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A bound on quantum chaos from Random Matrix Theory with Gaussian Unitary Ensemble

Sayantan Choudhury[a,1,2] **and Arkaprava Mukherjee**[b]

[a] *Quantum Gravity and Unified Theory and Theoretical Cosmology Group, Max Planck Institute for Gravitational Physics (Albert Einstein Institute), Am Mühlenberg 1, Potsdam-Golm 14476, Germany*

[b] *Department of Physical Sciences, Indian Institute of Science Education and Research Kolkata, Mohanpur, West Bengal 741246, India*

E-mail: sayantan.choudhury@aei.mpg.de, am16ms058@iiserkol.ac.in

Abstract: In this article, using the principles of Random Matrix Theory (RMT) with Gaussian Unitary Ensemble (GUE), we give a measure of quantum chaos by quantifying Spectral From Factor (SFF) appearing from the computation of two point Out of Time Order Correlation function (OTOC) expressed in terms of square of the commutator bracket of quantum operators which are separated in time scale. We also provide a strict model independent bound on the measure of quantum chaos, $-1/N(1-1/\pi) \leq \textbf{SFF} \leq 0$ and $0 \leq \textbf{SFF} \leq 1/\pi N$, valid for thermal systems with large and small number of degrees of freedom respectively. We have studied both the early and late behaviour of SFF to check the validity and applicability of our derived bound. Based on the appropriate physical arguments we give a precise mathematical derivation to establish this alternative strict bound of quantum chaos. Finally, we provide an example of integrability from GUE based RMT from *Toda Lattice model* to explicitly show the application of our derived bound on SFF to quantify chaos.

Keywords: Matrix Models, Random Systems, Thermal Field Theory

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Contents

1 Introduction

Quantum description of chaos has three important properties boundedness, exponential sensitivity and infinite recurrence. In the context of the study of dynamical systems the concept of quantum chaos describes quantum signatures of classically chaotic systems. Quantum chaos [1] can be formulated for two observable represented by hermitian operators $X(t)$ and $Y(t)$ using their commutator relation. This actually explains the perturbation effect of one operator $Y(t)$ on the measurement of other operator $X(t)$. Strength of this perturbation can be measured by formulating a time dependent function, defined by the following expression:

$$C(t) = -\frac{1}{Z}\mathrm{Tr}[e^{-\beta H}[X(t), Y(0)]^2] \tag{1.1}$$

at temperature $\beta = 1/T$ and T is the temperature of the system under consideration. Also the partition function of the system is defined as:

$$Z = \mathrm{Tr}\left[e^{-\beta H}\right], \tag{1.2}$$

where H representing the system Hamiltonian under consideration. Here we specifically assume that X and Y have zero one point function at finite temperature i.e.

$$\langle X(t)\rangle = \frac{1}{Z}\mathrm{Tr}\left[e^{-\beta H}X(t)\right] = 0, \tag{1.3}$$

$$\langle Y(t)\rangle = \frac{1}{Z}\mathrm{Tr}\left[e^{-\beta H}Y(t)\right] = 0, \tag{1.4}$$

where by the parenthesis symbol $\langle \cdots \rangle$ we represent the thermal average or expectation of a physical observable associated with the physical system under consideration. For this specific reason we use thermal two point function for measuring quantum chaos, which technically has been explained by the late time behaviour of the time dependent function $C(t)$, from which we can derive a generic bound on quantum chaos.

Another important observation is that, instead of using the commutator bracket of two quantum mechanical operators we actually use the square of the commutator bracket in the definition of observable $C(t)$, which represents the out-of-time-ordered correlation function in the present context. To know about the actual physical reason of this fact let us assume for the time being that we replace the commutator bracket by the Poisson bracket by considering the semi-classical approximation. In this situation the Poisson bracket shows an exponential growth with respect to time, $e^{\lambda t}$, where λ represents the *Lyapunov exponent* which quantifies chaos. Now if we take the thermal average over the previously mentioned commutator bracket representing two point function then both the contributions are cancelled in the semi-classical approximation and will not contribute to quantify quantum chaos. On the other hand, from the quantum perspective, the two point thermal averaged function, captures the effect of correlation between the two quantum Hermitian operators, which decay with respect to time in the large time limit and cannot characterise the chaotic behaviour at all. Instead of that if we consider the square of the commutator bracket which actually represents the four point function after transforming it to the Poisson bracket in the semi-classical limiting approximation it is perfectly consistent with the signature of the co-efficient as it takes only positive value, which implies no cancellation at all. After taking thermal average we get non trivial contribution using which one can quantify quantum chaos. In the same way, in the quantum picture the four point thermal averaged function, not decays exponentially with respect to time at the leading order approximation in the large time limiting region.

It has been previously shown that due to quantum effects two point function for chaos decrease to a particular constant value. In this connection, Lyapunov Exponent, λ_L, measure this effect and it entirely depends on system properties and the detail of the observable. For specific quench system it has been shown that Lyapunov exponent decay exponentially to a certain value. Thus a bound on Lyapunov Exponent can be treated as measure of quantum chaos. Using quantum field theory it has been shown that an universal bound [2] on the *Lyapunov exponent* exists:

$$\lambda_L \leq \frac{2\pi}{\beta}. \tag{1.5}$$

This bound is unique feature for all classes of out of equilibrium quantum field theory set up. In this article, we have also discussed similarly the saturation of chaos bound at late time scale using Random matrix theory (RMT) with Gaussian Unitray Ensemble (GUE) where the system under consideration has the unitary invarience. Additionally for completeness we have also discussed about the quantum chatoic behaviour in the early time scale as well. The asymptotic behaviour from both early and late time scales helps us to know about the complete chaotic behaviour of RMT with GUE. These class of system was previously discussed in the context of the quantization of classical chaotic systems, usually in the semi-classical or high quantum-number regimes. For this discussion, we construct Spectral From Factor (SFF) [3] from the mentioned GUE class of RMT, which is arising from two point thermal out-of- time-orderd correlation (OTOC) function in RMT.[1] This two point OTOC has been used to derive an alternative bound on quantum chaos.[2] This gives us an extra freedom and strong motivation to generalize our discussion for any quantum mechanical system with random interaction. This interaction has been included by a polynomial potential function of any general order.[3] By studying the late time behaviour of SFF from RMT we get its upper bound. Also it is important to note that, this approach of finding bound on SFF (or two point OTOC) to quantify quantum chaos is itself unique as it is valid for any arbitrary (infinite and finite) temperature of the quantum system under consideration. We also obtained a lower bound for SFF which depicts the minimal chaos a quantum system can have within the framework of GUE inspired RMT. For arbitrary random interaction where the specific mathematical structure of the interaction potential is unknown we can get a eigen value distribution from RMT with GUE. For simplicity the arbitrary random interaction potential can be expressed in terms of a general polynomial of different order. From this specific eigen value distribution one can explicitly compute the expression for the thermal partition function Z in the present context of discussion. For our computation of SFF (or two point thermal OTOC) and make further prediction for the chaotic behaviour of the quantum system under consideration from RMT we have used any arbitrary order of polynomial potential which characterise the random interaction. Quantifying quantum aspects of chaos through SFF using the principles of RMT is very useful when a specific type of interaction at the level of action of the quantum system under consideration is not exactly known, which is also true for our quantum system under consideration. To use the principles of RMT in the present context of discussion first of all we start with creating a Gaussian Unitary statistical ensemble (GUE), which includes all possible random interaction between the energy levels of many-body Hamiltonian with the various random matrices appearing in the GUE. If the many-body Hamiltonian is time-reversal in the present context, then the symmetric distribution of the potential will be invariant under the application of orthogonal transformation within the framework of GUE based RMT. Most importantly it is important to note that, in the thermodynamic limit (large N or $N \to \infty$) eigen value distribution of the random matrices showed a universal behaviour for GUE based RMT, characterized by the well known *Wigner's Semicircle law.* The results derived in this paper seemed to be applicable to a wide varied class of quantum

[1] *Important Note I*: studying the chaotic behaviour in both the early and late time scales actually helps us to check the validity and applicability of the derived bound on quantum chaos through SFF (or two point OTOC) computed from RMT.

[2] *Crucial Assumption*: if we restrict our assumption to just only to an unitary ensemble then one can explicitly show that the Spectral From Factor (SFF) as well as the two point OTOC could simply violate the bound derived in this paper. For this reason, we will only consider a specific class of statistical ensemble, which is the well known Gaussian Unitray Ensemble (GUE) to compute SFF and two point OTOC from RMT.

[3] *Important Note II*: here by the phrase"general" we actually pointing towards the fact that our analysis and derived bound is independent of the mathematical structure of the potentials appearing in the distribution of RMT. But it does not correspond to the independence over statistical ensemble appearing in RMT. We will explicitly show that, our derived bound will be only valid for GUE, not for any other ensembles.

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mechanical systems displaying chaotic behaviour. Even for many-body Hamiltonian (where N finite) the phenomena of quantum chaos can be devised in a more better way in systems where nearest neighbour spacing distribution (NNSD) of eigenvalues of the random matrices of the system explicitly show chaotic behaviour.

The plan of this paper is as follows. In section 2, we briefly review the basics of Random Matrix Theory (RMT) relevant for the quantification of quantum chaos. Next in section 3, we elaborately discuss about the role of out of time ordered correlation function (OTOC) in the context of RMT to quantify quantum chaos. Further, in section 4, we derive the expression for the two point Spectral Form Factor (SFF) from GUE based RMT from the thermal Green's function. Next, in section 5, we derive the saturation bound on two point SFF at the late time scale, which is very important to fix quantum chaos obtained from GUE based RMT set up. Further, in section 6, we study the early time behaviour of two point SFF to check the consistency and validity of our derived bound at the late time scale. Next, in section 7, we study the application of our derived bound by analysing the integrability issue from a specific GUE based RMT set up, *aka* Toda lattice model. Finally, in section 8, we conclude with some promising future prospects.

2 Random Matrix Theory (RMT) redux: quantifying quantum chaos

In this section, we explicitly discuss about the quantification of quantum chaotic behaviour using the principles of GUE based RMT [4, 5]. To illustrate our discussion let us start with Gaussian Unitary matrix ensemble (GUE) which is described as [6].a collection of large number of matrices are filled with random numbers picked arbitrarily from a Gaussian Unitary probability distribution. The corresponding partition function of this random system can be written in the basis of eigen value of the random matrix as:

$$Z = \prod_{i=1}^{N} \int d\lambda_i \; e^{-N^2 S(\lambda_1,\ldots\lambda_N)}, \tag{2.1}$$

where $S(\lambda_1, \ldots \lambda_N)$ is the required action describing the random system at temperature $\beta = 1/T$ and in the present context of discussion this thermal action is defined as:[4]

$$S(\lambda_1, \ldots \lambda_N) = \frac{1}{N} \sum_{i=1}^{N} V(\lambda_i) + \beta \sum_{i<j}^{N} \log |\lambda_i - \lambda_j|. \tag{2.2}$$

Here N represents the total number of eigen values appearing in the GUE based random distribution under consideration. Using standard formulation of time independent RMT, one can actually fix $\beta = 2$ for GUE. In the above mentioned action, in front of the random potential term $1/N$ scaling is particularly required to explicitly demonstrate that the eigenvalues of the random distribution scaled by a factor of $\sqrt{N}$. Further eextremizing the

[4] *Important Note III*: if the particle interaction is not specifically characterized by any mathematical function then at the level of the effective action this present approach is highly suitable for the computation of various physical observables including SFF (or two point thermal OTOC). If any information about the interaction is not known then action in usual notation can't be defined.

random action with respect to all eigen values, $\lambda_i \forall\ i = 1, \cdots, N$, we can get the required solutions in the present context. To compute the expression for the partition function one can use the method of resolvent, in which the resolvent is defined as follows:

$$O(x) = \frac{1}{N}\sum_{i=1}^{N}\frac{1}{(x-\lambda_i)}, \tag{2.3}$$

using which one can further write:

$$O^2(x) + \frac{1}{N}O'(x) = V'(x)O(x) - \rho(x)\,. \tag{2.4}$$

Here we express the density function $\rho(x)$ and the resolvent function $O(x)$ as:

$$\rho(x) = \sum_{i=1}^{N}\frac{V'(x)-V'(\lambda_i)}{(x-\lambda_i)}, \tag{2.5}$$

$$O(x) = \frac{1}{N}\frac{\Theta'(x)}{\Theta(x)}, \tag{2.6}$$

where $V(\lambda)$ represents the interaction random potential. It can be shown that the function $\Theta(x)$ has the following characteristic polynomial:

$$\Theta(x) = \prod_{i=1}^{N}(x-\lambda_i) = \det(x\,\mathbf{I} - \mathcal{R}). \tag{2.7}$$

Here $\mathcal{R}$ is the random matrix under consideration which have the eigenvalues $\lambda_i \forall\ i = 1, \cdots, N$. Here it is important to note that, the solution obtained in the large N limit is analogues to the solution obtained from the WKB approximation in Schrödinger equation, which is obviously a very very useful fact as far as the present computation is concerned. Considering the large N limiting approximation, we can neglect the term $\frac{1}{N}O'(x)$ and express the characteristic equation for $O(x)$ by the following simplified expression:

$$\overline{O}^2(x) - V'(x)\overline{O}(x) + \overline{\rho}(x) = 0 \tag{2.8}$$

where we have introduced two new quantities at the large N limit, $\overline{O}(x)$ and $\overline{\rho}(x)$, which are defined as follows:

$$\overline{O}(x) = \lim_{N\to\infty} O(x), \tag{2.9}$$

$$\overline{\rho}(x) = \lim_{N\to\infty} \rho(x). \tag{2.10}$$

Then the preferred algebraic solution of $\overline{O}(x)$ in the large N limit is given by the following expression:

$$\overline{O}(x) \equiv \overline{O}_{\pm}(x) = \frac{1}{2}\left[V'(x) \pm \sqrt{(V'(x))^2 - 4\overline{\rho}(x)}\right]. \tag{2.11}$$

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Here for our purpose $\overline{O}_+(x)$ is completely redundant and $\overline{O}_-(x)$ is the required physical solution. Further, it is important to note that, for our computation considering large N limiting approximation we can write:

$$\overline{\rho}(x) = \lim_{N\to\infty} \rho(x) \approx \rho(x) = V''(x), \tag{2.12}$$

where $\rho(x)$ is the density function of eigen values of the random distribution. Consequently, at large N the preferred solution of $\overline{O}(x)$ can be recast as:

$$\overline{O}(x) \equiv \overline{O}_-(x) = \frac{1}{2}\left[V'(x) - \sqrt{(V'(x))^2 - 4V''(x)}\right]. \tag{2.13}$$

On the other hand for finite N, using Schrödinger equation we get the corrected result of $\overline{O}(x)$:

$$\overline{O}(x) \equiv \overline{O}_-(x) = \frac{\mathcal{K}(x)}{2\sqrt{2}}\left[1 - \sqrt{1 - \frac{4\overline{\rho}(x)}{(\mathcal{K}(x))^2}}\right]. \tag{2.14}$$

where we introduce a new function $\mathcal{K}(x)$, which is defined in the finite N limit as:

$$\mathcal{K}(x) = \sqrt{4\widetilde{O}(x) + 1}\left[1 - \sqrt{\frac{16[(\widetilde{O}(x))^2 + V''(x)]}{(4\widetilde{O}(x) + 1)}}\right]^{\frac{1}{2}}. \tag{2.15}$$

Here, at finite N the resolvent $\widetilde{O}(x)$ is defined as:

$$\widetilde{O}(x) = \lim_{N\to\text{finite}} O(x). \tag{2.16}$$

Now, we consider the most general solution for the density function, which is represented by the following expression:[5]

$$\rho(\lambda) = \frac{1}{2\pi}\mathcal{R}(\lambda)\sqrt{-\Sigma(\lambda)} = \frac{1}{2\pi}\sum_{k=1}^{\infty} a_{n-k}\lambda^{(n-k+1)}\prod_{i=1}^{n}(\lambda - a_{2i-1})(\lambda - a_{2i}) \quad \forall \text{ general } n, \tag{2.18}$$

where both the functions, $\mathcal{R}(\lambda)$ and $\sigma(\lambda)$ are general polynomial in λ and in the present context it is defined as:

$$\mathcal{R}(\lambda) = \sum_{k=1}^{\infty} a_{n-k}\lambda^{(n-k+1)} \qquad \forall \text{ general } n, \tag{2.19}$$

$$\Sigma(\lambda) = \prod_{i=1}^{n}(\lambda - a_{2i-1})(\lambda - a_{2i}) \qquad \forall \text{ general } n. \tag{2.20}$$

Here for the computational purpose we actually consider n number of intervals on which the density function $\rho(\lambda)$ is supported and particularly a_{2i-1} and a_{2i} are appearing as the

[5]Here the density function satisfy the following normalization condition:

$$\int_{\text{supp }\mu} d\mu\ \rho(\mu) = 1\,. \tag{2.17}$$

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position of the end point. To demonstrate the significance of this result here we consider the simplest situation in which we fix, $n = 1$ and in that case we get the following simplified expression for the density function on a semi-circle:

$$\rho(\lambda) = \frac{1}{\pi}\sqrt{4a^2 - \lambda^2} \sum_{k=1}^{\infty} a_{1-k}\lambda^{2-k} \quad \text{for } n = 1. \tag{2.21}$$

In this context, the in a complex plane the expression for $O(\lambda + i0)$ is computed using *Residue theorem* as:

$$O(\lambda + i0) = \frac{1}{2}V^{'}(\lambda) - i\pi\overline{\rho}(\lambda). \tag{2.22}$$

Further substituting the expression for the density function mentioned in eq. (2.21) and taking the *Taylor series* expansion in the limiting situation, $\lambda \to \infty$ we get the following simplified result:

$$O(\lambda \to \infty) = \frac{1}{\lambda} + O\left(\lambda^2\right), \tag{2.23}$$

where all the coefficients appearing in the above expansion can be explicitly evaluated. This details of the computation and final outcomes will helps us further to explicitly compute the expression for SFF (or thermal two point OTOC) from GUE based RMT setup.

3 Out of time ordered correlation function (OTOC) from RMT

In this section, we briefly review about the construction of two and four point OTOC from GUE based RMT. [7–9] This will help us to futher connect with the computation of SFF and derive the bound on quantum chaos in the next sections of this paper. To serve this purpose, let us start our discussion with the well known two point correlation function for the GUE based RMT, which is described by the following expression:

$$\langle \mathcal{W}_1(0)\mathcal{W}_2(t)\rangle_{\rm GUE} \equiv \int dH\ \langle \mathcal{W}_1(0)\mathcal{W}_2(t)\rangle, \tag{3.1}$$

where the time evolution of the quantum operator $\mathcal{W}_2(t)$ can be expressed in terms of the following unitary similarity transformation defined in the *Heisenberg picture* as:

$$\mathcal{W}_2(t) = e^{-iHt}\mathcal{W}_2(0)e^{iHt}. \tag{3.2}$$

In this context, GUE integral Haar measure dH is actually represented by the system Hamiltonian H which describes the random distribution we have already mentioned earlier in our discussion. In GUE based RMT such Haar measure is remain invariant under the following unitary conjugation operation, which is explicitly defined by the following expression:

$$dH = d(\mathcal{U}H\mathcal{U}^{\dagger}) \qquad \forall\, \mathcal{U}, \tag{3.3}$$

were $\mathcal{U}$ is representing the unitary matrix in the present context, which helps us to perform the unitary conjugation operation on the matrix Haar measure in GUE based RMT.

Now, let us consider a very special situation where two quantum operators, $\mathcal{W}_1$ and $\mathcal{W}_2$ are described by the *Pauli operators* in the context of GUE based RMT. In such a specific situation, the GUE two point correlation function (OTOC) can be further simplified to the following expression:

$$\langle \mathcal{W}_1(0)\mathcal{W}_2(t)\rangle_{\rm GUE} = \begin{cases} \dfrac{\mathbf{S}_2(t)-1}{\mathcal{A}^2-1}\,, & \mathcal{W}_1 = \mathcal{W}_2 \\ 0\,, & \mathcal{W}_1 \neq \mathcal{W}_2 \end{cases}, \tag{3.4}$$

where $\mathbf{S}_2(t)$ represents the two point *Spectral Form Factor* (SFF) which we have explicitly derive for our system in the next section of this paper. For further simplification one can consider a physically justifiable the situation where the SFF is very large i.e. $\mathbf{S}_2(t) \gg 1$ and in this situation we have an additional constraint condition, given by:

$$\mathcal{W}_2(t) = \mathcal{W}_1^\dagger(t), \tag{3.5}$$

which we have to maintain always during our computation. For this specific situation the GUE two point correlation function (OTOC) for RMT can be expressed by the following simplified expression:

$$\langle \mathcal{W}_1(0)\mathcal{W}_2(t)\rangle_{\rm GUE} \sim \frac{\mathbf{S}_2(t)}{\mathcal{A}^2}. \tag{3.6}$$

Most importantly in this context, the factor $\mathcal{A}$ represents the 2^n dimensional Hilbert space in which we are performing the present computation.

In the same way the four point OTOC for the GUE based RMT can be expressed as:

$$\langle \mathcal{W}_1(0)\mathcal{W}_2(t)\mathcal{W}_3(0)\mathcal{W}_4(t)\rangle_{\rm GUE} = \int dH \int d\mathcal{U}\; \langle \mathcal{W}_1 U e^{-iHt}\mathcal{U}^\dagger \mathcal{W}_2 \mathcal{U} e^{iHt}\mathcal{U}^\dagger \mathcal{W}_3 \mathcal{U} e^{-iHt}\mathcal{U}^\dagger \mathcal{W}_4 \mathcal{U} e^{iHt}\mathcal{U}^\dagger\rangle, \tag{3.7}$$

Under same assumption as mentioned for two point OTOC, the four point OTOC for GUE based RMT can be further simplified to the following expression:

$$\begin{aligned}\langle \mathcal{W}_1(0)\mathcal{W}_2(t)\mathcal{W}_3(0)\mathcal{W}_4(t)\rangle_{\rm GUE} &\sim \langle \mathcal{W}_1\mathcal{W}_2\mathcal{W}_3\mathcal{W}_4\rangle \times \frac{\mathbf{S}_4(t)}{\mathcal{A}^4} \\ &\sim \langle \mathcal{W}_1\mathcal{W}_2\mathcal{W}_3\mathcal{W}_4\rangle \times \left[\frac{\mathcal{A}^2}{\pi^2 t^6} + \frac{1}{2\mathcal{A}^4} t(t-2)\right]\end{aligned} \tag{3.8}$$

where $\mathbf{S}_4(t)$ is the four point SFF for GUE based RMT, which is defined by the following expression:

$$\begin{aligned}\mathbf{S}_4(t) \equiv \langle Z(t)Z(t)Z^*(t)Z^*(t)\rangle_{\rm GUE} &= \int \mathcal{D}\lambda \sum_p e^{i\lambda_p t}\sum_j e^{i\lambda_j t}\sum_k e^{-i\lambda_k t}\sum_m e^{-i\lambda_m t} \\ &= \int \mathcal{D}\lambda \sum_{p,j,k,m} e^{i(\lambda_p+\lambda_j-\lambda_k-\lambda_m)t} \\ &\sim \frac{\mathcal{A}^6}{\pi^2 t^6} + \frac{1}{2} t(t-2).\end{aligned} \tag{3.9}$$

4 Two point Spectral Form Factor (SFF) from thermal Green's function of RMT

In this subsection our main focus is to explicitly compute the expression for two point SFF ($\mathbf{S}_2(t)$) for any arbitrary polynomial potential of GUE based random distribution of matrices. We will see that how this particular computation is very useful to quantify the quantum chaotic behaviour when we have no particular information about the interaction term at the level of system action. [10]

To demonstrate this, first of all consider a Thermofield Double State (TDS) which actually describes the canonical quantum mechanical state at finite temperature $\beta = 1/T$. The time evolution of the TDS can be described by the following expression:

$$|\Psi(\beta,t)\rangle = e^{-iHt}|\Psi(\beta,t=0)\rangle = \frac{1}{\sqrt{Z(\beta)}}\sum_n e^{-\left(it+\frac{\beta}{2}\right)E_n}|n\rangle_1 \otimes |n\rangle_2, \tag{4.1}$$

where 1 and 2 stands for two identical copies of the eigen quantum state of the Hamiltonian H, which are CPT conjugate of each other representing the degrees of freedom under consideration within this GUE based RMT set up. Also in this computation both the copies are entangled with each other, where the total Hamiltonian of the this bipartite quantum system can be represented as:

$$H = H_1 \otimes \mathbf{I}_2. \tag{4.2}$$

Here $|\Psi(\beta,0)\rangle$ represents the TDS at time $t=0$, which can be expressed as:

$$|\Psi(\beta,t=0)\rangle = \frac{1}{\sqrt{Z(\beta)}}\sum_n e^{-\frac{\beta}{2}E_n}|n\rangle_1 \otimes |n\rangle_2. \tag{4.3}$$

In this context, the thermal partition function $Z(\beta)$ is defined as:

$$Z(\beta) = \mathrm{Tr}\left[e^{-\beta H}\right] = \sum_n e^{-\beta E_n}. \tag{4.4}$$

Using this TDS one can further compute the *survival amplitude* or *overlap*, which can be expressed as:

$$\mathcal{M}(\beta,t) = \langle\Psi(\beta,0)|\Psi(\beta,t)\rangle = \frac{1}{Z(\beta)}\sum_n e^{-(it+\beta)E_n}. \tag{4.5}$$

Finally, using this *survival amplitude* or *overlap* one can further compute the *survival probability* or *fidelity* from the GUE based RMT, which can be expressed as:

$$\begin{aligned}\mathcal{P}(\beta,t) = |\mathcal{M}(\beta,t)|^2 &= \frac{1}{|Z(\beta)|^2}\left(\sum_{m,n,m\neq n} e^{-\beta(E_m+E_n)}e^{-it(E_m-E_n)} + \sum_n e^{-2\beta E_n}\right)\\ &= \frac{1}{|Z(\beta)|^2}\left(|Z(\beta+it)|^2 + |Z(2\beta)|^2\right)\\ &= \mathbf{S}_2(t) + \mathbf{N}(\beta),\end{aligned} \tag{4.6}$$

where the two point SFF $\mathbf{S}_2(t)$ is defined as:

$$\mathbf{S}_2(t) = \frac{1}{|Z(\beta)|^2} \sum_{m,n,m\neq n} e^{-\beta(E_m+E_n)} e^{-it(E_m-E_n)} = \frac{|Z(\beta+it)|^2}{|Z(\beta)|^2}. \tag{4.7}$$

Here also introduce a temperature dependent function $\mathbf{N}(\beta)$, which is defined as:

$$\mathbf{N}(\beta) = \frac{|Z(2\beta)|^2}{|Z(\beta)|^2} = \lim_{T\to\infty} \frac{1}{T} \int_{t=0}^{T} dt \; \mathcal{P}(\beta,t) = \widetilde{\mathcal{P}(\beta)}. \tag{4.8}$$

Here it is important to note that, for quantum systems which is characterised by a continuum spectrum, the *survival probability* or *fidelity* of an arbitrary initial quantum state vanishes exactly in the limiting situation, $t \to \infty$. Additionally, it is important to note that, for a finite quantum mechanical systems which is described by a discrete spectrum and not contain large number of degeneracies in the quantum spectrum, *survival probability* or *fidelity* ($\mathcal{P}(\beta,t)$) saturates to its infinite time average or expectation value $\widetilde{\mathcal{P}(\beta)}$ and randomly fluctuating around this infinite time average value.

Also, in this computation, E_n and E_m correspond to the n-th and m-th level of the quantum system under consideration within GUE based RMT set up. Here the Boltzmann factor $\beta = 1/T$, where T is the temperature associated to the system. Apart from Boltzmann factor the definition of two point SFF also involves physical time t. Additionally, it is important to note that, the formulation of two point SFF in terms of the TDS within GUE based RMT allows us to exactly map the unitarity constraints on the decay of most general type of quantum mechanical systems, which is usually expressed in terms of the decay of *survival probability* or *fidelity* (fidelity) to the spectral properties of quantum mechanical systems through two point SFF as explicitly shown here.

Now it important to note that, at very high temperature ($\beta = 1/T \to 0$) and low temperature ($\beta = 1/T \to \infty$) the two point SFF simplified to the following result:

$$\mathbf{S}_2(t) = \begin{cases} \displaystyle\sum_{m,n,m\neq n} e^{-it(E_m-E_n)}\,, & \beta = 1/T \to 0 \\ 0\,, & \beta = 1/T \to \infty \end{cases}, \tag{4.9}$$

It is also observed that in $t \to \infty$ limit the nearest neighbour energy spacings contribute only to quantify two point SFF. This implies that the concept of SFF in GUE based RMT also helps us to understand the dynamical behaviour of the quantum system and also very useful concept to analyse the discreteness appearing in quantum spectrum. Additionally, it is important to note that, chaotic system confronts *Wigner's formula* in a semi-circle which makes two point SFF a good theoretical observable to quantify quantum chaos.

In practical applications, two point SFF is averaged over a well known statistical ensemble i.e. GUE of random distribution of matrices. This is a very interesting feature of SFF appearing in GUE based RMT which can be directly connected to the quantify quantum chaos in the present context. Before going to discuss the further detail, we note that all distribution representing eigenvalues are different from each other in general but similar at small scales.

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Now, in the present context we define the thermal Green's function $\mathcal{G}(\beta,t)$, which is represented by the following expression:

$$\begin{aligned}\mathcal{G}(\beta,t) &= \frac{\langle|Z(\beta+it)|^2\rangle_{\rm GUE}}{\langle|Z(\beta)|^2\rangle_{\rm GUE}}\\ &= \frac{\int_{\text{supp }\overline{\rho}} d\lambda\; d\mu\; e^{-\beta(\lambda+\mu)}\; e^{-it(\lambda-\mu)}\langle\mathcal{J}(\lambda)\mathcal{J}(\mu)\rangle_{\rm GUE}}{\int_{\text{supp }\overline{\rho}} d\lambda\; d\mu\; e^{-\beta(\lambda+\mu)}\langle\mathcal{J}(\lambda)\rangle\langle\mathcal{J}(\mu)\rangle_{\rm GUE}}.\end{aligned}\tag{4.10}$$

Here, density of eigen values are defined as:

$$\mathcal{J}(\lambda)=\overline{\rho}(\lambda).\tag{4.11}$$

Now, one can decompose the total Green's function $\mathcal{G}$ in two parts (connected and disconnected part), as given by:

$$\mathcal{G}(\beta,t)=\mathcal{G}_{dc}(\beta,t)+\mathcal{G}_{c}(\beta,t),\tag{4.12}$$

where disconnected part of the Green's function $\mathcal{G}_{dc}$ and connected part of the Green's function $\mathcal{G}_c$ can be expressed as:

$$\begin{aligned}\mathcal{G}_{dc}(\beta,t) &= \left[\frac{\langle Z(\beta+it)\rangle\langle Z(\beta-it)\rangle}{\langle|Z(\beta)|^2\rangle}\right]\\ &= \frac{\int d\lambda\; d\mu\; e^{-\beta(\lambda+\mu)}\; e^{-it(\lambda-\mu)}\;\langle\mathcal{J}(\lambda)\rangle\langle\mathcal{J}(\mu)\rangle}{\int d\lambda\; d\mu\; e^{-\beta(\lambda+\mu)}\;\langle\mathcal{J}(\lambda)\rangle\langle\mathcal{J}(\mu)\rangle}.\end{aligned}\tag{4.13}$$

$$\begin{aligned}\mathcal{G}_{c}(\beta,t) &= \mathcal{G}(\beta,t)-\mathcal{G}_{dc}(\beta,t)\\ &= \left[\frac{\langle|Z(\beta+it)|^2\rangle_{\rm GUE}}{\langle|Z(\beta)|^2\rangle_{\rm GUE}}\right]-\left[\frac{\langle Z(\beta+it)\rangle\langle Z(\beta-it)\rangle}{\langle|Z(\beta)|^2\rangle}\right]\\ &= \frac{\int d\lambda\; d\mu\; e^{-\beta(\lambda+\mu)}\; e^{-it(\lambda-\mu)}\;\langle\mathcal{J}(\lambda)\mathcal{J}(\mu)\rangle_{\mathbf{c}}}{\int d\lambda\; d\mu\; e^{-\beta(\lambda+\mu)}\;\langle\mathcal{J}(\lambda)\rangle\langle\mathcal{J}(\mu)\rangle}.\end{aligned}\tag{4.14}$$

Here, we define the connected two-point function, which is given by the following expression:

$$\langle\mathcal{J}(\lambda)\mathcal{J}(\mu)\rangle_{\mathbf{c}}\equiv(\langle\mathcal{J}(\lambda)\mathcal{J}(\mu)\rangle-\langle\mathcal{J}(\lambda)\rangle\langle\mathcal{J}(\mu)\rangle).\tag{4.15}$$

To quantify this explicitly one can define the eigen value distribution function $\mathcal{J}(\lambda)$ in the neighbourhood of extremum of level density $(\overline{\rho}(\lambda))$ as:

$$\mathcal{J}(\lambda)=\widetilde{\mathcal{J}(\lambda)}+\delta\mathcal{J}(\lambda),\tag{4.16}$$

where $\widetilde{\mathcal{J}(\lambda)}$ represents the average value of the eigen value random distribution function over GUE and $\delta\mathcal{J}(\lambda)$ represents the quantum mechanical small fluctuation on $\widetilde{\mathcal{J}(\lambda)}$. Consequently the two point connected function can be expressed by the following simplified expression:

$$\langle\mathcal{J}(\lambda)\mathcal{J}(\mu)\rangle_{\mathbf{c}}=\langle\delta\mathcal{J}(\lambda)\delta\mathcal{J}(\mu)\rangle.\tag{4.17}$$

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Additionally, it is important to note that, the mean level distribution can be normalised in a semi circle using the following two constraints:

$$\frac{1}{N}\int_{-2a}^{2a} d\lambda\ \mathcal{J}(\lambda) = 1, \tag{4.18}$$

$$\int_{-2a}^{2a} d\lambda\ \rho(\lambda) = 1. \tag{4.19}$$

Here $\mathcal{J}(\lambda)$ represents the number of eigen values lying within the interval $(\lambda, \lambda + d\lambda)$ and it is proportional to $\mathcal{O}(\sqrt{N})$. Also, $\rho(\lambda)$ is the density function which can be determined by extremising the action of GUE based RMT, which is treated to be free from N and eigen values which are $\mathcal{O}(1)$. Now we need to find the specific point after which properties of SFF drastically changes. This points are identified as the *critical points* in the present context. Here the partition function is computed for general order polynomial as:

$$\rho(\lambda) = \frac{1}{\pi}\sqrt{4a^2 - \lambda^2}\sum_{k=1}^{n} a_{n-k}\lambda^{(n-k+1)} \qquad \forall \text{ general } n\,. \tag{4.20}$$

Now we calculate two point SFF by following the steps mentioned below:-

1. **Step I:**
   First, of all we compute the expectation of the partition function $Z(\beta+it)$ over GUE based RMT:

$$\begin{aligned}\langle Z(\beta+it)\rangle_{\text{GUE}} &= \frac{1}{\pi}\int_{-2a}^{2a} d\lambda\ e^{-it\lambda}\ e^{-\beta\lambda}\ \sqrt{4a^2-\lambda^2}\times\sum_{k=1}^{n} d_{n-k}\lambda^{-k+n+1} \\ &\equiv \sum_{k=1}^{n} a(-a^2)^{1-k}2^{-k+n+1}d_{n-k}\left((a^k(-a)^n-(-a)^k a^n)\Gamma\left(\frac{1}{2}(-k+n+2)\right)\times\right. \\ &\quad {}_1\tilde{F}_2\left(\frac{1}{2}(-k+n+2);\frac{1}{2},\frac{1}{2}(-k+n+5);-a^2(t-i\beta)^2\right)+a(\beta+it)((-a)^k a^n+a^k(-a)^n)\times \\ &\quad \left.\Gamma\left(\frac{1}{2}(-k+n+3)\right)\,{}_1\tilde{F}_2\left(\frac{1}{2}(-k+n+3);\frac{3}{2},\frac{1}{2}(-k+n+6);-a^2(t-i\beta)^2\right)\right).\end{aligned} \tag{4.21}$$

2. **Step II:**

   Secondly, we compute the expectation of the partition function $Z(\beta - it)$ over GUE based RMT:

$$\begin{aligned}\langle Z(\beta-it)\rangle_{\text{GUE}} &= \frac{1}{\pi}\int_{-2a}^{2a} d\lambda\ e^{it\lambda}\ e^{-\beta\lambda}\ \sqrt{4a^2-\lambda^2}\times\sum_{k=1}^{n} d_{n-k}\lambda^{-k+n+1} \\ &\equiv \sum_{k=1}^{n} a^3(-a^2)^{-k}2^{-k+n+1}d_{n-k}\left(i(e^{i\pi k}+e^{i\pi n})(t+i\beta)a^{k+n+1}\Gamma\left(\frac{1}{2}(-k+n+3)\right)\times\right. \\ &\quad {}_1\tilde{F}_2\left(\frac{1}{2}(-k+n+3);\frac{3}{2},\frac{1}{2}(-k+n+6);-a^2(t+i\beta)^2\right)+((-a)^k a^n-a^k(-a)^n)\times \\ &\quad \left.\Gamma\left(\frac{1}{2}(-k+n+2)\right)\,{}_1\tilde{F}_2\left(\frac{1}{2}(-k+n+2);\frac{1}{2},\frac{1}{2}(-k+n+5);-a^2(t+i\beta)^2\right)\right).\end{aligned}$$

3. **Step III:**
Then, we compute the expectation of the partition function $Z(\beta)$ over GUE based RMT:

$$\langle Z(\beta)\rangle_{\rm GUE} = \sum_{k=1}^{n} a^3(-a^2)^{-k}(-2^{-k+n+1})a_{n-k}\left(\beta((-1)^k+(-1)^n)a^{k+n+1}\Gamma\left(\frac{1}{2}(-k+n+3)\right)\times\right.$$
$${}_1\tilde{F}_2\left(\frac{1}{2}(-k+n+3);\frac{3}{2},\frac{1}{2}(-k+n+6);a^2\beta^2\right)+(a^k(-a)^n-(-a)^k a^n)\Gamma\left(\frac{1}{2}(-k+n+2)\right)\times$$
$$\left.{}_1\tilde{F}_2\left(\frac{1}{2}(-k+n+2);\frac{1}{2},\frac{1}{2}(-k+n+5);a^2\beta^2\right)\right).$$

4. **Step IV:**
At finite temperature the disconnected part of the Green's function $(G_{dc}(\beta,t))$ can be expressed as:

$$G_{dc}(\beta,t) = \frac{\langle Z(\beta+it)\rangle\langle Z(\beta-it)\rangle}{\langle |Z(\beta)|^2\rangle}, \tag{4.22}$$

where each of the components of the disconnected Green's function are computed earlier.

5. **Step V:**
The connected part of the Green's function $\mathcal{G}_c$ depends on the two point function $\langle \delta\mathcal{J}(\lambda)\delta\mathcal{J}(\mu)\rangle$. From GUE based RMT the exact functional form near the centre of spectrum of the eigen values of the random distribution can be expressed in the following form:

$$\langle \delta\mathcal{J}(\lambda)\delta\mathcal{J}(\mu)\rangle = -\frac{\sin^2[N(\lambda-\mu)]}{(\pi N(\lambda-\mu))^2} + \frac{1}{\pi N}\delta(\lambda-\mu) \tag{4.23}$$

which can be derived using the method of orthogonal polynomials for GUE.[6]

There are two parts appearing in the above mentioned kernel, which give different physical measures:

(a) $1/N^2$ part with sine squared function gives the ramp and have sub-dominant contribution in the integral kernel.

(b) $1/N$ part with Delta function gives the plateau and dominant contribution in the integral kernel.

6. **Step VI:**
Next, we get the following simplified expression for the connected part of the Green's function $\mathcal{G}_c$ as given by:

$$\mathcal{G}_c(t) = \frac{1}{N^2}\int d\lambda\, d\mu\, e^{-it(\lambda-\mu)}\left[-\frac{\sin^2[N(\lambda-\mu)]}{(\pi N(\lambda-\mu))^2} + \frac{1}{\pi N}\delta(\lambda-\mu)\right]. \tag{4.24}$$

[6] *Important Note IV*: the derived expression for this sine kernel is only valid for any polynomial potential measure whose matrix (operator) is of single trace. Various polynomial potentials change only the eigen value distribution near edges of the distribution.

To perform the integral we further substitute,

$$\lambda + \mu = E, \tag{4.25}$$

$$\lambda - \mu = \omega. \tag{4.26}$$

Since the integral over E gives trivial Diarc Delta function we choose our working region for which $E = 0$ (at hight temperature limit). Then the remaining integrand is only over ω and it finally gives:

$$\mathcal{S}(t) = N^2 \mathcal{G}_c(t) = \int_{-\infty}^{\infty} d\omega\; e^{-it\omega} \; \left[-\frac{1}{\pi^2}\frac{\sin^2[N\omega]}{(N\omega)^2} + \frac{1}{\pi N}\delta(\omega)\right]. \tag{4.27}$$

which gives us finally the following simplified expression:

$$\mathcal{S}(t) = \begin{cases} \dfrac{t}{(2\pi N)^2} - \dfrac{1}{N} + \dfrac{1}{(\pi N)}\,, & t < 2\pi N \\ \dfrac{1}{\pi N}\,, & t > 2\pi N \end{cases}, \tag{4.28}$$

7. **Step VII:**
Now to compute two point SFF we need to add both connected and disconnected part of the Green's function. Therefore, for different polynomial potential we get finally the following expression for two point SFF from GUE based RMT at finite temperature:

$$\mathbf{S}_2(\beta,t) \equiv \mathcal{G}(\beta,t) = \begin{cases} \mathcal{G}_{dc}(\beta,t) + \dfrac{t}{(2\pi N)^2} - \dfrac{1}{N} + \dfrac{1}{(\pi N)}\,, & t < 2\pi N \\ \mathcal{G}_{dc}(\beta,t) + \dfrac{1}{\pi N}\,, & t > 2\pi N \end{cases}, \tag{4.29}$$

where $\mathbf{S}_2(\beta,t)$ is defined with proper normalization.

8. **Step VIII:**
After substituting the expression for disconnected part we get the following expression for the two point SFF from GUE based RMT at finite temperature:

$$\begin{aligned}\mathbf{S}_2(\beta,t) \equiv \Bigg\{ &\sum_{k=1}^{n} a^3(-a^2)^{-k}(-2^{-k+n+1})a_{n-k} \\ &\times(\beta((-1)^k+(-1)^n)a^{k+n+1}\Gamma\left(\frac{-k+n+3}{2}\right) \\ &\times {}_1\tilde{F}_2\left(\frac{1}{2}(-k+n+3); \frac{3}{2}, \frac{1}{2}(-k+n+6); a^2\beta^2\right) \\ &+(a^k(-a)^n-(-a)^k a^n)\Gamma\left(\frac{-k+n+2}{2}\right) \\ &\times {}_1\tilde{F}_2\left(\frac{1}{2}(-k+n+2); \frac{1}{2}, \frac{1}{2}(-k+n+5); a^2\beta^2\right)\Bigg\}^{-2}\end{aligned}$$

$$
\begin{aligned}
&\times\Bigg\{\sum_{k=1}^{n} a(-a^2)^{1-k}2^{-k+n+1}b_{n-k} \\
&\qquad\times\left((a^k(-a)^n-(-a)^k a^n)\Gamma\left(\frac{-k+n+2}{2}\right)\right. \\
&\qquad\times{}_1\tilde{F}_2\left(\frac{-k+n+2}{2};\frac{1}{2},\frac{-k+n+5}{2};-a^2(t-i\beta)^2\right) \\
&\qquad+a(\beta+it)((-a)^k a^n+a^k(-a)^n) \\
&\qquad\left.\times\Gamma\left(\frac{-k+n+3}{2}\right)\,{}_1\tilde{F}_2\left(\frac{-k+n+3}{2};\frac{3}{2},\frac{-k+n+6}{2};-a^2(t-i\beta)^2\right)\right)\Bigg\} \\
&\times\sum_{k=1}^{n} a^3(-a^2)^{-k}2^{-k+n+1}d_{n-k} \\
&\times\left(i(e^{i\pi k}+e^{i\pi n})(t+i\beta)a^{k+n+1}\Gamma\left(\frac{1}{2}(-k+n+3)\right)\right. \\
&\qquad\times{}_1\tilde{F}_2\left(\frac{1}{2}(-k+n+3);\frac{3}{2},\frac{1}{2}(-k+n+6);-a^2(t+i\beta)^2\right) \\
&\qquad+((-a)^k a^n-a^k(-a)^n) \\
&\qquad\left.\times\Gamma\left(\frac{1}{2}(-k+n+2)\right)\,{}_1\tilde{F}_2\left(\frac{1}{2}(-k+n+2);\frac{1}{2},\frac{1}{2}(-k+n+5);-a^2(t+i\beta)^2\right)\right) \\
&+\begin{cases}\dfrac{t}{(2\pi N)^2}-\dfrac{1}{N}+\dfrac{1}{(\pi N)}\,, & t<2\pi N \\[2mm] \dfrac{1}{\pi N}\,, & t>2\pi N\,.\end{cases}
\end{aligned}
\tag{4.30}
$$

9. **Step IX:**
   Further taking the high temperature limit we get the following simplified expression for two point SFF from GUE based RMT:

$$
\begin{aligned}
\mathbf{S}_2(t)\equiv\frac{1}{N^2}&\Bigg\{\sum_{k=1}^{n} a(-a^2)^{1-k}2^{-k+n+1}b_{n-k} \\
&\times\left((a^k(-a)^n-(-a)^k a^n)\Gamma\left(\frac{1}{2}(-k+n+2)\right)\right. \\
&\times{}_1\tilde{F}_2\left(\frac{1}{2}(-k+n+2);\frac{1}{2},\frac{1}{2}(-k+n+5);-a^2t^2\right) \\
&+iat((-a)^k a^n+a^k(-a)^n) \\
&\left.\times\Gamma\left(\frac{1}{2}(-k+n+3)\right)\,{}_1\tilde{F}_2\left(\frac{1}{2}(-k+n+3);\frac{3}{2},\frac{1}{2}(-k+n+6);-a^2t^2\right)\right)\Bigg\}
\end{aligned}
$$

$$\times \left\{ \sum_{k=1}^{n} a^3(-a^2)^{-k} 2^{-k+n+1} d_{n-k} \right.$$
$$\times \left(((-a)^k a^n - a^k (-a)^n) \Gamma\left(\frac{1}{2}(-k+n+2)\right) \right.$$
$$\times {}_1\tilde{F}_2\left(\frac{1}{2}(-k+n+2); \frac{1}{2}, \frac{1}{2}(-k+n+5); -a^2t^2\right)$$
$$+ it(e^{i\pi k} + e^{i\pi n}) a^{k+n+1}$$
$$\left. \left. \times \Gamma\left(\frac{1}{2}(-k+n+3)\right) {}_1\tilde{F}_2\left(\frac{1}{2}(-k+n+3); \frac{3}{2}, \frac{1}{2}(-k+n+6); -a^2t^2\right) \right) \right\}$$
$$+ \begin{cases} \dfrac{t}{(2\pi N)^2} - \dfrac{1}{N} + \dfrac{1}{(\pi N)}, & t < 2\pi N \\ \dfrac{1}{\pi N}, & t > 2\pi N\,. \end{cases} \tag{4.31}$$

This equation for the nearest neighbor spacing distributions of random matrix models are not as simple as this. Also, at infinite temperature, it picks out contribution form the difference between nearest neighbor energy eigenvalues at very late time scale. Further averaging over Gaussian random matrices, two point SFF shows very particular behaviour at large N, particularly initial decay followed by a linear rise and then a saturation after critical point. This saturation can be related to saturation limit for large N which finally fix the *bound on quantum chaos.*

5 Bound on two point SFF from GUE based RMT

Now we will talk about the HypergeometricPFQregularized function which is appearing in the expression for two point SFF. This function can be related to well known Hypergeometric function of single variable by the following expression:

$${}_p\tilde{F}_q\left(a_1,\ldots,a_p;b_1,\ldots,b_q;z\right) = \frac{{}_pF_q\left(a_1,\ldots,a_p;b_1,\ldots,b_q;z\right)}{\prod_{k=1}^{q}\Gamma\left(b_k\right)}. \tag{5.1}$$

To derive the required bound on two point SFF from the GUE based RMT setup we first use the asymptotic behavior of regularized HypergeomtericPFQ function [11],[7] which is precisely given below:

$$\lim_{t\to\infty} {}_1\tilde{F}_2\left[A(t);B(t),C(t);a^2(\beta\pm it)^2\right] = 0 \;\; \forall\; k = 1,\cdots,n \tag{5.2}$$

[7] Mathematica Function Page for HypergeometricPFQ regularized function http://functions.wolfram.com/07.22.02.0001.01.

where $A(t), B(t)$ and $C(t)$ are time independent variables. Also n represents the highest order of the polynomial of the general random potential under consideration for this discussion. Now from eq. (4.31) it is important to note that every Hypergeomtric PFQ function has finite number in its first three argument as long as order of polynomial n is finite. Consequently, the asymptotic behaviour of the connected and disconnected part of the Green's function in the regime $t > 2\pi N$ can be expressed with finite N by the following expression:

$$\lim_{t\to\infty} \mathcal{G}_c(\beta, t) = \begin{cases} \dfrac{1}{\pi N}, & \beta \neq 0 \\ 0. & \beta = 0 \end{cases}, \tag{5.3}$$

$$\lim_{t\to\infty} \mathcal{G}_{dc}(\beta, t) = 0 \quad \forall\, t > 2\pi N,\ \forall\, \beta. \tag{5.4}$$

Finally, adding the contribution from the disconnected and connected part of the Green's function in the asymptotic limit we get the following simplified expression for two point SFF from GUE based RMT in the regime $t > 2\pi N$ for finite N:

$$\lim_{t\to\infty} \mathbf{S}_2(\beta, t > 2\pi N) = \begin{cases} \dfrac{1}{\pi N}, & \beta \neq 0 \\ 0. & \beta = 0 \end{cases}, \tag{5.5}$$

For $t \to \infty$ asymptotic limit till now we have only considered the part of two point SFF from GUE based RMT appearing only after the time scale, $t > 2\pi N$ with finite N. On the other hand, the main obstacle of taking $t \to \infty$ asymptotic limit in the regime, $t < 2\pi N$ with finite N is we get divergent contribution in the connected part of the Green's function from the term $t/(2\pi N)^2 \to \infty$. Also, the disconnected part of the Green's function behave same as eq. (5.4). Consequently, in the regime $t < 2\pi N$ with finite N the two point SFF for GUE based RMT can be computed as:

$$\lim_{t\to\infty} \mathbf{S}_2(\beta, t < 2\pi N) = \infty \quad \forall\, \beta. \tag{5.6}$$

Hence, combining both the contribution from connected and disconnected part of the total Green's function we get the following upper and lower bound on two point SFF for GUE based RMT in the regime, $t > 2\pi N$, as given by:

Quantum Chaos Saturation Bound I: $$0 \leq \mathbf{S}_2(\beta, t > 2\pi N) \leq \frac{1}{\pi N} \quad \forall\, 0 \leq \beta \leq \infty\,. \tag{5.7}$$

With large N asymptotic limit in the time regime, $t < 2\pi N$, gives finite contribution to the connected and disconnected part of the Green's function as given by:

$$\lim_{t\to\infty} \mathcal{G}_c(\beta, t) \simeq -\frac{1}{N}\left(1 - \frac{1}{\pi}\right), \quad \forall\, \beta \tag{5.8}$$

$$\lim_{t\to\infty} \mathcal{G}_{dc}(\beta, t) = 0 \qquad \forall\, \beta. \tag{5.9}$$

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Further adding both the contribution from connected and disconnected part of the Green's function for the asymptotic region $t < 2\pi N$ with large N, we get the following upper and lower bound on two point SFF from GUE based RMT, as given by:

$$\textbf{Quantum Chaos Saturation Bound II:} \quad -\frac{1}{N}\left(1-\frac{1}{\pi}\right) \leq \mathbf{SFF}(\beta, t < 2\pi N) \leq 0 \quad \forall\, 0 \leq \beta \leq \infty. \tag{5.10}$$

In figure 1 we have explicitly shown the time dependent behaviour of two point SFF from GUE based RMT at fixed finite temperatures. Additionally, we have explicitly depicted the saturation bound on quantum chaos both in the regime, $t > 2\pi N$ (for finite N) and $t < 2\pi N$ (for large N) respectively. We use a general n th order polynomial random potential to derive the saturation bound on quantum chaos. The symmetry property for $\lambda \to -\lambda$ is lost and non-vanishing support should be chosen [12]. Particularly for odd polynomial potential the resolvent method is not applicable. But for the even and general one it works perfectly well.

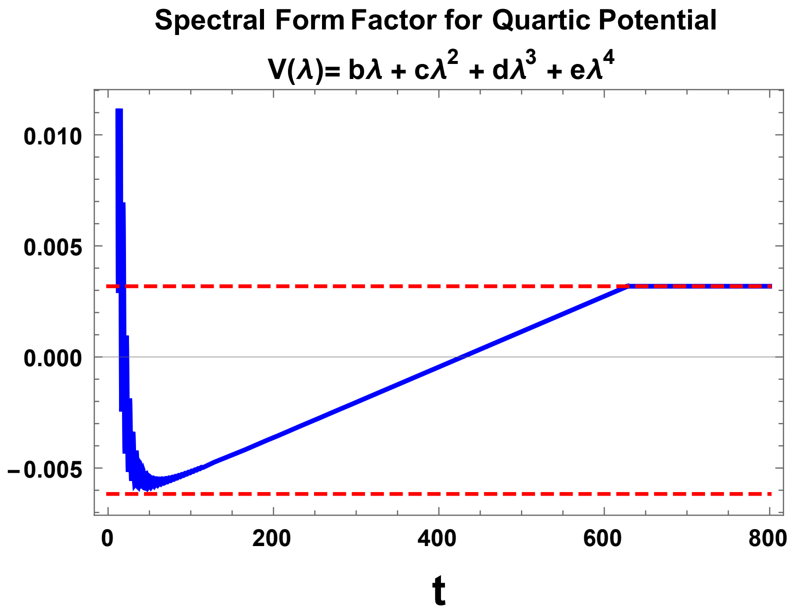
6 Early time behaviour of two point SFF from GUE based RMT

In this section we analyse the early time behaviour of two point SFF from GUE based RMT. To serve this purpose we start our discussion we use eq. (4.30) and eq. (4.31),in the limit $t \to 0$. To explain early time behaviour of quantum chaos from two point SFF from GUE based RMT we first use the asymptotic behavior of regularized HypergeomtericPFQ function [13], which is given below:

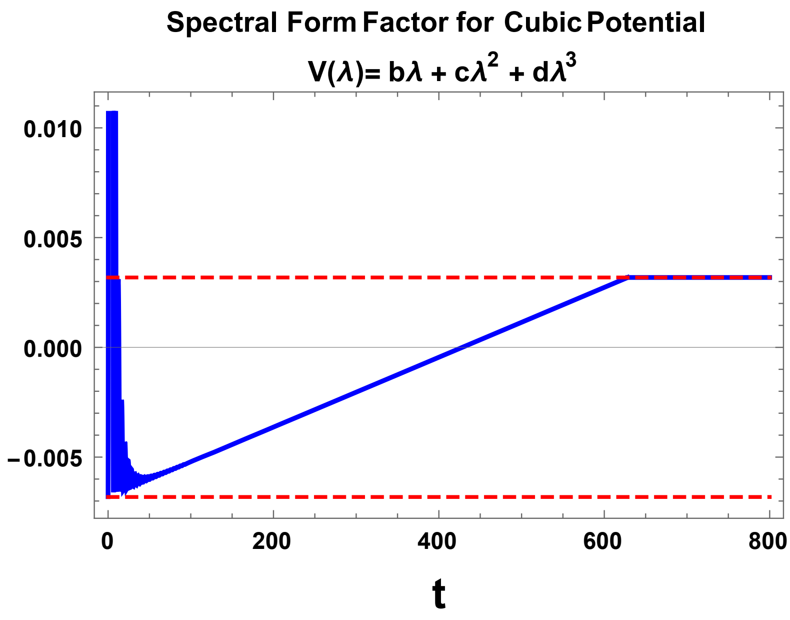
$$\lim_{t\to 0} {}_1\tilde{F}_2\left[A(t); B(t), C(t); a^2(\beta \pm it)^2\right] = 1 \quad \forall\; k = 1, \cdots, n \tag{6.1}$$

where $A(t), B(t)$ and $C(t)$ are time independent variables. Also n represents the highest order of the polynomial of the general random potential under consideration for this discussion. This gives the following simplified expression for two point SFF from GUE based RMT at early time scale (small t):

$$\begin{aligned}\mathbf{S}_2(\beta,t) = &\left\{\sum_{k=1}^{n} a^3(-a^2)^{-k}(-2^{-k+n+1})a_{n-k}\right.\\ &\quad\times\left(\beta((-1)^k+(-1)^n)a^{k+n+1}\Gamma\left(\frac{1}{2}(-k+n+3)\right)\right.\\ &\quad\quad\left.\left.+(a^k(-a)^n-(-a)^k a^n)\Gamma\left(\frac{1}{2}(-k+n+2)\right)\right)\right\}^{-2}\\ &\times\left[\sum_{k=1}^{n} a(-a^2)^{1-k}2^{-k+n+1}b_{n-k}\right.\\ &\quad\times\left((a^k(-a)^n-(-a)^k a^n)\Gamma\left(\frac{1}{2}(-k+n+2)\right)\right.\\ &\quad\quad\left.\left.+a(\beta+it)((-a)^k a^n+a^k(-a)^n)\Gamma\left(\frac{1}{2}(-k+n+3)\right)\right)\right]\end{aligned}$$



(a)



(b)

Figure 1. Behavior of SFF at finite temperature for different quartic and cubic) polynomial random potentials.

$$
\times\left[\sum_{k=1}^{n} a^3(-a^2)^{-k}2^{-k+n+1}d_{n-k}\right.
$$

$$
\times\left(((-a)^k a^n - a^k(-a)^n)\Gamma\left(\frac{1}{2}(-k+n+2)\right)\right.
$$

$$
\left.\left. + i(e^{i\pi k}+e^{i\pi n})(t+i\beta)a^{k+n+1}\Gamma\left(\frac{1}{2}(-k+n+3)\right)\right)\right]
$$

$$
+\begin{cases} \dfrac{t}{(2\pi N)^2}-\dfrac{1}{N}+\dfrac{1}{(\pi N)}\,, & t<2\pi N \\ \dfrac{1}{\pi N}\,, & t>2\pi N\,. \end{cases} \tag{6.2}
$$

Further, taking the high temperature limit, $\beta \to 0$ the two point SFF from GUE based RMT can be recast at early time scale as:

$$
\mathbf{S}_2(t) = \frac{1}{N^2}\left[\sum_{k=1}^{n} a(-a^2)^{1-k}2^{-k+n+1}b_{n-k}\right.
$$

$$
\times\left((a^k(-a)^n - (-a)^k a^n)\Gamma\left(\frac{1}{2}(-k+n+2)\right) + iat((-a)^k a^n\right.
$$

$$
\left.\left. + a^k(-a)^n)\Gamma\left(\frac{1}{2}(-k+n+3)\right)\right)\right]
$$

$$
\times\left[\sum_{k=1}^{n} a^3(-a^2)^{-k}2^{-k+n+1}d_{n-k}\right.
$$

$$
\times\left(((-a)^k a^n - a^k(-a)^n)\Gamma\left(\frac{1}{2}(-k+n+2)\right)\right.
$$

$$
\left.\left. + it(e^{i\pi k}+e^{i\pi n})a^{k+n+1}\Gamma\left(\frac{1}{2}(-k+n+3)\right)\right)\right]
$$

$$
+\begin{cases} \dfrac{t}{(2\pi N)^2}-\dfrac{1}{N}+\dfrac{1}{(\pi N)}\,, & t<2\pi N \\ \dfrac{1}{\pi N}\,, & t>2\pi N\,. \end{cases} \tag{6.3}
$$

Finally, at exactly $t = 0$ the two point SFF from GUE based RMT takes a finite value and it actually depends on the nature and highest degree of the polynomial potential of the random distribution. It is expressed by the following simplified result:

$$
\mathbf{S}_2(\beta\to 0, t=0) = \frac{1}{N^2}\left[\sum_{k=1}^{n} a(-a^2)^{1-k}2^{-k+n+1}b_{n-k}\right.
$$

$$
\left.\times\left((a^k(-a)^n - (-a)^k a^n)\Gamma\left(\frac{1}{2}(-k+n+2)\right)\right)\right]
$$

$$\times\left[\sum_{k=1}^{n} a^3(-a^2)^{-k}2^{-k+n+1}d_{n-k}\right.$$
$$\left.\times\left(((-a)^k a^n - a^k(-a)^n)\Gamma\left(\frac{1}{2}(-k+n+2)\right)\right)\right]$$
$$-\frac{1}{N}+\frac{1}{(\pi N)}, \qquad \forall\, t<2\pi N\,. \tag{6.4}$$

The last two terms are explicitly appearing due to the disconnected part of the Green's function appearing for the time regime, $t < 2\pi N$ at $t = 0$. For any finite order of the polynomial n this has some constant magnitude which depends on the co-efficient of the polynomial random potential.

In same way we can calculate the two point SFF from GUE based RMT for finite vale of β (finite temperature), which is expressed by the following simplified expression:

$$\mathbf{S}_2(\beta\to\text{finite}, t=0) = \left\{\sum_{k=1}^{n} a^3(-a^2)^{-k}(-2^{-k+n+1})a_{n-k}\right.$$
$$\times\left(\beta((-1)^k+(-1)^n)a^{k+n+1}\Gamma\left(\frac{1}{2}(-k+n+3)\right)\right.$$
$$\left.\left.+(a^k(-a)^n-(-a)^k a^n)\Gamma\left(\frac{1}{2}(-k+n+2)\right)\right)\right\}^{-2}$$
$$\times\left[\sum_{k=1}^{n} a(-a^2)^{1-k}2^{-k+n+1}b_{n-k}\right.$$
$$\times\left((a^k(-a)^n-(-a)^k a^n)\Gamma\left(\frac{1}{2}(-k+n+2)\right)\right.$$
$$\left.\left.+a(\beta)((-a)^k a^n+a^k(-a)^n)\Gamma\left(\frac{1}{2}(-k+n+3)\right)\right)\right]$$
$$\times\left[\sum_{k=1}^{n} a^3(-a^2)^{-k}2^{-k+n+1}d_{n-k}\right.$$
$$\times\left(((-a)^k a^n-a^k(-a)^n)\Gamma\left(\frac{1}{2}(-k+n+2)\right)\right.$$
$$\left.\left.-\beta(e^{i\pi k}+e^{i\pi n})a^{k+n+1}\Gamma\left(\frac{1}{2}(-k+n+3)\right)\right)\right]$$
$$-\frac{1}{N}+\frac{1}{(\pi N)}, \qquad \forall\, t<2\pi N\,. \tag{6.5}$$

In figure 2a, figure 2b, figure 2d and figure 2c we have shown the early time behaviour of two point SFF from GUE based RMT which shows increment upto a certain limiting point. In figure 1, the two point SFF from GUE based RMT near $t = 0$ has a finite large

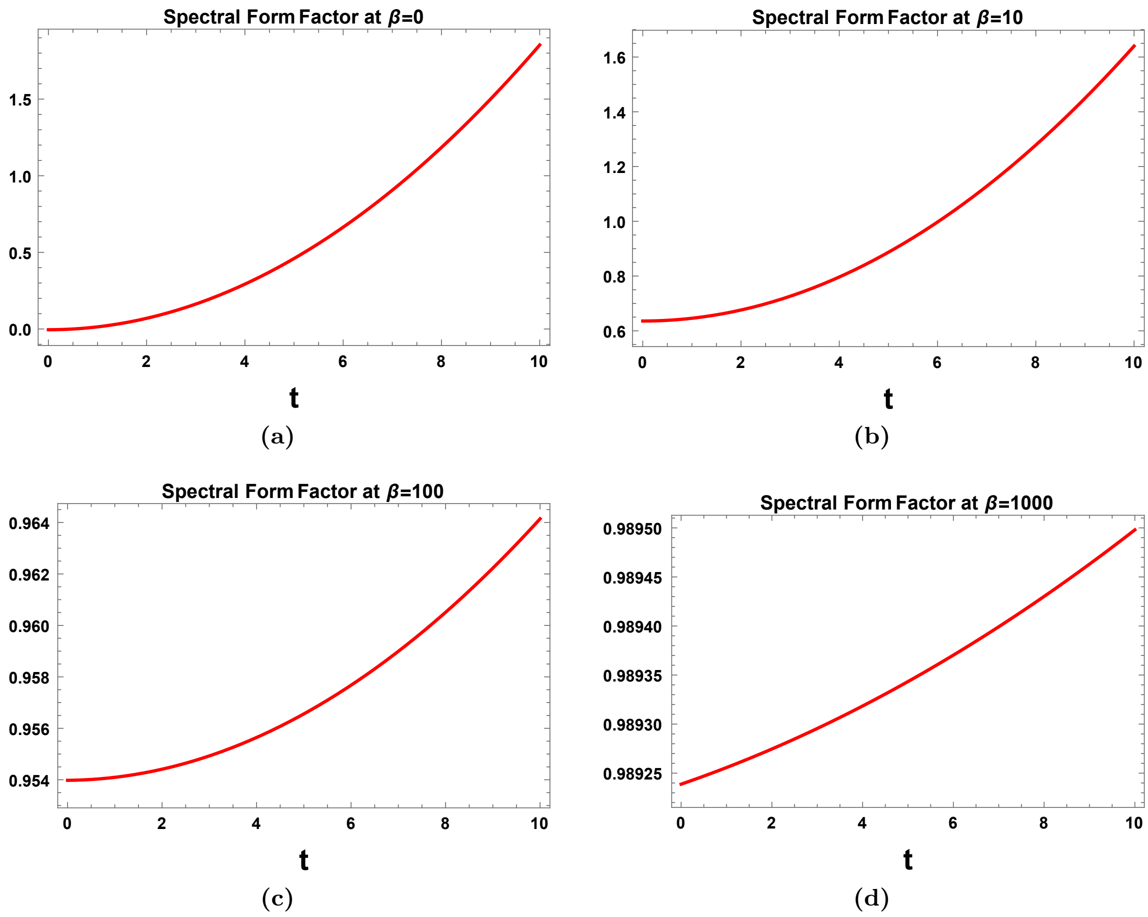


Figure 2. Early time behavior for cubic potential.

value. Starting from the constant value at $t = 0$,[8] one can observe that it actually increases near $t = 0$ exponentially to a finite value and then it merges to the normal behaviour of two point SFF from GUE based RMT and show the oscillatory behaviour and at last saturate at the value where we fix the bound on quantum chaos. In general early time behaviour has no effect to fix the saturation bound on quantum chaos from GUE based RMT. The saturation bound on two point SFF from GUE based RMT is defined previously at large time scale as no considerable effect is coming from the early time scale. But if we choose the value of N in such a way that we can come in the regime of early time chaos then we found that the disconnected part of the thermal Green's function is not zero for all time scales, where $t > 2\pi N$. This implies that, we have to wait until the time has reached infinity by tuning the parameters of the general n th degree polynomial of the random potential under consideration for this specific discussion. Further, at late time scale we found that again the value of two point SFF from GUE based RMT reached the saturation bound of quantum chaos. We explicitly observed that in the small time scale the value of two point

[8]For cubic potential this is $\frac{b_1 d_1}{a_1^2}$ which can be evaluated using method of resolvent and then putting only parameter inside the potential.

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SFF from GUE based RMT depicts almost exponential increase with small oscillation and then will reach the saturation bound at late time scales.

Also, we observe the following features by studying the behaviour of two point SFF from GUE based RMT at early time scale:

1. Value of the two point SFF from GUE based RMT at $t = 0$ has a particular nature. For any generic odd polynomial potential for the random distribution the value of two point SFF from GUE based RMT exactly same as appearing for any generic even polynomial potential except for the specific case appearing for $n = 1$ order polynomial potential.

2. We also observed that the early time chaos become dominant at very small value of number of degrees of freedom N.In this specific situation, the two point SFF from GUE based RMT increase from its zero point value and show exponential increment and as soon as the point $t = 2\pi N$ is crossed it actually converge to the same saturation bound of quantum chaos. So from this discussion it is very clearly appearing as the saturation bound on the two point SFF from GUE based RMT is dependent on only the choice of the value of number of degrees of freedom N.

3. As we go for larger values of β (small temperature) the growth of the two point SFF from GUE based RMT become approximately linear in nature. This can be explained based on the fact that the Hypergeometric functions are also function of the variable β. So, at $\beta \to \infty$ we found that the disconnected part of the thermal Green's function identically vanishes again. It is also observed that the increase in the two point SFF from GUE based RMT is due to the non vanishing contrbution appearing in the connected part of the thermal Green's function, which is actually linear in t.

7 Integrablity from GUE based RMT: Toda Lattice model

In this section our prime objective is to discuss the integrability issue from the GUE based RMT set up. This actually justifies the application of our derived analysis performed in this paper. To serve this purpose we need to consider the following steps:

1. Let us firsst consider an example of dispersionless Integrable system in the light of RMT set up [14], where the partition function of GUE of random matrices has a direct representation in Toda lattice hierarchy. Asymptotic expansion of the free energies given by the logarithm of the partition function leads to continuous limits of Toda and Pfaff lattice hierarchies. For more details see refs. [15–19]. The Toda lattice equation is given by the following expressions:

$$\frac{\partial a_n}{\partial t_1} = a_n(b_{n+1} - b_n) \tag{7.1}$$

$$\frac{\partial b_n}{\partial t_2} = a_n - a_{n-1} \qquad \forall\ n = 1, 2, 3. \tag{7.2}$$

The sequence of $\{\tau_n : n \geq 0\}$ τ-functions with $\tau_0 = 1$ fix the coefficients a_n b_n as given by:

$$a_n = \frac{\tau_{n+1}\tau_{n-1}}{\tau_n^2}, \tag{7.3}$$

$$b_n = \frac{\partial}{\partial t_1}\log\left(\frac{\tau_n}{\tau_n - 1}\right). \tag{7.4}$$

Therefore, Toda lattice equation in *Hirota bilinear* form can be simplified as:

$$\hat{\mathcal{D}}_1^2 \tau_n.\tau_n = 2\tau_{n+1}\tau_{n-1}, \tag{7.5}$$

where, $\hat{\mathcal{D}}_1$ is the usual *Hirota derivative* for a variable x_n, representing a flow-parameter for n-th member of Toda lattice hierarchy, can be written as:

$$\hat{\mathcal{D}}_n f.g := \left(\frac{\partial}{\partial x_n} - \frac{\partial}{\partial x_n'}\right) f(x_n)g(x_n')|_{x_n = x_n'}\,. \tag{7.6}$$

Now, using the well known Lax representation the hierarchy of the Toda lattice is defined as:

$$\frac{\partial L}{\partial t_k} = [L, B_k], \qquad B_k = [L^k]_-, \qquad k = 1, 2, 3, \ldots \tag{7.7}$$

Then τ_n-functions also satisfy KP-hierarchy [20, 21] and one can get the following sets of equations:

$$(\hat{\mathcal{D}}_k - h_k(\hat{D})\tau_{n+1}.\tau_n = 0 \qquad k = 2, 3, 4\,. \tag{7.8}$$

Here it is important to note that, for $k = 2$ this gives the non-linear *Schrödinger* equation which is interpreted as the second member of toda lattice hierarchy.

In continuous limiting approximation free energy term has been introduced explicitly, which is given by the following expression:

$$\tau_n(t;\hbar) = \exp\left(\frac{1}{\hbar^2}F(T_0, T) + \mathcal{O}(h^{-1})\right). \tag{7.9}$$

Here $\hbar$ is a small parameter and $T = (T_1, T_2, \ldots)$ is the slow variables with the following form:

$$T_k = \hbar t_k \quad \text{for } n \geq 1. \tag{7.10}$$

It is important to note that, for $\hbar \to 0$ gives a continuous limit of the lattice structure. The Free energy $F(T_0, T)$ is given as the following:

$$F(T_0, T) = \lim_{\hbar\to 0} \hbar^2 \log\left[\tau_k(\hbar^{-1}T;\hbar)\right], \tag{7.11}$$

which can be further represented by two point functions of corresponding topological quantum field theory (TQFT) as:

$$F_{mn} = \frac{\partial^2}{\partial T_m \partial T_n} \qquad m, n \geq 0. \tag{7.12}$$

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In the continuous limiting approximation, the spectral problem represented by the following equation:

$$L\phi = \lambda\phi, \tag{7.13}$$

with Lax operator and the corresponding eigen vector $\phi = (\phi_1, \phi_2, \dots)^T$ gives the spectral curve for Toda equation:

$$\lambda = \left(p(\lambda) + F_{01} + \frac{e^{F_0 0}}{p(\lambda)}\right), \tag{7.14}$$

where $p(\lambda)$ represents a quasi-momentum in the present context. The eigen vector can be represented by the WKB form at $\hbar \to 0$ limit, as given by:

$$\phi_n(t;\hbar) = \exp\left[\frac{1}{\hbar}S(T_0, T) + \mathcal{O}(1)\right], \tag{7.15}$$

where we introduce a function $S(T_0, T)$, which is defined as:

$$S(T_0, T) = \sum_{n=1}^{\infty} \lambda^n T_n + T_0 \log\lambda - \sum_{k=1}^{\infty} \frac{1}{k\lambda^k} F_k(T_0, T). \tag{7.16}$$

Special class of Toda lattice equation solution defines the partition function of GUE of random matrices. The τ-functions are taken to be the following form:

$$\tau_n(t) = Z_n^2(V_0(\lambda); t) = \int_{\Re} \dots \int_{\Re} \prod_{i<j} |\lambda_i - \lambda_j|^2 \exp\left[-\sum_{k=1}^{n} V_t(\lambda_k)\right], \tag{7.17}$$

with the following expression for the random potential:

$$V_t(\lambda_k) = V_0(\lambda_k) - \sum_{j=1}^{\infty} \lambda_k^j t_j. \tag{7.18}$$

Here$V_0(\lambda)$ is an even degree polynomial.

Now it is important to note that, for GUE we have the following expression:

$$\log\left[\frac{Z_N^{(2)}\left(\frac{N}{2}\lambda^2; NT\right)}{Z_N^{(2)}\left(\frac{N}{2}\lambda^2; 0\right)}\right] = \sum_{g\geq 0} e_g(T) N^{2-2g}, \tag{7.19}$$

where we define $e_g(T)$ by the following expression:

$$e_g(T) = \sum_{0\leq j_1, j_2, \dots} \kappa_g(j_1, j_2, \dots) \frac{T_1^{j_1} T_2^{j_2} T_3^{j_3} \dots}{j_1! j_2! \dots}. \tag{7.20}$$

A fundamental result of RMT is that for a quardatic potential the function $V_0(\lambda)$ and $\log(Z_N^{(2)})$ posses as asymptotic expansion in even powers of N, whose terms give generating partitioning ribbon graphs by genus. From this argument it can be explicitly shown that continuous assumption of Toda lattice is satisfied by GUE based RMT. From this calculated partition function we can further calculate the two point SFF for Toda Lattice within GUE.

Now, we know that Toda lattice system is a completely integrable system and in continuous limiting approximation the present idea can be easily related to GUE based RMT model.

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2. Now, to justify the applicability and validity of our calculated saturation bound on two point SFF we choose slight modification in Toda lattice system and from previously shown result on two point correlation function this bound has been explicitly justified in the present context. It has been shown, that after applying replica trick the partition function of GUE in its original and dual (σ model) representation satisfy Toda Lattice equation. For unitary ensemble with probability density function represented by the following expression:

$$P(H) = \exp\left[-\mathrm{Tr}\left(V(H)\right)\right], \tag{7.21}$$

with the following logarithmic confining potential, given by:

$$V(x) = \sum_{n=0}^{\infty} \ln[1 + 2q^{n+1}\ \sqrt{1+4x^2} + q^{2n+2}], \tag{7.22}$$

where we define the factors x and q by the following expressions:

$$x \equiv \frac{1}{2}\sinh\beta_{\chi}, \tag{7.23}$$

$$q \equiv e^{-\beta}\ \beta > 0. \tag{7.24}$$

This confining potential has been chosen for the calculation of two point correlation function for such model. Here it is important to note that, in the asymptotic limit, $|x| \to \infty$ the confining potential is behaving by the following fashion:

$$\lim_{|x|\to\infty} V(x) \propto \ln^2(|x|). \tag{7.25}$$

This model is very famous because of its eigenvalue distribution for $\beta < \pi^2$ is intermediate between *Wigner-Dyson* and *Poisson distribution* and explicitly have qualities of critical level statistics.

3. Now consider a matrix H, which can be expressed as:

$$H^2 = \frac{1}{4}\left(TT^{\dagger} + T^{\dagger}T - 2I\right), \tag{7.26}$$

where T is transfer matrix defining the disorder inside a conductor.

The replica partition function for this model has been defined by the following expression:

$$\begin{aligned} Z_{n,N}(\epsilon) &= \langle \det{}^n(\epsilon - H)\rangle_H \\ &= (C_M)^N \int dH\ \det{}^n(\epsilon - H)\prod_{k=1}^{M}\frac{1}{\det[(\mu_k + iH)(\mu_k - iH)]}, \end{aligned} \tag{7.27}$$

where we define, μ_k and C_M as:

$$\mu_k = \cosh\frac{k\beta}{2}, \tag{7.28}$$

$$C_M = \left\{\prod_{n=1}^{M} 4q^n\right\}^{-1} \quad \text{with } M \to \infty. \tag{7.29}$$

For contour integration at infinity and poles necessary transformation can be introduced.

For the two point correlation function $S_2(x, y)$ in the limit $N = M \to \infty$ we get:

$$S_2(x, y) = \frac{1}{Z_{0,N}(\epsilon)} \int_{-\infty}^{\infty} dH \ \mathrm{Tr}(\delta(x - H)) \ \mathrm{Tr}(\delta(y - H)) \ P(H) \,. \tag{7.30}$$

Further, Cauchy integration over poles and the sum over all possible position m and n gives the following result:

$$S_2(x, y) = \frac{1}{\pi^2} \sum_{m,n}^{M} \frac{\mu_n \mu_m}{(x^2 + \mu_n^2)(x^2 + \mu_m^2)(y^2 + \mu_n^2))(y^2 + \mu_m^2)} \times \prod_{k \neq m,n} \frac{(x - i\mu_k)(y - i\mu_k)(\mu_m + \mu_k)(\mu_n + \mu_k)}{(x + i\mu_k), (y - i\mu_k)(\mu_m - \mu_k)(\mu_n - \mu_k)}. \tag{7.31}$$

From the plot of two point correlation function $S_2[\xi - \eta]$ calculated exactly using Cauchy integration over poles gives the plot like figure [22] that we have plotted in this paper. The two point correlation function is exactly behaving like total thermal Green's function that we have derived earlier. Nature of the plot shows that it increases linearly upto a certain point and then saturate at a bound with very low oscillation tending to zero. It exactly matches with our prediction of bound of SFF from GUE based RMT.

8 Conclusion

From our detailed discussion using GUE based RMT we have derived a strict saturation bound on the two point SFF from GUE based RMT for both large N and finite N situations, which ultimately put stringent constraint on quantum chaos for randomly interacting system described by a general polynomial potential function. Using this approach we find that our predicted bound for quantum chaos from GUE based RMT is independent on the choice of temperature, which further implies that the derived result is universal in nature for GUE. Moreover, using the lower bound on the two point SFF from GUE based RMT one can precisely comment on the minimal chaotic nature of a random system from RMT set up. Also, this derived result can be further used to describe the out of equilibrium aspects in the context of primordial cosmology [10], specifically to quantify reheating temperature and particle creation phenomena in early universe cosmology. Also, in the context of quantum theory of black-hole [23–26] one can implement this result to explain many explored physical informations. Also for non-linear effects in redshift space this can be studied [27]. We have presented any computation for the bound on SFF from Gaussian Orthogonal Ensemble from RMT yet in this paper. One can study this in detail in future. Apart from that the role of integrability from RMT one can study in more detail for other models explicitly.

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