Some Remarks on the Impact of Computers on Mathematics and Physics *

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Abstract

A few programmable computing machines existed already in the 19-thirties and forties. Most of them were unique, constructed in the institutes of their users. Around 1955, larger numbers of commercially produced computers became available. In 1957, the German Science Foundation (DFG) bought three IBM-650 machines, one of them for the institute of Prof Collatz at University of Hamburg.

As soon as there was a larger number of scientists with computing experiences, in the years 1968-1974, researchers working in different fields of mathematics and physics met at international conferences with titles like ‘Computers in Mathematical Research’ (1968) or ‘The Impact of Computers on Physics’ (1972). There they told each other how much computers had already changed their work.

In this contribution, some of the experiences, observations and lines of thought will be retraced which accompanied this development or commented on it later on: expectations for the future and strategies of science policy in the thirties to sixties; reports on the already changed and still changing conditions of research work from the seventies, and retrospective comments from later years. Today, the words of the mathematician and computer scientist George E. Forsythe (1968) are still valid: “the question ‘What can be automated?’ is one of the most inspiring philosophical and practical questions of contemporary civilization.” It is a political question as well: *Should everything that can be automated actually get automated?* Which of the technologically possible developments are desirable, which ones should we try to block?

Zusammenfassung: Einzelne programmierbare Rechenmaschinen gab es schon in den 1930-er und 40-er Jahren. Sie waren zumeist Eigenbau der Institute ihrer Benutzer und Unikate. Ab etwa 1955 standen dann auf dem internationalen Markt kommerzielle Rechner in grösseren Stückzahlen zur Verfügung. Im Jahr 1957 kaufte die DFG drei IBM-650 Maschinen, eine davon für das Institut von Prof Collatz an der Universität Hamburg.

*slightly revised version of pp. 322–347 in: Wolfschmidt, Gudrun (ed.): *Vom Abakus zum Computer - Geschichte der Rechentechnik*, Begleitbuch zur Ausstellung in Hamburg, Hamburg: tredition 2019 (Nuncius Hamburgensis, Volume 21)

Sobald es dann einige Jahre später eine grössere Anzahl von Wissenschaftlern mit Rechner-Erfahrung gab, in den Jahren 1968-1974, bestand offenbar ein grosses Bedürfnis nach Diskussion, Erfahrungsaustausch und dem Ziehen erster Bilanzen: sie trafen sich auf internationalen Tagungen zu Themen wie 'Computers in Mathematical Research' (1968) und 'The Impact of Computers on Physics' (1972).

In diesem Beitrag sollen einige der Erfahrungen, Beobachtungen und Gedankengänge nachgezeichnet werden, die diese Entwicklung begleitet oder kommentiert haben: Hoffnungen und wissenschaftspolitische Erwägungen aus den 40-er bis 60-er Jahren, Berichte über den veränderten Arbeitsalltag aus den Jahren um 1970 und rückblickende Kommentare aus späteren Jahren. Auch heute noch ist die Bemerkung des Mathematikers und Informatikers George E. Forsythe (1968) aktuell: "the question 'What can be automated?' is one of the most inspiring philosophical and practical questions of contemporary civilization." Es ist aber auch eine politische Frage: *Wollen wir, daß alles automatisiert wird, das automatisiert werden kann?* Welche technologisch möglichen Entwicklungen sind wünschenswert, welche sollten wir zu blockieren versuchen?

1 Changes of Mathematics and Physics in History

All living scientific disciplines change in time in some way or another. It seems that many researchers felt in the years around 1970 that they were experiencing a time period of profound changes of their field. We start with some words of two eminent scientists on changes of mathematics and physics in general. Both, however, did not mention computers in these words.

Christoph J. Scriba (1929-2013), whom we honored on September 13, 2015 with a Memorial Meeting at University of Hamburg[1], discussed in 1968 in a survey article on *History of Mathematics*[2] several profound changes of mathematics and of the teaching of mathematics in the past, and he stressed that the education of young mathematicians should always change accordingly without much delay. Among other things, he wrote: 'In mathematics, it is extremely rare that known results are shown to be false. New results are added, and theorems are generalized quite often. Nevertheless, the growth of mathematical knowledge is not like constructing a house, where the foundations are laid first very carefully, and then floor by floor, structural level by structural level are added. Progress, evolution of mathematics always also means rearrangement of the whole construction, encluding its foundations, it means partial demolishment and reconstruction, incessant redecoration, replacement of out-dated furniture by new, maybe fashionable one which will also become out-dated. - Maybe it will be one of the special tasks of history of mathematics during the coming decades to study the interfaces where different cultures or epochs met each other. That could help to understand the processes of imparting and developing mathematics under strongly changing exterior conditions.' In his own words:

> Das Wachsen mathematischer Erkenntnis [...] ist nicht vergleichbar mit dem Bau eines Hauses, dessen Fundamente man zuerst sorgfältig im

[1] Wolfschmidt 2016 [52]
[2] Scriba 1968 [47]

> Boden verankert, um dann Stockwerk um Stockwerk darauf zu errichten [...] Fortschritt, Weiterentwicklung der Mathematik bedeutet immer auch Umgestaltung des Gesamtbaues bis in seine Grundlagen hinein, bedeutet teilweises Einreißen und Neuerrichten, bedeutet ständiges Renovieren, bedeutet Abstoßen veralteten Mobiliars zugunsten neuer - darunter auch modischer - Inneneinrichtung, die selbst wieder veralten wird.[3]
>
> Vielleicht gilt sogar überhaupt, daß es in den kommenden Jahrzehnten eine der besonderen Aufgaben der Mathematikgeschichte sein wird, die Nahtstellen zu betrachten, wo sich verschiedene Kulturen oder Epochen begegneten. Das könnte beitragen zur Klärung der Vorgänge, die sich bei der Vermittlung und Weiterbildung der Mathematik abspielen, wenn sie stark veränderten äußeren Bedingungen unterworfen wird.[4]

The Swedish plasma physicist and electrical engineer *Hannes Alfvén (1908 - 1995)* started his *Nobel Lecture* in 1970 with the words:

> The center of gravity of the physical sciences is always moving. Every new discovery displaces the interest and the emphasis. Equally important is that new technological developments open new fields for scientific investigation. To a considerable extent the way science takes depends on the construction of new instruments as is evident from the history of science.[5]

In his lecture Alfvén spoke about the impact of several new instruments on his field of science, but he did not mention computers. Nevertheless, he was very convinced at that time that computers would deeply affect *all of society*.[6]

In this text we shall consider the reaction of mathematicians and physicists to their new instrument computer during the first decades of the epoch of computers and automation, at a time when most people definitely did not have such far reaching visions of the impact of computers on society as Forsythe and Alfvén had, see section 4. This is a revised and substantially enlarged version of a 4-page paper.[7]

2 Early Computing Machines and Computations

2.1 Future importance of computers for mathematics? - 1946

In the year 1946 Princeton University celebrated its bicentennial existence[8] with several conferences. At the conference *Problems of Mathematics*[9] two eminent mathematicians

[3] Scriba 1968, p.11
[4] Scriba 1968, p.15
[5] Alfvén 1970, [1]
[6] Johannesson 1966, [28]
[7] Meyer-Spasche 2006, [36]
[8] Princeton University and Universität Göttingen were founded by the same *person*: Georg August, Kurfürst von Braunschweig/Lüneburg (Hannover), George II, King of England (*1683 in Hannover, +1760 in London).
[9] Bicentennial Conference 1947, [43]

spoke of the future importance of computers for mathematics, as J.G. Charney (1917-1981) reported in 1972:

> Hermann Weyl expressed concern that the store of mathematical substance which formed the basis for current generalizations was in danger of becoming exhausted without outside help, “be it even by such devilish devices as high-speed computing machines”. And John von Neumann remarked that the success of mathematics with the linear differential equations of electrodynamics and quantum mechanics had concealed its failure with the nonlinear differential equations of hydrodynamics, elasticity and general relativity. He expressed the hope that the computer-aided solution of a large store of problems in nonlinear continuum mechanics would indeed supply a basis for mathematical generalization.
>
> To him meteorology was *par excellence* the applied branch of mathematics and physics that stood the most to gain from high-speed computation. Earlier that year he had called a conference of meteorologists to tell them about the general-purpose electronic computer he was building at the Institute for Advanced Study and to seek their advice and assistance in designing meteorological programs for its use. The author [Charney] had the good fortune to attend and recalls that the response from the established figures was interested but less than enthusiastic.

It was not clear in those days which equations should be solved, and available observations, experiment and analysis were not sufficient for understanding fundamental atmospheric processes. It became clear at this conference that further theoretical work was necessary before computers could be used successfully.[10]

2.2 Some of the early computing machines

In 1946, some experience with analogue and digital calculating/computing machines was already available. We give here a short, very incomplete list of machines, mostly for reminding the reader which type of computer was available when. We focus on those machines which are often mentioned in the literature on history of numerical analysis. Many more details may be found in many other sources[11], especially in the exhibition *Vom Abakus zum Computer - Geschichte der Rechentechnik* which opened on 2015-05-13, the day of the Scriba Colloquium, in the Hamburg *Landesinstitut für Lehrerbildung und Schulentwicklung*, and in the accompanying book.[12] Many informations about mechanical calculating devices may be found in the *Rechnerlexikon*[13].

The announced list of early computing machines:
* the *differential analyser*, 1933+, was a mechanical analogue computer, first built by Vannevar Bush et al at MIT and then by Douglas Rayner Hartree[14] at University of

[10] J.G. Charney: Impact of computers on meteorology, p. 117 In: MacLeod 1972, [34]
[11] e.g. Oberquelle 2008, [42]; GWDG 2010, [24]; Hashagen 2010, [27]
[12] Wolfschmidt 2017, [53].
[13] Rechnerlexikon, [44]
[14] Froese Fischer 2003, chap.8, [20]; Buneman 1990, [10]

Figure 1: Prof Collatz (second from the right) with his group and their first digital computer, an IBM 650, at the computing center of Hamburg university (early in 1958). The leftmost person is Dr. J. Albrecht who taught the first programming courses at Hamburg university, starting summer semester 1958.

Manchester;
* the Zuse machines Z1 1936; Z2 1939; Z3 1941; and finally 1945 the Z4, which was used for computations at ETH Zürich[15] in the years 1949-1954;
* IBM ASCC 1944 , ENIAC 1946 - 1955;[16]
* G1 1952; G2 1954 ; G3 1961; constructed by Heinz Billing (*1914) et al;[17]
These were all individual, mostly unique machines, existing at most in very few copies.

Around 1950 industrial mass production of differently sized computers started, first with several, then with many copies, for instance
* IBM 650, at the time a ‘medium sized computer’, was available in the US since 1954, with 750 copies in 1957; in 1958 copies of it were installed at several European universities: Hamburg, Göttingen, Darmstadt, Hannover, Wien, ... More details about the development in other European countries were given by Brezinski and Wuytack in their introduction to their book[18];
* Big computers of the time were IBM 701, 702, 704, 705 and RR Univac (9, 11, 72, 73

[15] Stiefel 1955, [48]; Gutknecht 1990 [23]
[16] von Neumann 1955, [41]; Aspray 1989, [3]
[17] GWDG 2010, pp. 77ff, [24]
[18] Brezinski, Wuytack 2001, pp.1-40, [7]

and 37 copies of them, resp., in USA in 1958).[19]

Then numbers grew very fast. The following numbers of machines sold by the following companies were already installed in Germany on 1968-Juli-01: 2509 IBM, 429 BULL/GE, 378 UNIVAC, 291 SIEMENS, 240 ZUSE, ..., 55 TELEFUNKEN.[20] At the same time, there were more that 40 000 computers in the USA[21].

Figure 1 shows *Prof Lothar Collatz (1910 - 1990)* with his group and part of their new computer IBM 650.[22] A fotograph of the complete machine may be found, for instance, in the article by de Beauclair[23].

In those years, there was no general agreement on how these machines should be called. Some of the names in use were * arithmetic engine, * calculating machine, * high-speed calculating device, * stored-program computer, * automatic computing establishment, * digital computer, * Ziffernmaschine, * mathematische Maschine, * Rechenautomat; * Elektronengehirn[24], * electronic brain, * thinking machine.

2.3 Some early examples of computing

In their introduction to the book on the history of numerical analysis in the 20th century[25], C. Brezinski and L. Wuytack give a list of the first courses in numerical computing in 29 countries. This list was compiled with the help of *NA Digest* readers.[26] The dates vary widely: shortly after 1900 by *Carl Runge (1856-1927)* in Germany and *Edmund T. Whittaker (1873-1956)* in Great Britain[27], and in Venezuela in 1967 by Victor Pereyra, who had learned numerical analysis in lectures and courses in Argentina.

In the first years after World War II, there were clearly more computers and computer-experienced scientists in the US than in Europe. But everywhere there were high expectations for the future, and many discussions about strategies of science policy and about education of mathematics students concerning computers. When *Douglas Rayner Hartree (1897 - 1958)* moved from Manchester university to Cambridge university in 1947, he used his inaugural lecture to explain to his new colleagues his plans for his research in the next years[28]. It seems that his textbook[29] was the first publication

[19] de Beauclair 1983, p.75, [18]

[20] GWDG 2010, p.73, [24].

[21] Forsythe 1967, p.5, [22]

[22] http://www.math.uni-hamburg.de/home/collatz/gruppenfotos/

[23] de Beauclair 1983, Bild 5, p.73, [18]

[24] This probably was influenced by the logo of the very successful company *Brunsviga* selling mechanical desk-top calculators world-wide: it was a head with gear-wheels instead of a brain and the words *Gehirn von Stahl* (brain of steel)

[25] Brezinski, Wuytack 2001, pp.1-40, [7]

[26] `na-net`, initialized by Gene Golub around 1984 and connecting numerical analysts via ARPA and/or BITNET in the beginning, was probably one of the earliest international email-nets. At first, each member could send emails to any other member or to all members. Soon the number of members grew, and the emails to all turned out badly or even embarrassing too often. So the emails to all have to be sent to an editor first and are collected for a weekly NA Digest. The archive of NA Digests is available through netlib. The earliest digest archived there dates back to February 1987.[39]

[27] Whittaker and Robinson 1924, [49]

[28] Hartree 1947, [26]

[29] Hartree 1952, [25]

which used the term *Numerical Analysis* in today's meaning.[30] A good source for the discussions in the US in the early sixties are the 'Essays for COSRIMS'[31].

Germany profited from contacts to computer specialists in other countries: British computer specialists (among them *Alan Turing (1912 - 1954)*) visited Göttingen in the British Zone and had a colloquium about computers with Heinz Billing, Helmut Schreyer, Alwin Walther and Konrad Zuse as speakers. Heinz Billing told later that a discussion with a British colleague after this colloquium gave him important informations[32].

In the German state Nordrhein-Westfalen, there was an *Arbeitsgemeinschaft für Forschung* where leading mathematicians met for seminar talks and discussions. In 1954 there was a session with talks by John von Neumann and Eduard Stiefel.[33]

John von Neumann (1903 - 1957) gave an introductory talk about computers[34] and explained what was needed to run them: hardware requirements of the time and the human resources they had in Princeton for keeping the ENIAC running (3-5 engineers and mechanics (working in 2 1/2 to 3 shifts), 2-3 mathematicians, and 10 to 20 coders - altogether 15-30 persons). He also gave an example of a typical application, i.e. he discussed shortly the joint meteorological work with J. Charney and their meteorological group. An article focussing on the mathematical side of this work is given by Aspray[35]; see also subsection 3.1.

Eduard Stiefel (1909 - 1978) made clear that their conditions for running the Z4 at ETH Zürich were very different: the users of the Z4 had to do everything by themselves, without a supporting staff. If a mathematician wanted to solve a problem *just once, without a series of very similar problems to follow,* then it was much faster not to use the computer: for preparing a program for the Z4, the mathematician would need several times as much time as for solving the problem with paper and pencil or with help of a mechanical desktop calculator. The users did not have to do numerical calculations anymore, but this work was replaced by programming work, which required more work at the beginning and was much more of a bore.

In those years, the Z4 was the only computer in all of Switzerland, so many people from outside wanted to use it. The ETH people told the potential users that they will have to do the programming work by themselves. Stiefel made clear in his talk that the demand for computers in industry and engineering could grow substantially only if the programming of the computers would be simplified substantially. As a first step he suggested that there should be some international agreement between the numerics institutes about the terminology.[36] A later view on these pioneer days at the ETH Zürich is given by Gutknecht.[37]

Stiefel was not the only one who felt that need for simplifying programming in 1954. Not only that programming was difficult and boring at the same time. In addition, it

[30] Brezinski, Wuytack 2001, p.1, [7]
[31] Committee on Support of Research in the Mathematical Sciences, 1964, [16]
[32] GWDG 2010, pp.77ff, [24]
[33] Brandt 1955, Heft 45, [6]
[34] von Neumann 1955, [41]
[35] Aspray 1989, [3]
[36] Stiefel 1955, [48]
[37] Gutknecht 1990, [23]

was machine-dependent, i.e. every new computer required again to learn how to program it. And in those computing centers with a large programming staff, this staff was as expensive as the computer itself.

After some pioneering work by other people, several IBM employees (*John Warner Backus (1924 - 2007)* with his group) started in 1954 to develop the machine-independent programming language FORTRAN and, first for the IBM 704, a FORTRAN compiler.[38] The compiler still was machine dependent and to be bought together with the machine. But Standard-FORTRAN programs were machine-independent and thus portable.

Soon after that, two international groups, one in GAMM[39] and one in ACM[40] (including John Backus), developed also machine-independent programming languages. The two groups agreed to unify their attempts into one language. As a result, Algol 58 was created at an ACM-GAMM meeting at ETH Zürich in the summer of 1958.[41] Later on, discussions at IFIP[42] meetings followed[43].

In later years, big computers would typically have several compilers, one for Algol and at least one for Fortran, and maybe several others for other programming languages. So the users had a choice. Fortran and Algol were designed for numerical work and were the favorites for most of those doing numerical work. Fortran is still used today (2015), because every new Fortran version contains the previous Standard-Fortran version as a subset, so that old programs run on a new machine as well and also on machines in other countries. Algol, however, was created by academic people. They improved the language also by changing the logic of if-statements. Because of these changes in logic, automatic translation from one Algol dialect into another one was not possible. Since every new machine would have only one Algol compiler (the one for the newest Algol version), this search for elegance killed Algol around 1970.

In 1956, at the 59th session of the *Arbeitsgemeinschaft für Forschung*, Richard Courant and Ernst Peschl were the speakers. Peschl's talk focussed on complex numbers and theory of functions and was not related to computers.[44] *Richard Courant (1888 - 1972)* focussed on the mathematical side of the computational work of his institute at New York University. He gave several examples of important problems of applied mathematics of the time. He made clear that much deeper results are possible if the mathematical analysis of the problem is complimented by numerical work. He also gave advice about the education of mathematics students in Germany: in the US, the number of computers grew very suddenly very strongly – and now there were not enough mathematicians to make optimal use of these computers. So the number of mathematics students in Germany should be increased now, not to repeat this problem in Germany.[45]

[38] en.wikipedia, accessed on 2015-08-07, keyword Fortran, [51]

[39] Gesellschaft für Angewandte Mathematik und Mechanik – International Association for Applied Mathematics and Mechanics; founded in 1922

[40] Association for Computing Machinery; founded in 1947. SIAM, the Society for Industrial and Applied Mathematics, was founded in 1951

[41] en.wikipedia, accessed on 2015-08-07, keyword Algol 58, [51]

[42] International Federation of Information Processing, founded in 1960

[43] Backus et al 1963, [4]

[44] Brandt 1959, Heft 59, [6]

[45] Courant 1958, [17]

The education of mathematics students in Germany was discussed at several GAMM meetings; 1954 and 1957 during the annual GAMM meetings, in 1960 at a separate meeting. There are proceedings of the 1957 event, edited by Lothar Collatz, printed with financial support from the Siemens-Schuckert-Werke A.G..[46] This booklet gives a good idea of the mathematics education of students in those days, and of the mathematical skills needed in the companies of the experienced mathematicians who spoke. During the years 1954-57, the employment outlook changed fundamentally: while Diplom-Mathematiker had problems to find a job in 1954, there were more job offers than could be filled in 1957, and the numbers of job offers were expected to go up in the following years, because of computers - but the numbers of students for Diplom-Mathematik were declining. The meeting began with status reports of Prof. F. Reutter (TH Aachen) and Prof Collatz and Dr G. Bertram (Hamburg university) on the education at nine THs and 23 universities, resp.. At all THs all mathematics students did not only learn mathematics, but in addition they were trained to apply mathematics: they had also lectures in technical disciplines (like mechanics) and courses for 'mathematical practice', solving problems from technical disciplines by numerical and/or graphical methods and/or by using mathematical instruments like slide-rules or mechanical calculators. In most THs they also learned programming. At most universities, however, the education in mathematics was quite different: those students planning to become school teachers usually did not have to learn to apply mathematics at all, and those who would have liked to, often did not find the corresponding offers at their university. It became clear during that meeting that every mathematics student should learn how to work with a computer, and Collatz wrote this recommendation also into the *Studienführer* (student's guide) of Hamburg university. The development of computing and computers at German universities during the years 1870 to 1970 is described by Hashagen[47], but not very much how this influenced mathematics.

3 First Reviews of Changes: 1968 - 1974

After computers and their use had developed for several years and more and more scientists got experienced with them and observed that their way of working and thinking was changing because of the machines, they felt a need to reflect these new experiences in discussions with their colleagues. There was a series of international meetings and volumes of proceedings where mathematicians[48] and physicists[49] told each other how much their field had changed or still was changing. Only a few years later, computational work was considered completely normal, and there were enough scientists with computational experience for organizing moderately sized meetings on special fields of computational physics, for instance plasma physics and astrophysics with ca 125 participants[50].

[46] Collatz 1957, [15]
[47] Hashagen 2010, [27]
[48] Churchhouse and Herz 1968, [12]; AMS and MAA 1973, [2]
[49] Fernbach and Traub 1970, [19]; MacLeod 1972, [34]
[50] Biskamp 1976, [5]

Also, the new field of informatics/computer science started to develop in the 19-sixties as an interdisciplinary field with intersections with mathematics, electrical engineering and other fields of research. Many computer scientists of the first years were (numerical) mathematicians or physicists before. In 1967, there were about 30 computer science departments in the US and Canada. In Germany, *Friedrich L. Bauer (1924 - 2015)* gave the first official 'Informatik-Vorlesung' in 1967, at TU München. They called them 'EDV-Kurse' in previous years.[51] At Hamburg university, the 'Fachbereich Informatik' started work in 1971, but IT-work was done there since 1958.[52]

Computer scientists felt a need to define their new field of research, to examine the differences between computer science, mathematics and the other sciences, and to tell other scientists about this[53]:

> What is computer science anyway? This is a favorite topic in computer science department meetings. Just as with definitions in mathematics, there is less than total agreement and –moreover– you must know a good deal about the subject before any definition makes sense.[54]

In the following some of the experiences, observations and lines of thought will be retraced which are found in the cited references and which accompanied the changes of mathematics and physics. Also, we will consider several retrospective comments from later years.

3.1 Impact of computers on physics

Since the nonlinear differential equations mentioned by von Neumann in 1946 originate in physics or related fields, it seems adequate to consider the impact of computers on physics first.

At the first European conference on *Computational Physics* in 1972, physicists from many different fields of research met: atomic physics, fluid dynamics, geophysics, particle physics, plasma physics, statistical mechanics, astro physics, high-energy physics, industrial research and development, and so on. They talked about their research for non-specialists, focussing on how their way of working had changed through the influence of computers and will continue to change in the future. Here a few voices:

> Computers have brought a new situation for astronomers, since formerly there was no way to experiment with stars and now you can make your own star and watch it changing with time. [...] The real problem is to prove that these computer stars have anything to do with those stars we see in the sky.[55]

[51] de.wikipedia, accessed on 2015.08.05, keyword Friedrich L. Bauer, [50]

[52] Oberquelle 2008, [42]

[53] see for instance Forsythe 1967, [22]; Forsythe 1968, [21]; Knuth 1972 [30]; Knuth 1974, [29], and the references therein

[54] Forsythe 1967, p.1, [22]

[55] H.-C. Thomas, MPA, p.151 in: MacLeod 1972, [34]

Today, numerical simulations are also done in cases when experiments are possible. If an experiment is very longsome and expensive, previous optimization of the parameters speeds up the process and reduces costs. In addition, accompanying numerical simulations increase the theoretical understanding of the results obtained in the experiment.

> I would like to stress [...] that the aspects of high-energy physics where computers have been particularly useful, and where computers may well become vital for future break-throughs, are of two types [...] The first one concerns the study of complex physical systems [i.e. systems with many particles, degrees of freedom or variables]. [...] The second type concerns systems of a highly non-linear character. As we were reminded yesterday by Professor Charney, the vision of John von Neumann [in Princeton in 1946] [...] was that the highly non-linear problems of mathematics would get moving at last because of the possibilities offered by fast computers.[56]

Since the systems of partial differential equations treated in continuum mechanics, fluid dynamics, meteorology and plasma physics also describe high dimensional systems of non-linear equations, these fields also profit highly from the availability of computers.

> The computer has transformed bubble chamber physics from a qualitative science to a really quantitative science.[57]

And not only cloud chamber/bubble chamber physics, but also other branches of physics and of other sciences! As soon as research produces *quantitative* results, as soon as there are parameters (quantities!) that can be measured, mathematical modelling is possible: (differential) equations relating the various parameters to each other are formulated, and then measured results are compared to computed results. If the measured and the computed results agree to each other, there is a good chance that the mathematical model is adequate. It may be used as long as it does not produce contradictions. If discrepancies occur which cannot be traced back to other sources (e.g. errors in the computations or in the measurements), this will lead to a better mathematical model and a better theory.

> As well as opening up new areas of atomic physics, [...] the ready availability of powerful computers has had a profound influence on what is acceptable as a good theory.[58]

> The impact of computers on non-bubble chamber physics (NBC) has been quite large and will be even more so in the years to come. [...] For example, in my field of physics (NBC) [i.e. electronics experiments] an experiment can be divided into three main parts: (i) proposal; (ii) data taking; (iii) data analysis. Nowadays computers play an important role in all these three parts.[59]

[56] Van Hove, CERN/MPP, p.164 in: MacLeod 1972
[57] B. French, CERN, p.158 in: MacLeod 1972
[58] P.G. Burke, Queen's U Belfast, p.1 in: MacLeod 1972
[59] A. Zichichi, CERN, p. 160 in: MacLeod 1972, [34].

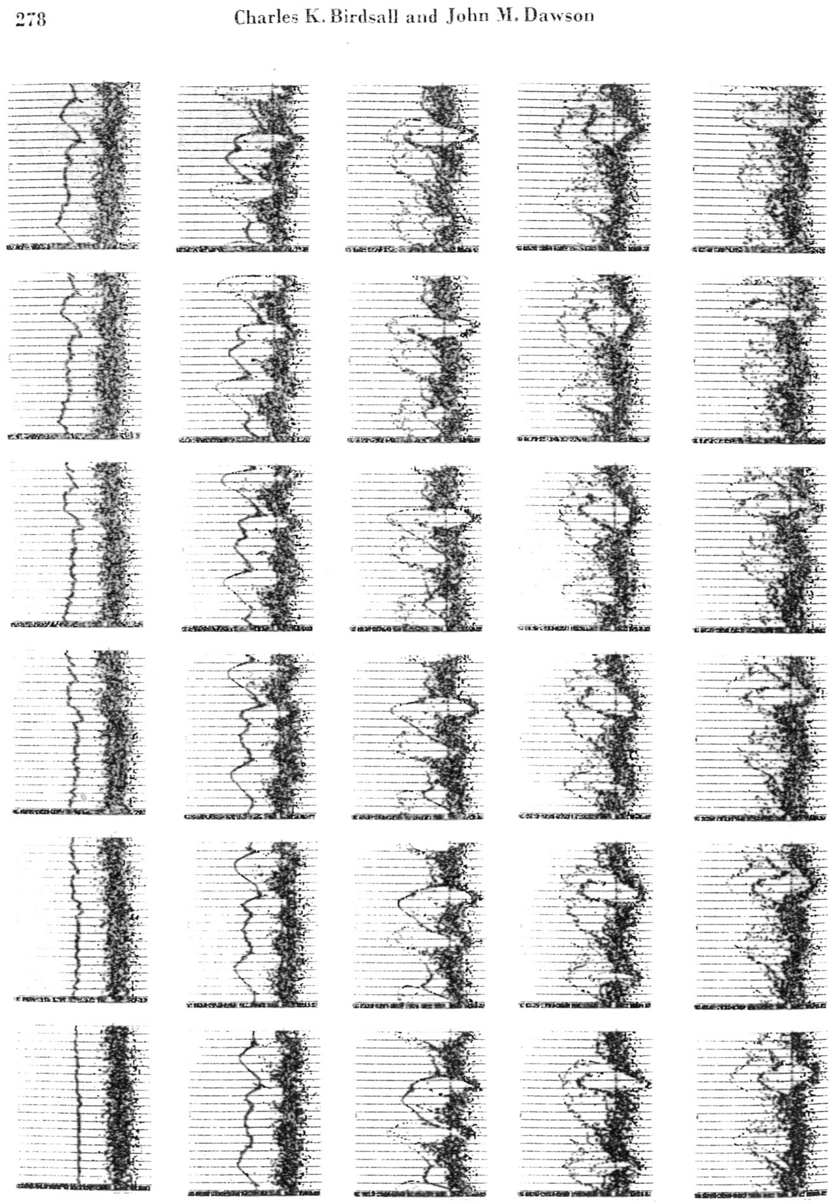

Figure 2: Example for numerical simulation of plasmas by particle methods. Computations by Birdsall and Dawson, 1970; to be read from bottom to top, from left to right. Every point shows a plasma particle. Phase space plots, velocity v versus position x (absissa), for weak cold beam (line in bottom left corner) interacting with a warm plasma.

Figure 2 shows a numerical simulation of the development in time of an interaction of a cold beam with a warm plasma. Computations by C.K. Birdsall and J.M. Dawson, using a particle method.[60]

3.2 Impact on Data Processing and Daily Life

At the CERN meeting in 1972, this was considered a bit exotic, but actually it was a glimpse into the future:

> [...] I have many friends whose first foreign language is FORTRAN and second is English. It's very often easier for people to communicate in FORTRAN than in English. [...] physicists tend to think of computers for all tasks, not just [...] theoretical calculations or analysis of data. The computer is such a common element in their lives - like a chair or a pencil - that they use computers for all sorts of everyday tasks, from cataloguing the references for a review paper to printing party invitations. [...] I have friends, even, who become physically uncomfortable if you deprive them of a computer for a few hours.[61]

3.3 Impact on Mathematics

In 1946, those computers available allowed fast calculations - about 10^3 to 10^5 times faster than the older mechanical calculating machines - but the quickly accessible memory in those computers was very small. Using external memories for storing and retrieving of intermediate results or data slowed down the execution of a program considerably. Goldstine and von Neumann described the situation in 1946 by writing

> ...in an automatic computing establishment there will be a "lower price" on arithmetic operations, but a "higher price" on storage of data, intermediate results, etc. Consequently, the "inner economy" of such an establishment will be very different from what we are used to now, and what we were uniformly used to since the days of Gauss. Therefore, new computing methods, or, to speak more fundamentally, new criteria for "practicality" and "elegance" will have to be developed.[62]

Goldstine and von Neumann made an important point. Though modern supercomputers have much larger internal memories and the prices for data storage went down dramatically, the internal memories of the biggest existing computers are still "too small" for certain users: the size of the problems which are treated grew faster than the computers. There exist methods and techniques today to treat big problems without loss of speed: where it is adequate, intermediate results are recomputed again and again instead of stored; large sparse systems of equations are not defined by storing

[60] Birdsall and Dawson, Fig. 2.19 in: Fernbach and Traub 1970, [19]; Meyer-Spasche 2012, [35].
[61] F.James, CERN, p.161 in: MacLeod 1972, [34]
[62] Aspray 1989, p.308f, [3]

their matrix, but by a subprogram which returns the product of this matrix with a given vector, etc.

These new criteria for practicality and elegance concerned Linear Algebra to a high degree:

> It is instructive to recall that in the 1940s linear algebra was dead as a subject for research; it was ready to be entombed in textbooks. Yet only a few years later, in response to the opportunities created by the availability of high-speed computers, very fast algorithms were found for the standard matrix operations that astounded those who thought there were no surprises left in this subject.[63]

The first mathematicians using computers were those who used numerical methods and who applied mathematical methods to problems from other sciences, so-called 'applied mathematicians'. Their field changed substantially: traditional numerical calculations, i.e. *Rechenkunst, Rechentechnik,* required that special features of the considered problems were taken into account and exploited whenever possible: otherwise these problems would either have been too difficult to be treated or too simple to be significant for the applications. This attention for seemingly unimportant details led 'pure' mathematicians to see their colleagues getting lost in details. Pure mathematicians who always want to generalize and clearly prefer a general theory to a collection of recipes or solved special cases, looked upon their colleagues with disgust - clearly underestimating the difficulty of the problems those were solving. In Collatz' words, already in 1957:

> Die heutige Angewandte Mathematik ist ja etwas ganz anderes als die Angewandte Mathematik vor vielleicht 50 Jahren, die als bloße Rezeptsammlung empfunden wurde. Die heutigen Anforderungen zwingen zum Verlassen der klassischen Methoden, man [...] ist auf numerische Methoden angewiesen und steht dann sofort vor einer Fülle schwieriger mathematischer Aufgaben.[64]

Through the availability of computers, numerical work of earlier times grew into *Numerical Analysis, Numerische Mathematik*.[65] Using computers as tools, it makes sense to compare different algorithms for solving the same problem (for instance solution of a linear system), to prove theorems about their properties and to develop criteria for judging the quality of numerical methods:
* systematic error analysis and error estimation;
* definition of condition numbers;
* definition of the order of convergence of an iterative method;
* find the number of necessary operations as a function of the size of the problem;
* define the complexity of algorithms;
* define the complexity of problems;
etc.

[63] Lax 1997, p. 205, [31]
[64] Collatz 1957, p.12, [15]
[65] Collatz 1990, [13]; Brezinski and Wuytack 2001, [7]; Nash 1990, [40]

There was no abrupt change, no discontinuity in the development: many traditional algorithms got adapted to the new conditions of work, were further developed and extended, and some became even more important than they were before, especially those which were invented at the time of mechanical desk calculators (for instance certain difference methods for the solution of differential equations, and also particle methods[66]). Other numerical methods got discarded because there are too many cases in which they fail under computer conditions, or simply because there are alternative methods of much better quality. Especially algorithms which involved the computation of determinants or of inverses of matrices were discarded. But also, new types of algorithms were and still are developed (e.g. Monte Carlo Methods[67]). A detailed historical review of the development of numerical methods in the time interval between 1900 and 1990 is given by Collatz, Nash, and by Brezinski and Wuytack[68].

Studying the quality of numerical methods often led and still leads to new questions/results in pure mathematics. Studies on the stability of difference methods, for instance, require much knowledge in matrix/operator theory. There is an intimate connection between the study of numerical methods and functional analysis. Already in 1964 Collatz wrote in the preface of his book *Funktionalanalysis und Numerische Mathematik*:

> [... The purpose of this book] is merely to point out the structural changes which numerical analysis has undergone, on the one hand as a result of the widespread use of large electronic computers, on the other hand through the development of abstract methods. The resulting picture of numerical analysis is quite different from the one of ten or twenty years ago. Just as in other areas of mathematics, a strong trend towards abstraction is apparent in numerical analysis. But at the same time, the boundaries between different mathematical disciplines disappear. It is for this reason that it is so difficult at present to decide whether functional analysis belongs to pure or to so-called applied mathematics. Functional analysis is a foundation for large segments of the two disciplines mentioned above and the author would be delighted to find that this book contributes to showing how absurd the distinction between "pure" and "applied" mathematics actually is; there is really no boundary that separates the two, there is only one mathematics, of which analysis, topology, algebra, numerical analysis, probability theory, etc., are merely some overlapping areas.[69]

Due to fast computers, the new field of *Experimental Mathematics* became feasible, with many different types of experiments, e.g.
* get new ideas for proving theorems by numerical simulations;
Example 1: though several solutions of the Korteweg de Vries equation were known from analytical investigations, it was a big surprize when numerical simulations suggested that there are additional solutions featuring eigenspeeds of solitary waves. After these

[66] Buneman 1990, [10]; Meyer-Spasche 2012, [35]
[67] Busch 2015, [11]; Brezinski and Wuytac 2001, p.23f, [7]; Aspray 1989, p.312ff, [3].
[68] Collatz 1990, [13]; Nash 1990, [40]; Brezinski and Wuytack 2001, [7].
[69] Collatz 1964, English edition 1966, [14]

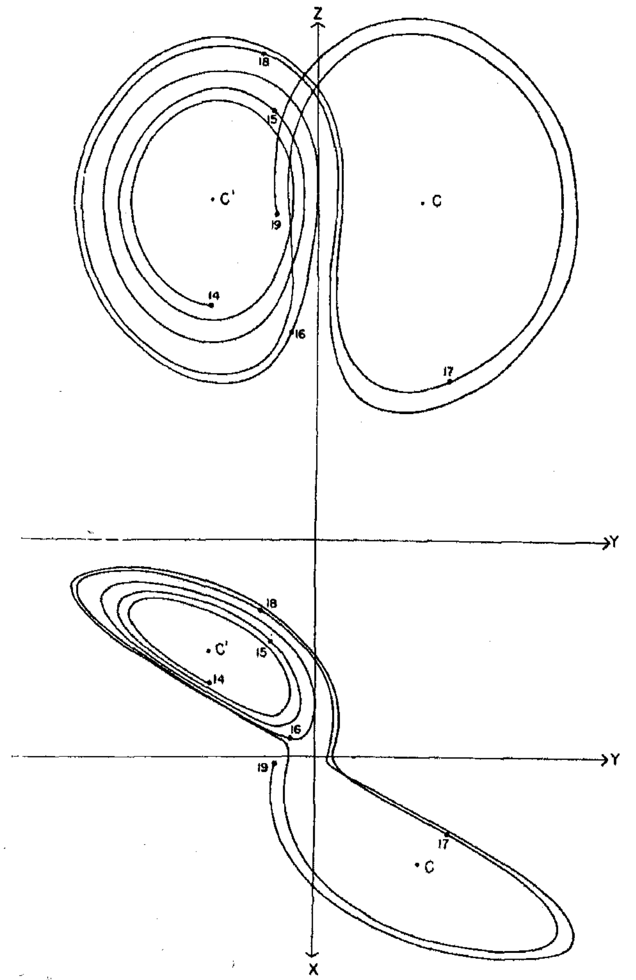

Figure 3: One of Ed Lorenz' original pictures of the Lorenz-Attractor (Lorenz 1963, Fig.2). Lorenz found this attractor by working with a system of 12 equations which he then condensed to 3 equations. Later on it was proved that three equations is the minimum for the existence of strange attractors of ordinary differential systems.

solutions were found in computations, their existence could also been proved in the traditional way with paper and pencil.[70]
Example 2: while *Ed Lorenz (1917-2008)* was testing the quality of certain numerical methods for weather prediction, he discovered the phenomenon of strange attractors in ordinary differential systems. This work could not have been done without a computer. One of Lorenz' 1963-pictures of the *Lorenz-Attractor* is shown in Figure 3.[71]
* check the validity of proofs of theorems by numerical simulations;
Example: as a byproduct of their numerical simulations of Taylor vortex flows, DiPrima, Hall showed in 1984 that a theorem by Yih of 1972 was wrong[72];
Another example was discussed by Meyer-Spasche in 1973[73]: programming a published algorithm for investigation of the eigenvalues of a given real matrix led to wrong results for matrices of a special structure. This algorithm relied on theorems by Howland and Senez (1970) and by Givens (1954) with the same error.

As we already observed in physics: also in mathematics the use of computers may improve the quality of theories, at times.

In the new field of *Scientific Computing*, old and new problems are solved which were too complex, too big and too nonlinear for traditional methods. In those years which are in the focus of this text, sometimes comparisons were given of times necessary for traditional treatment of a problem or treatment with the help of a computer. Today, such comparisons are impossible. Those problems treated by scientific computing today in, say, continuum mechanics, statistical mechanics, plasma physics, astrophysics etc, can only be treated on a computer, no way of treating them without a computer. This does not exclude, however, that some phenomena which were discovered in numerical simulations may lead to proven theorems later, as in Examples 1 and 2 or in many other cases.[74]

4 Conclusions, 2015

In this text we have looked at a few early, typical examples for the impact of computers on mathematics and physics. It is not surprizing that the fields of science affected first were those where lengthy calculations and/or collection and processing of data are involved. As we all know, many other changes followed, some of them quite deep, and not only affecting mathematics, physics and other sciences, but also affecting many aspects of our daily private lifes. Nowadays we are experiencing many ongoing automation processes, and others are to come and we even cannot guess what they will imply. There is still no final answer to the question *What can - or cannot - be automated?* The Stanford mathematician and computer scientist *George E. Forsythe (1917 - 1972)* began his invited talk at the IFIP 1968 Congress with the words:

[70] Lax 1970, [32]
[71] Lorenz 1963, [33]; Reeves and Lorenz 2014, [45]
[72] Meyer-Spasche 1999, p.27, [37]
[73] Meyer-Spasche 1973, p. 365 and p. 366, [38]
[74] e.g. Meyer-Spasche 1999, sections 4.4 to 4.6, [37].

> These years are witnessing an unparallelled growth and rate of growth of science and technology. It is unnecessary to remind members of the IFIP 68 that, as part of this growth, computing is rapidly invading almost every aspect of our intellectual and technological life. Indeed, the question "What can be automated?" is one of the most inspiring philosophical and practical questions of contemporary civilization.[75]

It is a political question as well: *Should everything that can be automated actually get automated?* Which of the technologically possible developments are desirable, which ones should we try to block?

Probably for making many people aware of this problem, Hannes Alfvén (under the pseudonym Olof Johannesson) published a booklet[76] in 1966 [sic!], describing how the development of computers could possibly lead to a global world society in which everything is automated and organized by computers. Finally computers even reproduce themselves and some computers service the others and prevent the whole system from breaking down for a second time. It is amazing to read this text today for the first time: in some respects Alfvén's predictions are surprizingly realistic (especially the development of computer technology for support of the daily public and private life).

In 1973, Alfvén's booklet provoked a one page article in a German newspaper[77]. The author *Klaus Brunnstein (1937-2015),* 'Professor für Anwendungen der Informatik' at Hamburg University since 1973, started by talking about Alfvén's saga and then explained that a public discussion is needed what people want: Computers only as a device for rationalisation of workflow and of improvements in productivity *or* computers as the main tool for a society whose active members are informing themselves? During the years following, Prof Brunnstein fought successfully for IT security, social accountability and the implementation of *Datenschutz* (information privacy) and *informationelle Selbstbestimmung* (informational self-determination) into German legislation, both as a professor of computer science and as a politician (FDP). During the years 1976 - 2004 he also was active at international meetings and in IFIP committees, elected IFIP president 2002-2004.[78]

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[76] Johannesson 1966, [28]

[77] Brunnstein 1973, in: Die Zeit Nr.41, [9]

[78] de.wikipedia, accessed on 2015-07-28, keyword Klaus Brunnstein, [50]; Homepage of Klaus Brunnstein, [8].

[79] Wolfschmidt 2015 [52, 53]

[80] e.g. Brunnstein 1973, [9]; Johannesson 1966, [28]; Knuth 1974, [29]

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