



## Supplement of

## The quasi-equilibrium framework revisited: analyzing long-term $CO_2$ enrichment responses in plant-soil models

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## Supplementary Materials

The following Supporting Information is available for this article:

Method S1 Supplementary equations used in this study

Figure S1 Illustration of effect of allocation flexibility on plant response to elevated  $CO_2$  and nitrogen deposition.

Figure S2 Sensitivity test of varying N uptake parameters on CO<sub>2</sub> fertilization effect.

Figure S3 Sensitivity test of varying priming effect parameters on the M-term  $CO_2$  fertilization effect.

**Table S1** Relationship of NPP and leaching N rate at different timescales

## Method S1. Supplementary equations used in this study

N:C ratio of root is linearly related to N:C ratio of leaf, as:

$$n_r = 0.7 n_f \tag{Eq. S1}$$

N:C ratio of wood, if variable, is also linearly related to N:C ratio of leaf, as:

$$n_w = 0.005 n_f$$
 (Eq. S2)

N:C ratio of leaf litter is related to leaf litter as:

$$n_{fl} = 0.5 n_f \tag{Eq. S3}$$

Following McMurtrie et al. (2000), nitrogen use efficiency is defined as:

$$NUE = 1/(a_f n_f + a_w n_w + a_r n_r)$$
 (Eq. S4)

**Table S1** Relationship of NPP (kg m<sup>-2</sup> yr<sup>-1</sup>) and leaching N rate (N<sub>loss</sub>, kg m<sup>-2</sup> yr<sup>-1</sup>) at different timescales, assuming explicit mineral N pool. VL400 is the very long term timescale under CO<sub>2</sub> = 400 ppm, and M<sub>800</sub>, L<sub>800</sub> and VL<sub>800</sub> are medium, long, and very long term equilibrium point under CO<sub>2</sub> = 800 ppm. Leaching rate is assumed at 0.05 yr<sup>-1</sup>, which is assumed to be the sum of all N lost from the system (i.e. leaching, denitrification, etc.). Detailed derivations and assumptions provided in Section 3.2 of the manuscript.

Timescale	NPP (kg m <sup>-2</sup> yr <sup>-1</sup> )	$N_{loss}$ (kg m <sup>-2</sup> yr <sup>-1</sup> )	
VL <sub>400</sub>	1.68		0.4
M <sub>800</sub>	1.79		0.23
L <sub>800</sub>	1.90	0	.395
VL <sub>800</sub>	1.91		0.4



**Figure S1** Illustration of effect of allocation flexibility on plant response to elevated CO<sub>2</sub> and nitrogen deposition, with a) as no coupling between allocation of leaf and wood, and b) linear coupling between leaf and wood allocation. No coupling assumes that allocation to wood ( $a_w$ ) = 0.6, and allocation to leaf ( $a_f$ ) = 0.2. Linear coupling assumes the same  $a_f$ , but  $a_w$  = 3 $a_f$ . The detailed derivations are shown in Medlyn and Dewar (1996). C400 and C800 are the photosynthetic constraint curves at CO<sub>2</sub> = 400 (aCO<sub>2</sub>) and 800 ppm (eCO<sub>2</sub>), respectively. L and L+10% are the long-term soil recycling constraint under ambient and ambient + 10% nitrogen deposition rate. Point A is the equilibrium point between C400 and L, point B is the equilibrium point between C800 and L, C is equilibrium point between C400 and L+10%, and D is equilibrium point between C800 and L+10%. The graph shows that linear coupling of  $a_f$  and  $a_w$  resulted in more responsive NPP to eCO<sub>2</sub> even if N deposition does not change, whereas no coupling between  $a_f$  and  $a_w$  has no effect of CO<sub>2</sub> on production, unless N deposition increases.



**Figure S2** Sensitivity test of varying nitrogen uptake coefficient on the  $CO_2$  fertilization effect (% change) at various equilibrium points. Nitrogen uptake coefficient are 0.2, 0.5, 1, 2 and 5 yr<sup>-1</sup>. I: instantaneous, M: medium term, L: long term, VL: very long term.



**Figure S3** Sensitivity test of varying priming effect parameters on the decomposition rate of slow soil organic matter pool ( $k_{slow}$ ). Parameters varied are allocation coefficient to rhizosphere ( $a_{rhizo}$ ), microbial carbon use efficiency ( $f_{cue}$ ), and scaling factor ( $k_m$ ). From left to right, value ranges are: 0.1, 0.2, 0.3, 0.4, and 0.5 for  $a_{rhizo}$ , 0.1, 0.2, 0.3, 0.4, and 0.5 for  $f_{cue}$ , and 1, 2, 3, 4, 5 for  $k_m$ . Default values are, Black horizontal line is the existing  $k_{slow}$  value, before introducing priming effect.