Gluon scattering in deformed $N = 4$ SYM [☆]

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Abstract

We consider gluon and gluino scattering amplitudes in large N β-deformed $\mathcal{N} = 4$ SYM with real β. A direct inspection of the planar diagrams shows that the scattering amplitudes to all orders in perturbation theory are the same as in the undeformed $\mathcal{N} = 4$ SYM theory. Using the dual σ-model description, we find the same equality at strong coupling to all orders in the σ-model loop expansion. Finally, we show that the same analysis holds for gluon scattering amplitudes in a three-parameter deformation of planar $\mathcal{N} = 4$ SYM that breaks all the supersymmetry.
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1. Introduction

Recently, a possible intriguing duality has been revealed in $\mathcal{N} = 4$ SYM theory at the planar limit. The planar MHV gluon scattering amplitudes $\mathcal{A}$ in the theory seem to have a dual description in terms of the expectation value of a Wilson loop, whose contour C consists of light-like segments, which are proportional to the light-like momenta k_i of the external gluons

$$\mathcal{A} = \frac{1}{N}\left\langle \operatorname{tr} P \exp\left(i \oint_C A_\mu(x)\,dx^\mu\right)\right\rangle + O\left(1/N^2\right). \tag{1.1}$$

A prescription, along these lines, for the computation of the planar gluon scattering amplitudes in $\mathcal{N} = 4$ SYM at strong coupling has been proposed in [1]. The kinematic factor of the color ordered planar scattering amplitude $\mathcal{A}$ was mapped to the computation of the Wilson loop, which using the gauge/gravity correspondence [2,3], translates to the construction of a minimal area string worldsheet ending on the loop. The exponential of this (regularized) area represents the gluon scattering ampli-

tude

$$\mathcal{A} \sim \exp\left(-\frac{\sqrt{\lambda}}{2\pi}\text{Area}\right), \tag{1.2}$$

where $\lambda = g^2 N$. The four-gluon scattering amplitude at strong coupling $\lambda \gg 1$ was computed in this way in [1], and was shown to be in agreement with the general structure conjectured by Bern et al. in [4]. Evidence for the duality at weak coupling has been given recently, at one-loop in [5,6], at two-loop in [7,8], and for its possible breakdown for six gluons in [9]. A study of quark scattering amplitudes has been done in [10,11].

A natural question to ask is whether there are other theories with less supersymmetry where this duality can hold. In this Letter we will consider gluon and gluino scattering amplitudes and the corresponding light-like Wilson loops in large N β-deformed $\mathcal{N} = 4$ SYM with real β (for a discussion of Wilson loops in the β-deformed theory see [12]). This deformation breaks the $\mathcal{N} = 4$ supersymmetry to $\mathcal{N} = 1$. We will further analyse a γ-deformation of $\mathcal{N} = 4$ SYM that breaks supersymmetry completely.

The Letter is organized as follows. In Section 2 we will analyze the planar gluon scattering amplitudes and Wilson loop in the weak coupling regime of the theory. We will argue, based on the observation that the β-deformed theory can be written as a non-commutative deformation of the $\mathcal{N} = 4$ theory, that to all orders in perturbation theory they have exactly the same

values as in $\mathcal{N} = 4$ SYM. In Section 3 we will analyze the strong coupling regime and use the σ-model description of the β-deformed theory and show that the same holds to all orders in the σ-model loop expansion. In Section 4 we will consider planar gluon scattering amplitudes and the corresponding Wilson loop expectation value in a three-parameter deformation of $\mathcal{N} = 4$ SYM that breaks all the supersymmetry. We will see that the same analysis done for the β-deformed theory applies also here.

In this Letter we assume that the deformation parameter β is real. One may also consider the case where β is complex. The resulting deformed theory was argued to be conformal [13] (for a recent discussion see [14]). On the perturbative field theory side, inspection of the planar diagrams shows that unlike the case with real β, here there is a dependence of the scattering amplitudes on the deformation parameter. However, it is straightforward to check that the dependence on β does not appear up to the two-loop order both in the gluon scattering amplitudes and in the Wilson loop. In the strong coupling regime, the dual supergravity background is of a warped type. It was obtained by employing an S-duality transformation [15], and we lack a simple σ-model description.

2. Weak coupling analysis

We will consider the β-deformation of $\mathcal{N} = 4$ SYM with $SU(N)$ gauge group at large N. In the following we will take the parameter β to be real. The deformed theory has $\mathcal{N} = 1$ supersymmetry and is conformally invariant [13,16,17]. The field content of the β-deformed theory is identical to that of $\mathcal{N} = 4$ SYM, i.e., it consists, in $\mathcal{N} = 1$ language, of a vector superfield $V \sim (A_\mu, \lambda_\alpha)$ and three chiral superfields $\Phi^I \sim (\phi^I, \psi^I_\alpha)$, $I = 1, 2, 3$, all transforming in the adjoint representation of the gauge group. The $SU(4)$ $\mathcal{R}$-symmetry of $\mathcal{N} = 4$ SYM is broken by the deformation to $U(1)_\mathcal{R} \times (U(1) \times U(1))_{\text{flavor}}$. The vector multiplet fields are neutral under the flavor symmetry, while the fields in the three chiral multiplets carry the charges $(Q^1, Q^2) = (0, -1), (1, 1)$ and $(-1, 0)$, respectively.

Written in terms of $\mathcal{N} = 1$ superfields, the action of the β-deformed theory is that of three adjoint chiral multiplets coupled to a vector multiplet and a superpotential

$$W = g \operatorname{tr}\left(q \Phi^1 \Phi^2 \Phi^3 - \bar{q} \Phi^1 \Phi^3 \Phi^2\right) = g_{abc} \Phi^a_1 \Phi^b_2 \Phi^c_3, \qquad (2.1)$$

where g is the gauge coupling, $q = e^{i\pi\beta}$ ($|q| = 1$ for real β) and

$$g_{abc} = g\left(q c_{abc} - q^{-1} c_{acb}\right), \qquad c_{abc} = \operatorname{tr}\left(T^a T^b T^c\right). \qquad (2.2)$$

The superpotential can be written in the form of a d-type and a f-type coupling as in [13],

$$W = 2g\left(\cos[\pi\beta] f_{abc} + i \sin[\pi\beta] d_{abc}\right) \Phi^a_1 \Phi^b_2 \Phi^c_3, \qquad (2.3)$$

where $f_{abc} = \operatorname{tr}(T^a[T^b, T^c])$, $d_{abc} = \operatorname{tr}(T^a\{T^b, T^c\})$. The latter term in W vanishes for $\beta = 0$ where one recovers the $\mathcal{N} = 4$ theory. Note that the $\mathcal{N} = 4$ SYM action and its β-deformation have the same propagators and chiral-vector vertices and differ only in the chiral vertex.

The Coulomb branch of the β-deformed theory is a solution to the F-term equations

$$\Phi^1 \Phi^2 = q \Phi^2 \Phi^1, \qquad \Phi^2 \Phi^3 = q \Phi^3 \Phi^2,$$
$$\Phi^3 \Phi^1 = q \Phi^1 \Phi^3. \qquad (2.4)$$

A useful observation is that the β-deformed action can be written using a noncommutative $*$-product between the matter fields defined as [15]

$$f * g = e^{i\pi\beta(Q^1_f Q^2_g - Q^2_f Q^1_g)} fg, \qquad (2.5)$$

where fg is the ordinary product and (Q^1, Q^2) are the $(U(1) \times U(1))_{\text{flavor}}$ charges of the matter fields (the vector multiplet is neutral under the flavor group). This has been employed in [17] with $\mathcal{N} = 4$ light-cone superspace to prove the finiteness of the β-deformed theory at large N.

By a direct inspection of Feynman diagrams as in [18–20], it is straightforward to see that the only modification of *planar* diagrams compared to $\mathcal{N} = 4$ SYM, is the multiplication by an overall phase, which depends only on the flavor charges of the fields on the external lines. In particular if all the fields on the external lines are gluons and gluinos, the phase is 1 and the scattering amplitudes are same as in the undeformed $\mathcal{N} = 4$ SYM theory. This holds to all orders in perturbation theory for any n-point function. The same argument can be employed in the computation of the expectation value of the Wilson loop in the planar limit of the β-deformed theory and shows that it agrees with the $\mathcal{N} = 4$ SYM result to all orders in perturbation theory.

3. Strong coupling analysis

We have argued in the previous section that at weak coupling the β deformation is invisible in the planar gluon and gluino scattering amplitudes to all orders in perturbation theory. In this section we will consider the β-independence of the scattering amplitudes at strong coupling. We will use the σ-model description of the dual supergravity background and show that it holds at strong coupling to all orders in the σ-model loop expansion.

As we discussed above, the prescription for the computation of the planar gluon scattering amplitudes in $\mathcal{N} = 4$ SYM at strong coupling was mapped using the gauge/gravity correspondence to the construction of a minimal area string worldsheet ending on the loop [1].

There are two ingredients in the computation that we will need to pay special attention to. The first one is that in order to map the scattering amplitude computation to the Wilson loop one at strong coupling, a T-duality on all four directions along the boundary of AdS_5 has been employed. The loop is the boundary of the T-dualized open string world sheet in AdS space, corresponding to the IR region in field theory. The second one is that the area of the minimal surface is infinite and needs regularization. The infinity corresponds to the infrared divergences in the field theory. One type of regularization introduced in [1], was in the form of a D3-brane placed at a large value of the AdS_5 radial coordinate Z_{IR}, and at an arbitrary

point in the internal S^5 space. In the field theory language it translates to going along the Coulomb branch. The Coulomb branch of the β-deformed theory (2.4) differs from that of $\mathcal{N} = 4$ SYM. Therefore, if we wish to employ a similar regularizaton procedure, we will have to place the D3-brane in a particular locus in the internal part of the space.

3.1. The dual supergravity description

The dual gravity background of the β-deformed theory at large N was constructed in [15]. The metric for real β is of the type $AdS_5 \times \mathcal{M}_5$, where $\mathcal{M}_5$ is a deformation of S^5. With the S^5 parameterized by three angular coordinates ϕ_i and three radial coordinates μ_i satisfying $\mu_1^2 + \mu_2^2 + \mu_3^2 = 1$, the metric on $\mathcal{M}_5$ takes the form

$$ds^2_{\mathcal{M}_5} = \sum_{i=1}^{3}(d\mu_i^2 + G\mu_i^2 d\phi_i^2)$$
$$+ \beta R^2 G\mu_1^2\mu_2^2\mu_3^2\left(\sum_{i=1}^{3} d\phi_i\right)^2, \qquad (3.1)$$

where $G^{-1} = 1 + \beta R^2(\mu_1^2\mu_2^2 + \mu_2^2\mu_3^2 + \mu_1^2\mu_3^2)$ and R is the S^5 radius. In addition to this deformation of the S^5-part of the metric and the field strength $F^{(5)}$, the deformed background has all bosonic Type IIb supergravity fields turned on. However, the modification of all the background fields depends completely on the compact part $\mathcal{M}_5$. This means, in particular, that the T-duality transformation in the AdS_5 part, that is used in order to map the scattering amplitude computation to a Wilson loop one is still valid in the β-deformed geometry.

The deformation of S^5 to $\mathcal{M}_5$ is obtained by TsT transformation: a combination of a T-duality along one of the circles, followed by a shift along the second circle and a second T-duality along the first circle. The flat directions corresponding to the field theory Coulomb branch are characterized by the points in $\mathcal{M}_5$ where two of the μ_i's vanish. Indeed, it is straightforward to see that a D3-brane that is moved in the radial direction of AdS_5 and is located at these points in the internal space experiences zero force. Thus, the analog of the regularization procedure of [1], requires a location of the D3-brane at this locus of points. Obviously, this is expected since the Coulomb branch of $\mathcal{N} = 4$ SYM differs from that of its deformation. Note also that along this locus the dependence of all the background fields on β disappears. We should stress, however, that we are computing the gluon scattering amplitude and the corresponding expectation value of the Wilson loop at the origin of the Coulomb branch. Moving the D3-brane along the Coulomb branch is only one type of regularization (dimensional regularization can be another method) and the fact that it can be located only at a certain locus in $\mathcal{M}_5$ is not a restriction on the validity of the results, but only on the use of this type of regularization.

The computation of the minimal surface in [1] was done at fixed position in the internal space and the solution does not depend on the coordinates of S^5. This minimal surface solution is also a solution in the β-deformed background and is inde-

pendent of the deformation parameter. Thus, the result of [1] carries over to the β-deformed case, and one gets exactly the same four-gluon scattering amplitude as that of $\mathcal{N} = 4$ SYM at strong coupling. Similarly, the n-gluon scattering amplitudes are independent of β at strong coupling and are the same as those of $\mathcal{N} = 4$ SYM.

$\frac{1}{\sqrt{\lambda}}$ corrections to the result of [1] are due to fluctuations around the classical minimal surface solution. To leading order in N they are σ-model loops on the world sheet with the disc topology. The σ-model corrections to [1] have been considered in [21]. This analysis uses the explicit form of the Green–Schwarz action in the $AdS_5 \times S^5$ background [22] to compute the fluctuation spectrum of the world-sheet fields.

As we discussed above, the β-deformed background is independent of β at those points where two of the μ_i's vanish. Let us fix the internal coordinate of the minimal surface solution to one of those points and consider the fluctuation spectrum around this solution. The procedure is the same as in [23] and uses a covariant background field expansion and Riemann normal coordinates. For the bosonic fluctuations along the tangent space of AdS_5 there are no changes compared to the undeformed case. The fluctuations tangential to the internal space is, in general, affected. However, due to the fact that the classical solution satisfies $\phi_i = \mu_i = \text{const}$ and furthermore, say, $\mu_1 = \mu_2 = 0$, the fluctuation determinant will again be as in the undeformed background geometry.[1] The same holds for the fermions. Thus, up to the quadratic order the action for the fermionic fluctuations is the same as in the undeformed case. This shows that the $\frac{1}{\sqrt{\lambda}}$ corrections to the strong coupling result are unmodified by the β-deformation, at least if we place the minimal surface which ends on the light-like Wilson loop at a point where two of the μ_i's vanish.

In the following we will show using the Green–Schwarz σ-model on the β-deformed background that the requirement of placing the solution at those special points corresponding to the Coulomb branch is not needed. Moreover, we will argue that the results are the same as in the undeformed $\mathcal{N} = 4$ SYM theory at strong coupling to all orders in the σ-model loop expansion.

3.2. The σ-model description

A Green–Schwarz σ-model describing a closed string propagating on the β-deformed background was constructed in [24]. The authors of [24] have shown that the deformation of the Green–Schwarz σ-model action can be replaced by a modification of the periodicity properties of the world-sheet fields. More specifically, they showed in general that the angular coordinates ϕ_i in the compact space, which were involved in the TsT-transformation are no longer periodic in $\sigma \to \sigma + 2\pi$. The difference $\phi(\sigma + 2\pi) - \phi(\sigma) \pmod{2\pi}$ is proportional to the deformation parameter and to the $U(1)$ Noether charges asso-

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[1] In general there are terms of the form $\sqrt{h}h^{ij} R_{\mu\nu\rho\sigma} \partial_i X_{cl}^\mu \partial_j X_{cl}^\rho \xi^\nu \xi^\sigma$ where h_{ij} is the world-sheet metric and ξ^μ the bosonic fluctuations. This term vanishes for $X_{cl} = \text{const}$.

ciated to the two isometries $\phi_i \to \phi_i + \text{const}$. In the following we will use the notations of [24].

For a purely bosonic background the action reads (F_α and B_α vanish)

$$\mathcal{L} = \text{tr}\big[\gamma^{\alpha\beta} \partial_\alpha G G^{-1} \partial_\beta G G^{-1}\big], \qquad (3.2)$$

where $G = \big(\begin{smallmatrix} g_a & 0 \\ 0 & g_s \end{smallmatrix}\big)$, and $g_a \in SU(2,2)$ and $g_s \in SU(4)$ provide a parametrization of AdS_5 and S^5. For us the relevant piece is g_s, which gives

$$\mathcal{L} \sim \gamma^{\alpha\beta} \sum_{i=1}^{3} \big(\mu_i^2 \partial_\alpha \phi_i \partial_\beta \phi_i + \partial_\alpha \mu_i \partial_\beta \mu_i\big). \qquad (3.3)$$

This implies that the Noether currents, whose charge are responsible for the non-periodicity of the fields depend only on $\partial^\alpha \phi$ and not on ϕ_i ($U^0_{\beta,i} = V^0_{\beta,i} = 0$)

$$J_i^\alpha \sim \partial^\alpha \phi_i. \qquad (3.4)$$

For constant ϕ_i, as is the case in the internal part of the minimal area solution, this vanishes. According to [24] this also means that the fermionic fields are still periodic. Combining these arguments we conclude that the analysis of the spectrum of fluctuations is unmodified by the deformation.

In fact, the calculation of the loop corrections to the lightlike Wilson loop in the σ-models of $AdS_5 \times S^5$ and its TsT deformation will give the same result to all orders. The reason being, that the classical solution dictates the periodicity of the fields and those are β-independent for the case at hand. Therefor, the corrections to any loop order in both σ-models are identical.

4. Nonsupersymmetric deformation

In this section we will consider planar gluon scattering amplitudes and the corresponding Wilson loop expectation value in a three-parameter deformation of $\mathcal{N} = 4$ SYM that breaks all the supersymmetry. The resulting nonsupersymmetric deformed theory is scale invariant in the planar limit [25]. We denote the three real deformation parameters by γ_i, $i = 1, 2, 3$ and refer to the deformed theory as γ-deformed. When all the deformation parameters are equal $\gamma_i = \beta$, $i = 1, 2, 3$, we obtain the β-deformed theory that we discussed previously. The γ-deformed theory has the same field content as $\mathcal{N} = 4$ SYM, with a modification of the $\mathcal{N} = 4$ SYM Yukawa and scalar quartic couplings by phase factors. These phases break the $SU(4)$ $\mathcal{R}$-symmetry to its Cartan subgroup, now being a flavor symmetry $U(1)^3_{\text{flavor}}$ of the deformed theory. The six scalars and all the Weyl fermions including the gaugino are charged under $U(1)^3_{\text{flavor}}$. The gauge field is not charged under the $U(1)^3_{\text{flavor}}$. Since unlike the β-deformed theory, the gauge field is neutral while the gaugino is charged under the flavor symmetry, supersymmetry is completely broken.

As in the β-deformed case, also the γ-deformed action can be written using a noncommutative $*$-product between the fields defined as

$$f * g = e^{i\pi\gamma_i \varepsilon^{ijk} Q_j^f Q_k^g} fg, \qquad (4.1)$$

where fg is the ordinary product and Q_i, $i = 1, 2, 3$ are the $U(1)^3_{\text{flavor}}$ charges of the fields. Again, by a direct inspection of Feynman diagrams it is straightforward to see that the only modification of planar diagrams compared to $\mathcal{N} = 4$ SYM, is the multiplication by an overall phase, which depends only on the flavor charges of the fields on the external lines. In particular if all the fields on the external lines are gluons, the phase is 1 and the scattering amplitudes are same as in the undeformed $\mathcal{N} = 4$ SYM theory. This holds to all orders in perturbation theory for any n-point function. Note, that unlike the β-deformed theory, here the gluino scattering amplitudes differ from those of the $\mathcal{N} = 4$ SYM since they are charged under the flavor symmetry group. Also, the same argument can be employed in the computation of the expectation value of the Wilson loop in the planar limit of the γ-deformed theory and shows that it agrees with the $\mathcal{N} = 4$ SYM result to all orders in perturbation theory.

The dual gravity background of the γ-deformed theory at large N was constructed in [26], using three TsT transformations. The metric is of the type $AdS_5 \times \mathcal{M}_5$, where $\mathcal{M}_5$ is a deformation of S^5

$$ds^2_{\mathcal{M}_5} = \sum_{i=1}^{3} \big(d\mu_i^2 + G\mu_i^2 d\phi_i^2\big)$$
$$+ R^2 G \mu_1^2 \mu_2^2 \mu_3^2 \bigg(\sum_{i=1}^{3} \gamma_i \, d\phi_i\bigg)^2, \qquad (4.2)$$

where $G^{-1} = 1 + R^2(\gamma_3^2 \mu_1^2 \mu_2^2 + \gamma_1^2 \mu_2^2 \mu_3^2 + \gamma_2^2 \mu_1^2 \mu_3^2)$ and R is the S^5 radius. Again, in addition to this deformation of the S^5-part of the metric and the field strength $F^{(5)}$, the deformed nonsupersymmetric background has all bosonic Type IIb supergravity fields turned on and affects only the compact part $\mathcal{M}_5$. Thus, the T-duality transformation in the AdS_5 part, that is used in order to map the scattering amplitude computation to a Wilson loop one is valid also in the γ-deformed geometry.

A Green–Schwarz σ-model describing a closed string propagating on the γ-deformed background was constructed in [24]. Again, the deformation of the Green–Schwarz σ-model action can be replaced by a modification of the periodicity properties of the world-sheet fields. The same arguments that we employed in the β-deformed case can be used here: the Noether currents, whose charge are responsible for the non-periodicity of the fields depend only on $\partial^\alpha \phi_i$ and vanish for constant ϕ_i, as is the case in the internal part of the minimal area solution. The fermionic fields are still periodic and the analysis of the spectrum of fluctuations is unmodified by the deformation. Therefore, the corrections to any loop order in the $AdS_5 \times S^5$ and the γ-deformed σ-models are identical.

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References

[1] L.F. Alday, J.M. Maldacena, JHEP 0706 (2007) 064, arXiv: 0705.0303 [hep-th].

[2] J.M. Maldacena, Adv. Theor. Math. Phys. 2 (1998) 231, hep-th/9711200, Int. J. Theor. Phys. 38 (1999) 1113.

[3] O. Aharony, S.S. Gubser, J.M. Maldacena, H. Ooguri, Y. Oz, Phys. Rep. 323 (2000) 183, hep-th/9905111.

[4] Z. Bern, L.J. Dixon, V.A. Smirnov, Phys. Rev. D 72 (2005) 085001, hep-th/0505205.

[5] J.M. Drummond, G.P. Korchemsky, E. Sokatchev, arXiv: 0707.0243 [hep-th].

[6] A. Brandhuber, P. Heslop, G. Travaglini, arXiv: 0707.1153 [hep-th].

[7] J.M. Drummond, J. Henn, G.P. Korchemsky, E. Sokatchev, arXiv: 0709.2368 [hep-th].

[8] J.M. Drummond, J. Henn, G.P. Korchemsky, E. Sokatchev, arXiv: 0712.1223 [hep-th].

[9] J.M. Drummond, J. Henn, G.P. Korchemsky, E. Sokatchev, arXiv: 0712.4138 [hep-th].

[10] Z. Komargodski, S.S. Razamat, arXiv: 0707.4367 [hep-th].

[11] J. McGreevy, A. Sever, arXiv: 0710.0393 [hep-th].

[12] C.S. Chu, D. Giataganas, JHEP 0710 (2007) 108, arXiv: 0708.0797 [hep-th].

[13] R.G. Leigh, M.J. Strassler, Nucl. Phys. B 447 (1995) 95, hep-th/9503121.

[14] F. Elmetti, A. Mauri, S. Penati, A. Santambrogio, D. Zanon, JHEP 0710 (2007) 102, arXiv: 0705.1483 [hep-th].

[15] O. Lunin, J.M. Maldacena, JHEP 0505 (2005) 033, hep-th/0502086.

[16] A. Mauri, S. Penati, A. Santambrogio, D. Zanon, JHEP 0511 (2005) 024, hep-th/0507282.

[17] S. Ananth, S. Kovacs, H. Shimada, JHEP 0701 (2007) 046, hep-th/0609149.

[18] T. Filk, Phys. Lett. B 376 (1996) 53.

[19] S. Minwalla, M. Van Raamsdonk, N. Seiberg, JHEP 0002 (2000) 020, hep-th/9912072.

[20] V.V. Khoze, JHEP 0602 (2006) 040, hep-th/0512194.

[21] M. Kruczenski, R. Roiban, A. Tirziu, A.A. Tseytlin, Nucl. Phys. B 791 (2008) 93, arXiv: 0707.4254 [hep-th].

[22] R.R. Metsaev, A.A. Tseytlin, Nucl. Phys. B 533 (1998) 109, hep-th/9805028.

[23] S. Forste, D. Ghoshal, S. Theisen, JHEP 9908 (1999) 013, hep-th/9903042.

[24] L.F. Alday, G. Arutyunov, S. Frolov, JHEP 0606 (2006) 018, hep-th/0512253.

[25] S. Ananth, S. Kovacs, H. Shimada, Nucl. Phys. B 783 (2007) 227, hep-th/0702020.

[26] S. Frolov, JHEP 0505 (2005) 069, hep-th/0503201.