

Theoretical predictions for the magnetic dipole moment of ^{229m}Th

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A recent laser spectroscopy experiment [J. Thielking *et al.*, *Nature*, (London) **556**, 321 (2018)] has determined for the first time the magnetic dipole moment of the 7.8 eV isomeric state ^{229m}Th . The measured value differs by a factor of approximately 5 from previous nuclear theory predictions based on the Nilsson model, raising questions about our understanding of the underlying nuclear structure. Here, we present a new theoretical prediction based on a nuclear model with coupled collective quadrupole-octupole and single-particle motions. Our calculations yield an isomer magnetic dipole moment of $\mu_{\text{IS}} = -0.35\mu_N$ in surprisingly good agreement with the experimentally determined value of $-0.37(6)\mu_N$, while overestimating the ground state dipole moment by a factor 1.4. The model provides further information on the states' parity mixing, the role and strength of the Coriolis mixing and the most probable value of the gyromagnetic ratio g_R and its consequences for the transition probability $B(M1)$.

Introduction. The persistent interest of the metrology and atomic and nuclear physics communities in the spectroscopic properties of the actinide nucleus ^{229}Th is related to the exceptionally low-energy 7.8 eV isomeric state ^{229m}Th [1, 2]. Vacuum ultraviolet (VUV) laser access to this nuclear state is believed to allow a number of applications such as a new “nuclear clock” frequency standard [3–5], the development of nuclear lasers in the optical range [6] or a more precise determination of the temporal variation of fundamental constants [7–9]. While so far direct laser excitation of the isomer remains illusive due to the poor knowledge of the exact transition frequency, recent experiments could demonstrate the existence of the isomer [10], and measure the isomer mean half-life in neutral Th atoms [11]. In 2018, laser spectroscopy experiments determined for the first time the magnetic dipole moment (MDM) of the nuclear isomeric state as $-0.37(6)\mu_N$ [12, 13], where μ_N stands for the nuclear magneton. This breakthrough is particularly important because it opens the possibility to experimentally probe the nuclear isomeric state via optical spectroscopy on the electronic hyperfine structure [14]. From the nuclear structure point of view, however, the measured MDM value poses a riddle: the only theoretical MDM prediction so far was providing the value $\mu_{\text{IS}} = -0.076\mu_N$ [15], which is about five times smaller than the measured value. This calls for new theoretical efforts to understand the physical mechanism behind the Th isomer.

From the theoretical point of view, predictions on ^{229m}Th have been provided based on three approaches. Dykhne and Tkalya [15] used the Nilsson model [16] to estimate the reduced transition probability $B(M1)$ and the isomer MDM, the former based on Alaga rules [17]. Ruchowska and co-workers [18] used the quasiparticle-plus-phonon model to predict the reduced transition probabilities $B(M1)$ and $B(E2)$ for the isomeric transition. Finally, the present authors have put forward a nuclear

model approach that takes into account the collective quadrupole-octupole vibration-rotation motion (typical for the nuclei in the actinide region) the motion of the unpaired nucleon within a reflection-asymmetric deformed potential with pairing correlations and the Coriolis interaction between the single nucleon and the core [19]. This model predicted the $B(M1)$ value for magnetic decay of the isomer in the limits of 0.006 – 0.008 Weisskopf units (W.u.), well below earlier deduced values of 0.048 W.u. [15, 20] and 0.014 W.u. [18], thus potentially offering an explanation for the recently reported experimental difficulties to observe the radiative decay of the isomer [21–23].

It is the purpose of this Letter to provide new nuclear structure predictions for the MDM of the ^{229}Th ground and isomeric states by extending the model approach in Ref. [19]. The model parameters are independent of MDM experimental data but rather rely on experimental energy levels and transition rates providing a further test of the model's predictive capability. Based on different models for the collective gyromagnetic ratio g_R , we obtain the isomeric-state MDM in the range $\mu_{\text{IS}} = -0.25\mu_N$ to $-0.35\mu_N$, well in agreement with the experimental values. The ground-state (g.s.) MDM value on the other hand is obtained in the range $\mu_{\text{GS}} = (0.53 - 0.66)\mu_N$. This overestimates the latest reported value of $0.36\mu_N$ deduced from state-of-the-art atomic structure calculations for Th^{3+} ions in Ref. [24] on the basis of electronic hyperfine splitting data [25] or the older experimental result $\mu_{\text{GS}} = 0.45\mu_N$ [26]. Our values for the g.s. MDM are similar to the prediction $\mu_{\text{GS}} = 0.54\mu_N$ based on a modified Wood-Saxon potential [27]. Our calculations further show that as a peculiarity, the single particle (s.p.) orbital for ^{229m}Th features very strong parity mixing. Furthermore, we find that whereas the effect of the Coriolis mixing is of crucial importance for the existence and strength of the isomeric transition, it is relatively weak

in the g.s. MDM and negligible in the isomeric MDM. Finally, a comparison of our theoretical prediction with the experimental results supports the consideration of a strong quenching of the collective gyromagnetic ratio g_R in ^{229}Th and in consequence a lower reduced transition probability $B(M1)$ than so far assumed.

Model approach. The model Hamiltonian is taken in the form [19]

$$H = H_{\text{s.p.}} + H_{\text{pair}} + H_{\text{qo}} + H_{\text{Coriol}}. \quad (1)$$

Here, $H_{\text{s.p.}}$ is the s.p. Hamiltonian of the deformed shell model (DSM) with a Woods-Saxon (WS) potential for axial quadrupole, octupole and higher-multipolarity deformations [28] providing the s.p. energies E_{sp}^K with given value of the projection K of the total and s.p. angular momentum operators \hat{I} and \hat{j} , respectively, on the intrinsic symmetry axis. H_{pair} is the Bardeen-Cooper-Schrieffer (BCS) pairing Hamiltonian [29]. This DSM-BCS part provides the quasi-particle (q.p.) spectrum ϵ_{qp}^K as implemented in Ref. [30]. H_{qo} represents a coherent QO motion (CQOM) of the even-even core as considered in Refs. [31, 32]. H_{Coriol} involves the Coriolis interaction between the core and the unpaired nucleon (see Eq. (3) in [32]). It is treated as a perturbation with respect to the remaining part of (1) and then incorporated into the QO potential of H_{qo} [19, 33]. The spectrum of (1) is then obtained through the solution of the CQOM problem [31, 32, 34] superposed to the q.p. spectrum obtained in the DSM-BCS problem. It has the form of quasi-parity-doublets (QPD) ensuing from the QO vibrations and rotations [35] built on q.p. bandhead (b.h.) states with given $K = K_b$ and parity π^b [19, 33].

The Coriolis perturbed wave function $\tilde{\Psi} \equiv \tilde{\Psi}_{nkIMK_b}^{\pi, \pi^b}$ corresponding to the Hamiltonian (1) reads

$$\tilde{\Psi} = \frac{1}{\tilde{N}_{I\pi K_b}} \left[\Psi_{nkIMK_b}^{\pi, \pi^b} + A \sum_{\nu \neq b} C_{K_\nu K_b}^{I\pi} \Psi_{nkIMK_\nu}^{\pi, \pi^b} \right], \quad (2)$$

where $K_\nu = K_b \pm 1, \frac{1}{2}$; $C_{K_\nu K_b}^{I\pi}$ are K -mixing coefficients, $\tilde{N}_{I\pi K_b}$ is a normalization factor and A is the Coriolis mixing constant [19]. The unperturbed QO core-plus-particle wave function in Eq. (2) has the form [33]

$$\begin{aligned} \Psi_{nkIMK}^{\pi, \pi^b}(\eta, \phi, \theta) &= \frac{1}{N_{K}^{(\pi^b)}} \sqrt{\frac{2I+1}{16\pi^2}} \Phi_{nkI}^{\pi, \pi^b}(\eta, \phi) \\ &\times \left[D_{MK}^I(\theta) \mathcal{F}_K^{(\pi^b)} + \pi \cdot \pi^b (-1)^{I+K} D_{M-K}^I(\theta) \mathcal{F}_{-K}^{(\pi^b)} \right], \end{aligned} \quad (3)$$

where $D_{MK}^I(\theta)$ are the rotation functions of the three Euler angles, $\Phi_{nkI}^{\pi, \pi^b}(\eta, \phi)$ are the QO vibration functions in radial (η) and angular (ϕ) coordinates with corresponding quantum numbers n and k (see [34, 35] for details) and $\mathcal{F}_{K_b}^{(\pi^b)}$ is the parity-projected component of the s.p. wave function of the b.h. state determined by DSM

[28]. The quantity $N_K^{(\pi^b)} = \left[\langle \mathcal{F}_K^{(\pi^b)} | \mathcal{F}_K^{(\pi^b)} \rangle \right]^{\frac{1}{2}}$ is the corresponding parity-projected normalization factor. Note that the projection is compulsory since for nonzero octupole deformation the s.p. wave function obtained in DSM is parity mixed, i.e., it contains components possessing both parities, $\mathcal{F}_K = \mathcal{F}_K^{(+)} + \mathcal{F}_K^{(-)}$. The corresponding expectation value of the s.p. parity operator is then $-1 \leq \langle \hat{\pi}_{\text{sp}} \rangle = \langle \mathcal{F}_K | \hat{\pi}_{\text{sp}} | \mathcal{F}_K \rangle \leq 1$. The projection $\mathcal{F}_{K_b}^{(\pi^b)}$ is taken with respect to the experimentally confirmed parity which is for the ground and isomeric states in ^{229}Th $\pi^b = +1$. In our case, the average $\langle \hat{\pi}_{\text{sp}} \rangle$ is 0.4014 for the ground state and 0.0101 for the isomeric state, respectively, showing that the parity mixing is very strong in the DSM solution for ^{229m}Th . We note that the parity-projection plus renormalization procedure plays a considerable role in the resulting MDM values.

The above CQOM-DSM-BCS formalism allows us to calculate the MDM in any state of the spectrum by using the complete Coriolis perturbed wave function (2). We consider the standard core plus particle magnetic dipole ($M1$) operator $\hat{M}1 = \sqrt{3/4\pi} \mu_N \left[g_R (\hat{I} - \hat{j}) + g_s \hat{s} + g_l \hat{l} \right]$, with \hat{s} and \hat{l} being the operators of the s.p. spin and orbital momenta ($\hat{l} + \hat{s} = \hat{j}$), respectively, and g_s the spin gyromagnetic factor. The MDM is determined by the matrix element $\mu = \sqrt{\frac{4\pi}{3}} \langle \tilde{\Psi}_{IIK} | \hat{M}1_0 | \tilde{\Psi}_{IIK} \rangle$ where $\hat{M}1_0$ is the zeroth spherical tensor component of $\hat{M}1$ taken after transformation to the intrinsic frame (see Chapter 9 of Ref. [36]). Thus we obtain the following MDM expression for a state with collective angular momentum I and parity π built on a q.p. b.h. state with $K = K_b$ and $\pi = \pi^b$:

$$\begin{aligned} \mu &= \mu_N g_R I + \frac{1}{I+1} \frac{1}{\tilde{N}_{I\pi K_b}^2} \left[K_b \frac{M_{K_b K_b}^{\pi^b}}{N_{K_b}^{(\pi^b)}} \right. \\ &+ 2A K_b \sum_{\substack{\nu \neq b \\ K_\nu = K_b = \frac{1}{2}}} \delta_{K_\nu K_b} C_{K_\nu K_b}^{I\pi} \frac{P_{K_\nu K_b}^b M_{K_\nu K_b}^{\pi^b}}{N_{K_\nu}^{(\pi^b)} N_{K_b}^{(\pi^b)}} \\ &\left. + A^2 \sum_{\substack{\nu_1, \nu_2 \neq b \\ K_{\nu_1}, K_{\nu_2} = K_\nu \\ = K_b \pm 1, \frac{1}{2}}} \delta_{K_{\nu_1} K_{\nu_2} K_\nu} C_{K_{\nu_1} K_b}^{I\pi} C_{K_{\nu_2} K_b}^{I\pi} \frac{P_{K_{\nu_1} K_{\nu_2}}^b M_{K_{\nu_2} K_{\nu_1}}^{\pi^b}}{N_{K_{\nu_1}}^{(\pi^b)} N_{K_{\nu_2}}^{(\pi^b)}} \right]. \end{aligned} \quad (4)$$

The complete expression would involve an additional so-called decoupling term applying for the case $K_b = 1/2$ [36], that we have disregarded in Eq. (4), since it is not applicable for the states under consideration here. Similarly, the second term in the brackets of Eq. (4) is also not relevant for our problem. We use $M_{K_\mu K_\nu}^{\pi^b} = (g_l - g_R) K_\nu \delta_{K_\mu K_\nu} \langle \mathcal{F}_{K_\mu}^{(\pi^b)} | \mathcal{F}_{K_\nu}^{(\pi^b)} \rangle + (g_s - g_l) \langle \mathcal{F}_{K_\mu}^{(\pi^b)} | \hat{s}_0 | \mathcal{F}_{K_\nu}^{(\pi^b)} \rangle$, where $g_l = 0(1)$ is the orbital gyromagnetic factor for neutrons (protons), $g_s = 0.6 g_s^{\text{free}}$ is the attenuated spin

gyromagnetic factor with $g_s^{\text{free}} = -3.826$ (5.586) for neutrons (protons) [29] and g_R is the collective gyromagnetic factor which will be discussed below. The factors $P_{K\nu_1 K\nu_2}^b$ involve the BCS occupation probabilities as shown in Ref. [19]. The third term in the brackets of Eq. (4) is important for the Coriolis mixing. One can easily check that in the case of missing Coriolis mixing, Eq. (4) appears in the usual form of the particle-rotor expression [29] and is consistent with the relevant limiting case.

Numerical results. We calculate the ^{229}Th ground and isomeric state MDM in Eq. (4) using the wave function (2). Following the procedure described in Ref. [19], we have chosen the quadrupole (β_2) and octupole (β_3) deformations entering DSM, the BCS pairing parameters, the collective CQOM parameters and the Coriolis mixing strength such that both states $5/2^+$ g.s. and the isomeric $3/2^+$ form a quasi-degenerate pair and the low-lying part of the ^{229}Th spectrum [37] is well reproduced. This set of parameters is the same as the one in Ref. [19] except for the Coriolis mixing constant A which was slightly shifted from 0.158 to 0.184 keV. The latter is due to fixing a minor numerical inaccuracy found in the Coriolis mixing coefficients. This leads to a negligible change in the other energies and slight change in the transition rates discussed below.

The parameter that needs special consideration is the collective gyromagnetic factor g_R . In Ref. [19] the $B(M1)$ values were obtained by using the phenomenological expression $g_R = Z/(Z + N)$ with Z and N the proton and neutron numbers, respectively, adopted on the basis of the liquid-drop-model [38]. However, it is well known that similarly to g_s (which is typically attenuated by the quenching factor 0.6 taking into account spin polarization effects [39]) in most deformed nuclei g_R is also lowered by 20-30% or more [36]. This effect has been proven both experimentally [40] and theoretically [41–43]. We note that the experimental determination of g_R is model dependent and in fact usually involves knowledge of MDM. In theory the lowering of g_R is explained through a suppressed relative contribution of the proton system to the total moment of inertia due to the pairing interaction [41, 42]. Thus in odd-mass nuclei the quenching of g_R is stronger when the odd particle is a neutron and weaker when it is a proton compared to the quenching in the adjacent even-even nuclei. In this basic approach g_R is usually calculated through the Inglis-Belyaev cranking procedure [44, 45] using [41, 42] the Nilsson deformed shell model [16] or applying a density-dependent Hartree-Fock plus BCS (HFBCS) model [46].

We introduce the quenching factor $q_R < 1$, such that $g_R = q_R Z/(Z + N)$ with $Z/(Z + N) = 0.393$ for ^{229}Th . We have used the following g_R reference values: $g_R = 0.28$ and 0.27 for ^{228}Th and ^{230}Th , respectively, obtained in the early cranked Nilsson approach [41]; 0.24 obtained for ^{230}Th in the cranked HFBCS calculation

[46]; and 0.31 ± 0.03 deduced in Ref. [47] on the basis of experimental measurements for the ground state MDM and $M1/E2$ mixing ratios for two ground state intraband transitions in ^{229}Th . Keeping in mind that for the odd-neutron nucleus g_R is lower compared to the neighboring even-even cases we consider the quenching factors $q_R = 0.7$ and 0.6 based on the cranked Nilsson and HFBCS calculations, respectively, and $q_R = 0.8$ given by the experimental estimate [47].

The MDM results are shown in Table I and compared with existing theoretical and experimental values. The previous nuclear theory approaches were based on the Nilsson model [15] and modified Woods-Saxon potential [27]. The experimental data on the MDM stems from laser spectroscopy measurements of the electronic hyperfine splitting of Th ions. The early work in Ref. [26] extracted the hyperfine constants A and B from Th^+ spectra and deduced based on rather simplified atomic structure matrix elements the ground state MDM of $\mu_{\text{GS}} = 0.45(4) \mu_N$. This value was corrected in Ref. [24] to $\mu_{\text{GS}} = 0.360(7) \mu_N$ based on a more recent measurement of the hyperfine structure of $^{229}\text{Th}^{3+}$ ions [25] and state-of-the-art atomic structure calculations based on the coupled-cluster model. The first experimental observation of the isomer electronic hyperfine splitting in $^{229}\text{Th}^{2+}$ was reported only recently [12]. Based on this measurement, an isomer MDM value of $\mu_{\text{IS}} = -0.37(6) \mu_N$ [12] or in the range of between -0.30 and $-0.40 \mu_N$ [13] was extracted. Note that Ref. [12] makes use of the ground state MDM value $\mu_{\text{GS}} = 0.360(7) \mu_N$ from Ref. [24] to extract the experimental value for μ_{IS} .

Regarding our model predictions, an overall observation is the decrease of both MDMs μ_{GS} and μ_{IS} (the latter going towards more negative values) with the decrease of g_R . As seen from Table I we have obtained for the ground state MDM the model values in the range between $\mu_{\text{GS}} = 0.655$ and $0.530 \mu_N$ for q_R taking the values 1.0, 0.8, 0.7 and 0.6. Comparing them to the experimental values in Refs. [26] and [24] we see that they overestimate the first one, $0.46(4) \mu_N$, by a factor between 1.42 and 1.15. The latter one is overestimated by a factor between 1.82 and 1.47. On the other hand our values for the isomer MDM which vary between $\mu_{\text{IS}} = -0.253$ and $-0.347 \mu_N$ essentially corroborate the experimental values in Refs. [12] and [13]. We see that the three values obtained at $q_R = 0.8, 0.7$ and 0.6 enter the error bar for the value of $\mu_{\text{IS}} = -0.37(6) \mu_N$ in [12]. The value at $q_R = 1$ underestimates (in absolute value) by a factor 0.84 the lower limit $-0.30 \mu_N$ of the values in Ref. [13]. Comparison with previous theoretical results shows that our calculations corroborate the g.s. MDM values $\mu_{\text{GS}} = 0.54 \mu_N$ obtained by Chasman et al [27] and disagree with the isomer MDM value $\mu_{\text{IS}} = -0.076 \mu_N$ obtained in [15]. As a further check, the model predicts a spectroscopic electric quadrupole moment $Q_{\text{GS}} = 2.79$ eb, close to the experimentally determined $3.11(16)$ eb [25]

TABLE I: Calculated ground state and isomer MDM μ_{GS} and μ_{IS} , respectively, obtained for ^{229}Th for several q_R (g_R quenching) values in comparison with previous nuclear theory predictions and experimentally deduced MDM values.

μ (μ_N)	q_R (this work)				nuclear theory		laser spectroscopy			
	1.0	0.8	0.7	0.6	Ref. [27]	Ref. [15]	Ref. [26]	Ref. [24]	Ref. [13]	Ref. [12]
μ_{GS}	0.654	0.591	0.559	0.528	0.54	–	0.46(4)	0.360(7)	–	–
μ_{IS}	-0.253	-0.300	-0.323	-0.347	–	-0.076	–	–	(-0.3)-(-0.4)	-0.37(6)

or 3.14(3) eb [48].

TABLE II: Predicted $B(M1)$ values (in W.u.) for several transitions involving yrast (yr) and excited (ex) QPD states of ^{229}Th obtained for four q_R values in comparison with available experimental data.

Decay	q_R				Exp. [49]
	1.0	0.8	0.7	0.6	
$\frac{3}{2}_{\text{ex}}^+ \rightarrow \frac{5}{2}_{\text{yr}}^+$	0.0081	0.0068	0.0062	0.0056	–
$\frac{7}{2}_{\text{yr}}^+ \rightarrow \frac{5}{2}_{\text{yr}}^+$	0.0096	0.0043	0.0025	0.0011	0.0110 (40)
$\frac{9}{2}_{\text{yr}}^+ \rightarrow \frac{7}{2}_{\text{yr}}^+$	0.0185	0.0097	0.0065	0.0038	0.0076 (12)
$\frac{9}{2}_{\text{yr}}^+ \rightarrow \frac{7}{2}_{\text{ex}}^+$	0.0144	0.0147	0.0149	0.0151	0.0117 (14)

While the variation of q_R (g_R) in Table I does not affect the energy levels and $B(E2)$ transition rates, it does change the predicted $B(M1)$ transition rates in Ref. [19]. This is illustrated in Table II where the model predictions for several $B(M1)$ values (including that of the isomer transition) obtained for the different q_R (g_R -quenching) values are given together with the available experimental data. The transition rates in the first column (with $q_R = 1.0$) are slightly different from the original calculation in [19] due to the already mentioned correction in the numerical mixing coefficients. We see that the isomer-decay $B(M1)$ value (first row) gradually decreases with the decrease of g_R in consistency with the MDM behavior. Faster decrease of $B(M1)$ with g_R is observed for the yrast intraband transitions $7/2^+ \rightarrow 5/2^+$ and $9/2^+ \rightarrow 7/2^+$ whereas the $B(M1)$ values for the yrast-excited interband transition $9/2_{\text{yr}}^+ \rightarrow 7/2_{\text{ex}}^+$ practically remain unaffected. Comparison with the experimental data shows that the first intraband transition $7/2^+ \rightarrow 5/2^+$ which is originally underestimated by the model goes even further down with g_R by an order of magnitude. On the other hand the other intraband transition $9/2^+ \rightarrow 7/2^+$ approaches rather well the corresponding experimental value for $q_R = 0.7$.

The dependence in Tables I and II suggests that, since the quenching of g_R is physically reasonable in the considered limits, we may state that the model predictions for μ_{GS} and μ_{IS} obtained with g_R attenuated below the

$Z/(Z+N) = 0.393$ value better approach the corresponding experimental values with μ_{IS} firmly entering the uncertainty bars. In addition, this result suggests that a further slight decrease of the predicted isomer-decay $B(M1)$ value as compared to Ref. [19] towards $B(M1) = 0.005$ W.u. is probable.

A word is due on the role of the Coriolis mixing for the calculated MDM values. The attenuation of g_R counteracts the mixing-effect on the connection between the two states. For example our numerical analysis shows that a slight raising of the Coriolis constant A from 0.184 keV used in this work to the value of 0.230 keV leads to a slight decrease in μ_{GS} at $q_R = 1.0$ from 0.654 to 0.643 μ_N with a negligible decrease in μ_{IS} in the fifth digit leaving the reported value of $-0.253 \mu_N$ unaffected. At the same time the isomer $B(M1)$ transition rate raises from 0.0081 to 0.0121 W.u. The predictions for the $E2$ transitions are also affected with the isomer $B(E2)$ raising from 29 to 43 W.u. Conversely by setting the Coriolis mixing strength zero as a limiting case, μ_{GS} slightly increases to 0.677 μ_N whereas μ_{IS} remains again unaffected. This analysis reveals some peculiarities of the present model mechanism in ^{229}Th . Whereas the Coriolis mixing strongly affects the isomer (interband) transitions being of crucial importance for their existence, it has only a small impact on the g.s. MDM and a completely negligible effect on the isomer MDM. The latter is explained by the circumstance, also checked numerically in wave function (2), that while the g.s. is mixed with the $I = 5/2^+$ isomer-based state, the isomer state has no $I = 3/2^+$ mixing-counterpart in the g.s. band due to the $K \leq I$ restriction. Thus, it appears that the connection between the ground and isomer state is mainly due to the admixture in the g.s. This effect enters the transition matrix elements and appears in the second power in the transition rates. We note that as the Coriolis mixing and the g_R quenching counteract in the isomer transition rates and act in the same direction suppressing μ_{GS} it may be possible to bring the latter closer to the experimental value without drastic change in the predicted transition rates. However, this would cause an overall change in all model observables, energy levels, $B(E2)$ and $B(M1)$ values, and would rather call for a full readjustment of the model parameters. Further refinements such as more precise tuning of the QO deformation parameters entering DSM, possible involvement

of hexadecapole deformation (suggested in Ref. [48]), or tuning of the spin-gyromagnetic quenching (similarly to q_R) as suggested in Ref. [47] could provide an even more reliable prediction of ^{229m}Th electromagnetic decay properties.

In conclusion, the CQOM-DSM-BCS model of the QPD spectrum and $B(M1)$ and $B(E2)$ transition rates in ^{229}Th provides a good description of the MDM in the isomeric state. Our prediction $\mu_{IS} = -0.35\mu_N$ is in good agreement with the experimentally determined value and largely differs from the previous prediction in Ref. [15] based on the Nilsson model. In the same time, the ground state MDM is overestimated by a factor of approx. 1.4. We conclude that our results provide ground for further refined consideration of the interplay between the nuclear dynamic modes which determine the electromagnetic properties of the isomeric state as a part of the entire ^{229}Th structure.

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