Stochastic gravitational wave background anisotropies: astrophysical dependencies in the LIGO/Virgo and LISA bands

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This letter characterizes the properties of the angular power spectrum in both the LIGO/Virgo and LISA frequency bands. It explores a wide range of self-consistent astrophysical models all calibrated such that they predict the same number of resolved mergers to fit the actual LIGO/Virgo number of detections during O1+O2 observations runs. It shows that the anisotropies depend on various ingredients of the astrophysical models, such as the black hole formation model, the distribution of initial binary parameters and the presence of a mass gap in the black hole mass distribution. This indicates that the anisotropies of the stochastic background contain additional information on sub-galactic processes, complementary to the isotropic background and individual mergers. This letters also presents the first predictions of the anisotropies in the LISA frequency band.

INTRODUCTION

The stochastic gravitational-wave (GW) background is generated by the superposition of various unresolved astrophysical and cosmological sources [1–9]. Based on the recent observations of merging black holes (BHs) and neutron star binaries by the Advanced LIGO and Advanced Virgo detectors [10–19], it is estimated that the stochastic background from unresolved stellar-mass binaries may be detected within a few years of operation of the LIGO-Virgo network [20, 21].

Since these astrophysical sources reside in galaxies which cluster on large scales, it is expected that the astrophysical gravitational waves background (AGWB) will be direction-dependent, and thus include an anisotropic component. The anisotropies of the AGWB, as first derived in Refs. [22, 23], and studied in Refs. [24–26] and in Refs. [27, 28] (see Ref. [29] for a critical analysis of these last two works) depend on three key ingredients: *i)* the underlying cosmology *ii)* the large scales structure or galaxy clustering and *iii)* the local astrophysics on sub-galactic scales. The standard cosmological model and its parameters are now constrained with a high precision, and the evolution and properties of the large scales structure are also well understood at the scales relevant for the AGWB (see Ref. [29] for details on this latter point). The sub-galactic astrophysics, in particular the formation and evolution of binary compact objects, is however less understood and more difficult to constrain with the usual electromagnetic observables. It follows that the AGWB can potentially provide new information on the relevant astrophysical processes, such as stellar evolution, BH formation rate and mass distribution, as well as the properties of the host galaxies of GW sources. This letter demonstrates the sensitivity of the AGWB anisotropies to sub-galactic astrophysical modeling. This work assumes the standard cosmological model including structure formation using the values of the cosmological parameters from *Planck* [30].

The angular average (i.e. monopole) of the AGWB signal is defined as the energy density in GW per logarithmic frequency interval in units of the critical density of the Universe: $\Omega_{\rm GW} = {\rm d}\rho_{\rm GW}/{\rm d}\ln f/\rho_c$. It can also be written as the sum of contributions from sources located at all the (comoving) distances r in the form $\Omega_{\rm GW}(f) \equiv \bar\Omega(f) = \int \partial_r\bar\Omega(f,r){\rm d}r$. Each astrophysical model predicts a functional dependence of $\partial_r\bar\Omega(f,r)$. In addition, in the Limber approximation, the general expression [22] of the angular power spectrum of the anisotropies of the AGWB reduces [29] to

$$C_\ell(f) \simeq \left(\ell+\tfrac{1}{2}\right)^{-1}\int {\rm d}k P(k)\left|\partial_r\bar\Omega(f,r)\right|^2 , \qquad (1)$$

where ℓ is the multipole in the spherical harmonic expansion, $P(k)$ is the galaxy power spectrum, and with the constraint $kr = \ell + 1/2$. Hence, the multipoles are sensitive to the shape of $\partial_r\bar\Omega$, and in particular to the low-redshift value for low ℓ (see Refs. [26, 29] for details and demonstration).

The latest analysis [21] of the first and second LIGO observing runs lead to the upper bound on the isotropic background of $\Omega_{\rm GW}(f = 25{\rm Hz}) < 4.8\times10^{-8}$, assuming a population of compact binary sources, and $\Omega_{\rm GW}(f = 25{\rm Hz}) < 6\times10^{-8}$ for a frequency-independent background. The anisotropic component is constrained by LIGO observations up to $\ell = 4$ [31] which result in upper limits on the amplitude in the range $\Omega_{\rm GW}(f = 25{\rm Hz}, \Theta) < 0.64 - 2.47 \times 10^{-8}$ sr^{-1} for a population of merging binary compact objects, where Θ denotes the angular dependence. Methods to measure and map the AGWB in the LIGO and LISA (Laser Interferometer Space Antenna) frequency ranges are discussed in Refs. [1–9]

ASTROPHYSICAL BUILDING BLOCKS

This *letter* computes the angular power spectrum of the anisotropies in the AGWB from merging stellar-mass binary BHs using the astrophysical framework described in Refs. [32–34], which we summarize below. All the details, as well as additional results, are provided in the companion article [26].

Technically, for each model, we compute the GW luminosity $\mathcal{L}_{\rm G}(z, \nu_{\rm G}, M_{\rm G})$ of a given galaxy as a function of halo mass $M_{\rm G}$ and redshift z. This goes in a series of steps. First, we calculate the SFR, $\psi(M_{\rm G}, t)$, and the stellar-to-halo mass ratio using a modified version of the abundance-matching relations of Ref. [35]. We use a Salpeter-like IMF [36] to describe the number of stars per unit total stellar mass formed, $\phi = {\rm d}N/{\rm d}M_* {\rm d}M_{{\rm tot},*} \propto M_*^{-p}$, where M_* is the mass of the star at formation. Massive stars collapse to BHs, and we assume that the remnant mass depends only on the mass of the progenitor star, M_*, and on its metallicity, Z. This is encoded in a function $m = g_s(M_*, Z)$, that needs to be computed for each model. We adopt the observational relation of Ref. [37] for the metallicity of the interstellar medium as a function of galaxy stellar mass and redshift. We also introduce a cut-off at high BH masses $M_{\rm co}$, that also depends and the astrophysical model.

Under these assumptions, the instantaneous BH formation rate at cosmic time t (or, equivalently, redshift z) for a galaxy with halo mass $M_{\rm G}$ in units of events per unit BH mass m is given by $\mathcal{R}_1(m, t) = \psi[M_{\rm G}, t]\phi(M_*) \times {\rm d}M_*/{\rm d}m$ where $M_*(m)$ and ${\rm d}M_*/{\rm d}m$ are deduced from the relation $m = g_s(M_*, Z)$. Then, we assume that only a fraction β of these BHs resides in binary systems that merge within the age of the Universe, so that the rate of formation of the latter is $\mathcal{R}_2(m, t) = \beta\mathcal{R}_1(m, t)$. The overall factor β is used to normalize our model with respect to the number of events observed by LIGO/Virgo. Following Ref. [34], the birth rate of binaries with component masses $(m, m' \le m)$ is $\mathcal{R}_{\rm bin}(m, m') = \mathcal{R}_2(m)\mathcal{R}_2(m')P(m, m')$ ($P(m, m')$ being a normalized distribution describing the association of mass m and m' in a BH binary system).

Since BH binaries are expected to circularize rapidly due to GW radiation reaction before reaching the LIGO/Virgo band (see e.g. Ref. [38]), we assume circular orbits in what follows. The distribution of the delay (or merger) time, $\tau_{\rm m}(m, m', a_{\rm f})$, between formation and mergers can be expressed through a distribution $f(a_{\rm f})$ of the semi-major axis at the formation of the binary and of the binary mass distribution. Then the birth rate of BH binaries (per unit mass squared per unit time and per units of $a_{\rm f}$) is $\mathcal{R}_f[m, m', a_{\rm f}, t] = \mathcal{R}_{\rm bin}(m, m')f(a_{\rm f})$ from which we deduce the merger rate at time t, $\mathcal{R}_{\rm m}[m, m', a_{\rm f}, t] = \mathcal{R}_f[m, m', a_{\rm f}, t - \tau_{\rm m}(m, m', a_f)]$. Finally, the quantity $\partial_r\bar{\Omega}$ is given by the total contribution of merging binary BHs in the entire galaxy population, weighted by the halo mass function ${\rm d}n/{\rm d}M_{\rm G}(M_{\rm G}, z)$ given in Ref. [39].

As reference astrophysical model we choose a Salpeter-like IMF with slope $p = 2.35$. The BH formation model $m = g_s(M_*, Z)$ was taken to be the "delayed"' model in Ref. [40] with a cutoff mass of $M_{\rm co} = 45M_\odot$ [41]. We assume $P(m, m') = $ cst for the distribution of masses in the binary, and a distribution of the semi-major axis *at formation* of $f(a_{\rm f}) \propto a_{\rm f}^{-1}$ between $a_{\rm f,min} = 0.014$ AU and $a_{\rm f,max} = 4000$ AU. The lower bound was chosen so as to ensure that the lightest BH binaries in our model $[(5, 5)M_\odot]$ merge within a Hubble time.

The normalization β is adjusted so as to match the number of detections by aLIGO/aVirgo during the O1+O2 observing runs [19]. We therefore require that all our models result in 10 detectable events over the span of the O1+O2 observation time. Our estimate of the detection rates follows Ref. [34], namely we calculate the signal-to-noise rate (SNR) for each binary BH merger produced in the model using the noise power spectral densities from Refs. [42, 43] and a correction factor following Ref. [44] to account for different source orientations. The GW strain is calculated using the PhenomB template [45] assuming zero spins. We define observed events as those with SNR > 8. The number of sources detectable during O1+O2 is given by multiplying the detection rate by the total observation time $T_{\rm obs} = 169.7$ days.

Again, we emphasize that while individual mergers are resolved only at low z, the AGWB depends on the whole redshift distribution up to high redshifts. Therefore, even though all the models are calibrated to predict the same number of resolved sources, their resulting AGWB may vary if the high-redshift population of sources differs among the models.

ASTROPHYSICAL MODELS

We consider a series of models, each varying from the reference model described above in one key aspect.

BH formation model. BH masses depend on the properties of their stellar progenitors (mostly their masses, chemical composition and rotation velocity e.g. Refs. [46–49]). The formation of *binaries* is further influenced by common envelope evolution in the case of isolated binaries, and various dynamical processes in the case of stellar clusters [50–52]. In this letter, we assume that binary formation process is encoded in the efficiency parameter β and the distribution of merger time delays. Furthermore, we restrict this study for simplicity to only one aspect of this complex problem, namely the evolution of isolated massive stars.

Our reference model uses the description of Ref. [40] which provides an analytic model for a neutrino-driven explosion and calculate the explosion energy, as well as the remnant mass, using numerical pre-collapse stellar models from [53]. Another set of stellar evolution models is provided in Ref. [49]. These models differ from those in Ref. [40] in two aspects. First, Ref. [49] uses a different set of pre-collapse stellar models which vary from Ref. [53] in their treatment of convection, mass-loss rate and angular momentum transport. Second, Ref. [49] assumed a constant explosion energy in the calculation of the remnant mass, contrary to Ref. [40]. As was shown in Ref. [34], these models predict different mass distributions of detectable BHs. In the following, the model *limongi* uses the model described in Ref. [49] without stellar rotation.

BH mass cutoff. Very massive stars (typically in the range $130-260M_\odot$) are unstable to electron-positron pair creation which may lead to a pair-instability supernova (PISN) that disrupts the entire star and prevents the formation of a BH [54]. The absence of BHs in the mass range $60-260M_\odot$ may provide an indirect confirmation of this effect. A cutoff in BH mass may be present at even lower masses due to pulsational PISN, where the instability causes short episodes of mass ejection followed by periods of quiescent evolution [55, 56]. As a result, the stellar mass is reduced below the limit of the onset of the instability. It was suggested in Ref. [57] that this process may lead to an excess of BHs around $\sim 40M_\odot$. Recent analysis of the LIGO/Virgo events detected during the O1+O2 observational runs [41] provides a tentative measurement of the BH mass cutoff at $M_{\rm co} = 45M_\odot$. In order to explore the sensitivity of the stochastic background to the PISN-induced mass cutoff we varied it from $M_{\rm co} = 40M_\odot$ [*dMco*-models] to $M_{\rm co} = 50M_\odot$ [*uMco*-models].

Stellar initial mass function. Our reference model assumes a Salpeter-like IMF with slope $p = 2.35$. Interestingly, some studies suggest that the IMF slope may not be universal (see e.g. the discussion in Ref. [58]), for example a recent hint to a more shallow IMF in the Large Magellanic Cloud [59]. In order to estimate the influence of the IMF we explore two models where the slope was taken to be $p = 2.6$ [*imfHi*-models] and $p = 2.1$ [*imfLo*-models].

Distribution of initial separations. The reference model assumes the initial semi-major axis of the BH binaries to be distributed according to $f(a_{\rm f}) \propto a_{\rm f}^{-1}$. This translates into a distribution of merger delay times of $f(t_{\rm delay}) \propto t_{\rm delay}^{-1}$, favoring short delay times. We also consider an extreme scenario [*aconst-model*] of flat distribution of the initial separations $f(a_{\rm f}) \sim$ const, which results in longer delay times.

Metallicity Metal-rich stars experience strong winds which can considerably reduce their masses. Therefore, the masses of the remnant BHs are very sensitive to the metallicity of the progenitors stars (e.g. Ref. [40]). This effect translates into a dependence on host galaxy mass and redshift, since low-mass and/or high-redshift galaxies typically contain less metals. To explore this dependence we considered a model with a constant metallicity of $10^{-3}Z_\odot$, which corresponds to an early stellar population.

RESULTS

Figure 1 depicts the AGWB monopole as a function of frequency (in LISA and LIGO/Virgo frequency bands), for the different astrophysical models explored in this letter. Fig. 2 presents the relative difference, $\delta C_\ell/C_\ell \equiv (C_\ell^{\rm mod} - C_\ell^{\rm ref})/C_\ell^{\rm ref}$ between the angular power spectra of each model (mod) and the reference model (ref) at the frequency of $f = 63$ Hz that lies in the LIGO band. It can be seen that the anisotropic component exhibits some differences, reaching 60% even for the low multipoles. Note also that the shape of the spectra differs from the reference model for the modified $f(a_{\rm f})$ [*aconst*] model, while the other models result in a renormalisation of the power independent of the multipole. This result indicates that some of these parameters can be independently constrained from AGWB anisotropies.

Figure 3 focuses on the prediction of the angular power spectrum of anisotropies in the LISA band, for the reference astrophysical model. Since the contribution to this frequency band from merging stellar-mass BHs is from the inspiralling part of the waveform, the frequency dependence of the angular power spectrum is exactly the same as of the monopole so that the relative anisotropies are frequency independent, see Ref. [26] for a detailed discussion of this latter point. The fractional difference between angular spectra of different astrophysical models in the LISA bands is very similar to that shown in Fig. 2 for the LIGO band. A full analysis is presented in the companion article [26].

CONCLUSIONS

This letter explored the astrophysical dependences of the anisotropies of the AGWB, considering the con-

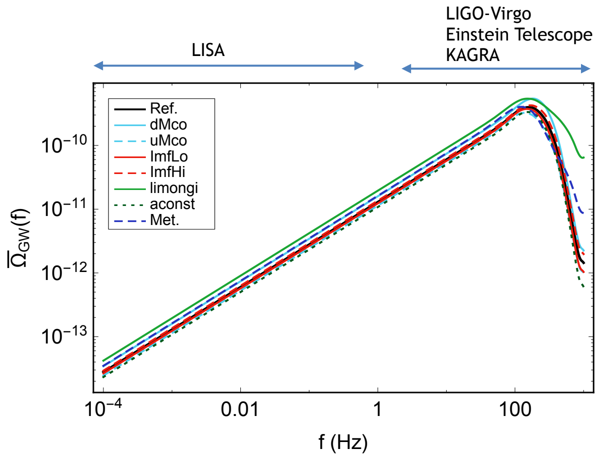

FIG. 1: Monopole of the energy density of the AGWB for the different astrophysical models in this letter, as a function of frequency. [*dMco*/*uMlco* for the BH mass cut-off; *imfHi*/*imfLo* for the IMF slope; *limongi* for the BH formation model; *aconst* for $f(a_{\rm f})$; *Met* for constant metallicity].

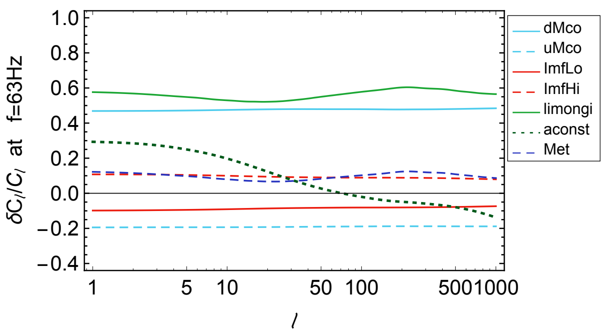

FIG. 2: Fractional difference between the angular power spectrum of anisotropies in different astrophysical models and in the reference model at the frequency $f = 63$ Hz.

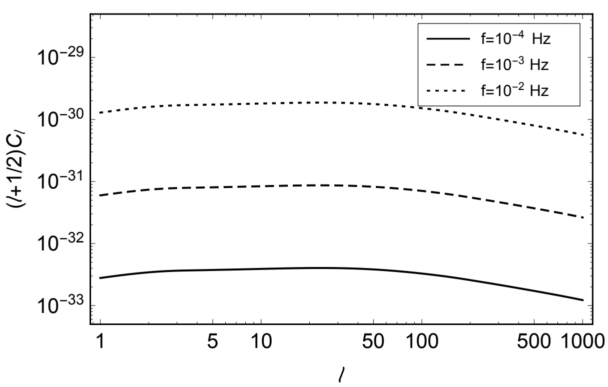

FIG. 3: Angular power spectrum of anisotropies for three frequencies in the LISA band. Multiplication by $\ell + 1/2$ emphasizes the large-scale behaviour of Eq. (1).

tribution of stellar-mass binary BH mergers in both the LIGO/Virgo and LISA frequency bands.

Our analysis concludes that the anisotropies of the AGWB are very sensitive to some astrophysical parameters such as the distribution of the initial semi-major axis of the binaries (essentially, the delay time distribution), the BH formation scenario and the metallicity of the progenitor stars. Quantitatively, we find that changing the BH cut-off mass $M_{\rm co}$ by $\sim 10\%$ results in a relative variation in anisotropies of up to 50% for small multipoles ($\ell < 10$), while using a consant low metallicity results in differences of order $\sim 20\%$ across the entire multipole range. These results hold for both LIGO and LISA frequency bands.

We demonstrate that the amplitude and shape of the angular power spectrum are very sensitive to the astrophysical modeling, in contrast with the conclusions of Ref. [60] (see Ref. [29] for a critical analysis of this work). We also show that some of the parameters do not affect the shape of the spectra but only their amplitude.

These results give the first characterization of the parameters that can in principle be constrained by this new observable. We showed that the monopole and the anisotropies of the AGWB give access to complementary information. While the former is sensitive to the integral over redshift of the astrophysical kernel describing sub-galactic physics, anisotropies at different angular separations are sensitive to the amplitude of this kernel at different redshifts. This can be easily seen from Eq. (1) where the Limber approximation has been used to relate distances to multipoles. This letter also gave the first study of the angular power spectrum of anisotropies in the LISA band for stellar-mass binary BH mergers.

Our work clarifies the role of the astrophysical assumptions that underpin the various predictions of the angular power spectra. We emphasize that since cosmological structure formation processes are well-understood, all the modelling uncertainty is related to the formation and evolution of GW sources. The anisotropies of the AGWB will therefore provide a new observational tool to study these physical processes.

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[1] B. Allen and A. C. Ottewill, Phys. Rev. **D56**, 545 (1997), gr-qc/9607068.
[2] N. J. Cornish, Class. Quant. Grav. **18**, 4277 (2001), astro-ph/0105374.
[3] S. Mitra, S. Dhurandhar, T. Souradeep, A. Lazzarini, V. Mandic, S. Bose, and S. Ballmer, Phys. Rev. **D77**, 042002 (2008), 0708.2728.
[4] E. Thrane, S. Ballmer, J. D. Romano, S. Mitra, D. Talukder, S. Bose, and V. Mandic, Phys. Rev. **D80**, 122002 (2009), 0910.0858.
[5] J. D. Romano, S. R. Taylor, N. J. Cornish, J. Gair, C. M. F. Mingarelli, and R. van Haasteren, Phys. Rev. **D92**, 042003 (2015), 1505.07179.
[6] J. D. Romano and N. J. Cornish (2016), 1608.06889.
[7] A. I. Renzini and C. R. Contaldi (2018), 1806.11360.

[8] B. P. Abbott et al. (Virgo, LIGO Scientific), Phys. Rev. Lett. **118**, 121102 (2017), 1612.02030.

[9] N. Christensen, arXiv e-prints arXiv:1811.08797 (2018), 1811.08797.

[10] F. Acernese, M. Agathos, K. Agatsuma, D. Aisa, N. Allemandou, A. Allocca, J. Amarni, P. Astone, G. Balestri, G. Ballardin, et al., Classical and Quantum Gravity **32**, 024001 (2015), 1408.3978.

[11] LIGO Scientific Collaboration, Classical and Quantum Gravity **32**, 074001 (2015), 1411.4547.

[12] B. P. Abbott et al. (Virgo, LIGO Scientific), Phys. Rev. Lett. **116**, 061102 (2016), 1602.03837.

[13] B. P. Abbott et al. (Virgo, LIGO Scientific), Phys. Rev. Lett. **116**, 241103 (2016), 1606.04855.

[14] B. P. Abbott et al. (Virgo, LIGO Scientific), Phys. Rev. **X6**, 041015 (2016), 1606.04856.

[15] B. P. Abbott et al. (Virgo, LIGO Scientific), Astrophys. J. **851**, L35 (2017), 1711.05578.

[16] B. P. Abbott et al. (VIRGO, LIGO Scientific), Phys. Rev. Lett. **118**, 221101 (2017), 1706.01812.

[17] B. P. Abbott et al. (Virgo, LIGO Scientific), Submitted to: Phys. Rev. Lett. (2017), 1709.09660.

[18] B. Abbott et al. (Virgo, LIGO Scientific), Phys. Rev. Lett. **119**, 161101 (2017), 1710.05832.

[19] The LIGO Scientific Collaboration and the Virgo Collaboration, arXiv e-prints arXiv:1811.12907 (2018), 1811.12907.

[20] B. P. Abbott et al. (Virgo, LIGO Scientific), Phys. Rev. Lett. **120**, 091101 (2018), 1710.05837.

[21] The LIGO Scientific Collaboration and the Virgo Collaboration, arXiv e-prints arXiv:1903.02886 (2019), 1903.02886.

[22] G. Cusin, C. Pitrou, and J.-P. Uzan, Phys. Rev. **D96**, 103019 (2017), 1704.06184.

[23] G. Cusin, C. Pitrou, and J.-P. Uzan, Phys. Rev. **D97**, 123527 (2018), 1711.11345.

[24] G. Cusin, I. Dvorkin, C. Pitrou, and J.-P. Uzan, Phys. Rev. Lett. **120**, 231101 (2018), 1803.03236.

[25] G. Cusin, R. Durrer, and P. G. Ferreira, Phys. Rev. **D99**, 023534 (2019), 1807.10620.

[26] G. Cusin, I. Dvorkin, C. Pitrou, and J.-P. Uzan, submitted to Phys. Rev. **D** (2019).

[27] A. C. Jenkins, M. Sakellariadou, T. Regimbau, and E. Slezak, Phys. Rev. **D98**, 063501 (2018), 1806.01718.

[28] A. C. Jenkins, R. O'Shaughnessy, M. Sakellariadou, and D. Wysocki (2018), 1810.13435.

[29] G. Cusin, I. Dvorkin, C. Pitrou, and J.-P. Uzan (2018), 1811.03582.

[30] N. Aghanim et al. (Planck) (2018), 1807.06209.

[31] The LIGO Scientific Collaboration and the Virgo Collaboration, arXiv e-prints arXiv:1903.08844 (2019), 1903.08844.

[32] I. Dvorkin, J.-P. Uzan, E. Vangioni, and J. Silk, Phys. Rev. **D94**, 103011 (2016), 1607.06818.

[33] I. Dvorkin, E. Vangioni, J. Silk, J.-P. Uzan, and K. A. Olive, Mon. Not. Roy. Astron. Soc. **461**, 3877 (2016), 1604.04288.

[34] I. Dvorkin, J.-P. Uzan, E. Vangioni, and J. Silk, Mon. Not. Roy. Astron. Soc. **479**, 121 (2018), 1709.09197.

[35] P. S. Behroozi, R. H. Wechsler, and C. Conroy, Astrophys. J. **770**, 57 (2013), 1207.6105.

[36] E. E. Salpeter, Astrophys. J. **121**, 161 (1955).

[37] X. Ma, P. F. Hopkins, C.-A. Faucher-Giguère, N. Zolman, A. L. Muratov, D. Kereš, and E. Quataert, Mon. Not. Roy. Astron. Soc. **456**, 2140 (2016), 1504.02097.

[38] M. E. Lower, E. Thrane, P. D. Lasky, and R. Smith, Phys. Rev. D. **98**, 083028 (2018), 1806.05350.

[39] J. Tinker, A. V. Kravtsov, A. Klypin, K. Abazajian, M. Warren, G. Yepes, S. Gottlöber, and D. E. Holz, Astrophys. J. **688**, 709 (2008), 0803.2706.

[40] C. L. Fryer, K. Belczynski, G. Wiktorowicz, M. Dominik, V. Kalogera, and D. E. Holz, Astrophys. J. **749**, 91 (2012), 1110.1726.

[41] The LIGO Scientific Collaboration and the Virgo Collaboration, arXiv e-prints arXiv:1811.12940 (2018), 1811.12940.

[42] L. S. Collaboration, lIGO-G1401390.

[43] L. S. Collaboration, lIGO-G1801952.

[44] L. S. Finn and D. F. Chernoff, Phys. Rev. D **47**, 2198 (1993), gr-qc/9301003.

[45] P. Ajith, M. Hannam, S. Husa, Y. Chen, B. Brügmann, N. Dorband, D. Müller, F. Ohme, D. Pollney, C. Reisswig, et al., Physical Review Letters **106**, 241101 (2011), 0909.2867.

[46] E. O'Connor and C. D. Ott, Astrophys. J **730**, 70 (2011), 1010.5550.

[47] I. Mandel and S. E. de Mink, Mon. Not. Roy. Astron. Soc. **458**, 2634 (2016), 1601.00007.

[48] P. Marchant, N. Langer, P. Podsiadlowski, T. M. Tauris, and T. J. Moriya, Astron. Astrophys. **588**, A50 (2016), 1601.03718.

[49] M. Limongi, *Supernovae from Massive Stars* (2017), p. 513.

[50] M. Dominik, K. Belczynski, C. Fryer, D. E. Holz, E. Berti, T. Bulik, I. Mandel, and R. O'Shaughnessy, Astrophys. J. **759**, 52 (2012), 1202.4901.

[51] K. Belczynski, A. Heger, W. Gladysz, A. J. Ruiter, S. Woosley, G. Wiktorowicz, H.-Y. Chen, T. Bulik, R. O'Shaughnessy, D. E. Holz, et al., Astron. Astrophys. **594**, A97 (2016), 1607.03116.

[52] C. L. Rodriguez, S. Chatterjee, and F. A. Rasio, Phys. Rev. D **93**, 084029 (2016), 1602.02444.

[53] S. E. Woosley, A. Heger, and T. A. Weaver, Reviews of Modern Physics **74**, 1015 (2002).

[54] D. Kasen, S. E. Woosley, and A. Heger, Astrophys. J. **734**, 102 (2011), 1101.3336.

[55] S. E. Woosley, S. Blinnikov, and A. Heger, Nature **450**, 390 (2007), 0710.3314.

[56] S. E. Woosley and A. Heger, in *Very Massive Stars in the Local Universe*, edited by J. S. Vink (2015), vol. 412 of *Astrophysics and Space Science Library*, p. 199, 1406.5657.

[57] C. Talbot and E. Thrane, Astrophys. J. **856**, 173 (2018), 1801.02699.

[58] M. R. Krumholz, Physics Reports **539**, 49 (2014), 1402.0867.

[59] F. R. N. Schneider, O. H. Ramírez-Agudelo, F. Tramper, J. M. Bestenlehner, N. Castro, H. Sana, C. J. Evans, C. Sabín-Sanjulián, S. Simón-Díaz, N. Langer, et al., Astron. Astrophys. **618**, A73 (2018), 1807.03821.

[60] A. C. Jenkins, R. O'Shaughnessy, M. Sakellariadou, and D. Wysocki (2018), 1810.13435.