Physics Letters B 785 (2018) 631–632

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Corrigendum

Corrigendum to “Coupling the SO(2) supergravity through dimensional reduction” [Phys. Lett. B 96 (1–2) (1980) 89–93]

A. Chamseddine [a,b,*], H. Nicolai [c]

[a] Physics Department, American University of Beirut, Lebanon
[b] I.H.E.S., Bures-sur-Yvette, France
[c] Max-Planck-Institut fuer Gravitationsphysik (Albert-Einstein-Institut), Muehlenberg 1, D-14476 Potsdam, Germany

ARTICLE INFO

Article history:
Available online 4 August 2018

Due to renewed interest in $D = 5$ supergravity and its quartic fermionic terms, fuelled by very recent work on supersymmetric quantum cosmology [1], we would like to correct an error in eq. (4) of the above paper [2] (to which we also refer for our notations and further explanations[1]). In 1.5 order formalism [3] the correct Lagrangian of pure $D = 5$ supergravity reads

$$\begin{aligned}\mathcal{L} = &-\frac{1}{4}eR(\omega) - \frac{i}{2}e\Big[\bar{\psi}_M\Gamma^{MNP}\overrightarrow{D}_N\left(\frac{\omega+\hat{\omega}}{2}\right)\psi_P \\ &- \bar{\psi}_M\overleftarrow{D}_N\left(\frac{\omega+\hat{\omega}}{2}\right)\Gamma^{MNP}\psi_P\Big] \\ &- \frac{1}{4}eF^{MN}F_{MN} - \frac{\sqrt{3}}{8}ie\bar{\psi}_M X^{MNPQ}\psi_N(F_{PQ}+\hat{F}_{PQ}) \\ &- \frac{1}{6\sqrt{3}}\epsilon^{MNPQR}F_{MN}F_{PQ}A_R \qquad (0.1)\end{aligned}$$

with the 4-component Dirac vector spinor ψ_M. The $(3\omega - \hat{\omega})/2$ in eq. (4) of [2] must thus be replaced by $(\omega + \hat{\omega})/2$. The terms with $\hat{\omega}$ and $\hat{F}$ account for all higher order fermionic terms in 1.5 order formalism, as they do in $D = 11$ supergravity [4]. Here the spin connection ω is determined by its equation of motion

$$\omega_{MAB} = \overset{\circ}{\omega}_{MAB}(e) + \kappa_{MAB} \qquad (0.2)$$

with the contorsion tensor

$$\begin{aligned}\kappa_{MAB} = &-\frac{i}{2}\bar{\psi}_Q\Gamma^{QR}{}_{MAB}\psi_R + \frac{i}{2}(\bar{\psi}_M\Gamma_B\psi_A - \bar{\psi}_A\Gamma_B\psi_M) \\ &+ \frac{i}{2}(\bar{\psi}_B\Gamma_M\psi_A - \bar{\psi}_A\Gamma_M\psi_B) \\ &- \frac{i}{2}(\bar{\psi}_M\Gamma_A\psi_B - \bar{\psi}_B\Gamma_A\psi_M) \qquad (0.3)\end{aligned}$$

The supercovariant spin connection and field strength are

$$\begin{aligned}&\hat{\omega}_{MAB} \equiv \omega_{MAB} + \frac{i}{2}\bar{\psi}_Q\Gamma^{QR}{}_{MAB}\psi_R, \\ &\hat{F}_{MN} \equiv F_{MN} + \frac{\sqrt{3}i}{2}(\bar{\psi}_M\psi_N - \bar{\psi}_N\psi_M) \qquad (0.4)\end{aligned}$$

From (0.1) one obtains the supercovariant Rarita–Schwinger equation ($\equiv$ eq. (14) of [2])

$$\Gamma^{MNP}D_N(\hat{\omega})\psi_P + \frac{\sqrt{3}}{4}X^{MNPQ}\psi_N\hat{F}_{PQ} = 0 \qquad (0.5)$$

The Lagrangian in second order formalism, which was not explicitly written out in [2], is obtained in the usual way by substituting the second order spin connection into the Lagrangian (0.1), with the result

$$\begin{aligned}\mathcal{L} = &-\frac{1}{4}eR(\overset{\circ}{\omega}(e)) - \frac{i}{2}e\Big[\bar{\psi}_M\Gamma^{MNP}\overrightarrow{D}_N\left(\overset{\circ}{\omega}(e)\right)\psi_P \\ &- \bar{\psi}_M\overleftarrow{D}_N\left(\overset{\circ}{\omega}(e)\right)\Gamma^{MNP}\psi_P\Big] \\ &- \frac{1}{4}eF^{MN}F_{MN} - \frac{\sqrt{3}}{4}ie\bar{\psi}_M X^{MNPQ}\psi_N F_{PQ} \\ &- \frac{1}{6\sqrt{3}}\epsilon^{MNPQR}F_{MN}F_{PQ}A_R + \mathcal{L}_{\text{quartic}} \qquad (0.6)\end{aligned}$$

where

DOI of original article: https://doi.org/10.1016/0370-2693(80)90218-X.
* Corresponding author.
E-mail addresses: chams@aub.edu.lb (A. Chamseddine), hermann.nicolai@aei.mpg.de (H. Nicolai).

[1] In particular, our metric signature is $(+----)$, and thus $\bar{\psi}_M \equiv \psi_M^\dagger\Gamma^0$. Furthermore, $\Gamma^{ABCDE}_{\alpha\beta} = \epsilon^{ABCDE}\delta_{\alpha\beta}$ with $\epsilon^{01234} = \epsilon_{01234} = 1$.

https://doi.org/10.1016/j.physletb.2018.05.029

$$\begin{aligned}\mathcal{L}_{\text{quartic}}
&= \frac{1}{4} e \Big[\left(\overline{\psi}_M \Gamma^N \psi_N - \overline{\psi}_N \Gamma^N \psi_M \right) \left(\overline{\psi}^M \Gamma^P \psi_P - \overline{\psi}_P \Gamma^P \psi^M \right) \\
&\quad - \frac{1}{4} \left(\overline{\psi}_M \Gamma_N \psi_P - \overline{\psi}_P \Gamma_N \psi_M \right) \left(\overline{\psi}^M \Gamma^N \psi^P - \overline{\psi}^P \Gamma^N \psi^M \right) \\
&\quad - \frac{1}{2} \left(\overline{\psi}_M \Gamma_N \psi_P - \overline{\psi}_P \Gamma_N \psi_M \right) \left(\overline{\psi}^M \Gamma^P \psi^N - \overline{\psi}^N \Gamma^P \psi^M \right) \\
&\quad + \overline{\psi}_M \Gamma^{MNPQ} \psi_N \overline{\psi}_P \psi_Q + \frac{3}{2} \left(\overline{\psi}_M \psi_N - \overline{\psi}_N \psi_M \right) \overline{\psi}^M \psi^N \Big]\end{aligned}$$

This can now be compared to [5] (which, however, uses symplectic Majorana spinors rather than Dirac spinors).

Acknowledgements

We would like to thank T. Damour, P. Spindel and A. Van Proeyen for discussions and correspondence.

References

[1] T. Damour, P. Spindel, Phys. Rev. D 90 (2014) 103509;
T. Damour, P. Spindel, Phys. Rev. D 95 (2017) 126011.
[2] A. Chamseddine, H. Nicolai, Phys. Lett. B 96 (1980) 89.
[3] A. Chamseddine, P. West, Nucl. Phys. B 129 (1977) 39;
P.K. Townsend, P. van Nieuwenhuizen, Phys. Lett. B 67 (1977) 439.
[4] E. Cremmer, B. Julia, J. Scherk, Phys. Lett. B 76 (1978) 409.
[5] E. Cremmer, Supergravities in 5 dimensions, in: A. Salam, E. Sezgin (Eds.), Supergravities in Diverse Dimensions, Cambridge University Press, 1980.