numero ſecundę Rotę, cum numero ipſius colini, oportet nunc ſcire modum compoſitionis, ſeu coniunctionis harum rerum, hoc eſt duorum colinorum concentricorũ (ſed de ijs ſatis iam ſuperius dictum fuit) duarum Rotarum concentricarum cum Tabula, cum Zodiaco, & cum indice, ſeu Oſtenſore, cuius quidem Oſtenſoris medietas tantummodo nobis ſufficiet. Sciendum igitur nunc eſt quod cum primus colinus reuoluat totam primam Rotam, ſpacio temporis.12.horarum, oportet vt eius axis, ſeu arbor voluat oſtenſorem, Zodiacumq́;, eodem temporis ſpacio, & quia Rota hęc inalterabis eſt, propter eius coniunctionem cum ſuo colino, & nos oporteat indicem Zodiacumq́;, quotidie ferè, dirigere, ſuisq́; locis collocare, ideo nos oportet, indicem, Zodiacum, & primam Rotam, ita cum axe, ſeu arbore coniungere, vt poſſimus dicta omnia efficere. Pars igitur Arboris, ſeu axis dicti, quæ ingredi debet in prima Rota, ſit rotunda, & contigua ipſi Rotæ, non autem continua, vel cum Rota conſolidata. Pars verò quę per foramen Zodiaci, ſeu Araneę tranſibit, ſit quadrata vſque ad ſummitatē ipſius axis (tali ſpiſſitudine, vt in claui ipſius horologij ingredi poſſit) & ita foramen ipſius Araneę, quadratum ſit, Oſtenſor autem circa axem, compoſitus ſit tali ordine, vt circa paruum circulum volui poſſit, qui paruus circulus habeat quadratum foramen, per quod tranſeat axis, qui axis aliquantulum emineat ſupra oſtenſorē. Sub Aranea vero vel Zodiaco, locata erit Tabula, vt nũc dicemus, ſed ſciendum eſt prius, quod inter Tabulam, & ſuam ſecundam Rotam, aliam laminam immobilem interpoſitam eſſe oportet, quę circulare foramen habeat, per qđ quędam breuis fiſtula tranſeat circundans axem & coniungens Tabulã cum ſua Rota, cuius quidem fiſtulæ ſuperficies concaua, rotunda ſit, ſuperficies verò extrinſeca, non tota, niſi ea pars, quę ſecundam Rotam ingreditur, vt in rotundo foramine ipſius Rotę, dicta fiſtula volui poſſit, pars vero extrinſeca quę Tabulam ingredi debet, ſit quadrata. Tabula vero quatuor paruiſſima foramina habeat in extremitati-

bus

bus linearum,meridianę, & verticalis,vt acu mediante volui poſſit, prout oportebit.

Perfectum igitur cum fuerit opus hoc, te oportet ſcire modum ipſo vtendi. Quapropter quotieſcunque volueris,aſpice Solis locum in Zodiaco,Ephemeridibus mediantibus, idem dico de vnoquoque reliquorum planetarum. Inuento poſtea Solis loco in noſtro Zodiaco horologij, manu mediante,volue oſtenſorem, ita, vt linea fiduciæ tranſeat per gradum Solis, deinde, claui ipſius horologij mediante,volue indicem, ita cum Zodiaco coniunctum, vt linea fiducię, punctum,ſeu partem horę oſtendat in limbo horologii, quę quidem hora notanda eſt ſi fuerit ex ijs quę incipiunt à meridie vſque ad mediam noctem, vel à media nocte vſque ad meridiem, tunc acu ſupradicta mediante, poſita in aliquo illorum quatuor foraminum, circunuoluenda eſt Tabula, ita, vt extremitas lineę meridianę ſupra orizontem, ex ęquo incidat inter duodecimam horam, & lineam fiducię, computum incipiendo à duodecima hora, ſi vero dicta indicis hora fuerit ex ijs quę incipiũt à media nocte & deſinunt poſtea in meridie, oportebit, acu mediante, circunuoluere Tabulam,quouſque punctum extremum meridianæ ſub terra,medio loco exiſtat inter duodecimã horam, & horam oſtenſam à linea fiducię. Quo facto continuo videbis erectam cęli figuram. & quia vidiſti loca planetarum in Ephemeridibus, videbis etiam eorum loca accidentalia in domibus ſcilicet accidentalibus, ſi aliquas fixarum in Aranea deſiderabis,accipere poteris Ocu. ♉, cor. ♌, ſpi. ♍, Liram,Aquilam, & Arcturum, dum locus fuerit capax. Nec te moueat, quod oportebit lineam fiducię ſupra gra. Solis quotidie collocare, quod nihil refert. Nam oportet etiam quotidie cordam fuſo circunuoluere.

## DE DEMONSTRATIONIBVS PROPOSITIONVM Mathematicarum,nec non de Aſtrologia Iudiciaria.

### *Illuſtriſſ. D. Volfardo Aiſeſtain.*

NIHIL mihi gratius & iucundius afferri potuit tuis literis, quibus te cupidum oſtendis ſciendi rationem, quare ego non vna methodo ad omnes propoſitiones demonſtrandas vſus ſim, hoc eſt, quare non omnia ea Eucl. Theoremata citem in vnaquaque propoſitione, quę ad eã demonſtrandam faciũt, quemadmodum in mea Gnomonica vidiſti me aliquando omiſiſſe. Reſpondeo ꝙ mathematicę demonſtrationes, hominibus Euclidis Elementa poſſidentibus, non indigent aliqua citatione numerorum Theorematum ipſius Euclidis, & ſi aliquando vſus ſum aliqua citatione eorundem, hoc feci propter conſuetudinem noſtri temporis, vel etiam ad faciliorem intelligentiam illorum, quibus ſcribebam. Sed omnia quamuis minima citare, vt faciũt nonnulli, mihi, nimis laborioſum,ſuperfluumq́; videtur, preſertim ijs (vt dixi) qui memoria tenent prima Elementa. Hęc igitur eſt vna ratio. Alia, quia multoties, ita coniuncta eſt ſpeculatio cum ipſa concluſione, vt mihi ſępius viſum ſit ſuperfluum, aliquid de ipſa theoria ſcribere. In iis enim, quę dum puer eram ſcripſi, videbis ſcrupuloſam illam methodum, ſed poſtea, non niſi in arduis propoſitionibus me nihil eſſentiale prętermittere.

Circa vero id de quo me interrogas, ſcilicet, vtrum putem omnia vera eſſe, ea quę ſcripta reperiuntur in libris Aſtrologiæ iudiciarię. Reſpondeo quod non, imo

puto

puto plurima falſa eſſe. Nam illa multitudo partium, vt pars vitę, pars Hylech, pars
futurorum, & reliquę omnium domorum cœleſtium, ſalua parte fortunę, ſunt merę
nugę. Idem dico de faciebus, ſiue decanis, de terminis, & de gradibus ipſis, vt pu-
ta azemenis, puteis, vacuis, fumoſis, & de reliquis. De Domibus vero, Exaltationi
bus, nec non triplicitatibus, experientia cõfirmat ea vera eſſe. Idẽ affirmo de Domi
bus accidentalibus, rationalibus tamen, non autẽ de Domibus Campani, & Gazuli.
Obſeruationes etiam complexionum ſeu influentiarum ipſorum Planetarum rectè
factæ ſunt, quę etiam à coloribus ipſorum Planetarum ferè iudicari poſſunt. Con-
iunctiones aſpectusq́; ipſorum inuicem, ſimiliter mirabilia faciunt, & ex maiori par
te, ea, quę de iſtis ſcribuntur vera ſunt. Reuolutiones annuę ſimiliter, cum Domino
anni. Dominum verò orbis Diuiſoremq́; non approbo, nam hic pendet à termino,
ille verò ab hora. Nouenarias autem Dodecathemoria, Alfridarias, & multa iis ſi
milia omnia nego. Antiſcia, vera ſunt, ideſt influunt, malos tamen effectus, alia
plus alia verò minus, prout aliqua eorum ſunt tetragona, alia verò trigona, alia ma-
gna, alia parua, magna ſunt, vt Arietis cum Virgine, & Librę cum Piſcibus, parua ve
rò, debiliaq́; Geminorum cum Cancro, & Sagittarij cũ Capricorno. Sed difuſius
hęc oĩa videbis in meo illo particulari tractatu, de quo tibi aliàs dixi, in quo multa
videbis, quę omnia ab experientia, ex multis à me obſeruatis, comprobata ſunt,
quem quidem tractatum cum quibuſdam alijs meis ſpeculationibus in lucem prode
re cupio, ſi fieri poterit, antequam ad directionem mei Horoſcopi cum corpore
Martis Anęretę perueniam, quę quidem directio circa annum milleſimum quin-
genteſimum nonageſimum ſecundum eueniet.

FINIS.

# ERRATA CORRIGITO IVXTA INFRASCRIPTAM TABVLAM.

Reliquos verò errores, qui orthographiam respiciunt, benignus lector corrigat, qui sciat multos errores in editione irrepsisse, quod non paucos dies morbo fuerim detentus dum præsens opus excuderetur.

| Pag. | Lin. | Errata | Correcta |
|---|---|---|---|
| 3 | 29 | æqualis | æquali |
| 8 | 35 | maius | maior |
| 9 | 15 | in vnitate superficialis, erit ac | in vnitate, supreficialis erit, ac |
| 11 | 1 | prouenients | prouenientem |
| 11 | 8 | futurum | futurus |
| 11 | 31 | illa nihil aliud sunt | illud nihil aliud est |
| 11 | 38 | diuidemus | diuidamus |
| 12 | 1 | tertiæ, sint | tertiæ sint |
| 12 | 2 | productum | productus |
| 19 | 30 | prouenients | prouenientem |
| 19 | 30 | productum æquale | productus æqualis |
| 21 | 27 | eadem | eædem |
| 24 | 18 | est | esset |
| 26 | 39 | est | esset |
| 41 | 24 | qua | quæ |
| 41 | 31 | distinguendæ | distinguendo |
| 59 | 5 | subsequens | subsequentem |
| 61 | 3 | hæc via tenenda | hanc viam tenere |
| 61 | 3 | fuit | fuerit |
| 61 | 45 | numerum quæsitum | numerus quæsitus |
| 62 | 30 | quantum est dimidiā occupatam | quanta est dimidia occupata |
| 64 | 4 | speculari | considerari |
| 64 | 15 | totq; sunt termini | totq; esse terminos |
| 64 | 18 | hoc est numerum | hoc est per numerum |
| 64 | 19 | primo quod vnus est | primo qui vnus est |
| 66 | 18 | minimum | minimus |
| 67 | 13 | maximum terminum addendū | maximus terminus addendus |
| 72 | 1 | numerum | numerus |
| 72 | 8 | singulos itinere | singulos in itinere |
| 74 | 7 | itinerarium | itinerantium |
| 78 | 21 | morum | modum |
| 78 | 24 | noueratius | nouenarius |
| 81 | 10 | iuncta | iunctæ |
| 88 | 35 | armonicæ | harmonicæ |
| 91 | 6 | Quare argumētādo permutando | Quare permutando |
| 95 | 27 | vitium | vicium |
| 97 | 14 | summa | summam |
| 97 | 30 | illud verò quod con | illud verò con |
| 98 | 22 | desideraremus | desiderauerimus |
| 103 | 23 | dispositis facit | dispositis, tantum facit |
| 104 | 39 | dispositis, facit | dispositis, tantum facit |
| 107 | 21 | eccedit | excedit |
| 110 | 41 | habuerit eius cerebrum | habuerit cerebrum |
| 113 | 46 | sufficeret | suffecisset |
| 116 | 8 | quarum secundam | quorum secundum |
| 116 | 8 | primæ, tertiam | primi, tertium |
| 116 | 9 | secundæ | secundi |
| 130 | 23 | ducenda | ducendus |
| 133 | 10 | lineam qua | lineam, quæ |
| 133 | 11 | dratam | dratum |
| 134 | 16 | inueniet | inuenimus |
| 137 | 22 | distans | distantius |
| 137 | 45 | Notissimum igitur primum | Notissimum primum |
| 139 | 15 | est | sit |
| 139 | 28 | duas | duos |
| 141 | 5 | comperuisse | comperisse |
| 142 | 19 | quā | ac |
| 143 | 17 | linea | lineam |
| 144 | 23 | patebit, si quis | patebit, quòd si quis |
| 145 | 18 | constante | constantem |
| 146 | 20 | pastam | massam |
| 146 | 22 | pastam | massam |
| 148 | 13 | pponit nō cōcludit melius aūt | proponit nō concludit, melius aūt |
| 14 | 41 | prætergradiatur | prætergredietur |
| 152 | 6 | videtur | videri |
| 153 | 22 23 | quia libra | quia cum libra |
| 153 | 23 | materiales, cum sustineantur | materiales sustineantur |
| 153 | 24 | existente, vnde aliqua | existente, aliqua |
| 154 | 23 | futurum | effecturam |
| 155 | 25 | carta | charta |
| 156 | 23 | sufficere | sufficeret |

| Pag. | Lin. | Errata | Correcta |
|---|---|---|---|
| 158 | 26 | versa | versam |
| 158 | 26 | sit | sint |
| 162 | 22 | cindendæ | scindendæ |
| 163 | 7 | oppositus | oppositum |
| 164 | 24 | tanta | tantæ |
| 164 | 37 | adiunctæ nobis essent duæ aliæ | adiuncti nobis essent duo alii |
| 164 | 39 | sestineret | sustineret |
| 164 | 40 | dicta | dicti |
| 164 | 41 | aliquam | aliquem |
| 165 | 3 | sufficiant | sufficiat |
| 165 | 4 | subsesquialter | subsesquialterum |
| 186 | 32 | finit | finitam |
| 187 | 29 | propinquæ | propinqui |
| 148 | 30 | philosophi supra | philosophi, proximè supra |
| 148 | 31 | contingeret mense | contingeret vt mense |
| 187 | 34 | regionis)non considerans | regionis)in quo non considerauit |
| 190 | 11 | lenas | lenes |
| 190 | 23 | lenis | leuis |
| 191 | 10 | aliud quadratum | alius quadratus |
| 191 | 15 | qualis | qui |
| 193 | 17 | cum | tum |
| 193 | 36 | fit | facit |
| 193 | 40 | pleno vbi | pleno posuerimus, vbi |
| 195 | 10 | valent | valent apud ipsos |
| 196 | 19 | phylosophia | philosophia |
| 196 | 20 | pellendo | plendo |
| 198 | 18 | sunt etiam | esse etiam |
| 204 | 12 | principium | principum |
| 204 | 18 | indigna | indignas |
| 204 | 21 | diffundetur, crescat | diffundatur, & crescat |
| 205 | 16 | aliquando ser | aliquando me ser |
| 205 | 19 | anno præter, necessitatē, gignitur | anno, præter necessitatē gignitur |
| 206 | 21 | plenilunium, quod | plenilunium fieri, quod |
| 207 | 26 | inchoet annus | inchoetur annus |
| 209 | 9 | soli, quod punctum | solis, cuius punctum |
| 210 | 10 | calebrandi | celebrandis |
| 212 | 33 | Inuenta | Inuenti |
| 212 | 33 | duæ | duo |
| 214 | 9 | classi | classis |
| 214 | 10 | Inter Eximias | Quia inter Eximias |
| 214 | 26 | reperiretur | reperietur |
| 214 | 34 | necessario sit futurum, vt | necessarium sit, vt |
| 215 | 37 | quod si velimus | si velimus |
| 215 | 39 | potest vno eodemq; | vno eodemq; |
| 215 | 45 | sit circulus | sit verè circulus |
| 217 | 2 | Quod cum verum | Quod si verum |
| 217 | 4 | nulla est ratio | nulla esset ratio |
| 221 | 31 | inueniemus | designabimus |
| 2 2 | 20 | falli | fallat |
| 225 | 20 | si quæ | siq; |
| 225 | 25 | chuum | chaium |
| 226 | 14 | oleum effundebat | effundebatur |
| 227 | 5 | euadit | euadet |
| 231 | 36 | supradicta, minutæ | supradictæ minutæ |
| 232 | 21 | progrediuntur | progredi |
| 234 | 15 | altiori cælo | altius cælum |
| 239 | 12 | dum | cum |
| 241 | 41 | afferens | cogitans |
| 245 | 25 | diametri | diametros |
| 248 | 42 | sit & ob id | sit, ob id |
| 249 | 42 | alium numerum | alio numero |
| 250 | 13 | um voluerimus | cum inuestigare voluerimus |
| 251 | 1 | inuenerimus | inuenimus |
| 252 | 6 | tros | ter |
| 252 | 6 | alia verò diametro | alius verò diameter |
| 252 | 8 | te id non | te non |
| 252 | 12 | duabus | duobus |
| 255 | 16 | in numero | numero |
| 255 | 25 | cum non videat si | cum non videat quod si |
| 255 | 36 | suo centro | suis centris |
| 255 | 39 | mille milliaria | mille milliariorum |
| 255 | 41 | trecenta mille | tercenties mille |

| Pag. | Lin. | Errata | Correcta |
|---|---|---|---|
| 257 | 5 | *quem* | *quod* |
| 257 | 11 | *quem* | *quod* |
| 257 | 18 | *quod* | *nam* |
| 258 | 15 | *ictbus* | *ictbus* |
| 258 | 15 | *eundem esse futurum* | *idem esset* |
| 258 | 27 | *debet, quanto* | *debet in bonis fidutis situ, quanto* |
| 258 | 41 | *quod* | *qui* |
| 262 | 17 | *erunt* | *supponuntur* |
| 263 | 22 | *a K. facta sit* | *a K. sit* |
| 263 | 22 | *semidiameter esse minus* | *semidiameter minus* |
| 265 | 4 | *cognita* | *cogniti* |
| 272 | | *in figura vbi est P.* | *ponatur. R* |
| 272 | 18 | *E. i. ad. E. i.* | *E. I. ad. E. i.* |
| 273 | 3 | *licet* | *liceret* |
| 274 | 7 | *te* | *tibi* |
| 275 | 10 | *diuidere* | *diuidendo* |
| 276 | 47 | *accipiemus* | *accipiamus* |
| 278 | 8 | *Mereta* | *Moeta* |
| 280 | 25 | *diminuta, quarta* | *diminuta seu defectiua, quarta* |
| 281 | 36 | *grauissimum* | *grauissimo* |
| 281 | | *in fine pag. vnamquanque Ziffram accommoda sub vnoquoque alphabeti characterem ad priores Ziffras prius addendo. 1. Idem dico in pag. 282. & inter. D. et E. pone. b.* | |
| 285 | 24 | *antea* | *ante* |
| 289 | 11 | *quanta pars* | *quantam partem* |
| 289 | 21 | *quanta pars* | *quantam partem* |
| 289 | 42 | *est* | *erit* |
| 290 | 10 | *fatile* | *facilis* |
| 290 | 19 | *aequalis* | *aequale* |
| 290 | 43 | *circumscriptibilis* | *circumscribentis* |
| 291 | 1 | *ad* | *Quod* |
| 292 | 33 | *detractis* | *detracti* |
| 294 | 34 | *anguli contingentia solidisq;* | *anguli contingentia solidiq;* |
| 295 | 24 | *chorda* | *chordam* |
| 295 | 32 | *fixum* | *fixus* |
| 297 | 1 | *distincta, procedenda* | *distincta sunt, procedendo* |
| 299 | 32 | *reflexum* | *reflexum* |
| 299 | 34 | *flexum* | *flexum* |
| 302 | 14 | *eisdem* | *eisdem* |
| 304 | 3 | *ta* | *tus* |
| 305 | 21 | *Idem facere* | *Idem possumus facere* |
| 312 | 37 | *dupla* | *dupla* |
| 313 | 6 | *quotiescunque* | *quis* |
| 315 | 23 | *retrogradandum* | *retrogradiendum* |
| 316 | 22 | *vbi* | *tibi* |
| 324 | 41 | *si constitueremus* | *si non constitueremus* |
| 325 | 12 | *simili ad* | *simili. Ad* |
| 326 | 9 | *longare* | *ducere* |
| 329 | 5 | *ipsi ellipsi* | *ipsius ellipsis* |
| 333 | 15 | *cogitetur* | *cogitemus* |
| 333 | 17 | *vnde pro genera* | *vnde ex genera* |
| 333 | 20 | *aequalem esse longitudini* | *aequalem longitudini* |
| 333 | 21 | *ea* | *eam* |
| 333 | 21 | *minor* | *minorem* |
| 333 | 23 | *nor* | *norem* |
| 333 | 24 | *maior* | *maiorem* |
| 333 | 25 | *maior* | *maiorem* |
| 333 | 37 | *communneq;* | *communiq;* |
| 335 | 26 | *reflectit* | *reflectitur* |
| 335 | 47 | *& remotiori* | *vel remotiori* |
| 336 | 19 | *speculi superficiem* | *speculi planam superficiem* |
| 336 | 22 | *reflectit* | *reflectitur* |
| 337 | 8 | *vnam tantummodo imaginem* | *vna tantummodo imago* |
| 337 | 22 | *ipsa* | *ipsi* |
| 337 | 42 | *qua* | *qui* |
| 338 | 14 | *protracta* | *protracte* |
| 338 | 26 | *ratiotinare* | *apta* |
| 340 | 23 | *Alius modus* | *Alium modum* |
| 341 | 9 | *aequale* | *aequalem* |
| 343 | 23 | *videbitur* | *videri* |
| 344 | 5 | *hoc est* | *hoc est quod* |
| 344 | 1 | *punctus* | *punctum* |
| 345 | 6 | *ei* | *cum altitudine* |
| 345 | 19 | *ipsum. repertum* | *ipsum, vt repertum* |
| 347 | 16 | *reflectit* | *reflectitur* |
| 352 | 38 | *opinauerunt* | *opinati sunt* |
| 353 | 8 | *parabolam* | *parabolam* |
| 353 | 15 | *hac* | *hic* |
| 353 | 18 | *parallelam* | *parallela* |
| 356 | 15 | *muri, &* | *muri, in orizontali q p er* |
| 356 | 33 | *mediante. cognoscere* | *mediate. cuius axis ad orizontem erectus sit.a.u. cognoscere* |
| 360 | 13 | *duplum* | *duplus* |
| 360 | 28 | *vnde iste circulus* | *iste circulus* |
| 361 | 36 | *quae* | *qui* |
| 364 | 23 | *diameter* | *diametrum* |
| 367 | 25 | *prolongando* | *producendo* |
| 369 | 2 | *aequalem* | *aequale* |
| 370 | 4 | *sinus* | *sinum* |
| 370 | 30 | *summa* | *summam* |
| 372 | 1 | *caeterarum* | *caeteris* |
| 372 | 36 | *qui qui* | *qui* |
| 378 | 11 | *hec* | *ha* |
| 382 | 41 | *prolongatis* | *productis* |
| 383 | 3 | *qua* | *qui* |
| 383 | 13 | *genabis* | *ginaberis* |
| 383 | 20 | *triangulisq;* | *triangulisq;* |
| 383  384 | | *in prima parabola obliqua extremitatem inferiorem diametri signabis charactere. d. in secunda vero charactere. r.* | |
| 384 | 29 | *maior proportio* | *maiorem proportionem* |
| 384 | 37 | *proportio habebit* | *proportionem habebit* |
| 385 | 5 | *c.b.* | *e.b.* |
| 386 | 22 | *b.e.* | *.b.c.* |
| 386 | 29 | *positum* | *positus* |
| 385 | 31 | *b.c.* | *.b.e.* |
| 386 | 33 | *& proportionalitate* | *& ex proportionalitate* |
| 386 | 34 | *antecedens in* | *antecedens. A B. in* |
| 386 | 40 | *quod* | *qua* |
| 388 | 2 | *B.)* | *B.)* |
| 388 | 8 | *b.cum duplo b.e.* | *b.d.cum duplo b.e.* |
| 388 | 45 | *b.e.* | *b.c.* |
| 388 | 45 | *b.e.* | *.b.c.* |
| 388 | 46 | *ad.a b.* | *ad.d.b. cum e.b.* |
| 390 | 25 | *diuisa* | *diuisas* |
| 392 | 3 | *sit paraboles a.* | *sit paraboles a. b.c.* |
| 392 | 27 | *aliqua reliquarum* | *aliquas reliquarum* |
| 393 395 397 | | *in parabola vbi sub g est i. pone r. & supra fundum minus dicatur cubus minor (leautur dua vero lineae sine characterib. de* | |
| 394 | 18 | *vt.y ad.m n.* | *vt p.ad.m.n.* |
| 394 | 33 | *x.o.* | *n.o.* |
| 394 | 34 | *i k.ad.i.k.ad. f g.* | *i.k.ad f g.* |
| 395 | 11 | *quintas* | *quinta* |
| 396 | 1 | *d b.* | *g.b.* |
| 397 | | *Vbi rubrica dicit. Defensio nostra contra Antonium Bergam, & Alexandrum Piccol.* | *dicatur. contra Anto. Bergam & Alex. Piccol. atq; defensio nostra contra Excell. Augusti. Michelem* |
| 399 | 32 | *timue* | *timem* |
| 400 | 39 | *vnum tantum numerum medium* | *vnus tantum numerus medius* |
| 402 | 19 | *diametrum* | *semidiametrum* |
| 403 | 7 | *sua* | *sua* |
| 403 | 9 | *iam supra* | *extractatis* |
| 403 | 17 | *quodquid* | *quodquam* |
| 404 | 37 | *milliaria* | *milliaribus* |
| 406 | 19 | *trabuchum quadratum* | *trabuchus quadratus* |
| 410 | 38 | *Qua fors* | *qua forte* |
| 410 | 41 | *quamlibet* | *qualibet* |
| 411 | 27 | *dicens* | *dicat* |
| 411 | 28 | *eodem libro* | *vt eodem libro* |
| 412 | 24 | *non dependeat?* | *dependeat?* |
| 413 | 2 | *vt supra* | *vt alias* |
| 413 | 22 | *millia vices* | *millibus vicibus* |
| 415 | 32 | *fenestra* | *fenestra* |
| 416 | 15 | *operatio* | *operationem* |

F I N I S.