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Twisted Superspace

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Abstract

We formulate the ten-dimensional super-Yang–Mills theory in a twisted superspace with $8+1$ supercharges. Its constraints do not imply the equations of motion and we solve them. As a preliminary step for a complete formulation in a twisted superspace, we give a superspace path-integral formulation of the $\mathcal{N}=2,\ d=4$ super-Yang–Mills theory without matter. The action is the sum of a Chern–Simons term depending on a super-connection plus a BF-like term. The integration over the superfield B implements the twisted superspace constraints on the super-gauge field, and the Chern–Simons action reduces to the known action in components.

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1 Introduction

Superspace formulations of supersymmetric theories are often very efficient for practical computations and proofs of non-renormalization theorems. However, a complete superspace path-integral formulation requires that the supersymmetry algebra admits a functional representation on the fields, but the latter is believed not to exist for maximal supersymmetry. This has lead to several proposals for restricting the whole super-Poincaré algebra to subalgebras that admit a functional representation on the fields. For instance, the maximally supersymmetric Yang–Mills theory has been formulated within $\mathcal{N} = 3$ harmonic superspace [1]. There are in fact severe restrictions for getting such off-shell closed representations. In six dimensions, to maintain the full Poincaré invariance, one must reduce the $\mathcal{N} = 2$ super-Poincaré symmetry to the $\mathcal{N} = 1$ one. From dimension seven and above, there is no non-trivial subalgebra that includes the whole Poincaré algebra. As a consequence, a superspace path-integral formulation of maximally supersymmetric Yang–Mills theories in higher dimensions must presumably give up manifest Lorentz invariance.

In [2] we have shown that $SO(1,1) \times Spin(7) \subset SO(1,9)$ is the biggest subgroup of the ten-dimensional Lorentz group that can be preserved for obtaining an off-shell closed supersymmetric algebra of the $\mathcal{N} = 1, d = 10$ supersymmetric theory. We introduced for this theory $SO(1,1) \times Spin(7) \subset SO(1,9)$ invariant constraints for the curvatures of superfields depending of $1 + 8$ fermionic coordinates, as a hint for a possible off-shell superspace description. A superspace action was also proposed, which is reminiscent of harmonic superspace formulations. Part of this paper is devoted to solve these constraints.

The maximally supersymmetric theory in ten dimensions is a chiral model with a gauge anomaly that spoils its quantization. A consistent approach implies in fact its coupling to supergravity at the quantum level. However, its chiral anomaly often disappears after dimensional reduction to lower dimensions. A better understanding of the Yang–Mills supersymmetry in ten dimensions could be also helpful for improving that of supergravity. It is thus a relevant question to investigate a possible superspace off-shell formulation of the pure ten-dimensional supersymmetric Yang–Mills theory. It is admitted that the most efficient way for describing the extended supersymmetry in superspace is by using harmonic superspace formulations, which depend on the harmonics of a compact homogenous space G/H [3]. In these formulations of supersymmetric theories, the group G acts linearly on the harmonic variables. The latter must therefore be a subgroup of $Spin(7)$ in order to permit a functional representation of $8+1$ supersymmetry charges on the fields. This group admits compact cossets, if one chooses the invariant subgroup H as an invariant subgroup of $Spin(7)$. As a matter of fact, power counting forbids that the action be a $SO(1,1) \times Spin(7)$ covariant integral over full superspace, and the introduction of an harmonic superspace formulation is required. Its status is yet quite unclear when one uses twisted representations of the Lorentz group. Thus, for defining a complete twisted superspace path-integral formulation, we will reduce ourselves to the case of the $\mathcal{N}=2, d=4$ theory, which does not require the introduction of harmonics variables.

The $SO(1,1) \times Spin(7)$ invariant formulation of the ten-dimensional Yang–Mills theory is closely related to its formal dimensional reduction down to eight euclidean dimensions [2]. The latter is formally very close to the twisted $SU(2) \times SU(2)$ invariant formulation of the $\mathcal{N}=2$ super-Yang–Mills theory without matter in four dimensions. In this case the gauge-invariant action can be written in term of the super-connections, and this model is thus very interesting to understand path-integral on a twisted superspace. In this publication, we study this four-dimensional case in great details. We extend our results to the ten-dimensional case, whenever it does not require important conceptual modifications. We will ignore through the paper the problems associated with unitarity and the doubling of fermions in four and eight-dimensional euclidean space. This is justified within the context of describing the ten-dimensional structure.

For describing the twisted super-Yang–Mills theory, three types of superfields are involved, namely the super-connection, the super-ghosts, and Lagrange multipliers superfields for the superspace constraints. Interestingly, there is no need to introduce a prepotential in the four-dimensional case, and the theory can be formulated in terms of the geometrical superspace connections, which have strictly positive canonical dimensions. This property indicates a close link between the superspace and the components formulations of the theory. Moreover, the gauge-fixing in superspace is a rather simple superfield generalization of the usual Landau gauge-fixing action. Its decomposition in components turns out to be equivalent to a supersymmetric gauge-fixing involving shadow fields, which generalize those introduced in [4]. This results hold true in any dimensions. The gauge-fixing action involves super-antighosts fields with negative canonical dimensions. However, in four dimensions, antighost Ward identities in superspace ensure the stability of the action and imply that these fields are multiplicatively renormalized with the same factor as the super-gauge field along the ordinary space directions.

The part of the action that implements the constraints turns out to be a source of new technical questions. It is similar to a BF system, where F stands for the components of the super-curvature that define the constraints. Because of Bianchi identities, the auxiliary superfields B possess a set of zero modes that must be taken into account in the super-Feynman rules. The complete gauge-fixing of the BF component of the action requires the introduction of an infinite tower of ghosts and ghosts for ghosts. This problem is reminiscent of the infinite set of auxiliary fields required in the harmonic-superspace formulation of the theory. We will solve these technical subtleties in a more detailed forthcoming publication [5].

The classical action is a Chern–Simons-like action for the superspace connection along the scalar odd coordinate. It reduces to the gauge-invariant action after integration over the auxiliary fields.

Our superspace construction followed a determination of twisted scalar and vector supersymmetric transformations that close without the use of equations of motion and gauge transformations, using scalar and vector shadows directly in component formalism [5]. The latter construction helped us to select a convenient gauge in superspace for solving the covariant constraints.

The paper is organized as follows. In the first section we define the $\mathcal{N}=2$ twisted superspace, and its generalization in higher dimensions. Then we define and solve the twisted super-Yang–Mills constraints in four dimensions and generalize the results in ten dimensions, with an obvious application in eight dimensions. In the last section, we construct the action for the $\mathcal{N}=2, d=4$ theory. We explain the problem associated to the gauge invariance of the Lagrange multipliers that enforce the covariant constraints on the supercurvature, but postpone to a forthcoming publication the definition of the corresponding gauge-fixing action.

Let us end this introduction with earlier references for the idea of twisted superspace. A superspace for the $\mathcal{N}=2, d=4$ twisted super-Yang–Mills vector multiplet is constructed in [6] for matter couplings to the Donaldson–Witten theory and topological symmetry breaking. It involves the full set of supersymmetry generators so that one cannot get the action as an integral over the whole superspace. Its projection in components gives the Donaldson–Witten action by using a Wess–Zumino gauge. Analogous results are in [9], with a truncation of an $\mathcal{N}=4$ twisted algebra using a Dirac–Kähler twist. A generalization that involves central charges is presented in [10], together with a coupling to a twisted hypermultiplet. All twisted formulations, including ours, have been inspired by the methodology of [7, 8] for superspace constraints in extended supersymmetry. No path-integral formulation in twisted superspace had been proposed so far.

2 Twisted superspace set-up

2.1 The $\mathcal{N}=2$, $d=4$ case

Let us recall the basic features of the twisted formulation of $\mathcal{N}=2$ super-Yang–Mills theory [11]. It is defined in a four-dimensional euclidean space with the manifest invariance reduced to $L' = SU(2)' \times SU(2)_R$, where $SU(2)'$ is the diagonal subgroup of $SU(2)_L \times SU(2)_I$, and $SU(2)_I$ is the internal symmetry group associated to $\mathcal{N}=2$ supersymmetry. The vector multiplet in representations of L' is made of the gauge field A_μ, two commuting scalar fields Φ and $\bar\Phi$, an anticommuting vector Ψ_μ, an anticommuting anti-selfdual 2-form $\chi_{\mu\nu^-}$, an anticommuting scalar η, and a commuting auxiliary field $G_{\mu\nu^-}$. These fields transform under the scalar and vector anticommuting generators $\delta \equiv \epsilon^{\alpha\imath} Q_{\alpha\imath}$ and $\delta_\mu \equiv i\sigma_\mu^{\dot\alpha\imath} Q_{\dot\alpha\imath}$. The invariance under the action of these 5 generators completely determines the classical action of the theory, which is nothing but the super-Yang–Mills action, in twisted form [4]. In order to recover the complete super-Poincaré symmetry with 8 generators, one must introduce the anti-selfdual generator $\delta_{\mu\nu^-} \equiv \sigma_{\mu\nu}^{\alpha\imath} Q_{\alpha\imath}$. The $\delta_{\mu\nu^-}$ invariance is an additional symmetry of the action, which is obtained for free from the requirement of δ and δ_μ symmetry. Moreover the absence of trivial anomalies for the tensor symmetry shows that forgetting about the tensor symmetry does not introduce ambiguities in the renormalization program [5]. Therefore, as long as we only consider correlation functions of the fields, the scalar and vector supersymmetry generators unambiguously determine the theory to be invariant by the action of all the supersymmetry generators, including the tensor generator $\delta_{\mu\nu^-}$.

To express the scalar and vector supersymmetry in terms of superspace derivatives, we complete the four-dimensional space by five anticommuting coordinates, a scalar one θ and a vector one ϑ^μ ($\mu = 1\cdots 4$). We define as follows the superspace differential operators $\mathbb{Q}$ and $\mathbb{Q}_\mu$, whose action on superfields provide component by component a linear realization of the scalar δ and vector δ_μ supersymmetry generators

$$
\begin{aligned}
\mathbb{Q} &\equiv \frac{\partial}{\partial\theta} + \vartheta^\mu \partial_\mu, \qquad\qquad \mathbb{Q}_\mu \equiv \frac{\partial}{\partial\vartheta^\mu} \\
\mathbb{Q}^2 = 0, \qquad &\{\mathbb{Q}, \mathbb{Q}_\mu\} = \partial_\mu, \qquad \{\mathbb{Q}_\mu, \mathbb{Q}_\nu\} = 0
\end{aligned} \tag{1}
$$

A general superfield $\mathbb{S}_A$ is a polynomial expansion in (θ, ϑ^μ)

$$
\mathbb{S}_A = \mathbf{S}^0_A + \theta \mathbf{S}^\theta_A = S_A + \vartheta^\mu S_{A\mu} + \vartheta^\mu\vartheta^\nu S_{A\mu\nu} + \cdots + \theta S^\theta_A + \theta\vartheta^\mu S^\theta_{A\mu} + \cdots \tag{2}
$$

Here the index A stands for the L' representation of the superfield and $\mathbb{S}_A$ carries $\sharp(A) \times 2^5$ components, where $\sharp(A)$ is the dimension of the corresponding L' representation.

The covariant superspace derivatives and their anticommuting relations are

$$
\begin{aligned}
\nabla \;&\equiv \frac{\partial}{\partial\theta} \qquad\qquad \nabla_\mu \equiv \frac{\partial}{\partial\vartheta^\mu} - \theta\partial_\mu \\
\nabla^2 = 0 \qquad &\{\nabla, \nabla_\mu\} = -\partial_\mu \qquad \{\nabla_\mu, \nabla_\nu\} = 0
\end{aligned} \tag{3}
$$

They anticommute with the supersymmetry generators.

A connection superfield $(\mathbb{C}, \mathbf{\Gamma}_\mu, \mathbb{A}_\mu)$ valued in the adjoint of the gauge group of the theory can be defined in correspondence with the set of the superspace derivatives $(\nabla, \nabla_\mu, \partial_\mu)$. This provides the following gauge covariant superderivatives

$$
\hat\nabla \equiv \nabla + \mathbb{C}, \quad \hat\nabla_\mu \equiv \nabla_\mu + \mathbf{\Gamma}_\mu, \quad \hat\partial_\mu \equiv \partial_\mu + \mathbb{A}_\mu \tag{4}
$$

and the corresponding covariant superspace curvatures

$$
\begin{aligned}
\mathbb{F}_{\mu\nu} &\equiv [\hat\partial_\mu, \hat\partial_\nu] \qquad & \mathbf{\Phi} &\equiv \hat\nabla^2 \\
\mathbf{\Psi}_\mu &\equiv [\hat\nabla, \hat\partial_\mu] \qquad & \mathbb{L}_\mu &\equiv \{\hat\nabla, \hat\nabla_\mu\} + \hat\partial_\mu \\
\boldsymbol{\chi}_{\mu\nu} &\equiv [\hat\nabla_\mu, \hat\partial_\nu] \qquad & \bar{\mathbf{\Phi}}_{\mu\nu} &\equiv \tfrac{1}{2}\{\hat\nabla_\mu, \hat\nabla_\nu\}
\end{aligned} \tag{5}
$$

so that

$$
\begin{aligned}
\mathbb{F}_{\mu\nu} &= \partial_\mu\mathbb{A}_\nu - \partial_\nu\mathbb{A}_\mu + [\mathbb{A}_\mu, \mathbb{A}_\nu] \qquad & \mathbf{\Phi} &= \nabla\mathbb{C} + \mathbb{C}^2 \\
\mathbf{\Psi}_\mu &= \nabla\mathbb{A}_\mu - \partial_\mu\mathbb{C} - [\mathbb{A}_\mu, \mathbb{C}] \qquad & \mathbb{L}_\mu &= \nabla\mathbf{\Gamma}_\mu + \nabla_\mu\mathbb{C} + \{\mathbf{\Gamma}_\mu, \mathbb{C}\} + \mathbb{A}_\mu \\
\boldsymbol{\chi}_{\mu\nu} &= \nabla_\mu\mathbb{A}_\nu - \partial_\nu\mathbf{\Gamma}_\mu - [\mathbb{A}_\nu, \mathbf{\Gamma}_\mu] \qquad & \bar{\mathbf{\Phi}}_{\mu\nu} &= \nabla_{\{\mu}\mathbf{\Gamma}_{\nu\}} + \mathbf{\Gamma}_{\{\mu}\mathbf{\Gamma}_{\nu\}}
\end{aligned} \tag{6}
$$

These different objects can be assembled into an extended exterior differential

$$
\Delta \equiv d + \nabla d\theta + \nabla_{d\vartheta} \equiv dx^\mu\partial_\mu + d\theta\nabla + d\vartheta^\mu\nabla_\mu \tag{7}
$$

and the extended connection

$$
\mathcal{A} \equiv \mathbb{A} + \mathbb{C}d\theta + \mathbf{\Gamma} \equiv \mathbb{A}_\mu dx^\mu + \mathbb{C}d\theta + \mathbf{\Gamma}_\mu d\vartheta^\mu \tag{8}
$$

Since $(d + \nabla d\theta + \nabla_{d\vartheta} + d\theta i_{d\vartheta})^2 = 0$, (where i is the Cartan contraction operator, e.g. $i_{d\vartheta}dx^\mu \equiv d\vartheta^\mu$), we define the following extended curvature superfield 2-form $\mathcal{F}$

$$
\mathcal{F} \equiv (d + \nabla d\theta + \nabla_{d\vartheta} + d\theta i_{d\vartheta})\mathcal{A} + \mathcal{A}^2 = \mathbb{F} + \mathbf{\Psi}d\theta + \boldsymbol{\chi} + \mathbf{\Phi}d\theta d\theta + \mathbb{L}d\theta + \bar{\mathbf{\Phi}} \tag{9}
$$

where $\mathbb{F} \equiv \frac{1}{2}\mathbb{F}_{\mu\nu}dx^\mu dx^\nu$, $\mathbf{\Psi} \equiv \mathbf{\Psi}_\mu dx^\mu$, $\boldsymbol{\chi} \equiv \boldsymbol{\chi}_{\mu\nu}d\vartheta^\mu dx^\nu$, $\mathbb{L} \equiv \mathbb{L}_\mu d\vartheta^\mu$, $\bar{\mathbf{\Phi}} \equiv \bar{\mathbf{\Phi}}_{\mu\nu}d\vartheta^\mu d\vartheta^\nu$. The Bianchi identity implies the following constraints on the components of $\mathcal{F}$

$$
\begin{aligned}
&(d + d\theta\nabla + \nabla_{d\vartheta} + d\theta i_{d\vartheta})(\mathbb{F} + \mathbf{\Psi}d\theta + \boldsymbol{\chi} + \mathbf{\Phi}d\theta d\theta + \mathbb{L}d\theta + \bar{\mathbf{\Phi}}) + \\
&\qquad\qquad [\mathcal{A}, \mathbb{F} + \mathbf{\Psi}d\theta + \boldsymbol{\chi} + \mathbf{\Phi}d\theta d\theta + \mathbb{L}d\theta + \bar{\mathbf{\Phi}}] = 0
\end{aligned} \tag{10}
$$

The super-gauge transformations of the extended connection $\mathcal{A}$ and curvature $\mathcal{F}$ are

$$\mathcal{A} \to e^{-\alpha}(\Delta + \mathcal{A})e^{\alpha}, \quad \mathcal{F} \to e^{-\alpha}\mathcal{F}e^{\alpha} \tag{11}$$

where the gauge superparameter α can be any given general superfield valued in the Lie algebra of the gauge group. The "infinitesimal" gauge transformation is $\delta\mathcal{A} = \Delta\alpha + [\mathcal{A}, \alpha]$.

2.2 Higher dimensions

The formalism for the scalar and vector supersymmetry generalizes directly to the euclidean eight-dimensional case, by extending the eight-dimensional space-time with nine fermionic coordinates and considering a reduction of the Wick rotated Lorentz group $SO(8)$ to $Spin(7)$, with all previous equations remaining formally identical. One can further "oxidise " the eight-dimensional theory into the $\mathcal{N} = 1, d = 10$ theory. This has already been described in [2], and we shall only summarise the equations that are relevant for the following. (One can go from four to six dimensions in an analogous way).

The $\mathcal{N} = 1$, $d = 10$ superspace is made of ten bosonic coordinates x^m and nine fermionic ones θ and ϑ^i. The x^m $(m = 0, \cdots 9)$ split into euclidean eight-dimensional coordinates x^i and light-cone coordinates x^+ and x^-, so that a general ten-dimensional form splits as $\mathbb{A}_m dx^m = \mathbb{A}_i dx^i + \mathbb{A}_+ dx^+ + \mathbb{A}_- dx^-$. The Grassmann coordinates θ and ϑ^i are scalar and vector, the latter being identified with the spinorial representation $\mathbf{8}$ of $Spin(7)$. The covariant superspace derivatives are defined as $\nabla \equiv \frac{\partial}{\partial\theta} - \theta\partial_+$ and $\nabla_i \equiv \frac{\partial}{\partial\vartheta^i} - \theta\partial_i - \vartheta_i\partial_-$, with

$$\nabla^2 = -\partial_+, \qquad \{\nabla, \nabla_i\} = -\partial_i, \qquad \nabla_{\{i}\nabla_{j\}} = -\delta_{ij}\partial_- \tag{12}$$

Super-curvatures are defined by the analogue of Eq.(9) for ten dimensions

$$\begin{aligned}(d + d\theta\nabla + \nabla_{d\vartheta} + d\theta i_{(\partial_+ + d\vartheta + |d\vartheta|^2\partial_-)})(\mathbb{A} + \mathbb{C}d\theta + \mathbf{\Gamma}) + (\mathbb{A} + \mathbb{C}d\theta + \mathbf{\Gamma})^2 \\ = \mathbb{F} + \mathbf{\Psi}d\theta + \boldsymbol{\chi} + \mathbf{\Phi}d\theta d\theta + \mathbb{L}d\theta + \bar{\mathbf{\Phi}}\end{aligned} \tag{13}$$

where $\mathbb{F} \equiv \frac{1}{2}\mathbb{F}_{mn}dx^m dx^n, \mathbf{\Psi} \equiv \mathbf{\Psi}_m dx^m, \boldsymbol{\chi} \equiv \boldsymbol{\chi}_{in} d\vartheta^i dx^n, \mathbb{L} \equiv \mathbb{L}_i d\vartheta^i$ and $\bar{\mathbf{\Phi}} \equiv \bar{\mathbf{\Phi}}_{ij} d\vartheta^i d\vartheta^j$. One has in particular[1]

$$\mathbf{\Phi} \equiv \hat{\nabla}^2 + \hat{\partial}_+, \qquad \mathbb{L}_i \equiv \{\hat{\nabla}, \hat{\nabla}_i\} + \hat{\partial}_i, \qquad \bar{\mathbf{\Phi}}_{ij} \equiv \hat{\nabla}_{\{i}\hat{\nabla}_{j\}} + \delta_{ij}\hat{\partial}_-. \tag{14}$$

[1]We have analogous notations $\mathbf{\Phi}$ and $\bar{\mathbf{\Phi}}_{\alpha\beta}$ for the curvatures of the different $\mathcal{N} = 1$ and $\mathcal{N} = 2$ cases, in six (respectively four) dimensions $(\alpha, \beta \hat{=} \mu, \nu)$, and ten (respectively eight) dimensions $(\alpha, \beta \hat{=} i, j)$. However, after dimensional reduction and once the constraints $\mathbf{\Phi}^{\mathcal{N}=1} = \bar{\mathbf{\Phi}}_{ij}^{\mathcal{N}=1} = 0$ are imposed, we have the correspondence $\mathbb{A}_+ \to \mathbf{\Phi}^{\mathcal{N}=2}$ and $\mathbb{A}_- \to \bar{\mathbf{\Phi}}^{\mathcal{N}=2}$.

3 Constraints and their resolution

3.1 The $\mathcal{N}=2$, $d=4$ case

To eliminate superfluous degrees of freedom and to make contact with the component formulation, we must impose superspace gauge covariant constraints, as follows

$$\mathbb{L}_\mu = 0, \quad \bar{\mathbf{\Phi}}_{\mu\nu} = \frac{1}{4}\delta_{\mu\nu}\bar{\mathbf{\Phi}}_\sigma{}^\sigma \equiv \delta_{\mu\nu}\bar{\mathbf{\Phi}}, \quad \chi_{[\mu\nu]_+} = 0. \tag{15}$$

The super-gauge symmetry defined in Eq.(11) allows us to simplify the resolution of the constraints. We partially fix super-gauge invariance by setting to zero all antisymmetric components $(\frac{\partial}{\partial\vartheta^{[\mu}}\cdots\frac{\partial}{\partial\vartheta^\sigma}\Gamma_{\rho]})|_0$ and $(\frac{\partial}{\partial\theta}\frac{\partial}{\partial\vartheta^{[\mu}}\cdots\frac{\partial}{\partial\vartheta^\sigma}\Gamma_{\rho]})|_0$ of $\mathbf{\Gamma}_\mu$, including $\mathbf{\Gamma}_\mu|_0$, as well as the first component $\mathbb{C}|_0$ of $\mathbb{C}$.[2] In this gauge, the remaining gauge invariance reduces to that of the component formalism ($\alpha = \alpha|_0$). The details of the procedure will be found in [5]. After solving the constraints in this particular super-gauge, we will reintroduce the super-gauge invariance by a general gauge transformation depending on new fields that stand for the longitudinal components.

We start with $\mathbf{\Gamma}_\mu$. The constraint Eq.(15) on $\bar{\mathbf{\Phi}}_{\mu\nu}$ and its Bianchi identity leave its ϑ^μ independent trace components unconstrained. We define them as $\bar{\mathbf{\Phi}}|_0 \equiv \bar{\Phi}$ and $(\frac{\partial}{\partial\theta}\bar{\mathbf{\Phi}})|_0 \equiv \eta$. Using the definition of $\bar{\mathbf{\Phi}}_{\mu\nu}$ in terms of $\mathbf{\Gamma}_\mu$ and its Bianchi identity, we then obtain

$$\mathbf{\Gamma}_\mu = \vartheta_\mu\bar{\Phi} + \theta(\vartheta_\mu\eta + \vartheta_\mu\vartheta^\rho\partial_\rho\bar{\Phi}), \quad \bar{\mathbf{\Phi}} = \bar{\Phi} + \theta(\eta - \vartheta^\mu\partial_\mu\bar{\Phi}) \tag{16}$$

The constraint $\mathbb{L}_\mu = 0$ allows us to express $\mathbb{A}_\mu$ in terms of $\mathbf{\Gamma}_\mu$ and $\mathbb{C}$. It is convenient to parametrize the superfield $\mathbb{C}$ as

$$\mathbb{C} \equiv \tilde{A} + \theta(\tilde{\Phi} - \tilde{A}^2) \quad \rightarrow \quad \mathbf{\Phi} = \tilde{\Phi} + \theta[\tilde{\Phi}, \tilde{A}] \tag{17}$$

where $\tilde{\Phi}$ and $\tilde{A}$ are general functions in ϑ variables, except that $\tilde{A}|_0 = 0$ as it is required by our special gauge choice. Moreover, we define $(\frac{\partial}{\partial\vartheta^\mu}\tilde{A})|_0 \equiv A_\mu$ and $\tilde{\Phi}|_0 \equiv \Phi$. We can then determine $\mathbb{A}_\mu$ as

$$\mathbb{A}_\mu = \frac{\partial}{\partial\vartheta^\mu}\tilde{A} + \cdots - \theta\Big(\frac{\partial}{\partial\vartheta^\mu}\tilde{\Phi} + \cdots\Big) \tag{18}$$

The explicit content of $\tilde{\Phi}$ and $\tilde{A}$ is determined through the resolution of the anti-selfdual constraint on the $\chi_{\mu\nu}$ curvature. We first observe that the Bianchi identities and the constraint $\mathbb{L}_\mu = 0$ imply

$$\chi_{\mu\nu} = -\delta_{\mu\nu}\big(\nabla\bar{\mathbf{\Phi}} + [\mathbb{C}, \bar{\mathbf{\Phi}}]\big) + \chi_{[\mu\nu]} \equiv -\delta_{\mu\nu}\eta + \chi_{[\mu\nu]} \tag{19}$$

[2]We use the standard notation $|_0$ for expressing that all fermionic coordinates are set to zero.

This allows one to express $\boldsymbol{\eta}$ and $\boldsymbol{\chi}_{[\mu\nu]}$ in terms of $\tilde{\Phi}$, $\tilde{A}$ and $\bar{\Phi}$ and η,

$$\begin{aligned}\boldsymbol{\eta} &= \eta + \vartheta^\mu \partial_\mu \bar{\Phi} + [\tilde{A}, \bar{\Phi}] + \cdots \\ \boldsymbol{\chi}_{[\mu\nu]} &= \frac{\partial}{\partial\vartheta^\mu}\frac{\partial}{\partial\vartheta^\nu}\tilde{A} + \cdots + \theta\left(\frac{\partial}{\partial\vartheta^\mu}\frac{\partial}{\partial\vartheta^\nu}\tilde{\Phi} + \cdots\right)\end{aligned} \tag{20}$$

The component $(\frac{\partial}{\partial\vartheta^\mu}\tilde{\Phi})|_0$ is not constrained. We define $(\frac{\partial}{\partial\vartheta^\mu}\tilde{\Phi})|_0 \equiv -\Psi_\mu$ and we solve the constraint $\boldsymbol{\chi}_{[\mu\nu]+} = 0$, component by component. From the θ-independent part, we get

$$\tilde{A} = \vartheta^\mu A_\mu - \frac{1}{2}\vartheta^\mu\vartheta^\nu \chi_{\mu\nu} + \frac{1}{3!}\vartheta^\mu\vartheta^\nu\vartheta^\rho \epsilon_{\mu\nu\rho}{}^\sigma D_\sigma \bar{\Phi} - \frac{1}{4!}\vartheta^\mu\vartheta^\nu\vartheta^\rho\vartheta^\sigma \epsilon_{\mu\nu\rho\sigma}[\bar{\Phi}, \eta] \tag{21}$$

and the part proportional on θ gives us that

$$\begin{aligned}\tilde{\Phi} &= \Phi - \vartheta^\mu \Psi_\mu - \frac{1}{2}\vartheta^\mu\vartheta^\nu(F_{\mu\nu} + G_{\mu\nu}) + \frac{1}{3!}\vartheta^\mu\vartheta^\nu\vartheta^\rho\big(3D_\mu\chi_{\nu\rho} - \epsilon_{\mu\nu\rho}{}^\sigma(D_\sigma\eta - [\bar{\Phi}, \Psi_\sigma])\big) \\ &\quad - \frac{1}{4!}\vartheta^\mu\vartheta^\nu\vartheta^\rho\vartheta^\sigma\left(2\epsilon_{\mu\nu\rho\sigma}D_\lambda D^\lambda \bar{\Phi} - 6\chi_{\mu\nu}\chi_{\rho\sigma} + 2\epsilon_{\mu\nu\rho\sigma}\eta^2 - \epsilon_{\mu\nu\rho\sigma}[\bar{\Phi}, [\bar{\Phi}, \Phi]]\right)\end{aligned} \tag{22}$$

where χ, G are anti-selfdual 2-forms and $F = dA + A^2$. As a result, the general solution of the constrained superfields in the chosen Wess–Zumino-like gauge can be written in term of the known component fields of the theory, with the auxiliary field required for the functional representation of the supersymmetry algebra.

The general solution to the constraints (15) can now be obtained by application of a general super-gauge transformation, which we parametrize as follows[3]

$$e^{\boldsymbol{\alpha}} = e^{\theta\vartheta^\mu\partial_\mu} e^{\tilde{\gamma}} e^{\theta\tilde{c}} = e^{\tilde{\gamma}}\big(1 + \theta(\tilde{c} + e^{-\tilde{\gamma}}\vartheta^\mu\partial_\mu e^{\tilde{\gamma}})\big) \tag{23}$$

where $\tilde{\gamma}$ and $\tilde{c}$ are respectively commuting and anticommuting functions of ϑ^μ and the coordinates x^μ, with the condition $\tilde{\gamma}|_0 = 0$. The superfield connections $\mathbb{C}$, $\boldsymbol{\Gamma}$ and their curvatures then have the following expressions

$$\begin{aligned}\mathbb{C} &= \tilde{c} + e^{-\tilde{\gamma}}\left(\vartheta^\mu\partial_\mu + \tilde{A}\right)e^{\tilde{\gamma}} + \theta\left(e^{-\tilde{\gamma}}\tilde{\Phi}e^{\tilde{\gamma}} - \left(\tilde{c} + e^{-\tilde{\gamma}}\left(\vartheta^\mu\partial_\mu + \tilde{A}\right)e^{\tilde{\gamma}}\right)^2\right) \\ \boldsymbol{\Phi} &= e^{-\tilde{\gamma}}\tilde{\Phi}e^{\tilde{\gamma}} + \theta\left(\left[e^{-\tilde{\gamma}}\tilde{\Phi}e^{\tilde{\gamma}}, \tilde{c}\right] + e^{-\tilde{\gamma}}\left[\tilde{\Phi}, \vartheta^\mu\partial_\mu + \tilde{A}\right]e^{\tilde{\gamma}}\right) \\ \boldsymbol{\Gamma}_\mu &= e^{-\tilde{\gamma}}\left(\frac{\partial}{\partial\vartheta^\mu} + \vartheta_\mu\bar{\Phi}\right)e^{\tilde{\gamma}} + \theta\bigg(e^{-\tilde{\gamma}}\left(\vartheta_\mu\eta + \vartheta_\mu\vartheta^\rho\partial_\rho\bar{\Phi}\right)e^{\tilde{\gamma}} \\ &\qquad - \left[e^{-\tilde{\gamma}}\left(\frac{\partial}{\partial\vartheta^\mu} + \vartheta_\mu\bar{\Phi}\right)e^{\tilde{\gamma}}, \tilde{c} + e^{-\tilde{\gamma}}\vartheta^\mu\partial_\mu e^{\tilde{\gamma}}\right]\bigg) \\ \bar{\boldsymbol{\Phi}} &= e^{-\tilde{\gamma}}\bar{\Phi}e^{\tilde{\gamma}} + \theta\left(e^{-\tilde{\gamma}}\left(\eta - \vartheta^\mu\partial_\mu\bar{\Phi}\right)e^{\tilde{\gamma}} + \left[e^{-\tilde{\gamma}}\bar{\Phi}e^{\tilde{\gamma}}, \tilde{c} + e^{-\tilde{\gamma}}\vartheta^\mu\partial_\mu e^{\tilde{\gamma}}\right]\right)\end{aligned} \tag{24}$$

[3]The gauge transformation is chosen in such a way as to recover the transformation laws computed in components.

and

$$\mathbb{A}_\mu = e^{-\tilde{\gamma}} \left(\partial_\mu + \frac{\partial}{\partial \vartheta^\mu} \tilde{A} - \vartheta_\mu \left(\eta - \vartheta^\nu \partial_\nu \bar{\Phi} - \left[\tilde{A}, \bar{\Phi} \right] \right) \right) e^{\tilde{\gamma}} + \theta(\cdots) \tag{25}$$

One can check that the supersymmetry transformations of the connection superfields reduce in components to the known twisted transformation laws of the $\mathcal{N}$=2 super-Yang–Mills theory in the Wess–Zumino gauge. This is obtained for $\tilde{\gamma} = \tilde{c} = 0$ and redefining the supersymmetry transformations by adding appropriated field-dependent super-gauge transformations such that these fields are left invariant.

3.2 Higher dimensions

We now consider the $\mathcal{N} = 1$, $d = 10$ theory, which also encodes the case $\mathcal{N} = 2$, $d = 8$. The constraints Eq.(15) become

$$\boldsymbol{\Phi} = \mathbb{L}_i = \bar{\boldsymbol{\Phi}}_{ij} = 0, \quad \boldsymbol{\chi}_{ij} - \boldsymbol{\chi}_{ji} + \frac{1}{3} \Omega_{ij}{}^{kl} \boldsymbol{\chi}_{kl} = 0. \tag{26}$$

where Ω_{ijkl} is the octonionic eight-dimensional $Spin(7)$-invariant 4-form [2]. Proceeding along the same line as for the resolution of the constraints in four dimensions, we get the gauge-fixed solution (once again we refer the reader to [5] for more details)

$$\mathbb{A}_- = A_- + \theta(\eta - \vartheta^i \partial_i A_-) \tag{27}$$

which gives the solution to $\nabla_{d\vartheta} \boldsymbol{\Gamma} + \boldsymbol{\Gamma}^2 = -|d\vartheta|^2 \mathbb{A}_-$ as $\boldsymbol{\Gamma}_i = -\vartheta_i \mathbb{A}_-$. Then, by introducing the functions $\tilde{A}$ and $\tilde{A}_+$ of ϑ^i to parametrize $\mathbb{C}$, and by using the constraints $\boldsymbol{\Phi} = \mathbb{L}_i = 0$ and the Bianchi identities, one can write $\mathbb{A}_+$, $\mathbb{A}_i$ and $\boldsymbol{\chi}_{ij}$ in terms of $\mathbb{C}$ and $\boldsymbol{\Gamma}_i$. Eventually, the anti-selfdual constraint on $\boldsymbol{\chi}_{[ij]}$ permits one to completely determine the component field content of each superfield. The expansion of $\tilde{A}$ and $\tilde{A}_+$ is in fact

$$\begin{aligned} \tilde{A} &= \vartheta^i A_i - \frac{1}{2} \vartheta^i \vartheta^j \chi_{ij} - \frac{1}{3!} \vartheta^i \vartheta^j \vartheta^k \Omega_{ijk}{}^l F_{l-} + \cdots , \\ \tilde{A}_+ &= A_+ - \vartheta^i \Psi_i - \frac{1}{2} \vartheta^i \vartheta^j (F_{ij} + G_{ij}) + \cdots . \end{aligned} \tag{28}$$

By introducing the fields $\tilde{c}$ and $\tilde{\gamma}$, one can reenforce the super-gauge invariance and get the following expression for the ten-dimensional superfield $\mathbb{C}$

$$\mathbb{C} = \tilde{c} - e^{-\tilde{\gamma}} (\vartheta^i \partial_i + \tilde{A}) e^{\tilde{\gamma}} - \theta e^{-\tilde{\gamma}} (\partial_+ + \tilde{A}_+) e^{\tilde{\gamma}} - \theta (\tilde{c} - e^{-\tilde{\gamma}} (\vartheta^i \partial_i + \tilde{A}) e^{\tilde{\gamma}})^2. \tag{29}$$

The supersymmetry transformation laws of the ten-dimensional super-Yang–Mills in components in the Wess–Zumino gauge [2] are then recovered in an analogous way as in the four dimensional case.

4 Action in superspace

4.1 The gauge invariant part

The action for $\mathcal{N} = 2$, $d = 4$ super-Yang–Mills can be written as an integral over the twisted superspace in terms of superfields, with the above covariant constraints, which can be imposed by mean of Lagrange multipliers.

To get its equivariant part, we observe from the Bianchi identity $\nabla\mathbf{\Phi} + [\mathbb{C}, \mathbf{\Phi}] = 0$ that the gauge invariant function $\mathrm{Tr}\,\mathbf{\Phi}^2$ is θ independent. Therefore, its components of highest order in ϑ^μ can be used to write the equivariant part of the action. The latter can be expressed as a full superspace integral of a Chern–Simons-like term

$$\mathcal{S}_{EQ} = \int d^4\vartheta \,\mathrm{Tr}\, \mathbf{\Phi}^2 = \int d^4\vartheta \, d\theta \,\mathrm{Tr} \left(\mathbb{C} \nabla \mathbb{C} + \frac{2}{3}\mathbb{C}^3 \right) \tag{30}$$

One can check that this action reproduces the known action for super-Yang–Mills in components. Notice that the superfield $\mathbb{C}$ has a positive canonical dimension, which is an interesting point for its renormalization properties.

Unfortunately this formula does not generalize to higher dimensions. However, the one-loop invariant counter-terms involved in the eight-dimensional theory can be expressed as simple integrals over superspace

$$\int d^8\vartheta \,\mathrm{Tr}\, \mathbf{\Phi}^4 \qquad \int d^8\vartheta \,\mathrm{Tr}\, \mathbf{\Phi}^2 \,\mathrm{Tr}\, \mathbf{\Phi}^2 \tag{31}$$

The constraints can be covariantly implemented by the following superspace integral depending on auxiliary Lagrange multipliers superfields

$$\begin{aligned}
\mathcal{S}_C &= \int d^4\vartheta \, d\theta \,\mathrm{Tr} \left(\mathbb{B}^{(\mu\nu)} \bar{\mathbf{\Phi}}_{\mu\nu} + \bar{\mathbf{\Psi}}^{[\mu\nu]+} \boldsymbol{\chi}_{\mu\nu} + \bar{\mathbb{K}}^\mu \mathbb{L}_\mu \right) \\
&= \int d^4\vartheta \, d\theta \,\mathrm{Tr} \Big(\mathbb{B}^{(\mu\nu)} \big(\nabla_\mu \mathbf{\Gamma}_\nu + \mathbf{\Gamma}_\mu \mathbf{\Gamma}_\nu \big) + \bar{\mathbf{\Psi}}^{[\mu\nu]+} \big(\partial_\mu \mathbf{\Gamma}_\nu + \nabla_\mu \mathbb{A}_\nu + [\mathbb{A}_\mu, \mathbf{\Gamma}_\nu] \big) \\
&\qquad\qquad + \bar{\mathbb{K}}^\mu \big(\nabla \mathbf{\Gamma}_\mu + \nabla_\mu \mathbb{C} + \{\mathbf{\Gamma}_\mu, \mathbb{C}\} + \mathbb{A}_\mu \big) \Big)
\end{aligned} \tag{32}$$

where $\mathbb{B}^{(\mu\nu)}$ is symmetric traceless and $\bar{\mathbf{\Psi}}^{[\mu\nu]+}$ is antisymmetric selfdual. The superfields $\bar{\mathbb{K}}_\mu$ and $\mathbb{A}_\mu$ can be trivially integrated, giving rise to a simple substitution of $\mathbb{A}_\mu$ by minus $\nabla\mathbf{\Gamma}_\mu + \nabla_\mu\mathbb{C} + \{\mathbf{\Gamma}_\mu, \mathbb{C}\}$. The resolution of the constraints is such that the formal integration over the auxiliary superfields $\mathbb{B}^{(\mu\nu)}$ and $\bar{\mathbf{\Psi}}^{[\mu\nu]+}$ leads to the non-manifestly supersymmetric formulation of the theory in components, without introducing any determinant contribution in the path-integral. However, $\mathbb{B}^{(\mu\nu)}$ and $\bar{\mathbf{\Psi}}^{[\mu\nu]+}$ admit a large class of zero modes that must be considered in the manifestly supersymmetric superspace Feynman rules. They can be summarized by the following invariance of the action

$$\begin{aligned} \delta^{\rm zero}\mathbb{B}^{(\mu\nu)} &= \hat{\nabla}_\sigma\big(\lambda^{(\sigma\mu\nu)} - \frac{1}{3}\hat{\nabla}\varphi^{\sigma(\mu,\nu)}\big) - \hat{\partial}_\sigma\varphi^{\sigma(\mu,\nu)} \\ \delta^{\rm zero}\bar{\Psi}^{[\mu\nu]+} &= \hat{\nabla}_\sigma\varphi^{[\mu\nu]+,\sigma} \end{aligned} \tag{33}$$

where $\lambda^{(\sigma\mu\nu)}$ is a superfield in the rank three symmetric traceless representation and $\varphi^{[\mu\nu]+,\sigma}$ is in the irreducible representation defined by firstly taking the symmetric traceless component in the two last indices and then projecting on the antisymmetric selfdual component on the two first indices. These gauge transformations are themselves invariant by a redefinition of the superfields $\lambda^{(\sigma\mu\nu)}$ and $\varphi^{[\mu\nu]+,\sigma}$ by a gauge transformation involving a superfield in the rank four symmetric traceless representation and another one in the rank four irreducible representation defined by firstly taking the symmetric traceless component in the three last indices and then projecting on the antisymmetric selfdual component on the two first indices. As a matter of fact, the gauge-fixing of this gauge invariance requires the introduction of an infinite set of ghosts including the ghosts for ghosts, the ghosts for ghosts for ghosts and so on.

4.2 The BRST symmetry and the gauge-fixing action in superspace.

To fix the super-gauge invariance, one first introduces a Fadeev–Popov ghost superfield Ω and a BRST differential s that anticommutes with Δ. As indicated by the super-gauge transformations (11) and their infinitesimal version, the BRST symmetry is defined as

$$s\,\mathcal{A} = -\Delta\Omega - [\mathcal{A},\Omega], \qquad s\,\mathcal{F} = -[\Omega,\mathcal{F}], \qquad s\,\Omega = -\Omega^2, \tag{34}$$

One also needs a Fadeev–Popov antighost superfield $\bar{\Omega}$ and its Lagrange multiplier superfield $\mathbb{B}$. In fact, the BRST transformation laws of the super-connection, super-ghost and super-antighost follow from the following generalization of the horizontality equation Eq.(9), which involves both the anti-BRST operator $\bar{s}$ and the BRST operator s

$$(\Delta + d\theta\, i_{d\vartheta} + s + \bar{s})(\mathcal{A}+\Omega+\bar{\Omega}) + (\mathcal{A}+\Omega+\bar{\Omega})^2 = \mathcal{F}, \tag{35}$$

This equation implies the degenerate equation $s\,\bar{\Omega} + \bar{s}\,\Omega + [\Omega,\bar{\Omega}] = 0$. It is solved by the introduction of the Lagrange multiplier superfield $\mathbb{B}$, so that one gets

$$s\,\bar{\Omega} = \mathbb{B}, \qquad s\,\mathbb{B} = 0, \qquad \bar{s}\,\Omega = -\mathbb{B} - [\Omega,\bar{\Omega}] \tag{36}$$

A fully invariant gauge-fixing action can then be written as

$$
\begin{aligned}
\mathcal{S}_{GF} = s\ \bar{s} \int d^4\vartheta\, d\theta\, \mathrm{Tr}\, \Big(\mathbb{A}_\mu \mathbb{A}^\mu\Big) &= s \int d^4\vartheta\, d\theta\, \mathrm{Tr}\, \Big(\bar{\mathbf{\Omega}}\, \partial^\mu \mathbb{A}_\mu\Big) \\
&= \int d^4\vartheta\, d\theta\, \mathrm{Tr}\, \Big(-\mathbb{B}\partial^\mu \mathbb{A}_\mu + \bar{\mathbf{\Omega}}\partial^\mu \hat{\partial}_\mu \mathbf{\Omega}\Big)
\end{aligned} \tag{37}
$$

We will display in [5] the precise component fields content of the above superfields. Going down to the components formalism and considering only Green functions with no external legs of the additional fields, we can integrate them out from closed loops by formal gaussian integrations that compensate each other. These formal integrations can be shown to be rigourously exact by the use of antighost like Ward identities. Eventually, no interactions occurring from the gauge-fixing contribute except for the b, A_μ and $\bar{\Omega}, \Omega$ terms, and Eq.(37) shrinks in component formalism to the usual gauge-fixing term in components, involving the Fadeev–Popov ghost, the anti-ghost and its Lagrange multiplier

$$
\mathcal{S}_{GF} \approx \int d^4x\ \mathrm{Tr}\ \Big(-b\, \partial^\mu A_\mu + \bar{\Omega}\, \partial^\mu D_\mu \Omega\Big) \tag{38}
$$

where $b \equiv \frac{1}{24}\varepsilon^{\mu\nu\sigma\rho}(\frac{\partial}{\partial\theta}\frac{\partial}{\partial\vartheta^\mu}\frac{\partial}{\partial\vartheta^\nu}\frac{\partial}{\partial\vartheta^\sigma}\frac{\partial}{\partial\vartheta^\rho}\mathbb{B})|_0$, $\bar{\Omega} \equiv \frac{1}{24}\varepsilon^{\mu\nu\sigma\rho}(\frac{\partial}{\partial\theta}\frac{\partial}{\partial\vartheta^\mu}\frac{\partial}{\partial\vartheta^\nu}\frac{\partial}{\partial\vartheta^\sigma}\frac{\partial}{\partial\vartheta^\rho}\bar{\mathbf{\Omega}})|_0$ and $\Omega \equiv \mathbf{\Omega}|_0$.

One has also to write a gauge-fixing action for the action of constraints. The gauge invariance (33) can be written in terms of the BRST operator, thanks to the introduction of the ghosts $\bar{\mathbf{\Psi}}^{(1,0)\mu\nu,\sigma}$ and $\mathbb{B}^{(1,0)\mu\nu\sigma}$. As discussed in the previous section, the BRST transformations are themselves subject to a gauge invariance and one has to introduce an infinite tower of ghosts for ghosts to correctly gauge-fix the theory. We define the commuting ghosts $\bar{\mathbf{\Psi}}^{(n,0)\mu\nu,\cdots}$ in the rank $n+2$ irreducible representation obtained by applying the symmetric traceless projector on the $n+1$ last indices and then the antisymmetric selfdual projector to the two first indices, as well as the anticommuting ghost $\mathbb{B}^{(n,0)\mu\nu\cdots}$ in the rank $n+2$ symmetric traceless representation. The BRST transformations are the following

$$
\begin{aligned}
s\, \bar{\mathbf{\Psi}}^{(n,0)\mu\nu,\cdots} &= \hat{\nabla}_\sigma \bar{\mathbf{\Psi}}^{(n+1,0)\mu\nu,\cdots\sigma} - [\mathbf{\Omega}, \bar{\mathbf{\Psi}}^{(n,0)\mu\nu,\cdots}] \\
s\, \mathbb{B}^{(n,0)\mu\nu\cdots} &= \hat{\nabla}_\sigma \Big(\mathbb{B}^{(n+1,0)\mu\nu\cdots\sigma} - \tfrac{1}{n+3}\hat{\nabla}\bar{\mathbf{\Psi}}^{(n+1,0)\sigma(\mu,\nu,\cdots)}\Big) - \hat{\partial}_\sigma \bar{\mathbf{\Psi}}^{(n+1,0)\sigma(\mu\,\nu,\cdots)} - \{\mathbf{\Omega}, \mathbb{B}^{(n,0)\mu\nu\cdots}\} \\
s\, \bar{\mathbb{K}}^\mu &= \hat{\nabla}_\nu \hat{\nabla}_\mu \bar{\mathbf{\Psi}}^{(1,0)\mu\nu,\sigma} - \{\mathbf{\Omega}, \mathbb{K}^\mu\}
\end{aligned} \tag{39}
$$

where $\bar{\mathbf{\Psi}}^{(0,0)\mu\nu}$ and $\mathbb{B}^{(0,0)\mu\nu}$ are simply $\bar{\mathbf{\Psi}}^{\mu\nu}$ and $\mathbb{B}^{\mu\nu}$. The BRST operator is nilpotent modulo the constraints, that is modulo the equations of motion of the fields $\bar{\mathbf{\Psi}}^{\mu\nu}$, $\mathbb{B}^{\mu\nu}$ and

$\bar{\mathbb{K}}^\mu$. The Batalin–Vilkovisky formalism permits one to solve this problem, by introducing antifields as sources for the BRST transformations.[4]

We have not yet worked out the gauge-fixing of this BF system. Even if it shares similarities with a standard bosonic BF model, the choice of gauge-functions cannot be defined by naively replacing the space derivative of the bosonic case by the anticommuting vector covariant derivative ∇_μ. It seems that the free case can be worked out, by introducing transverse projectors for the auxiliary fields, but more work is yet required for a complete procedure. It will be described in the forthcoming publication [5], as well as a practical way for doing computations that takes into account the existence of the infinite tower of ghosts in loops.

Despite our present ignorance of the gauge-fixing of the BF system that enforces the covariant constraints, we thus propose as a defining superspace action the following integral over the twisted superspace

$$\mathcal{S} = \mathcal{S}_{EQ} + \mathcal{S}_C + \mathcal{S}_{GF} + \mathcal{S}_{CGF} \tag{40}$$

The four-dimensional expressions (32) and (37) of $\mathcal{S}_C$ and $\mathcal{S}_{GF}$ can be extended to eight and ten dimensions. It is not clear however if these expressions are relevant in higher dimensions, where the introduction of a prepotential is required in order to write the equivariant part of the action.

5 Conclusion

By using twisted variables, one can reexpress the $\mathcal{N}=2$ supersymmetry algebra in such a way that the theory is determined by a subalgebra of the super-Poincaré algebra. We have seen the existence of a corresponding twisted superspace, with coordinates $(x^\mu, \theta, \vartheta^\mu)$. The result generalizes in higher dimensions. Quite interestingly, the constraints on the super-curvatures are such that they donnot imply the equations of motion. This property makes it plausible that one can obtain a superspace path-integral formulation of maximally supersymmetric theories. We have shown in this publication that a twisted superspace path-integral formulation of the $\mathcal{N}=2$ super-Yang–Mills theory does exist in four dimensions.

This theory is formulated as a Chern–Simons term for the classical action plus a BF term for expressing the covariant constraints in superspace. Despite the fact that

[4]However, we have not yet determined the rank of the system, that is the maximal order at which the antifields have to appear in the action.

the gauge-fixing of the BF part requires the introduction of an infinite tower of ghosts and ghosts for ghosts, we hope that it will exhibit a general structure for a compact resumation of the ghost contributions. Here, we have solved explicitly the constraints and verified that the theory reduces to the usual Yang–Mills theory in components, after integration of the superspace longitudinal components of the super-gauge fields and their corresponding Faddeev–Popov ghosts.

These superspace longitudinal components and the corresponding ghost fields will be interpreted in a forthcoming more detailed publication [5] as shadow fields that are required to write a manifestly supersymmetric gauge-fixing action in components, without introducing supersymmetry gauge parameters. As a matter of fact, the direct introduction of these shadows in component formalism was our starting point for the study of the full superfields content. These properties make the twisted superspace formulation of the theory very close to that in components.

We will also define in [5] the superspace path-integral and its Feynman rules, which implies a BRST invariant breaking of the gauge invariance of the Lagrange multipliers fields for the constraints and their infinite tower of ghosts, with suitable choices of gauge functions. The definition of transverse projectors in superspace will permit one to define perturbation theory in the free abelian case. The construction of the gauge-fixing functions from the knowledge of these projectors will then permit a well-defined gauge-fixing procedure in the non-abelian case.

Finally, it must be understood that the construction of a twisted superspace for the $\mathcal{N} = 2$ supersymmetric theory is not an attempt for an alternative to its harmonic superspace formulation. Rather, it is a preliminary construction, as an example of a non-manifestly Lorentz invariant superspace-path-integral that can be generalized in ten dimensions, but must be completed within an harmonic superspace path-integral formulation for a complete description of the ten-dimensional super-Yang–Mills theory. Eventually, one expects the full Lorentz invariance to be recovered for the on-shell amplitudes.

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References

[1] A. Galperin, E. Ivanov, S. Kalitsyn, V. Ogievetsky and E. Sokatchev, "Unconstrained off-shell $\mathcal{N} = 3$ supersymmetric Yang–Mills theory," Class. Quant. Grav. **2** (1985) 155.

[2] L. Baulieu, N. Berkovits, G. Bossard and A. Martin, "Ten-dimensional super-Yang–Mills with nine off-shell supersymmetries," Phys. Lett. B **658** (2008) 249, `0705.2002 [hep-th]`.

[3] A. S. Galperin, E. A. Ivanov, V. I. Ogievetsky and E. S. Sokatchev, "Harmonic Superspace," Cambridge University Press (2001)

[4] L. Baulieu, G. Bossard and S. P. Sorella, "Shadow fields and local supersymmetric gauges," Nucl. Phys. B **753** (2006) 273, `[hep-th/0603248]`.

[5] L. Baulieu, G. Bossard and A. Martin, in preparation.

[6] M. Alvarez and J.M.F. Labastida, " Topological matter in four-dimensions", Nucl. Phys. **B437** (1995) 356, [`hep-th/9404115`].

[7] R. Grimm, M. Sohnius, J. Wess, " Extended supersymmetry and gauge theories," Nucl. Phys. **B133** (1978) 275.

[8] M. Sohnius, " Bianchi identities for supersymmetric gauge theories," Nucl. Phys. **B136** (1978) 461.

[9] J. Kato, N. Kawamoto, A. Miyake, " $\mathcal{N} = 4$ twisted superspace from Dirac–Kähler twist and off-shell susy invariant actions in four dimensions," Nucl. Phys. **B721** (2005) 229, [`hep-th/0502119`].

[10] J. Kato, A. Miyake, " Topological hypermultiplet on $\mathcal{N} = 2$ twisted superspace in four dimensions," Mod. Phys. Lett. **A21** (2006) 2569, [`hep-th/0512269`].

[11] F. Fucito, A. Tanzini, L.C.Q. Vilar, O.S. Ventura, C.A.G. Sasaki, S.P. Sorella, "Algebraic renormalization: Perturbative twisted considerations on topological Yang–Mills theory and on $\mathcal{N} = 2$ supersymmetric gauge theories," Lectures given at 1st School on Field Theory and Gravitation, Vitoria, Brazil, 15-19 Apr 1997, `[hep-th/9707209]`.