

# A Field Guide to Spin Liquids

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Spin liquids are collective phases of quantum matter which have eluded discovery in correlated magnetic materials for over half a century. Theoretical models of these enigmatic topological phases are no longer in short supply. In experiment there also exist plenty of promising candidate materials for their realisation. One of the central challenges for the clear diagnosis of a spin liquid has been to connect the two. From that perspective, this review discusses characteristic features in experiment, resulting from the unusual properties of spin liquids. This takes us to thermodynamic, spectroscopic, transport, and other experiments on a search for traces of emergent gauge fields, spinons, Majorana Fermions and other fractionalised particles.

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## I. THE QUEST FOR SPIN LIQUIDS

The search for spin liquids as fundamentally new states of matter is a long-running quest<sup>1-4</sup>. Their occurrence in insulating magnets appears to be greatly facilitated by

frustrated interactions for which the corresponding classical spin systems display a large ground state degeneracy because the local energetics cannot be minimized in a unique way<sup>5-10</sup>. For a long time, the central and defining concept involved was a negative one – a (ground) state without any magnetic order – in contrast to the prevailing phases with spontaneous symmetry breaking characterised by local order parameters. The rejuvenated interest in resonating valence bond (RVB) physics through high-temperature superconductors<sup>11</sup> focused attention on topological properties<sup>12-15</sup>.

Initially, theoretical ideas were centred around wavefunctions, e.g. of the RVB type, but it took considerable time until microscopic Hamiltonians realising such states at isolated points<sup>16</sup> or extended bona fide quantum spin liquid (QSL) phases were established<sup>2</sup>. The advent of exactly soluble model Hamiltonians<sup>16,17</sup> has allowed to gain an unprecedented understanding of ground- and excited state properties of QSLs. Nowadays, the theory community has developed a remarkable capacity to invent elaborate schemes with a plethora of different phenomenologies but arguably with little guidance from experiment. At the same time, while the target space of interesting models has exploded, the arsenal of methods for their detection has not grown commensurably. Nonetheless, there has been a sustained materials physics effort covering a huge number of magnetic compounds, and many a promising candidate systems have been unearthed, some of which have benefitted from an intense research effort for clarifying their properties in considerable detail.

The aim of this review is to contribute towards redressing this balance. In particular, we discuss the rich phenomenology of spin liquids in order to make connection to past and future experiments.

Before we embark on this, we would like to motivate this programme with a few words about how it fits in with the broader research landscape of modern condensed matter physics. The search for spin liquids nowadays forms part of the grand challenge of understanding existence, scope and nature of physics beyond the standard theory of Landau and spontaneous symmetry breaking.

As such, they fall in the field of topological condensed matter physics under the headings of long-range entangled phases and topological order.

To this date, we have only one outstanding established class of experimental systems (at least in terms of materials science and beyond one dimension) exhibiting a topologically ordered quantum phase with fractionalized excitations: this is the fractional quantum Hall effect. The Laughlin state and its even more elaborate brethren<sup>18,19</sup> have attracted much attention, most recently fuelled by the dream of realising a quantum computer topologically protected against decoherence<sup>20</sup>. This is nicely reviewed in Ref.<sup>21</sup>.

QSLs have the added attraction of accessing the vast space of possible materials provided by the combinatorial richness of the periodic table, the presence of sometimes large exchange energy scales, as well as a high degree of tuneability and being amenable to experimental probes, e.g. through the application of magnetic fields and the use of neutron scattering. The central goal for the foreseeable future therefore is an unambiguous identification of a QSL phase. For this, new experiments, including new probes, may be needed, accompanied by a reliable theoretical analysis framework.

Our approach here is to support this quest, but not by providing a complete introduction to quantum spin liquids for the expert. Rather, we present a compendium of ideas to provide a broader overview, which can also act as a guide for the newcomer. Many reviews are available with material which we do not cover here, e.g. comprehensive reviews on spin liquids<sup>22</sup> and frustrated magnetism more generally<sup>23</sup>; about spin ice<sup>24</sup>; introductory reviews to quantum spin liquids<sup>4,25-27</sup>; an overview over spin ice<sup>28</sup> and Kitaev QSLs<sup>29</sup> from a fractionalisation perspective, as well as a nice review of classical emergent Coulomb gauge fields<sup>30</sup>; and works focusing on the materials aspects of pyrochlores<sup>31</sup>, herbertsmithite<sup>32</sup> and Kitaev material candidates<sup>33,34</sup>.

Physics is an experimental science. However, while discoveries are mostly driven by experiments, the resulting insights are naturally preserved in the language of theory. The history of the search for spin liquids is therefore naturally intertwined with what phenomena one excludes and includes under this heading. Before we move on to the core of this field guide, we provide a brief review of the background taxonomy.

The remaining sections are then devoted to spin liquid phenomenology. The ultimate ambition – to provide a textbook, not unlike standard solid state physics textbooks on conventional phases, on the behaviour of such topological phases – is a step too far for us, and we make a selection of phenomena which we feel are particularly instructive and/or realistically attainable.

## A. What is a spin liquid

What is certainly true is that the meaning of the term has shifted over the years. This is not an uncommon state of affairs, driven not only by the human tendency to adapt a definition to the requirements of the moment, but also by the fact that as the understanding of the subtleties of the phenomenon advances, refinements to the concepts follow.

A constitutive concept for a spin liquid is the absence of magnetic order of a system of interacting spins at temperatures smaller than the interaction scale. This encodes the idea of a phase beyond the Landau paradigm (which covers all forms of magnetic order), as well as the intuition that a 'liquid' should be different from a 'solid'. In this sense, other forms of ordering of the spin degrees of freedom – such as nematic orders<sup>38,39</sup> – also a priori disqualify a system from being classified as a spin liquid.

This negative definition, about the absence of something, continues to be the most ubiquitous and intuitive. Besides its practical limitation, to which we return below, it is nowadays considered to be somewhat too broad. For instance, it includes models – interesting in their own right – which are considered somewhat too simple. One is a quantum paramagnet, such as the kagome lattice Ising model in a transverse field, which is connected continuously to a high-temperature (classical) paramagnetic phase. Another is the Shastry-Sutherland model, whose two spins per structural unit cell form a dimer at low temperature, producing a simple inert state which again is straightforwardly connected to a high-temperature paramagnet but which can also display protected edge excitations of triplons at low temperatures<sup>40</sup>. One calls this broader class of disordered magnets 'cooperative paramagnets', to distinguish them from magnets disordered by thermal fluctuations at high temperatures, although this nomenclature is by no means universally used.

A more 'modern', positive definition involves listing conditions which a phase should meet to qualify as a spin liquid. This derives from advances in our understanding of what phases beyond the Landau paradigm can look like, and applies these to spin systems.

Akin to the Mermin-Wagner theorem<sup>41</sup>, which forbids spontaneous breaking of continuous symmetry (SSB) at finite temperatures in dimensions  $d \leq 2$ , there is a rigorous result for spin systems with short range interactions. For systems with half-odd integer spin per unit cell, hence proper Mott insulators, and without symmetry breaking, the Lieb-Schultz-Mattis theorem states<sup>42</sup> that the ground state is either unique with gapless excitations or degenerate with a gap to excitations. It establishes to some level of mathematical rigour<sup>43,44</sup> the possibility of gapless QSLs or gapped ones with topological order.

Perhaps the crispest is to demand that the magnet should at low temperatures be described by a topological field theory, such as Chern-Simons theory<sup>45</sup>. On the one hand, this is very restrictive, a priori ruling out gapless spin liquids, and spin liquids with some additional

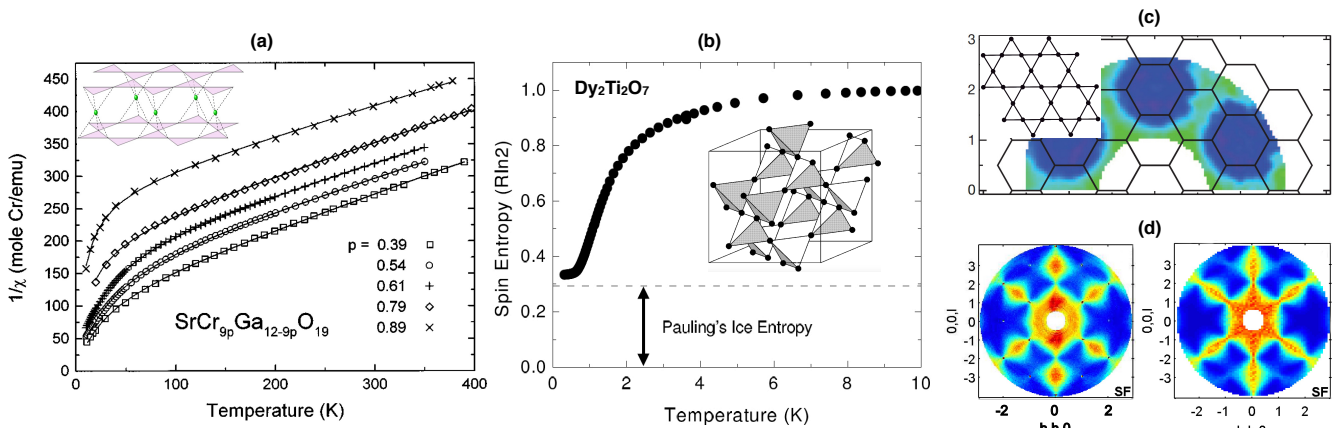


FIG. 1. (Color online) (a) The Curie-Weiss temperature for various levels of dilution in  $\text{SCGO}_x$  is much larger than the featureless regime of the susceptibility (adapted from Ref.<sup>35</sup>). The schematic lattice structures are shown in the insets. (b) The residual entropy of spin ice and the frustrated pyrochlore lattice (after Ref.<sup>25</sup>). (c) Broad features in inelastic neutron scattering of the kagome lattice material herbertsmithite<sup>36</sup>. (d) The emergent gauge field of 3d spin ice gives rise to pinch points in the static structure factor (left panel, neutron scattering experiment; right panel, theory; adapted from Ref.<sup>37</sup>).

ordered degrees of freedom<sup>46</sup>. On the other hand, this exclusion is not arbitrary—trying to braid quasiparticles in the presence of gapless Goldstone modes of a ferromagnet, like for  $\text{SU}(2)$  quantum Hall Skyrmions<sup>47</sup>, does present an obstacle for envisaged quantum computation experiments<sup>21</sup>.

However, in practise to qualify as a spin liquid, it may be enough to bear in mind that 'some subset' of the degrees of freedom should look 'essentially topological'. This is more or less the attitude we will take for the remainder of this review. For the purposes of a field guide, we will therefore be looking for *fractionalised excitations* and *emergent gauge fields*. The latter two are intimately linked together as standard spin flip excitations always lead to integer changes of the total spin. Therefore, excitations labeled by 'fractions' of such quantum numbers, e.g. quasiparticles carrying half-integer spin, need to be created in even numbers and once separated we can think of the emergent background gauge field as taking care of the global constraint. The connection holds particularly in higher dimensional spin liquids, which are the focus of our field guide, but in  $d = 1$  fractionalisation may appear much simpler via domain wall excitations of ordered states.

## B. How to tell one, as a matter of principle...

The characterisation of a topological state of matter proceeds most easily via its global properties. Given the importance that numerical simulations have played in advancing the field, some of these – while appearing rather complex – are comparatively straightforwardly diagnosed numerically.

The topological order discovered by Wen and Niu posits that topological states have a degeneracy which

depends on the genus of the surface they live on<sup>48</sup>. For instance, the Laughlin state at filling fraction  $\nu = 1/3$  is non-degenerate on a sphere, and threefold degenerate on a torus. This is intimately connected with the existence of fractionalised quasiparticles. When a pair of Laughlin quasiparticles of charge  $\pm e/3$  is created from a ground state, and one member of the pair moves around a non-contractible loop of the torus before annihilating the other, the system moves from one ground state to another. Only once three such particles, and hence an electron with unit charge, have made such a trajectory does the system return to the original ground state. Indeed, the connections between quantum Hall physics and quantum spin liquids can become remarkably detailed, such as in transfers of wavefunctions between the two settings, see e.g.<sup>49</sup>.

Observing the topological ground state degeneracy can be challenging<sup>50,51</sup> but numerical methods like DMRG are well-suited for extracting an equally reliable quantifier for gapped QSLs. In analogy to long range order of conventional phases the *long-range entanglement* can serve as an order parameter of topological phases<sup>52–54</sup>. The entanglement entropy of a ground state wave function can be calculated from a reduced density operator with one part of the total degrees of freedom of a bipartitioned system traced out (with a smooth boundary of length  $L$  separating the two regions). For gapped phases it follows a universal scaling form

$$S = cL - \gamma + \dots \quad (1)$$

The first term with a non-universal prefactor  $c$  is the 'area' law common to all gapped phases but the second term  $\gamma$  quantifies the long-range entanglement (it is independent of the length of the boundary of the partition). It is only nonzero in a topological phase and is directly related to the emergent gauge structure of the QSL phase,

e.g. in a  $Z_2$  QSL  $\gamma = \ln 2^{54}$ . More detailed analyses can then yield information on the properties of fractionalised excitations and edge spectra<sup>55</sup>.

### C. ... and in practice

Above, we called these diagnostics 'comparatively straightforward' because the experimental situation is considerably less promising. The entanglement entropy – in particular, any subleading contribution to it – does not correspond to any natural measurement on a many-body quantum spin system. Also, putting even a two-dimensional magnet on a manifold of nontrivial topology sounds like a thought experiment par excellence, even more so than the idea of diagnosing a spin stiffness for a conventionally ordered magnet by twisting boundary conditions.

The core aim of this review is to address precisely the question how to move forward from here. Lacking a silver bullet or smoking gun (or whatever alternative martial metaphor one prefers to use) in experimental reality, one needs to make do with the probes that actually exist, and think about how best to employ and combine them for an unambiguous identification of spin liquids.

### D. The role of universality

Before we turn to this in more detail, we would like to raise an additional 'ideological' point of fundamental importance which may at times be somewhat underappreciated when making contact between theory and experiment.

The tension arises from the need in theory to devise precise definitions. One of the most successful of these is the idea of universality, which is intimately related to the success of the developments of the concepts of symmetry breaking and encoded in the renormalisation group half a century ago. Phases and phase transitions have properties which are independent of microscopic details of the Hamiltonian – these properties are called universal.

The question then is: how much of this universality is visible – and where/how – in an actual experiment. The fundamentalist answer is: a priori, nothing. A case in point is the existence of Goldstone modes accompanying the breaking of a continuous symmetry, in the limit of long wavelengths and low frequencies. Obviously, the limit of low frequencies will be cut off by a finite energy resolution – not only due to Heisenberg's uncertainty relations – of any conceivable experimental probe. On top of these, innumerable other limitations, many of them based on nothing less than the second law of thermodynamics, will always be with us. We have to live with them (but can at times perhaps, see Sec. V, even turn them to our advantage).

In practise, this observation is not just a complaint around the edges. As we will argue below, many of the

most striking manifestations of spin liquid behaviour in fact are non-universal in the sense that they could be altered without leaving the phase; or conversely, that proximate phases may exhibit the behaviour we are interested in essentially just as characteristically as the pristine version.

As a poster child of this, we would like to adduce the fractionalised Heisenberg chain, see Fig.3 (a). The agreement between theory and experiment is striking, up to considerable detail of the structure factor at high energies, including subtle intensity variations with wavevector and frequency. However, none of these are universal. Close inspection of the universal part of the spectrum at low frequencies reveals the opposite<sup>56</sup>: due to the residual coupling between neighbouring chains, they undergo an ordering transition into a *different phase* with different, 'conventional' universal behaviour of a long range ordered 3d magnet.

A fundamentalist 'universalist' perspective therefore leaves two non-palatable alternatives: either one has the low-temperature, conventional ordered phase; or, above the ordering transition, a phase continuously connected to a boring high-temperature paramagnet. Both miss the remarkable, and in our minds convincing, evidence for fractionalisation 'in practise' in this compound. As is so often the case, while the worldview of the fundamentalist is deceptively simple, much of what makes real life interesting lies in the grey areas, the appreciation of which requires an open mind.

## II. HOW TO START LOOKING ...

A time-honoured way of making a first cut at the diagnosis of spin liquidity in a candidate material is via studies of thermodynamic properties, Sect. III, in part because these are relatively easy to carry out locally in a laboratory. The first chore is to establish the absence of magnetic ordering in a strongly interacting (low-temperature) regime.

A popular measure for its existence is the frustration parameter  $f = |\Theta_{CW}|/T_f^{57,58}$ , which is defined as the ratio of two quantities. One is the Curie(-Weiss) temperature extracted from a straightforward fit to the high-temperature susceptibility, which to first order in a high-temperature expansion is given by  $\chi = C/(T - \Theta_{CW})$ . This expression applies to an insulating magnet the size and nature of whose magnetic moments determine the Curie constant  $C$ , and whose interactions determine the size of  $\Theta_{CW}$ . The second quantity,  $T_f$ , is the location of any non-analyticity (divergence, cusp, ...) in  $\chi$ , indicating a residual ordering tendency, or spin freezing which is commonly encountered in frustrated magnets. The regime in temperature  $\Theta_{CW} \gg T > T_f$ , the cooperative paramagnetic regime, is then a natural place to start looking for a spin liquid, and it is well defined provided  $f$  is sufficiently large, see Fig.1 (a) for a classic example.



If appropriate measurements are possible and available (e.g. thanks to the availability of a neutron source), the absence of ordering can be confirmed by verifying that no magnetic Bragg peaks appear when cooling down the system.

The challenge in practice lies in the need to eliminate the presence of less obvious ordering tendencies (such as multipolar or distortive order), which are more elusive in neutron scattering; and also not to miss any features in the specific heat, which is nonspecific in the type of orderings it picks up, other than requiring the corresponding non-analytic features of the phase transition to be discernible above its smooth background temperature dependence. Also, it may be polluted by ‘incidental’ phase transitions, such as those of the lattice which have little bearing on its magnetism.

Further valuable insights can be gleaned in spectroscopic experiments, Sec. IV and Fig.3. These can provide considerably more detailed information than the purely macroscopic thermodynamic ones: inelastic neutron scattering has made tremendous technological progress in the past few years, and now routinely provides data in  $d + 1$  (wavevector+frequency) space. Other probes, such as Raman scattering, and NMR lack wavevector dependence and provide a local response averaged over the full system. Crucially, alternative probes each couple differently to the magnetic degrees of freedom, and therefore, they come with different selection rules for probing the quasiparticles (or non-quasiparticle excitations) of the material thereby providing complementary evidence, as we discuss below.

One central motivation for the use of finite-frequency probes lies in the fact that the ground states of topological systems are at first sight unspectacular. While gapless spin liquids should generically come with algebraically decaying ground state correlations, gapped spin liquids (just like the Laughlin charge liquid) really look featureless. It is the fractionalised quasiparticles which provide a local indication of the topological physics involved: instead of the magnons in an ordered magnet, one looks for spinons, holons, monopoles and the like. (An alternative to studying such finite-frequency responses lies in enlisting the help of disorder to nucleate these excitations already in the ground state, Sec. V.)

For such excitations, kinematic considerations can play an important role: broad responses for a scattering experiment involving the creation of several particles, thereby weakening the restrictions imposed by energy and momentum conservation, are taken as a prime indicator of novel spin liquid physics. In the case where the excitations of the emergent gauge field are very heavy – as in the case of Kitaev’s QSL, where they do not move at all – they can effectively remove momentum conservation as a kinematic constraint, as their energy is barely momentum dependent<sup>59</sup>, Fig.1(c). In other cases, there may be direct evidence for the emergent gauge field, as is provided by the pinchpoints in spin ice<sup>28</sup>, see Fig.1(d).

## A. ... and where

The search for spin liquid compounds has been going on for a long time, and many compounds have yielded interesting insights and phenomena. To conclude this introduction, we mention some materials which have shaped our own thinking – this article tries to synthesize the resulting insights, rather than discuss and model each system – or indeed, all systems – in specific detail.

Kagome-based lattices are prime examples for geometric frustration and very prominent for quasi two-dimensional compounds. The jarosites<sup>60</sup> represent a family of compounds with varying degrees of further-neighbour interactions and disorder. Beyond this, two particularly well-studied systems are volborthite<sup>61</sup>, which is now believed to have an important spatial anisotropy<sup>62,63</sup> and herbertsmithite<sup>64,65</sup>. The latter is in some ways an outstanding candidate<sup>32</sup>. Next in line are triangular compounds, where a family of organic systems (‘dmits’) are particularly prominent<sup>66</sup>, as discussed in the thermodynamics section below.

In the last few years, candidate materials<sup>67,68</sup> for Kitaev spin liquids have received intense attention<sup>29,33,34</sup>, with a combination of mapping out the Hamiltonian and understanding the resulting consequences keeping a large community busy. These include the ‘Kitaev Iridates’  $A_2 \text{IrO}_3$  with  $A=\text{Na, Li}$ <sup>69,70</sup>, as well as  $\alpha\text{-RuCl}_3$ <sup>71,72</sup>, see Fig.3(b+c).

Historically very important – as the founding material of highly frustrated magnetism – has been SCGO<sup>57</sup>, a kagome-triangle-kagome trilayer, which may alternatively be viewed as a slab of the pyrochlore lattice of corner-sharing tetrahedra. The pyrochlore lattice in turn hosts the Ising magnet known as spin ice<sup>24,28</sup>, most prominently on  $\text{Dy}/\text{Ho}_3\text{Ti}_2\text{O}_7$ , the only generally acknowledged fractionalised magnetic material in three dimensions. Many other compounds exist on this lattice, such as a more ‘quantum’ version with  $Pr$  ions<sup>73</sup>, as well as a large class of spinel compounds, including the well-studied Cr-spinels<sup>74</sup> with, like SCGO, isotropic  $S = 3/2$  moments. Diluting a quarter of sites of that lattice in turn yields the hyperkagome lattice, with  $\text{Na}_4\text{Ir}_3\text{O}_3$  the most prominent exponent<sup>75</sup>.

Newcomers are constantly added to this list, most recently  $\text{Ca}_{10}\text{Cr}_7\text{O}_{28}$ <sup>76</sup>,  $\text{Ba}_3\text{NiSb}_2\text{O}_9$ <sup>77</sup>,  $\text{YbMgGaO}_4$ <sup>78</sup>,  $\text{Cu}_2\text{IrO}_3$ <sup>79</sup> and  $\text{H}_3\text{LiIr}_2\text{O}_6$ <sup>80</sup>.

## III. THERMODYNAMICS AND TRANSPORT

A key target for diagnosing spin liquids is the experimental identification of fractionalized excitations at low energies. Thermodynamic and transport measurements are complementary in this endeavour, the former probing the necessary low energy density of states (DOS) and the latter the mobility of the excitations. The absence of standard Goldstone modes from conventional symmetry breaking phases, e.g. spin waves of an ordered magnet,

can be deduced from the the absence of non-analyticities in thermodynamic observables. In practice many material candidates have preemptive symmetry breaking instabilities leading to non-analyticities from sub-leading interactions. It prevents a true low temperature liquid phase for example due to weak interlayer couplings of quasi-two-dimensional materials, but as long as the frustration parameter  $f$  is small enough the correlated paramagnetic regime at intermediate temperatures is a good starting point in the search for QSL physics.

Candidate spin liquids often exhibit a *spectral weight downshift* of the specific heat as part of their refusal to order. This can in the most extreme cases go as far as apparent violations of the third law of thermodynamics. This happens in spin ice, Fig.1 (b), where upon cooling a 'residual entropy' is measured, which indicates that even at the lowest temperatures, the system continues to explore an exponentially large number of states. This in particular sets cooperative paramagnetism apart from, say, dimensionality-induced destruction of ordering. While purely one-dimensional spin systems such as a  $S = 1/2$  Heisenberg antiferromagnetic chain do not order at any nonzero temperature, they nonetheless are close to an ordered state and often lose most of their entropy already upon cooling through  $\Theta_{CW}$ .

### A. Thermodynamics

Even though characteristic correlations of a QSL are only expected at temperatures well below  $\Theta_{CW}$ , the high temperature thermodynamic response of a candidate material already contains useful information. Details of a microscopic description are obtained by a comparison of the temperature scale at which (and the form of how) the magnetic susceptibility deviates from Curie-Weiss behaviour to that calculated in a high-temperature expansion of a putative spin Hamiltonian<sup>86,87</sup>. The angular magnetic field dependence of  $\chi$  with respect to crystal orientation is in principle able to detect spin-anisotropic interactions<sup>88</sup>. The goal is to invert macroscopic measurements to microscopic descriptions which however is only attainable as long as the low energy spin Hamiltonian and the magnetic  $g$ -tensor are sufficiently simple.

A defining feature of QSLs are fractionalized magnetic excitations which, after subtracting other contributions to the heat capacity (mainly from phonons, but at times, in particular at the lowest energies, nuclear spin), can be probed via the low temperature dependence of thermodynamic observables. In a gapless QSL information about a low energy power-law DOS,  $N(\omega) \propto \omega^\alpha$ , can be readily extracted because the specific heat is able to directly probe the exponent  $\alpha$

$$\frac{C_V}{T} = \frac{1}{T} \frac{\partial}{\partial T} \int d\omega \omega N(\omega) n(\omega, T) \propto T^\alpha. \quad (2)$$

Hence, in conjunction with the dimensionality of the system detailed information about the low energy dispersion

can be inferred such as the presence of emergent Fermi surfaces, Dirac points or nodal lines, all of which have been proposed in model QSLs<sup>89-91</sup>. Of course, the procedure rests on assumptions like the presence of weakly interacting quasi-particles whose thermal distribution only depends on their individual energies, for example  $n(\omega, T)$  being the Fermi-Dirac or Bose-Einstein distribution functions.

Another caveat is that these power laws are not necessarily fixed in all circumstances. For instance, in a honeycomb system with slowly varying bond disorder the resulting hopping problem is equivalent to particles in a random gauge field. Famously, this problem gives rise to a density of states with a power increasing *continuously* with disorder strength<sup>92</sup>. Such strains may quite conceivably be present in, say, organic systems with relatively low lattice rigidities.

Another intrinsic complication for a straightforward interpretation of thermodynamic data could be the presence of very different energy scales making it hard to estimate the right scaling regime. For example, the Dirac spectrum of the honeycomb Kitaev QSL with  $N(\omega) \propto \omega$  would simply predict  $C_V/T \propto T$  which turns out to be only observable at extremely low temperatures but instead over a large temperature window a metallic like  $C_V/T \propto T^0$  appears<sup>93,94</sup>. The reason is that spin flip excitations fractionalize into Majorana fermions and flux excitations. The latter have a small gap which is only a fraction of the total magnetic energy scale. At all but the lowest temperatures the presence of thermally excited fluxes destroys the Dirac spectrum of the Majorana fermions changing the low energy DOS to roughly a constant  $N(\omega) \propto \omega^0$ . This particular example shows, on the one hand, the difficulties of drawing reliable conclusions from thermodynamic measurements. On the other hand, it highlights how a close comparison of microscopic calculations and experimental data could be used in principle to extract complementary information about the fractionalized excitations of a QSL.

### B. Longitudinal transport: thermal versus charge

In the absence of mobile charge carriers in magnetic insulating materials thermal transport experiments can probe the mobility of elementary excitations. Ideally the heat flow along a temperature gradient contains information about the velocity,  $v_{\mathbf{k}}$ , and mean-free path,  $l_{\text{MFP}}$ , of fractionalized quasi-particles of a QSL with dispersion  $E_{\mathbf{k}}$ , e.g. as obtained for the longitudinal thermal conductivity  $\kappa$  in a semi-classical Boltzmann calculation

$$\kappa = \frac{\partial}{\partial T} \int d^d k l_{\text{MFP}} E_{\mathbf{k}} |v_{\mathbf{k}}| n(E_{\mathbf{k}}, T). \quad (3)$$

In a number of low dimensional QSL candidate materials such purely magnetic contributions have been observed, e.g. in triangular<sup>82,95</sup>, kagome<sup>85</sup> and Kitaev honeycomb<sup>96,97</sup> systems. However, in general it is very hard

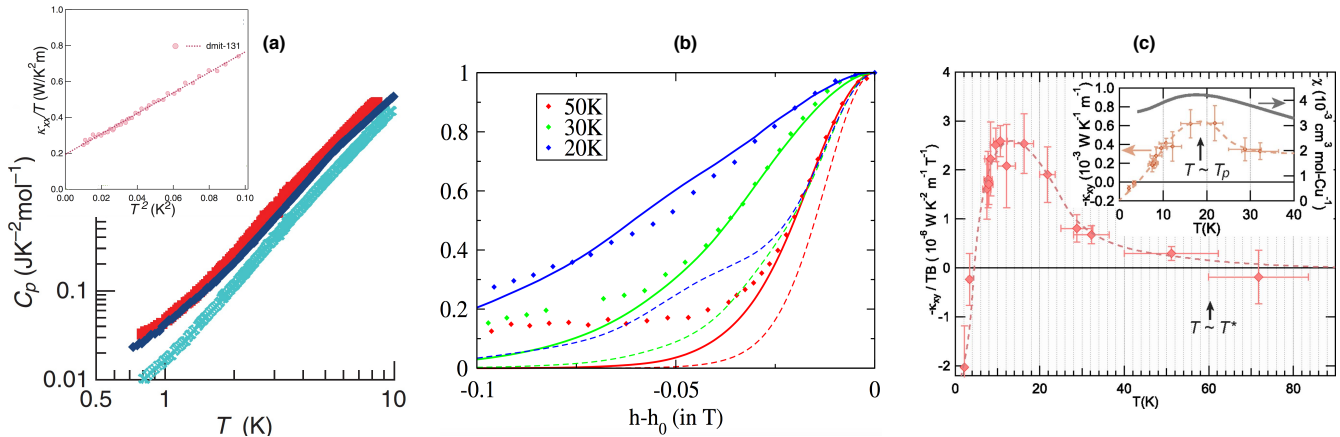


FIG. 2. (Color online) (a) Linear specific heat<sup>81</sup> and residual linear in temperature longitudinal heat conductivity<sup>82</sup> (inset) in triangular organic compound 'dmit'. (b) Linewidth in NMR experiment of SCGO due to orphan spins and their interactions<sup>83,84</sup>. (c) Evidence for a thermal Hall effect in the kagome volborthite<sup>85</sup>.

to get rid of the spurious phonon contribution. Moreover, in the presence of sizeable spin-phonon couplings magnetic excitations scatter of phonon contributions and vice versa which makes it hard to disentangle the two. These problems could in principle be overcome by studying directly the *spin current* transport which is measurable via the inverse spin Hall effect as demonstrated with insulating ordered magnets<sup>98</sup>. This has recently been proposed for QSL materials<sup>99</sup> but more theoretical work and experiments are needed to show whether spin transport measurements can be turned into a new tool for studying QSLs.

### C. Bulk-boundary correspondence and quantization of currents

A crowning achievement would be the observation of *quantised* transport signatures directly related to topological invariants, to rank alongside the famously quantised Hall conductivity of the fractional quantum Hall effect. Again, for insulating magnets, these cannot be charge transport, but nevertheless a quantization of the thermal Hall effect<sup>100</sup>,  $\kappa_{xy}$ , has been predicted in certain types of QSLs with broken time reversal symmetry<sup>101</sup>. The origin of the quantization<sup>17</sup> can be illustrated for a chiral QSL with  $\nu$  chiral edge modes (their one dimensional dispersions labeled by momentum  $q$  connecting zero energy with the gapped bulk states) and at temperatures below the bulk gap  $\Delta$ . The current is simply determined by the product of energy, thermal occupation (here for fermionic spinons obeying Fermi-Dirac statistics) and their velocity

$$\begin{aligned} I_{xy} &= \nu \int_0^{\Delta} \epsilon(q) n(\epsilon) v(q) \frac{dq}{2\pi} \\ &= \nu \int_0^{\infty} \epsilon(q) \frac{1}{1 + e^{\epsilon(q)/T}} \frac{d\epsilon}{dq} \frac{dq}{2\pi} = \nu \frac{\pi}{24} T^2. \end{aligned} \quad (4)$$

Recent experiments have observed signatures of a thermal Hall effect in disordered magnetic insulators, e.g. in kagome Volbortite<sup>85</sup>, pyrochlore compound  $\text{Tb}_2\text{Ti}_2\text{O}_7$ <sup>102</sup> and the Kitaev candidate  $\alpha\text{-RuCl}_3$ <sup>103</sup>. However, the unambiguous confirmation of the quantised prefactor of the temperature dependence is missing because it is yet again difficult to disentangle from, say, more prosaic types of heat transport. In addition, not every spin liquid comes with quantised transport coefficient, and at any rate, different spin liquids may not be distinguishable in this way alone. Nevertheless, the resulting problems of uniqueness and completeness of a classification scheme can safely be deferred until a time that such quantised transport has unambiguously been detected in at least two compounds.

### D. Unconventional phase transitions

Despite the identification of a distinguishing characteristic of a QSL via its topological properties – the long-range entanglement of its ground state – this can be of limited practical use as even some of the most paradigmatic states – among them classical spin ice, the  $\mathbb{Z}_2$  gauge theories or the Kitaev honeycomb model – are only zero temperature phases and no phase transition occurs when cooling down from the simple high temperature paramagnet. Nevertheless, the behaviour of short length and time scales [and here short could mean logarithmic in system size<sup>104</sup>] can still be governed by the fractionalized excitations of the zero temperature QSL<sup>105</sup>.

The phase transitions out of topological phases can be of considerable autonomous importance. Since the first non-symmetry breaking phase transition beyond the Landau paradigm identified by Wegner for lattice gauge theories<sup>106,107</sup>, many other interesting proposals have been made. We mention these only in passing since pinning these down in detail is even more challenging than identifying the relevant phase in itself.

The basic attraction of such transitions is that they reflect the exotic nature of the emergent degrees of freedom. For instance, the Kasteleyn transition<sup>108</sup> is an asymmetric transition on account of the string-like nature of an emergent U(1) gauge field: when a string has a negative free energy per unit length, it is suppressed by a Boltzmann factor *in systems size* as it needs to span the entire system. Hence, none – not even in the form of fluctuations – are present until the sign of its free energy changes, upon which there is a totally conventional continuous onset of string density. In spin ice, where the topological spin state is most reliably established, (a thermally rounded version of) this transition has indeed been observed<sup>109–111</sup>.

Spin ice also hosts a liquid-gas transition with a critical endpoint of the emergent magnetic monopoles as zero-dimensional defects in a three-dimensional topological phase<sup>112</sup>. These form a Coulomb liquid which can then be treated with methods imported from electrochemistry such as Debye-Hückel theory and its extensions<sup>113–115</sup>. This is an instance of non-trivial collective behaviour of the emergent degrees of freedom, which in itself remains a largely unexplored aspect of the field, Sec. V.

Much beautiful theory has been developed regarding such unusual phase transitions, including the identification of unusual signatures such as anomalously large anomalous exponents. A particular case in point is the possibility of deconfined quantum criticality<sup>116,117</sup>, where the critical point with deconfined excitations separates two symmetry-breaking confined phases<sup>2</sup>.

#### IV. SPECTROSCOPY

The absence of Bragg peaks in zero frequency measurements probing static correlations is an alternative indicator for ruling out conventional symmetry-breaking phases in spin liquid candidate materials. The generic situation of the correlations not only lacking a long-range ordered component, but also being numerically short-range means that, in reciprocal space, all features are broad.

Inelastic scattering experiments at nonzero frequency have the big advantage of also probing excited states beyond the asymptotic low energy regime of thermodynamic measurements. The apparent disadvantage that these are 'non-universal' is remedied by the prospect of identifying concrete spin liquids in actual experiments. Many experimental probes with ever increasing frequency resolution are available. Each of these has its well-developed strengths and also well-known set of shortcomings, to discuss all of which would go beyond the scope of a simple review such as this, so that we will emphasize the points specific to spin liquids in the following.

First of all, just like in the thermodynamic probes, one thing to fundamentally look out for is an unusual temperature dependence of dynamical correlations<sup>118</sup>, see Fig.3 (c) for a recent example. There should again be a cooperative paramagnetic regime in which interactions are

strong but response functions change little as the temperature is lowered.

#### A. Inelastic neutron scattering and quasi-particle kinematics

The method of choice for measuring the basic spin correlation functions – both static and dynamic – is inelastic neutron scattering whose cross-section is given by

$$\frac{d\sigma}{d\Omega dE} \propto F(q) \left( \delta_{\alpha\beta} - \frac{q^\alpha q^\beta}{q^2} \right) \times \sum_{\mathbf{r}_i, \mathbf{r}_j} e^{i\mathbf{q}\cdot(\mathbf{r}_i - \mathbf{r}_j)} \int dt dt' e^{-i\omega(t-t')} \langle S_{\mathbf{r}_i}^\alpha(t) S_{\mathbf{r}_j}^\beta(t') \rangle. \quad (5)$$

From the sum rules connecting static correlations to frequency integrated dynamical ones it is apparent that the absence of static Bragg peaks in spin liquids goes along with the spectral weight being found elsewhere at nonzero frequencies.

A central role in pursuing spin liquids and their concomitant fractionalisation is played by selection rules. Most simply put, if scattering involves a two-body process – e.g. a neutron spin flip creating a magnon – the twin constraints of energy and momentum conservation can lead to a sharp response in the form of a single line of energy versus wavevector transfer,  $\omega(\mathbf{q})$  which represents the magnon dispersion relation. This simple situation is fortuitous in that the selection rules for neutron scattering off magnons permit precisely such a matrix element. The combined facts that neutrons are well-matched to length/energy scales *and* quantum numbers of magnons underpins some of their phenomenal success in the field of magnetism.

A broader response can therefore have a number of different origins. Firstly, there may not be such simple scattering processes available, such as in Raman scattering, where zero wavevector transfer  $q = 0$  requires the creation of multiple magnetic excitations. Secondly, there may not be a simple magnon available, but rather only fractionalised excitations which have to be created together, thereby rendering the scattering process a many-particle one. Thirdly, there may not be a quasiparticle description of the low-energy spectrum in the first place, so that no dispersion relation  $\omega(q)$  exists even as a matter of principle. This latter situation, especially with view to gapless spin liquids, is perhaps the least understood at this point in time.

Hence, while the existence of a broad dynamic scattering response is a good indicator of a correlated paramagnetic regime, it is itself not a sufficient piece of evidence for fractionalized excitations of a spin liquid. Nevertheless, given the large amount of information available in a full  $\omega(q)$  map, it opens up the possibility of providing a more microscopic modelling of the *non-universal* features discussed above.

For the Heisenberg chain, the kinematics of fractionalisation is most beautifully illustrated already in the three parabolas which denote the possible energy-momentum combinations of two independent domain walls obtained from flipping a single spin in an ordered chain. The rather nontrivial intensity map – the kinematically allowed processes take place with very different intensities – follows from an actual enumeration of the matrix elements involved, see Fig.3(a).

In high dimensions, an analogous picture may e.g. apply to a  $Z_2$  spin liquid where a triplet excitation can be decomposed into a pair of  $S = 1/2$  spinons, which would appear as monomers in a quantum dimer model<sup>119</sup>. For this to be straightforwardly visible, it would be necessary for the individual spinon to act like a coherent free particle, which is likely the case in the limit of low spinon excitation energies. However, it is at present unclear – and one of the most interesting questions – over what energy range such long-lived well-defined fractionalised quasiparticles will exist.

This case of symmetric fractionalisation into two particles is a particularly simple scenario. In addition, many other emergent particles are possible, and it is a priori straightforward to come up with mean-field parton constructions with a wide variety of different fractionalisation schemes<sup>89</sup>. One which is not uncommon is to end up with a spin flip corresponding to the creation of a gauge-charged particle, and an excitation of the emergent gauge field itself, e.g. as happens in the Kitaev models<sup>120,121</sup>. This fractionalisation can be very asymmetrical: the flux may be very heavy, that is to say, have a very small bandwidth. It can thus take up an arbitrary amount of momentum at almost constant energy, and thereby render momentum conservation essentially inoperative<sup>59</sup>. This then leads to features in reciprocal space which are so broad that it is hard to infer much about the dispersion of the light particle. Nevertheless, key information about the energy of flux excitations and the DOS of the light fractionalized particles can in principle be inferred from the INS response<sup>120–122</sup>.

### B. Light scattering

Light can interact with the purely magnetic low energy degrees of freedom of Mott insulators via the virtual hopping processes, which determine the magnetic exchange constant themselves. This has been shown for example to lead to a nonzero electric polarisability. It gives rise to an AC optical conductivity in Mott insulators<sup>127</sup> signatures of which have been analysed for a number of QSLs<sup>128–131</sup>. Moreover, optical absorption can directly couple via magnetic dipole excitations to spin flips and thus becomes sensitive to the zero momentum structure factor. Such experiments on a  $\alpha$ - $\text{RuCl}_3$  have been recently interpreted as indications of a magnetic field induced QSL<sup>132–134</sup>.

Alternatively, higher order photon process can induce virtual electron-hole pairs causing double spin flip

excitations<sup>135</sup>. Such dynamical Raman response of kagome<sup>136,137</sup> and Kitaev QSLs<sup>138,139</sup> has been analysed theoretically. Interestingly, the difference in matrix elements compared to INS permits a more direct coupling to certain types of fractionalized quasi-particles<sup>105</sup> and the polarization dependence of this zero momentum probe contains additional information<sup>140</sup>. Hence, Raman measurements on pyrochlores<sup>141</sup>, herbertsmithite<sup>142</sup> and two-<sup>126</sup> and three-dimensional<sup>143</sup> Kitaev candidate materials have been interpreted in terms of the spinon DOS of QSLs but it is difficult to separate the inevitable phonon contribution to Raman scattering – we return to this point below, Sec. IV E. Finally, with a further increase in energy resolution resonant inelastic X-ray scattering (RIXS) will be a promising new tool for probing QSL excitations including their momentum dependence<sup>144,145</sup>.

### C. Local probes

Nuclear magnetic resonance (NMR) experiments probe the local magnetic fields of the spin degrees of freedom in insulators via the hyperfine interaction with nuclear levels. They are a powerful tool in the study of spin liquids<sup>146</sup>. For example, an NMR frequency, which remains sharp and does not split when cooling to low temperature, rules out the presence of static magnetism with inequivalent magnetic sites or a static disordered state. Even more information is obtained from the relaxation time  $1/T_1T$  directly sensitive to the local magnetic susceptibility (in the zero frequency limit) related to the magnetic DOS. In a gapped QSL an Arrhenius type behaviour is expected but gapless QSLs would again lead to characteristic power law behaviour as a function of temperature similar to the specific heat but without the parasitic phonon contributions.

An alternative probe of local magnetic fields is the relaxation of spin-polarized muons deposited in a candidate material. These  $\mu\text{SR}$  experiments are able to reliably distinguish between the presence of static moments due to conventional LRO, which leads to long lived oscillations of the polarization, or dynamical moments of spin liquids, which lead to a quick decay without oscillations<sup>147</sup>. The main advantage of  $\mu\text{SR}$  is its high sensitivity but a straightforward interpretation of the data can be complicated because the positively charged muon interacts with the lattice altering the local magnetic environment.

Unfortunately, experiments with both local *and* controlled spatial resolution are missing for magnetic insulators. For weakly correlated electronic materials thanks to scanning tunnelling microscopy (STM) and angle resolved photo emission spectroscopy (ARPES) a hallmark signature of topological systems – the bulk-boundary correspondence – could be confirmed shortly after its prediction, e.g. of the surface Dirac cone of three dimensional topological insulators<sup>148</sup> or of Majorana zero energy modes in superconducting wires<sup>149</sup>. Of course, for a direct confirmation of topological surface states in spin

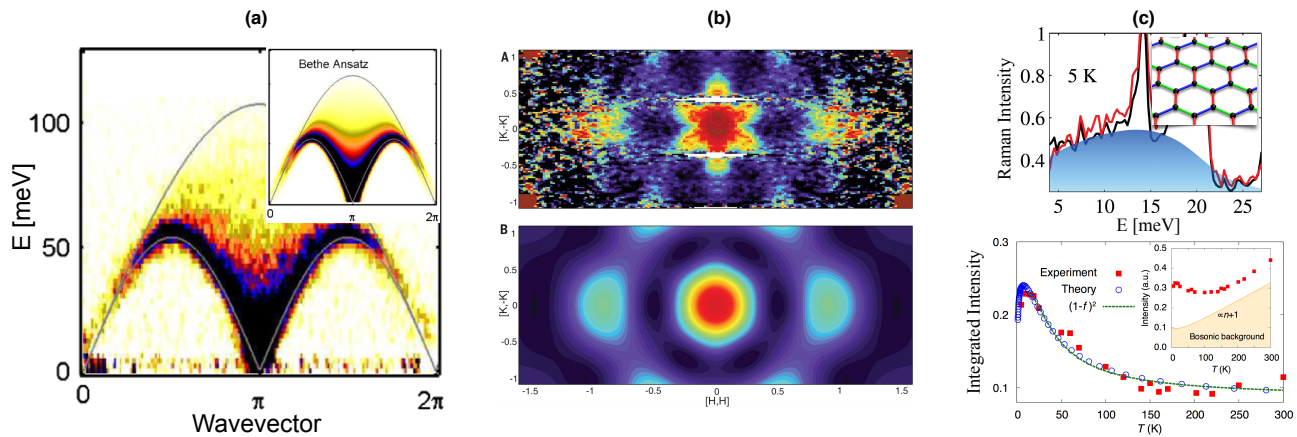


FIG. 3. (Color online) Comparison of inelastic scattering experiment and theory. (a) Fractionalised spinons in the  $S = 1/2$  Heisenberg chain material  $\text{KCuF}_3$ <sup>123,124</sup>. (b) Finite frequency neutron structure factor of the proximate spin liquid in  $\alpha\text{-RuCl}_3$ <sup>125</sup>. (c) Fermionic nature of excitations in  $\text{RuCl}_3$  as evidenced in Raman scattering<sup>105,126</sup>.

liquid candidates *without* charged quasi-particles similar measurement tools are highly desirable. In that context it will be promising to explore new directions for example spin noise spectroscopy, scanning SQUID magnetometry<sup>150</sup>, Raman microscopes<sup>151</sup> or inelastic scanning tunnelling microscopy<sup>152</sup> on spin liquid candidate thin films all with *spatial* resolution.

#### D. Bound states of fractionalized quasiparticles

Contrary to the intuition developed from the discussion of INS experiments, fractionalised quasiparticles need not have a broad, fully continuous spectrum. Instead, they may form bound or localised states. Indeed, their quantum numbers may look a lot like that of the unfractioalised spin flip; we remind the reader that the statement of deconfinement of fractionalised particles refers to the energy cost of their separation being bounded; this does not preclude the possibility of discrete composite states with a finite binding energy. Again, in  $d = 1$  there is a celebrated and experimentally established instance of this<sup>153</sup>, when at the magnetic field tuned critical point the exchange field leads to a discrete part of the spectrum from bound pairs of domain wall excitations. In higher dimension, analogous phenomena have been theoretically proposed. In spin ice, like in the  $d = 1$  case, application of a field can lead to bound states of monopoles with a characteristic spectrum<sup>154</sup>.

More exotically, the possibility of the gauge field degree of freedom being involved in a ground state is present in non-Abelian spin liquids. This is analogous to the case of a  $p_x + ip_y$  superconductor<sup>155</sup> where vortices host Majorana fermion bound states, e.g. the two Majoranas of a pair of vortices lead to a fermionic bound state whose energy goes exponentially to zero with vortex separation. Of course, the long-term goal is the controlled manipulation of the degenerate ground state manifold for braid-

ing in the context of topological quantum computation<sup>21</sup>. Slightly less ambitious would be the observation of such a flux-Majorana bound state which has been shown to lead, for example, to a sharp contribution in the spin structure factor<sup>121,122</sup> in the non-Abelian phase of the Kitaev honeycomb QSL in which a spin flip introduces a pair of nearest neighbour fluxes binding a pair of Majoranas below the gapped continuum response. Alternatively, already in the Abelian but *anisotropic* Kitaev phases<sup>156,157</sup>, the leading response can be a sharp delta-function corresponding to the addition of a pair of emergent gauge fluxes.

#### E. Statistics

The quantum statistics of quasiparticles is a very fundamental property – it affects the many-body density of states even for non-interacting particles. This is already evidenced by the (conventional) Fermi sphere and its suppressed heat capacity, and thus in principle accessible in thermodynamic measurements already, Sec. III. The temperature dependence of dynamical scattering experiments contains information about the *thermal distribution functions* which are qualitatively different for quasiparticles with different quantum statistics, e.g. Fermi-Dirac versus Bose-Einstein. For example, in the context of Raman experiments on the Kitaev candidate material  $\alpha\text{-RuCl}_3$  a close comparison between the T-dependence of the high energy Raman response and experimental data arguably points to the presence of spin fractionalisation in terms of fermionic excitations<sup>105</sup>.

In general, fractionalised quasiparticles in topological phases can have unusual exchange statistics of Anyons due to the relative phases picked up when emergent particles of different type are interchanged. Such braiding operations are particularly of interest given their much-appreciated potential in procuring a framework

for fault-tolerant topological quantum computing<sup>20</sup>. Directly probing the exchange statistics of emergent particles in a quantum spin liquid is a tall order, which at present – given the difficulties in doing the same even in the much more controlled setting of quantum interferometers in the quantum Hall effect – seems not too close at hand.

However, there are nonetheless qualitative features to look out for. For instance, a scattering process which creates a set of fractionalised particles via a local interaction is sensitive to the statistics of the particles generated together, as their relative wavefunction influences the matrix elements for the process in question: for a pointlike interaction, the creation of two Fermions, say, is inhibited by the vanishing of their wavefunction as the pair moves close together. Hence, under rather general conditions for gapped QSLs the onset of the INS response is dominated by the long range statistical interaction between Anyonic quasiparticles leading to a universal power law dependence as a function of frequency<sup>158</sup>. Similarly, a promising direction will be ambitious experiments for noise spectroscopy directly probing statistics or measuring emergent quantum numbers of fractionalized quasiparticles.

## V. DISORDER AND DEFECT PHYSICS

The presence of disorder – defects, vacancies, impurities and the like – is an unavoidable fact of life in condensed matter systems. Indeed, in many candidate spin liquids, understanding the role of disorder is a crucial step towards the identification of the physics involved, see e.g.<sup>64,65,73,159</sup>.

Besides being a nuisance, however, disorder can also be used as a probe. The basic idea is that disorder can make fractionalisation physics visible in the ground state that would otherwise require probing excitations. As a simple illustration, consider the following picture. A vacancy in a two-dimensional system can be thought of as inserting a microscopically tiny hole into the plane – in a sense, it changes the topology of the plane into that of an annulus. This hole can then have effective degrees of freedom. One instance could occur in a non-Abelian spin liquid, where (well-separated) vacancies can host Majorana zero modes at zero energy, as described in Sec.IV E. Also, not unlike impurities in a semiconductor, a vacancy can host a localised fractionalised excitation – such as a magnetic monopole – which would otherwise require an activation energy in the bulk<sup>160</sup>.

In this sense, disorder physics is closely related to our discussion of dynamical probes of fractionalised degrees of freedom, upon replacing dynamical probes by probes of the disorder sites. Local probes can then be used to resolve the signal coming from the defects. While in a bulk probe, a signal from a small density of defects, unless it is singularly large, is easily swamped by the bulk signal, a local probe like NMR can detect the defect response

at a frequency separated from that of the bulk. Again, in one dimension, edge defects have provided a beautiful picture of the physics of the gapped Haldane chain<sup>161</sup>. This kind of study has been carried out in some detail for gauge-charged vacancy degrees of freedom<sup>83</sup> following detailed NMR measurements on SCGO<sup>84</sup>, see Fig.2 (b). These orphan spins<sup>35</sup> establish an oscillating spin texture decaying like a Coulomb law; modelling this has led to the conclusion that the level of disorder in SCGO is likely higher than that determined from the controllable non-stoichiometry only. The spin liquid response to disorder can thus be used to inverse-infer properties of the composition of the material itself.

The response to disorder can also be very intuitive. For a spinon Fermi surface, one can develop an analogy to the response of a Fermi liquid to disorder: an impurity can induce Friedel oscillations, leading to a disturbance surrounding the impurity modulated at the Fermi wavevector, which in turn can give rise to RKKY-type interactions between impurities<sup>162</sup>.

As mentioned above, collective defect physics is a huge field in its own right, so far little studied. A systematic understanding of the many-body state of a finite density of defects embedded in a topological spin liquid seems like a particularly promising direction for the discovery of surprising new phenomena<sup>159,163–166</sup>. This topic is beyond the scope of the present article, and arguably merits a stand-alone treatment.

## VI. DISCUSSION AND FUTURE DIRECTIONS

After several decades of searching for quantum spin liquids in magnetic materials we are still awaiting the unambiguous sighting of this elusive state of matter. Encouragingly, recent years have seen a flurry of discoveries of new candidate materials and novel indicative signatures of liquidity, which raise the hopes that this long search will come to a successful conclusion in the not too distant future. One of the great attractions of finding a topological phase in a magnetic system is their high tunability. For instance, magnetic fields of a few tens of teslas in strength are becoming increasingly routinely available. They potentially push 'proximate' spin liquid candidates<sup>72</sup> in the desired direction<sup>167</sup> and the impressive advances in the energy-wavevector resolution of neutron experiments, for example, enable a visualisation of the field induced melting of conventional long range magnetism<sup>168</sup>. A magnetic field can not only add a Zeeman term to the Hamiltonian, but also acts in the presence of spin-orbit interactions as a versatile probe, even mimicking an effectively staggered field on different sublattices which can change the effective dimensionality of the emergent gauge field<sup>28</sup>.

In the absence of uniquely and individually compelling features of spin liquidity in a particular material, an investigation of how a particular feature (gap size, mode dispersion, continuum bandwidth, bound state energy)



changes as a function of tuning parameter (external fields, pressure, composition, strain) will be a crucial ingredient for assessing the validity of a particular interpretation of experimental data in terms of a spin liquid under the merciless action of Ockham’s razor. In that context, a joint effort of both theory and experiment continues to be called for pinning down the rich phenomenology of spin liquids.

Besides such more detailed model-based input, methodological progress is also on the horizon. High on the wishlist are improved local probes, especially with a resolution approaching the lattice scale, which is currently elusive for, say, SQUID-based devices measuring local field distributions. These could then be used to probe boundary modes or impurity susceptibilities even in insulating magnets, in the hope of emulating their huge success in electronic systems with charge degrees of freedom via STM and ARPES. Another exciting development for probing fractionalized excitations lies in the realm of non-equilibrium techniques, e.g. pump-probe measurements<sup>169,170</sup> or spin echo and noise spectroscopy. The non-equilibrium physics of topological phases is a nascent field that will surely hold numerous surprises for the patient explorer.

Our field guide is also subject to continual extension on account of the search for new materials. These may arise in the form of bulk materials, metal organic frameworks<sup>171</sup>, metallic Kondo systems<sup>172</sup>, or in ‘artificial’ settings such as nanostructured/thin film samples. In addition, there remains the promise<sup>173,174</sup> of one day realising new phases of spin systems in analogue cold atomic quantum simulators.

In conclusion of this little field guide, it is worth recalling that in physics, the most interesting phenomena are often the ones which are not anticipated at all, so that the most basic suggestion remains to produce and analyse experimental data with an open mind.

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