Annual Review of Condensed Matter Physics

Thermodynamics in Single-Electron Circuits and Superconducting Qubits

J.P. Pekola¹ and I.M. Khaymovich^{2,3}

¹Quantum Technology Finland Centre of Excellence, Department of Applied Physics, Aalto University School of Science, 00076 Aalto, Finland; email: jukka.pekola@aalto.fi

²Max Planck Institute for the Physics of Complex Systems, 01187 Dresden, Germany; email: ivan.khaymovich@pks.mpg.de

³Institute for Physics of Microstructures, Russian Academy of Sciences, 603950 Nizhny Novgorod, GSP-105, Russia

Annu. Rev. Condens. Matter Phys. 2019. 10:193–212

The *Annual Review of Condensed Matter Physics* is online at conmatphys.annualreviews.org

<https://doi.org/10.1146/annurev-conmatphys-033117-054120>

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Keywords

stochastic thermodynamics, fluctuation relations, Maxwell's Demon, quantum open systems, heat engines and refrigerators

Abstract

Classical and quantum electronic circuits provide ideal platforms to investigate stochastic thermodynamics, and they have served as a stepping stone to realize Maxwell's Demons with highly controllable protocols. In this article, we first review the central thermal phenomena in quantum nanostructures. Thermometry and basic refrigeration methods are described as enabling tools for thermodynamics experiments. Next, we discuss the role of information in thermodynamics that leads to the concept of Maxwell's Demon. Various Maxwell's Demons realized in single-electron circuits over the past couple of years are described. Currently, true quantum thermodynamics in superconducting circuits is a focus of attention, and we end the review by discussing the ideas and first experiments in this exciting area of research.

MD: Maxwell's
Demon

1. INTRODUCTION

1.1. Thermal Properties of Quantum Nanostructures

Thermodynamics of small systems, where fluctuations play a key role, is a topic of intense experimental interest currently. Theoretical framework has been laid over the past decades (1–10), but first realizations in the laboratory have come up much later (11–28; Ribezzi, private communication). In this article, we present experiments on electronic circuits, which present highly favorable systems for the experimental studies of stochastic thermodynamics. In contrast to soft-matter systems, where fluctuation relations were first investigated experimentally, electronic circuits present certain advantages. First, the circuits are stable, meaning that experiments can be repeated many times under essentially identical conditions: This allows for large statistics; normally up to 10^6 experimental realizations can be achieved as compared to typically 10^2 – 10^3 repetitions in soft-matter experiments. Second, the Hamiltonian of the circuits is usually very simple, and the system is governed by it accurately; furthermore, electrons can be monitored one by one, and coupling to heat baths can also be modeled precisely. Finally, the experiments are usually performed at low temperatures, which means that circuits made from superconducting metals behave quantum mechanically: They thus present an ideal test bed to investigate thermodynamics of open quantum systems.

Our main interest in most of this review is the electron system in the nanostructures. We deal primarily with conductors formed of ordinary metals, in which even the smallest structures (volume of $\sim 10^{-22}$ m³ can be realized by standard electron-beam lithography based nanofabrication) contain about 10^8 conduction electrons. This large number means that it is possible to assign a well-defined temperature to such a conductor of electrons, if this subsystem is in local equilibrium. It is quite fortunate that this is indeed the case in most experiments at subkelvin temperatures, like the ones presented in this review. First, under these conditions the electrons are effectively thermally isolated from the phonon bath and locally from other conductors, typically either by superconducting leads or tunnel barriers. Another important point is that even in clean elemental metals, the electron–electron scattering rate γ_{e-e} is as large as 10^9 s⁻¹. This rate is larger than a typical injection rate γ of nonequilibrium carriers in the experiments that we discuss. For instance, $\gamma \sim 1$ s⁻¹ in the Szilard's engine (17, 18), and $\gamma \sim 10^6$ s⁻¹ in the autonomous Maxwell's Demon (MD) experiments (20). The local temperature of the electrons T_e can then differ from that of the phonon bath T because the electron–phonon relaxation rate γ_{ep} scales as T^3 in ordinary metals and assumes a value of about $\gamma_{ep} \sim 10^5$ s⁻¹ at a typical operating temperature of $T = 0.1$ K. Putting all above together means that electrons follow Fermi–Dirac distribution, where their temperature T_e is then determined by external biasing conditions and coupling to other conductors.

In the scenario illustrated above, one can then distinguish two regimes of operation in terms of the response to external injection of heat. The most common and straightforward scheme corresponds to quasi-stationary power $\dot{Q}$, which results in a steady-state temperature change δT_e with respect to that under equilibrium conditions. In this kind of bolometric detection, the signal, i.e., the temperature change, depends on the thermal conductance G_{th} to the bath, which is most often given by the electron–phonon coupling. We have for small changes of temperature, $\delta T_e/T_e \ll 1$, $\delta T_e = \dot{Q}/G_{th}$. It is worth noting the basic but often overlooked feature of bolometric detection: The steady-state temperature does not depend on the heat capacity C_e of the electron system. On the contrary, the temporal evolution of T_e is governed by the heat capacity and G_{th} . In the particular case of abrupt absorption of energy Q in the electron system, the local temperature changes—again the linear regime is assumed—instantaneously (within the time $\sim \gamma_{e-e}^{-1}$) by the amount $\delta T_e = Q/C_e$. Following this transient, the local temperature then returns back to the equilibrium value exponentially with the relaxation time $\tau = C_e/G_{th}$ as $\delta T_e(t) = \frac{Q}{C_e} e^{-t/\tau}$ (see **Figure 1a**). Detecting such energy quanta thermally, i.e., calorimetry, is described later in

Figure 1

(a) Schematics of calorimeter setup. Upper panel shows a normal-metal resistor absorbing photons from an “artificial atom” (qubit, etc.), temperature of which is measured in real time via voltage $V(t)$ (bottom panel). (b) Energy diagram of the NIS junction at a temperature T . The energies are considered with respect to the Fermi energy of the normal metal normalized to the superconducting gap. Solid lines show the density of states in the normal metal $n_N(E)$ and superconductor $n_S(E)$, occupation numbers are described by filled areas. (c) The same NIS energy diagram as in panel b, but under a finite voltage bias V demonstrating the refrigeration principle. Electrons tunnel from the normal metal at energies above the Fermi level to the superconductor, cooling the former and heating up the latter. (d) The schematics of the SEB based on two Coulomb-blockaded islands (1 and 2) tunnel-coupled to each other (gray area) and voltage-biased V_g with respect to each other via capacitive couplings $C_{g1,2}$ to the leads. The tunnel junction is characterized by the resistance R_T and the capacitance C . The electron tunneling between islands is shown by black arrows together with the charge ne of the left island. The charge detector shown by gray dashed lines is formed by an SET. Abbreviations: NIS, normal metal–insulator–superconductor; SEB, single-electron box; SET, single-electron transistor.

Section 4.2 of this review. One more ingredient to add to this simplified picture of temperature variation of the electron subsystem is the issue of fundamental energy fluctuations. Due to the coupling to the bath, the energy of the electron system fluctuates around its mean value by $\langle \delta E^2 \rangle = k_B T_e^2 C_e$. The frequency spectrum of these fluctuations extends up to τ^{-1} . This noise can be facilitated, for instance, by the fluctuating impact of phonons interacting with electrons. Such fluctuating energy then translates into fluctuating effective temperature of the electron system with $\langle \delta T_e^2 \rangle = k_B T_e^2 / C_e$, which sets the fundamental detection limit of temperature. In a practical sense this temperature noise can be as large as 1 mK at $T_e = 100$ mK for a small metallic conductor described in the beginning of this section. For electron-phonon-limited heat transport, the fluctuations then extend up to the cut-off frequency $\gamma_{e-p} \sim 10^5$ Hz. Measuring temperature in the DC and AC regimes by a few particular thermometers is described in the next section.

The focus of this review is in measurements of work and heat in small systems. The mesoscopic electronic systems at low temperatures provide a nearly unique setup to detect small amounts of heat directly via temperature variations as described above. Heat is then the energy that is distributed to the electron gas. The detectability of temperature variations is a favorable consequence of the combination of the effective isolation of the system from the bath and the extremely small heat capacity. The heat capacity is determined by the nearly free Fermi gas at a temperature of $\sim 10^{-6} T_F$, where T_F is the Fermi temperature. However, work in these systems is the change of the energy of the few degrees of freedom on which the external force is applied to. This can be given by the charging state of the single-electron circuit (electron number on the box) or the level populations in a quantum bit (qubit). We discuss these issues in detail in Sections 2 and 4.

1.2. Thermometry

Temperature of the electron system is coded in its energy distribution of the Fermi–Dirac form, i.e., $f(E) = 1/(1 + e^{E/k_B T_e})$, where E is the energy of an electron with reference at the Fermi level.

Qubit: quantum bit

DOSs: densities of states

NIS: normal metal–insulator–superconductor

It is therefore necessary to find sensitive probes of the distribution for thermometry. The probe should also lead to as little back-action as possible; that is, it should not disturb the system by driving it into nonequilibrium.

Tunnel probes provide a convenient measurement of temperature. The rate of tunneling in a junction from electrode 1 to 2 typically through an insulating barrier is given by

$$\Gamma_{1 \rightarrow 2}(\delta E) = \frac{1}{e^2 R_T} \int dE n_1(E) n_2(E + \delta E) f_1(E) [1 - f_2(E + \delta E)]. \quad 1.$$

Here, R_T is the resistance of the tunnel contact, $n_i(E)$ are the normalized (with the corresponding normal state value) densities of states (DOSs), and $f_i(E)$ are the distributions in electrodes $i = 1, 2$. Here, δE is the energy gain in the tunneling event, which is determined by the voltage bias eV in the absence of charging effects; the case of Coulomb effects is discussed separately. Furthermore, we assume an electron–hole symmetry to be satisfied for DOSs $n_i(-E) = n_i(E)$ and distribution functions $f_i(-E) = 1 - f_i(E)$ in both electrodes leading to $\Gamma_{1 \rightarrow 2}(\delta E) = \Gamma_{2 \rightarrow 1}(\delta E)$, and omit the rate subscript where it does not cause a confusion.

For the sake of our arguments below, let us take a normal metal–insulator–superconductor (NIS) junction, where 1 is an “S” with Bardeen–Cooper–Schrieffer DOS $n_S(E) = |\text{Re}(E/\sqrt{E^2 - \Delta^2})|$ and 2 is an “N” with $n_N(E) = 1$. Here, Δ is the superconducting gap. Then for a voltage-biased junction $\Gamma_{S \rightarrow N}(eV) = \frac{1}{e^2 R_T} \int dE n_S(E) f_S(E) [1 - f_N(E + eV)]$, with the electron charge $-e < 0$. The average charge current $I(V) = e [\Gamma_{S \rightarrow N}(eV) - \Gamma_{N \rightarrow S}(-eV)] = -I(-V)$ from N to S is then given in this system by

$$I(V) = \frac{1}{2eR_T} \int dE n_S(E) [f_N(E - eV) - f_N(E + eV)]. \quad 2.$$

For thermometry this equation implies that current through the NIS junction depends only on the temperature of N (29), i.e., its distribution $f_N(E)$, since $n_S(E)$ is temperature independent for practical purposes when the temperature is much lower than the critical temperature T_C of the superconductor (see **Figure 1b**). This makes an NIS junction a very favorable choice for thermometry, working down to a few-millikelvin range of temperatures (30). Current or conductance can be monitored in either of the two steady-state conditions or, more recently, also to follow fast changes of temperature.

1.3. Refrigeration

Another important result for an NIS junction yields the average heat current $\dot{Q}(V)$ into the normal electrode N as

$$\dot{Q}(V) = \frac{1}{e^2 R_T} \int dE (E - eV) n_S(E) [f_N(E - eV) - f_S(E)]. \quad 3.$$

The implications of this relation are that due to the gap Δ and the singularities in the DOS of S, the power is negative for $|eV| < \Delta$; i.e., the biased junction refrigerates N (and heats S; see **Figure 1c**), and for large biases $|eV| \gg \Delta$ it Joule heats both the N and S electrodes (31–34). Both Equations 2 and 3 are useful in what follows.

2. SINGLE-ELECTRON CIRCUITS: WORK, HEAT, AND ENTROPY AND FLUCTUATION RELATIONS

2.1. Single-Electron Box

SEB: single-electron box

Up to now, we have assumed tunneling with no Coulomb effects included. We noted earlier, however, that the energy gain in tunneling can be generalized to δE instead of eV , which is essential when the additional energy of one electron counts. Let us focus on a single-electron box (SEB), as depicted in **Figure 1d**. It consists of a tunnel junction with capacitance C , where the rates are governed by the formulae presented above Equation 1, and of a gate capacitance $C_g = C_{g1} + C_{g2}$ and voltage V_g applied on it. We denote the total capacitance $C_\Sigma = C + C_g$. Irrespective of the type of the electrodes of the tunnel contact, the electrostatic energy $U(n, n_g)$ of the SEB with n extra electrons on it is that of a capacitor with¹

$$U(n, n_g) = E_C(n - n_g)^2. \quad 4.$$

Here, $E_C = e^2/2C_\Sigma$ is the charging energy, and $n_g = C_g V_g/e$ is the polarization charge in units of e induced by the gate voltage. The polarization charge n_g is a continuous variable, whereas n assumes naturally only integer values. (Yet n can be both negative and positive, since as noted before, the total number of electrons is $\sim 10^8$ or larger, but the Coulomb energy fixes its value typically to within few adjacent values.)

2.2. Work, Heat, and Entropy

The SEB can be viewed as a system in which the work on it is done by the gate voltage source, and the bath is that formed of the electron gas (that is eventually coupled to the “superbath” of phonons). It is now quite straightforward to write the expressions of work W and heat Q to the bath in a driven process. This is a realization specific quantity, because we are dealing with stochastic thermodynamics, where standard thermodynamic results emerge as averages over many realizations. The heat is the irreversible part of the energy that is associated with inelastic transitions from the point of view of the Coulomb energy. For simplicity, and to represent a typical experimental situation, we focus on the situation in which n can assume values 0 and 1 only, when the gate parameter is within $0 \leq n_g \leq 1$. This happens when the temperature of the electrons is low enough, $k_B T_e \ll E_C$. Based on Equation 4, Q_i , the loss of energy to the bath in the transition occurring at gate position $n_{g,i}$, where an electron tunnels into (+) or out from (−) the box, is given by $Q_i = \pm E_C(2n_{g,i} - 1)$. If there are several transitions within the measurement protocol, the total heat is then

$$Q = \sum_i \pm E_C(2n_{g,i} - 1). \quad 5.$$

To find the total work W performed to the system, we need to add the change in internal energy of the system in the particular realization; i.e.,

$$W = \Delta U + Q, \quad 6.$$

where $\Delta U = U(n^f, n_g^f) - U(n^i, n_g^i)$. Here superscripts i and f refer to the initial and final configurations, respectively. For some experiments to be presented, the protocol runs from $n_g^i = 0$ to

¹This charging energy is modified due to the so-called parity effect when one or both islands are superconducting and the energy gap in the low-temperature limit shifts up the energy of the state with odd electrons.

$n_g^f = 1$, and in this case the internal energy change simplifies into $\Delta U = (1 - n^i - n^f)E_C$. The dissipated work, $W_d \equiv W - \Delta F$, appearing in some fluctuation relations, is related to Equation 6 through the equilibrium free energy difference $\Delta F = F(n_g^f) - F(n_g^i)$ between the end points of the drive, $F(n_g^k) = -k_B T \ln\{\sum_n \exp[-U(n, n_g^k)/k_B T]\}$, and $k = i, f$. In the equilibrium state of the system, i.e., when $\Delta F = \Delta U - T \Delta S_{\text{sys}}$, the dissipated work defines the total entropy production on the trajectory,

$$W_d/T = \Delta S \equiv \Delta S_{\text{sys}} + \Delta S_{\text{bath}} \quad 7.$$

as the sum of system $\Delta S_{\text{sys}} = -\ln(P_{n_f}^f/P_{n_i}^i)$ and bath $\Delta S_{\text{bath}} = Q/T$ entropy productions.

In general, one can assign the bath entropy production to each i th transition between n_{i-1} and n_i in an experiment as soon as the tunneling rates at the particular values of control parameter(s), i.e., gate voltage, are known:

$$\Delta S_{\text{bath}} = \sum_i \ln \frac{\Gamma_{n_{i-1} \rightarrow n_i}(n_{g,i})}{\Gamma_{n_i \rightarrow n_{i-1}}(n_{g,i})}. \quad 8.$$

This expression summed with ΔS_{sys} is particularly useful for determining the total entropy production $\Delta S = \Delta S_{\text{sys}} + \Delta S_{\text{bath}}$ in the situations when the precise temperature of the bath is unknown or this reservoir is in nonequilibrium. The system entropy $S_{\text{sys}} = -\ln P_n$ is still determined by the instantaneous probability distribution P_n .

The presented expressions allow us to discuss thermodynamics in a SEB. From the point of view of primary value of the measurements, one should, however, keep in mind that measuring the transitions in a SEB is not a completely direct measurement of heat and work, but it is in line with the majority of measurements of thermodynamics in small systems. We have presented in Section 1.1 ways to measure heat bolometrically or calorimetrically, which are the most direct ways of determining heat experimentally. It is very rare, however, that heat can be measured directly: In this respect the recent experiment on MD in a single-electron circuit is an exception (20). In what follows, we relate the transition statistics (in theory and experiment) to common fluctuation relations and to the analysis of MDs. The basic SEB is naturally the simplest single-electron circuit to consider, and more involved circuits are applied in experiments. The relevant expressions are given along the way as such circuits are presented.

2.3. Fluctuation Relations

Fluctuation relations governing small systems even far from equilibrium conditions are currently under active investigation (1–8). This is because various nano- and microscale systems have become experimentally feasible over recent years. Although fluctuation relations often serve just as sanity checks of the experiment, they also provide means to extract parameters of the studied system and, as advertised since early experiments, the equilibrium free energy of the system (26–28). The recent studies of MDs and information-powered refrigerators yield interesting connections of information and energy via the newly found generalized fluctuation relations (9, 10).

The general fluctuation relation (see References 6–8, and references therein),

$$P(\Delta S)/P_R(-\Delta S) = e^{\Delta S/k_B}, \quad 9.$$

relates the probability distribution $P(\Delta S)$ of stochastic entropy production ΔS in the forward process with the one $P_R(-\Delta S)$ in the reversed process.² This expression leads simply to the well-known expectation value of the exponent of the entropy production in repeated experiments with a given protocol as

$$\langle e^{-\Delta S/k_B} \rangle = 1. \quad 10.$$

These expressions have their counterparts for driven systems either initialized in equilibrium with the temperature T , where ΔS is replaced by W_d/T (5), or under steady-state drive conditions, where in Equation 9 the probabilities in numerator and denominator correspond to the same probability distribution (4, 35–38).³

An alternative way to quantify the events of negative entropy production is to address its negative record statistics. For stationary processes there have been several universal bounds derived for the statistics of the events of negative entropy production (40–42). However, charge counting in single-island devices, such as an SEB or a single-electron transistor (SET), is not applicable to address the above-mentioned observables. Indeed, these devices, being a perfect toolbox for quantitative experimental studies of stochastic thermodynamics (22), do not provide information on the direction of electron tunneling, a feature that is key to characterize entropy production in the system in nonequilibrium steady state in contact with several reservoirs. To access the negative record statistics, one needs single-electron devices with multiple islands allowing measurement of the charge tunneling direction (43–45) and thus providing a test bench for results of steady-state thermodynamics. In this review, we focus mostly on single-island devices that are the simplest ones allowing us to realize MDs.

SET: single-electron transistor

TLS: two-level system

3. CLASSICAL MAXWELL'S DEMONS IN SINGLE-ELECTRON CIRCUITS

3.1. Introduction

Fluctuation relations, Equations 9 and 10 considered above, hint that the second law of thermodynamics is satisfied only on average and should be “violated” in a certain amount of realizations due to thermal fluctuations of the bath(s) coupled to the system. This fact opens a way to utilize these thermal fluctuations challenging the second law by a thought experiment by Maxwell: A human being, called Maxwell’s Demon, by measuring and controlling a microstate of the system, can extract some work from the System by decreasing a system stochastic entropy S_{sys} without direct energy costs. This paradoxical example has initiated long-standing debates of the concept of MD and of the role of the information in statistical physics and feedback control (see, e.g., References 46–48, and references therein).

Leo Szilard has given a more quantitative example of an information-mediated engine that relates a bit of information gathered by an observer with the maximal amount of work possible to extract from a system. The main idea of this Szilard engine can be explained using an example of a classical two-level system (TLS) coupled to a single bath (see **Figure 2a**). Consider the TLS initialized in the state with maximal average entropy $\langle S_{\text{sys}} \rangle = k_B \ln 2$ corresponding to equal probabilities of both states $P_n = 1/2$, with $n = 0, 1$. The actual state of the system n is a stochastic

²There are some other variants for the same probability distribution $P(\Delta S)$ either in steady-state conditions (3, 4, 35) or in periodically driven systems (36).

³Some relations between generally and periodically driven or steady-state regimes are currently under investigations; see, e.g., Reference 39.

Figure 2

(a) The original proposal of a Szilard's engine in terms of a classical two-level system (shown as a particle in a two-well potential). (b) The top trace shows the applied gate voltage signal $n_g = C_g V_g / e$ providing a feedback. The bottom time trace is the measured excess electron location signaled by the SET current I_{det} . The corresponding values of n are shown in panel b. Panel b adapted from Reference 17. Abbreviation: SET, single-electron transistor.

variable governed by bath thermal fluctuations. After an ideal measurement of the state n , one can freeze the TLS in it, e.g., by tilting the potential such that it makes the excited state nearly unreachable, $P_n = 1$, $P_{1-n} = 0$ [see upper (lower) leg of **Figure 2a** for $n = 0(1)$]. In this process the detector stores one bit of information as $n = 0$ or 1 . The averaged system entropy after the measurement becomes zero, $\langle S_{\text{sys}} \rangle = 0$. To close the loop one can adiabatically return the system to the energy-level crossing point with the equilibrium distribution $P_0 = P_1 = 1/2$ and extract on average the work $\langle W \rangle = k_B T \langle \Delta S_{\text{sys}} \rangle = T \ln 2$. As the TLS and the bath constitute a thermally isolated system $\Delta S \geq 0$, the process behind a Szilard engine is nothing else but a conversion of heat $-\langle Q \rangle \leq k_B T \ln 2$ extracted on average from a single bath into work. This process seems to be a realization of a perpetual mobile of the second kind.

To resolve this obvious paradox, one should take into account the observer's thermodynamics, the amount of the information stored in its memory, and an unavoidable need to erase this information. As imposed by Landauer's principle (49, 50), the energy dissipation needed to erase one bit of information is bounded by $\langle Q_{\text{det}} \rangle \geq k_B T \ln 2$. This principle restores, in particular, the second law for the Szilard's engine,

$$\langle Q_{\text{det}} \rangle + \langle Q \rangle \geq 0. \quad 11.$$

In more rigorous analysis, one should take into account errors in the measurement: In particular, the outcome of the measurement m being a stochastic variable is correlated with the actual state of the system n , but it does not necessarily coincide with it. To quantify this correlation in

the information theory the mutual information is introduced,

$$I_M(n|m) = \ln P_{n,m} - \ln P_n - \ln P_m, \quad 12.$$

where $P_{n,m}$, P_n , and P_m are joint and marginal distributions of the actual state n and of the measurement outcome m .⁴ The average mutual information $\langle I_M \rangle = \sum_{m,n} P_{n,m} I_M(n|m)$ is nonnegative and has the upper bound of the Shannon's entropy $\langle S_{\text{sys}}/k_B \rangle$ obtained in the completely correlated case $m = n$. $I_M(n|m)$ plays an important role in the information theory and, in particular, appears in the generalization of the Jarzynski equality Equation 10 for a feedback-controlled system as follows (10):

$$\langle e^{-\beta(W-\Delta F)-I_M} \rangle = 1. \quad 13.$$

Note that the standard Jarzynski equality is violated in this case $\langle e^{-\beta(W-\Delta F)} \rangle = \gamma_E$ with $0 < \gamma_E < 2$. Consequently, the mutual information also places a lower (negative) bound on the average work dissipated in the system under feedback control (9),

$$\langle W - \Delta F \rangle \geq -k_B T \langle I_M \rangle. \quad 14.$$

3.2. Szilard's Engine

In the past decade several realizations of MDs have been demonstrated in various systems ranging from colloidal particles (11, 19) and photons (24) to complicated objects such as DNA molecules and organic polymers (Ribezi, private communication). Single-electronic devices, being an easy realization of a classical few-level system, provide an opportunity to realize the concept of information-mediated work extraction (see References 17 and 18 for seminal experiments), which thereafter can be used for electronic cooling (20, 55) and squeezing of shot noise (56). Another advantage of single-electronic devices is their stability and robustness, allowing one to repeat a drive protocol many times and collecting large statistics in one sample. Overall, despite all simplicity of single-electronic devices, they can work as a benchmark for testing various concepts of stochastic thermodynamics (12, 13) and counting statistics (57), and they can even simulate multifractality of the critical wave functions in the vicinity of the Anderson transition (21).

The first MD in single-electron devices realized in References 17 and 18 uses an SEB as a feedback-controlled system with the very same protocol of a Szilard's engine as that discussed above and shown in **Figure 2a**. The initial state of the maximal average system entropy is realized at the control parameter $n_g = 1/2$, where the energies $U(0, n_g)$ and $U(1, n_g)$ (Equation 4) are degenerate. The measurement of the microscopic state n performed by a capacitively coupled SET working as a charge detector (see **Figure 1d**) gives the outcome m . The feedback freezing the SEB in its ground state is applied by rapid driving of n_g toward the m value. Eventually n_g is slowly returned to the degeneracy point; see several realizations of this protocol in **Figure 2b**.

To determine the work (Equation 6) equal to the heat (Equation 5) in each realization of this cyclic process, the SET is constantly monitoring the charge state n reconstructing charge trajectories $n(t)$. Repeating this process many times, one can extract the distribution $P(W)$ of applied work (**Figure 3a**), which concentrates mostly on the negative side close to the ideal value $-k_B T \ln 2$ (the *top two insets* show the corresponding process). Due to the not fully adiabatic slow part of the drive, cycle-to-cycle fluctuations of work form continuous $P(W)$. A small bump far on the positive side of the distribution is related to errors in measurement, $m \neq n$, and/or delay in feedback control,

⁴The detailed analysis of errors, optimization of the feedback, and the higher-order tunneling processes in this system have been investigated, e.g., in References 51–54.

Figure 3

(a) Distribution $P(W)$ of experimentally observed work under feedback control is shown as a histogram. The numerical results (black solid line) are in good agreement with the experimental data. Insets show sketches of two processes corresponding to two peaks in $P(W)$. (b) Test of the standard Jarzynski equality (Equation 10) and the generalized one (Equation 13) with mutual information. The error bars include the statistical error, as well as the uncertainty in the measured value of $E_C/k_B T$. Figure adapted from References 17 and 18.

which send the system to the excited state with excess charging energy dissipated eventually to the bath (see the *two right insets* of **Figure 3a**). The optimization of the control parameter value n_g reached in the fast part of the drive gives the overall average work $\langle W \rangle = -0.75 k_B T \ln 2$ quite close to its ideal value, meaning that up to 75% of the collected information can be extracted as work from the bath.

To uncover the role of the mutual information (Equation 12) in this experiment, the error probability $P(m \neq n) \equiv \epsilon \leq 1/2$ is efficiently tuned by changing the SET detector bandwidth. Finite ϵ determines the mutual information as follows: $I_M(n = m) = \ln[2(1 - \epsilon)] \geq 0$ and $I_M(n \neq m) = \ln(2\epsilon) \leq 0$. Because of the applied feedback the standard Jarzynski equality is violated $\langle e^{-\beta(W - \Delta F)} \rangle = \gamma_{JE}$ with the parameter $0 < \gamma_{JE} < 2$ decreasing with the error probability ϵ , whereas its generalized version (Equation 13) including the mutual information is still satisfied with experimental accuracy of 8% (**Figure 3b**). The average mutual information $\langle I_M \rangle = -\epsilon \ln \epsilon - (1 - \epsilon) \ln(1 - \epsilon)$ also places the upper bound (Equation 14) on the average extracted $W - \Delta F$, showing that the feedback efficiency is decreasing as errors grow.

3.3. Autonomous Single-Electron Maxwell's Demon

In all previous examples, feedback-controlled systems were the focus of consideration, with the role of the Demon itself played by an external agent, whose thermodynamics related to the information gathering and erasure was implicitly assumed to be valid. Some aspects of MD thermodynamics have been observed in various mesoscopic systems on the level of verification of Landauer's principle (14–16, 23, 25), but it has been done separately from the actual Demon's operation. All fluctuation relations for work and entropy production (see, e.g., Equations 9, 10, and 13) describe only the system keeping the Demon's dynamics beyond the scope. Another issue with most above-mentioned MD experiments is that the heat extracted from the bath is measured indirectly (owing to fast relaxation rates) and not transformed into some useful work such as an increase of the free energy or the charging of a battery.

To access both the Demon's thermodynamics and the effect of the extracted work, it is natural to implement an MD to the same configuration as a feedback-controlled System and design it in such a way that the information processing and feedback are carried out autonomously (58–60).

Figure 4

(a) Operation principle of an autonomous Maxwell's Demon. Due to the potential applied by the Demon the electrons in the System are always tunneling against this potential, and therefore the System cools down. (b) Energy diagram of autonomous Maxwell's Demon setup built of a System SET (shown in *black*) under voltage bias V coupled to a Demon's SEB (shown in *red*). Finite electronic temperature of the System provides electrons that can overcome the energy cost $J - eV/2$, cooling down the System at the same time. The energy J is dissipated by the Demon as it reacts (subpanels *i* and *iii*), changing the projected energy cost experienced by the electron tunneling in the System from $-J - eV/2$ to $J - eV/2$. Panel *a* adapted from Reference 20. Abbreviations: SEB, single-electron box; SET, single-electron transistor.

On one hand, this allows one to observe the heat dissipation related to the memory erasure in the standalone MD and compare it with the mutual information gained. On the other hand, the autonomous character of the Demon's operation allows for increasing the operating frequencies of the System and the Demon and directing the heat extracted from the bath to some useful work with a noticeable power, which can be directly measured. Note that here we restrict our consideration to autonomous MDs without a separate memory register to access directly the heat dissipated because of information erasure.

In the seminal experiment on an autonomous MD (20), a biased SET plays a role of a feedback-controlled system in which an electron can tunnel between the Coulomb-blockaded island and two leads changing the charge state between $n = 0$ and 1 and cooling or heating the corresponding junction depending on the difference in effective chemical potentials in accordance with Joule's law. The main idea of the autonomous MD behind this is the following (see **Figure 4a**). As soon as something is happening in the System that can be observed, there is a faster detector that reacts immediately and gives the feedback to the monitored System. When an electron tunnels in the System uphill into the island, the Demon reacts immediately. Due to a tunneling event in the Demon, the Demon moves down the level on the System SET island, via an attractive electrostatic potential, trapping the electron there. In this case, to escape from the System island the electron again has to tunnel uphill, and the Demon closes the loop by applying a repulsive potential on the island, moving the island level up again. Overall the System cools down, whereas the detector should in fact heat up due to continuous memory erasure.

In the actual experiment, the Demon is made by an unbiased SET (equivalent to SEB) with N excess electrons on the island, capacitively coupled to the System SET.⁵ The resistance of the

⁵The configuration resembles theoretical proposals of heat-to-current convertors on quantum dots (61, 62) realized in several experiments up to now (63, 64).

detector junctions R_{det} determining its operating frequency by tunneling rates is made much smaller than the one in the System R_s , $R_{\text{det}} \ll R_s$.⁶ This allows MD to measure and apply feedback faster than the System evolves. The energy of the supersystem of two coupled single-electronic devices,

$$U(n, N, n_g, N_g) = E_{C,s}(n - n_g)^2 + E_{C,d}(N - N_g)^2 + 2J(n - n_g)(N - N_g), \quad 15.$$

contains electrostatic energies (Equation 4) of the System itself and the Demon as the first two terms, with the corresponding polarization charges $0 < n_g$ and $N_g < 1$ in units of e . The latter term describes the mutual Coulomb interaction between subsystems. The expressions for the corresponding Coulomb energies $E_{C,s}$, $E_{C,d}$, and J are given in the **Supplemental Text**.

As the Demon is assumed to be autonomous, the supersystem is put in steady-state conditions by keeping both control parameters n_g and N_g constant. A rather small constant bias voltage $eV \ll E_{C,s}, E_{C,d}$ applied to the System makes the overall heat dissipation rate in the Demon $\dot{Q}_{\text{det}}$ and in the System $\dot{Q}_s$ satisfy Joule's law $\dot{Q}_{\text{det}} + \dot{Q}_s = IV$, where I is the steady-state System current. To measure the work extracted by the Demon, one can direct this work, e.g., to the cooling of the System that is in contact just with one bath. The resulting steady-state heat flowing in both subsystems $\dot{Q}_{\text{det}}$ and $\dot{Q}_s$ can be directly measured via changes in the temperatures of the System T_L and T_R , and in detector T_{det} electrodes.⁷

The values of control parameters n_g and N_g can significantly change the mutual dynamics of the System and the MD. Polarization charge n_g of the biased System SET controls the current I flowing through the System, suppressing it at integer values and putting it to the maximum at half-integers. Unlike this, in the unbiased Demon SET, the parameter N_g governs mostly the coupling of the System and MD, tuning the amplitude $2J \min(N_g, 1 - N_g)$ of the mutual electrostatic energy variation as n changes between 0 and 1 and N is in the corresponding ground state.

In the case of $N_g = 0$, the biased System SET is decoupled from the feedback. The bias value determines the dominant tunneling direction of electrons. The parameter n_g governs the current flowing through it by changing the effective energy level of an electron in the SET island (**Figure 5e, 1–3**). If this level is in between electrode biases, the electron tunneling in both junctions occurs with a positive energy gain leading to heat dissipation in both electrodes. Otherwise, if the level is slightly above the upper bias or slightly below the lower one, an electron has to tunnel uphill in one of two junctions, absorbing the energy from thermal fluctuations of the bath and, thus, cooling the corresponding electrode down.⁸ Nevertheless, due to Joule's law $\dot{Q}_s = IV > 0$, the total heat dissipated in the System is nonnegative at any n_g . This effect of the cooling in the normal SET has been discovered in Reference 65, demonstrated in Reference 66, and reproduced in Reference 20 for $N_g = 0$ (see **Figure 5a,b**).

However, one can cool down both electrodes of the System by switching on the feedback. In this case the temperatures of the System electrodes have negative deviations while the Demon heats up significantly (see **Figure 5c,d**). As each tunneling event in the System happens with heat absorption from the bath, the System current is suppressed in comparison to a case without any feedback. To get the qualitative idea of this process, we consider the simplest case of maximal coupling between subsystems, $N_g = 1/2$, and current flowing through the System, $n_g = 1/2$ (see

⁶Here, for simplicity, we consider both detector (Demon) and System with equal left and right tunnel junctions.

⁷For thermal isolation of the System and Demon from the remaining world, superconducting leads were used in the experiment.

⁸Note that at the same time, the current through the device is suppressed, as thermally-activated tunneling is exponentially slower than that with positive energy gain.

Figure 5

Maxwells Demon operation with the bias voltage $V = 20 \mu\text{V}$. (a,b) the measurement of System current I , System $T_{L,R}$ and Demon's T_{det} temperatures at operation point $N_g = 0$, where the Demon is not interacting with the System. Measured data is shown on the left, and simulated data on the right. Color code is given in the y-axis labels. One-sided cooling can be observed in the System (65, 66); however, the total heat generated is positive. (c,d) In the feedback regime at $N_g = 0.5$, both T_L and T_R drop below the base temperature value, showing that the System is thoroughly cooled down by the feedback operation by the Demon. Heat is generated in the Demon in turn. Panel e shows the energetics at the different operation points indicated by black circled numbers in panels a and c. Figure adapted from Reference 20.

Figure 5e, ●. We also assume temperatures to be quite small $k_B T_L, k_B T_R, k_B T_{\text{det}} \ll E_{C,s}, E_{C,d}$ with respect to the charging energies. In this case n and N are limited to two possible values, 0 and 1, and the electrostatic energy (Equation 15) depends on these variables only via the last term,

$$\delta U(n, N) = U(n, N, 1/2, 1/2) - E_{C,s}/4 - E_{C,d}/4 = J(2n - 1)(2N - 1)/2. \quad 16.$$

This forms a doubly degenerate ground state, (0, 1) and (1, 0), and excited state, (0, 0) and (1, 1), with the corresponding energies $\delta U = J/2, -J/2$ and steady-state probabilities $P_{n,N}$,

$$P_{0,1} = P_{1,0} \equiv P_g/2, \quad 17a.$$

$$P_{0,0} = P_{1,1} \equiv P_e/2. \quad 17b.$$

Here, $P_g = \Gamma_+ / (\Gamma_+ + \Gamma_-)$, $P_e = \Gamma_- / (\Gamma_+ + \Gamma_-)$, and the upper (lower) sign in $\Gamma_{\pm} = \Gamma_s(\pm J_+) + \Gamma_s(\pm J_-) + \Gamma_d(\pm J)$ corresponds to the tunneling to the ground (excited) state. The System (detector) tunneling rates $\Gamma_{s(d)}$ are given by Equation 1 with normal-metal DOSs $n_1 = n_2 = 1$

and R_T substituted by $R_{s(d)}$. The energy gains $J_{\pm} = J \pm eV/2$ in tunneling events in the System are subject to the bias voltages of the left $V/2$ and right $-V/2$ lead, respectively (for complete energetics see **Figure 5c,d**). Due to a small detector resistance $R_{\text{det}} \ll R_s$ and a weak δE -dependence of the rates $\Gamma_{s(d)}(\delta E)$ at $\delta E > 0$, the relaxation rate to the ground state is mostly dominated by the tunneling in the detector $\Gamma_+ \simeq \Gamma_d(J)$. At the same time it is mostly the tunneling in the System that brings the total supersystem to the excited state $\Gamma_- \simeq \Gamma_s(-J_-) \ll \Gamma_+$ owing to the exponential suppression of thermally activated rates $\Gamma_{s(d)}(-\delta E) = e^{-\delta E/k_B T_{s(d)}} \Gamma_{s(d)}(\delta E)$. Ideally, when all System tunneling events contribute only to the excitation rate Γ_- , while tunneling in the detector relaxes the total supersystem back, the System itself should absorb the energy of $2J - eV$ from the bath and MD has to dissipate $2J$ per each electron transferred from left-to-right electrode. Thus, the ideal heat generation rates in the System and in the Demon take the forms $\dot{Q}_s^{\text{ideal}} = -(2J/e - V)I$ and $\dot{Q}_d^{\text{ideal}} = 2JI/e$. Here and further, we consider $J - eV/2 > 0$.⁹ Note that in any case the average System current is flowing toward bias $2I/e = P_g [\Gamma_s(-J_-) - \Gamma_s(-J_+)] + P_e [\Gamma_s(J_+) - \Gamma_s(J_-)] > 0$.

Figure 5c,d shows that temperatures T_L and T_R of both System leads goes below the bath temperature value T simultaneously, whereas the System current is suppressed compared to the case of $N_g = 0$. The best efficiency on the order of 50%, $\dot{Q}_{\text{det}} \simeq 0.5 \dot{Q}_{\text{det}}^{\text{ideal}}$, is experimentally achieved near the degeneracy point of the System, $n_g = 1/2$, where the maximal current is flowing through the System.

Although energetically this device follows Joule's law, it is the information flow between the System and MD that permits the decrease of System entropy leading to its cooling. According to Reference 59, the mutual information rates place an upper bound to the System heat absorption rate $-\dot{Q}_s \leq k_B T_s \dot{I}_{M,d}$ and the lower bound to the heat rate generated in MD $\dot{Q}_d \geq k_B T_{\text{det}} \dot{I}_{M,d}$. As the process is cyclic, the average mutual information is conserved and the contribution to its rate governed by the System tunneling is $\dot{I}_{M,s} = -\dot{I}_{M,d}$. The inequality $\dot{Q}_d \geq k_B T_{\text{det}} \dot{I}_{M,d}$ is saturated in the limit $R_{\text{det}} \ll R_s$, as in this case the probability distribution has a thermal form $\ln(P_g/P_e) = 2J/k_B T_{\text{det}}$ (see the **Supplemental Text** for details). In the experiment the above-mentioned inequality $\dot{Q}_d \geq k_B T_{\text{det}} \dot{I}_{M,d}$ differs from equality only by 15%, which verifies the theoretical predictions for MD thermodynamics.

4. THERMODYNAMICS IN OPEN QUANTUM SYSTEMS

4.1. Stochastic Thermodynamics of Qubits

Stochastic thermodynamics of superconducting qubits is expected to be based on the detection of single microwave photons or, in some schemes, on measurement of the state of the qubit itself (see, e.g., References 67 and 68, and references therein). An important difference, as compared to classical stochastic thermodynamics, is that the measurement back-action naturally has a more fundamental meaning in quantum mechanics. In what follows, we discuss the most obvious state measurement of the qubit and calorimetric detection of heat, the latter of which is most compatible with the spirit of traditional thermodynamics.

Quantum trajectories provide a basis to understand stochastic dynamics of qubits (69). On one hand, they are a computational tool, but on the other, they offer a way to represent the true dynamics in a single realization of an experiment. By averaging many such trajectories under identical experimental conditions and realizations, one obtains what is familiar from the standard master equation approach for the density matrix of an open quantum system. Thus, this method based on quantum jumps looks like an ideal tool to treat problems in stochastic quantum

⁹The general (nonideal) case is considered in detail in References 20 and the **Supplemental Text**.

processes; furthermore, associating heat to jumps equivalent to the energy released in a transition process ($\pm$ the energy splitting of the qubit), it is a means to master the energy exchanges in quantum thermodynamic problems. This approach was harnessed in References 70 and 71. In Reference 70, particular attention was given to a qubit driven at its resonance frequency over a few periods in each realization. The Jarzynski fluctuation relation was found to be fulfilled in general in the limit of coupling the qubit weakly to its equilibrium environment (a bath at a given temperature), bringing confidence to the physical correctness of the method. Further theoretical studies confirm this conclusion under different driving protocols (72–75).

4.2. Potential Detection of Single Microwave Photons by Nanocalorimetry

A way to detect quantum trajectories for thermodynamics experiments in a laboratory would be a calorimetry measurement of a mesoscopic bath (76). A possible scenario is to couple a small metallic or semiconducting absorber as a termination of a coplanar resonator of a superconducting (transmon) qubit, as realized in several experiments up to now (77, 78). Yet, in order to be able to detect the quantum jumps, one needs to fulfil two conditions not realized in the earlier qubit setups in this context: (a) One needs to measure the temperature of the absorber fast (79–81), with a bandwidth exceeding the thermal relaxation rate of the electrons in the absorber, typically the electron–phonon rate, which is of the order of tens of microseconds at low ($\lesssim 100$ mK) temperatures; (b) the measurement is to be made at low temperature to decrease thermal noise and the heat capacity of the absorber; and (c) the absorber needs to be made physically small in order to secure small enough heat capacity C_e for sufficient thermal signal upon the photon absorption event.

To be more quantitative, the energy resolution $\delta\epsilon$ of a calorimeter is given by $\delta\epsilon = \sqrt{G_{\text{th}}C_eS_T}$, where G_{th} is the thermal conductance of the absorber to the bath, and S_T is the spectral density of temperature noise at low frequencies, determined either by the measurement setup or by natural temperature fluctuations. In the latter case, which presents the fundamental lower bound when the detector is ideal, the energy resolution is given by $\delta\epsilon = \sqrt{k_B C_e T}$ at temperature T . For realistic parameters, we may take the free-electron heat capacity of a metallic (e.g., copper) absorber of volume 10^{-21} m³, at a temperature of 10 mK, which yields $\delta\epsilon/k_B \lesssim 0.1$ K, which is a few times smaller than the energy of the photons emitted/absorbed by a typical superconducting qubit. Therefore, calorimetric single microwave photon detection looks feasible although challenging with a modest (2–10) signal-to-noise ratio. Further improvements can be expected by using absorbers with lower electron density (semiconductors, two-dimensional conductors), i.e., with lower heat capacity.

4.3. Qubit-Based Quantum Heat Engines and Refrigerators

A quantum heat engine is a quantum-mechanical device converting the heat flow between hot and cold reservoirs to useful work (see, e.g., Reference 82 for a review). The refrigerator is a kind of an inverted heat engine transferring heat from cold to hot reservoir by means of work performed on it. Recently, several theoretical proposals of quantum heat engines (82–90) and refrigerators (91–96) have been suggested, and a few experimental realizations are given (97). Like for the classical heat engines and refrigerators, the efficiency of their quantum counterparts is limited by the Carnot efficiency (83, 98). However, intrinsically quantum phenomena such as entanglement add some specifics to the consideration of quantum systems. Although typically the quantum nature of heat engines diminishes their efficiency (92, 95), it may, however, squeeze the fluctuations of charge (56) and heat (Ptaszynski, private communication) properties.

Figure 6

(a) Schematics of a quantum refrigerator. The cold (R_1) and hot (R_2) resistors are shown by blue and red colors, respectively. (b) Heat and work flow schematics of the system. The resistors R_k and frequencies $\omega_k \simeq (L_k C_k)^{-1/2}$, $k = 1, 2$, correspond to the circuit elements in panel a. (c) Qubit energy diagram with the flux drive $\Phi(t)$ performing the work W to the system. (d) The normalized cooling power $\Pi_1 = \hbar P_1 / E_0^2$ of the cold reservoir R_1 versus dimensionless driving frequency $\Omega = 2\pi \hbar f / E_0$ showing different regimes of the quantum Otto refrigerator, namely, a (nearly) adiabatic regime with the quadratic $\Pi_1 \sim \Omega^2$, an ideal Otto cycle with $\Pi_1 \sim \Omega$, and a high-frequency regime with quantum oscillations in Π_1 . The inset shows the zoomed small frequency limit corresponding to the quadratic in Ω cooling power. The parameters are the couplings $g = 4E_0^2 M_i^2 / (\hbar \Phi_0^2 R_i) = 1$; the quality factors $Q = \sqrt{L_i / C_i} / R_i = 30$; the temperatures $k_B T_i / E_0 = 0.3$, with $i = 1, 2$; and the minimal qubit splitting $\hbar \omega_1 / 2E_0 = 0.15$. Here $\Phi_0 = h/2e$ is the superconducting flux quantum and E_0 is the overall energy scale of the qubit. This figure is based on the analysis and numerics in Reference 95.

In this section, we address quantum heat engines and refrigerators based on qubits. An early work on the qubit-based quantum heat engine (93) considers a cyclic quantum Otto refrigerator consisting of a flux qubit inductively coupled to two low- (ω_1) and high- (ω_2) frequency LC -resonators with resistors $R_{1,2}$ as heat baths placed at high (T_1) and low (T_2) temperatures, respectively (**Figure 6a**). The mutual inductances are $M_{1,2}$, respectively. The small damping $R_k \ll (L_k / C_k)^{1/2}$ of LC -resonators together with a strong coupling brings the selectivity in qubit relaxation; namely, the qubit is coupled to k th resonator only in the vicinity of the corresponding frequency $\omega_k \simeq (L_k C_k)^{-1/2}$. The adiabatic drive realized as a time-dependent flux $\Phi(t)$ with a period $1/f$ tunes the qubit energy splitting ΔE between the resonator frequencies ω_k (**Figure 6c**) and performs work on the system (**Figure 6b**). As the qubit relaxation in the resonance is quite strong, the population of qubit levels at $\Delta E = \omega_k$ follows the Gibbs distribution with the corresponding k th resistor temperature T_k . In this approximation, the refrigerator works provided the temperature of the cold resistor $T_1 > \omega_1 T_2 / \omega_2$ is not too small.

The paper (95) is devoted to the detailed analysis of operating the Otto refrigerator in different frequency regimes. There, using the standard quantum master equation written in the instantaneous eigenbasis (99, 100) and ignoring pure dephasing due to intentionally large relaxation rates close to $\Delta E = \omega_k$, the authors have found three distinct regimes: (a) (nearly) adiabatic regime, (b) ideal Otto cycle, and (c) high-frequency regime. The (nearly) adiabatic regime shows the quadratic deviation of the powers dissipated to the hot and cold reservoirs with f and the negative influence of quantum coherence on the engine performance for any drive profile $\Phi(t)$ (see **Figure 6d, inset**). In the ideal Otto cycle regime, when the qubit thermalizes both at $\Delta E = \omega_1$ and ω_2 and its population is kept intact between resonances $\omega_1 < \Delta E < \omega_2$, the cooling power is

approximately linear in f in agreement with Reference 93. The hallmark of the high-frequency limit is the coherent qubit oscillations present in the f -dependence of the powers dissipated in the two baths (see **Figure 6**). This regime spreads in the range of $\omega_1 \lesssim f \lesssim \omega_2$. Here, the qubit population in the adiabatic leg of the drive (when the qubit is decoupled from both reservoirs) varies in time due to driving-induced coherent oscillations. The analysis at even higher frequencies may not be rigorous, as the drive frequency can exceed the inverse bath correlation time. Considering the Otto refrigerator based on several qubits one can demonstrate that the correlation of the noises affecting both qubits separately can only diminish the efficiency of the refrigerator (96).

Recently the quantum heat valve effect¹⁰ has been reported (78), where the heat transport has been considered between galvanically uncoupled systems, namely coplanar waveguide resonators, each terminated by a normal-metal mesoscopic resistor, only capacitively coupled via a superconducting transmon qubit. The reservoir-to-reservoir heat flux is shown to depend on the interplay between the qubit-resonator and the resonator-reservoir couplings, yielding qualitatively dissimilar results in different coupling regimes.

5. FEEDBACK SYSTEMS: QUANTUM MAXWELL'S DEMON

Over the past few years the idea of MD presented in Section 3 has been applied to quantum systems. In principle, an experiment analogous to the Szilard's engine for electrons (17, 18) can be implemented in a system consisting of a qubit, as proposed in Reference 100.

The key element in this kind of a quantum MD is the type of measurement and feedback. First, to avoid additional thermodynamic costs one has to perform a nondemolition quantum measurement of the qubit state such as the measurement of a qubit energy itself or its curvature. Second, unlike the classical case in a qubit system there is an avoided level crossing in the vicinity of the degeneracy point (cf. the insets of **Figure 3a**). It is this level anticrossing that causes the change of the conditional feedback control. Indeed, if the qubit is found in an excited state after the measurement, one must apply additional π -pulse to extract the energy and drive the system to the ground state. Only after this, one can change the control parameter, detuning the qubit away from the degeneracy point. Third, by optimizing the protocol of the control parameter change, one can avoid additional excitations of the qubit due to Landau-Zener transitions (100).

Different experimental realizations have been reported recently (102–104). However, direct measurement of heat extracted from the bath is still elusive and demanding.

DISCLOSURE STATEMENT

The authors are not aware of any affiliations, memberships, funding, or financial holdings that might be perceived as affecting the objectivity of this review.

ACKNOWLEDGMENTS

We thank T. Ala-Nissila, D.V. Averin, M. Arzeo, J. Bergli, M. Campisi, L. Casparis, Y.-C. Chang, C.D. Chen, T. Faivre, R. Fazio, A.V. Feshchenko, C. Flindt, Y.M. Galperin, S. Gasparinetti, F. Giazotto, D.S. Golubev, W. Guichard, T.T. Heikkilä, F.W.J. Hekking, M. Helle, F. Jülicher, B. Karimi, J.V. Koski, V.E. Kravtsov, A. KUPIAINEN, A. KUTVONEN, M.M. Leivo, A. Luukanen,

¹⁰This effect is based on the photon-mediated heat transfer quantization discussed in Reference 101. For more detailed discussion of heat quantization, please see the **Supplemental Text**.

V.F. Maisi, D. Maradan, Y. Masuyama, M. Meschke, M. Möttönen, P. Muratore-Ginanneschi, Y. Nakamura, I. Neri, A.O. Niskanen, M. Palma, J.T. Peltonen, M. Ribezzi-Crivellari, F. Ritort, É. Roldán, A. Ronzani, T. Sagawa, O.-P. Saira, A.M. Savin, K. Schwieger, A. Shnirman, J. Senior, S. Singh, P. Solinas, S. Suomela, T. Tanttu, A.V. Timofeev, K.L. Viisanen, Y. Yoon, and D.M. Zumbühl for insightful discussions and related collaborations. J.P.P. acknowledges partial support through the Academy of Finland, Project Nos. 272218 and 312057, by the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation program under grant agreement No. 742559 (SQH), I.M.K. is supported by the Russian Science Foundation Grant No. 17-12-01383 (Secs. 3 and 4), by the German Research Foundation (DFG) Grant No. KH 425/1-1 and the Russian Foundation for Basic Research.

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