Gravitational waves from compact dark matter objects in the solar system

C. J. Horowitz*

*Center for Exploration of Energy and Matter and Department of Physics,
Indiana University, Bloomington, IN 47405, USA*

M. A. Papa†

*Max Planck Institute for Gravitational Physics (Albert Einstein Institute), D-30167 Hannover, Germany and
Department of Physics, University of Wisconsin, Milwaukee, WI 53201, USA*

S. Reddy‡

*Institute for Nuclear Theory and Department of Physics,
University of Washington, Seattle, WA 98195.*

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Dark matter could be composed of compact dark objects (CDOs). We find that a close binary of CDOs orbiting *inside* solar system bodies can be a loud source of gravitational waves (GWs) for the LIGO and VIRGO detectors. An initial search of data from the first Advanced LIGO observing run (O1), sensitive to $h_0 \approx 10^{-24}$, rules out close binaries orbiting near the center of the Sun with GW frequencies (twice the orbital frequency) between 50 and 550 Hz and CDO masses above $\approx 10^{-9} M_\odot$.

Dark matter, or one component of it, could be composed of compact dark objects (CDOs). These objects are assumed to have small non-gravitational interactions with normal matter and could be primordial black holes, see for example [1]. Microlensing observations rule out most of dark matter being made of CDOs with masses between 10^{-7} and $15 M_\odot$ [2–5]. In this paper, we focus on CDOs with masses below $10^{-7} M_\odot$ that are not black holes in order to avoid destroying solar system bodies.

In a previous paper we explored CDOs in the Galaxy [6], while the LIGO-Virgo collaboration has searched for gravitational waves (GWs) from binaries with masses in the range $0.2 - 1\ M_\odot$ [7]. It may be difficult to detect GWs from very low CDOs at typical Galactic distances. Therefore, we consider GWs from CDOs in the solar system. Note that there has been considerable discussion of particle dark matter in the Sun, see for example [8–10]. Close binaries of CDOs, orbiting *inside* solar system objects, can be loud sources of GWs for the LIGO/ VIRGO detectors. At these much shorter distances, far weaker GW signals may be detectable. Furthermore, these weaker signals only require much smaller radiated powers and therefore the systems may have much longer lifetimes. This could further increase their detection probability.

Dark matter in the solar system, and in particular inside Saturn's orbit, is constrained by accurate tracking of spacecraft [11]. We avoid these limits by focusing on CDOs *inside* solar system bodies such as the Sun, Jupiter, or Earth. In this case the mass of the CDOs would have already been included as a (tiny) part of the observed mass of the body. In addition, it may be difficult to rule out a number of CDOs at large distances from the Sun where constraints on dark matter in the outer solar system are likely weaker.

Perhaps occasionally a distant CDO of mass m_D would move into the inner solar system and enter a solar system body. When this occurs, it is possible that dissipation from unconstrained non-gravitational interactions could trap the CDO in the body and perhaps over a longer time scale bring it towards the center of the body. Here it may form a close binary with a second trapped CDO. Furthermore, the lifetime of this binary against GW radiation of a given frequency scales as $m_D^{-5/3}$. This lifetime can be very long for low mass objects, and ranges from millions of years for $m_D = 10^{-8} M_\odot$ to longer than the Hubble time for $m_D = 10^{-11} M_\odot$.

We start by estimating the rate of CDO-solar system object collisions assuming dark matter is composed of low mass CDOs. If the mass density of dark matter near the solar system is $\rho \approx 6 \times 10^{-22}$ kg/m³ [12], the number density of CDOs is $n_D = \rho/m_D \approx 3 \times 10^{-42} \text{m}^{-3} \times (10^{-10} M_\odot/m_D)$. Note that n_D increases as m_D decreases. The rate of CDO collisions with a given body is $\sigma_i v_d n_D$ where v_d is the velocity of the CDO. For simplicity we consider a single value $v_d \approx 220$ km/s that is adequate for a first rough estimate. The collision cross section is $\sigma_i = \pi R_i^2 [1 + (v_{ei}/v_d)^2]$ for a body of radius R_i and escape velocity v_{ei}. The total number of collisions during the solar system's lifetime $T_{SS} = 5 \times 10^9$y is $N_i \approx T_{SS}\sigma_i v_d n_D$. For the Sun the number of collisions is,

$$N_\odot \approx 1.4 \times \left(\frac{10^{-10} M_\odot}{m_D} \right). \tag{1}$$

If m_D is $10^{-10} M_\odot$ or less, there is a good chance that the Sun has suffered one or more collisions with a CDO during its main sequence lifetime. For the planets the number of collisions is in general smaller because of their smaller size. For Jupiter $N_J \approx 0.2 \times (10^{-12} M_\odot/m_D)$, while for the Earth $N_e \approx 1 \times 10^{-3} \times (10^{-12} M_\odot/m_D)$. The number of collisions could be higher if the solar system

passed through a region of higher dark matter density in the past. Alternatively, if the Sun somehow has an "Oort cloud" of CDOs at large distances, this cloud could be a source of additional collisions. Finally, CDOs could act as seeds to start the formation of condensed objects and eventually the planets from the solar nebula. This would naturally explain the presence of CDOs in the planets or other solar system objects.

What happens to a CDO when it collides with the Sun? It is possible that there are non-gravitational interactions between the CDO and the conventional matter in the Sun that could help trap the object into an orbit that eventually decays to be inside the Sun. Note that these non-gravitational interactions are poorly constrained. If m_D is low enough and CDOs can be trapped after they collide, then it is likely the Sun would now contain one or more CDO. Two or more CDOs in the Sun could move towards the center (as energy is dissipated) where they may find each other and form a close binary. This binary system will radiate GWs.

Two CDOs orbiting each other at angular frequency ω will be separated by a distance $r = (2Gm_D/\omega^2)^{1/3}$. They will radiate GWs at frequency $f_{GW} = \omega/\pi$ that is twice the rotational frequency. LIGO is insensitive to f_{GW} below about 10Hz. Therefore two CDOs must come within a distance $[2Gm_D/(10\pi\text{Hz})^2]^{1/3}$. This is 300 m for $m_D = 10^{-10}M_\odot$. To avoid touching, each spherical CDO must have an average density greater than $3\pi f_{GW}^2/G$. This minimum density is 1.4×10^{10} g/cm^3 for $f_{GW} = 10$ Hz or 5.6×10^{12}g/cm^3 at 200 Hz. These densities are large but still less than nuclear density $\approx 3 \times 10^{14}$ g/cm^3. The density of CDOs is unknown.

In summary, to generate GWs in LIGO's band a solar system object must contain two or more CDO that are very dense and in a very close binary orbit. Note that a space based GW detector such as LISA, that is sensitive at lower frequencies [13], could detect a single CDO moving in the Sun. This would not require two objects in a close binary orbit and it would not require the CDO to be very dense. We will discuss this further in a later publication.

Two equal mass CDOs in a circular orbit will produce a GW strain h_{jk} of,

$$h_{jk} = \frac{2G}{c^4 d}\frac{d^2}{dt^2}I_{jk}^{TT}\,. \tag{2}$$

Here d is the distance to the source and I_{jk}^{TT} is the transverse traceless quadrupole moment [14]. The intrinsic strain amplitude $h_0 \propto h_{xx} - h_{yy}$ is,

$$h_0 = \frac{4G}{c^4 d}m_D r^2 \omega^2\,. \tag{3}$$

Using Kepler's law $r^3 = 2Gm_D/\omega^2$ to eliminate r yields,

$$h_0 = \frac{2^{8/3}\pi^{2/3}}{c^4 d}(Gm_D)^{5/3}f_{GW}^{2/3}\,. \tag{4}$$

For objects inside the Sun with $d = 1.5 \times 10^{11}$ m, we have,

$$h_0 = 1.4 \times 10^{-9}\left(\frac{m_D}{M_\odot}\right)^{5/3}\left(\frac{f_{GW}}{200\text{Hz}}\right)^{2/3}\,. \tag{5}$$

The strain amplitude h_0 in Eq. 5 is dramatically larger than many galactic GW sources because the distance d is very small, only 10^{-8} kpc.

Since the signal amplitude is potentially so large we perform an initial very simple search on data from the first Advanced LIGO observing run (O1) [15, 16] in the frequency band 50-550 Hz. We assume that the center of mass of the binary is nearly at rest with respect to the Sun. If the center of mass of the binary were also at rest with respect to the gravitational wave detectors, the gravitational wave signal at the detectors would be a pure tone at the frequency f_{GW}. However, because of the relative motion between the CDOs and the detectors due to the rotation of the Earth, the signal at each detector is Doppler-modulated in a way that varies in time depending on the relative velocity between the CDOs center of mass and the detector. The modulation has a periodicity of a day and the maximum Doppler shift for a signal at a frequency f_{GW} is $10^{-6}f_{GW}$.

The frequency spacing of a Fourier transform of data covering a period of time T_{FFT}, is $\delta f = \frac{1}{T_{\text{FFT}}}$. Any signal whose instantaneous frequency does not vary by more than δf during T_{FFT} will appear like a monochromatic signal, i.e. with the signal power concentrated at the frequency bin(s) corresponding to the signal frequency [17]. We will refer to this as the "signal peak".

The longer T_{FFT} is, the higher is the signal peak. As the T_{FFT} increases and the δf decreases, because the signal frequency varies, there will come a point when the signal's instantaneous frequency during T_{FFT} is not confined to one or two bins, but moves over more bins. As a result the signal power is not concentrated in a single bin but spread over more and hence decreases in amplitude in all of them compared to what it would be if the frequency did not vary. For frequency variations due to the Doppler shift of the Earth's spin, the maximum T_{FFT} such that the instantaneous frequency does not shift by more than half a frequency bin is [18, 19]

$$T_{\text{FFT}}^{\max} = \frac{4.7 \times 10^4}{\sqrt{f_{GW}}}\text{s}. \tag{6}$$

For $f_{GW} = 550$ Hz this yields $T_{\text{FFT}}^{\max} = 2.0 \times 10^3$ s; hence we take Fourier transforms of the LIGO O1 data over time intervals that are 1800 s long. This will give maximum sensitivity to a signal during each half hour, without having to perform any demodulation.

The instantaneous frequency of the signal does not move by more than half a frequency bin during each half hour, but it does in general move over the course of the observation time. With a frequency resolution

of $\delta f = 1/1800$ Hz, the signal frequency f_0 such that $f_0 \times 10^{-6} = \delta f$ is about 555 Hz. Signals with frequencies higher than this, would appear in different bins depending on the observation time. An optimal detection procedure would require tracking of such peaks and would be more involved than what we do here. Therefore 550 Hz is the highest frequency we consider.

The FFTs are prepared following the standard procedure used for the Einstein@Home searches (see for instance [20]), including a procedure that eliminates very loud time-domain disturbances from the data [21]. For this reason we use the standard name for these FFTs computed over short time intervals, i.e. SFTs (Short timebaseline Fourier Transforms)[22].

In total we produce 3507 SFTs from the Hanford detector data (LHO) and 2889 from the Livingston detector (LLO). We indicate the SFT data with $\tilde{x}_{\alpha,k}^I$, where α is the SFT order-number, k is the frequency index and $I = H$ or L indicates the detector. The conventions for the SFT data are given in [22].

For each detector I and each SFT α we compute the power spectral density as a function of frequency k

$$p_{\alpha,k}^I = \frac{2}{T_{\text{SFT}}} |\tilde{x}_{\alpha,k}^I|^2, \tag{7}$$

with $T_{\text{SFT}} = 1800$ s, and its average over SFTs:

$$p_k^I = \frac{1}{N^I} \sum_\alpha p_{\alpha,k}^I, \tag{8}$$

with N^I being 3507 and 2889 for the Hanford and Livingston detector respectively. Figure 1 shows p_k^I for the two LIGO detectors.

A loud CDO signal will appear as a peak in the power spectral density of both detectors at the same frequency. Since the noise level in the detectors varies with frequency, we do not set a fixed threshold but rather we identify signal peak candidates as outlier values of the *normalized* average power spectral density. We normalize the average power spectral density in each detector with the average of the running median power spectral densities, with a window of 21 bins. This detection statistic has an expected value of 1.0 and a standard deviation of $\simeq 0.024$ for Gaussian stationary noise data. Since the detector noise is neither Gaussian not stationary, we do not expect that the normalized average power spectral density values (shown in Fig. 2) generally follow the predicted Gaussian-noise behaviour. This makes it hard to assess what an outlier is.

As shown in Figs. 1 and 2, the power spectral density of the O1 data displays a number of lines. This is well known and was studied in depth by the LIGO team [23]. As a result a large part of these lines have been identified as due to local disturbances acting on the instruments. We exclude from our analysis any frequency bin that is in the range identified by [23] and references therein to
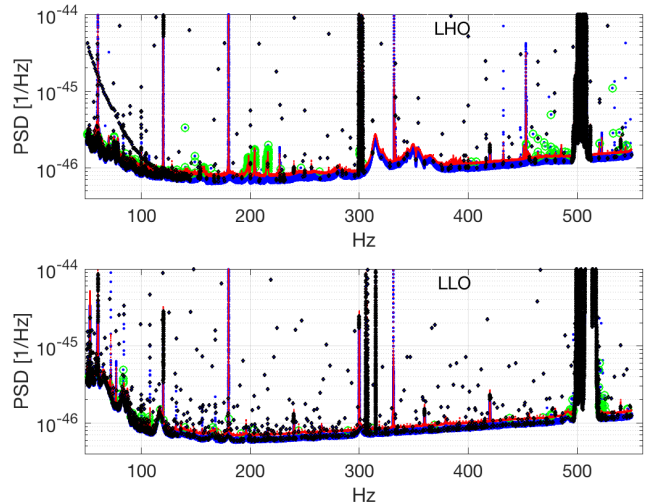

FIG. 1. (Color online) Power spectral density (PSD) for the Hanford (LHO) and Livingston (LLO) detector, estimated with the data used for this search. The blue points are the original data set. The black points indicate frequencies that are vetoed based on known line lists [23]. The red line is the threshold used to select outliers. The green circles indicate the outliers which fall at unvetoed frequencies.

LOSC pages. On this data we further only consider sets of 90 bins whose average of the normalized power spectral density (after removing the highest value) is within 2.5 standard deviations of the Gaussian-noise expectations. These cuts remove between 12% (LLO) and 18% (LHO) of the total number of bins. For the remaining frequencies we set a threshold corresponding to 8 Gaussian standard deviations and cluster together bins that are not more distant than 3 frequency bins. We find 83 such clusters in the LLO data and over 466 in the LHO data, reflecting the different noise conditions of the two detectors. We require that a signal be present in both detectors at similar frequencies and look for clusters in the LIGO and Hanford data that overlap in frequency. We find none and conclude that there is no clear detection of this kind of signal. We proceed to set upper limits on the intrinsic amplitude of such gravitational wave signals h_0^{UL} (Fig. 3a), and then translate these into limits on the CDOs mass (Fig. 3b).

A loud monochromatic signal of amplitude h_0 optimally oriented with respect to the detector will produce a peak in the power spectral density of about $p_0 = h_0^2 T_{\text{SFT}}$. The frequency of the peak will be at the frequency of the signal. If the detector were not optimally oriented with respect to the signal, as is generally the case, the signal amplitude spectral peak will be lower than p_0 by a factor of a few, and this factor will be slightly different in each detector. For a signal coming from the Sun, on average a reduction by a factor of $\simeq 7.8$ is expected with respect
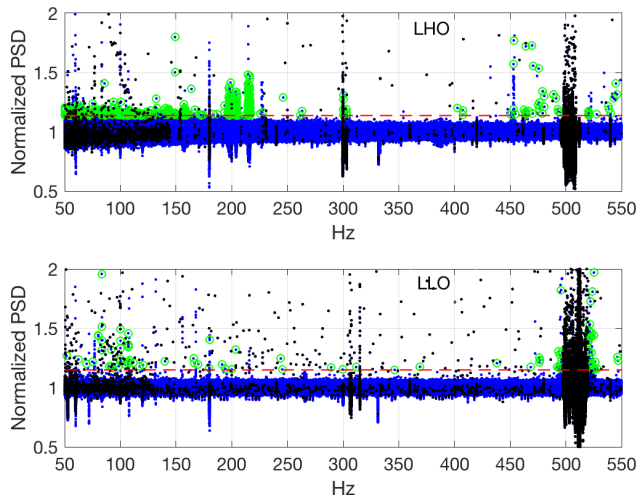
FIG. 2. (Color online) Normalised average power spectral densities. The blue points are the original data set. The black points indicate frequencies that are vetoed based on known line lists [23]. The magenta dashed line is the 8 σ threshold. The green circles indicate the points above threshold that come from frequency bands whose statistic is within the accepted range (the mean is within 2.5σs of the Gaussian noise value.).
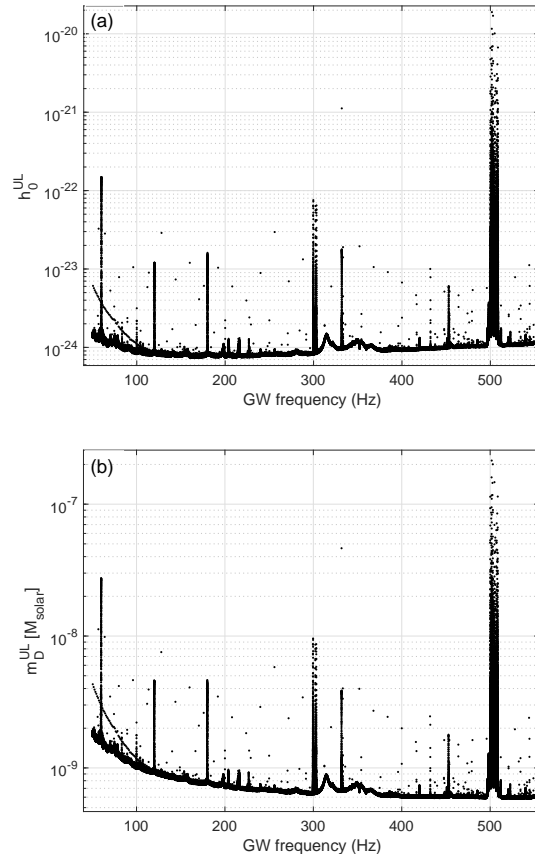
FIG. 3. Intrinsic gravitational amplitude upper limits (a) and upper limits on the mass of compact dark matter object (CDO) binaries orbiting at the center of the Sun (b), as a function of the gravitational wave signal frequency. Based on this search, we can exclude values above these curves.

to optimal orientation.

We translate the observed power spectral density values into intrinsic gravitational wave amplitude upper limits h_0^{UL} at different frequencies as follows:

$$h_0^{UL}(f_k) = \sqrt{\frac{15.6 \ \max_I[p_k^I]}{T_{SFT}}} = \frac{1}{10.7}\sqrt{\max_I[p_k^I]}, \quad (9)$$

where the factor of 15.6 includes an additional reduction in signal power for higher confidence for the least favourable polarizations and inclinations of the source. These upper limits are plotted as a function of the gravitational wave frequency in Fig. 3a. We translate them into upper limits on the mass of CDOs, obtaining the curve shown in Fig. 3b. For many frequencies the limit is less than $10^{-9}M_\odot$ and the most stringent upper limit on the CDO mass is $5.8 \times 10^{-10} \ M_\odot$ at $\simeq 525.5$ Hz.

We have excluded from the search frequency bins that appear polluted by spurious noise that we could not model. We did this, because we would not have been able to assess the significance of a signal candidate from such frequencies. We are however able to place upper limits on the signal amplitude, even at these frequencies. At frequencies with large spectral lines the upper limits are not as constraining as they are in "quieter" bands, but they are still valid.

This is a very simple search that has reached a modest sensitivity depth [24] of ~ 10 Hz$^{-1/2}$. Other searches for continuous signals from known objects typically reach

sensitivity depths of [25] a few hundred Hz$^{-1/2}$ and broad parameter space searches reach a depth of several tens Hz$^{-1/2}$. In the future, we plan to perform more sophisticated searches for CDOs that will likely reach comparable sensitivity depths and may probe CDO masses below $10^{-10}M_\odot$.

In addition to the Sun, it is interesting to search for GWs from CDOs in the Earth and Jupiter. Jupiter's gravity influences much of the solar system and many objects, such as Comet Shoemaker Levy 9, have collided with it. We plan to search for GWs from CDOs in Jupiter. We also plan to search for GWs from CDOs near the center of the Earth. Because of the very small distance to the source (that may be less than a GW wavelength) this search may be sensitive to the lowest CDO masses.

In conclusion, dark matter could be composed of compact dark objects (CDOs). A close binary of CDOs orbiting *inside* solar system bodies can be a loud source of gravitational waves (GWs). We have performed an ini-

tial search for GWs from the Sun, using data from the first Advanced LIGO observing run (O1), that reached a sensitivity of $h_0 \approx 10^{-24}$. This search rules out close binaries of CDOs orbiting near the center of the Sun with GW frequencies (twice the orbital frequency) between 50 and 550 Hz and CDO masses above $\approx 10^{-9} M_\odot$.

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* horowit@indiana.edu
† maria.alessandra.papa@aei.mpg.de
‡ sareddy@uw.edu

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