

# Gravitino condensate in $N = 1$ supergravity coupled to the $N = 1$ supersymmetric Born-Infeld theory

Ryotaro Ishikawa <sup>a</sup> and Sergei V. Ketov <sup>a,b,c,d</sup>

<sup>a</sup> Department of Physics, Tokyo Metropolitan University,  
1-1 Minami-ohsawa, Hachioji-shi, Tokyo 192-0397, Japan

<sup>b</sup> Max Planck Institute for Gravitational Physics (Albert Einstein Institute),  
Science Park Potsdam-Golm, Am Mühlenberg 1, D-14476, Potsdam-Golm, Germany

<sup>c</sup> Research School of High-Energy Physics, Tomsk Polytechnic University,  
2a Lenin Avenue, Tomsk 634050, Russian Federation

<sup>d</sup> Kavli Institute for the Physics and Mathematics of the Universe (WPI),  
The University of Tokyo Institutes for Advanced Study, Kashiwa, Chiba 277-8583, Japan

ishikawa-ryotaro@ed.tmu.ac.jp, ketov@tmu.ac.jp

## Abstract

The  $N = 1$  supersymmetric Born-Infeld theory coupled to  $N = 1$  supergravity in four spacetime dimensions is studied in the presence of a cosmological term with spontaneous supersymmetry breaking. The consistency is achieved by compensating a negative contribution to the cosmological term from the Born-Infeld theory by a positive contribution originating from the gravitino condensate. This leads to an identification of the Born-Infeld scale with the supersymmetry breaking scale. The dynamical formation of the gravitino condensate in supergravity is reconsidered and the induced one-loop effective potential is derived. Slow roll cosmological inflation with the gravitino condensate as the inflaton (near the maximum of the effective potential) is viable against the Planck 2018 data and can lead to the inflationary (Hubble) scale as high as  $10^{12}$  GeV. Uplifting the Minkowski vacuum (after inflation) to a de Sitter vacuum (dark energy) is possible by the use of the alternative Fayet-Iliopoulos term. Some major physical consequences of our scenario to reheating are briefly discussed also.

# 1 Introduction

The gravitino condensate and the gravitino mass gap in  $N = 1$  supergravity [1] coupled to the Volkov-Akulov field [2] in four spacetime dimensions arise as the one-loop effect due to the quartic gravitino interaction coming from the gravitino contribution to the spacetime (con)torsion [3, 4]. This is similar to the Nambu-Jona-Lasinio model [5] of the dynamical generation of electron mass and the formation of Cooper pairs near the Fermi surface in superconductivity. The dynamical gravitino mass also leads to a positive contribution to the vacuum energy and, hence, the dynamical supersymmetry breaking too [6]. Given the standard (reduced) Planck mass as the only (dimensional) coupling constant, the gravitino mass gap should be of the order of Planck scale also, which prevents phenomenological applications of the gravitino condensate to physics under the Planck scale.

However, the effective scale of quantum gravity may be considerably lower than its standard value associated with the (reduced) Planck mass  $M_{\text{Pl}} = 1/\sqrt{8\pi G_N} \approx 2.4 \times 10^{18}$  GeV. It may happen because the effective strength of gravity can depend upon either large or warped extra dimensions in braneworld, or the dilaton expectation value in string theory, or both these factors together [7, 8, 9].<sup>1</sup> The negative results of the Large Hadron Collider (LHC) searches for copious production of black holes imply that the low-scale gravity models may have to be replaced by the high-scale gravity (or supergravity) models, whose effective Planck scale  $\tilde{M}_{\text{Pl}}$  is much higher the TeV scale but is still under the standard scale  $M_{\text{Pl}}$ , i.e.

$$1 \text{ TeV} \ll \tilde{M}_{\text{Pl}} \ll M_{\text{Pl}} . \quad (1)$$

This can be of particular importance to the early Universe cosmology, where the Newtonian limit does not apply, as well as for high-energy particle physics well above the electro-weak scale.

On the other hand, it is expected that Maxwell electrodynamics does not remain unchanged up to the standard Planck scale, because of its internal problems related to the Coulomb singularity and the unlimited values of electro-magnetic field. This motivated Born and Infeld to propose the non-linear vacuum electrodynamics known in the literature as Born-Infeld (BI) theory [11] with the Lagrangian (in flat spacetime)

$$\mathcal{L}_{\text{BI}} = -M_{\text{BI}}^4 \sqrt{-\det(\eta_{\mu\nu} + M_{\text{BI}}^{-2} F_{\mu\nu})} = -M_{\text{BI}}^4 - \frac{1}{4} F^2 + \mathcal{O}(F^4) , \quad (2)$$

where  $\eta_{\mu\nu}$  is Minkowski metric,  $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ , and  $F^2 = F^{\mu\nu} F_{\mu\nu}$ . The constant term on the right-hand-side of Eq. (2) can be ignored in flat spacetime. The BI theory has the new scale  $M_{\text{BI}}$  whose value cannot exceed the GUT scale where electro-magnetic interactions merge with strong and weak interactions. On the other hand, we need  $M_{\text{BI}} < \tilde{M}_{\text{Pl}}$  in order to ignore quantum gravity corrections. The BI theory naturally emerges (i) in the bosonic part of the open superstring effective action [12], (ii) as part of Dirac-Born-Infeld (DBI) effective action of a D3-brane [13], and (iii) as part of Maxwell-Goldstone action describing partial supersymmetry breaking of  $N = 2$  supersymmetry to  $N = 1$  supersymmetry [14, 15].<sup>2</sup> The peculiar non-linear structure of the BI theory is responsible for its electric-magnetic self-duality, taming the Coulomb self-energy of a point-like electric charge, and causal wave propagation (no shock waves and no superluminal propagation) — see e.g., Refs. [20, 21] and the references therein for a review and non-abelian extensions of BI theory.

<sup>1</sup>The effective Planck scale may also be dynamically generated [10].

<sup>2</sup>See Refs. [16, 17, 18, 19] for possible extensions of BI theory to extended supersymmetry and higher dimensions.

In a curved spacetime with metric  $g_{\mu\nu}$  the BI action is usually defined as the difference between two spacetime densities,

$$S_{\text{BI,standard}} = M_{\text{BI}}^4 \int d^4x \left[ \sqrt{-\det(g_{\mu\nu})} - \sqrt{-\det(g_{\mu\nu} + M_{\text{BI}}^{-2} F_{\mu\nu})} \right], \quad (3)$$

where the first term has been added "by hand" in order to eliminate the cosmological constant arising from the second term and in Eq. (2). In this paper we propose the gravitino condensation as the origin and the mechanism of such cancellation in the supergravity extension of the BI theory with spontaneously broken SUSY.

The  $N = 1$  (rigid) supersymmetric extension of BI theory [22] has the second (non-linearly realized) supersymmetry [14] and is self-dual [23]. The  $N = 1$  supersymmetric BI theory coupled to  $N = 1$  supergravity, as the locally supersymmetric extension of Eq. (3), was constructed in Ref. [24]. In the case of spontaneously broken supersymmetry, the fermionic superpartner ("photino") of the BI electro-magnetic field becomes massive and can be identified with the Akulov-Volkov field (goldstino) up to a field redefinition because of the uniqueness of the goldstino action [25, 26].

The supersymmetric BI theory coupled to supergravity can be applied to a description of the cosmological inflation (of the Starobinsky type [27]) and the dark energy (as the positive cosmological constant) [28, 29] by using the alternative (new) Fayet-Iliopoulos (FI) terms [30, 31, 32, 33, 34] without gauging the R-symmetry.<sup>3</sup>

In this paper we reconsider a formation of the gravitino condensate from the above perspective.

Our paper is organized as follows. In Sec. 2 we consider a supersymmetrization of the BI theory (2) instead of (3) in supergravity and relate the BI scale to the spontaneous supersymmetry (SUSY) breaking scale. In Sec. 3 we derive the dynamical gravitino condensate arising from the one-loop effective action of pure supergravity, and study the induced scalar potential. Slow roll inflation with the gravitino condensate playing the role of inflaton is numerically studied in Sec. 4. Uplifting the Minkowski vacuum to a de Sitter vacuum by the use of the alternative FI term is proposed in Sec. 5. Our conclusion is Sec. 6. We use the supergravity notation of Ref. [1].

## 2 Cosmological constant and spontaneous SUSY breaking

The manifestly supersymmetric extension of the BI action (3) minimally coupled to supergravity in curved superspace of the (old-minimal) supergravity (in a superconformal gauge) with the *vanishing* cosmological constant reads (see e.g., [20, 32])

$$S_{\text{SBI}}[V] = \frac{1}{4} \left( \int d^4x d^2\theta \mathcal{E} W^2 + \text{h.c.} \right) + \frac{1}{4} M_{\text{BI}}^{-4} \int d^4x d^2\theta d^2\bar{\theta} E \frac{W^2 \bar{W}^2}{1 + \frac{1}{2}A + \sqrt{1 + A + \frac{1}{4}B^2}}, \quad (4)$$

$$A = \frac{1}{8} M_{\text{BI}}^{-4} (\mathcal{D}^2 W^2 + \text{h.c.}), \quad B = \frac{1}{8} M_{\text{BI}}^{-4} (\mathcal{D}^2 W^2 - \text{h.c.}),$$

where  $\mathcal{E}$  is the chiral (curved) superspace density,  $E$  is the full (curved) superspace density,  $\mathcal{D}^\alpha$  are the covariant spinor derivatives in superspace,  $W^\alpha$  is the chiral gauge-invariant field strength,

$$W_\alpha = -\frac{1}{4} (\bar{\mathcal{D}}^2 - 4\mathcal{R}) \mathcal{D}_\alpha V, \quad (5)$$

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<sup>3</sup>In Refs. [35, 36, 37], the massive vector supermultiplet, unifying inflaton (scalaron) and goldstino, with the DBI and FI terms, was used together with the chiral Polonyi multiplet as the "hidden sector" for spontaneous SUSY breaking, with massive gravitino as the Lightest SUSY Particle (LSP) representing dark matter.

of the gauge real scalar superfield pre-potential  $V$ ,  $\mathcal{R}$  is the chiral (scalar curvature) supergravity superfield,  $W^2 = W^\alpha W_\alpha$  and  $\mathcal{D}^2 = \mathcal{D}^\alpha \mathcal{D}_\alpha$ . In supergravity, spacetime metric  $g_{\mu\nu}$  is replaced by vierbein  $e_\mu^a$  and is extended to an off-shell supermultiplet  $(e_\mu^a, \psi_\mu, B, b_\mu)$ , where  $\psi_\mu$  is (Majorana) gravitino field, whereas the complex scalar  $B$  and the real vector field  $b_\mu$  are the supergravity auxiliary fields.<sup>4</sup> The gauge vector (photon) field  $A_\mu$  is extended in SUSY to an off-shell (real) gauge vector multiplet (or the superfield)  $V$  with the field components

$$V = (C, \beta, H, A_\mu, \lambda, D) , \quad (6)$$

where  $\lambda$  is the Majorana fermion called photino,  $D$  is the auxiliary field, while the rest of the fields  $(C, \chi, H)$  are the super-gauge degrees of freedom that are ignored in what follows.

Adding a negative cosmological constant to Eq. (4), as in Eq. (2), in the setup with unbroken (linearly realized) local SUSY is known to be impossible in supergravity, unless this cosmological constant is compensated by another positive contribution [6]. Indeed, in order to cancel the SUSY variation of the cosmological constant multiplied by  $\sqrt{-\det(g_{\mu\nu})} = e$  due to  $\delta_{\text{susy}} e_\mu^a = \frac{1}{2} \tilde{M}_{\text{Pl}}^{-1} (\bar{\varepsilon} \gamma^a \psi_\mu)$  with the infinitesimal SUSY parameter  $\varepsilon(x)$ , we have to add the photino-gravitino mixing term

$$+ \frac{ie}{\sqrt{2}} \frac{M_{\text{BI}}^2}{\tilde{M}_{\text{Pl}}} (\bar{\lambda} \psi_\mu) \quad (7)$$

to the Lagrangian, and simultaneously demand the supersymmetric variation of photino  $\lambda$  as

$$\delta_{\text{susy}} \lambda = \sqrt{2} M_{\text{BI}}^2 \varepsilon + \dots , \quad (8)$$

where the dots stand for the other field-dependent terms. Hence, the photino  $\lambda$  should be identified with the goldstino of spontaneously broken local SUSY. Moreover, the goldstino field  $\lambda$  can then be gauged away or eaten up by the gravitino  $\psi_\mu$  that was introduced as the gauge field of local SUSY with the transformation law  $\delta_{\text{susy}} \psi_\mu = \tilde{M}_{\text{Pl}} D_\mu \varepsilon + \dots$ . As the immediate consequence of this observation in our setting, we conclude that the BI scale has to be identified with the SUSY breaking scale,

$$M_{\text{BI}} = M_{\text{SUSY}} . \quad (9)$$

The positive contribution  $M_{\text{BI}}^4$  to the cosmological constant in supergravity comes together (by SUSY) with the gravitino mass term having the mass parameter  $m^2 = \frac{1}{3} M_{\text{BI}}^4 / \tilde{M}_{\text{Pl}}^2$ , and the related modification of the gravitino SUSY transformation law as  $\delta_{\text{susy}} \psi_\mu = \tilde{M}_{\text{Pl}} (D_\mu \varepsilon + \frac{1}{2} m \gamma_\mu) + \dots$ . In turn, this also implies (by SUSY) the presence of the goldstino mass term in the Lagrangian with the same mass parameter  $m$  [6]. By construction, the full action also includes the non-trivial self-interaction terms of goldstino, that should be related to the standard Akulov-Volkov contribution [2]

$$S_{\text{AV}} = -M_{\text{BI}}^4 \int d^4 x e \det[\delta_\nu^\mu + \frac{i}{2} M_{\text{BI}}^{-4} (\bar{\lambda} \gamma^\mu \partial_\nu \lambda)] \quad (10)$$

by the field redefinition [25, 26]. The same conclusions are supported by the superconformal tensor calculus in supergravity [38].

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<sup>4</sup>The auxiliary fields of the supergravity multiplet do not play a significant role in our investigation and are ignored below.

### 3 Gravitino condensate (one-loop) effective action

In addition to compensating the negative cosmological constant of the initial BI theory, the gravitino condensate has its own dynamics governed by the one-loop generated kinetic term and scalar potential.

The pure supergravity action  $S_{\text{SUGRA}}$  besides the usual Einstein-Hilbert and Rarita-Schwinger terms also has the quartic gravitino coupling,

$$\mathcal{L}_{\text{quartic}} = \frac{11}{16} \tilde{M}_{\text{Pl}}^{-2} [(\bar{\psi}_\mu \psi^\mu)^2 - (\bar{\psi}_\mu \gamma_5 \psi^\mu)^2] - \frac{33}{64} \tilde{M}_{\text{Pl}}^{-2} (\bar{\psi}^\mu \gamma_5 \gamma_\nu \psi^\mu)^2, \quad (11)$$

originating from the spacetime (con)torsion in the covariant derivative of the gravitino field in its kinetic term, in the second-order formalism for supergravity [1].

Since the pure supergravity action is invariant under the local SUSY, whose gauge field is  $\psi_\mu$ , one can choose the (physical) gauge condition  $\gamma^\mu \psi_\mu = 0$ , which implies  $(\bar{\psi}_\mu \Sigma^{\mu\nu} \psi_\nu) = -\frac{1}{2} \bar{\psi}_\mu \psi^\mu$ , in the notation  $\Sigma_{\mu\nu} = \frac{1}{4} [\gamma^\mu, \gamma^\nu]_-$ , and rewrite the (non-chiral) quartic gravitino term in Eq. (11) as

$$\mathcal{L}_{\text{quartic}} = \sqrt{11} \tilde{M}_{\text{Pl}}^{-1} \rho (\bar{\psi}_\mu \Sigma^{\mu\nu} \psi_\nu) - \rho^2, \quad (12)$$

where the real scalar field  $\rho$  has been introduced. As is clear from Eq. (12), the gravitino condensate leads to the non-vanishing Vacuum Expectation Value (VEV),  $\langle \rho \rangle \equiv \rho_0 \neq 0$ , whereas  $\rho_0$  contributes to the gravitino mass.

The one-loop effective potential  $V_{1\text{-loop}}(\rho)$  of the scalar field  $\rho$  arises after quantizing the gravitino sector and summing up the one-loop gravitino graphs contributing to the propagator of  $\rho$  in supergravity. Hence, the  $V_{1\text{-loop}}(\rho)$  should be of the Coleman-Weinberg-type, the scalar field  $\rho$  itself should become physical, with its propagator having a pole at  $p^2 = -M_c^2$ , where  $M_c$  is the gravitino condensate mass. The results of such calculation are controversial in the literature [3, 4, 39, 40, 41]. We went through those papers and performed our calculations along the same lines. We confirm that the gravitino condensate is real (i.e., is quantum stable), while the one-loop integration over gravitons can be ignored. The 1-loop effective potential in pure supergravity is given by

$$\begin{aligned} V_{1\text{-loop}}(\rho) &= \rho^2 - \frac{4}{(2\pi)^4} \int^\Lambda d^4 p \ln \left( 1 + 11 \tilde{M}_{\text{Pl}}^{-2} \frac{\rho^2}{p^2} \right) \\ &= \rho^2 + \frac{1}{8\pi^2} \left\{ \frac{121\rho^4}{\tilde{M}_{\text{Pl}}^4} \ln \left( 1 + \frac{\tilde{M}_{\text{Pl}}^2 \Lambda^2}{11\rho^2} \right) - \frac{11\rho^2 \Lambda^2}{\tilde{M}_{\text{Pl}}^2} - \Lambda^4 \ln \left( 1 + \frac{11\rho^2}{\tilde{M}_{\text{Pl}}^2 \Lambda^2} \right) \right\}, \end{aligned} \quad (13)$$

where the UV-cutoff scale  $\Lambda$  has been introduced in order to regularize the divergent integral.<sup>5</sup> In addition, the logarithmic scaling of the wave function renormalization in the one-loop approximation yields the factor proportional to  $\ln \left( \frac{\Lambda^2}{\mu^2} \right)$ , where  $\mu$  is the renormalization scale. Hence, the canonical (physical) scalar  $\phi$  is given by [40]

$$\phi = \text{const.} \sqrt{\ln \left( \frac{\Lambda^2}{\mu^2} \right)} \tilde{M}_{\text{Pl}}^{-1} \rho \equiv \tilde{w} M_{\text{Pl}} \sigma, \quad (14)$$

where we have introduced the dimensionless (renormalization) constant  $\tilde{w}$  as the parameter. We also use the other dimensionless quantities

$$\sigma = \tilde{M}_{\text{Pl}}^{-2} \rho, \quad \tilde{M}_{\text{Pl}}^{-1} \Lambda = \tilde{\Lambda} \quad \text{and} \quad \tilde{M}_{\text{Pl}}^{-1} M_{\text{BI}} = \alpha, \quad (15)$$

<sup>5</sup>The first line of Eq. (13) differs by the sign from the result of Ref. [4], but the second line is very different.

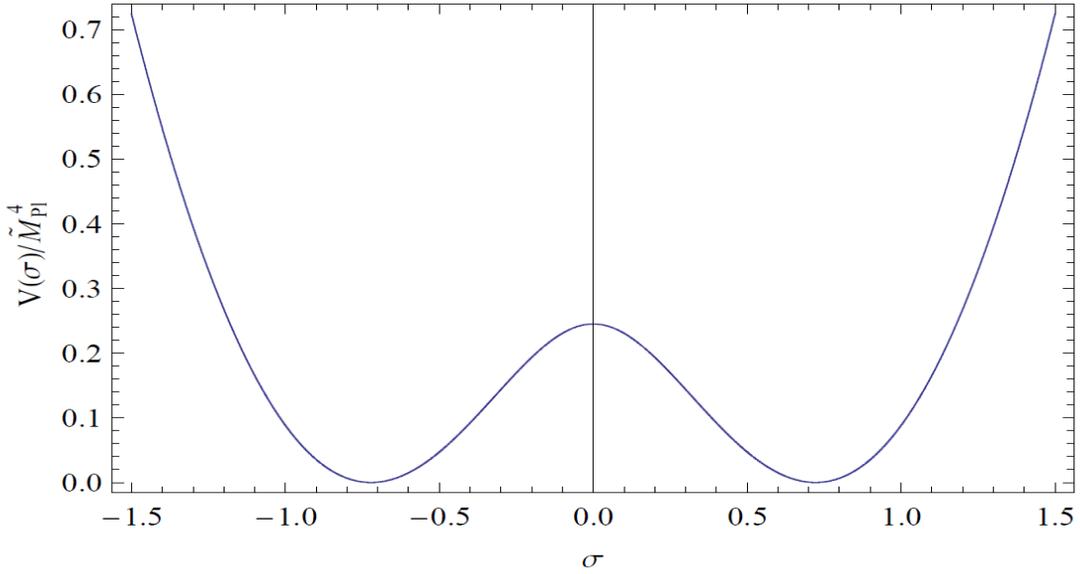


Figure 1: The profile of the  $V(\sigma)$  function in Eq. (16).

which allow us to rewrite the full scalar potential as

$$V(\sigma)\tilde{M}_{\text{Pl}}^{-4} = \sigma^2 - \frac{1}{8\pi^2} \left\{ \tilde{\Lambda}^4 \ln \left( 1 + \frac{11\sigma^2}{\tilde{\Lambda}^2} \right) - 121\sigma^4 \ln \left( 1 + \frac{\tilde{\Lambda}^2}{11\sigma^2} \right) + 11\sigma^2\tilde{\Lambda}^2 \right\} + \alpha^4, \quad (16)$$

where we have added the contribution of the first term on the right-hand-side of Eq. (2).

The scalar potential (16) has the double-well shape and is bounded from below, see Fig. 1, provided that

$$\tilde{\Lambda}^2 > \frac{4\pi^2}{11} \approx 3.59, \quad \text{or} \quad \tilde{\Lambda} > \frac{2\pi}{\sqrt{11}} \approx 1.89. \quad (17)$$

There is a local maximum at  $\rho = \sigma = 0$  with the positive height  $M_{\text{BI}}^4$ . A similar potential near its maximum was used for describing slow-roll inflation with the inflaton field  $\phi$  [40], see the next Sec. 4 for more. There are also two stable Minkowski vacua at  $\rho_c \neq 0$ .

According to the previous Section, supersymmetry requires the scalar potential (16) to vanish at the minimum, i.e.  $V(\sigma_c) = 0$ . In addition, according to Eq. (12), the  $\rho_c \neq 0$  determines the gravitino condensate mass

$$m_\phi = \sqrt{11}\rho_c/\tilde{M}_{\text{Pl}} = \sqrt{11}\tilde{M}_{\text{Pl}}\sigma_c. \quad (18)$$

The non-vanishing values of  $\rho_c$  and  $\sigma_c$  are determined by the condition  $dV/d(\sigma^2) = 0$  that yields a transcendental equation,

$$121\sigma_c^2 \ln \left( 1 + \frac{\tilde{\Lambda}^2}{11\sigma_c^2} \right) = 11\tilde{\Lambda}^2 - 4\pi^2 > 0. \quad (19)$$

The hierarchy between the inflationary scale  $H_{\text{inf}}$ , the BI scale  $M_{\text{BI}}$ , the SUSY breaking scale  $M_{\text{SUSY}}$ , the (super)GUT scale  $M_{\text{GUT}}$ , the effective gravitational scale  $\tilde{M}_{\text{Pl}}$  and the Planck scale

$M_{\text{Pl}}$  in our approach reads

$$H_{\text{inf.}} \ll M_{\text{BI}} = M_{\text{SUSY}} \approx M_{\text{GUT}} \approx \tilde{M}_{\text{Pl}} \ll M_{\text{Pl}} \quad , \quad (20)$$

where "much less" means the 2-3 orders of magnitude "less" (in GeV), and "approximately" means the same order or magnitude, see the next Section for our numerical estimates. As regards the GUT scale, we take  $M_{\text{GUT}} \approx \mathcal{O}(10^{15})$  GeV.

## 4 Gravitino condensate as inflaton

A slow-roll inflation induced by gravitino condensation in supergravity was proposed and studied by Ellis and Mavromatos in Ref. [40]. Since our induced scalar potential differs from that of Ref. [40], we reconsider this inflation in this Section, by using  $\tilde{\Lambda}$  and  $\tilde{w}$  as the phenomenologically adjustable parameters.

A slow roll is possible near the maximum of the scalar potential (16). Since the height of the potential at the maximum is related to the inflationary Hubble scale  $H_{\text{inf.}}$  by Friedmann equation,

$$V_{\text{max.}} = 3M_{\text{Pl}}^2 H_{\text{inf.}}^2 \quad , \quad (21)$$

the value of  $H_{\text{inf.}}/M_{\text{Pl}}$  is suppressed by the factor  $(\tilde{M}_{\text{Pl}}/M_{\text{Pl}})^2$ . On the other hand, the inflationary Hubble scale is related to the Cosmic Microwave Background (CMB) tensor-to-scalar ratio  $r$  as

$$\frac{H_{\text{inf.}}}{M_{\text{Pl}}} = 1.06 \cdot 10^{-4} \sqrt{r} \quad . \quad (22)$$

In turn, the  $r$  is restricted by Planck (2018) measurements [43] as  $r < 0.064$  (with 95% CL), which implies  $H_{\text{inf.}} < 6 \cdot 10^{13}$  GeV. Therefore, the ratio  $(\tilde{M}_{\text{Pl}}/M_{\text{Pl}})$  should be of the order  $10^{-2} \div 10^{-3} \ll 1$  for viable inflation. This justifies our setup in the Introduction (Sec. 1). We define the dimensionless parameter  $\gamma$  as  $(\tilde{M}_{\text{Pl}}/M_{\text{Pl}}) \equiv 10^{-3}/\gamma$ , where  $\gamma$  is of the order one.

In our numerical calculations, we have chosen the cutoff scale  $\tilde{\Lambda} = 3$ , so that the restriction (17) is satisfied. Then Eqs. (16) and (19) imply

$$V_{\text{max}} \tilde{M}_{\text{Pl}}^{-4} = 0.245 \quad \text{and} \quad \sigma_{\text{cr.}} = \tilde{w}^{-1}(\phi_{\text{cr.}}/M_{\text{Pl}}) = 0.722 \quad . \quad (23)$$

In turn, this implies

$$m_{3/2} = 2.39 \tilde{M}_{\text{Pl}} \quad \text{and} \quad m_{\text{cond.}} = \sqrt{8/11} m_{3/2} = 2.038 \tilde{M}_{\text{Pl}} \quad . \quad (24)$$

We numerically studied the running of the slow inflationary parameters  $\varepsilon = \frac{1}{2} M_{\text{Pl}}^2 (V'/V)^2$  and  $\eta = M_{\text{Pl}}^2 (V''/V)$  with respect to the inflaton field  $\phi$  for the values of the parameter  $\gamma$  as 0.1, 0.5 and 1, and found that  $\varepsilon$  is always under  $\mathcal{O}(10^{-4})$  so that it can be ignored within the errors of Planck 2018 data. Then the value of the scalar index  $n_s = 1 - 6\varepsilon + 2\eta = 0.9649 \pm 0.0042$  (with 68% CL) [43] can be reached with  $\eta = -0.0177$  at the horizon crossing by using the parameter  $\tilde{w}$  of the order one. There are no additional constraints on the parameters  $\gamma$  and  $\tilde{w}$  from demanding the e-folding number

$$N_e = -\frac{1}{M_{\text{Pl}}^2} \int_{\phi_{\text{ini.}}}^{\phi_{\text{end}}} \frac{V}{V'} d\phi \quad (25)$$

to be between 50 and 60, as is desired for viable inflation, when assigning the inflaton field  $\phi/M_{\text{Pl}}$  to run somewhere between 0 and 5 during inflation. The running of the slow-roll parameter  $\eta$  is displayed in Fig. 2.

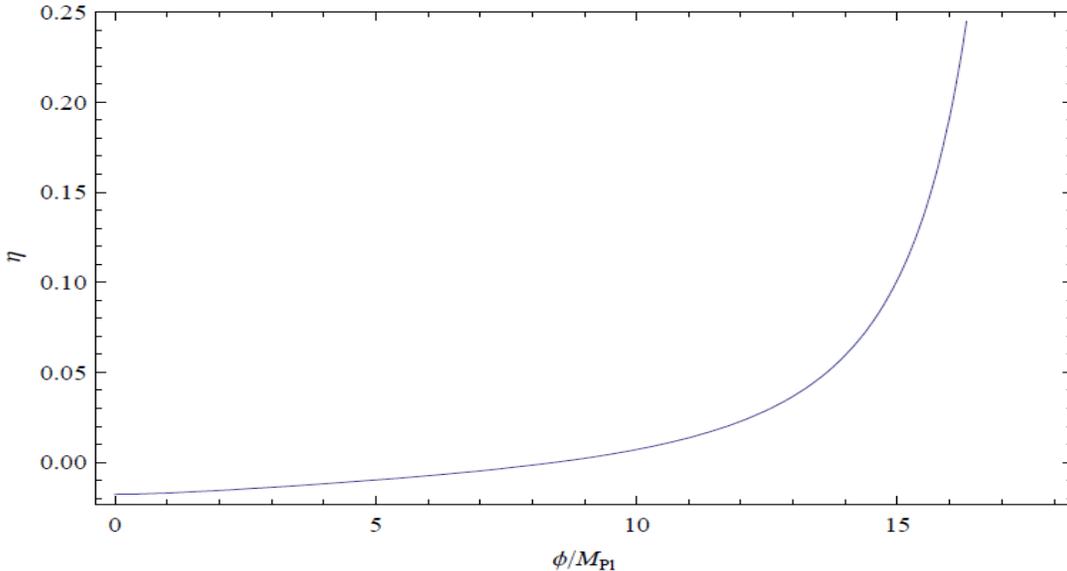


Figure 2: The running of the slow-roll parameter  $\eta$  for  $\gamma = 0.5$  and  $\tilde{w} = 13$ .

In summary, our results qualitatively agree with those of Ref. [40], but quantitatively allow considerably higher values of  $\varepsilon$  and  $r$  up to the order  $\mathcal{O}(10^{-4})$  contrary to  $\mathcal{O}(10^{-8})$  of Ref. [40], with the Planckian values of the inflaton  $\phi$  during inflation, contrary to its sub-Planckian values of  $\mathcal{O}(10^{-3})M_{\text{Pl}}$  in Ref. [40]. Hence, the inflationary scale  $H_{\text{inf}}$  can be as high as  $10^{12}$  GeV contrary to  $10^{10}$  GeV in Ref. [40].

## 5 Adding the FI term

In order to uplift the Minkowski vacuum to a de Sitter vacuum (dark energy) in our approach, we need an extra tool of spontaneous SUSY breaking. In the BI theory (without chiral matter) coupled to supergravity such tool can be provided by the (new) FI terms [31, 32, 33, 34] that do not require the gauged R-symmetry, unlike the standard FI term [30] whose extension to supergravity is severely restricted [42].

The (abelian) gauge vector multiplet superfield  $V$  can be decomposed into a sum of the reduced gauge superfield  $\mathcal{V}$  including the gauge field  $A_\mu$ , and the nilpotent gauge-invariant goldstino superfield  $\mathcal{G}$  that contains only goldstino  $\lambda$  and the auxiliary field  $D$  [32],

$$V = \mathcal{V} + \mathcal{G}, \quad \mathcal{G}^2 = 0. \quad (26)$$

The simplest examples of the goldstino superfield are given by [31, 32]

$$\mathcal{G}_1 = -4 \frac{W^2 \bar{W}^2}{\mathcal{D}^2 W^2 \bar{\mathcal{D}}^2 \bar{W}^2} (\mathcal{D}W) \quad (27)$$

and

$$\mathcal{G}_2 = -4 \frac{W^2 \bar{W}^2}{(\mathcal{D}W)^3}, \quad (28)$$

respectively, in terms of the standard  $N = 1$  gauge superfield strength

$$W_\alpha = -\frac{1}{4} (\bar{\mathcal{D}}^2 - 4\mathcal{R}) \mathcal{D}_\alpha V , \quad (29)$$

where  $\mathcal{R}$  is the chiral scalar curvature superfield. The  $W_\alpha$  obeys Bianchi identities

$$\bar{\mathcal{D}}_{\dot{\beta}} W_\alpha = 0 \quad \text{and} \quad \bar{\mathcal{D}}_{\dot{\alpha}} \bar{W}^{\dot{\alpha}} \equiv \bar{\mathcal{D}} \bar{W} = \mathcal{D}^\alpha W_\alpha \equiv \mathcal{D} W . \quad (30)$$

The field components are given by  $W_\alpha| = \lambda_\alpha$ ,  $\mathcal{D}W| = -2D$ , and  $\mathcal{D}_{(\alpha} W_{\beta)}| = i(\sigma^{ab})_{\alpha\beta} F_{ab} + \dots$ . The difference between the superfields  $\mathcal{G}_1$  and  $\mathcal{G}_2$  is only in the gauge sector, while it is not essential for our purposes here.

The extra FI term with the coupling constant  $\xi \neq 0$  is given by

$$S_{\text{FI}} = \xi \int d^4x d^4\theta E G , \quad (31)$$

where  $E$  is the supervielbein (super)determinant [1]. This FI term is manifestly SUSY- and gauge-invariant, does *not* include the higher spacetime derivatives of the field components, but leads to the inverse powers of the auxiliary field  $D$  (up to the fourth order) in the non-scalar sector of the theory.<sup>6</sup> Integrating out the auxiliary field  $D$  leads to the *positive* contribution to the cosmological constant

$$V_\xi = \frac{1}{2} \xi^2 > 0 . \quad (32)$$

Matching  $V_\xi$  with the observed cosmological constant allows us to include a viable description of the dark energy into our approach.

## 6 Conclusion

The gravitino condensate can be considered as a viable candidate for inflaton in supergravity, when assuming the effective (quantum) gravity scale to be close to the (super)GUT scale that is also close to the SUSY breaking scale in our approach, with all scales close to  $10^{15}$  GeV. Actually, in this scenario we have the hyper-GUT where *all* fundamental interactions merge, including gravity. At the same time, it is the weak point of our calculations, because we ignored (other) quantum gravity corrections.

The inflationary (Hubble) scale is well below the GUT scale, and can be as large as  $10^{12}$  GeV. The gravitino mass is above the inflationary scale, so that there is no gravitino overproduction problem in the early Universe. The constraints from proton decay and the Big Bang Nucleosynthesis (BBN) are very weak because of high-scale supersymmetry. Then SUSY is not a solution to the hierarchy problem with respect to the electro-weak scale. This is similar to the setup studied in Refs. [44, 45, 46]. Our scenario is consistent with the known Higgs mass of about 125 GeV after taking into account the extreme possible values of the gaugino mixing parameter  $\tan \beta$  in the context of SUSY extensions of the Standard Model [47].

As regards reheating after inflation, the inflaton (gravitino condensate) field decays into other matter and radiation, which is highly model-dependent, as usual. Unlike Ref. [37], the inflaton as the gravitino condensate cannot decay into gravitinos because Eq. (24) leads to the kinematical constraint  $2m_{3/2} > m_{\text{cond}}$ . It also implies that gravitino cannot be a dark matter particle in this scenario. A detailed study of reheating requires the knowledge of couplings of gravitino and gravitino condensate to the Standard Model particles, which is beyond the scope of this paper.

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<sup>6</sup>The limit  $\xi \rightarrow 0$  does not lead to a well defined theory, so that  $\langle D \rangle = \xi$  must be non-vanishing.

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