

1 Nonlinear dynamics of Shear Alfvén fluctuations in Divertor Tokamak Test facility 2 plasmas

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10 Following the analysis on linear spectra of shear Alfvén fluctuations excited by en-
11 ergetic particles (EPs) in the Divertor Tokamak Test (DTT) facility plasmas [T.
12 Wang *et al.*, Phys. Plasmas **25**, 062509 (2018)], in this work, nonlinear dynamics
13 of the corresponding mode saturation and the fluctuation induced EP transport is
14 studied by hybrid magnetohydrodynamic-gyrokinetic simulations. For the reversed
15 shear Alfvén eigenmode driven by magnetically trapped EP precession resonance in
16 the central core region of DTT plasmas, the saturation is mainly due to radial de-
17 coupling of resonant trapped EPs. Consistent with the wave-EP resonance structure,
18 EP transport occurs in a similar scale to the mode width. On the other hand, passing
19 EP transport is analyzed in detail for toroidal Alfvén eigenmode in the outer core
20 region, with mode drive from both passing and trapped EPs. It is shown that pass-
21 ing EPs experience only weak redistributions in the weakly unstable case; and the
22 transport extends to meso-scale diffusion in the strongly unstable case, due to orbit
23 stochasticity induced by resonance overlap. Here, weakly/strongly unstable regime is
24 determined by Chirikov condition for resonance overlap. This work then further illu-
25 minates rich and diverse nonlinear EP dynamics related to burning plasma studies,
26 and the capability of DTT to address these key physics.

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27 I. INTRODUCTION

28 In Ref. 1, we have investigated linear dynamics of shear Alfvén fluctuations excited by en-
29 ergetic particles (EPs) in the recently proposed next generation tokamak device, the Divertor
30 Tokamak Test (DTT) facility,² which mainly aims at studying viable divertor configurations
31 for the demonstration power plant (DEMO).³ Despite the practical objectives of DTT, we
32 have shown that many substantial physics can be explored in DTT core plasmas,¹ including
33 fundamental issues related to burning plasma operations.^{4,5} In particular, the EPs produced
34 by nuclear fusion reactions and/or auxiliary heating methods can drive Alfvénic fluctuations
35 unstable via wave-particle resonant interactions, as their characteristic dynamic frequencies
36 are in the magnetohydrodynamic (MHD) range. Depending on the intensity of EP drive and
37 resonance condition, the collective fluctuations of shear Alfvén waves (SAWs) could exist as
38 Alfvén eigenmodes (AEs)⁶ inside frequency gaps of SAW continuous spectrum (continuum),
39 or as energetic particle continuum modes (EPMs).⁷ Following the linear analysis presented
40 in Ref. 1, in this paper, we focus on the nonlinear saturation of the Alfvénic fluctuations and
41 the fluctuation induced EP transport in DTT plasmas, as the assessment of EP confinement
42 property is crucial in the next generation tokamak experiments. In fact, only a small frac-
43 tion of EP loss could be tolerated in burning plasma devices without severely damaging the
44 plasma facing components. Thus, a deeper comprehension of these key physics is not only
45 important for the success of DTT, but also of practical interest for burning plasma studies,
46 such as in the International Thermonuclear Experimental Reactor (ITER)⁸⁻¹⁰ and DEMO.³

47 Targeting self-sustained nuclear fusion in the next generation tokamaks, the physics un-
48 derstanding of SAW-EP dynamics in toroidal plasmas has significantly improved in the last
49 several decades, and is reviewed in a few recent publications^{5,9,11-15} from the perspective
50 of theoretical and experimental research as well as numerical simulation. In general, the
51 nonlinear saturation of SAW fluctuation may follow two routes, i.e., nonlinear wave-wave
52 and wave-EP interactions.^{5,16} In this paper, we focus on the latter route by means of hybrid
53 MHD-gyrokinetic code (HMGC)^{17,18} simulations, due to the crucial role played by EPs in the
54 multi-scale dynamics of fusion plasmas.⁴ Adopting the theoretical framework of the general
55 fishbone-like dispersion relation,^{5,19,20} nonlinear dynamics and saturation of a single toroidal
56 mode number SAW fluctuation can be understood as two fundamental mechanisms, namely,
57 resonance detuning and radial decoupling.^{5,21-24} Briefly speaking, resonance detuning is due

58 to the nonlinear shift in the wave-EP phase, and it is ubiquitous in wave-particle resonant in-
 59 teractions. On the other hand, radial decoupling is due to the finite nonlinear excursion of EP
 60 orbits with respect to the likewise finite localized mode structure in nonuniform plasmas.^{5,24}
 61 Thus, in a realistic plasma, the complex behavior underlying the nonlinear interplay between
 62 SAW fluctuation and EPs depends on the relative importance of the two mechanisms. As
 63 shown theoretically^{5,24} and by recent numerical simulations,^{23,25–31} the saturation mechanism
 64 is determined by the relative ordering of nonlinear EP orbit excursion to the perpendicu-
 65 lar (with respect to equilibrium magnetic field) fluctuation wavelength and/or equilibrium
 66 nonuniformity; and it can be reflected by the relative scale lengths of wave-EP power trans-
 67 fer, mode structure and effective resonance condition.^{23,28,29,31} For two paradigmatic cases,
 68 typically in the marginally unstable limit, nonlinear EP orbit excursion is restricted by the
 69 effective resonance condition, and is much smaller than the perpendicular fluctuation wave-
 70 length; that is, the resonant EP response is similar to that of a uniform plasma. Hence,
 71 in this regime, resonance detuning outweighs radial decoupling and, when only resonance
 72 detuning is considered, the saturated fluctuation amplitude scales quadratically with respect
 73 to the linear growth rate of the mode,^{32,33} consistent with that predicted by wave-particle
 74 trapping mechanism typical of a 1-D beam-plasma system.³⁴ Meanwhile, in the strongly
 75 unstable regime with non-perturbative EP response, the EP orbit excursion is compara-
 76 ble with the perpendicular fluctuation wavelength, radial nonuniformity becomes essential
 77 for the resonant EP response, and radial decoupling is therefore crucially important.^{5,24} As
 78 shown by previous numerical simulations and predicted by analytic models, the scaling of
 79 saturation amplitude versus mode linear growth rate could be linear in this regime.^{28,29,31}
 80 In all cases, the non-perturbative EP response and plasma nonuniformity can introduce
 81 additional twists in the complex behaviors underlying wave-EP power exchange, and allow
 82 enhanced fluctuation levels with respect to the predicted quadratic and/or linear scaling.^{4,5}
 83 Thus, the proper description of saturation mechanism generally requires accounting for the
 84 self-consistent interplay of mode structures and EP transport, as extensively discussed in
 85 the recent comprehensive review paper by Chen and Zonca.⁵

86 In this work, some of the key integrated physics aspects of burning plasmas are addressed
 87 for the DTT reference scenario. DTT plasmas can be generally divided into a central core
 88 region, characterized by low magnetic shear and coherent fluctuation induced redistributions
 89 of magnetically trapped EPs; and an outer core region, with finite magnetic shear and

90 predominant diffusive losses of passing EPs due to resonance overlap.

91 Due to the similarity to ITER of DTT dimensionless parameters relative to both supra-
92 thermal and core plasma components, the Alfvénic fluctuation spectrum resonantly excited
93 by EPs is characterized by toroidal mode numbers $n \sim O(10)$ ¹ and, therefore, by micro-
94 scales that are of the same order of the meso-scale structures spontaneously formed by drift
95 wave turbulence. Thus, DTT core plasmas can access operation regimes where complex
96 behavior will mimic those of reactor relevant fusion plasmas, with EPs acting as mediators
97 of cross scale couplings.^{4,5} This work, in particular, will address and illuminate the rich
98 variety of spatiotemporal scales self-consistently generated in DTT plasmas, and resulting
99 from nonlinear interplay of Alfvénic fluctuations and EP sources of various strength.

100 EP transport will be analyzed in phase space, since AEs excited in the central core region
101 by magnetically trapped EPs are characterized by very different resonance structures and
102 corresponding spatiotemporal scales than AEs due to both trapped and passing EPs in the
103 outer core region.¹ The different behaviors will be discussed by means of test particle Hamil-
104 tonian mapping techniques^{23,27,31} to illuminate the nonlinear evolution of phase space zonal
105 structures^{5,24} and, ultimately, its impact on EP transport, characterized by both coherent
106 nonlinear redistributions as well as diffusive radial fluxes.

107 This paper is organized as follows. The simulation model and main simulation parameters
108 are presented in Sec. II. In Sec. III, we analyze the nonlinear dynamics of two types of SAW
109 fluctuations interacting resonantly with EPs; namely, reversed shear Alfvén eigenmodes
110 (RSAEs) resonantly excited by trapped EPs in the central core region, and toroidal Alfvén
111 eigenmodes (TAEs) destabilized by both trapped as well as passing EPs in the DTT outer
112 core. In this work, we focus on the initial mode saturation in order to illustrate and discuss
113 the underlying physics, and to illuminate the richness of diverse nonlinear behaviors that
114 can be expected in the DTT reference scenario. More complicated long time scale nonlinear
115 evolutions are intentionally left to further and more detailed analyses to be carried out as
116 future work. Section IV gives the final summary and discussion.

117 II. NUMERICAL MODEL AND SIMULATION PARAMETERS

118 In this paper, we recall the numerical model of the DTT reference scenario considered
119 in Ref. 1. Here, we only summarize the most important numerical aspects, while a full

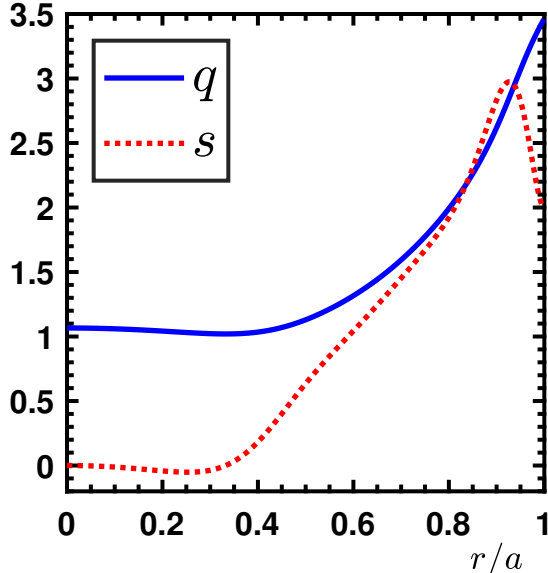


FIG. 1. Radial profiles of equilibrium safety factor q and magnetic shear s .

120 description and discussion of the adopted model are given in Ref. 1.

121 The simulation code HMGC^{17,18} is based on the hybrid MHD-gyrokinetic model,³⁵ and fo-
 122 cuses on the self-consistent interplay between thermal plasma components and fast/energetic
 123 particles in simplified tokamak geometry. In this study, the bulk plasma fluctuations are
 124 described by a set of $O(\epsilon^3)$ -reduced MHD equations³⁶ in the limit of zero pressure, where
 125 $\epsilon \equiv a/R_0$ is the inverse aspect ratio, with a and R_0 the minor and major tokamak radii,
 126 respectively. The EP response is accounted for by solving nonlinear Vlasov equation in
 127 the drift-kinetic limit with particle-in-cell method, and enters in the MHD equations non-
 128 perturbatively via the pressure coupling formulation.³⁵ Therefore, finite Larmor radius effect
 129 is neglected, but finite magnetic drift orbit width effect is fully taken into account.^{19,20,37,38}
 130 Consistent with Ref. 1, in this work, we investigate a single toroidal mode number n in
 131 each simulation case, while multi- n simulations will be part of future work (see Ref. 39 for a
 132 recent publication on this subject). Thus, MHD nonlinear mode-mode coupling is neglected,
 133 but EP nonlinearities are self-consistently retained. We also emphasize, as anticipated in
 134 the Introduction, that single- n simulations do not necessarily imply studying the nonlinear
 135 dynamics of isolated resonances. Quite the contrary, we will be able to discuss various
 136 features of both isolated (Sec. III A) as well as overlapped (Sec. III B) resonances.

137 A shifted circular equilibrium with $\epsilon = 0.18$ is adopted in this paper, along with ITER-
 138 like EP parameters.^{1,40} Figure 1 shows the radial profiles of equilibrium safety factor q

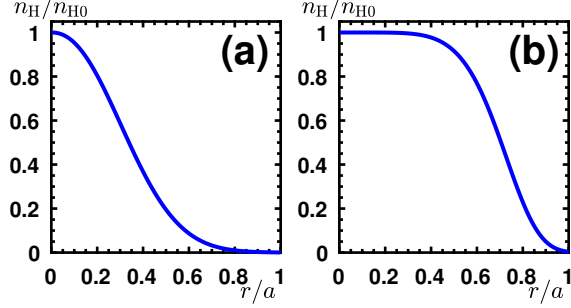


FIG. 2. Radial profiles of normalized EP density n_H . The two profiles are used for, respectively, (a) the central core region, and (b) the outer core region.

140 and the corresponding magnetic shear $s \equiv r q' / q$, where “prime” indicates derivative with
 141 respect to the minor radius coordinate r . As articulated in Ref. 1, the structure of the
 142 adopted equilibrium suggests a subdivision into a central core region with q marginally
 143 above unity and vanishing s , and an outer core region with larger q and finite s . The two
 144 regions are investigated separately in this paper, adopting EPs with different radial pressure
 145 profiles (cf. Fig. 2). EPs are assumed to be fusion born alpha particles characterized by
 146 an isotropic slowing-down distribution function. The pressure drive of EPs is controlled by
 148 the normalized radial profile of EP density n_H/n_{H0} , shown in Fig. 2 for, respectively, the
 149 central and outer core region. Here, n_{H0} indicates the value of n_H on the magnetic axis,
 150 and is normalized to on-axis bulk ion density n_{i0} to control the intensity of EP drive (cf.
 151 Ref. 1). Other important parameters are $\rho_H/a = 0.01$, $v_H/v_{A0} = 1.80$, with $\rho_H \equiv v_H/\Omega_H$ the
 152 EP Larmor radius, $v_H \equiv \sqrt{E_0/m_H}$ the characteristic EP birth speed, Ω_H the EP cyclotron
 153 frequency, E_0 the alpha particle birth energy, m_H the EP mass, and v_{A0} the on-axis Alfvén
 154 speed. Note that $\rho_H/a = 0.01$ is significantly smaller than in present day tokamaks;⁵ and
 155 the smaller EP orbit width in DTT is crucial to determine the linear¹ as well as nonlinear
 156 dynamics of resonantly excited Alfvénic fluctuations. In particular, the range of excited
 157 toroidal mode numbers is such that the peculiar role of EPs in cross-scale coupling becomes
 158 evident, as to be shown in Sec. III.

159 In this paper, in order to illuminate the nonlinear physics anticipated in the Introduction,
 160 we focus on the two representative cases discussed in detail in Ref. 1, namely, $n = 4$ RSAE
 161 case in the central core region (Sec. III A), and $n = 6$ TAE case in the outer core region
 162 (Sec. III B). The selected two cases can well represent the peculiar features of each region,

163 while the nonlinear dynamics of other modes with different toroidal mode numbers can be
 164 predicted following their linear properties¹ and the analysis in this paper. In all simulations,
 165 poloidal harmonic m is retained in the interval $[3, 14]$ for $n = 4$, and in the interval $[5, 21]$
 166 for $n = 6$.

167 III. NONLINEAR DYNAMICS

168 In this section, after briefly reviewing the linear spectra reported in Ref. 1, the nonlinear
 169 saturation of SAW fluctuations and the associated EP transport in DTT plasmas are inves-
 170 tigated by HMGC simulations. As indicated above, we analyze $n = 4$ RSAE fluctuations
 171 for the central core region in Sec. III A, and $n = 6$ TAE fluctuations for the outer core re-
 172 gion in Sec. III B. Normalized EP pressure profiles with variable intensity assumed as initial
 173 conditions in the two cases are shown in Fig. 2(a) and (b), respectively. The two cases are
 174 characterized by very different wave-EP resonance structures.¹ The dominant destabilization
 175 mechanism for the RSAE fluctuations is the precession resonance with magnetically trapped
 176 EPs; meanwhile, the wave-EP power transfer for the TAE fluctuations consists of compa-
 177 rable contributions from the precession resonance with trapped EPs, and several transit
 178 harmonic resonances with passing EPs. Different mechanisms of EP transport, suggested
 179 by the wave-EP resonance conditions and relevant spatial scales, are also investigated in
 180 detail in this section.

181 A. RSAE nonlinear dynamics

182 A series of $n = 4$ RSAE cases with n_{H0}/n_{i0} in the interval $[0.0004, 0.0030]$ are analyzed.
 183 The unstable fluctuations are characterized by very similar mode structures, but have dif-
 184 ferent spectral properties. The mode real frequencies ω_r and linear growth rates γ_L are
 185 shown in Fig. 3, where the upward frequency shift due to the non-perturbative effect of EPs
 186 is evident.¹ Moreover, γ_L scales almost linearly with n_{H0} , suggesting very low instability
 187 threshold of EP drive as a result of low background damping. For a reference case with
 188 $n_{H0}/n_{i0} = 1.5 \times 10^{-3}$, several dominant poloidal harmonics of scalar potential fluctuation
 189 $\delta\varphi_{m,n}$ in the linear growth stage are shown in Fig. 4, along with the corresponding power
 190 spectrum in the (r, ω) plane. Consistent with the equilibrium q profile and the structure of
 192

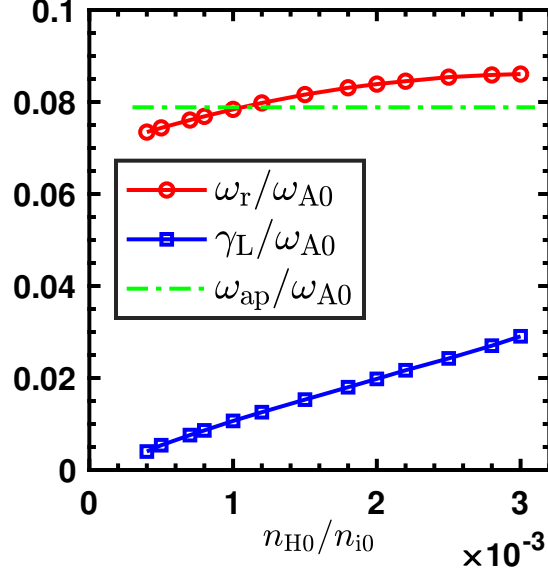


FIG. 3. RSAE real frequencies ω_r (circles) and linear growth rates γ_L (squares) shown as functions of EP on-axis density n_{H0} . Here, on-axis Alfvén frequency ω_{A0} is used for normalization. In addition, the RSAE accumulation point frequency ω_{ap} is indicated as a horizontal dash-dotted line.

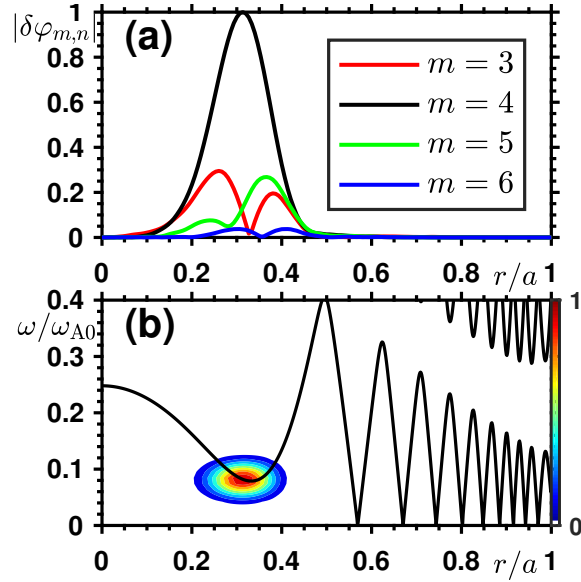


FIG. 4. Radial structures of several dominant Fourier decomposed poloidal harmonics of scalar potential fluctuation $\delta\varphi_{m,n}$ [frame (a)], and the intensity contour plot of the corresponding power spectrum in the (r, ω) plane [frame (b)] shown in arbitrary units for the RSAE case with $n_{H0}/n_{i0} = 1.5 \times 10^{-3}$ in the linear stage. The solid curves in frame (b) represent SAW continua.

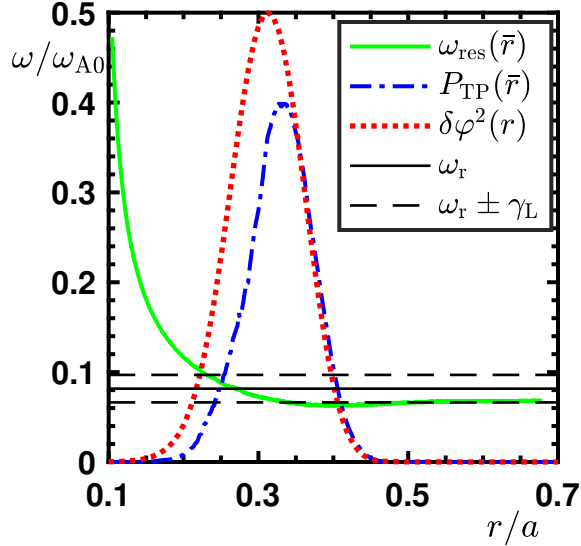


FIG. 5. For the reference case in the linear stage, the test particle precession resonance frequency $\omega_{\text{res}}(\bar{r}, \ell = 0)$ (solid curve) [cf. Eq. (A3)] is shown with respect to the orbit averaged radial coordinate \bar{r} . ω_r and $\omega_r \pm \gamma_L$ are indicated as, respectively, horizontal solid and dashed lines to illustrate the resonance condition [cf. Eq. (A5)]. In addition, the integrated test particle power transfer $P_{\text{TP}}(\bar{r})$ (dash-dotted curve), as well as the radial mode structure, denoted by $\delta\varphi^2(r) = \int |\delta\varphi_{m,n}|^2 d\theta d\phi$ (dotted curve), are shown in arbitrary units.

193 SAW continuum, the $m = 4$ harmonic is dominant, and the mode is radially localized near
 194 the surface with minimum q value.

195 As a useful tool in analyzing wave-EP resonant interactions, test particle method is
 196 extensively applied to illustrate the resonance condition as well as the nonlinear dynamics of
 197 mode saturation and EP transport. Test particles are chosen as “representative” of resonant
 198 EPs, which can be readily identified from wave-EP power transfer.²³ Further details about
 199 test particle selection are given in Appendix A (interested readers may also refer to Ref. 23
 200 for an exhaustive description), while here we only emphasize that the test particle population
 201 is characterized by two constants of the perturbed motion, M and C , corresponding to a
 202 reduced phase space grid (of trapped particles) with significant wave-EP power transfer in
 203 the linear stage. Here, M is the magnetic moment and C , given by Eq. (A1), is an invariant
 204 constructed from the extended phase space Hamiltonian.^{5,24,41} For the reference case, the
 205 linear resonance structure is shown in Fig. 5, along with radial mode structure and flux

207 surface integrated test particle power transfer. We can see that the effective wave-particle
 208 power transfer is limited by the resonance condition on the inner side, and by the radially
 209 localized mode structure on the outer side. The radial width of power transfer (denoted by
 210 Δr_{power}) is similar to that of the mode structure (denoted by Δr_{mode}); but it remains much
 211 smaller than the effective resonance width Δr_{res} [cf. Eq. (A5) and the discussion below] or
 212 equilibrium pressure gradient scale length [cf. Fig. 2(a)]. Furthermore, similar structures
 213 can also be found for the lower growth rate cases reported in Fig. 3 due to the flat ω_{res} profile.
 214 Meanwhile, for higher growth rate cases, power transfer is entirely limited by the finite mode
 215 width as Δr_{res} becomes broader. In general, $\Delta r_{\text{power}} \lesssim \Delta r_{\text{mode}} \ll \Delta r_{\text{res}}$ for all considered
 216 cases, suggesting that radial decoupling is the dominant mechanism for mode saturation,
 217 consistent with the strongly non-perturbative EP response. In the weakly unstable limit
 218 where $\Delta r_{\text{power}} < \Delta r_{\text{mode}}$, however, resonance detuning and radial decoupling may both play
 219 important roles and should be treated on the same footing.

220 The nonlinear evolution of long-lived EP phase space zonal structures can be illustrated
 221 by test particle Hamiltonian mapping technique via kinetic Poincaré plots,^{23,27,31,41} which
 222 represent wave-particle phase shift (resonance detuning) and particle orbit excursion (ra-
 223 dial decoupling) on the same footing. In the kinetic Poincaré plot, each test particle's last
 224 completed orbit (when the particle crosses the equatorial plane at the outmost radial co-
 225 ordinate, i.e., poloidal angle $\theta = 0$) is represented by a marker in the (Θ, P_ϕ) plane. Here,
 226 $\Theta = \omega t_0 - n\phi_0$ is the wave-particle phase at $\theta = 0$, where t_0 and ϕ_0 indicate the corresponding
 227 values of time and toroidal angle; P_ϕ is the toroidal angular momentum given by Eq. (A2).
 228 Figure 6 shows the kinetic Poincaré plots for 3 successive time frames of the reference case,
 229 corresponding to, respectively, the linear stage, early nonlinear stage and saturation, as indi-
 230 cated in the energy evolution plot Fig. 6(d). Here, note that P_ϕ is used to represent the test
 231 particle radial distribution, with larger P_ϕ corresponding to smaller \bar{r} (orbit averaged radial
 232 coordinate) and vice versa. In order to show more intuitively the mode width, the radial
 233 mode structure is shown in Fig. 6(a) by mapping the test particle \bar{r} into P_ϕ coordinates. In
 234 the linear stage with negligible fluctuation amplitude, P_ϕ is conserved and the particles with
 235 $P_\phi = P_{\phi\text{res}}$ (defined by $\omega_{\text{res}} = \omega$) stay constant in phase. Other particles with $P_\phi > P_{\phi\text{res}}$ (red
 236 markers) and $P_\phi < P_{\phi\text{res}}$ (blue markers) get a finite phase change after each bounce orbit, and
 237 thus, these markers drift along Θ in negative and positive direction, respectively. When the
 238 fluctuation amplitude grows to a finite value, P_ϕ varies due to wave-particle interaction, and
 239

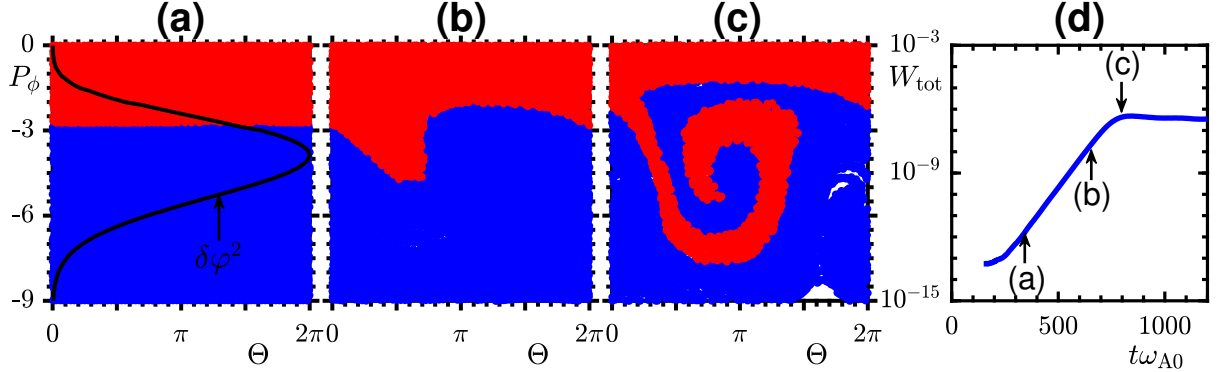


FIG. 6. Kinetic Poincaré plots of test particles shown in the (Θ, P_ϕ) plane for three successive times of the reference case [frames (a)-(c)]. Frame (d) indicates the three considered times in the time evolution of the total (kinetic plus magnetic) perturbed field energy W_{tot} . In frames (a)-(c), Θ is modulo 2π , and P_ϕ is normalized to $m_H av_H$. The test particle marker is colored by the particle's initial P_ϕ value: red for $P_\phi > P_{\phi\text{res}}$ and blue otherwise. In addition, radial mode structure $\delta\varphi^2(r)$, given in Fig. 5, is also shown in frame (a) by mapping \bar{r} into P_ϕ coordinates.

240 an island-like structure naturally forms around the $P_{\phi\text{res}}$, with increasing island width as the
 241 fluctuation amplitude grows. The mode eventually saturates when the phase space structure
 242 extends over the region of effective linear wave-particle power transfer. As clearly shown in
 243 Fig. 6(c), the resonant particles sample nearly the whole mode structure during their nonlin-
 244 ear orbit excursion, suggesting that radial decoupling is a crucial element of the saturation
 246 mechanism. As a quantitative assessment of the saturation mechanism, Fig. 7 compares the
 247 averaged resonant test particle orbit radial excursion, Δr_{orbit} , with Δr_{mode} and Δr_{power} for
 248 the cases reported in Fig. 3. We can see that Δr_{orbit} is indeed similar to Δr_{power} , and is
 249 comparable with Δr_{mode} for most of the cases. Furthermore, as shown in Fig. 6, the mixing
 250 of particles from $P_\phi > P_{\phi\text{res}}$ (smaller \bar{r}) with $P_\phi < P_{\phi\text{res}}$ (larger \bar{r}) suggests a net outward
 251 particle flux due to the radial inhomogeneity of the EP distribution function. The outward
 252 flux can also be shown from C conservation since, as the particles lose energy to the wave,
 253 P_ϕ decreases (r increases) and more particles move outward than inward during the mode
 255 growth stage. Figure 8 shows the distortion of test particles density profile at saturation for
 256 the reference case. Consistent with the previous analysis, mode saturation is reached when
 257 the width of particle redistribution is comparable with the radial region of power transfer,
 258 since the resonant EP drive is significantly reduced. Here, we emphasize that the fluctua-

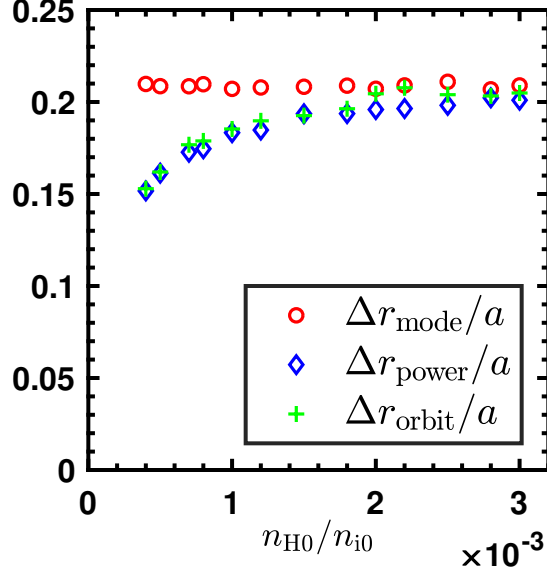


FIG. 7. For the cases reported in Fig. 3, the averaged resonant test particle orbit radial excursion Δr_{orbit} at saturation is compared with mode radial width Δr_{mode} and power transfer radial width Δr_{power} in the linear stage. Here, Δr_{orbit} is calculated as the largest variation of the particle’s equatorial plane radial coordinate, and is averaged over the resonant test particle population. Δr_{mode} and Δr_{power} are measured as the radial width of the region where the corresponding quantity is larger than the 10% of the peak value (cf. Fig. 5).

259 tion induced EP transport indeed occurs on meso-spatial scales [$\sim O(10^{-1}a)$], intermediate
 260 between macro-scales such as the equilibrium profiles [$\sim O(10^0a)$], and micro-scales of char-
 261 acteristic EP orbit width [$\sim O(10^{-2}a)$]. As anticipated in Ref. 1, the meso-scales reflect
 262 the properties of the fluctuation spectrum and the relevant toroidal mode number.^{4,5} Note
 263 that the clear distortion of test particle density profile shown in Fig. 8 only reflects the
 264 considered (M, C) slice of EP distribution function, while the overall EP radial distribution,
 265 obtained by averaging the EP response over all the (M, C) slices representing the entire
 266 phase space, is almost unchanged, due to the fact that most of the EPs are not resonant
 267 and, thus, experience a much weaker effect of the fluctuations.

269 Figure 9 shows the saturated fluctuation amplitude versus γ_L for all considered cases. We
 270 can observe that the scaling is clearly different from quadratic and is close to linear, as ex-
 271 pected from radial decoupling being the dominant saturation mechanism. In the low growth
 272 rate limit, where resonance detuning may also become important, we find a slight deviation
 273 towards a steeper scaling. The approximately linear scaling is consistent with theoretical

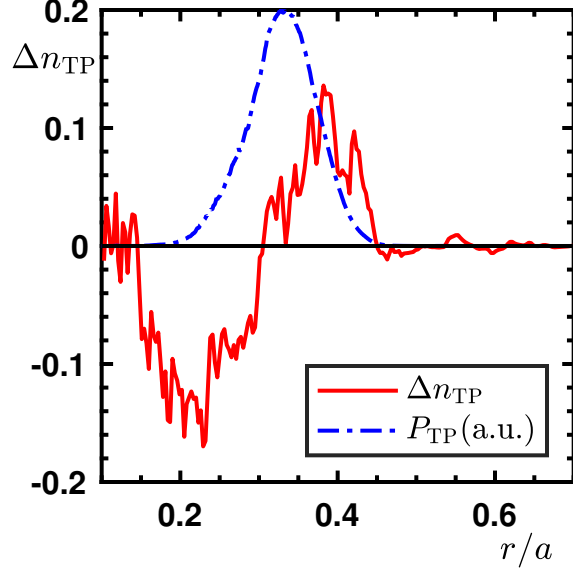


FIG. 8. The distortion of test particle density radial profile Δn_{TP} (solid curve) shown in arbitrary units for the reference case. Δn_{TP} is calculated as the difference of test particle density at saturation with respect to the linear stage, and is normalized to the test particle density at mode peak. The integrated wave-particle power transfer radial profile P_{TP} (dash-dotted curve) in the linear stage, reported in Fig. 5, is also shown in arbitrary units for comparison.

274 understanding^{5,24} and previous numerical simulations.^{28,29,31} In our simulations, however,
 275 the scaling deviates from linear in the high growth rate limit. This could be due to the fact
 276 that, in the strongly driven cases, the mode structures and frequencies are self-consistently
 278 modified with the non-perturbative EP redistribution.^{4,5,24} As an example, Fig. 10 shows
 279 the time evolutions of mode frequency and fluctuation radial peak location (representative
 280 of the mode structure) of a strongly unstable case. The self-consistent modulation of mode
 281 frequency and mode structure becomes evident when the fluctuation grows to an appreciable
 282 amplitude approaching saturation. Simulation results suggest that fluctuations are further
 283 enhanced in this self-consistent non-perturbative process, and saturate at a higher amplitude
 284 than the predicted linear scaling, which assumes constant mode frequency and frequency-
 285 independent mode structure.^{28,29,31} From Fig. 10, it is also interesting to note that, on longer
 286 time scale of the strongly unstable case with clear non-perturbative wave-EP interactions,
 287 the frequency chirping shows non-adiabatic features, as the mode structure is strongly mod-
 288 ified. In such conditions, it is expected that the non-perturbative EP redistributions may
 289 become secular and characterized by avalanches,^{42,43} which are important issues in burning

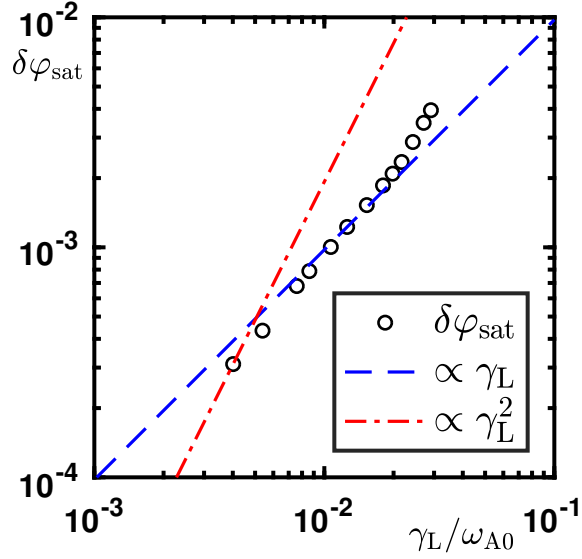


FIG. 9. For the cases reported in Fig. 3, the fluctuation amplitude (largest value in radial direction) at the initial saturation $\delta\varphi_{\text{sat}}$, is shown as a function of the linear growth rate γ_L in logarithmic scale. Here, $\delta\varphi_{\text{sat}}$ is in units of E_0/e_H (e_H is the EP charge). Dashed lines corresponding to linear and quadratic scaling are also indicated.

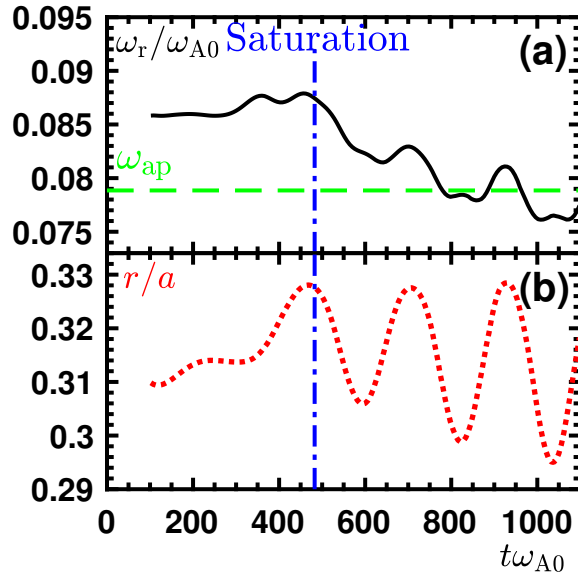


FIG. 10. Time evolutions of RSAE real frequency (maximum power intensity in the frequency spectrum) [frame (a)] and peak location of the radial mode structure [frame (b)] for a strongly unstable case with $n_{H0}/n_{i0} = 3.0 \times 10^{-3}$. The saturation time (vertical dash-dotted line) and the RSAE accumulation point frequency ω_{ap} (horizontal dashed line) are also indicated.

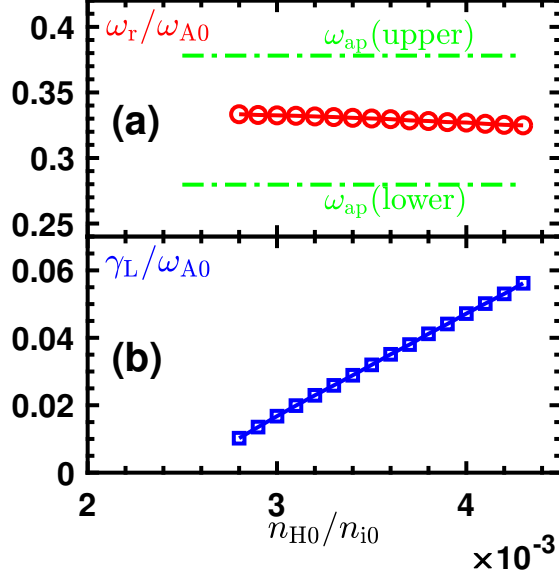


FIG. 11. TAE real frequencies ω_r [frame (a)] and linear growth rates γ_L [frame (b)] shown as functions of EP on-axis density n_{H0} . In addition, the upper and lower TAE accumulation point frequencies ω_{ap} (closest to mode peak) are indicated as horizontal dash-dotted lines to illustrate the importance of non-local coupling with SAW continuum.

290 plasma physics studies.^{4,5,24} Thus, a more detailed analysis of these behaviors is worthwhile
 291 being pursued and will be continued in future work, since it is beyond the intended scope of
 292 this paper, which mainly aims at illuminating the diverse and rich nonlinear physics that can
 293 be investigated in DTT. Here, we just note that the nonlinear dynamics of strongly unstable
 294 cases further addresses the importance of self-consistent treatment of mode structure and
 295 EP nonlinear evolutions, especially in the next generation tokamak relevant conditions.

296 B. TAE nonlinear dynamics

297 Contrary to the weakly damped RSAE fluctuations analyzed above, the TAE fluctuations
 298 in the outer core region experience heavy damping due to strong coupling with the SAW
 299 continuum.^{44–46} Therefore, larger values of EP density are applied to drive the TAE fluctua-
 300 tions unstable. Figure 11 shows ω_r and γ_L of the $n = 6$ TAE fluctuations with n_{H0}/n_{i0} in the
 302 range of $[0.0028, 0.0043]$, where the relatively high destabilization threshold (with respect
 303 to the RSAE fluctuations in the central core region) can be clearly observed from the trend
 304 of γ_L . First, we focus on a weakly unstable case with $n_{H0}/n_{i0} = 3.0 \times 10^{-3}$ (we will refer

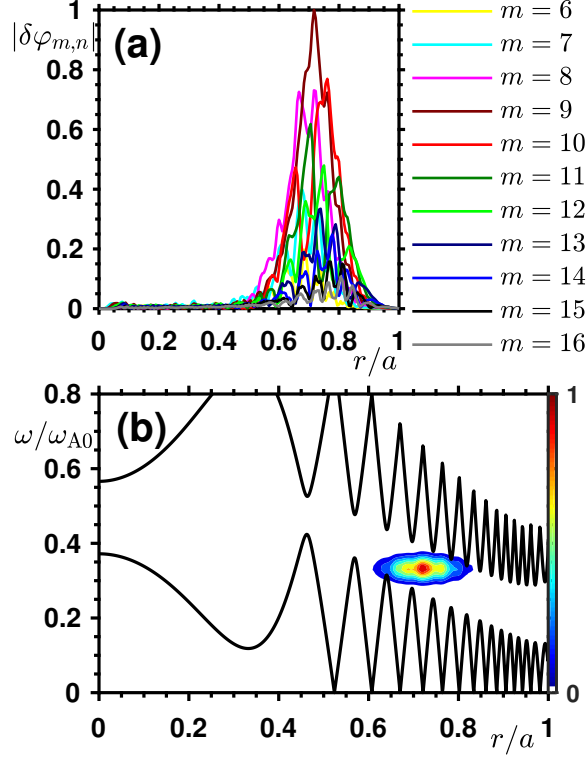


FIG. 12. Radial TAE mode structure for several Fourier decomposed poloidal harmonics of scalar potential fluctuation $\delta\varphi_{m,n}$ [frame (a)], and the intensity contour plot of the corresponding power spectrum in the (r, ω) plane [frame (b)] for the “weak TAE case” with $n_{H0}/n_{i0} = 3.0 \times 10^{-3}$ in the linear stage. The solid curves in frame (b) represent SAW continua.

305 to this case as “weak TAE case” in the following), whose mode structure in the linear stage
 306 is shown in Fig. 12. The mode structure appears as a broad radial envelope consisting of
 308 a wide range of coupled poloidal harmonics, as expected from the equilibrium profiles and
 309 the structure of SAW continua. As discussed in Ref. 1 and introduced above, the peculiar
 310 interest for the TAE fluctuations is that different types of EPs in the velocity space distribu-
 311 tion provide finite mode drive via their respective resonances: precession/transit resonances
 312 for trapped/passing EPs. On account of the fact that the TAE fluctuations are typically
 313 localized in the outer core region, they may be more of a concern for EP confinement, due
 314 to their potential effect of causing significant outward EP flux and consequently, of dam-
 315 aging the plasma facing components. Furthermore, the radial structure of wave-passing EP
 316 power transfer appears as several isolated peaks, as shown in Fig. 13(a) by test particle
 318 analysis. Here, for simplicity, only co-passing EPs are analyzed in detail, as the behaviors

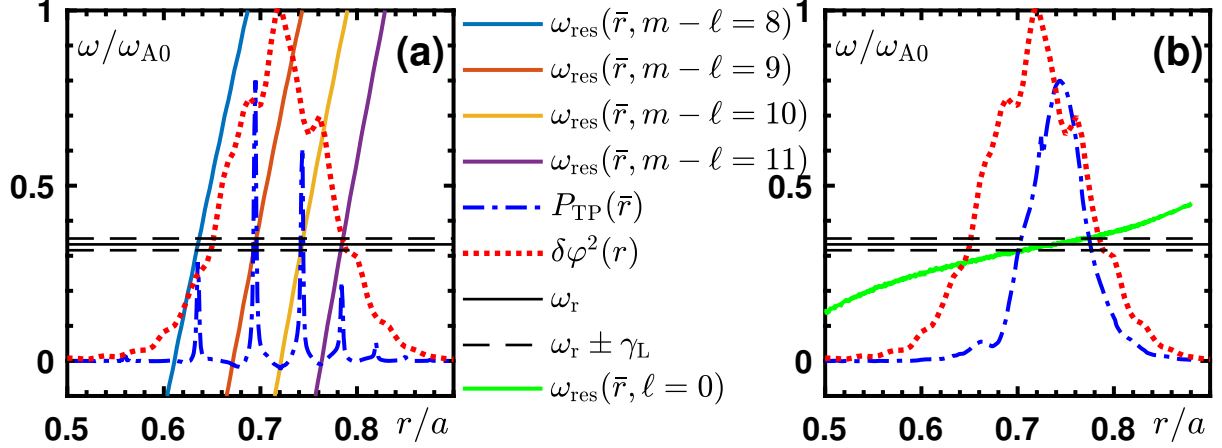


FIG. 13. Linear structures of several transit harmonic resonances by co-passing particles [frame (a)] and the precession resonance by trapped particles [frame (b)] for the weak TAE case. Analogous to Fig. 5, the radial profiles of test particle power transfer $P_{\text{TP}}(\bar{r})$ in each analysis and the radial mode structure $\delta\varphi^2(r)$ are also indicated. Note that, in frame (a), the transit resonances are identified by the effective resonance harmonic $m - \ell$.

319 of co- and counter-passing EPs are very similar due to similar resonance structures. Note
 320 that in Fig. 13(a), the transit resonances are identified by the effective resonance harmonic
 321 $m - \ell$ from the particle's perspective, since both the poloidal harmonic m and “bounce”
 322 harmonic ℓ enter in the expression of transit resonance frequency as the combination $m \pm \ell$
 323 [cf. Eq. (A4)], with minus/plus sign for co-/counter-passing particle, respectively. However,
 324 at each resonant radius characterized by $m - \ell$ in Fig. 13(a), multiple poloidal harmonics of
 325 the mode are excited via the corresponding transit harmonic resonances, which are weighted
 326 differently by finite orbit width bounce averaging, consistent with the mode structure shown
 327 in Fig. 12(a). Thus, toroidal mode number and finite normalized (with respect to plasma
 328 minor radius) EP orbit width play crucial roles. In addition, we note that the character-
 329 istic scale length of the radial separation of the transit harmonic resonances is $1/nq'$; that
 330 is, the meso-scale of drift wave turbulence, typically characterized by much higher toroidal
 331 mode number.^{1,4,5,22,24} On the other hand, the resonance structure of trapped EPs, shown
 332 in Fig. 13(b), is similar to the RSAE case analyzed in Sec. III A. We can see that Δr_{power} of
 333 passing particles is limited by the resonance condition, and is much narrower than Δr_{mode} .
 334 The situation is not as clear for trapped particles, however, it is still legitimate to expect
 335 that resonance detuning, more than radial decoupling, is a relevant saturation mechanism

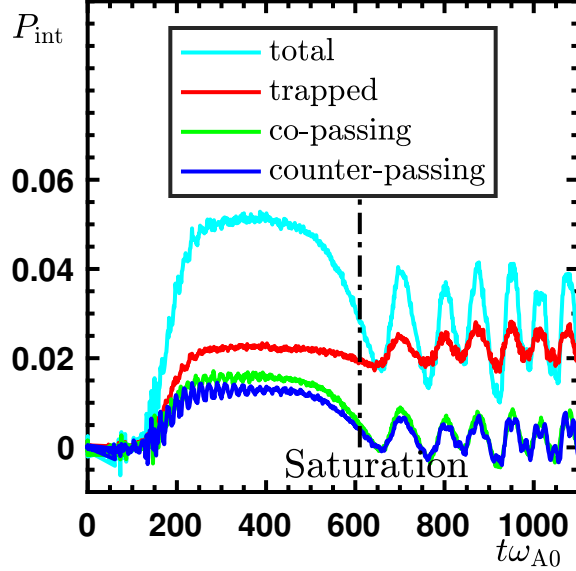


FIG. 14. Time evolution of phase space integrated power transfer for the weak TAE case. The power transfer is normalized to the sum of instantaneous kinetic and magnetic energy density. The saturation time is indicated as a vertical dash-dotted line.

336 for the weak TAE case.³¹

337 Due to the different resonance structures of trapped and passing EPs and, in particular,
 338 the resonant interaction length scales, they play different roles in the nonlinear saturation
 339 of TAE fluctuations. Figure 14 shows the time evolution of phase space integrated power
 340 transfer for the weak TAE case. We can observe that, mode saturation is due to significant
 341 reduction of passing particle drive. Meanwhile, trapped particle drive is still kept in a
 342 significant level, suggesting that the strong damping also plays an important role in mode
 343 saturation, as the residual trapped particle drive approximately balances the dissipation.
 344 Thus, the passing particle resonance plays a more crucial role in mode saturation for the
 345 weak TAE case. Figure 15 shows the kinetic Poincaré plots of both passing and trapped
 346 particles in the linear stage and at saturation, with different colors denoting the transition
 347 across the resonances shown in Fig. 13. We can observe that the mode saturates when
 348 the resonant passing EP orbit excursion matches the narrow radial width of linear power
 349 transfer. In fact, since $\Delta r_{\text{power}}(\text{passing})$ is very small, the phase space structure of passing
 350 particles can be clearly seen on the zoomed scale in Fig. 15(c) only, which is shown as an
 351 expanded insert of the phase space region affected by one single resonance in Fig. 15(b).
 352 Moreover, the redistribution of resonant passing particles is very weak and localized around
 353
 354

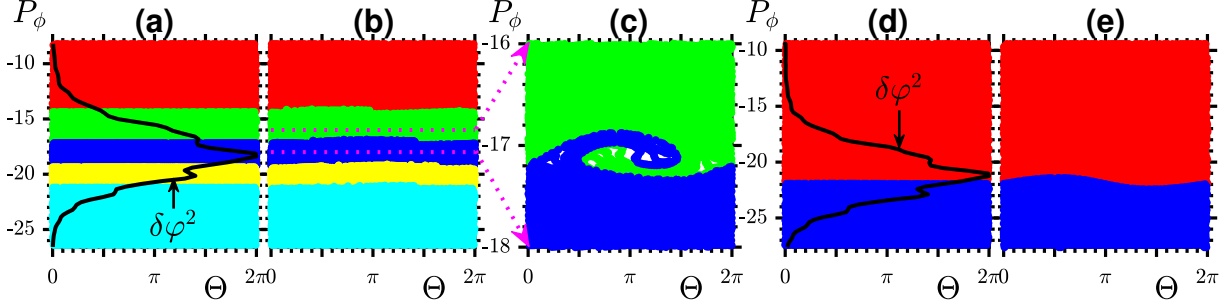


FIG. 15. Kinetic Poincaré plots of co-passing [frames (a)-(c)] and trapped test particles [frames (d)-(e)] for the weak TAE case, where frames (a) and (d) refer to the linear stage and frames (b) and (e) to the saturation time (see Fig. 14). As labeled on the figure, frame (c) is a zoom of frame (b) to more clearly visualize the region affected by one single resonance therein. Analogous to Fig. 6, the test particle markers are colored by their initial values of P_ϕ , with different colors denoting the transition across the resonances shown in Fig. 13. In addition, radial mode structure is shown in frames (a) and (d) by mapping \bar{r} into P_ϕ coordinates.

355 $P_{\phi_{\text{res}}}$; no interaction of adjacent resonances takes place, since the perturbation of equilibrium
356 particle orbits is exceedingly small. (Note that, this is the criterion of “weak drive”.) On the
357 other hand, trapped particle nonlinear transport is intrinsically nonlocal [cf. Fig. 13(b)].^{5,24}
358 However, as a result of the low fluctuation amplitude, trapped particle transport also occurs
359 on a much smaller scale compared to the mode width, similar to the case in Fig. 6(b). The
360 fluctuation induced particle transport, thus, does not have significant impact on the power
361 transfer by trapped EPs. The relative ordering of resonant particle response length scale
362 and the mode width suggests that resonance detuning is indeed the dominant saturation
363 mechanism for the weak TAE case, in contrast to the RSAE cases with similar growth
364 rate that is regulated by radial decoupling mechanism. Therefore, our simulation results
365 suggest that the underlying mechanism of mode saturation and EP nonlinear dynamics is
366 not only determined by the linear growth rate but, more crucially, by the wave-EP resonance
367 structures, where the saturation mechanism is reflected by the relative ordering of the scale
368 lengths of mode structure, wave-EP resonant power transfer and nonlinear EP transport.^{4,5,23}
369 Furthermore, the clear diversity of RSAE fluctuations in the central core region and TAE
370 fluctuations in the outer core region also illustrates the capability of DTT to address a
371 variety of nonlinear EP physics related with burning plasma studies.

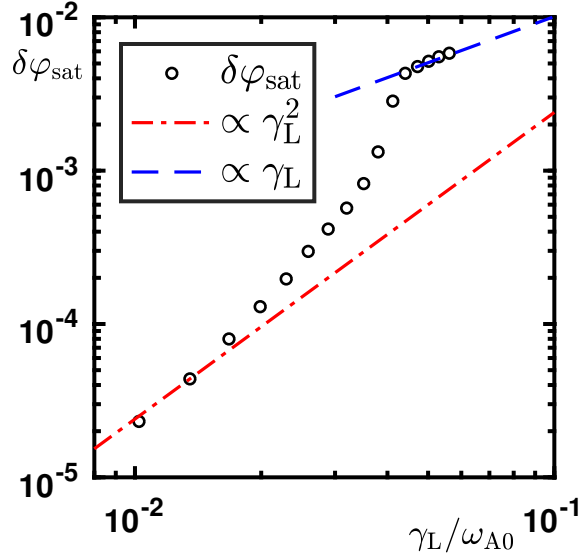


FIG. 16. For the cases reported in Fig. 11, the saturated fluctuation amplitude $\delta\varphi$ is shown with respect to γ_L in logarithmic scale. Quadratic and linear scalings in different regimes are indicated by dashed lines.

373 Figure 16 shows the saturated fluctuation amplitude with respect to γ_L for all TAE cases
 374 reported in Fig. 11. In the low growth rate limit, the scaling is close to quadratic, confirming
 375 that resonance detuning is indeed the main saturation mechanism. Moreover, in the higher
 376 growth rate cases, the scaling first becomes higher than quadratic, and reduces to approx-
 377 imately linear in the strongly unstable limit. It suggests that qualitative and quantitative
 378 differences of wave-EP nonlinear dynamics take place with increasing EP drive, due to equi-
 379 librium geometry, plasma nonuniformity and non-perturbative EP response.^{5,18,24,26} As an
 380 example of the strongly unstable regime, we look at the case with $n_{H0}/n_{i0} = 4.0 \times 10^{-3}$
 381 (in the following, we refer to this case as “strong TAE case”). The strong TAE case shows
 382 similar linear mode and resonance structures compared to those of the weakly unstable case
 383 shown in Figs. 12 and 13. The power transfer widths of both passing and trapped particles
 384 are larger in the strong TAE case as $\Delta r_{\text{power}}(\text{passing}) \ll \Delta r_{\text{power}}(\text{trapped}) \simeq \Delta r_{\text{mode}}$, due
 385 to increased γ_L . (Note that the several transit harmonic resonances are still well separated
 386 in the linear stage.) However, as shown in Fig. 17, the mode time evolution has different
 388 features. We note that, after the linear growth stage, during $t \sim 240 - 280\omega_{A0}^{-1}$, the total
 389 power transfer decreases, mostly due to a rapid reduction of passing EP drive, since the
 390 trapped EP drive is not significantly impacted in this stage. Different from the weak TAE

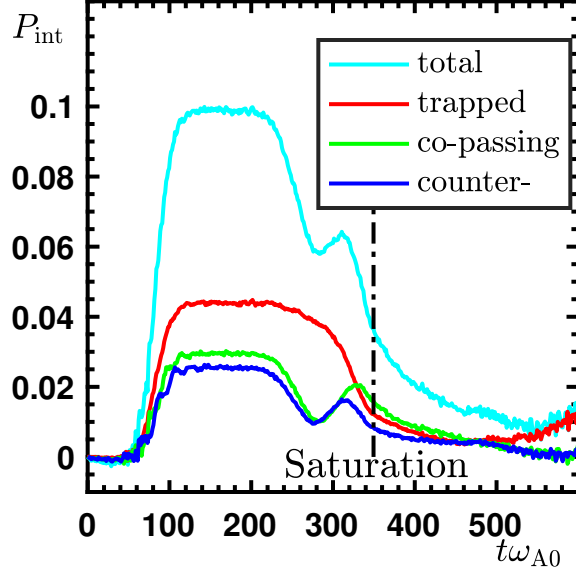


FIG. 17. Time evolution of and integrated power transfer for the “strong TAE case”. The saturation time is indicated as a vertical dash-dotted line.

391 case, the mode keeps growing due to less affected trapped EP drive and residual passing EP
 392 contribution. Then, a short second growth stage follows by an interestingly strengthened
 393 passing EP drive; meanwhile, trapped EP drive starts clearly decreasing. At $t \sim 350\omega_{A0}^{-1}$,
 394 the mode eventually saturates with significant decrease of both passing and trapped EP
 395 drive.

396 More details underlying this complicated time evolution can be illustrated by test particle
 397 analysis, where we focus on the novel nonlinear dynamics of passing EPs. Figure 18 shows
 398 kinetic Poincaré plots of co-passing test particles at four times of the strong TAE case. Here,
 399 in order to see the nonlinear dynamics more clearly, the test particles are distributed around
 400 one single resonance with $m - \ell = 9$ at $P_{\phi\text{res}} \simeq -17.10m_{\text{H}}av_{\text{H}}$. In addition, linear $P_{\phi\text{res}}$ with
 401 $m - \ell = 8 \div 11$ are also indicated in P_{ϕ} coordinate. The dynamics in the early nonlinear
 402 stage [Fig. 18(b)] is similar to the weak TAE case analyzed above, that is, the passing
 403 particles are radially redistributed around $P_{\phi\text{res}}$, and the power transfer by passing particles
 404 decreases correspondingly. However, since the fluctuation strength keeps increasing mostly
 405 due to magnetically trapped EP drive, the resonant island extends and passing particle
 406 transport becomes nonlocal, as the particles are distributed to an increasingly wider region.
 407 At $t \sim 300\omega_{A0}^{-1}$ [Fig. 18(c)], we can observe that the particle distribution is strongly distorted,
 408 where a substantial part of particles are radially transported on a radial scale comparable
 409

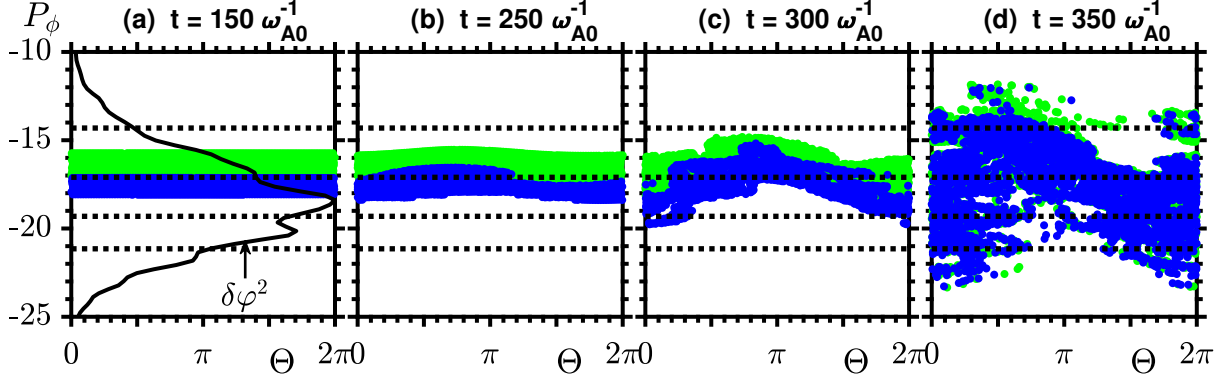


FIG. 18. Kinetic Poincaré plots of co-passing test particles at four times of the strong TAE case with test particles initialized around the $m - \ell = 9$ resonance. The four times refer to the linear stage [frame (a)], two times of the nonlinear growth stage [frames (b) and (c)] and saturation time [frame (d)]. The radial mode structures is shown in frame (a) by mapping \bar{r} into P_ϕ coordinates. In all frames, linear $P_{\phi_{\text{res}}}$ with $m - \ell = 8 \div 11$ are indicated by horizontal dotted lines.

410 with separation of adjacent resonances ($1/nq'$). Thus, the expanding resonant islands, whose
 411 characteristic widths scale as $\sqrt{\delta\varphi}$, are effectively overlapping. Since the wave-passing EP
 412 resonant interaction scale length is very narrow in the linear stage, the meso-scale transport
 413 and increasing resonant island width allow the wave to more effectively extract energy from
 414 particles, including the ones that are not resonant in the linear stage, as they still retain
 415 a significant amount of free energy.²⁴ In fact, all particles with significantly modified phase
 416 space orbits [cf. Fig. 18(d)] are resonant in this stage, and their small but finite resonant
 417 drive contributes to the increase in the integrated power transfer of passing particles shown
 418 in Fig. 17. This enhanced mode drive is also responsible for the stronger (than quadratic)
 419 scaling of saturation amplitude with γ_L in Fig. 16.^{5,18,26} Eventually at mode saturation
 420 [Fig. 18(d)], we notice that the radially redistributed particles almost sample the whole
 421 mode structure, with the corresponding decrease of power transfer.

422 It is also interesting to further address the mechanism of resonant passing particle trans-
 423 port at mode saturation. As shown in Fig. 18(d), the particle transport is significantly
 424 enhanced in this stage due to the high fluctuation amplitude, and it is difficult to identify
 425 any corresponding phase space structure. This happens because the particles are trans-
 426 ported on a spatial scale larger than or comparable with the radial resonance separation
 427 $1/nq'$, with the consequent overlap of adjacent resonances (Chirikov condition). Thus, as

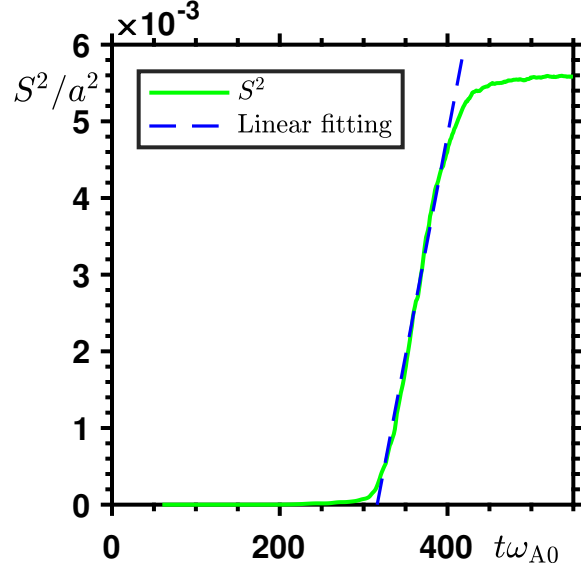


FIG. 19. Time evolution of the mean squared displacement S^2 for a group of resonant passing particles initialized with $P_\phi = P_{\phi_{\text{res}}} \simeq -17.10m_{\text{H}}av_{\text{H}}$ and evenly distributed on the flux surface.

428 particles receive random “kicks” from overlapping resonances, their nonlinear orbits become
 429 stochastic in this stage. As a result of orbit stochasticity, the nature of passing particle
 430 transport becomes diffusive. In order to illustrate the transition from localized redistribu-
 432 tion to meso-scale diffusive transport more clearly, Fig. 19 shows the time evolution of the
 433 mean squared displacement S^2 for a group of resonant passing test particles initialized with
 434 $P_\phi = P_{\phi_{\text{res}}}$ and evenly distributed on the flux surface. Here, test particle’s outer equato-
 435 rial plane radial coordinate r_0 is used to represent the particle’s radial position, and S^2 is
 436 calculated as

$$437 \quad S^2(t) = \langle (r_0(t) - \langle r_0(t) \rangle)^2 \rangle,$$

438 where $\langle \dots \rangle$ stands for averaging over the test particle population. Thus, S^2 is the char-
 439 acteristic relative radial separation of the test particles, and is representative of particle
 440 transport length scales. In the linear stage, $S^2 \simeq 0$ since all particles have the same r_0 .
 441 In the early nonlinear stage, the particles are re-distributed around $P_{\phi_{\text{res}}}$, and the collective
 442 particle transport remains coherent with relatively low value of S^2 . When fluctuation ampli-
 443 tude reaches a threshold value, and the scale of particle transport increases to the resonance
 444 separation length scale, we notice that S^2 increases significantly. When this occurs, S^2
 445 scales roughly linearly with time, suggesting that the nature of particle transport is indeed

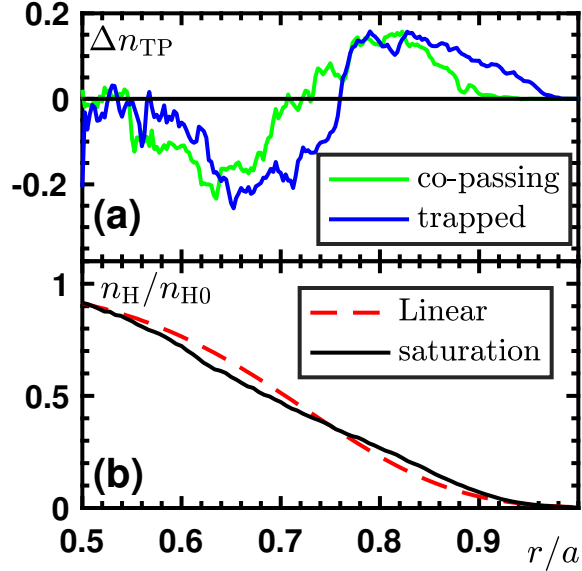


FIG. 20. The distortion of test particle density profile Δn_{TP} [frame (a)] shown in arbitrary unit for the strong TAE case. The calculation and normalization of Δn_{TP} are analogous to Fig. 8. Frame (b) shows the comparison of integrated EP density n_{H} in the linear stage (dashed curve) and at saturation (solid curve).

446 diffusive as anticipated above. The diffusion rate D can be estimated by the slope of $S^2(t)$
 447 as

$$448 \quad D \simeq \frac{1}{2} \frac{dS^2}{dt}.$$

449 After mode saturation, S^2 also reaches a steady state value characterized by the mode
 450 envelope width, as the particles are roughly evenly distributed within the mode location:
 451 $S_{\text{sat}}^2 \sim 1/12(\Delta r_{\text{mode}})^2 \simeq 5.2 \times 10^{-3}a^2$. Thus, the finite mode width becomes effective in
 452 preventing the particles from being transported further out.

453 The meso-scale EP transport and high saturation amplitude also result in significant EP
 454 redistribution in the strong TAE case. Figure 20(a) shows the distortion of test particle
 455 density profile at saturation for both co-passing and trapped particles. We can see that
 456 at mode saturation, the outward particle fluxes of both types of particles occur on similar
 457 scales comparable with the mode width, consistent with the significant reduction of power
 458 transfer in both channels. Since a substantial portion of the EPs are resonant, the integrated
 459 EP density n_{H} also exhibits a clear outward EP flux at saturation, as shown in Fig. 20(b).
 460 Thus, the TAE fluctuation induced EP loss could be more crucial for the assessment of
 461

462 EP confinement, and such a problem is worthwhile being further analyzed in future works,
463 taking into account the whole Alfvén fluctuation spectrum self-consistently. Furthermore,
464 since the finite mode width is the more effective factor in regulating the nonlinear transport
465 of both types of particles, radial decoupling is the dominant saturation mechanism for the
466 strong TAE case, confirmed also by the approximately linear scaling of saturation amplitude
467 in the high growth limit of Fig. 16.

468 IV. SUMMARY AND DISCUSSION

469 In this paper, following the previous work in Ref. 1, we have analyzed the nonlinear dy-
470 namics of shear Alfvén fluctuation saturation and the corresponding fluctuation induced EP
471 transport in DTT plasmas. The simulations address two particular cases, namely, $n = 4$
472 RSAE fluctuations and $n = 6$ TAE fluctuations for the central and outer core regions of
473 DTT plasmas, respectively. These cases can be considered as typical paradigms to illus-
474 trate the rich and diverse physics due to resonant excitation of Alfvénic fluctuations by
475 supra-thermal particles in DTT and, more generally, in reactor relevant fusion plasmas. In
476 particular, we focus on the mode saturation mechanism and on the relative importance of
477 resonance detuning versus radial decoupling, exploring the properties of EP transport as
478 coherent redistribution and/or diffusive transport. Test particle method and Hamiltonian
479 mapping technique are extensively used to illustrate the wave-EP resonant interactions and
480 the nonlinear evolution of EP phase space zonal structures.

481 The nonlinear saturation of RSAE fluctuations in the central core region, dominated by
482 trapped particle precession resonance, is consistent with previous theoretical and numerical
483 studies. By analyzing the linear mode and resonance structures, nonlinear EP orbit ex-
484 cursion as well as saturation amplitude, we show that the prevalent saturation mechanism
485 is radial decoupling, which plays an important role even when the fluctuation is close to
486 marginal instability. This is consistent with theoretical understanding, where the effect of
487 resonance detuning is expected to be much weaker for trapped than passing particles, due
488 to the flatter radial profile of the resonance frequency. Moreover, the radial scale of res-
489 onant EP re-distribution is generally comparable with the mode radial width. Thus, the
490 fluctuation induced trapped EP transport expectedly occurs on meso-spatial scales for a
491 wide range of reference equilibria and corresponding plasma stability. In fact, the relevant

492 spatio-temporal scales of nonlinear wave-EP dynamics are controlled by equilibrium geom-
493 etry, plasma nonuniformity and perturbative versus non-perturbative EP response; and are
494 ultimately reflected by the features of the fluctuation spectrum and the corresponding most
495 unstable toroidal mode number. The scaling of saturated fluctuation amplitude with respect
496 to the linear growth rate is, in general, approximately linear. Meanwhile, in high growth
497 rate limit, self-consistent modulations of mode structure and frequency are observed, which
498 maximize wave-EP power transfer and contribute to the enhanced saturation level in this
499 regime. In addition, the longer time scale evolution with strong drive suggests non-adiabatic
500 frequency sweeping and secular EP transport, similar to those of strongly unstable EPs.
501 A detailed investigation of longer time scale nonlinear dynamics is beyond the scope of this
502 work, and will be the subject of a future publication. In fact, we note that the strongly
503 unstable RSAEs discussed in this work are in the relevant parameter regime for burning
504 plasma physics studies.^{1,40}

505 The nonlinear dynamics of TAE fluctuations in the outer core region can be quite different
506 depending on the strength of mode drive. The TAE fluctuations are driven collectively by
507 both magnetically trapped and passing EPs, which show very different resonant interaction
508 length scales due to their resonance condition: wave-particle power transfer with passing
509 EPs is characterized by much finer scale than trapped EPs. Thus, trapped and passing EPs
510 exhibit different nonlinear transport length scales and play independent or synergetic roles
511 in mode saturation under various stability regimes. For sufficiently low linear growth rate,
512 low amplitude saturation is observed, mainly due to the nonlinear reduction of passing EP
513 drive, while trapped EP drive is essentially unaltered. EP radial redistribution in this case
514 is local in phase space; thus, transit resonances are radially well separated and transport
515 effects on the EP density profile are negligible. Meanwhile, for stronger linear growth rate,
516 with sufficiently high fluctuation level and corresponding enhanced EP radial excursion,
517 transit resonances may overlap. Phase space orbits become stochastic and passing EP radial
518 transport is diffusive over the length scale of the mode width. The fluctuation saturates with
519 meso-scale redistributions over the whole radial mode structure for both resonant trapped
520 and passing EPs; and the overall EP flux is reflected by a significant distortion of the EP
521 density profile. This suggests that TAEs in the outer core region may be a more serious
522 concern than RSAEs in the central core for the limits they may impose on plasma operations
523 to avoid global EP losses. Furthermore, note that for single- n simulations reported in this

524 paper, large power input and the contribution of trapped EP drive are necessary to cause
 525 transit resonance overlap and diffusive transport of passing EPs. For realistic scenarios
 526 with multi- n modes excited simultaneously, the stochasticity threshold is much lower with
 527 much more resonances.⁴⁷ Thus, EP diffusive transport by spontaneously excited multi- n
 528 TAEs could occur at much lower EP concentration, and will be further explored in future
 529 studies. The present work also suggests that transition to resonance overlap and diffusive
 530 EP transport is connected with equilibrium geometry and plasma nonuniformity as well
 531 as non-perturbative EP response. In fact, stronger EP drive causes the saturation level
 532 to be enhanced over the quadratic scaling with the linear growth rate to be expected for
 533 resonance detuning. The scaling finally reduces to approximately linear in the high growth
 534 rate limit with nonlinear EP transport comparable to the mode width, suggesting that radial
 535 decoupling should be expected for strongly driven TAEs in the outer core region.

536 In summary, by further investigating the DTT reference scenario assumed in Ref. 1, the
 537 present work confirms the anticipations on the rich and diverse physics that is expected in
 538 DTT core plasmas. The characterizing element is the Larmor radius normalized to plasma
 539 minor radius, $\rho^* \equiv \rho_L/a$, for both EPs as well as thermal plasmas. In particular, the ratio
 540 of these two fundamental parameters, which is controlled by the characteristic EP energy in
 541 units of the critical energy, plays a fundamental role. In DTT plasmas and, more generally,
 542 in reactor relevant conditions, the micro-scales of Alfvénic instabilities resonantly excited
 543 by EPs are of the same order of meso-scale structures due to drift wave turbulence. This
 544 is one crucial reason why EPs are considered mediators of cross scale couplings, with their
 545 predominant contribution to the local power balance further emphasizing their unique role.

546 Another important physics process illuminated by the present work is the nonlocal trans-
 547 fer of energy and momentum in phase space, due to the peculiar role of magnetically trapped
 548 and passing EPs. This *channeling* in phase-space, which may involve different mode num-
 549 bers, and the general properties of the fluctuation spectrum discussed in this work, confirm
 550 the importance of looking at transport processes in phase space when dealing with collision-
 551 less fusion plasmas; that is, the importance of phase space zonal structures.

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 561 Computing infrastructure and its staff.⁴⁸

562 **Appendix A: Test particle selection**

563 Following Ref. 23, the test particle selection is introduced in this appendix. In the
 564 equilibrium magnetic field, the particle toroidal angular momentum P_ϕ and energy E are
 565 constants of motion. However, in the presence of a finite amplitude fluctuation, due to wave-
 566 particle interactions (e.g., $\mathbf{E} \times \mathbf{B}$ drift), the particle orbit is perturbed and conservations
 567 of P_ϕ and E are broken. Following Refs. 5, 24, and 41, for a single- n mode with constant
 568 frequency, a new quantity can be constructed from the extended phase space Hamiltonian,

569
$$C \equiv \omega P_\phi - nE, \tag{A1}$$

570 which is conserved in addition to the magnetic moment M . In the physics model of HMGC,

571
$$P_\phi \simeq m_H R U + e_H R_0 (\psi_{\text{eq}} - \psi_{\text{eq}0}) / c \tag{A2}$$

572 at the leading order.²³ Here, R is the major radius coordinate, U is the parallel (to the
 573 equilibrium magnetic field) velocity, e_H is the EP charge, ψ_{eq} is the equilibrium magnetic flux
 574 function, and $\psi_{\text{eq}0}$ is the value of ψ_{eq} on the magnetic axis. Moreover, $E = m_H U^2 / 2 + M \Omega_H$.

575 Given the conservation properties of M and C , we could then only look at a single resonant
 576 “slice” (M_0, C_0) of the EP distribution function, since linear and nonlinear evolution of the
 577 considered slice is independent of others. The selected slice is identified from a reduced
 578 phase space grid (r, M, U) with significant wave-EP power transfer in the linear stage, and
 579 is sampled by a group of test particles, which are initialized with M_0 , C_0 and properly

580 (e.g., uniformly) distributed in r , θ , ϕ directions, where θ and ϕ are respectively, poloidal
 581 and toroidal angles. Test particles are evolved in the electromagnetic field stored from the
 582 self-consistent simulation and, thus, are representative of the dynamic behavior of physical
 583 particles with the same phase space coordinates. The test particle characteristic resonance
 584 frequency, ω_{res} , can be computed as²⁴

$$585 \quad \omega_{\text{res}}(\bar{r}, M_0, C_0, \ell) = n\omega_d + \ell\omega_b \quad (\text{A3})$$

586 for magnetically trapped particles, and as

$$587 \quad \omega_{\text{res}}(\bar{r}, M_0, C_0, \ell) = n\omega_d + \ell\omega_b + (n\bar{q} - m)\sigma\omega_b \quad (\text{A4})$$

588 for passing particles. Here, \bar{r} stands for the orbit averaged particle radial coordinate (playing
 589 the same role as P_ϕ for fixed M_0 and C_0); $\omega_d = (\Delta\phi/2\pi - \sigma\bar{q})\omega_b$ is the toroidal precession
 590 frequency, where $\Delta\phi$ is the change of ϕ over the period of a bounce orbit, $\tau_b = \oint d\theta/\dot{\theta}$,
 591 $\sigma = \text{sgn}(U)$, \bar{q} is the weighted safety factor integrated along the particle orbit;²⁴ ℓ is the
 592 bounce harmonic; and $\omega_b = 2\pi/\tau_b$ is bounce/transit frequency for trapped/passing particles.
 593 Furthermore, for a mode with finite linear growth rate γ_L , the condition for effective resonant
 594 power transfer could be given as

$$595 \quad |\omega - \omega_{\text{res}}(r, M_0, C_0, \ell)| \lesssim \gamma_L. \quad (\text{A5})$$

596 That is, significant wave-particle resonant interaction can take place when the frequency
 597 difference is of the order of γ_L . Eq. (A5) could be solved with respect to r or P_ϕ , yielding
 598 the resonance width $\Delta r_{\text{res}}(\omega, M_0, C_0, \ell)$ or $\Delta P_{\phi\text{res}}(\omega, M_0, C_0, \ell)$.

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