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Gravitational wave bounds on dirty black holes

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Detection of gravitational waves has provided a new way to test black hole (BH) models. We show how simple constraints can be obtained for models that go beyond vacuum Einstein gravity solutions of binary BH mergers. Generic stationary metrics, termed dirty BHs in the literature, are not vacuum solutions of the Einstein equations. These models are, however, general enough to describe BHs surrounded by matter fields. Gravitational wave constraints already rule out certain parts of parameter space for these solutions, including certain parameters describing objects without horizons that have recently been studied in the context of pseudo-complex general relativity.

KEYWORD

gravitational waves – black hole physics – methods: observational – relativistic processes – binaries: general

1 | INTRODUCTION

Einstein's theory of general relativity (GR) is one of the most successful scientific theories of all times. In the second decade of the twentieth century it replaced Newton's theory of gravity as the dominant paradigm to describe gravitational physics. While Newtonian gravity successfully described a wide range of terrestrial and solar system observations, careful measurements backed by detailed calculations were able to show that the solar system is not described by Newtonian gravity.

One interesting aspect of Einstein's theory is that it predicts its own incompleteness through the celebrated singularity theorems. These theorems provide a number of conditions, which if fulfilled in our Universe, imply that Einstein's theory cannot be a complete description of gravitation. Although quantum effects are expected to modify gravity with Planck-scale curvature, the singularity theorems do not actually depend on quantum mechanics.

While it is widely expected that Planck-scale curvature will be attained at the Big Bang and the centers of black holes (BHs), BH entropy, and the information paradox suggest that vacuum GR may even fail profoundly on horizon scales. Because of this, a complete theoretical understanding of how BHs behave on horizon scales is still lacking.

With the detections of coalescing binary BHs with gravitational waves (Abbott et al. 2016a, 2016c, 2016d, 2017a, 2018a) by the Advanced LIGO detectors (Aasi et al. 2015; Abbott et al. 2013, 2016b; Abramovici et al. 1992; Harry 2010), we now have observational tools to probe horizon-scale physics for dynamical BHs. In a certain sense, BH gravitational wave data is now ahead of the theoretical tools needed to exploit the observations. If there are modifications to vacuum Einstein gravity on horizon scales, then gravitational wave data may be sensitive to them. But there is still great uncertainty as to what form any such modifications should take (Healy et al. 2012; Hirschmann et al. 2017; Okounkova et al. 2017).

In the absence of specific model predictions, a number of generic tests have been performed on gravitational wave data looking for deviations from Einstein's GR. So far, the data has been found consistent with GR (Abbott et al. 2016a, 2016d, 2016e, 2017a, 2017b).

To emphasize the role of universal geometric properties in BH physics, the concept of a dirty BH was introduced by Visser (1992). These were not intended to be exact solutions of certain theories with certain matter filed content, but rather spacetime metrics with sufficient generality to describe generic BHs. The versions that we will focus on here are those describing stationary solutions. Stationary solutions, such as the Kerr metric, are expected to be reasonably accurate descriptions for a large number of BHs in our Universe and even in the case of binary BH inspirals, numerical simulations of vacuum Einstein gravity support the idea that the near-horizon geometry of inspiralling BHs are well described by stationary solutions at sufficient accuracy for the current generation of gravitational wave detectors (Gupta et al. 2018).

Dirty BHs can also encompass solutions of non-Einstein theories. One such example is pseudo-complex general relativity (pcGR) (Hess & Greiner (2009), following Moffat (1979), Mann & Moffat (1981, 1982), Kelly & Mann (1986), and Einstein (1945, 1948). One of the features believed to be associated with pcGR is a strong modification of near-horizon physics. The theory has been used to justify the nonexistence of horizons in gravitational collapse (Hess et al. 2010; Hess & Boller 2018). The relationship between pcGR and gravitational waves has been previously studied in Hess (2016, 2017) and Nielsen & Birnholtz (2018).

2 | MODEL AND SPACETIME METRIC

To obtain the functional form of the metric functions in a specific theory, we should solve the equations of motion. However, in situations where this is difficult, or the exact form of the equations of motion or couplings to matter fields is unknown, one can also proceed more generically and study spacetime metrics that contain certain symmetries and undetermined metric functions whose form would ultimately be determined by a full solution. We adopt here a dirty BH metric of the following form:

$$g_{tt} = -\left(1 - \frac{\psi}{\Sigma}\right) \quad , \quad g_{rr} = \frac{\Sigma}{\Delta} \quad , \quad g_{\theta\theta} = \Sigma,$$
$$g_{\phi\phi} = \left((r^2 + a^2) + \frac{a^2\psi}{\Sigma}\sin^2\theta\right)\sin^2\theta,$$
$$g_{t\phi} = g_{\phi t} = -a\frac{\psi}{\Sigma}\sin^2\theta, \tag{1}$$

with $\Sigma = r^2 + a^2\cos^2\theta$ and $\Delta = r^2 + a^2 - \psi(r)$. If the function $\psi(r)$ is chosen to satisfy $\psi = 2Mr$, then this solution is just the Kerr solution of vacuum GR with mass M and specific angular momentum $a = \chi M$. However, more general functional forms are possible. A metric of this form was derived within the context of pcGR (Caspar et al. 2012). In the work of Caspar et al. (2012), $\psi(r)$ was taken to be of the form $\psi = 2m(r)r$, where

$$m(r) = M - \frac{B}{2r^n} = M\,g(r), \quad g(r) = \left[1 - b\left(\frac{M}{r}\right)^n\right]. \tag{2}$$

Here, b is a new dimensionless parameter controlling the modification from the Kerr solution. Its value in the vacuum GR Kerr solution is zero. Quasi-normal modes for such dirty BHs were studied by Medved et al. (2004).

An interesting feature of this metric is that the parameter b can be chosen large enough such that the metric does not contain a horizon. The Killing horizons of the Killing vector field $k^a = \delta^a_t - \Omega\delta^a_\phi$ are located at the zeroes of $r^2 + a^2 - 2m(r)r = 0$, where $\Omega = g_{t\phi}/g_{\phi\phi}$. No horizons exist when this equation does not have real positive solutions, which occurs for b greater than $b_{\max_\mathrm{H}} = \Upsilon^n_{\max_\mathrm{H}}\left(1 - \frac{\chi^2}{2\Upsilon_{\max_\mathrm{H}}} - \frac{\Upsilon_{\max_\mathrm{H}}}{2}\right)$, where $\Upsilon_{\max_\mathrm{H}} = (n + \sqrt{n^2 - (n^2-1)\chi^2})/(n+1)$. Thus for sufficiently large b, BHs can be said to be nonexistant (Hess et al. 2010; Hess & Boller 2018). This limiting value is largest when $\chi = 0$; in the $n = 2$ case, it takes the value 16/27.

Equatorial circular orbits have four velocities, u^a, given by

$$u^a = \frac{\mathrm{d}t}{\mathrm{d}\lambda}\delta^a_t + \frac{\mathrm{d}\phi}{\mathrm{d}\lambda}\delta^a_\phi, \tag{3}$$

where λ parameterizes the orbital path and can be chosen to be the proper time in the case of time-like orbits. For these orbits to be geodesic (Hess & Greiner 2009; Schönenbach et al. 2014), bound only by gravity, we should additionally solve the geodesic equation $u^a\nabla_a u^b = 0$. For the metric (1), the only nontrivial equation of the four geodesic equations is the r-component. This condition suffices to determine the functions $\frac{\mathrm{d}t}{\mathrm{d}\lambda}$ and $\frac{\mathrm{d}\phi}{\mathrm{d}\lambda}$ up to an overall normalization as a function of the r-coordinate. Since the orbital frequency observed asymptotically is given by $\omega = \mathrm{d}\phi/\mathrm{d}t$, the geodesic equation for the metric components of (1) gives $(\omega a - 1)^2(m - m' r) - \omega^2 r^3 = 0$, where $'$ denotes an r-derivative. In the limit of $m' = 0$, this gives the expected behavior for the Kerr spacetime (Bardeen et al. 1972), and further in the Schwarzschild limit of $a = 0$, it reduces to the familiar Kepler-like relation between r and ω (Abbott et al. 2017b).

3 | BOUNDS FROM THE PRE-MERGER INSPIRAL PHASE

In order to solve the two-body problem for two objects of comparable mass orbiting each other, more is needed than just the one-body metric (1). In Newtonian gravity, the two-body problem of bound gravitational orbits is solved by the Keplerian orbits. These Keplerian orbits, along with the Einstein quadrupole formula for gravitational wave emission, can be used to infer the basic properties of the binary source of GW150914 (Abbott et al. 2017b).

Beyond the Newtonian order, post-Newtonian (PN) corrections to the orbits also impact the gravitational wave signal for relativistic systems. These relativistic corrections are regulated (Blanchet 2014; Blanchet et al. 1995; Cutler & Flanagan 1994) by the PN parameter $x \sim (v/c)^2$, the dimensionless spins, and the mass ratio $q = M_1/M_2$, with $M = M_1 + M_2$ being the total mass and $\mu = M_1M_2/M = Mq/(1+q)^2$ the reduced mass. For the dirty BH metric 1, we can add the parameter b, which regulates the relative strength of the modification to the function ψ. In the Newtonian and PN regimes, $x \sim M/r \sim (M\omega)^{2/3}$. From the factor $g(r)$ in (2), it can be seen that every appearance of b involves suppression by at least the nth pN order. Thus the leading order correction to the Newtonian frequency and phase evolutions depends on the modification, and we approximate the b-dependent PN term to a leading order in b.

We maintain that the leading radiation effect is quadrupolar and that the relevant time derivatives of the mass monopole (total energy) and mass dipole (total momentum) vanish at the leading order. In the wave zone, we expect the same relation between the metric perturbation and the source quadrupole as in vacuum GR, and so we expect the same wave polarizations and multipole decomposition. We therefore treat the generation of gravitational waves as governed by a modified quadrupole formula

$$\dot{E}_{\rm GW} = -\frac{32}{5}\frac{G}{c^5}\mu^2 r^4 \omega^6 g^{\varrho}(r), \quad (4)$$

where we allow for a possible deviation from the GR quadrupole formula with a subleading term $g^{\varrho}(r)$. Examining the energy carried by the waves far away suggests adopting $\varrho = 0$, while we note that Hess (2016) used $\varrho = 1$ for regulating this emission.

The output of gravitational waves removes orbital energy from the system, and we assume it descends through quasi-circular orbits. Hess (2016) considered an orbital energy of the form

$$E_{\rm orb} = -\frac{Gm_1(r)m_2(r)}{2r}, \quad (5)$$

and we generalize it to an expansion to second order in b:

$$E_{\rm orb} = -\frac{GM\mu}{2r}\left[1 - b\left(\frac{M}{r}\right)^n Q + b^2\left(\frac{M}{r}\right)^{2n} Q_2\right], \quad (6)$$

where $Q = \frac{1+q^n}{(1+q)^n}$ and $Q_2 = \frac{q^n}{(1+q)^{2n}}\mathfrak{f}(n, \epsilon_1, \epsilon_2)$ depend on the mass ratios and distributions.[1] We note that $Q = 1$, $Q_2 = 0$ corresponds to the model of Hess (2016, 2017). We also introduce, for the deviation from vacuum GR, the shorthand

$$\widetilde{g}(r) = 1 - g(R) = b\left(\frac{M}{r}\right)^n, \quad \frac{d\widetilde{g}(r)}{dr} = -\frac{n}{r}\widetilde{g}(r), \quad (8)$$

such that to leading order in the deviation

$$\frac{dE_{\rm orb}}{dr} = \frac{GM\mu}{2r^2}[1 - (n+1)Q\widetilde{g}(r)], \quad (9)$$

which can be used in the equation for the energy balance equation $\dot{E}_{\rm GW} = \dot{E}_{\rm orb} = E'_{\rm orb}\dot{r}$ to find

$$-\frac{32}{5}\frac{G}{c^5}\mu^2 r^4\omega^6 g^{\varrho}(r) = \frac{GM\mu}{2r^2}[1 - (n+1)Q\widetilde{g}(r)]\dot{r}. \quad (10)$$

We note that for large b, the gravitational potential may have a minimum at finite $r = M^n\sqrt{(n+1)bQ}$. At this point, the energy balance approximation will certainly fail. Before reaching that point, the orbital angular velocity ω can be eliminated from (10) by noting that, for quasi-circular orbits, there is a relation between ω and r given, similar to Kepler's third law, by

$$\omega^2 = \frac{GM}{r^3}[1 - (n+1)Q\widetilde{g}(r)], \quad (11)$$

Using (10), this results in

$$\dot{r} = -\frac{64}{5}\frac{G^3M^2\mu}{r^3c^5}[1 - (n+1)Q\widetilde{g}(r)]^2 g^{\varrho}(r), \quad (12)$$

which can be solved numerically to obtain the orbit.

To obtain the phase evolution of the orbital motion and of the gravitational wave, it is useful to work in the frequency domain in the PN framework (to leading and next-to-leading orders). Differentiating the Keplerian relation (11) with respect to time yields after some algebra

$$[3 - n(n+1)Q\widetilde{g}(r)]\frac{\dot{r}}{r} = -2\frac{\dot{\omega}}{\omega}. \quad (13)$$

and we can change from r to ω using Equations (11) and (13)

$$r = \left[\frac{GM}{\omega^2}\right]^{1/3}\left[1 - \frac{(n+1)}{3}Q\widetilde{g}(\omega)\right], \quad (14)$$

$$\dot{r} = -\frac{2}{3}\frac{\dot{\omega}}{\omega}r\left[1 + \frac{n(n+1)}{3}Q\widetilde{g}(\omega)\right], \quad (15)$$

with $\widetilde{g}(\omega) = \widetilde{g}(r(\omega)) = b(M\omega)^{2n/3}$; $g^{\varrho}(r) = 1 - \varrho\widetilde{g}(\omega)$. Hence instead of (12) we have

$$\dot{\omega} = \frac{96}{5}\frac{(G\mathcal{M})^{5/3}}{c^5}\omega^{11/3}[1 - \mathfrak{B}_{nq\varrho}(\omega)], \quad (16)$$

$$\dot{f} = \frac{96}{5}\frac{\pi^{8/3}(G\mathcal{M})^{5/3}}{c^5}f^{11/3}[1 - \mathfrak{B}_{nq\varrho}(\omega)], \quad (17)$$

as the new chirp equations, with the standard chirp mass $\mathcal{M} = (M^2\mu^3)^{1/5}$ and with $\widetilde{g}(\omega)$ and numerical prefactors collected into the modification at n-PN $\mathfrak{B}_{nq\varrho}(\omega)$

$$\mathfrak{B}_{nq\varrho}(\omega) = \left(\frac{(n+2)(n+1)}{3}Q + \varrho\right) b(M\omega)^{2n/3}. \quad (18)$$

We note also (17) in terms of the gravitational wave frequency $f = \omega/\pi$ (twice the orbital frequency), with $\mathfrak{B}_{nq\varrho}(f) = \mathfrak{B}(\omega = \pi f)$.

This $\mathfrak{B}_{nq\varrho}(f)$ modification can be compared to known bounds on PN coefficients from gravitational wave observations. Using (17) in the integrals for the time and for the phase (compare Cutler & Flanagan 1994)[2]

$$t = t_c + \int\frac{df}{\dot{f}} = t_c - \frac{5c^5(\pi f)^{-8/3}}{256(G\mathcal{M})^{5/3}}\left[1 - \frac{4}{n-4}\mathfrak{B}_{nq\varrho}(f)\right], \quad (19)$$

$$\phi = 2\pi\int f\,dt = 2\pi\int\frac{f}{\dot{f}}df = -\frac{c^5}{16(\pi G\mathcal{M}f)^{5/3}}\left[1 - \frac{5}{2n-5}\mathfrak{B}_{nq\varrho}(f)\right], \quad (20)$$

[1]The form factor

$$\mathfrak{f}(n, \epsilon_1, \epsilon_2) = n\sum_{k=1}^{n+2}(-1)^k\frac{\Gamma(n+k-2)\Gamma(n+2-k)}{\Gamma(n+2)\Gamma(n-1)} \cdot \left[x^{-(n+2-k)}\sum_{m=0}^{1}(-1)^{m(n-1)}(x+(-1)^m)^{-(n+k-2)}\right]\Bigg|_{\epsilon_1}^{1-\epsilon_2} \quad (7)$$

can also depend on the how the singularities near $r = 0$ are regularized, but we neglect such complexities here. In the simplest model, $\mathfrak{f}(n, \epsilon_1, \epsilon_2) = 1$.

[2]These forms must be trivially modified to apply to $n = 2.5, 4$ where the integrals for ϕ and t, respectively, give the logarithms of πMf rather than its powers.

TABLE 1 Pseudo-complex general relativity (pcGR) post-Newtonian (PN) coefficients: a pcGR modification of order *n* introduces an *n*-PN modification to the GR phase evolution. The final column gives the approximate 90% credible intervals on the deviations from the vacuum Einstein gravity values obtained from the two loudest events in the first advanced LIGO observing run (Abbott et al. 2016a)

n	$\Upsilon_{\max_H}$	$b_{\max_H}$	$p_n^{\text{pc-GR}}$	p_n^{GR}	δ_ϕ (%)	Range (δ_ϕ)
1	1	0.5	20/9	6.44	34	(−20%, 5%)
2	4/3	16/27	320/27	46.2	26	(−130%, 15%)
3	1.5	27/32	−225/8	−652	4.3	(−110%, 10%)

and then applying the stationary phase approximation (Cutler & Flanagan 1994), we obtain

$$\Psi = 2\pi f t_c - \phi_c - \pi/4 + \frac{3}{128(\pi G\mathcal{M}f)^{5/3}} \times \left[1 + \frac{20}{(n-4)(2n-5)}\mathfrak{B}_{nq\varrho}(f)\right]. \quad (21)$$

This form can be compared directly with the expected PN coefficients in vacuum GR of Buonanno et al. (2009) (following Iyer & Will 1993; Will & Wiseman 1996; and Blanchet & Faye 2000), and with the limits set on deviations from them by the observed gravitational waves in the inspiral regime in Abbott et al. (2016a, 2016e) (based on Arun et al. 2006; Li et al. 2012; Mishra et al. 2010; Talmadge et al. 1988; Yunes & Pretorius 2009). This comparison is summarized in Table 1 for the leading pcGR PN terms for $n = 1$, 2, 3 and the corresponding GR PN phase coefficients of orders 1, 2, and 3. All coefficients are calculated for the fiducial equal mass nonspinning case ($q = 1, a = 0$); the pcGR coefficients are calculated for the critical $b_{\max_H}$ value of Equation (2), where the horizon vanishes, and for $\varrho = 0$. The table also compares to the 90% credible intervals set on the relative deviations $(p_n^{mod\text{-GR}} - p_n^{GR})/p_n^{GR}$ established for the two loudest events in LIGO's first observation run O1. These observational bounds are obtained by varying individually the PN phase coefficients in models of vacuum GR. Here we compare them to the leading-order deviations from dirty BHs. Although generically there will also be next-to-leading order corrections, which will complicate a direct comparison with observational results, we have ignored this subtle issue here, since any deviation that does occur will have a dominant effect at the leading order.

The limits on the deviations of PN coefficients can be translated into limits on b, which for $n = 1$, 2, and 3 are $|b| \leq 0.85$, 2.96, 118 respectively. This suggests that $b(\pi Mf)^{2n/3}$ is indeed a small parameter throughout the system's evolution and that modifications from the dirt of dirty BHs should not produce large deviations from the standard vacuum GR PN inspiral. In particular, these should not largely affect the chirp mass $\mathcal{M}$. For the GW150914 data, estimating the chirp mass directly from f and $\dot{f}$ at different inspiral times using the Newtonian approximation (0PN, $b = 0$) shows that it remains nearly constant up to a frequency of ∼150 Hz (Abbott et al. 2017b) and is equal to roughly 30 solar masses. This corresponds to $(\pi Mf)^{2/3} \sim 0.17$, which is consistent with treating $\mathfrak{B}_{nq\varrho}(f)$ only at leading order but is inconsistent with the much larger modified chirp mass (and correspondingly higher redshift) estimated in Hess (2016, 2017) from the late part of the orbit.

The pcGR model of Hess (2016, 2017) can be considered under the dirty BH formalism as the case $n = 2$, $Q = 1$, $Q_2 = 0$, $\varrho = 1$. For the critical value of $b = 16/27$, which delimits the horizon-forming solutions, the leading coefficient p_2^{GR} changes from 320/27 to 800/27, which changes δ_ϕ from 26 to 65% of the GR value. As Table 1 indicates, this value is well beyond the range observed and reported by LIGO's O1 results. Thus, at least for the parameter values and approximations adopted here, these events are not consistent with horizonless objects in pcGR.

4 | CONCLUSIONS

Fully simulating binary BH mergers is difficult. Even in vacuum Einstein gravity, the efforts of a full community have been necessary to build models sufficient for comparison with gravitational wave data. In situations that contain matter, quantum effects, or non-Einstein gravity, very little is known about how full simulations should look.

We have shown how generic dirty BH models can be used as a first indication of the types of effects these complications will induce. In this work, we have focused on the inspiral regime, when the two objects are still orbiting each other. It is also possible to extend this to other regimes observable with gravitational waves, such as the post-merger phase (Cardoso et al. 2016; Nielsen et al. 2018; Nielsen & Birnholtz 2018; Westerweck et al. 2018). While we have compared our results to those of inspiralling, heavy BH-like objects (Abbott et al. 2016a), it is also possible to use constraints from the inspiral of neutron stars, such as the system GW170817 (Abbott et al. 2018b). Tests with gravitational waves are complementary to observations with other techniques such as accretion disk studies and imaging of supermassive BHs.

Models with a well-behaved Newtonian limit require the chirp mass of the inspiralling system to be broadly consistent with the values found using vacuum Einstein's GR, and this bounds both the total mass and luminosity distance to be broadly consistent with those found using Einstein GR.

We have discussed the model in terms of a dimensionless parameter b and power index n in Equation (2). For sufficiently large values of b, the objects can be horizonless. We find that horizonless objects in the $n = 1$ case are already ruled out, independent of other solar system constraints (Will 2006).

The critical horizonless case with $n = 2$ and $b = 16/27$ discussed in Hess (2016) is in disagreement with the gravitational wave data. For $n = 3$, the leading-order correction from pcGR is a 3PN term, which is less tightly constrained by current LIGO observations. Higher terms at 4PN and beyond are not yet fully calculated in GR, so a direct comparison with these terms is not yet possible.

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