On on-shell closure of maximal superconformal symmetry and its application

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Theories with the maximal supersymmetry play important roles in the present day string/M theory. It is believed that the maximal supersymmetry implies powerful non-renormalisation theorems. However, the consequence of the maximal supersymmetry is difficult to explore. This is in marked contrast to theories with a smaller supersymmetry such as the $D = 4$, $\mathcal{N} = 1$ supersymmetry, where one can use superfield techniques to prove non-renormalisation theorems.

This difference between the maximally supersymmetric theories and theories with a smaller supersymmetry can be traced back to the following basic property of supersymmetric theories: when one computes the (anti-)commutator of the action of the supersymmetry on (physical) fields, one obtains terms which are proportional to the equation of motion. This is called as the on-shell closure of supersymmetry. For systems with a smaller supersymmetry, one can introduce auxiliary fields, such that the (anti-)commutator closes without using the equation of motion, thereby achieving the "off-shell closure". These auxiliary fields are essential to formulate the theories in terms of the (off-shell) superfield, which in turn enables one to prove non-renormalisation theorems. However, as number of supersymmetries increases, it becomes more difficult to find auxiliary fields achieving the off-shell closure, and in particular for the maximal supersymmetry, it has not been possible to find them.

It therefore seems worthwhile to seek for an alternative approach to deal with non-renormalisation of supersymmetric theories which does not rely on the use of the auxiliary fields. In the talk, I described an ongoing work with Y. Kazama (University of Tokyo) in this direction. Our idea is to look for a direct connection between the on-shell closure and non-renormalisation theorems: instead of viewing the on-shell closure as an undesirable property preventing the introduction of superfields, we try to use the on-shell closure to prove non-renormalisation theorems.

To achieve this, it is necessary to have the explicit form of the term in the (anti-)commutator which is proportional to the equation of motion. Surprisingly, this has not been appeared in the literature, for the important case of maximal superconformal symmetry of $D = 4$, $\mathcal{N} = 4$ Super Yang-Mills. In the talk, I outlined our calculation of the anti-commutator. The next step is to invent a method to exploit the information obtained from this calculation (namely, the tensor coefficients multiplying the equation of motion appearing in the anti-commutator). We believe that there is a new class of Ward identities corresponding to symmetries with the on-shell closure. In the talk, I showed this new Ward identities for a simplified case. Although the power of these Ward identities still remains to be uncovered, we hope that our work will lead to a new approach for studying properties of supersymmetric theories.