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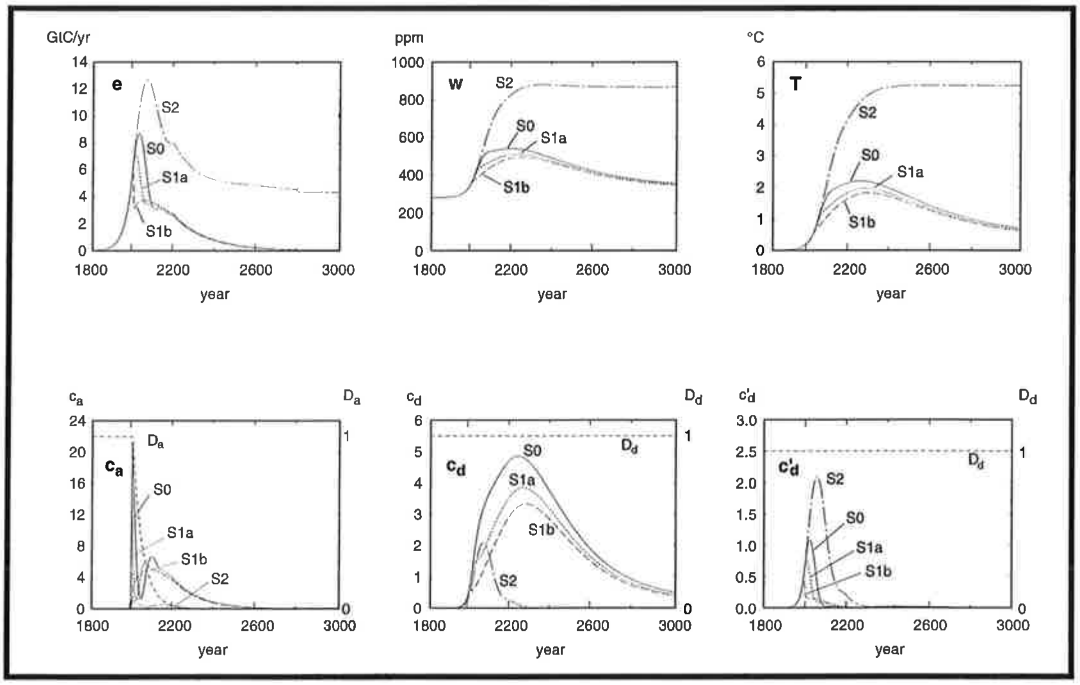

OPTIMIZATION OF CO_2 EMISSIONS USING COUPLED INTEGRAL CLIMATE RESPONSE AND SIMPLIFIED COST MODELS. A SENSITIVITY STUDY

by

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Optimization of CO_2 emissions using coupled integral climate response and simplified cost models. A sensitivity study

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ABSTRACT

A cost-benefit analysis for greenhouse warming based on a structurally simplified globally integrated coupled climate-economic costs model SIAM (Structural Integrated Assessment Model) is used to compute optimal paths of global CO_2 emissions which minimize the net sum of climate damage and mitigation costs. The climate model is represented by a linearized inpulse-response model calibrated against a coupled ocean-atmosphere general circulation climate model and a three-dimensional global carbon-cycle model. The cost terms are represented by strongly simplified expressions designed for the study of the sensitivity of the computed optimal emission paths with respect to critical input assumptions. These include the discount rates assumed for mitigation and damage costs, the inertia of the socio-economic system, and the dependence of climate damages on the change in temperature and the rate of change of temperature. Different assumptions regarding these parameters are believed to be the origin of the marked divergences of existing cost-benefit analyses based on more sophisticated economic models.

The long memory of the climate system implies that very long time horizons of several hundred years are needed to optimize CO_2 emissions on time scales relevant for a policy of sustainable development. Cost-benefit analyses over shorter time scales of a century or two can lead to dangerous underestimates of the long term climatic impact of increasing greenhouse-gas emissions. However, the necessary draw-down of CO_2 emissions to very low levels needed to avert a major long term global warming need not be implemented on the short term, but can be realized as a gradual transition over many decades and even centuries. The transition nevertheless becomes less costly the sooner the necessary mitigation policies are initiated, the long time horizon still leaving room for later adjustments. Short term energy conservation alone is insufficient and can be viewed only as a useful measure in support of the necessary long term transition to carbon-free energy technologies.

Optimal emission paths limiting long term global warming to sustainable development levels are recovered only if climate damage costs are not significantly discounted. Discounting of climate damages at normal economic rates yields emission paths which are only weakly reduced relative business as usual scenarios, producing unacceptably high global warming levels in the long term. While these solutions are logically consistent with the assumption that global warming damages in the distant future are indeed of negligible concern today, a commitment to sustainable development may be regarded as a willingness-to-pay-today value assessment which to first order does not depend on the time horizon of climate change and therefore should not be discounted.

To translate our general conclusions into quantitative cost estimates required by decision makers, the present exploratory study needs to be extended using more detailed disaggregated climate damage and mitigation cost estimates and more realistic socio-economic models, including multi-actor interactions, inherent variability, the role of uncertainty and adaptive control strategies.

1 Introduction

The definition and implementation of an effective international climate protection policy is one of the central issues facing decision makers today. A basic difficulty in arriving at a common policy is the global nature of the problem, combined with the relatively small contribution of any individual nation to the global anthropogenic climate forcing. This invites a free-rider approach – a tendency which is reinforced by divergent national interests.

This basic game-theoretical difficulty is compounded by insufficient scientific information on the impact of climate change on the ecology, economy and societal conditions. The uncertainty provides individual actors with a wide range of possible scenarios from which they can select and promote those which further their particular interests. To establish a level game theoretical playing field it is therefore important that the present uncertainties regarding the impact of climate change are reduced. To provide a rational basis for decision making, the costs for adapting to climate change need to be assessed further in relation to the abatement costs of reducing greenhouse gas emission levels.

The scientific basis for such integrated assessment studies is still far from complete and varies strongly for the different components of the integrated climate-socio-economic system. The Scientific Assessment of Working Group 1 of the Intergovernmental Panel on Climate Change (IPCC) has provided valuable summaries of our present ability to predict anthropogenic climate change (IPCC, 1990a, 1992,1994). These reports provided an important background for the negotiations at the 1993 Rio Summit and the 1995 Berlin Climate Conference. A parallel assessment of the socio-economic impact of anthropogenic climate change, together with analyses of the mechanisms for the transmission of scientific information into the political arena, the decision making processes and the implementation of policy decisions through appropriate market or regulatory instruments, would be similarly beneficial. However, our understanding in this field has not yet advanced to a stage in which general scientific consensus statements can be presented (cf. summaries in IPCC, 1992b). To narrow the present divergences of existing analyses, extensive interdisciplinary research by climate, ecological and economic modellers, with support from the social sciences, is needed.

In the present paper we attempt to contribute to this interaction by investigating the origin of some of the marked divergences found in previous cost-benefit analyses. Our approach is to combine a climate response model calibrated against sophisticated state-of-the-art climate models with a relatively simple, structurally transparent climate-damage and abatement costs model designed to illuminate the impact of the various assumptions which we believe lie at the core of the divergent results. By means of this Structural Integrated Assessment Model (SIAM) we are then able to distinguish between the relatively robust conclusions which are only weakly dependent on such assumptions and the more sensitive results, whose dependence on the critical input pararameters can then be systematically explored.

We purposely chose much simpler abatement cost expressions in this study than used in most previous greenhouse cost analyses (cf. Reilly *et al*, 1987, Nordhaus, 1991, 1993, Nordhaus and Yang, 1995, Manne and Richels, 1991, 1995, Peck and

Teisberg, 1992, Michaelis, 1994, Tahvonen *et al* 1994, 1995, Beltratti, 1995, Richels and Edmonds, 1995, and the more complete list of references and discussion in Cline, 1992, and Fankenhaus, 1995). The wide divergences in the conclusions of previous cost-benefit analyses using more sophisticated multi-sectoral economic models do not arise from differences in the internal details of the models, but can be attributed at a much more elementary level to different basic input assumptions, such as the dependence of the climate damage costs on climate change and the rate of change of climate, the discount rates for climate damage and mitigation costs, the inherent inertia of the economic system, the indigenous rate of technical development, or the adaptablility of energy technology in response to imposed mitigation measures. An expert poll conducted by Nordhaus (1994) revealed a very wide range of opinions on the magnitudes and impacts of these processes among economists, social scientists and climate researchers. Before embarking on a detailed description of interactions between different sectors of the economy, it therefore appears appropriate to investigate first the impact of these basic assumptions on the computed optimal CO_2 emission paths in a general framework, independent of model details. We believe this is best achieved using structurally highly simplified cost function expressions designed to illuminate the fundamental cause-effect relations.

A fundamental property of both climate and the socio-economic system is the wide range of time scales involved. The major climate sub-systems atmosphere, ocean and biosphere relevant for anthropogenic climate change vary on time scales – excluding short weather time scales – from weeks to millennia. Ice sheets and geological processes involve still longer time scales. Economic and societal adjustment processes similarly cover time scales from weeks to several decades or even centuries. This implies that realistic integrated Global Environment and Society (GES) models used for cost-benefit analyses must be conceived from the outset as dynamical models. Moreover, the impact of climate change in response to man's activities must be considered over time horizons compatible with the natural time constants of the coupled GES systems, i.e. over several hundred years. This far exceeds usual economic planning horizons, but is an unavoidable consequence of the dynamics of the GES system if the challenge of sustainable development is to be faced.

A novel feature of our approach is the introduction of a simple linearized integral impulse-response climate model. This clarifies the impact of the long climatic time scales on the optimal emissions solution. The climate model is calibrated using the outputs of a state-of-the-art climate model consisting of a coupled ocean-atmosphere general circulation model and a three-dimensional global carbon cycle model. The impulse-response climate model is then coupled to a structurally highly simplified economic climate-damage and abatement costs model.

The analysis is restricted to an idealized single-world system whose evolution is governed by a single decision maker representing the collective decisions of the world community. Multi-actor models constructed with the same basic building blocks as presented in this paper, but allowing for different climate-damage and abatement costs as well as the divergent political goals and strategies of different economic regions, are considered in Hasselmann and Hasselmann (1996).

Following the standard cost-benefit approach, the optimal climate protection strategy is defined as the time-dependent path for the control variables of the integrated climate-socio-economic system which minimizes the total climate-change related costs, consisting of the sum of the time-integrated global mitigation and climate-damage costs. We shall regard as control variable only the emissions of CO_2, but shall discuss briefly also the impact of other greenhouse gases.

An alternative approach which is sometimes pursued is to define *a priori* a permissible climate change 'corridor' within which the climate state trajectory is constrained to remain. The optimal emissions path is then defined as the path which minimizes the economic abatement costs under this constraint, ignoring the climate damage costs within the corridor. One can follow this approach one step further by prescribing instead of a climate-change limit a ceiling on the atmospheric CO_2 concentration (cf. Richels and Edmonds, 1995, Wigley *et al*, 1996, and the discussion in Manne and Richels, 1995). The usual motivation for prescribing *a priori* limits for the climate change or CO_2 concentration is the notorious difficulty of assessing climate damage costs, including intangible values such as the protection of species or the 'quality of the environment'. However, the corridor approach hides rather than avoids the issue of quantifying climate damage costs. Formally, the corridor approach is equivalent to minimizing the sum of climate-damage and emission-abatement costs under the assumption that the damage costs are zero within the allowed climate-change or CO_2 corridor and immediately become very large – in excess of any conceivable mitigation costs – as soon as one leaves the corridor. We prefer a more continuous representation of the climate damage costs within and outside the corridor. Independent of the details of the climate-damage cost function, however, a rational determination of the acceptable size of the corridor inevitably leads to the problem of assessing climate impacts in relation to mitigation costs: the trade-off between climate change impacts and mitigation efforts – independent of the value units in which these are measured – is the central issue of the climate protection problem and cannot be circumvented by the *ad hoc* introduction of arbitrary climate change or CO_2 concentration ceilings.

For the political implementation of abatement measures it may neverthess be expedient to define CO_2 concentration targets and devise market control or other regulatory mechanisms for meeting these targets – in accordance, for example, with the approach adopted in the Framework Convention on Climate Change. However, the definition of the concentration targets should be based on prior cost-benefit analyses based on all components of the cost budget.

The paper is organized as follows: Following a discussion of the general structure of GES models in Section 2, the construction of simple linearized integral impulse-response climate models from the simulation results of complex nonlinear climate models is described in Section 3. The coupling of the impulse-response climate model to an idealized climate damage and mitigation costs model, and the application of this elementary GES model to the single-actor greenhouse-gas optimization problem, is presented in Section 4. A series of sensitivity experiments with model is described in Section 5. The results are summarized in Section 6 and placed in the perspective of more complete GES models in the concluding Section 7.

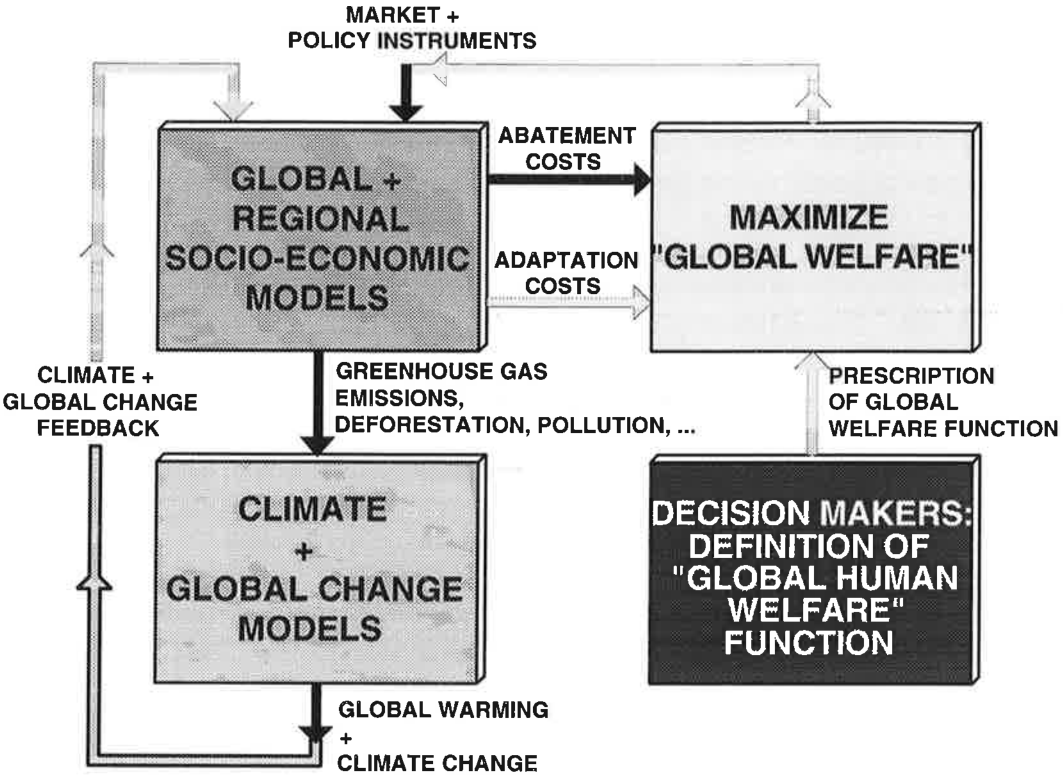

Figure 1: Interactions and sub-systems of an integrated Global Environment and Society (GES) model (from Hasselmann, 1991).

2 Structure of GES models

Figure 1 (from Hasselmann, 1991) shows the basic elements und interactions within a GES model. It is assumed in this simplified scheme that negotiations lead to the cooperative definition of a global welfare function. This assigns appropriate weights to the welfare values and interests of individual nations and distributes the burdens of an optimized global climate protection policy in accordance with some accepted rules. Once the cooperative global welfare function and burden sharing have been agreed upon, the optimization task reduces essentially to a single-actor dynamic optimization problem in which the available market and policy instruments are applied to minimize the time-integrated, appropriately discounted net climate damage and mitigation costs.

A more detailed representation of the same set of interactions, consisting again of a single global climate system and a single international negotiation box, but with the socio-economic system disaggregated into separate units representing different economic regions, is discussed in the context of the more general multi-actor greenhouse-gas optimization problem in Hasselmann and Hasselmann (1996).

In either case – cooperative agreement on a global welfare function or the more general game-theoretical situation – the dynamical system will generally be too complex for analytic investigations and will need to be studied by numerical simulation techniques. Unfortunately, there appear to be available today no suitable set of sub-system models which could be combined in a reasonably realistic GES model for such dynamical optimization studies. There exist a number of sophisticated climate models based on coupled general circulation models (CGCMs) of the atmosphere and ocean, which have been well validated against the present climate (cf. IPCC, 1992, Cubasch et al, 1992), as well as similarly sophisticated and realistic three-dimensional ocean-atmosphere carbon cycle models (Maier-Reimer and Hasselmann, 1987, Maier-Reimer, 1993, Sarmiento *et al*, 1992,). However, these are far too costly in computer time to be applied in dynamical optimization studies, which will normally require a large number of integrations using some iterative optimization algorithm. Similarly, realistic economic models, although less demanding on computer resources and still highly simplified with respect to the societal components and the interactions between the climate and economic systems, are generally too cumbersome for applications in iterative optimization studies.

It is therefore not surprising that most of the dynamical optimization studies carried out to date have been single-actor investigations based on simplified box-type climate models and strongly aggregated economic models (Nordhaus, 1991, 1993, Peck and Teisberg, 1992, Michaelis, 1994, Tahvonen *et al* 1994, 1995, Beltratti, 1995). Greenhouse cost studies using more sophisticated disaggregated economic models (cf. references quoted above and Cline, 1992, Fankhauser, 1995) have normally been carried out in the scenario mode, rather than as optimization computations. We limit ourselves here also to optimization studies using single-actor models, but with the goal, as outlined above, of clarifying the sensitivity of the computed optimal emission paths with respect to critical input assumptions, rather than on providing quantitative cost estimates of particular emission paths.

In the following section we describe, as a basic building block which can be used also for the development of more realistic GES models, a general technique for projecting the simulation results of sophisticated CGCM climate models onto simpler but none the less geographically and dynamically realistic climate models. The models are formulated as linear integral response models and are sufficiently economic in computer time to be used in iterative optimization integrations.

3 Projection of CGCM climate models onto linear integral-response climate models

General approach

Although the climate system and its detailed model representation in terms of CGCMs are inherently strongly nonlinear, the response of the climate system, as of any differentiable nonlinear system, to small external forcing is to first order linear. As external forcing we consider in this paper the annual anthropogenic emissions $e(t)$ of CO_2. Since CO_2 is well mixed in the atmosphere, $e(t)$ can be represented as a single scalar function of time. Although CO_2 contributes only about 60% of the anthropogenic radiative forcing of all greenhouse gases, we restrict the discussion here to CO_2, since models of non-CO_2 greenhouse gases are generally less well developed. Also, the sources and sinks of these gases are often poorly known, so that the mechanisms for controlling their atmospheric concentrations are not well defined. It must therefore be kept in mind that the following projections of future climate change represent systematic underestimates of the real climate change. However, we shall attempt to provide first order estimates of the impact of non-CO_2 greenhouse gases later.

In the linear approximation, the response of the perturbed climate state $\mathbf{x}(t)$, consisting, in a discretized model representation, of the perturbation vector of all climate variables at all model gridpoints, to an arbitrary, sufficiently small emission function $e(t)$ can be represented in the general integral form

$$\mathbf{x}(t) = \int_{t_0}^{t} \mathbf{R}(t - t')e(t')dt', \tag{1}$$

where the climate impulse-response function $\mathbf{R}(t-t')$ represents the climate response at time t to a unit δ-function emission at time t'. It is assumed that the forcing and climate perturbation are zero prior und up to the initial time $t_0 :\ e(t) = \mathbf{x}(t) = 0$ for $t \leq t_0$.

The first-order linear response approximation can be generalized to nonlinear response relations in which the linear kernel $\mathbf{R}(t) \equiv \mathbf{R}_1(t_1)$ is replaced by a series expansion in terms of higher order nonlinear kernels $\mathbf{R}_2(t_1, t_2), \mathbf{R}_3(t_1, t_2, t_3), \ldots$ occurring in quadratic, cubic, ... integrals over the emission. However, noting that a doubling of the CO_2 concentration corresponds to an increase in radiative forcing of about $4\,W/m^2$, or little more than 1% of the mean incident solar radiation of $340W/m^2$, the linear form will be adequate for many applications. We discuss the limitations of the linearization approximation in more detail below.

The dimension of $\mathbf{R}(t)$ in eq.(1) is the same as that of $\mathbf{x}(t)$. Thus the linear response can be represented with the same geographical resolution and with respect to the same set of variables (temperature, humidity, precipitation, ocean currents, etc) as a full coupled ocean-atmosphere general circulation climate model. The response function can be determined empirically from numerical climate response experiments with realistic three-dimensional carbon cycle or CGCMs (Maier-Reimer and Hasselmann, 1987, Cubasch et al, 1992, Hasselmann et al., 1993). In practice, it will normally be convenient to reduce the number of degrees of freedom of $\mathbf{R}(t)$ by expanding the response function with respect to some set of base functions, such as the empirical orthogonal functions (EOFs) of the CGCM climate response simulations. However, it is important to recognize that the linearized form (1) implies no loss of information in the representation of the climate state relative to the complete nonlinear system, but represents simply a reduction of the full nonlinear dynamics to the first-order linearized response, which is always permissible for small external forcing.

The present approach appears preferable to the usual construction of simplified climate models in the form of empirical box models with a small number of degrees of freedom. These loose the detailed information on the climate state and therefore cannot be readily constrained to conform to the detailed linearized dynamics of a more realistic CGCM climate model.

The formulation of the climate response in terms of a response integral rather than in the traditional form of a differential equation for a box model has further advantages: it is not limited to simple low-order differential equations, but applies generally for differential equations of arbitrary order; it is easy to fit to the data; and it enables a direct determination of the gradient of the cost function (cf. next section and Appendix), without solving a Hamiltonian problem in terms of the adjoint model. The last advantage does not come to bear, however, if an automatic adjoint model and functional derivative compiler is used, as in our applications below. This can be applied equally well to differential or integral representations of the system dynamics (Giering, in preparation).

A simple climate model

In the applications of this paper we shall use a strongly aggregrated climate model in which the climate state vector $\mathbf{x}$ is reduced to a single climate variable T representing the global mean (surface) temperature. The model consists of two sub-systems:

1. A *carbon cycle model.*

This describes the evolution of the atmospheric CO_2 concentration w in response to the CO_2 emission $e(t)$,

$$w(t) = \int_{t_0}^{t} R_w(t-t')e(t')dt' \tag{2}$$

where $R_w(t-t')$ is the impulse response of the concentration at time t for a unit δ-function emission pulse at time t' and it is assumed, as in (1), that $e(t) = w(t) = 0$ for $t \leq t_0$. We shall chose $t = t_0$ later as the pre-industrial date 1800 (the exact date is immaterial, since $e(t)$ is assumed to be zero in the pre-industrial epoch).

Time in this paper is in units of years. To retain the same carbon units GtC (Gigatons carbon) for w and the emissions e (in GtC/yr), w represents in all equations the total carbon in the atmosphere. However, we shall present results for w in the figures later in the usual units of *ppm*. The conversion factor is $w\,[GtC] = 2.123\,w\,[ppm]$. The present atmospheric CO_2 concentration is $358\,ppm$, corresponding to an atmospheric carbon content of $760\,GtC$, while the preindustrial concentration was $w_0 = 280\,ppm = 594\,GtC$.

Initially, all of the emissions enter the atmosphere, so that

$$R_w(t_0) = 1. \tag{3}$$

$R_w(\infty)$ defines the fraction of the emissions which is retained in the atmosphere in the asymptotic equilibrium state. If the ocean sink alone is considered, this is approximately 14 %; if the uptake of CO_2 by dissolution in the upper layers of the ocean sediments is also included, the long-term atmospheric retention factor may fall to about 7 % (Maier-Reimer, 1993). The increased storage of CO_2 in the terrestrial biosphere through CO_2 fertilization and the significantly slower loss of CO_2 through sedimentation in the ocean is not included in these estimates.

Invoking eq.(3), the time derivative of eq.(2) (which will be needed to couple the CO_2 model to the following temperature response model) is given by

$$\frac{dw}{dt} \equiv \dot{w}(t) = \int_{t_0}^{t} \dot{R}_w(t-t')e(t')dt' + e(t) \tag{4}$$

In an analysis of the response of a nonlinear three-dimensional global ocean carbon cycle model to various CO_2-emission levels, Maier-Raimer and Hasselmann (1987) found that the model response could be fitted to a linear relation of the form (1) quite well for an increase in the CO_2 level up to a factor of two. For a stronger emission level producing a four-fold increase in the CO_2 concentration, the linear response underestimated the atmospheric concentration predicted by the full model by about 30%. This was due primarily to the nonlinear decrease of the solubility of CO_2 in sea water with increasing CO_2 concentration. A relatively simple nonlinear extension of the linear response form to allow for the nonlinearities (and temperature dependence) associated with the solution of CO_2 in sea-water has recently been proposed by Joos *et al* (1995).

2. A *global temperature response model.*

The general linear response of the change $T(t)$ of the global mean temperature induced by a change w in the CO_2 concentration is given by

$$T(t) = \int_{t_0}^{t} \hat{R}_T(t-t')w(t')dt', \tag{5}$$

where the temperature impulse-response function $\hat{R}_T(t-t')$ represents the change in the global mean temperature produced at time t by a unit δ-function change in the atmospheric CO_2 concentration at time t'.

It is more convenient to rewrite (5) in terms of the rate of change $\dot{w}$ of the CO_2 concentration instead of w. This is because a δ-function input in the emissions generates a step-function response in the concentration (cf. eq.(2)), i.e a δ-function response in the derivative of the concentration, rather than in the concentration itself. Integrating (5) in parts, we obtain

$$T(t) = \int_{t_0}^{t} R_T(t-t')\dot{w}(t')dt', \tag{6}$$

where the response function

$$R_T(t-t') = \int_{t'}^{t} \hat{R}_T(t-t'')dt'' \tag{7}$$

represents the change in the global mean temperature produced at time t by a unit step-function increase in the atmospheric CO_2 concentration at time t'.

Because of the inertia of the climate system, the instantaneous response to a step-function change in CO_2 concentration is zero (cf. eq. (7)),

$$R_T(0) = 0, \tag{8}$$

while $R_T(\infty)$ represents the asymptotic equilibrium response of the (thermodynamic) climate system to a unit increase in the CO_2 concentration.

The generalization of this simple one-parameter climate model to more complex climate-state models, including, for example, regional temperature distributions represented by the first few EOFs of CGCM climate response experiments, or additional information such as regional changes in sea-level or precipitation patterns as well as temperature patterns, is basically straightforward. Such models could be readily constructed, in accordance with the general form (1), from existing data generated by CGCM climate-response simulations. However, for illustrative purposes we restrict the model here to a single climate variable representing the global mean temperature. The critical elements of our optimization analysis concern in fact not so much the detailed description of the predicted climate change, as the estimation of the resulting climate-damage costs. As long as these are not better assessed, there is little point in being too specific about the details of the climate change.

In the applications discussed in Hasselmann and Hasselmann (1996) involving simultaneous multi-actor greenhouse gas emission optimization strategies, it would be more appropriate to consider different climate impact functions for different actors. This can be achieved within the framework of the present model by simply assigning different regional impact factors to the single global climate variable T. To the extent that the climate impact for a given region can be characterized by the average temperature change over the region, this can, in fact, be justified by the results of numerical global warming simulations with coupled CGCMs (Cubasch *et al*, 1992). The response of the global temperature distribution is dominated in these simulations by the first EOF, implying that the average temperature response for any region can indeed be related to the global mean temperature by a time-independent scaling factor.

The linear response relation between the temperature change and the change of the CO_2 concentration can be modified in accordance with the more accurate logarithmic dependence between the radiative greenhouse forcing and the CO_2 concentration by replacing $\dot{w}$ by $d(\ln w)/dt$ in (5). This introduces no significant complications in the numerical examples considered in the following section. However, the difference between the linear and logarithmic formulation is small for small forcing (which we assume), and for the present illustrative purposes, the linear relation (5) has the advantage (see below) of yielding a net linear climate response to the emissions in accordance with (1).

Linear-response-fitting exercises for coupled ocean-atmosphere CGCM global warming simulations (Hasselmann et al, 1993) suggest that, as in the case of the linearized carbon cycle model, the linearized temperature response relation is applicable for climate changes associated with CO_2 concentration increases up to about double the pre-industrial level, i.e. for a temperature rise up to about $3°C$. The linear response relations should not be used beyond this range also because the temperature feedback on the CO_2 model (increasing temperature decreases the CO_2 solubility of sea-water and thus increases the atmospheric retention factor) has not been included in the CO_2 response relation (2) (however, this effect is incorporated in the general nonlinear impulse-response relation of Joos *et al*, 1995).

Combining the carbon cycle and global temperature response models, the net response of the 'climate' T to the emission $e(t)$ can now be written

$$T(t) = \int_{t_0}^{t} dt' R_T(t-t') \left\{ e(t') + \int_{t_0}^{t'} dt'' \dot{R}_w(t'-t'')e(t'') \right\} \tag{9}$$

Noting that

$$\int_{t_0}^{t} dt' \int_{t_0}^{t'} dt'' = \int_{t_0}^{t} dt'' \int_{t''}^{t} dt' \tag{10}$$

this may be expressed as

$$T(t) = \int_{t_0}^{t} R(t-t')e(t')dt', \tag{11}$$

in accordance with the form (1), where

$$R(t) = R_T(t) + \int_{0}^{t} R_T(t-t')\dot{R}_w(t')dt'. \tag{12}$$

At $t = t_0$ we have

$$T(t_0) = \dot{T}(t_0) = R(t_0) = 0. \tag{13}$$

The net temperature impulse response function $R(t)$, or *global warming response* (to be disinguished from the global warming 'potential' or 'commitment', defined by IPCC (1990a) as integrated radiative warming variables) represents the temperature increase at time t due to a unit δ-function CO_2 input into the atmosphere at time $t = 0$, allowing for both the thermal inertia of the ocean-atmosphere climate system and the slow decay of the atmospheric CO_2 concentration through the transfer of CO_2 from the atmosphere to other components of the carbon cycle.

Numerical values

The response functions R_w and R_T have been determined empirically from numerical response experiments using realistic three-dimensional models of the global carbon cycle (Maier-Reimer and Hasselmann, 1987, Maier-Reimer, 1993) and the coupled ocean-atmosphere climate system (Hasselmann et al, 1993). It was found that the response curves could be closely fitted by sums of exponentials in the form

$$R_w = A_0^w + \sum_j A_j^w \exp(-t/t_j^w) \tag{14}$$

$$\begin{aligned} R_T &= w_0^{-1} \sum_j A_j^T [1 - \exp(-t/t_j^T)] \\ &= w_0^{-1} R_T', \end{aligned} \tag{15}$$

where R_T' represents the temperature response to a step-function doubling of the the CO_2 concentration at time $t = 0$ relative to the pre-industrial value. The empirically fitted amplitude factors A_j^w, A_j^T and time constants t_j^w, t_j^T for various response models are listed in Table 1.

The CO_2 response model RW1 was fitted to the response of the original inorganic 3d ocean carbon cycle model of Maier-Reimer and Hasselmann (1987) and yields an

Model	A_0^w	A_1^w	t_1^w	A_2^w	t_2^w	A_3^w	t_3^w	A_4^w	t_4^w
RW0	0.07	0.648	258.5	0.101	71.9	0.097	17.6	0.084	1.6
RW1	0.142	0.241	313.8	0.323	79.8	0.206	18.8	0.088	1.7

Model	A_1^T	t_1^T	A_2^T	t_2^T	A_3^T	t_3^T
RT0	1.21	2.1	0.759	12.0	0.531	138.6
RT1	2.5	36.8	-	-	-	-
RT2	0.8	2.9	0.3	40.0	1.4	300

Table 1: Top part: amplitudes A_j^w and time constants t_j^w for the CO_2 response models RW0 (Maier-Reimer, 1993) and RW1 (Maier-Reimer and Hasselmann, 1987). Bottom part: amplitudes A_j^T and time constants t_j^T for the temperature response function R_T' for the models RT0 (baseline case), RT1 (single time constant model of Hasselmann *et al*, 1993) and RT2 (modification of RT0 with long time constant term)

asymptotic atmospheric retention factor of 14%. The modified form RW0, which we shall take as our baseline model, was derived from a fit (Maier-Reimer, private communication) to the response of a more recent 3d organic carbon cycle model (Maier-Reimer, 1993), including an additional sediment pool whose CO_2 uptake reduces the asymptotic atmospheric retention factor to 7%. Other impulse response functions for different CO_2 models are presented in the background report of Enting *et al* (1994) for IPCC Working Group 1.

Various temperature response function were considered by Hasselmann *et al* (1993) in their analysis and correction of cold start errors in CGCM global warming simulations. These are incurred when, to save computing costs, the climate is initialized as an equilibrium state at some relatively recent starting time, ignoring the delayed impact (global warming response) of the CO_2 which has already been emitted prior to the start of the model integration. They found that the global mean temperature response computed directly from an experiment in which the CO_2 level was suddenly increased by a factor of two was initially larger but asymptotically smaller than the equilibrium response inferred from transient response experiments in which the CO_2 level was increased gradually. They attributed this to nonlinearities in the response of the ocean mixed layer to a sudden CO_2 step- function doubling: the rapid initial warming tends to stabilize the upper mixed layer of the ocean, inhibiting the subsequent penetration of heat into the deeper ocean.

To investigate the impact of different time delay characteristics of the temperature response function, we considered three models, listed in Table 1. All models were normalized to yield the same asymptotic equilibrium temperature $2.5°C$ for a CO_2 doubling. The baseline model RT0 represents a fit to the 800-year transient response computed with the Hamburg Large Scale (LSG) global ocean circulation model, which was coupled to an atmospheric energy balance model, for a very small step-function increment in the CO_2 concentration (Mikolajewicz and Maier-Reimer, personal communication). The model RT1 corresponds to the single time-constant fit of Hasselmann *et al* (1993) to the global warming simulation of Cubasch *et al*

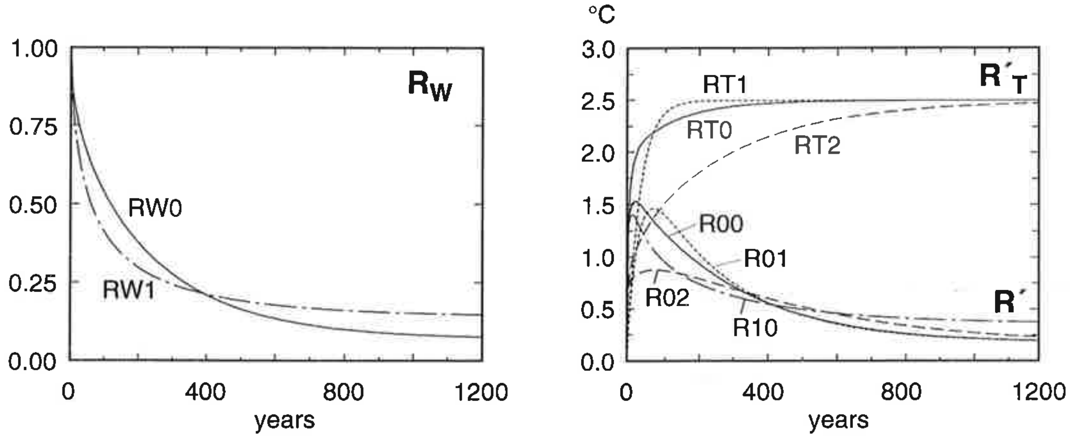

Figure 2: Left panel: Response functions R_W representing the atmospheric retention factor for a unit δ-function emission of CO_2 at time $t = 0$, as given by the CO_2-response models RW0 (full curve) and RW1 (dotted curve). Right panel: Temperature response functions $R'_T = w_0 R_T$ and $R' = w_0 R$ for a step-function doubling of the CO_2 concentration at time $t = 0$ for the R'_T models RT0 (full line), RT1 (dotted) and RT2 (dashed) and the resultant R' models R00 (full line), R10 (dotted), R01 (dashed) and R02 (dash-dotted).

(1992) for IPCC Scenario A. Model RT2, finally, was obtained by fitting the temperature impulse response model to a 100-year CGCM simulation for a sudden CO_2 doubling (Cubasch *et al*, 1992). It reproduces the principal short-term response characteristics of model RT0, but with smaller amplitude, and is augmented by an additional long time-constant term representing heat storage in the deep ocean. This term is probably exaggerated for typical slowly increasing transient global warming simulations, which are better represented by the models RT0 and RT1. However, it is reasonable for a sudden CO_2 doubling because of the inhibition of heat transfer into the deep ocean by the nonlinear response of the mixed layer. The model has been included to investigate the sensitivity of cost-benefit analyses with respect to the details of the climate model.

Figure 2 shows the various carbon cycle and temperature response functions R_w, $R'_T = w_0 R_T$ (left and right panels, respectively), together with the net temperature response function $R' = w_0 R$ (right panel) for the model combinations R00 = (RW0,RT0), R10 = (RW1,RT0), R01 = (RW0,RT1) and R02 = (RW0,RT2). The temperature response functions R'_T, R' represent the response to a step-function doubling of the atmospheric CO_2 concentration at time $t = 0$, which is then either retained at a constant level (in the case of R'_T), or (in the case of R') is allowed to relax back to an asymptotic value representing 7% (model R00) or 14% (model R10) of the initial level, in accordance with the carbon cycle response (14).

The response curves illustrate – as indicated by the analytical expressions – that the net climate response to CO_2 emissions cannot be characterized by a single time constant. In all models, after a rapid temperature rise in the first few years as the upper mixed layer of the ocean warms, the net response function for the global mean temperature increases more slowly as the warming penetrates into the main ocean thermocline, reaching its maximum value of about $1°C - 1.5°C$ after about a decade or two (compared with the asymptotic temperature response of $2.5°C$ for a CO_2 doubling without subsequent CO_2 losses from the atmosphere), after which the temperature gradually relaxes back over a period of several hundred years to its asymptotic equilibrium value of $2.5 \times 0.07 = 0.175°C$, for models R00, R01 and R02, or $2.5 \times 0.14 = 0.35°C$ for model R10. The initial fast response is governed by the temperature response of the ocean-atmosphere system, while the later relaxation stages are determined by slow response terms in both the carbon cycle and the climate system.

Although there are clearly differences in detail between the different carbon cycle and temperature response models, all model combinations shown in Fig. 2 exhibit rather similar qualitative features. It was found that the computed optimal emission paths presented below did not dependend sensitively on the choice of model combination shown in Fig. 2, and that our general conclusions applied for all climate models considered: the climate model is not a critical element in integrated assessment studies (ignoring possible instabilities of the climate system, which are excluded in the models considered). Accordingly, we shall present results later only for the baseline model R00.

For the optimization of greenhouse-gas emission paths, both the near-time and far-time climate response characteristics must be considered. In particular, if the mandate of sustainable development is taken seriously, the socio-economic impact of the long-term climate response over several hundred years should not be ignored or severely attenuated through the application of exponential discount factors designed to model economics or intertemporal societal preferences over the short term. Furthermore, in keeping with the multi-time scale nature of the climate system, the dynamical properties of the ecological and economic response to climatic change should also be modelled in terms of several different time constants reflecting different dynamical processes in the coupled ecological-socio-economic system. We shall attempt to follow this principle later in the formulation of simplified expressions for the climate damages and mitigation costs in our sensitivity studies.

The need to consider climate impact over long time horizons of several hundred years has been stressed by several authors, in particular Cline (1992). He points out that the limitation to a time span of only one hundred years, as in the IPCC reports (IPCC 1990a, 1992), can lead to a dangerous underestimation of the long-term greenhouse warming impact. However, in considering longer term climate impacts, it is important also to apply realistic climate response models. It is often assumed that the asymptotic atmospheric retention factor for CO_2 emissions is about 50%, in accordance with the observed retention factor in recent decades. This leads to an incorrect over-estimate of the long term global warming response. The recent atmospheric retention values of the order of 50% are the result of a continual exponential increase in CO_2 emissions in the last decades. This has been too rapid for the large but very slow deep-ocean CO_2 sink to become effective. The incorrect assumption that half of the emissions are retained asymptotically in the atmosphere yields for a CO_2 pulse corresponding to, say, an initial CO_2 doubling, a long term global warming response which is half as large as the equilibrium warming for a doubled CO_2

concentration, or $2.5/2 = 1.25°C$. However, for a finite CO_2 pulse (or for constant rather than exponentially growing emissions) the asymptotic atmospheric retention factor is of the order of only 7-14% (eq. (14, table 1). Thus the global warming response for a δ-function emission pulse corresponding to an initial CO_2 doubling is not constant, but, as indicated by Figure 2, attains a maximum after a few decades and decreases continually thereafter, approaching a relatively low asymptotic equilibrium value of $0.07 \times 2.5 = 0.175°C$ (for model R00) or $0.14 \times 2.5 = 0.35°C$ (for model R10).

We note in conclusion that the existence of a small but non-zero asymptotic CO_2-response level $R_w(\infty)$ implies that for a finite asymptotic temperature rise, the total emissions must remain finite, i.e. the asymptotic emission level must approach zero. This is indeed the case in the optimal solutions derived below (with the exception of simulation S2, in which only the rate of change of temperature, not the temperature change itself enters in the climate damage cost expression). In practice, of course, finite total emissions are assured by the finite resources of fossil fuel.

4 The optimization problem

Cost functions

We combine now our global climate model with a simple globally integrated economic climate-damage and abatement-costs model to form a coupled climate-economic model. We adopt the same level of global aggregation as in the similar studies of Nordhaus (1991,1993), Tahvonen *et al* (1994,1995) or Beltratti (1995). There are, however, two main differences in our approach relative to previous studies: the use of a general integral impulse-response climate model, which illustrates more clearly the memory properties of the climate system and enables a direct calibration of the model in terms of CGCM global warming simulations, and the introduction of a structurally highly simplified abatement-costs model.

The resulting GES model involves two levels of aggregration of basically different quality: 1) the climate model; here our input information for the aggregrate climate state (the global mean temperature) is relatively reliable, and we have merely introduced a linear approximation, valid for small perturbations, of the basically well defined nonlinear system to arrive at a numerically readily tractable system; 2) the economic climate-damage and greenhouse-gas abatement costs. Since the climate-impact relations are poorly known, we have assumed strongly simplified expressions for the climate-damage costs, and, for the reasons stated earlier, have also considered only structurally highly idealized expressions for the mitigation costs. These are introduced in order to focus on the differences in the basic assumptions which have lead to the marked divergences in the conclusions of earlier cost-benefit analyses based on more sophisticated economic models. As background for the application of more detailed economic models, it appears necessary to clarify first the origin of the present divergences. Despite these simplifications, the important effects of inertia have been included in the expressions for both climate damage and mitigation costs.

We repeat that the purpose of our exercise is not to generate quantitative cost calculations, but to study the sensitivity of the coupled GES system and the computed optimal emission paths with respect to different input assumptions and parameters. Our goal is to distinguish between relatively robust and more sensitive conclusions of the optimization analysis and to clarify the role of the characteristic climatic and economic time scales in governing the short and long term properties of the optimal emission paths. The same basic model, but disaggregated into several interacting sub-systems, is applied also in Hasselmann and Hasselmann (1996) in the discussion of the multi-actor greenhouse-gas emission problem.

The global economy is represented as a two-parameter system dependent on the total CO_2 emissions and the climate state. It is assumed that there exists a global welfare function W, which has been agreed upon by all actors involved, and which depends solely on $e(t)$ (including its first and second derivatives, to represent the effects of economic inertia) and $T(t)$ (including its first derivative, to model climate impacts, for example in the ecology, governed by the rate of change of climate). The common goal of all actors, represented by a single actor in this idealized cooperative scenario, is to maximize W.

For the case that climate damages are ignored, the optimal solution, yielding a welfare value W_A, will be some 'business as usual' (BAU) path $e_A(t)$, corresponding to, say, the IPCC Scenario A (IPCC, 1990a). How this optimal reference path excluding climate damage costs is attained is irrelevant for the following. If climate-damage costs C_d are included, the optimal solution will be a diminished emission path which reduces the climate-damage costs but incurs some abatement costs C_a. The optimal emission path is then the path which maximizes the net welfare

$$W = W_A - C \tag{16}$$

or minimizes the additional costs

$$C = C_a + C_d \tag{17}$$

relative to the BAU path. We use the term 'cost' here as a synonym for loss of welfare. The distinction between costs and welfare loss is immaterial for the present optimization problem provided welfare depends monotonically on costs. In general, this cannot be assumed if the concept of welfare includes non-monetary quality-of-life factors. However, for the present idealized single-actor problem, there is no need to be more specific in distinguishing between costs and negative welfare.

We assume that both cost contributions can be expressed as integrals over the *specific* costs $c_a(t)$, $c_d(t)$ in the form

$$\begin{aligned} C_a &= \int_{t_0}^{t_h} c_a(e(t), \dot{e}(t), \ddot{e}(t), t)dt & (18)\\ C_d &= \int_{t_0}^{t_h} c_d(T(t), \dot{T}(t), t)dt & (19) \end{aligned}$$

We can chose a finite time horizon t_h for the total cost definition or consider the case $t_h \rightarrow \infty$. The integrals converge for $t_h \rightarrow \infty$ if exponential discount factors are introduced. Time has been included explicitly as a separate variable in the specific cost functions c_a, c_d to allow for such discount factors, which may be chosen differently for the abatement and damage costs.

Costs and discount factors are assumed to be inflation adjusted. We shall be concerned only with the ratios of abatement and climate-damage costs, defined as additional costs relative to a non-specified business-as-usual welfare value W_A. Thus all costs are defined only to within an arbitrary constant scaling factor. We make no attempt to introduce an absolute scaling with respect to, say, GDP. Our interest lies in establishing the form of the optimal emission paths for various input assumptions regarding the relative magnitudes and forms of the cost functions. For this analysis the absolute cost values are irrelevant. However, we note that most quantitative cost estimates suggest that the mitigation and damage costs for optimal emission paths are generally of the same order and lie in the range of one to a few percent (this does not apply for estimates of the climate damage costs for the uncontrolled BAU emission path, however, which vary more widely).

We ignore cross-coupling of the climate and emission variables in the cost expressions. A change in emissions, producing a change in the structure of the socio-economic system, may be expected to affect the vulnerability of the system to climate change. Similarly, a change in climate will presumably have some impact on the abatement costs. For example, the costs for transferring from fossil fuel to solar energy will be increased if the cloud cover is increased. However, these effects are regarded as of higher order and are neglected.

In addition to e, first and second time derivatives $\dot{e}$ and $\ddot{e}$ are included in the specific abatement-cost function in order to penalize rapid changes in the emissions, thereby ensuring a smooth transition from the reference BAU emission path $e_A(t)$ to alternative reduced-emission paths without discontinuities in the emission and its time derivative. In a more sophisticated economic model, these inertia effects would, of course, be achieved by introducing capital investments. However, to demonstrate the sensitivity of the computed optimal emission paths with respect to the effects of economic inertia, we prefer to represent the dependence of the abatement costs on the first and second derivatives of the emissions in the simplest possible manner, without the camouflaging details of a more complex economic model.

With the same philosophy, we assume a particularly simple dependence of the mitigation costs on the deviation of the emissions from the prescribed optimal climate-insensitive BAU path. As simplest mathematical expression which captures the principal properties of the abatement costs which may be anticipated from a more detailed economic model we set

$$c_a = \left\{ (\frac{1}{r} - r)^2 + \tau_1^2 \dot{r}^2 + \tau_2^4 \ddot{r}^2 \right\} D_a(t) \tag{20}$$

where $r = e/e_A$, τ_1 and τ_2 are time constants and

$$D_a(t) = \exp(-t/\tau_a) \tag{21}$$

is the abatement-cost discount factor, characterized by an abatement-cost discount time constant τ_a (inverse annual discount factor).

The first term in the form (20) has the property that any positive or negative departure from the reference BAU emission path e_A incurs costs which are quadratic in the deviations $\delta r = r - 1$ for small δr, $(\frac{1}{r} - r)^2 \approx 4(\delta r)^2$ and approach infinity both for $r \to 0$ and $r \to \infty$. The quadratic dependence on the first and second derivatives of $e(t)$ is the simplest way of parametrizing economic inertia in the model. We have not included a 'no regrets' feature to model market imperfections which would yield an initial decrease in the costs for an initial decrease in emissions.

The use of a prescribed BAU emission path as reference in the abatement costs expression follows Nordhaus (1991, 1993) and Tahvonen *et al* (1994, 1995). It can be argued that this is unrealistic. The introduction of abatement measures will necessarily induce changes in technology. This will result in continually changing – presumably continually lowered – reference BAU emission curves, if these are continually updated. Thus the BAU curves should be defined ideally with respect to a running reference time, allowing for technological changes already induced by mitigation measures in the past. However, the optimization problem becomes more complex if this is taken into account, and there exist little data to define such a dynamical set of BAU emission curves. In the interest of transparency, we shall therefore use a fixed BAU reference curve. In practice, this simplification is probably not too serious, as the impacts of uncertainties in the future mitigation costs are exponentially discounted (see also the later discussion).

For the specific climate-damage costs we take the simple form

$$c_d = \left\{ \left(\frac{T}{T_c}\right)^2 + \left(\frac{\dot{T}}{\dot{T}_c}\right)^2 \right\} D_d(t) \tag{22}$$

where

$$D_d(t) = \exp(-t/\tau_d) \tag{23}$$

is the climate-damage costs discount factor, with discount time constant τ_d, and T_c, $\dot{T}_c$ are scaling constants. Thus we assume that climate damages are incurred not only through a change in the temperature itself but also through the rate at which the temperature changes: the adjustment of the ecology and human activities to climate change is more difficult the faster the change. The incurred climate damages are assumed to be independent of the sign of the temperature change, although we will be concerned only with positive changes. The quadratic dependencies reflect also the general view that climate damage costs increase nonlinearly with climate change.

We have made use of the freedom to choose an arbitrary common normalization constant in the definition of the cost functions by setting the coefficient of the first term of the abatement cost function (20) equal to unity. This establishes the significance of the constants T_c, $\dot{T}_c$ in the damage cost function in relation to the abatement costs: T_c and $\dot{T}_c$ represent critical values of the temperature and rate of change of temperature, respectively, for which the climate-damage costs become comparable with the abatement costs for the case that the emissions are reduced by approximately 50% (r=0.5) relative to the BAU case. Thus the parameters T_c and $\dot{T}_c$ may be regarded as defining a critical (soft shouldered) elliptical window or corridor in the climate phase space $T_c, \dot{T}_c$ within which the climate-damage costs are less than or of the same order as the mitigation costs at an abatement level of order $r = O(0.5)$. Outside the corridor the climate damage costs are greater than the mitigation costs at this abatement level.

Scenario	*Figure*	*Parameter settings*
SA	3	business-as-usual (BAU)
SB	3	modified business-as-usual
SF	3	frozen emissions at 1990 level after 2000
SG	3	reduced emissions frozen at 80% of 1990 level after 2000
$S0$	4	baseline reduced-emissions run: baseline climate model R00, cost-function parameters: $T_c = 1°C$, $\dot{T}_c = 0.02°C$/yr $\tau_1 = \tau_2 = 100$yrs $\tau_a = 50$ yrs, $\tau_d = \infty$ yrs
$S1a, b$	4	same as $S0$ but with reduced abatement-cost inertial terms (run $S1a$, $\tau_1 = \tau_2 = 50$yrs) or zero inertial terms (run $S1b$, $\tau_1 = \tau_2 = 0$)
$S2$	4	same as $S0$, but with temperature rate-of-change term $\dot{T}_c$ only in climate-damage costs
$S3a, b$	5	same as $S0$ but with abatement-cost discount time constant changed from $\tau_a = 50$yrs to $\tau_a = 25$yrs (S3a) and $\tau_a = 100$yrs (S3b)
$S4a, b, c, d$	6	same as $S0$ but with finite climate-damage cost discount time constants $\tau_d = 100$yrs (S4a), 50yrs (S4b), 35yrs (S4c) and 25yrs (S4d)
$S5$	7	same as $S0$ but with damage costs enhanced by various factors γ

Table 2: Emission scenarios

The minimal-cost solution can be found numerically by a method of steepest descent (e.g. a conjugate gradient technique, cf. Press et al, 1986). This requires computing the gradient of the cost with respect to the control function, i.e the emissions $e(t)$. For a climate model expressed in integral response form, the gradient can be computed explicitly (cf. Appendix). However, in the numerical results presented below the gradient was computed automatically using a general numerical functional derivative compiler developed by Giering (1995). This had the advantage of immediately providing the gradient whenever the climate model was modified.

5 Sensitivity experiments

In all computations we have taken as our reference climate-independent BAU emission scenario $e_A(t)$ for the computation of the abatement costs simply a linear increase for the first 205 years, from 1995 until 2200, growing from 6.3 GtC/yr in 1995 at an initial growth rate of 2.5 %/year to 38 GtC/yr in 2200. This is consistent with the upper and lower bounds of the emission projections by different energy models (Nordhaus and Yohe, 1983, Reilly et al, 1987, Manne and Richels, 1991, cf. Table 2.1 in Cline, 1992). After 205 years, the emissions have simply been frozen at the 38 GtC/yr level. This is based in part on the tentative longer-term projections of these authors, who assume a continual decrease of the emission growth rate beginning in the next century (although they do not consider projections significantly longer then 200 years), but is basically arbitrary. A constant long-term emissions level will clearly not be attainable indefinitely because of limited fossil fuel resources. Nevertheless, we have not used a decreasing long term projection for our reference level in computing the abatement cost, as the relevant information would be speculative, and – more importantly – our optimal emission scenarios are found to be insensitive to the form of $e_A(t)$ beyond a few hundred years, provided a modest discount factor, with a time constant of the order of 50 or 100 years, is applied to the abatement costs. (This assumes, however, as discussed below, that a smaller discount rate is applied to the climate damage costs in order to obtain optimal emission paths which are consistent with limited global warming).

The simulations were repeated with a BAU scenario in which the linear increase of e_A was extended to 800 years. Despite the major (and clearly unrealistic) increase in the BAU reference emission level and the corresponding CO_2 concentration over the longer term, the differences in the computed optimal emission paths were minimal, since the changes in the BAU path became effective at a late time when the abatement costs were already strongly discounted. Nevertheless, to place the BAU scenario in a more general perspective we compare the BAU climate projections (run SA) below with a modified business-as-usual Scenario (run SB), in which the emissions decline linearly after 200 years, and two frozen emission scenarios (runs SF and SG).

Prior to 1995 we have introduced a spin-up period, beginning with the pre-industrial state, which we set at $t_0 = 1800$. For the spin-up period we assume an exponential emissions growth function

$$e_A(t) = 6.3 \exp\left[(t - t_0 - 195)/t_s\right] \tag{24}$$

where $195 = t(\text{today}) - t_0 = 1995 - 1800$ corresponds to the length of the spin-up period. The emissions spin-up time constant was determined as $t_s = 35$ years from the condition that the carbon cycle model (14) must reproduce the 1995 CO_2 concentration $w(1995) = 358\,ppm$ for the given pre-industrial concentration $w_0 = w(1800) = 280\,ppm$. By coincidence, this also almost satisfies the condition for a continuous derivative in the transition from exponential to linear growth in 1995, which would require $t_s = 40$ years.

All computations have been carried out with a discretization time step of $\Delta t = 5$ years from the year 1800 over a period of 1200 years, up to the year 3000. However, the emissions were allowed to adjust freely only over 805 years, from 1995 to 2800, and were then frozen at the level $e(2800)$ for the last 200 years. The time span is clearly unrealistically long for economic predictions, but, as is apparent from Fig. 2 and the results shown in the following figures, is nevertheless appropriate for assessing long-term climate impacts relevant for a sustainable development policy. The set of computations for different parameter combinations is listed in Table 2. The results are shown in Figs. 3-7.

The BAU scenario

The CO_2 emissions and resultant concentrations and global warming for the reference BAU scenario (SA, full curves) are shown together with other scenarios in which the emissions are prescribed in Fig. 3. The evolution is depicted both for the full 1000 year horizon (with an additional initial 200 year spin-up period) and for a 200 year horizon to illustrate the dangers of designing sustainable development strategies only over short horizons. The BAU scenario can be interpreted quantitatively only for the first 100-150 years. Thereafter, the CO_2 concentrations and temperatures greatly exceed the limits of our linear response model. However, the order-of-magnitude prediction that the CO_2 concentrations will grow to some ten times the present value in the course of several hundred years may be expected to remain valid. In fact, this is presumably an underestimate, since it ignores the positive feedbacks of the decreasing solubility of CO_2 in the ocean with increasing temperature and increasing CO_2 concentrations (these effects are included in the above-mentioned nonlinear response model of Joos *et al*, 1995). The linearized temperature response, on the other hand, is strongly exaggerated for higher temperature increases. If the usual logarithmic dependence of the radiative forcing on changes in the CO_2 concentration is assumed instead of our linear relation, the temperature response for a ten-fold increase in the CO_2 level is estimated to be of the order of $8°C$ (cf. logarithmic temperature scale on the right side of the top-right panel of Fig. 3; the scale is normalized by setting the equilibrium temperature response to a CO_2 doubling at $2.5°C$ for both the linear and the logarithmic case). However, at these temperatures other nonlinearities besides the radiative forcing dependence on the CO_2 concentration will become important – including possible instabilities, for example through a breakdown of the North Atlantic circulation. For these extreme climate changes reliable predictions cannot be made even with complex nonlinear three-dimensional carbon cycle and coupled atmosphere-ocean general circulation models, since one enters then a climate regime for which there exists no previous experience or data.

The full severity of the business-as-usual climate-change impact becomes apparent only in the long-term perspective over several hundred years. However, the monotonic increase in the second half of the next millennium depends on the presumably unrealistic assumption of a continual constant emission level of 38 GtC/yr after 200 years. We have accordingly shown in Fig. 3 also a modified business-as-usual Scenario, more consistent with the estimated fossil fuel reserves, which assumes a linear decrease of the emission level after attaining a maximum value of 38 GtC in the year 2200 down to zero in the year 3000. The climate change is dramatic also for this scenario.

Although it is useful to remind oneself of the drastic climatic impact of a laissez-faire climate policy, the BAU climate prediction and thus the limitations of the present linearized climate-response model, together with our questionable long-term emissions assumption, are, in fact, irrelevant for the present study. We shall need to refer to the BAU emission curve only to compute the abatement costs for the determination of optimal reduced-emission scenarios, all of which – assuming a rational climate-protection strategy consistent with a policy of sustainable devel-

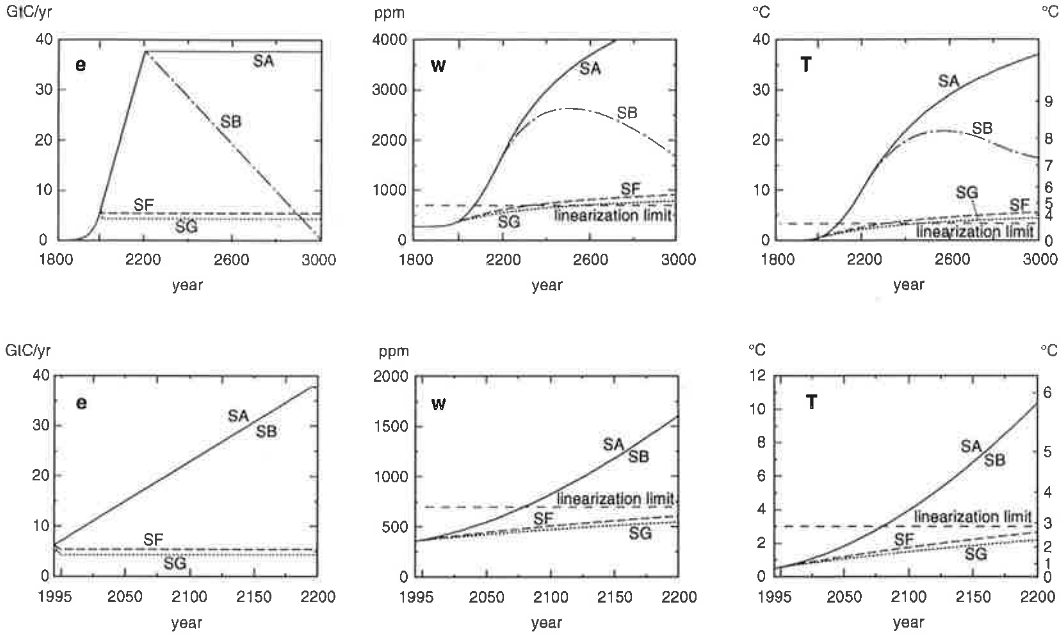

Figure 3: CO_2 emissions, computed CO_2 concentrations and global warming (from left to right) for the time periods 1800-3000 (top) and 1995-2200 (bottom) for the BAU scenario (SA, full curves), modified BAU scenario (SB, dashed-dotted curves), frozen emissions at 1990 levels after the year 2000 (SF, dashed curves) and 20% reduced emissions relative to the 1990 level after 2000 (SG, dotted curves). The linear model is not applicable above the indicated dashed levels. The logarithmic T scale on the right ordinate axis of the top-right panel indicates the order-of-magnitude temperature response allowing for the logarithmic dependency of the radiative forcing on the CO_2 concentration.

opment – yield significantly smaller climate changes lying more or less within the linear climate-response range.

The frozen emissions scenarios

The Rio climate convention recommended as first target towards a long-term climate stabilization policy the freezing of CO_2 emissions at the 1990 levels by the year 2000. The evolution of CO_2 concentrations and the global mean temperature for this scenario SF, assuming that the 1990 emission level is maintained after 2000, is shown in Fig. 3 together with an alternative scenario SG in which the emissons are frozen at a slightly lower level of 80% of the 1990 levels, as proposed by some more concerned countries. Although the medium term global warming is significantly reduced in the frozen emission scenarios, the long term temperature rise is still large. Thus they can be regarded only as effective in gaining time for the implementation of longer term abatement measures, which, as shown below, require a stronger reduction of CO_2 emission levels and a transition to carbon-free energy technologies.

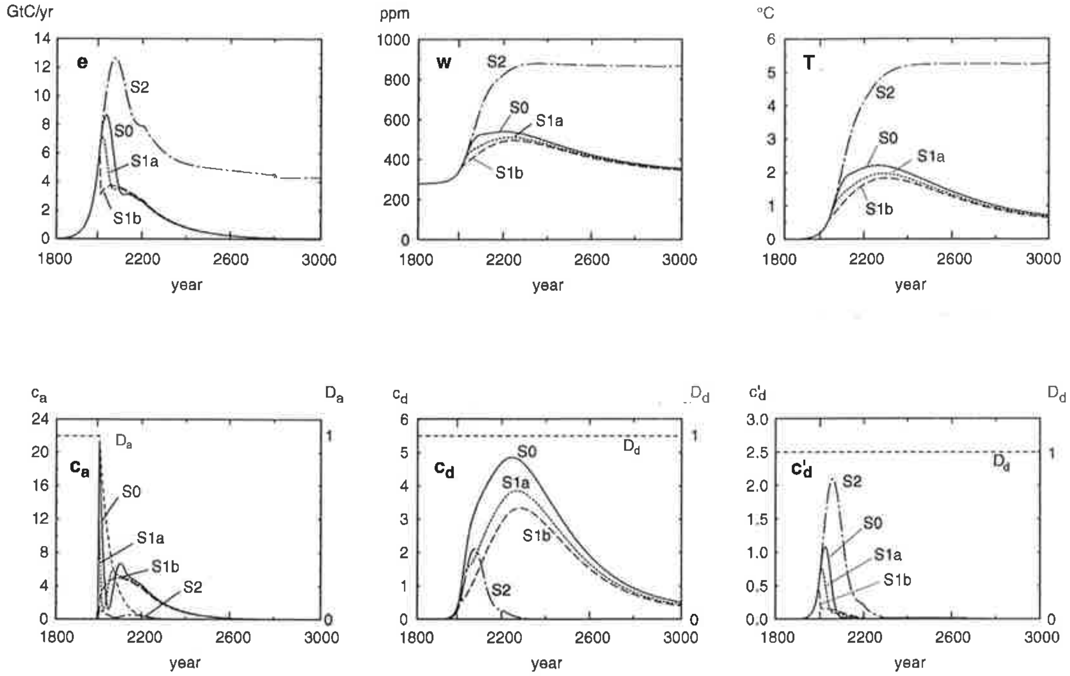

Figure 4: Evolution over the period 1800-3000 of: (top, left to right) CO_2 emissions, CO_2 concentrations, global mean temperature and (bottom, left to right) specific abatement costs c_a, specific damage costs c_d and the contribution to the specific damage cost c_d' from the rate of change of temperature, for (cf. Table 2): the baseline reduced-emissions scenario $S0$ (full curves), the same run with reduced or zero inertial terms in the abatement-cost function (run S1a, dotted curves, and run S1b, dashed curves, respectively) and a modified baseline run in which the climate-damage costs are assumed to depend only on $\dot{T}$ (run $S2$, dash-dotted curves). Also shown in the lower panels are the exponential abatement and damage cost discount factors D_a and D_d.

The baseline scenario S0

A baseline reduced-emissions computation $S0$ (Fig. 4) was carried out for the cost-function parameters values $T_c = 1°C$, $\dot{T}_c = 0.02°C/\text{yr}$ and $\tau_1 = \tau_2 = 100\text{yrs}$, with discount time constants $\tau_a = 50$ years and $\tau_d = \infty$. The impact of different parameter settings and different discount factors is explored in the runs $S1 - S4$ (Figs. 4-6) and $S5$ (Fig. 7).

The critical temperature $T_c = 1°C$ and rate of change of temperature $\dot{T}_c = 0.02°C/\text{yr}$ ($1°C$ increase in 50 years) for the climate-damage cost function of the standard scenario $S0$ are representative of typical values which have been quoted in the literature. They lead for Scenario $S0$ to a maximum temperature increase of $T_{max} = 2.2°C$ (cf. Fig. 4). The decrease in temperature beyond the year 2200 results from discounting the abatement costs while applying no discounting factor to the climate damage costs (discount factors are discussed further below): one can more easily afford to reduce emissions over the long time to reduce damage costs.

Economic inertia

The choice of the economic-inertia coefficients τ_1 and τ_2 was found to be uncritical in a broad band of values. They act mainly in the initial stages, ensuring that the emissions reduction is not discontinuous at the start time $t = 1995$ of the control path. Thus initially the emissions follow the BAU path (see also the more detailed discussion in Wigley *et al*, 1996). However, the long-term impact of economic inertia remains small, as demonstrated by a comparison in Fig. 4 of the baseline scenario $S0$ with runs in which the inertial terms were reduced ($S1a$) or set equal to zero ($S1b$).

Impact of temperature change and rate of change of temperature

The principal contribution to the climate-damage costs was found to stem from the temperature change itself rather than the rate-of-change of temperature (cf. net climate-damage costs c_d and the contribution c'_d incurred by the rate of change of temperature depicted in Fig. 4). This is demonstrated also by the optimal emissions scenario $S2$ (also shown in Fig. 4), in which the climate-damage costs were represented only by a single term depending on the rate of change of temperature. The maximal temperature increases to $6°C$ within 300 years and then remains at this level. The results of Tahvonen *et al* (1994,1995), who considered only this $\dot{T}$-dependent term in their climate-damage costs, should therefore be regarded only as illustrative (as pointed out by the authors). Adopting the usually quoted critical values T_c and $\dot{T}_c$, our model indicates, for the typical time constants of climate change, that the climate damage costs will be dominated by the temperature change itself rather than the rate of change of temperature. However, for quantitative projections this point needs closer scrutiny with respect to the different types of climate damage.

Discount rates for mitigation costs

The most critical and also most controversial terms in the cost functions are the discount factors. It has been argued that the discount rates for mitigation and climate damage costs should be treated differently. We accordingly study their impacts first separately, returning later, however, to the question of their interrelation.

Since our simple abatement costs model does not distinguish between the separate effects of growth in wealth, return on capital, endogenous technological development and other processes normally included in a more detailed economic model, our discount factor for the mitigation costs represents the net impact of all of these processes combined. Our choice of the abatement cost discount time constant $\tau_a = 50$ years (2% per year) for the baseline scenario is at the lower range of (inflation adjusted) discount factors proposed in greenhouse-gas abatement studies (cf. Nordhaus, 1991, 1993). Figure 5 shows the impact of decreasing the time constant τ_a to 25 years (Scenario S3a), and also the effect of doubling τ_a to 100 years (Scenario S3b). A shorter discount time scale implies that one can afford to apply mitigation measures earlier, reducing global warming, while for a larger time constant it is more economic to delay abatement measures, with a resultant increase in global warming.

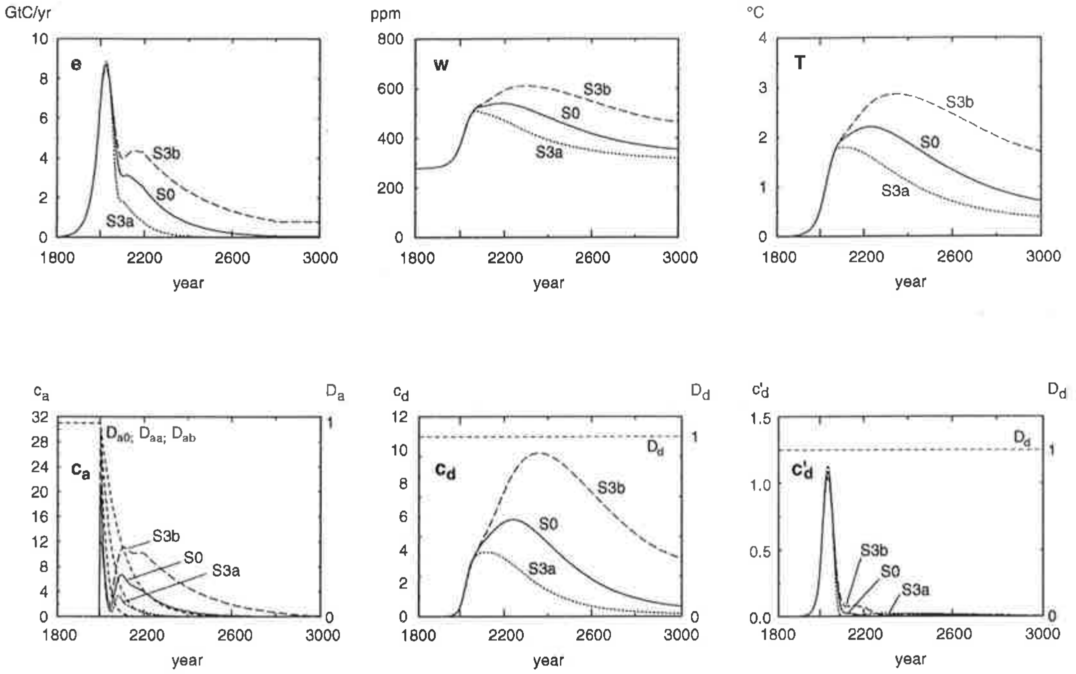

Figure 5: Impact of changed abatement cost discount time constants $\tau_a = 25$yrs (S3a, dotted curves) and $\tau_a = 100$yrs (S3b, dashed curves) compared with baseline case $\tau_a = 50$yrs (Scenario $S0$, full curves; cf.Table 2 and layout of Fig. 4).

The value of τ_a is seen to have a strong influence on the computed optimal emission paths. However, this applies for a fixed disount rate for the climate damage costs (which we have set to zero in our baseline scenario S0 and Scenarios S3a,b). Since we are concerned only with the ratio of climate damage to mitigation costs, parallel changes in the discount rates for both types of costs tend to offset one another. This is discussed further below.

Discount rates for climate damage costs

More controversial than the discount rate for mitigation costs has been the proper intertemporal treatment of climate damage costs. According to the traditional economic view, climate damage costs are economic costs just as any other costs, and should accordingly be discounted at the same rate as mitigation costs. This is based on the concept that climate damages can be countered by appropriate engineering measures, such as building higher dams in response to rising sea levels, or other economic adjustments. Thus there is no difference in principle between the economic efforts required to respond to or to limit climate change.

An alternative view is that a deterioration of future living conditions through an irreversible change in climate represents a loss in welfare which to first order is independent of the period in the future when the climate change actually takes place. Future sustainable development is perceived as a non-time-degradable commitment to which one should assign a time-independent welfare value. In this view climate

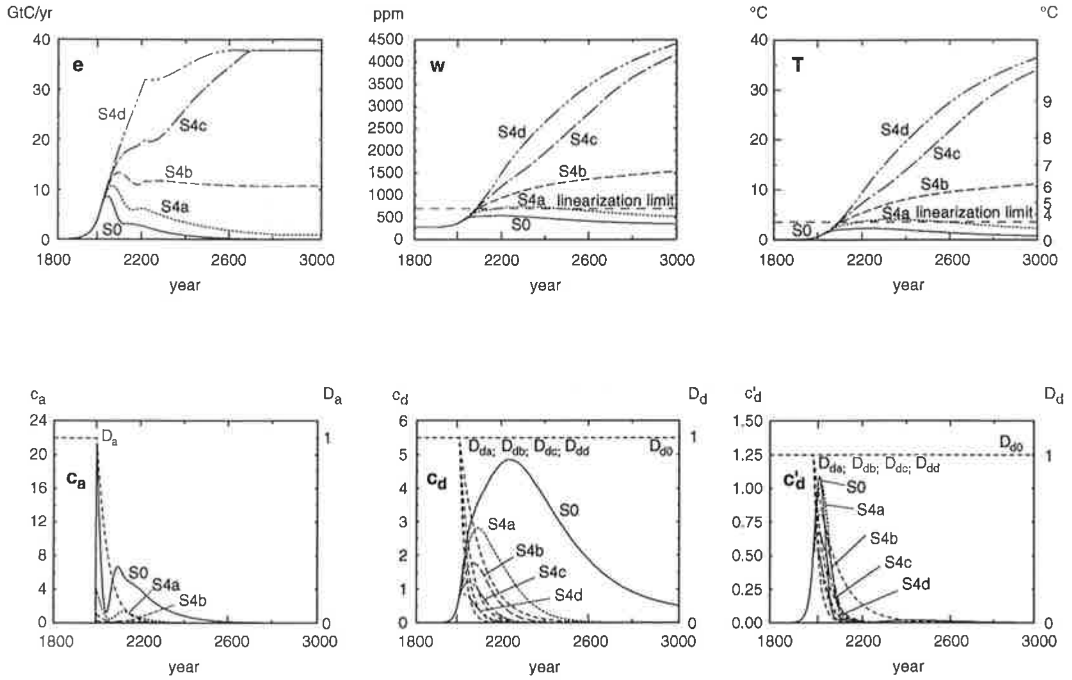

Figure 6: Comparison of the baseline case $S0$ without climate damage-cost discounting (full curves) with scenarios assuming finite discount time constants $\tau_d = 100$yrs (simulation $S4a$, dotted curves),$\tau_d = 50$yrs ($S4b$, dashed curves), $\tau_d = 35$yrs ($S4c$, dashed-dotted curves) and $\tau_d = 25$yrs ($S4d$, dashed-double-dotted curves); cf.Table 2 and layout of Fig. 4).

damage represents a basically different quality-of-life or welfare loss than abatement costs. The preservation of an habitable planet for future generations is accepted as a legacy which must be honored today, regardless of the time horizon over which our present actions will affect future living conditions.

Following this second line of reasoning, we have introduced no discounting of damage costs in our baseline reduced-emissions run $S0$. The underlying value judgements are, of course, debatable. The application of the same or comparable discount factors to both mitigation and climate-damage costs (e.g. Nordhaus, 1991,1993, Beltratti, 1995) yields basically different conclusions, as discussed below.

For political decision making, however, it is irrelevant which of these theoretical assessments of the future impact of climate change is 'correct'. Relevant for the computation of an optimal emission path – at least in a democratic society – is only the public and politically transmitted perception of the value of a future stable climate. It would be an instructive sociological exercise to ascertain whether a significant irreversible climate change resulting from the present activities of mankind which is predicted to create major existential problems for generations far in the future, well beyond the normal economic discounting time horizon, is regarded by the public and politicians as a serious problem which requires remedial action today. Investigations by Kempton *et al* (1995) suggest that this is probably the case.

Different assumptions regarding the rate of damage-cost discounting can be readily explored with our model. Scenarios $S4a, b, c, d$ (Fig. 6) show as examples the effect of introducing finite damage-costs discount time constants of 100, 50, 35 and 25 years, respectively. The maximal CO_2 concentrations and temperatures increase markedly, particularly for the last two cases. The climate changes implied by these temperature increases – noting that regional temperature changes, for example over continents, can be significantly higher than the global mean temperature rise – implies a dramatic change in the living conditions of our planet. However, this occurs only after several hundred years, when the climate-damage costs have been discounted by one or two orders of magnitude.

Ratio of climate damage and abatement cost discount rates

The character of the solutions depends critically on the ratio of the climate-damage and abatement cost discount factors. In all cases considered so far except Scenarios $S4b, c$ and d, the discount time constant was higher for the climate damage costs than for the abatement costs, and the long term temperature increase for the optimal emissions path remained limited. If this inequality holds, the discounted specific abatement costs become exponentially small compared with the discounted specific climate damage costs for large times, and the most cost effective path is one in which the emissions approach zero asymptotically (except for Scenario S2, in which the damage costs depended only on $\dot{T}$).

The form of the solution changes completely if the opposite inequality $\tau_d < \tau_a$ holds (Scenarios $S4c, d$). In this case, the climate damage costs are discounted more rapidly than the mitigation costs, and it becomes more cost effective to revert to the business as usual scenario asymptotically. Although the non-discounted specific climate damages grow with the square of the temperature, this is more than offset by the more effective exponential discount factor for the damage costs, and $e(t) \rightarrow e_A(t)$ as $t \rightarrow \infty$. The asymptotic CO_2 concentrations and temperatures of Scenarios $S4c, d$ are accordingly the same as for the BAU scenario (Fig. 3).

If $\tau_d = \tau_a$ (Scenario $S4b$), neither cost term is discounted more rapidly than the other. (However, the discounted climate damage costs are reduced by a more or less constant factor relative to the discounted abatement costs because of the time lag of climate change relative to the emissions.) In this case, the optimal emissions path remains at a relatively high level between the BAU path and zero emissions (cf. Fig. 6).

The global warming levels of the optimal path solutions of Fig. 6 – even for the case $S4a$ with $\tau_d = 100\text{yrs} > \tau_a = 50\text{yrs}$ – are considerably higher than the solutions obtained assuming zero discount rates for the climate-damage costs. The temperature increases exceed most estimates of the limits of global warming acceptable for sustainable development. Thus if one subscribes to the ethical commitment of preserving a habitable planet for future generations, these solutions cannot be accepted. It follows that to the extent that there exists a public commitment to this principle, the societal intertemporal preference relations describing the present and future costs of adapting to or mitigating climate change cannot be expressed in terms of standard economic discount factors appropriate for, say, the short-term return on capital investment or intertemporal expenditure preferences for consumer goods. Rather, the willingness to pay for the well being of future generations is analagous to the willingness to contribute to public education, development aid or other societal actions which do not directly benefit the individual. Thus it appears more appropriate to determine the intertemporal values attached by society to the mitigation of future climate change empirically by assessing the public willingness to pay for such measures.

We conclude from these examples that the computed optimal emission paths are highly sensitive to the relative values of the discount rates for climate damage and mitigation costs, and that solutions qualitatively consistent with the requirement of sustainable development are obtained only if the climate damage discount time constants are larger than the discount time constants for abatement costs.

Impact of other greenhouse gases or modified mitigation/damage cost ratios

Our greenhouse-warming simulations have been carried out for CO_2 emissions only and are thus overly optimistic. To allow for the comparable climatic impact of other greenhouse gases such as methane and chlorofluorocarbons (CFCs), our computed optimal CO_2 emission paths need to be reduced. To gain a qualitative estimate of the influence of the non-CO_2 greenhouse gases, we assume that they can be reduced in parallel with, and at the same relative costs as, the CO_2 concentrations. The computed CO_2 concentrations may then be regarded to first order simply as a proxy for the equivalent greenhouse CO_2 concentration, representing the net effect of all greenhouse gas concentrations (cf. IPCC, 1990a). Assuming a fixed ratio γ between the equivalent and true CO_2 concentrations, the effect of the non-CO_2 greenhouse gases can then be represented by simply replacing the temperature T computed for the true CO_2 emissions path by the temperature $T_{eqiv} = \gamma T$. Since the damage cost function depends quadratically on the temperature (cf. eq.(22)), this corresponds to an increase of the damage cost function by a factor γ^2. The mitigation costs, on the other hand, increase by a factor of only γ. Thus the ratio of climate damage to mitigation costs is increased by a net factor γ.

The impact is shown in Fig. 7. The curves can also be interpreted as showing generally the effect of a change γ in the ratio of climate damage to mitigation costs. The impacts are smaller than may have been anticipated intuitively. This can be explained by two effects. Firstly, a relative increase of the climate-damage costs by a factor γ implies a decrease of the critical climate temperature T_c (and the critical rate of change of temperature $\dot{T}_c$) by a factor of only $\gamma^{-1/2}$ (eq. (22)). Thus to reduce the climate damage costs to the same level as in the CO_2 only case, the emissions need to be decreased by a factor of only $\gamma^{-1/2}$. Secondly, while for these emission values the climate-damage costs are at the same level as in the CO_2-only case, the abatement costs, because of the lower emission levels, are higher. For the optimal-emissions solution, in which a balance is attained between the mitigation and damage costs, the abatement costs will therefore be lower and the emission levels higher than these values. Hence the reduction in emission levels for the solution including both CO_2 and non-CO_2 greenhouse gases will be still smaller than the factor $\gamma^{-1/2}$.

However, if we adopt the alternative assumption that the non-CO_2 greenhouse

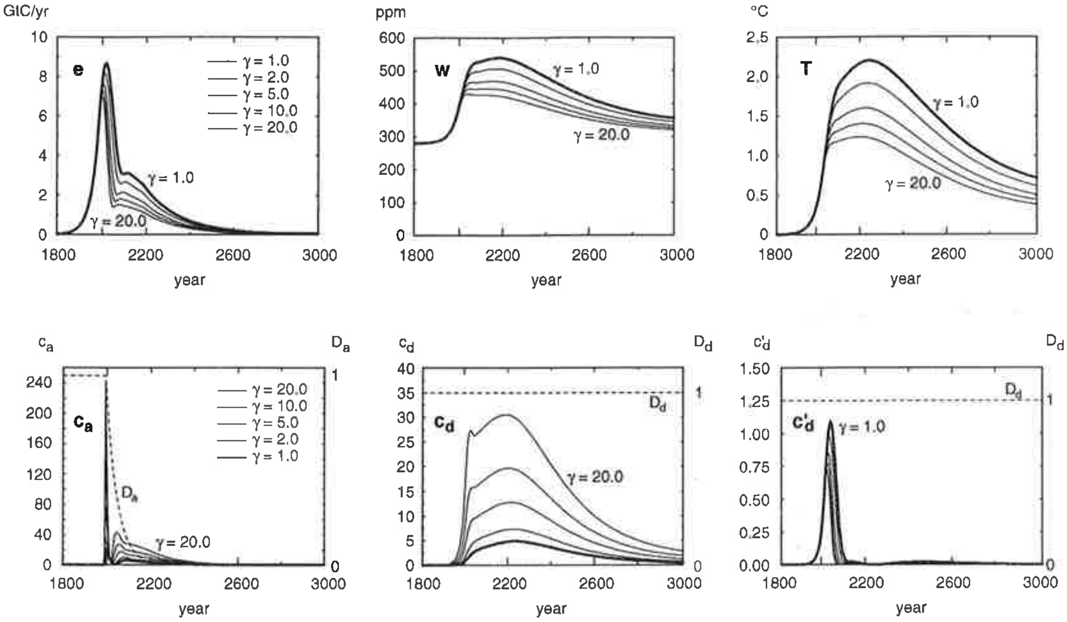

Figure 7: Impact of a change in the ratio of climate-damage to mitigation costs by a factor γ. Non-CO_2 greenhouse gases can be modelled qualitatively by values $\gamma > 1$ (e.g. $\gamma = 2$ if they contribute the same radiative forcing as CO_2). Results for the baseline scenario $S0$ ($\gamma = 1$) are shown as thick full curves (cf.Table 2 and layout of Fig. 4).

gases cannot be readily reduced, the reduction in CO_2 emission levels needed to counteract the effect of the increasing concentrations of other greenhouse gases can be considerably larger than computed for the CO_2-only case. This situation is discussed in the context of non-cooperative actors in the n-actor climate mitigation problem in Hasselmann and Hasselmann (1996).

6 Conclusions

The purpose of this study was not to provide quantitative monetary estimates of costs and benefits of optimal CO_2 emission strategies to assist decision makers in determining, say, the proper level of carbon taxes, but rather to clarify the basic input assumptions and cause-and-effect relations which are presumably responsible for the pronounced divergencies in existing cost-benefit analyses. This has enabled us to discriminate between conclusions which represent relatively robust consequences of the dynamics of the climate system and predictions which depend critically on controversial input assumptions.

To this end we introduced a simple impulse-response climate model, calibrated against state-of-the-art CGCM climate and three dimensional global carbon cycle models, and highly idealized but structurally transparent expressions for the climate damage and mitigation costs. For the determination of optimal emission paths only the relative levels of climate damage and mitigation costs, not the absolute cost values, are relevant.

The principal conclusions of our investigation can be summarized as follows:

- Since the global warming response for CO_2 emissions extends over several hundred years (Fig. 2), the costs associated with the climatic impact of present and future CO_2 emissions must be optimized over horizons far beyond normal economic discounting time scales.

- If, as in many studies, climate damage costs are discounted at standard economic discount rates, the optimal CO_2 emission paths are only weakly reduced relative to the Business as Usual scenario. The resultant long-term climate warming remains very large and sustainable development is not attained. This is logically consistent: by discounting climate damage costs, it is assumed that the maintenance of a habitable climate far in the future is of negligible present value. It is questionable, however, whether this scenario corresponds to the value assigned by society to sustainable development: an 'optimal' emissions path leading to major global warming is probably not acceptable by the general public. While subscribing to the principle that an optimal climate protection strategy should be determined through a cost-benefit analysis in which an attempt is made to moneterize all costs, we suggest that the monetary value of the asset 'a habitable planet for future generations' should be ascertained on the basis of willingness-to-pay criteria. This would presumably reveal different intertemporal value assignments for the principle of sustainable development than the normal discount relations used to model societal time preference relations associated with, say, the deferred purchase of consumer goods.

- A necessary condition for global warming to remain below an acceptable bound is that the discount rate for mitigation costs is greater than the discount rate for climate damage costs. In practice, optimal CO_2 emission paths yielding acceptable global warming are obtained only if the discount rate of climate damages is very small or zero. Our baseline scenario accordingly assumes a zero discount rate for climate damage costs. In all solutions yielding limited global warming, CO_2 emissions must be drawn down significantly by a factor of at least a half over a few centuries, with a continual decrease thereafter. The rate of reduction for the optimal path depends sensitively on the assumed discount rate for the mitigation costs.

- Because of the inclusion of economic inertia in the mitigation cost function, CO_2 emissions are not immediately reduced in our baseline optimal emissions path, but rise for a few decades before declining. However, even when the inertial terms are omitted, allowing the emissions to adjust immediately to a new level at no economic rate-of-change cost penalty, the optimal emission paths exhibit no immediate drastic draw-down. Moreover, the long-term climate response does not differ significantly for the cases with and without economic inertia. We conclude that an effective climate mitigation strategy must focus on the long-term transition to energy technologies with zero or very low CO_2 emissions. Short term reductions through energy saving, although high on the present political agenda, are insufficient on their own and can be viewed only as a useful auxilliary measure in support of the necessary long-term technological transition process.

- On the other hand, the technological restructuring can be carried out without dramatic dislocations in the course of many decades or a century. This should not be interpreted to imply that there is no urgency in the implementation of policies initiating the necessary gradual transition to lower CO_2-emission levels: any delay, permitting a non-regulated continuation along the business-as-usual path, incurs the need for larger, more costly adjustments later. Moreover, in initiating the transition, the inertia not only of the economy but also of the political process must be taken into account. The computed delay in the drawdown of CO_2 emissions for our baseline scenario was based on a simple parametrization of the transition costs associated with economic inertia only – assuming an optimal reduction policy can be immediately implemented politically. Our results are thus overly optimistic regarding the time pressures of adjusting the complete socio-economic system and should not be interpreted as implying the existence of a time cushion for delaying implementation decisions.

- Another simplification resulting in too optimistic emission scenarios is the limitation to CO_2 emissions, ignoring the comparable global warming contributions of non-CO_2 greenhouse gases. To the extent that the abatement of non-CO_2 greenhouse gases can be achieved at a relative cost similar to that of CO_2 emissions, the impact of non-CO_2 greenhouse gases can be accounted for to first order by simply increasing the climate damage costs by an appropriate factor. This leads to somewhat lower but not drastically reduced optimal CO_2 emission paths. As the ratio of climate damage to abatement costs is an arbitrary free parameter in our analysis, our general conclusions are not affected by this modification. However, the problem is more severe if the non-CO_2 greenhouse gases cannot be effectively abated (see discussion in Hasselmann and Hasselmann, 1996).

- For the time scales of climate change corresponding to the optimal CO_2 emission paths, climate damages due to the rate of change of temperature are an order of magnitude smaller than the damages due to the change in temperature itself. However, these estimates are based on global critical climate damage thresholds of $T_c = 1°C$ for temperature and $\dot{T}_c = 0.2°C/\text{decade}$ for the rate of change of temperature, which need to be differentiated more carefully with regard to the type of climate damage.

A number of general implications can be drawn from these conclusions. Although our sensitivity analysis was based on structurally highly simplified cost models and needs to be quantified in monetary units using more realistic economic models, most of the practical policy implications of our structural analysis are independent of the details of such models. In practice, more realistic economic models necessarily involve assumptions, for example regarding future technological development, whose uncertainties largely mask the quantitative predictive potential of the models.

The central dilemma for decision makers highlighted by our analysis is the time scale mismatch between the multi-century climate response to present and future CO_2 emissions on the one hand and typical economic and policy planning horizons of a few years to a decade on the other hand. It is obviously not realistic to plan CO_2 emissions centuries into the future. Our computed optimal emission paths are meaningful only in the sense that they identify the time scales and orders of magnitude of the emission reductions required to stablize climate. The optimal paths will depend in detail on evolving energy technology and other factors which cannot be predicted over long time horizons. Short and medium term policy decisions can establish only the necessary framework favoring a gradual transition to a path of continually decreasing emissions. Long-term policies will necessarily by limited to establishing effective monitoring mechanisms and periodically adjusting regulatory mechanisms in accordance with continually updated projections.

Much of the discussion on the reduction of CO_2 emissions has revolved around instruments for internalizing climate damage costs, for example through carbon taxes or tradable emission permits. However, our computations indicate that the encouragement of energy efficiency through these measures alone will be insufficient to attain the goal of stabilizing climate. To achieve the necessary transition to carbon-free energy technologies, a push-pull approach will presumably be needed, including both penalties for CO_2 emisssions and rewards for the development of alternative energy technologies.

Although realistic climate protection measures are necessarily limited in their immediate impact on CO_2 emissions to time scales which are short relative to the natural time span of the global warming problem, so that their immediate influence on long-term climate evolution is small, a far-sighted policy can nevertheless induce a negative curvature in the emissions curve which, if upheld into the future, would have a significant long term impact. From this viewpoint, the principal role of more realistic economic models would be to study the impact of the available instruments for controlling climate emissions in the politically viable short and medium time scales on the first and second time derivatives of the CO_2 emissions curve. From these studies one could then derive realistic (moving) targets for the first two time derivatives, defined from the perspective of the major long term reduction of CO_2 emissions mandated by climate model predictions. The performance of the economy in response to the applied regulatory instruments would need to be continually monitered, and the targets and control mechanisms periodically updated.

7 Outlook

The implementation of a long-term monitoring and continually retuned regulatory policy requires more realistic modelling tools than are presently available. The realization of an effective climate protection policy within an international framework, for example, raises a number of complex issues involving decision making between several actors with different values and goals, which cannot be adequately addressed with the single-actor economic models considered here. However, we suggest that before embarking on complex multi-actor game-theoretical analyses using sophisticated multi-regional, multi-sectoral economic models, it would be useful, in keeping with the philosophy of the present approach, to carry out a general system-analytical study using a structurally highly simplified multi-actor model (cf. Hasselmann and Hasselmann, 1996).

In addition to the restriction to a single actor and the simplification of the economics, there are a number of other basic limitations of the present model which need to be addressed. For example, a realistic model would need to simulate also the inherent internal variability of the system. This is an essential dynamical feature of both climate and the socio-economic system. It has been shown (Hasselmann, 1976) that long-term fluctuations in the climate system can be generated by the stochastic forcing exerted by short-term random weather fluctuations acting on the slow components of the system (the oceans, biosphere and cryosphere), in analogy with the Brownian motion of heavy molecules excited by random collisions with lighter molecules. Stochastic forcing may be expected to produce also slow fluctuations in the socio-economic system, which similarly contains both slow elements, for example in the form of energy technology or the cultural values of a society, and more rapidly fluctuating components, such as business cycles, societal fads and other short-term adjustment processes. A realistic representation of the interactions between the different spectral frequency bands of the natural variability spectrum is an important test of our understanding of the dynamics of the GES system, and our ability to properly represent the response of the system to external anthropogenic forcing.

A consideration of natural variability is important also because the impact of anthropogenic global climate change must be weighed against the impacts of the inherent internal variability of the GES system. The skepticism which is occasionally expressed with regard to the need for a climate protection strategy can probably be attributed in good part to the intuitive feeling that the effects of the (unpredictable) inherent variability of the socio-economic system will always outweigh the impact of the predicted climate change. For the rational analysis of such assessments one will need GES models which are able to simulate both the response to external anthropogenic forcing and the internal variability of the system.

A more realistic GES model will also need to include societal components, particularly with regard to the establishment of the mitigation and climate-damage cost functions and the representation of the decision-making module in Figure 1. For the political decision-making process, the 'true' costs are less relevant than the 'perceived costs' (Stehr and v.Storch, 1995). The transmission of scientific predictions of future climate change, as well as rational assessments of the ensuing climate-damage or mitigation costs, into the political arena involves the creation of a 'social construct' of climate change and climate-change impact. This product of the media, interest groups and the public awareness and education need not be closely correlated with scientific perceptions. A significant portion of the population in the US, for example, perceives as dangers attributed to global warming the unrelated problem of the pollution of the atmosphere by health-threatening gases or the (entirely negligible) depletion of oxygen in the atmosphere (Kempton *et al*, 1995). In a similar poll conducted in Germany, 80% of the persons interviewed believed that global warming and the ozone hole were directly related.

In this context, the concept of a predefined cost function dependent only on the state of the economy and the climate may also be questioned. Social values change with time, as evidenced by the recent increase in the public concern over threats to the environment (cf. also Turner, 1995). Our inderstanding of climate change also evolves with time. The non-stationarity of the 'social construct' of climate change on longer time scales of several hundred years is well illustrated by the medieval example of Stehr and v. Storch (1995), in which a severe climate degradation in 14'th century England was successfully reversed (in the perception of the time) by a 'mitigation' policy of public penitence initiated by the archbishop of Canterbury.

Thus both our scientific assessment of climate change and climate-change impact, and the transmission of this understanding into a 'climate construct' serving as the basis of policy decisions, should be viewed as evolving entities. Our present assessment and the resultant policy decisions may well be regarded as inadequate and inappropriate by future generations. A further aspect which should be included in more detailed integrated assessment studies is therefore the problem of decision making under uncertainty. This would need to include the probabilistic assessment of risk and the impact of an anticipated future reduction of uncertainty on the timing of decisions. The time scale and uncertainty dilemma non-withstanding, however, we have no choice but to accept our present understanding as the basis for defining and implementing policies which – although subject to continual later revision – must nevertheless be designed to shape the future far beyond the societal horizon which we can confidently perceive or anticipate today.

Despite the limitations of the present study and the non-monetary, illustrative nature of our simulations, we believe that several general features of the optimal emission-path solutions we have presented will survive later improved insights and more quantitative treatments. These concern, in particular, the long time scales of the climate response, the general time history and order of magnitude of the reduction in CO_2 emissions required to avert a major global warming, and the need to express the commitment to long term sustainable development in non-discounted 'willingness-to-pay' present values in order to obtain meaningful optimal emission solutions from cost-benefit analyses which do indeed satisfy the requirement of sustainable development.

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A Appendix: Computation of the cost gradient

We derive the gradient $g(t)$ of the cost in the following using continuous functional-derivative notation. In practice, however, the cost C, eq. (17), is computed as a discrete sum rather than the integrals (18), (19), and the functional derivative $\delta C/\delta e = g(t)$ becomes a normal gradient vector whose components are indicated by a discrete time index.

Applying the definitions (18), (19), the variation of C yields

$$\begin{aligned}\delta C &= \int_{t_0}^{t_h} \left(\frac{\partial c_a(e,\dot{e},t)}{\partial e}\delta e + \frac{\partial c_a(e,\dot{e},t)}{\partial \dot{e}}\delta \dot{e} + \frac{\partial c_a(e,\ddot{e},t)}{\partial \ddot{e}}\delta \ddot{e} \right. \\ &\quad \left. + \frac{\partial c_d(T,\dot{T},t)}{\partial T}\delta T + \frac{\partial c_d(T,\dot{T},t)}{\partial \dot{T}}\delta \dot{T} \right) dt \end{aligned} \tag{A 1}$$

or, substituting the variational relations

$$\delta T(t) = \int_{t_0}^{t} R(t-t')\delta e(t')dt' \tag{A 2}$$

$$\delta \dot{T}(t) = \int_{t_0}^{t} \dot{R}(t-t')\delta e(t')dt' + R(0)\delta e(t) \tag{A 3}$$

for the model equations (1), and removing the time derivatives on δe by partial differentiation:

$$\begin{aligned}\delta C &= \int_{t_0}^{t_h} \left(\frac{\partial c_a}{\partial e} - \frac{d}{dt}\frac{\partial c_a}{\partial \dot{e}} + \frac{d^2}{dt^2}\frac{\partial c_a}{\partial \ddot{e}} \right) \delta e(t) dt \\ &\quad + \left[\frac{\partial c_a}{\partial \dot{e}}\delta e(t) - \frac{d}{dt}\frac{\partial c_a}{\partial \ddot{e}}\delta e(t) \right]_0^{t_h} + \left[\frac{\partial c_a}{\partial \ddot{e}}\delta \dot{e}(t) \right]_0^{t_h} \\ &\quad + \int_{t_0}^{t_h} dt \frac{\partial c_d}{\partial T} \int_{t_0}^{t} dt' R(t-t')\delta e(t') \\ &\quad + \int_{t_0}^{t_h} dt \frac{\partial c_d}{\partial \dot{T}} R(0)\delta e(t) + \int_{t_0}^{t_h} dt \frac{\partial c_d}{\partial \dot{T}} \int_{t_0}^{t} dt' \dot{R}(t-t')\delta e(t') \end{aligned} \tag{A 4}$$

Applying the relation (10) to the double integrals and invoking eq.(13), we obtain then for the gradient $g(t) = \delta C/\delta e(t)$

$$g(t) = \left(\frac{\partial c_a}{\partial e} - \frac{d}{dt}\frac{\partial c_a}{\partial \dot{e}} + \frac{d^2}{dt^2}\frac{\partial c_a}{\partial \ddot{e}} \right) + \int_{t}^{t_h} \left\{ \frac{\partial c_d}{\partial T}(t')R(t'-t) + \frac{\partial c_d}{\partial \dot{T}}(t')\dot{R}(t'-t)dt' \right\} \tag{A 5}$$

We have dropped in (A 5) the terms resulting from the perturbations in (A 4) at the endpoints of the interval. These yield δ-function expressions which impose in effect the boundary conditions

$$e(t) = 0 \quad \text{at } t = 0, t_h \quad (\text{if } c^a \text{ depends on } \dot{e}) \tag{A 6}$$

$$\dot{e}(t) = 0 \quad \text{at } t = 0, t_h \quad (\text{if } c^a \text{ depends on } \ddot{e}) \tag{A 7}$$

If these boundary conditions are not satisfied, the contributions to the abatement cost c_a at the endpoints of the interval would become infinite if the dependence on $\dot{e}$ or $\ddot{e}$ is quadratic, as we have assumed. However, in the discretized practical implementation there is no need to impose the boundary conditions (A 6),(A 7) explicitly; they are satisfied automatically by the minimal-cost solution in the limit of a very small discretization increment.

References

[1] Beltratti, A. (1995) Sustainable growth: models and implications, 296-311, in *Environmental Economics*, ed. G.Boero and A.Silverstone, 325 pp.

[2] Bryan, K., F.G.Komro S.Manabe and M.J.Spelman (1982), Transient climate response to increasing atmospheric carbon dioxide, Science **215**, 56-58

[3] Cline,W.R.(1992), The economics of global warming, Inst.Internat.Econ., 399 pp.

[4] Cubasch, U. K.Hasselmann, H.Höck, E.Maier-Reimer, U.Milolajewicz, B.D.Santer and R.Sausen (1992), Time-dependent greenhouse warming computations with a coupled ocean-atmosphere model, Climate Dynamics, **8**, 55-69

[5] Enting,I.G., T.M.L. Wigley and M. Heimann (1994) Future emissions and concentrations of carbon dioxide: key ocean/atmosphere/land analyses, CSIRO Techn.Paper **31**, 120 pp.

[6] Fankhauser,S. (1995), Valuing climate change, Earthscan, London, 180 pp.

[7] Hasselmann, K. (1976) Stochastic climate models. Part 1: Theory. Tellus, **28**, 473-485.

[8] Hasselmann, K. (1991) How well can we predict the climate crisis? Conf. on Environmental Scarcity: The international dimension, 5-6 July, Kiel (ed. H.Siebert), Symposien- und Konferenzbände des Instituts für Weltwirtschaft an der Universität Kiel, J.C.B. Mohr (Paul Siebeck), Tübingen, 165-183.

[9] Hasselmann,K., R.Sausen, E.Maier-Reimer and R.Voss (1993) On the cold start problem in transient simulations with coupled atmosphere- ocean models, Climate Dynamics, **9**, 53-61.

[10] Hasselmann, K. and S.Hasselmann (1996), Multi-actor optimization of greenhouse-gas emission paths using coupled integral climate response and economic models, Proc.Potsdam Symp."Earth System Analysis: Integrating Science for Sustainability", 1994

[11] IPCC (1990a) Climate Change. The IPCC Scientific Assessment (Houghton,J.T. G.J.Jenkins, J.J.Ephraums, eds), Cambridge University Press, Cambridge, pp

[12] IPCC (1990b) Climate Change. The IPCC Impact Assessment. Report from Working Group II, Canberra, Australian Government Publishing Service.

[13] IPCC (1992), Climate Change 1992. The supplementary report to the IPCC Assessment (J.T.Houghton, B.A.Callander, S.K.Varney. eds), Cambridge University Press, Cambridge, pp

[14] IPCC (1994), Climate Change 1994. Radiative forcing of climate change and an evaluation of the IPCC IS92 emission scenarios (J.T. Houghton, L.G.Meira Filho, J.Bruce, Hoesung Lee, B.A.Callander, E.Haites, N.Harris and K.Maskell, eds.). Cambridge University Press, Cambridge, 339 pp.

[15] Joos, F., M.Bruno, R.Fink, U.Siegenthaler, T.F.Stocker, C. Le Quéré, J.L.Sarmiento (1995) An efficient and accurate representation of complex oceanic and biospheric models of anthropogenic carbon uptake (submitted to Tellus)

[16] Kempton, W., J.S. Boster and Jennifer A. Hartley, 1995: Environmental values in American Culture. MIT Press, Cambridge MA and London, ISBN 0-262-11191-8, 320 pp.

[17] Maier-Reimer, E. and K.Hasselmann (1987), Transport and storage of CO_2 in the ocean - an inorganic ocean-circulation carbon cycle model, Climate Dynamics, **2**, 63-90.

[18] Maier-Reimer, E. (1993), The biological pump in the greenhouse, Global and Planetary Climate Change, **8**, 13 -15.

[19] Manne, A.S. and R.G.Richels (1991), Global CO_2 emission reductions: The impacts of rising energy costs, The Energy Journal,**12**, 88-107.

[20] Manne, A.S. and R.G.Richels (1995), The greenhouse debate: Economic efficiency, burden sharing and hedging strategies, The Energy Joournal,**16**, 4.

[21] Michaelis,P. (1994) The cost of stabilising the atmospheric concentration of greenhouse gases, EU workshop on 'Designing economic policy for management and of natural resources and the environment, Sept. 7-9, 1994, Rethymon, Crete

[22] Mikolajewicz.U., B.Santer and E.Maier-Reimer (1990), Ocean response to greenhouse warming, Nature, **345**, 589-593.

[23] Nordhaus, W.D.(1991), To slow or not to slow: the economics of the greehnouse effect, Econ.J.**101**, 920 -937.

[24] Nordhaus, W.D.(1994), Expert opinion on climate change, American Scientist,**82**,45-51.

[25] Nordhaus, W.D.(1993), Rolling the 'DICE': An optimal transition path for controlling greenhouse gases, Resource and Energy Economics, **15**, 27 - 50.

[26] Nordhaus, W.D. and G.W.Yohe (1983) Future carbon dioxide emissions from fossil fuels, in **Changing Climate**, 87-153, National Research Council, National Academy Press, Washington.

[27] Nordhaus, W.D. and Z.Yang (1995) A regional dynamic general equilibrium model of optimal climate-change policy. Draft report, Yale University

[28] Peck, S.C. and C.J.Teisberg (1992) CETA: a model for carbon emissions trajectory assessment, Energy Journal **13** 55-77.

[29] Press,W.H., B.P.Flannery, S.A.Teukolsky and W.T.Vetterling (1986) Numerical Recipes, Camb.Univ.Press, 818 pp.

[30] Reilly,J.M, J.A.Edmonds, R.H.Gardner and A.L.Brenkert (1987), Uncertainty analysis of the IEA/ORAU CO_2 emissions model, The Energy Journal, **8**, 1-29.

[31] Richels, R.G. and J.Edmonds (1995), The economics of stabilizing CO_2 emissions, Energy Policy, **23**, 4/5

[32] Rotmans, J, H.de Boois and R.Swart (1990) An integrated model for the assessment of the greenhouse effect: the Dutch approach, Climate Change **16**, 331-335.

[33] Sarmiento, J.L., J.C.Orr and U.Siegenthaler (1992) A perturbation simulation of CO_2 uptake in an ocean general circulation model, J.Geophys.Res. **97**, 3621-3624

[34] Stehr, N. and H. von Storch, 1995: The social construct of climate and climate change. Clim. Res. it 5, 99-110.

[35] Tahvonen O., H. von Storch and J. von Storch, 1994: Economic efficiency of CO2 reduction programs, Clim. Res. **4**, 127-141

[36] Tahvonen O., H. von Storch and J. von Storch, 1995: Atmospheric CO2 accumulation and problems in dynamically efficient emission abatement. G. Boero and Z.A. Silbertston (ed.): Environmental Economics. St. Martin's Press ISBN 0-312-12579-8, 234-265

[37] Turner, R.K., 1995: An ecological economics approach to coastal zone management. Proc. of Changing Estuarine and Coastal Environments. GKSS Research Centre Geesthacht, 8-12 October 1995, 122-141

[38] Wigley,T.M.L., R.Richels and J.A.Edmonds (1996) Economic and environmental choices in the stabilization of atmospheric CO_2 emissions (1996), Nature, **379**, 240-243.