

# Elucidating the Quantum Dynamics of Intramolecular Double Hydrogen Transfer in Porphycene

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## Abstract

The making and breaking of H-bonds on highly anharmonic potential energy surfaces (PES) in proton and hydrogen transfer reactions require a full-dimensional quantum mechanical treatment of not only electrons, but also of nuclei. Here we demonstrate that dealing with this complexity is necessary for achieving predictive simulations that can solve puzzling properties of these reactions by addressing the intramolecular double hydrogen transfer (DHT) in porphycene. We combine high level density-functional theory calculations and full-dimensional path-integral ring polymer methods. Our quantum dynamics simulations show that although below 100 K the concerted tunneling pathway dominates, between 100 K and 300 K there is a competition between concerted and stepwise pathways when nuclear quantum effects are included. We obtain reaction rates

in excellent agreement with experiment, thus confirming our determination of the tunneling pathways and of the anharmonic mode couplings that play a role in this reaction. We also reproduce the puzzling and controversial N-H stretching band of porphycene with remarkable accuracy. We relate the red-shifted position and large width of this peak with the strengthening of the hydrogen bonds due to the delocalization of the hydrogens within the cage, highlighting the role of NQEs in vibrational fingerprints. These results verify the appropriateness of our general theoretical approach and provide a framework for a deeper physical understanding of hydrogen transfer dynamics in complex systems.

## Introduction

Hydrogen and proton transfer reactions are involved in a myriad of chemical and biological processes.<sup>1</sup> What has hindered a detailed understanding and control over these processes is the lack of a quantitative description of the dynamics involved in these reactions. The inherent difficulty to simulate such processes arises from the need to describe the breaking and forming of hydrogen bonds in addition to the hydrogen transfer. In these processes, electronic polarization and anharmonicities of the potential energy surface (PES) cause a considerable coupling among different degrees of freedom in the system, leading to complex vibrational energy transfers with the intra- or inter-molecular environment.<sup>2</sup> In addition, due to the light mass of hydrogen, nuclear quantum effects (NQEs) such as tunneling and zero-point energy (ZPE) play a crucial role in hydrogen bonding and transfer reactions at remarkably high temperatures,<sup>3-7</sup> and even heavy atoms (e.g., carbon or oxygen) can impact the tunneling process.<sup>8,9</sup> Therefore, theoretical approaches based on empirical potentials and/or a dimensionality reduction of the PES, as well as a classical treatment of nuclear dynamics are inadequate to describe hydrogen transfer reactions. Instead, an all-atom, all-electron quantum simulation is required.

Intramolecular hydrogen transfer reactions, in particular the tautomerization of por-

phyrin derivatives, has been extensively investigated as a prototype of double hydrogen transfer (DHT) reactions.<sup>10</sup> Porphycene, the first synthesized structural isomer of free-base porphyrin,<sup>11</sup> has emerged as a unique example of DHT in a multidimensional anharmonic PES, where strong hydrogen bonds are formed in the molecular cavity.<sup>10,12</sup> The presence of the hydrogen bonds results in a low reaction barrier and a high DHT rate ( $\approx 10^{11} \text{ s}^{-1}$ )<sup>13</sup> compared to other derivatives, e.g. porphine ( $\approx 10^4 \text{ s}^{-1}$ ).<sup>14</sup> The DHT reaction, which in this case connects the two degenerate *trans* tautomeric states, can occur either through a stepwise or concerted mechanism, as schematically shown in Fig. 1. In the former case, the hydrogen atoms are transferred sequentially and the reaction pathway involves an intermediate *cis* tautomeric state and happens through a first-order saddle point. In the latter case, two hydrogen atoms are transferred simultaneously through a second-order saddle point. At cryogenic temperatures in the gas-phase (or in a helium droplet), the vibrational levels exhibit a tunneling splitting of  $4.4 \text{ cm}^{-1}$ ,<sup>15-17</sup> indicating that the *trans-trans* tautomerization takes place via coherent tunneling through the concerted mechanism.<sup>18,19</sup> However, at higher temperatures and/or in condensed phases, the coupling to the environment leads to quick decoherence of the wave-function and the quantum state decays exponentially, such that the reaction dynamics can be described as a rate process.<sup>20</sup> In these cases, which are more relevant for biological or technological applications, it is not straightforward to clarify which mechanism governs the DHT reaction, how temperature affects the reaction rates, and the character of the vibrational modes that play a role.

Vibrational spectroscopy is a powerful tool that can shed light on nuclear dynamics and the PES. The hydrogen stretching mode is an important spectroscopic probe of hydrogen bonding and transfer. Its vibrational band in hydrogen bonding systems exhibits extremely complex spectral features, such as a considerable broadening and emergence of satellite vibrations. In porphycene, the characterization of the N-H stretching mode ( $\nu_{\text{N-H}}$ ) related to the *trans-trans* tautomerization has been particularly controversial, because of its apparent absence in the experimental Raman and IR spectra, even if theoretical calculations in the

harmonic approximation predict a very intense band.<sup>21</sup> This suggests that a strongly anharmonic PES, intermode couplings, and NQEs play a decisive role in the static and dynamic properties of the inner hydrogen atoms of porphycene. Although the broadening of  $\nu_{\text{N-H}}$  in porphycene resulting from the intermode coupling was examined by a classical nuclear dynamics approach,<sup>21</sup> the agreement between simulations and experiment could not be fully achieved.

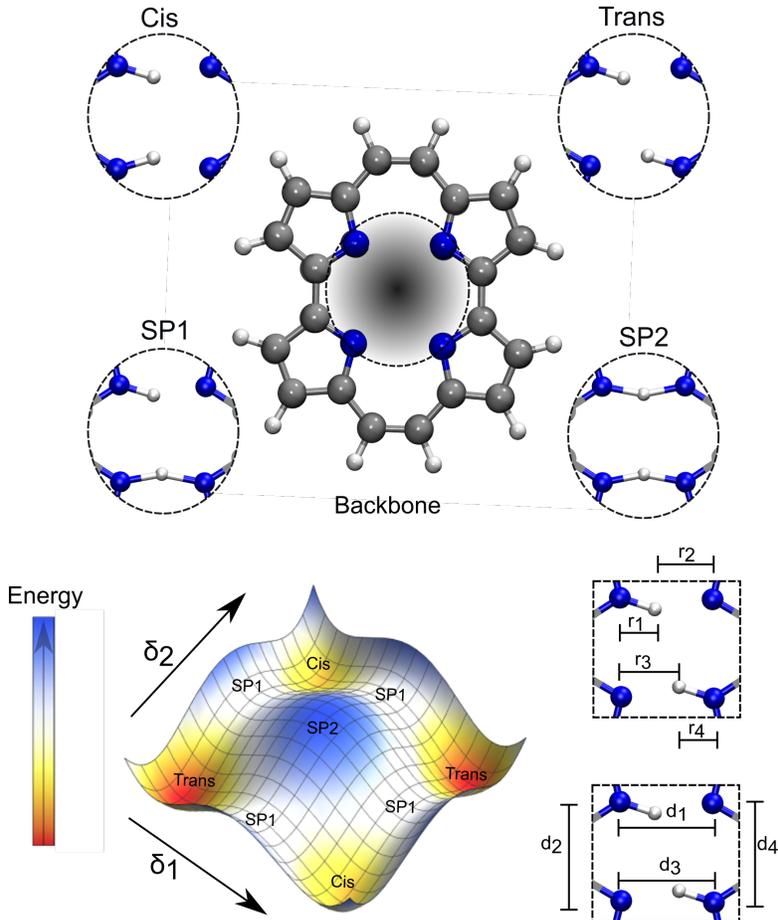


Figure 1: Characterization of the *cis* and *trans* isomers as well as a schematic PES of the porphycene molecule. SP1 and SP2 stand for first order and second order saddle points and  $r$  and  $d$  are distances between atoms as marked in the figure. The hydrogen transfer coordinates are given by  $\delta_1 = r_1 - r_2$  and  $\delta_2 = r_3 - r_4$ .

The situation outlined above has hampered the elucidation of the hydrogen-bonding structure and tautomerization dynamics in porphycene for a long time. Many studies re-

lying on a dimensionality-reduction of the PES have been performed,<sup>19,22-24</sup> but its full-dimensional quantum dynamics still remains elusive. Here we demonstrate that all-atom and all-electron quantum simulations can fully reveal the equilibrium and dynamical properties of this molecule in the gas phase and explain a range of experimental observations. We assess in detail the open questions regarding the hydrogen bonding geometry, the hydrogen transfer dynamics, and the nature of the  $\nu_{\text{N-H}}$  band observed in the IR spectrum. We show that the hydrogen transfer mechanism in porphycene follows non-intuitive pathways at given temperatures, which underlines the decisive role of NQEs. The quantitative agreement with experiment highlights the reliability and generality of our theoretical approach, providing a reliable path for the study of hydrogen transfer reactions and vibrational dynamics in hydrogen-bonding systems.

## Results and discussion

We first analyze the equilibrium properties of the porphycene molecule obtained from density-functional theory (DFT-B3LYP+vdW) calculations and *ab initio* path-integral molecular dynamics (PIMD) simulations (see Methods). In Fig. 2a we show the free energy profile projected on the two hydrogen transfer coordinates  $\delta_1$  and  $\delta_2$  defined in Fig. 1 for the PIMD simulation at 290 K. As expected, the *trans* isomer is the most energetically stable and the free energy barrier for hydrogen transfer is around  $2k_B T$ , in agreement with previous studies where NQEs were also included.<sup>25,26</sup> For comparison, in the classical nuclei *ab initio* molecular dynamics (MD) simulation at the same temperature, the hydrogen transfer reaction is a rare event with an effective barrier above  $4k_B T$ , as shown in the SI.

The stationary points of the free-energy surface can be assigned to the *cis*, *trans*, first order (SP1) and second order saddle point (SP2) configurations of the molecule. While the exact boundary value to distinguish the different states is arbitrary, the *cis/trans* population ratio is quite robust for reasonable values of  $0.1 < \delta_{1,2} < 0.3$ . We found that the *trans*

population is larger than the *cis* population by a factor 7. Correcting for the fact that the B3LYP functional underestimates the energy difference between *trans* and *cis* by a factor of 0.58 compared to CCSD(T), this factor can be revised to almost 30, which corresponds to a total *cis* population of only  $\sim 3\%$ .

The 1D PIMD free-energy projections along the  $q_1$  and  $q_2$  coordinates (defined along the lines  $\delta_1 = -\delta_2$  and  $\delta_1 = \delta_2$ , respectively) is shown in Fig. 2b at 290 K and 100 K. These projections exhibit two interesting features. First, the effective energy difference between the wells, which within a first approximation would determine the *cis* and *trans* populations, does not change between 290 and 100 K. This means that in this temperature range only the lowest vibrational modes involving the NH groups are populated and consequently the relative population is mostly governed by ZPE. The wells along  $q_2$  (*cis*), however, are considerably softened towards the barrier when the temperature is reduced while the wells along  $q_1$  (*trans*) are not. This effectively increases the probability of *cis* isomers at lower temperatures. Second, lowering the temperature has the surprising effect of decreasing the effective free energy barrier by 10%. This effect can be traced back to fluctuations in the cage dimensions. In Fig. 2c we show the probability density of the N-N distances  $d_1$  and  $d_3$  defined in Fig. 1. The quantum distribution at 290 K is broader and shifted to higher values in comparison to the one at 100 K. The cage expansion at higher temperatures leads to an increase in the effective barrier height for the hydrogen transfer reaction and is directly translated to the free energy profile. The expansion/contraction of the cage can be understood from the permanent dipole present in the *cis* local minima, which acts to pull the N atoms together. The *trans* minima, on the other hand, have no permanent dipole due to symmetry and thus a larger cage. The smaller average N-N distances at (PIMD) lower temperatures correlates with an increased amount of quasi-*cis* conformations with respect to *trans*, and is consistent with the strengthening of the hydrogen bonds observed from previous calculations on a reduced PES and NMR experiments.<sup>23</sup> In the classical-nuclei simulations, the *cis*-like geometries are not visited at all, and the cage is much larger.

At this point, it is worth noting that a reduction of the effective free energy barrier along the chosen reaction coordinate does not necessarily result in an *increase* on the hydrogen transfer rate. Within transition state theory (TST), a probabilistic term that depends on the effective barrier height is multiplied by a dynamical correction factor (also called transmission coefficient). It has been shown that the dynamical factor can have a much stronger temperature dependence behaviour than the probabilistic one.<sup>27</sup> Consequently, rates and barrier heights are not always trivially correlated and rate trends obtained based exclusively on free energy barriers should be taken with precaution. In fact, for this system it is observed experimentally that the rates *decrease* with decreasing temperatures<sup>20</sup> and this is also what we find in our calculations shown below.

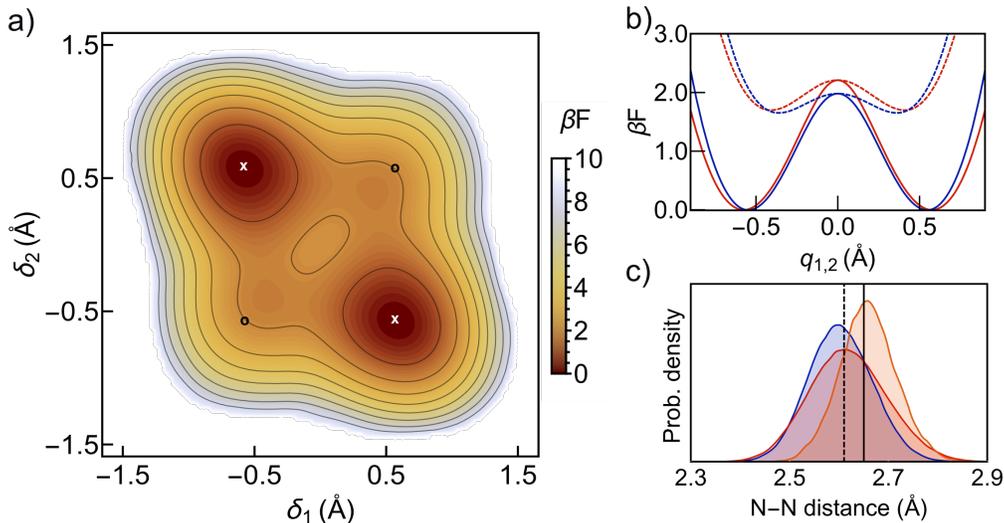


Figure 2: a) Effective free energy profile of the porphycene molecule obtained from PIMD simulations at 290K, projected on the  $\delta_1$  and  $\delta_2$  coordinates. The contour lines are separated by  $1 k_b T$  and the x and o symbols mark the position of the *trans* and *cis* isomers, respectively, when optimized at the potential energy surface. b) Effective free energy projections along  $q_1$  ( $\delta_1 = -\delta_2$ ) (solid lines) and  $q_2$  ( $\delta_1 = \delta_2$ ) (dashed lines) directions. Red and blue lines correspond to PIMD simulations at 290K and 100K respectively. c) Nitrogen-nitrogen distance probability density obtained from PIMD simulations at 100K (blue), PIMD simulations at 290K (red) and MD simulation at 290K (orange). Vertical lines indicate the minimum energy geometry values for the *trans* isomer (solid) and *cis* isomer (dashed).

The results discussed above show how NQEs impact equilibrium properties of porphycene and the importance of an accurate electronic-structure PES. Building up on these results, we

can unravel the quantum dynamics of the hydrogen transfer reactions, which we address with ring-polymer instanton theory<sup>28,29</sup> and thermostatted ring-polymer molecular dynamics.<sup>30</sup>

We examined two instanton pathways, namely, the one related to the *trans-trans* concerted pathway and the one related to the *trans-cis(-trans)* stepwise pathway. With these pathways, we can compute the thermal rate as  $k = 2k_{\text{step}} + k_{\text{conc}}$ ,<sup>22</sup> where  $k_{\text{step}}$  and  $k_{\text{conc}}$  are the rate constants corresponding to the concerted and stepwise mechanisms, respectively. The factor 2 takes into account the two possible *cis* intermediates. The ratios  $R_s = 2k_{\text{step}}/k$  and  $R_c = k_{\text{conc}}/k$  show the relative contribution of each pathway to the total observed DHT rate. Our best estimates are presented in Table 1. These values are in good agreement with the experimental values measured in solution. Direct comparison between our gas-phase calculation and experimental results in solution is supported by the observation of negligible solvent dependency.<sup>31</sup> The tunneling enhancement factor, also reported in Table 1, is defined as  $\kappa_{\text{tun}} = k/k_{\text{TST}}$ , where  $k \equiv k_{\text{inst}}$  is the rate obtained with the instanton method and  $k_{\text{TST}}$  is the rate obtained with the Eyring TST. Tunneling effects thus increase the rates by almost 2 and 3 orders of magnitude at 150 K and 100 K, respectively. More interestingly, at  $T=150$  K the concerted and stepwise mechanisms have a comparable contribution to the total rate, while at  $T=100$  K the concerted path accounts for 95% of the rate. Note that if we neglect NQEs, the scenario is completely different and at both temperatures the stepwise fully dominates.

Table 1: Our calculated DHT rates  $k$ , compared to experimental  $k_{\text{exp}}$  data from Ref.,<sup>20</sup> as well as the tunneling enhancement factor  $\kappa_{\text{tun}}$ , the contribution from the concerted path to the rate  $R_c = k_{\text{conc}}/k$ , and the calculated kinetic isotopic effect  $\text{KIE} = k^{\text{HH}}/k^{\text{DD}}$

	$T$ (K)	$k$ ( $10^{11} \text{s}^{-1}$ )	$k_{\text{exp}}$ ( $10^{11} \text{s}^{-1}$ )	$R_c$	$\kappa_{\text{tun}}$	KIE
HH	150	2.190	$1.62 \pm 0.04$	0.63	70	
HH	100	0.630	$1.09 \pm 0.08$	0.95	1135	
DD	100	0.0025	$0.03 \pm 0.02$	0.59	471	250

This result can be understood in terms of the mechanisms predicted by the instanton pathways. In Fig. 3a and b we show the minimum energy path (MEP) for the concerted *trans-trans* path and the stepwise *trans-cis* path, respectively, at 150 and 100 K. The con-

certed path has a higher barrier than the stepwise one. Analyzing the tunneling paths themselves, we note that the stepwise path requires that the initial *trans* geometry reaches an energy comparable to the intermediate *cis* geometry, which is around 100 meV higher than the minimum, in order for tunneling to take place. From that point, the tunnelling path follows quite closely the MEP and has essentially no temperature dependence between 100 and 150 K. In contrast, in the concerted path, due to symmetry, the tunneling is possible already from the bottom of the well. This creates a competition between the microcanonical tunnelling probability and the thermal population. The first component increases with an energy increase since at a higher point on the PES the tunneling path is shorter. The second component is Boltzmann-like and decreases exponentially with an energy increase. The final instanton geometry results from a subtle balance of these two components and is fulfilled differently at different temperatures. At 100 K the tunnelling path starts at 25 meV above the minimum and reaches points above 350 meV. At 150 K, since more thermal energy is available, the path starts at 64 meV but does not reach such high values, keeping closer to the MEP.

At a low temperature, only the concerted mechanism is possible since there is not enough thermal energy available to reach the high energy conformation at which the stepwise path takes place. The concerted mechanism presents a larger effective path length because two particles need to tunnel and consequently the microcanonical tunnelling probability is always smaller than for the stepwise case (See SI). For this reason, increasing the available thermal energy strongly favors the stepwise mechanism. Indeed above the crossover temperature of this system  $T_c = \hbar\omega^*/2\pi k_B \approx 300$  K, where tunneling can be neglected, the stepwise mechanism is dominant. Our calculated DHT mechanisms at different temperatures are resumed schematically in Fig.3d.

The competition of both mechanisms below  $T_c$  is compatible with previous 2D-model studies.<sup>18</sup> However, it goes against the common experimental interpretation that this reaction happens exclusively through the concerted path at any temperature.<sup>32,33</sup> In particular,

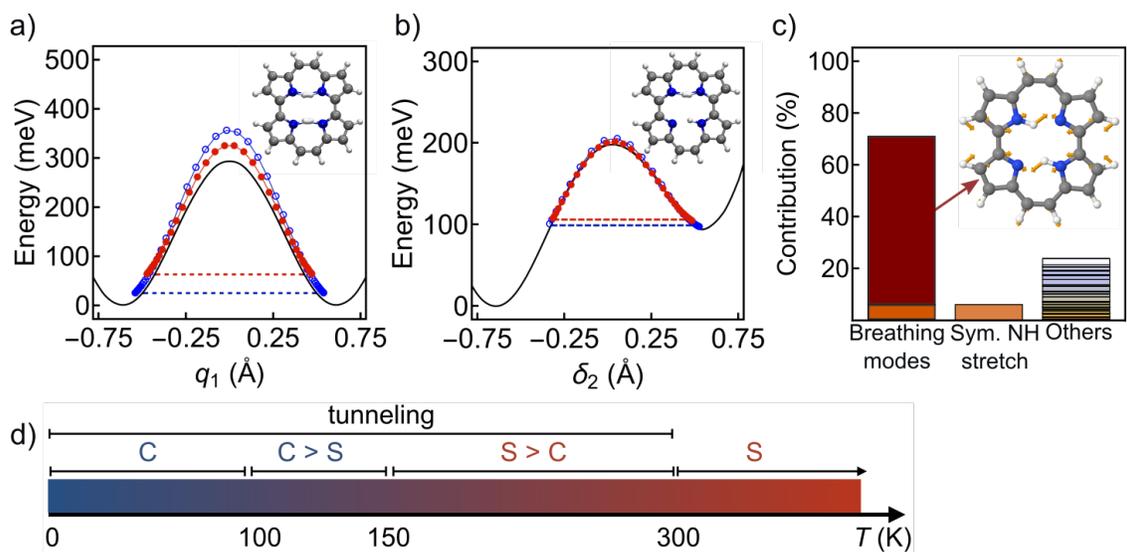


Figure 3: a) Potential energies of different instanton imaginary-time slices along the *trans-trans* path at 100 K (open blue circles) and 150 K (filled red circles). The black energy profile marks the ground state potential energy surface and the dashed lines mark the minimum slice energy. b) Same as in a) for the *trans-cis* path. c) Decomposition of the instanton turning point displacement of the *trans-trans* path into the minimum energy *trans* geometry normal modes. The inset shows the breathing mode at  $197\text{ cm}^{-1}$  ( $2A_g$ ) with largest contribution. d) Competition of different transfer mechanisms at different temperatures: C denotes the concerted mechanism and S the stepwise mechanism.

the lack of *cis* isomers observed in fluorescence anisotropy studies is interpreted as an indirect evidence for the concerted mechanism.<sup>13,31</sup> We propose that this is not a conclusive observation. The energy difference between the two isomers results in a very small *cis* population and due to the low barrier height for the *cis-trans* reaction, any transient *cis* isomer will have a very short lifetime. Therefore, anisotropy measurements performed so far cannot rigorously confirm the mechanism in this particular scenario. Another possible experimental strategy to discern the DHT mechanism is to monitor kinetic isotope effects (KIE) upon substitution of two (DD) internal hydrogens by deuterium.<sup>14</sup> However, due to the lack of well defined  $\nu_{\text{HH}}$  and  $\nu_{\text{DD}}$  bands, it has been difficult to discriminate and quantify the isotopic purity of any sample. In order to provide a reference for experimental results, we report in Table 1 the DHT rates and the KIE for the DD isotopologues of porphycene at 100 K. KIEs from instanton theory benefit from error cancellations<sup>34</sup> that make them typically extremely reliable. Nonetheless, our calculated KIE is 10 times higher than what was reported from experiments.<sup>20</sup> Our result would be consistent with a 2.4% of HH contamination in the sample. This small contamination cannot be quantified in conventional experimental analyses that measure molecular ensembles. Additionally, the DHT mechanism itself changes upon isotopic substitution, with deuteration strongly favoring the stepwise path as reflected by  $R_c$ . This situation is different for porphyrin, where there is no competition between the different pathways already for HH, such that it was possible to confirm the dominance of the stepwise mechanism in these experiments.<sup>14</sup>

Our full dimensional description allows us to go one step further and understand mode-coupling in this DHT in detail. We use the *trans* isomer normal modes to decompose the displacement between the geometry from where the tunneling takes place (the instanton turning point) of the concerted path and the global minimum. This decomposition was also employed in Ref.<sup>35</sup> to analyze mode-specific contributions to tunneling splittings in porphycene, arriving at similar conclusions. Our decomposition, shown in Fig. 3c, shows that more than 65% of the displacement is given by the breathing mode of the molecular

cage at around  $200\text{ cm}^{-1}$  ( $0.56\text{ kcal/mol}$ ). The coupling of the DHT to this low-energy mode implies a temperature-dependence of this reaction rate at low-temperatures that can look like an Arrhenius behavior, despite the massive contribution from tunneling. Indeed, this result is in excellent agreement with the observed temperature dependence of the rate measured experimentally at low temperatures in Refs.<sup>20,33</sup> (consistent with a barrier of  $0.50\text{ kcal/mol}$ ) and their previous suggestion that this mode contributes to the DHT.

The signature of anharmonic intermode coupling and quantum contributions in the DHT is directly reflected in the vibrational fingerprints that appear in an IR spectrum. We compare in Fig. 4 the IR spectra obtained with the quantum harmonic approximation (QH), the (anharmonic) time-correlation formalism with classical-nuclei dynamics (CL) at  $290\text{ K}$ , and the same formalism with quantum-nuclei dynamics (QM) at the same temperature (see Methods). We also present the experimental IR spectrum obtained at  $293\text{ K}$  in KBr from Ref.<sup>21</sup>

Our calculated QH and CL spectra are in very good agreement with previous simulations.<sup>21</sup> As discussed there, (classical-nuclei) anharmonicities play an important role for  $\nu_{\text{NH}}$  at around  $2900\text{ cm}^{-1}$  (easily assignable from normal mode analysis) and coupling to low-frequency vibrations considerably broaden the peak. However, the band is still present and shows a different shape than the experimentally measured spectrum at  $290\text{ K}$  from Ref.<sup>21</sup> With the inclusion of NQEs, this peak gets red-shifted by  $300\text{ cm}^{-1}$  and broadened, producing a very weak signal, remarkably similar to the experimental line shape. We only observe a rigid shift of  $\approx 80\text{ cm}^{-1}$  between our simulations and experiment, which likely originates from the B3LYP+vdW PES. We corroborated the NH-stretch character of this peak by comparing spectra from hydrogenated and deuterated molecules (see SI). The overtone peaks just below  $2000\text{ cm}^{-1}$  that were also observed and discussed in Ref.<sup>21</sup> are present in our CL and QM simulations, albeit with a much lower intensity than the experimental bands.

Our results show that NQEs are essential to explain quantitatively the softening of the  $\nu_{\text{NH}}$  and solve the apparent absence of the  $\nu_{\text{NH}}$  band in the experimental spectrum.<sup>21,36</sup>

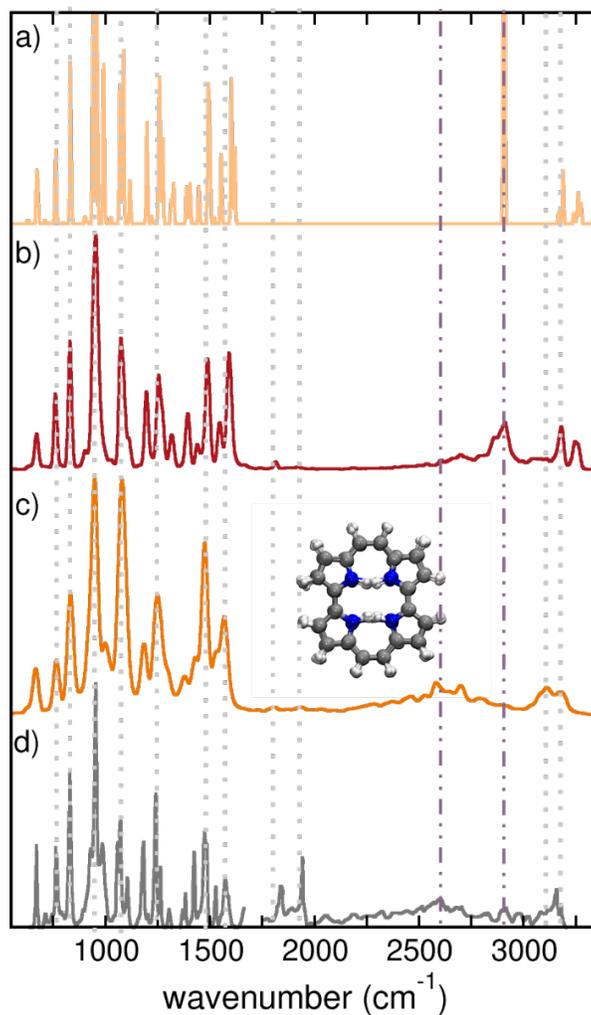


Figure 4: Porphycene IR spectra obtained through the (a) harmonic approximation, (b) Fourier transform of the dipole autocorrelation function from DFT-B3LYP+vdW trajectories with classical nuclei at 290 K, (c) Fourier transform of the dipole autocorrelation function from DFT-B3LYP+vdW trajectories with quantum nuclei at 290 K (thermostatted ring-polymer molecular dynamics. Inset shows superposed snapshots of this simulation. (d) Experimental spectra as reported in Ref.<sup>21</sup> with a rigid blue shift of  $70\text{ cm}^{-1}$  above  $1750\text{ cm}^{-1}$ . The harmonic spectrum was artificially broadened with Gaussian functions for better visualization. Dotted lines serve as guides to the eye and dot-dashed lines mark the positions of the NH stretch peaks.

A model that is capable of reproducing such lineshapes in DHT reactions coupled to low frequency vibrations has been discussed in the literature.<sup>37</sup> The substructure of this band is consistent with Davydov coupling between the two NH-stretch vibrations and an anharmonic coupling to the low-frequency skeletal mode. One can relate this lineshape also to the N-N distance distributions shown in Fig. 2c. For the MD simulations, the distribution is centered at the minimum-energy-geometry value, such that  $\nu_{\text{NH}}$  is broadened but presents no energy shift. On the contrary, the PIMD distribution is shifted to smaller distances and it is considerably broader. The inclusion of NQEs allows the N-H modes to explore higher energy conformations, which in this case are mainly distributed in the direction of the bond. This relates to a strengthening of the H-bond, which is the reason for the mode-softening (red shift), and also a strengthening of the coupling with the backbone modes (broadening). These results underline the importance of properly accounting for NQEs in deriving geometrical rules of thumb for an assessment of the strength of hydrogen-bonds.<sup>38</sup>

## Conclusions

The accurate and quantitative description of hydrogen-bonding structures and hydrogen transfer reactions is related to important properties and functions of solid state organic crystals, e.g. ferroelectricity, conductivity, and magnetism,<sup>39-41</sup> biological systems,<sup>42,43</sup> and hydrogen compounds.<sup>44</sup> The difficulty in understanding and controlling these mechanisms lies in the lack of computational methodologies that are predictive for high dimensional anharmonic systems where the quantum nature of both electronic and nuclear degrees of freedom is important. In this paper we have presented a combination of techniques joining high level density-functional theory simulations and path-integral based approximations to nuclear quantum dynamics that provides a deep physical understanding of these processes. We validated the predictive power of our approach through comparison with experimental data.

In particular, we have shown that in an accurate potential energy surface, NQEs involved in hydrogen transfer events can *qualitatively* change dynamical properties. For the paradigmatic case of the intermolecular DHT in porphycene, we obtain an excellent agreement between our calculated ring-polymer instanton rate and experiment. From our simulations, we are able to reveal how the relative contribution from the stepwise and the concerted DHT mechanisms depends on temperature, presenting similar contributions at 150 K. In addition we showed which skeletal low-frequency vibrational mode mostly couples to the DHT coordinate, leading to an Arrhenius-like temperature dependence for the rate even when the DHT is dominated by tunneling. This coupling and the delocalization of the hydrogens within the cage, mostly in the direction of the (strong) H-bonds also fully explains the apparent absence of the NH stretch band in the IR spectrum of this molecule.

Our results show how NQEs can strengthen NH $\cdots$ N bonds, modify the N-N distances, increase coupling to other modes, and induce peak-shifts of up to 300 cm $^{-1}$  unambiguously confirm their importance in describing these systems and deriving geometrical rules of thumb to assess the strength of H-bonds in more complex systems. We expect that with this methodology, it will be possible to address important problems also in biology, related, for example, to enzymatic reactions through low-barrier H-bonds<sup>42</sup> and unusual fingerprints of NH $\cdots$ N bonds in proteins.<sup>43</sup> The approaches presented here can pave the way for modeling functional materials that take advantage of hydrogen transfer events and will even aid the search for new classes of systems in which quantum dynamics largely contributes to both structure and dynamics.

## Methods

We performed CCSD(T) calculations extrapolated to the infinite basis-set limit to obtain benchmarks for the energetics of local minima and saddle points of porphycene (see more details in SI). We then compared less expensive exchange-correlation (xc) functionals within

density-functional theory (DFT) to these benchmarks, and concluded that the B3LYP functional<sup>45</sup> including pairwise van der Waals corrections (vdW)<sup>46</sup> presented the best agreement with our reference for both energetic and geometrical properties. We thus chose this functional to perform all further electronic-structure calculations in this work. The Orca package<sup>47</sup> was used to perform CCSD(T) simulations and the FHI-aims<sup>48</sup> package for DFT simulations.

The nuclear degrees of freedom were sampled according to the B3LYP+vdW PES with MD and PIMD through the i-PI program<sup>49,50</sup> in connection with FHI-aims. The PIMD simulations were performed coupled to the colored noise PIGLET thermostat.<sup>51</sup> We were therefore able to use 6 and 12 beads for 290 K and 100 K simulations, respectively.

Thermal rates were calculated with instanton rate theory.<sup>28,29</sup> The instanton path represents the optimal tunneling pathway at a given temperature and can be determined for the full high-dimensional problem. We used the ring-polymer approach<sup>52-54</sup> where the path is discretized and represented by  $P$  beads in a harmonic ring polymer,<sup>55</sup> through our recent implementation in the i-PI code.<sup>50</sup> We include all the 108 porphycene degrees of freedom in the path optimization. We have achieved convergence with 192 replicas at both 100 and 150 K (see SI) and we also included a correction factor to the instanton action, that linearly scales the action to match the CCSD(T) reference barrier, similar to.<sup>56</sup>

Restarting an instanton calculation with higher number of replicas can be computationally expensive due to the necessity of calculating all Hessians for the new amount of replicas. In order to overcome this bottleneck, we have derived and implemented in i-PI a ring-polymer expansion of the Hessian, given by  $\mathcal{H}_{jm}^{(k)} = \sum_{s=1}^P H_{jm}^{(s)} T(P', P)_{ks}$ .  $P$  and  $P'$  are the old (smaller) and new (larger) number of beads,  $H_{jm}^{(s)}$  is the  $jm$  matrix element of the Hessian corresponding to the  $s$ -th old replica and  $\mathcal{H}_{jm}^{(k)}$  is the  $jm$  matrix element of the Hessian corresponding to the  $k$ -th new replica. The  $P' \times P$  matrix  $T(P, P')$  is the same transformation matrix used in other contraction/expansion approaches and given in.<sup>57</sup> This procedure completely removes the necessity of calculating *any* new Hessians during the op-

timization, after their calculation for the first ring-polymer geometry at a much reduced number of beads.

The IR spectra were computed from the Fourier transform of the dipole-dipole autocorrelation function. The classical correlation function was computed running 4 different NVT 10 ps trajectories and the quantum correlation function was computed using thermostated ring polymer molecular dynamics<sup>30</sup> coupled to generalized Langevin equation (TRPMD+GLE) thermostats.<sup>58</sup> In this case we ran 7 different 10 ps trajectories using 16 beads. In all cases the starting configurations were taken from uncorrelated thermalized structures. In all cases, we used a 0.5 fs time step for the integration of the equations of motion.

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## Supporting Information Available

The following files are available free of charge.

See supporting information for further information regarding convergence tests, instanton rates, and vibrational spectra simulations, this material is available free of charge via the Internet at <http://pubs.acs.org>.

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