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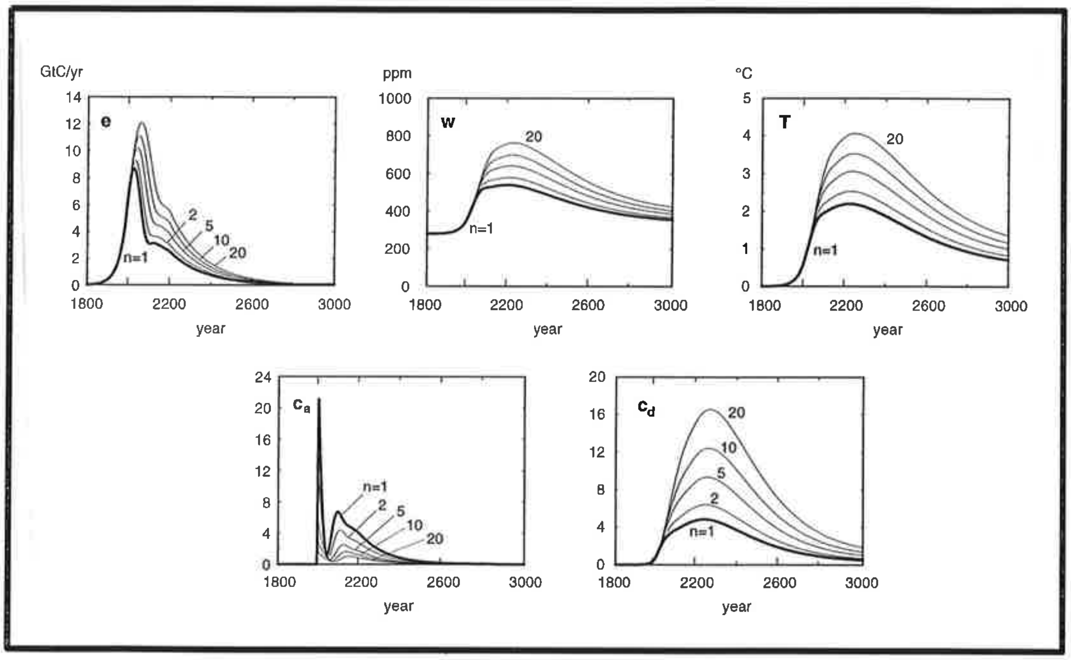

MULTI-ACTOR OPTIMIZATION OF GREENHOUSE GAS EMISSION PATHS USING COUPLED INTEGRAL CLIMATE RESPONSE AND ECONOMIC MODELS

by

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Multi-actor optimization of greenhouse gas emission paths using coupled integral climate response and economic models

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ABSTRACT

The simple Structural Integrated Assessment Model (SIAM) applied previously to optimization studies for the abatement of CO_2 emissions for an integrated global economy with a single decision maker is extended to the multi-actor case. The general non-cooperative multi-actor optimization problem is formulated both for the case of non-trading actors, in which the coupling between actors is limited to the jointly modified global climate, and for trading actors, where the control variables of individual actors also directly affect the welfare of other actors. Numerical examples are presented for both classes of interaction. In the non-trading case, the optimal CO_2 emission paths are less affected by the change from a single-actor cooperative strategy to a non-cooperative multi-actor strategy than may have been anticipated intuitively. For a modest number of identical actors (5 - 20), the abatement of the individually optimized emission paths is generally weakened but is of the same order as in the single-actor case. For the case of a single actor undertaking mitigation measures in the presence of $(n-1)$ actors pursuing a 'business-as-usual' policy, the lone mitigator even enhances his (or her) abatement measures for $n < 10$. To illustrate the impact of trade, a two-actor fossil fuel supplier-consumer model is considered. The conflicting goals of the two actors can lead to an effective neutralization of the consumer's attempts to mitigate climate change through reduced fossil fuel consumption by the supplier, who has a motivation to stimulate consumption by reducing the fossil fuel price.

1 Introduction

A major challenge facing mankind today is the development of an effective climate protection strategy to avert or mitigate a major global climate warming. There is a broad scientific consensus that an unimpeded increase in greenhouse-gas emissions at current growth rates will produce a global mean temperature increase within the next century of the order of $3°C$, leading to a climate regime beyond the historical experience of mankind (cf. reports of the Intergovernmental Panel on Climate Change, IPCC, 1990, 1992, 1994, 1995). If emission levels continue to rise, climate model projections predict a still more drastic gobal warming in the following centuries of at least twice this magnitude (cf. Cline, 1992, Hasselmann *et al*, 1996 – referred to in the following as HHGOS).

Since it will be impossible to completely avoid greenhouse warming – indeed, an estimated global warming of the order of $0.5-1$°C has already taken place today – an optimal climate-protection strategy must seek to minimize the net impact of climate change by minimizing the total costs incurred through the abatement of greenhouse-gas emissions and the adaptation to the residual climate warming. 'Costs' are interpreted here in the general sense of 'welfare loss', including both direct economic costs and quality-of-life factors which, although difficult to quantify economically, must nevertheless be considered in political value judgements and trade-off decisions.

The rational determination of an optimal greenhouse-gas emission strategy re-

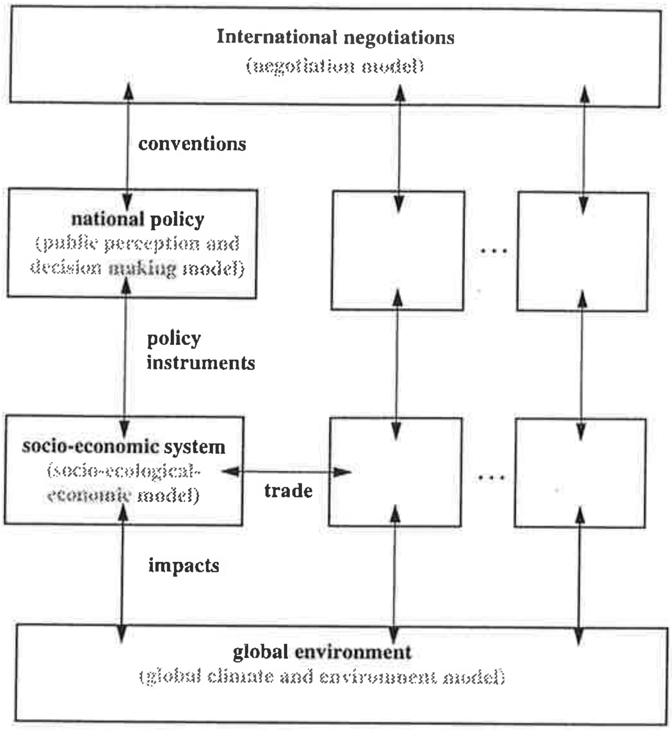

Figure 1: Interactions and sub-systems of an integrated Global Environment and Society (GES) model, as in Figure 1, HHGOS, but broken down into separate interacting actors.

It describes the same set of interactions between the climate system, the socio-economic system and the policy making sub-systems depicted for the single-actor case in Figure 1 of HHGOS (see also Hasselmann, 1991), but broken down now into individual columns representing different political-economic regions or sectors. The disaggregation can refer to either different geographical regions or different sectors within the economy (e.g. consumers and suppliers of fossil fuels, or private and government sectors).

Different actors interact through trade and negotiations, and are coupled through global climate change, to which all contribute, but which affects each actor differently. Each actor seeks to optimize his individual welfare function. Whether the outcome is a non-cooperative Nash equilibrium (assuming such an equilibrium exists) or an optimal cooperative solution is an open question and depends on the type of interactions between the various sub-systems, the negotiation rules (possibilities of retribution, reward, etc) and the negotiation strategies of the individual actors. We will consider various alternatives.

As basic building blocks for our multi-actor GES model we use the climate impulse-response and structurally simplified economic-cost modules of the single-actor SIAM model developed in HHGOS. Since realistic climate models based on coupled ocean-atmosphere general circulation models (CGCMs) (cf. IPCC, 1992, for the climate damage costs, composed of a term which depends quadratically on the change in temperature and a similar second term which depends quadratically on the rate of change of temperature; and a structurally highly simplified abatement costs expression. The costs incurred at any given time (specific costs) are integrated over all time, from the present to infinity, to determine the total costs. Future specific costs are discounted at rates which may be chosen differently for climate damage and abatement costs.

In the following we summarize the principal features of the SIAM model, together with the main conclusions of the single-actor HHGOS study relevant for the present n-actor investigation. A more detailed description is given in HHGOS.

The general impulse-response climate model

The climate module of the SIAM model is based on the principle that, although the climate system and sophisticated climate models are inherently strongly nonlinear, the response of the climate system, as of any differentiable nonlinear system, to small external forcing is to first order linear. As external greenhouse forcing the model considers only the annual emissions $e(t)$ of CO_2. This represents about 60% of the total anthropogenic greenhouse forcing today, and – in the absence of abatement measures – is projected to contribute a larger fraction in the future.

In the linearized approximation, the change $\mathbf{x}(t)$ of the climate state relative to an initial state at some time t_0 (which we take as some time in the pre-industrial period) to an arbitrary, sufficiently small emission function $e(t)$ can be represented in the general integral form

$$\mathbf{x}(t) = \int_{t_0}^{t} \mathbf{R}(t-t')e(t')dt', \tag{1}$$

where the climate impulse-response function $\mathbf{R}(t-t')$ represents the climate response at time t to a unit δ-function emission at time t' and the climate state consists, in a discretized model representation, of the vector of all climate variables at all model gridpoints.

Since a doubling of the CO_2 concentration corresponds to an increase in radiative forcing of about $4W/m^2$, or little more than 1% of the global mean incident solar radiation of $340W/m^2$, the linear form is adequate for most applications concerned with optimal emission scenarios which lead to an acceptable stabilized climate. However, for extreme climate change scenarios, corresponding to unregulated or only weakly regulated CO_2 emissions, the linear approximation is no longer applicable and can be used only as a rough guide (after application of first-order nonlinear correction factors) to the order of magnitude of the predicted climate change. Since we shall be concerned mainly with optimized emission scenarios which avoid unsustainable climate change regimes, the limitations of the linearization approximation will not be serious.

We note that the dimension of $\mathbf{R}(t)$ in eq.(1) is the same as that of $\mathbf{x}(t)$: the linearized form (1) implies no loss of information in the representation of the climate state relative to the complete nonlinear system, either in terms of geographical resolution or with regard to the set of climate variables (temperature, humidity,

$R_w(\infty)$ defines the fraction of the emissions which is retained in the atmosphere in the asymptotic equilibrium state. If the ocean sink alone is considered, the retention factor is approximately 15 %; if the dissolution of CO_2 in the upper layers of the ocean sediments is also included, the long-term atmospheric retention factor may fall (Maier-Reimer, 1993) to about 7 %. The increased storage of CO_2 in the terrestrial biosphere through CO_2 fertilization and the significantly slower loss of CO_2 through sedimentation in the ocean is not included in these estimates.

Invoking eq.(3), the time derivative of eq.(2) (which will be needed to couple the CO_2 model to the temperature response model defined below) is given by

$$\frac{dw}{dt} \equiv \dot{w}(t) = \int_{t_0}^{t} \dot{R}_w(t-t')e(t')dt' + e(t) \tag{4}$$

In an analysis of the response of a nonlinear three-dimensional global ocean carbon cycle model to various CO_2-emission levels, Maier-Raimer and Hasselmann (1987) found that the model response could be fitted to a linear relation of the form (1) quite well for an increase in the CO_2 level up to a factor of two. For stronger emission levels, producing a four-fold increase in the CO_2 concentration, the linear response underestimated the atmospheric concentration predicted by the full model by about 30%. This was due primarily to the nonlinear decrease of the solubility of CO_2 in sea water with increasing CO_2 concentration. A relatively simple nonlinear extension of the linear response form to allow for the nonlinearities associated with the solution of CO_2 in sea-water has recently been proposed by Joos *et al* (1995).

2. A *global temperature response model*

This describes the change $T(t)$ of the global mean temperature induced by the rate of change $\dot{w}$ in the CO_2 concentration,

$$T(t) = \int_{t_0}^{t} R_T(t-t')\dot{w}(t')dt', \tag{5}$$

where the temperature impulse response function $R_T(t-t')$ represents the change in the global mean temperature produced at time t by a unit step-function increase in the atmospheric CO_2 concentration at time t'.

Since the climate system has inertia, the instantaneous climate response to a sudden change in CO_2 concentration is zero,

$$R_T(0) = 0. \tag{6}$$

At the other end of the time scale, $R_T(\infty)$ represents the asymptotic equilibrium response of the ocean-atmosphere system to a unit increase of the atmospheric CO_2 concentration.

The generalization of this simple one-parameter climate model to more complex climate-state models, including, for example, regional temperature distributions, modified precipitation patterns and sea level change is basically straightforward. Such models could be readily constructed, in accordance with the general form (1), from existing data generated by CGCM climate-response simulations. However, for in accordance with the form (1), where

$$R(t) = R_T(t) + \int_0^t R_T(t-t')\dot{R}_w(t')dt'. \tag{10}$$

The net temperature impulse response or *global warming response* function $R(t)$ represents the temperature increase at time t due to a unit δ-function CO_2 input into the atmosphere at time $t = 0$. It reflects the net effect of both the thermal inertia of the ocean-atmosphere climate system and the decrease of the atmospheric CO_2 concentration through the transfer of CO_2 from the atmosphere to other components of the carbon cycle. ($R(t)$ should not be confused with the 'global warming potential' or 'commitment' defined in IPCC, 1990, as a measure of the time integrated radiative forcing.)

Numerical values

The response functions R_w and R_T have been determined empirically from numerical response experiments using realistic three-dimensional models of the global carbon cycle (Maier-Reimer and Hasselmann, 1987, Maier-Reimer, 1993) and the coupled ocean-atmosphere climate system (Hasselmann *et al*, 1993). A number of different models were intercompared in HHGOS. It was found that the differences had little impact on the computed optimal emission scenarios. We shall accordingly use only their baseline model R00 (cf.Fig. 2):

$$\begin{aligned} R_w &= 0.07 + 0.648\exp(-t/258.5) + 0.101\exp(-t/71.9) \\ &\quad + 0.097\exp(-t/17.6) + 0.084\exp(-t/1.6), \end{aligned} \tag{11}$$

$$\begin{aligned} R_T &= w_0^{-1}\{1.21(1-\exp(-t/2.1)) + 0.759(1-\exp(-t/12)) \\ &\quad + 0.531(1-\exp(-t/138.6))\} \\ &= w_0^{-1}R'_T. \end{aligned} \tag{12}$$

The normalized function $R'_T = w_0 R_T$ represents the temperature response to a CO_2 doubling relative to the pre-industrial CO_2 concentration w_0.

The response curves in Fig. 2 demonstrate (in accordance with the analytical expresssions (11), (12)) that the net climate response to a δ-function CO_2 emissions pulse cannot be characterized by a single time constant. Following a rapid temperature rise in the first few years as the upper mixed layer of the ocean warms, the net response function for the global mean temperature increases more slowly as the warming penetrates into the deeper ocean. After a few decades, the CO_2 transfer from the atmosphere into the ocean causes the temperature to gradually relax back, over a period of several hundred years, to an asymptotic equilibrium value of $2.5 \times 0.07 = 0.175°C$. For the optimization of greenhouse-gas emission paths, both the near-time and far-time climate response characteristics must be considered.

The cost function

In the SIAM model the impulse-response climate model is coupled with a simple economic costs module. This consists of the sum of the costs C_a incurred through

The integrals converge for the assumed infinite time horizon provided suitable discount factors are introduced.

Cross-coupling of the climate and emission variables in the cost expressions is not considered. Although a change in the socio-economic system induced by a change in emissions will presumably modify the sensitivity of the system to climate change, and a change in climate will conversely have some impact on the effectivity of abatement measures, these effects are ignored.

The first and second time derivatives $\dot{e}$ and $\ddot{e}$ of the emissions are included in the specific abatement-cost function in order to parametrize the effects of economic inertia.

As simplest mathematical expression which captures the principal properties of the abatement costs anticipated from a more detailed economic model the SIAM model sets

$$c_a = \left\{ (\frac{1}{r} - r)^2 + \tau_1^2 \dot{r}^2 + \tau_2^4 \ddot{r}^2 \right\} D_a(t) \tag{17}$$

where τ_1 and τ_2 are time constants, $r = e/e_A$, and

$$D_a(t) = \exp(-t/\tau_a) \tag{18}$$

is an abatement-cost discount factor, characterized by an abatement-cost discount time constant τ_a (inverse annual discount rate).

The first term in the form (17) has the property that any positive or negative departure from the reference BAU emission path e_A incurs costs which are quadratic in the deviations $\delta r = r - 1$ for small δr, $(\frac{1}{r} - r)^2 \approx 4(\delta r)^2$, and approach infinity for $r \rightarrow 0$ and $r \rightarrow \infty$. The second and third terms depending on the time derivatives of $e(t)$ penalize rapid changes in the emissions relative to the BAU emissions curve $e_A(t)$. They prevent the occurence of discontinuities in $e(t)$ and $\dot{e}(t)$. For $e_A(t)$, a simple linear growth form from a 1995 emission level of 6.3 GtC/yr up to a level of 38 GtC/yr in the year 2200 was assumed, with frozen emissions at this level thereafter (cf. Fig.3).

For the climate damage costs the SIAM model assumes

$$c_d = \left\{ \left(\frac{T}{T_c}\right)^2 + \left(\frac{\dot{T}}{\dot{T}_c}\right)^2 \right\} D_d(t) \tag{19}$$

where

$$D_d(t) = \exp(-t/\tau_d) \tag{20}$$

is the climate damage costs discount factor, with discount time constant τ_d, and T_c, $\dot{T}_c$ are constants. Climate damages are assumed to arise not only through a change in the temperature itself but also through the rate of change of temperature. The quadratic dependencies reflect the general view that climate damage costs increase nonlinearly with climate change and that costs are incurred through climate changes of either sign.

For the optimization problem, only the ratio of the cost functions is relevant. The freedom to choose an arbitrary normalization constant has been used to set the coefficient of the first term of the abatement cost function (17) equal to unity. This

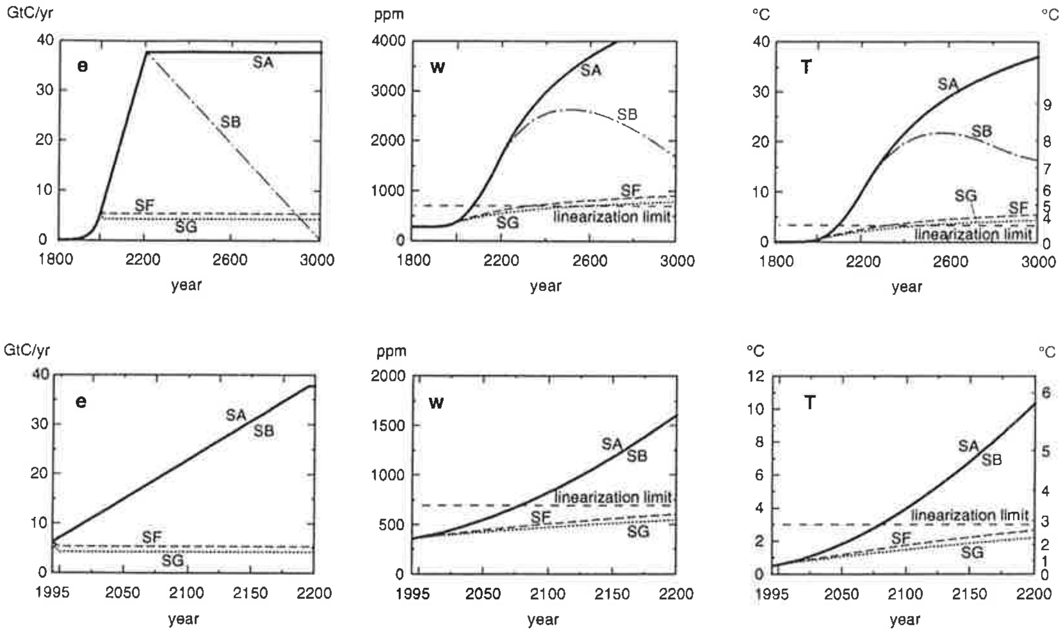

Figure 3: CO_2 emission paths, computed CO_2 concentrations and global warming (from left to right) for the time periods 1800-3000 (top) and 1995-2200 (bottom) for the BAU scenario (SA, full curves, cf. HHGOS), modified BAU scenario (SB, dashed-dotted curves), frozen emissions at 1990 levels after the year 2000 (SF, dashed curves) and 20% reduced emissions relative to the 1990 level after 2000 (SG, dotted curves). The linear model is not applicable above the indicated levels (dashed lines). The logarithmic T scales on the right ordinate axes of the panels on the right indicate the order-of-magnitude temperature response allowing for the logarithmic dependency of the radiative forcing on the CO_2 concentration. (Reproduced from HHGOS).

4. Optimal emission paths yielding acceptable global warming limits are obtained only if the discount rate for climate damages is set at a significantly lower level than the discount rate for abatement costs (cf. Fig. 4). Discounting climate damages at normal economic discount rates implies that negligible present value (in the sense of willingness to pay today) is attached to sustainable development over time scales of a few centuries, and that there is therefore no incentive to avoid long term global warming.

In the following sections we investigate the impact of multi-actor interactions on these results.

3 Non-trading multi-actor models

We consider first the simplest case in which the interdependence between actors occurs only through global climate change, with each actor seeking to optimize his (or her) individual welfare function independently of the actions of the other actors. The more general case of actor interdependence through both climate and trade,

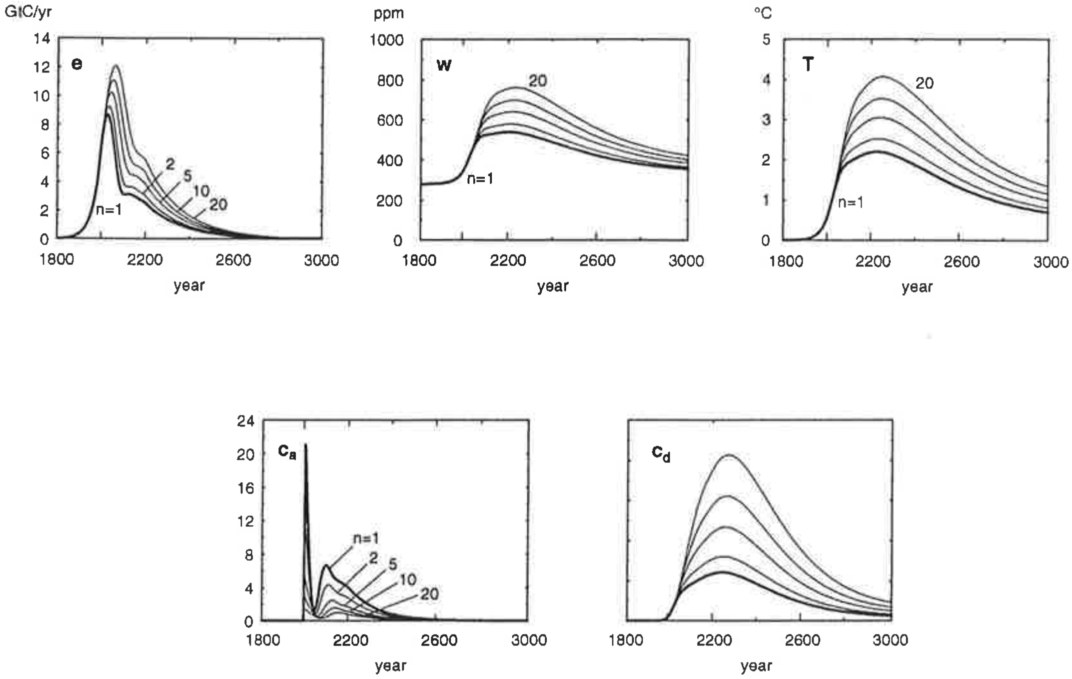

Figure 5: Optimal CO_2 emission paths, CO_2 concentrations and global warming (top, left to right) and specific *per capita* abatement and climate damage costs c_a, c_d, respectively, (bottom, left to right) for the n-identical actor Nash equilibrium solution (computed for the parameters of the baseline reduced-emissions scenario S0 of HHGOS, full curves, also shown in figure 4).

To determine the variation of the *per capita* climate damage costs $C_{di} = C_d(T, \dot{T}, t)$ induced by a variation δe_i, we need to determine the variation δT of the global mean temperature (and similarly $\delta\dot{T}$) induced by δe_i. Here the partial derivative $\partial e/\partial e_i = 1$ of the total emissions with respect to the emissions e_i is relevant, as given by the left hand part of eq. (23):

$$\frac{\delta T}{\delta e_i} = \frac{\delta T}{\delta e}\frac{\partial e}{\partial e_i} = \frac{\delta T}{\delta e} \tag{25}$$

The net result of both transformations is that in the expression for the gradient with respect to e_i of the net *per capita* costs, the ratio of the *per capita* climate damage to abatement costs is reduced by a factor n^{-1} relative to the single-actor case. Thus the Nash equilibrium for the case of n identical non-cooperating actors is given by the solution of the single-actor problem with the climate damage costs reduced by a factor n^{-1}. In effect, each actor considers only his own contribution to the climate damage costs and thus acts as though the climate damage costs are a factor n^{-1} smaller than they in fact are when summed over all actors. In the limit of a large number of actors, one obtains the free-rider solution: no one carries out abatement measures.

However, for a moderate number of actors, of the order of 10 or 20, the non-cooperative Nash equilibrium emission paths do not differ as strongly from the co-

For the $(n-1)$ BAU actors with prescribed emissions $e_A(t)/n$, the distinction between different actors is irrelevant. We may thus replace the set of $(n-1)$ BAU actors simply by a single 'rest of the world' BAU actor whose emissions are given by $[(n-1)/n]e_A(t)$. However, to retain an analagous notation to the previous example, we shall continue to refer to '$(n-1)$ BAU actors'.

Regarding costs again as *per capita* costs, the expression for the abatement costs for actor 1 remains the same as in the global single-actor case (eq.(17)), with the global emission abatement factor $r = e/e_A$ replaced by the individual abatement factor $r_1 = e_1/e_{1A} = ne_1/e_A$ of actor 1. However, the climate damage costs are now dominated by the prescribed emissions of the other $(n-1)$ BAU actors.

The total emissions are given by

$$e = \frac{n-1}{n} e_A + e_1. \tag{26}$$

Since the climate change in our model depends linearly on emissions, the global mean temperature T and rate of change of temperature $\dot{T}$, which determine the climate damage costs, are given by

$$\begin{aligned} T &= \frac{n-1}{n} T_A + T_1, \\ \dot{T} &= \frac{n-1}{n} \dot{T}_A + \dot{T}_1 \end{aligned} \tag{27}$$

where T_A and T_1 represent the climate response to the emissions e_A and e_1, respectively, as defined by eqs. (9, 10). Substitution of (27) into (19) defines the damage costs as a function of $e_1(t)$, for given $e_A(t)$. Adding to these the abatement costs of actor 1, which are also defined as a function of e_1, the total costs of actor 1, given the BAU emissions of the remaining actors, are thus expressed in terms of the emissions of actor 1 and can be minimized with respect to $e_1(t)$.

The resulting optimal paths are depicted in Fig.6 for various values of n. If the number of non-cooperative BAU actors is less than 10, the single mitigator compensates for the lacking abatement policy of the other actors by enhancing his own abatement measures. However, as n increases beyond 10, it becomes more difficult for the single mitigator to influence the climate damage costs, and ultimately, for very large n, actor 1 resigns and also undertakes no abatement measures. However, the asymptotic solution is approached more slowly than may have been anticipated intuitively.

It is surprising also, in view of the relatively small contribution of the single mitigator to the total climate damages, that for $n < 10$ it is economical for him to enhance his abatement efforts. This is a consequence of the assumed nonlinear dependence of the climate damage costs on the climate change. The background temperature increase caused by the BAU actors amplifies the climate change impact of the single mitigator. This can be seen by separating the total climate damage costs into the contributions from the $(n-1)$ BAU actors and the residual contribution from the mitigator (including the nonlinear interaction term with the BAU actors).

According to eq.(27), the total climate damage costs (ignoring, for simplicity, incurred if all actors adopted the same emissions path as actor 1 in a cooperative scenario. The ratio

$$\frac{c_d^{(1)}}{c_d^{(n)}} \approx \frac{2(n-1)}{n^2}\frac{T_A}{nT_1} \tag{31}$$

contains a first factor $2(n-1)/n^2 \approx 2/n$, which becomes small for large n, as anticipated, and a second amplification factor $T_A/nT_1 = T_A/T_0$, where $T_0 = nT_1$ is the temperature change for the equivalent cooperative emissions path $e = ne_1$. For the baseline cooperative optimal emissions path, a typical value of T_A/T_0, averaged over the first part of the emissions path, is of order 5. Thus the cross-over point at which the additional damage costs incurred by a single mitigator become comparable to the total damage costs of the cooperative solution is near $n = 10$, as indeed found.

For policy makers this simple result has important implications. Although actors who regard climate change as a potential hazard which should be mitigated through appropriate abatement measures will naturally strive to achieve binding international agreements on joint actions, the frequently heard argument that there is no incentive for reducing emissions on an individual basis does not appear to be valid. Our model suggests that self-interest should motivate at least the larger industrial countries to undertake unilateral mitigation actions which are comparable to the measures they wish to realize ultimately in an international agreement. However, this conclusion is strongly dependent on the assumed nonlinear nature of the climate damages and could be modified if trading interactions are taken into account (e.g. problems of competition and leakage), as discussed in the following section.

4 Multi-actor models with trade

Although the non-trading multi-actor model discussed in the previous section is useful for investigating certain aspects of the multi-actor problem, it clearly represents an unrealistic simplification of the true situation. The coupling between different actors will in general be governed not only by changes in global climate, but also by trade. Moreover, in attempting to maximize his individual welfare expression, each actor will normally not ignore the actions of other actors. We consider again only the non-cooperative case, so that direct negotiations are excluded. Although not communicating directly, each actor will nevertheless endeavour to anticipate the response of other players to his own actions, and will generally develop a foresighted strategy accordingly.

To model the general case, we assume that there exists a general multi-actor economic model in which the actions of each player i are described by a set of control variables $v_{i\alpha}(t), \alpha = 1, 2, \ldots$ including, in addition to the CO_2 emissions, various other economic control factors such as the prices of commodities, tariffs or, in place of CO_2 emissions, a CO_2 tax or some other regulatory instrument. In the following, it will be convenient to discretize the time variable t and introduce the control vector $\mathbf{v}_i = (v_{i,\alpha,t})$ representing the set of paths of all control variables of actor i.

The individual welfare functions W_i which each actor i strives to maximize will depend generally not only on the individual control parameters $\mathbf{v}_i$ of that actor,

4.1 The Cournot-Nash equilibrium

In text-book treatments of coupled economic optimization problems (e.g. Binmore, 1992), the interactions between players are often regarded as single-shot games. Each player i specifies his (or her) control variables $\mathbf{v}_i$ once and for all. A player's choice is based on some assumption about the control variables of the other players, but the possible 'response' of the other players to his own choice is irrelevant, since the other players have no opportunity to respond, once they have chosen their control variables $\bar{\mathbf{v}}_i$. Thus the Nash equilibrium (also termed the Cournot-Nash equilibrium in this context) is given by the solution of the simultaneous set of equations

$$\frac{\partial W_i}{\partial \mathbf{v}_i} = 0, \quad i = 1, \ldots, n. \tag{36}$$

This approach corresponds to the examples discussed in the previous section and other recent analyses of the multi-actor problem (Tahvonen, 1993, Nordhaus and Yang, 1996). However, as pointed out, this model is in general not very realistic. For example, in the two-player fossil-fuel supplier-consumer model considered below, it will be found that there exists no Cournot-Nash equilibrium. In a time dependent interactive dynamic control problem, the actors clearly do have an opportunity to react to the choice of control variables of the other actors, and will adjust their control variables accordingly. Thus in chosing their control variables, they will anticipate the response of the other actors to the control strategy, and will optimize their strategy taking this response into account.

4.2 The self-consistent interactive Nash equilibrium

If the response of the other players is included in the optimization analysis, the simultaneous solutions of (36) no longer represent simultaneous local extrema (or turning points) of W_i with respect to the control variables $\mathbf{v}_i$. Although a variation $\delta\mathbf{v}_i$ induces no direct variation of W_i, since $(\partial W_i/\partial \mathbf{v}_i)\delta\mathbf{v}_i = 0$, it generates a variation $\delta\bar{\mathbf{v}}_i$ in the other control variables through the coupling relation (34). This produces an indirect variation in W_i through terms $(\partial W_i/\partial \mathbf{v}_j)\delta\mathbf{v}_j$. The necessary condition for a maximum of W_i, taking the response of the other actors into account, is therefore the stationarity condition

$$\frac{dW_i}{d\mathbf{v}_i} = \frac{\partial W_i}{\partial \mathbf{v}_i} + \sum_{j \neq i} \frac{\partial W_i}{\partial \mathbf{v}_j} \mathbf{M}_{ji} = 0. \tag{37}$$

Thus the Nash equilibrium in the interactive case is given by the solution of the set of equations (37), which must be solved simultaneously with the defining equations (35) for the response matrix. The two sets of equations are coupled and can normally be solved only iteratively: to determine the solutions of (37), one needs the response matrix, but the response matrix can be evaluated from (35) only when the solutions of (37) have been found.

4.3 The conjectured response Nash equilibrium

If the actors have no reliable information on the welfare functions of the other actors – or are not able to carry out the fully coupled optimization analysis for all actors

Nash equilibrium.

Intuitively, one may expect such straightforward iterative constructions to converge for both the conjectured response and the internally consistent Nash equilibrium solutions, provided the solutions exist. However, general conditions for the existence of Nash equilibrium solutions in these cases, and the convergence properties of appropriate numerical algorithms for their construction have not, to our knowledge, been studied.

Numerically, the derivation of internally consistent response matrices for the dynamical interactive optimization problem is not a simple exercise. The computation of the response matrix $\mathbf{M}_{ij}$ involves the determination of the second-derivative matrix $\partial^2 W_i/\partial \mathbf{v}_j \partial \mathbf{v}_i$ obtained by differentiating the partial gradient $\partial W_i/\partial \mathbf{v}_i$ of the welfare function W_i with respect to the set of control vectors $\mathbf{v}_j$, $j \neq i$, to be carried out in the high-dimensional space resulting from the discretization of the time axis. The computation must then be repeated many times to obtain internally consistent response matrices through the iterative adjustment of the conjectured response matrices to the response matrices computed from the resultant solutions.

In practice, however, the determination of the internally consistent interactive Nash equilibrium is probably an academic exercise. The construction of the solution – if it indeed exists – will be as elusive for the actors in a real situation as for the theoretical analyst. To define an iterative joint optimization algorithm (which may be regarded as a mathematical proxy of the real joint adjustment procedure, which also may or may not converge), it is sufficient that each actor has a model of the response of the other actors to his own strategy, regardless of whether or not the model corresponds to the actual response computed *a posteriori* for the resultant jointly optimized solution. The real situation is probably best studied by numerical experiments, for example, by considering the sensitivity of the mutual optimization exercise with respect to the conjectured marginal response matrices.

5 Two-actor interactions between fossil-fuel suppliers and consumers

A realistic multi-actor game theoretical model of global climate-protection strategies must consider not only different climate damage and abatement cost functions for different fossil fuel consumers, but also the different roles of fossil fuel suppliers and consumers. Strategies of fuel consumers for reducing climate damage costs by reducing emissions can well be counteracted by fuel suppliers, who may respond by reducing fuel prices to enhance fuel consumption (cf. Blank and Ströbele, 1994, Richels *et al*, 1996). As simple but illustrative example of an interactive multi-player model we consider a two-player system consisting of a single world fossil fuel producer and a single world fossil fuel consumer. Thus we assume that all economic actors generating greenhouse gas emissions agree on a joint optimal emission-path strategy and act as a single player, while all fossil-fuel producing actors form a cartel (e.g. OPEC), also acting as a single player.

To describe the interactions between fossil fuel suppliers and consumers we need to introduce as control variable, in addition to the consumption of fossil fuel (exsome assessment of the impact of marginal changes in his emission path on marginal changes in the price path of the fuel supplier. Thus the appropriate definition of the Nash equilibrium in this case is the interactive Nash equilibrium, based on the conjectured or internally consistent interaction response matrices. We shall consider only the simpler case of the conjectured response Nash equilibrium.

Assume, for example, that the fossil fuel supplier 1 anticipates that the response $\delta e(t)$ of the fuel consumer 2 to a change $\delta p(t)$ in fuel price can be represented by an instantaneous marginal response coefficient γ,

$$\delta p(t) \rightarrow \delta e(t) = -\gamma\, \delta p(t). \tag{40}$$

The form (40) corresponds to a diagonal response matrix $M_{2,s;1,t} = -\gamma\delta_{ts}$ with respect to the suppressed time indices t, s in eq.(35), where δ_{ts} denotes the Kroneker symbol. The marginal response coefficient γ has the dimension [emissions/price] and can be a function of time, $\gamma = \gamma(t)$, but is assumed otherwise to be independent of p and e. (It is more convenient for the present example to assume a state-independent response coefficient γ rather than a constant elasticity coefficient $p\delta e/e\delta p$.)

The maximization of the fuel supplier's earnings, eq.(38), yields then the condition

$$\begin{aligned} \delta W_1 &= \sum_t \alpha \left\{\delta p(t)e(t) + \delta e(t)(p(t) - p_e(t))\right\} D_a(t)\Delta t \\ &= \sum_t \alpha\delta p(t) \left\{e(t) - \gamma(p(t) - p_e(t))\right\} D_a(t)\Delta t = 0, \end{aligned} \tag{41}$$

or

$$e(t) - \gamma(p(t) - p_e(t)) = 0, \tag{42}$$

so that

$$p(t) = p_e(t) + e(t)/\gamma. \tag{43}$$

Thus the fossil-fuel supplier can immediately determine his maximal-earnings price from eq. (43), given the level of emissions (fuel consumption) $e(t)$ of the fuel consumer. His earnings for this optimal price is then given by (cf. eqs (38),(43))

$$W_1 = \sum_t \alpha\gamma^{-1}e^2 D_a \Delta t, \tag{44}$$

or, expressed in terms of non-dimensional variables,

$$W_1 = \sum_t \beta r^2 D_a \Delta t, \tag{45}$$

where $r = e/e_A$ and

$$\beta = \alpha\gamma^{-1}e_A^2 \tag{46}$$

The coefficients α and γ occur in (44) and in all following dimensionless relations only in the non-dimensional combination $\alpha\gamma^{-1}e_A^2 = \beta$. In the numerical examples presented below we shall therefore assume that the dimensional parameters γ and α scale with the BAU emissions (the only relevant externally prescribed dimensional variable) such the non-dimensional parameter β remains constant.

supplier-consumer problem, the emissions are controlled by only one of two actors, the fossil fuel consumer, who will chose the BAU emission path e_A to maximize his own welfare $W_2 = W - W_1$, rather than the total welfare W. If we retain the same expression (17) for the abatement costs as in the single-actor case, W_2 will not be maximized for $e = e_A$, since for this path $\delta W_2/\delta e = -\delta W_1/\delta e = -2\beta e_A^{-1} D_a \Delta t \neq 0$ (cf. eq. 45).

Through the presence of the additional fuel cost term W_1 in the welfare expression of the fuel consumer, the optimal emissions path is depressed relative to the single-actor BAU path $e_A(t)$ in the unregulated emissions case. If we wish to recover the original single-actor BAU path also for the two-actor case, this depression factor must be appropriately neutralized. This can be achieved, for example, by rescaling the ratio $r = e/e_A$, $r \rightarrow \tilde{r} = \lambda r$, in the abatement costs expression (17). The rescaling factor $\lambda\,(< 1)$ can be chosen such that one recovers $r = 1$ as the optimal solution for the BAU case.

Introducing the scaling factor λ into the climate damage costs, the net costs for actor 2 in the unregulated case, i.e. without climate damage costs, are given by (cf. eqs. (17),(45))

$$C_2 = C_a(\lambda r) + W_1 = \sum_t \left\{ \left(\frac{1}{\lambda r} - \lambda r \right)^2 + \tau_1^2 (\lambda \dot{r})^2 + \tau_2^4 (\lambda \ddot{r})^2 + \beta r^2 \right\} D_a \Delta t \quad (52)$$

The necessary condition for the minimization of C_2 is accordingly (noting that the first and second time derivative terms of r do not contribute to the gradient if λ is adjusted such that $r = 1$, i.e. $\dot{r} = \ddot{r} = 0$, for the optimal solution)

$$\frac{\delta C_2}{\delta r} = 2\lambda \left(\lambda r - \frac{1}{\lambda r} \right) \left(1 + \left(\frac{1}{\lambda r} \right)^2 \right) + 2\beta r = 0, \quad (53)$$

or

$$(\lambda r)^4 - 1 + \lambda^2 r^4 \beta = 0, \quad (54)$$

which yields

$$r = \left(\lambda^4 + \beta \lambda^2 \right)^{-1/4}. \quad (55)$$

Thus to recover the BAU solution $r = 1$ we must set

$$\lambda^4 + \beta \lambda^2 = 1, \quad (56)$$

or (taking the relevant positive real root)

$$\lambda = \left\{ -\frac{\beta}{2} + \left(\frac{\beta^2}{4} + 1 \right)^{1/2} \right\}^{1/2} \quad (57)$$

With this modification, the welfare of the fossil fuel consumer is given by

$$W_2 = W_A - \tilde{C}_a - C_d - W_1, \quad (58)$$

fossil fuel suppliers respond to reductions in fossil fuel use with large decreases in the fossil fuel price. This stimulates consumption, thereby counteracting the abatement measures.

The impact of the fuel costs on the computed optimal emission paths is seen to increase with β, but at a less than linear rate. This is an artifact of our side condition that in the absence of climate damage costs, the BAU optimal emission curve $e_A(t)$ should always be the same, independent of the interaction parameter β. Our adjustment of the abatement cost expression to satisfy this condition (eqs. (52) - (62)) has the effect that, as β is increased, the abatement costs grow more rapidly with decreasing emission reduction factor $r = e/e_A$. Thus the economic system becomes 'stiffer', and the costs of counteracting the negative effects of climate change by reducing emissions are increased. This cross-coupling of two opposing effects would presumably be avoided in a more realistic economic model, including a specific description not only of the costs incurred through a deviation from the BAU path, as in the present analysis, but also of the basic economics determining the reference economic BAU path itself.

Specific implications for policy can clearly be derived from interactive optimization analyses of the type presented here only if they are based on more realistic economic models than we have used. However, we anticipate that the general qualitative conclusions of the present study, as well as the general methodological approach, will carry over to more detailed quantitative models.

6 Conclusions

The implementation of a global climate protection strategy with optimized emissions of CO_2 and other greenhouse gases in the real world of many interacting, interdependent decision makers with diverse interests and different assessments of climate change impact is a complex multi-actor problem. The optimization problem has been investigated with quantitative models so far only in two limiting cases, both of which are rather far from reality: full cooperation, assuming an agreement has been reached on joint mitigation goals, for which the problem reduces to the single-actor case; and the fully non-cooperative n-actor problem, which ignores all negotiatory aspects, including the various options of forming partial alliances. Despite these limitations, the two limiting cases are useful in identifying basic features of the problem and defining a space of possibilities which may span some of the key conclusions of more realistic multi-actor models.

The principal results of single-actor optimization computations, using the same basic model as in this study, have been summarized in HHGOS and need not be repeated here. The main conclusions of the present n-actor investigation is that, while non-cooperative strategies generally yield reduced abatement measures and a stronger global warming, the impact of non-cooperative optimization on the individual emission paths is smaller than may have been anticipated intuitively. In particular, there is no reason, based on our model simulations, to postpone mitigation measures until an international climate protection agreement has been achieved; individual emission abatement, although in a modified form, is cost effective also in licher Beirat Globale Umweltveränderungen (German Advisory Council on Global Change).

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