

Gravitational-wave luminosity distance in quantum gravity

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Dimensional flow, the scale dependence of the dimensionality of spacetime, is a feature shared by many theories of quantum gravity (QG). We present the first study of the consequences of QG dimensional flow for the luminosity distance scaling of gravitational waves in the frequency ranges of LIGO and LISA. We find generic modifications with respect to the standard general-relativistic scaling, largely independent of specific QG proposals. We constrain these effects using two examples of multimessenger standard sirens, the binary neutron-star merger GW170817 and a simulated supermassive black-hole merger event detectable with LISA. We apply these constraints to various QG candidates, finding that the quantum geometries of group field theory, spin foams and loop quantum gravity can give rise to observable signals in the gravitational-wave spin-2 sector. Our results complement and improve GW propagation-speed bounds on modified dispersion relations. Under more model-dependent assumptions, we also show that bounds on quantum geometry can be strengthened by solar-system tests.

Introduction. Quantum gravity (QG) includes any approach aiming at unifying General Relativity (GR) and quantum mechanics consistently, so as to keep gravitational ultraviolet (UV) divergences under control [1, 2]. Any such approach can be either top-down or bottom-up, depending on whether it prescribes a specific geometric structure at the Planck scale, or it starts from low energies and then climbs up to higher energy scales. The former class includes string theory, nonlocal QG, and nonperturbative proposals as Wheeler–DeWitt canonical gravity, loop QG, group field theory, causal dynamical triangulations, causal sets, and noncommutative spacetimes. The latter class contains asymptotic safety and the spectral approach to noncommutative geometry. Such variety of QG theories leads to many cosmological consequences which are currently under investigation [3].

Given the recent direct observations of gravitational waves (GW) [4–10], opening a new era in GW and multimessenger astronomy, new opportunities are arising to test theories beyond GR. In general, QG may affect both the *production* [11, 12] and the *propagation* of GWs [11, 13–15] in ways that differ from those obtained from modified-gravity models for dark energy. While QG aims at regularizing UV divergencies in a framework applying the laws of quantum mechanics to the gravitational force, one might hope that yet-to-be developed connections between UV and infrared regimes of gravity can lead to a consistent theory of dark energy from QG.

On one hand, one may believe that QG theories can leave no signature in GWs, arguing that quantum correc-

tions will be suppressed by the Planck scale. Such a conclusion is reached by considering the leading-order perturbative quantum corrections to the Einstein–Hilbert action. Since these corrections are quadratic in the curvature and proportional to the Planck scale $\ell_{\text{Pl}} \approx 10^{-35} \text{ m} = 5 \times 10^{-58} \text{ Mpc}$, they are strongly subdominant at energy or curvature scales well above ℓ_{Pl} . For instance, for a Friedmann–Lemaître–Robertson–Walker (FLRW) universe, there are only two scales for building dimensionless quantities, ℓ_{Pl} and the Hubble radius H^{-1} . Therefore, quantum corrections should be of the form $(\ell_{\text{Pl}} H)^n$, where $n = 2, 3, \dots$. Today, quantum corrections are as small as $(\ell_{\text{Pl}} H_0)^n \sim 10^{-60n}$, and any late-time QG imprint is Planck-suppressed and undetectable.

On the other hand, these considerations are not necessarily correct. One may consider nonperturbative effects going beyond the simple dimensional argument quoted above. Indeed, in the presence of a third intermediate scale $L \gg \ell_{\text{Pl}}$, quantum corrections may become $\sim \ell_{\text{Pl}}^a H^b L^c$ with $a - b + c = 0$, and not all these exponents are necessarily small. Such is the case, for instance, of loop quantum cosmology with anomaly cancellation (a mini-superspace model motivated by loop quantum gravity), where quantum states of spacetime geometry may be endowed with a mesoscopic effective scale [16]. These and other QG inflationary models can leave a sizable imprint in the early universe [3]. However, there are very few and not fully developed models of fundamental-QG dark energy [3]; such models modify UV physics, but have also long-range effects.

In this Letter, we consider a long-range nonperturbative mechanism, *dimensional flow*, namely the change of spacetime dimensionality found in most QG candidates [17–19]. We argue that this feature of QG, already used as a direct agent in QG inflationary models [20–23], can also have important consequences for the propagation of GWs over cosmological distances. We identify QG predictions shared by different quantization schemes, and determine a model-independent expression, Eq. (5), for the luminosity distance of GWs propagating in a dimensionally changing spacetime in QG. Testing this expression against current LIGO-Virgo data, mock LISA data, and solar-system tests, allows us to constrain the spacetime dimensionality of a representative number of QG theories. We mainly focus on the spin-2 GW sector and on specific opportunities of GW experiments to test QG scenarios, assuming that the other dynamical sectors (e.g. spin-0 and spin-1) are not modified by QG corrections. Our results suggest that group field theory/spin foams/loop quantum gravity (GFT/SF/LQG), known to affect both the UV limit of gravity and cosmological inflationary scales, can also modify late-time GWs, due to effects that have not been previously considered. We also compare our results with complementary constraints on modified dispersion relations, and discuss possible implications of the Hulse–Taylor pulsar. Finally, we also take into consideration some different type of model-dependent bounds to QG theories, particularly from solar-system experiments.

Dimensional flow. The fact that the dimensionality of spacetime experienced by a quantum field might depend on the energy scale has important implications for the field dynamics. We illustrate this phenomenon by considering a metric perturbation propagating on a QG spacetime, effectively emerging from some fundamental dynamics that we do not need to specify here. In Isaacson shortwave approximation [28], a gravitational wave is a high-frequency spin-2 perturbation $h_{\mu\nu} = h_+ e_{\mu\nu}^+ + h_\times e_{\mu\nu}^\times$ over a background metric $g_{\mu\nu}^{(0)} = g_{\mu\nu} - h_{\mu\nu}$ and is described by the two polarization modes $h_{+,\times}$ (with $e_{\mu\nu}^{+,\times}$ being the polarization tensors). We make the following technical assumptions, valid for the main QG theories, that will be the basis for our arguments.

- (i) There is a continuum limit of the QG theory to a spacetime with a continuous integrodifferential structure.
- (ii) The effective dynamics of a high frequency GW over a spacetime distorted by QG effects can be characterized by a spacetime measure $d\varrho(x)$ and a kinetic term $\mathcal{K}(\partial)$. Both can be deformed by QG effects unrelated to perturbative curvature corrections. The perturbed action for a small perturbation $h_{\mu\nu}$ over a background $g_{\mu\nu}^{(0)}$ is

$$S = \frac{1}{2\ell_*^{2\Gamma}} \int d\varrho \sqrt{-g^{(0)}} [h_{\mu\nu} \mathcal{K} h^{\mu\nu} + O(h_{\mu\nu}^2) + \mathcal{J}^{\mu\nu} h_{\mu\nu}], \quad (1)$$

where the prefactor makes the action dimensionless, $\mathcal{J}^{\mu\nu}$

is a generic source term, and the $O(h_{\mu\nu}^2)$ terms play no role at small scales. The modes $h_{+,\times}/\ell_*^\Gamma$, where ℓ_* is a characteristic scale of the geometry, are dimensionally and dynamically equivalent to a scalar field.

The measure defines a geometric observable, the Hausdorff dimension $d_H(\ell) := d \ln \varrho(\ell) / d \ln \ell$, describing how volumes scale with their linear size ℓ . In a classical spacetime, $d_H = 4$.

(iii) Spacetime is dual to a well-defined momentum space characterized by a measure $\tilde{\varrho}(k)$ with Hausdorff dimension d_H^k , in general different from d_H . The kinetic term is related to d_H^k and to another geometric observable, the spectral dimension $d_S(\ell) := -d \ln \mathcal{P}(\ell) / d \ln \ell$, where $\mathcal{P}(\ell) \propto \int \tilde{\varrho}(k) \exp[-\ell^2 \tilde{\mathcal{K}}(-k^2)]$ and the function $\tilde{\mathcal{K}}$ is the dispersion relation \mathcal{K} rescaled by a length power. It is not difficult to see that $d_S = 2d_H^k / [\mathcal{K}]$ [24], with square brackets indicating the scaling dimension.

(iv) $d_S \neq 0$ at all scales. The case of geometries where $d_S = 0$ at short scales must be treated separately [27].

We now have the tools to express the scaling of φ in terms of geometric observables: $[h_{+,\times}/\ell_*^\Gamma] = \Gamma(\ell)$, where

$$\Gamma(\ell) := \frac{d_H(\ell)}{2} - \frac{d_H^k(\ell)}{d_S(\ell)}. \quad (2)$$

In the GR limit, $d_H = d_H^k = d_S = 4$ and $\Gamma = 1$. Equation (2) applies to many concrete proposals for QG, each with its own characteristic motivation and level of theoretical robustness. The predictions of representative theories at small (Γ_{UV}) and intermediate scales (Γ_{meso}) are found in Tab. I. Scales at which QG corrections are important belong to the UV regime, whereas intermediate scales where the corrections to GR are small but non-negligible belong to the mesoscopic one.

	Γ_{UV}	$\Gamma_{meso} \gtrsim 1$
GFT/SF/LQG [29–31]	$[-3, 0)$	yes
Causal dynamical triangulations [32]	$-2/3$	
κ -Minkowski (other) [33, 34]	$[-1/2, 1]$	
Stelle gravity [35, 36]	0	
String theory (low-energy limit) [37, 38]	0	
Asymptotic safety [39]	0	
Hořava–Lifshitz gravity [40]	0	
κ -Minkowski bicross-product ∇^2 [34]	3/2	yes
κ -Minkowski relative-locality ∇^2 [34]	2	yes
Padmanabhan nonlocal model [41, 42]	2	yes

TABLE I. The value of Γ_{UV} for different QG theories. Theories with a near-IR parameter $\Gamma_{meso} \gtrsim 1$ are indicated in the second column.

Given a spacetime measure ϱ , a kinetic operator \mathcal{K} , and a compact source \mathcal{J} , the Green function $G(r)$ of the modes h (subscripts omitted) in radial coordinates and Euclidean signature in the absence of curvature is $G(r) = \langle h(r)h(0) \rangle \sim (\ell_*^2/r^2)^\Gamma$. This scaling is consistent

with the one in ordinary spacetime with D directions, where $\Gamma = D/2 - 1$. It is valid for any length range where Γ is approximately constant. Around a homogeneous background, for each polarization mode we obtain

$$h(t, r) \sim f_h(t, r) (\ell_*/r)^\Gamma, \quad [f_h] = 0. \quad (3)$$

Equation (3) schematically describes the distance scaling of the amplitude of GW radiation emitted by a binary system and observed in the local wave zone, a region of space larger than the system size, but smaller than any cosmological scale. The function f_h depends on the source and on the type of correlation function (advanced or retarded), but the key point is that we can express h as the product of a dimensionless function f_h and a power-law distance behavior which is fairly general in QG, since it is based only on the scaling properties of the measure and the kinetic term.

Gravitational waves. We now apply these results focussing on the specific case of gravitational waves propagating over cosmological distances. To investigate the propagation of GWs on a flat FLRW background, we work on a conformally flat metric, where $t \rightarrow \tau$ is conformal time and r is the comoving distance of the GW source from the observer. Therefore, we multiply r by the scale factor $a_0 = a(\tau_0)$ in the right-hand side of Eq. (3). In order to express Eq. (3) in terms of a physical observable, we assume that the source has an electromagnetic counterpart. Recall that the luminosity distance of an object emitting electromagnetic radiation is defined as the power per flux unit, $d_L^{\text{EM}} := \sqrt{L/(4\pi F)}$ and, on a flat FLRW background, $d_L^{\text{EM}} = (1+z) \int_{\tau(z)}^{\tau_0} d\tau = a_0^2 r/a$, where $z = a_0/a - 1$ is the redshift. We assume that QG corrections to d_L^{EM} are negligible at large scales. Absorbing redshift factors into f_h , we express Eq. (3) as

$$h(z) \sim f_h(z) \left[\frac{\ell_*}{d_L^{\text{EM}}(z)} \right]^\Gamma. \quad (4)$$

The details (chirp mass, spin, etc) of the source are all encoded into the dimensionless function $f_h(z)$.

The final step is to generalize relation (4), which is only valid for a plateau in dimensional flow, to all scales. We argue that the correct expression to adopt is

$$h \propto \frac{1}{d_L^{\text{GW}}}, \quad d_L^{\text{GW}} = d_L^{\text{EM}} \left[1 + \varepsilon \left(\frac{d_L^{\text{EM}}}{\ell_*} \right)^{\gamma-1} \right], \quad (5)$$

with $\varepsilon = \mathcal{O}(1)$, and $\gamma \neq 0$ is a scale parameter.

In fact, suppose that QG introduces only one fundamental length scale ℓ_* close to the Planck scale. This is sufficient to trigger a nontrivial dimensional flow and the scaling of distances takes a universal form of the type of Eq. (5). In this case, $\gamma = \Gamma_{\text{UV}}$. For a scale close to the end of the flow, the modified relation has again two contributions [44]: however, in this case $\gamma = \Gamma_{\text{meso}}$ is a mesoscopic-scale parameter close to one.

Although the structure of Eq. (5) is expected to be generic in QG, the coefficient ε cannot be determined universally, since it depends on the details of the transient regime. In general, it can be either a random variable with zero average (in “fuzzy” spacetimes with intrinsic measurements uncertainty) or a number. Suppose it is a number: since also ℓ_* is a free parameter, we can set the coefficient to be $\varepsilon = \mathcal{O}(1)$ without loss of generality. However, the case with $\gamma \approx 1$ is subtle since we can not recover GR unless ε vanishes. This implies that ε must have a γ dependence: the simplest choice such that $\varepsilon(\gamma \neq 1) = \mathcal{O}(1)$, $\varepsilon(\gamma = 1) = 0$, and recovering the pure power law Eq. (4) on any plateau with $\gamma = \Gamma$, is $\varepsilon = \gamma - 1$. If we also allow for a sign ambiguity for ε , we are able to encompass also the case of fuzzy spacetimes where ε randomly fluctuates around zero (from observations one can get only upper or lower bounds on the quantum correction). The net result is Eq. (5) with $\varepsilon = \pm(\gamma - 1)$.

Equation (5) is our key result for analyzing the phenomenological consequences of QG dimensional flow for the propagation of GWs. Its structure resembles the GW luminosity-distance relation expected in some models with large extra-dimensions [9, 45, 46], where gravity classically “leaks” into a higher dimensional space. However, we emphasize that Eq. (5) is based on a feature of most QG proposals, dimensional flow, and does not rely on realizations in terms of classical extra dimensions.

The left-hand side of Eq. (5) is the strain measured in a GW interferometer. The right-hand side features the luminosity distance measured for the optical counterpart of the standard siren. Therefore, observations can place constraints on the two parameters ℓ_* and γ in a model-independent way, by constraining the ratio $d_L^{\text{GW}}(z)/d_L^{\text{EM}}(z)$ as a function of the redshift of the source. Our analysis is based on two standard sirens, the binary neutron-star merger GW170817 observed by LIGO-Virgo and the Fermi telescope [8], and a simulated $z = 2$ supermassive black hole merging event that could be observed by LISA [24–26]. There are three cases to consider:

(a) $0 > \gamma - 1$ leads to an upper bound on ℓ_* of cosmological size, namely $\ell_* < (10^1 - 10^4)$ Mpc. Hence we cannot constrain the deep UV limit of quantum gravity, since $\ell_* = \mathcal{O}(\ell_{\text{Pl}})$. This is expected in QG theories with $\Gamma_{\text{UV}} < 1$ (Tab. I) on the tenet that deviations from classical geometry occur at microscopic scales unobservable in astrophysics.

(b) $0 < \gamma - 1 = \mathcal{O}(1)$: there is a lower bound on ℓ_* of cosmological size. Therefore, if Eq. (5) is interpreted as valid at *all* scales of dimensional flow and $\gamma = \Gamma_{\text{UV}}$, this result rules out the three models not included in the previous case: κ -Minkowski spacetime with ordinary measure and the cross-product or relative-locality Laplacians and Padmanabhan’s nonlocal model of black holes.

(c) $0 < \gamma - 1 \ll 1$: Eq. (5) is valid in a near-IR regime and $\gamma = \Gamma_{\text{meso}}$ is very close to 1 from above. The resulting upper bound on γ is shown in Fig. 1. For the smallest

QG scales, the bound saturates to

$$0 < \Gamma_{\text{meso}} - 1 < 0.02. \quad (6)$$

Examining Eq. (2), we conclude that case (c) is realized

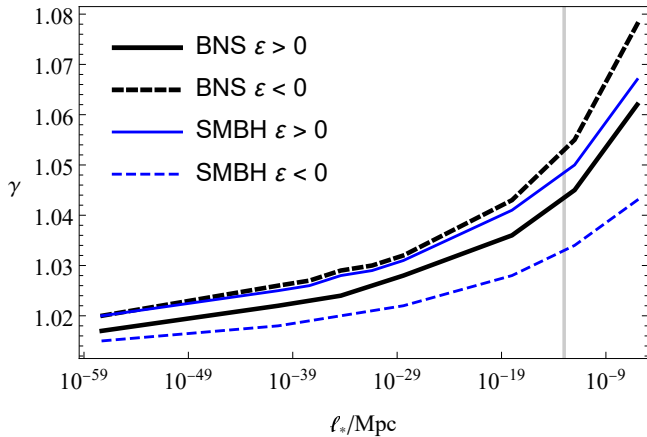


FIG. 1. Upper bounds on γ for ℓ_* fixed between 1 Mpc and the Planck scale $\ell_{\text{Pl}} = 5 \times 10^{-58}$ Mpc for the LIGO-Virgo observed binary neutron-star merger GW170817 (BNS) and a simulated LISA supermassive black hole (SMBH).

only for geometries with a spectral dimension reaching $d_S \rightarrow 4$ from above. The only theories in our list that do so are those where $\Gamma_{\text{UV}} > \Gamma_{\text{meso}} > 1$ (the last three in Tab. I: κ -Minkowski spacetime with ordinary measure and bicross-product or relative-locality Laplacians and Padmanabhan's model [24]) or $\Gamma_{\text{meso}} > 1 > \Gamma_{\text{UV}}$ (GFT/SF/LQG [30]). However, we exclude observability of the models with $\Gamma_{\text{UV}} > \Gamma_{\text{meso}} > 1$, since they predict $\Gamma_{\text{meso}} - 1 \sim (\ell_{\text{Pl}}/d_L^{\text{EM}})^2 < 10^{-116}$ [24]. Thus, only GFT, SF or LQG could generate a signal detectable with standard sirens. Here d_S runs from small values in the UV, but before reaching the limit $d_S^{\text{IR}} = 4$ it overshoots the asymptote and decreases again: hence $\Gamma_{\text{meso}} > 1 > \Gamma_{\text{UV}}$. It would be interesting to find realistic quantum states of geometry giving rise to such a signal, with the construction of simplicial complexes as in Ref. [30].

Complementary constraints. Dimensional flow is also influenced by modifications of the dispersion relation $\mathcal{K}(-k^2) = -\ell_*^{2-2d_{\text{H}}^k/d_S} k^2 + k^{2d_{\text{H}}^k/d_S}$ of the spin-2 graviton field, and this fact has been used to impose constraints on QG theories exhibiting dimensional flow using the LIGO-Virgo merging events [11, 13, 14]. However, the limits obtained this way are weaker than the ones we have found here because the GW frequency is much lower than the Planck frequency. One gets either very weak bounds on ℓ_* or, setting $\ell_*^{-1} > 10$ TeV (LHC scale), a bound $n = d_{\text{H}} - 2 - 2\Gamma < 0.76$ [14], for $d_{\text{H}}^{\text{meso}} \approx 4$ corresponding to $\Gamma_{\text{meso}} - 1 > -0.38$. This can constrain models such as the second and third in Tab. I, but not those such as GFT/SF/LQG for which Eq. (6) holds.

Additional constraints on the spin-2 sector can arise from observations of the Hulse–Taylor pulsar [52]. If the spacetime dimension deviates from four roughly below scales $l_{\text{pulsar}} = 10^6 \text{ km} \approx 10^{-13} \text{ Mpc}$, then the GW emission from this source is expected to be distinguishable from GR. However, it is difficult to analyze the binary dynamics and GW emission in higher-dimensional spacetimes [53] and it is consequently more complicated to set bounds from binary pulsar systems. We will thus leave these investigations for future work. We point out, however, that at scales below $\ell_* = l_{\text{pulsar}}$ (the vertical line in Fig. 1), our results could be largely improved by stronger constraints from the dynamics of compact objects.

Finally, stronger but model-dependent bounds can arise in scenarios that affect other sectors besides the dynamics of the spin-2 graviton field. To have an idea of the constraints that can arise when other sectors become dynamical in QG, we consider a case where the effective scalar Newtonian potential $\Phi \sim h_{00}$ experiences QG dimensional flow: then the bound (6) can be strengthened by solar-system tests. In fact, Eq. (3) can describe Φ in a regime where Γ is approximately constant, while choosing subhorizon distances $d_L^{\text{EM}} = r$ in Eq. (5) we get a multiscale expression. Thus, in four dimensions

$$\Phi \propto -\frac{1}{r} \left(1 \pm \frac{\Delta\Phi}{\Phi} \right), \quad \frac{\Delta\Phi}{\Phi} = |\gamma - 1| \left(\frac{r}{\ell_*} \right)^{\gamma-1}. \quad (7)$$

This result, different from but complementary [24] to what found in the effective field theory approach to QG, applies to the nonperturbative GFT/SF/LQG theories with $\gamma > 1$ at mesoscopic scales. Assuming that photon geodesics are not modified at those scales, GR tests within the solar system using the Cassini bound impose $\Delta\Phi/\Phi < 10^{-5}$ [48, 49], implying

$$0 < \Gamma_{\text{meso}} - 1 < 10^{-5}, \quad (8)$$

which is stronger than the limit obtained from GWs. However, this result relies on model-dependent assumptions on the scalar sector, independent of our previous arguments on the propagation of spin-2 GWs, and should be taken *cum grano salis*. We emphasize that in QG the dynamics of spin-0 fields and the Newtonian potential Φ can be far from trivial. Precisely for GFT/SF/LQG, the classical limit of the graviton propagator is known [50], but corrections to it and to the Newtonian potential are not [51]. Therefore, we cannot compare Eq. (7) with the full theory, nor do we know whether quantum states exist giving rise to such a correction.

Conclusions. Quantum gravity can modify both the production and the propagation of gravitational waves. We derived the general equation (5) describing model-independent modifications due to nonperturbative QG on the GW luminosity distance associated with long distance propagation of GWs. We have then shown that departures from classical GR due to QG effects can be

in principle testable with LIGO and LISA detections of merging events. Solar-system tests of the Newtonian potential Φ lead to stronger constraints than the ones imposed from GW data, but rely on model-dependent assumptions on the dynamics of the scalar Newtonian potential Φ . Focussing on the spin-2 field only, there are several directions that remain to be explored. For instance, time delays in gravitational lensing might be another place where to look for propagation effects beyond GR within LISA sensitivity. Moreover, also the details of the astrophysical systems giving rise to GW signals should be studied, in order to understand the consequences of a QG geometry on the production of GWs in the high-curvature region surrounding compact objects.

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